10/02/2020

Conditional Probability

1) We have two boxes A and B. A box contains 100 gold coins. Box B contains 50 gold and 50 silver coins. You randomly choose a box to open and then randomly choose a ~~po~~ coin from that box. If the coin you chose is gold then what is the probability that you chose box A.

2) If a single card is drawn from a standard deck of playing cards and it is a face card then what is the probability that ~~a~~ it is a King.

4K
4Q
4J
12

3) A factory production line is manufacturing bolts using 3 machines A, B and C ~~of the~~. Of the total output machine A is responsible for 25%, machine B is for 35% and m/c C for the rest. It is known from previous experience with the machines that 5% of the output from A is defective 4% from m/c B and 2% from m/c C. A bolt is chosen ~~from~~ at random from the production line and found to be defective. What is the probability that it came from m/c A, B and C.

4.) An engineering company advertises a job in 3 papers A, B and C. It is known that these papers attract undergraduate ~~readi~~ engineering readerships in the proportion of 2 : 3 : 1. The probabilities that an engineering undergraduates sees and replies to the job advertisement in these papers are 0·002, 0·001 & 0·005 ~~resp~~ respectively. Assume that the undergraduate sees only 1 job

$\frac{2}{6}$ $\frac{3}{6}$ $\frac{1}{6}$ $\frac{1}{3}$, $\frac{1}{2}$, $\frac{1}{6}$

advertisement –

i) If the engineering company receives only 1 reply to its advertisements calculate the probability that the applicant has seen the job advertised in place A, B and C.

ii) If the company receives two replies what is the probability that both applicants saw the job advertised in paper A.

Baye's Theorem :-

$$\boxed{P(A/B) = \frac{P(B/A)\ P(A)}{P(B)}}$$

P(A/B)
The likelihood of event A occuring, given that B is true.

P(B/A) → likelihood of event B occuring, given that A is true.

P(A) & P(B) probabi

Extended version of Baye's theorem :-

$$\cancel{P(A/B) = \frac{P(B/A)\ P(A)}{P(B/A)\ P}}$$

$$P(A/B) = \frac{P(B/A)\ P(A)}{P(B/A)\ P(A) + P(C/A) \cdot P(C) + P(D/A)\ P(D)}$$

(Q-1)

(Q-3)

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$$P(F/T) = \frac{P(T/F)\,P(F)}{P(T/F)\,P(F) + P(T/M)\,P(M)}$$

Q-1) $$P(A/G) = \frac{P(G/A)\,P(A)}{P(G)}$$

$$= \frac{1 \cdot 1/2}{150/200} = \frac{1/2}{3/4}$$

$$\Rightarrow \frac{1}{2} \times \frac{4}{3} \Rightarrow \frac{2}{3}$$

(when one event is dependent on other event).

Q-3) Let, D = bolt is defective

A = bolt from A
B = bolt from B
C = bolt from C

$$P(A) = \frac{1}{4} = 0.25$$

$$P(B) = 0.35$$

$$P(C) = 0.40$$

$P(D/A) = 0.05$, $P(D/B) = 0.04$, $P(D/C) = 0.02$

$$P(A/D) = \frac{P(D/A)\,.\,P(A)}{P(D/A).\,P(A) + P(D/B).\,P(B) + P(D/C).\,P(C)}$$

$$= \frac{0.05 \times 0.25}{0.05 \times 0.25 + 0.04 \times 0.35 + 0.02 \times 0.40}$$

$$\Rightarrow \frac{0.0125}{0.0125 + 0.0140 + 0.0080} = \frac{0.0125}{0.0345} = 0.362$$

$$P(B/D) = \frac{P(D/B) \cdot P(B)}{P(D/A)\,P(A) + P(D/B) \cdot P(B) + P(D/C)\,P(C)}$$

$$= \frac{0.04 \times 0.35}{0.0345} = \frac{0.0140}{0.0345} = 0.4058$$

$$P(C/D) = \frac{P(D/C) \cdot P(C)}{P(D/A)\,P(A) + P(D/B)\,P(B) + P(D/C)\,P(C)}$$

$$= \frac{0.0080}{0.0345} = 0.2318$$

Q-2) $P(\text{King}/\text{Face}) = \dfrac{P(F/K) \cdot P(K)}{P(F)}$

$P(K) = \dfrac{1}{13}$ $\quad P(F) = \dfrac{12}{52} = \dfrac{3}{13}$

$\Rightarrow \dfrac{1 \cdot 1/13}{3/13} \Rightarrow 1 \times \dfrac{1}{13} \times \dfrac{13}{3} = \dfrac{1}{3}$ Ans.

If two event A and B are independent

then, $P(A \cap B) = P(A) \cdot P(B)$

If $A_1, A_2, \ldots, A_n$ are independent then

$$P(A_1 \cup A_2 \cup A_3 \ldots \cup A_n) = 1 - [(1 - P(A_1)) \cdot (1 - P(A_2)) \cdot (1 - P(A_3)) \ldots (1 - P(A_n))]$$

Ex: Suppose that the probability of being killed in a single flight is $P_c = \dfrac{1}{4 \times 10^6}$ based on available statistics. Assume the different flights are independent. If a businessman takes 20 flights per year what is the probability that he is killed in a plane crash within next 20 years (Lets assume he will not die because of another reasons in next 20 years)

400 $\frac{1}{10^4}$

Sol^n Probability of being killed $= P_c = \dfrac{1}{4 \times 10^6}$

Probability of survival $= P_S = 1 - P_c$

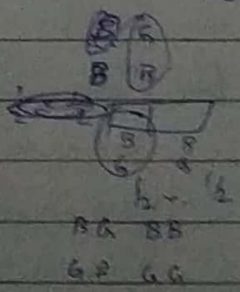
In next 20 years, he takes 400 flights then →
$P_S \times P_S \times P_S \times P_S \times \ldots$ 400 times

$$\Rightarrow (1 - P_c)^{400}$$

Probability of survival for next 20 years,

$$\Rightarrow \left(1 - \frac{1}{4 \times 10^6}\right)^{400}$$

Q→ A couple has 2 children the older of which is a boy what is the probability that they have 2 boys?



Sol^n→ A → both children are boys.
B → the older child is boy.

$$P(A/B) = \frac{P(B/A) \cdot P(A)}{P(B)} = \frac{1 \cdot \left(\frac{1}{2} \times \frac{1}{2}\right)}{1/2}$$

$$\Rightarrow \frac{1}{4} \times 2 \Rightarrow \frac{1}{2} \text{ Ans}$$

(Last wala)

Q-4) A = candidate from A
B = candidate from B
C = candidate from C
R = candidate Replies.

(A) $P(A/R) = \dfrac{P(R/A) \cdot P(A)}{P(R/A) \cdot P(A) + P(R/B)\,P(B) + P(R/C) \cdot P(C)}$

$P(R/A) = 0.002$

$P(R/B) = 0.001$

$P($

Q→ Two basketball players play a game in which they alternately shoot a basketball at a hoop. The first one to make a basket wins the game. On each shot Player 1 (the one who shots first) has probability P_1 of success while Player 2 has probability P_2 of success. The shots are assumed to be independent.

i) Find $P(W_1)$ the probability that Player 1 wins the game

ii) For what values of P_1 and P_2 is this a fair game i.e., each player has a 50% chance of wining the game.

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Q- A box contains 3 coins, ~~two~~ 2 regular coins & 1 fake two headed coin.

i) You pick a coin at random and toss it. What is the probability that it lands heads up.

ii) and gets head, what is the probability that it is the two headed coin.

Sol^n i) ~~$P(H_1) = \frac{1}{2}$~~ ~~$P(H_2) = \frac{1}{2}$~~ ~~$P(H_3) = 1$~~

$$P\,Q = \frac{2}{3} \times \frac{1}{2} + \frac{2}{3} \times 1 \Rightarrow \frac{1}{6} + \frac{1}{3} \Rightarrow \frac{1+2}{6} = \frac{3}{6}$$

$$\cancel{\frac{2}{3} + \frac{2}{3}}$$

$$\frac{2}{3} \times \frac{1}{2} + \frac{1}{3} \times 1 \Rightarrow \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$$

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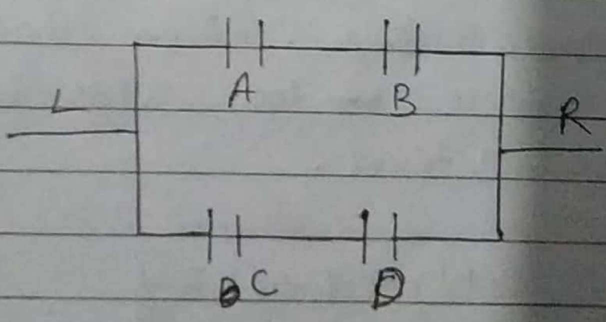
Test

Q→ The probability that each relay close in the circuit shown below is P. Assuming that each relay functions independently of the others. Find the probability that current can flow from L to R.



A & B close
C & D close
A, B & D close
A, B, C close
C, D & A close
C, D & B close
A, B, C & D close

Q-2) A garbage m/c keeps a box of good springs to use as replacements on customer's car. The box contains 5 springs. A collegue thinking that the springs are for scrap, put 3 faulty springs into the box. The m/c picks two springs out of the box while servicing a car. Find the probability that

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(i) The first spring draw is faulty

(ii) The second spring draw is faulty.

Solⁿ 1:- ~~Probability that relay not close = 1-P~~
~~Probability of relay to be close = P~~

$P((A \cap B) \cup (C \cap D)) =$

$P(A \cap B) + P(C \cap D) - P((A \cap B) \cap (C \cap D))$

$\Rightarrow p^2 + p^2 - p^4$

$= 2p^2 - p^4.$

Solⁿ 2 (i) Total springs = 8

Total faulty springs = 3

$P = \frac{3}{8}$

$\cancel{1 - \frac{3}{8} = \frac{5}{8}}.$

~~(ii) $\frac{4}{7} + \frac{2}{7} \Rightarrow \frac{8}{49}$~~

(ii) $\frac{5}{8} \times \frac{3}{7} + \frac{3}{8} \times \frac{2}{7} \Rightarrow \frac{21}{56} = \frac{3}{8}$

Q-3) A coin is tossed until a head appears or until it has been tossed 3 times. Given that head does not occur on the first toss. What is the probability that coin is tossed 3 times.

Solⁿ $P(\text{Head}) = \frac{1}{2}$ $P(\text{not head}) = \frac{1}{2}$

Event A = coin is tossed 3 times. $= \frac{1}{2}$

Event B = No head on first toss. $= \frac{1}{2}$

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$1 - \frac{1}{2} = \frac{1}{2}$ Date Page

Conditional Probability

$$\boxed{P(A/B) = \frac{P(A \cap B)}{P(B)}}$$

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$P(B) = P(TH) + P(TTH) + P(TTT)$$
$$= \frac{1}{4} + \frac{1}{8} + \frac{1}{8} = \frac{1}{2}$$

$$P(A) = \{TTH, TTT\}$$
$$= \frac{1}{8} + \frac{1}{8} = \frac{1}{4}$$

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{2/8}{1/2} = \frac{1}{4} \times 2 = \frac{1}{2}$$

Q- Assume that certain school contains equal no. of female and male students. 5% of the male population is football players find the probability that a randomly selected student is a football player male.

Soln F = Football player
M = Male player

$P(F/M)$

$\cancel{P(F)} = \frac{5}{100} = \frac{1}{20}$

$P(M) = 1/2$ $\quad P(F/M) = \frac{P(F \cap M)}{P(M)}$

$= \frac{1}{20}$ ~~$\times 2 = P(F \cap M) \Rightarrow \frac{1}{10} = 0.1$~~

$$P(F/M) = \frac{P(F \cap M)}{P(M)}$$

$$\Rightarrow \frac{1}{20} = \frac{P(F \cap M)}{1/2} \Rightarrow P(F \cap M) = \frac{1}{40}$$

$$\Rightarrow P(F \cap M) = \underline{0.025}$$

Q→) What is the probability that a positive integer selected at random from the set of positive integers not exceeding 100 is divisible by either 2 or 5.

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

E_1 : ~~Div~~ No. divisible by 2

E_2 : No. divisible by 5

$$P(E_1) = \frac{50}{100} \qquad P(E_2) = \frac{20}{100} \qquad P(E_1 \cap E_2) = \frac{10}{100}$$

$$\Rightarrow P(E_1 \cup E_2) = \frac{50}{100} + \frac{20}{100} - \frac{10}{100}$$

$$\Rightarrow \frac{60}{100} = \frac{6}{10} = \frac{3}{5} \text{ Ans}$$

12/02/2020 Prove by Induction

A Prove by mathematical induction that P(n) is true for every positive integer n consists of two steps

(i) Basic step

The proposition P(1) is ~~true~~ shown to be true.

(ii) Inductive step:

The proposition P(k) has to be true,

$P(K) \longrightarrow P(K+1)$

It is true for every positive integer k

~~K is inductive hypothesis~~.

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where k is known as inductive hypothesis.

Q-1) Using mathematical induction prove that the sum of the first n odd positive integers is n^2.

Sol^n-1) $P(n) = n^2$

$\cancel{1^2 + 2^2 + 3^2 + \dots + (n-1)^2 + n^2}$

$\cancel{1^2 + 3^2 + 5^2 + \dots + (2n-1)^2 = n^2}$

$1 + 3 + 5 + \dots + (2n-1) = n^2$

(i) Basic step :-

$P(n)$ $\cancel{= P(1) = 2(1) - 1 \Rightarrow 1 =}$

Let $n = 1$

then $P(n) = 2(1) - 1 = 1 = n^2$

So, $P(1)$ is true.

(ii) Now,

~~Let $n = k$~~

~~$P(k) = 1 + 3 + 5 + \dots + (2k-1) = k^2$ be true~~

~~Now,~~

Lets assume $P(k)$ is true, i.e.,

$1 + 3 + 5 + \dots + (2k-1) = k^2$ is true.

Now,

Consider $P(k+1)$, the sum of first k+1 odd positive integers,

$\Rightarrow \cancel{1 + 3 + 5 + \dots + 2(k+1) - 1}$

$\Rightarrow \cancel{1 + 3 + 5 + \dots +}$

$P(k+1): 1 + 3 + 5 + \dots + (2k-1) + [2(k+1) - 1]$

$\Rightarrow k^2 + 2k + 1 \Rightarrow (k+1)^2$

Proved

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Q→ Prove for all $n >= 1$, $n^3 + 2n$ is a multiple of 3.

Sol^n $P(n) : n^3 + 2n$ is multiple of $3 \Rightarrow 3n$.

Now, $P(n)$

for $n = 1$,

$n^3 + 2n = 1^3 + 2 \times 1 = 3$

which is divisible by 3 So,

$P(1)$ is true.

Now, Let's assume $P(k)$ is true,

$\Rightarrow \underbrace{k^3 + 2k}$ is multiple of 3, $\Rightarrow 3x$

$\Rightarrow 3(k^3 + 2k)$ $3($

Consider $P(k+1)$ is also true,

$P(k+1)$:

$(k+1)^3 + 2(k+1)$

$= k^3 + 1 + 3k(k+1) + 2k + 2$

$= k^3 + 1 + 3k^2 + 3k + 2k + 2$

$= k^3 + 3 + 3k^2 + 3k + 2k$.

$\Rightarrow \underbrace{k^3 + 2k} + 3 + 3k^2 + 3k$

$\Rightarrow k^3 + 2k + 3(k^2 + k + 1)$

$\Rightarrow 3x + 3(k^2 + k + 1)$

$\Rightarrow 3(x + k^2 + k + 1)$

which is a multiple of 3

So, $P(k+1)$ is true.

Hence, $n^3 + 2n$ is a multiple of 3 for all $n \geq 1$

Q→A Prove by mathematical induction that,

$1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$ for all $n \in N$

~~Sol^n Let, P(n) be the statement,~~

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Q-13 Prove that

$1+2+2^2+\dots+2^n = 2^{n+1}-1$ for all non-negative integer n.

Sol^n: Let $P(n)$ be the statement,

$P(n): 1^2+2^2+3^2+\dots+n^2 = \frac{n}{6}(n+1)(2n+1)$

Now, for $n=1$

$\frac{n(n+1)(2n+1)}{6} = \frac{1}{6}(1+1)(2\times1+1) = \frac{1}{6}\times2\times3 = 1$

So, $P(1)$ is true,

Now, Lets assume $P(K)$ is true,

So,

$1^2+2^2+3^2+\dots+K^2 = \frac{K}{6}(K+1)(2K+1)$

$\Rightarrow \frac{K+1}{6}($ K.

Now,

Let $P(K+1)$ is also true when $P(K)$ is true

$\Rightarrow 1^2+2^2+3^2+\dots+K^2+(K+1)^2$

$\Rightarrow \frac{K}{6}(K+1)(2K+1)+(K+1)^2$

$\frac{K+1}{6}(K+2)(2(K+1)+1)$

$(K+2)(2K+3)$

$\Rightarrow \frac{K(K+1)(2K+1)+6(K+1)^2}{6}$

$\Rightarrow \frac{(K+1)}{6}[K(2K+1)+6K+6] \Rightarrow \frac{(K+1)}{6}[2K^2+K+6K+6]$

$\Rightarrow \cancel{\frac{(K+1)}{6}[2K^2+}\ \frac{(K+1)}{6}[2K^2+7K+6]$

$2K^2+4K+3K+6$

$\frac{(K+1)}{6}[2K^2+4K+3K+6]$

$\frac{(K+1)}{6}[2K(K+2)+3(K+2)] \Rightarrow \frac{(K+1)}{6}(K+2)(2K+3)$

Proved

Q→ Using mathematical induction for $n > 1$

$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \dots + \frac{1}{\sqrt{n}} > \sqrt{n}$$

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Sol^n→ $1 + 2 + 2^2 + \dots + 2^n = 2^{n+1} - 1$

Let P(n) be the statement,

So,

for $n = 1$

$P(n) : 1$

$2^{n+1} - 1 \Rightarrow 2^{1+1} - 1 \Rightarrow 4 - 1 \Rightarrow 3$

Q→ To prove the inequality $n < 2^n$.