Spin splitting in the quantum Hall effect of disordered GaAs layers with strong overlap of the spin subbands

S. S. Murzin,[1,2] M. Weiss,[2] D. A. Knyazev,[1] A. G. M. Jansen,[2,3] and K. Eberl[4]
[1]*Institute of Solid State Physics RAS, 142432, Chernogolovka, Moscow District, Russia*
[2]*Grenoble High Magnetic Field Laboratory, Max-Planck-Institut für Festkörperforschung and Centre National de la Recherche Scientifique, Boîte Postale 166, F-38042, Grenoble Cedex 9, France*
[3]*Service de Physique Statistique, Magnétisme, et Supraconductivité, Département de Recherche Fondamentale sur la Matière Condensée, CEA-Grenoble, 38054 Grenoble Cedex 9, France*
[4]*Max-Planck-Institut für Festkörperforschung, Postfach 800 665 D-70569, Stuttgart, Germany*
(Received 27 December 2004; published 29 April 2005)

The spin-resolved quantum Hall effect is observed in the magnetotransport data of strongly disordered GaAs layers with low electron mobility $\mu \approx 2000\ \mathrm{cm}^2/\mathrm{V\,s}$, in spite of the fact that the spin-splitting energy is much smaller than the level broadening. Experimental results are explained in the frame of scaling theory of the quantum Hall effect, applied independently to each of the two spin subbands.

DOI: 10.1103/PhysRevB.71.155328 PACS number(s): 71.70.Ej, 71.30.+h, 73.43.−f

In an electron system with a small g factor, strong disorder broadens and suppresses the spin-splitted structure in the electron spectrum in an applied magnetic field. Therefore, spin splitting with energy separation $E_s = g\mu_B B$ (μ_B is the Bohr magneton and B the magnetic field) does not show up in the kinetic and thermodynamic properties of strongly disordered three-dimensional (3D) bulk electron systems. However, for a 2D system, the scaling theory for diffusive interference (i.e., localization) effects leads to a quite unexpected conclusion: spin splitting can arise in the magnetoquantum transport data even when spin splitting is absent in the density of states for spin-splitting energy E_s very small compared to the Landau-level broadening Γ. For this situation with $E_s \ll \Gamma$, the spin-splitted quantum Hall effect (QHE) with odd integer Hall-conductance plateaus at $G_{xy} = (2i+1)e^2/h$ and corresponding minima in the diagonal conductance (per square) G_{xx} could appear at sufficiently low temperature due to the localization of the electronic states in between the extended states of the strongly overlapping spin-up and spin-down Landau levels.[1,2] This localization should occur at very low temperatures when the coherence length of the electrons becomes larger than their localization length. However, spin splitting due to localization effects was not observed in disordered 2D GaAs systems with mobilities $\sim 1000\ \mathrm{cm}^2/\mathrm{V\,s}$ (Ref. 3) when $E_s \ll \Gamma$, because even lower temperatures than those used in the experiments would have been necessary. Higher mobility samples generally do show spin splitting[4,5] because of an exchange enhanced g factor.[6,7] Strong disorder should suppress this enhancement of spin-splitting[8] as observed experimentally.[10]

In the present work, we observed the manifestation of spin splitting due to localization effects in the magnetoconductance of a strongly disordered system, a heavily Si-doped GaAs layer with a low electron mobility $\mu \approx 2000\ \mathrm{cm}^2/\mathrm{V\,s}$. For these layers, essential exchange enhancement of the g factor is not expected. The special interest of our samples resides in the fact that the Zeeman energy, with $E_s/k_B = |g\mu_B B/k_B| \approx 4$ K using $g = -0.5$ for GaAs at a magnetic field of $B = 12$–13 T, is much smaller than the level broadening Γ ($\Gamma/k_B \gtrsim 100$ K) resulting in a strong overlap of the two spin subbands. We analyzed the scaling properties of the transport data of this electron system assuming that the conductances of the different spin subbands are renormalized independently for variations due to diffusive interference effects. Such an approach is justified in the absence of spin-flip scattering, at least for noninteracting electrons. Our experimental data are in accordance with such an analysis.

The investigated heavily Si-doped n-type GaAs layers sandwiched between undoped GaAs were prepared by molecular-beam epitaxy. The number given for a sample corresponds to the thickness d of the conducting doped layers with $d = 34$, 40, and 50 nm. The Si-donor concentration is $1.5 \times 10^{17}\ \mathrm{cm}^{-3}$. Hall bar geometries of width 0.2 mm and length 2.8 mm were etched out of the wafers. A phase-sensitive ac technique was used for the magnetotransport measurements down to 40 mK with the applied magnetic field up to 20 T perpendicular to the layers.

The electron densities per square as derived from the slope of the Hall resistance R_{xy} in weak magnetic fields (0.5–3 T) at $T = 4.2$ K are $N_s = 3.9$, 4.6, and $5.0 \times 10^{11}\ \mathrm{cm}^{-2}$ for samples 34, 40, and 50, respectively. The bare high-temperature mobilities μ_0 are about 2000, 2200, and $2400\ \mathrm{cm}^2/\mathrm{V\,s}$. Because of the rather large quantum corrections to the conductance, even in zero magnetic field at 4.2 K, we used for determining the mobility the approximate relation $\mu_0 = R_{xy}/BR_{xx}$ at the intersection point of the $R_{xx}(B)$ curves for different temperatures.

The characteristic energy scales of our samples with not more than two size-quantized energy levels are as follows. The Fermi energy at zero magnetic field $E_F/k_B \approx 200$ K, the splitting of the size quantization $E_{sq}/k_B = 3(\pi\hbar/d)^2/2mk_B \approx 100$–$200$ K (for our thinnest sample with $E_{sq}/k_B \approx 200$ K, the second subband is occupied due to disorder), $\hbar/\tau k_B \approx 100$ K (τ is the transport relaxation time at zero magnetic field), the Landau-level energy broadening $\Gamma/k_B \geqslant \hbar/\tau k_B \approx 100$ K, and cyclotron energy $\hbar\omega_c/k_B \approx 250$ K at the magnetic field $B = 12$–13 T. For our layers thicker than the magnetic length $l_B = \sqrt{\hbar/eB} = 7$ nm at 13 T, the Coulomb energy

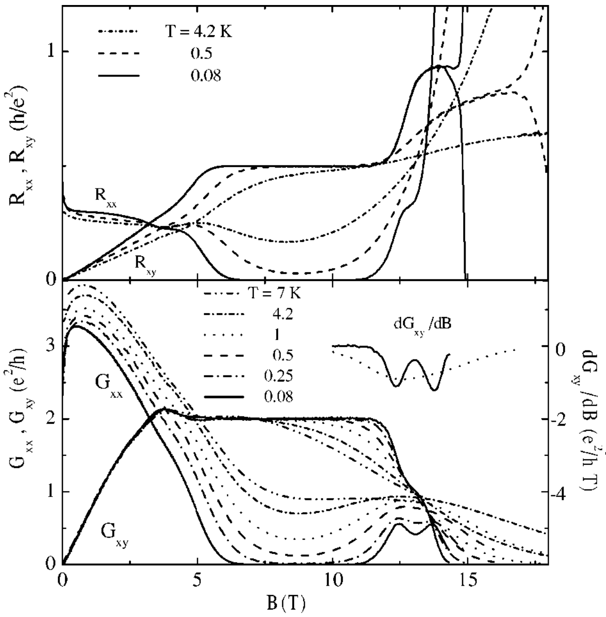

FIG. 1. Magnetic field dependence of the diagonal (R_{xx}, per square) and Hall (R_{xy}) resistance, of the diagonal (G_{xx}) and Hall (G_{xy}) conductance, and of the derivative dG_{xy}/dB for sample 40 in a magnetic field perpendicular to the heavily doped GaAs layer (thickness 40 nm) at different temperatures, showing spin splitting at 13 T where $G_{xy}\approx 1$. For the lowest temperature R_{xy} is shown for two directions of the magnetic field.

scale $E_C/k_B = e^2/\epsilon d k_B \approx 30-40$ K is d/l_B times smaller than for thin 2D systems.[6] With $\Gamma > E_C$, the exchange enhancement of the g factor should be suppressed completely by disorder[8] leading to $E_s \ll \Gamma$.

Magnetotransport data for the three samples are rather similar. In Fig. 1, the diagonal (R_{xx}, per square) and Hall (R_{xy}) resistance (both given in units of h/e^2) and the diagonal (G_{xx}) and Hall (G_{xy}) conductance as calculated from resistance have been plotted for sample 40 at the two opposite field orientations. The absolute values of the Hall resistance R_{xy} differ strongly for the two orientations at the highest fields, probably because of an admixture of R_{xx} to R_{xy} when $R_{xx} \gg R_{xy}$. The Hall conductance G_{xy} in the field range of $B = 0.5-4$ T does not depend on temperature, unlike the Hall resistance R_{xy}. This is in accordance with the theory of quantum corrections in the diffusive transport due to electron-electron interaction.[9]

At low temperatures, the curves $R_{xy}(B)$ and $G_{xy}(B)$ of Fig. 1 show a wide QHE plateau from ≈ 6 up to ≈ 11.5 T with $G_{xy}=2$ accompanied by exponentially small values of R_{xx} and G_{xx} at low temperatures $T \lesssim 0.3$ K.[11] At the lowest temperature, the derivative $\partial R_{xx}/\partial B$, the diagonal conductance G_{xx}, and the derivative $|\partial G_{xy}/\partial B|$ show minima at $B\approx 13$ T, which we ascribe to development of the spin-resolved QHE. The minima occur at a field where, at all temperatures, the Hall conductance $G_{xy}\approx 1$. However, at $B\approx 13$ T, the filling factor $\nu \equiv N_s h/eB \approx 1.5$, which is much larger than the expected value 1 for the QHE of high-mobility 2D electron systems. Above 13 T, R_{xy} reveals an additional plateau at a value close to 1 at the lowest temperature where $R_{xx} > R_{xy}$, possibly indicating a quantized Hall-insulator state.[12,13]

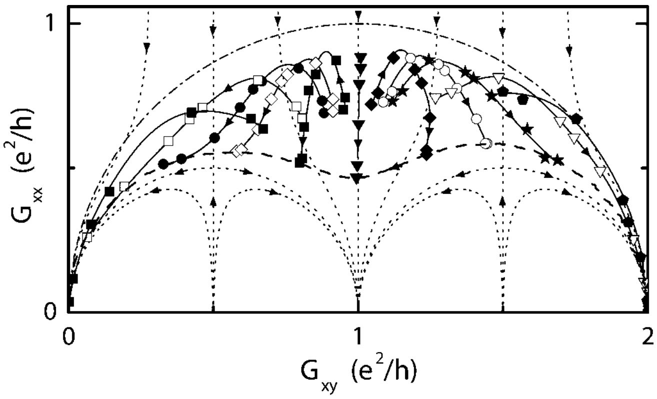

FIG. 2. Flow diagram of the $(G_{xx}(T), G_{xy}(T))$ data points for sample 34 with decreasing temperature (arrows) from 12 down to 0.1 K. Different symbols connected by solid lines are for different magnetic fields from 9 to 14 T. The dashed line follows the (G_{xx}, G_{xy}) data points from 9 to 14 T at the lowest temperature. The dotted lines show the flow of (G_{xx}, G_{xy}) with increasing coherence length for a totally polarized electron system according to scaling theory (Ref. 16) and the dash-dotted line a two times larger semicircle than Eq. (2).

The spin-splitting energy E_s in our experiments can be estimated from the magnetic field difference ΔB between the spin-splitting maxima of $G_{xx}(B)$ and $|\partial G_{xy}/\partial B|$. For the case of constant electron density N_s, one finds

$$\frac{\partial N_s}{\partial E_F} E_s + \frac{\partial N_s}{\partial B} \Delta B = 0, \qquad (1)$$

where E_F is the Fermi energy, $\partial N_s/\partial E_F \sim N_s/\Gamma$, and $\partial N_s/\partial B \sim -N_s/B$. Therefore, $E_s/k_B \sim \Gamma \Delta B/B k_B \approx 10$ K is much smaller than $\Gamma/k_B \approx 100$ K, leading to a vanishing spin-splitted structure in the density of states. Moreover, an additional argument for the absence of a spin-splitting structure in the density of states which would explain the spin-splitting QHE structure is given by the fact that, if one would have $E_s > \Gamma$ with a minimum in the density of states, the spin-splitting structure should become apparent already in the conductance at temperatures of the order of $\Gamma/k_B \approx 100$ K, whereas experimentally it develops only at $T \sim 0.1$ K. The very low temperatures for the observation of the spin splitting point to a different mechanism such as localization for the explanation of the phenomenon.

The scaling treatment of the QHE is graphically presented by a flow diagram for the coupled evolution of the diagonal (G_{xx}) and Hall (G_{xy}) conductance components with increasing coherence length.[14,15] Recent developments[16] of the scaling theory based on symmetry arguments resulted in a calculation of the exact shape of the flow lines $G_{xx}(G_{xy})$ for a totally spin-polarized electron system as plotted in Fig. 2 with dotted lines for $0 \leq G_{xy} \leq 2$. The different quantum Hall phases ($i=0,1,\dots$) in the flow diagram are separated by the vertical lines $G_{xy}=i+1/2$. At sufficiently low temperatures, the (G_{xx}, G_{xy}) data flow on a separatrix in the form of a semicircle

$$G_{xx}^2 + [G_{xy} - (i + 1/2)]^2 = 1/4. \quad (2)$$

Critical points can be found at $(G_{xy}^c, G_{xx}^c) = (i+1/2, 1/2)$. The same critical positions were found in microscopic descriptions of the QHE for the case of noninteracting electrons.[17,18]

In Fig. 2, we have plotted the experimental flow lines showing the temperature evolution of the points $(G_{xy}(T), G_{xx}(T))$ of the conductances for sample 34 at different magnetic fields with temperature ranging from ≈ 10 down to ≈ 0.1 K. For sample 40, the flow diagram is rather similar to the one for sample 34. For these samples, at the magnetic fields where the spin splitting is observed, the flow lines move upwards and then downwards for decreasing temperatures. The lines cross each other for data at different magnetic fields, in contrast to the theoretical prediction for the case of a totally spin-polarized electron system. For sample 50, the flow lines do not show the upward trend and are not crossing each other. For low temperatures (below 3 K), the flow diagrams are very similar for all three samples: the flow lines approach the semicircles according to Eq. (2). Linear extrapolation of $G_{xx}(T)$ and $G_{xy}(T)$ from 0.5 to 0 K at the two fields where $G_{xx}(B)$ has maxima (for sample 40 at 12.5 and 13.7 T, see Fig. 1) results in values $G_{xx} = 0.5 \pm 0.02$ and $G_{xy} = 0.5 \pm 0.05$ and 1.5 ± 0.05. These critical values are the same as predicted for a totally spin-polarized electron system. At the lowest temperatures, the magnetic-field driven dependence $G_{xx}(G_{xy})$ is not far from the semicircles [Eq. (2)], as shown in Fig. 2.

In the absence of spin-flip scattering, the conductances of the different spin subbands are renormalized independently, at least for the case of noninteracting electrons. Since the temperature dependence of the magnetoconductance is not known for a single spin-polarized band, it is impossible to estimate accurately the flow lines for the total conductance from the flow lines for the single polarized bands because the summation $G_{ij}(T) = G_{ij}^-(T) + G_{ij}^+(T)$ involves different positions on the spin-polarized flow lines at the same temperature. The indices + and − correspond to the majority and minority spin subsystems with larger and smaller Hall conductances, respectively. Nevertheless, we can draw some conclusions on the scaling properties of the total conductance G_{ij}.

For weak spin splitting $g\mu_B B \ll \hbar/\tau \lesssim E_F$, the bare (nonrenormalized) conductances $G_{ij}^{0\pm}$ for the two spin subbands as measured at high temperatures,

$$G_{ij}^{0\pm} = \frac{G_{ij}^0}{2} \pm \frac{g\mu_B B}{4} \frac{\partial G_{ij}^0}{\partial E}, \quad (3)$$

differ weakly from each other because $g\mu_B B \partial G_{ij}^0/\partial E \sim G_{ij}^0 g\mu_B B/\Gamma \ll G_{ij}^0$. Here $G_{ij}^0 = G_{ij}^{0-} + G_{ij}^{0+}$. The QHE with total Hall conductance $G_{xy} = 1$ should arise when one subsystem is in the insulator state $[(G_{xy}^-, G_{xx}^-) \to (0,0)$ for $T \to 0]$ and the other is in the QHE state $[(G_{xy}^+, G_{xx}^+) \to (1,0)]$. This occurs in a narrow magnetic field range where $G_{xy}^{+0} > 1/2$ but $G_{xy}^{-0} < 1/2$. At the critical value of $G_{xy}^{-0} = 1/2$ and $G_{xy}^{+0} > 1/2$, $(G_{xy}^-, G_{xx}^-) \to (1/2, 1/2)$ and $(G_{xy}^+, G_{xx}^+) \to (1,0)$, therefore the total conductance $(G_{xy}, G_{xx}) \to (3/2, 1/2)$. Similarly, at the critical value of $G_{xy}^{+0} = 1/2$, the total conductance $(G_{xy}, G_{xx}) \to (1/2, 1/2)$. Thus, the critical points are the same as for the case of a totally spin-polarized electron system, in accordance with experimental results. This differs from the results of Ref. 19 predicting essentially different positions of the critical points whose exact position depends on the amount of spin splitting. Note that these results[19] have been obtained on the basis of a postulated symmetry group in order to include spin splitting, without giving any microscopic picture for the scaling behavior.

At low enough temperatures, when the spin-splitted QHE is rather well developed so that in the QHE minimum $G_{xx} \approx 0$, one can argue that the flow lines should follow the lines derived for the case of a totally spin-polarized electron system. In the minimum of G_{xx} holds $(G_{xy}^-, G_{xx}^-) = (0,0)$ and $(G_{xy}^+, G_{xx}^+) = (1,0)$, i.e., the minority subsystem does not contribute to conductance and the majority subsystem contributes only the quantum value $G_{xy} = 1$ to the Hall conductance. At lower magnetic fields, the + subsystem contributes only to the Hall conductance the value 1 as before, and the total conductance $(G_{xy}, G_{xx}) = (G_{xy}^- + 1, G_{xx}^-)$. Similarly, at higher magnetic fields $(G_{xy}, G_{xx}) = (G_{xy}^+, G_{xx}^+)$. At the lowest temperatures, the total conductance (G_{xy}, G_{xx}) is expected to flow along the same lines as derived for a single spin-polarized electron system. Therefore, G_{xx} as a function of G_{xy} flows for a changing magnetic field close to the semicircles given by Eq. (2), in accordance with experimental data below 0.1 K.

With $G_{xy}^{0\pm}$ far away from $1/2$, the conductances of the different spin subbands flow approximately in the same way, and, therefore, the flow lines for the total conductance should be twice as elongated compared to those calculated for a spin-polarized electron system.[16] In agreement with this, the outermost experimental flow lines approach the large semicircle in Fig. 2 and all data are inside this semicircle.

With $G_{xy}^{0\pm}$ close to $1/2$ or G_{xy}^0 close to 1 at the spin-splitted QHE structure, the temperature dependence of the total conductance can be essentially different depending on the value of G_{xx}^0. For $G_{xx}^0 > 1$ or $G_{xx}^{0\pm} > 1/2$, the two spin-polarized flow lines go down and the total conductance G_{xx} decreases for decreasing temperature. For $G_{xx}^0 \approx 1$ or $G_{xx}^{0\pm} \approx 1/2$, G_{xx} stays constant followed by a decrease at lower temperatures, as shown in Fig. 3 for sample 50. For $G_{xx}^0 < 1$ or $G_{xx}^{0\pm} < 1/2$, the conductances $G_{xx}^{0\pm}$ first increase and then decrease for decreasing temperature resulting in the observed nonmonotonic temperature dependence of G_{xx} for samples 34 and 40 in Fig. 3.

As mentioned above, for the totally spin-polarized electron system, the flow lines should not cross each other[15] in contrast to our experimental data. For the case of two different spin projections, we will show now that the flow lines starting in the region in between the large semicircle and the two smaller semicircles shown in Fig. 2 can cross each other. Consider the line ending at the critical point $(1/2, 1/2)$ as a reference line. Starting slightly at the left from the starting point of this line, and knowing that this line should end at (0,0), the crossing is unavoidable. Starting at the right from the starting point of this reference line, and knowing that this line should end at (1,0), leads also to a crossing point.

The spin-resolved QHE should develop at very low temperatures, where the phase-coherence length increases above

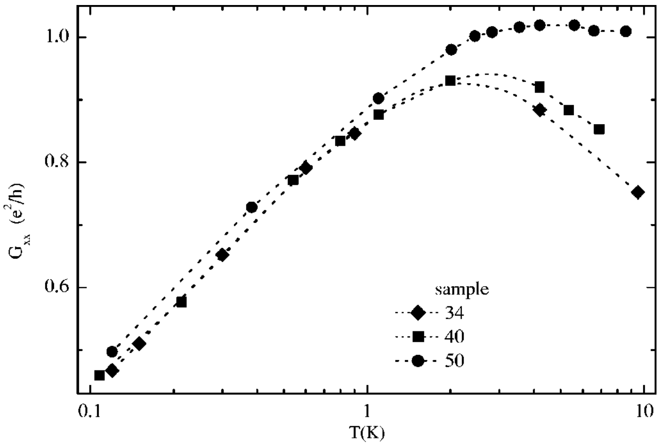

FIG. 3. Temperature dependence of the diagonal (G_{xx}) conductance, for sample 34 (diamonds), 40 (squares), and 50 (circles) at magnetic fields $B=11.8$, 13.1, and 13.1 T, correspondingly, where the minimum due to spin splitting is observed.

the localization lengths $\xi^{\pm}$ of the two spin systems, supposing that the samples are sufficiently homogeneous with the variation of G_{xy}^0 along the samples essentially smaller than the difference $(g\mu_B B\partial G_{xy}^0/\partial E)$ between spin-up and spin-down contributions. For the $G_{xy}=1$ QHE, the localization lengths $\xi^{\pm}$ are getting very large because the electronic states at the Fermi level are not far from the extended states of both spin systems $(|1/2-G_{xy}^{0\pm}|\sim g\mu_B B\partial G_{ij}^0/\partial E \ll 1)$. Therefore, for the observation of the QHE with $G_{xy}=1$, a much lower temperature (or a larger coherence length) is necessary than for the QHE with $G_{xy}=2$. In previous experiments mostly at smaller magnetic fields,[3,20] the spin splitting was not observed, probably because smaller $g\mu_B B\partial G_{ij}^0/\partial E$ leads to a larger $\xi^{\pm}$. In our samples, the spin splitting is observed only at low temperatures $T\lesssim 0.1$ K, which is much smaller than $\Gamma/k_B \gtrsim 100$ K, and even than $E_s/k_B\approx 4$ K.

Usually, for small Landau-level broadening $\Gamma \ll \hbar\omega_c$, the integer QHE effect with $G_{xy}=\nu$ occurs at magnetic fields corresponding to an integer filling factor $\nu=N_s h/eB$ for the free-electron Hall conductance $G_{xy}^0=(eN_s/B)/(e^2/h)$ normalized with respect to e^2/h. For the spin-splitted structure at $B\approx 13$ T, the filling factor $\nu\approx 1.5$ is essentially larger compared to the expected value of 1. When Γ becomes comparable with $\hbar\omega_c$, G_{xy}^0 will be smaller than the normalized Hall conductance $(eN_s/B)/(e^2/h)$ (Ref. 21) leading to $G_{xy}^0 < \nu$ and explaining the observation of a filling factor $\nu > 1$ at the spin-splitting minimum where G_{xy}^0 should be close to 1 according to the scaling theory.

The assumption about independent renormalization of the conductances of the two spin subbands is undoubtedly valid for noninteracting electrons in the absence of spin-flip scattering. Although electron-electron interaction is important in real systems, the experimental study of the flow diagram on samples 34, 40, and other thinner layers[20] shows good quantitative agreement with the predicted flow lines[16] for half the measured conductance values in the field range below 6 T, where there is not any manifestation of spin splitting and, therefore, $G_{ij}/2=G_{ij}^+=G_{ij}^-$. This gives support for our model of independent spin-band contributions.

In summary, we observed the spin-resolved quantum Hall effect in heavily doped *n*-type GaAs layers with disorder broadening much larger than the spin splitting energy, without any signature of spin splitting in the density of states. Our results are in accordance with the scaling treatment of the quantum Hall effect, applied independently to the two spin subbands. Namely, the magnetic field position for the QHE is imposed by the occurrence of the Hall quantum value $G_{xy}\approx 1$, where at all temperatures the Hall conductance $G_{xy}\approx 1$ while the filling factor $\nu > 1$. Several features in the (G_{xy},G_{xx}) flow diagrams, like the observed critical values $G_{xx}^c=0.5\pm 0.02$ and $G_{xy}^c=0.5\pm 0.05$ and 1.5 ± 0.05, and the anomalous shapes of the flow lines, can be deduced from an independent summation of the contributions of the two spin bands. The spin splitting is well observed at temperatures $T\approx 0.1$ K much smaller than all other energy scales determining the electron spectrum. Therefore, probably localization is at the origin of the observed spin-resolved QHE.

This work is supported by the Russian Foundation for Basic Research. We would like to thank B. Lemke for her help in the preparation of the samples.

[1] D. E. Khmel'nitskiĭ, Helv. Phys. Acta **65**, 164 (1992).
[2] V. Kagalovsky, B. Horovitz, and Y. Avishai, Phys. Rev. B **55**, 7761 (1997), and references therein.
[3] H. W. Jiang, C. E. Johnson, K. L. Wang, and S. T. Hannahs, Phys. Rev. Lett. **71**, 1439 (1993); T. Wang, K. P. Clark, G. F. Spencer, A. M. Mack, and W. P. Kirk, *ibid.* **72**, 709 (1994); J. F. Hughes, J. T. Nicholls, J. E. F. Frost, E. H. Linfield, M. Pepper, C. J. B. Ford, D. A. Ritchie, G. A. C. Jones, E. Kogan, and M. Kaveh, J. Phys.: Condens. Matter **6**, 4763 (1994); C. H. Lee, Y. H. Chang, Y. W. Suen, and H. H. Lin, Phys. Rev. B **58**, 10 629 (1998); C. F. Huang, Y. H. Chang, C. H. Lee, H. T. Chou, H. D. Yeh, C.-T. Liang, Y. F. Chen, H. H. Lin, H. H. Cheng, and G. J. Hwang, *ibid.* **65**, 045303 (2001).
[4] D. Shahar, D. C. Tsui, and J. E. Cunningham, Phys. Rev. B **52**, R14 372 (1995).
[5] V. T. Dolgopolov, A. A. Shashkin, A. V. Aristov, D. Schmerek, W. Hansen, J. P. Kotthaus, and M. Holland, Phys. Rev. Lett. **79**, 729 (1997), and references therein.
[6] T. Ando and Y. Uemura, J. Phys. Soc. Jpn. **37**, 1044 (1974).
[7] A. P. Smith, A. H. MacDonald, and G. Gumbs, Phys. Rev. B **45**, R8829 (1992).
[8] M. M. Fogler and B. I. Shklovskii, Phys. Rev. B **52**, 17 366 (1995); G. Murthy, *ibid.* **64**, 241309(R) (2001).
[9] B. L. Al'tshuler and A. G. Aronov, in *Electron-Electron Interaction in Disordered Systems*, edited by A. L. Efros and M. Pollak (North-Holland, Amsterdam, 1987).
[10] L. W. Wong, H. W. Jiang, E. Palm, and W. J. Schaff, Phys. Rev. B **55**, R7343 (1997).

[11] S. S. Murzin, M. Weiss, A. G. M. Jansen, and K. Eberl, Phys. Rev. B **64**, 233309 (2001).
[12] E. Shimshoni and A. Auerbach, Phys. Rev. B **55**, 9817 (1997).
[13] M. Hilke, D. Shahar, S. H. Song, D. C. Tsui, Y. H. Xie, and Don Monroe, Nature (London) **395**, 675 (1998).
[14] A. M. M. Pruisken, in *The Quantum Hall Effect*, edited by R. E. Prange and S. M. Girvin (Springer-Verlag, New York, 1990).
[15] D. E. Khmel'nitskii, Pis'ma Zh. Eksp. Teor. Fiz. **38**, 454 (1983) [JETP Lett. **38**, 552 (1984)]; Phys. Lett. **106A**, 182 (1984).
[16] B. P. Dolan, Nucl. Phys. B **554**, 487 (1999).
[17] Y. Huo, R. E. Hetzel, and R. N. Bhatt, Phys. Rev. Lett. **70**, 481 (1993).
[18] Igor Ruzin, and Shechao Feng, Phys. Rev. Lett. **74**, 154 (1995).
[19] B. P. Dolan, Phys. Rev. B **62**, 10 278 (2000).
[20] S. S. Murzin, M. Weiss, A. G. M. Jansen, and K. Eberl, Phys. Rev. B **66**, 233314 (2002).
[21] T. Ando, A. B. Fowler, and F. Stern, Rev. Mod. Phys. **54**, 437 (1982).