Assignment 3: Sampling variability and a taste of inference

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Due Saturday, July 15 at 8am

Please upload the PDF that you obtain by knitting the Rmd file that contains your R code and your text answering other questions. So this uploaded file will also show any output that R produces in addition to your code.

You should not touch the starter code as it will print out the necessary data frames and results for grading purposes.

```
set.seed(123)
```

Part 1. Summary statistics and loss minimization

For this exercise, we will verify for ourselves some properties of the sample mean, sample median, and sample mode.

To start, let's download some data on baseball players, including their salary, on-base percentage, number of runs, hits, etc. Specifically, we will be looking into the `salary` column.

```
baseball_data = read.table(url('http://web.stanford.edu/class/stats191/data/baseball.table'), header=T)
baseball_data
```

```
##    salary batting   obp run hit double triple hr rbi walk strike.out
## 1    3300   0.272 0.302  69 153     21      4 31 104   22        80
## 2    2600   0.269 0.335  58 111     17      2 18  66   39        69
## 3    2500   0.249 0.337  54 115     15      1 17  73   63       116
## 4    2475   0.260 0.292  59 128     22      7 12  50   23        64
## 5    2313   0.273 0.346  87 169     28      5  8  58   70        53
## 6    2175   0.291 0.379 104 170     32      2 26 100   87        89
## 7     600   0.258 0.370  34  86     14      1 14  38   15        45
## 8     460   0.228 0.279  16  38      7      2  3  21   11        32
## 9     240   0.250 0.327  40  61     11      0  1  18   24        26
## 10    200   0.203 0.240  39  64     10      1 10  33   14        96
## 11    177   0.262 0.283   7  38      5      0  0  10    5        18
## 12    140   0.222 0.307  21  45      9      0  6  22   19        56
## 13    117   0.227 0.280   4   5      2      0  1   3    2         1
## 14    115   0.261 0.370   1   6      0      0  0   2    4         3
## 15   2600   0.300 0.368  69 141     22      3 19  75   53        64
## 16   1907   0.225 0.292  60 130     22      1 13  73   50       100
## 17   1190   0.255 0.321  39 108     22      8  3  26   42        61
## 18    990   0.290 0.349  59 141     30      2 16  64   42       102
## 19    925   0.246 0.323  22  81     14      0  6  26   22        26
## 20    365   0.208 0.265  12  35     11      1  0  15   14        30
## 21    302   0.238 0.347  83 134     15      4 10  51   95       151
```

```
## 22      300    0.267 0.310  73 149       23       9  6  39   34           89
## 23      129    0.353 0.435  16  48       12       2  2  18   20           22
## 24      111    0.213 0.222   4  13        4       1  1   8    1           18
## 25     6100    0.302 0.391 102 174       44       6 18 100   90           67
## 26     4125    0.260 0.321  69 150       23       1 19  96   55           74
## 27     3213    0.255 0.347  45  71        7       5  1  17   39           47
## 28     2319    0.259 0.349 108 146       34       4 38 117   78          120
## 29     2000    0.223 0.307  43  84       10       4 10  40   36           56
## 30     1600    0.225 0.310  44  96        9       3  0  31   50           69
## 31     1394    0.258 0.381  58 108       23       0  4  51   83           50
## 32      935    0.275 0.351  40  70       16       4  4  21   30           42
## 33      850    0.327 0.424  60 141       14       3  0  54   75           38
## 34      775    0.272 0.300  18  62       14       2  5  35    9           19
## 35      760    0.241 0.320  33  84       16       2  6  36   40           53
## 36      629    0.293 0.355  32  79       17       0  1  30   24           43
## 37      275    0.257 0.315  51  96       10       1  6  34   33           57
## 38      120    0.225 0.330   7  20        2       0  0   5   14           19
## 39     2567    0.196 0.297  36  56       12       0 12  42   41           66
## 40     2500    0.252 0.309  66 137       33       1 28  81   48           93
## 41     2350    0.294 0.367  84 158       27       6 21  92   67          100
## 42     2317    0.297 0.391  48  73       13       5  3  12   37           20
## 43     2000    0.258 0.288  46  86        7       4 12  40   12           57
## 44      715    0.205 0.268  21  36        5       3  1  11   16           36
## 45      660    0.272 0.304  38  82       21       3  9  49   14           49
## 46      650    0.243 0.344  20  45       12       0  0  15   30           30
## 47      260    0.337 0.413  13  32        5       0  0  12   13           14
## 48      250    0.228 0.238  12  36        4       1  1  11    2           26
## 49      200    0.240 0.300  51  92       16       3 13  50   31           73
## 50      180    0.298 0.378  18  45       10       2  6  21   17           26
## 51      180    0.249 0.313  38  81       11       4  1  20   29           45
## 52     5150    0.292 0.410  95 149       28       5 25 116  107           73
## 53     4450    0.265 0.355  87 130       24       7 17  83   71           85
## 54     2000    0.265 0.306  53 133       16       6  3  54   30           56
## 55     1850    0.289 0.351  25  97       11       2  3  41   33           27
## 56     1192    0.295 0.363  19  65       17       1  1  29   21           32
## 57      875    0.270 0.330  96 164       32       8 16  67   52           99
## 58      825    0.246 0.324  45  62       12       2  7  24   28           39
## 59      525    0.288 0.366  24  47        7       0  7  24   18           23
## 60      367    0.273 0.344  23  51       11       2  4  23   19           34
## 61      325    0.239 0.328  16  26        1       1  4  18   14           15
## 62      320    0.244 0.281   7  20        4       0  1  11    5           17
## 63      150    0.275 0.373  83 113       17       2 10  50   64           81
## 64      113    0.340 0.381  15  36        7       0  0   7    7           17
## 65      113    0.250 0.280   2   6        0       2  0   1    1            8
## 66     2425    0.272 0.326  41 116       12       1  8  70   37           46
## 67     2367    0.219 0.268  34  82       13       2  9  33   23           86
## 68     2050    0.240 0.357  37  88       11       4  1  26   67           48
## 69     2000    0.285 0.380  96 157       30       3  3  50   83           36
## 70     1617    0.307 0.368  55 100       16       5  6  34   32           53
## 71     1167    0.240 0.300  29  58        8       4  6  36   22           34
## 72      992    0.264 0.319  38 121       24       5  2  57   36           63
## 73      400    0.227 0.260  21  47       10       2  1  15   10           29
## 74      315    0.305 0.360  69 173       40       6  8  77   50          113
## 75      315    0.280 0.353  76 158       36       3 11  81   62           94
```

```
## 76      230   0.251 0.301   83 142     23     15  9  69   41        114
## 77      135   0.216 0.316   28  58      7      2  5  28   39         33
## 78      130   0.243 0.322   38  45      8      3  5  17   18         45
## 79     3150   0.319 0.363   94 187     34      8 22  86   43         70
## 80     2785   0.297 0.371   81 119     10      1  0  26   47         40
## 81     2700   0.251 0.338  101 141     35      3 32 105   71        104
## 82     2400   0.253 0.313   32  67     12      0 11  45   25         31
## 83     1750   0.275 0.377   58  97     19      1  7  44   50         64
## 84     1262   0.320 0.368   41  98     17      2  3  32   23         19
## 85      940   0.259 0.358   49  91     14      3 11  54   54         59
## 86      555   0.275 0.377   67 109     25      1 21  87   65         81
## 87      500   0.249 0.296   36  88      9      2  0  27   22         63
## 88      370   0.188 0.243   13  30      7      0  5  14   11         42
## 89      350   0.241 0.316   46  99     25      0  6  44   44         48
## 90      148   0.242 0.284    7  23      6      0  4  23    6         20
## 91      146   0.251 0.296   32  68     16      1 12  50   17         48
## 92     4350   0.302 0.378   88 140     27      4 20  69   55         64
## 93     2833   0.280 0.359   51 101     12      2  6  35   46         39
## 94     2833   0.256 0.346   71 136     36      0 28  91   73        107
## 95     2750   0.301 0.354   91 175     35      3 26  88   44         79
## 96     1550   0.281 0.342   66 119     13      3  3  32   37         71
## 97     1300   0.234 0.301   13  36      5      0  6  19   16         39
## 98     1000   0.260 0.323   36  65     10      0 11  39   23         46
## 99      750   0.267 0.321   20  72     15      2  3  31   23         38
## 100     430   0.318 0.374   72 152     33      1 14  59   46         61
## 101     260   0.216 0.265   21  58     11      0 11  41   18         53
## 102     120   0.286 0.303   11  18      1      0  0   3    1         15
## 103    1500   0.253 0.312   65 145     30      3 13  80   46         85
## 104    1458   0.295 0.358   79 161     23      4  4  46   53         71
## 105    1210   0.285 0.331   84 170     28     10  8  54   42         65
## 106     935   0.262 0.319   44 121     20      7  4  50   40         49
## 107     800   0.214 0.290   38  72     12      1 11  38   36         92
## 108     395   0.218 0.327   26  31      3      1  1  11   24         17
## 109     350   0.294 0.387   79 163     26      4 15  82   75        116
## 110     285   0.254 0.320   51 120     28      9 13  69   40        101
## 111     139   0.243 0.270   27  61     13      2  9  36    9         74
## 112     133   0.153 0.227   11  18      6      0  1   7   12         41
## 113    4050   0.265 0.361   86 134     22      4 28  99   75        125
## 114    3600   0.235 0.353   39  67     10      0 11  33   48         92
## 115    3333   0.296 0.401  112 182     13      5  2  38  108         79
## 116    2183   0.264 0.353   39  91     16      2  8  40   47         32
## 117    1150   0.294 0.338   29  86     15      3  2  40   17         46
## 118     840   0.287 0.349   59 123     16      1  3  38   37         32
## 119     587   0.278 0.355   24  60     11      2  2  20   25         24
## 120     400   0.222 0.296   21  38      8      5  2  19   18         52
## 121     350   0.248 0.286   24  56      5      2  2  13   12         35
## 122     135   0.268 0.293    3  15      4      0  1   5    2         12
## 123     135   0.214 0.250    1   3      1      0  0   1    0          5
## 124     135   0.195 0.345   10  22      2      0  0   3   25         32
## 125    4075   0.278 0.396   84 147     19      1 31 106  105        135
## 126    3300   0.267 0.296   60 155     22      3 17  87   23        114
## 127    2387   0.317 0.355   69 158     27     11  4  62   34         19
## 128    2100   0.272 0.337   81 162     27      5  4  38   55         74
## 129    1750   0.265 0.322   44 102     17      1  6  51   33         56
```

```
## 130    805   0.262 0.315  51  94       12       1 21  49    27           66
## 131    725   0.194 0.277  25  34       12       2  2  22    19           15
## 132    687   0.217 0.319  41  74       16       0 12  44    51           77
## 133    200   0.228 0.295  26  84       16       0 10  47    31           90
## 134    130   0.286 0.444   0   2        0       0  0   0     2            3
## 135    125   0.269 0.345   4   7        2       1  0   5     3            4
## 136    120   0.246 0.267   5  14        2       0  0   9     1            9
## 137    117   0.275 0.298   6  25        3       0  1   6     2           14
## 138    114   0.243 0.308  13  26        7       2  2   8     9           27
## 139   4275   0.301 0.359  84 170       32       7 29 116    51           91
## 140   3563   0.312 0.357  67 155       30       3  4  43    34           74
## 141   2000   0.268 0.310  72 158       24       5 34  98    33          128
## 142   1600   0.262 0.352  74 129       24       5 19  48    63           95
## 143   1533   0.221 0.283  23  51        8       4  1  12    20           33
## 144    940   0.175 0.216  14  29        4       1  3  17     9           60
## 145    850   0.238 0.273  31 104       16       2  5  41    18           61
## 146    650   0.264 0.325  51  92       10       6  0  18    30           31
## 147    360   0.225 0.271  16  40        9       0  0  19     9           22
## 148    150   0.123 0.188  12  13        3       0  2   8     7           26
## 149    145   0.246 0.358  41  55        5       3  1  15    36           30
## 150    140   0.240 0.306  14  31        7       1  2  14    12           25
## 151    109   0.115 0.148   0   3        1       0  0   2     1            6
## 152    109   0.120 0.185   0   3        0       0  0   1     2           11
## 153   3415   0.227 0.307  29  40        9       1 10  28    16           29
## 154   2100   0.323 0.374  99 210       46       5 34 114    53           46
## 155   1300   0.278 0.321  57 135       22       1  5  43    28           45
## 156   1200   0.263 0.373  57 127       17       2 16  70    84          108
## 157   1025   0.260 0.313  82 158       27      10 19  59    47          115
## 158    730   0.216 0.253  24  62       11       1  0  14    15           31
## 159    687   0.233 0.326  45  74       16       0 23  61    41           99
## 160    380   0.204 0.255  29  42        9       0  7  18    13           49
## 161    365   0.230 0.336  40  59       12       3  2  27    36           44
## 162    175   0.243 0.304  36  83       15       0 11  31    29           61
## 163    150   0.233 0.302  40  91       17       2 16  45    40           82
## 164    150   0.278 0.316  15  59        7       0  2  22    12           19
## 165    140   0.269 0.303  32  58       12       1 13  33    11           51
## 166   3050   0.300 0.350  76 163       26       6  9  83    43           35
## 167   2900   0.249 0.374  75 120       18       1 26  87    96          133
## 168   2850   0.229 0.303  54 105       24       1 16  70    49           72
## 169   2700   0.332 0.421  93 181       42       2  8  51    89           32
## 170   2400   0.231 0.291  45 107       23       2  5  48    37           53
## 171   2300   0.251 0.314  56 119       33       3 14  56    39           81
## 172   1650   0.262 0.312  45 116       25       3  4  41    26           55
## 173   1600   0.283 0.349  87 175       42       2  5  60    60           53
## 174   1075   0.258 0.318  64 107       22       3  8  40    35           86
## 175    550   0.225 0.272  17  38        6       1  1   4    11           40
## 176    350   0.263 0.271  10  30        8       0  0   9     1           16
## 177    340   0.295 0.375  69 141       21       1 11  71    61           66
## 178    155   0.260 0.339  21  57       12       0  4  32    26           43
## 179    145   0.457 0.486   6  16        4       2  0   7     2            2
## 180    950   0.262 0.307  30 111       13       2  0  31    26           27
## 181    700   0.243 0.279  23  69       14       0  1  24    14           67
## 182    550   0.224 0.274  28  94       21       1  1  44    27           54
## 183    525   0.217 0.264  10  40        9       0  0   7     8           24
```

```
## 184    525   0.288 0.346  80 171      28       2 11  69   48           74
## 185    333   0.243 0.297  46  99      18       7  9  45   30           85
## 186    325   0.258 0.324  29  57       8       2  8  25   23           54
## 187    300   0.209 0.293   9  28       5       1  0  11   15           12
## 188    200   0.284 0.310  22  73      14       0  5  30   10           43
## 189    175   0.282 0.323  60 130      31       2 26  95   25           99
## 190    158   0.249 0.309  30  70      12       3  4  22   24           57
## 191    145   0.264 0.293  29  83      15       1  0  30   15           45
## 192   4500   0.261 0.347 102 163      25       0 44 133   78          151
## 193   3358   0.263 0.387  85 132      17       2 31  89  101          131
## 194   2400   0.248 0.320  57  93      20       0  9  55   37           39
## 195   2200   0.279 0.391  94 131      26       2 23  78   90           45
## 196   2017   0.179 0.314  64  60      14       2 25  64   89          175
## 197   1567   0.284 0.371  87 160      28       4 17  72   79           95
## 198   1050   0.247 0.306  65 114      14       9  6  52   36           60
## 199    375   0.237 0.351  23  46      10       1  7  29   35           40
## 200    355   0.260 0.293  18  66       6       0  4  21   12           26
## 201    300   0.259 0.309  65 144      36       3 21  91   40          149
## 202    230   0.257 0.335  77 122      15       7  3  33   52           92
## 203    150   0.289 0.325  29  46      13       2  5  17    9           24
## 204    120   0.291 0.341  19  37       5       0  2  11   10           25
## 205    109   0.216 0.341  37  61       7       5  0  19   54           91
## 206   3453   0.325 0.399 133 216      32      13 17  75   77           62
## 207   3200   0.260 0.332  66 131      20       4 10  77   54           79
## 208   2530   0.289 0.319  57 146      19       4  5  68   26           33
## 209   2167   0.213 0.282  48  77      16       2 11  38   35           71
## 210   1150   0.286 0.320  63 149      27       4  2  47   27           34
## 211    750   0.283 0.337  71 117      13       6  8  54   34           55
## 212    500   0.244 0.319  81 132      24       5 27  98   62          125
## 213    395   0.311 0.361  64 125      15       6  1  57   33           38
## 214    300   0.206 0.262  14  51      18       1  1  28   17           26
## 215    284   0.265 0.350  28  62      11       3  1  25   29           21
## 216    230   0.238 0.272  53 106      18       3 15  59   22          107
## 217    109   0.364 0.364   2   4       1       0  1   1    0            4
## 218   5300   0.316 0.397  78 153      35       3 31 100   65          121
## 219   3620   0.288 0.339  64 169      35       0  9  68   46           42
## 220   2500   0.268 0.308  52 122      20       0 24  77   25           49
## 221   2167   0.225 0.312  37  64      12       0 17  48   36           80
## 222   2150   0.267 0.333  68 130      22       2 20  69   45           77
## 223   1950   0.247 0.343  67 119      15       4 12  49   67           84
## 224   1300   0.285 0.321  67 140      23       2 19  80   26           40
## 225    360   0.245 0.322  19  45      11       1  1  15   18           43
## 226    280   0.230 0.257  34 106      23       1 12  53   16           75
## 227    255   0.220 0.333  69 110      14       1 23  63   83          128
## 228    147   0.242 0.288  35  72      12       4  3  23   15           52
## 229    125   0.238 0.336  43  76      19       4  3  34   48           57
## 230    109   0.308 0.310   4   8       1       0  0   3    1            3
## 231   3742   0.273 0.330  89 174      42       3 33 108   49          112
## 232   3633   0.252 0.308  58 108      18       2 20  65   31           70
## 233   2983   0.295 0.354  88 188      41      11  9  69   57           86
## 234   2383   0.282 0.342 110 181      40      10 17  60   55          135
## 235   2300   0.262 0.326  75 149      27       4 28  86   56          109
## 236   1375   0.250 0.342  37  72      15       0 12  48   36           76
## 237   1000   0.244 0.271  22  71      17       0  5  36   11           45
```

```
## 238    1000    0.234 0.274  41 104       18        3  0  29   24          107
## 239     900    0.216 0.318  20  40        5        1  1  21   29           21
## 240     775    0.250 0.364  27  60       12        1  2  24   44           44
## 241     600    0.243 0.286  27  85        6        2  0  27   22           49
## 242     387    0.256 0.353  64 116       30        1 17  68   68           84
## 243     117    0.275 0.361  17  44        7        0  4  20   19           43
## 244    2700    0.246 0.293  58 144       22        1 18  66   33          104
## 245    2317    0.238 0.322  48  85       11        1 16  50   44           62
## 246    1650    0.296 0.352  92 179       28        8  2  50   52           74
## 247     640    0.263 0.321  32  65       10        2  2  26   11           55
## 248     190    0.258 0.295  38  94       14        1  3  20   14           26
## 249     170    0.200 0.258  11  23        7        0  1  13    8           34
## 250     135    0.211 0.274   5  12        2        0  0   3    3            2
## 251     130    0.203 0.313   7  14        4        0  0   3   10           17
## 252     130    0.293 0.354   8  17        7        0  0   9    6           12
## 253    3650    0.285 0.323  63 159       27        0 25  86   32           62
## 254    3575    0.304 0.345  85 198       38        2 10  56   41           38
## 255    3500    0.268 0.359 102 163       20        6  5  50   83           68
## 256    2500    0.259 0.358  71 106       22        5 18  66   62           86
## 257    1900    0.273 0.284  52 143       20        3  3  49   11           38
## 258    1350    0.241 0.299  42 111       25        0 18  74   32           86
## 259     740    0.284 0.367  92 172       25        1 23 100   80           67
## 260     650    0.246 0.310  25  41       13        0  5  22   15           42
## 261     620    0.318 0.453 104 178       31        2 32 109  138          112
## 262     565    0.274 0.304  72 161       14       13  0  49   26           58
## 263     235    0.241 0.313  37  55        2        3  0  18   20           21
## 264     225    0.165 0.218  15  19        4        1  1   5    7           25
## 265     200    0.281 0.386  37  63       16        3  6  31   38           40
## 266     145    0.251 0.361  42  61       10        2  3  25   39           48
## 267     120    0.217 0.250   4   5        0        0  0   0    1            7
## 268     109    0.225 0.333  71   8       16        4  0   3   14           12
## 269    4200    0.301 0.360  79 166       34        3 21  96   52           66
## 270    3417    0.259 0.322  65 135       32        1 16  74   49           46
## 271    3100    0.255 0.327  77 129       40        2 10  61   58           75
## 272    1650    0.301 0.333  47 113       22        3  2  47   20           35
## 273    1150    0.272 0.338  59 132       19        2  9  62   47           38
## 274     900    0.250 0.279  11  57       10        0  1  23   11           46
## 275     740    0.277 0.330  34  74       18        2 13  41   17           52
## 276     575    0.280 0.336  41  77       22        1  4  23   23           44
## 277     425    0.252 0.301  15  35        5        1  5  22   10           23
## 278     400    0.188 0.243  20  36        9        1  2  18   15           40
## 279     287    0.277 0.320  24  51        9        0  2  13   11           42
## 280     275    0.261 0.289  86 164       28        9  8  64   24           99
## 281     183    0.217 0.283  45  80       16        4  5  34   30           75
## 282     170    0.251 0.315  22  58        8        0  3  31   23           42
## 283     153    0.216 0.267  20  51        7        0  1  17   16           45
## 284     109    0.271 0.328  74 161       22        6 12  58   49          133
## 285    3100    0.284 0.373  72 131       20        1 20  89   67           48
## 286    2992    0.319 0.352  92 195       29        6 15  89   31           78
## 287    2800    0.277 0.385  84 148       34        1 29  93   95          117
## 288    2500    0.311 0.336  54 137       28        1 10  69   14           22
## 289    1933    0.265 0.310  52 108       23        3  8  42   26           72
## 290    1200    0.279 0.322  36 102       20        0  6  36   21           55
## 291    1075    0.310 0.363  79 137       27        8 18  74   34           79
```

```
## 292    950   0.286 0.361  34  73      14      1  2  19   30          21
## 293    450   0.303 0.401  21  50      10      1  6  23   24          25
## 294    360   0.281 0.351  78 159      24      6  1  50   59          40
## 295    175   0.286 0.378  35  57       7      2  5  20   30          35
## 296    155   0.283 0.327  15  39       7      1  7  26    9          31
## 297   5000   0.307 0.357 110 203      44      5 25 116   56          91
## 298   3250   0.268 0.400 105 126      17      1 18  57   96          73
## 299   2825   0.201 0.330  62  97      22      0 22  75   93         116
## 300   2600   0.276 0.346  86 158      33      0 25  85   58         113
## 301   2050   0.274 0.312  50 125      31      1  6  67   22          70
## 302   1583   0.295 0.383  76 144      25      1 20  90   72          67
## 303   1150   0.063 0.063   0   1       0      0  0   1    0           2
## 304   1100   0.238 0.290  38  70      14      4  0  28   18          43
## 305    760   0.226 0.286  15  30       8      1  0  13   12          14
## 306    525   0.261 0.321  16  53       4      0  1  17   16          28
## 307    500   0.249 0.385  32  51      10      1  1  20   47          25
## 308    205   0.249 0.336  33  46       8      0  3  21   23          42
## 309    185   0.238 0.289  21  56       5      1  0  21   14          37
## 310    115   0.235 0.268   9  16       5      0  2   4    3          11
## 311   3750   0.256 0.338  52  95      13      1 27  69   43          57
## 312   2188   0.248 0.300  58 139      29      3 17  88   44          61
## 313   2167   0.254 0.332  95 160      34      6  3  57   72          63
## 314   2167   0.194 0.286  38  63       8      1  8  32   34          49
## 315   2050   0.327 0.399  76 179      42      1 22 100   71          82
## 316   1445   0.244 0.337  64  99      14      4 27  77   53         117
## 317    875   0.305 0.347  35  54       6      2  6  23   10          27
## 318    560   0.307 0.405  98 167      35      1 14  52   84          72
## 319    360   0.230 0.302  42  98      16      4  1  41   45          37
## 320    162   0.247 0.286  16  44      13      0  2  22    9          38
## 321    133   0.205 0.272  11  23       2      0  4   9   11          24
## 322    109   0.216 0.285  38  87      12      0 19  51   35         117
## 323   4300   0.266 0.359 115 152      32      1 44 122   78         152
## 324   3850   0.322 0.389 115 203      49      3 26  88   68          72
## 325   2387   0.341 0.408 108 201      27      3 15  78   65          78
## 326    950   0.278 0.377  76 113      17      2 17  49   58          70
## 327    675   0.271 0.339  21  54       8      1  2  20   21          25
## 328    600   0.252 0.283  44 136      18      4  9  44   25          84
## 329    287   0.264 0.321  78 144      34      1 27 102   42         118
## 330    230   0.269 0.332  46 106      22      0 20  69   33          93
## 331    215   0.194 0.270   8  28       3      2  1  11   16          23
## 332    183   0.213 0.312  36  57       8      3  2  26   39          32
## 333    170   0.111 0.138   3   3       0      0  0   1    1           7
## 334    160   0.264 0.318  24  48       7      0  1  22   15          18
## 335    142   0.187 0.281  38  50       9      2 15  37   32          98
## 336    140   0.264 0.270  24  74      16      0  3  27    5          42
## 337    109   0.258 0.395   6   8       1      0  1   6    7          11
##     stolen.base error free.agent.eligible free.agent.1991 arbitr.elgible.
## 1             4     3                   1               0               0
## 2             0     3                   1               1               0
## 3             6     5                   1               0               0
## 4            21    21                   0               0               1
## 5             3     8                   0               0               1
## 6            22     4                   1               0               0
## 7             0    10                   1               0               0
```

```
## 8             2     3                   0               0               0
## 9            14     2                   0               0               0
## 10           13     6                   0               0               0
## 11            2     7                   0               0               0
## 12            3     3                   0               0               0
## 13            0     0                   0               0               0
## 14            0     0                   0               0               0
## 15           31     7                   1               0               0
## 16            2    14                   1               0               0
## 17            2     8                   0               0               1
## 18           14     6                   1               0               0
## 19            2     5                   1               0               0
## 20            2     6                   0               0               0
## 21           56    27                   0               0               0
## 22           76     6                   0               0               0
## 23            0     5                   0               0               0
## 24            0     0                   0               0               0
## 25            2    15                   1               1               0
## 26           10     7                   1               1               0
## 27           37     3                   1               0               0
## 28           30    31                   1               0               0
## 29            7     4                   1               0               0
## 30            8    15                   1               1               0
## 31            1     5                   1               0               0
## 32           15     3                   1               1               0
## 33            4    20                   1               1               0
## 34            0     3                   0               0               1
## 35            2    14                   0               0               1
## 36            2     0                   0               0               0
## 37           13     8                   0               0               0
## 38            1     2                   0               0               0
## 39            5     8                   0               0               1
## 40            1     5                   1               0               0
## 41            7     3                   1               0               0
## 42           24     4                   1               0               0
## 43            5     9                   1               1               0
## 44            7     3                   1               0               0
## 45            0     9                   0               0               1
## 46            3     4                   1               0               0
## 47            0     0                   0               0               0
## 48            0     2                   0               0               0
## 49            9     3                   0               0               0
## 50            1     8                   0               0               0
## 51           13     6                   0               0               0
## 52           43     3                   0               0               1
## 53           10     1                   1               0               0
## 54            7     9                   0               0               1
## 55            2     1                   1               1               0
## 56            1     5                   1               0               0
## 57           10    24                   0               0               1
## 58           17     6                   1               0               0
## 59            2     3                   0               0               0
## 60            9     1                   0               0               0
## 61            3     2                   0               0               0
```

```
## 62             4    2                   0          0          0
## 63             8   12                   0          0          0
## 64             3    6                   0          0          0
## 65             0    1                   0          0          0
## 66             4   16                   1          1          0
## 67             5    9                   1          0          0
## 68             1    9                   1          0          0
## 69            35    8                   1          0          0
## 70            16    2                   1          0          0
## 71            15    5                   1          0          0
## 72             9    7                   0          0          1
## 73            12    2                   0          0          0
## 74            20    3                   0          0          0
## 75            17   25                   0          0          0
## 76            44    6                   0          0          0
## 77            14    1                   0          0          0
## 78            15    6                   0          0          0
## 79            10   24                   1          0          0
## 80            72    3                   1          1          0
## 81            34    6                   0          0          1
## 82             0    3                   1          0          0
## 83             9    5                   1          0          0
## 84             2   15                   0          0          1
## 85             5   17                   0          0          1
## 86             8    7                   0          0          0
## 87             3   18                   0          0          1
## 88             1    8                   0          0          1
## 89             1    4                   0          0          0
## 90             1    3                   0          0          0
## 91             0    8                   0          0          0
## 92            24   15                   1          0          0
## 93             5    7                   1          0          0
## 94            12    2                   0          0          1
## 95            19   12                   0          0          1
## 96            26   10                   0          0          1
## 97             0    6                   0          0          1
## 98            11    5                   0          0          1
## 99             0    5                   0          0          0
## 100           10    9                   0          0          0
## 101            0   11                   0          0          0
## 102            1    2                   0          0          0
## 103            4   23                   0          0          1
## 104           19   11                   0          0          1
## 105           34    5                   0          0          1
## 106            9   10                   0          0          0
## 107            1    3                   1          1          0
## 108           16    0                   0          0          0
## 109            7   12                   0          0          0
## 110           10    5                   0          0          0
## 111            4   18                   0          0          0
## 112            1    1                   0          0          0
## 113           10    5                   1          0          0
## 114           14    3                   1          0          0
## 115           38    0                   1          0          0
```

```
## 116          4    7                    1                0                0
## 117          2    3                    1                0                0
## 118         12   20                    1                0                0
## 119          1    4                    0                0                1
## 120          0    2                    1                0                0
## 121          2   11                    1                1                0
## 122          1    0                    0                0                0
## 123          1    1                    0                0                0
## 124          3   10                    0                0                0
## 125          4   14                    1                0                0
## 126          8   14                    0                0                1
## 127          8    3                    1                0                0
## 128         23   20                    0                0                1
## 129          3   18                    1                1                0
## 130          5    2                    0                0                1
## 131          5    8                    0                0                1
## 132          9    9                    1                1                0
## 133          2    2                    0                0                0
## 134          0    0                    0                0                0
## 135          0    0                    0                0                1
## 136          2    2                    0                0                0
## 137          0    7                    0                0                0
## 138          1    1                    0                0                0
## 139          4    4                    1                0                0
## 140         17    6                    1                0                0
## 141          5   16                    0                0                1
## 142         14   11                    1                0                0
## 143          3   11                    0                0                1
## 144          0    3                    1                0                0
## 145          3    0                    0                0                1
## 146         21    4                    0                0                0
## 147          1    4                    0                0                0
## 148          3    3                    0                0                0
## 149         13    0                    0                0                0
## 150          0    0                    0                0                0
## 151          0    3                    0                0                0
## 152          0    1                    0                0                0
## 153          4    8                    1                1                0
## 154          6   11                    1                0                0
## 155          6    1                    1                0                0
## 156          0   11                    0                0                1
## 157         16    3                    1                0                0
## 158          0    7                    0                0                1
## 159          0    0                    0                0                0
## 160          0    4                    0                0                0
## 161         12    3                    0                0                0
## 162          0    1                    0                0                0
## 163          1    7                    0                0                0
## 164          1    3                    0                0                0
## 165          1    2                    0                0                0
## 166         15    3                    0                0                1
## 167          0    0                    1                0                0
## 168          1    3                    1                0                0
## 169          1   12                    1                0                0
```

```
## 170           8    5                 1           0           0
## 171           6    2                 1           0           0
## 172          11    5                 0           0           1
## 173           6   14                 0           0           1
## 174           4   24                 0           0           1
## 175           4    5                 1           1           0
## 176           0    3                 0           0           0
## 177           1    9                 0           0           0
## 178           2    6                 0           0           0
## 179           0    2                 0           0           0
## 180           5   12                 0           0           1
## 181           0    5                 0           0           0
## 182           2    7                 1           1           0
## 183           0    4                 0           0           0
## 184           3   27                 0           0           0
## 185           4    7                 0           0           0
## 186           6    3                 0           0           0
## 187           0    1                 1           1           0
## 188           3    8                 0           0           0
## 189           3    9                 0           0           0
## 190          10    1                 0           0           0
## 191           2    9                 0           0           0
## 192           0    8                 1           0           0
## 193           3    6                 1           0           0
## 194          11    9                 1           0           0
## 195           4    4                 1           0           0
## 196           1    7                 1           0           0
## 197          10    8                 1           0           0
## 198          15    3                 1           1           0
## 199           1    1                 1           1           0
## 200           2    3                 1           0           0
## 201          12   23                 0           0           0
## 202          41    6                 0           0           0
## 203          10    2                 0           0           0
## 204           2    2                 0           0           0
## 205          29    6                 1           0           0
## 206          19    6                 1           0           0
## 207           6    2                 1           0           0
## 208           5    4                 0           0           1
## 209          13    9                 1           0           0
## 210           4   12                 1           1           0
## 211          14   17                 0           0           0
## 212           2    2                 0           0           0
## 213          16    1                 0           0           0
## 214           0    3                 1           1           0
## 215           4   11                 1           0           0
## 216          14    7                 0           0           0
## 217           0    0                 0           0           0
## 218           6    7                 1           1           0
## 219           2    5                 1           0           0
## 220           3    6                 1           0           0
## 221           1    0                 1           0           0
## 222          32    4                 0           0           1
## 223           6   12                 1           1           0
```

```
## 224          0   3                   0                0                1
## 225          3  15                   0                0                0
## 226          3  15                   0                0                0
## 227          5   6                   0                0                0
## 228         12  18                   0                0                0
## 229         10   5                   0                0                0
## 230          0   0                   0                0                0
## 231         20   8                   1                0                0
## 232         12  13                   1                0                0
## 233         53  15                   0                0                1
## 234         33   1                   1                0                0
## 235          7   2                   1                1                0
## 236          4   2                   1                0                0
## 237          0   4                   0                0                1
## 238          7  19                   0                0                1
## 239          0   3                   1                0                0
## 240          0   0                   1                0                0
## 241          5  22                   1                1                0
## 242          0   5                   0                0                0
## 243          0  14                   0                0                0
## 244          5  17                   1                0                0
## 245          3   5                   1                0                0
## 246         48   5                   0                0                1
## 247          7   3                   0                0                0
## 248          4  11                   0                0                0
## 249          1   3                   0                0                0
## 250          0   4                   0                0                0
## 251          0   1                   0                0                0
## 252          1   1                   0                0                0
## 253          2  10                   1                0                0
## 254         31  10                   1                0                0
## 255         51   3                   1                0                0
## 256          0   6                   1                1                0
## 257         21  21                   0                0                1
## 258          1   6                   1                1                0
## 259          2  18                   0                0                1
## 260          0   4                   0                0                1
## 261          1   2                   0                0                0
## 262         26   2                   0                0                0
## 263         11  10                   0                0                0
## 264          0   0                   0                0                0
## 265          1  10                   0                0                0
## 266         14   2                   0                0                0
## 267          1   1                   0                0                0
## 268         25   0                   0                0                1
## 269          2   8                   1                1                0
## 270          6   2                   1                0                0
## 271          2   1                   1                0                0
## 272          5   5                   1                1                0
## 273         26  17                   0                0                1
## 274          0   1                   1                0                0
## 275          1   3                   0                0                1
## 276         14  10                   0                0                0
## 277          1   0                   0                0                1
```

```
## 278           2    2          1          1          0
## 279          15    4          0          0          0
## 280          20    3          0          0          0
## 281          17   16          0          0          0
## 282           2    6          0          0          0
## 283           3   12          0          0          0
## 284          23   17          1          1          0
## 285           4    8          1          0          0
## 286          11    6          1          0          0
## 287           5    0          1          0          0
## 288           1    8          1          1          0
## 289          11    9          0          0          1
## 290           1   11          1          1          0
## 291          13    7          0          0          1
## 292           2    3          0          0          1
## 293           0    2          0          0          0
## 294          25   18          0          0          0
## 295           5    7          0          0          0
## 296           3    1          0          0          0
## 297          16    7          0          0          1
## 298          58    8          1          0          0
## 299           2    4          1          0          0
## 300           6    1          1          0          0
## 301           2   15          0          0          1
## 302           0    1          1          0          0
## 303           0    0          1          1          0
## 304          20    3          1          0          0
## 305           0    5          0          0          1
## 306           0    6          1          0          0
## 307           2    3          1          1          0
## 308          12    3          0          0          0
## 309           3   11          0          0          0
## 310           3    1          0          0          0
## 311           2    6          1          0          0
## 312           0    5          1          0          0
## 313          28   18          0          0          1
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## 315          18    4          0          0          1
## 316           0    5          0          0          1
## 317          16    2          0          0          1
## 318           0   15          0          0          0
## 319           7   13          0          0          0
## 320           0    7          0          0          0
## 321           0    2          0          0          0
## 322           0    2          1          0          0
## 323          26    9          1          0          0
## 324           4   12          0          0          1
## 325          36   14          1          0          0
## 326           1    0          1          1          0
## 327           2   11          1          1          0
## 328          11   21          1          1          0
## 329           4    6          0          0          0
## 330           0    6          0          0          0
## 331           1    1          0          0          0
```

```
## 332           8    15                   0               0               0
## 333           0     0                   0               0               0
## 334           0     7                   0               0               0
## 335           0     9                   0               0               0
## 336           0    10                   0               0               0
## 337           0     0                   0               0               0
##     arbitr.1991
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## 2             0
## 3             0
## 4             0
## 5             0
## 6             0
## 7             0
## 8             0
## 9             0
## 10            0
## 11            0
## 12            0
## 13            0
## 14            0
## 15            0
## 16            0
## 17            0
## 18            0
## 19            0
## 20            0
## 21            0
## 22            0
## 23            0
## 24            0
## 25            0
## 26            0
## 27            0
## 28            0
## 29            0
## 30            0
## 31            0
## 32            0
## 33            0
## 34            0
## 35            1
## 36            0
## 37            0
## 38            0
## 39            0
## 40            0
## 41            0
## 42            0
## 43            0
## 44            0
## 45            0
## 46            0
## 47            0
```

```
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## 49                0
## 50                0
## 51                0
## 52                0
## 53                0
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## 55                0
## 56                0
## 57                1
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## 59                0
## 60                0
## 61                0
## 62                0
## 63                0
## 64                0
## 65                0
## 66                0
## 67                0
## 68                0
## 69                0
## 70                0
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## 93                0
## 94                0
## 95                0
## 96                0
## 97                0
## 98                1
## 99                0
## 100               0
## 101               0
```

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## 103                0
## 104                0
## 105                0
## 106                0
## 107                0
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## 110                0
## 111                0
## 112                0
## 113                0
## 114                0
## 115                0
## 116                0
## 117                0
## 118                0
## 119                0
## 120                0
## 121                0
## 122                0
## 123                0
## 124                0
## 125                0
## 126                1
## 127                0
## 128                0
## 129                0
## 130                0
## 131                0
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## 147                0
## 148                0
## 149                0
## 150                0
## 151                0
## 152                0
## 153                0
## 154                0
## 155                0
```

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## 156                0
## 157                0
## 158                0
## 159                0
## 160                0
## 161                0
## 162                0
## 163                0
## 164                0
## 165                0
## 166                0
## 167                0
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## 202                0
## 203                0
## 204                0
## 205                0
## 206                0
## 207                0
## 208                0
## 209                0
```

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## 212                0
## 213                0
## 214                0
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## 216                0
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## 218                0
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## 245                0
## 246                1
## 247                0
## 248                0
## 249                0
## 250                0
## 251                0
## 252                0
## 253                0
## 254                0
## 255                0
## 256                0
## 257                0
## 258                0
## 259                0
## 260                0
## 261                0
## 262                0
## 263                0
```

```
## 264                  0
## 265                  0
## 266                  0
## 267                  0
## 268                  0
## 269                  0
## 270                  0
## 271                  0
## 272                  0
## 273                  0
## 274                  0
## 275                  0
## 276                  0
## 277                  0
## 278                  0
## 279                  0
## 280                  0
## 281                  0
## 282                  0
## 283                  0
## 284                  0
## 285                  0
## 286                  0
## 287                  0
## 288                  0
## 289                  0
## 290                  0
## 291                  0
## 292                  0
## 293                  0
## 294                  0
## 295                  0
## 296                  0
## 297                  1
## 298                  0
## 299                  0
## 300                  0
## 301                  0
## 302                  0
## 303                  0
## 304                  0
## 305                  0
## 306                  0
## 307                  0
## 308                  0
## 309                  0
## 310                  0
## 311                  0
## 312                  0
## 313                  0
## 314                  0
## 315                  0
## 316                  0
## 317                  0
```

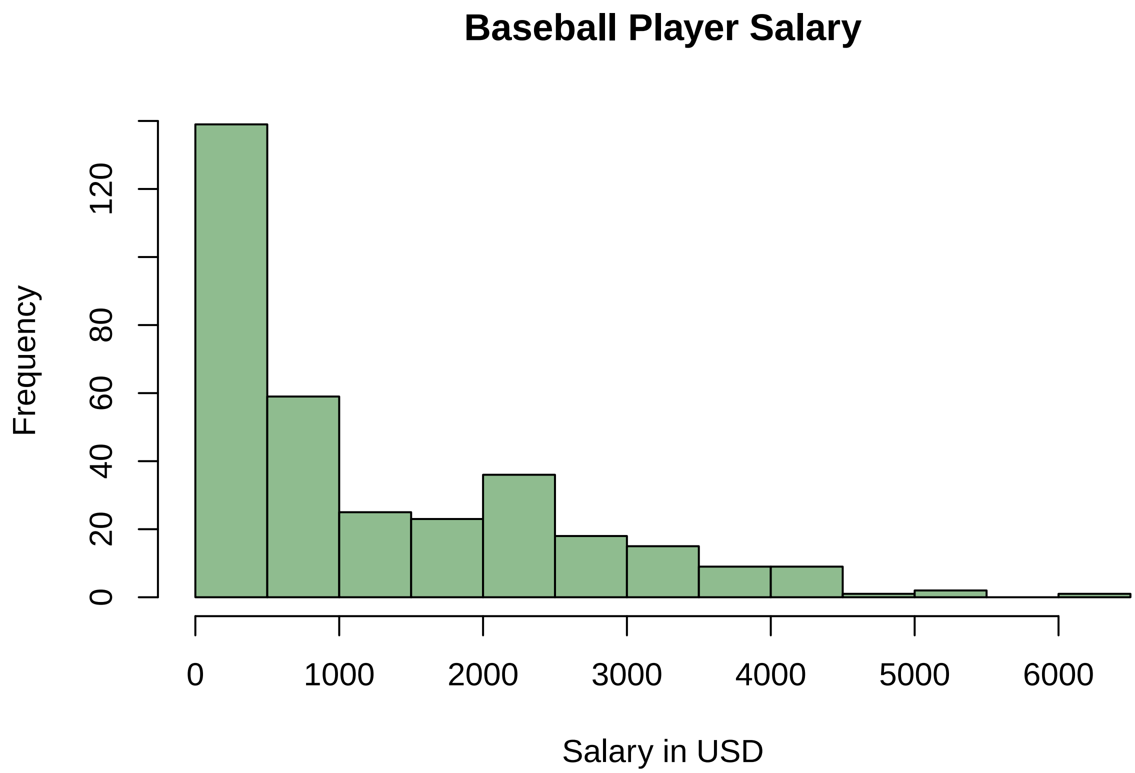
```
## 318             0
## 319             0
## 320             0
## 321             0
## 322             0
## 323             0
## 324             1
## 325             0
## 326             0
## 327             0
## 328             0
## 329             0
## 330             0
## 331             0
## 332             0
## 333             0
## 334             0
## 335             0
## 336             0
## 337             0
```

Exercise 1.1

Make a histogram of the `salary` column. Your plot should be appropriately titled and axes are appropriately labeled. You should not label each bin because the bin labels will make the plot unreadable. Briefly describe the data using the histogram.

```r
par(mfrow=c(1,1))

### YOUR CODE HERE
hist(baseball_data$salary, main = "Baseball Player Salary", xlab = "Salary in USD", col = "darkseagreen
```




```r
### END OF YOUR CODE
```

YOUR EXPLANATION HERE. The plot is incredibly right skewed.

Exercise 1.2

Now, write a function named `compute_mse` that computes the mean euclidean distance between each salary point and some input x. This means, your function should take in one argument, `x`, and output (you probably have to knit this file first to read the equation)

$$f(x) = \frac{1}{n}\|\boldsymbol{s}_i - x\|_2^2 \tag{1}$$

$$= \frac{1}{n}\sum_{i=1}^{n}(\mathrm{s}_i - x)^2 \tag{2}$$

where $\boldsymbol{s} \in \mathbb{R}^n$ denotes the vector of salaries, and s_i denotes the ith player's salary.

This function is also called the **mean squared error** because it measure the average of the squared errors. (In this sense, the mean squared error is an empirical risk, meaning it is the average loss on an observed dataset; the loss function in this case is the squared error loss/euclidean distance.)

```r
### YOUR CODE HERE
mse = 0
compute_mse <- function(x){
  for(i in 1:length(baseball_data$salary)){
```

```r
    mse = mse+(baseball_data$salary[i] - x)^2
  }
  mse = mse/length(baseball_data$salary)
  return(mse)
}
compute_mse(mean(baseball_data$salary))
```

```
## [1] 1533070
```

```r
### END OF YOUR CODE
```

Exercise 1.3

Now, you will write another function named `compute_mad` that computes the mean absolute deviation between each salary point and some input x. This means, your function should take in one argument `x` and output this time:

$$
\begin{aligned}
f(x) &= \frac{1}{n}\|\boldsymbol{s}_i - x\|_1 &\quad (3)\\
&= \frac{1}{n}\sum_{i=1}^{n}|\mathrm{s}_i - x| &\quad (4)
\end{aligned}
$$

```r
### YOUR CODE HERE
mad = 0
compute_mad <- function(x){
    for(i in 1:length(baseball_data$salary)){
    mad = mad+abs(baseball_data$salary[i] - x)
    }
  mad = mad/length(baseball_data$salary)
  return(mad)
}

compute_mad(median(baseball_data$salary))
```

```
## [1] 954.3858
```

```r
### END OF YOUR CODE
```

Exercise 1.4

Our goal is to create two plots: one visualizing the mean squared error (MSE) as a function of x and the mean absolute deviation (MAD) as a function of x. To choose our input x, we will evenly grid an interval as follows: You will create a sequence, starting from 0 and ending at 5000 that is evenly gridded by 50,000 points. Name this vector `x_vec`.

```r
### YOUR CODE HERE
x_vec = seq(from = 0, to = 5000, length.out = 50000)
### END OF YOUR CODE
```

Now, we will get the MSE and MAD evaluated for each one of those x's in `x_vec` by calling the `compute_mse` and `compute_mad` functions you wrote in Exercise 1.3.

Hint. `compute_mse` expects the input to be a number instead of a vector. To compute the MSE for each of the `x` values in `x_vec`, check out the `sapply` function.

```
### YORU CODE HERE
?sapply
```

```
## starting httpd help server ... done
```

```
MSE = sapply(x_vec, compute_mse)
MAD = sapply(x_vec, compute_mad)
### END OF YOUR CODE
```

Exercise 1.5

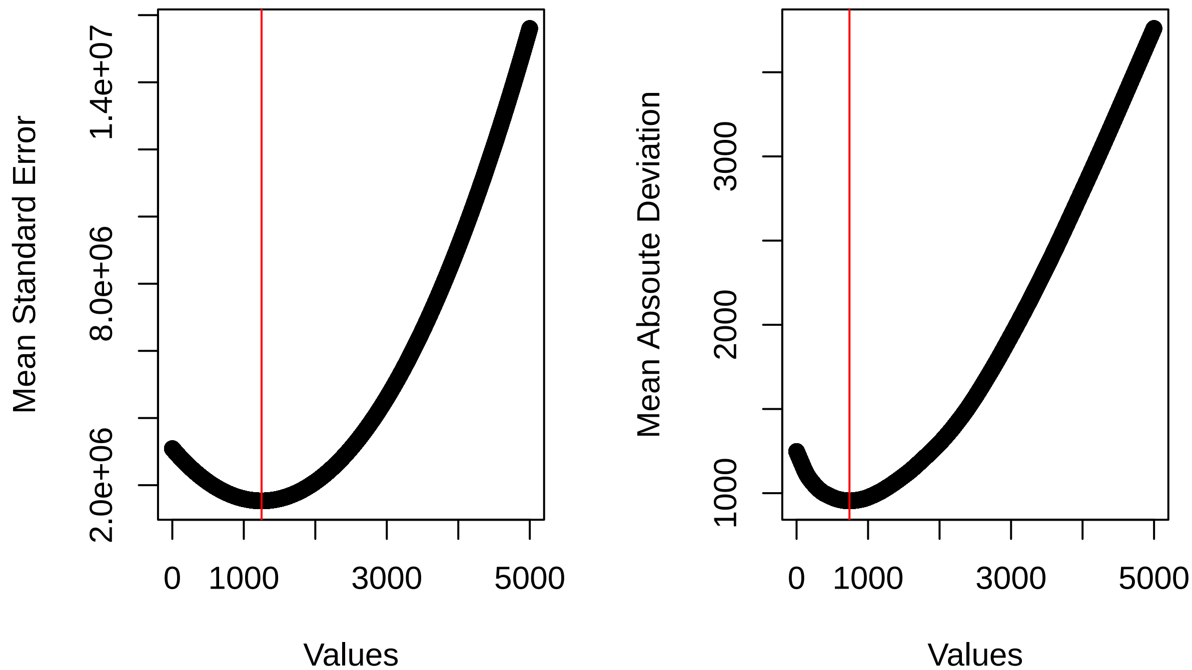
Before we create our plots, there is just one more thing remaining: we will compute the mean and median of `salary` and store them respectively in `mean_salary` and `med_salary`.

Then, having created our vectors of x and y values, we can now plot side-by-side two line plots. Your x-axis should be values in `x_vec`, and your y-axis should be the MSE and MAD that you computed, respectively. You should title and label the axes accordingly. We will overlay the MSE plot with a vertical line at `mean_salary` and the MAD plot with a vertical line at `med_salary`. You should color both vertical lines **red**.

Hint. To overlay the plot with a line, check out the `abline` function.

What do you notice in the plots?

```
### YOUR CODE HERE
par(mfrow=c(1,2))
mean_salary = mean(baseball_data$salary)
med_salary = median(baseball_data$salary)
#Plots the graphs
plot(x_vec, MSE, xlab = "Values", ylab = "Mean Standard Error")
abline(v = mean_salary, col = 'red')
plot(x_vec, MAD, xlab = "Values", ylab = "Mean Absoute Deviation")
abline(v = med_salary, col = 'red')
```




```
### END OF YOUR CODE
```

YOUR EXPLANATION HERE. The MSE plot grows much faster than the MAD plot as the distances are being squared.

Exercise 1.6

Recall that if the average sales price of 10 homes is one million dollars, then each of the 10 houses was sold on average for one million dollars.

On the other hand, if the median sales price of 10 homes is one million dollars, then we know that at least 5 homes sold for one million dollars **or more**, and at least 5 homes sold for one million dollars **or less**.

From the histogram of the data, do you think it is more reasonable that we use the **mean** or the **median** as the summary statistic to describe our data?

YOUR EXPLANATION HERE.

We should use the median as it is more robust to skewed data.

Part 2. A central limiting phenominon

Exercise 2.1

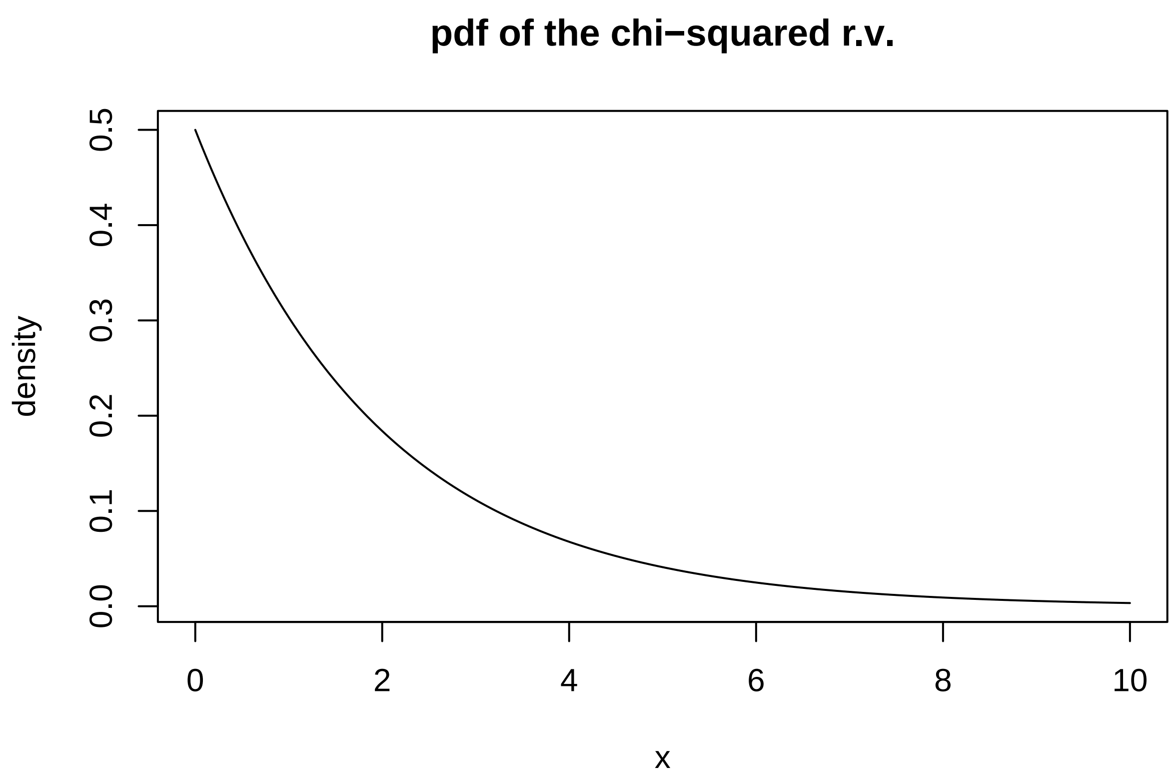
In this problem, we will explore a puzzling phenomenon that underpins the theory of "normal approximation" and the central limit theorem. The central limit theorem is a cornerstone to the field of statistics. We will see this theorem in action.

We begin by generating some samples from the chi-squared distribution with 2 degrees of freedom (denoted χ^2_2, the subscript represents the degrees of freedom which is in our case 2). But before that, let us first see what the probability density function (pdf) of a chi-squared random variable (abbreviated r.v. henceforth) looks like.

You will generate a evenly spaced sequence, starting from 0, ending at 10; the length of this sequence should be 1000. Name this sequence `x`.

```
par(mfrow=c(1,1))
### YOUR CODE HERE
x = seq(from = 0, to = 10, length.out = 1000)
### END OF YOUR CODE

## compute the pdf of chisq rv on the grid generated above
plot(x, dchisq(x, df=2), type='l', main='pdf of the chi-squared r.v.',
     xlab='x', ylab='density') # df is degrees of freedom
```



Briefly describe the density using the terminologies from the lecture on **Describing the Data**.

YOUR EXPLANATION HERE. The data is right skewed and the density curve reflects that. The density curve is still unimodal however.

Exercise 2.2

Below, you will generate $n = 4$ data points from the chi-squared distribution with 2 degrees of freedom. Store your generated samples in the vector `cs_samples`. Then, plot a histogram of your samples. Your plot should be appropriately titled; your axes should also be appropriately labeled. Finally, compute the mean of your samples, print it out, and store it in a variable called `cs_samples_mean`.

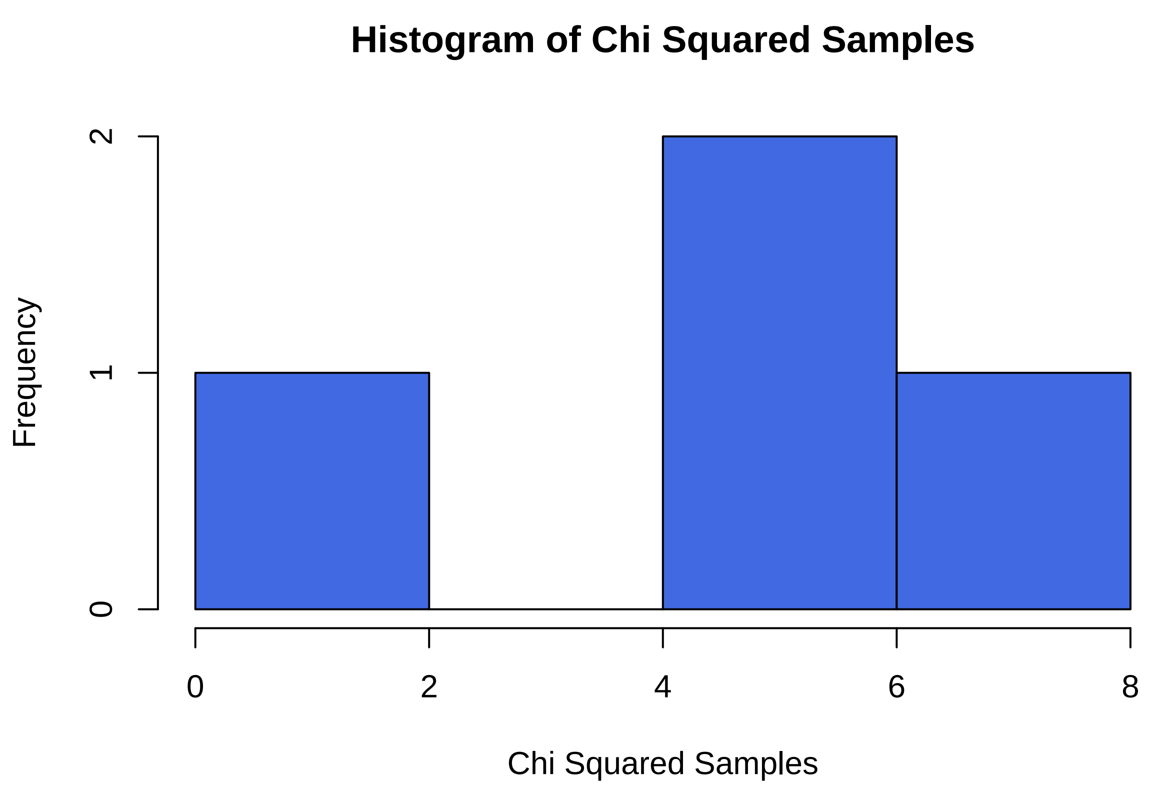
Hint. The `rchisq` function could be of use.

```
?par(mfrow=c(1,1))
### YOUR CODE HERE
## generate n=4 samples from a chisq dist. with 2 df
cs_samples = rchisq(4, df = 2)
### END OF YOUR CODE

print(cs_samples)
```

```
## [1] 4.896192 1.758056 5.334637 7.771851
```

```
### YOUR CODE HERE
## plot a histogram of your samples
hist(cs_samples, main = "Histogram of Chi Squared Samples", xlab = "Chi Squared Samples", col = "royalbl
```




```
### END OF YOUR CODE

### YOUR CODE HERE
## compute mean of your samples
mean(cs_samples)
```

```
## [1] 4.940184
```

```
### END OF YOUR CODE
```

Exercise 2.3

Now, you will write a function that performs the tasks outlined in Exercise 2.2. Your should name your function `simulate_chisq`. This function should take in 1 argument, n, that governs the size of the sample to be generated. This function should return one thing: the sample mean of n samples.

```
### YOUR CODE HERE
simulate_chisq <- function(n){
  nmean = mean(rchisq(n, df = 2))
  return(nmean)
}
### END OF YOUR CODE

simulate_chisq(n = 10)
```

```
## [1] 1.162978
```

Exercise 2.4

Having written the function, let's repeat the procedure in Exercise 2.2 10,000 times. You will write a function called `simulate_means` that takes in one argument `n` (the sample size) and returns 3 things: the 10,000 sample means themselves, the average of the 10,000 sample means, and the standard deviation of the sample means. Inside `simulate_means`, you will write a `for` loop that, in each iteration, calls the `simulate_chisq` function that you wrote in Exercise 2.3.

Then, test your function by executing the function with $n = 4$, same n as that used in Exercise 2.2.

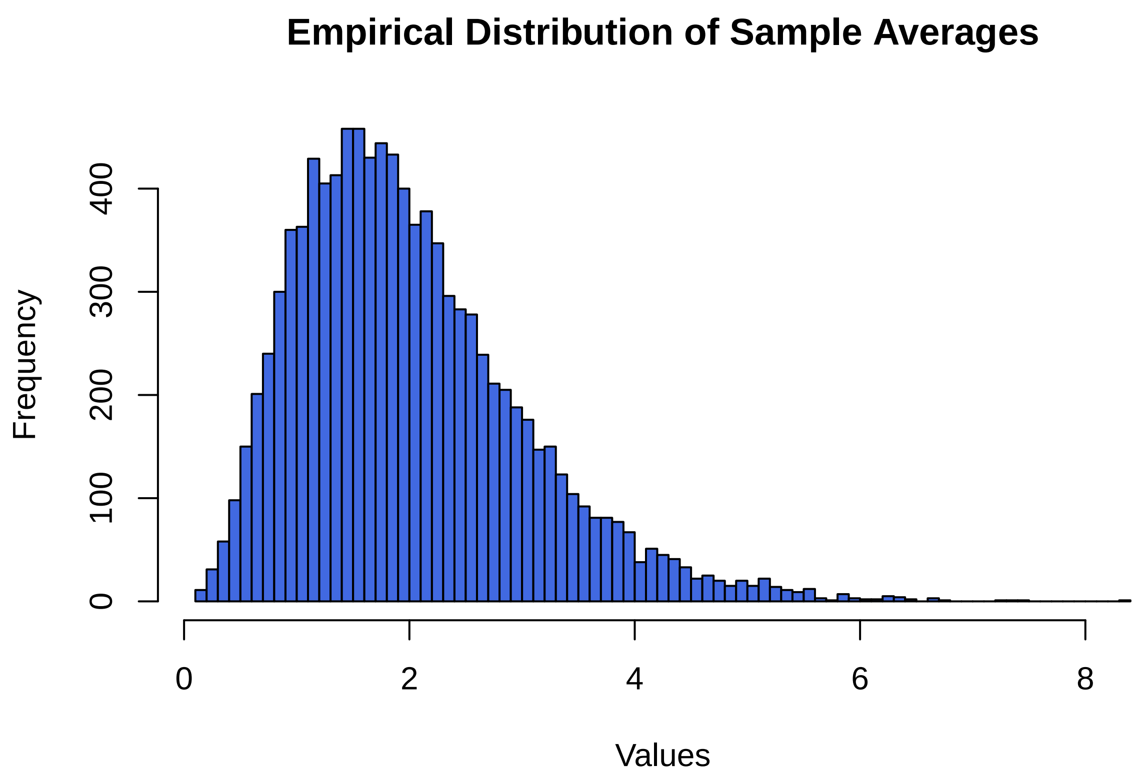
Finally, plot a histogram of the empirical distribution of the **sample averages** with appropriately chosen axes labels and title, and **100 bins**.

Describe what you see in the histogram using terminologies from the lecture on "Describing the Data".

Hint. To create a numeric vector of a fixed length, check out the `numeric` function.

Hint. For returning multiple values at once, look into **lists**.

```
par(mfrow=c(1,1))
### YOUR CODE HERE
n = 4
simulate_means = function(n) {
  means = numeric(10000)
  for (i in 1:10000){
    means[i] <- simulate_chisq(n)
  }
    return(list(means, mean(means), sd(means)))
}
hist1 = simulate_means(4)
hist(hist1[[1]], breaks = 100, main = "Empirical Distribution of Sample Averages", xlab = "Values", col
```




```
### END OF YOUR CODE
```

YOUR EXPLANATION HERE. Thw data is once again right skewed due to the outliers. The data is still unimodal.

Exercise 2.5

Now, you will call the function `simulate` that you wrote in Exercise 2.4, except for this time, your sample size for each of the 10,000 trials will be $n = 100$ instead.

Then, plot the histograms visualizing the empirical distributions of your generated sample averages from Exercise 2.4 and Exercise 2.5 side-by-side, each with 100 bins. As always, your plots should be appropriate titled and labeled.

What do you notice in the histogram?

```
### YOUR CODE HERE
## generate a vector of length 10,000 of sample means
simulate_means(100)
```

```
## [[1]]
##     [1] 1.761664 1.879394 1.646637 2.149854 1.776589 1.924151 2.120258 2.416655
##     [9] 2.112758 2.075300 1.997033 2.155894 2.142548 2.012455 2.197417 1.943588
##    [17] 2.013692 2.038648 1.888839 2.142622 1.870539 1.604036 1.794976 1.839945
##    [25] 1.783458 1.989950 1.907686 2.154660 2.082237 2.251767 1.772826 2.013468
```

```
##  [33] 2.293242 1.990839 2.134209 2.207893 1.985481 1.733169 2.092087 2.046576
##  [41] 1.937983 2.050953 2.211680 1.950862 1.959793 1.786219 2.056270 2.292607
##  [49] 1.831399 2.146449 1.826458 2.138193 2.116143 2.057002 2.176421 2.168480
##  [57] 1.938326 2.161323 2.037448 2.181212 1.750087 1.721960 1.821086 2.281396
##  [65] 1.861157 1.917335 1.984149 1.821844 1.793698 1.839230 1.903767 1.948784
##  [73] 1.489149 2.144958 1.914482 2.037557 2.420703 1.907264 1.836311 2.210972
##  [81] 1.823498 1.614752 1.878593 2.122677 1.971187 1.863808 2.223956 2.082607
##  [89] 1.849508 1.661096 2.320391 2.256348 2.267288 1.882483 1.866454 1.959991
##  [97] 1.886069 2.078155 1.937565 2.003126 2.274969 2.241281 1.717012 2.056462
## [105] 1.937536 2.054398 1.912760 2.165983 2.221220 2.118266 1.918412 2.253673
## [113] 1.790393 2.030413 1.942834 2.046110 1.790921 1.969684 2.437050 2.034734
## [121] 2.276452 1.964791 2.023363 1.859190 2.122276 1.982143 1.759745 1.985786
## [129] 2.490892 2.091731 2.029109 2.452872 2.267475 1.838467 1.907151 1.862864
## [137] 1.912369 1.614235 2.014217 1.700443 2.011130 1.953927 2.061281 1.669276
## [145] 1.781161 1.936619 1.820787 2.034227 2.340169 1.878032 1.976083 1.807491
## [153] 2.224198 2.001927 1.918071 2.019081 2.045036 1.925607 2.136905 2.334860
## [161] 2.179948 2.039398 2.109785 2.062881 2.027704 1.836714 2.012856 1.910485
## [169] 1.983655 2.066040 1.709640 2.016547 1.783730 1.852133 2.088508 1.806238
## [177] 1.948776 2.111088 2.082982 2.250645 1.635962 1.767719 2.031740 1.857687
## [185] 2.023761 2.329765 2.064173 1.923421 1.842608 2.024889 1.482535 1.879499
## [193] 2.416202 1.981372 1.938887 1.808236 2.080789 1.782027 1.857787 2.250841
## [201] 2.132881 2.149599 1.973692 1.930467 1.794613 2.288585 1.723694 1.995969
## [209] 2.002688 1.854745 1.703643 2.254718 1.923380 2.281220 2.247639 1.893614
## [217] 2.088246 1.805669 2.034086 2.154858 2.147072 1.961004 2.135557 1.696895
## [225] 1.983920 2.088707 1.995148 1.820761 2.343674 1.858966 2.006109 1.966220
## [233] 1.957210 2.041841 2.275382 2.132981 2.190847 2.047305 1.993701 2.493764
## [241] 1.954607 1.835743 2.109891 2.252501 2.261872 1.752718 1.903022 2.173970
## [249] 1.820947 2.316471 1.874340 2.134720 2.127826 1.834530 1.736989 1.946689
## [257] 1.982420 2.079510 1.965302 2.218953 1.838807 1.817774 2.044621 1.497925
## [265] 2.247363 1.988948 1.965951 1.730873 1.700740 2.266683 1.835189 2.085862
## [273] 2.496858 1.800729 1.935258 2.212555 1.753465 2.354022 1.893531 2.158415
## [281] 1.816797 2.203051 2.011279 1.972527 1.660170 2.097447 1.963046 2.060842
## [289] 1.604777 2.115851 2.000103 1.838141 1.913126 1.824929 1.918441 1.749938
## [297] 2.141063 2.105459 1.771112 1.790496 1.954255 2.244318 1.904615 2.010356
## [305] 2.403251 1.829659 1.738935 1.668470 2.152032 1.662541 1.912488 2.098249
## [313] 1.570612 1.977699 2.091806 2.077782 1.657974 2.117930 1.939220 2.012029
## [321] 1.881749 2.051229 2.038969 2.069893 1.945235 1.731684 1.977642 2.084752
## [329] 2.062761 2.177695 1.975678 1.990067 2.165289 2.136366 2.140559 2.115358
## [337] 2.437781 1.832336 1.991718 2.136417 2.186765 2.252978 1.946229 2.074675
## [345] 1.850426 2.330614 1.888718 2.297641 1.948345 2.089885 2.168354 1.667141
## [353] 2.590056 2.338494 2.010147 2.247700 1.948187 2.213867 1.897803 2.347720
## [361] 1.860543 2.258723 1.601712 1.859630 2.136719 1.679985 2.093489 1.575270
## [369] 1.949487 1.972581 2.119148 2.215597 1.888545 2.035533 2.033447 1.704670
## [377] 1.953944 2.224705 1.996209 1.715438 1.950882 2.200820 2.061903 2.595409
## [385] 2.341570 1.654868 2.001930 1.757903 1.963395 2.287379 2.088029 2.013852
## [393] 1.875878 1.944439 1.683796 2.063469 2.331866 2.114345 1.918342 2.222252
## [401] 2.126028 2.451641 2.331601 1.990413 2.135650 1.850071 2.196181 1.858454
## [409] 1.853283 1.878785 2.211732 1.761798 2.086270 1.918153 1.922123 2.011038
## [417] 1.969662 2.127288 2.069957 1.935186 2.480673 1.991211 2.605037 2.124564
## [425] 1.694092 2.277228 2.279455 1.800415 1.757208 2.321236 1.999781 1.841303
## [433] 1.927708 2.076376 2.027740 2.424904 2.314007 1.815939 2.365668 1.847373
## [441] 2.166196 1.974126 1.810660 2.220877 1.818911 2.488234 1.914813 1.983879
## [449] 1.743523 1.816244 1.807020 1.999073 2.205445 1.805181 2.027112 1.763714
## [457] 1.871600 1.941333 1.886099 2.083962 1.852591 2.199454 1.912724 1.861490
```

```
## [465] 2.064527 1.901125 1.885474 1.760078 2.295169 1.973250 2.061636 1.849255
## [473] 2.014973 2.004288 2.080247 1.874339 2.490850 1.905962 1.996672 2.337305
## [481] 2.104998 1.828261 2.116830 2.263256 1.982254 1.802936 2.210946 1.951732
## [489] 1.905854 1.915763 2.082745 1.816122 2.149131 1.979885 2.416460 2.194645
## [497] 1.710111 2.161356 2.011627 2.130349 1.984741 1.900853 2.144499 1.944078
## [505] 1.982477 2.100402 2.096435 2.324863 1.916103 2.081104 1.669562 2.151967
## [513] 1.877422 2.019369 2.348327 1.932441 2.040212 1.825638 2.113658 2.024489
## [521] 2.212310 1.968402 2.058823 1.670051 2.035164 2.176727 1.963560 2.396794
## [529] 1.839690 1.858247 2.327679 2.290490 1.938382 2.010639 1.995577 2.252642
## [537] 1.911315 2.242532 2.046772 1.939729 2.481341 2.041479 1.924134 2.239656
## [545] 1.826114 2.246130 1.995444 2.075716 2.167111 1.715500 2.286217 2.005970
## [553] 2.011649 2.085696 1.967560 1.893785 1.828113 2.114327 2.113481 1.802873
## [561] 2.121692 1.844596 1.936705 2.551261 1.804732 1.941113 2.014915 2.076523
## [569] 1.773693 1.730316 1.769846 2.186762 2.023928 1.948680 2.035741 1.896219
## [577] 2.011973 1.593176 2.052986 1.802812 2.142738 1.886345 2.176237 1.912469
## [585] 1.991743 1.929088 1.811488 2.376924 1.882548 1.715424 1.691101 1.885714
## [593] 2.035326 2.188269 1.558878 2.201481 2.164295 2.053962 2.062859 2.129895
## [601] 2.149709 1.991892 2.330862 2.197395 2.066036 2.057969 2.044837 1.962976
## [609] 1.978757 1.896615 2.157987 1.720638 1.935733 2.011548 2.597944 1.867491
## [617] 2.098774 2.025627 1.835035 1.847280 2.073044 2.105627 2.168073 2.292366
## [625] 1.932286 2.187030 1.758167 2.347237 2.137468 1.829550 2.018453 1.946454
## [633] 2.126660 2.176859 2.046768 1.678866 1.898100 1.770963 1.953990 1.760600
## [641] 2.010608 1.906355 2.196623 1.722886 2.515300 1.888341 1.957702 2.043368
## [649] 1.901088 1.850081 2.054997 2.083582 2.121424 2.081472 2.223129 1.936391
## [657] 2.040603 1.908142 1.944024 1.715657 1.829894 2.090440 1.923695 1.784274
## [665] 1.925193 2.212146 2.391180 2.069541 2.077719 1.925189 2.137608 1.818189
## [673] 1.873085 2.025697 1.671186 2.377746 2.106448 1.880036 1.864782 2.044437
## [681] 1.765153 2.105459 1.806158 1.813680 1.871375 2.475638 2.025534 2.127437
## [689] 1.725364 1.926567 2.103202 2.202880 1.988866 1.826012 2.235883 2.032187
## [697] 2.178983 1.853155 2.009367 1.913645 1.906902 1.980139 1.668330 1.945050
## [705] 2.086412 1.686835 2.148427 1.723574 2.239983 2.080229 2.112166 2.052501
## [713] 2.058591 1.928557 2.002713 2.083878 1.831706 2.251850 1.852783 1.809143
## [721] 1.771894 2.013967 2.031792 2.086360 2.271979 1.972597 1.955589 1.946356
## [729] 2.173408 1.980725 2.177336 2.030999 2.084468 2.063274 1.802000 1.775292
## [737] 2.335365 1.744835 1.823000 1.744410 2.254705 2.040507 2.160943 2.225189
## [745] 1.682464 1.728557 1.930699 1.742785 2.016952 1.797141 2.005436 2.234347
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## [9505] 1.775931 2.218800 1.789400 1.633209 2.017620 2.381102 2.195489 1.904278
## [9513] 1.903223 1.944413 2.234685 2.136367 2.003536 1.969879 1.914833 1.856201
## [9521] 1.775688 2.219580 1.699984 2.071111 1.932314 1.957121 2.118194 2.253291
## [9529] 2.128153 1.769682 2.051949 1.704474 2.021645 2.291699 2.009203 2.129518
```
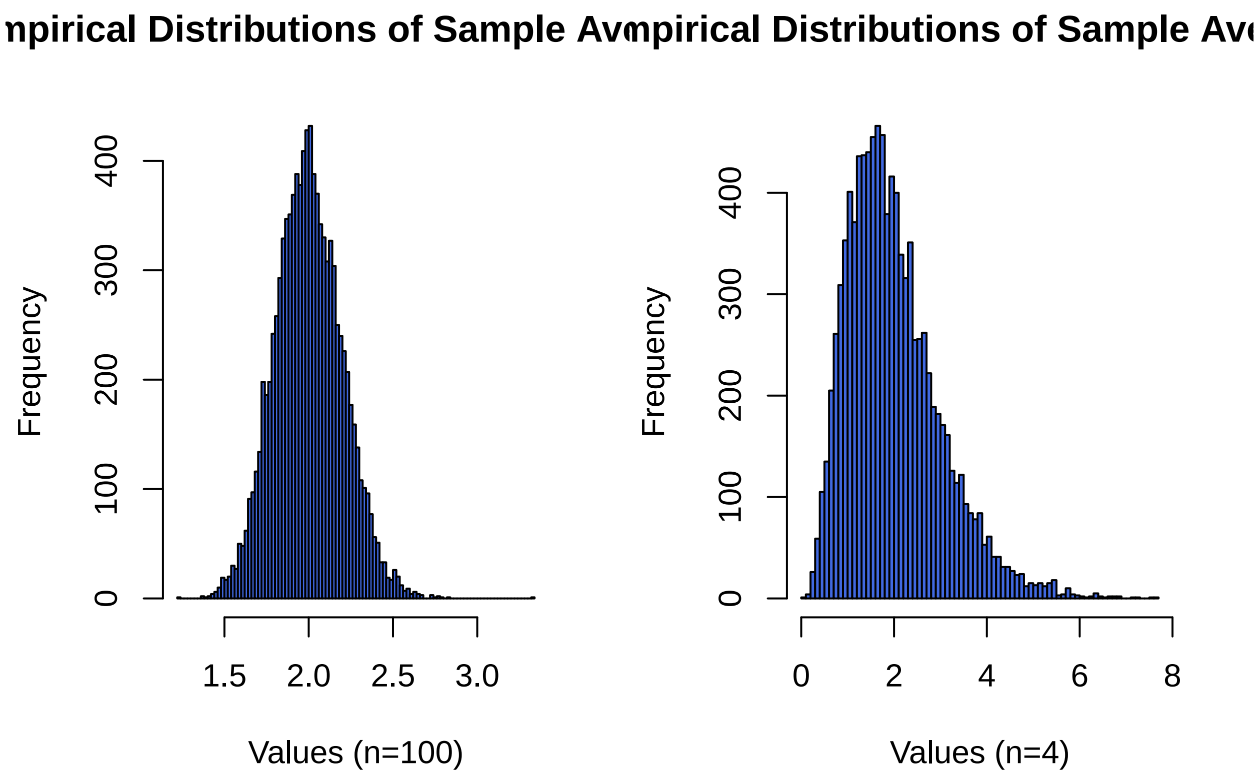

```
## [9537] 1.886147 2.089589 2.266018 1.897672 2.181255 2.044960 2.131336 1.756068
## [9545] 2.342231 2.265944 2.186502 2.076707 1.874266 1.663103 1.983252 1.983965
## [9553] 2.000151 1.900307 1.883932 2.034045 2.290604 1.834968 2.206935 1.683101
## [9561] 1.945485 2.005885 2.001203 2.227538 1.823302 2.171937 2.448585 2.581903
## [9569] 1.712134 1.852589 2.399193 2.044703 1.574125 2.033719 2.291859 2.154133
## [9577] 2.180075 1.811324 2.208352 2.061732 1.934941 1.479011 2.123360 2.059508
## [9585] 1.897111 1.905309 2.275030 2.357889 1.793705 2.018310 1.927852 2.061274
## [9593] 1.887916 1.867009 1.992619 1.842865 2.200384 1.689247 1.807805 2.182440
## [9601] 2.364892 2.188287 2.101169 2.181121 1.899704 2.441076 1.794159 1.808640
## [9609] 2.150012 1.996350 2.060705 2.126583 2.209339 1.899468 2.141796 1.919178
## [9617] 1.867967 1.895089 1.811624 2.142154 2.092605 2.090264 1.824095 2.034216
## [9625] 2.177591 2.075300 2.118754 2.373551 1.971570 2.039655 2.261723 2.297970
## [9633] 2.005025 1.925226 1.818755 1.955694 1.640064 1.658088 2.628433 1.924981
## [9641] 2.407667 1.852681 2.002686 2.163398 1.710834 2.087020 2.234616 2.027334
## [9649] 2.082897 1.836406 2.200875 2.195344 2.055662 1.809276 2.018891 1.887110
## [9657] 2.584120 1.792323 2.639664 2.145443 1.904576 2.091269 1.898543 1.552843
## [9665] 1.945268 1.879954 2.053773 2.193460 2.146228 1.896541 1.509675 2.041487
## [9673] 1.897617 2.122044 1.889885 2.269612 2.191098 1.726045 2.074803 2.059050
## [9681] 1.932747 2.021279 1.836077 2.161503 1.888026 2.009042 1.683834 1.678867
## [9689] 1.687623 1.886707 1.622514 1.671426 2.152787 1.863049 1.742258 1.797380
## [9697] 2.079700 2.465413 1.765699 1.924369 2.422116 1.984236 2.093627 1.854168
## [9705] 2.010654 1.584144 1.989070 2.351873 2.150619 1.894447 2.009743 2.277389
## [9713] 1.984135 1.935431 1.827157 2.080157 1.937238 2.066179 1.990830 1.843531
## [9721] 1.825907 1.815491 2.020156 2.106553 2.269309 2.299780 1.873588 2.217105
## [9729] 2.043968 2.145922 2.024125 1.797582 1.955108 1.861603 1.927971 2.155084
## [9737] 2.290367 2.022128 1.898386 1.955921 2.188738 2.467051 2.151882 2.058101
## [9745] 2.050890 1.891027 1.600113 1.969885 1.891438 2.165520 1.708804 1.928543
## [9753] 1.688226 1.590277 1.760432 1.780440 1.890294 2.228940 1.568321 1.758151
## [9761] 2.154621 2.336178 1.923467 2.109634 1.992475 2.307700 1.762702 1.703590
## [9769] 1.991809 2.040836 1.900066 2.357561 1.823142 1.997638 1.690506 2.046612
## [9777] 2.103304 2.120878 1.808094 1.973175 2.209861 2.208041 1.987502 1.842070
## [9785] 2.025668 1.896136 2.165274 1.870270 1.964197 2.207371 1.857086 2.298826
## [9793] 1.920384 2.085635 1.954075 2.194510 1.899807 2.189856 2.305391 2.057534
## [9801] 1.881153 2.227565 2.441286 1.732719 2.029259 1.836338 2.084631 1.864378
## [9809] 2.108312 1.913233 2.104326 2.359669 1.789216 1.940187 2.024232 1.813840
## [9817] 2.140053 2.076428 2.054920 1.889622 1.617481 2.138865 2.201112 2.066376
## [9825] 1.755943 2.361696 1.816152 2.138476 2.227437 2.072118 1.824326 1.994048
## [9833] 2.215353 1.704622 2.174338 2.073559 2.182399 1.944382 1.896616 1.904376
## [9841] 1.958408 1.910785 2.008128 1.848305 2.247355 2.135118 2.042948 2.041372
## [9849] 2.022498 1.931095 2.473913 1.811193 2.076280 1.735787 2.134477 2.129656
## [9857] 1.894590 2.262482 1.986775 2.396180 2.208578 2.380779 2.068977 2.140460
## [9865] 2.211110 2.128197 1.956884 2.076741 2.302355 1.750638 2.377685 2.162420
## [9873] 1.850014 2.062995 2.098120 2.015580 1.864955 2.111813 1.778081 1.995845
## [9881] 2.176560 2.213865 2.109884 1.671811 1.877843 1.884082 1.980384 2.467108
## [9889] 2.009276 1.862253 1.991900 2.041282 2.228686 1.953835 1.986400 2.054970
## [9897] 2.123049 2.162824 1.901155 1.942224 1.826006 2.149153 2.112287 1.479916
## [9905] 1.881267 2.109939 2.056364 2.140365 2.168455 1.872556 1.927929 1.850095
## [9913] 1.692379 2.153219 1.921170 2.284060 2.045260 1.996043 1.920423 1.877427
## [9921] 2.038841 2.517282 2.312931 2.215911 2.194676 2.070930 2.131020 1.661541
## [9929] 2.068316 1.961978 2.115746 2.103991 1.666427 2.170194 1.831215 2.194612
## [9937] 2.008291 2.240480 2.292361 1.971411 1.868797 1.754308 2.028788 2.202292
## [9945] 1.839404 1.695279 2.006592 1.661562 2.140900 2.034840 1.816046 1.981263
## [9953] 1.948515 1.664369 2.169829 1.803567 1.697519 2.075323 1.968623 2.180188
## [9961] 1.988857 2.003060 1.851885 2.396468 2.361836 1.911153 2.142446 2.107145
```

```
##  [9969] 1.691166 1.947348 1.805194 2.244592 2.001377 1.938676 1.750166 2.072623
##  [9977] 2.128095 1.658230 1.963519 1.955881 1.896130 2.048015 2.009082 1.908293
##  [9985] 2.192902 2.232194 2.108312 2.380433 1.989175 2.158389 2.182244 2.160323
##  [9993] 1.848962 2.242037 1.968704 1.565554 1.951376 1.938853 2.094504 2.165974
##
## [[2]]
## [1] 2.000711
##
## [[3]]
## [1] 0.2010436
```

```r
### END OF YOUR CODE

par(mfrow=c(1,2))

### YOUR CODE HERE
## plot the 2 histograms below
hist2 = simulate_means(100)
hist(hist2[[1]], breaks = 100, main = "Empirical Distributions of Sample Averages", xlab = "Values (n=1
hist1 = simulate_means(4)
hist(hist1[[1]], breaks = 100, main = "Empirical Distributions of Sample Averages", xlab = "Values (n=4
```




```r
### END OF YOUR CODE
```

YOUR EXPLANATION HERE. The increased n value yielded a more normalized histogram while the smaller n still contained skew.

Exercise 2.6

Repeat the sampling procedure outlined in Exercise 2.4 for $n = 4, 9, 16, 25, 36, 100, 400, 900, 1600$.

You will output three things:

1. Save your resulting mean and standard deviations in a 9 x 3 matrix; each row of this matrix stores the corresponding **n, mean, sd** values to that `n`. For example, the first row should be **4, mean of 10,000 averages of samples each of sample size 4, sd of 10,000 averages of samples each of sample size 4**. The second row should be **9, mean of 10,000 averages of samples each of sample size 9, sd of 10,000 averages of samples each of sample size 9**. So, put it another way: the first column should be **4, 9, 16, 25, 36, 100, 400, 900, 1600**. The second column should be the corresponding means of 10,000 averages. The third column should be the corresponding standard deviations of 10,000 averages. The matrix `sample_result` has been created for you already; you will only need to fill in the entries.
2. For each n, generate a histogram of the 10,000 sample means. You should title your plot $n =$ 4/9/16/25/36/100/400/900/1600 (the appropriate value of n corresponding to the plot) and label your axes appropriately. Your histogram should have 100 bins.
3. Save the samples for each n in a list called `all_samples`. You can read more about lists here.

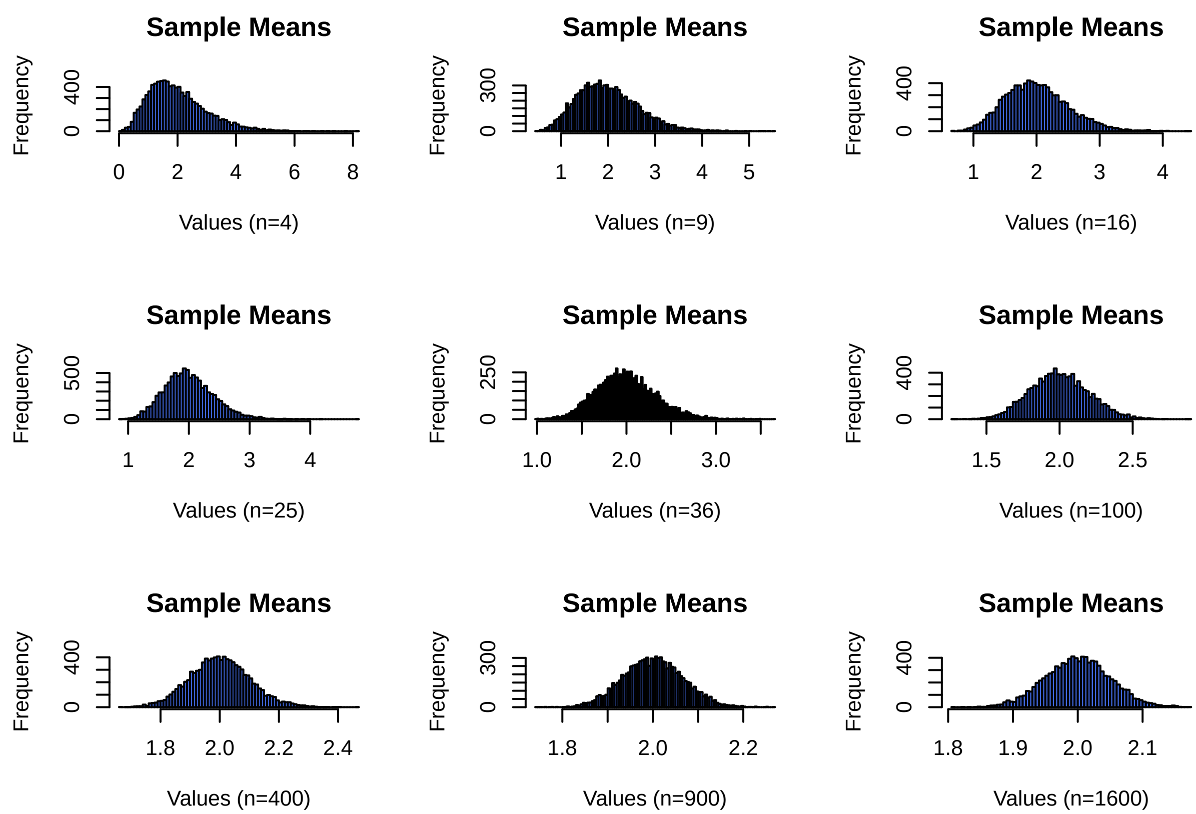
Hint. You will use a `for` loop and populate the matrix iteratively. To create a vector, use the `c()` command; for example, if I want to create a vector with entries 2, 3, I would write `c(2, 3)`. To concatenate a string with a integer, look into the `paste` or `sprintf` functions.

```
par(mfrow=c(3, 3))

sample_result = matrix(0, nrow=9, ncol=3)
all_samples = list()


### YOUR CODE HERE
index = c(4, 9, 16, 25, 36, 100, 400, 900, 1600)
for (i in 1:9){
  sample_result[i, 1] = index[i]
  sample_result[i, 2] = simulate_means(index[i])[[2]]
  sample_result[i, 3] = simulate_means(index[i])[[3]]
}
for (j in 1:9){
  all_samples[[j]] = simulate_means(index[j])[[1]]
}
#print(all_samples)

### END OF YOUR CODE

colnames(sample_result) = c('n', 'mean', 'sd')
print(sample_result)
```

```
##          n     mean         sd
##  [1,]    4 2.003751 1.00670337
##  [2,]    9 1.997914 0.66842822
##  [3,]   16 2.000455 0.50321532
##  [4,]   25 2.001170 0.39982754
##  [5,]   36 2.001656 0.33252052
```

```
## [6,]  100 1.997520 0.19984100
## [7,]  400 2.001383 0.10010689
## [8,]  900 1.999277 0.06679219
## [9,] 1600 1.999470 0.04976303
```

```r
hist(simulate_means(4)[[1]], breaks = 100, main = "Sample Means", xlab = "Values (n=4)", col = "royalblu
hist(simulate_means(9)[[1]], breaks = 100, main = "Sample Means", xlab = "Values (n=9)", col = "royalblu
hist(simulate_means(16)[[1]], breaks = 100, main = "Sample Means", xlab = "Values (n=16)", col = "royalb
hist(simulate_means(25)[[1]], breaks = 100, main = "Sample Means", xlab = "Values (n=25)", col = "royalb
hist(simulate_means(36)[[1]], breaks = 100, main = "Sample Means", xlab = "Values (n=36)", col = "royalb
hist(simulate_means(100)[[1]], breaks = 100, main = "Sample Means", xlab = "Values (n=100)", col = "roya
hist(simulate_means(400)[[1]], breaks = 100, main = "Sample Means", xlab = "Values (n=400)", col = "roya
hist(simulate_means(900)[[1]], breaks = 100, main = "Sample Means", xlab = "Values (n=900)", col = "roya
hist(simulate_means(1600)[[1]], breaks = 100, main = "Sample Means", xlab = "Values (n=1600)", col = "r
```



Exercise 2.7

In theory, we know that what the true mean of a chi-squared r.v. should be. The expected value (or the theoretical mean) of a chi-squared r.v. is equal to its degrees of freedom. So, for our chi-squared r.v. with 2 degrees of freedom, we know that the theoretical mean should be 2. If you are curious, you can read more about the chi-squared distribution here.

Make 3 line plots overlayed with the points using the columns of the matrix `sample_result`. As always, you should label the axes accordingly. The plots will be respectively:

1. **y-axis**: `mean` of sample averages against **x-axis** sample size `n`.

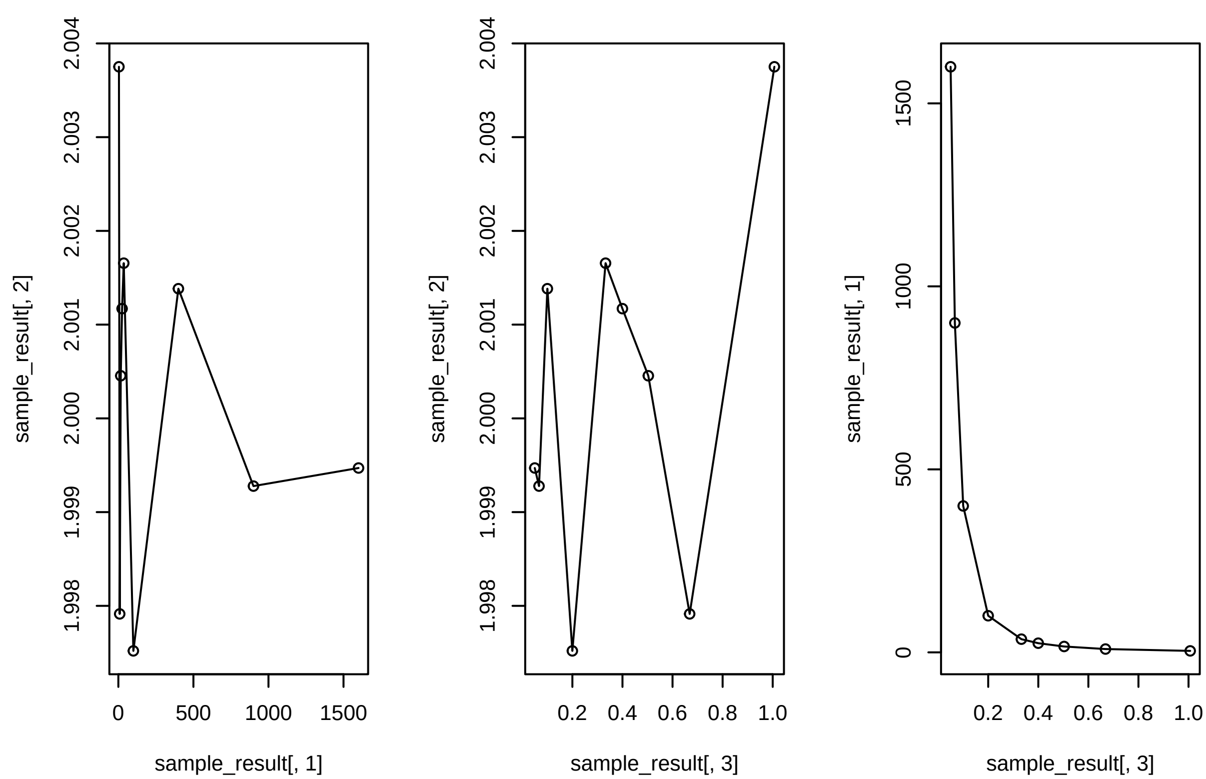
2. **y-axis**: `sd` of sample averages against **x-axis** sample size `n`.
3. **y-axis**: `sd` of sample averages against **x-axis** `mean` of sample averages.

Comment on what you observe (whether or not you observe a clear pattern; if so, what is the pattern?) for each plot as we increase the sample size from $n = 4$ to $n = 1600$.

Hint. You can overlay a plot with points using the `points` function. You probably have to sort your values for one of the plots in order for the line segments to be connected correctly. To sort all the rows of a data frame according to values specified in a column, you can use the `order` function and apply it directly to the rows of a data frame like `data[order(something), ]`.

Remark. Recall that each of the summary statistics (mean, sd) is calculated on the same number of trials, 10,000. The difference here is that the sample size of the individual samples for which we calculated the mean is different.

```r
par(mfrow=c(1,3))

### YOUR CODE HERE
plot(sample_result[,1], sample_result[,2],type = "l")
points(sample_result[,1], sample_result[,2])

plot(sample_result[,3], sample_result[,2],type = "l")
points(sample_result[,3], sample_result[,2])

plot(sample_result[,3], sample_result[,1],type = "l")
points(sample_result[,3], sample_result[,1])
```



```r
### END OF YOUR CODE
```

Exercise 2.8

Now, we will first see what the pdf of a normal distribution looks like. Then, for each of your samples corresponding to different n's, you will standardize the samples by first subtracting 2 from all the samples, and then dividing each sample by $1/\sqrt(n)$ for their respective n.

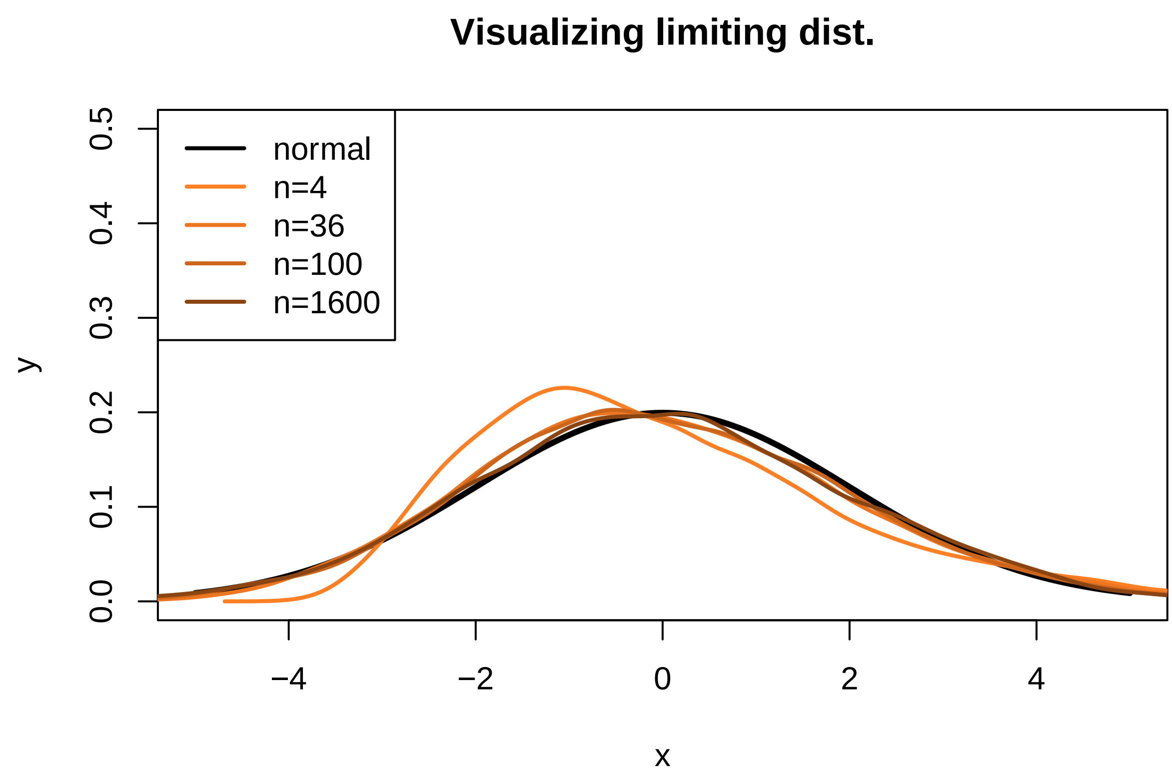
Then, you will overlay the density plots of the **standardized samples** corresponding to $n = 4, 36, 100, 1600$ on top of the density plot of the normal distribution, which has been created already for you. This means, you will create **one** plot with the normal pdf and the densities for $n = 4, 36, 100, 1600$.

You should use the colors `chocolate1`, `chocolate2`, `chocolate3`, and `chocolate4` corresponding each to $n = 4, 36, 100, 1600$, respectively. For clarity, you will set `lwd=2` for each of the density plots so that the lines are thicker.

What do you observe from the plot? As a refresher, you can revisit Exercise 2.1 in which we visualized the distribution of a chi-squared r.v. with 2 degrees of freedom - the distribution we sampled from.

Hint. You should not `for` loop again. All of your samples have been saved in `all_samples`.

```r
par(mfrow=c(1,1))
x = seq(-5, 5, length.out=1000)
y = dnorm(x, mean = 0, sd = sqrt(2*2))
plot(x, y, type='l', col='black',
     main='Visualizing limiting dist.',
     lwd=3,
     ylim = c(0, 0.5))
samples = list()
### YOUR CODE HERE

lines(density((simulate_means(4)[[1]] - 2)*sqrt(4)), col = "chocolate1", lwd = 2)
lines(density((all_samples[[5]] - 2)*sqrt(36)), col = "chocolate2", lwd = 2)
lines(density((all_samples[[6]] - 2)*sqrt(100)), col = "chocolate3", lwd = 2)
lines(density((all_samples[[9]] - 2)*sqrt(1600)), col = "chocolate4", lwd = 2)

### END OF YOUR CODE

legend('topleft',
       lty=1,
       lwd=2,
       legend=c('normal', 'n=4', 'n=36', 'n=100', 'n=1600'),
       col=c('black', 'chocolate1', 'chocolate2', 'chocolate3', 'chocolate4'))
```



YOUR EXPLANATION HERE. Increasing n leads to a density curve that is more than more close to a perfect normal distribution curve due to the CLT.

Part 3. A bootstrapped confidence interval

In this exercise, we will work with the Claridge data found in the `boot` library. You will first install the `boot` library. Then, we will look up what the `claridge` dataset is and load in the data.

```
require(boot)
```

```
## Loading required package: boot
```

```
?claridge
data("claridge")
```

Exercise 3.1

This exercise concerns the **sample correlation** of the variables in this dataset. Recall that the sample correlation is a measure of how dependent two variables are.

(The difference between the **sample correlation** and the **population correlation** is that: we can assume that the `claridge` data is a **sample** from a much larger **population** that has **population correlation** ρ, and we will estimate the true population correlation coefficient ρ using the sample correlation (denoted $\hat{\rho}$). Usually in Statistics, we put a ˆ to denote estimators.

Upon looking up the `claridge` dataset, you will see that there are two columns in the dataset: `dnan` and `hand`.

You will start by computing the sample Pearson correlation coefficient between `dnan` and `hand` using the `cor` function and print it out.

```
### YOUR CODE HERE
cor(claridge$dnan, claridge$hand, method = 'pearson')
```

```
## [1] 0.5087758
```

```
### END OF YOUR CODE
```

Exercise 3.2

Now we want to form an interval that gives us a sense of how much variability there is for the sample correlation using repeated sampling.

We can do so using the bootstrap procedure. You will start by creating a vector filled with `NA`s called `boot_cor_vec` with length 10,000 using the `rep` function. It is important that we populate this vector with `NA`s so if any entry is not updated, we will get a warning. Then, you will obtain 10,000 bootstrap resamples of the `claridge` data. For each bootstrap resample, you will compute the sample correlation and store that in `boot_cor_vec`.

When you create the bootstrap sample, you should resample with replacement **entire rows** of the `claridge` data. Briefly explain the reason why resampling `dnan` and `hand` independently will produce the incorrect distribution for the correlation coefficient.

Hint. Each one of your bootstrap sample should have the same number of samples as the original dataset.

```
set.seed(123)

### YOUR CODE HERE
boot_cor_vec = rep(NA, length.out = 10000)
for (i in 1:10000){
  bootstrap_sample <- claridge[sample(1:nrow(claridge), nrow(claridge), replace = TRUE), ]
  boot_cor_vec[i] <- cor(bootstrap_sample$dnan, bootstrap_sample$hand, method = 'pearson')
}

### END OF YOUR CODE
```

YOUR EXPLANATION HERE. The re sampling would create direct x and y vectors whose cos(theta) of their dot products would not yield the same value as the dot product of the vectors re sampled together.

Exercise 3.3

The central limit theorem (CLT) does not always apply to our data. Even when it does, the sample size that is required for the CLT to be an adequate approximation could vary a lot depending on the statistic being computed.

Recall that we can use the bootstrapped samples to estimate the standard error of our sampling distribution. Let us suppose for a moment that the CLT does indeed apply and is a good approximation for the sampling distribution of the sample Pearson correlation coefficient. If that is the case, we can use the bootstrapped standard error and the CLT to form a 95% confidence interval for the population correlation.

Hint. In order to find the 97.5% quantile of a normal distribution, you can use the `qnorm` function.

```r
### YOUR CODE HERE
boot_mean = mean(boot_cor_vec)
boot_se = (sd(boot_cor_vec)/sqrt(length(boot_cor_vec)))
upper_interval = boot_mean+(qnorm(0.975)*boot_se)
lower_interval = boot_mean-(qnorm(0.975)*boot_se)
upper_interval
```

```
## [1] 0.4675308
```

```r
lower_interval
```

```
## [1] 0.459494
```

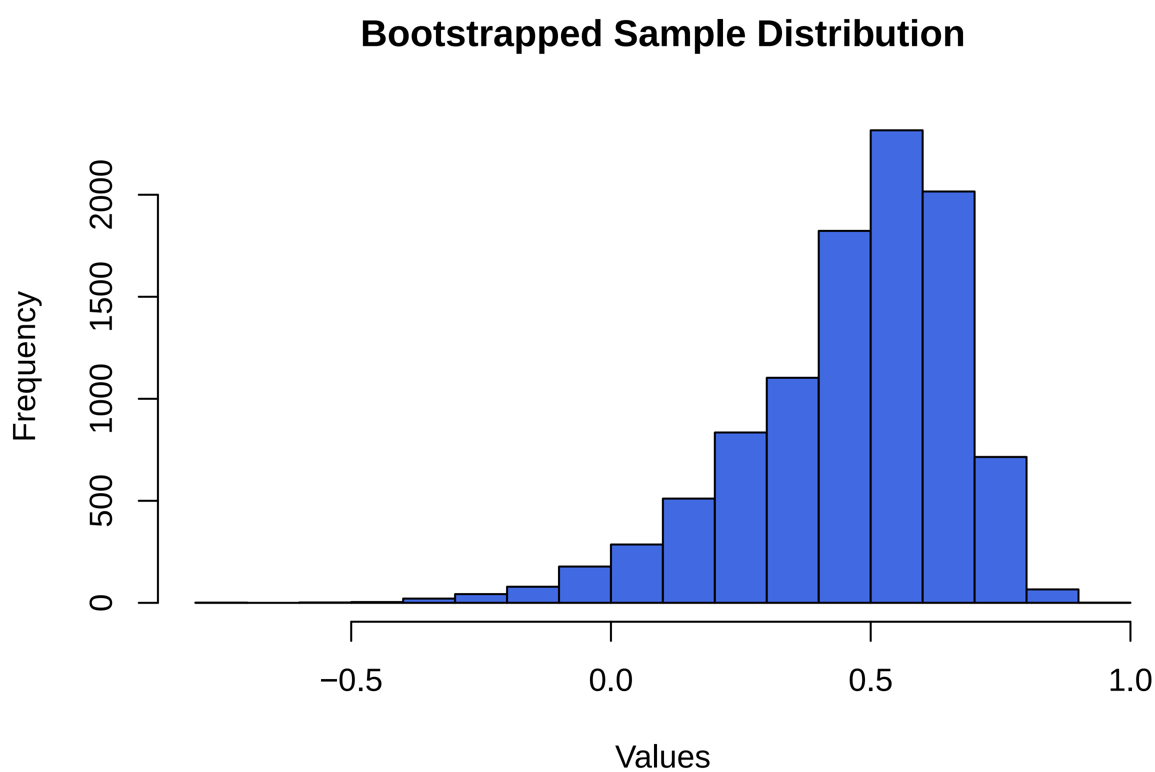
```r
### END OF YOUR CODE
```

Exercise 3.4

First, before we form a confidence interval for the population correlation using the bootstrapped samples, we will plot a histogram of `boot_cor_vec`, the bootstrapped samples. Your plot should be adequately titled with appropriate axes labels.

From the histogram, does the sampling distribution look normal?

```r
### YOUR CODE HERE
hist(boot_cor_vec, main = "Bootstrapped Sample Distribution", xlab = "Values", col = "royalblue")
```



```r
### END OF YOUR CODE
```

YOUR EXPLANATION HERE. The sampling distribution does not look normal as it has a left skew.

Exercise 3.5

Now, we will form a confidence interval for the population correlation using the **bootstrap quantiles**. Below, compute a 95% confidence interval by finding the 2.5% and 97.5% quantiles of `boot_cor_vec`, the bootstrapped distribution of the sample correlation coefficient. You can find the quantile using the `quantile` function.

Briefly describe how this interval compares to your answer in Exercise 3.3. From the histogram of the sampling distribution of the sample correlation coefficients, which interval do you think is more reasonable and why?

```r
### YOUR CODE HERE
hint = quantile(boot_cor_vec, 0.975)
lint = quantile(boot_cor_vec, 0.025)
hint
```

```
##     97.5%
## 0.7565131
```

```r
lint
```

```
##        2.5%
## -0.04431399
```

```r
### END OF YOUR CODE
```

YOUR EXPLANATIONS HERE. This interval is much larger as the data is more spread out. In this case, this interval is most reasonable as the previous one is too small to account for such a large percent of the