CMPE 257 Machine Learning Spring 2019

HW#3 Due April 8th, 11:59 PM, on Canvas

1. (15 points) Problem 8.2 from the textbook e-Chapters

Problem 8.2 Consider a data set with three data points in $\mathbb{R}^2$:

$$\mathrm{X} = \begin{bmatrix} 0 & 0 \\ 0 & -1 \\ -2 & 0 \end{bmatrix} \quad \mathbf{y} = \begin{bmatrix} -1 \\ -1 \\ +1 \end{bmatrix}$$

Manually solve (8.4) to get the optimal hyperplane $(b^*, \mathbf{w}^*)$ and its margin.

$$\begin{aligned} &\underset{b,\mathbf{w}}{\text{minimize:}} && \tfrac{1}{2}\mathbf{w}^{\mathrm{T}}\mathbf{w} && (8.4) \\ &\text{subject to:} && y_n(\mathbf{w}^{\mathrm{T}}\mathbf{x}_n + b) \geq 1 && (n = 1, \cdots, N). \end{aligned}$$

2. (15 points) Problem 8.4 from the textbook e-Chapters

Problem 8.4 Set up the dual problem for the toy data set in Exercise 8.2. Then, solve the dual problem and compute $\boldsymbol{\alpha}^*$, the optimal Lagrange multipliers.

Exercise 8.2

Consider the data below and a 'hyperplane' $(b, \mathbf{w})$ that separates the data.

$$\mathrm{X} = \begin{bmatrix} 0 & 0 \\ 2 & 2 \\ 2 & 0 \end{bmatrix} \quad \mathbf{y} = \begin{bmatrix} -1 \\ -1 \\ +1 \end{bmatrix} \quad \mathbf{w} = \begin{bmatrix} 1.2 \\ -3.2 \end{bmatrix} \quad b = -0.5$$

3. (45 points) Problem 8.5 from the textbook e-Chapters (a) (b) (c) (d). Submit ipynb files.

Problem 8.5 [Bias and Variance of the Optimal Hyperplane]
In this problem, you are to investigate the bias and variance of the optimal hyperplane in a simple setting. The input is $(x_1, x_2) \in [-1, 1]^2$ and the target function is $f(\mathbf{x}) = \mathrm{sign}(x_2)$.

The hypothesis set $\mathcal{H}$ contains horizontal linear separators $h(\mathbf{x}) = \mathrm{sign}(x_2 - a)$, where $-1 \leq a \leq 1$. Consider two algorithms:

Random: Pick a random separator from $\mathcal{H}$.
SVM: Pick the maximum margin separator from $\mathcal{H}$.

(a) Generate 3 data point uniformly in the upper half of the input-space and 3 data points in the lower half, and obtain g_{Random} and g_{SVM}.

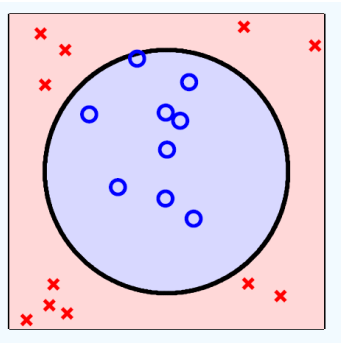
(b) Create a plot of your data, and your two hypotheses.

(c) Repeat part (a) for a million data sets to obtain one million Random and SVM hypotheses.

(d) Give a histogram of the values of a_{Random} resulting from the random algorithm and another histogram of a_{SVM} resulting from the optimal separators. Compare the two histograms and explain the differences.

4. (45 points) Problem 8.13 from the textbook e-Chapters (a) (b) (c), and

(d) Use SVM to find the classifier and compare the results with (a) and (c). Submit ipynb files.



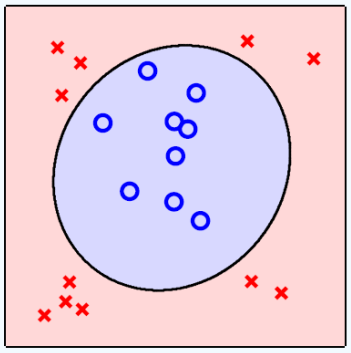
Problem 8.13 The data for Figure 8.6(b) are given below:

$y_n = +1$	$y_n = -1$
	$(0.491, 0.920)$
$(-0.494, 0.363)$	$(-0.892, -0.946)$
$(-0.311, -0.101)$	$(-0.721, -0.710)$
$(-0.0064, 0.374)$	$(0.519, -0.715)$
$(-0.0089, -0.173)$	$(-0.775, 0.551)$
$(0.0014, 0.138)$	$(-0.646, 0.773)$
$(-0.189, 0.718)$	$(-0.803, 0.878)$
$(0.085, 0.32208)$	$(0.944, 0.801)$
$(0.171, -0.302)$	$(0.724, -0.795)$
$(0.142, 0.568)$	$(-0.748, -0.853)$
	$(-0.635, -0.905)$

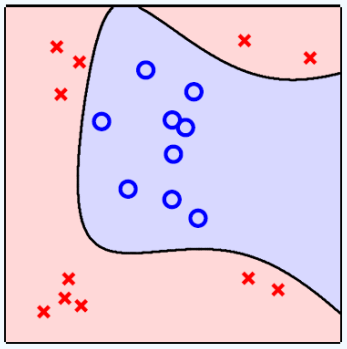
Use the data on the left with the 2nd and 3rd order polynomial transforms Φ_2, Φ_3 and the pseudo-inverse algorithm for linear regression from Chapter 3 to get weights $\tilde{\mathbf{w}}$ for your final final hypothesis in $\mathcal{Z}$-space. The final hypothesis in $\mathcal{X}$-space is:

$$g(\mathbf{x}) = \text{sign}(\tilde{\mathbf{w}}^{\mathrm{T}}\Phi(\mathbf{x}) + \tilde{b}).$$

(a) Plot the classification regions for your final hypothesis in $\mathcal{X}$-space. Your results should look something like:



Φ_2



Φ_3

(b) Which of fits in part (a) appears to have overfitted?

(c) Use the pseudo-inverse algorithm with regularization parameter $\lambda = 1$ to address the overfitting you identified in part (c). Give a plot of the resulting classifier.

Submission instructions:

- Please read the submission instructions on Canvas for naming conventions. Please use meaningful names for variables and file names.
- Discussions are encouraged, but do not copy your answers from external sources or each other.
- Please cite all the sources you used in your submission.
- For questions on the textbook problems, you can check out the book forum: http://book.caltech.edu/bookforum/