

## **CMPE 257 Machine Learning Spring 2019 HW#1**

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**Question 2:** - What types of Machine Learning, if any, best describe the following scenarios:

**a.**

**Answer:-** None. Machine is not learning from the data.

**b.**

**Answer:-** Supervised Learning. With the help of given data, algorithm will try to model relationship dependencies between input and output. To be specific it is a binary classification problem.

**c.**

**Answer:** - Supervised Learning. With the help of given data, algorithm will try to model relationship dependencies between input and output. To be specific it is a multiclass classification problem.

**d.**

**Answer:** - Unsupervised Learning. We do not have output data to model relationship.

**e.**

**Answer:** - Reinforcement learning. Agent is learning its behaviour from the reward feedback by performing actions on the environment.

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**Question 3:**

**Answer:** We know that,  $P(A|B) = P(A \cap B)/P(B)$

where,  $P(A \cap B)$  is the probability of 2 kittens in a box =  $\frac{1}{2}$

and,  $P(B)$  is the probability of the white kitten in a box =  $\frac{3}{4}$

So,  $P(A|B) = (\frac{1}{2} / \frac{3}{4}) = \frac{2}{3}$

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**Question 4:**

**a.**

**Answer:** Given,  $\mu = 0.65$ .

Probability that any marble we draw is not red =  $1 - 0.65 = 0.35$

As each draw is independent (with replacement), the probability to draw one such sample is-

$$P = (0.35)^{10} = 0.00002759$$

**b.**

**Answer:** Probability that at least one  $v=0$  exists among 1000 independent samples will be equal to  $(1 - P[\text{no sample has } v=0])$ .

Probability that no marble drawn is red = 0.35

Probability that no marble drawn is red in a sample =  $(0.35)^{10}$

Probability that at least one marble is red =  $1-(0.35)^{10}$

probability that (at least) one of the samples has  $v = 0$  will be  $(1-(1-(0.35)^{10})^{1000})$  which is equal to 0.0272

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#### Question 5.

**Answer: -**

Considering the remaining combinations {101, 110, 111} we will get 8 possible target functions as described below: -

$X_n$	Target Functions $f(x_n)$							
	$f(x_1)$	$f(x_2)$	$f(x_3)$	$f(x_4)$	$f(x_5)$	$f(x_6)$	$f(x_7)$	$f(x_8)$
1 0 1	0	0	0	0	1	1	1	1
1 1 0	0	0	1	1	0	0	1	1
1 1 1	0	1	0	1	0	1	0	1

a.  $g$  returns 1 for all three points.

**Answer: -** As per table above and “score” formula in question,  $f(x_8)$  is the only target function agreeing with hypothesis on all 3 points.  $f(x_4)$ ,  $f(x_6)$ ,  $f(x_7)$  are the 3 functions agreeing with hypothesis on exactly 2 points.  $f(x_2)$ ,  $f(x_3)$ ,  $f(x_5)$  are the 3 target functions agreeing with hypothesis on exactly 1 point.  $f(x_1)$  is the only target function agreeing with hypothesis on 0 points.

So,

$$\text{Score} = (1*3) + (3*2) + (3*1) + (1*0) = 12$$

b.  $g$  returns 0 for all three points.

**Answer: -**  $f(x_1)$  is the only target function agreeing with hypothesis on all 3 points.  $f(x_2)$ ,  $f(x_3)$ ,  $f(x_5)$  are the 3 functions agreeing with hypothesis on exactly 2 points.  $f(x_4)$ ,  $f(x_6)$ ,  $f(x_7)$  are the 3 target functions agreeing with hypothesis on exactly 1 point.  $f(x_8)$  is the only target function agreeing with hypothesis on 0 points.

So,

$$\text{Score} = (1*3) + (3*2) + (3*1) + (1*0) = 12$$

c.  $g$  is the XOR function applied to  $x$ , i.e., if the number of 1s in  $x$  is odd,  $g$  returns 1; if it is even,  $g$  returns 0.

**Answer: -**  $f(x_2)$  is the only target function agreeing with hypothesis on all 3 points.  $f(x_1)$ ,  $f(x_4)$ ,  $f(x_6)$  are the 3 functions agreeing with hypothesis on exactly 2 points.  $f(x_3)$ ,  $f(x_5)$ ,  $f(x_8)$  are the 3 target functions agreeing with hypothesis on exactly 1 point.  $f(x_7)$  is the only target function agreeing with hypothesis on 0 points.

So,

$$\text{Score} = (1*3) + (3*2) + (3*1) + (1*0) = 12$$

d.  $g$  returns the opposite of the XOR function.

**Answer: -**  $f(x_7)$  is the only target function agreeing with hypothesis on all 3 points.  $f(x_3)$ ,  $f(x_5)$ ,  $f(x_8)$  are the 3 functions agreeing with hypothesis on exactly 2 points.  $f(x_1)$ ,  $f(x_4)$ ,  $f(x_6)$  are the 3 target

functions agreeing with hypothesis on exactly 1 point.  $f(x_2)$  is the only target function agreeing with hypothesis on 0 points.

So,

$$\text{Score} = (1*3) + (3*2) + (3*1) + (1*0) = 12$$

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**Question 6.** The weight update rule is  $w(t+1) = w(t) + x(t)y(t)$

(a) Show that  $y(t)w^T(t)x(t) < 0$ .

**Answer.** Given,  $x(t)$  is misclassified by  $w(t)$ ,

means  $w^T(t)x(t)$  &  $y(t)$  has the opposite signs. Therefore, product of these two will always be negative.

Hence,  $y(t)w^T(t)x(t) < 0$ .

(b) Show that  $y(t)w^T(t+1)x(t) > y(t)w^T(t)x(t)$ .

**Answer.** According to the update rule:

$$w(t+1) = w(t) + y(t)x(t)$$

$$w^T(t+1) = w^T(t) + x^T(t)y(t) \quad // \text{take transpose of the whole equation.}$$

$$y(t)w^T(t+1)x(t) = y(t)w^T(t)x(t) + y(t)x^T(t)y(t)x(t) \quad // \text{multiply } y(t)x(t) \text{ on both sides.}$$

$$y(t)w^T(t+1)x(t) = y(t)w^T(t)x(t) + y^2(t)|x(t)|^2$$

Since  $y^2(t)|x(t)|^2 > 0$ ,

Therefore,  $y(t)w^T(t+1)x(t) > y(t)w^T(t)x(t)$

(c) As far as classifying  $x(t)$  is concerned, argue that the move from  $w(t)$  to  $w(t+1)$  is a move "in the right direction."

**Answer.** We have already proved-

$$y(t)w^T(t)x(t) < 0,$$

$$\text{And } y(t)w^T(t+1)x(t) > y(t)w^T(t)x(t)$$

the update rule keep on increases  $y(t)w^T(t)x(t)$  until it reaches 0, where  $\{x(t), y(t)\}$  is perfectly classified by  $w(t)$ .

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**Question 7.** Given, the perceptron in 2D:  $h(x) = \text{sign}(w^T x)$

Where  $w = [w_0, w_1, w_2]^T$  &  $x = [1, x_1, x_2]^T$ .

(a). We express the line by the equation:  $x_2 = ax_1 + b$ . //equation 1

According to the given data:

$$w_0 + w_1x_1 + w_2x_2 = 0$$

$$x_2 = (-w_1/w_2)x_1 + (-w_0/w_2) \quad // \text{equation 2}$$

On comparing equation 1 & 2.

Slope  $a = -w_1/w_2$  & Intercept  $b = -w_0/w_2$

**(b).** First,  $w = [1, 2, 3]^T$

So the equation will be:

$$1 + 2x_1 + 3x_2 = 0$$

$$2x_1 + 3x_2 = -1 \quad // \text{ equation 1}$$

If  $x_1=0$ , then  $x_2=-1/3$

& if  $x_2=0$ , then  $x_1=-1/2$

Second,  $w = -[1, 2, 3]^T$

So the equation will be:

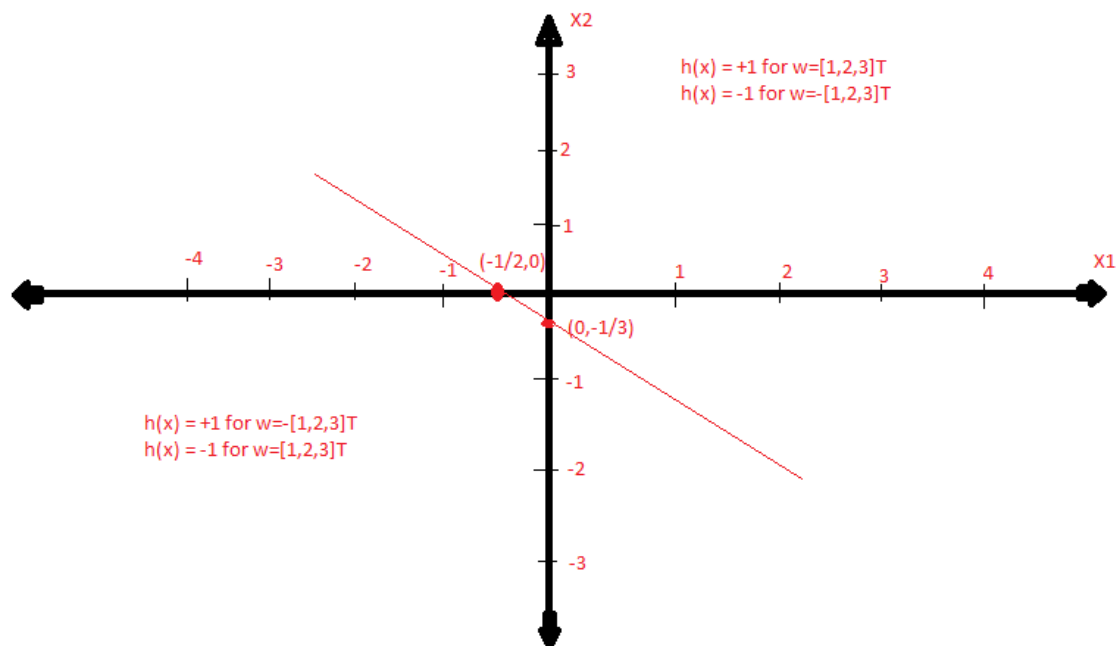
$$-1 - 2x_1 - 3x_2 = 0$$

Or  $1 + 2x_1 + 3x_2 = 0$

$$2x_1 + 3x_2 = -1 \quad // \text{ equation 1}$$

If  $x_1=0$ , then  $x_2=-1/3$

& if  $x_2=0$ , then  $x_1=-1/2$



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**Question 8.** ipynb file attached.

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**Question 9.** ipynb file attached.

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**Question 10. Choice of Error Measure:** Fingerprint Verification:  $f$  is  $+1$ , if it is authentic user, and  $f$  is  $-1$  if it is unauthentic user. Now, error could be of two types: false accept and false reject.

		$g$	
		$+1$	$-1$
$f$	$+1$	Correct	false Reject
	$-1$	false Accept	Correct

**Fingerprint Verification for Supermarket:**

		$g$	
		$+1$	$-1$
$f$	$+1$	0	10
	$-1$	1	0

$$E_{in} = 1/N \sum_{n=1 \text{ to } N} [10 * \text{false reject} + 1 * \text{false accept}]$$

**Fingerprint Verification for CIA Fingerprint:**

		$g$	
		$+1$	$-1$
$f$	$+1$	0	1
	$-1$	1000	0

$$E_{in} = 1/N \sum_{n=1 \text{ to } N} [1 * \text{false reject} + 1000 * \text{false accept}]$$


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