LQR-based Cooperative Path Following of Multiple UAVs with Collision Avoidance

Nishant Mohanty, Akshath Singhal, T.K.Dan and P.B. Sujit

***Abstract*— This paper presents a Linear Quadratic Regulator (LQR) based approach for cooperative path following problem for multiple UAVs along with collision avoidance. The goal of each vehicle, in a fleet, is to track a given desired trajectory and coordinate with other vehicles to be in a desired formation, while avoiding collision with each other. We model the cooperative path following problem as an infinite horizon LQR problem which is solved to obtain the velocity and steering control commands. Numerical simulations for different geometric paths are evaluated to verify the efficacy of our proposed method.**

I. INTRODUCTION

Unmanned Aerial Vehicles (UAVs) have been used in various applications like reconnaissance, search, and rescue, mapping, etc. In some of these applications, the vehicles may need to travel as a team while maintaining safety distance and avoiding collision. When the team needs to maintain a desired geometric pattern then the problem is addressed as a formation control problem [1]. Often the team may be given individual objectives, for example, each vehicle needs to follow a given path while performing aerial mapping along the route. The objective of following a desired path without any temporal constraints is the path following problem [2]. When the vehicles need to follow a pre-specified path while maintaining a desired geometric pattern (possibly even time-varying formations) is the cooperative path following problem [3].

The cooperative path following problem has been an active area of research for over a decade. Several controllers have been synthesized to achieve cooperative path following [4]–[7] and the references therein. Most of the controllers designed in these articles are nonlinear controllers. In this article, we are interested in developing cooperative path following controllers that are optimal in the sense of minimizing the cross-track error with respect to the path while maintaining a given formation and avoiding collisions. We develop an infinite horizon LQR formulation for the cooperative path following problem. The controller is simple to design and provides optimal control effort. The proposed controller is evaluated against complex paths having both straight line and circular trajectories.

The organization of rest of the paper is as follows: Section II comprises of the formulation of the trajectory generation and error dynamics of the problem, followed by the formulation of the optimal guidance law in Section III. Section IV entails the collision avoidance algorithm followed by simulation results of the guidance law implementation in Section V. The concluding remarks have been presented in Section VI.

Nishant Mohanty and T.K. Dan are with Department of Electronics and Communication Engineering, National Institute of Technology, Rourkela

Akshath Singhal and P.B. Sujit are with IIIT Delhi, New Delhi 110020, India. Email: `(akshaths,sujit)@iiitd.ac.in`

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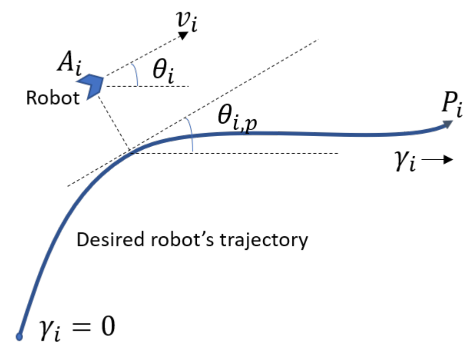


Fig. 1: Formulation Geometry.

II. PROBLEM FORMULATION

For a fleet of n UAVs, the aim is to control the movements of each individual, based on the data received from the neighboring UAVs only. The following formulation assumes that there are m number of nearest neighbors for a single UAV.

A. Trajectory Geometry

Considering the i^{th} UAV, A_i from the fleet of n. At certain time t, it is moving at a velocity of $v_i(t)$, along a path P_i, in a 2-dimensional plane, as shown in Fig. 1. The parameterized curve 'P_i' is the desired trajectory represented with a solid blue line. Let γ_i be the path parameter, which is a variable increasing along the curve 'P_i'. For the i^{th} vehicle, at time t it's coordinate $\mathbb{Q}_i(t)$ on the desired path and its parameter can be related as,

$$\mathbb{Q}_i(t) = P_i(\gamma_i(t)) \tag{1}$$

Thus, when the parameter γ_i increases with time, it implies $P_i(\gamma_i(t))$ follows the reference path for UAV 'i'. The desired position of the vehicle at time t can be expressed as $P_{i,d}(\gamma_i(t))$. Next, the vehicle is expected to move with the assigned velocity $v_i(\gamma_i)$, at each point γ_i on the path P_i. The relationship between the actual speed given by $\dot{\mathbb{Q}}(t)$ and the speed profile obtained from the generated trajectory can be determined by deriving the following expression [8],

$$\dot{\mathbb{Q}}_i(t) = \dot{P}_i(\gamma_i(t)) = \frac{dP_i(\gamma_i(t))}{d\gamma_i(t)}\frac{d\gamma_i(t)}{dt} = \dot{\mathbb{Q}}_{p,i}(t)\dot{\gamma}_i(t) \tag{2}$$

where $\dot{\mathbb{Q}}_{p,i}(t)$ is the desired velocity according to the trajectory-generation algorithm and $\dot{\mathbb{Q}}_i(t)$ is the current velocity.

B. Error Formulation

Path following can be defined as a problem where the vehicles are required to follow the given reference paths with assigned velocities. Let the normal distance between the i^{th} UAV and it's desired trajectory be defined as '$position$ $error$' given by $d_i(t)$ and the lateral velocity of the vehicle be $v_{i,l}(t)$ [9]. Then the relation between $d_i(t)$ and $v_{i,l}(t)$ can be expressed as:

$$v_{i,l}(t) = \dot{d}_i(t) = v_i(t)\sin(\theta_i(t) - \theta_{p,i}(t)) \tag{3}$$

where, $v_i(t), \theta_i(t)$ and $\theta_{p,i}(t)$ are the speed, heading angle, and desired path angle of the i^{th} UAV respectively. From equation (3), the real speed equals the required speed profile obtained during trajectory generation if $\dot{\gamma}_i(t) = 1$. This ensures that the mission is being executed at the desired pace. Thus we define the velocity error, given by $z(t)$ at any time t as,

$$z_i(t) = \dot{\gamma}_i(t) - 1 \tag{4}$$

In order to achieve the path following the position error, velocity error and the lateral velocity must converge to neighbourhood of zero.

Motivated by the work of [8], the following formulation is done to achieve time coordination for cooperative task. As mentioned earlier, the desired position assigned to the i^{th} vehicle at time t is given by $P_i(\gamma_i(t))$, where P_i is the geometric path produced by the trajectory-generation algorithm. This formulation implies that the desired path assigned to each vehicle is parameterized by $\gamma_i(t)$. So, it can be said that if

$$\gamma_i(t) - \gamma_j(t) = 0, \qquad \forall i, j \in \{1, ..., n\}, \qquad i \neq j \tag{5}$$

then, at time t, all the vehicles are coordinated. As mentioned earlier, an UAV has m such vehicle as neighbors. Thus the coordination error vector can be defined as:

$$\xi_i(t) = [\xi_{i,1}(t), \ldots, \xi_{i,m}(t)]^T \tag{6}$$

where $\xi_{i,j}(t) = \gamma_i(t) - \gamma_j(t)$ is the coordination error between i^{th} and j^{th} vehicle.

In order to achieve cooperative movement along with the fleet, the coordination error vector of an individual must converge to neighbourhood of zero. To achieve the objective of cooperative path following, a control law vector u needs to be formulated for a UAV in order to minimize the cooperation error vector comprising of d, $\dot{d}$, ξ and z.

III. LQR Formulation

This paper, formulates the problem at hand as an infinite horizon regulator problem where the controller's objective is to minimize the cooperation error vector discussed earlier. A brief idea about the infinite horizon LQR and the solution to the aforesaid problem has been presented in the following discussion.

A. Trajectory tracking and time coordination in Standard Form

The cooperation error vectors for an individual can be represented as differential equations as,

$$\dot{d}(t) = v_l(t) = v(t)\sin(\theta(t) - \theta_p(t)) \tag{7}$$

$$\begin{aligned}\dot{v}_l(t) &= v(t)(\dot{\theta}(t) - \dot{\theta}_p(t))\cos(\theta(t) - \theta_p(t)) \\ &\quad + \dot{v}(t)\sin(\theta(t) - \theta_p(t))\end{aligned} \tag{8}$$

$$\dot{\xi}(t) = [(\dot{\gamma}(t) - \dot{\gamma}_1(t)), ..., (\dot{\gamma}(t) - \dot{\gamma}_m(t))]^T \tag{9}$$

$$\dot{z}(t) = \ddot{\gamma}(t) \tag{10}$$

where $\dot{x}$ stands for derivative of x. The control variables angular velocity, $u_1(t) = \dot{\theta}(t)$ and linear velocity, $u_2(t) = v(t)$. So, from equation (2) $\dot{\gamma}(t)$ can be defined as,

$$\dot{\gamma}(t) = \frac{\dot{\mathbb{Q}}(t)}{\dot{\mathbb{Q}}_p(t)} = \frac{u_2(t)}{v_d(t)} \tag{11}$$

where, $v_d(t)$ is the desired speed at time t along the trajectory and is assumed to be constant in the absence of information about trajectory. As the formulations have been done from the point of view of a single UAV, it assumed that $\dot{\gamma_1}(t) = \dot{\gamma}_m(t) = 1$, for the i^{th} UAV. Therefore, the time coordination error equation (9) can be reduced to,

$$\dot{\xi}(t) = \beta(\dot{\gamma}_i(t) - 1) \ = \beta(\frac{u_2(t)}{v_d} - 1) \tag{12}$$

where $\beta = [1, 1, ...1_m]^T$. We define the double derivative of $\gamma_i(t)$ as:

$$\ddot{\gamma}_i(t) = -K(\dot{\gamma}_i(t) - 1) \tag{13}$$

where K is a positive constant. From equation (10) and (13), we obtain,

$$\dot{z}_i(t) = \ddot{\gamma}_i(t) = -Kz_i(t) \tag{14}$$

Now, to reduce and linearize equation (8), in the absence of information about the trajectory, we assume $\dot{\theta_p}$ (the desired angular velocity) to be 0, i.e, $\dot{\theta_p} = 0$. Using equation (8) and substituting for $\dot{\theta_p}$, along with small angle approximation, i.e, $\theta - \theta_p \to 0$, we have,

$$\dot{v}_l(t) = v(t)\dot{\theta}(t) = u_2(t)u_1(t) \tag{15}$$

As these terms involve non-linear expressions, we use Taylor series about the base value to linearize them. This is given as,

$$f(x, y) \approx f(\overline{x}, \overline{y}) + \frac{\partial f(x, y)}{\partial x}|_{\overline{x},\overline{y}}.X(t) + \frac{\partial f(x, y)}{\partial y}|_{\overline{x},\overline{y}}.Y(t) \tag{16}$$

and, $X(t)$ and $Y(t)$ are defined as deviation variables about their respective base values and are given as,

$$X(t) = x(t) - \overline{x} \ ; \ Y(t) = y(t) - \overline{y} \tag{17}$$

where $\overline{x}$ and $\overline{y}$ are the base values or the steady state values of variable x and y respectively. In steady state, $X(t) =$

$x(t) - \overline{x} \to 0$. Using equation (17) in the above form of expansion (16) we get,

$$\dot{v}_r(t) = \overline{u}_1\overline{u}_2 + \overline{u}_2(u_1(t) - \overline{u}_1) + \overline{u}_1(u_2(t) - \overline{u}_2) \quad (18)$$
$$= \overline{u}_1\overline{u}_2 + \overline{u}_2 U_1(t) + \overline{u}_1 U_2(t) \quad (19)$$

Let $u = [U_1, U_2]^T$, where u is the optimal control law vector. The linearized system of equation can be represented as

$$\begin{aligned} \dot{d}(t) &= v_l(t) \\ \dot{v}_l(t) &= \overline{u}_1\overline{u}_2 + \overline{u}_2 U_1(t) + \overline{u}_1 U_2(t) \\ \dot{\xi}(t) &= \beta(\frac{u_2(t)}{v_d} - 1) \\ \dot{z}_i(t) &= -Kz(t) \end{aligned} \quad (20)$$

In steady state the above set of equations (20) are reduced to

$$\begin{aligned} 0 &= \overline{v}_l(t); & 0 &= \overline{u}_1\overline{u}_2 \\ 0 &= \beta(\frac{\overline{u}_2(t)}{v_d} - 1); & 0 &= -K\overline{z}(t) \end{aligned} \quad (21)$$

Subtracting set of equations (21) from (20), and representing them in terms of deviation variable, we get

$$\begin{aligned} \dot{D}(t) &= V_l(t) \\ \dot{V}_d(t) &= \overline{u}_2 U_1(t) + \overline{u}_1 U_2(t) \\ \dot{E}(t) &= \frac{\beta U_2(t)}{v_d} \\ \dot{Z}_i(t) &= -KZ(t) \end{aligned} \quad (22)$$

Now, representing the set of equations (22) in the standard form, i.e, $\dot{x} = Ax + Bu$, we have,

$$x = \begin{bmatrix} D \\ V_r \\ E \\ Z \end{bmatrix}, A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -K \end{bmatrix}, B = \begin{bmatrix} 0 & 0 \\ \overline{u}_2 & \overline{u}_1 \\ 0 & \beta/v_d \\ 0 & 0 \end{bmatrix} \quad (23)$$

B. LQR Solution

In infinite-horizon, linear quadratic regulator (LQR) problem we minimize

$$J = \frac{1}{2}\int_{t_0}^{\infty} [x^T Q x + u^T R u] \; dt \quad (24)$$

where, state x and control u are subject to the linear system dynamics $\dot{x} = Ax + Bu$. $Q \geq 0$ and $R > 0$ are design parameters available to us for obtaining the desired control, where, Q is for penalizing the errors and R for the control inputs. The optimal solution for the feedback control is,

$$u = -R^{-1}B^T P x \quad (25)$$

where, u is the control input vector and P is a matrix obtained by solving the algebraic Riccati equation. The Riccati equation is,

$$A^T P + PA - PBR^{-1}B^T P + Q = 0 \quad (26)$$

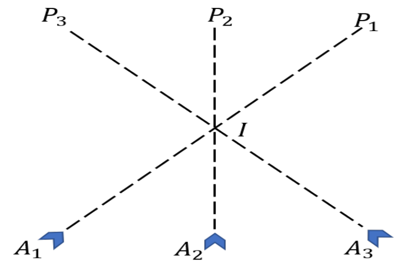


Fig. 2: Collision prone scenario along the reference path.

Now that A, B, R and Q matrix are known, using equation (26) and (23) we get P. Using the value of P in equation (25), the feedback control or the optimal guidance law can be derived.

IV. Intersecting Trajectory Condition

A. Collision Avoidance

The above methodology would ensure cooperative path following of multiple UAVs. But cooperation, could lead to collision in case of intersecting trajectories. Fig. 2 depicts such a scenario. The aim of the vehicles is to follow the straight line assigned to them and simultaneously cooperate along y axis. But this would lead to collision between the individuals at some time t, at the intersection point I. The collision can be avoided by following a certain strategy, i.e, one UAV slows down for the other to go first. As the vehicles stay on the desired trajectory their position error does not increase. But the coordination error and the velocity error increases. For smooth functioning of the mission assigned to the UAVs, collision avoidance need to be achieved at the expense of coordination. However, these errors must be constrained as much as possible in order to achieve the best possible cooperation. Collision avoidance can be ensured if the following condition is always met, i.e,

$$r_{i,j} \geq r^*_{i,j}, \qquad \forall t > 0 \quad (27)$$

where, the distance between two vehicles i and j is denoted as $r_{i,j}$ and safety distance between two agents i and j is given by $r^*_{i,j}$. In order to avoid collision we regulate the cooperation error (5). For this purpose we present a regulation function H_c that would help us achieve the collision avoidance task. The regulation function is chosen such that $H_c(t)$ increases when the distance between two agents approaches the safety radius, $r^*_{i,j}$. We define $H_c(t)$ as [10],

$$H_c(t) = F_c(r_{i,j}(t) - r^*_{i,j}(t)) \quad (28)$$

where F_c is a positive monotonically decreasing function. This implies that, the regulation function increases when the difference between $r_{i,j}$ and $r^*_{i,j}$ decreases, i.e, when the distance between two vehicles approaches safety distance. For achieving such control with a safe maneuvering coordination offset, we redefine the coordination error between i^{th} and j^{th} vehicle as,

$$\xi_{i,j}(t) = \gamma_i(t) - \gamma_j(t) + H_c(t) \quad (29)$$

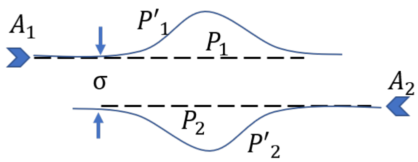


Fig. 3: Attempting a collision free movement with position error regulation in a deadlock prone scenario.

In this case, the concerned UAV does not require to have knowledge about all the vehicles in the fleet for avoiding collision. The above given formulation ensures that if a particular UAV considers another UAV as its neighbour, then it will always be able to avoid collision at the expense of coordination.

B. Priority Order

The above mentioned method helps in avoiding collision when the generated trajectories intersect. This subsection, presents a way to set priority about which agent would go first in collision scenario. For achieving such control we redefine the coordination error as:

$$\xi_{i,j}(t) = \gamma_i(t) - \gamma_j(t) + (-1)^p H_c(t) \tag{30}$$

We know that $\gamma_i(t) > 0$ and $\gamma_j(t) > 0$ at any time $t > 0$. As mentioned earlier $H_c(t)$ always gives a positive value. Assuming $p = 1$, in order to decrease $\xi_{i,j}(t)$, the LQR controller will try to increase the first part of the equation, i.e., $\gamma_i(t) - \gamma_j(t)$. This leads to increase in rate of change of $\gamma_i(t)$ and decrease in rate of change of $\gamma_j(t)$ (as velocity is one of the control variable). So, we infer that agent i is given higher priority over j and vice-versa happens when we set $p = 0$.

C. Deadlock Avoidance

There are certain scenarios, in which, if we try to avoid collision by regulating coordination error, the vehicles will ultimately stop and make deadlock. Fig.3, depicts a situation, where, the perpendicular distance between two parallel paths, given by σ, is less than safety distance $r^*_{i,j}$. In such scenario, collision avoidance has to be achieved by moving out of the desired path to a safer path given by P'. This comes at an expense of position error. For this purpose we present a regulation function H_p that helps us avoid deadlock.

The regulation function needs to be chosen such that $H_p(\gamma(t))$ gradually increases when the parameter $\gamma_i(t)$ does not move forward because of a deadlock [10]. The modified position error equation is stated as,

$$d'(t) = d(t) + H_p(t) \tag{31}$$

where, $d'(t)$ is the new position error.

V. Results and Discussion

This section evaluates the performance of the proposed LQR based method. All the simulations are done in MATLAB. The simulated motion of the vehicles, is presented for standard nonlinear geometries, i.e. a circle and a complex trajectory that includes both linear and non linear trajectory. Simulations for collision and deadlock avoidance have also been conducted. All the UAVs have the linear and angular velocity constraints as $10m/sec < u_1 < 30m/sec$ and $-0.5rad/sec < u_2 < 0.5rad/sec$ respectively. The design parameter Q and R are taken to be as identity matrix.

Topic	UAV	Desired trajectory eqn	Coordinates (x,y) [m]	Heading angle [rad]	Desired speed (v_d) [m/sec]
Circular, Collision free trajectory	1	$x^2 + y^2 = 45^2$	(-20,40)	0	14.13
	2	$x^2 + y^2 = 60^2$	(0,65)	0	18.84
	3	$x^2 + y^2 = 75^2$	(20,80)	0	23.56
Complex, Collision free trajectory	1	$x^2+y^2 = 45^2, y < -31.6$ $x = -31.6 \quad , y \geq -31.6$	(31.6,-31.6)	$7\pi/8$	14.13
	2	$x^2+y^2 = 60^2, y < -42.2$ $x = -42.2 \quad , y \geq -42.2$	(42.2,-42.2)	$-7\pi/8$	18.84
	3	$x^2+y^2 = 75^2, y < -49.3$ $x = -49.3 \quad , y \geq -49.3$	(49.3,-49.3)	$-7\pi/8$	23.56
Collision Avoidance	1	$x = y$	(-50,-50)	$\pi/4$	24.03
	2	$x = 0$	(0,50)	$\pi/2$	17.00
	3	$x = -y$	(50,-50)	$3\pi/4$	24.03
Deadlock Avoidance	1	$x = 0$	(0,70)	$-\pi/2$	17.00
	2	$x = 0$	(0,-70)	$\pi/2$	17.00

TABLE I: Initial configurations and parameters

A. Cooperation along circular collision free trajectory:

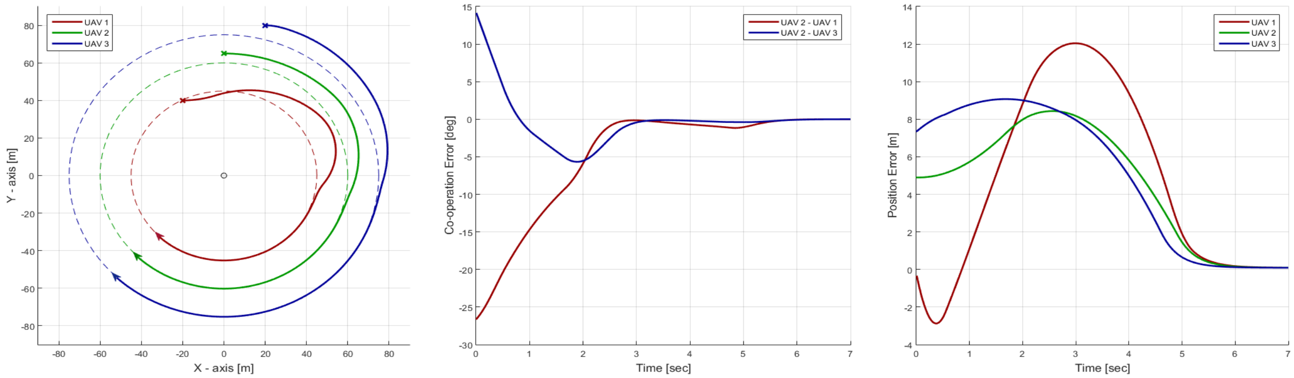
This section, deals with a scenario of a circular geometrical path. The mission for the vehicles is to coordinate along the radial axis. The simulated motion of the 3 cooperating UAVs is shown in 4a. The values taken for the corresponding simulations have been given in table I.

Though initialised elsewhere, the vehicles converge to their prescribed path as shown by their trajectories in Fig. 4a. All the vehicles can also be seen coordinating along the radial axis after a certain amount of time i.e, $7\ sec$. Their coordination errors can be seen in Fig. 4b, where the coordination errors of UAV_1 and UAV_3 with respect to UAV_2 is given. The position errors of all the three vehicles can be seen in the plots in Fig. 4c. To further validate the formulation, UAV's motion along a complex trajectory, i.e, a combination of both linear and circular paths, while coordinating along radial axis is shown in Fig. 5a. The corresponding cooperation and position error is depicted in Fig. 5b and Fig. 5c respectively. The vehicles initial configuration is as shown in table I. The coordination and position errors can be seen to have a spike when the trajectory changes from linear to non linear.

B. Cooperation along linear collision prone trajectory:

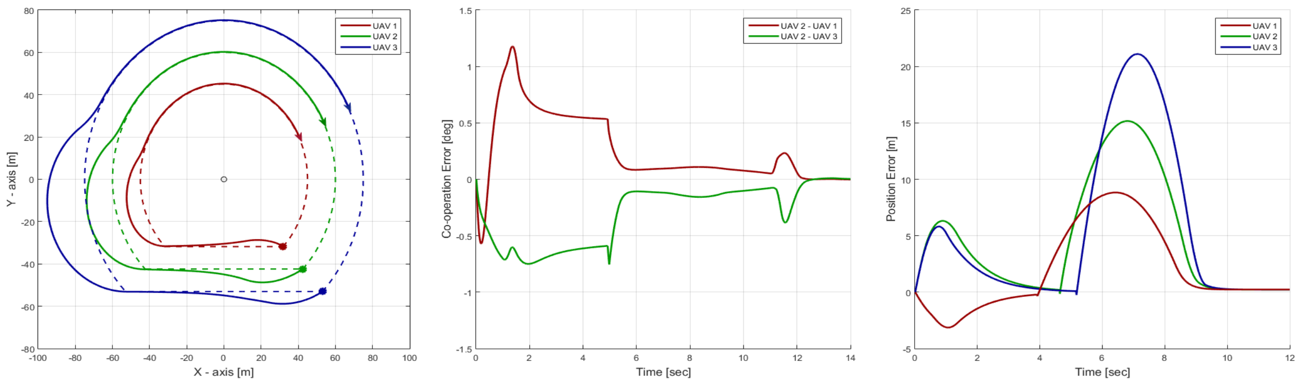
The simulated motion of three UAVs in a scenario depicted in collision avoidance section (refer Fig. 2) is presented in Fig.6. The straight line equations of these paths are $y = x$, $x = 0$, $y = -x$ respectively. These three straight lines would intersect at (0,0). The mission assigned to the UAVs is to coordinate along the y-axis and avoid collision at the expense of coordination whenever necessary. The coordination error equation for the i^{th} vehicle, in this case, is given as,

$$\xi_{i,j}(t) = \gamma_i(t) - \gamma_j(t) + (-1)^p.H_c(t) \tag{32}$$



(a) Path followed (b) Coordination error (c) Position error

Fig. 4: Three UAVs following a circular trajectory while cooperating along radial axis.



(a) Path followed (b) Coordination error (c) Position error

Fig. 5: Three UAVs following a complex trajectory while cooperating along radial axis.

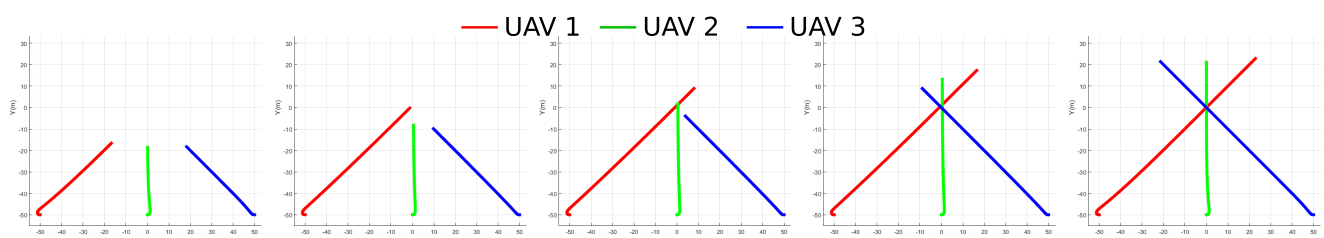


Fig. 6: Frame wise representation of three agent collision avoidance motion control

Here we take the regulation function, $H_c(t)$ as,

$$H_c(t) = \frac{K_c}{r_{i,j}(t) - r_{s,i,j}} \tag{33}$$

where, K_c is a positive constant. Note that this function can be replaced with any other function satisfying the conditions for H_c while implementation. In Fig. 6 it can be seen that after the start of mission the UAVs have reached a steady state in the first frame where they are coordinating along the y-axis. After a certain time the distance between them decreases, hence approaching safety distance. Thus in further frames, it can be seen that Agent 1 moves first, then agent 2 and after that agent 3. Fig. 7 gives the coordination error plot between the pair of (UAV_1, UAV_2) and (UAV_1, UAV_2). A sudden rise in the coordination error can be seen in

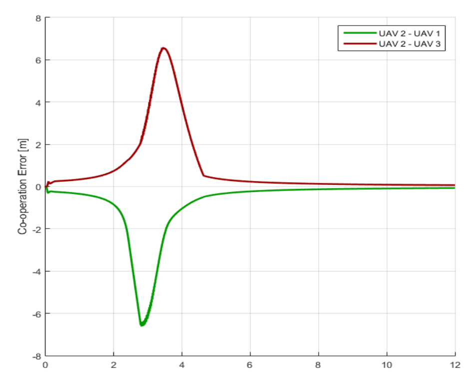


Fig. 7: Coordination errors of the UAVs with respect to their neighbours for collision avoidance scenario.

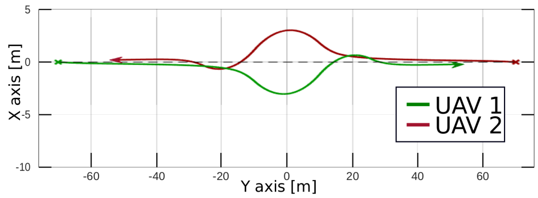


Fig. 8: Motion of two vehicle moving along the desired trajectory while avoiding deadlock.

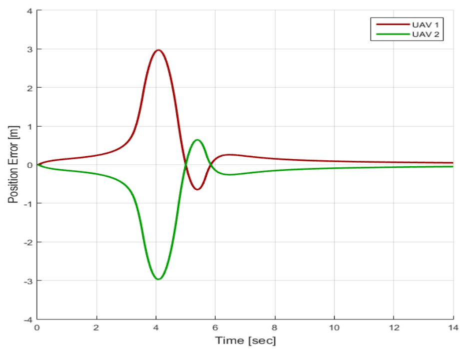


Fig. 9: Position errors of the UAVs in deadlock avoidance scenario.

Fig.7 when distance between the UAVs approaches the safety distance. This case, demonstrates the movement with priority order of the UAVs set as 1,2,3.

C. Deadlock Avoidance:

In the scenario depicted in Fig.3, the simulated motion of the UAVs can be seen in Fig. 8. The distance between the two path γ is taken to be zero here. The straight line equation for the desired trajectory is $x = 0$ for both. UAV_1, depicted by red, is moving towards negative y-direction and UAV_2, depicted by green, is moving along the positive y-direction. The mission assigned to the vehicles is to move along the prescribed path and swap places with each other. As discussed earlier, this is achieved by avoiding collision at the expense of position error. The position error for the i^{th} vehicle, in this case, can be given as,

$$d_i(t) = R(t)\sin(\theta_i - \theta_{p,i}) + H_p(t) \tag{34}$$

where $R(t)$ is the distance between the initial position of the vehicle and the current position, and all other notation mean the same as before. Here we take $H_p(t)$ as,

$$H_p(t) = K_p(|\gamma_i(t) - \eta(t)|) \tag{35}$$

where K_p is also a positive constant and $\eta(t)$ is the desired path parameter $\gamma(t)$ at time t. And $H_p(t)$ increases when $\eta(t)$ becomes much greater than $\gamma_i(t)$. The function $\eta(t)$ must be taken such that its derivative, $\dot{\eta}(t)$ is less than 1, else this would lead to increase in velocity error, which the LQR based controller would try to suppress. This would lead to instability and undesirable outcomes. In Fig. 8, we can see that after the start of the mission the vehicles move along the prescribed path and move an equal distance along the axis. But as the distance between them decreases the regulation of coordination error to avoid collision takes effect and ends up creating a deadlock situation. With regulation of position error, it can be seen that the UAVs successfully avoids the collision and are able to achieve their mission. In Fig. 9, we can see the position errors of agent 1 and 2 in red and green respectively. The sudden rise in error corresponds to certain time t when the agents are reaching a deadlock. The error plots are symmetrical as it should be in the ideal case because the regulation function used is the same for both the agents.

VI. Conclusion

We present the formulation of control input to the system as a solution to the time coordination and path following problem for a fleet of UAVs. Collision avoidance strategies are shown in the case of intersecting trajectories. The solution to the deadlock condition arising due to the strict following of the above method is given. Infinite horizon LQR formulation is adapted as a solution to the cooperative path following problem after linearizing the error dynamics. Simulation results involving three vehicles are conferred to justify the proposed approach and the errors can be seen to be very low.

References

[1] K.-K. Oh, M.-C. Park, and H.-S. Ahn, “A survey of multi-agent formation control,” *Automatica*, vol. 53, pp. 424–440, 2015.

[2] A. P. Aguiar and J. P. Hespanha, “Trajectory-tracking and path-following of underactuated autonomous vehicles with parametric modeling uncertainty,” *IEEE transactions on automatic control*, vol. 52, no. 8, pp. 1362–1379, 2007.

[3] R. Ghabcheloo, A. P. Aguiar, A. Pascoal, C. Silvestre, I. Kaminer, and J. Hespanha, “Coordinated path-following in the presence of communication losses and time delays,” *SIAM journal on control and optimization*, vol. 48, no. 1, pp. 234–265, 2009.

[4] F. Vanni, A. P. Aguiar, and A. M. Pascoal, “Cooperative path-following of underactuated autonomous marine vehicles with logic-based communication,” *IFAC Proceedings Volumes*, vol. 41, no. 1, pp. 107–112, 2008.

[5] E. Xargay, V. Dobrokhodov, I. Kaminer, A. M. Pascoal, N. Hovakimyan, and C. Cao, “Time-critical cooperative control of multiple autonomous vehicles: Robust distributed strategies for path-following control and time-coordination over dynamic communications networks,” *IEEE Control Systems Magazine*, vol. 32, no. 5, pp. 49–73, 2012.

[6] F. C. Rego, N. T. Hung, C. N. Jones, A. M. Pascoal, and A. P. Aguiar, “Cooperative path-following control with logic-based communications: Theory and practice,” *Navigation and Control of Autonomous Marine Vehicles*, p. 187, 2019.

[7] R. P. Jain, A. P. Aguiar, and J. Sousa, “Self-triggered cooperative path following control of fixed wing unmanned aerial vehicles,” in *2017 International Conference on Unmanned Aircraft Systems (ICUAS)*. IEEE, 2017, pp. 1231–1240.

[8] V. Cichella, I. Kaminer, V. Dobrokhodov, E. Xargay, R. Choe, N. Hovakimyan, A. P. Aguiar, and A. M. Pascoal, “Cooperative path following of multiple multirotors over time-varying networks,” *IEEE Transactions on Automation Science and Engineering*, 2015.

[9] A. Ratnoo, P. Sujit, and M. Kothari, “Adaptive optimal path following for high wind flights,” *Proceedings of the 18th World Congress, The International Federation of Automatic Control*, 2011.

[10] K. Sakurama and K. Nakano, “Deadlock-free path-following control for collision avoidance of multiple robots,” *Joint 48th IEEE Conference on Decision and Control and 28th Chinese Control Conference*, 2009.