★★ INDEX ★★

Exam Seat No.

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who has worked for the year 2019-2020 in the Computer Laboratory.

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Teacher In-Charge

Head of Department

Examiner

Date :

PRACTICAL NO-1.

Topic - LIMITS & CONTINUITY

i) $$\lim_{x \to a} \left[\frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \right].$$

$$\lim_{x \to a} \left[\frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \times \frac{\sqrt{a+2x} + \sqrt{3x}}{\sqrt{a+2x} + \sqrt{3x}} \times \frac{\sqrt{3a+x} + 2\sqrt{x}}{\sqrt{3a+x} + 2\sqrt{x}} \right].$$

$$\lim_{x \to a} \frac{(a+2x-3x)(\sqrt{3a+x} + 2\sqrt{x})}{(3a+x-4x)(\sqrt{a+2x} + \sqrt{3x})}$$

$$\lim_{x \to a} \frac{(a-x)(\sqrt{3a+x} + 2\sqrt{x}))}{(a-x)(\sqrt{a+2x} + \sqrt{3}\sqrt{x}))}$$

$$= \frac{1}{3} \lim_{x \to a} \frac{\sqrt{3a+a} + 2\sqrt{a}}{\sqrt{a+2a} + \sqrt{3a}}.$$

$$= \frac{1}{3} \times \frac{\sqrt{4a} + 2\sqrt{a}}{\sqrt{3a} + \sqrt{3a}}$$

$$= \frac{1}{3} \frac{2\sqrt{a} + 2\sqrt{a}}{\sqrt{3a} + \sqrt{3a}}.$$

$$= \frac{1}{\cancel{3}} \times \frac{4\sqrt{a}}{2\sqrt{3}\sqrt{a}}$$

$$= \boxed{\frac{2}{3\sqrt{3}}}$$

2) $\lim_{y\to 0}\left[\dfrac{\sqrt{a+y}-\sqrt{a}}{y\sqrt{a+y}}\right]$

$= \lim_{y\to 0}\left[\dfrac{\sqrt{a+y}-\sqrt{a}}{y\sqrt{a+y}} \times \dfrac{\sqrt{a+y}+\sqrt{a}}{\sqrt{a+y}+\sqrt{a}}\right]$

$= \lim_{y\to 0} \dfrac{a+y-a}{y\sqrt{a+y}(\sqrt{a+y}+\sqrt{a})}$

$= \lim_{y\to 0} \dfrac{y}{y\sqrt{a+y}(\sqrt{a+y}+\sqrt{a})}$

$= \lim_{y\to 0} \dfrac{1}{\sqrt{a+0}(\sqrt{a+0}+\sqrt{a})}$

$= \dfrac{1}{\sqrt{a}(\sqrt{a}+\sqrt{a})}$

$= \dfrac{1}{\sqrt{2}(2\sqrt{a})} = \dfrac{1}{2a}.$

3) $\lim_{x\to \pi/6} \dfrac{\cos x - \sqrt{3}\sin x}{\pi - 6x}$

By substituting $x - \dfrac{\pi}{6} = h$

where $x = h + \dfrac{\pi}{6}$.

where $h \to 0$

$\lim_{h\to 0} \dfrac{\cos\left(h+\frac{\pi}{6}\right) - \sqrt{3}\sin\left(h+\frac{\pi}{6}\right)}{\pi - 6\left(h+\frac{\pi}{6}\right)}$ using $\cos(A+B) = \cos A\cos B - \sin A\sin B$

$\lim_{h\to 0} \dfrac{\cosh\cdot\cos\frac{\pi}{6} - \sinh\cdot\sin\frac{\pi}{6} - \sqrt{3}\sinh\cdot\cos\frac{\pi}{6} + \cosh\cdot\sin\frac{\pi}{6}}{\pi - 6\left(\frac{6h+\pi}{6}\right)}$

$\cos\frac{\pi}{6} = \cos 30 = \sqrt{3}/2$, $\sin\pi/6 = \sin 30 = \frac{1}{2}$

$\lim_{h\to 0} \dfrac{\cosh\cdot\frac{\sqrt{3}}{2} - \sinh\frac{1}{2} - \sqrt{3}\left(\sinh\frac{\sqrt{3}}{2} + \cosh\cdot\frac{1}{2}\right)}{\pi - 6h + \pi}$

$\lim_{h\to 0} \dfrac{\cos\cancel{\frac{\sqrt{3}}{2}h} - \sin\frac{h}{2} - \sin\frac{3h}{2} - \cos\cancel{\frac{\sqrt{3}}{2}h}}{-6h}$

$\lim_{h\to 0} \dfrac{-\sin 4h/2}{-6h}$

$\lim_{h\to 0} \dfrac{\sin 4h}{3+2h}$

$\frac{1}{3}\lim_{h\to 0}\dfrac{\sinh}{h} = \frac{1}{3}\times 1 = \frac{1}{3}$

4) $\lim_{x\to\infty} \left[\frac{\sqrt{x^2+5}-\sqrt{x^2-3}}{\sqrt{x^2+3}-\sqrt{x^2+1}}\right]$

By rationalizing numerator and denominator both

$$\lim_{x\to\infty} \left[\frac{\sqrt{x^2+5}-\sqrt{x^2-3}}{\sqrt{x^2+3}-\sqrt{x^2+1}} \times \frac{\sqrt{x^2+5}+\sqrt{x^2-3}}{\sqrt{x^2+5}+\sqrt{x^2-3}} \times \frac{\sqrt{x^2+3}+\sqrt{x^2+1}}{\sqrt{x^2+3}+\sqrt{x^2+1}}\right]$$

$$\lim_{x\to\infty} \left[\frac{(x^2+5-x^2+3)(\sqrt{x^2+3}+\sqrt{x^2+1})}{(x^2+3-x^2-1)(\sqrt{x^2+5}+\sqrt{x^2-3})}\right]$$

$$\lim_{x\to\infty} \frac{\overset{4}{\cancel{8}}(\sqrt{x^2+3}+\sqrt{x^2+1})}{\cancel{2}(\sqrt{x^2+5}+\sqrt{x^2-3})}$$

$$4\lim_{x\to\infty} \frac{\sqrt{x^2\left(1+\frac{3}{x^2}\right)}+\sqrt{x^2\left(1+\frac{1}{x^2}\right)}}{\sqrt{x^2\left(1+\frac{5}{x^2}\right)}+\sqrt{x^2\left(1-\frac{3}{x^2}\right)}}$$

After Applying limit we get
$= 4$

5) $f(x) = \frac{\sin 2x}{\sqrt{1-\cos 2x}}$, for $0 < x \le \frac{\pi}{2}$

$= \frac{\cos x}{\pi - 2x}$, for $\frac{\pi}{2} < x < \pi$ $\Big\}$ at $x = \frac{\pi}{2}$

$f(\pi/2) = \frac{\sin 2(\pi/2)}{\sqrt{1-\cos 2(\pi/2)}}$ $\therefore f(\pi/2) = 0$

f at $x = \pi/2$ define.

$$\lim_{x\to\pi/2^+} f(x) = \lim_{x\to\pi/2^+} \frac{\cos x}{\pi - 2x}$$

By substituting method,

$x - \pi/2 = h$

$x = h - \pi/2$

where $h \to 0$

$$\lim_{h\to 0} \frac{\cos(h+\pi/2)}{\pi - 2(h+\pi/2)}$$

$$\lim_{h\to 0} \frac{\cos(h+\pi/2)}{\pi - 2(2\pi+\pi/2)}$$

$$\lim_{h\to 0} \frac{\cos(h+\pi/2)}{-2h}$$ using $\cos(A+B) = \cos A\cdot\cos B - \sin A\cdot\sin B$

$$\lim_{h\to 0} \frac{\cos h\cdot\cos\pi/2 - \sin h\cdot\sin\pi/2}{-2h}$$

$$\lim_{h\to 0} \frac{\cos h\cdot 0 - \sin h}{-2h}$$

$$\lim_{h\to 0} \frac{\cancel{-}\sin h}{\cancel{-}2h}$$

$$\frac{1}{2}\lim_{h\to 0} \frac{\sin h}{h} = \frac{1}{2}$$

$$\lim_{x\to\pi/2^-} f(x) \lim_{x\to\pi/2} = \frac{\sin 2x}{\sqrt{1-\cos 2x}}$$

Using $\sin 2x = 2\sin x \cdot \cos x$.

$$\lim_{x\to \pi/2^-} \frac{2\sin x \cdot \cos x}{\sqrt{2\sin^2 x}}$$

$$\lim_{x\to \pi/2^-} \frac{2\sin x \cdot \cos x}{\sqrt{2}\,\sin x}$$

$$\lim_{x\to \pi/2^-} \frac{2\cos x}{\sqrt{2}}$$

$$\frac{2}{\sqrt{2}} \cdot \lim_{x\to \pi/2} - \cos x$$

$\therefore$ LHD $\neq$ RHD

$\therefore f$ is not cts at $x = \pi/2$.

ii) $f(x) = \dfrac{x^2-9}{x-3}$ $\quad 0 < x < 3$
$= x+3$ $\quad 3 \le x \le 6$
$= \dfrac{x^2-9}{x+3}$ $\quad 6 \le x < 9$

at $x=3$ & $x = 6$

at $x = 3$

(i) $f(3) = \dfrac{x^2-9}{x-3} = 0$.

f at $x=3$ define.

30

$$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} (x+3) = 6$$

$$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} \frac{x^2-9}{x-3} = (x+3)$$

LHL = RHL

f is continous at $x=3$

for $x=6$

$$f(6) = \frac{x^2-9}{x+3} = \frac{36-9}{6+3} = \frac{27}{9} = 3$$

$$\lim_{x\to 6^+} \frac{x^2-9}{x+3}$$

$$\lim_{x\to 6^+} (x-3) = 6-3 = 3$$

$$\lim_{x\to 6^-} x+3 = 3+6 = 9$$

$\therefore$ LHL $\neq$ RHL.

function is not continous

Now $\lim_{x\to 0} f(x) = f(0)$.

(iii) f has removable discontinuity at $x=0$.

$$f(x) = \frac{(e^{3x}-1)\sin\left(\frac{\pi x}{180}\right)}{x^2}$$

$$\lim_{x\to 0} \frac{e^{3x}-1}{x} \lim_{x\to 0} \frac{\sin(\pi x/180)}{x}$$

$$3\lim_{x\to 0} \frac{e^{3x}-1}{3x} \lim_{x\to 0} \frac{\sin\left(\frac{\pi x}{180}\right)}{x}$$

$$3\lim_{x\to 0} \frac{e^{3x}-1}{3x} \lim_{x\to 0} \sin\left(\frac{\pi x/180}{x}\right)$$

$$3 \log e \cdot \frac{\pi}{180_{60}} = \frac{\pi}{60} = f(0)$$

f is continous at $x=0$

$$\text{(vi)}\ f(x) = \begin{cases} \dfrac{1-\cos 4x}{x^2} & , x<0 \\ = k & , x=0 \end{cases} \Bigg\}\ \text{at } x=0$$

f is continuous at x = 0

$$\lim_{x\to 0} f(x) = f(0).$$

$$\lim_{x\to 0} \frac{1-\cos 4x}{x^2} = k.$$

$$\lim_{x\to 0} \frac{\cancel{1-\cos 4}\ 2\sin^2 2x}{x^2} = k$$

$$2\lim_{x\to 0} \frac{\sin^2 2x}{x^2} = k.$$

$$2\lim_{x\to 0} \left(\frac{\sin 2x}{x}\right)^2 = k$$

$$2\lim_{x\to 0} \left(\frac{\sin 2x}{x}\right)^2 = k$$

$$2(2)^2 = k$$

$$k = 8$$

~~ii) f(x) = sec~~

$$\text{ii) } f(x) = (\sec^2 x)^{\cot^2 x}$$

Using $\tan^2 x = 1 + \sec^2 x$

$$\therefore \sec^2 x = 1 + \tan^2 x$$

$$\therefore \cot^2 x = \frac{1}{\tan^2 x}$$

$$\lim_{x\to 0} (\sec^2 x)^{\cot^2 x}$$

$$\lim_{x\to 0} (1+\tan^2 x)^{\frac{1}{\tan^2 x}}$$

w.k.t

$$\lim_{x\to 0} (1+px)^{1/px} = e$$

$$= e$$

$$k = e$$

$$\text{iii) } f(x) = \begin{cases} \dfrac{\sqrt{3} - \tan x}{\pi - 3x} & , x \neq \pi/3 \\ = k & , x = \pi/3 \end{cases} \Bigg\}\ \text{at } x = \pi/3$$

$$x - \pi/3 = h$$

$$x = h + \pi/3$$

when $h \to 0$

$$f(\pi/3 + h) = \frac{\sqrt{3} - \tan(\pi/3 + h)}{\pi - 3(\pi/3 + h)}$$

$$\lim_{h\to 0} \frac{\sqrt{3} - \tan(\pi/3 + h)}{\pi - 3(\pi/3 + h)}$$

Using $(\tan A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

$$\lim_{h\to 0} \frac{\sqrt{3} - \dfrac{\tan \pi/3 + \tanh}{1 - \tan \pi/3 \cdot \tanh}}{\pi - \cancel{\pi} - 3h}$$

Using $\tan \pi/3 = \tan 60° = \sqrt{3}$

$$\lim_{h\to 0} \frac{(\sqrt{3}-\sqrt{3}\times\sqrt{3}.\tan h) - (\sqrt{3}+\tan h)}{\frac{1-\tan \pi/3 \cdot \tan h}{-3h}}$$

$$= \lim_{h\to 0} \frac{(\sqrt{3} - 3\tan h - \sqrt{3} - \tan h)}{\frac{1-\sqrt{3}\tan h}{-3h}}$$

$$\cancel{= \lim_{h\to 0} (\sqrt{3} - 3t}$$

$$= \frac{4}{3} \lim_{h\to 0} \frac{\tan h}{h} \lim_{h\to 0} \frac{1}{(1-\sqrt{3}\tan h)}$$

$$= \frac{4}{3}\left(\frac{1}{1}\right) = \frac{4}{3}$$

7 (i) $f(x) = \frac{1-\cos 3x}{x \cdot \tan x}$, $x \neq 0$; $= 9$, $x = 0$ at x

$$f(x) = \frac{1-\cos 3x}{x\tan x}$$

$$\lim_{x\to 0} \frac{2\sin^2 3/2x}{x \cdot \tan x}$$

$$\lim_{x\to 0} \frac{\frac{2\sin^2 x}{x^2} \cdot x^2}{\frac{x\tan x}{x^2} \cdot x^2}$$

32

$$= 2 \lim_{x\to 0} \frac{(3/2)^2}{1} = 2\times 9/4 = 9/2$$

$\lim_{x\to 0} f(x) = 9/2$, $9 = f(0)$

f is not cts at $x=0$.

Redefine function

$$f(x) = \begin{cases} \frac{1-\cos 3x}{x\tan x}, & x \neq 0 \\ = 9/2, & x=0 \end{cases}$$

(ii) $f(x) = \frac{e^{x^2} - \cos x}{x^2}$, $x = 0$

is continuous at $x=0$

$\therefore$ given f is cts at $x=0$

$$\lim_{x\to 0} f(x) = f(0)$$

$$\lim_{x\to 0} \frac{e^{x^2} - \cos x}{x^2} = f(0)$$

$$\lim_{x\to 0} \frac{e^{x^2}-1}{x} + \lim_{x\to 0} \frac{1-\cos x}{x^2}$$

$$\log e + \lim_{x\to 0} \left(\frac{2\sin^2 x/2}{x}\right)^2$$

multiply with 2

$\longrightarrow 1 + 2\times 1/4 = 3/2 = f(0)$

9) $f(x) = \frac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x}$, $x \neq \pi/2$

$f(0)$ is cts at $x = \pi/2$.

$\lim_{x \to \pi/2} \frac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x} \times \frac{\sqrt{2} + \sqrt{1+\sin x}}{\sqrt{2} + \sqrt{1+\sin x}}$

$\lim_{x \to \pi/2} \left(\frac{2 - 1 + \sin x}{\cos^2 x (\sqrt{2} + \sqrt{1+\sin x})} \right)$

$\lim_{x \to \pi/2} \frac{1 + \sin x}{1 + \sin^2 x (\sqrt{2} + \sqrt{1+\sin x})}$

$\lim_{x \to \pi/2} \frac{(1+\sin x)}{(1 - \sin x)(1 + \sin x)(\sqrt{2} + \sqrt{1+\sin x})}$

$\lim_{x \to \pi/2} = \frac{1}{(1 - \sin x)(\sqrt{2} + \sqrt{1+\sin x})}$

$= \frac{1}{2(\sqrt{2} + \sqrt{2})}$

$= \cancel{\frac{1}{4\sqrt{2}}}$

$\therefore f(\pi/2 = \frac{1}{4\sqrt{2}}$

4/11/19

2. Derivative

Show that the following function defined from R to R are differentiable

i) $\cot x$

$\longrightarrow f(x) = \cot x$

$f(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$

$= \lim_{x \to a} \frac{\cot x - \cot a}{x - a}$

$= \lim_{x \to a} \frac{1/\tan x - 1/\tan a}{x - a}$

$= \lim_{x \to a} \frac{\tan a - \tan x}{(x-a)\tan x \cdot \tan a}$

put $x - a = h$

$x = a + h$

as $x \to a$, $h \to a$.

$\longrightarrow f(h) = \lim_{h \to 0} \frac{\tan a - \tan(a+h)}{(a+h-a)\tan(a+h)\tan a}$

$= \lim_{h \to 0} \frac{\tan a - \tan(a+h)}{h \times \tan(a+h)\tan a}$

~~formula - tan(A-B) = tanA - tan~~

formula - $\boxed{\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \cdot \tan B}}$

$\tan A - \tan B = \tan(A-B) \mid (1 + \tan A \cdot \tan B)$

$$= \lim_{h\to 0} \frac{\tan(a-a-h) - (1+\tan a + \tan(a+h))}{h \times \tan(a+h)\ \tan a}$$

$$= \lim_{h\to 0} -\frac{\tan h}{h} \times \frac{1+\tan a \cdot \tan(a+h)}{\tan(a+h)\ \tan a}$$

$$= -1 \times \frac{1+\tan^2 a}{\tan^2 a} = \frac{-\sec^2 a}{\tan^2 a}$$

$$= \frac{-1}{\cancel{\cos^2 a}} \times \frac{\cancel{\cos^2 a}}{\sin^2 a}$$

$$= -\operatorname{cosec}^2 a$$

$\therefore f'(a) = -\cos^2 a$

f is differentiable $\forall a \in R$

ii) cosec x

$f(x) = \operatorname{cosec} x$

$$\Longrightarrow f'(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$$

$$= \lim_{x\to a} \frac{\operatorname{cosec} x - \operatorname{cosec} a}{x-a}$$

$$= \lim_{x\to a} \frac{\frac{1}{\sin x} - \frac{1}{\sin a}}{x-a}$$

$$= \lim_{x\to a} \frac{\sin a - \sin x}{(x-a)\ \sin a \cdot \sin x}$$

put $x-a=h$

$x = a+h$

as $x \to a$, $h \to 0$

$$\Longrightarrow f'(h) = \lim_{h\to 0} \frac{\sin a - \sin(a+h)}{(a+h-a)\ \sin a \cdot \sin(a+h)}$$

formula $\rightarrow$

$$\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right) \cdot \sin\left(\frac{C-D}{2}\right)$$

$$= \lim_{h\to 0} \frac{2\cos\left(\frac{a+(a+h)}{2}\right) \cdot \sin\left(\frac{a-(a+h)}{2}\right)}{h \times (\sin a \cdot \sin(a+h))}$$

$$= \lim_{h\to 0} -\frac{\sin\frac{h}{2}}{\frac{h}{2}} \times \frac{1}{2} \times \frac{2\cos\left(\frac{2a+h}{2}\right)}{\sin a \cdot \sin(a+h)}$$

$$= -\frac{1}{\cancel{2}} \times \frac{\cancel{2}\cos\left(\frac{a+0}{2}\right)}{\sin(a+0)}$$

$$= -\frac{\cos a + 0}{\sin a} = -\cot a \cdot \operatorname{cosec}$$

iii) sec x

$f(x) = \sec x$

$$\Longrightarrow f'(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$$

$$= \lim_{x\to a} \frac{\sec x - \sec a}{x-a}$$

$$= \lim_{x\to a} \frac{\frac{1}{\cos x} - \frac{1}{\cos a}}{x-a}$$

$$= \lim_{x\to a} \frac{\cos a - \cos x}{(x-a)\ \cos a \cdot \cos x}$$

$x = a + h$

as $x \to a$, $h \to 0$

$$\Rightarrow f(h) = \lim_{h \to 0} \frac{\cos a - \cos(a+h)}{\cancel{fx}\, h \times \cos a \cdot \cos(a+h)}$$

formula $\rightarrow$ $-2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

$$= \lim_{h \to 0} \frac{-2\sin\left(\frac{a+a+h}{2}\right) \sin\left(\frac{a-a+h}{2}\right)}{h \times \cos a \cdot \cos(a+h)}$$

$$= \frac{-1}{2} \times \frac{-2\sin\left(\frac{2a+0}{2}\right)}{\cos a \cdot \cos(a+0)}$$

$$= \frac{-1}{\cancel{2}} \times \cancel{-2} \frac{\sin a}{\cos a \times \cos a}$$

$= \tan a \cdot \sec a$

Q2) If $f(x) = 4x + 1$, $x \leq 2$

$= x^2 + 5$ $\quad x > 0$, at $x = 2$, then

find function is differentiable or not

Soln $\rightarrow$

LHD.

$$\cancel{f(2^-)} = \lim_{x \to 2^-} \frac{f(x) - f(2)}{x - 2}$$

$$= \lim_{x \to 2^-} \frac{4x + 1 - (4(2)+1)}{x - 2}$$

$$= \lim_{x \to 2^-} \frac{4x + 1 - 9}{x - 2}$$

$$= \lim_{x \to 2^-} \frac{4(x-2)}{(x-2)} = 4$$

$\boxed{f(2^-) = 4}$

RHD – $f(2^+) = \lim_{x \to 2^+} \frac{x^2 + 5 - 9}{x - 2}$

$$= \lim_{x \to 2^+} \frac{x^2 - 4}{x - 2}$$

$$= \lim_{x \to 2^+} \frac{(x+2)\cancel{(x-2)}}{\cancel{(x-2)}}$$

$= 2 + 2 = 4$

$f(2^+) = 4$ $\quad \therefore \boxed{LHD = RHD}$

f is differentiable at $x = 2$

Q3 If $f(x) = 4x + 7$, $x < 3$

$= x^2 + 3x + 1$, $x \geq 3$ at $x = 3$ then

find f is differentiable or not ?

Soln –

$$\lim_{x \to 3^+} \frac{f(x) - f(3)}{x - 3}$$

$$= \lim_{x \to 3^+} \frac{x^2 + 3x + 1 - (3^2 + 9 + 1)}{x - 3}$$

$$= \lim_{x \to 3^+} \frac{x^2 + 3x - 18}{x - 3}$$

$$= \lim_{x \to 3^+} \frac{x^2 + 6x - 3x - 18}{x - 3}$$

$$= \lim_{x \to 3^+} \frac{(x+6)(x-3)}{(x-3)} = 3 + 6 = 9$$

$\boxed{f(3^+) = 9}$

LHD $\rightarrow f(3^-) = \lim_{x\to 3^-} \frac{f(x) - f(3)}{x-3}$

$= \lim_{x\to 3^-} \frac{4x+7-19}{x-3}$

$= \lim_{x\to 3^-} \frac{4x-12}{x-3}$

$= \lim_{x\to 3^-} \frac{4\cancel{(x-3)}}{\cancel{(x-3)}}$

$\therefore f(3^-) = 4$

$\therefore$ LHD $\neq$ RHD

f is not differentiable at $x = 3$

Q4) If $f(x) = 8x-5$, $x \leq 2$
$= 3x^2 - 4x + 7$ $x > 2$ at $x=2$ then find f is differentiable or not

Soln $\rightarrow$ $f(2) = 8 \times 2 - 5 = 11$

RHD $\lim_{x\to 2^+} \frac{f(x) - f(2)}{x-2}$

$= \lim_{x\to 2^+} \frac{3x^2 - 4x + 7 - 11}{x-2}$

$= \lim_{x\to 2^+} \frac{3x^2 - 6x + 2x - 4}{x-2}$

$= \lim_{x\to 2^+} \frac{(3x+2)(x-2)}{(x-2)}$

$= 3\times 2 + 2 = 8$

$f(2^+) = 8$

LHD $f(2^-) = \lim_{x\to 2^-} \frac{f(x)-f(2)}{x-2}$

$= \lim_{x\to 2^-} \frac{8x-5-11}{x-2}$

$= \lim_{x\to 2^-} \frac{8x-16}{x-2}$

$= \lim_{x\to 2^-} \frac{8(x-2)}{(x-2)}$

$\boxed{f(2^-) = 8}$

LHD = RHD

$\therefore f$ is differentiable at $x=3$.

AK 11/12/19

Practical No-3
Application of Derivative

① Find the interval in which function is increasing or decreasing

i) $f(x) = x^3 - 5x - 11$.

Soln → f is increasing iff

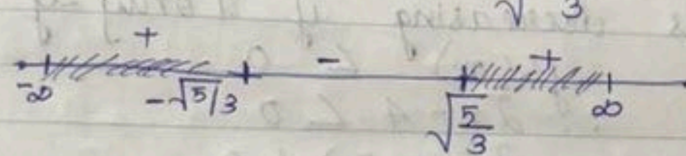
$$f'(x) > 0$$

$$\because f(x) = x^3 - 5x - 11$$

$$\therefore f'(x) = 3x^2 - 5$$

$$3x^2 - 5 > 0$$

$$x = \pm\sqrt{\frac{5}{3}}$$



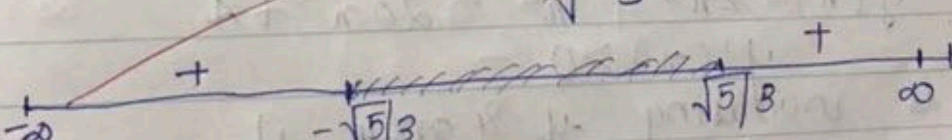
$$\therefore x \in \left(-\infty, -\sqrt{\frac{5}{3}}\right) \cup \left(\sqrt{\frac{5}{3}}, \infty\right)$$

Now f is ~~increasing~~ decreasing iff

$$f'(x) < 0$$

$$\therefore 3x^2 - 5 < 0$$

$$\therefore x = \pm\sqrt{\frac{5}{3}}$$



$$\therefore x \in \left(-\sqrt{\frac{5}{3}}, \sqrt{\frac{5}{3}}\right).$$

b] $f(x) = x^2 - 4x$

f is increasing iff $f'(x) > 0$

$\therefore f(x) = x^2 - 4x$

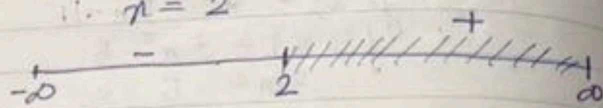
$f'(x) = 2x - 4$

$2x - 4 > 0$

$2(x-2) > 0$

$x - 2 > 0$

$\therefore x = 2$



$\therefore x \in (2, \infty)$

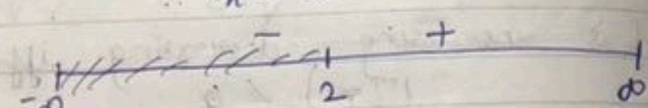
Now f is decreasing if & only if $f'(x) < 0$

$\therefore 2x - 4 < 0$

$\therefore 2(x-2) < 0$

$x - 2 < 0$

$\therefore x = 2$



$\therefore x \in (-\infty, 2)$

c] $f(x) = 2x^3 + x^2 - 20x + 4$

$f'(x) > 0$

f is increasing if & only if $f'(x) > 0$

$\therefore f(x) = 2x^3 + x^2 - 20x + 4$

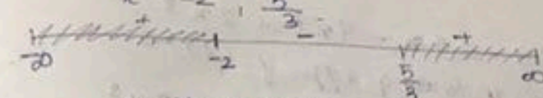
$\therefore f'(x) = 6x^2 + 2x - 20$

$\therefore 6x^2 + 2x - 20 > 0$

$\therefore 6x(x+2) - 10(x+2) > 0$

$\therefore (x+2)(6x-10) > 0$

$x = -2, \frac{5}{3}$



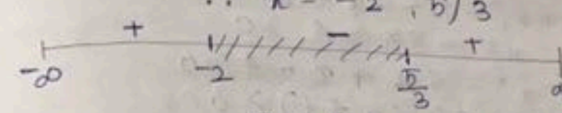
$\therefore x \in (-\infty, -2) \cup (\frac{5}{3}, \infty)$

f is decreasing iff $f'(x) < 0$

$\therefore 6x^2 + 2x - 20 < 0$

$(x+2)(6x-10) < 0$

$\therefore x = -2, 5/3$



$x \in (-2, \frac{5}{3})$

d] $f(x) = x^3 - 27x + 5$

Soln → f is increasing iff $f'(x) > 0$

$\therefore f(x) = x^3 - 27x + 5$

$\therefore f'(x) = 3x^2 - 27$

$\therefore 3x^2 - 27 > 0$

$\therefore 3(x^2 - 9) > 0$

$\therefore x^2 - 9 > 0$

$\therefore x = 3, -3$

$\therefore x \in (-\infty, -3) \cup (3, \infty)$

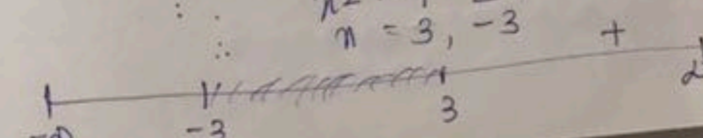
Now f is decreasing iff $f'(x) < 0$

$\therefore 3x^2 - 27 < 0$

$\therefore 3(x^2 - 9) < 0$

$\therefore x^2 - 9 < 0$

$\therefore x = 3, -3$



$x \in ($

e] $f(x) = 69 - 24x - 9x^2 + 2x^3$

<u>Soln</u> - f is increasing iff $f'(x) > 0$

$\therefore f(x) = 69 - 24x - 9x^2 + 2x^3$

$\therefore f'(x) = -24 - 18x + 6x^2$

$\therefore -(-6x^2 + 18x + 24) > 0$

$\therefore$ ~~$-((x+4)(x-1))$~~

$\therefore +6(-4 - 3x + x^2) > 0$

$\therefore x^2 - 3x - 4 > 0$

$\therefore x^2 - 4x + x - 4 > 0$

$\therefore (x-4)(x+1) > 0$

$\therefore x = 4, -1$

$-\infty \quad + \quad -1 \quad - \quad 4 \quad + \quad \infty$

$x \in (-\infty, -1) \cup (4, \infty)$

Now f is decreasing iff $f'(x) < 0$

$\therefore -24 - 18x + 6x^2 < 0$

$\therefore 6(-4 - 3x + x^2) < 0$

$\therefore (x-4)(x+1) < 0$

$\therefore x = 4, -1$.

$-\infty \quad + \quad -1 \quad 4 \quad + \quad \infty$

$x \in (-1, 4)$

Q2] Find the intervals in which function is concave upwards & concave downwards

a] $y = 3x^2 - 2x^3$

<u>Soln</u> $\rightarrow$ $y = f(x)$

$\therefore f(x) = 3x^2 - 2x^3$

$\therefore f'(x) = 6x - 6x^2$

$\therefore f''(x) = 6 - 12x$

$\therefore f$ is concave upward iff $f''(x) > 0$

$\therefore 6 - 12x > 0$

$\therefore 6(1 - 12x) > 0$

$\therefore (1 - 12x) > 0$

$\therefore -(2x - 1) > 0$

$-\infty \quad + \quad 1/2 \quad - \quad \infty$

$x \in (-\infty, \frac{1}{2})$

$\therefore f$ is concave downwards iff $f''(x) < 0$

$\therefore 6(1 - 12x) < 0$

$\therefore -(2x - 1) < 0$

$-\infty \quad - \quad 1/2 \quad \infty$

$x \in (\frac{1}{2}, \infty)$

b] $y = x^4 - 6x^3 + 12x^2 + 5x + 7$

<u>Soln</u> $\rightarrow$ $\therefore y = f(x)$

$\therefore f(x) = x^4 - 6x^3 + 12x^2 + 5x + 7$

$\therefore f'(x) = 4x^3 - 18x^2 + 24x + 5$

$\therefore f''(x) = 12x^2 - 36x + 24$

$\therefore f$ is concave upward iff $f''(x) > 0$

$\therefore 12x^2 - 36x + 24 > 8$

$\therefore 12(x^2 - 3x + 2) > 0$

$\therefore x^2 - 3x + 2 > 0$

$\therefore (x^2 - 2)(x - 1) > 0$

$x = 2, 1$

$-\infty$ $+$ 1 $-$ 2 $+$ ∞

$x \in (-\infty, 1) \cup (2, \infty)$

$\therefore f$ is concave downward iff $f''(x) < 0$

$\therefore 12x^2 - 36x + 24 < 0$

$\therefore 12(x^2 - 3x + 2) < 0$

$\therefore x^2 - 3x + 2 < 0$

$\therefore (x - 2)(x - 1) < 0$

$x = 2, 1$

$-\infty$ $+$ 1 $-$ 2 $+$ $+\infty$

$x \in (1, 2)$

c] $y = x^3 - 27x + 5$

<u>Soln</u>

$\therefore y = f(x)$

$\therefore f(x) = x^3 - 27x + 5$

$\therefore f'(x) = 3x^2 - 27$

$f''(x) = 6x$

$\therefore f$ is concave upward if & only if $f''(x) > 0$

$\therefore 6x > 0$

$\therefore x > 0$

$\therefore x = 0$

$-\infty$ $-$ 0 $+$ ∞

$x \in (0, \infty)$

$\therefore f$ is concave downward iff $f''(x) < 0$

$6x < 0$

$x < 0$

$x = 0$

$-\infty$ $-$ 0 $+$ ∞

$\therefore x \in (-\infty, 0)$

d] $y = 69 - 24x - 9x^2 + 2x$

<u>Soln</u> $\rightarrow$ $\therefore y = f(x)$

$\therefore f(x) = 69 - 24x - 9x^2 + 2x^3$

$\therefore f'(x) = -24 - 18x + 6x^2$

$\therefore f''(x) = -18 + 12x$

$\therefore f$ is concave upward iff $f''(x) > 0$

$\therefore -18 + 12x > 0$

$\therefore 6(2x - 3) > 0$

$\therefore 2x - 3 > 0$

$x = 3/2$

$-\infty$ $-$ $3/2$ $+$

$\therefore x \in (+\frac{3}{2}, \infty)$

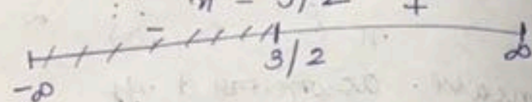
$\therefore f$ is concave downwards iff $f''(x) < 0$

$\therefore -18 + 12x < 0$

$\therefore 6(2x-3) < 0$

$\therefore 2x - 3 < 0$

$\therefore x = 3/2$



$\therefore x \in \left(-\infty, \frac{3}{2}\right)$

e] $y = 2x^3 + x^2 - 20x + 4$.

Soln − $y = f(x)$

$\therefore y(x) = 2x^3 + x^2 - 20x + 4$

$f'(x) = 6x^2 + 2x - 20$

$f''(x) = 12x + 2$

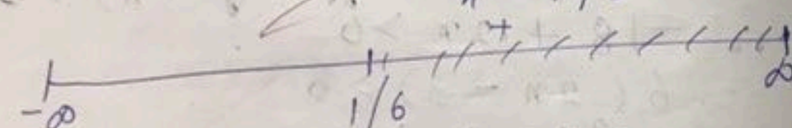
$\therefore f$ is concave upwards iff

$f''(x) > 0$

$\therefore 12x + 2 > 0$

$\therefore 6x + 1 > 0$

$\therefore x = 1/6$



$x \in (-1/6, \infty)$

41

$\therefore f$ is concave downward iff

$f''(x) < 0$

$\therefore 12x + 2 < 0$

$\therefore 2(6x+1) < 0$

$\therefore 6x + 1 < 0$

$\therefore x = -1/6$

$\therefore x \in \left(-\infty, -\frac{1}{6}\right)$

18

Practical-4

Applications of derivatives & newton's method

Q1 Find max & min value of following

i) $f(x) = x^2 + \frac{16}{x^2}$

ii) $f(x) = 3.5x^3 + 3x^5$

iii) $f(x) = x^3 - 3x^2 - 1 \quad [-1/2, 4]$

iv) $f(x) = 2x^3 - 3x^2 - 12x + 1 \quad [-2, 3]$

Q2 Find the root of the following eqn by newton

i) $f(x) = x^3 - 3x^2 - 55x + 95 \quad (x_0 = 0)$

ii) $f(x) = x^3 - 4x - 9 \quad$ in $[2, 3]$

iii) $f(x) = x^3 - 18x^2 - 10x + 17$ in $[1, 2]$

Soln

Q1) i) $f(x) = x^2 + \frac{16}{x^2}$

$f'(x) = 2x - 32/x^3$

Now consider $f'(x) = 0$

$2x - 32/x^3 = 0$

$2x = 32/x^3$

$2x^4 = 32/2$

$x^4 = 16$

$x = \pm 2$

$f''(x) = 2 + 96/x^4$

$f''(x) = 2 + 96/24$

$= 2 + 96/16$

$= 2 + 6$

$= 8 > 0$

$\therefore f$ has max value at $x = 2$

$f(2) = 2^2 + 16/2^2$

$= 4 + 16/4$

$= 8$

$\therefore f''(-2) = 2 + 96/(-2)^4$

$= 2 + 6$

$= 8 > 0$

$\therefore f$ has max value at 2

Function reaches minimum value at $x = 2$ & $x = -2$

ii) $f(x) = 3.5x^3 + 3x^5$

$f'(x) = 15x^2 + 15x^4$

consider $f'(x) = 0$

$15x^2 + 15x^4 = 0$

$15x^4 = 15x^2$

$x^2 = 1$

$x = \pm 1$

$f''(x) = -30x + 60x^3$

$f(1) = -30 + 60$

$= 30 > 0$

$f(1) = 3.5(1)^3 + 3(1)^5 \quad f$ has maximum

$= 6.5 = 1$

42

$= 5$

f has max value 5 at $x = -1$ & has min value 1 at $x = 1$.

iii) $f(x) = x^3 - 3x^2 + 1$

$f'(x) = 3x^2 - 6x$

Consider $f'(x) = 0$

$3x^2 - 6x = 0$

$3x(x-2) = 0$

$3x = 0$ or $x - 2 = 0$

$x = 0$ or $x = 2$

$f''(x) = 6x - 6$

$f''(0) = 6(0) - 6$

$= 6 < 0$ ∴ f has max value at $x = 0$

$f''(2) = 6(2) - 6$

$= 12 - 6 = 6 > 0$

∴ f has mini value at $x = 2$

$f(x) = (x)^3 - 3(2)^2 + 1$

$= 8 - 3(4) + 1 = -3$

∴ f has max value 1 at $x = 0$ & f has min ~~value~~ -3 at $x = 2$

iv) $f(x) = 2x^3 - 3x^2 - 12x + 1$

$f'(x) = 6x^2 - 6x - 12$

consider $f'(x) = 0$

∴ $6(x^2 - x - 2) = 0$

$x^2 - x - 2 = 0$

$x(x+1) - 2(x+1) = 0$

$x = 2$ or $x = -1$

$f''(x) = 12x - 6$

$f''(2) = 12(2) - 6 = 24 - 6$

$= \cancel{18 > 0}$ 18

∴ f has mini value at $x = +2$

$f(2) = 2(2)^3 - 3(2)^2 - 12(2) + 1$

$= 16 - 12 - 24 + 1$

$= -19$

$f''(-1) = 12(-1) - 6$

$= -12 - 6$

$= -18 < 0$

∴ f has max value at $x = -1$

$f(-1) = 2(-1)^3 - 3(-1)^2 - 12(1) + 1$

$= -2 - 3 + 12 + 1$

$= 8$

f has max value at $x = -1$ & mini value at $x = 2$

Q2

1) $f(x) = x^3 - 3x^2 - 55x + 95$, $x_0 = 0$

$f'(x) = 3x^2 - 6x - 55$

By Newton's method

$x_{n+1} = x_n - f(x_n) / f'(x_n)$

$x_1 = x_0 - f(x_0) / f'(x_0)$

$x_1 = 0 + 9.5 / 55$

$x_1 = 0.1727$

$f(x_1) = (0.1727)^3 - 3(0.1727)^2 - 55(0.1727) + 9.5$

$= 0.0051 - 0.0895 - 9.4985 + 9.5$

$= -0.0829$

$f''(x_1) = 3(0.1727)^2 - 6(0.1727) - 55$

$= -55.9467$

$x_2 = x_1 - f(x_1) / f'(x_1)$

$f(x_2) = (0.1712)^3 - 3(0.1712)^2 - 55(0.1712)$

$= 0.0011$

$f''(x_2) = 3(0.1712)^2 - 6(0.1712) - 55$

$= -55.9393$

$x_3 = x_2 \cdot f(x_2) / f'(x_2)$

$= 0.1712$

The root of eqn is 0.1712.

ii) $f(x) = x^3 - 4x - 9$

$f'(x) = 3x^2 - 4$

$f(2) = 2^3 - 4(2) - 9$

$= 8 - 0 - 9 = -9$

$f(3) = 3^3 - 4(3) - 9$

$= 27 - 12 - 9 = 6$

Let $x_0 = 3$ be initial approximation ~~tay~~

$\therefore$ By Newton's Method

$x_{n+1} = x_n - f(x_n) / f'(x_n)$

$x_1 = x_0 f(x_0) / f'(x_0)$

$= 3 - 6/23$

$= 2.7392$

$f(x_1) = (2.7392)^3 - 4(2.7392) - 9$

$= 20.5528 - 10.9568 - 9$

$= 0.596$

$f'(x_1) = 3(2.7392)^2 - 4$

$= 22.5096 - 4$

$= 18.5096$

$x_2 = x_1 - f(x_1) / f'(x_1)$

$= 2.7392 - 0.596 / 18.5096$

$= 2.7071$

$f(x_2) = (2.7071)^3 - 4(2.7071)$

$= 0.01072$

$f'(x_2) = 3(2.7071)^2 - 4$

$= 17.9851$

$2.7071 = \frac{0.0102}{17.9851} = 2.7015$

$f(x_3) = (2.7015)^3 - 4(2.7015) - 9$
$= (9.7)58 - 10.806 - 9 = -0.901$

$f(3) = 3(2.7015)^3 - 4 = 17.8943$

$x_4 = 2.7015 + 0.0901 \,/\, 17.8943$
$= 2.7065.$

3) $f(x) = x^3 - 18x^2 - 10x + 17 \; [1, 2]$

$f'(x) = 3x^2 - 3.6x - 10$

$f(1) = (1)^3 - 1.8(2)^2 - 10(1) + 17$
$= -1.8 - 10 + 17$
$= 6.2$

$f(2) = (2)^3 - 1.8(2)^2 - 10(2) + 17$
$= 8 - 7.2 - 20 + 17 = 2.2$

Let $x_0 = 2$ be initial approximation by Newton's Method

$x_{n+1} = x_n - f(x_n) \,/\, f'(x_n)$

$x_1 = x_0 - f(x_2) \,/\, f'(x_2)$
$= 2 - 2.2 \,/\, 5.2$
$= 2 - 0.4230 = 1.597.$

$f(x_1) = (1.5777)^3 - 1.8(1.5777)^2 - 10(1.5777) + 17.$
$= 3.9219 - 44.764 - 15.77 + 17.$
$= 0.6755.$

$f'(x) = 3(1.577)^2 - 3.6(1.577) - 10$
$= 2.4608 - 5.6772 - 10$
$= 8.2167.$

$x_2 = x_1 - f(x_1) \,/\, f'(x_1)$
$= 1.577 + 0.6755 \,/\, 8.2164$
$= 1.6592.$

$f(x_2) = (1.6592)^3 - 1.8(1.6592)^2 - 10(1.6592) + 17$
$= 0.0204$

$f'(x_2) = 3(1.6592)^2 - 3.6(1.6592) - 10$
$= -7.7143$

$x_3 = x_2 - f(x_2) \,/\, f'(x_2)$
$= 1.6592 \,/\, 0.0204 \,/\, 7.7143$
$= 1.6592 + 0.0026$
$= 1.6618.$

$f(x_3) = (1.6618)^3 - 1.8(1.6619)^2 - 10(1.6618) + 17$
$= 0.0004.$

$f'(x_2) = 3(1.6618)^2 - 3.6(1.6618) - 10$
$= -7.677.$

$x_4 = x_3 - f(x_3) \,/\, f'(x_3)$
$= 1.8618 + \frac{0.0004}{7.7697}$
$= 1.6618.$

Practical-5

Aim - Integration.

1) Solve the following

1) $\frac{dx}{\sqrt{x^2+2x-3}}$

2) $\int (4e^{3x+1})dx$

3) $\int (2x^2 - 3\sin x + 5\sqrt{x})\, dx$

4) $\int \frac{x^3+3x+4}{\sqrt{x}}\, dx$.

5) $\int t^7 \sin(2t^5)\, dx$

6) $\int \sqrt{x}(x^2-1)dx$

7) $\int 1/x^3 \sin(1/x^2)dx$

8) $\int \frac{\cos x}{3\sqrt{\sin^2 x}}\, dx$

9) $\int e^{\cos^2 x} \sin 2x\, dx$

10) $\int \left(\frac{x^2-2x}{8x^3-3x^2+1}\right) dx$

1) $\int \frac{1}{x^2+2x-3}\, dx$.

$= \int \frac{1}{\sqrt{x^2+2x-3}}\, dx = \int \frac{1}{\sqrt{x^2+2x+1-4}}$

$a^2+2ab+b^2 = (a+b)^2$

$= \int \frac{1}{\sqrt{(x+1)^2-4}}\, dx$

Put $x+1 = t$ where $t=1$, $t = x+1$

$dx = 1|t \times dt$

$I = \frac{1}{\sqrt{t^2-4}}\, dt$.

using

\# $\int \frac{1}{\sqrt{x^2-a^2}}\, dx = \ln(|x+\sqrt{x^2+a^2}|)$

$= \ln(|t+\sqrt{t^2-4}|)$

$= \ln(|x+1\ \sqrt{(x+1)^2-4}|)$

$= \ln(|x+1 + \sqrt{x^2+2x-3}|)$

$= \ln(|x+1+\sqrt{x^2+2x-3}|) + c$

2) $\int (4e^{3x+1})\, dx$.

$= \int 4e^{3x}\, dx + \int 1\, dx$.

$= 4\int e^{3x}\, dx + \int 1\, dx$ # $\int e^{x}dx = 1/x^{*} e^{dx}$

$= \frac{4e^{3x}}{3} + x$.

$= \frac{4e^{3x}}{3} + x + c$

3) $\int 2x^2 - 3\sin(x) + 5\sqrt{x}\, dx$

$\int 2x^2 - 3\sin(x) + 5x^{1/2}\, dx$

$\sqrt[n]{a^m} = a^{m/n}$

$\int 2x^2 dx - \int 3\sin(x)\, dx + \int 5x^{1/2} dx$

$= \frac{2x^3}{3} + 3\cos x + \frac{\cos\sqrt{x}}{3} + c$

$= \frac{2x^3 + \cos\sqrt{x}}{3} + 3\cos x + c$

4) $\int \frac{x^3+3x+4}{\sqrt{3}}\, dx = \int \frac{x^3+3x+4}{x^{1/2}}\, dx$

split the denominator.

$= \int \frac{x^3}{x^{1/2}} + \frac{3x}{x^{1/2}} + 4/x^{1/2}\, dx.$

$= \int x^{5/2} + 3x^{1/2} + 4/x^{1/2}\, dx.$

$= \int x^{5/2} dx + \int 3x^{1/2} dx + \int 4/x^{1/2} dx$

$= \frac{x^{5/2+1}}{5/2+1} = \frac{2x^3\sqrt{x}}{7} + 2x\sqrt{x} + 8\sqrt{x} + c.$

~~$t = \int t^4 \sin(2t^5)\, dt.$~~

~~$\int t^4 \sin$~~

47

5) $\int t^7 x \sin(2t^4)\, dt.$

put $u = 2t^4$ $\quad du = 2 \times t^3$

$= \int t^7 x\sin(2t^4) \times \frac{1}{4t^3}\, dx.$

$= \int t^4 \sin(2t^4) \times 1/2 \times 2u\, dx.$

$= \int t^4 \sin(u^4) \times \frac{1}{3}\, dx = \frac{t^4 \times \sin(2t^4)\, dx}{8}.$

Substitute t^4 with $u/2$.

$= \int \frac{u/2 \times \sin(u)}{8}\, dx.$

$= \int \frac{4 \times \sin(u)}{2} / 8\, du$

$= \int \frac{4 \times \sin(u)}{16}\, dx.$

$= 1/16 \int 4x \sin(u)\, dx.$

$\int u\, dv = uv - \int v\, du$

where $u = 4$

$dv = \sin(u) \times dx$

$du = 1\, du$ $\quad v = -\cos(u).$

$= \frac{1}{6}(4 \times (-\cos(u))) - \int -\cos(u)\, dx.$

$= \int \cos x\, dx = \sin(x)$

$= 1/16 \times (4x(-\cos(u)) + \sin(u))$

$= -\frac{t^4 \times \cos(2t^4)}{8} + \frac{\sin(2t^4)}{16} + c$

6) $\int \sqrt{x}\,(x^2-1)\,dx$

$= \int \sqrt{x}\; x^2 - \sqrt{x}\,dx$

$= \int x^{1/2} \times x^2 - x^{1/2}\,dx$

$= \int x^{5/2} - x^{1/2}\,dx$

$= \int x^{5/2}\,dx - \int x^{1/2}\,dx$.

$I_1 = \frac{x^{5/2+1}}{5/2+1} = \frac{x^{7/2}}{7/2} = \frac{2x^{7/2}}{7} = \frac{2\sqrt{x^7}}{7} = \frac{2x^3\sqrt{x}}{7}$

$I_1 = \frac{x^{1/2}+1}{1/2+1} = \frac{x^{3/2}}{3/2} = \frac{2x^{3/2}}{3/2}$.

$= \frac{2x^3\sqrt{x}}{7} + \frac{2\sqrt{x^3}}{3} + c$

8) $\int \frac{\cos x}{\sqrt[3]{\sin(x)^2}}\,dx = \int \frac{\cos x}{\sin x^{2/3}}\,dx$.

Put $t = \sin(x)$

$\cancel{t = \cos x}$.

$= \int \frac{\cos(x)}{\sin(x)^{3/2}} \times \frac{1}{\cos x}\,dt$.

$= \frac{1}{\sin x^{3/2}} \frac{\cos x}{dt} = \frac{1}{t^{2/3}}\,dt$

$I = \int \frac{1}{t^{2/3}}\,dt = \frac{-1}{(2/3-1)\,t^{2/3-1}} = \frac{-1}{1/3\;\; 2/3^{-1}}$

$= \frac{1}{\sqrt[3]{t^{-1}}} = 3t^{1/3}$

Returns ubsitution $t = \sin(x)$

$= 3\sqrt[3]{\sin(x)} + c$

7) $\int \frac{1}{x^3} \sin\left(\frac{1}{x^2}\right) dx = I$

$\frac{1}{x^2} = t$

$\therefore \frac{-2}{x^3} = \frac{dt}{dx}$.

$\therefore \frac{dx}{x^3} = \frac{-1}{2}\,dt$

$I = \frac{-1}{2} \int \sin t\,dt$

$= \frac{-1}{2} \times -\cos t + c$

$= \frac{\cos t}{2} + c$

$I = \frac{\cos(x^2)}{2} + c$

9) $\int e^{\cos^2 x} \sin 2x\,dx$

$\cancel{I = \int e}$ Put $\cos^2 x = t$

$\therefore -2\cos^2 x = t$

$\therefore \sin 2x\,dx = -dt$

$I = -\int e^t\,dt$

$I = \cancel{-e^t} + c$

$I = -e^{\cos^2 x} + c$

(10) $\int \frac{x^2-2x}{x^3-3x^2+1}\,dx$

Put $x^3 - 3x^2 + 1 = dt$

$I = \int \frac{x^2-2x}{x^3-3x^2+1} \times \frac{1}{3x^2-3x2x}\,dt$

$= \int \frac{x^2-2x}{x^3-3x^2+1} \times \frac{1}{3x^2-6x}\,dt$

$= \int \frac{x^2-2x}{x^3-3x^2+1} \times \frac{1}{3(x^2-2x)}\,dt$

$= \int \frac{1}{x^3-3x^2+1} \times 1/3\,dt$

$= \int \frac{1}{3(x^3-3x^2+1)}\,dt = \int 1/3\,dt$

$= 1/3 \int 1/t\,dt \quad \int 1/x\,dx = \ln(x)$

$= 1/3 \times \ln|t| + c$

$= 1/3 \times \ln(|x^3-3x^2+1|) + c$

Practical 6

Application of integration & numerical Integration

1) Find the length of curve.
$x = t - \sin t,\ y = 1-\cos t,\ t = [0, 2\pi]$

Sol - $l = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$

$= \int_0^{2\pi} \sqrt{(1-\cos t)^2 + (\sin t)^2}\,dt$

$= \int_0^{2\pi} \sqrt{1-2\cos t + \cos^2 t + \sin^2 t}\,dt$

$= \int_0^{2\pi} \sqrt{1-2\cos t+1}\,dt$

$= \int_0^{2\pi} \sqrt{2 \cdot 2\cos t}\,dt$

$= \int_0^{2\pi} 2\left|\sin\frac{t}{2}\right| dt \quad \ldots \sin^2\frac{t}{2} = \frac{1-\cos t}{2}$

$= \int_0^{2\pi} 2\sin\frac{t}{2}\,dt$

$= \left(-4\cos\left(\frac{t}{2}\right)\right)_0^{2\pi} = (-4\cos\pi) - (-4\cos 0)$

$= 4 + 4$

$= 8$

(2) $y = \sqrt{4-x^2}$ $\quad x \in [-2, 2]$

Soln $\frac{dy}{dx} = \frac{-2x}{2\sqrt{4-x^2}} = \frac{-x}{\sqrt{4-x^2}}$

$$l = 2\int_0^2 \sqrt{1+\left(\frac{-x}{\sqrt{4-x^2}}\right)^2}\, dx$$

$$= 2\int_0^2 \sqrt{\frac{1+x^2}{4-x^2}}\, dx$$

$$= 4\int_0^2 \frac{1}{4-x^2}\, dx$$

$$= 4\int_0^2 \left(\sin^{-1}\left(\frac{x}{2}\right)\right)_0^2$$

$$= 2\pi$$

(3) $y = x^{3/2}$ in $[0, 4]$

Soln : $y'(x) = \frac{3}{2} x^{1/2}$

$$[f'(x)]^2 = \frac{9}{4}x$$

$$l = \int_a^b \sqrt{1+[a'(x)]^2}\, dx$$

$$= \int_0^x \sqrt{1+\frac{9}{4}x}\, dx$$

put $y = 1 + \frac{9}{4}x$, $dy = \frac{9}{4} dx$

$$l = \int_4^{1+\frac{9}{4}x} \frac{4}{9}\sqrt{4}\, du = \left[\frac{4}{9}\cdot\frac{2}{3}(4^{3/2})\right]_1^{+\frac{9}{4}x}$$

50

$$= \frac{8}{27}\left[\left(1 + \frac{9x}{4}\right) - 1\right]$$

$x = 3\sin t$, $y = 3\cos t$ $\quad t \in (0, 2\pi)$

$\frac{dx}{dt} = 3\cos t$ $\quad \frac{dy}{dt} = -3\sin t$

$$L = \int_0^{2\pi} \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{(3\cos t)^2 + (-3\sin t)^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{9\sin^2 t + 9\cos^2 t + dt}$$

$$= \int_0^{2\pi} \sqrt{9\sin^2 t} + 9\cos^2 t\, dt$$

$$= \int_0^{2\pi} 3\sqrt{x}\, dt$$

$$= 3\int_0^{2\pi} dt$$

$$= 3[x]_0^{2\pi}$$

$$= 3(2\pi - 0)$$

$L = 6\pi$ units

(5) $x = \frac{1}{6}y^3 + \frac{1}{2y}$ on $y \in (1,2)$

$\therefore \frac{dx}{dy} = \frac{y^2}{2} - \frac{1}{2y^2}$

$\frac{dx}{dy} = \frac{y^4-1}{2y^2}$

$L = \int_1^2 \sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy$

$= \int_1^2 \sqrt{1+\frac{(y^4-1)^2}{4y^4}}\,dy$

$= \int_1^2 \sqrt{\frac{(y^4+1)^2}{(2y^2)^2}}\,dy$

$= \int_1^2 \frac{y^4+1}{2y^2}\,dy$

$= \frac{1}{2}\int_1^2 y^2\,dy + \frac{1}{2}\int^2 y^{-2}\,dy$

$= \frac{1}{2}\left[\frac{y^3}{3} - \frac{y^{-2}}{1}\right]_1^2$

$= \frac{1}{2}\left[\frac{8}{3} - \frac{1}{2} - \frac{1}{3} + 1\right]$

$= \frac{1}{2}\left[\frac{7}{3} - \frac{1}{2}\right]$

$= \frac{1}{2}\left[\frac{17}{6}\right]$

$L = \frac{17}{12}$ unite

Q2 Using Simpson's Rule Solve the following

(1) $\int_0^2 e^{x^2}dx$ with $n=4$

$[0,0.5],[0.5,1],[1,1.5],[1.5,2]$.

By simpson's Rule.

$\int_0^2 e^{x}dx = \frac{1/2}{3}(y_0 + 4y_1 + 2y_2 + 4y_3 + y_4)$

$= \frac{1/2}{3}\left(e^{0^2} + 4e^{(0.5)^2} + 2e^{(1)^2} + 4e^{(1.5)^2} + e^{(2)^2}\right)$

$= 17.35362645$.

(2) $\int_0^4 x^2\,dx$ $n=4$

$L = \frac{4-0}{4} = 1$

x	0	1	2	3	4
y	0	1	4	9	16

$\int_0^1 x^2\,dx = \frac{L}{3}\left((y_0+y_4) + 4(y_1+y_3) + 2y_2\right)$

$= \frac{1}{3}[0+16+4(1+9)+2\times 4]$

$= \frac{1}{3}[16+4(10)+8]$

$= \frac{64}{3}$

18

$$\int_0^{7} x^2 dx = 21.333$$

(3) $\int_0^{\pi/3} \sqrt{\sin x}\, dx$ with $n = 6$

$$h = \frac{\pi/3 - 0}{6} = \frac{\pi}{18}$$

x	0	$\pi/18$	$2\pi/18$	$3\pi/18$	$4\pi/18$	$5\pi/18$	$6\pi/18$
y	0	0.4167	0.5848	0.7071	0.8017	0.8152	0.9306

$$\int_0^{\pi/3} \sqrt{\sin x}\, dx = \frac{h}{3}\left[(y_0 + y_6) + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4 + y_6)\right]$$

$$= \frac{\pi/18}{3}\left[1.3473 + 4(1.999) + 2(1.3865)\right].$$

$$= \frac{\pi}{54}\left[1.3473 + 7.996 + 2.773\right]$$

$$= \frac{\pi}{54} \times 12.1163$$

$$\int_0^{\pi/3} \sqrt{\sin}\, dx = 0.7049.$$

AV 08/01/2020

52

Practical-7

Differential Equation

1] $x\frac{dy}{dx} + y = e^x$

$$\frac{dy}{dx} + \frac{1}{x}y = \frac{e^x}{x}$$

$P(x) = \frac{1}{x}$ $\qquad Q(x) = \frac{e^x}{x}$

$$I.F = e^{\int 1/(x) dx}$$

$$= e^{\int 1/x\, dx}$$

$$= e^{\ln x}$$

$$I.F = x$$

$$y(I.F) = \int Q(x)(I.F)dx + c$$

$$= \int \frac{e^x}{x} \cdot x \cdot dx + c$$

$$= \int e^x dx + c$$

$$= xy = e^x + c$$

Q2] $e^{2x}\frac{dy}{dx} + 2e^{x}y = 1$

$\frac{dy}{dx} + 2\frac{e^x}{e^x}y = \frac{1}{e^x}$ ($\div$ by e^x)

$\frac{dy}{dx} + 2y = \frac{1}{e^x}$

$\frac{dy}{dx} + 2y = e^{-x}$

$P(x) = 2 \qquad Q(x) = e^{-x}$

$\int P(x) = dx$

$IF = e^{\int 2 dx}$

$= e^{2x}$

$y = (I \cdot F) \cdot \int Q(x)(I \cdot F) dx + c$

$= e^{2x} \int e^{-x} + 2x dx + c$

$= \int e^{x} dx + c$

$y = e^{2x} = e^{x} + c$

iii) $x\frac{dy}{dx} = \frac{\cos x}{x} = 2y$

$x\frac{dy}{dx} = \frac{\cos x}{x} - 2y$

$\therefore \frac{dy}{dx} \cancel{\#} + \frac{2y}{x} = \frac{\cos x}{x^3}$

$P(x) = 2(x) \qquad Q(x) = \frac{\cos x}{x^3}$

$I \cdot F = e^{\int P(x) dx}$

$= e^{\int 2/x \, dx}$

$= e^{2/x}$

$= \ln x^2$

$y(I \cdot F) = \int Q(x)(I \cdot F) dx + c$

$\int \frac{\cos x}{x^3} - x^2 dx + c = \int \cos x + c$

$x^2 y = \sin x + c$

iv) $x\frac{dy}{dx} + 3y \cdot \frac{\sin x}{x^2}$

$= x\frac{dy}{dx} + 3y \cdot \frac{\sin x}{x^3}$ ($\div$ by x on b.s)

$P(x) = 3/x \qquad Q(x) = \sin x / x^3$

$= e^{\int P(x) dx} = e^{\int 3/x \, dx}$

$= e^{3/x}$

$= e^{\ln x^3}$

$I \cdot F = x^3$

$y(I \cdot F) = \int Q(x)\,(I \cdot F)\,dx + c$

$= \int \frac{\sin x}{x^3} \cdot x^3\,dx + c$

$= \int \sin x\,dx + c$

$= \cancel{x^3 y} = -\cos x + c$

v) $e^{2x}\frac{dy}{dx} + 2e^{2x}y = 2x$

$\frac{dy}{dx} + 2y = \frac{2x}{e^{2x}}$

$P(x) = 2 \quad Q(x) = 2x/e^{2x} \cdot 2xe^{-2x}$.

$I \cdot F = e^{\int P(x)\,dx}$

$= e^{\int 2\,dx} = e^{2x}$

$y(I \cdot F) = \int Q(x)\,(I \cdot F)\,dx + c$

$= \int 2x\,e^{2x}\,dx + c$

$= \int 2x\,dx + c$

$ye^{2x} = x^2 + c$

vi) $\sec^2 x \tan y\,dx + \sec^2 y \tan x\,dy = 0$

$\sec^2 x \cdot \tan y\,dx - \sec^2 y \cdot \tan x\,dy$

$\frac{\sec^2 x}{\tan x}\,dx = \frac{-\sec^2 y\,dx}{\tan x}$

$\int \frac{\sec^2 x\,dx}{\tan x} = -\int \frac{\sec^2 y\,dy}{\tan y}$

$\log|\tan x| = -\log|\tan y| + c$

$\log|\tan x \cdot \tan y| = e$

$\tan x \cdot \tan y = e^c$

vii) $\frac{dy}{dx} = \sin^2(x-y+1)$

Put $x - y + 1 = v$

Differentiating on B.S

$x = y + 1 \cdot v$

$1 - \frac{dy}{dx} = \frac{dy}{dx}$

$\frac{1 - dv}{dx} = \frac{dy}{dx}$

$1 - \frac{dv}{dx} = \sin^2 x$

$\frac{dv}{dx} = 1 - \sin^2 v$

$\frac{dv}{dx} = \cos^2 v$

$\frac{dv}{\cos^2 v} = dx$

$\int \sec^2 v\,dv = \int dx$

$\tan v = x + c$

$\tan(x+y-1) = x + c$

$$\frac{dy}{dx} = \frac{2x+3y-1}{6x+9y+6}$$

Put $2x+3y = v$

$$2 + 3\frac{dy}{dx} = \frac{dv}{dx}$$

$$\frac{dy}{dx} = \frac{1}{3}\left(\frac{dv}{dx} - 2\right)$$

$$\frac{1}{3}\left(\frac{dv}{dx} - 2\right) = \frac{1}{3}\left(\frac{(v-1)}{(v+2)}\right)$$

$$\frac{dv}{dx} = \frac{v-1}{v+2} + 2$$

$$\frac{dv}{dx} = \frac{v-1+2v+4}{v+2}$$

$$= 3v+3/v+2$$

$$= 3(v+1)/v+2$$

$$\int \frac{(v+2)}{(v+1)}\, dv = 3\, dx$$

$$\int \frac{v+1}{v}\, dx + \int \frac{1}{v+1}\, dv = 3x$$

$$= v + \log|x| - 3x + c$$

$$= 2x + 3y + \log|2x+3y+1| \cdot 3x + c$$

$$= 3y = x - \log|2x+3y+1| + c$$

PRACTICAL-8

Euler's Method

Q1] $\frac{dy}{dx} = y + e^x - 2$ $y(0)-2$ $h = 0.5$

find $y(2)$ - ?

soln- $f(x) = y + e^x - 2$ $x_0 = 2\ y(0)$ $h = 0.5$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	2	1	2.5
1	0.5	2.5	2.1487	3.5743
2	1	3.5743	4.2926	5.7205
3	1.5	5.7205	8.2021	9.8215
4	2	9.8215		

$\therefore y(2) = 9.8215$

Q2] $\frac{dy}{dx} = 1 + y2$, $y(0) = 1$ $h = 0.2$ fin d $y(1)$

$x_0 = 0$, $y_0 = 0$, $h = 0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	0	1	0.2
1	0.2	0.2	1.04	0.408
2	0.4	0.408	1.1664	0.6412
3	0.6	0.6412	1.411	0.9234
4	0.8	0.9234	1.8526	1.2939
5	1	1.2939		

$Y(1) = 1.2939$

Q3) $\frac{dy}{dx} = \sqrt{\frac{x}{y}}$ $y(0)=1$ $h=0.2$ find $y(1)$-?

$x_0 = 0$ $y(0)=1$ $h=0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	1	0	1
1	0.2	1	0.4472	1.0894
2	0.4	1.0894	0.6059	1.2105
3	0.6	1.2105	0.7040	1.3513
4	0.8	1.3513	0.7969	1.5051
5	1	1.5051		

$\therefore \boxed{y(1) = 1.5051}$

Q4 $\frac{dy}{dx} = 3x^2+1$ $y(1)=2$ find $y(2)$ $h=0.5$

$h=0.25$

$y_0 = 2$ $x_0 = 1$ $h=0.5$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	7.754	4
1	1.5	4	7.75	7.875
2	2	7.875		

$y(2) = 7.875$

(2) $y_0 = 2$ $x_0 = 1$ $h=0.25$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	3
1	1.25	3	5.6875	4.4218
2	1.5	4.4218	59.6569	19.3360
3	1.75	19.3360	1122.6426	299.9960
4	2	299.9960		

$y(2) = 299.9960$

Q5] $\frac{dy}{dx} = \sqrt{xy} + 2$ $y(1)=1$ $h=0.2$

$x_0 = 1$ $y_0 = 1$ $h=0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	1	3	3.6
1	1.2	3.6		

$y(1.2) = 3.6$

Practical - 9

Topic - Limits & Partial Order Derivations

Evaluate the following limits.

$$\lim_{(x,y)\to(-4,-1)} \frac{x^3-3x+y^2-1}{xy+5}$$

$$\lim_{(x,y)\to(-4,-1)} \frac{x^3-3x+y^2-1}{xy+5}$$

Apply limit

$$= \frac{(-4)^3-3(-4)+(-1)^2-1}{(-4)(-1)+5} = \frac{-64+12+1-1}{4+5} = \frac{-52}{9}$$

$$\lim_{(x,y)\to(2,0)} \frac{(y+1)(x^2+y^2-4x)}{x+3y}$$

$$\lim_{(x,y)\to(2,0)} \frac{(y+1)(x^2+y^2-4x)}{x+3y}$$

Apply limit

$$= \frac{(0+1)((2)^2+(0)^2-4(2))}{2+3(0)} = \frac{1(4+0-8)}{2}$$

$$= \frac{4-8}{2} = \frac{4}{2} = 2.$$

$$\lim_{(x,y,z)\to(1,1,1)} \frac{x^2-y^2z^2}{x^3-x^2y^2}.$$

$$= \lim_{(x,y,z)\to(1,1,1)} \frac{x^2-y^2z^2}{x^3-x^2y^2}$$

Apply limit

$$= \frac{(1)^2 - (1)^2 (1)^2}{(1)^3 - (1)^2 (1)(1)} = \frac{1-1}{1-1} = 0$$

∴ limit does not exist

Q2

(i) $f(x,y) = xy\, e^{x^2+y^2}$

$\cancel{f_x = y(e^{x}+y^2}$

$f_x = y(e^{x^2+y^2}) + xy(e^{x^2+y^2} \cdot 2x)$

$= y \cdot e^{x^2+y^2} + 2x^2 y\, e^{x^2+y^2}$

$f_y = x(1 \cdot e^{x^2+y^2}) + xy(e^{x^2+y^2} \cdot 2y)$

$= x \cdot e^{x^2+y^2} + 2xy^2 \cdot e^{x^2+y^2}$

$f_x = ye^{x^2+y^2} + 2x^2y \cdot e^{x^2+y^2}$

$f_y = xe^{x^2+y^2} + 2xy^2\, e^{x^2+y^2}$.

ii] $f(x,y) = e^x \log y$

$f_x = \log y\, e^x$

$f_y = e^x \cdot \sin y$.

$f_y = -\sin y\, e^x$.

iii] $f(x,y) = x^3y^2 - 3x^2y + y^3 + 1$

$f_x = y^2\, 3x^2 - 3y^2 x + 6 + 0$

$= 3x^2y^2 - 6xy$.

$f_y = x^3\, 2y - 3x^2 + 3y^2$

$= 2x^3 y - 3x^2 + 3y^2$

Using definition find value of f_x, f_y at $(0,0)$ for

$f(x,y) = \frac{2x}{1+y^2}$

$f_x(0,b) = \lim_{h \to 0} \frac{f(a+h, b) - f(a,b)}{h}$

$f_y(0,b) = \lim_{h \to b} \frac{f(a, b+h) - f(a,b)}{h}$

According to given $(a,b) = (0,0)$

$f_x(0,0) = \lim_{h \to 0} \frac{f(h,0) - f(0,0)}{h}$

$= \lim_{h \to 0} \frac{2h - 0}{h} = 2$

$f_y(0,0) = \lim_{h \to 0} \frac{f(0,h) - f(0,0)}{h}$

$= \lim_{h \to 0} \frac{0 - 0}{h} = 0$

$f_x = 2, \quad f_y = 0$

$f_x(0,0) = \lim_{h \to 0} \frac{f(h,0) - f(0,0)}{h}$

$= \lim_{h \to 0} \frac{2h - 0}{h} = 2$

$f_y(0,0) = \lim_{h \to 0} \frac{f(0,h) - f(0,0)}{h}$

$= \lim_{h \to 0} \frac{0 - 0}{h} = 0$

$f_x = 2, f_y = 0$.

Q4] Find all 2nd order partial derivatives of f.
Use verify whether $f_{xy} = f_{yx}$

i] $f(x,y) = \frac{y^2 - xy}{x^2}$

$f_{xx} = \frac{\partial^2 f}{\partial x^2} = \frac{d^2 f}{dx^2}$

$f_{yy} = \frac{d^2 f}{dy^2}$

Applying uv rule.

$f_x = \frac{x^2(0-y) - (y^2 - xy)2x}{x^4}$

$= \frac{x^2 y - 2xy^2 + 2x^2 y}{x^4}$

$f_x = \frac{x^2 y - 2xy^2}{x^4}$

$f_{xx} = \frac{x^4(2xy - 2y^2) - (x^2 y - 2xy^2)(4x^3)}{x^8}$

$= \frac{2x^5 y - 2x^4 y^2 - (4x^5 y - 8x^4 y^2)}{x^8}$

$= \frac{2x^5 y - 2x^4 y^2 - 4x^5 y + 8x^4 y^2}{x^8}$

$= \frac{-2x^5 y + 6x^4 y^2}{x^8}$

$= \frac{6x^4 y^2 - 2x^5 y}{x^8}$

$f_{xx} = \frac{6y^2 - 2xy}{x^4}$

$f_y = \frac{1}{x^2}(2y - x) \quad \therefore f_y = \frac{2x}{x^3}$

$f_{yy} = \frac{1}{x^2} 2 = \frac{2}{x^2}$

$f_{xy} = \frac{2y - x}{x^2} = \frac{x^2(-1) - (2y - x)(2x)}{x^4}$

$= \frac{x^2 - 4xy + 2y^2}{x^4}$

$= \frac{x^2 - 4xy}{x^4} = \frac{x - 4y}{x^3}$

$\therefore f_{xy} = \frac{x - 4y}{x^3}$

$\therefore f_{yx} = \frac{x^2 y - 2xy}{x^4}$

$= \frac{x^2 - 4xy}{x^4}$

$= \frac{x - 4y}{x^3}$

$= f_{xy}$

$\therefore f_{yx} = f_{xy}$
Hence, verified.

Q5] Find the linearization of $f(x,y)$ at given point

i] $f(x,y) = \sqrt{x^2+y^2}$ at $(1,1)$

$f_x = \frac{1}{2\sqrt{x^2+y^2}} \cdot 2x$ $\qquad f_y = \frac{2y}{2\sqrt{y^2+x^2}}$

$= \frac{x}{\sqrt{x^2+y^2}}$ $\qquad = \frac{y}{\sqrt{x^2+y^2}}$

$f_x(1,1) = \frac{1}{\sqrt{2}}$ $\qquad f_y = \frac{1}{\sqrt{2}}$

$\therefore L(x,y) = f(1,1) + f_x(1,1)(x-1) + f_y(1,1)(y-1)$

$= \sqrt{2} + \frac{x-1}{\sqrt{2}} + \frac{y-1}{\sqrt{2}}$

$= \frac{x+y}{\sqrt{2}}$

ii] $f(x,y) = 1-x+y\sin x$ at $\frac{\pi}{2}$.

$f(\pi/2, 0) = 1 - \frac{\pi}{2} + 0 + \sin\frac{\pi}{2}$

$f(\frac{\pi}{2}, 0) = \frac{2-\pi}{2}$

$f_x = -1+y\cos x$ $\qquad f_y = \sin x$

$f_x(\frac{\pi}{2}, 0) = -1 + 0\sin\frac{\pi}{2}$

$f_y(\frac{\pi}{2}, 0) = \sin\frac{\pi}{2}$

$L(x,y) = f(\frac{\pi}{2}, 0) + f_x(\frac{\pi}{2}, x)(x-\pi/2) + f_y(\frac{\pi}{2}, 0)(y-0)$

$= \frac{2-\pi}{2} + (-1)(x - \frac{\pi}{2}) + 1(y)$

$= 1 - \frac{\pi}{2} + x + \frac{\pi}{2} + y$

$L(x,y) = 1-x+y$.

iii] $f(x,y) = \log x + \log y$.

$f(1,1) = \log 1 + \log 1$

$= 0$

$f(x) = \frac{1}{x}$ $\qquad f_y = \frac{1}{y}$

$f_x(1,1) = 1$ $\qquad f_y(1,1) = 1$

$L(x,y) = f(1,1) + f_x(1,1)(x-1) + f_y(1,1)(y-1)$

$= 0 + 1(x-1) + 1(y-0)$

$L(x,y) = x+y-2$

21/01/2020

PRACTICAL-10

Q) Find the directional derivative of the following function at a given point & in the direction of given vector.

1) $f(x,y) = x + 2y - 3$ $\quad a = (1,-1) \quad u = 3i - j$

$|\bar{u}| = \sqrt{(3)^2 + (-1)^2} = \sqrt{9+1} = \sqrt{10}$

unit vector along u is $\frac{u}{|u|} = \frac{1}{\sqrt{10}}(3,-1)$

$= \left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$f(a+hu) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$f(a) = f(1,-1) = (1) + 2(-1) - 3 = 1-2-3 = -4$

$f(a+hu) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$= f = \left(1 + \frac{3}{\sqrt{10}}\right), \left(-1 - \frac{h}{\sqrt{10}}\right) - 3$

$f(a+hu) = \left(1 + \frac{3}{\sqrt{10}}\right) + 2\left(-1 - \frac{h}{\sqrt{10}}\right) - 3$

$= 1 + \frac{3}{\sqrt{10}} - 2 - \frac{2h}{\sqrt{10}}$

$f(a+hu) = -4 + \frac{h}{\sqrt{10}}$

:a

$$D_u f(a) = \lim_{h\to 0} \frac{f(a+hu) - f(a)}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{-4} + \cancel{h}/\sqrt{10} \cancel{+4}}{\cancel{h}}$$

$$\boxed{D_u f(a) = \frac{1}{\sqrt{10}}}$$

ii) $f(x) = y^2 - 4x + 1 \quad a = (3, 4), \quad v = i + 5j$

Here, $u = i + 5j$ is not a unit vector

$$|\bar{u}| = \sqrt{(1)^2 + (5)^2} = \sqrt{26}$$

unit vector along u is $\frac{u}{|u|} = \frac{1}{\sqrt{26}}(1, 5)$

$$= \left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$$

$f(a) = f(3, 4) = (4)^2 - 4(3) + 1 = 5$

$f(a+hu) = f(3, 4) + h\left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$

$$= f\left(3 + \frac{h}{\sqrt{26}}, 4 + \frac{5h}{\sqrt{26}}\right)$$

$$f_{xy}(a+hu) = \left(4 + \frac{5h}{\sqrt{26}}\right)^2 - 4\left(3 + \frac{h}{\sqrt{26}}\right) + 1$$

$$= 16 + \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - 12 - \frac{4h}{\sqrt{26}} + 1$$

$$= \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - \frac{4h}{\sqrt{26}} + 5$$

$$= \frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5$$

$$D_u f(a) = \lim_{h\to 0} \frac{\frac{25h^2}{\sqrt{26}} + \frac{36h}{\sqrt{26}} \cancel{+5} \cancel{-5}}{h}$$

$$\frac{\cancel{h}\left(\frac{25h}{26} + \frac{36}{\sqrt{26}}\right)}{\cancel{h}}$$

$$D_u f(a) = \frac{25h}{26} + \frac{36}{\sqrt{26}}$$

iii) $2x + 3y \quad a = (1, 2), \quad v = (3i + 4j)$

$u = 3i + 4j$ is not unit vector

$$|\bar{u}| = \sqrt{(3)^2 + (4)^2} = \sqrt{25} = 5$$

Unit vector along u is $\frac{u}{|u|} = \frac{1}{5}(3, 4)$

$$= \left(\frac{3}{5}, \frac{4}{5}\right)$$

$f(a) = f(1, 2) = 2(1) + 3(2) = 8$

$f(a+ha) = f(1, 2) + h\left(\frac{3}{5}, \frac{4}{5}\right)$

$$= f\left(1 + \frac{3h}{5}, 2 + \frac{4h}{5}\right)$$

$$f(a+hu) = 2 + \frac{6h}{5} + 6 + \frac{12h}{5}$$

$$= \frac{18h}{5} + 8$$

$Df(2a) = \lim_{h \to 0} \dfrac{\frac{18h}{5} + 8 - 8}{h}$

$= \dfrac{18}{5}$

Q2. Find the gradient value for the following fund at given point

(i) $f(x,y) = x^y + y^x$, $a = (1,1)$

$f_x = y \cdot x^{y-1} + y^x \log y$

$f_y = x^y \log x + x y^{x-1}$

$\therefore f(x,y) = (f_x, f_y)$

$= (y \cdot x^{y-1} + y^x \cdot \log y,\ x^y \log x + x y^{x-1})$

$f(1,1) = (1+0, 1+0)$

$= (1,1)$

(ii) $f(x,y) = (\tan^{-1} x) \cdot y^2$ $\quad a = (1,-1)$

$f_x = \dfrac{1}{1+x^2} \cdot y^2$

$f_y = 2y \cdot \tan^{-1} x$

$\therefore f(x,y) = (f_x, f_y)$

$= \left(\dfrac{y^2}{1+x^2},\ 2y \tan^{-1} x\right)$

$f(1,-1) = \left(\dfrac{1}{2},\ \tan^{-1}(1)(-2)\right)$

$= \left(\dfrac{1}{2},\ \dfrac{\pi}{4}(-2)\right).$

$= \left(\dfrac{1}{2},\ \dfrac{-\pi}{2}\right)$

(iii) $f(x,y,z) = xyz - e^{x+y+z}$, $a = (1,-1,0)$

$f_x = yz \cdot e^{x+y+z}$

$f_y = xz \cdot e^{x+y+z}$

$f_z = xy \cdot e^{x+y+z}$

$f(x,y,z) = f_x, f_y, f_z$

$= yz \cdot e^{x+y+z},\ xz - e^{x+y+z},\ xy \cdot e^{x+y+z}$

$f(1,-1,0) = \left((-1)(0) - e^{(1-1+0)},\ (1)(0) - e^{(1-1-0)},\ (1)(-1)\, e^{(1+(-1)+0)}\right)$

$= (0 - e^0,\ 0 - e^0,\ -1e^0)$

$= (-1,\ -1,\ -2)$

Q3 Find the eqn of tangent & normal to each of the following using curves at given point.

(i) $x^2 \cos y + e^{xy} = 2$ at $(1,0)$

$f_x = \cos y \, 2x + e^{xy} y$

$f_y = x^2(-\sin y) + e^{xy} \cdot x$

$(x_0, y_0) = (1,0)$ $\quad x_0 = 1, y_0 = 0$

eqn of tangent

$f_x(x - x_0) + f_y(y - y_0) = 0$

$f_x(x_0, y_0) = \cos 0 \; 2(1) + e^0 \, 0$

$= 1(2) + 0 = 2$

$f_y(x_0, y_0) = (1)^2 (-\sin 0) + e^0 \cdot 1$

$= 0 + 1 \cdot 1 = 1$

$2(x-1)+1(y-0)=0$

$2x-2+y=0$

$2x+y-2=0$ It is required Eqn of Tangent

Equⁿ of Normal

$= ax+by+c=0$

$= bx+ay+d=0$

$1(1)+2(y)+d=0$ at $(1,0)$

$1+2y+d=0$

$=1+2(0)+d=0$

$\therefore d+1=0$

$d=-1$

(ii) $x^2+y^2-2x+3y+2=0$ at $(2,-2)$

$f_x = 2x+0-2+0+0$

$= 2x-2$

$f_y = 0+2y-0+3+0$

$= 2y+3$

$(x_0,y_0) = (2,-2)$ $\therefore x_0 = -2, y_0 = 2$

$f_x(x_0,y_0) = 2(2)-2 = 2$

$f_y(x_0,y_0) = 2(-2)+3 = -1$

eqn of tangent

$f_x(x-x_0) + f_y(y-y_0) = 0$

$2(x-2)+(-1)(y+2)=0$

$2x-2-y-2=0$

$2x-y-4=0 \longrightarrow$ eqn of tangent

eqn of Normal

$= ax+by+c=0$

$bx+ay+d=0$

$-1(x)+2(y)+d=0$

$-x+2y+d=0$ at $(2,-2)$

$\cdot\ -2+2(-2)+d=0$

$-2-4+d=0$

$-6+d=0$

$\therefore \boxed{d=6}$

Q4] Find the eqn of tangent & normal line to each of following surface.

(i) $x^2-2yz+3y+xz=1$ at $(2,1,0)$

$f_x = 2x-0+0+z$

$f_x = 2x+z$

$f_y = 0-2z+3+0$

$f_y = 2x+3$

$f_z = 0-2y+0+x$

$f_z = -2y+x$

$(x_0, y_0, z_0) = (2,1,0)$ $\therefore x_0 = 2, y_0 = 1, z_0 = 0$

$f_x(x_0,y_0,z_0) = 2(2)+0 = 4$

$f_y(x_0,y_0,z_0) = 2(0)+3 = 3$

$f_z(x_0,y_0,z_0) = -2(1)+2 = 0$

eqn of Tangent

$f_x(x_0 - x_0) + f_y(y_0 - y_0) + f_z(z_0 - z_0) = 0$

$f_x(x_0 - x_0) + f_y(y_0 - y_0) + 0(z - 0) = 0$

$= 4(x - 2) + 3(y - 1) + 0(z - 0) = 0$

$= 4x - 8 + 3y - 3 = 0$

$4x + 3y - 11 = 0 \rightarrow$ eqn of tangent

eqn of Normal at $(4, 3, -11)$:

$\frac{x - x_0}{f_x} = \frac{y - y_0}{f_y} = \frac{z - z_0}{f_z}$

$= \frac{x-2}{4} = \frac{y-1}{3} = \frac{z+11}{0}$ ✓

(ii) $3xyz - x - y + z = -4$ at $(1, -1, 2)$

$f_x = 3yz - 1 - 0 + 0 + 0$

$= 3yz - 1$

$f_y = 3xz - 1$

$f_z = 3xy + 1$ $(x_0, y_0, z_0) = (1, -1, 2)$

$f_x(x_0, y_0, z_0) = 3(-1)(2) - 1 = -7$

$f_y(x_0, y_0, z_0) = 3(1)(2) - 1 = 5$

$f_z(x_0, y_0, z_0) = 3(1)(-1) + 1 = -2$

eqn of tangent:

$-7(x - 1) + 5(y + 1) - 2(z - 2) = 0.$

$-7x + 7 + 5y + 5 - 2z + 4 = 0$

$-7x + 5y - 2z + 16 = 0 \rightarrow$ eqn of tangent

eqn of Normal at $(-7, 5, -2)$

$\frac{x - x_0}{f_x} = \frac{y - y_0}{f_y} = \frac{z - z_0}{f_z}$

$= \frac{x-1}{-7} = \frac{y+1}{5} = \frac{z-2}{-2}$ //

(b) Find local maxima & minima for the following

(i) $f(x, y) = 3x^2 + y^2 - 3xy + 6x - 4y$

$f_x = 6x + 0 - 3y + 6 - 0$

$= 6x - 3y + 6$

$f_y = 0 + 2y - 3x + 0 - 4y$

$= 2y - 3x - 4$

$f_x = 0$

$6x - 3y + 6 = 0$

$3(2x - y + 2) = 0$

$2x - y + 2 = 0$

$2x - y = -2$ — ①

$f_y = 0$

$2y - 3x - 4 = 0$

$2y - 3x = 4$ — ②

Multiplying eqn ① with 2

$4x - \cancel{2y} = -4$

$\cancel{2y} - 3x = 4$

$x = 0$

Substitute value of x in eqn ①

$2(0) - y = -2$

$-y = -2$ $\boxed{y = 2}$

$\therefore$ Critical points are $(0, 2)$

$r = f_{xx} = 6$

$t = f_{yy} = 2$

$s = f_{xy} = -3$

Here $r > 0$

$rt - s^2$

$= 6(2) - (-3)^2$

$= 12 - 9$

$= 3 > 0$

f has maximum at $(0, 2)$

$3x^2 + y^2 - 3xy + 6x - 4y$ at $(0, 2)$

$3(0)^2 + (2)^2 - 3(0)(2) + 0 - 4(2)$

$= 0 + 4 - 0 + 0 - 8$

$= -4$

(ii) $f(x, y) = 2x^4 + 3x^2 y - y^2$

$f_x = 8x^3 + 6xy$

$f_y = 3x^2 - 2y$

$f_x = 0$

$\therefore 8x^3 + 6xy = 0$

$2x(4x^2 + 3y) = 0$

$4x^2 + 3y = 0$ — ①

$f_y = 0$

$3x^2 - 2y = 0$ — ②

Multiplying eqn ① with 3

② with 4

$\cancel{12x^2} + 9y = 0$

$-\cancel{12x^2} \mp 8y = 0$

$11y = 0$ $\boxed{y = 0}$

Substitute values of y in eqn ①

$4x^2 + 3(0) = 0$

$4x^2 = 0$

$\boxed{x = 0}$

Critical pts is $(0, 0)$

$r = f_{xx} = 24x^2 + 6x$

$t = f_{yy} = 0 - 2 = -2$

$s = f_{xy} = 6x - 0 = 6x - 6(0) = 0$

r at $(0, 0)$

$= 24(0) + 6(0) = 0$

$\therefore r = 0$

$rt - s^2 = 0(-2) - (5)^2$

$= 0 - 0 = 0$

$r = 0$ & $rt - s^2 = 0$

$f(x, y)$ at $(0, 0)$

$2(0)^4 + 3(0)^2(0) - (0)$

$= 0 + 0 - 0$

$= 0$

(iii) $f(x, y) = x^2 - y^2 + 2x + 8y - 70$

$f_x = 2x + 2$

$f_y = -2y + 8$

$f_x = 0$

$\therefore 2x + 2 = 0$

$x = -1$

$f_y = 0$

$-2y + 8 = 0$

$y = \frac{-8}{-2}$

$\boxed{y = 4}$

$\therefore$ Critical point is $(-1, 4)$

$r = f_{xx} = 2$

$t = f_{yy} = -2$

$s = f_{xy} = 0$

$r > 0$

$rt - s^2 = 2(-2) - (0)^2$

$= -4 - 0$

$= -4 < 0$

$f(x,y)$ at $(-1, 4)$

$(-1)^2 - (4)^2 + 2(-1) + 8(4) - 70$

$1 + 16 - 2 + 32 - 70$

$17 + 30 - 70 = 37 - 70$

$= 33$ //

AK 05/02/2020