A study of Hedging Strategies using Spot rates

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Abstract:

The spot rates can be used to hedge fixed income securities. The objective of this project is to compare the hedging strategies implemented using Modified duration-matching barbell strategy, Macaulay duration-matching barbell strategy, Simple regression-based hedging, and Regression-based hedging using multiplicative portfolios. This project picks the optimal strategy to hedge the excess return of 3-year and 7-year maturity zero-coupon bonds based on the root-mean square hedging errors(RMSHE). Also, we look at the time series variation in the spot rates using the yield curve factors, level, slope, and curvature.

Keywords: spot rates, zero-coupon bond, hedging error, duration, barbell strategy, regression

1. Introduction

Treasury bond also known as T-bond are issued by U.S. government. They are considered as the safest investments in the world. The bonds differ according to their maturity and coupon payments. The holder of the treasury bond receives coupon payments every six month until the maturity period. The bond holder can hold the bond until it matures or sell before the maturity date. If you hold the bond, you will receive the coupon payments in addition to the face value over the life of the bond.

The bonds can be priced with the help of spot rate treasury curve. The spot rate treasury curve can be constructed using Treasury spot rates rather than yields. One needs to plot the spot rates to obtain the resulting spot rate treasury curve. The yield to maturity for zero-coupon bonds can be calculated using the spot rate treasury curve which is used to discount a single cash flow at maturity. Since bonds have multiple cash flows using a single interest rate to discount the coupon payments received at different time is not correct. Therefore, each coupon payment is discounted using corresponding treasury spot rate to obtain an error free valuation.

The holder of treasury bonds has to deal with interest rate risk. This risk can be reduced using different hedging strategies. In our project, we define four different portfolios each using a different hedging strategy.

The purpose of this project is to familiarize with the spot rate data, the properties of the spot rates and implement and evaluate various hedging strategies. We used the spot rate equation proposed by Gurkaynak, Sack, and Wright (2006) to generate continuously compounded spot rates by using parameter estimates, namely, β_0, β_1, β_2, β_3, τ_1, and τ_2 and also determined their statistical and time series properties. The spot rates were used to calculate the price of zero coupon bonds of different maturities, later price of bonds was used to compute returns and excess return on the bonds. The information was then later utilized to perform four different hedging strategies and to calculate the respective root-mean squared hedging error for each strategy and then evaluated the performance of the hedging strategies accordingly.

2. Data Description

The daily data of spot rates from Federal Reserve is used and the end of month values for each month are used for the analysis.

Table 1: Summary statistics of the 1-year, 5-year and 10-year spot rates.

	$y_t(1)$	$y_t(5)$	$y_t(10)$
Mean	5.10466	5.79914	6.24153
Std dev	3.30695	2.98971	2.73522
Skewness	0.51633	0.50198	0.62430
Kurtosis	0.24991	0.06135	0.16972

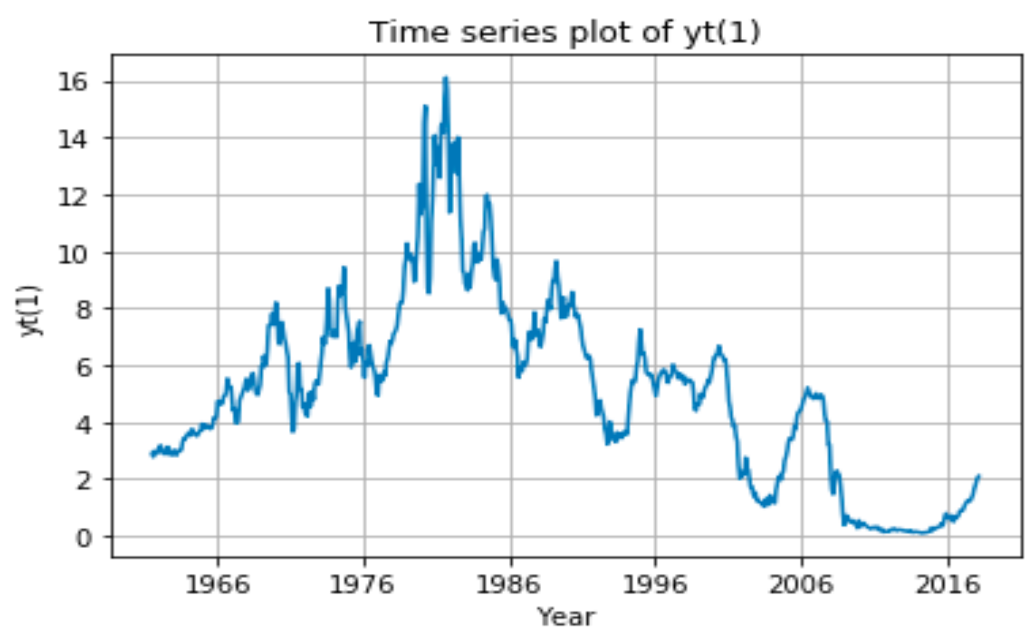
The mean of the 10-year spot rates is higher than the 1-year and 5-year spot rates. Also, the dispersion in the 10-year spot rates are considerably lower than the 1-year and 5-year spot rates. The longer maturity spot rates are less fluctuating. The 5-year spot rates are closest to a normal distribution. As the time to maturity increases, the skewness increases but the kurtosis decreases which means that spot rates for longer maturities increase more than spot rates for shorter maturities in general but the frequency with which they take extreme values is less than that for shorter maturity spot rates.

Table 2: Autocorrelation coefficients at lag 1,2,3, and 4 of the 1-year, 5-year and 10-year spot rates.

	$y_t(1)$	$y_t(5)$	$y_t(10)$
γ_1	0.99068	0.99337	0.99347
γ_2	0.97826	0.98538	0.98643
γ_3	0.96733	0.97854	0.98043
γ_4	0.95741	0.97216	0.97498

Table 3: Autocorrelation matrix of the 1-year, 5-year and 10-year spot rates.

	$y_t(1)$	$y_t(5)$	$y_t(10)$
$y_t(1)$	1.00000	0.97283	0.93859
$y_t(5)$	0.97283	1.00000	0.98958
$y_t(10)$	0.93859	0.98958	1.00000



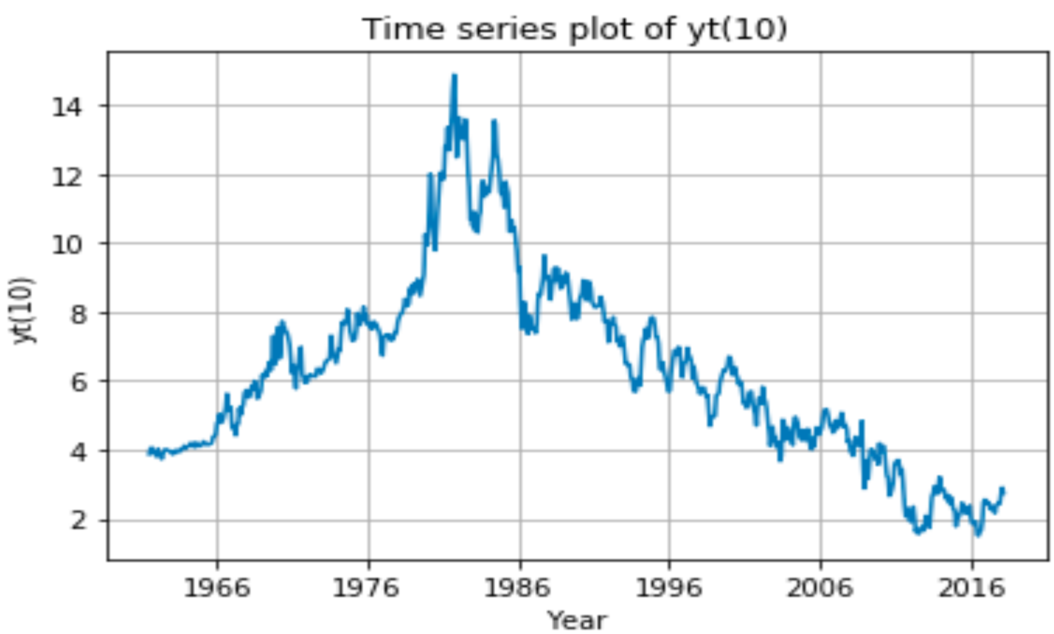


Figure 1 – Time series plots of yt(1), $y_t(5)$ and $y_t(10)$

Investing in longer duration bonds have generated higher returns in general. The only exception being a period between 1976-1986 where the short-term spot rates were higher than the long-term spot rates.

2.1 Data Transformation

The following steps are followed to transform the data as required.

i) The spot rates at every time t for maturity n, is calculated using the following formulae.

$$y_t(n) = \beta_{0,1} + \beta_{1,t}\frac{1-\exp\left(-\frac{n}{\tau_{1,t}}\right)}{\frac{n}{\tau_{1,t}}} + \beta_{2,t}\left[\frac{1-\exp\left(-\frac{n}{\tau_{1,t}}\right)}{\frac{n}{\tau_{1,t}}} - \exp\left(-\frac{n}{\tau_{1,t}}\right)\right]$$

$$+ \beta_{3,t}\left[\frac{1-\exp\left(-\frac{n}{\tau_{2,t}}\right)}{\frac{n}{\tau_{2,t}}} - \exp\left(-\frac{n}{\tau_{2,t}}\right)\right]$$

ii) The price of the zero-coupon bond of maturity n is calculated as,

$$P_t(n) = \exp\left(-y_t(n) * n\right)$$

$$P_t(0) = 1$$

iii) The return on the bonds of maturity n, for holding the bond for one month is calculated as,

$$RET_{t+\Delta}(n) = \frac{P_{t+\Delta}(n-\Delta)}{P_t(n)} - 1$$

iv) The excess return on the bond is calculated as,

$$ER_{t+\Delta}(n) = RET_{t+\Delta}(n) - RET_{t+\Delta}(\Delta)$$

v) The yield curve factors, yield, slope, and curvature are calculated as,

$$Level_t = y_t\left(\frac{1}{4}\right)$$

$$\mathrm{Slope}_t = y_t(8) - y_t\left(\frac{1}{4}\right)$$

$$\mathrm{Curvature}_t = [y_t(8) - y_t(2)] - [y_t(2) - y_t\left(\frac{1}{4}\right)]$$

3. Methodology

3.1 Barbell Strategy

A barbell strategy is one in which half the portfolio is made up of long-term bonds and the other half of short-term bonds. When we construct a hedge using a barbell portfolio we match the duration of our assets, i.e., short-term and long-term bonds with our liabilities which is a bond with maturity date between the short-term and long-term bonds.

$$D(L) = w(s)D(s) + (1 - w(s))D(l)$$

where,

D(L) is duration of liability

D(s) is duration of short-term bond

D(l) is duration of long-term bond

w(s) is weight of short-term bond

w(l) is weight of long-term bond

3.1.1 Modified-duration-matching barbell strategy

Modified duration, D^*, is a relative measure of interest rate sensitivity. It is the percentage change in the price of a bond, or of a portfolio of bonds, for a unit change in a given yield.

$$D^* = -\frac{dP}{dy}\frac{1}{P}$$

Modified duration of a zero-coupon bond is given by the formula,

$$D^* = \frac{1}{P}\frac{nM}{(1+y)^{n+1}} = \frac{n}{1+y}$$

where,

n is time to expiration of zero-coupon bond

M is maturity value

P is price of zero-coupon bond

3.1.2. Macaulay-duration-matching barbell strategy

The Macaulay duration, D, can be viewed as the economic balance point of a group of cash flows. Since there is only one cash flow in a zero-coupon bond which happens at the maturity of the bond the Macaulay duration of an n-year zero-coupon bond is n.

$$D = n$$

3.2 Simple regression-based hedging

The excess return of a bond at time t for maturity n is regressed on the excess return of a bond with maturities of 1 year, 5 years and 10 years. The corresponding coefficients from the regression is used as the hedging portfolio weights. The regression equation is defined as,

$$ER_t(n_h) = w_t(1)ER_t(1) + w_t(5)ER_t(5) + w_t(10)ER_t(10) + u_t(n_h)$$

where $u_t(n_h)$ is the error term.

3.3 Regression-based hedging with multiplicative portfolios

The excess return of a bond at time t for maturity n is regressed on the excess return of a bond multiplied by the yield curve factors, level, slope, and curvature of maturities 1 year, 5 years and 10 years. The corresponding coefficients from the regression is used as the hedging portfolio weights. The regression equation is defined as,

$$ER_t(n_h) = \theta_t(1; X_{t-\Delta})ER_t(1) + \theta_t(5; X_{t-\Delta})ER_t(5) + \theta_t(10; X_{t-\Delta})ER_t(10) + u_t(n_h)$$

where $u_t(n_h)$ is the error term and $\theta_t(n; X_{t-\Delta})$ depends on X:

$$\theta_t(n; X_{t-\Delta}) = a_t(n) + b_t(n)Level_{t-\Delta} + c_t(n)Slope_{t-\Delta} + d_t(n)Curvature_{t-\Delta}$$

3.4 Hedging Errors

The out of sample hedging errors for the respective hedging strategy is calculated as,

$$\varepsilon_{t+\Delta} = ER_{t+\Delta}(n_h) - [w_t(1)ER_{t+\Delta}(1) + w_t(5)ER_{t+\Delta}(5) + w_t(10)ER_{t+\Delta}(10)$$

And the root-mean square hedging error (RMSHE) is defined as,

$$RMSHE = \{\frac{1}{T}\sum([\epsilon_{t+\Delta}(n_h)]^2)\}^{0.5}$$

4. Results

4.1 Regression of Spot rates

The spot rates of 1-year, 5-year and 10-year are regressed on the yield curve factors in order to explain the time series variation.

Table 4: OLS regression summary of $y_t(1)$.

Dep Variable	$y_t(1)$		R-squared:	0.998
Model:	OLS		F-statistic:	1.49E+05
Method:	Least Squares		Durbin-Watson:	0.613
	coefficient	Std error	t	P>|t|
Intercept	0.0592	0.016	3.669	0
level	0.9911	0.002	471.299	0
slope	0.2831	0.004	63.63	0
curvature	-0.4571	0.007	-64.748	0

Table 5: OLS regression summary of $y_t(5)$.

Dep Variable	$y_t(5)$		R-squared:	0.999
Model:	OLS		F-statistic:	3.11E+05
Method:	Least Squares		Durbin-Watson:	0.389
	coefficient	Std error	t	P>|t|
Intercept	-0.0702	0.01	-6.943	0
level	1.0073	0.001	764.656	0
slope	0.8466	0.003	303.768	0
curvature	-0.2397	0.004	-54.21	0

Table 6: OLS regression summary of $y_t(10)$.

Dep Variable	$y_t(10)$		R-squared:	0.999
Model:	OLS		F-statistic:	3.91E+05
Method:	Least Squares		Durbin-Watson:	0.409
	coefficient	Std error	t	P>|t|
Intercept	0.0663	0.008	8.041	0
level	0.9937	0.001	925.168	0
slope	1.0568	0.002	465.079	0
curvature	0.1097	0.004	30.413	0

In normal markets, as the time increases so does the yield. This is intuitive because interest-rate risk tends to increase with maturity, and investors demand compensation for this risk. Conceptually, the yield curve depends on expected future changes in the short rate as well as the risk premium for the long-maturity bond reflecting the perceived risk of investing in them. The longer the maturity of the bond, the higher the risk to the investor, and so the higher the yield. This is evident from our results, as we move on from $y_t(1)$ to $y_t(10)$, the slope increases by 0.7737.

Till the 10-year spot rate, the negative curvature indicates that the function is convex. This confirms the idea that interest rates reflect the perceived risks of investing in longer-term debt instruments. However, there is a switch point at the 10-year spot rate: the curvature becomes positive and therefore the function is concave. It turns out that the final structure is perfectly suitable for the "normal times" where, as Nelson-Svensson-Siegel showed, there is an asymptotic level that cannot be knocked by the long-term interest rate even if maturities still keep increasing (rate cannot be positively infinite!)

The three yield factors (level, slope and curvature) explain the 99%-time series variation in spot rate. However, the Durbin-Watson stat tends to be systematically close to zero which indicates a positive autocorrelation among residuals which still may have explanatory power.

4.2 Hedging Errors analysis

The hedging errors for the excess return on bonds of 3-year and 7-year maturity for the four different strategies are calculated for the out of the sample data using the hedging portfolio weights from the most recent T months. The portfolio weight for the excess return on r-year maturity bond, $w_t(5) = 0$ for the barbell strategies.

Table 6: RMSE of the four hedging strategies.

	RMSE	
Strategy	$ER_t(3)$	$ER_t(7)$
Modified-duration-matching	134642.83	134463.62
Modified-duration-matching	141153.44	140974.23
Simple regression	1.876579	50.646422
Multiplicative regression	4.817427	229.781841

The relative effectiveness of our proposed hedging strategies is gauged using root-mean squared error(RMSE), a natural measure of the success of the optimal hedging strategy. The (RMSE) serves to aggregate the magnitudes of the errors in predictions for various times into a single measure of predictive power. The results confirm that the Simple regression-based hedging strategy is the optimal strategy to hedge the excess return for both the 3-year and 7-year maturity bonds. The Macaulay duration-matching hedging strategy has the maximum RMSE compared to other 3 strategies in both cases because Macaulay duration does not take the spot rates into account.

References

Gurkaynak, Sack, and Wright (2006). The U.S. Treasury Yield Curve: 1961 to the Present, Finance and Economics Discussion Series Divisions of Research & Statistics and Monetary Affairs Federal Reserve Board, Washington, D.C.