Fixed Income Securities and Credit Risk

Final Project

April 9, 2018

1 Objective

The purpose of this project is to familiarize yourself with the spot rate data, the time-series projects of the spot rates and their relations to the yield-curve factors, and implement and evaluate various hedging strategies.

2 Instructions

- By May 7, 2018, 5:30 PM EDT: please turn in a hard copy of the report.
- This is a group project with group size of 4 or below. Writing and analysis should be entirely your own group's work.

3 Background Readings

- Chapter 6 of Tuckman and Serrat (2012).
- Working paper by Gurkaynak, Sack, and Wright (2006); see below.

4 Data

1. Obtain the Gurkaynak-Sack-Wright dataset: http://www.federalreserve.gov/pubs/feds/2006/200628/200628abs.html. Their 2006 working paper is also available from the same web page.
2. Construct the data series over as long a time period as your dataset allows, sort the data in ascending order, and only keep end-of-month observations. Suppose there are $T_0 = 2T$ (T an integer) months in the data.

3. Equation (22) of Gurkaynak, Sack, and Wright (2006) describes how to use their parameter estimates, namely, β_0, β_1, β_2, β_3, τ_1, and τ_2 (in the last six columns of the spreadsheet), to generate n-year *continuously compounded* spot rates at any given date t:

$$\begin{aligned} y_t(n) = {} & \beta_{0,t} + \beta_{1,t}\frac{1-\exp(-\frac{n}{\tau_{1,t}})}{\frac{n}{\tau_{1,t}}} \\ & + \beta_{2,t}\left[\frac{1-\exp(-\frac{n}{\tau_{1,t}})}{\frac{n}{\tau_{1,t}}} - \exp(-\frac{n}{\tau_{1,t}})\right] + \beta_{3,t}\left[\frac{1-\exp(-\frac{n}{\tau_{2,t}})}{\frac{n}{\tau_{2,t}}} - \exp(-\frac{n}{\tau_{2,t}})\right]. \end{aligned} \quad (1)$$

Find the spot rates for the following maturities: 1/12, 1/4, $1-1/12$, 1, 2, $3-1/12$, 3, $5-1/12$,5, $7-1/12$, 7, 8, $10-1/12$, and 10.

4. The time-t price of n-year zero-coupon bond with face value \$1 is

$$P_t(n) = \exp[-y_t(n) \times n], \quad (2)$$

and $P_t(0) = 1$. The time-$(t+\Delta)$ return on a n-year bond is

$$\mathrm{RET}_{t+\Delta}(n) = \frac{P_{t+\Delta}(n-\Delta)}{P_t(n)} - 1, \quad (3)$$

and its excess return is

$$\mathrm{ER}_{t+\Delta}(n) = \mathrm{RET}_{t+\Delta}(n) - \mathrm{RET}_{t+\Delta}(\Delta). \quad (4)$$

Set $\Delta = 1/12$ and form excess returns for $n = 1$, 3, 5, 7, and 10.

5. Form the three "yield-curve factors" $X_t = (\mathrm{Level}_t, \mathrm{Slope}_t, \mathrm{Curvature}_t)^\top$, where

$$\mathrm{Level}_t = y_t(1/4), \quad (5)$$

$$\mathrm{Slope}_t = y_t(8) - y_t(1/4), \quad (6)$$

$$\mathrm{Curvature}_t = [y_t(8) - y_t(2)] - [y_t(2) - y_t(1/4)]. \quad (7)$$

5 Hedging Strategies and Errors

5.1 Hedging Strategies

Suppose we want to hedge a bullet portfolio with maturity n_h, using the following strategies:

- Modified-duration-matching barbell strategy: At time t, use 1-year and 10-year zero-coupon bonds to match the modified duration of the bullet portfolio. Suppose the hedging portfolio weights are $w_t(1)$ and $w_t(10)$, respectively. This is the case in Handout 4.

- Macaulay-duration-matching barbell strategy: At time t, use 1-year and 10-year zero-coupon bonds to match the Macaulay duration of the bullet portfolio. Suppose the hedging portfolio weights are $w_t(1)$ and $w_t(10)$, respectively.

- Simple regression-based hedging: At time t, use the data of the most recent T months to run the following regression:

$$\mathrm{ER}_t(n_h) = w_t(1)\mathrm{ER}_t(1) + w_t(5)\mathrm{ER}_t(5) + w_t(10)\mathrm{ER}_t(10) + u_t(n_h), \tag{8}$$

  where $u_t(n_h)$ is the error term. The hedging portfolio weights are $w_t(1)$, $w_t(5)$ and $w_t(10)$.

- Regression-based hedging with multiplicative portfolios: At time t, use the data of the most recent T months to run the following regression:

$$\mathrm{ER}_t(n_h) = \theta_t(1; X_{t-\Delta})\mathrm{ER}_t(1) + \theta_t(5; X_{t-\Delta})\mathrm{ER}_t(5) + \theta_t(10; X_{t-\Delta})\mathrm{ER}_t(10) + u_t(n_h), \tag{9}$$

  where $u_t(n_h)$ is the error term, and $\theta_t(n; X_{t-\Delta})$ depends on X:

$$\theta_t(n; X_{t-\Delta}) = a_t(n) + b_t(n)\mathrm{Level}_{t-\Delta} + c_t(n)\mathrm{Slope}_{t-\Delta} + d_t(n)\mathrm{Curvature}_{t-\Delta}. \tag{10}$$

  The hedging portfolio weights are

$$w_t(1) = \theta_t(1; X_t), \tag{11}$$

$$w_t(5) = \theta_t(5; X_t), \tag{12}$$

$$w_t(10) = \theta_t(10; X_t). \tag{13}$$

5.2 Hedging Errors

Suppose we hold a hedging portfolio from t to $t+\Delta$ (i.e., a month), the time $t+\Delta$ hedging error is

$$\epsilon_{t+\Delta} = \mathrm{ER}_{t+\Delta}(n_h) - [w_t(1)\mathrm{ER}_{t+\Delta}(1) + w_t(5)\mathrm{ER}_{t+\Delta}(5) + w_t(10)\mathrm{ER}_{t+\Delta}(10)]; \tag{14}$$

obviously $w_t(5) = 0$ for the barbell strategies.

6 Guidelines

Structure your report around the following items:

1. What are the time-series properties of the spot rates $y_t(1)$, $y_t(5)$, and $y_t(10)$? Report their summary statistics, including mean, standard deviation, skewness, kurtosis, and the first four autocorrelation coefficients, and the correlation matrix of the spot rates. Comment on your results. Also plot them and comment on the time series patterns.

2. Can the three yield curve factors explain the time-series variation in spot rates? Regress $y_t(1)$ on a constant and X_t and comment on the regression statistics. Perform the same analysis for $y_t(5)$ and $y_t(10)$.

3. Perform the "out-of-sample" hedging exercises where $n_h = 3$. Split the sample into two halves, such that $T_0 = 2T$. Begin with the T-th month, calculate $w_T(1)$, $w_T(5)$, and $w_T(10)$ for each hedging strategy, and save $\epsilon_{T+\Delta}(n_h)$. Move forward and repeat the process each month, and calculate the root mean squared hedging error (RMSHE) for each strategy:
$$\text{RMSHE} = \left[\frac{1}{T}\sum \epsilon_{t+\Delta}(n_h)\right]^{0.5}. \tag{15}$$
Report your results and evaluate the performance of the hedging strategies.

4. Now consider the case where $n_h = 7$. Report your results and evaluate the performance of the hedging strategies. Is it more challenging to hedge longer-maturity bonds?

7 Format

Assemble your report in the following order:

1. Title page: It should include date, and name(s) and email(s) of the team member(s).

2. Main text: Limit to 10 or fewer pages. The report should demonstrate your thoughtfulness, rigor, technical proficiency, and ability to tell the story. Clearly state and justify the assumptions you make. Acknowledge any limitations or biases in your analysis.

3. (Optional) Appendix: Technical or institutional details.

4. References: If you use information from external sources, cite them clearly.

5. Tables and Figures: You may include tables and figures and reference them in the main text of the report. Tables and figures should include descriptive captions and legends. They should be as self-contained as possible; do not copy tables and figures directly from Excel or statistical packages. Number tables and figures sequentially.

Your report must adhere to the following formatting guidelines:

1. Single-sided letter-size paper.
2. 1" margins top, bottom, left, and right.
3. 12-point serif font, such as Times New Roman, Book Antiqua, and Georgia.
4. 10 or fewer pages in the main text.
5. Double spacing.
6. Each page should be numbered, in the bottom center of the page.