Math 6205 - Numerical Methods for Financial Derivatives
Fall 2019
HW 9

Output:

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In [1]: runfile('/Users/AkshayPatil/Desktop/NMFD/hw9_PatilAkshay/main.py', wdir='/Users/AkshayPatil/
NMFD/hw9_PatilAkshay')


===============The output will be displayed in 3 minutes and 30 seconds====================

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  Finite Difference Methods (FDMs) for European Call Option

             FDM    Algorithm  European Call  Error_Call
0         Explicit  Explicit      23.686678    0.040491
1         Implicit    Thomas      23.690786    0.036383
2         Implicit       SOR      23.690801    0.036368
3  Crank-Nicholson    Thomas      23.708322    0.018847
4  Crank-Nicholson       SOR      23.708322    0.018847

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 Finite Difference Methods (FDMs) for European Put Option

             FDM    Algorithm  European Put  Error_Put
0         Explicit  Explicit     22.701779   0.040274
1         Implicit    Thomas     22.705261   0.036792
2         Implicit       SOR     22.705774   0.036279
3  Crank-Nicholson    Thomas     22.723110   0.018943
4  Crank-Nicholson       SOR     22.723112   0.018941

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 Finite Difference Methods (FDMs) for American Options

             FDM    Algorithm  American Call  American Put
0         Explicit  Explicit      23.698560     22.841751
1         Implicit   Brennan      23.702505     22.831401
2         Implicit      PSOR      23.703203     22.835309
3  Crank-Nicholson   Brennan      23.720096     22.853696
4  Crank-Nicholson      PSOR      23.720130     22.854883


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 2. Closed-form solution for European Options

    European Call  European Put
0      23.727169     22.742053

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Analysis:

The objective of this Python program is to compute the prices of European and American calls and puts using Finite Difference Methods (FDMs) and the closed form solutions. Every method has its pros and cons. We have used Explicit, Implicit and Crank-Nicholson discretization methods to solve using differing algorithms such as Thomas, Brennan-Schwartz, SOR and PSOR. We know that American options are costlier than European counterparts and we validated using different methods. Also, we have calculated the absolute difference compared to the closed form, Black Scholes solution.

In explicit method, we discretize the heat equation. Once we discretize the heat equation, we create the mesh for space and time and then use the mesh as a tridiagonal system. The tridiagonal system can be solved by using algorithms such as Thomas and SOR for European options and Brennan-Schwartz and PSOR for American options

Thomas and Brennan-Schwartz algorithm is a simplified version of the Gaussian elimination. This algorithm involves two core substitutions, a forward loop and a backward loop in which we calculate a multiplier and use it as required. This algorithm can be used to solve any generate tridiagonal matrix.

SOR and PSOR algorithms are an extension of Gauss-Seidel algorithm which is an extension of Jacobi algorithm. They are iterative method for solving linear system of equations. SOR algorithm is very useful is pricing European style options, while PSOR algorithm is very useful in pricing American Style options. We use a relaxation parameter and a convergence condition so that the iterative solution converges. This algorithm can be used to solve any general tridiagonal matrix.