UNC CHARLOTTE

AN APPLICATION OF TAGUCHI L9 METHOD IN BLACK SCHOLES MODEL FOR EUROPEAN CALL OPTION

A report submitted to Dr. Shaoyu Li

Applied Statistics 2

Spring 2019

Author: Akshay D Patil

ABSTRACT:

The Black Scholes Model (BSM) is an important tool in financial economics in order to measures the option price at some future date at some defined price. This study is based on the design of experiment (DOE) on option pricing model at one period. DOE is a set of relations between the inputs and outputs variables. This study is based on an application of Taguchi orthogonal array L9, in which the four parameters of BSM for European call option, is varied at three different levels. The aim of the experiment is to get the more and maximum realistic information regarding the input and output variables. The four parameters of BSM at one period are; the underlying asset price S_0*, the strike price* K*, the interest rate* r *and the volatility* σ*. The main aims of this study are: a) to determine which parameter impacts more or less on European call option at one period, b) the percentage contribution of each parameter and c) it discusses the section of the parameters for obtaining the best combination d) and whether the value of a call option follows a certain distribution.*

KEYWORDS: BSM, Taguchi method, ANOM, ANOVA, Mean Effect Plot, Residual Plot.

INTRODUCTION

European call option is a derivative that gives the keeper the rights but not obligation to buy the underlying asset at the defined price at expiry date. In order to estimate the value of call option at time t=0 the Black Scholes formula is used. Black and Scholes developed a formula for European option without dividend paying in 1973. The parameters that are necessary to estimate the value of European call option using Black Scholes formula at one period are: *the underlying asset price* S_0*, the strike price* K*, the interest rate* r *and the volatility* σ. For an owner of a European call option, it is better to know which input parameters effects more/less on call option. In order to know about the input parameters effects on response variable, Design of Experiment approach can be applied. The DOE gives the relation between the input and output variables.

There are two main approaches to DOE, Full Factorial design (FFD) and the Taguchi's method. FFD is a set of an experiment whose design consists of more than one factors each with discrete possible level and whose experiment units takes all possible combinations of all those levels across all such factors. For example, if there are K factors each at 3 levels, FFD has 3^K runs. This for 4 factors at 3 levels it would take 81 trials runs.

The Taguchi method is a statistical tool developed by Genier Taguchi (1940's) a Japanese engineer, proposed a model for experiment design. Its main goal is to study the whole process parameters with only minimum balanced trials, called orthogonal array. These standard arrays stipulates the way of conducting the minimal number of experiments which could give the full information of all the factors that affect the performance parameter. The crux of the orthogonal arrays method lies in choosing the level combinations of the input design variables for each experiment. The main aim of DOE (Taguchi's method) is to look which parameter (input) effects more on the response variable. Instead of having all possible experiment like FFD, Taguchi model provides a minimum number of experiments. In case of 4 factors and 3 levels, it would take 9 trials runs. The experiments are not randomly generated but they are based on judgmental sampling. It reduces time, resources and cost.

In this paper, the Taguchi method is used in order to do an experiment on European call option at one period. The report looks at the effects of four parameters in the BSM-European call option at one period, the underlying asset price S_0, the strike price K, the interest rate r and the volatility σ. The Taguchi's orthogonal array L9(3^4) is used in order to estimate the factors that influence the performance criteria and also which factors are more important than others. The Analysis of Mean (ANOM) and Analysis of variance (ANOVA) is used in order to get the objectives of this paper.

OBJECTIVES:

The objectives of this study are:

1. To identify the best level for each parameter.
2. To measures which factors are more important than others.
3. To check whether the value of a call option follows a certain distribution.
4. To measure the percentage contribution of each parameter.

METHODOLOGY:

EUROPEAN CALL OPTION

European call option: “It is a contract that gives rights to the owner, but not obligation to buy the underlying asset (S_0) at a specified price (strike price K) within a specified time (T). It will not exercise before the maturity date. The buyer of the call option believes that the price of an underlying asset goes up in a future date. In this case, the buyer of the call option will decide whether to exercise or not because he is having the rights. At the expiry date T, there are two possibilities a) if the price of the underlying asset S_T is less than strike price K. Then the call option is not exercised and b) if the price of an underlying asset S_T at maturity time is greater than the strike price K, then he will exercise it, i.e. the holder buys the underlying asset at price K and sells it to the market at a price S_T”. From above both cases, the payoff at maturity date is:

$$\max\,(S_T - K, 0)$$

The Black and Scholes developed a formula in order to estimate the values of European call and put option at time in 1973.

The Black-Scholes formula for European call option without dividend paying is:

$$\mathrm{f}(\mathrm{t}, S_T) = \mathrm{S_t} * \mathrm{N}(D_1) - \mathrm{K} * \mathrm{e}^{-\mathrm{r}(\mathrm{T}-\mathrm{t})} * \mathrm{N}(D_2) \qquad [1]$$

Where N (*) is the standard cumulative distribution function

$$D_1 = \frac{[\ln\left(\frac{\mathrm{S_0}}{\mathrm{K}}\right) + \left(\mathrm{r} - 1/2\sigma^2\mathrm{T}\right)]}{\sigma\sqrt{\mathrm{T}}} \text{ and } D_2 = D_1 - \sigma\sqrt{\mathrm{T}}$$

“S_T is the price of an underlying asset at time t, K is the strike price, r is the risk-free rate of the interest, $(\mathrm{T} - \mathrm{t})$ is the time to maturity, σ is the volatility of the return of the underlying asset, N (*) is the cumulative distribution function of the standard normal distribution”.

Taguchi's Method

The aim of this paper is to do an experiment on the European call option at one period at three levels. The Taguchi's method (L9 (3^4)) is used to run the trials. In Taguchi's method, only 9 experiments are used instead of 81 as per FFD. The data in Table 1 gives the complete information to measure the value of the call option by using Black Scholes formula. Finally, the ANOM is used to look which factor effects more or less and ANOVA gives the percentage contribution of the variables on European call option.

DATA FOR CALCULATING THE CALL OPTION VALUE BY USING BSM

Levels|Parameters	S_0	K	r	σ
1	130	100	5%	20%
2	140	105	6%	21%
3	150	110	7%	22%

Table 1

In order to develop a DOE by using Taguchi method. The following points are necessary:

1. Define the response variable, in this study the response variable is the value of the European call option.
2. Select the input variables, there are four input variables that are: underlying asset, strike price, interest rate, and volatility in order to estimate the value of call option.
3. Select the number of levels, in this study we choose three levels as shown in table 1.
4. Select the orthogonal array, it is based on the 2nd and 3rd point. In this study, we want to conduct an experiment in order to understand the influence of four independent with each having three set values on a call option, then L9 orthogonal array might be the right choice. It allows us to consider a selected subset of combinations of multiple factors at multiple levels.
5. Assigning the four independent variables to each column
6. Conduct the experiment
7. Analysis the data (the call option)

The experiment runs with four parameters at three levels are determined by using the Taguchi L9 orthogonal array. The experiment layouts for call option process parameters by using Taguchi L9 approach shown in Table 2. These experiments are not randomly selected but it is based on well-defined procedure or sampling.

TAGUCHI'S L9(3^4) ORTHOGONAL ARRAY FOR EUROPEAN CALL OPTION USING BSM

Experiment	S_0	K	r	σ	$f(t, S_T)$
1	130	100	0.05	0.20	35.44027
2	130	105	0.06	0.21	32.18835
3	130	110	0.07	0.22	29.25158
4	140	100	0.06	0.22	46.18057
5	140	105	0.07	0.20	42.44147
6	140	110	0.05	0.21	36.32111
7	150	100	0.07	0.21	56.86112
8	150	105	0.05	0.22	50.45923
9	150	110	0.06	0.20	46.71759

Table 2

RESULT, ANALYSIS AND DISCUSSION

Regression Equation:

$$\boldsymbol{Call\ value = -10.79 + 0.953 * S_0 - 0.873 * K + 105.56 * r + 21.53 * \sigma}$$

$$\boldsymbol{R\text{-}sq = 99.98\% \qquad Adj\ R\text{-}Sq = 99.88\%}$$

In this model R-sq value is 99.98% which indicates that the fit of the experimental data is satisfactory. The R-sq is approximately equal to 1 that means the regression line perfectly fits the data.

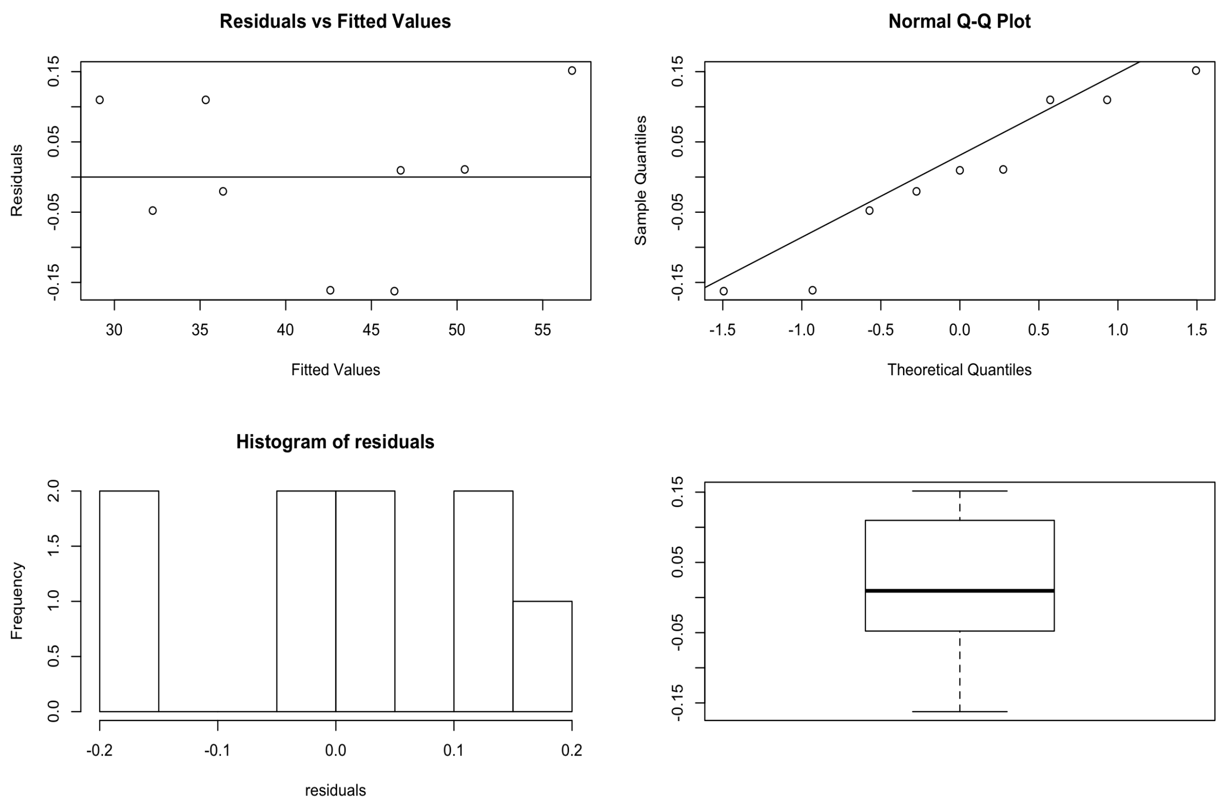

Figure 1

RESIDUAL PLOTS FOR CALL OPTION VALUE

The following points explain the figure 1

a. In normal probability plot, there is no outliers exist which means that the values of call options (data) follow a normal distribution and the factors are influencing the response.
b. In versus fits plot (fitted value vs. residual), the plot indicates that the relationship between the data is non-linear and the variance is constant.
c. The Histogram indicates that there is no outlier exists and the data are not skewed.

Analysis of Mean (ANOM)

The response mean (ANOM) is the average response for each combination of control parameters (factors) levels in a statistic Taguchi method. The aim of this method is to identify which parameter effects more on European call option and also it determines the best combination where the European call option gets maximum value. For each parameter, the average of the response (call option value) at each level of the parameter is calculated and tabulated below in Table 3. The delta identifies the size of effect by the taking the difference between the highest and the lowest value of average for a parameter and the rank in the response Table 3 helps us to identify which parameter effects more. The parameter with the highest delta value is given rank 1, the parameter with the second highest delta is given rank 2, and so on.

Response Table for Mean (ANOM)

Level	S_0	**K**	**r**	**σ**
1	32.2934	**46.16065**	40.7402	41.53311
2	41.64772	41.69635	41.6955	41.79019
3	**51.34598**	37.43009	**42.85139**	**41.96379**
Range	19.05258	8.73056	2.111187	0.430683
Rank	1	2	3	4

Table 3

The selected numbers (bold) in Table 4 are the maximum in every column, as per range we set the ranking for all the parameters (higher range=rank 1 and so on). The ANOM gives us an idea about which parameter affect more on option pricing. Table 4 clearly shows that which parameter affects more. The rank indicates that the underlying asset affects more and the volatility affects less on call option. The bolded values are maximum in every column and we also conclude that the best combination is $\boldsymbol{S_0 3 * K1 * r3 * \sigma 3}$ provide us maximum value of call option.

ANALYSIS OF MEAN (ANOVA)

In order to measure the percentage contributions of each independent variable in European call option using BSM. An application of ANOVA is used in order to estimate the percentage of each parameter. It shows the relation between the response variable and the predictor variable. In order to estimate the percentage contribution, we need regression model known as ANOVA. It is defined as "sum of squares of a parameter by total sum of squares". Table 4 shows the contribution of each parameter.

The percentage contribution of the parameters that are shown in Table 4 can be calculated as

$$percentage\ contribution = \frac{sum\ of\ square\ of\ a\ parameter}{total\ sums\ os\ squares}$$

ANOVA TABLE

Source	Adj SS	Adj MS	F-Value	P-Value	Percentage contribution	Rank
$\boldsymbol{S}$	544.501	544.501	21277.24	1.325 e-08	81.77%	1
$\boldsymbol{K}$	114.334	114.334	4467.78	3.001 e-07	17.17%	2
$\boldsymbol{r}$	6.686	6.686	261.25	8.571e-05	1.004%	3
$\boldsymbol{\sigma}$	0.278	0.278	10.87	0.03001	0.0424%	4
Error	0.102	0.026				
Total	665.901					

Table 4

The P-value of all the parameters are less than ($\alpha = 0.05$), because of this we can conclude that there is a statistically significant differ. So, all the factors are effects on call option differently

The percentage (%) numbers represent that the underlying asset price at time t=0, the strike price, the interest rate and the volatility have a significant effect on pricing of European call option using BSM. It can be observed in Table 4 that the underlying asset price at time S_0, the strike price K, the interest rate r and volatility σ effects the call option by 81.77%, 17.17%, 1.004% and 0.0424% respectively are shown in Table 4. (The percentage contribution will vary with the change in data set.)

CONCLUSION

This study investigated that which input parameter effects more/less on European call option. In general, 81 trials were supposed to be conducted. However, only 9 trials were done with the help of Taguchi method.

The conclusion of this study is summarized below:

a. The values of a call option follow a normal distribution because the values approximately in a straight line and no outlier exist.

b. The ANOM is being used in order to identify the best level for every four parameters. The best combination in this study is $\boldsymbol{S_0 3} * \boldsymbol{K1} * \boldsymbol{r3} * \boldsymbol{\sigma 3}$. This combination gives the maximum value of European call option as compared to all other possible combinations.

c. The ANOM table shows us that the underlying asset S0 is having higher delta, so it is given as rank 1. It means that the underlying asset affects the European call option more.

d. The percentage contribution of the underlying asset price S0 at time $t = 0$, the strike price K, the interest rate r and volatility effects the call option by 81.77%, 17.17%, 1.004% and 0.0424% respectively.

In this paper, the Taguchi L9 orthogonal array was successfully applied in order to identify which parameter effects more on European call option using Black Scholes Model.

Ranks

Rank = 1	Rank = 2	Rank = 3	Rank = 4
Underlying Asset (S)	Strike Price (K)	Interest Rate (r)	Volatility (σ)

==

```
#Applied Statistics 2 - Project Code

#Read Data
> L9_array = read.table('l9_ortho_array.txt',header=T)
> L9_array
    s   k    r    v        y
1 130 100 0.05 0.20 35.44027
2 130 105 0.06 0.21 32.18835
3 130 110 0.07 0.22 29.25158
4 140 100 0.06 0.22 46.18057
5 140 105 0.07 0.20 42.44147
6 140 110 0.05 0.21 36.32111
7 150 100 0.07 0.21 56.86112
8 150 105 0.05 0.22 50.45923
9 150 110 0.06 0.20 46.71759
>

"y = Call option value, s= Underlying asset price, k = Strike Price, r = Risk free
 interest Rate, v = Volatility."

> y=L9_array$y
> s=L9_array$s
> k=L9_array$k
> r=L9_array$r
> v=L9_array$v
> y
[1] 35.44027 32.18835 29.25158 46.18057 42.44147 36.32111 56.86112 50.45923 46.71759
> s
[1] 130 130 130 140 140 140 150 150 150
> k
[1] 100 105 110 100 105 110 100 105 110
> r
[1] 0.05 0.06 0.07 0.06 0.07 0.05 0.07 0.05 0.06
> v
[1] 0.20 0.21 0.22 0.22 0.20 0.21 0.21 0.22 0.20

#Regression

> ls=lm(y~s+k+r+v)
> summary(ls)

Call:
lm(formula = y ~ s + k + r + v)

Residuals:
        1         2         3         4         5         6         7         8 
 0.109849 -0.047726  0.109849 -0.162417 -0.161147 -0.020382  0.151591  0.010826 
        9 
 0.009556 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
```

```
(Intercept) -10.790549   2.180413  -4.949  0.00777 **
s             0.952629   0.006531 145.867 1.32e-08 ***
k            -0.873056   0.013062 -66.841 3.00e-07 ***
r           105.559333   6.530797  16.163 8.57e-05 ***
v            21.534167   6.530797   3.297  0.03001 *
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.16 on 4 degrees of freedom
Multiple R-squared:  0.9998,    Adjusted R-squared:  0.9997
F-statistic:  6504 on 4 and 4 DF,  p-value: 7.088e-08

#ANOVA
> anova(ls)
Analysis of Variance Table

Response: y
          Df Sum Sq Mean Sq   F value    Pr(>F)
s          1 544.50  544.50 21277.238 1.325e-08 ***
k          1 114.33  114.33  4467.781 3.001e-07 ***
r          1   6.69    6.69   261.253 8.571e-05 ***
v          1   0.28    0.28    10.872   0.03001 *
Residuals  4   0.10    0.03
Total      8 665.9   665.83
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
>


##Residual Analysis
residuals= ls$res
fits = ls$fits
plot(fits,residuals,main = 'Residuals vs Fitted Values', xlab='Fitted Values',
 ylab='Residuals')
abline(h=0)

#To check for outliers and Distribution of values of call option.
par(mfrow=c(2,2))
fits = ls$fitted
plot(fits,residuals,main = 'Residuals vs Fitted Values', xlab='Fitted Values',
 ylab='Residuals')
abline(h=0)
qqnorm(residuals)
qqline(residuals)
hist(residuals)
boxplot(residuals)
```