1-1.5-27

AI24BTECH11002 - K.AKSHAY TEJA

Question:

Show that the points
$$\mathbf{P} = \begin{pmatrix} -2 \\ 3 \\ 5 \end{pmatrix}$$
, $\mathbf{Q} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ and $\mathbf{R} = \begin{pmatrix} 7 \\ 0 \\ -1 \end{pmatrix}$ are collinear.

Solution:

Variable	Description
Point P	$\begin{pmatrix} -2\\3\\5 \end{pmatrix}$
Point Q	$\begin{pmatrix} 1\\2\\3 \end{pmatrix}$
Point R	$\begin{pmatrix} 7 \\ 0 \\ -1 \end{pmatrix}$

TABLE 0: Coordinates of Points P, Q and R

Points P, Q and R are collinear if

$$rank \begin{pmatrix} \mathbf{P} & \mathbf{Q} & \mathbf{R} \end{pmatrix}^{\mathsf{T}} = 2 \tag{0.1}$$

$$\implies \begin{pmatrix} -2 & 3 & 5 \\ 1 & 2 & 3 \\ 7 & 0 & -1 \end{pmatrix} \xrightarrow{R_2 \leftarrow 2R_2 + R_3} \begin{pmatrix} -2 & 3 & 5 \\ 0 & 7 & 11 \\ 7 & 0 & -1 \end{pmatrix} \tag{0.2}$$

$$\stackrel{R_3 \leftarrow 2R_3 + 7R_1}{\longleftrightarrow} \begin{pmatrix} -2 & 3 & 5 \\ 0 & 7 & 11 \\ 0 & 21 & 33 \end{pmatrix} \stackrel{R_3 \leftarrow R_3 - 3R_2}{\longleftrightarrow} \begin{pmatrix} -2 & 3 & 5 \\ 0 & 7 & 11 \\ 0 & 0 & 0 \end{pmatrix}$$
(0.3)

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Points P, Q and R

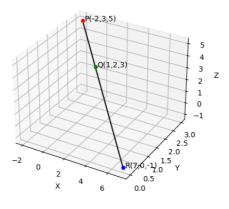


Fig. 0.1: Plot of Points P, Q and R