2023-Apr-11 Shift-2

AI24BTECH11002 - K. Akshay Teja

1) The angle of elevation of the top P of a tower from the feet of one person standing due South of the tower is 45° and from the feet of another person standing due West of the tower is 30°. If the height of the tower is 5 meters, then the distance ($in\,meters$) between the two persons is equal to

a) 10 b) $5\sqrt{5}$ c) $\frac{5}{2}\sqrt{5}$ d) 5

2) Let a, b, c and d be positive real numbers such that $a + b + c + d = 11$. If the maximum value of $a^5b^3c^2d$ is 3750β, then the value of β is

a) 55 b) 108 c) 90 d) 110

3) Let $f : R \to R$ be a continuous function satisfying $\int_0^{\frac{\pi}{2}} f(\sin 2x)\sin x\,dx + \alpha \int_0^{\frac{\pi}{4}} f(\cos 2x)\cos x\,dx = 0$, then the value of α is

a) $-\sqrt{3}$ b) $\sqrt{3}$ c) $-\sqrt{2}$ d) $\sqrt{2}$

4) Let f and g be two functions defined by $f(x) = \begin{cases} x+1, & x < 0 \\ |x-1|, & x \geq 0 \end{cases}$ and $g(x) = \begin{cases} x+1, & x < 0 \\ 1, & x \geq 0 \end{cases}$

Then $(gof)(x)$ is

a) continuous everywhere but not differentiable at $x = 1$
b) continuous everywhere but not differentiable exactly at one point
c) differentiable everywhere
d) not continuous at $x = -1$

5) If the radius of the largest circle with center $(2, 0)$ inscribed in the ellipse $x^2 + 4y^2 = 36$ is r, then $12r^2$ is equal to

a) 69 b) 72 c) 115 d) 92

6) Let the mean of 6 observations $1, 2, 4, 5, x$, and y be 5 and their variance be 10. Then their mean deviation about the mean is equal to

a) $\frac{7}{3}$ b) $\frac{10}{3}$ c) $\frac{8}{3}$ d) 3

7) Let $A = \{1, 3, 4, 6, 9\}$ and $B = \{2, 4, 5, 8, 10\}$. Let R be a relation defined on $A \times B$ such that $R = \{((a_1, b_1), (a_2, b_2)) : a_1 \leq b_2 \text{ and } b_1 \leq a_2\}$. Then the number of elements in the set R is

a) 52 b) 160 c) 26 d) 180

8) Let P be the plane passing through the points $(5, 3, 0)$, $(13, 3, -2)$, and $(1, 6, 2)$. For $\alpha \in \mathbb{N}$, if the distances of the points $A(3, 4, \alpha)$ and $B(2, \alpha, a)$ from the plane P are 2 and 3 respectively, then the positive value of a is

a) 5 b) 6 c) 4 d) 3

9) If the letters of the word MATHS are permuted and all possible words so formed are arranged as in a dictionary with serial number, then the serial number of the word THAMS is

a) 102 b) 103 c) 101 d) 104

10) f four distinct points with position vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ and $\vec{d}$ are coplanar, then $\left[\vec{a}\ \vec{b}\ \vec{c}\right]$ is equal to

a) $\left[\vec{d}\ \vec{c}\ \vec{a}\right] + \left[\vec{b}\ \vec{d}\ \vec{a}\right] + \left[\vec{c}\ \vec{d}\ \vec{b}\right]$
b) $\left[\vec{d}\ \vec{b}\ \vec{a}\right] + \left[\vec{a}\ \vec{c}\ \vec{d}\right] + \left[\vec{d}\ \vec{b}\ \vec{c}\right]$
c) $\left[\vec{a}\ \vec{d}\ \vec{b}\right] + \left[\vec{d}\ \vec{c}\ \vec{a}\right] + \left[\vec{d}\ \vec{b}\ \vec{c}\right]$
d) $\left[\vec{b}\ \vec{c}\ \vec{d}\right] + \left[\vec{d}\ \vec{a}\ \vec{c}\right] + \left[\vec{d}\ \vec{b}\ \vec{a}\right]$

11) The sum of the coefficients of three consecutive terms in the binomial expansion of $(1 + x)^{n+2}$, which are in the ratio 1 : 3 : 5, is equal to

a) 63 b) 92 c) 25 d) 41

12) Let $y = y(x)$ be the solution of the differential equation $\frac{dy}{dx} + \frac{5}{x\left(x^5+1\right)}y = \frac{\left(x^5+1\right)^2}{x^2}$, $x > 0$. If $y(1) = 2$, then $y(2)$ is equal to

a) $\frac{693}{128}$ b) $\frac{637}{128}$ c) $\frac{697}{128}$ d) $\frac{679}{128}$

13) The converse of $\sim (p \wedge q) \implies r$ is

a) $(p \vee (\sim q)) \implies (\sim r)$
b) $((\sim p) \vee q) \implies r$
c) $(\sim r) \implies ((\sim p) \wedge q)$
d) $(\sim r) \implies p \wedge q$

14) If the 1011th term from the end in the binomial expansion of $\left(\frac{4x}{5} - \frac{5}{2x}\right)^{2022}$ is 1024 times the 1011th term from the beginning, then $|x|$ is equal to

a) 8 b) 12 c) 10 d) 15

15) If the system of linear equations
$7x + 11y + \alpha z = 13$
$5x + 4y + 7z = \beta$
$175x + 194y + 57z = 361$ has infinitely many solutions, then $\alpha + \beta + 2$ is equal to:

a) 3 b) 6 c) 5 d) 4