

4-4.2-13

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Question:

Show that the vectors $\mathbf{a} = \hat{i} - 2\hat{j} + 3\hat{k}$, $\mathbf{b} = 2\hat{i} + 3\hat{j} - 4\hat{k}$, $\mathbf{c} = \hat{i} - 3\hat{j} + \hat{k}$ are not coplanar.

Solution: The given vectors are coplanar if

Vector	Description
Vector A	$\begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$
Vector B	$\begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix}$
Vector C	$\begin{pmatrix} 1 \\ -3 \\ 1 \end{pmatrix}$

TABLE 0: Position Vectors

$$\text{Rank}(\mathbf{A} \ \mathbf{B} \ \mathbf{C})^T \neq 2 \quad (0.1)$$

$$\Rightarrow \begin{pmatrix} 1 & -2 & 3 \\ 2 & 3 & -4 \\ 1 & -3 & 5 \end{pmatrix} \xleftrightarrow{R_2 \leftarrow R_2 - 2R_1} \begin{pmatrix} 1 & -2 & 3 \\ 0 & 7 & -10 \\ 1 & -3 & 5 \end{pmatrix} \quad (0.2)$$

$$\xleftrightarrow{R_3 \leftarrow R_3 - R_1} \begin{pmatrix} 1 & -2 & 3 \\ 0 & 7 & -10 \\ 0 & -1 & 2 \end{pmatrix} \xleftrightarrow{R_2 \leftarrow \frac{1}{7}R_2} \begin{pmatrix} 1 & -2 & 3 \\ 0 & 1 & -\frac{10}{7} \\ 0 & -1 & 2 \end{pmatrix} \quad (0.3)$$

$$\xleftrightarrow{R_3 \leftarrow R_3 - R_2} \begin{pmatrix} 1 & -2 & 3 \\ 0 & 1 & -\frac{10}{7} \\ 0 & 0 & \frac{4}{7} \end{pmatrix} \quad (0.4)$$

The rank of the matrix is 3. Therefore the given vectors are not coplanar.

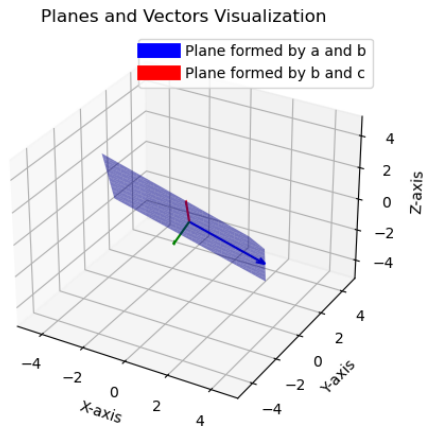


Fig. 0.1: Line and Vectors