1-1.5-27

AI24BTECH11002 - K.AKSHAY TEJA

Question:

Show that the points $\mathbf{P} = \begin{pmatrix} -2 \\ 3 \\ 5 \end{pmatrix}, \mathbf{Q} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ and $\mathbf{R} = \begin{pmatrix} 7 \\ 0 \\ -1 \end{pmatrix}$ are collinear.

Solution:

Variable	Description
P	$(-2, 3, 5)$
Q	$(1, 2, 3)$
R	$(7, 0, -1)$

TABLE 0: Coordinates of points P, Q and R

Points $\mathbf{P}, \mathbf{Q}$and$\mathbf{R}$ are collinear if

$$\text{rank}\begin{pmatrix} \mathbf{P} & \mathbf{Q} & \mathbf{R} \end{pmatrix}^{\top} = 2 \tag{0.1}$$

$$\implies \begin{pmatrix} -2 & 3 & 5 \\ 1 & 2 & 3 \\ 7 & 0 & -1 \end{pmatrix} \xleftrightarrow{R_2 \to 2R_2 + R_3} \begin{pmatrix} -2 & 3 & 5 \\ 0 & 7 & 11 \\ 7 & 0 & -1 \end{pmatrix} \tag{0.2}$$

$$\xleftrightarrow{R_3 \to 2R_3 + 7R_1} \begin{pmatrix} -2 & 3 & 5 \\ 0 & 7 & 11 \\ 0 & 21 & 33 \end{pmatrix} \xleftrightarrow{R_3 \to R_3 - 3R_2} \begin{pmatrix} -2 & 3 & 5 \\ 0 & 7 & 11 \\ 0 & 0 & 0 \end{pmatrix} \tag{0.3}$$

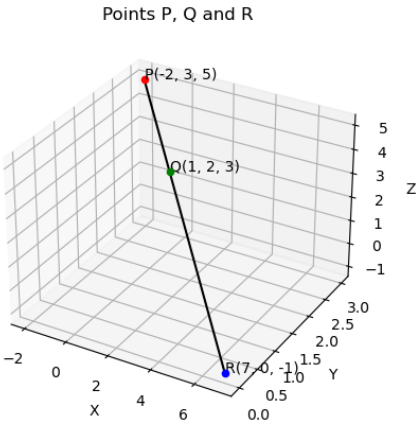

Fig. 0.1: Plot of points P, Q and R