2023-Jan-30 Shift-2

AI24BTECH11002 - K. Akshay Teja

1) Consider the following statements:
P : I have fever
Q : I will not take medicine
R : I will take rest
The statement "If I have fever, then I will take medicine and I will take rest" is equivalent to:

a) $((-P) \vee -Q) \wedge ((-P) \vee R)$
b) $((-P) \vee -Q) \wedge ((-P) \vee -R)$
c) $(P \vee Q) \wedge ((-P) \vee R)$
d) $(P \vee -Q) \wedge (P \vee -R)$

2) Let A be a point on the x-axis. Common tangents are drawn from A to the curves $x^2 + y^2 = 8$ and $y^2 = 16x$. If one of these tangents touches the two curves at Q and R, then $(QR)^2$ is equal to

a) 64
b) 76
c) 81
d) 72

3) Let q be the maximum integral value of p in $[0, 10]$ for which the roots of the equation $x^2 - px + \frac{5}{4}p = 0$ are rational. Then the area of the region $(x, y) : 0 \le y \le (x - q)^2, 0 \le x \le q$ is

a) 243
b) 25
c) $\frac{125}{3}$
d) 164

4) If the functions $f(x) = \frac{x^3}{3} + 2bx + a\frac{x^2}{2}$ and $g(x) = \frac{x^3}{3} + ax + bx^2$, $a \neq 2b$, have a common extreme point, then $a + 2b + 7$ is equal to

a) 4
b) $\frac{3}{2}$
c) 3
d) 6

5) The range of the function $f(x) = \sqrt{3 - x} + \sqrt{2 + x}$ is

a) $\left[\sqrt{5}, \sqrt{10}\right]$
b) $\left[2\sqrt{2}, \sqrt{11}\right]$
c) $\left[\sqrt{5}, \sqrt{13}\right]$
d) $\left[\sqrt{2}, \sqrt{7}\right]$

6) The solution of the differential equation $\frac{dy}{dx} = -\left(\frac{x^2+3y^2}{3x^2+y^2}\right)$, $y(1) = 0$ is

a) $\log_e |x + y| - \frac{xy}{(x+y)^2} = 0$
b) $\log_e |x + y| + \frac{xy}{(x+y)^2} = 0$
c) $\log_e |x + y| + \frac{2xy}{(x+y)^2} = 0$
d) $\log_e |x + y| - \frac{2xy}{(x+y)^2} = 0$

7) Let $x = \left(8\sqrt{3} + 13\right)^{13}$ and $y = \left(7\sqrt{2} + 9\right)^9$. If $[t]$ denotes the greatest integer $\le t$, then

a) $[x] + [y]$ is even
b) $[x]$ is odd but $[y]$ is even
c) $[x]$ is even but $[y]$ is odd
d) $[x]$ and $[y]$ are both odd

8) A vector v in the first octant is inclined to the x-axis at 60°, to the y-axis at 45° and to the z-axis at an acute angle. If a plane passing through the points $\left(\sqrt{2}, -1, 1\right)$ and (a, b, c) is normal to $\vec{v}$, then

a) $\sqrt{2}a + b + c = 1$
b) $a + b + \sqrt{2}c = 1$
c) $a + \sqrt{2}b + c = 1$
d) $\sqrt{2}a - b + c = 1$

9) Let f, g and h be the real valued functions defined on $\mathbb{R}$ as $f(x) = \begin{pmatrix} \frac{x}{|x|}, & x \neq 0 \\ 1, & x = 0 \end{pmatrix}$ $g(x) = \begin{pmatrix} \frac{\sin(x+1)}{(x+1)}, & x \neq -1 \\ 1, & x = -1 \end{pmatrix}$ and $h(x) = 2[x] - f(x)$, where $[x]$ is the greatest integer $\leq x$. The value of $\lim_{x \to -1} g(h(x-1))$ is

a) 1
b) $\sin(1)$
c) -1
d) 0

10) The number of ways of selecting two numbers a and b, where $a \in \{2, 4, 6, \ldots, 100\}$ and $b \in \{1, 3, 5, \ldots, 99\}$ such that 2 is the remainder when $a + b$ is divided by 23 is

a) 186
b) 54
c) 108
d) 268

11) If P is a 3×3 real matrix such that $P^T = aP + (a-1)I$, where $a > 1$, then

a) P is a singular matrix
b) $\left|\text{Adj } P\right| > 1$
c) $\left|\text{Adj } P\right| = 1$
d) $\left|\text{Adj } P\right| < 1$

12) Let $\lambda \in \mathbb{R}, \vec{a} = \lambda\hat{i} + 2\hat{j} - 3\hat{k}, \vec{b} = \hat{i} - \lambda\hat{j} + 2\hat{k}$. If $\left(\vec{a} + \vec{b}\right) \times \left(\vec{a} \times \vec{b}\right) = 8\hat{i} - 40\hat{j} - 24\hat{k}$, then $\left|\lambda\left(\vec{a} + \vec{b}\right) \times \left(\vec{a} - \vec{b}\right)\right|^2$ is equal to

a) 140
b) 132
c) 144
d) 136

13) Let $\vec{a}$ and $\vec{b}$ be two vectors. Let $\left|\vec{a}\right| = 1, \left|\vec{b}\right| = 4$ and $\vec{a} \cdot \vec{b} = 2$. If $\vec{c} = 2\vec{a} - 3\vec{b}$, then the value of $\vec{b} \cdot \vec{c}$ is

a) -24
b) -48
c) -84
d) -60

14) Let $a_1 = 1, a_2, a_3, a_4, \ldots$ be consecutive natural numbers. Then $\tan^{-1}\left(\frac{1}{1+a_1a_2}\right) + \tan^{-1}\frac{1}{1+a_2a_3} + \ldots + \tan^{-1}\left(\frac{1}{1+a_{2021}a_{2022}}\right)$ is equal to

a) $\frac{\pi}{4} - \cot^{-1}(2022)$
b) $\cot^{-1}(2022) - \frac{\pi}{4}$
c) $\tan^{-1}(2022) - \frac{\pi}{4}$
d) $\frac{\pi}{4} - \tan^{-1}(2022)$

15) The parabolas: $ax^2 + 2bx + cy = 0$ and $dx^2 + 2ex + fy = 0$ intersect on the line $y = 1$. If a, b, c, d, e, f are positive real numbers and a, b, c are in G.P., then

a) d, e, f are in A.P.
b) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in G.P.
c) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in A.P.
d) d, e, f are in G.P.