

# 2020-Sep-3 Shift-2

AI24BTECH11002 - K. Akshay Teja

- 1) If  $x^3 dy + xy dx = x^2 dy + 2y dx$ ;  $y(2) = e$  and  $x > 1$ , then  $y(4)$  is equal to:
- a)  $\frac{\sqrt{e}}{2}$       b)  $\frac{3}{2}\sqrt{e}$       c)  $\frac{1}{2} + \sqrt{e}$       d)  $\frac{3}{2} + \sqrt{e}$
- 2) Let  $A$  be a  $3 \times 3$  matrix such that  $\text{adj} A = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 0 & 2 \\ 1 & -2 & -1 \end{pmatrix}$  and  $B = \text{adj}(\text{adj} A)$ . If  $|A| = \lambda$  and  $|(B^{-1})^T| = \mu$ , then the ordered pair,  $(|\lambda|, \mu)$  is equal to:
- a)  $(9, \frac{1}{81})$       c)  $(3, \frac{1}{81})$       b)  $(9, \frac{1}{9})$       d)  $(3, 81)$
- 3) Let  $a, b, c \in \mathbb{R}$  be such that  $a^2 + b^2 + c^2 = 1$ , if  $a \cos \theta = b \cos(\theta + \frac{2\pi}{3}) = c \cos(\theta + \frac{4\pi}{3})$ , where  $\theta = \frac{\pi}{9}$ , then the angle between the vectors  $a\hat{i} + b\hat{j} + c\hat{k}$  and  $b\hat{i} + c\hat{j} + a\hat{k}$  is:
- a)  $\frac{\pi}{2}$       b)  $\frac{2\pi}{3}$       c)  $\frac{\pi}{9}$       d) 0
- 4) Suppose  $f(x)$  is a polynomial of degree four, having critical points at  $(-1, 0, 1)$ . If  $T = \{x \in \mathbb{R} \mid f(x) = f(0)\}$ , then the sum of squares of all the elements of  $T$  is:
- a) 6      b) 2      c) 8      d) 4
- 5) If the value of the integral  $\int_0^{\frac{1}{2}} \frac{x^2}{(1-x^2)^{\frac{3}{2}}} dx$  is  $\frac{k}{6}$ , then  $k$  is equal to:
- a)  $2\sqrt{3} + \pi$       c)  $3\sqrt{2} - \pi$       b)  $3\sqrt{2} + \pi$       d)  $2\sqrt{3} - \pi$
- 6) If the term independent of  $x$  in the expansion of  $\left(\left(\frac{3}{2}\right)x^2 - \frac{1}{3x}\right)^9$  is  $k$ , then  $18k$  is equal to:
- a) 5      b) 9      c) 7      d) 11
- 7) If a triangle  $ABC$  has vertices  $A(-1, 7)$ ,  $B(-7, 1)$ , and  $C(5, -5)$ , then its orthocentre has coordinates:
- a)  $(-3, 3)$       c)  $\left(\frac{3}{5}, -\frac{3}{5}\right)$       b)  $\left(-\frac{3}{5}, \frac{3}{5}\right)$       d)  $(3, -3)$
- 8) Let  $e_1$  and  $e_2$  be the eccentricities of the ellipse,  $\frac{x^2}{25} + \frac{y^2}{b^2} = 1$  (where  $b < 5$ ) and the hyperbola,  $\frac{x^2}{16} - \frac{y^2}{b^2} = 1$  respectively, satisfying  $e_1 e_2 = 1$ . If  $\alpha$  and  $\beta$  are the distances between the foci of the ellipse and the foci of the hyperbola respectively, then the ordered pair  $(\alpha, \beta)$  is equal to:
- a)  $(8, 12)$       c)  $\left(\frac{20}{3}, 12\right)$       b)  $\left(\frac{24}{5}, 10\right)$       d)  $(8, 10)$
- 9) If  $z_1, z_2$  are complex numbers such that  $\text{Re}(z_1) = |z_1 - 1|$ ,  $\text{Re}(z_2) = |z_2 - 1|$  and  $\arg(z_1 - z_2) = \frac{\pi}{6}$ , then  $\text{Im}(z_1 + z_2)$  is equal to:
- a)  $2\sqrt{3}$       b)  $\frac{2}{\sqrt{3}}$       c)  $\frac{1}{\sqrt{3}}$       d)  $\frac{\sqrt{3}}{2}$
- 10) The set of all real values of  $\lambda$  for which the quadratic equations,  $(\lambda^2 + 1)x^2 - 4\lambda x + 2 = 0$  always have exactly one root in the interval  $(0, 1)$  is:
- a)  $(-3, -1)$       c)  $(1, 3)$       b)  $(2, 4)$       d)  $(0, 2)$
- 11) Let the latus rectum of the parabola  $y^2 = 4x$  be the common chord to the circles  $C_1$  and  $C_2$ , each of them having radius  $2\sqrt{5}$ . Then, the distance between the centres of the circles  $C_1$  and  $C_2$  is:
- a) 8      b)  $8\sqrt{5}$       c)  $4\sqrt{5}$       d) 12
- 12) The plane which bisects the line joining the points  $(4, -2, 3)$  and  $(2, 4, -1)$  at right angles also passes through the point:

- a)  $(0, -1, 1)$                       c)  $(4, 0, -1)$   
 b)  $(4, 0, 1)$                       d)  $(0, 1, -1)$

13)  $\lim_{x \rightarrow a} \frac{(a+2x)^{\frac{1}{3}} - (3x)^{\frac{1}{3}}}{(3a+x)^{\frac{1}{3}} - (4a)^{\frac{1}{3}}}$  is equal to:

- a)  $\frac{2}{9} \left( \frac{4}{3} \right)$                       c)  $\left( \frac{2}{3} \right) \left( \frac{2}{9} \right)^{\frac{1}{3}}$   
 b)  $\frac{2}{3} \left( \frac{4}{3} \right)$                       d)  $\left( \frac{2}{9} \right) \left( \frac{2}{3} \right)^{\frac{1}{3}}$

14) 14. Let  $x_i$  ( $1 \leq i \leq 10$ ) be ten observations of a random variable  $X$ . If  $\sum_{i=1}^{10} (x_i - p) = 3$  and  $\sum_{i=1}^{10} (x_i - p)^2 = 9$  where  $0 \neq p \in \mathbb{R}$ , then the standard deviation of these observations is:

- a)  $\frac{7}{10}$                       b)  $\frac{9}{10}$                       c)  $\sqrt{\frac{3}{5}}$                       d)  $\frac{4}{5}$

15) The probability that a randomly chosen 5-digit number is made from exactly two digits is:

- a)  $\frac{134}{10^4}$                       b)  $\frac{121}{10^4}$                       c)  $\frac{135}{10^4}$                       d)  $\frac{50}{10^4}$