2014-PH-27-39

AI24BTECH11002 - K. Akshay Teja

1) An electron in the ground state of the hydrogen atom has the wave function:

$$\psi\left(\overrightarrow{r}\right) = \frac{1}{\sqrt{\pi a_0^3}} e^{-\left(\frac{r}{a_0}\right)}$$

where a_0 is a constant. The expectation value of the operator $\hat{Q} = z^2 - r^2$, where $z = r\cos\theta$, is (Hint: $\int_0^\infty e^{-ar} r^n dr = \frac{\Gamma(n)}{\alpha^{n+1}} = \frac{(n-1)!}{\alpha^{n+!}}$

a) $-\frac{a_0^2}{2}$ b) $-a_0^2$ c) $-\frac{3a_0^2}{2}$ d) $-2a_0^2$

2) For Nickel, the number density is 8×10^{23} atoms/cm^3 and the electronic configuration is $1s^2 2s^2 2p^6 3s^2 3p^6 3d^8 4s^2$. The value of the saturation magnetization of Nickel in its ferromagnetic state is:
(Given the value of Bohr magneton $\mu_B = 9.21 \times 10^{-21}$Am^2)
3) A particle of mass m is in a potential given by

$$V(r) = -\frac{a}{r} + \frac{ar_0^2}{3r^3}$$

where a and r_0 are positive constants. When disturbed slightly from its stable equilibrium position, it undergoes simple harmonic oscillation. The time period of oscillation is:

a) $2\pi\sqrt{\frac{mr_0^3}{2a}}$ b) $2\pi\sqrt{\frac{mr_0^3}{a}}$ c) $2\pi\sqrt{\frac{2mr_0^3}{a}}$ d) $4\pi\sqrt{\frac{mr_0^3}{a}}$

4) The donor concentration in a sample of n-type silicon is increased by a factor of 100. The shift in the position of the Fermi level at 300K, assuming the sample to be non-degenerate is ($k_B T = 25$ meV at 300K)
5) A particle of mass m is subjected to a potential:

$$V(x, y) = \frac{1}{2}m\omega^2\left(x^2 + y^2\right), -\infty \leq x \leq \infty, -\infty \leq y \leq \infty$$

. The state with energy $4\hbar\omega$ is g-fold degenerate. The value of g is:
6) A hydrogen atom is in the state:

$$\psi = \sqrt{\frac{8}{21}}\psi_{200} - \sqrt{\frac{3}{7}}\psi_{310} + \sqrt{\frac{4}{21}}\psi_{321},$$

where n, l, m in ψ_{nlm} denote the principal, orbital, and magnetic quantum numbers, respectively. If $\overrightarrow{L}$ is the angular momentum operator, the average value of L^2 is h^2
7) A planet of mass m moves in a circular orbit of radius r_0 in the gravitational potential $V(r) = -\frac{k}{r}$, where k is a positive constant. The orbital angular momentum of the planet is:

a) $2r_0km$ b) $\sqrt{2r_0km}$ c) r_0km d) $\sqrt{r_0km}$

8) The moment of inertia of a rigid diatomic molecule A is 6 times that of another rigid diatomic molecule B. If the rotational energies of the two molecules are equal, then the corresponding values of the rotational quantum numbers J_A and J_B are:

a) $J_A = 2,\ J_B = 1$
b) $J_A = 3,\ J_B = 1$
c) $J_A = 5,\ J_B = 0$
d) $J_A = 6,\ J_B = 1$

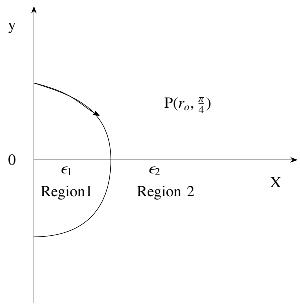
9) The value of the integral:

$$\oint_C \frac{z^2}{e^z + 1}\, dz,$$

where C is the circle $|z| = 4$, is:

a) $2\pi i$ b) $2\pi^2 i$ c) $4\pi^3 i$ d) $4\pi^2 i$

10) A ray of light inside Region 1 in the xy-plane is incident at the semicircular boundary that carries no free charges. The electric field at the point $P(r, \pi/4)$ in plane polar coordinates is $\overrightarrow{E_1} = 7e_0\hat{e}_r - 3e_0\hat{e}_\theta$where $\hat{e}_r$ and $\hat{e}_\theta$ are the unit vectors. The emerging ray in Region 2 has the electric field $\overrightarrow{E_2}$ parallel to the x-axis. If ϵ_1 and ϵ_2 are the dielectric constants of Region 1 and Region 2 respectively, then $\frac{\epsilon_1}{\epsilon_2}$ is



11) The solution of the differential equation:

$$\frac{d^2y}{dt^2} - y = 0$$

subject to the boundary conditions $y(0) = 1$ and $y(\infty) = 0$, is:

a) $\cos t + \sin t$ b) $\cosh t + \sinh t$ c) $\cos t - \sin t$ d) $\cosh t - \sinh t$

12) Given that the linear transformation of a generalized coordinate q and the corresponding momentum p,

$$Q = q + 4ap$$

$$P = q + 2p$$

is canonical, the value of the constant a is:

13) The value of the magnetic field required to maintain non-relativistic protons of energy 1 MeV in a circular orbit of radius 100 mm is:
(Given: $m_p = 1.67 \times 10^{-27}$ kg, e = 1.6 $\times 10^{-19}$ C)