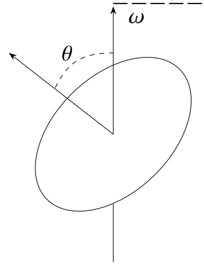
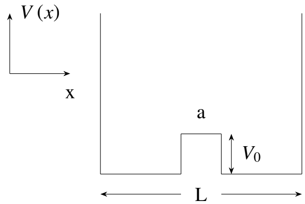
2018-PH-53-65

AI24BTECH11002 - K. Akshay Teja

1) For the transformation
$$Q = \sqrt{2q}\, e^{-1+2\alpha} \cos p,\ P = \sqrt{2q}\, e^{\alpha-1} \sin p$$
(where α is a constant) to be canonical, the value of α is ______. (2018)
2) Given
$$\frac{d^2 f(x)}{dx^2} - 2\frac{df(x)}{dx} + f(x) = 0,$$
and boundary conditions $f(0) = 1$ and $f(1) = 0$, the value of $f(0.5)$ is ______ (up to two decimal places). (2018)
3) The absolute value of the integral
$$\int \frac{5z^3 + 3z^2}{z^2 - 4}\, dz,$$
over the circle $|z - 1.5| = 1$ in the complex plane, is _______ (up to two decimal places).(2018)
4) A uniform circular disc of mass m and radius R is rotating with angular speed ω about an axis passing through its center and making an angle $\theta = 30°$ with the axis of the disc. If the kinetic energy of the disc is $\alpha m \omega^2 R^2$, the value of α is _______ (up to 2 decimal places). (2018)



5) The ground state energy of a particle of mass m in an infinite potential well is E_0. It changes to $E_0\left(1 + \alpha \times 10^{-3}\right)$ when there is a small potential bump of height $V_0 = \frac{\pi^2 \hbar^2}{50mL^2}$ and width $a = \frac{L}{100}$, as shown in the figure. The value of α is _______ (up to two decimal places). (2018)



6) An electromagnetic plane wave is propagating with an intensity $I = 1.0 \times 10^5$ Wm^2 in a medium with $\varepsilon = 3\varepsilon_0$ and $\mu = \mu_0$. The amplitude of the electric field inside the medium is _______$\times 10^3$ Vm^{-1} (up to one decimal place). (2018)
$$\left(\varepsilon_0 = 8.85 \times 10^{-12} C^2 N^{-1} m^2,\ \mu_0 = 4\pi \times 10^{-7}\, N \cdot A^{-2},\ c = 3 \times 10^8\, ms^{-1}\right)$$
7) A microcanonical ensemble consists of 12 atoms with each taking either energy 0 state or energy ε state. Both states are non-degenerate. If the total energy of this ensemble is 4ε, its entropy will be ________k_B (up to one decimal place), where k_g is the Boltzmann constant. (2018)

8) A two-state quantum system has energy eigenvalues $\pm\varepsilon$ corresponding to the normalized states $|\psi\pm\rangle$. At time $t = 0$, the system is in quantum state $\frac{1}{\sqrt{2}}(|\psi_+\rangle + |\psi_-\rangle)$. The probability that the system will be in the same state at $t = \frac{h}{(6\varepsilon)}$ is ______ (up to two decimal places). (2018)
9) An air-conditioner maintains the room temperature at 27° C while the outside temperature is 47° C. The heat conducted through the walls of the room from outside to inside due to temperature difference is 7000 W. The minimum work done by the compressor of the air-conditioner per unit time is ______ W. (2018)
10) Two solid spheres A and B have the same emissivity. The radius of A is four times the radius of B, and the temperature of A is twice the temperature of B. The ratio of the rate of heat radiated from A to that from B is ______. (2018)
11) The partition function of an ensemble at a temperature T is:

$$Z = \left(2\cosh\left(\frac{\varepsilon}{k_B T}\right)\right)^N$$

where k_B is the Boltzmann constant. The heat capacity of this ensemble at $T = \frac{\varepsilon}{k_B}$ is XNk_B, where the value of X is ______ (up to two decimal places). (2018)
12) An atom in its singlet state is subjected to a magnetic field. The Zeeman splitting of its 650 nm spectral line is 0.03 nm. The magnitude of the field is ______ Tesla (up to two decimal places).

$$\left(e = 1.60 \times 10^{-19}\, C,\ m_e = 9.11 \times 10^{-31}\, kg,\ c = 3.0 \times 10^{8}\, ms^{-1}\right)$$

(2018)
13) The quantum effects in an ideal gas become important below a certain temperature T_0 when the de Broglie wavelength corresponding to the root mean square thermal speed becomes equal to the inter-atomic separation. For such a gas of atoms of mass 2×10^{-26} kg and number density 6.4×10^{25} m^{-3}, T_Q = _______$\times 10^{-3}$ K (up to one decimal place). (2018)

$$\left(k_B = 1.38 \times 10^{-23}\, J/K,\ h = 6.6 \times 10^{-34}\, J \cdot s\right)$$