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## AI24BTECH11002 - K. Akshay Teja

1) An electron in the ground state of the hydrogen atom has the wave function:

$$\psi\left(\overrightarrow{r}\right) = \frac{1}{\sqrt{\pi a_0^3}} e^{-\left(\frac{r}{a_0}\right)}$$

where  $a_0$  is a constant. The expectation value of the operator  $\hat{Q} = z^2 - r^2$ , where  $z = r \cos \theta$ , is (Hint:  $\int_0^\infty e^{-ar} r^n dr = \frac{\Gamma(n)}{a^{n+1}} = \frac{(n-1)!}{a^{n+1}}$ 

a)  $-\frac{a_0^2}{2}$ 

b)  $-a_0^2$ 

c)  $-\frac{3a_0^2}{2}$ 

- d)  $-2a_0^2$
- 2) For Nickel, the number density is  $8 \times 10^{23}$  atoms/cm<sup>3</sup> and the electronic configuration is  $1s^22s^22p^63s^23p^63d^84s^2$ . The value of the saturation magnetization of Nickel in its ferromagnetic state is:

(Given the value of Bohr magneton  $\mu_B = 9.21 \times 10^{-21} \text{Am}^2$ )

3) A particle of mass m is in a potential given by

$$V(r) = -\frac{a}{r} + \frac{ar_0^2}{3r^3}$$

where a and  $r_0$  are positive constants. When disturbed slightly from its stable equilibrium position, it undergoes simple harmonic oscillation. The time period of oscillation is:

- a)  $2\pi \sqrt{\frac{mr_0^3}{2}}$
- b)  $2\pi \sqrt{\frac{mr_0^3}{a}}$  c)  $2\pi \sqrt{\frac{2mr_0^3}{a}}$
- d)  $4\pi \sqrt{\frac{mr_0^3}{m^2}}$
- 4) The donor concentration in a sample of n-type silicon is increased by a factor of 100. The shift in the position of the Fermi level at 300K, assuming the sample to be non-degenerate is  $(k_B T = 25 \text{ meV at } 300 \text{K})$
- 5) A particle of mass m is subjected to a potential:

$$V(x,y) = \frac{1}{2}m\omega^2(x^2 + y^2), -\infty \le x \le \infty, -\infty \le y \le \infty$$

. The state with energy  $4\hbar\omega$  is g-fold degenerate. The value of g is:

6) A hydrogen atom is in the state:

$$\psi = \sqrt{\frac{8}{21}} \psi_{200} - \sqrt{\frac{3}{7}} \psi_{310} + \sqrt{\frac{4}{21}} \psi_{321},$$

where n, l, m in  $\psi_{nlm}$  denote the principal, orbital, and magnetic quantum numbers, respectively. If  $\overrightarrow{L}$  is the angular momentum operator, the average value of  $L^2$  is  $h^2$ 

7) A planet of mass m moves in a circular orbit of radius  $r_0$  in the gravitational potential  $V(r) = -\frac{k}{r}$ , where k is a positive constant. The orbital angular momentum of the planet is:

- a)  $2r_0km$
- b)  $\sqrt{2r_0km}$
- c)  $r_0km$
- d)  $\sqrt{r_0 km}$
- 8) The moment of inertia of a rigid diatomic molecule A is 6 times that of another rigid diatomic molecule B. If the rotational energies of the two molecules are equal, then the corresponding values of the rotational quantum numbers  $J_A$  and  $J_B$  are:
  - a)  $J_A = 2$ ,  $J_B = 1$

c)  $J_A = 5$ ,  $J_B = 0$ 

b)  $J_A = 3$ ,  $J_B = 1$ 

d)  $J_A = 6$ ,  $J_B = 1$ 

9) The value of the integral:

$$\oint_C \frac{z^2}{e^z + 1} \, dz,$$

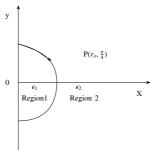
where C is the circle |z| = 4, is:

a)  $2\pi i$ 

b)  $2\pi^2 i$ 

c)  $4\pi^3 i$ 

- d)  $4\pi^2 i$
- 10) A ray of light inside Region 1 in the *xy*-plane is incident at the semicircular boundary that carries no free charges. The electric field at the point  $P(r, \pi/4)$  in plane polar coordinates is  $\overrightarrow{E_1} = 7e_0\hat{e}_r 3e_0\hat{e}_\theta$  where  $\hat{e}_r$  and  $\hat{e}_\theta$  are the unit vectors. The emerging ray in Region 2 has the electric field  $\overrightarrow{E_2}$  parallel to the *x*-axis. If  $\epsilon_1$  and  $\epsilon_2$  are the dielectric constants of Region 1 and Region 2 respectively, then  $\frac{\epsilon_1}{\epsilon_2}$  is



11) The solution of the differential equation:

$$\frac{d^2y}{dt^2} - y = 0$$

subject to the boundary conditions y(0) = 1 and  $y(\infty) = 0$ , is:

- a)  $\cos t + \sin t$
- b)  $\cosh t + \sinh t$
- c)  $\cos t \sin t$
- d)  $\cosh t \sinh t$
- 12) Given that the linear transformation of a generalized coordinate q and the corresponding momentum p,

$$Q = q + 4ap$$

$$P = q + 2p$$

is canonical, the value of the constant a is:

13) The value of the magnetic field required to maintain non-relativistic protons of energy 1 MeV in a circular orbit of radius 100 mm is:

(Given:  $m_p = 1.67 \times 10^{-27}$  kg,  $e = 1.6 \times 10^{-19}$  C)