2021-Aug-26 Shift-2

AI24BTECH11002 - K. Akshay Teja

1) Let $[t]$ denote the greatest integer less than or equal to t. Let $f(x) = x - [x]$, $g(x) = 1 - x + [x]$, and $h(x) = \min\{f(x), g(x)\}$, $x \in (-2, 2)$. Then h is:
   a) continuous in $(-2, 2)$ but not differentiable at more than four points in $(-2, 2)$.
   b) not continuous at exactly three points in $(-2, 2)$.
   c) continuous in $(-2, 2)$ but not differentiable at exactly three points in $(-2, 2)$.
   d) not continuous at exactly four points in $(-2, 2)$.

2) Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$. Then $A^{2025} - A^{2020}$ is equal to:

   a) $A^6 - A$ b) A^5 c) $A^5 - A$ d) A^6

3) The local maximum value of the function $f(x) = \left(\frac{2}{x}\right)^{x^2}, x > 0$ is:

   a) $\left(2\sqrt{e}\right)^{\frac{1}{e}}$ b) $\left(\frac{4}{\sqrt{e}}\right)^{\frac{e}{4}}$ c) $(e)^{\frac{2}{e}}$ d) 1

4) If the value of the integral $\int_0^5 \frac{x+[x]}{e^{x-[x]}}\, dx = \alpha e^{-1} + \beta$, where $\alpha, \beta \in \mathbb{R}$, and $5\alpha + 6\beta = 0$, and $[x]$ denotes the greatest integer less than or equal to x; then the value of $(\alpha + \beta)^2$ is equal to:

   a) 100   b) 25   c) 16   d) 36

5) The point $P\left(-2\sqrt{6}, \sqrt{3}\right)$ lies on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ having eccentricity $\frac{\sqrt{5}}{2}$. If the tangent and normal at P to the hyperbola intersect its conjugate axis at the points Q and R respectively, then QR is equal to:

   a) $4\sqrt{3}$ b) 6   c) $6\sqrt{3}$ d) $3\sqrt{6}$

6) Let $y(x)$ be the solution of the differential equation $2x^2 dy + (e^y - 2x)\, dx = 0, x > 0$. If $y(e) = 1$, then $y(1)$ is equal to:

   a) 0   b) 2   c) $\log_e 2$ d) $\log_e (2e)$

7) Consider the two statements:
   $(S1)$: $(p \to q) \vee (\neg q \to p)$ is a tautology.
   $(S2)$: $(p \wedge \neg q) \wedge (\neg p \vee q)$ is a fallacy.
   Then:

   a) only $(S1)$ is true.
   b) both $(S1)$ and $(S2)$ are false.
   c) both $(S1)$ and $(S2)$ are true.
   d) only $(S2)$ is true.

8) The domain of the function $\operatorname{cosec}^{-1}\left(\frac{1+x}{x}\right)$ is:

a) $\left(-1,-\frac{1}{2}\right] \cup (0,\infty)$ b) $\left[-\frac{1}{2},0\right) \cup [1,\infty)$ c) $\left(-\frac{1}{2},\infty\right) - \{0\}$ d) $\left[-\frac{1}{2},\infty\right) - \{0\}$

9) A fair die is tossed until a six is obtained. Let X be the number of required tosses. Then the conditional probability $P(X \geq 5 \mid X > 2)$ is:

a) $\frac{125}{216}$ b) $\frac{11}{36}$ c) $\frac{5}{6}$ d) $\frac{25}{36}$

10) If $\sum_{r=1}^{50} \tan^{-1} \frac{1}{2r^2} = p$, then the value of $\tan p$ is:

a) $\frac{101}{102}$ b) $\frac{50}{51}$ c) 100 d) $\frac{51}{50}$

11) Two fair dice are thrown. The numbers on them are taken as λ and μ, and a system of linear equations is constructed as: $x+y+z=5, x+2y+3z=\mu, x+3y+\lambda z=1$. If p is the probability that the system has a unique solution and q is the probability that the system has no solution, then:

a) $p=\frac{1}{6}$ and $q=\frac{1}{36}$ b) $p=\frac{5}{6}$ and $q=\frac{5}{36}$ c) $p=\frac{5}{6}$ and $q=\frac{1}{36}$ d) $p=\frac{1}{6}$ and $q=\frac{5}{36}$

12) The locus of the midpoints of the chords of the hyperbola $x^2-y^2=4$, which touch the parabola $y^2=8x$, is:

a) $y^3(x-2)=x^2$ b) $x^3(x-2)=y^2$ c) $y^2(x-2)=x^3$ d) $x^2(x-2)=y^3$

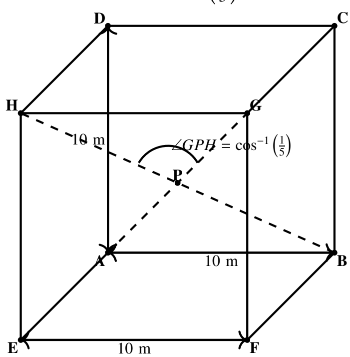
13) The value of $2\sin\left(\frac{\pi}{8}\right)\sin\left(\frac{3\pi}{8}\right)\sin\left(\frac{5\pi}{8}\right)\sin\left(\frac{6\pi}{8}\right)\sin\left(\frac{7\pi}{8}\right)$ is:

a) $\frac{1}{4\sqrt{2}}$ b) $\frac{1}{4}$ c) $\frac{1}{8}$ d) $\frac{1}{8\sqrt{2}}$

14) If $\left(\sqrt{3}+i\right)^{100} = 2(p+iq)^{99}$, then p and q are roots of the equation:

a) $x^2-\left(\sqrt{3}-1\right)x-\sqrt{3}=0$
b) $x^2+\left(\sqrt{3}+1\right)x+\sqrt{3}=0$
c) $x^2+\left(\sqrt{3}-1\right)x-\sqrt{3}=0$
d) $x^2-\left(\sqrt{3}+1\right)x+\sqrt{3}=0$

15) A hall has a square floor of dimension 10m ×10m (see the figure) and vertical walls. If the angle $\angle GPH$ between the diagonals AG and BH is $\cos^{-1}\left(\frac{1}{5}\right)$, then the height of the hall (in meters) is:



a) 5 b) $2\sqrt{10}$ c) $5\sqrt{3}$ d) $5\sqrt{2}$