9-9.2-22

AI24BTECH11002 - K.AKSHAY TEJA

Question:
Find the area bounded by the curve $x^2 = 4y$ and the line $x - 2 = 4y$.
Solution:

Equation	Description
$x^2 = 4y$	Parabola(conic)
$x + 2 = 4y$	Line

TABLE 0: Final Information

The given curve can be expressed as a conic of parameters

$$v = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} u = \begin{pmatrix} 0 \\ -2 \end{pmatrix} f = 0 \tag{0.1}$$

The given line parameters are

$$h = \begin{pmatrix} 0 \\ \frac{1}{2} \end{pmatrix} m = \begin{pmatrix} 1 \\ \frac{1}{4} \end{pmatrix} \tag{0.2}$$

From (**??**), the points of intersection of line and conic:

$$\mathbf{x_1} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \mathbf{x_2} = \begin{pmatrix} -1 \\ \frac{1}{4} \end{pmatrix} \tag{0.3}$$

As you can see in the figure, the area bounded by the curve $x^2 = 4y$ and the line $x-2 = 4y$ is given by

$$\int_{-1}^{2} \left(\frac{x + 2 - x^2}{4} \right) dx = \frac{9}{8} \tag{0.4}$$

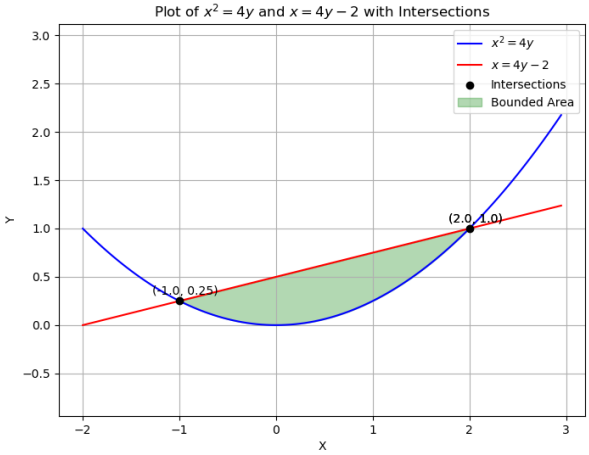

Fig. 0.1: Line and Vectors