

- 53) For the transformation

$$Q = \sqrt{2q} e^{-1+2\alpha} \cos p, P = \sqrt{2q} e^{\alpha-1} \sin p$$

(where α is a constant) to be canonical, the value of α is _____.

(2018)

- 54) Given

$$\frac{d^2 f(x)}{dx^2} - 2 \frac{df(x)}{dx} + f(x) = 0,$$

and boundary conditions $f(0) = 1$ and $f(1) = 0$, the value of $f(0.5)$ is _____ (up to two decimal places).

(2018)

- 55) The absolute value of the integral

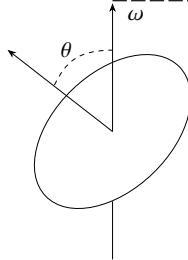
$$\int \frac{5z^3 + 3z^2}{z^2 - 4} dz,$$

over the circle $|z - 1.5| = 1$ in the complex plane, is _____ (up to two decimal places).

(2018)

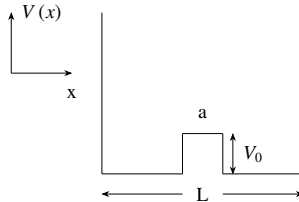
- 56) A uniform circular disc of mass m and radius R is rotating with angular speed ω about an axis passing through its center and making an angle $\theta = 30^\circ$ with the axis of the disc. If the kinetic energy of the disc is $\alpha m \omega^2 R^2$, the value of α is _____ (up to 2 decimal places).

(2018)



- 57) The ground state energy of a particle of mass m in an infinite potential well is E_0 . It changes to $E_0(1 + \alpha \times 10^{-3})$ when there is a small potential bump of height $V_0 = \frac{\pi^2 \hbar^2}{50mL^2}$ and width $a = \frac{L}{100}$, as shown in the figure. The value of α is _____ (up to two decimal places).

(2018)



- 58) An electromagnetic plane wave is propagating with an intensity $I = 1.0 \times 10^5 \text{ Wm}^2$ in a medium with $\epsilon = 3\epsilon_0$ and $\mu = \mu_0$. The amplitude of the electric field inside the medium is _____ $\times 10^3 \text{ Vm}^{-1}$ (up to one decimal place).

(2018)

$$(\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^2, \mu_0 = 4\pi \times 10^{-7} \text{ N} \cdot \text{A}^{-2}, c = 3 \times 10^8 \text{ ms}^{-1})$$

- 59) A microcanonical ensemble consists of 12 atoms with each taking either energy 0 state or energy ϵ state. Both states are non-degenerate. If the total energy of this ensemble is 4ϵ , its entropy will be _____ k_B (up to one decimal place), where k_B is the Boltzmann constant.

(2018)

- 60) A two-state quantum system has energy eigenvalues $\pm\epsilon$ corresponding to the normalized states $|\psi_{\pm}\rangle$. At time $t = 0$, the system is in quantum state $\frac{1}{\sqrt{2}}(|\psi_{+}\rangle + |\psi_{-}\rangle)$. The probability that the system will be in the same state at $t = \frac{h}{(6\epsilon)}$ is _____ (up to two decimal places). (2018)
- 61) An air-conditioner maintains the room temperature at 27°C while the outside temperature is 47°C . The heat conducted through the walls of the room from outside to inside due to temperature difference is 7000 W . The minimum work done by the compressor of the air-conditioner per unit time is _____ W . (2018)
- 62) Two solid spheres A and B have the same emissivity. The radius of A is four times the radius of B , and the temperature of A is twice the temperature of B . The ratio of the rate of heat radiated from A to that from B is _____. (2018)
- 63) The partition function of an ensemble at a temperature T is:

$$Z = \left(2 \cosh \left(\frac{\epsilon}{k_B T} \right) \right)^N$$

where k_B is the Boltzmann constant. The heat capacity of this ensemble at $T = \frac{\epsilon}{k_B}$ is XNk_B , where the value of X is _____ (up to two decimal places). (2018)

- 64) An atom in its singlet state is subjected to a magnetic field. The Zeeman splitting of its 650 nm spectral line is 0.03 nm . The magnitude of the field is _____ Tesla (up to two decimal places).

$$(e = 1.60 \times 10^{-19}\text{ C}, m_e = 9.11 \times 10^{-31}\text{ kg}, c = 3.0 \times 10^8\text{ ms}^{-1})$$

(2018)

- 65) The quantum effects in an ideal gas become important below a certain temperature T_0 when the de Broglie wavelength corresponding to the root mean square thermal speed becomes equal to the inter-atomic separation. For such a gas of atoms of mass $2 \times 10^{-26}\text{ kg}$ and number density $6.4 \times 10^{25}\text{ m}^{-3}$, $T_0 = \text{_____} \times 10^{-3}\text{ K}$ (up to one decimal place). (2018)

$$(k_B = 1.38 \times 10^{-23}\text{ J/K}, h = 6.6 \times 10^{-34}\text{ J} \cdot \text{s})$$