9-9.2-38

AI24BTECH11002 - K.AKSHAY TEJA

Question:

Find the area bounded by the curve $y = x|x|$, x-axis and the ordinates $x = -1$ and $x = 1$.

Solution:

Range	Equation of conic
$x \geq 0$	$y = x^2$
$x < 0$	$y = -x^2$

TABLE 0: Information

For $x \geq 0$, the equation of the conic and its parameters are given by

$$y = x^2 \tag{0.1}$$

$$v = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, u = \begin{pmatrix} 0 \\ -\frac{1}{2} \end{pmatrix}, f = 0 \tag{0.2}$$

For $x < 0$, the equation of the curve is given by

$$y = -x^2 \tag{0.3}$$

$$v = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, u = \begin{pmatrix} 0 \\ \frac{1}{2} \end{pmatrix}, f = 0 \tag{0.4}$$

As you can see in the figure, the area bounded by the curve $y = x|x|$ in between the lines $x = -1$ and $x = 1$ is given by

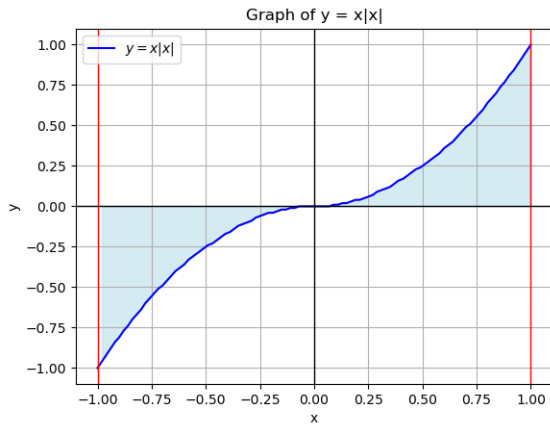$$\left| \int_{-1}^{0} -x^2 dx \right| + \left| \int_{0}^{1} x^2 dx \right| = \frac{2}{3} \tag{0.5}$$

Fig. 0.1: Graph of $y = x|x|$