2024-Feb-1 Shift-1

AI24BTECH11002 - K. Akshay Teja

1) If 3, a, b, x are in A.P, and 2, a-1, b+1 are in G.P. Then arithmetic mean of a, b and c is

a) 11 b) 10 c) 9 d) 13

2) The value of $\int_0^{\frac{\pi}{4}} \frac{x dx}{\sin^4(2x)+\cos^4(2x)}$ is equal to

a) $\frac{\pi^2}{16\sqrt{2}}$ b) $\frac{\pi^2}{64}$ c) $\frac{\pi^2}{32}$ d) $\frac{\pi^2}{8\sqrt{2}}$

3) If $A = \begin{bmatrix} \sqrt{2} & 1 \\ -1 & \sqrt{2} \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, C = ABA^T$, then $|X|$ is equal to

a) 729 b) 283 c) 27 d) 23

4) If $3, 7, 11, \ldots, 403 = AP_1$ $2, 5, 8, \ldots, 401 = AP_2$ Find sum of common term of AP_1 and AP_2

a) 3366 b) 6699 c) 9999 d) 6666

5) $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{8\sqrt{2}\cos x}{(1+e^{\sin x})(1+\sin^4 x)} dx = a\pi + b\log\left(3+2\sqrt{2}\right)$ then find a + b.

a) 4 b) 6 c) 8 d) 2

6) If $(t+1)\,dx = \left(2x + (t+1)^3\right)\,dt$ and $x(0) = 2$, then $x(1)$ is equal to

a) 5 b) 12 c) 6 d) 8

7) Five people are distributed in four identical rooms. A room can also contain zero people. Find the number of ways to distribute them.

a) 47 b) 53 c) 43 d) 51

8) $5f(x) + 4f\left(\frac{1}{x}\right) = x^2 - 4$ and $y = 9f(x)x^2$. If y is a strictly increasing function, find the interval of x.

a) $\left(-\infty, \frac{-1}{\sqrt{5}}\right) \cup \left(\frac{-1}{\sqrt{5}}, 0\right)$
b) $\left(-\frac{-1}{\sqrt{5}}, 0\right) \cup \left(0, \frac{-1}{\sqrt{5}}\right)$
c) $\left(0, \frac{-1}{\sqrt{5}}\right) \cup \left(\frac{-1}{\sqrt{5}}, \infty\right)$
d) $\left(-\sqrt{\frac{2}{5}}, 0\right) \cup \left(\sqrt{\frac{2}{5}}, \infty\right)$

9) If the hyperbola $x^2 - y^2\operatorname{cosec}^2\theta = 5$ and the ellipse $x^2\operatorname{cosec}^2\theta + y^2 = 5$ has eccentricities e_H and e_e respectively, and $e_H = \sqrt{7}e_e$, then θ is equal to:

a) $\frac{\pi}{3}$ b) $\frac{\pi}{6}$ c) $\frac{\pi}{2}$ d) $\frac{\pi}{4}$

10) A bag contains 8 balls (*blackandwhite*). If four balls are chosen without replacement and 2 white (W) and 2 black (B) balls are found, then the probability that the number of white and black balls are the same in the bag is equal to:

a) $\frac{1}{7}$ b) $\frac{2}{7}$ c) $\frac{3}{5}$ d) $\frac{1}{2}$

11) If two circles $x^2 + y^2 = 4$ and $x^2 + y^2 - 4\lambda x + 9 = 0$ intersect at two distinct points, then find the range of λ.

a) $\left(-\infty, -\frac{13}{2}\right) \cup \left(-\frac{13}{2}, \infty\right)$
b) $\left(-\infty, -\frac{13}{8}\right) \cup \left(-\frac{13}{2}, \infty\right)$
c) $\left[-\frac{13}{8}, \frac{13}{8}\right]$
d) $\lambda \in \left(\frac{3}{2}, \infty\right)$

12) If $S = \{x \in R : 3\left(\sqrt{3} + \sqrt{2}\right)^x + \left(\sqrt{3} - \sqrt{2}\right)^x = \frac{10}{3}\}$, then the number of elements in set S is

a) Zero b) 1 c) 2 d) 3

13) $f(x) = \begin{cases} e^x, & x < 0 \\ \ln x, & x > 0 \end{cases}$ $g(x) = \begin{cases} e^x, & x < 0 \\ x, & x > 0 \end{cases}$ The $gof : A \to R$ is

a) Onto but not one-one
b) Into and many-one
c) Onto and one-one
d) Into and one-one

14) If $\tan A = \frac{1}{\sqrt{x^2+x+1}}$, $\tan B = \frac{\sqrt{x}}{\sqrt{x^2+x+1}}$ and $\tan C = \frac{1}{\sqrt{x\left(x^2+x+1\right)}}$, then A + B =

a) 0 b) $\pi - C$ c) $\frac{\pi}{2} - C$ d) None

15) $\lim_{x\to 0} \frac{\cos^{-1}\left(1-\{x\}^2\right)\sin^{-1}(1-\{x\})}{\{x\}-\{x\}^3}$ where {} is fractional part function. If L.H.L = L and R.H.L = R, then the correct relation between L and R is

a) $\sqrt{2}R = 4L$ b) $\sqrt{2}L = 4R$ c) $R = l$ d) $R = 2L$