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AI24BTECH11002 - K. Akshay Teja

1) Let A and B be two similar square matrices of order two. If 1 and -2 are the eigenvalues of A, then the Trace of B is

a) -2
b) -1
c) 1
d) 2

2) The root of ax + b = 0 (a, b constants), can be found by the Newton-Raphson method with a minimum of

a) 1 iteration
b) 2 iterations
c) 3 iterations
d) an undeterminable number of iterations

3) The solution $u(x,t)$ of the one-dimensional heat equation,

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}, x \in \mathbb{R}$$

with a Gaussian initial condition,

a) travels with finite constant wave-speed
b) travels with finite variable wave-speed
c) spreads in both directions, with the magnitude of the peak increasing with time
d) spreads in both directions, with the magnitude of the peak decreasing with time

4) Let C be the boundary of the square given by $0 \le x \le 1$, $0 \le y \le 1$. Then $\oint_C (x\,dy - y\,dx)$ equals

a) -2
b) 0
c) 1
d) 2

5) Let the eigenvalues of a square matrix A of order two be 1 and 2. The corresponding eigenvectors are $\begin{pmatrix} 0.6 \\ 0.8 \end{pmatrix}$ and $\begin{pmatrix} 0.8 \\ -0.6 \end{pmatrix}$, respectively. Then, the element $\mathrm{A}(2,2)$ is

a) -0.48
b) 0.48
c) 1.36
d) 1.64

6) Let $y_1(x)$ and $y_2(x)$ be two linearly independent solutions of

$$\frac{d^2y}{dx^2} + \frac{6}{x}\frac{dy}{dx} + q(x)\,y = 0, x \in (1,3)$$

where $q(x)$ is a continuous function in $(1,3)$. If the Wronskian of $y_1(x)$ and $y_2(x)$ at $x = 1$, denoted by $W(y_1, y_2)(1)$, is 1, then $W(y_1, y_2)(2)$ is

a) $\frac{1}{2^6}$
b) $\frac{1}{2^3}$
c) $\frac{1}{2}$
d) 2

7) Simpson's $\frac{1}{3}$ rule applied to $\int_{-1}^{1} \left(3x^2 + 5\right) dx$ with sub-interval $h = 1$, will give

a) the exact result
b) error between 0.01% to 0.1%
c) error between 0.1% to 1.0%
d) error > 1.0%

8) The probability that a six-sided dice is thrown n times without giving a '6', even once, is

a) $\left(\frac{5}{6}\right)^n$ b) $\frac{5}{6}\left(\frac{1}{6}\right)^n$ c) $\frac{(n-1)!}{n!}\left(\frac{1}{6}\right)^n$ d) $1-\frac{(n-1)!}{n!}\left(\frac{5}{6}\right)^n$

9) If a complex function $f(z) = u(x, y) + iv(x, y)$ is analytic, then

a) $i\frac{\partial u}{\partial x} + \frac{\partial v}{\partial x} = \frac{\partial u}{\partial y} + i\frac{\partial v}{\partial y}$ b) $i\frac{\partial u}{\partial x} + \frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y} - i\frac{\partial v}{\partial y}$ c) $\frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x} = -i\frac{\partial u}{\partial y} + \frac{\partial v}{\partial y}$ d) $\frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x} = i\frac{\partial u}{\partial y} - \frac{\partial v}{\partial y}$

10) Let $\overrightarrow{u} = -\omega y\hat{i} + \omega x\hat{j}$ and $\overrightarrow{v} = \omega z\hat{j} - \omega y\hat{k}$ be two given vectors, where ω is a constant. Then $div\left(\overrightarrow{u} \times \overrightarrow{v}\right)$ equals

a) 0 b) $2\omega^2 y$ c) $4\omega^2 y$ d) $-4\omega^2 y$

11) The infinite series $\sum_{m=1}^{\infty} \frac{(-1)^m x^2}{\left(1+x^2\right)^m}$ is

a) Divergent for all x
b) Convergent only for $x \geq 1$
c) Convergent for all x
d) Divergent only for $-1 \leq x \leq 1$

12) Let $f(x)$ be continuous and satisfy $m \leq f(x) \leq M$ in $1 \leq x \leq 10$. Then, $\mu = \frac{\int_1^{10}\left(f(x)x^2\right)dx}{\int_1^{10}\left(x^2\right)dx}$ satisfies

a) $\mu \leq 333m$ b) $333\mu \geq M$ c) $m \leq \mu \leq M$ d) $m \leq \mu \leq \frac{M}{333}$