2020-Sep-3 Shift-2

AI24BTECH11002 - K. Akshay Teja

1) If $x^3dy + xydx = x^2dy + 2ydx$; $y(2) = e$ and $x > 1$, then $y(4)$ is equal to:

   a) $\frac{\sqrt{e}}{2}$ b) $\frac{3}{2}\sqrt{e}$ c) $\frac{1}{2}+\sqrt{e}$ d) $\frac{3}{2}+\sqrt{e}$

2) Let A be a 3×3 matrix such that $\text{adj}A = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 0 & 2 \\ 1 & -2 & -1 \end{pmatrix}$ and $B = \text{adj}(\text{adj}A)$. If $|A| = \lambda$ and $|(B^{-1})^T| = \mu$, then the ordered pair, $(|\lambda|, \mu)$ is equal to:

   a) $\left(9, \frac{1}{81}\right)$
   b) $\left(9, \frac{1}{9}\right)$
   c) $\left(3, \frac{1}{81}\right)$
   d) $(3, 81)$

3) Let $a, b, c \in \mathbb{R}$ be such that $a^2 + b^2 + c^2 = 1$, if $a\cos\theta = b\cos\left(\theta + \frac{2\pi}{3}\right) = c\cos\left(\theta + \frac{4\pi}{3}\right)$, where $\theta = \frac{\pi}{9}$, then the angle between the vectors $a\hat{i} + b\hat{j} + c\hat{k}$ and $b\hat{i} + c\hat{j} + a\hat{k}$ is:

   a) $\frac{\pi}{2}$ b) $\frac{2\pi}{3}$ c) $\frac{\pi}{9}$ d) 0

4) Suppose $f(x)$ is a polynomial of degree four, having critical points at $(-1, 0, 1)$. If $T = \{x \in \mathbb{R} \mid f(x) = f(0)\}$, then the sum of squares of all the elements of T is:

   a) 6 b) 2 c) 8 d) 4

5) If the value of the integral $\int_0^{\frac{1}{2}} \frac{x^2}{(1-x^2)^{\frac{3}{2}}}$ is $\frac{k}{6}$, then k is equal to:

   a) $2\sqrt{3} + \pi$
   b) $3\sqrt{2} + \pi$
   c) $3\sqrt{2} - \pi$
   d) $2\sqrt{3} - \pi$

6) If the term independent of x in the expansion of $\left(\left(\frac{3}{2}\right)x^2 - \frac{1}{3x}\right)^9$ is k, then $18k$ is equal to:

   a) 5 b) 9 c) 7 d) 11

7) If a triangle ABC has vertices $A(-1, 7)$, $B(-7, 1)$, and $C(5, -5)$, then its orthocentre has coordinates:

   a) $(-3, 3)$
   b) $\left(-\frac{3}{5}, \frac{3}{5}\right)$
   c) $\left(\frac{3}{5}, -\frac{3}{5}\right)$
   d) $(3, -3)$

8) Let e_1 and e_2 be the eccentricities of the ellipse, $\frac{x^2}{25} + \frac{y^2}{b^2} = 1$ (where $b < 5$) and the hyperbola, $\frac{x^2}{16} - \frac{y^2}{b^2} = 1$ respectively, satisfying $e_1e_2 = 1$. If α and β are the distances between the foci of the ellipse and the foci of the hyperbola respectively, then the ordered pair (α, β) is equal to:

   a) $(8, 12)$
   b) $\left(\frac{24}{5}, 10\right)$
   c) $\left(\frac{20}{3}, 12\right)$
   d) $(8, 10)$

9) If z_1, z_2 are complex numbers such that $\text{Re}(z_1) = |z_1 - 1|$, $\text{Re}(z_2) = |z_2 - 1|$ and $\arg(z_1 - z_2) = \frac{\pi}{6}$, then $\text{Im}(z_1 + z_2)$ is equal to:

   a) $2\sqrt{3}$ b) $\frac{2}{\sqrt{3}}$ c) $\frac{1}{\sqrt{3}}$ d) $\frac{\sqrt{3}}{2}$

10) The set of all real values of λ for which the quadratic equations, $\left(\lambda^2 + 1\right)x^2 - 4\lambda x + 2 = 0$ always have exactly one root in the interval $(0, 1)$ is:

   a) $(-3, -1)$
   b) $(2, 4)$
   c) $(1, 3)$
   d) $(0, 2)$

11) Let the latus rectum of the parabola $y^2 = 4x$ be the common chord to the circles C_1 and C_2, each of them having radius $2\sqrt{5}$. Then, the distance between the centres of the circles C_1 and C_2 is:

a) 8 b) $8\sqrt{5}$ c) $4\sqrt{5}$ d) 12

12) The plane which bisects the line joining the points $(4, -2, 3)$ and $(2, 4, -1)$ at right angles also passes through the point:

a) $(0, -1, 1)$
b) $(4, 0, 1)$
c) $(4, 0, -1)$
d) $(0, 1, -1)$

13) $\lim_{x\to a} \frac{(a+2x)^{\frac{1}{3}} - (3x)^{\frac{1}{3}}}{(3a+x)^{\frac{1}{3}} - (4a)^{\frac{1}{3}}}$ is equal to:

a) $\frac{2}{9}\left(\frac{4}{3}\right)$
b) $\frac{2}{3}\left(\frac{4}{3}\right)$
c) $\left(\frac{2}{3}\right)\left(\frac{2}{9}\right)^{\frac{1}{3}}$
d) $\left(\frac{2}{9}\right)\left(\frac{2}{3}\right)^{\frac{1}{3}}$

14) Let $x_i \, (1 \le i \le 10)$ be ten observations of a random variable X. If $\sum_{i=1}^{10} (x_i - p) = 3$ and $\sum_{i=1}^{10} (x_i - p)^2 = 9$ where $0 \ne p \in \mathbb{R}$, then the standard deviation of these observations is:

a) $\frac{7}{10}$ b) $\frac{9}{10}$ c) $\sqrt{\frac{3}{5}}$ d) $\frac{4}{5}$

15) The probability that a randomly chosen 5-digit number is made from exactly two digits is:

a) $\frac{134}{10^4}$ b) $\frac{121}{10^4}$ c) $\frac{135}{10^4}$ d) $\frac{50}{10^4}$