2022-June-29 Shift-1

AI24BTECH11002 - K. Akshay Teja

16) If the constant term in the expansion of $\left(3x^3 - 2x^2 + \frac{5}{x^5}\right)^{10}$ is $2^k \cdot l$, where l is an odd integer, then the value of k is equal to:

a) 6 b) 7 c) 8 d) 9

17) $\int_0^5 \cos\left(\pi\left(x - \left[\frac{x}{2}\right]\right)\right) dx$, where $[t]$ denotes the greatest integer less than or equal to t, is equal to :

a) -3 b) -2 c) 2 d) 0

18) Let PQ be a focal chord of the parabola $y^2 = 4x$ such that it subtends an angle of $\frac{\pi}{2}$ at the point $(3, 0)$. Let the line segment PQ be also a focal chord of the ellipse $E : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a^2 > b^2$. If e is the eccentricity of the ellipse, then the value of $\frac{1}{e^2}$ is equal to:

a) $1 + \sqrt{2}$ b) $3 + 2\sqrt{2}$ c) $1 + 2\sqrt{3}$ d) $4 + 5\sqrt{3}$

19) Let the tangent to the circle $C_1 : x^2 + y^2 = 2$ at the point $M(-1, 1)$ intersects the circle $C_2 : (x-3)^2 + (y-2)^2 = 5$ at two distinct points A and B. If the tangents to C_2 at the points A and B intersect at N, then the area of the triangle ANB is equal to:

a) $\frac{1}{2}$ b) $\frac{2}{3}$ c) $\frac{1}{6}$ d) $\frac{5}{3}$

20) Let the mean and the variance of 5 observations x_1, x_2, x_3, x_4, x_5 be $\frac{24}{5}$ and $\frac{194}{25}$ respectively. If the mean and variance of the first 4 observation are $\frac{7}{2}$ and a respectively, then $(4a + x_5)$ is equal to

a) 13 b) 15 c) 17 d) 18

21) Let $S = \{z \in \mathbb{C} : |z-2| \le 1, z(1+i) + \bar{z}(1-i) \le 2\}$. Let $|z - 4i|$ attains minimum and maximum values, respectively, at $z_1 \in S$ and $z_2 \in S$. If $5\left(|z_1|^2 + |z_2|^2\right) = \alpha + \beta\sqrt{5}$, where α and β are integers, then the value of $\alpha + \beta$ is equal to

22) Let $y = y(x)$ be the solution of the differential equation $\frac{dy}{dx} + \frac{\sqrt{2}y}{2\cos^4 x - \cos 2x} = xe^{\tan^{-1}(\sqrt{2}\cot 2x)}, \quad 0 < x < \frac{\pi}{2}$ with $y\left(\frac{\pi}{4}\right) = \frac{\pi^2}{32}$. If $y\left(\frac{\pi}{3}\right) = \frac{\pi^2}{18}e^{-\tan^{-1}(\alpha)}$, then the value of $3\alpha^2$ is equal to

23) Let d be the distance between the foot of perpendiculars of the points P$(1, 2, -1)$ and Q$(2, -1, 3)$ on the plane $-x + y + z = 1$. Then d^2 is equal to

24) The number of elements in the set $S = \left\{\theta \in [-4\pi, 4\pi] : 3\cos^2 2\theta + 6\cos 2\theta - 10\cos^2 2\theta + 5 = 0\right\}$ is

25) The number of solutions of the equation $2\theta - \cos^2\theta + \sqrt{2} = 0$ in R is equal to

26) $50\tan\left(3\tan^{-1}\left(\frac{1}{2}\right) + 2\cos^{-1}\left(\frac{1}{\sqrt{5}}\right)\right) + 4\sqrt{2}\tan\left(\frac{1}{2}\tan^{-1}\left(2\sqrt{2}\right)\right)$

27) Let $c, k \in R$. If $f(x) = (c+1)x^2 + \left(1 - c^2\right)x + 2k$ and $f(x+y) = f(x) + f(y) - xy, \ \forall x, y \in R$, then the value of $\left|2\left[f(1) + f(2) + f(3) + \cdots + f(20)\right]\right|$ is equal to

28) Let $H : \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1, a > 0, b > 0$, be a hyperbola such that the sum of lengths of the transverse and the conjugate axes is $4\left(2\sqrt{2} + \sqrt{14}\right)$. If the eccentricity of H is $\frac{\sqrt{11}}{2}$, then the value of $a^2 + b^2$ is equal to

29) Let $P_1 : \vec{r} \cdot \left(2\hat{i} + \hat{j} - 3\hat{k}\right) = 4$ be a plane. Let P_2 be another plane which passes through the points $(2, 3, 2)$, $(2, -2, 3)$, and $(1, -4, 2)$. If the direction ratios of the line of intersection of P_1 and P_2 are $16, \alpha, \beta$, then the value of $\alpha + \beta$ is equal to

30) Let b_1, b_2, b_3, b_4 be a 4-element permutation with $b_i \in \{1, 2, 3, \ldots, 100\}$ for $1 \leq i \leq 4$ and $b_i \neq b_j$ for $i \neq j$, such that either b_1, b_2, b_3 are consecutive integers or b_2, b_3, b_4 are consecutive integers. Then the number of such permutations b_1, b_2, b_3, b_4 is equal to