# 1-1.5-27

AI24BTECH11002 - K.AKSHAY TEJA

**Question:**

Show that the points $\mathbf{P} = \begin{pmatrix} -2 \\ 3 \\ 5 \end{pmatrix}, \mathbf{Q} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ and $\mathbf{R} = \begin{pmatrix} 7 \\ 0 \\ -1 \end{pmatrix}$ are collinear.

**Solution:**

| Variable | Description |
|----------|-------------|
| P | $(-2, 3, 5)$ |
| Q | $(1, 2, 3)$ |
| R | $(7, 0, -1)$ |

TABLE 0: Coordinates of points $P, Q$ and $R$

Points $\mathbf{P}, \mathbf{Q}$ and $\mathbf{R}$ are collinear if

$$\text{rank} \begin{pmatrix} \mathbf{P} & \mathbf{Q} & \mathbf{R} \end{pmatrix}^{\top} = 2 \tag{0.1}$$

$$\implies \begin{pmatrix} -2 & 3 & 5 \\ 1 & 2 & 3 \\ 7 & 0 & -1 \end{pmatrix} \xleftarrow{R_2 \to 2R_2 + R_3} \begin{pmatrix} -2 & 3 & 5 \\ 0 & 7 & 11 \\ 7 & 0 & -1 \end{pmatrix} \tag{0.2}$$

$$\xleftarrow{R_3 \to 2R_3 + 7R_1} \begin{pmatrix} -2 & 3 & 5 \\ 0 & 7 & 11 \\ 0 & 21 & 33 \end{pmatrix} \xleftarrow{R_3 \to R_3 - 3R_2} \begin{pmatrix} -2 & 3 & 5 \\ 0 & 7 & 11 \\ 0 & 0 & 0 \end{pmatrix} \tag{0.3}$$
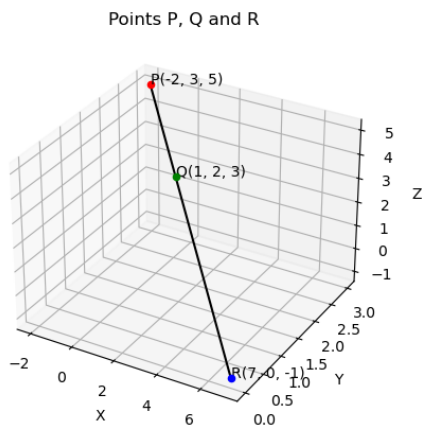
Fig. 0.1: Plot of points P, Q and R