Assignment on Collinearity of Points

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Outline

Problem Statement

Show that the points

$$P = \begin{pmatrix} -2 \\ 3 \\ 5 \end{pmatrix}, \quad Q = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \quad R = \begin{pmatrix} 7 \\ 0 \\ -1 \end{pmatrix} \tag{1.1}$$

are collinear.

Variable Description

Define the coordinates for each point as follows:

$$P = \begin{pmatrix} -2 \\ 3 \\ 5 \end{pmatrix}, \quad Q = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \quad R = \begin{pmatrix} 7 \\ 0 \\ -1 \end{pmatrix} \tag{2.1}$$

Variable	Description
Point P	$\begin{pmatrix} -2 \\ 3 \\ 5 \end{pmatrix}$
Point Q	$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$
Point R	$\begin{pmatrix} 7 \\ 0 \\ -1 \end{pmatrix}$

Table: Coordinates of Points P, Q, and R

Collinearity Condition

Points P, Q, and R are collinear if:

$$\text{rank} \begin{pmatrix} P & Q & R \end{pmatrix}^{\top} = 2 \tag{2.2}$$

Matrix Formulation and Row Reduction

Write the coordinates as a matrix:

$$\begin{pmatrix} -2 & 3 & 5 \\ 1 & 2 & 3 \\ 7 & 0 & -1 \end{pmatrix} \tag{2.3}$$

Perform row operations:

$$\xleftarrow{R_2 \leftarrow 2R_2 + R_3} \begin{pmatrix} -2 & 3 & 5 \\ 0 & 7 & 11 \\ 7 & 0 & -1 \end{pmatrix} \tag{2.4}$$

$$\xleftarrow{R_3 \leftarrow 2R_3 + 7R_1} \begin{pmatrix} -2 & 3 & 5 \\ 0 & 7 & 11 \\ 0 & 21 & 33 \end{pmatrix} \tag{2.5}$$

$$\xleftarrow{R_3 \leftarrow R_3 - 3R_2} \begin{pmatrix} -2 & 3 & 5 \\ 0 & 7 & 11 \\ 0 & 0 & 0 \end{pmatrix} \tag{2.6}$$

Conclusion

Since the matrix has a rank of 2, the points P, Q, and R are collinear.

Visualization

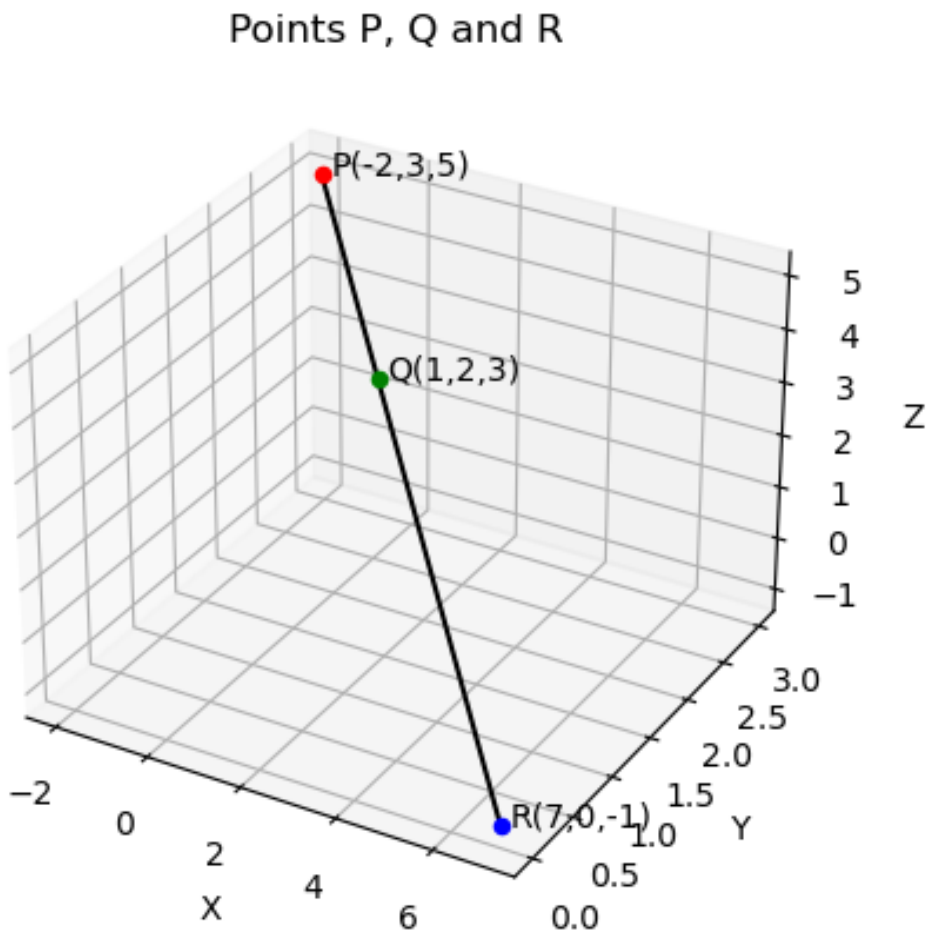

Figure: Plot of Points P, Q, and R

Generating points on line using C I

```c
#include<stdio.h>

int main(){
        double P[]={-2,3,5},Q[]={1,2,3},R[]={7,0,-1};

        printf("P %.2lf %.2lf %.2lf\n",P[0],P[1],P[2]);
        printf("Q %.2lf %.2lf %.2lf\n",Q[0],Q[1],Q[2]);
        printf("R %.2lf %.2lf %.2lf\n",R[0],R[1],R[2]);
        int numberOfValues=100;

        double
        ↪  x_values[numberOfValues],y_values[numberOfValues],z_values[numberO

        for(int i=0;i<numberOfValues;i++){
                double t=(double)i/numberOfValues;
                x_values[i]=P[0]+t*(R[0]-P[0]);
                y_values[i]=P[1]+t*(R[1]-P[1]);
                z_values[i]=P[2]+t*(R[2]-P[2]);
        }
           for (int i = 0; i < numberOfValues; i++) {
```

Generating points on line using C II

```
20          printf("%.2f %.2f %.2f\n", x_values[i], y_values[i], z_values[i]);
21      }
22
23      return 0;
24  }
25
```

Plotting the figure using Python I

```
1  import numpy as np
2  import matplotlib.pyplot as plt
3  from mpl_toolkits.mplot3d import Axes3D
4  import subprocess
5
6  result = subprocess.run(['./code'],stdout = subprocess.PIPE,text=True)
7  output = result.stdout.strip().split('\n')
8
9  P = np.fromstring(output[0].replace('P ',''),sep=' ')
10 Q = np.fromstring(output[1].replace('Q ',''),sep=' ')
11 R = np.fromstring(output[2].replace('R ',''),sep=' ')
12
13 store=np.genfromtxt(output[3:],delimiter='')
14 x_values,y_values,z_values = store.T
15
16
17 fig = plt.figure()
18 ax = fig.add_subplot(111, projection='3d')
19
20 ax.scatter(*P, color='r', label='P')
```

Plotting the figure using Python II

```
21 ax.scatter(*Q, color='g', label='Q')
22 ax.scatter(*R, color='b', label='R')
23
24 ax.text(P[0]+0.2,P[1],P[2],'P(-2,3,5)',color='black', ha='left')
25 ax.text(Q[0]+0.2,Q[1],Q[2],'Q(1,2,3)',color='black', ha='left')
26 ax.text(R[0]+0.2,R[1],R[2],'R(7,0,-1)',color='black', ha='left')
27
28 ax.plot(x_values,y_values,z_values,color='k',label='Line through P,Q,R')
29
30 ax.set_xlabel('X')
31 ax.set_ylabel('Y')
32 ax.set_zlabel('Z')
33
34 plt.title('Points P, Q and R')
35 plt.grid(True)
36 plt.savefig('/home/akshay-teja-kondi/gvv/Assignment3/fig/fig.png')
```