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Assignment - 1

1. Network Queuing Simulation :-

Part B :- (a). At each time instant the user communicates with probability = p. (Given)

Let the probability of x bits generates = p_x
∴ weight of p_x in $E[X] = x p_x p$.

Then, $E[X] = a p_a p + b p_b p + c p_c p + d p_d p \Rightarrow E[X] = (2p_a + 4p_b + 6p_c + 8p_d)p$

$E[X^2] = a^2 p_a p + b^2 p_b p + c^2 p_c p + d^2 p_d p$
$= (4p_a + 16p_b + 36p_c + 64p_d)\,p.$

(i). $p_a = p_b = p_c = p_d = 0.25$

$E[X] = (2+4+6+8)(0.25)p$
$\boxed{E[X] = 5p}$

$E[X^2] = (4+16+36+64)(0.25)p = 30p \Rightarrow \boxed{E[X^2] = 30p}$

$Var[X] = E[X^2] - (E[X])^2$
$= 30p - 25p^2$

$\boxed{Var[X] = 5p(6-5p)}$

(ii). $p_a = p_d = 0, \; p_b = p_c = 0.5$

$E[X] = (2(0) + 4(0.5) + 6(0.5) + 8(0))\,p$
$= 5p$

$E[X^2] = (4(0) + 16(0.5) + 36(0.5) + 64(0))\,p$
$= 26p$

$Var[X] = E[X^2] - (E[X])^2.$
$= 26p - 25p^2$
$= (26 - 25p)p$

(iii) $p_a = p_d = 0.5$, $p_b = p_c = 0$

$E[X] = (2(0.5) + 4(0) + 6(0) + 8(0.5))p$
$= 5p$

$E[X^2] = (4(0.5) + 16(0) + 36(0) + 64(0.5))p$
$= 34p$

$Var[X] = E[X^2] - (E[X])^2$
$= 34p - 25p^2$

$Var[X] = (34 - 25p)p$

(iv) $p_a = p_b = p_c = 0$, $p_d = 1$

$E[X] = 8p$

$E[X^2] = 64p$

$Var[X] = E[X^2] - (E[X])^2$
$= 64p - 64p^2$

$Var[X] = (1-p)(64p)$

(b). Given, network capacity, $R = 5$ bit/sec

∴ at $p = 1$,

(i). $E[X] = 5p = 5$ bits/sec
(ii) $E[X] = 5p = 5$ bits/sec
(iii) $E[X] = 5p = 5$ bits/sec
(iv). $E[X] = 8p = 8$ bits/sec.

We can see from above that $E[X] > R$(5 bits/sec) in part (iv), only where $p_a = p_b = p_c = 0$, $p_d = 1$. Therefore in this case only the incoming communication traffic exceeds the network capacity for $p = 1$.