MATH-505A: Homework # 3

Due on Friday, September 12, 2014

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Contents

Exercise # 1.7

(1)

Given: Two roads $r1_{AB}$, $r2_{AB}$ connectinng points A and B and $s1_{BC}$, $s2_{BC}$ connecting B and C.
Let $p(AB)$ denote the probability that path between A$\longrightarrow$B is open and let $p(AB^c)$ denote the probability that there is no open road b/w A and B. Alternativels $p(AB)$ denotes that road(s) between A and B are open. **To find:**$Y = P(AB|AC^c)$.
Y is equal to the probability that road between A and B is open AND still the path between A and C is closed $\implies$ Path between B and C is closed AND between A and B is open
p(AB) = Path b/w A,B is open = 1 - Path b/w A,B is closed $= 1 - p * p$
Thus

$$p(AB) = 1 - p^2 \tag{1}$$

Also,

$$p(AB) = p(BC) \tag{2}$$

$p(AC^C)$ = 1 - Probability A,C is open = 1 - Probability AB is open AND BC is open. Thus,

$$p(AC^c) = 1 - p(AB)p(AC) = 1 - (1-p^2)^2 \tag{3}$$

$$p(AB \cap AC^C) = p(AC^C|AB)p(AB) = p(BC^C)p(AB) = p^2(1-p^2) \tag{4}$$

$$p(AB|AC^c) = \frac{P(AB \cap AC^c)}{p(AC^c)} = \frac{p(AC^C|AB)p(AB)}{p(AC^c)} = \frac{p^2(1-p^2)}{1-(1-p^2)^2} \tag{5}$$

Part 2: Additional direct road from A to C. Find $p(AB|AC^c)$:
$p(AC^c|AB)$ = Probability that A,C is closed given A,B are open = Probability A,C(direct) are closed AND B,C are closed

$$p(AC^C|AB) = p * p(BC^c)p(AB) \tag{6}$$

where the extra p in 6 as compared to 4 is because the direct path A,C should be blocked too.

$$p(AC^c) = 1 - (1-p^2)^2(1-p) \tag{7}$$

where the extra $(1-p)$ factor in 7 as compared to 3 accounts for the fact that direct path AC is open.
Thus, for part 2:

$$p(AB|AC^c) = \frac{p^3(1-p^2)}{1-(1-p^2)^2(1-p)} \tag{8}$$

(2)

$$p(2K \cap 1A) = \frac{\binom{4}{2} * \binom{4}{1} * \binom{52-4-4}{10}}{\binom{52}{13}} = \frac{24 * 44! * 13!}{10! * 52!} = 1.357 * 10^{-9} \tag{9}$$

$p(1A|2K) = \frac{p(1A \cap 2K)}{p(2K)}$

$$p(2K) = \frac{\binom{4}{2} * \binom{52-4}{11}}{\binom{52}{13}} \tag{10}$$

Exercise # 1.8

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