CSCI-567: Assignment #3

Due on Friday, October 16, 2015

Saket Choudhary
2170058637

Contents

Problem 4

Given: $k_1(.,.)$ and $k_2(.,.)$ are kernel function. Thus, for any vector $y \in \mathbf{R}$, $y^TKy \geq 0$ where $K_{ij} = k(x_i, x_j)$ Mercer's theorem requires K to be positive semi-definite.

Problem 4: (a)

$k_3(x,x') = a_1k_1(x,x') + a_2k_2(x,x')$ where$a_1, a_2 \geq 0$
Since $k_1(x,x')$ is positive definite, $\forall y \in \mathbf{R}$,

$$\begin{aligned} y^TK^{(1)}y \geq 0, \\ \text{where} \\ K^{(1)}_{ij} = k_1(x_i, x'_j) \end{aligned} \tag{4a.1}$$

Similarly,

$$\begin{aligned} y^TK^{(2)}y \geq 0, \\ \text{where} \\ K^{(2)}_{ij} = k_2(x_i, x'_j) \end{aligned} \tag{4a.2}$$

Thus, from (4a.1) and (4a.2), we get

$$\begin{aligned} y^T(K^{(1)} + K^{(2)})y \geq 0\ \forall y \in \mathbf{R} \implies \\ y^TK^{(3)}y \geq 0\ \forall y \in \mathbf{R} \\ \text{where} \\ K^{(3)}_{ij} = k_3(x_i, x'_j) \end{aligned}$$

Problem 4: (b)

$k_4(x,x') = f(x)f(x')$ Let $K^{(4)}_{ij} = k_4(x_i, x_j) = f(x_i)f(x'_j)$
Since $f(x)$ is a real valued function, consider $K^{(4)}$

$$\begin{aligned} K^{(4)} = \begin{bmatrix} f(x_1)f(x'_1) & f(x_1)f(x'_2) & \cdots & f(x_1)f(x'_n) \\ \vdots & & & \\ f(x_n)f(x'_1) & f(x_n)f(x'_2) & \cdots & f(x_n)f(x'_n) \end{bmatrix} \\ K^{(4)} = F(\vec{x})_{n\times 1}F(\vec{x})^T_{1\times n} \\ \text{where} \\ F(x)^T_{1\times n} = \begin{pmatrix} f(x_1) \\ f(x_2) \\ \vdots f(x_n) \end{pmatrix} \end{aligned}$$

Now consider $y^TK^{(4)}y = y^TF(x)F(x)^Ty = y^TF(x)(y^TF(x))^T = ||y^TF(x)||_2^2 \geq 0$x‘
Thus, $k_2(.,.)$ is a valid kernel function!.

Problem 4 continued on next page. . .

Problem 4: (c)

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