MATH-501: Homework # 1

Due on Wednesday, February 11, 2015

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Contents

Problem # 1

1a

$f(x) = atan(x)$ on interval $[a, b] = [-4.9, 5.1]$
$f(a) = atan(-4.9) = 0.7854$ and $f(b) = atan(5.1) = -1.3695$
Since $atan(x)\ \epsilon\ C[-4.9, 5.1]$ and $f(-4.9)f(5.1) < 0$ the conditions required for bisection method to converge are satisfied.
The Number of iterations is given by $M = \lceil log_2(\frac{b-a}{2\delta}) \rceil$ where δ = Absolute error = 10^-2
Hence $M = \lceil log_2(\frac{10}{2*10^{-2}}) \rceil = 9$

1b

f(-4.9) = -1.369
f(5,1) = 1.377
$c_0 = \frac{a+b}{2} = 0.1$ and $f(c_0) = 0.0997 \implies f(c_0)f(a) < 0$ Hence $c_1 = \frac{c_0+a}{2} = -2.4$ and $f(c_1) = -1.176$ $\implies f(c_1)f(c_0) < 0$
Hence $c_2 = \frac{c_1+c_0}{2} = -1.15$ and $f(c_2) = -0.855 \implies f(c_2)f(c_0) < 0$ so
$c_3 = \frac{c_2+c_0}{2} = -0.525$ and $f(c_3) = -0.05254$ and $f(c_3)f(c_0) < 0$

1c

ϵ	Number of iteration	Solution	k=$\lceil log_2(\frac{b-a}{2\delta}) \rceil$
10^{-2}	9	0.00234375	9
10^{-4}	12	-9.76562500003553e-05	12
10^{-8}	22	9.53674312853536e-08	22
10^{-16}	52	-4.44089209850063e-16	52
10^{-32}	104	-9.86076131526265e-32	104
10^{-64}	212	-3.03858167864314e-64	212
10^{-128}	424	-4.61648930889287e-128	424

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2a

$x0 = 5$
Iteration : 1 —— x2 = 3.6266 —— x1 = 2.32486
Iteration : 2 —— x2 = 1.16027 —— x1 = 3.6266
Iteration : 3 —— x2 = 2.32486 —— x1 = 1.16027
Iteration : 4 —— x2 = 0.300819 —— x1 = 2.32486
Iteration : 5 —— x2 = 1.16027 —— x1 = 0.300819
Iteration : 6 —— x2 = 0.00861099 —— x1 = 1.16027
Iteration : 7 —— x2 = 0.300819 —— x1 = 0.00861099
Iteration : 8 —— x2 = 2.12823e-07 —— x1 = 0.300819

$x_s = 2.12823149138563e - 07$
$g(x_s) = x_s - atan(x_s) = 3.20284333180532e - 21$
x0 = -5
Iteration : 1 —— x2 = -3.6266 —— x1 = -2.32486
Iteration : 2 —— x2 = -1.16027 —— x1 = -3.6266
Iteration : 3 —— x2 = -2.32486 —— x1 = -1.16027
Iteration : 4 —— x2 = -0.300819 —— x1 = -2.32486
Iteration : 5 —— x2 = -1.16027 —— x1 = -0.300819
Iteration : 6 —— x2 = -0.00861099 —— x1 = -1.16027
Iteration : 7 —— x2 = -0.300819 —— x1 = -0.00861099
Iteration : 8 —— x2 = -2.12823e-07 —— x1 = -0.300819

$x_s = -2.12823149138563e - 07$ $g(x_s) = x_s - atan(x_s) = -3.20284333180532e - 21$
x0 = 1
Iteration : 1 —— x2 = 0.214602 —— x1 = 0.00320628
Iteration : 2 —— x2 = 1.0987e-08 —— x1 = 0.214602

$x_s = 1.09870240240853e - 08$
$g(x_s) = x_s - atan(x_s) = 0$
$x0 = -1$
Iteration : 1 —— x2 = -0.214602 —— x1 = -0.00320628
Iteration : 2 —— x2 = -1.0987e-08 —— x1 = -0.214602

$x0 = 0.1$
$x_s = 1.21263429527819e - 11$ $g(x_s) = x_s - atan(x_s) = 0$

2b

The number of iterations reduce as we approach the exact solution.
TODO: Expand

3

3a

$|x_{k+1} - \sqrt{a}| \leq \frac{1}{2}|x_k - \sqrt{a}|$
Extending we get.
$|x_{k+1} - \sqrt{a}| \leq \frac{1}{2}|x_k - \sqrt{a}| \leq \frac{1}{4}|x_{k-1} - \sqrt{a}| ... \leq \frac{1}{2^{k+1}}|x_0 - \sqrt{a}|$

3b

$x0 = 1.1$
Iteration : 1 —— x2 = 1.00455 —— x1 = 1.00001
$x_s = 1.00454545454545$
$x_0 = 2$
Iteration : 1 —— x2 = 1.25 —— x1 = 1.025 Iteration : 2 —— x2 = 1.0003 —— x1 = 1.25 Iteration : 3 —— x2 = 1.025 —— x1 = 1.0003 Iteration : 4 —— x2 = 1 —— x1 = 1.025
$x_s = 1.00000004646115$
$x0 = 5$
Iteration : 1 —— x2 = 2.6 —— x1 = 1.49231 Iteration : 2 —— x2 = 1.08121 —— x1 = 2.6 Iteration : 3 —— x2 = 1.49231 —— x1 = 1.08121 Iteration : 4 —— x2 = 1.00305 —— x1 = 1.49231 Iteration : 5 —— x2 = 1.08121 —— x1 = 1.00305 Iteration : 6 —— x2 = 1 —— x1 = 1.08121
$x_s = 1.00000463565079$
$x0 = 10$
Iteration : 1 —— x2 = 5.05 —— x1 = 2.62401 Iteration : 2 —— x2 = 1.50255 —— x1 = 5.05 Iteration : 3 —— x2 = 2.62401 —— x1 = 1.50255 Iteration : 4 —— x2 = 1.08404 —— x1 = 2.62401 Iteration : 5 —— x2 = 1.50255 —— x1 = 1.08404 Iteration : 6 —— x2 = 1.00326 —— x1 = 1.50255 Iteration : 7 —— x2 = 1.08404 —— x1 = 1.00326 Iteration : 8 —— x2 = 1.00001 —— x1 = 1.08404
$x_s = 1.00000528956427$
$x0 = 50$
Iteration : 1 —— x2 = 25.01 —— x1 = 12.525 Iteration : 2 —— x2 = 6.30242 —— x1 = 25.01 Iteration : 3 —— x2 = 12.525 —— x1 = 6.30242 Iteration : 4 —— x2 = 3.23054 —— x1 = 12.525 Iteration : 5 —— x2 = 6.30242 —— x1 = 3.23054 Iteration : 6 —— x2 = 1.77004 —— x1 = 6.30242 Iteration : 7 —— x2 = 3.23054 —— x1 = 1.77004 Iteration : 8 —— x2 = 1.1675 —— x1 = 3.23054 Iteration : 9 —— x2 = 1.77004 —— x1 = 1.1675 Iteration : 10 —— x2 = 1.01202 —— x1 = 1.77004
$x_s = 1.01201564410353$

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