MATH-578B: Assignment # 3

Due on Thursday, October 22, 2015

Saket Choudhary
2170058637

Contents

Problem 1 **3**

Problem 2 **4**

Problem 4 **4**

Problem 1

The coverage c depends on the position x as: $c = \frac{NL_x}{G}$ where L_x is the expected length of of clones covering x.
Probability any position x to be covered by atleast one clone = 1- Probability that it is sequenced by atleast one clone.
Probability that position x is not sequenced = Probability of zero clones starting in $(x-L, x]$ = No arrivals in the interval $(x - L, x] = e^{-c(x)}$
Probability that it is sequenced $= 1 - e^{-c(x)}$ where $c(x)$ represents that c is a function of x.
$C \sim \Gamma(\alpha, \beta)$

$$f(c) = \frac{c^{\alpha-1}e^{-c/\beta}}{\beta^\alpha \Gamma(\alpha)}$$

Thus,

$$P(N_h = k) = \int_0^\infty e^{-ch}\frac{(ch)^k}{k!} \times \frac{c^{\alpha-1}e^{-c/\beta}}{\beta^\alpha \Gamma(\alpha)} dc$$

Problem 2

Given: $\lim_{n\to\infty}(1 - F(b\log(n) + x/a)) = G(x)$

$$\lim_{n\to\infty}(1 - F(b\log(n) + x/a)) = G(x)$$
$$\lim_{n\to\infty} F(b\log(n) + x/a) = 1 - G(x)/n$$

$$\begin{aligned}
P(a(max_i X_i - b\log(n)) \leq x) &= P(max_i X_i \leq x/a + b\log(n)) \\
&= P(X_1 \leq x/a + b\log(n))P(X_2 \leq x/a + b\log(n))\dots P(X_n \leq x/a + b\log(n)) \\
&= (F(x/a + b\log(n)))^n \\
&= \lim n_i nfty(1 - G(x)/n)^n \\
&= \lim_{n\to\infty} e^{n\log(1-G(x)/n)} \\
&= e^{-G(x)}
\end{aligned}$$

Choosing a, b for $G(x) = e^{-x}$ given $X_i \sim exponential(\lambda)$
$f(x|\lambda) = \lambda e^{-}\lambda x \implies F(x) = 1 - e^{-\lambda x}$
Now,

$$\begin{aligned}
\lim n \to \infty 1 - G(x)/n &= F(b\log(n) + x/a) \\
&= 1 - e^{-\lambda(b\log(n)+x/a)} \\
e^{-x}/n &= e^{-\lambda(b\log(n)+x/a)} \\
-x &= \log(n) + -\lambda(b\log(n) + x/a) \\
x(-1 + \lambda/a) &= \log(n) - b\lambda\log(n)
\end{aligned}$$

Thus, $a = \lambda$ and $b = \frac{1}{\lambda}$

Problem 4

Minimal neighborhood set $J_{i,j}$ such that $\{i', j' \in J^c_{i',j'}\}$ are independent of $Y_{i,j}$ is given by: $\{(i', j') : |i - i'| \leq tor|j - j'| \leq t\}$
Now,

$$\begin{aligned}
b1 &= \sum_{i\in I}\sum_{jinJ_i} E(X_i)E(X_j) \\
&= p^t \sum_{j\in J_i} E(X_j) + \sum_{i=2}^{n-t+1}(1-p)p^t \sum_{j\in J_i} E(X_j) \\
&= (n - t + 1)p^t(2t + 1)p^t \times 2 + (n - t + 1)^2(1 - p)^2 p^{2t}(4t + 2) \\
&= p^{2t}(n - t + 1)(4t + 2)(1 + (n - t + 1)(1 - p)^2)
\end{aligned}$$