# MATH-505A: Assignment #

Due on Friday, August 29, 2014

*10:30am*

**Saket Choudhary**
**2170058637**

# Contents

# Exercise # 1.2

## (3)

At the start of the tournament we have $2^n$ players to begin with. At each round there will be **one** winner emerging from each of the pairs while the other gets 'knocked out'. One possible configuration for the first round of the tournament would be: $Player_1$ v/s $Player_2$; $Player_3$ v/s $Player_4$;...;, $Player_(2^n-1)$ v/s $Player(2^n)$. At the end of first round, there are exactly $\frac{2^n}{2} = 2^{n-1}$ winners and an equal number of knocked out players.
At round 1 the set of $2^n-1$ pairs can be represented as :$P_1, P_2, P_3, P_4, .., P_{2^n-1}$. The total number of such pairs is $2^n$ divided by 2 since each pair has 2 players. The outcome of first round can generate two values for each of these pairs depending on who amongst the two players is the winner. For e.g. $Player_1$ can win while playing in $P_1$ or $Player_2$ can, Thus total number of such configurations for the round 1 would be $2*2*2*...*(2^n-1)$ times which is equal to $2^{2^{n-}}$. Now at round 2 we would have $2^{n-2}$ pairs of players to play with and the possible configuration for choosing a winner of such a configuration is $2^{2^{n-2}}$
Thus, the sample space representing how the winners are chosen (or the knocked out persons are knocked out) can be calculated by multiplying configurations as obtained in $round_1$ $round_2$, ... $round_n$ by the **rule of product** as: $2^{2^{n-1}} * 2^{2^{n-2}} * .... * 2^1 = X$

$$log_2 X = 2^{n-1} + 2^{n-2} + ... + 1$$
$$log_2 X = \frac{2^{n-1+1}-1}{2-1}$$

Thus $X = 2^{2^n-1}$

## 5

## 5: (a)

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
Let $x \in A \cup (B \cap C) \implies x \in A \; OR \; x \in B \cap C$. Case 1: $xinA$ Then $x \in (A \cup B) \; AND \; x \in (A \cup C)$. That is given $x$ is contained in $A$ it is for sure contained in union of $A$ and $B$, and also in the union of $A$ with $C$. From the definition of intersection, this would imply: $x \in (A \cup B) \cap (A \cup C)$
Case 2: $x \in (B \cap C)$ Then $x \in B \; AND \; x \in C \implies x \in (A \cap B) \; AND \; x \in (A \cap C)$ where $A$ can be any set, since $B \subseteq (A \cap B)$
Thus from both the cases we get: $x \in (A \cup B) \cap (A \cup C)$
This implies

$$A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C) \tag{1}$$

Now consider a $y \in (A \cup B) \cap (A \cup C) \implies y \in (A \cup B) \; AND \; y \in (A \cup C)$. This implies $x$ belongs to $A$ $OR \; B \; AND \; A \; OR \; C$ Two cases again: Case 1: $x \in A \implies x \in A \cup (B \cap C)$ as $A \subseteq (A \cup (B \cap C))$
Case 2: $x \in B \; AND \; x \in C \implies x \in (B \cap C) \cup A$ as $(B \cap C) \subseteq (A \cup (B \cap C))$
Thus from both the cases we draw the same conclusion: $x \in (A \cup (B \cap C)) \implies (A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C)$
From 1 and , it is implied that:
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ **Ans. TRUE**

### 5: (b)

$A \cap (B \cap C) = (A \cap B) \cap C$
Let $x \in (A \cap (B \cap C)) \implies x \in A\ AND\ x \in B\ AND\ xinC$, which can be easily regrouped as $()x \in A\ AND\ x \in B)\ AND\ x \in C$, which is same as $x \in (A \cap B) \cap C$.
Another approach would be what we used in part (a) above to show that the $L.H.S$ and $R.H.S$ are subsets of each other. However the $AND$ solution is straight forward, since there are no $OR's$ involved.
**Ans. TRUE**

### 5: (c)

$(A \cup B) \cap C = A \cup (B \cap C)$
From part (a) of this problem, we proved that the following equation is true:
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
Substituting the $R.H.S$ of  as the $L.H.S$ of  we get:
$(A \cup B) \cap C = (A \cup B) \cap (A \cup C)$
Comparing  and  we see, that for  to be always true, the following should hold:
$C = A \cup C$
**which will only be true if $A \subseteq C$.**

## Exercise # 1.3

### 1

### 3

### 5

## Exercise # 1.4

### 2