MATH-578A: Homework # 1

Due on Tuesday, March 10, 2015

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Contents

Question # 1

Definition: $SP(i) = \max\ k < i$ such that $P[1..k] = P[i-k+1..i]$

String: CACGCAACGA
NOTE: Iteration indexed at 0. So SP[0] = 0(By Definition) and hence the loop iterations start from 1 and go till n-1=9 ;

Iteration	$SP[i]$	All other SP values examined	# of times inner while loop executed
1	0	-	0
2	1	-	0
3	0	$SP[0]$	1
4	1	-	0
5	2	-	0
6	1	$SP[0]$	1
7	1	-	0
8	1	$SP[0]$	1
9	0	-	0

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Question #2

$S = CACGGCACGG$
NOTE: Indexing starts from 0. By definition $Z[0] = |S| = 10$
The 'cases' are choosen out of:
Case 1. $k > r$. The index for which Z value is being calculated is greater than the right most ending of all the previous(till $k-1$) Z boxes calculated. Since this is as good as having no pre-calculated Z scores, this case leads to explicit character comparison(starting at k) till a mismatch occurs.
Case 2. $k \leq r$ The current position k is inside one of the previoulsy calculated Z boxes. Hence there exists a correpsonding position $k' = k - l + 1$ where l is the left ending of the Z box with it's right ending at r, such that $S[k`] = S[k]$. There is a corresponding one to one match for $S[k'..r - l + 1]$ with $S[k..r]$ and we define this to be another box β with $\beta = r - k + 1$ and hence $Z[k]$ can be caculated utilising this information.
The following three cases arise:
Case 2a. $Z'_k < |\beta|$ So starting at k' the length of largest substring that matches the prefix of S is less than size of that β box starting at k'. Since this β box appears starting from k too and $Z'_k < |\beta|$ implies $Z_k = Z'_k$. Total comparisons:

1. Comparison: $k \leq r$
2. Assignment/Calculation: $k' = k - l + 1$
3. Lookup: Z'_k
4. Assignment/Caculation: $|\beta| = r - k + 1$
5. Comparison: $Z'_k < |\beta|$
6. Assigment: $Z_k = Z'_k$

No character comparisons are involved.
Case 2b. $Z'_k > |\beta|$ So the substring starting at k' matches a prefix of S and has length equal to the β box. If we call the box with it's leftmost end=l and rightmost end=r as α, then we know that $S[r+1] \neq S[|\alpha|+1]$ otherwise α would not have been the largest such box. Thus, $Z_k = \beta$ Thus no character comparisons involved in this case too.
The comparisons involved:

1. Comparison: $k \leq r$
2. Assignment/Calculation: $k' = k - l + 1$
3. Lookup: Z'_k
4. Assignment/Caculation: $|\beta| = r - k + 1$
5. Comparison: $Z'_k > |\beta|$
6. Assigment: $Z_k = Z'_k$

Case 2c. $Z'_k = |\beta|$
The substring starting at k might have a matching prefix in S, and hence explicit character comparions are required from $r+1$ to $q \geq r+1$ till the first mismatch occurs. These iterations are bound by $O(|S|)$ since the maximum possible mismatches are $O(|S|)$.
The comparisons involved:

1. Comparison: $k \leq r$
2. Assignment/Calculation: $k' = k - l + 1$
3. Lookup: Z'_k
4. Assignment/Caculation: $|\beta| = r - k + 1$

[Question #2] continued on next page...

Question # 3

Algorithm 1 Calculate $y = x^n$

Input: r

Output: f =0

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