CSCI-570: Homework # 1

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HW1

(2: Ch#1 Ex#1)

False Consider the following example:
m ranks women as: $rank_w > rank_{w'}$
m' ranks women as: $rank'_w > rank_w$
w ranks men as: $rank_{m'} > rank_m$
w' ranks men as: $rank_m > rank_{m'}$

In such a case there is no possible stable matching where the men m or m' can be paired up with their top preferences.

(3: Ch#1 Ex#2)

True Consider the statement to be false. In that case m pairs up with woman w' such that $w' \neq w$ and hence w pairs up with m' such that $m' \neq m$. Now preference wise m is ranked at the top for w and vice-versa(in on of the instances, preferences do not change with instances). So given that the pairs in this instance are (m, w') and (m', w) man m would have preferred w' over w and w would have preferred m' over m. But this contradicts the fact that m and w are at the top of each other's list.

(4)

False
Consider this case:

$m \longrightarrow w > w'$
$m' \longrightarrow w' > w$
$w \longrightarrow m' > m$
$w' \longrightarrow m > m'$

There is one possible pairing: $(m, w); (m', w')$ if men propose. However if women propose $(m', w); (m, w')$ is also one possible configuration. So the $G - S$ algorithm still gives **unique** solutions if **only men** or **only women** propose. So even though the male and female versions produce two independent outputs, the output from either of them is still unique!

(5)

A stable matching will not always exist. Consider the case of simple cyclic permutations:
$a \longrightarrow b > c > d$

$b \longrightarrow c > d > a$

$c \longrightarrow d > a > b$

$d \longrightarrow a > b > c$

They cannot settle down with their first choices since $(a, b); (c, d)$ is unstable as b prefers c to be its room mate and d prefers a followed by b to be its roommate.

(6: Ch#1, Ex#3)

Let the n shows of A have a rating given by $\{A_1, A_2, ..., A_n\}$ and that of B have $\{B_1, B_2, ..., B_n\}$. Consider a simpler case A_1, A_2, A_3, and B_1, B_2, B_3 such that $A = \{1, 3, 5\}$ and $B = \{2, 4, 6\}$ would result in a configuration as B, B, A and if however B now changes it configuration to $\{4, 2, 6\}$ the new configuration would be $\{B, A, B\}$ clearly violating the requirements of stability.

(7: Ch#1, Ex#4)

Let H represent the set of hospitals and S represent the set of students

while *there is a hospital $h \epsilon H$ with atleast one spot empty that hasn't been offered to $s \epsilon E$* **do**
  hire the next best valid student s' as in the preference list of h **if** s' *free* **then**
    s' gets hired by h, number of open spots reduce by 1
  else
    if s' *ranks h higher to its current employer* h' **then**
      s' leaves h';
      h becomes occupied; h' has 1 less student
    else
      s' remaiins with originla h' and h is still free and moves on to the next student on its preference list
    end
  end
end

To prove it's correctness:

First type of instability:

s is assigned h, s' is assigned no hospital and h prefers s':

Let as assume First type of staibility exists. Since h prefers s', it must have tried to hire s' before it hired s, but s' could have refused since there was another hospital h'' which was higher ranked than s' But at the end s' lands up with no hospital which is a CONTRADICTION.

Second Type of instability:

s assigned h; s' assigned h', h prefers s' to s and s' prefers h to h'.

Let us assume such an instability exists. Since h prefers s to s', it must have tried to hire s' at some point to which s' refused leading to hire of s or alternative s' was hired but later on waas asked by another hospital h'' which was higher on its prefernce list leading to s' leaving h, But now it is with h' which is lower ranked than h which is clearly a CONTRADICTION since s' is supposed to be shifting to higher ranked hospitals(like the women in the G-S problem/solution)

(8)

After stable matching terminated man $m1$ changed his mind to marry woman $w2$ though he was already married to $w1$ Original preference list : $w1 > w2$ New preference list: $w2 > w1$ Let the original pairing be $(m1, w1)$ and $(m2, w2)$. New pairings : ?
Case1: If $w2$ prefers $m2$ over $w1$ the change in preference of $m1$ does not matter, as even if he now asks $w2$ he would be refused since she is already engaged with a person ranked higher.

Case2: If $w2$ prefers $m1$ over $m2$ and now $m1$ also prefers $w2$ over $m1$, then when it is $m1$'s turn he would ask $w2$ instead of $w1$ and stands a chance to get engaged initially(when $w2$ divorves $m2$) Now $m2$ is free, $w1$ is free and the G-S algorithm starts to run again with $m2$. $m2$ will not stsrt from the top of his list but should make an efficient choice to start from where he was left off by $w2$, because he is going to go further down the ladder(he would be rejected again by all women who were above $w2$ in his list even if he chososes to ask them again) so he 'initiates' the 'G-S' by asking a woman w' who was ranked just below to $w2$. The procedure then otherwise contiues like normal G-S, though $w1$ is free and would get engaged as soon as $m2$ or any other person proposes her.

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