MATH-505A: Homework # 4

Due on Friday, September 19, 2014

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Contents

Exercise # 2.1

(1)

Given: X is a random variable $\implies$

$$\{\omega \in \Omega : X(\omega) \leq x\} \in \mathcal{F} \ \forall x \in R \tag{1}$$

Part A) To Prove: aX is a random variable
Consider $Y = aX$, then since equation 1 holds:
Case1: $a \geq 0$
Then $\{\omega \in \Omega : aX(\omega) \leq x'\} \in \mathcal{F} \ \forall x' \in R$ where $x' = ax$
Case2: $a \leq 0$
Then $\{\omega \in \Omega : aX(\omega) \geq x'\} \forall x' \in R$ where $x' = ax \implies \cup\{\{\omega \in \Omega : aX(\omega) \leq x''\}\}^c \in \mathcal{F}$ where $x'' = x'$
Case3: $a \ is \ 0$
Then, $aX = 0$
Case i: $x < 0$
$\{\omega \in \Omega : aX(\omega) = \phi\} \in \mathcal{F}$
Case ii: $x \geq 0$
$\{\omega \in \Omega : aX(\omega) = \Omega\} \in \mathcal{F}$
Thus from all the above cases,

(2)

For part 1, $Y' = aX$ is also a random variable:
To Prove: $Y = Y' + b$ is a ranodm variable where Y' is a random variable and b is a constant.
Since Y' is a random variable: $\{\omega \in \Omega : Y(\omega) \leq y\} \in \mathcal{F} \ \forall y \in R$ and so, $\{\omega \in \Omega : Y(\omega) + b \leq y'\} \in \mathcal{F} \ \forall y' \in R$ where $y' = y + b$
Since $\{\omega \in \Omega : Y(\omega) + b \leq y'\} \in \mathcal{F} \ \forall y' \in R$, $Y' + b$ is a random variable $\implies aX + b$ is a random variable

(3)

$p(H) = p$; $p(T) = 1 - p$
Tossing a coin n times is a binomial process(each individual toss is a bernoulli process) and let A be the event such that k out of n tosses are heads and this can occur in $\binom{n}{k}$ ways with probability p^k. There would also be $n - k$ tails and the probability for that is $(1-p)^{n-k}$. Thus,:
$p(A) = \binom{n}{k} p^k * (1-p)^{n-k}$
For a fair coin, $p = \frac{1}{2}$ and hence p(A) $= \binom{n}{k} (\frac{1}{2})^k (\frac{1}{2})^{n-k} = \binom{n}{k} (\frac{1}{2})^n$

(4)

(5)

Exercise # 2.3

(1)

(2)

(3)

(4)

(5)