MATH-578B: Assignment # 2

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Contents

Prob. I

Define $h(w)$ to be an indicator function:

$$h(x) = \begin{cases} 1 & x \in A \\ -1 & x \notin A \end{cases}$$

Now consider $E[h(W)]$:

$$\begin{aligned} E[h(W)] &= P(W \in A) \times 1 + P(W \notin A) \times -1 \\ &= P(W \in A) - (1 - P(W \in A)) \\ &= 2P(W \in A) - 1 \end{aligned} \tag{1.1}$$

Similarly,

$$E[h(Z)] = 2P(Z \in A) - 1 \tag{1.2}$$

where $A \in Z^+$
From (1.1), (1.2)

$$\begin{aligned} E(h(W)) - E(h(Z)) &= 2P(W \in A) - 2P(Z \in A) \\ |E(h(W)) - E(h(Z))| &= 2|P(W \in A) - P(Z \in A)| \\ max_{h:||h||=1}|E(h(W)) - E(h(Z))| &= 2max_{A \in Z^+}|P(W \in A) - P(Z \in A)| \\ &= 2|P(W = 0) - P(Z = 0) + P(W = 1) - P(Z = 1) + \cdots| \\ &= \sum_{k \geq 0} |P(W = k) - P(Z = k)| \end{aligned}$$