MATH-505A: Homework # 3

Due on Friday, September 12, 2014

Saket Choudhary
2170058637

Contents

Exercise # 1.7

(1)

Given: Two roads $r1_{AB}$, $r2_{AB}$ connectinng points A and B and $s1_{BC}$, $s2_{BC}$ connecting B and C.
Let $p(AB)$ denote the probability that path between A$\longrightarrow$B is open and let $p(AB^c)$ denote the probability that there is no open road b/w A and B. Alternativels $p(AB)$ denotes that road(s) between A and B are open. **To find:**$Y = P(AB|AC^c)$.
Y is equal to the probability that road between A and B is open AND still the path between A and C is closed $\implies$ Path between B and C is closed AND between A and B is open
p(AB) = Path b/w A,B is open = 1 - Path b/w A,B is closed = $1 - p * p$
Thus

$$p(AB) = 1 - p^2 \tag{1}$$

Also,

$$p(AB) = p(BC) \tag{2}$$

$p(AC^C)$ = 1 - Probability A,C is open = 1 - Probability AB is open AND BC is open. Thus,

$$p(AC^c) = 1 - p(AB)p(AC) = 1 - (1 - p^2)^2 \tag{3}$$

$$p(AB \cap AC^C) = p(AC^C|AB)p(AB) = p(BC^C)p(AB) = p^2(1 - p^2) \tag{4}$$

$$p(AB|AC^c) = \frac{P(AB \cap AC^c)}{p(AC^c)} = \frac{p(AC^C|AB)p(AB)}{p(AC^c)} = \frac{p^2(1 - p^2)}{1 - (1 - p^2)^2} \tag{5}$$

Part 2: Additional direct road from A to C. Find $p(AB|AC^c)$:
$p(AC^c|AB)$ = Probability that A,C is closed given A,B are open = Probability A,C(direct) are closed AND B,C are closed

$$p(AC^C|AB) = p * p(BC^c)p(AB) \tag{6}$$

where the extra p in 6 as compared to 4 is because the direct path A,C should be blocked too.

$$p(AC^c) = 1 - (1 - p^2)^2(1 - p) \tag{7}$$

where the extra $(1 - p)$ factor in 7 as compared to 3 accounts for the fact that direct path AC is open.
Thus, for part 2:

$$p(AB|AC^c) = \frac{p^3(1 - p^2)}{1 - (1 - p^2)^2(1 - p)} \tag{8}$$

(2)

$$p(2K \cap 1A) = \frac{\binom{4}{2} * \binom{4}{1} * \binom{52-4-4}{10}}{\binom{52}{13}} = \frac{24 * 44! * 13!}{10! * 52!} = 1.357 * 10^{-9} \tag{9}$$

$p(1A|2K) = \frac{p(1A \cap 2K)}{p(2K)}$

$$p(2K) = \frac{\binom{4}{2} * \binom{52-4}{11}}{\binom{52}{13}} \tag{10}$$

Exercise # 1.7 continued on next page...

(4)

To prove/disprove: $p(x|C) > p(y|C) AND p(x|C^c) > p(y|C^c) \implies p(x) > p(y)$

$$p(x|C) - p(y|C) > 0 \tag{11}$$

$$p(x|C^c) - p(y|C^c) > 0 \tag{12}$$

$$p(x) = p(x|C)p(C) + p(x|C^c)p(C^c) \tag{13}$$

Also,

$$p(y) = p(y|C)p(C) + p(y|C^c)p(C^c) \tag{14}$$

Consider $p(x) - p(y)$:

$$p(x) - p(y) = (p(x|C) - p(y|C))p(C) + (p(x|C^c) - p(y|C^c))p(C^c) \tag{15}$$

From the 12, **??** and 15:

$$p(x) - p(y) > 0 \forall x, y \tag{16}$$

Thus, x is always prefered over y.

(5)

Let X_i represent the i^{th} ccard draw
Given: $X_k > X_i, \ \forall i \in [1, k-1] and k \in [1, m]$
$p(X_k = m | X_k > X_i) = \frac{p(X_k = m \cap X_k > X_i)}{p(X_k > X_i)} = \frac{p(X_k = m)}{p(X_k > X_i)} = \frac{\frac{1}{m}}{\frac{1}{k}}$
Where the equality in the last step comes from the fact that the probability of choosing cards such that $p(X_k > X_i)$ is simply to choose the largest card, i.e. k among the rest i.
Thus $p(X_k = m | X - k > X_i) = \frac{k}{m}$.

Exercise # 1.8

1

(a)

2

(a)