MATH-505A: Homework # 1

Due on Friday, August 29, 2014

10:30am

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Contents

Exercise # 1.2

(3)

At the start of the tournament we have 2^n players to begin with. At each round there will be **one** winner emerging from each of the pairs while the other gets 'knocked out'. One possible configuration for the first round of the tournament would be: $Player_1$ v/s $Player_2$; $Player_3$ v/s $Player_4$;...;, $Player_(2^n - 1)$ v/s $Player(2^n)$. At the end of first round, there are exactly $\frac{2^n}{2} = 2^{n-1}$ winners and an equal number of knocked out players.
At round 1 the set of $2^n - 1$ pairs can be represented as :$P_1, P_2, P_3, P_4, .., P_{2^n-1}$. The total number of such pairs is 2^n divided by 2 since each pair has 2 players. The outcome of first round can generate two values for each of these pairs depending on who amongst the two players is the winner. For e.g. $Player_1$ can win while playing in P_1 or $Player_2$ can, Thus total number of such configurations for the round 1 would be $2 * 2 * 2 * ... * (2^n - 1)$ times which is equal to $2^{2^{n-}}$. Now at round 2 we would have 2^{n-2} pairs of players to play with and the possible configuration for choosing a winner of such a configuration is $2^{2^{n-2}}$
Thus, the sample space representing how the winners are chosen (or the knocked out persons are knocked out) can be calculated by multiplying configurations as obtained in $round_1$ $round_2$, ... $round_n$ by the **rule of product** as: $2^{2^{n-1}} * 2^{2^{n-2}} * .... * 2^1 = X$

$$log_2 X = 2^{n-1} + 2^{n-2} + ... + 1$$
$$log_2 X = \frac{2^{n-1+1} - 1}{2-1}$$

Thus $X = 2^{2^n - 1}$

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5: (a)

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
Let $x \in A \cup (B \cap C) \implies x \in A\ OR\ x \in B \cap C$. Case 1: $xinA$ Then $x \in (A \cup B)\ AND\ x \in (A \cup C)$. That is given x is contained in A it is for sure contained in union of A and B, and also in the union of A with C. From the definition of intersection, this would imply: $x \in (A \cup B) \cap (A \cup C)$
Case 2: $x \in (B \cap C)$ Then $x \in B\ AND\ x \in C \implies x \in (A \cap B)\ AND\ x \in (A \cap C)$ where A can be any set, since $B \subseteq (A \cap B)$
Thus from both the cases we get: $x \in (A \cup B) \cap (A \cup C)$
This implies

$$A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C) \tag{1}$$

Now consider a $y \in (A \cup B) \cap (A \cup C) \implies y \in (A \cup B)\ AND\ y \in (A \cup C)$. This implies x belongs to A $OR\ B\ AND\ A\ OR\ C$ Two cases again: Case 1: $x \in A \implies x \in A \cup (B \cap C)$ as $A \subseteq (A \cup (B \cap C))$
Case 2: $x \in B\ AND\ x \in C \implies x \in (B \cap C) \cup A$ as $(B \cap C) \subseteq (A \cup (B \cap C))$
Thus from both the cases we draw the same conclusion: $x \in (A \cup (B \cap C)) \implies (A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C)$
From 1 and , it is implied that:
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ **Ans. TRUE**

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5: (b)

$A \cap (B \cap C) = (A \cap B) \cap C$
Let $x \in (A \cap (B \cap C)) \implies x \in A\ AND\ x \in B\ AND\ xinC$, which can be easily regrouped as $()x \in A\ AND\ x \in B)\ AND\ x \in C$, which is same as $x \in (A \cap B) \cap C$.
Another approach would be what we used in part (a) above to show that the $L.H.S$ and $R.H.S$ are subsets of each other. However the AND solution is straight forward, since there are no $OR's$ involved.
Ans. TRUE

5: (c)

$(A \cup B) \cap C = A \cup (B \cap C)$
From part (a) of this problem, we proved that the following equation is true:
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
Substituting the $R.H.S$ of as the $L.H.S$ of we get:
$(A \cup B) \cap C = (A \cup B) \cap (A \cup C)$
Comparing and we see, that for to be always true, the following should hold:
$C = A \cup C$
which will only be true if $A \subseteq C$.

Exercise # 1.3

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Exercise # 1.4

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