CSCI-567: Assignment #4

Due on Monday, November 9, 2015

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Contents

Problem 1

Problem 1: (a) Gradient Calculation

$$L(y_i, \hat{y_i}) = \log(1 + \exp(-y_i\hat{y_i}))$$

$$g_i = \frac{\partial L(y_i, \hat{y_i})}{\partial \hat{y_i}}$$

$$\boxed{g_i = \frac{-y_i \exp(-y_i\hat{y_i})}{1 + \exp(-y_i\hat{y_i})}}$$

Problem 1: (b) Weak Learner Section

$$h^* = \min_{h \in H} \big(\min_{\gamma \in R} \sum_{i=1}^{n} (-g_i - \gamma h(x_i))^2 \big)$$

$$\implies \frac{\partial h^*}{\partial \gamma} = 0$$

$$\implies 2\sum_{i=1}^{n} (-g_i - \gamma h(x_i))(-h(x_i)) = 0$$

$$\boxed{\hat{h} = -\frac{\sum_{i=1}^{n} g_i h(x_i)}{\sum_{i=1}^{n} h(x_i)^2}}$$

Also check if it is indeed minimum with a second derivative test:

$$\frac{\partial^2 h^*}{\partial \gamma^2} = 2\sum_{i=1}^{n} h(x_i)^2 > 0$$

Since the second derivative is positive definite, $ga\hat{m}ma$ is indeed where the minima occurs.

Problem 1 continued on next page. . .

Problem 1: (c) Step Size Selection

$$\alpha^* = \arg\min_{\alpha \in R} \sum_{1}^{n} L(y_i, \hat{y}_i + \alpha h^*(x_i))$$

Newton's approximation:

$$\alpha_1 = \alpha_0 - \frac{f(\alpha_0)}{f'(\alpha_0)}$$

We start from $\alpha_0 = 0$ and hence:

$$\begin{aligned}
f(\alpha_0) &= \sum_{i=1}^{n} \log(1 + \exp(-y_i \hat{y}_i)) \\
f'(\alpha_0) &= \sum_{i=1}^{n} \frac{\partial L}{\partial \alpha} \\
&= \sum_{i=1}^{n} \frac{-y_i h^*(x_i) \exp\left(-y_i(\hat{y}_i + \alpha h^*(x_i))\right)}{1 + \exp\left(-y_i(\hat{y}_i + \alpha h^*(x_i))\right)} \\
&= \sum_{i=1}^{n} \frac{-y_i h^*(x_i) \exp(-y_i \hat{y}_i)}{1 + \exp(-y_i \hat{y}_i)}
\end{aligned}$$

Thus,

$$\alpha_1 = \frac{\sum_{i=1}^{n} (\log(1 + \exp(-y_i \hat{y}_i)))}{\sum_{i=1}^{n} \frac{-y_i h^*(x_i) \exp(-y_i \hat{y}_i)}{1 + \exp(-y_i \hat{y}_i)}}$$

Problem 2

Problem 2: (a)

Primal form:

$$\min_{w} ||w||^2$$
$$\text{such that } |y_i - (w^T x_i + b)| \leq \epsilon$$

Problem 2: (b)

$$\min_{w,\epsilon_i} \frac{1}{2}||w||^2 + C\sum_i \epsilon_i$$

$$\text{such that } (w^T x_i + b) - y_i \leq n_i + \epsilon_i \text{ (positive deviation)}$$

$$\text{and } y_i - (w^T x_i + b) \leq p_i + \epsilon_i \text{ (negative deviation)}$$

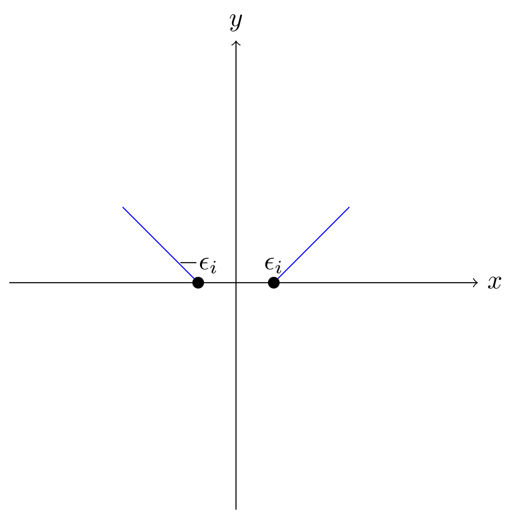
$$n_i \geq 0$$

$$p_i \geq 0$$

Also, the slackness loss needs further constraints:

$$n_i = \begin{cases} 0 & |n_i| < \epsilon_i, \\ |n_i| - \epsilon & otherwise \end{cases}$$

$$p_i = \begin{cases} 0 & |p_i| < \epsilon_i, \\ |p_i| - \epsilon & otherwise \end{cases}$$



So essentially n_i, p_i are non zero, only above the two blue lines

Problem 2 continued on next page...

Problem 2: (c)

$$
\begin{aligned}
L = \frac{1}{2}||w||^2 &+ C\sum_i (p_i + n_i) \\
&- \sum_i (\eta_i p_i + \eta_i' n_i) \\
&- \sum_i \alpha_i(\epsilon + p_i - (y_i - (w^T x_i + b))) \\
&- \sum_i \beta_i(\epsilon + n_i + (y_i - (w^T x_i + b)))
\end{aligned}
$$

Conditions :

$$
\begin{aligned}
\alpha_i &\geq 0 \\
\beta_i &\geq 0 \\
\eta_i &\geq 0 \\
\eta_i^* &\geq 0
\end{aligned}
$$

Dual Form:

$$
\begin{aligned}
\Delta_w L &= 0 \\
&= w - \sum_i \alpha_i x_i + \sum_i \beta_i x_i = 0 \\
\Delta_b L &= 0 \\
&= \sum_i \alpha_i - \sum_i \beta_i = 0 \\
\Delta_{p_i} L &= 0 \\
&= C - \sum \eta_i - \sum_i \alpha_i = 0 \\
\Delta_{n_i} L &= 0 \\
&= C - \sum \eta_i' - \sum_i \beta_i = 0
\end{aligned}
$$

Thus, w is given by:

$$w = \sum_i \alpha_i x_i - \sum_i \beta_i x_i$$

depends only on the support vectors.
This reduces the optimisation to:

$$\begin{aligned}
\max f &= \frac{1}{2}\sum_{i,j}(\alpha_i - \beta_i)x_i^T x_j(\alpha_j - \beta_j) + p_i(C - \sum_i \eta_i - \sum_i \alpha_i) \\
&+ n_i(C + \sum_i \eta_i' - \sum_i \beta_i) \\
&+ \epsilon(-\sum_i \alpha_i - \sum_i \beta_i) \\
&+ \sum_i y_i(\alpha_i - \beta_i) \\
&= \frac{1}{2}\sum_{i,j}(\alpha_i - \beta_i)x_i^T x_j(\alpha_j - \beta_j) - \epsilon(\sum_i(\alpha_i + \beta_i)) \\
&+ \sum_i y_i(\alpha_i - \beta_i)
\end{aligned}$$

$$\text{such that} \sum_i(\alpha_i - \beta_i) = 0$$

$$\text{and } \alpha_i, \beta_i \in [0, C]$$

Problem 2: (d)

Using Kernel transformation:

$$w = \sum_i (\alpha_i - \beta_i)\phi(x_i)$$

this happens because $x_i' x_j$ gets mapped onto by an equivalent kernel function $k(x_i, x_j) = \phi^T(x_i)\phi(x_j)$ and the objective function is:

$$\max_f = \frac{1}{2}\sum_{i,j}(\alpha_i - \beta_i)k(x_i, x_j)(\alpha_j - \beta_j) - \epsilon(\sum_i(\alpha_i + \beta_i))$$

Problem 3.3

Problem 3.3: (a)

C	Tr. Dataset 1(t)	Tr. Dataset 2(t)	Training Dataset 3(t)	CV
4^{-6}=0.000244	0.606156	0.400791	0.346078	0.578976
4^{-5}=0.000977	0.379495	0.477559	0.498054	0.907001
4^{-4}=0.003906	0.529940	0.495841	0.534946	0.926001
4^{-3}=0.015625	0.516236	0.577324	0.561874	0.935501
4^{-2}=0.062500	0.512287	0.529517	0.554510	0.945006
4^{-1}=0.250000	0.630195	0.663459	0.657651	0.943010
4^{0}=1.000000	0.746649	0.601710	0.563063	0.939003
4^{1}=4.000000	0.633370	0.572011	0.595552	0.942501
4^{2}=16.000000	0.674677	0.681010	0.698041	0.943503

As seen from the table. the time seems to increase with C and the CV increases too. C determines the tradeoff between objective function complexity and the overall loss. When C is small, there are chances of overfitting, this is evident from low CV values for lower C(because the generalisation error is high)
In terms of time complexity, if
The larger the value of C, the more is the penalisation and hence smaller the ϵ_i would be

Problem 3.3: (b)

Based on lowest cross validation error. $C = 4^2$

Problem 3.3: (c)

With $C = 16$, test accuracy = 0.943500

Problem 3.4

Problem 3.4: (a)

'libsvm' gives 0.9455 as its accuracy which is pretty close to

Problem 3.4: (b)

'libsvm' gives 0.9455 as its accuracy which is pretty close to

Problem 3.5

Problem 3.5: (a)

Problem 3.5: (b)