MATH-650 Assignment 6

Saket Choudhary (USCID: 2170058637) ([skchoudh@usc.edu](mailto:skchoudh@usc.edu))

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# Problem 19

pH.data <- read.csv('case0702.csv', header=T)  
logT <- log(pH.data$Time)  
pH.data$logT <- logT  
n <- nrow(pH.data)

Simple linear regression model for is given by:

### Part (a)

fit <- lm(pH~logT, data=pH.data)  
s <- summary(fit)  
b0 <- fit$coefficients[1]  
b1 <- fit$coefficients[2]  
se0 <- s$coefficients[3]  
se1 <- s$coefficients[4]  
t0 <- s$coefficients[5]  
t1 <- s$coefficients[6]  
p0 <- s$coefficients[7]  
p1 <- s$coefficients[8]

Thus, and

### Part (b)

Xbar <- mean(log(pH.data$Time))#1.190  
sx2 <- var(log(pH.data$Time))#0.6344  
mu <- b0 +b1\*log(5)

Thus,

### Part (c)

sigmahat <- 0.08226  
se = sigmahat \*sqrt(1/n+(log(5)-Xbar)^2/(n-1\*sx2))

# Problem 25

### Part (a)

$$
\begin{align\*}
\frac{\partial SS}{\partial \beta\_0} &= -2\sum\_{i=1}^N (Y\_i-\beta\_0-\beta\_1X\_i) =0\\
\implies \sum\_{i=1}^N Y\_i &= \sum\_{i=1}^N (\beta\_0 +\beta\_1X\_i)\\
\implies \sum\_{i=1}^N Y\_i &= \beta\_0n +\beta\_1\sum\_{i=1}^N X\_i\tag{1}
\end{align\*}
$$

$$
\begin{align\*}
\frac{\partial SS}{\partial \beta\_1} &= -2\sum\_{i=1}^N (Y\_i-\beta\_0-\beta\_1X\_i)X\_i = 0\\
\implies \sum\_{i=1}^N Y\_iX\_i &= \sum\_{i=1}^N(\beta\_0+\beta\_1X\_i)X\_i\\
\implies \sum\_{i=1}^N Y\_iX\_i &= \beta\_0\sum\_{i=1}^NX\_i+\sum\_{i=1}^N\beta\_1 X\_i^2\tag{2}
\end{align\*}
$$

### Part (b)

$$
\begin{align\*}
\hat{\beta\_0} &= \bar{Y}-\hat{\beta\_1}\bar{X} \tag{3}
\end{align\*}
$$

And,

Thus,

$$
\begin{align\*}
\hat{\beta\_1}\sum\_{i=1}^N X\_i^2-n \hat{\beta\_1} \bar{X}^2 &=\sum\_{i=1}^N X\_iY\_i -n\bar{X}\bar{Y}\\
\hat{\beta\_1}\sum\_{i=1}^N X\_i^2 + n\bar{X}\bar{Y} -n \hat{\beta\_1} \bar{X}^2 &= \sum\_{i=1}^N X\_iY\_i\\
\hat{\beta\_1}\sum\_{i=1}^N X\_i^2 + n\bar{Y}(\bar{X} - \hat{\beta\_1} \bar{X}) &= \sum\_{i=1}^N X\_iY\_i \tag{4}\\
\end{align\*}
$$

Substituting (3) in (4), we get

$$
\begin{align\*}
\hat{\beta\_1}\sum\_{i=1}^N X\_i^2 + n\bar{X}(\bar{Y} - \hat{\beta\_1} \bar{X}) &= \sum\_{i=1}^N X\_iY\_i\\
\hat{\beta\_1}\sum\_{i=1}^N X\_i^2 + n\bar{X}(\hat{\beta\_0}) \bar{X} &= \sum\_{i=1}^N X\_iY\_i\\
\hat{\beta\_1}\sum\_{i=1}^N X\_i^2 + \hat{\beta\_0} \sum\_{i=1}^N X\_i &= \sum\_{i=1}^N X\_iY\_i\tag{5}\\
\end{align\*}
$$

Thus, (5) is same as (2)

Now, From (3)

$$
\begin{align\*}
\hat{\beta\_0} &= \bar{Y}-\hat{\beta\_1}\bar{X}\\
\bar{Y} &= \hat{\beta\_0} + \hat{\beta\_1}\bar{X}\\
\frac{\sum\_{i=1}^N Y\_i}{N} &= \hat{\beta\_0} + \hat{\beta\_1}\frac{\sum\_{i=1}^N X\_i}{N}\\
\sum\_{i=1}^N Y\_i &= n\hat{\beta\_0} + \hat{\beta\_1}\sum\_{i=1}^N X\_i \tag{6}
\end{align\*}
$$

Thus, (6) is same as (1)

Consider second order differentials:

Thus, the values of indeed guarantee a minima since the Hessian is positive definitee.

### Part (c)

# Problem 13

Intercept = 0.3991 Standard Error = 0.1185 df = 22 $t\_{0.975,22} = 2.073

Upper limit = 0.3991 + 2.073 \* 0.1185 = 0.6447505 Lower limit = 0.3991 - 2.073 \* 0.1185 = 0.1534495

Thus, the 95% CI for Intercept is [0.1534495, 0.6447505]

Also following is the R code:

intercept <- 0.3991  
se <- 0.1185  
df <- 22  
t975 <- qt(0.975,df)  
limit.upper <- intercept + t975\*se  
limit.lower <- intercept - t975\*se  
limit.upper

## [1] 0.644854

limit.lower

## [1] 0.153346