Day 3

$\boxed{\text{Maths Algorithms}}$

1. Base Conversion → Base-2 to Base-10

→ $(101)_2 \Rightarrow (\quad)_{10}$

2 1 0

$(2^0 \times 1) + (2^1 \times 0) + (2^2 \times 1)$

$= 1 + 0 + 4$

$= 5$

→ $(101)_4 \Rightarrow (17)_{10}$

$4^0 \times 1 + 4^1 \times 0 + 4^2 \times 1$

$1 + 0 + 16 = 20$

$$\begin{array}{r|l} 4 & 17 \quad 1 \\ 4 & 4 \quad 0 \\ 4 & 1 \quad 1 \\ & 0 \end{array}$$

17 — base 4 rep.

base 10

base-b → base-x

$(201)_3 \Rightarrow (23)_8$

$\cancel{2^0 \times 1 + 2^1 \times 0 + 2^2 \times 2}$

$3^0 \times 1 + 3^1 \times 0 + 3^2 \times 2$

$= 1 + 0 + 18 = (19)_{10}$

/x

$$\begin{array}{r|l} 8 & 19 \quad 3 \\ 8 & 2 \quad 2 \\ & 0 \end{array}$$

$\sqrt{36}$ $\sqrt{48}$

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(2) N ⇒ prime or not;

$2 \to N$ — $O(n)$

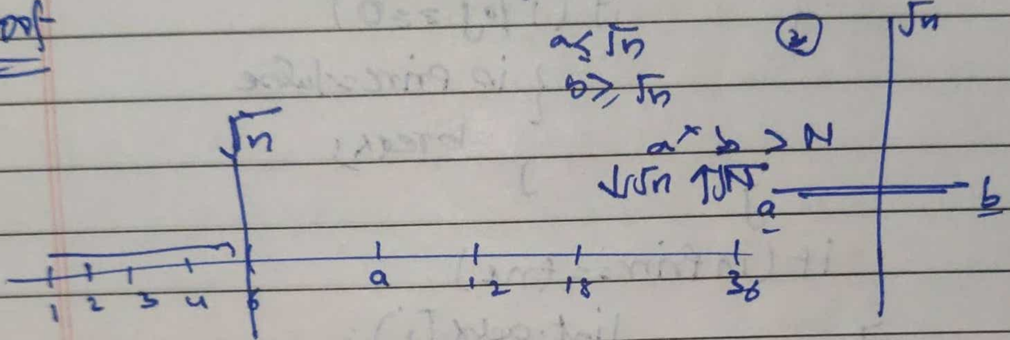
$2 \to N/2$ — $O(n)$

$2 \to \sqrt{n}$ — $O(\sqrt{n})$ All factors are in pair

(36) ⇒ (1×36), (2×18), (3×12), (4×9), (6×6), ~~(12×3)~~

(36×1) (18×2) (12×3) (9×4)

Proof



1. $N \% a == 0$
2. $\frac{N}{a} = b$

$\boxed{a \times b = N}$

~~1~~, ~~2~~, ~~3~~, ~~4~~, 5, ~~6~~, 7, ~~8~~, ~~9~~, ~~10~~, 11, ~~12~~, 13

i=5 i=i+6

Amortized $O(\sqrt{n})$

2 < i++

(3) All factors of a Number.

```
for(i=1; i*i<=N; i++)        O(√n)
{ if (N%i==0)
        list.add(i);

  if(N/i != i)
        list.add(N/i);
}
```

1 1 11

(4) Find all prime 1 to N

```
list = []
for( i=2 ; i*i <= N ; i++ )
{   ~~is prime (i)~~
    bool isPrime = true;

    for( j = 2; j <= √i ; j++)
    {       if (i%j == 0)
            { is Prime = false
              break;
            }
    }
    if (isPrime == true)
            list.add[i];
}
```

$O(n\sqrt{n})$

(5) Sieve of ~~Grat~~ Eratosthenes

① Assuming all are prime

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
All false	F	F	F	~~F~~	~~F~~	F	~~F~~	F	~~F~~	~~F~~	~~F~~	F	~~F~~	F	~~F~~	~~F~~	~~F~~	F	~~F~~
i=2					T		T		T		T		T		T		F		
i=3				T						T			T			T			
i=4																			

$\sqrt{16} = 4$

to Remember

```
for( i=2, i*i<=n ; i++)
{       if (isPrime[i] == false)
        {
            for (j = 2*i ; j<=n ; j=j+i)
            {  isPrime[j] = True;
        }   }
}
```

n log log n

(29)

5) Prime factor of N → 12 → 2 × 2 × 3

```
for (i = 2 ; i*i <= n ; i++)
{
        while ( n % i == 0)
              list.push(i);
                n = N / i
  }
if (N > 1)
   list.push(N);
```

$O(\sqrt{n})$

Best for 2^N / 0 while loop run log n

42
→ 21   i = 2   [2]
→ 7   i = 3   [2, 3]
→ n > 1   push 7   [2, 3, 7]

50
→ 25   i = 2
→ 5   i = 5   (2, 5, 5)
→ 1

2_{C_1} $\frac{2}{1 \times 1} = 2$

$3_{C_2} = \frac{3}{1} = 3$

1 1 1

6) Coeff of N_{CR}

$$\frac{n!}{(n-r)! \times R!}$$

$\frac{\cancel{4} \times 3 \times 2}{2 \times 1}$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

pascal Δ

pt[][]

r, j

n, i					
0	1				
1	1	1			
2	1	2	1		
3	1	3	3	1	
4	1	4	6	4	1

if (i == j or j == 0) val = 1

4_{C_2}

n_{C_R} ⇒ nth row & rth col^n

n ≤ 20

→ Build 21 × 21 table

col=0 ⇒ val=1

pt[0][0] = 1

$O(n^2)$ for(i=1 ; i ≤ N ; i++) ✓ Rows.
{ pt[i][0] = 1; pt[i][i] = 1

for(j=1 ; j < i ; j++)

pt[i][j] = pt[i-1][j-1] + pt[i-1][j]

HW 1 → if n = 10 , fixed , u can build 1D Table

→ But if n ranges from 0 ≤ N ≤ 20 , better make [21×21]

^nC_r ⇒ n^{th} row
r^{th} col^n

☆ $\boxed{^nC_r = {^nC_{n-r}}}$

$\boxed{^nC_r = \dfrac{n!}{(n-r)!\, r!}}$

Representation overflow (27)

7) Distributive Property

wrong / correct

1.) $(a+b)\ \%\ c = ((a\ \%\ c) + (b\ \%\ c))\ \%\ c$

2.) $(a \times b)\ \%\ c = ((a\ \%\ c) \times (b\ \%\ c))\ \%\ c$

3.) $(a-b)\ \%\ c = ((a\ \%\ c) - (b\ \%\ c))\ \%\ c$

4.) $\boxed{(a/b)\ \%\ c = ((a\ \%\ c) / (b\ \%\ c))\ \%\ c}$ ✗ Not possible

All CF of (a & b) is same for (b % a, a)

8) Euclidean Algo ⇒ GCD

⇒ gcd(a, b) = gcd(a − b, b) O(b−a)

log(min(A, B))

```
int gcd( int a, int b)
{   if (a= =0)
        return b;
    else ret (b%a, a);
```

Assum a is smaller
↓
converge to 0.
↓
as b%a, Chose b→a

Dry Run

a = 28, b = 48

20, 28

8, 20

4, 8

0, 4 (gcd)

a = 15
b = 12 % a 12.

a = 12
b = 15

it will rearrange as a to smaller

a = 168 b = 90

90 % 168.

~~78~~ ~~168~~

a interchange as smaller.

90	168
78	90
12	78
6	12
0	6 GCD

$$LCM = \frac{a \times b}{gcd(a, b)}$$

gcd(a, b, c, d) → Make for loop find hcf of two numbers using Euclid

a) Binary Exponentiation

(a) $a^b = a \times a \times a \ldots$ (b times) $O(b)$ – Brute force

(b) $a^b = a^{b/2} \times a^{b/2}$

$= a^{b/4} \times a^{b/4}$

$a^1 = a$

$a^0 = 1$

$a^7 = a^3 \times a^3 \times a$

$a^8 = a^4 \times a^4$

Eg. $2^8 = 2^4 \times 2^4$

$2^4 = 2^2 \times 2^2$

$2^2 = 2^1 \times 2^1$

$2^8 = (2^2)^{8/2} = (4^2)^{4/2}$

$= 4^4 \quad 16^2$

iterative method a^b

```
Long res = 1;
while (b>0)
{ if (b%2 == 1)
      res = res*a;

   a = a*a;

   b = b/2;
}
return res;
```

2^7 $2^3 \times 2^4$

$b=3>0$ $b=1>0$

res = 1×2 res = 2×4 = 8 res = 8×16

a = 4 a = 4×4 = 16 a = 16×16

$b = 7/2 = 3$ $b = 3/2 = 1$ b = 0

Stop

16
8

128

16
16

2^{10}

res = 1	res = 1	res = 4	res = 4	res = 4×256 = 1024
a = 2	a = 4	a = 16	a = 256	a = 256^2
b = 10	b = 5	b = 2	b = 1	b = 0

$2^{10} = 4^5 = 16^2$

Stop
ret res

$\Theta(\log b)$ exponent part.

10 Catylyan Number

$C_n = \cancel{(2n)!} \quad \frac{{}^{2n}C_n}{n+1} = \frac{(2n)!}{(n+1)!\,(2n-n)!\,n!}$

$= \frac{(2n)!}{(n+1)!\,(n!)}$

Series ⇒ 1, 1, 2, 5, 14, 42

$C_{n+1} = \sum_{i=0}^{n} C_i C_{n-i}$

$C_3 = C_0C_3 + C_1C_2 + C_2C_1 + C_3C_0$

1. The no of valid parathesen expression the consist of N right paranthes & N left paranthes is C_N

1, 1, 2, 5, 14

Eg N=1

() ⇒ 1

N=2

(()) ⇒ 2

()()

N=3

((()))

(()())

()()() ⇒ 5

()(())

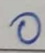
(()) ()

2. A rooted BT with one root node where each node has either $\boxed{0 \text{ or } 2}$ branches

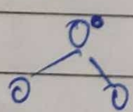
explicitly mentioned

An internal node has 2 branches & leaf Node has 0. Count How many rooted BTs are there with N interal nodes.

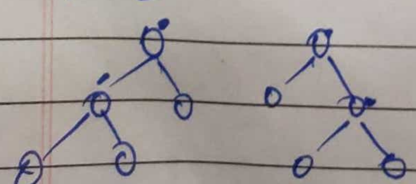
N=0



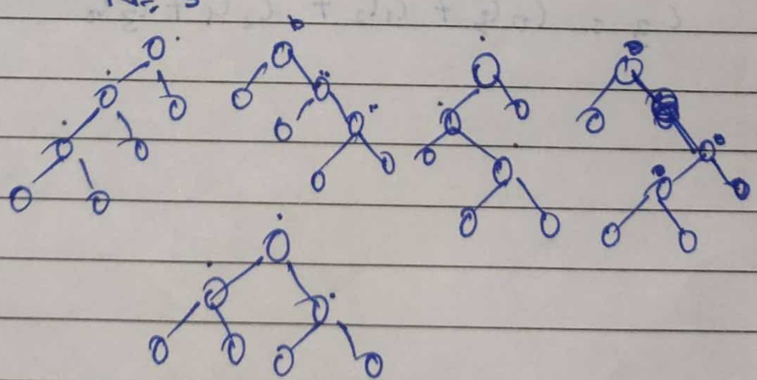
N=1



N=2



N=3



HW (3) Count no. of ways to divide circle using N non-intersecting chords (2N points on circle)

(4) ~~C_N~~ is the no. of diff ways (N+1) factor can be completely ()ised. (U canot () just one value)

N=1
(ab)

N=2
abc
((ab)c)
(a(bc))

N=3
(((ab) c) d)   (a(b(cd)))
((ab)(cd))   (a((bc) d))
((a(bc)) d)   ((a)(bcd))

Same as 1 → presented in diff way.

(5) Count of structually unique BSTs with N keys.

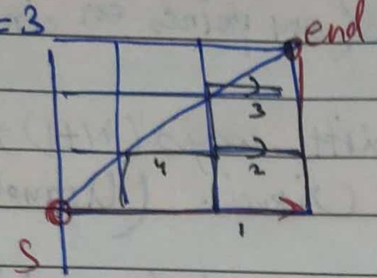
(6)

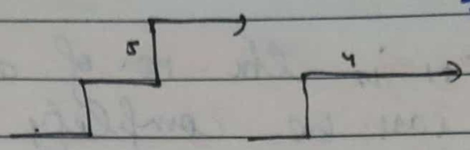
any diagonal (one-end to another w/o using main diagonal)

⑥ How many ways bottom left → top right to reach start → end on (N×N) matrix

C_n

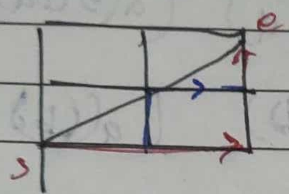
N=3



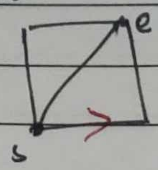


5

N=2



N=1



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⑦ All possible Binary Trees with given inorder traversal ⇒ C_n