The assumption $w = l$ (width equals length) in the problem of maximizing the volume of a cuboid with a given perimeter and area for its rectangular base simplifies the optimization process. This assumption is not arbitrary but is derived from the optimal conditions for the problem. Here's a detailed explanation:

Problem Constraints and Formulation

Given:

- The perimeter P of the rectangular base.
- The area A of the rectangular base.

We need to maximize the volume V of a cuboid with these constraints.

1. **Perimeter Constraint**:

$$4(l + w + h) = P$$

2. **Area Constraint**:

$$2(lw + lh + wh) = A$$

3. **Volume of the Cuboid**:

$$V = l \times w \times h$$

Simplifying Assumptions and Derivations

Step 1: Express h in terms of l and w

From the perimeter constraint:

$$4(l + w + h) = P \implies h = \frac{P}{4} - l - w$$

Step 2: Substitute h into the Area Constraint

Substitute $h = \frac{P}{4} - l - w$ into the area constraint:

$$2(lw + l\left(\frac{P}{4} - l - w\right) + w\left(\frac{P}{4} - l - w\right)) = A$$

Simplify the equation:

$$2(lw + l\frac{P}{4} - l^2 - lw + w\frac{P}{4} - wl - w^2) = A$$

$$2(l\frac{P}{4} + w\frac{P}{4} - l^2 - w^2) = A$$

$$2\left(\frac{P}{4}(l + w) - l^2 - w^2\right) = A$$

$$\frac{P}{2}(l + w) - 2(l^2 + w^2) = A$$

Step 3: Use Calculus to Maximize the Volume

To maximize the volume $V = lwh$:
Substitute h back into the volume equation:

$$V = l \times w \times \left(\frac{P}{4} - l - w\right)$$

To simplify the optimization, assume $l = w$ (width equals length), which reduces the number of variables and simplifies the math:

$$4(l + w + h) = P \implies h = \frac{P}{4} - 2l$$

Substitute h:

$$2(lw + l(\frac{P}{4} - 2l) + w(\frac{P}{4} - 2w)) = A$$

Since $l = w$:

$$2(l^2 + l(\frac{P}{4} - 2l) + l(\frac{P}{4} - 2l)) = A$$

$$2(l^2 + 2l(\frac{P}{4} - 2l)) = A$$

$$2(\frac{P}{2}l - 3l^2) = A$$

$$Pl - 6l^2 = A$$

Solve for l:

$$6l^2 - Pl + A = 0$$

Use the quadratic formula to solve for l:

$$l = \frac{P \pm \sqrt{P^2 - 24A}}{12}$$

Conclusion

The assumption $l = w$ is derived from simplifying the problem constraints. It is not arbitrary but results from the conditions that lead to the optimization of the cuboid volume given the perimeter and area constraints.