# Basic Control Manual

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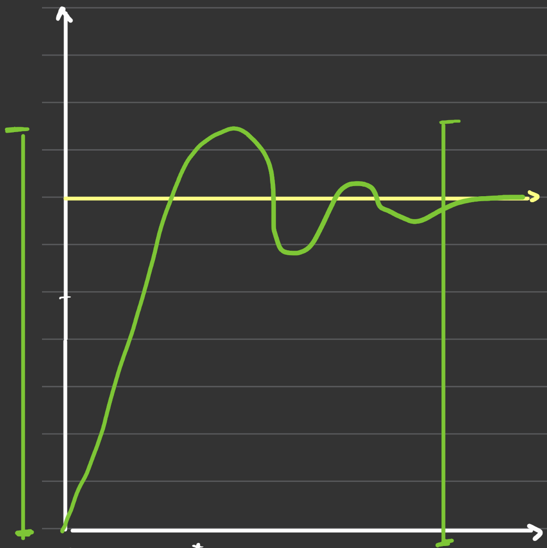
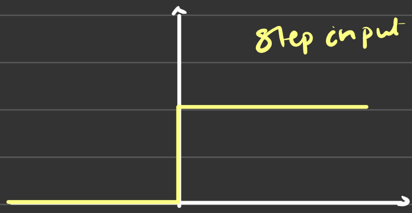
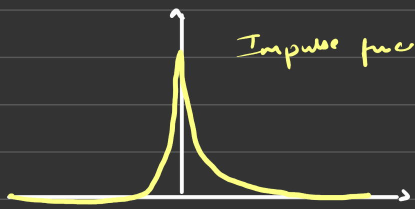
# Control System:





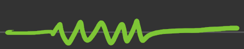

transient prd
- ↳ rise time
- ↳ overshoot
- ↳ time for overshoot

Steady state prd
- ↳ % error
- ↳ time taken to settle down

} important parameters.

**Transfer function:** = $\frac{\text{Output}}{\text{Input}}$ . $H(s) = \frac{R(s)}{C(s)}$

(s) is laplace domain conversion

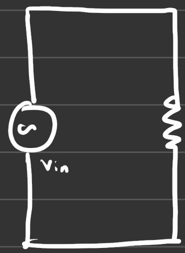


time domain : R    Laplace domain : R

time domain : $L\frac{dI}{dt}$    Laplace domain : LS

time domain : $\frac{1}{C}\int I\,dt$    Laplace domain : $\frac{1}{CS}$

## assume a R circuit :

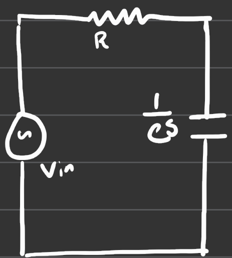



$V = IR$ ; in laplace ; $V = IR$

$\Rightarrow$ transfer fnc = $\left[\frac{V}{I} = R\right] \Rightarrow$

and as we know its a constant linear fnc.

## RC :



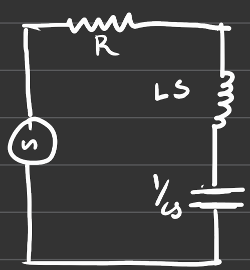

$$V_c = \left[\frac{V_{in}}{R + 1/cs}\right] 1/cs \Rightarrow \left[\frac{V_{in}}{R + cs}\right]$$

$$\Rightarrow \frac{V_c}{V_{in}} = \frac{\text{output}}{\text{input}} = \frac{1}{R + cs} = \left[\frac{1}{R + cs}\right]$$

@ $s \to 0$ $H(s) \to 1/R$ @ $s \to \infty$ $H(s) \to 0$

$\Rightarrow$ This is the characteristics of a low pass filter.

## RLC :

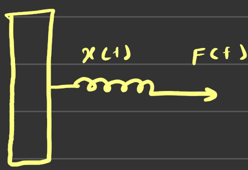
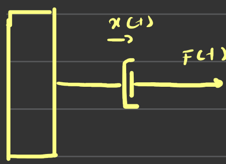
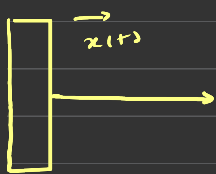
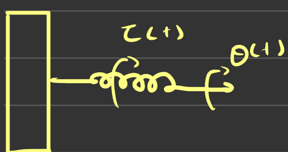
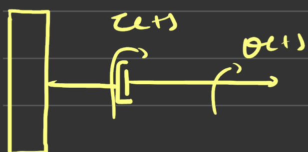
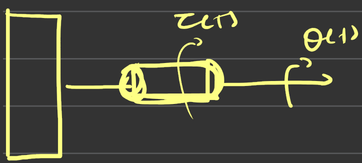



$$V_R = \left[\frac{V_{in}}{R + Ls + 1/cs}\right] R = \frac{V_{in}\, cs}{Rcs + LCs^2 + 1}$$

$$\Rightarrow \frac{V_{in}[s/L]}{s^2 + \frac{R}{L}s + 1/LC}$$

| we can see this is an oscillatory responce.

## More transfer [Mechanical systems]

| Component | F-v | F-x | Impedance |
|---|---|---|---|
|  | $K\int v(t)\,dt$ | $K\,x(t)$ | $K$ |
|  | $f_v\,(v(t))$ | $f_v \frac{dx(t)}{dt}$ | $f_v s$ |
|  | $M\frac{dv}{dt}$ | $M\frac{d^2x(t)}{dt^2}$ | $Ms^2$ |
|  | $K\int_0^t w(\tau)\,d\tau$ | $K\,\theta(t)$ | $K$ |
|  | $D\,w(t)$ | $D\left(\frac{d\theta(t)}{dt}\right)$ | $Ds$ |
|  | $J\frac{dw(t)}{dt}$ | $J\frac{d^2\theta(t)}{dt^2}$ | $Js^2$ |

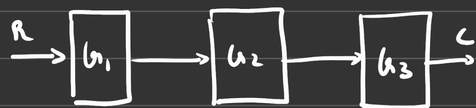
## Zeroes and Poles of a transfer func:

given a transfer func $\frac{(s-a)(s-b)}{(s-c)(s-d)}$ $s = a, b$ are zeroes
$s = c, d$ are poles

Zeroes are points where transfer function is 0
Poles are points where transfer func is $\infty$

## Block diagrams:

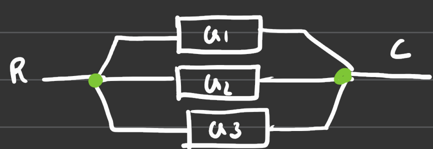
Serial blocks ⇒




all $G_1, G_2, G_3$ are individual transfer func

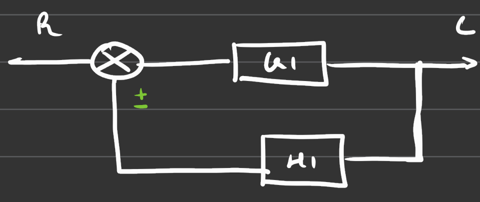
$$= \frac{C}{R} = G_1 G_2 G_3$$

Parallel blocks ⇒



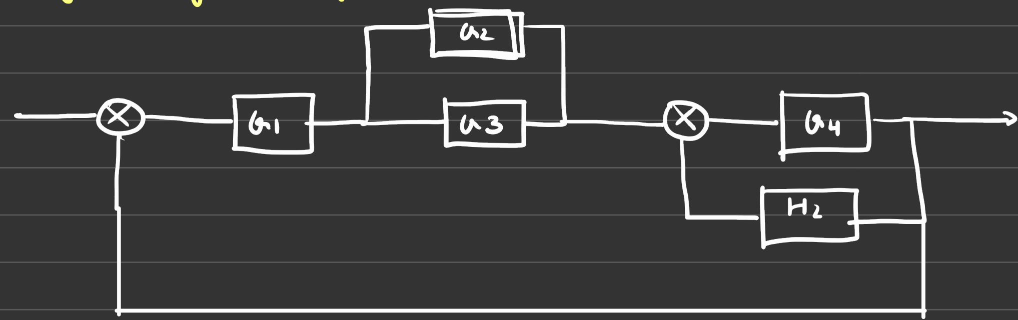

$$= \frac{C}{R} = [G_1 \pm G_2 \pm G_3]$$

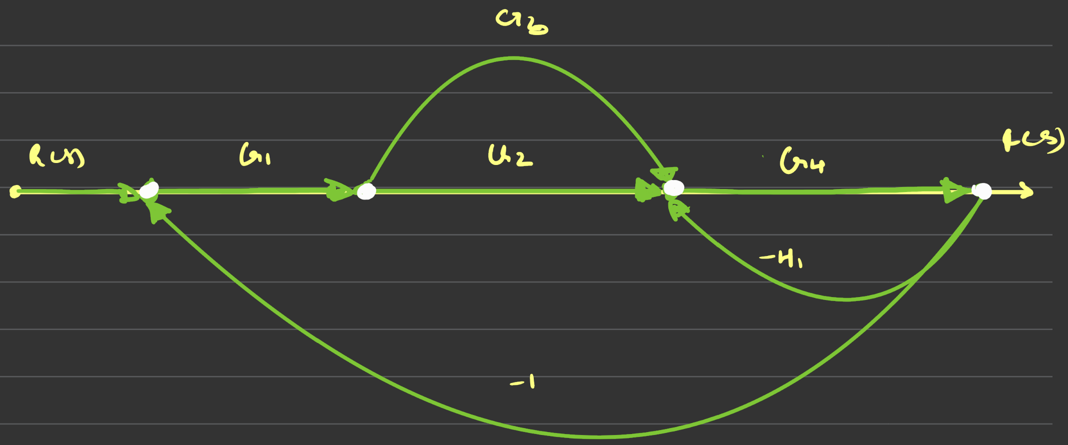
Feed back ⇒




$$\frac{C}{R} = \frac{G_1}{1 \mp H_1 G_1}$$

# Signal flow graph:




## Mason's gain formula

TF of any system $= \sum_{i=1}^{n} \frac{T_i \Delta_i}{\Delta}$

where N = no of 'forward' paths -> i.e. doesn't touch the same node more than once.

$T_i$ = gain of the $i^{th}$ forward path

$\Delta$ = 1 − ( $\Sigma$ loop gain) + ( $\Sigma$ 2 non touching loops) ... etc

$\Delta_i$ = gain of all loops touching path $i$

## Stability analysis with poles and zeroes:

a system's performance is normally analysed by giving an "impulse". If the system responds in a bounded manner, its stable, else its unstable.

before that we define "order of a system"

the highest degree of the denominator of a transfer func is defined as the order of the system

eg: $\frac{a}{s+a}$ is a first order system

$\frac{a}{s^2+as+b}$ is a second order system

Mostly we need to analyse systems only until second order because any higher order systems can be approximated as 2nd order

## Relationship of poles on system behaviour

### (a) First order system response to unit signal

(i). $\frac{1}{s}\left(\frac{1}{(s+a)}\right) = F(s) = \frac{1}{a}\left[\frac{1}{s} - \frac{1}{s+a}\right]$

$\uparrow$ unit input $\qquad \uparrow$ system

$= \mathcal{L}^{-1}\left[\frac{1}{a}\left[\frac{1}{s}\right] - \frac{1}{a}\left[\frac{1}{s+a}\right]\right]$

$= \frac{1}{a}\mathcal{L}^{-1}\left[\frac{1}{s}\right] - \frac{1}{a}\mathcal{L}^{-1}\left[\frac{1}{s+a}\right]$

$= \frac{1}{a}[u(t)] - \frac{1}{a}e^{-at}$

$\Rightarrow \frac{1}{a}[1 - e^{-at}]$ $\quad$ @ $t \to 0 \to f(t) = 0$

$\Rightarrow$ Bounded input ; $\quad$ @ $t \to \infty \to f(t) = 1/a$

Bounded output $\Rightarrow$ system response is <u>stable</u>

(ii) $\frac{1}{s}\left(\frac{1}{s-a}\right) \Rightarrow f(t) = \frac{1}{a}[1 - e^{at}]$

$\Rightarrow$ we can clearly see the response isnt bounded $\Rightarrow$ unstable <u>system</u>

(ii) different 2nd order system responses:

→ unit signal

(i) Both poles same and -ve

$F(s) = \frac{1}{s}\left(\frac{9}{(s+3)^2}\right) = \mathcal{L}^{-1}F(s) = f(t) = 1 - 3te^{-3t} - e^{-3t}$

@ $t \to 0$ $f(t) = 0$

@ $t \to \infty$ $f(t) = 1$

⇒ system is bounded, stable

(ii) Both poles are imaginary

$F(s) = \frac{1}{s}\left(\frac{a^2}{s^2+a^2}\right) = 1 - \cos at$

⇒ system is oscillatory about 1

(iii) Both poles +ve

(iv) Both poles complex with -ve real part.

We can observe a general trend here :

i). +ve poles are <u>unstable</u>

ii). -ve poles are <u>stable</u>

iii). purely imaginary poles <u>oscillatory</u>

iv). complex poles with -ve real = <u>Damped osc</u>

⇒ With these points we can create a Map