Day34_Advanced_Statistics

July 2, 2025

Day 34 – Advanced Regression and Model Evaluation

Welcome to **Day 34** of the statistics for machine learning series!

In this notebook, we focus on some of the most important and practical topics in advanced statistics — especially for machine learning applications:

- **Hypothesis Testing** – How to test assumptions with data
- **Type I and Type II Errors** – Understanding errors in decision-making
- **P-value and Significance** – How to decide when a result is meaningful
- **Linear Regression with Statistical Understanding** – Not just how to fit a line, but how to interpret it
- **ANOVA (Analysis of Variance)** – Breaking down variability using SSR, SSE, and SST
- **Regression Table Interpretation** – Reading output from `statsmodels` to assess model strength

These tools not only help us build better predictive models but also explain **why** a model works (or doesn't). Understanding these concepts is essential before jumping into more complex ML algorithms.

By the end of this notebook, you'll be able to: - Build and evaluate multiple regression models - Understand and calculate error metrics - Interpret full regression output with confidence

Let's dive in and build some statistical muscle

1 Hypothesis Testing

1.1 What is Hypothesis Testing?

Hypothesis testing is a statistical method used to make decisions or judgments about a population based on a sample.

We **start with a claim** (called a **null hypothesis**, H0) and test whether we should keep or reject it based on sample data.

1.2 Real-Life Example:

Suppose a juice company claims that their bottles contain **500ml** of juice on average.

You suspect they might be underfilling. So you collect a sample of bottles and check whether there's enough **evidence to reject their claim**.

1.3 Step-by-Step Manual Solution

Let's say you measured the volume (in ml) of 10 random juice bottles:

Data:
`[495, 498, 499, 502, 500, 496, 493, 497, 501, 494]`

1.3.1 Set Hypotheses

- **H0 (Null Hypothesis)**: The average volume is 500ml (= 500)

- **H1 (Alternative Hypothesis)**: The average volume is not 500ml (500)

This is a **two-tailed test** because we are checking for "different", not "greater" or "less".

1.3.2 Calculate Sample Statistics

- Sample size (n) = 10

- Sample mean ($\bar{x}$) = 497.5

- Sample standard deviation (s)   2.95

- Population mean ( ) = 500

1.3.3 Calculate the t-statistic

Use this formula:

`t = (x̄ -  ) / (s / √n)`

Substitute values:

`t = (497.5 - 500) / (2.95 / √10)   -2.67`

1.3.4 Find the critical t-value

For 95% confidence and 9 degrees of freedom (n - 1 = 9), the **t-critical value**   ±2.262

1.3.5 Compare

Since -2.67 < -2.262 → **Reject H0**

Conclusion: There is evidence that the bottles do **not** contain 500ml on average.

[1]:
```python
# Python Code to Perform the Test
```

```python
from scipy import stats
import numpy as np

# Step 1: Sample data
sample = [495, 498, 499, 502, 500, 496, 493, 497, 501, 494]

# Step 2: Perform t-test
t_stat, p_val = stats.ttest_1samp(sample, 500)

# Step 3: Print results
print("Sample Mean:", np.mean(sample))
print("T-statistic:", round(t_stat, 3))
print("P-value:", round(p_val, 4))

# Step 4: Decision
alpha = 0.05
if p_val < alpha:
    print(" Result: Reject H0 → Bottles are not 500ml on average")
else:
    print(" Result: Fail to Reject H0 → No strong evidence against 500ml claim")
```

```
Sample Mean: 497.5
T-statistic: -2.611
P-value: 0.0282
 Result: Reject H0 → Bottles are not 500ml on average
```

1.4 Summary

- Hypothesis testing helps us decide whether to accept or reject a claim based on sample data.
- We use a **t-test** when sample size is small and population standard deviation is unknown.
- A **low p-value ($<$ 0.05)** means we **reject the null hypothesis**.
- This is how we bring real-world doubt to data-driven decisions!

This method is commonly used in: - Product quality testing
- Medicine (testing if a drug works)
- A/B Testing (marketing and web experiments)

2 Type I and Type II Errors (with Confusion Matrix in ML)

2.1 What Are Type I and Type II Errors?

In hypothesis testing or classification problems, we often make two kinds of mistakes:

- **Type I Error (False Positive)**: You rejected the null hypothesis, but it was actually true.
- **Type II Error (False Negative)**: You failed to reject the null hypothesis, but it was actually false.

2.2 In Machine Learning: Classification

When we build a **classification model** (e.g., predicting spam vs not spam), these errors appear inside the **confusion matrix**.

2.3 What is a Confusion Matrix?

The confusion matrix is a 2x2 table used to evaluate classification models.

	Predicted Positive	Predicted Negative
Actual Positive	True Positive (TP)	False Negative (FN)
Actual Negative	False Positive (FP)	True Negative (TN)

2.4 Relating to Errors:

- **Type I Error = False Positive (FP)**
  → You predicted positive, but it's actually negative.
  → Example: A healthy person diagnosed with disease
- **Type II Error = False Negative (FN)**
  → You predicted negative, but it's actually positive.
  → Example: A sick person diagnosed as healthy

2.5 Real-World Example: COVID Test

Person Status	Test Result	Meaning
Sick	Positive (TP)	Correct detection
Sick	Negative (FN)	**Type II Error** (missed case)
Healthy	Positive (FP)	**Type I Error** (false alarm)
Healthy	Negative (TN)	Correct rejection

[2]:
```python
# Python Code Example: Confusion Matrix and Error Counts


from sklearn.metrics import confusion_matrix

# True values (actual class labels)
y_true = [1, 0, 1, 1, 0, 1, 0]

# Predicted values from the model
y_pred = [1, 1, 1, 0, 0, 1, 0]

# Create the confusion matrix
cm = confusion_matrix(y_true, y_pred)

# Extract values
TP = cm[1, 1]
```

```python
FN = cm[1, 0]
FP = cm[0, 1]
TN = cm[0, 0]

# Print matrix and error types
print("Confusion Matrix:")
print(cm)
print("\nTrue Positives (TP):", TP)
print("False Negatives (FN):", FN, "← Type II Error")
print("False Positives (FP):", FP, "← Type I Error")
print("True Negatives (TN):", TN)
```

```
Confusion Matrix:
[[2 1]
 [1 3]]

True Positives (TP): 3
False Negatives (FN): 1 ← Type II Error
False Positives (FP): 1 ← Type I Error
True Negatives (TN): 2
```

2.6 Summary

- **Type I Error = False Positive (FP)**
  - You wrongly predicted **positive**
  - Serious in medical diagnosis (healthy flagged as sick)
- **Type II Error = False Negative (FN)**
  - You wrongly predicted **negative**
  - More dangerous in critical systems (missed a real case)

2.7 Bonus Tip: When to Worry About Which?

- In **medicine**: Type II is worse (missing a sick patient)
- In **spam detection**: Type I is worse (important email goes to spam)
- In **fraud detection**: Type II is worse (you miss a fraud)

Always adjust your model to minimize the more **costly error** based on real-world consequences.

2.8 Note:

In machine learning, especially in classification problems, Type I and Type II errors are part of a tool called the **confusion matrix**.

The confusion matrix is a 2x2 table that shows how many predictions were correct and how many were wrong — and it helps us calculate important metrics like **accuracy**, **precision**, **recall**, and **F1-score**.

We will study the **confusion matrix in detail later** when we learn about classification models in machine learning.

3 P-Value and Significance

3.1 What is a P-Value?

The **p-value** is a number that tells us how likely it is to get the result we observed, or something more extreme, **if the null hypothesis (H0) were true**.

In simple words:
> P-value helps us decide whether to believe or doubt the claim (null hypothesis).

3.2 Rule of Thumb

- If **$p < 0.05$** → **Reject H0** → Result is **statistically significant**
- If **p 0.05** → **Fail to reject H0** → No strong evidence against H0

3.3 Real-World Example

A new medicine claims to reduce blood pressure more than the standard one.

- You give the new medicine to 30 people.
- You measure the average reduction.
- You perform a test and get **$p = 0.03$**

Interpretation:
- Since $p = 0.03 < 0.05$ → You **reject H0**
- You have enough evidence to say the new medicine **works better**.

3.4 Manual Concept (How P-Value Works)

Let's say we test if a sample of bottles contains 500ml of juice on average.

- H0: = 500ml

- H1: 500ml

- After performing a t-test, you get **$p = 0.042$**

Now compare: - If your alpha (significance level) is 0.05
- Since **$0.042 < 0.05$**, you **reject H0**

This means the average is likely **not** 500ml.

[4]:
```python
# Python Example Using Scipy

# Let's use the same juice bottle data again.

from scipy import stats

# Sample data (juice bottle volumes)
sample = [495, 498, 499, 502, 500, 496, 493, 497, 501, 494]

# Perform one-sample t-test
```

```
t_stat, p_val = stats.ttest_1samp(sample, 500)

print("T-statistic:", round(t_stat, 3))
print("P-value:", round(p_val, 4))

# Decision based on p-value
alpha = 0.05
if p_val < alpha:
    print(" Reject H0 → Juice volume is statistically different from 500ml")
else:
    print(" Fail to Reject H0 → No strong evidence against 500ml claim")
```

```
T-statistic: -2.611
P-value: 0.0282
 Reject H0 → Juice volume is statistically different from 500ml
```

3.5 Summary

- P-value tells us **how surprising** the data is **if H0 is true**
- It is used to decide whether to reject H0
- Common thresholds: 0.05 (95% confidence), 0.01 (99%)

Lower p-value → **stronger evidence** against the null hypothesis

3.6 Tip:

P-value is not the probability that H0 is true. It's the probability of seeing your result **if** H0 is true.

4 Linear Regression

4.1 What is Linear Regression?

Linear regression is a statistical method to model the relationship between a **dependent variable (Y)** and an **independent variable (X)** using a straight line.

The goal is to **predict values of Y** based on X.

4.2 Formula:

`Y = mX + c`

Where: - Y = dependent (target) variable
- X = independent (input) variable
- m = slope of the line (how much Y changes with X)
- c = intercept (Y value when X = 0)

4.3 Real-Life Example

A teacher wants to predict **exam scores** based on the number of **hours studied**.

Hours Studied	Score
1	50
2	55
3	65
4	70
5	75

Can we draw a line that shows how score changes with study hours?

4.4 Manual Steps

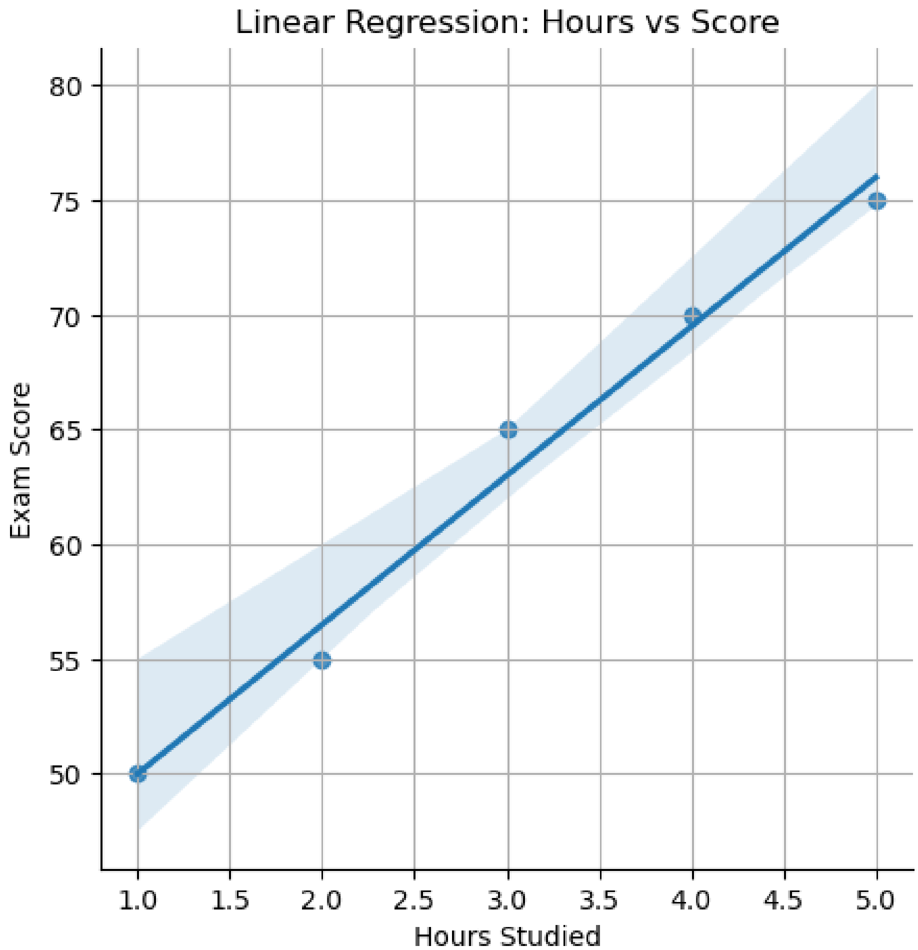
1. Plot the data points
2. Fit a straight line (best fit)
3. Use the formula Y = mX + c
4. Predict new values using the line

[5]:
```python
# Python Implementation (with Seaborn Plot)


import pandas as pd
import seaborn as sns
import matplotlib.pyplot as plt

# Sample dataset
data = {
    'Hours': [1, 2, 3, 4, 5],
    'Score': [50, 55, 65, 70, 75]
}
df = pd.DataFrame(data)

# Regression plot
sns.lmplot(x='Hours', y='Score', data=df)
plt.title("Linear Regression: Hours vs Score")
plt.xlabel("Hours Studied")
plt.ylabel("Exam Score")
plt.grid(True)
plt.show()
```



4.5 Interpretation:

- The line shows the **trend** between Hours and Score.
- We can now **predict scores** for new values of hours (e.g., 6 hours).

4.6 Summary:

- Linear regression finds a line that best fits the data.
- Useful for **prediction**, **trend analysis**, and **feature relationships**.
- The fit can be evaluated using **MSE, MAE, RMSE, and R^2**, which we'll cover next.

5 Multiple Linear Regression

5.1 What is Multiple Linear Regression?

Multiple Linear Regression is used when we want to predict a target variable using **two or more independent variables**.

5.2 Formula:

```
Y = b0 + b1*X1 + b2*X2 + ... + bn\*Xn
```

5.3 Real-Life Example:

Predicting **house prices** using: - Area (sqft) - Number of Bedrooms - Age of the house

[6]:

```python
# Python Code:

import pandas as pd
from sklearn.linear_model import LinearRegression

# Create dataset
data = {
    'Area': [1000, 1500, 1700, 1300, 1600],
    'Bedrooms': [2, 3, 3, 2, 3],
    'Age': [10, 5, 3, 8, 4],
    'Price': [300000, 400000, 420000, 330000, 410000]
}

df = pd.DataFrame(data)
X = df[['Area', 'Bedrooms', 'Age']]
y = df['Price']

# Train model
model = LinearRegression()
model.fit(X, y)

# Display results
print("Intercept:", model.intercept_)
print("Coefficients:", model.coef_)
```

```
Intercept: 100000.00000000029
Coefficients: [ 1.00000000e+02  5.00000000e+04 -1.29153671e-11]
```

5.4 Evaluation Metrics (MSE, MAE, RMSE, R^2)

5.4.1 Evaluation Metrics for Regression

5.4.2 Why Metrics Matter

After training a regression model, we measure how well it predicts. These are the most common metrics:

Metric	What it Measures	Notes
MAE	Mean Absolute Error	Average of absolute errors
MSE	Mean Squared Error	Penalizes large errors more
RMSE	Root Mean Squared Error	Same units as target

Metric	What it Measures	Notes
R^2	Coefficient of Determination	How well model explains the data (0 to 1)

[7]:
```
# Python Code:

from sklearn.metrics import mean_absolute_error, mean_squared_error, r2_score
import numpy as np

# Predict using model
predicted = model.predict(X)
actual = y  # from earlier

# Calculate metrics
mae = mean_absolute_error(actual, predicted)
mse = mean_squared_error(actual, predicted)
rmse = np.sqrt(mse)
r2 = r2_score(actual, predicted)

# Print metrics
print("MAE:", round(mae, 2))
print("MSE:", round(mse, 2))
print("RMSE:", round(rmse, 2))
print("R² Score:", round(r2, 3))
```

```
MAE: 0.0
MSE: 0.0
RMSE: 0.0
R² Score: 1.0
```

5.5 Regression Table (with statsmodels)

5.5.1 Why Use statsmodels?

`statsmodels` gives you a full regression output with:

- Coefficients
- Standard errors
- t-values
- p-values
- R-squared
- Confidence intervals

This helps in **interpreting the importance** and **reliability of predictors**.

5.5.2 Key Terms in the Output:

Term	Meaning	
coef	Change in Y for one-unit increase in X	
std err	Standard error of the estimate	
t	t-statistic for hypothesis testing	
P>	t	p-value — if < 0.05, the variable is statistically significant
[0.025, 0.975]	95% confidence interval for the coefficient	
R-squared	How well the model explains the variance (closer to 1 is better)	

[8]:
```python
# Python Code:

import statsmodels.api as sm

# Add constant to X for intercept
X_sm = sm.add_constant(X)
model_sm = sm.OLS(y, X_sm).fit()

# Show full regression summary
print(model_sm.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                  Price   R-squared:                       1.000
Model:                            OLS   Adj. R-squared:                  1.000
Method:                 Least Squares   F-statistic:                 3.574e+26
Date:                Wed, 02 Jul 2025   Prob (F-statistic):           3.89e-14
Time:                        18:22:26   Log-Likelihood:                 94.618
No. Observations:                   5   AIC:                            -181.2
Df Residuals:                       1   BIC:                            -182.8
Df Model:                           3
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const           1e+05   1.75e-07   5.72e+11      0.000       1e+05       1e+05
Area         100.0000   6.54e-11   1.53e+12      0.000     100.000     100.000
Bedrooms        5e+04   1.44e-08   3.48e+12      0.000       5e+04       5e+04
Age         5.821e-11   8.34e-09      0.007      0.996   -1.06e-07    1.06e-07
==============================================================================
Omnibus:                          nan   Durbin-Watson:                   0.214
Prob(Omnibus):                    nan   Jarque-Bera (JB):                0.561
Skew:                           0.639   Prob(JB):                        0.755
Kurtosis:                       1.971   Cond. No.                     1.73e+05
==============================================================================
```

```
Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
[2] The condition number is large, 1.73e+05. This might indicate that there are
strong multicollinearity or other numerical problems.
```

```
C:\Users\Lenovo\anaconda3\Lib\site-packages\statsmodels\stats\stattools.py:74:
ValueWarning: omni_normtest is not valid with less than 8 observations; 5
samples were given.
  warn("omni_normtest is not valid with less than 8 observations; %i "
```

5.5.3 How to Read the Output:

- **coef**: Effect of each feature on the target
- **P>|t|**: Probability that the coefficient is zero (if $p < 0.05$, it's significant)
- **R-squared**: Percentage of variance explained (closer to 1 is better)
- **[0.025, 0.975]**: 95% confidence interval for each coefficient

6 ANOVA: SSR, SSE, SST in Regression

6.1 What is ANOVA in Regression?

ANOVA (Analysis of Variance) in regression helps us **understand how much of the variation** in the dependent variable is **explained by the model** and how much is **left as error**.

6.2 Components:

- **SST (Total Sum of Squares)**: Total variation in the dependent variable

- **SSR (Regression Sum of Squares)**: Variation explained by the regression model

- **SSE (Error Sum of Squares)**: Variation not explained (residual/error)

Relationship:

```
SST = SSR + SSE
```

6.3 Real-Life Example:

Imagine you're predicting house prices. ANOVA tells you: - How much of the price difference is explained by your model (SSR) - How much is still unexplained error (SSE)

[9]:
```python
# Python Example:
import numpy as np

# Actual and predicted values
actual = np.array([300000, 400000, 420000, 330000, 410000])
predicted = model.predict(X)

# SST: Total Sum of Squares
```

```python
sst = np.sum((actual - np.mean(actual))**2)

# SSE: Error Sum of Squares
sse = np.sum((actual - predicted)**2)

# SSR: Regression Sum of Squares
ssr = sst - sse

print("SST (Total):", round(sst, 2))
print("SSR (Explained):", round(ssr, 2))
print("SSE (Unexplained):", round(sse, 2))
print("Check: SST = SSR + SSE →", round(ssr + sse, 2))
```

```
SST (Total): 11480000000.0
SSR (Explained): 11480000000.0
SSE (Unexplained): 0.0
Check: SST = SSR + SSE → 11480000000.0
```

7 Final Wrap-up

In this notebook, we have covered almost all the **important statistics concepts** required for understanding and applying machine learning — from basic ideas like hypothesis testing to **advanced regression analysis**, ANOVA, and model evaluation metrics.

Today, we focused especially on the **advanced elements** like multiple regression, model diagnostics, and interpreting statistical outputs — which are often skipped but are **crucial for serious ML practice**.

Some additional statistical concepts (like advanced probability, chi-square, or specific ML-focused techniques) will be covered **while we apply machine learning algorithms in practice**.

You're now well-prepared to move forward with ML projects, confident in the statistical foundation behind them!