MA323 Lab-3

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Q1. The number of calculations can be calculated using the probability

(P(U) ≤ f(Y)/cg(Y))

Where Y is the random variable generated for the distribution g(x) and U is the random variable from the distribution U(0,1).

Here P(U) represents the acceptance probability of Y and f(x) is the required distribution.

Now, the probability of acceptance is

$$\begin{aligned}
P\left(U \leq \frac{f(Y)}{cg(Y)}\right) &= \int_{-\infty}^{\infty} P\left(cg(Y)U \leq f(Y) | Y = t\right) g(t)dt \\
&= \int_{-\infty}^{\infty} P\left(U \leq \frac{f(t)}{cg(t)} | Y = t\right) g(t)dt \\
&= \int_{-\infty}^{\infty} P\left(U \leq \frac{f(t)}{cg(t)}\right) g(t)dt \\
&= \int_{-\infty}^{\infty} \frac{f(t)}{cg(t)} \times g(t)dt \\
&= \frac{1}{c}\int_{-\infty}^{\infty} f(t)dt \\
&= \frac{1}{c}.
\end{aligned}$$

Thus, the expected number of iterations before the first occurrence of P(U) ≤ f(Y)/cg(Y) can be calculated using the geometric distribution with probability P(X=k) = (1-p)^(k-1) * p

Where p = P(U) ≤ f(Y)/cg(Y)

Hence the expectation of X is

$$E(X) = \sum_{k=1}^{\infty} k(1-p)^{k-1}p$$

$$= p\sum_{k=1}^{\infty} k(1-p)^{k-1}$$

$$= p\left(1 + 2(1-p) + 3(1-p)^2 + 4(1-p)^3 + \cdots\right)$$

$$= p\left(\sum_{k=1}^{\infty}(1-p)^{k-1} + \sum_{k=2}^{\infty}(1-p)^{k-1} + \sum_{k=3}^{\infty}(1-p)^{k-1} + \cdots\right)$$

$$= p\left(\frac{1}{1-(1-p)} + \frac{1-p}{1-(1-p)} + \frac{(1-p)^2}{1-(1-p)} + \cdots\right)$$

$$= 1 + (1-p) + (1-p)^2 + \cdots$$

$$= \frac{1}{1-(1-p)}$$

$$= \frac{1}{p}$$

Therefore, the expected number of iterations is $1/(P(U) \leq f(Y)/cg(Y)) = c$

The sample mean of the distribution is
Sample mean = 0.3325253819208145
The actual expectation of the function f(x) can be calculated as:

E[X] = ∫f(x)dx for 0<x<1

$$=\int_0^1 20(x)(1-x)^3 dx$$

$$=\int_0^1 20(x - 3x^2 + 3x^3 - x^4)dx$$

$$= (10x^2 - 20x^3 + 15x^4 - 4x^5)\Big|_0^1$$

= 1/3

Expectation of f(x) = 0.3333333333333333

As observed, the average value of the sample is close to the expectation of f(x)

Estimated value of P(0.25<=X<=0.75): 0.6253
Exact value of P(0.25<=X<=0.75) can be calculated as:

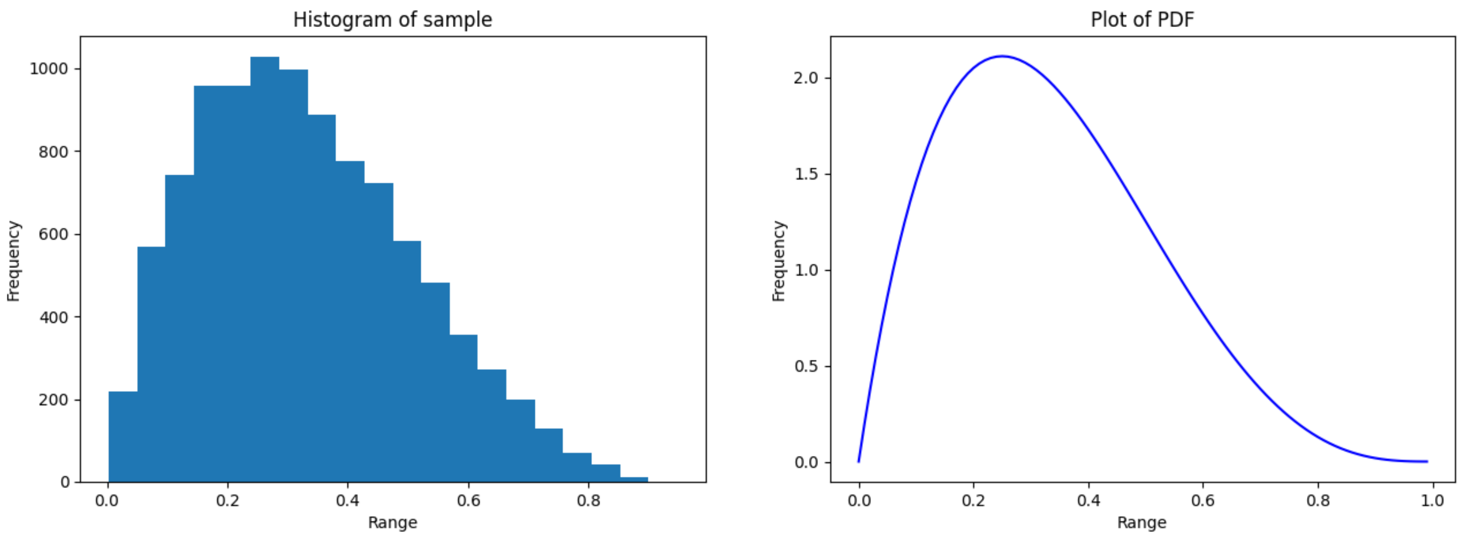
$$P(0.25<=X<=0.75) = \int_{0.25}^{0.75} 20(x)(1-x)^3 dx$$

$$= (10x^2 - 20x^3 + 15x^4 - 4x^5)\Big|_{0.25}^{0.75}$$

= 79/128
Exact value of P(0.25<=X<=0.75): 0.6171875
Again, these values are similar

The average of all the counts is
Average number of iterations = 9.9311
Actual Average number of iterations = 10

Plot of the histogram of the sample vs the PDF of sample



As observed, the histogram converges to the plot on the right as sample size approaches infinity.

For c = 20:

Sample mean = 0.33434576490864604

Estimated value of P(0.25<=X<=0.75): 0.6173

Average number of iterations = 29.5031

Actual Average number of iterations = 20

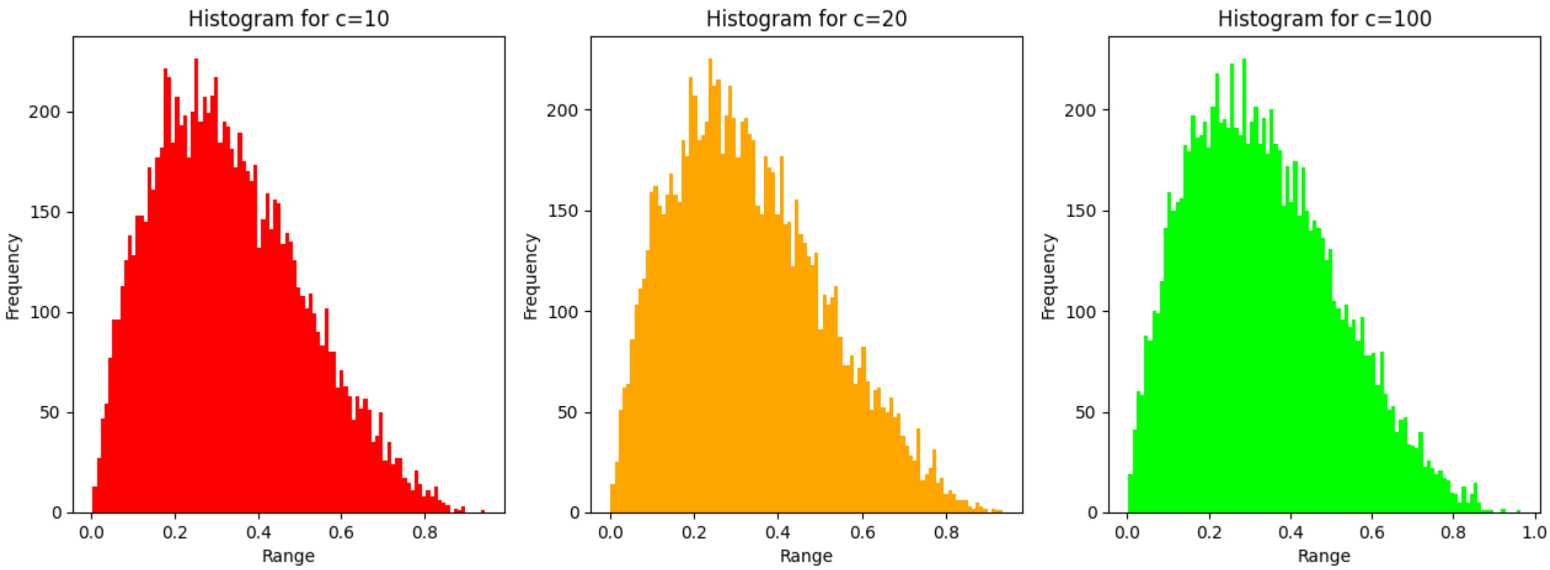
For c = 100:

Sample mean = 0.32906831724380287

Estimated value of P(0.25<=X<=0.75): 0.5998

Average number of iterations = 130.5057

Actual Average number of iterations = 100



For values c = 10, 20, 100, the histogram plots are similar although probabilities of acceptance are different.