Problem 1 Compute the frequency response of a moving average filter. Its impulse response is given by

$$h[n] = \begin{cases} 1/M & , \ 0 \le n \le M-1 \\ 0 & , \ \text{otherwise} \end{cases}$$

Problem 2 Consider a causal LTI discrete-time system with real impulse response $h[n]$. Compute the steady-state response for the sinusoidal input

$$x[n] = A \cdot \cos(\Omega_0 n + \phi)$$

where $\Omega_0 = \omega_0 \cdot T_\mathrm{s}$ is the normalized radian frequency.

Problem 3 Consider a causal and stable LTI discrete-time system with real impulse response $h[n]$. Compute the response to the causal exponential sequence

$$x[n] = \mathrm{e}^{\mathrm{j} n \Omega} \cdot u[n]$$

where $\Omega = \omega \cdot T_\mathrm{s}$ is the normalized radian frequency and $u[n]$ is the unit step sequence. Identify and discuss the different parts of the solution.

Problem 4 Prove the bilateral z-transforms and the region of convergence as listed in appendix B:

a) $x[n] = \alpha^n \cdot u[n]$

b) $x[n] = \cos(\Omega_0 \cdot n) \cdot u[n]$

c) $x[n] = r^n \cdot \cos(\Omega_0 \cdot n) \cdot u[n]$

Problem 5 Determine the bilateral z-transform in a closed form and the region of convergence of the following series:

a) $x[n] = \begin{cases} \alpha^n & , \ M \le n \le N-1 \\ 0 & , \ \text{otherwise} \end{cases}$

b) $x[n] = 7 \cdot \left(\frac{1}{3}\right)^n \cdot u[n] - 6 \cdot \left(\frac{1}{2}\right)^n \cdot u[n]$

Check the initial- and final-value theorems if possible.

Problem 6 Determine the inverse causal z-transforms:

a) $X(z) = \dfrac{z \cdot (z+2)}{(z-0.2) \cdot (z+0.6)}$

b) $X(z) = \dfrac{2 + 0.8 \cdot z^{-1} + 0.5 \cdot z^{-2} + 0.3 \cdot z^{-3}}{1 + 0.8 \cdot z^{-1} + 0.2 \cdot z^{-2}}$

c) $X(z) = \dfrac{3 \cdot z^2 + 2 \cdot z - 10}{z^3 - 5 \cdot z^2 + 8z - 4}$ with $p_1 = 1$

Check the initial- and final-value theorems.

Answers

Problem 1

$$H\left(e^{j\Omega}\right) = \frac{1}{M} \cdot \frac{\sin(M\Omega/2)}{\sin(\Omega/2)} \cdot e^{-j(M-1)\Omega/2}$$

Problem 2

$$y[n] = A \cdot \left|H\left(e^{j\Omega_0}\right)\right| \cdot \cos\left(\Omega_0 n + \phi + \varphi(\Omega_0)\right) \quad ; \quad \varphi(\Omega_0): \text{phase response}$$

Problem 3

$$y[n] = H\left(e^{j\Omega}\right) \cdot e^{jn\Omega} - \left(\sum_{k=n+1}^{\infty} h[k] \cdot e^{-jk\Omega}\right) \cdot e^{jn\Omega} \quad ; \quad n \geq 0$$

Problem 4

see appendix B

Problem 5

a) $X(z) = \dfrac{\alpha^M \cdot z^{-M} - \alpha^N \cdot z^{-N}}{1 - \alpha \cdot z^{-1}}$

ROC is the entire z-plane, except: 1) $z = 0$ for $0 \leq M < N$; 2) $z = 0$ and $z = \infty$ for $M < 0 < N$; 3) $z = \infty$ for $M < N < 0$.

b) $X(z) = \dfrac{z\left(z - \frac{3}{2}\right)}{\left(z - \frac{1}{3}\right) \cdot \left(z - \frac{1}{2}\right)}$; $\text{ROC}: |z| > \dfrac{1}{2}$

Problem 6

a) $x[n] = 2.75 \cdot (0.2)^n \cdot u[n] - 1.75 \cdot (-0.6)^n \cdot u[n]$

b) $x[n] = -3.5 \cdot \delta[n] + 1.5 \cdot \delta[n-1] +$
$(2.75 + j0.25) \cdot (-0.4 + j0.2)^n \cdot u[n] + (2.75 - j0.25) \cdot (-0.4 - j0.2)^n \cdot u[n]$

c) $x[n] = 2.5 \cdot \delta[n] - 5 \cdot u[n] + 2.5 \cdot 2^n \cdot u[n] + 1.5 \cdot n \cdot 2^n \cdot u[n]$
$\{x[n]\} = \{0, 3, 17, 51, \ldots\}$ with $n = 0, 1, 2, 3, \ldots$

or $x[n] = -5 \cdot u[n-1] + 4 \cdot n \cdot 2^n \cdot u[n] - 5 \cdot (n-1) \cdot 2^{n-1} \cdot u[n-1]$