Probability and Statistics

3 – Discrete Random Variables

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Discrete Random Variables

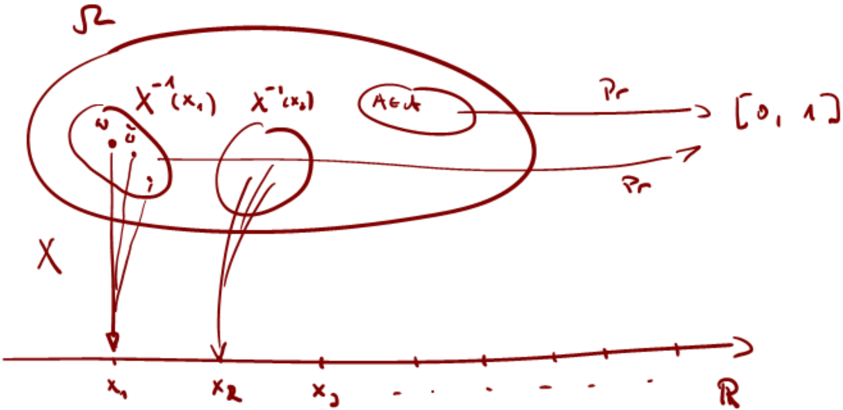
Definition (3.1)

Let Ω be a sample space and $\mathcal{A} \subseteq \mathcal{P}(\Omega)$ a set of events with a probability measure $\Pr : \mathcal{A} \to \mathbb{R}$. Given $(\Omega, \mathcal{A}, \Pr)$, a mapping $X : \Omega \to \mathbb{R}$ is called a *discrete random variable* if

$$\boxed{X^{-1}(x) \in \mathcal{A}} \qquad \text{for all } x \in \mathbb{R}$$

and there exist countable (finite or infinite) many distinct real numbers $(x_i)_{i \in I}$ such that:

$$\sum_{i \in I} \Pr(X^{-1}(x_i)) = 1$$



Preimage of x : $X^{-1}(x) = \{\omega \in \Omega \mid X(\omega) = x\} \in \mathcal{A}$

Probability Distribution (Probability Mass Function)

Definition (3.1)

If X is a discrete random variable, its *distribution* (also called *probability mass function* (pmf)) is defined by:

$$p_X : \mathbb{R} \to [0,1], \qquad p_X(x) \;:=\; \Pr(X = x) \;:=\; \Pr(X^{-1}(x)) \qquad \text{for all } \; x \in \mathbb{R}$$