Problem 1 Determine the location of the notch frequency of the FIR notch filter given by

$$H(z) = 1 + \sqrt{2} \cdot z^{-1} + z^{-2}.$$

Compute the response to the input signal $x[n] = \cos(n \cdot \pi / 4)$.

Problem 2 Design a first-order highpass filter with a normalized 3-dB cutoff frequency at 0.25 radian/sample.

Problem 3 Show that the causal FIR transfer function

$$H(z) = \frac{1}{1+\alpha} \cdot \left(1 + \alpha \cdot z^{-1}\right); \ \alpha > 0$$

is a bounded-real (BR) function.

Problem 4 The frequency response of a causal digital filter is given

$$H(z) = 1 - 2 \cdot z^{-1} + 3 \cdot z^{-2} - 3 \cdot z^{-4} + 2 \cdot z^{-5} - z^{-6}.$$

Determine the magnitude and phase responses.

Problem 5 A causal LTI FIR discrete-time system is characterized by an impulse response

$$h[n] = a_1 \cdot \delta[n] + a_2 \cdot \delta[n-1] + a_3 \cdot \delta[n-2] + a_4 \cdot \delta[n-3] + a_5 \cdot \delta[n-4] + a_6 \cdot \delta[n-5].$$

For what values of the impulse response samples will its frequency response have a constant group delay?

Problem 6 A type-II real-coefficient FIR filter (symmetric impulse response, N odd) with the transfer function $H(z)$ has the following zeros: $z_1 = 1$, $z_2 = -1$, $z_3 = 0.5$, $z_4 = 0.8 + j$.

a) Determine the locations of the remaining zeros of $H(z)$ having the lowest order.

b) Determine the transfer function of the filter.

Problem 7 Estimate the order and the group delay of a linear-phase lowpass FIR filter with the following specifications: passband edge $f_{pass} = 1.8$ kHz, stopband edge $f_{stop} = 2$ kHz, peak passband ripple $a_{pass} = 0.1$ dB, minimum stopband attenuation $a_{stop} = 35$ dB, and sampling rate $f_s = 12$ kHz.

Problem 8 Estimate the order of a linear-phase bandpass FIR filter with the following specifications: passband edges $f_{pass,1} = 0.35$ kHz and $f_{pass,2} = 1$ kHz, stopband edges $f_{stop,1} = 0.3$ kHz and $f_{stop,2} = 1.1$ kHz, passband ripple $\delta_{pass} = 0.002$, stopband ripple $\delta_{stop} = 0.01$, and sampling rate $f_s = 10$ kHz.

Answers

Problem 1

$\Omega_0 = 0.75 \cdot \pi = 135°$

$y[n] = 2 \cdot \sqrt{2} \cdot \cos\left((n-1) \cdot \pi / 4\right)$

Problem 2

$$H(z) = \frac{0.8884 \cdot (1 - z^{-1})}{1 - 0.7767 \cdot z^{-1}}$$

Problem 3

Problem 4

$\tilde{H}(\Omega) = 6 \cdot \sin(\Omega) - 4 \cdot \sin(2 \cdot \Omega) + 2 \cdot \sin(3 \cdot \Omega)$

$\varphi(\Omega) = -3 \cdot \Omega \pm \frac{\pi}{2}$

Problem 5

$a_1 = a_6, a_2 = a_5, a_3 = a_4$

or

$a_1 = -a_6, a_2 = -a_5, a_3 = -a_4$

Problem 6

a) $z_5 = 1$, $z_6 = 2$, $z_7 = 0.8 - \mathrm{j}$, $z_8 = 0.488 - \mathrm{j} \cdot 0.610$, $z_9 = 0.488 + \mathrm{j} \cdot 0.610$,

b) $H(z) = \prod_{i=1}^{9} \left(1 - z_i \cdot z^{-1}\right)$

Problem 7

KAISER: order ≈ 99, group delay ≈ 4.13 ms

BELLANGER: order ≈ 108, group delay ≈ 4.5 ms

Problem 8

KAISER: order ≈ 466