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Information Fusion – New Research Concepts

Combination Techniques for Uncertain Information in Measurement and Signal Processing

5. Fusion: Research and Application

Information Fusion Prof. Dr.-Ing. Volker Lohweg

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5.1 Fusion methods – Motivation

- Increase of complex (distributed) technical systems worldwide
  - Increase of Multi-sensor Information Fusion approaches
  - 24/7 supervision of process parameters by observation of different physical, chemical, biological, etc. quantities
  - Measurement of system's health
  - Goals
    - Enabling cost-effective production
    - Improving reliability of production and quality of products by observing more than one single quantity

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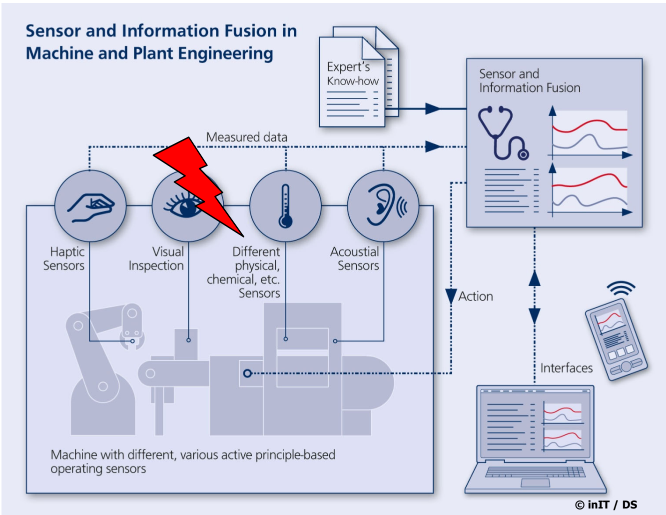
5.1 Fusion methods – Motivation



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5.1 Fusion methods – Motivation

- **Constraints**
  - Conflicting sources
    - Inappropriately parameterised/defective/aged sensors, etc.
    - Large influence on fusion process
  - Non-stationary process
    - Measurements prone to uncertainty
- **Our basic approach**
  - Uncertainty: Fuzzy Membership Functions [Zadeh 65]
  - Conflict handling: Evidential Aggregation Rules [Shafer 76]
  - Specitivity of ill-known information [Dubois & Prade 93]
  - Psychologically inspired: Decisions are negotiated starting with small groups (2) [Li & Lohweg 08]

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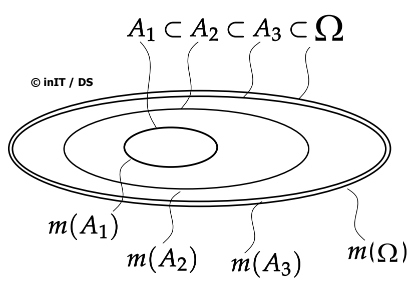
5.1 Fusion methods – Nested Sets and Memberships

- **Interconnections between Possibility and Dempster-Shafer Theory**
  - Fundamental prerequisite:
    Nested sets on the frame of discernment Ω

$$A_1 \subset A_2 \subset \cdots \subset A_n (= \Omega),\ A_k \in \Omega,\ m(A_i) \in [0,1]$$



Possibility measure

$$\Pi(A) = \mathrm{Pl}(A) = \sum_{B \cap A \neq \emptyset} m(B)$$

Necessity measure

$$\mathrm{Nec}(A) = \mathrm{Bel}(A) = \sum_{B \subset A} m(B) = 1 - \Pi(A^C)$$

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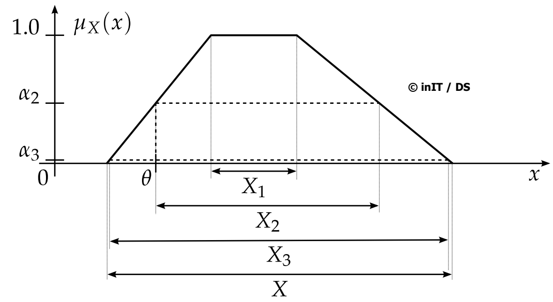
5.1 Fusion methods – Nested Sets and Memberships

- **Interconnections between Possibility and Fuzzy Set Theory**
  - Fuzzy Membership Function

$$\mu_X : X \to [0,1],\; X_k = \{\theta \mid \mu_X(\theta) \geq \alpha_k, \theta \in X, \alpha_k \in [0,1]\}$$



Possibility distribution

$\pi : \Omega \to [0,1]$

$\Pi(A) = \max_{\theta \in A} \pi(\theta) \forall \theta \in \Omega$

For nested sets

$\pi(\theta) = \Pi(\theta) = \mathrm{Pl}(\theta) = \mu_X(\theta)$

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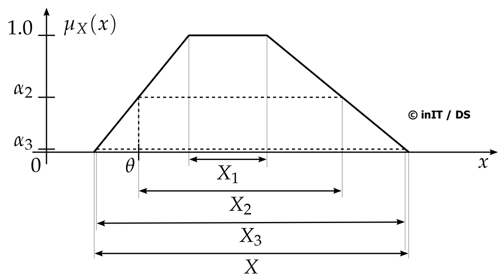
5.1 Fusion methods – Nested Sets and Memberships

- **Interconnections between Dempster-Shafer and Fuzzy Set Theory**
  - With $X_k = A_k \Rightarrow m(X_k) = m(A_k)$ and $\sum_{k=1}^{n} m(X_k) = 1$

$$\mu_X(x) = \sum_{x \in X_k} m(X_k), \quad m(X_k) = \alpha_k - \alpha_{k+1}$$

$$\forall k,\, \alpha_{k+1} = 0$$



$\alpha_1 = 1$

$\alpha_2 = \mu_X(\theta)$

$\alpha_3 = \varepsilon > 0,\, \varepsilon << 1$

$(\alpha_4 = 0)$

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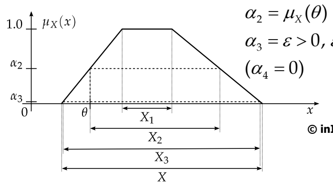
5.1 Fusion methods – Nested Sets and Memberships

$$\lim_{\varepsilon\to 0} m(X_1) = \lim_{\varepsilon\to 0}(\alpha_1 - \alpha_2) = 1 - \alpha_2$$
$$\lim_{\varepsilon\to 0} m(X_2) = \lim_{\varepsilon\to 0}(\alpha_2 - \alpha_3) = \lim_{\varepsilon\to 0}(\alpha_2 - \varepsilon) = \alpha_2$$
$$\lim_{\varepsilon\to 0} m(X_3) = \lim_{\varepsilon\to 0}(\alpha_3 - \alpha_4) = \lim_{\varepsilon\to 0} \varepsilon - 0 = 0$$
$$\Rightarrow \lim_{\varepsilon\to 0} \mu_X(\theta) = \lim_{\varepsilon\to 0} \sum_{\theta\in X_i} m(X_i) = \lim_{\varepsilon\to 0}\left(m(X_3) + m(X_2)\right) = m(X_2)$$



$\alpha_1 = 1$
$\alpha_2 = \mu_X(\theta)$
$\alpha_3 = \varepsilon > 0, \varepsilon << 1$
$(\alpha_4 = 0)$

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$$\lim_{\varepsilon\to 0} m(\theta) = \mu_X(\theta) \;\; \forall \varepsilon < \alpha_2 \leq 1$$

A **one-to-one relationship** is obtained
regardless of the membership function's shape, as long it is **normal**

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5.1 Fusion methods – Fuzzy-Pattern-Classifiers

Modified-Fuzzy-Pattern-Classifier (MFPC); kernel-based Machine

- The **Modified-Fuzzy-Pattern-Classifier** (MFPC) is a hardware optimized derivate of Bocklisch's Fuzzy Pattern Classifier (FPC) [Boc87]. Lohweg et al. examined both, the FPC and the MFPC, in detail [LDM04]. MFPC's general concept of simultaneously calculating a number of membership values and aggregating these is borrowed from the original FPC. Lohweg's intention which led to the MFPC in the form of an optimized structure was to create a pattern recognition system on a Field Programmable Gate Array (FPGA) that can be applied in high-speed industrial applications.

[BP86] S. F. Bocklisch and U. Priber, "A parametric fuzzy classification concept," Proc. International Workshop on Fuzzy Sets Applications, 147–156, Akademie-Verlag, Eisenach, Germany, March 1986.

[LDM04] Lohweg, V.; Diederichs, C; and Mueller, D.: Algorithms for Hardware-Based Pattern Recognition, EURASIP Journal on Applied Signal Processing, Volume 2004 (2004), Issue 12, Pages 1912-1920

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5.1 Fusion methods – Fuzzy-Pattern-Classifiers

- The originally proposed FPC membership function μ(*m*, **p**) →I, namely parameterizable unimodal potential function, of the form

$$\mu(m, \mathbf{p}) = \frac{A}{1 + d(m, \mathbf{p})} (\Rightarrow A \in I) \text{ with } d(m, \mathbf{p}) = \left(\frac{|m - S|}{C}\right)^D$$

- is not suitable for hardware implementations since many of the limited resources are used to implement the multiplication, thus not suitable for use with MFPC. **p** = (*S, C, D*) is a parameter vector defining the membership function's properties, namely mean value (*S*), width (*C*) and steepness of its edges (*D*).

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5.1 Fusion methods – Fuzzy-Pattern-Classifiers

- Th distance d(*m*, **p**) is the distance measure of the inspected feature *m* with regard to the properties of the membership function, i.e. how far is the measured feature *m* away from its mean value *S*.

- In other words, the distance measure represents how large is the dissimilarity to an ideal member of a class, represented by the membership function. A defines the membership function's maximum value with *A* = μ(*S*, **p**), where d(*S*, **p**) = 0.

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5.1 Fusion methods – Fuzzy-Pattern-Classifiers

- The much more hardware efficient membership function used for the MFPC is defined as:

$$\mu_{\mathrm{MFPC}}(m, \mathbf{p}) = 2^{-d(m,\mathbf{p})} \in I$$

- being also a parameterizable unimodal potential function, can be computed without the need of any arithmetic multiplications. Having 2 in the base, $\mu_{MFPC}(m, \mathbf{p})$ can be calculated by logical shifts. The distance measure d $(m, \mathbf{p})$ is the same as used in the standard approach.

$$d(m, \mathbf{p}) = \left(\frac{|m - S|}{C}\right)^D$$

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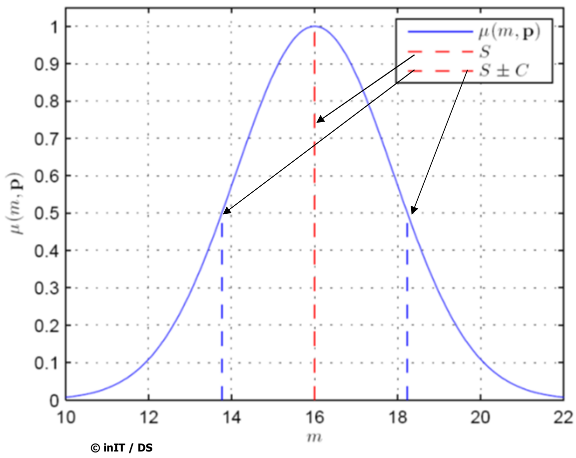
5.1 Fusion methods – Fuzzy-Pattern-Classifiers

- The MFPC membership function's parameters can be adjusted by an expert, but are more practically obtained automatically during a learning phase. During this phase, the features *m* are extracted from *N* typical members of a class.



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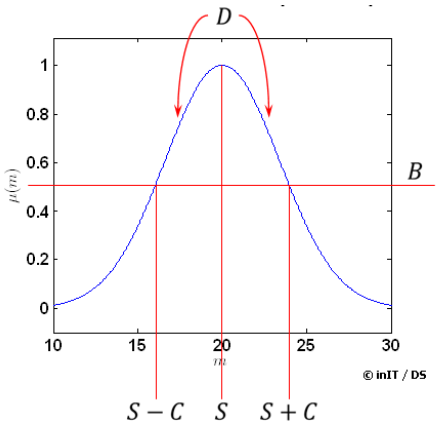
5.1 Fusion methods – Fuzzy-Pattern-Classifiers

- Prototype fuzzy membership function μ_{MFPC} from Modified-Fuzzy-Pattern-Classifier (MFPC) (Lohweg, 2003)



- Hardware-optimisation through $2^{-d} \propto \mathrm{e}^{-\alpha d}$
- Parameters
  - S: mean value
  - C: width
    - p_{C_e}: width adjustment
  - D: steepness of the edges
  - B: class boundary membership

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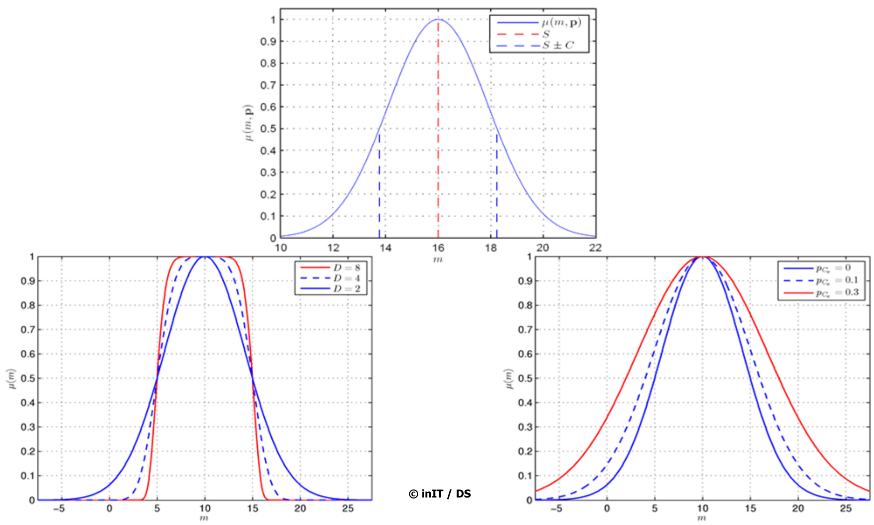
5.1 Fusion methods – Fuzzy-Pattern-Classifiers



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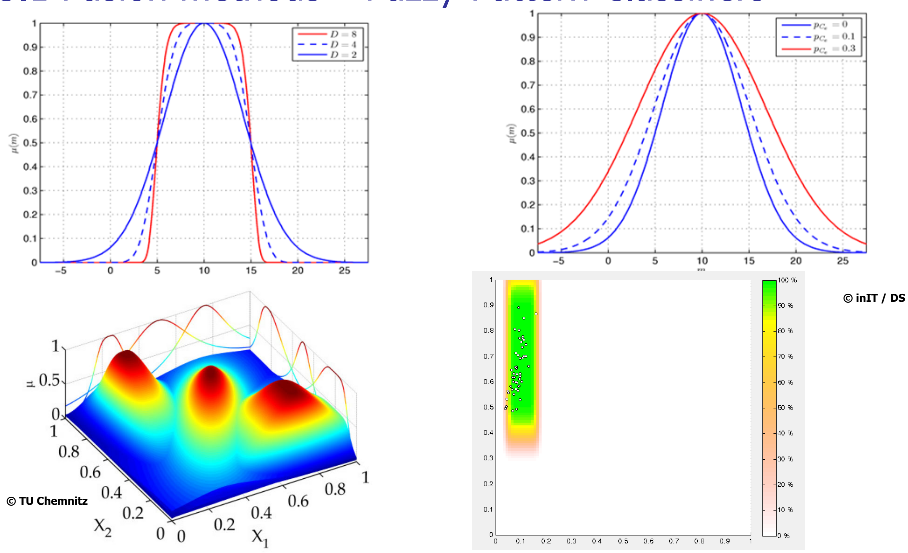
5.1 Fusion methods – Fuzzy-Pattern-Classifiers



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5.1 Fusion methods – Fuzzy-Pattern-Classifiers

Object Features **(physical world)**	Feature Membership Values **[0,1]**	Object Membership Value

Fuzzy-Pattern-Classifier

-5.92 →
25.6 →
0.0005 →
3.89 →
2 →

$h(\mathbf{m},\mathbf{p})$ → 0.725

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Membership Functions (parameterized in learning phase)

Aggregation Operator h

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5.1 Fusion methods – Fuzzy-Pattern-Classifiers

- For each feature, the parameters are then calculated in the following way [DL03]: Modified Fuzzy-Pattern-Classifier (MFPC)
  - Distance function for each feature m_i

$$d_i(m;\mathbf{p}) = \left(\frac{|m-S_i|}{C_i}\right)^{D_i}$$

  - Definition of C by learning

$$C_i = \tfrac{1}{2}\cdot\left[\max(m_i) - \min(m_i)\right] + C_{E,i}$$

S: average $S_i = \frac{1}{N}\sum_{j=0}^{N-1} m_j$

S: Min/Max (in case of compact data sets)

$$S_i = \tfrac{1}{2}\cdot\left(\max(m_i) + \min(m_i)\right)$$

  - Definition of the slope
    - empirical, e.g. D = 2, 4 or 8 or by use of the „Petker, Mönks"-approach

[DL03]: Diederichs, Carsten; Lohweg, Volker: An Image-Processing-System-On-Chip Based on Nonlinear Generalized Circular Transforms and Fuzzy Pattern Classification. In: IEEE-EURASIP Workshop on Nonlinear Signal and Image Processing EURASIP, Grado-Trieste, Italy, Jun 2003

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5.1 Fusion methods – Fuzzy-Pattern-Classifiers

- The math including p_{Ce}

$$S_i + C_i = \max(m_i) \rightarrow S_i = \frac{1}{2}\left[\max(m_i) - \min(m_i)\right] + \min(m_i) \Rightarrow$$

$$C_i = \frac{1}{2}\left[\max(m_i) - \min(m_i)\right] + C_E,$$

$$C_E \overset{!}{=} p_{Ce}\cdot\left[\max(m_i) - \min(m_i)\right];\; 0 \le p_{Ce} \le 1$$

$$C_i = \frac{1}{2}\left[\max(m_i) - \min(m_i)\right] + p_{Ce}\cdot\left[\max(m_i) - \min(m_i)\right]$$

$$C_i = \frac{1}{2}\left[\max(m_i) - \min(m_i)\right] + 2p_{Ce}\cdot\frac{1}{2}\left[\max(m_i) - \min(m_i)\right]$$

$$C_i = \left(1+2p_{Ce}\right)\cdot\underbrace{\frac{1}{2}\left[\max(m_i) - \min(m_i)\right]}_{\Delta(m_i)} = \left(1+2p_{Ce}\right)\cdot\Delta(m_i)$$

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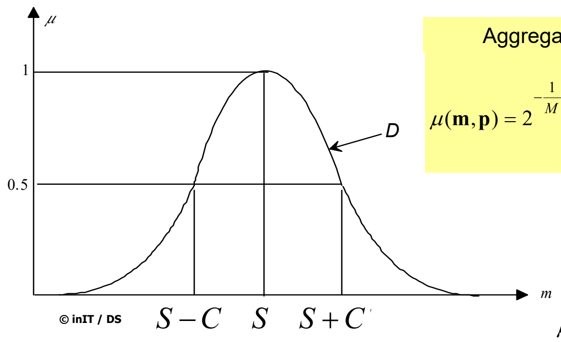
5.1 Fusion methods – Fuzzy-Pattern-Classifiers



Aggregated Membership Function

$$\mu(\mathbf{m},\mathbf{p}) = 2^{-\frac{1}{M}\sum_{i=1}^{M}\left(\frac{|m_i-S_i|}{C_i}\right)^{D_i}} = \left[\prod_{i=1}^{M} 2^{-\left(\frac{|m_i-S_i|}{C_i}\right)^{D_i}}\right]^{\frac{1}{M}}$$

- Geometric Mean

$$\mu(\boldsymbol{m};\,{}^{k}\boldsymbol{p}) = 2^{-\frac{1}{M}\sum_{i=0}^{M-1} d(m_i;\,{}^{i}\boldsymbol{p})}$$

- Aggregation operator

$$x_{\text{geo}} = \left(\prod_{i=0}^{N-1} x_i\right)^{\frac{1}{N}}$$

- Defuzzification

$$c = \begin{cases} 1, & \mu(\boldsymbol{m};\,{}^{k}\boldsymbol{p}) \geq \mu_t \\ 0, & \mu(\boldsymbol{m};\,{}^{k}\boldsymbol{p}) < \mu_t \end{cases}$$

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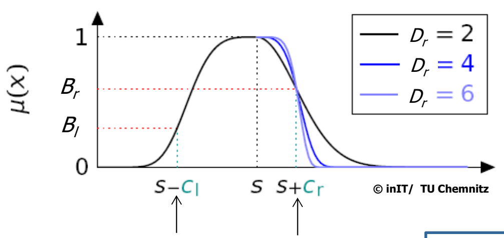
5.1 Fusion methods – Fuzzy-Pattern-Classifiers

- Extended Approach



$$\mu(d) = 2^{-\mathrm{ld}(\frac{1}{B})d} \rightarrow \mu(d=1) = 2^{-\mathrm{ld}(\frac{1}{B})}$$

$$\mathrm{ld}\,\mu(d=1) = -\mathrm{ld}(\frac{1}{B}) \rightarrow \mu(d=1) = B.$$

$$0 \leq B \leq 1,\ B \in \mathbb{R}$$

$$d_{l/r} = \left(\frac{|m-S|}{C_{l/r}}\right)^{D_{l/r}} \overset{!}{=} d = 1$$

$$\mu_{\text{PMFPC}}(m,\mathbf{p}) = \begin{cases} 2^{-\mathrm{ld}\left(\frac{1}{B_l}\right)\left(\frac{|m-S|}{C_l}\right)^{D_l}}, & m \leq S \\ 2^{-\mathrm{ld}\left(\frac{1}{B_r}\right)\left(\frac{|m-S|}{C_r}\right)^{D_r}}, & m > S \end{cases}$$

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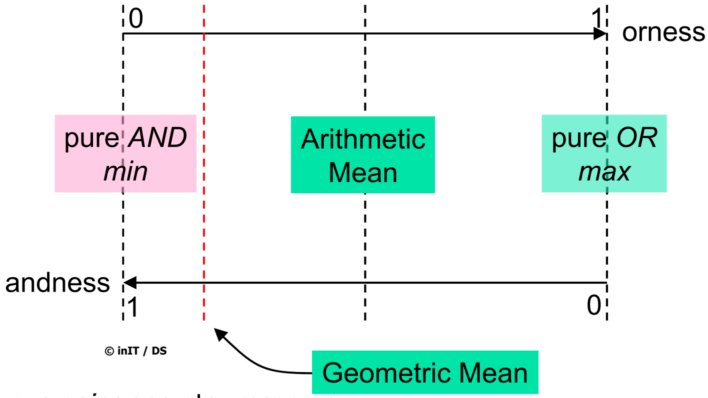
5.1 Fusion methods – Fuzzy-Pattern-Classifiers



- Fuzzy *averaging* operator measure
  - Degree to which an operator behaves like the *min* or *max* operator
  - Value between 0 and 1
  - Geometric Mean's andness ~0.666, depending on dimension

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5.1 Fusion methods – Fuzzy-Pattern-Classifiers

- Since the MFPC aggregates fuzzy membership functions, it can be interpreted as also a fuzzy aggregation operator. Its properties are derived in the next Section.

- The MFPC's aggregating fuzzy membership functions belong to the class of fuzzy averaging operators. Any fuzzy averaging operator $h : \mathrm{I}^n \rightarrow \mathrm{I}$ satisfies the following Axioms [KY95]:

[KY95]: Klir, G.J. and Yuan, B.: Fuzzy sets and fuzzy logic – Theory and applications, Prentice Hall, 1995

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5.1 Fusion methods – Fuzzy-Pattern-Classifiers

Axiom H 1. Monotonicity

For any pair $(a_1, a_2, \ldots, a_n)$ *and* $(b_1, b_2, \ldots, b_n)$ *of* n*-tuples such that* $a_i, b_i \in I$ *for all* $i \in \mathbb{N}_n$, *if* $a_i \leq b_i$ *for all* $i \in \mathbb{N}_n$, *then*

$$h(a_1, a_2, \ldots, a_n) \leq h(b_1, b_2, \ldots, b_n).$$

Axiom H 2. Continuity

$h(a_1, a_2, \ldots, a_n)$ *is a continuous function so that any arbitrary small change* ε *in any* a_i, $i \in \mathbb{N}_n$ *results also only in a small change in* $h(a_1, \ldots, a_i + \varepsilon, \ldots, a_n)$. *In other words*

$$\lim_{\varepsilon \to 0} h(a_1, \ldots, a_i + \varepsilon, \ldots, a_n) = h(a_1, \ldots, a_i, \ldots, a_n).$$

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5.1 Fusion methods – Fuzzy-Pattern-Classifiers

Axiom H 3. Symmetry

For any permutation p *on* $\mathbb{N}_n = \{1, \ldots, n\}$,

$$h(a_1, a_2, \ldots, a_n) = h(a_{p(1)}, a_{p(2)}, \ldots, a_{p(n)}).$$

Axiom H 4. Idempotency

If $a_i = a \; \forall i \in \mathbb{N}_n$, *then*

$$h(a_1, a_2, \ldots, a_n) = a.$$

5.1 Fusion methods – Fuzzy-Pattern-Classifiers

- Thus, if the MFPC actually was an averaging operator, it necessarily satisfes Axioms H 1 - H 4. Before proving that MFPC satisfies these Axioms, its formal description is rewritten in the following way:

$$
\begin{aligned}
h_{\mathrm{MFPC}}(\mathbf{m},\mathbf{p}) &= 2^{-\frac{1}{M}\sum_{i=1}^{M} d_i(m_i,\mathbf{p}_i)} \\
&= \left(2^{-\sum_{i=1}^{M} d_i(m_i,\mathbf{p}_i)}\right)^{\frac{1}{M}} \\
&= \left(2^{-d_1(m_1,\mathbf{p}_1)} \cdot 2^{-d_2(m_2,\mathbf{p}_2)} \cdots 2^{-d_M(m_M,\mathbf{p}_M)}\right)^{\frac{1}{M}} \\
h_{\mathrm{MFPC}}(\mathbf{m},\mathbf{p}) &= \left(\prod_{i=1}^{M} 2^{-d_i(m_i,\mathbf{p}_i)}\right)^{\frac{1}{M}} .
\end{aligned}
$$

5.1 Fusion methods – Fuzzy-Pattern-Classifiers

Proof. **Monotonicity**

Let $a_i \leq b_i$ with $a_i, b_i \neq 0 \ \forall \mathrm{i} \in \mathbb{N}_{\mathrm{n}}$. Then

$$
\begin{aligned}
h_{\mathrm{GM}}(a_1,a_2,\ldots,a_n) &\overset{!}{\leq} h_{\mathrm{GM}}(b_1,b_2,\ldots,b_n) \\
\left(\prod_{i=1}^{n} a_i\right)^{\frac{1}{n}} &\overset{!}{\leq} \left(\prod_{i=1}^{n} b_i\right)^{\frac{1}{n}} \\
(a_1 \cdot a_2 \cdots a_n)^{\frac{1}{n}} &\overset{!}{\leq} (b_1 \cdot b_2 \cdots b_n)^{\frac{1}{n}} \\
a_1^{\frac{1}{n}} \cdot a_2^{\frac{1}{n}} \cdots a_n^{\frac{1}{n}} &\overset{!}{\leq} b_1^{\frac{1}{n}} \cdot b_2^{\frac{1}{n}} \cdots b_n^{\frac{1}{n}} \\
1 &\overset{!}{\leq} \frac{b_1^{\frac{1}{n}} \cdot b_2^{\frac{1}{n}} \cdots b_n^{\frac{1}{n}}}{a_1^{\frac{1}{n}} \cdot a_2^{\frac{1}{n}} \cdots a_n^{\frac{1}{n}}} \\
1 &\leq \underbrace{\left(\frac{b_1}{a_1}\right)^{\frac{1}{n}}}_{\geq 1} \cdot \underbrace{\left(\frac{b_2}{a_2}\right)^{\frac{1}{n}}}_{\geq 1} \cdots \underbrace{\left(\frac{b_n}{a_n}\right)^{\frac{1}{n}}}_{\geq 1}
\end{aligned}
$$

$\square$

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5.1 Fusion methods – Fuzzy-Pattern-Classifiers

Proof. **Continuity**
Let $\varepsilon \in \mathbb{R}$ be a small change in an arbitrary a_i, $i \in \mathbb{N}_n$. Then

$$\begin{aligned}
\lim_{\varepsilon \to 0} h_{\mathrm{GM}}(a_1, \ldots, a_i + \varepsilon, \ldots, a_n) &\overset{!}{=} h_{\mathrm{GM}}(a_1, \ldots, a_i, \ldots, a_n) \\
\lim_{\varepsilon \to 0} h_{\mathrm{GM}}(a_1, \ldots, a_i + \varepsilon, \ldots, a_n) &= \lim_{\varepsilon \to 0} (a_1 \cdots a_i + \varepsilon \cdots a_n)^{\frac{1}{n}} \\
&= \lim_{\varepsilon \to 0} a_1{}^{\frac{1}{n}} \cdots (a_i + \varepsilon)^{\frac{1}{n}} \cdots a_n{}^{\frac{1}{n}} \\
&= a_1{}^{\frac{1}{n}} \cdots a_i{}^{\frac{1}{n}} \cdots a_n{}^{\frac{1}{n}} \\
&= h_{\mathrm{GM}}(a_1, \ldots, a_i, \ldots, a_n)
\end{aligned}$$

□

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5.1 Fusion methods – Fuzzy-Pattern-Classifiers

Proof. **Symmetry**
Let $\mathbf{a} = (a_1, \ldots, a_{j-1}, a_j, a_{j+1}, a_{j+2}, \ldots, a_n)$ and $\mathbf{a}' = (a_1, \ldots, a_{j-1}, a_{j+1}, a_j, a_{j+2}, \ldots, a_n)$ be a permutation of $\mathbf{a}$. Then $h_{\mathrm{GM}}(\mathbf{a}) \overset{!}{=} h_{\mathrm{GM}}(\mathbf{a}')$:

$$\begin{aligned}
h_{\mathrm{GM}}(\mathbf{a}) &= (a_1 \cdots a_{j-1} \cdot a_j \cdot a_{j+1} \cdot a_{j+2} \cdots a_n)^{\frac{1}{n}} \\
h_{\mathrm{GM}}(\mathbf{a}') &= (a_1 \cdots a_{j-1} \cdot a_{j+1} \cdot a_j \cdot a_{j+2} \cdots a_n)^{\frac{1}{n}} \\
&= (a_1 \cdots a_{j-1} \cdot a_j \cdot a_{j+1} \cdot a_{j+2} \cdots a_n)^{\frac{1}{n}} \\
&= h_{\mathrm{GM}}(\mathbf{a})
\end{aligned}$$

Since multiplication is commutative, each aggregation of every permutation of $\mathbf{a}$ yields the same result as the aggregation of $\mathbf{a}$. □

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5.1 Fusion methods – Fuzzy-Pattern-Classifiers

Proof. **Idempotency**
Let $a_i = a \; \forall i \in \mathbb{N}_n$. Then

$$\begin{aligned} h_{\mathrm{GM}}(a_1, a_2, \ldots, a_n) &= \left(\prod_{i=1}^{n} a_i \right)^{\frac{1}{n}} \\ &= (\underbrace{a \cdot a \cdots a}_{n \text{ times}})^{\frac{1}{n}} \\ &= (a^n)^{\frac{1}{n}} \\ &= a \end{aligned}$$

□

Since all Axioms H 1 – H 4 are satisfied by the geometric mean operator, which is the base of MFPC aggregation, MFPC is proved to be a fuzzy averaging operator.

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5.1 Fusion methods – Fuzzy-Pattern-Classifiers

For what do we need this all?

- We have now a solution for measuring the basic belief assignment (BBA) directly from measured data
- We are able to train the parameter vector **p** for an (P)MFPC
- We are able to get rid of physical units (SI-system)
- We are able to aggregate features with *h*
- The aggregation operator *h* is adaptable

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5.1 Fusion methods – Two-Layer-Conflict-Solving

- The Base: Psychological concepts
  - Psychologically, as clearly stated in "Decision making has been traditionally studied at three levels:
    - individual,
    - group and
    - organizational."
  - This equals to say that decision is made at three layers, in which conflict is unavoidable to be considered and solved: individual is the basic element that holds conflict; group has a larger range which includes conflict while organization is the largest.

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5.1 Fusion methods – Two-Layer-Conflict-Solving

- Because of the counter-intuitive results of DST and other alternatives have limited assistance as remedies, thus a *Two-Layer Conflict Solving* (TLCS) data fusion approach is suggested, which includes two layers to combine pieces of evidence [LL08].

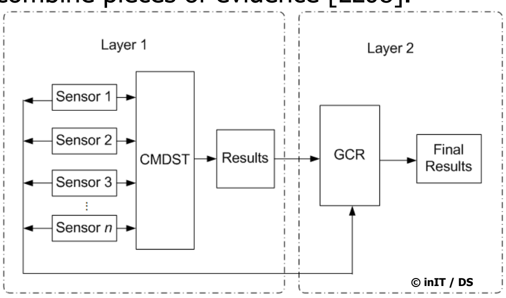

Fig. 1

[LL08]: Li, Rui; Lohweg, Volker: A Novel Data Fusion Approach using Two-Layer Conflict Solving, 2008 IAPR Workshop on Cognitive Information Processing; June 9-10, Santorini, Greece Jun 2008

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5.1 Fusion methods – Two-Layer-Conflict-Solving

- In Figure 1, layer 1 is regarded as working at the individual level because ***Conflict Modified Dempster-Shafer theory*** **(CMDST)** is an approach which combines every two sensors' data so that conflict is sort of considered and solved between individuals.

- After receiving the results from the previous layer, layer 2 collects sensors' original knowledge and fuses them with combined results from CMDST, hence conflict is further resolved at a group level. The following section introduces the first layer →CMDST.

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5.1 Fusion methods – Two-Layer-Conflict-Solving

- In such a way, humans believe that conflict can be optimally solved, although it is impossible to totally eliminate its negative impacts.
- Therefore, a TLCS data fusion algorithm is suggested and studied in this lecture slides.
- It could become three layers if several groups of sensors are considered in a larger system. The Figure 2 below depicts the scheme of TLCS.

individual group organisatorial

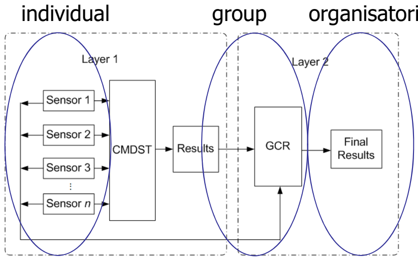

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Fig. 2

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5.1 Fusion methods – Two-Layer-Conflict-Solving

- Based on the idea of DST, CMDST computes the conflict in a different manner as shown in the formula (cf. Figure 2):

$$k_{\mathrm{cm}} = \sum_{(i,j)\in\mathbb{M}} \sum_{(k,l)\in\mathbb{A}} m_i(A_k)\cdot m_j(A_l)\,,$$

- with $\mathbb{M} = \{(i,j) \mid i \neq j\}$ being the index set addressing the available sensors/experts, $\mathbb{A} = \{(k,l) \mid A_k \cap A_l = \varnothing, k < l\}$ addressing the observed conflicting propositions/attributes and $i, j, k, l \in \mathbb{N}$. Note that the latter constraint on $\mathbb{A}$ implies that the pair wise possible combinations of two attributes are considered only once – since $(A_k, A_l) = (A_l, A_k)$ – and non-conflicting combinations ($k = l$) are omitted. Thus, conflicts are calculated between every **two sensors** instead of all the sensors together (which is used in DST). This difference can be seen from the condition of summation.

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5.1 Fusion methods – Two-Layer-Conflict-Solving

- Within this definition, conflicts are calculated between every two sensors instead of all the sensors together (which is used in DST), this difference can be seen from the condition of summation in DST

$$A_1 \cap \ldots \cap A_n = \varnothing$$

- … and CMDST

$$A_1 \cap A_2 = \varnothing, A_1 \cap A_3 = \varnothing, \cdots, \quad A_1 \cap A_n = \varnothing, \cdots, A_{n-1} \cap A_n = \varnothing$$

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5.1 Fusion methods – Two-Layer-Conflict-Solving

- Hint: In some older publications the following formula is found:

$$k_{\mathrm{cm}} = \sum_{\substack{A_1\cap A_2=\varnothing,\, A_1\cap A_3=\varnothing,\cdots,\\ A_1\cap A_n=\varnothing,\cdots,\, A_{n-1}\cap A_n=\varnothing}} \prod_{i=1}^{n} m_i(A_k).$$

- This formula is mathematically unclear because of its indexing.

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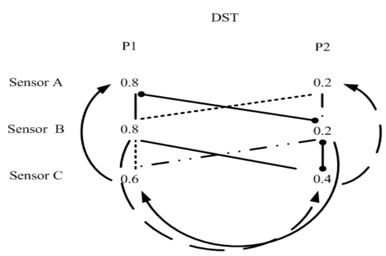
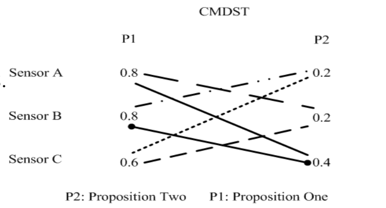
5.1 Fusion methods – Two-Layer-Conflict-Solving

- DST vs CMDST – Conflict computing

$$A(P1)\cdot B(P2)\cdot C(P2) = 0.8\cdot 0.2\cdot 0.4 = 0.064.$$



$$A(P1)\cdot B(P2) + A(P1)\cdot C(P2) = 0.8\cdot 0.2 + 0.8\cdot 0.4 = 0.48.$$



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5.1 Fusion methods – Two-Layer-Conflict-Solving

- It can be readily seen that the total conflict in CMDST is likely to be larger than one. The reason for choosing $\binom{n}{2}$ (**binomial coefficient**) is that there are $\binom{n}{2}$ possible combinations for calculating conflicts (n is the number of sensors). Thus, K_{cm} (K in DST) is:

$$K_{\mathrm{cm}} = \frac{1}{\binom{n}{2} - k_{cm}} = \frac{1}{B_c(n) - \sum_{(i,j)\in\mathbb{M}} \sum_{(k,l)\in\mathbb{A}} m_i(A_k)\cdot m_j(A_l)},$$

$$B_c(n) = \binom{n}{2} = \frac{n!}{2(n-2)!} = \frac{1\cdot 2\cdot 3\cdot\dots\cdot n}{2\left(1\cdot 2\cdot 3\cdots n-2\right)} = \frac{1}{2}\cdot n\cdot(n-1).$$

$$\oplus_{i=1}^{n} m_i(A) = K_{\mathrm{cm}} \sum_{(i,j)\in\mathbb{S}} m_i(A)\cdot m_j(A) = CMDST(A). \quad \text{joined mass}$$

$$\mathbb{S} = \left\{\forall(i,j)\in(\mathbb{N}^{+}\leqslant n)\exists i:\{1,\dots,n-1\}, j:\{i+1,\dots,n\} \middle| i\neq j, i<j\right\}.$$

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5.1 Fusion methods – Two-Layer-Conflict-Solving

- As pointed out, decision making is also studied in a group level. Hence, Group Conflict Redistribution (GCR, layer 2 in Figure 1) acts as group conflict solving strategy, solving conflict in a larger extent compared to individual level (CMDST). Distinguishing from layer 1 (CMDST), GCR combines sensors' propositions in a group manner which means all sensors shall participate in this procedure.

$$m(A) = \frac{\sum_{A_1\cap\dots\cap A_n = A} m_i(A) + \left(\binom{n}{2} + \left|\log\left(\binom{n}{2} - k_{\mathrm{cm}}\right)\right|\right)\cdot CMDST(A)}{n + \binom{n}{2} + \left|\log\left(\binom{n}{2} - k_{\mathrm{cm}}\right)\right|}.$$

joined mass

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5.1 Fusion methods – Two-Layer-Conflict-Solving

Maximum Conflict $K_{\text{cm}} \to \infty$

$$m(A)=\frac{\sum_{A_1\cap\ldots\cap A_n=A} m_i(A)+\left(\binom{n}{2}+\left|\log\left(\binom{n}{2}-k_{\text{cm}}\right)\right|\right)\cdot CMDST(A)}{n+\binom{n}{2}+\left|\log\left(\binom{n}{2}-k_{\text{cm}}\right)\right|}.\quad K_{\text{cm}}=\frac{1}{\binom{n}{2}-k_{cm}},\ \frac{1}{\binom{n}{2}}\le K_{\text{cm}}\le\infty$$

$$m(A)=\frac{\sum_{A_1\cap\ldots\cap A_n=A} m_i(A)+\left(\binom{n}{2}+\left|\log(K_{\text{cm}}^{-1})\right|\right)\cdot CMDST(A)}{n+\binom{n}{2}+\left|\log(K_{\text{cm}}^{-1})\right|}=\frac{\sum_{A_1\cap\ldots\cap A_n=A} m_i(A)+\binom{n}{2}\cdot CMDST(A)+\left|\log(K_{cm}^{-1})\right|\cdot CMDST(A)}{n+\binom{n}{2}+\left|\log(K_{\text{cm}}^{-1})\right|}$$

$$m(A)=\frac{\frac{1}{\left|\log(K_{\text{cm}}^{-1})\right|}\cdot\left[\sum_{A_1\cap\ldots\cap A_n=A} m_i(A)+\binom{n}{2}\cdot CMDST(A)\right]+CMDST(A)}{\frac{1}{\left|\log(K_{\text{cm}}^{-1})\right|}\cdot\left[n+\binom{n}{2}\right]+1};\quad \lim_{K_{\text{cm}}\to\infty}\frac{1}{\left|\log(K_{\text{cm}}^{-1})\right|}\to 0$$

$$\lim_{K_{\text{cm}}\to\infty} m(A)=CMDST(A) \qquad \blacksquare$$

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5.1 Fusion methods – Two-Layer-Conflict-Solving

- To achieve intuitive results also for two sources of information (sensors, cognition) and to simplify the fusion process in a way that it is an additive function of all inputs, the following TLCS approach of a subsequent work was modified [LM10]. In the following sections the *Balanced TLCS* (BalTLCS) is introduced.
- The approach of the normalised CMDST version is based on the normalised conflicting factor which is defined as follows:

$$^{N}K_{\text{cm}}=\frac{B_c(n)}{B_c(n)-k_{\text{cm}}+\varepsilon}<\infty$$

$$0\le k_{\text{cm}}\le B_c(n) \qquad \forall\varepsilon>0,\forall n>1,n\in\mathbb{N},k_{\text{cm}}=0\exists L\in\mathbb{R}:L=(1-{}^{N}K_{\text{cm}})<\varepsilon$$

[LM 10] Lohweg, Volker; Mönks, Uwe: Sensor Fusion by Two-Layer Conflict Solving. In: The 2nd International Workshop on Cognitive Information Processing 14-16 June, 2010, Elba Island (Tuscany), Italy, Jun 2010.

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5.1 Fusion methods – Two-Layer-Conflict-Solving

- Range $\dfrac{B_c(n)}{B_c(n)+\varepsilon} \le {}^N K_{cm} \le B_c(n)\cdot\varepsilon^{-1} < \infty$
- The non-conflicting part is coupled with the conflicting part according to:

$$^{\mathrm{N}}\mathrm{CMDST}(A) = \oplus_{i=1}^{n} m_i(A) = \frac{^N K_{cm}}{B_c(n)} \sum_{(i,j)\in\mathbb{S}} m_i(A)\cdot m_j(A)$$

$$\mathbb{S} = \left\{ \forall (i,j) \in (\mathbb{N}^+ \leqslant n) \exists i : \{1,\ldots,n-1\}, j : \{i+1,\ldots,n\} \middle| i \ne j, i < j \right\}.$$

[LM 10] Lohweg, Volker; Mönks, Uwe: Sensor Fusion by Two-Layer Conflict Solving. In: The 2nd International Workshop on Cognitive Information Processing 14-16 June, 2010, Elba Island (Tuscany), Italy, Jun 2010.

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5.1 Fusion methods – Balanced TLCS (BalTLCS)

- The ***Balanced Group Conflict Redistribution*** **(BalGCR)** part combines the sensors' propositions in a group manner which implies that in this case all sensors will participate additively in this procedure.
- The intention is to utilise the inverse of the normalised conflict factor as a control parameter. If no conflict has occurred, mainly the $^{\mathrm{N}}$CMDST-fusion result should contribute to the overall result. If the conflict is high, then all different information sources have to be taken into account. None of the information sources is allowed to dominate the other ones. The maximum weight of each sensor is 1/n.

[LM 10] Lohweg, Volker; Mönks, Uwe: Sensor Fusion by Two-Layer Conflict Solving. In: The 2nd International Workshop on Cognitive Information Processing 14-16 June, 2010, Elba Island (Tuscany), Italy, Jun 2010.

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5.1 Fusion methods – Balanced TLCS (BalTLCS)

- As the set must be complete regarding the sensors, all information sources (hypotheses) must be covered in a set. Formally, BalGCR fusion consists of two parts. The first one describes the non-conflicting part:

$$m_{\mathrm{nc}}(A) = {}^{N}K_{\mathrm{cm}}^{-1} \cdot {}^{\mathrm{N}}\mathrm{CMDST}(A) = \frac{1}{B_c(n)} \sum_{(i,j)\in\mathbb{S}} m_i(A) \cdot m_j(A),$$

$$\mathbb{S} = \left\{ \forall (i,j) \in (\mathbb{N}^{+} \leqslant n) \exists i : \{1,\ldots,n-1\}, j : \{i+1,\ldots,n\} | i \neq j, i < j \right\}.$$

- The above mentioned Eq. tends to zero in heavy conflicts.
- The second part characterises the conflicting part.

[LM 10] Lohweg, Volker; Mönks, Uwe: Sensor Fusion by Two-Layer Conflict Solving. In: The 2nd International Workshop on Cognitive Information Processing 14-16 June, 2010, Elba Island (Tuscany), Italy, Jun 2010.

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5.1 Fusion methods – Balanced TLCS (BalTLCS)

- Therefore, it is proposed here that all sensors supporting a certain hypothesis have to be averaged to fulfill the above mentioned statements. Furthermore, the average value has to be controlled (e.g. by the normalised conflicting factor):

$$m_c(A) = \frac{k_{\mathrm{cm}}}{B_c(n)} \cdot \frac{1}{n} \sum_{i=1}^{n} m_i(A)$$

- It can be recognised that, in the case of maximum conflict, the average value of all sensory hypotheses is determined.

[LM 10] Lohweg, Volker; Mönks, Uwe: Sensor Fusion by Two-Layer Conflict Solving. In: The 2nd International Workshop on Cognitive Information Processing 14-16 June, 2010, Elba Island (Tuscany), Italy, Jun 2010.

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5.1 Fusion methods – Balanced TLCS (BalTLCS)

- Both parts are additively connected and generate the fusion result:

$$m(A) = m_c(A) + m_{nc}(A) = \frac{k_{\mathrm{cm}}}{B_c(n)} \cdot \frac{1}{n}\sum_{i=1}^{n} m_i(A) + \frac{1}{B_c(n)} \sum_{(i,j)\in\mathbb{S}} m_i(A) \cdot m_j(A),$$
$$\mathbb{S} = \left\{ \forall (i,j) \in (\mathbb{N}^{+} \leqslant n) \exists i : \{1,\ldots,n-1\}, j : \{i+1,\ldots,n\} | i \neq j, i < j \right\}.$$

- The Eq. describes the fusion approach according to the BalTLCS. All masses are fused regarding their pro-and-con-hypotheses. Finally, the sum of final fused results remains always '1', if all sensors deliver acceptable results in the sense of an adequately functioning sensor. As the fusion process is based on averaging, the final result is insensitive to noise effects.

[LM 10] Lohweg, Volker; Mönks, Uwe: Sensor Fusion by Two-Layer Conflict Solving. In: The 2nd International Workshop on Cognitive Information Processing 14-16 June, 2010, Elba Island (Tuscany), Italy, Jun 2010.

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5.2 Fusion methods – Examples

- Example 5.2-2: **DST vs TLCS vs BalTLCS**

	Sunny	*Rainy*
Expert A	0.6	0.4
Expert B	0.7	0.3
Expert C	0.6	0.4

	Sunny	*Rainy*
DST	0.84	0.16

The results seem to be quite intuitive! Is the combining rule redundant? Of course not! Mathematically, DST emphasizes the majority opinion, which brings robustness in some applications, e.g. classification.
However, ...

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5.2 Fusion methods – Examples

Expert A → A(Hypothesis) → A(Sunny) → A(S)
Expert B → B(Hypothesis) → B(Sunny) → B(S)
Expert C → C(Hypothesis) → C(Sunny) → C(S)
...

- Example 5.2-2: **DST** vs TLCS vs BalTLCS

$$k_c = \sum_{A_1 \cap \ldots \cap A_n = \varnothing} \prod_{i=1}^{n} m_i(A_k).$$

$$A(Sunny) \cap B(Sunny) \cap C(Rainy) = \varnothing$$
$$A(Sunny) \cap B(Rainy) \cap C(Sunny) = \varnothing$$
$$A(Rainy) \cap B(Sunny) \cap C(Sunny) = \varnothing$$
$$A(Rainy) \cap B(Rainy) \cap C(Sunny) = \varnothing$$
$$A(Rainy) \cap B(Sunny) \cap C(Rainy) = \varnothing$$
$$A(Sunny) \cap B(Rainy) \cap C(Rainy) = \varnothing$$

Conflict calculation:

$$k_c = A(S)B(S)C(R) + A(S)C(S)B(R) + B(S)C(S)A(R) + A(R)B(R)C(S) + \ldots$$
$$\ldots + A(R)C(R)B(S) + A(S)B(R)C(R)$$
$$= 0.6 \cdot 0.7 \cdot 0.4 + 0.6 \cdot 0.6 \cdot 0.3 + 0.7 \cdot 0.6 \cdot 0.4 + 0.4 \cdot 0.3 \cdot 0.6 + 0.4 \cdot 0.4 \cdot 0.7 + \ldots$$
$$\ldots + 0.6 \cdot 0.3 \cdot 0.4 = 0.168 + 0.168 + 0.108 + 0.072 + 0.112 + 0.072 = 0.7.$$

n: number of experts / sensors

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5.2 Fusion methods – Examples

- Example 5.2-2: **DST** vs TLCS vs BalTLCS

$$K = \frac{1}{1-k_c} = \frac{1}{1 - \sum_{A_1 \cap A_n = \varnothing} \prod_{i=1}^{n} m_i(A_k)},$$

$$\oplus_{i=1}^{n} m_i(A) = K \sum_{A_1 \cap \ldots \cap A_n = A} \prod_{i=1}^{n} m_i(A_k),$$

$$Sunny(A,B,C) = A(S)B(S)C(S) = 0.6 \cdot 0.7 \cdot 0.6 = 0.252;$$
$$Rainy(A,B,C) = A(R)B(R)C(R) = 0.4 \cdot 0.3 \cdot 0.4 = 0.048.$$

$$m(Sunny) = \frac{Sunny(A,B,C)}{1-k_c} = \frac{0.252}{1-0.7} = 0.84.$$

$$m(Rainy) = \frac{Rainy(A,B,C)}{1-k_c} = \frac{0.048}{1-0.7} = 0.16.$$

$$Bel(Sunny) = 0.84 \quad Pl(Sunny) = 1 - Bel(Rainy) = 1 - 0.16 = 0.84$$

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5.2 Fusion methods – Examples

- Example 5.2-2: DST vs **TLCS** vs BalTLCS

$$k_{\mathrm{cm}} = \sum_{(i,j)\in\mathbb{M}} \sum_{(k,l)\in\mathbb{A}} m_i(A_k)\cdot m_j(A_l). \qquad K_{\mathrm{cm}} = \frac{1}{\binom{n}{2} - k_{cm}} = \frac{1}{B_c(n) - \sum_{(i,j)\in\mathbb{M}} \sum_{(k,l)\in\mathbb{A}} m_i(A_k)\cdot m_j(A_l)},$$

$$\oplus_{i=1}^{n} m_i(A) = K_{\mathrm{cm}} \sum_{(i,j)\in\mathbb{S}} m_i(A)\cdot m_j(A).$$

$$m(A) = \frac{\sum_{A_1\cap\ldots\cap A_n = A} m_i(A_i) + \left(\binom{n}{2} + \left|\log\left(\binom{n}{2} - k_{\mathrm{cm}}\right)\right|\right)\cdot CMDST(A)}{n + \binom{n}{2} + \left|\log\left(\binom{n}{2} - k_{\mathrm{cm}}\right)\right|}. \qquad \binom{n}{k} = \frac{n!}{k!(n-k)!}.$$

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5.2 Fusion methods – Examples

- Example 5.2-2: DST vs **TLCS** vs BalTLCS

Conflict calculation

$$\begin{aligned} k_{\mathrm{cm}} &= A(S)B(R) + A(R)B(S) + A(S)C(R) + A(R)C(S) + B(S)C(R) + \ldots \\ &\ldots + C(S)B(R) = 0.6\cdot 0.3 + 0.4\cdot 0.7 + 0.6\cdot 0.4 + 0.4\cdot 0.6 + 0.7\cdot 0.4 + 0.3\cdot 0.6 \\ &= 1.4. \end{aligned}$$

Non-conflicting part:

$$\begin{aligned} Sunny(A,B,C) &= A(S)B(S) + A(S)C(S) + B(S)C(S) \\ &= 0.6\cdot 0.7 + 0.6\cdot 0.6 + 0.7\cdot 0.6 = 0.42 + 0.36 + 0.42 = 1.2, \\ Rainy(A,B,C) &= A(R)B(R) + A(R)C(R) + B(R)C(R) \\ &= 0.4\cdot 0.3 + 0.4\cdot 0.4 + 0.3\cdot 0.4 = 0.4. \end{aligned}$$

First layer (CMDST)

$$m(Sunny) = \frac{Sunny(A,B,C)}{3 - k_{cm}} = \frac{1.2}{3-1.4} = \frac{1.2}{1.6} = 0.75,$$

$$m(Rainy) = \frac{Rainy(A,B,C)}{3 - k_{cm}} = \frac{0.4}{3-1.4} = \frac{0.4}{1.6} = 0.25. \qquad \binom{3}{2} = 3$$

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5.2 Fusion methods – Examples

- Example 5.2-2: DST vs **TLCS** vs BalTLCS

Second layer (GCR)

$$m(Sunny) = \frac{A(sunny) + B(Sunny) + C(Sunny) + (3 + |\log(3 - k_{cm})|) \cdot CMDST(Sunny)}{3 + 3 + |\log(3 - k_{cm})|}$$

$$m(Sunny) = \frac{0.6 + 0.7 + 0.6 + (3 + |\log(3 - k_{cm})|) \cdot 0.75}{3 + 3 + |\log(3 - k_{cm})|} = 0.69$$

$$m(Rainy) = \frac{A(Rainy) + B(Rainy) + C(Rainy) + (3 + |\log(3 - k_{cm})|) \cdot CMDST(Rainy)}{3 + 3 + |\log(3 - k_{cm})|}$$

$$m(Rainy) = \frac{0.4 + 0.3 + 0.4 + (3 + |\log(3 - k_{cm})|) \cdot 0.25}{3 + 3 + |\log(3 - k_{cm})|} = 0.31$$

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5.2 Fusion methods – Examples

- Example 5.2-2: DST vs TLCS vs **BalTLCS**

$$k_{\mathrm{cm}} = \sum_{(i,j)\in\mathbb{M}} \sum_{(k,l)\in\mathbb{A}} m_i(A_k) \cdot m_j(A_l), \quad B_c(n) = \binom{n}{2} = \frac{n!}{2(n-2)!} = \frac{1}{2}n(n-1)$$

$$m(A) = m_c(A) + m_{nc}(A) = \frac{k_{\mathrm{cm}}}{B_c(n)} \cdot \frac{1}{n}\sum_{i=1}^{n} m_i(A) + \frac{1}{B_c(n)} \sum_{(i,j)\in\mathbb{S}} m_i(A) m_j(A)$$

$$m_c(A) = \frac{k_{\mathrm{cm}}}{B_c(n)} \cdot \frac{1}{n}\sum_{i=1}^{n} m_i(A)$$

$$m_{nc}(A) = \frac{1}{B_c(n)} \sum_{(i,j)\in\mathbb{S}} m_i(A) m_j(A)$$

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5.2 Fusion methods – Examples

- Example 5.2-2: DST vs TLCS vs **BalTLCS**

Conflict calculation

$$k_{\text{cm}} = A(S)B(R) + A(R)B(S) + A(S)C(R) + A(R)C(S) + B(S)C(R) + \ldots$$
$$\ldots + C(S)B(R) = 0.6 \cdot 0.3 + 0.4 \cdot 0.7 + 0.6 \cdot 0.4 + 0.4 \cdot 0.6 + 0.7 \cdot 0.4 + 0.3 \cdot 0.6$$
$$= 1.4.$$

$$B_c(n) = \binom{3}{2} = 3$$

$$m_c(Sunny) = \frac{k_{\text{cm}}}{B_c(n)} \cdot \frac{1}{n} \sum_{i=1}^{n} m_i(Sunny) = \frac{1.4}{3} \cdot \frac{1}{3}\left(0.6 + 0.7 + 0.6\right) = 0.296$$

$$m_c(Rainy) = \frac{k_{\text{cm}}}{B_c(n)} \cdot \frac{1}{n} \sum_{i=1}^{n} m_i(Rainy) = \frac{1.4}{3} \cdot \frac{1}{3}\left(0.4 + 0.3 + 0.4\right) = 0.171$$

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5.2 Fusion methods – Examples

- Example 5.2-2: DST vs TLCS vs **BalTLCS**

Non-conflicting part:

First layer (CMDST)

$$Sunny(A,B,C) = A(S)B(S) + A(S)C(S) + B(S)C(S)$$
$$= 0.6 \cdot 0.7 + 0.6 \cdot 0.6 + 0.7 \cdot 0.6 = 0.42 + 0.36 + 0.42 = 1.2,$$
$$Rainy(A,B,C) = A(R)B(R) + A(R)C(R) + B(R)C(R)$$
$$= 0.4 \cdot 0.3 + 0.4 \cdot 0.4 + 0.3 \cdot 0.4 = 0.4.$$

$$m_{nc}(A) = \frac{1}{B_c(n)} \sum_{(i,j) \in \mathbb{S}} m_i(A) \cdot m_j(A)$$

$$m_{nc}(Sunny) = \frac{Sunny(A,B,C)}{3} = \frac{1.2}{3} = 0.40,$$
$$m_{nc}(Rainy) = \frac{Rainy(A,B,C)}{3} = \frac{0.4}{3} = 0.1\bar{3}.$$

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5.2 Fusion methods – Examples

- Example 5.2-2: DST vs TLCS vs **BalTLCS**

Second layer (GCR)

$$m(S) = m_c(S) + m_{nc}(S) = \frac{k_{\mathrm{cm}}}{B_c(n)} \cdot \frac{1}{n} \sum_{i=1}^{n} m_i(S) + \frac{1}{B_c(n)} \sum_{(i,j)\in\mathbb{S}} m_i(S) \cdot m_j(S) = 0.696$$

$$m(R) = m_c(R) + m_{nc}(R) = \frac{k_{\mathrm{cm}}}{B_c(n)} \cdot \frac{1}{n} \sum_{i=1}^{n} m_i(R) + \frac{1}{B_c(n)} \sum_{(i,j)\in\mathbb{S}} m_i(R) \cdot m_j(R) = 0.304$$

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5.2 Fusion methods – Examples

- Example 5.2-2: DST vs TLCS vs BalTLCS

	Sunny	*Rainy*	*Stormy*
Expert A	0.99	0.01	0
Expert B	0	0.01	0.99
Expert C	0.99	0.01	0

Zadeh's example

	Sunny	*Rainy*	*Stormy*	*S,R,St*
Yager	0	0.000001	0	**0.999999**
Murphy	0.66	0.01	0.33	0
DST	**0**	**1**	**0**	**0**
TLCS	**0.82**	**0.005**	**0.165**	**0**
BalTLCS	**0.771**	**0.007**	**0.222**	**0**

However, ...
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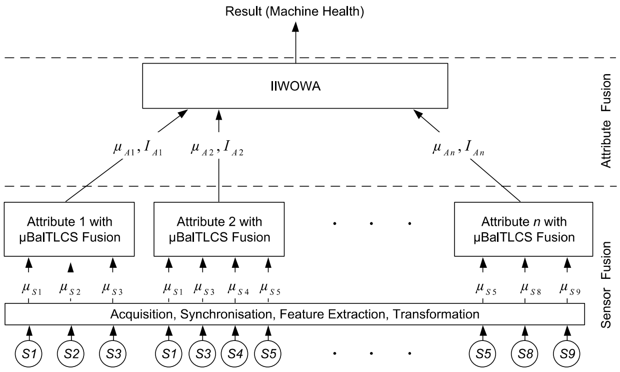
5.2 Fusion methods – Examples



- **Attribute Layer Fusion (Fuzzy Averaging)**
  - Implicative Importance Weighted Ordered Weighted Averaging (IIWOWA) [Larsen 99]
    - Importance $I_{An} = 1 - \frac{k_{cm}}{B_c(n)}$

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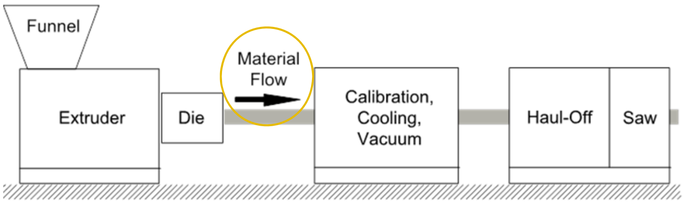
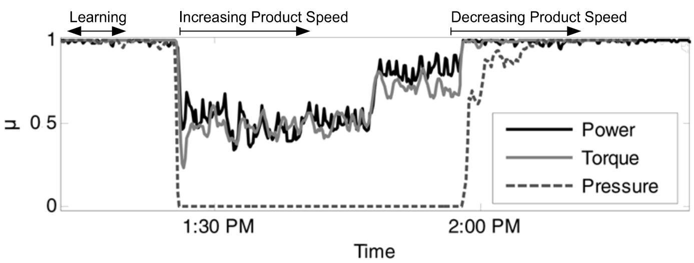
5.2 Fusion methods – Examples

- **Condition Monitoring on a Pipe Extrusion Line**



Attribute **Material Flow** = Power, Torque, Pressure



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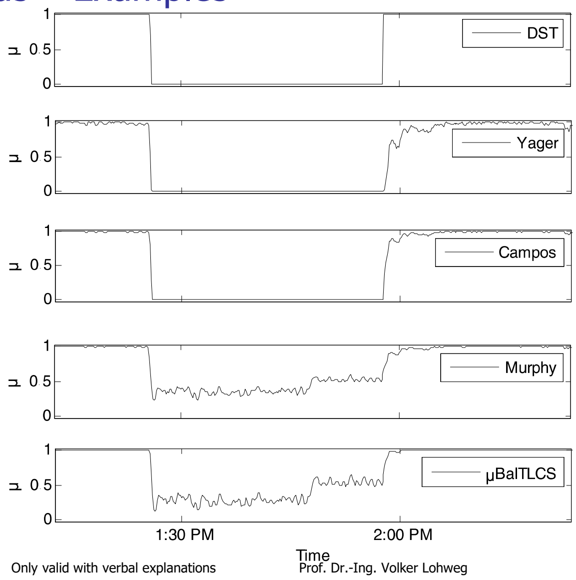
5.2 Fusion methods – Examples

Attribute **Material Flow** =
Power, Torque, Pressure



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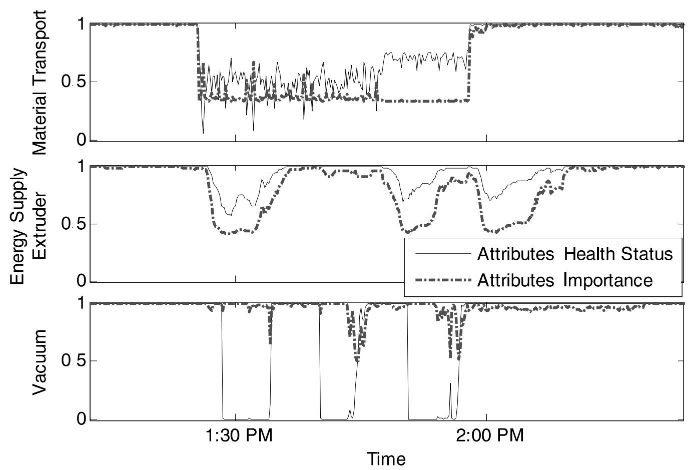
5.2 Fusion methods – Examples

- **Fused Attributes and Their Importances**



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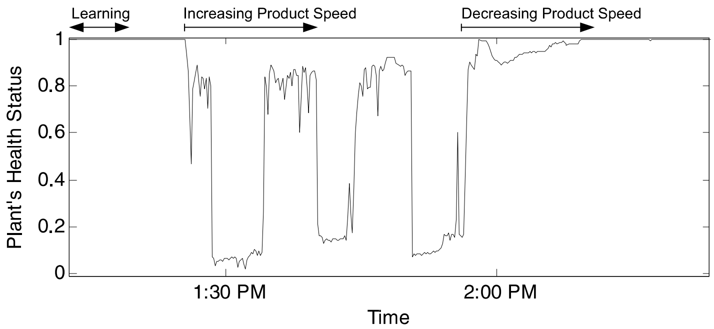
5.2 Fusion methods – Examples

- **Production Line's Health**
  - Fusion Result: Machine Health



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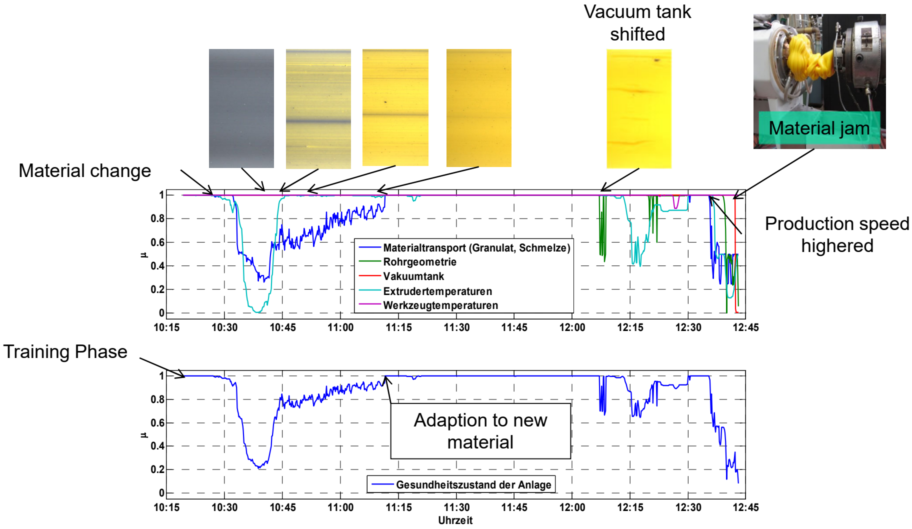
5.2 Fusion methods – Examples



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5.2 Fusion methods – Examples

- **Novel interrelationship between Fuzzy Membership functions and Evidential masses**
  - Machine-learnable
  - Comfortable to use
- **Multi-Layer Information Fusion approach (µBalTLCS)**
  - Conflict reduction
- **Research for Industry 4.0, and Health**

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5.2 References

L.A. Zadeh, "A mathematical theory of evidence (book review)," AI Magazine, 55(81-83), 1984.

L.A. Zadeh, "A simple view of the Dempster-Shafer theory of evidence and its implication for the rule of combination," AI Magazine, 7:85-90,1986.

S.A. Sunita, "Order effects and memory for evidence in individual versus group decision making in auditing," Journal of Behavioral Decision Making, Vol. 12, Issue 1, pp. 71 – 88, 1999.

R. Lipshitz, G. Klein and J. Orasanu, et al, "Taking Stock of Naturalistic Decision Making," Journal of Behavioral Decision Making, 14:331-352, 2001.

Li, Rui; Lohweg, Volker: A Novel Data Fusion Approach using Two-Layer Conflict Solving. In: 2008 IAPR Workshop on Cognitive Information Processing, June 9-10, Santorini, Greece, 2008.

Lohweg, Volker; Mönks, Uwe: Sensor Fusion by Two-Layer Conflict Solving. In: The 2nd International Workshop on Cognitive Information Processing 14-16 June, 2010, Elba Island (Tuscany), Italy, Jun 2010.

Mönks, Uwe; Lohweg, Volker; Petker, Denis: Fuzzy-Pattern-Classifier Training with Small Data Sets. In: IPMU 2010 - International Conference on Information Processing and Management of Uncertainty in Knowledge Based Systems 28 Jun 2010 - 02 July 2010, Dortmund, Germany, Jun 2010.

Volker Lohweg and Uwe Mönks (2010). Fuzzy-Pattern-Classifier Based Sensor Fusion for Machine Conditioning, Sensor Fusion and its Applications, Ciza Thomas (Ed.), ISBN: 978-953-307-101-5, InTech, Available from: http://www.intechopen.com/articles/show/title/fuzzy-pattern-classifier-based-sensor-fusion-for-machine-conditioning

Voth, Karl; Glock, Stefan; Mönks, Uwe; Türke, Thomas; Lohweg, Volker: Multi-sensory Machine Diagnosis on Security Printing Machines with Two Layer Conflict Solving. In: SENSOR+TEST Conference 2011, 7 – 9 June 2011, Nürnberg, Germany , Jun 2011.

Glock, Stefan; Voth, Karl; Schaede, Johannes; Lohweg, Volker: A Framework for Possibilistic Multi-source Data Fusion with Monitoring of Sensor Reliability. In: World Conference on Soft Computing, San Francisco, CA, USA, May 23-26, 2011 May 2011.

Information Fusion Only valid with verbal explanations Prof. Dr.-Ing. Volker Lohweg 69

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5.2 References

G. Shafer, *A Mathematical Theory of Evidence*. Princeton University Press, Princeton, 1976

L. A. Zadeh, "Fuzzy sets," *Inf. and Control*, vol. 8, no. 3, pp. 338–353, 1965

D. Dubois and H. Prade, "Fuzzy sets and probability: misunderstandings, bridges and gaps," Fuzzy *Systems, 2nd IEEE International Conference on*, vol. 2, pp. 1059–1068, 1993.

H. L. Larsen, "Importance Weighted OWA Aggregation of Multicriteria Queries," *18th International Conference of the North American Fuzzy Information Processing Society*, New York, pp. 740–744, 1999.

Fritze, Alexander; Mönks, Uwe; Lohweg, Volker: A Support System for Sensor and Information Fusion System Design. In: *3rd International Conference on System-Integrated Intelligence - New Challenges for Product and Production Engineering Paderborn*, Germany, Jun 2016.

Ehlenbröker, Jan-Friedrich; Mönks, Uwe; Lohweg, Volker: Sensor Defect Detection in Multisensor Information Fusion. In: *Journal of Sensors and Sensor Systems*, ISSN 2194-8771 Copernicus Publications, Aug 2016.

Mönks, Uwe; Dörksen, Helene; Lohweg, Volker; Hübner, Michael: Information Fusion of Conflicting Input Data. In: *Sensors*, ISSN 1424-8220 MDPI AG (Multidisciplinary Digital Publishing Institute), Aug 2016.

Fritze, Alexander; Mönks, Uwe; Holst, Christoph-Alexander; Lohweg, Volker: An Approach to Automated Fusion System Design and Adaptation. In: *Sensors 17*, no. 3: 601; doi: 10.3390/s17030601, Mar 2017.

Holst, Christoph-Alexander; Lohweg, Volker: Conflict-based Feature Selection for Information Fusion Systems. In: *22th IEEE International Conference on Emerging Technologies and Factory Automation (ETFA 2017),* Limassol, Cyprus Sep 2017.

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5.2 References

Holst, Christoph-Alexander; Lohweg, Volker: A Conflict-Based Drift Detection And Adaptation Approach for Multisensor Information Fusion. In: *23rd IEEE International Conference on Emerging Technologies and Factory Automation (ETFA)* Torino, Italy, Sep 2018.

Holst, Christoph-Alexander; Lohweg, Volker: Feature Fusion to Increase the Robustness of Machine Learners in Industrial Environments. In: *at - Automatisierungstechnik 67* (10) S.: 853-865, De Gruyter, Sep 2019.

Dörksen, Helene; Lohweg, Volker: Margin-based Refinement for Linear Discriminant Analysis.
In: *4th European Conference on Data Analysis (ECDA2017)* submitted to Journal Archives of Data Sciences, Wroclaw, Poland , Sep 2017.

Dörksen, Helene; Lohweg, Volker: Multivariate Gaussian Feature Selection. In: *European Conference on Data Analysis (ECDA2018) Paderborn*, Germany, Jul 2018.

Dörksen, Helene; Lohweg, Volker: Linear Classification of Badly Conditioned Data. In: *23rd IEEE International Conference on Emerging Technologies and Factory Automation (ETFA) Torino*, Italy, Sep 2018. Best ETFA-Paper in Automation

Dörksen, Helene; Lohweg, Volker: Margin-based Refinement for Linear Discriminant Analysis. In: *Archives of Data Science, Series A* (Online First)(Band 4, Heft 1, online) Apr 2020.

Further and actual literature is found here:
https://www.hs-owl.de/init/veroeffentlichungen/publikationen/a/filteron/8/author.html

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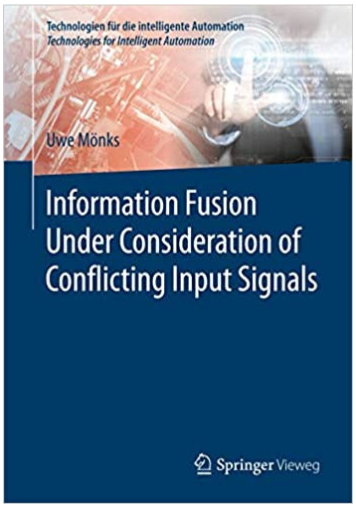
5.2 References



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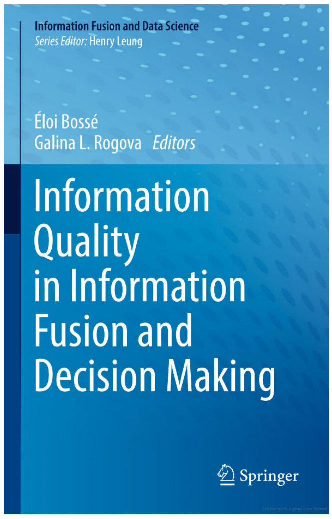

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