Problem 1 Proof, if the following systems are linear, time invariant, causal, and memoryless.

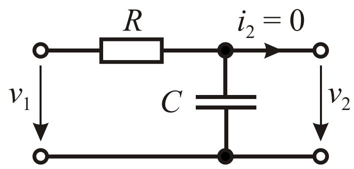
a) $y(t) = \sin\left[x(t-1)\right]$; x,t real b) $y(t) = a \cdot x(t) - b \cdot \dfrac{\mathrm{d}\,y(t)}{\mathrm{d}\,t}$; a,b,x,t real

c) $y(t) = a \cdot t^2 + x(t+3)$; a,x,t real

Problem 2 Which of the given unit impulse responses characterizes a stable LTI system?

a) $h(t) = \cos(t) \cdot u(t)$ b) $h(t) = \mathrm{e}^{at} \cdot u(t)$; a real

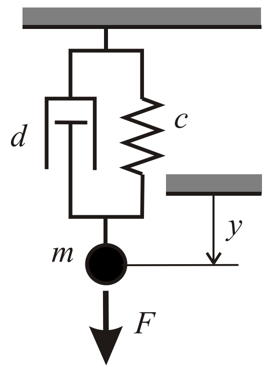
Problem 3 The given RC circuit shall be analyzed by solving the system's differential equation under the initial rest condition. The capacitor requires a steady voltage v_2.



a) Compute $y(t) = v_2(t)$ for $x(t) = v_1(t) = \dfrac{1\,\mathrm{Vs}}{T} \cdot \mathrm{rect}\left(\dfrac{t-T/2}{T}\right)$

b) Compute $x(t) = v_1(t)$ and $y(t) = v_2(t)$ for $T \to 0$ and interpret the result.

Problem 4 A mechanical system shall be analyzed. Fixed at a ceiling is a spring (c), a damper (d) and a mass (m). An external force F stimulates the system. The spring is characterized by the relation $F_c = c{\cdot}y$. The distance y is measured from the relaxed position of the spring. A spring is able to store energy according to $E_c = c{\cdot}y^2/2$. The damper is characterized by $F_d = d{\cdot}v$, where v is the velocity of the moving part. The damper dissipates energy.



a) Derive a differential equation between the stimulation force F and the mass position y. Is the system linear?

b) Derive a state-space representation.

Problem 5 Let $x(t)$ be an input signal whose FOURIER transform is

$$X(\mathrm{j}\omega) = \delta(\omega) + \delta(\omega - \pi) + \delta(\omega - 5)$$

and let $h(t) = u(t) - u(t-2)$

be the unit impulse response of a system.

a) Is $x(t)$ periodic?

b) Compute the output signal $y(t)$. Is $y(t)$ periodic?

Problem 6 Derive the frequency response of the circuit in problem 3 from the differential equation.

Problem 7 Consider a causal LTI system with frequency response

$$H(\mathrm{j}\omega) = \frac{1}{3 + \mathrm{j}\omega}$$

For a particular input $x(t)$ this system is observed to produce the output signal

$$y(t) = \mathrm{e}^{-3t} \cdot u(t) - \mathrm{e}^{-4t} \cdot u(t)$$

a) Determine and plot the magnitude and the phase response.

b) Which properties of $h(t)$ can be derived from the magnitude and the phase response?

c) Determine the input signal $x(t)$.

Answers

Problem 1

linear, time invariant, causal, memoryless

a) no, yes, yes, no ; b) yes, yes, yes, no ; c) no, no, no, no

Problem 2

a) not stable ; b) stable for $a < 0$

Problem 3

a) $v_2 = 0$ for $t < 0$; $v_2 = \frac{1\,\mathrm{Vs}}{T} \cdot \left(1 - \mathrm{e}^{-t/\tau}\right)$ for $0 \le t < T$; $v_2 = \frac{1\,\mathrm{Vs}}{T} \cdot \left(\mathrm{e}^{T/\tau} - 1\right) \cdot \mathrm{e}^{-t/\tau}$ for $t > T$

b) $x(t) = 1\,\mathrm{Vs} \cdot \delta(t)$; $y(t) = 1\,\mathrm{Vs} \cdot h(t) = u(t) \cdot \frac{1\,\mathrm{Vs}}{\tau} \cdot \mathrm{e}^{-t/\tau}$

Problem 4

a) $c \cdot y + d \cdot \frac{\mathrm{d}\,y}{\mathrm{d}\,t} + m \cdot \frac{\mathrm{d}^2\,y}{\mathrm{d}\,t^2} = m \cdot g + F$; no

b) $\begin{bmatrix} \frac{\mathrm{d}\,z_1}{\mathrm{d}\,t} \\ \frac{\mathrm{d}\,z_2}{\mathrm{d}\,t} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{c}{m} & -\frac{d}{m} \end{bmatrix} \cdot \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} \cdot x$; $x = m \cdot g + F$; $\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} y \\ \frac{\mathrm{d}\,y}{\mathrm{d}\,t} \end{bmatrix}$

$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + 0 \cdot x$

Problem 5

a) no ; b) $y(t) = \frac{1}{\pi}\left(1 + \frac{\sin 5}{5} \cdot \mathrm{e}^{\mathrm{j}5(t-1)}\right)$; yes

Problem 6

$$H(\omega) = \frac{1}{1 + \mathrm{j}\omega RC}$$

Problem 7

a) $\left|H(\omega)\right| = \frac{1}{\sqrt{9 + \omega^2}}$; $\varphi(\omega) = -\arctan\left(\frac{\omega}{3}\right)$

b) $h(t)$ real with $h_\mathrm{e} \neq 0$ and $h_\mathrm{o} \neq 0$

c) $x(t) = \mathrm{e}^{-4t} \cdot u(t)$