Problem 1 Is the down-sampler a linear and time-invariant system?

Problem 2 Is the M-point moving average filter a stable system? Its input-output relation is given by:

$$y[n] = \frac{1}{M}\sum_{i=0}^{M-1} x[n-i]$$

Problem 3 Consider a causal LTI discrete-time system with the impulse response $h[n] = \beta^n \cdot u[n]$, where $|\beta| < 1$. Determine the output sequence $y[n]$ for the causal input sequence $x[n] = \alpha^n \cdot u[n]$ with $|\alpha| < 1$. $u[n]$ is the unit step sequence.

Problem 4 A discrete-time system is characterized by the following difference equation:

$$y[n] - 2^n \cdot y[n+1] + 3 \cdot y^2[n+2] = 4 \cdot x[n] - 2 \cdot x[n+1]$$

Check, whether the system is linear, time-invariant, causal, and memoryless.

Problem 5 A causal discrete-time LTI system is characterized by the following difference equation:

$$y[n] + y[n-1] - 6 \cdot y[n-2] = x[n]$$

Compute the impulse response. Is the system stable?

Problem 6 A causal discrete-time LTI system is characterized by the following difference equation:

$$y[n] - \frac{1}{2} \cdot y[n-1] = x[n]$$

a) Compute the impulse response and check for stability.
b) Determine the solution for $n \geq 0$ for the step input $x[n] = 8 \cdot u[n]$ with initial condition $y[0] = 0$.

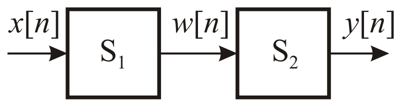
c) Determine the solution for $n \geq 0$ for $x[n] = \left(\frac{1}{2}\right)^n \cdot u[n]$ with initial condition $y[0] = 0$.

Problem 7 Consider the cascade of the causal LTI systems S_1 and S_2.

$$S_1: \; w[n] = \frac{1}{2} \cdot w[n-1] + x[n] \quad ; \quad S_2: \; y[n] = \alpha \cdot y[n-1] + \beta \cdot w[n]$$

The difference equation relating $x[n]$ and $y[n]$ is:

$$y[n] = -\frac{1}{8} \cdot y[n-2] + \frac{3}{4} \cdot y[n-1] + x[n]$$



Determine α and β and the impulse response $h[n]$.

Problem 8 a) Show that the sequence $x[n] = z^n$, where z is a complex constant, is an eigenfunction of an LTI discrete-time system. b) Is the sequence $x[n] = z^n \cdot u[n]$ with $u[n]$ being the unit step sequence also an eigenfunction of an LTI discrete-time system?

Answers

Problem 1
linear: yes ; time-invariant: no

Problem 2
yes

Problem 3

$$y[n] = \sum_{k=0}^{n} \alpha^k \cdot \beta^{n-k}$$

Problem 4
linear: no ; time-invariant: no ; causal: yes; memoryless: no

Problem 5
$h[n] = 0.4 \cdot 2^n + 0.6 \cdot (-3)^n$ with $n \geq 0$; not stable

Problem 6

a) $h[n] = \left(\frac{1}{2}\right)^n \cdot u[n]$

b) $y[n] = 16 \cdot \left\{1 - \left(\frac{1}{2}\right)^n\right\} \cdot u[n]$

c) $y[n] = n \cdot \left(\frac{1}{2}\right)^n \cdot u[n]$

Problem 7

$\alpha = 1/4$; $\beta = 1$; $h[n] = \left\{2 \cdot \left(\frac{1}{2}\right)^n - \left(\frac{1}{4}\right)^n\right\} \cdot u[n]$

Problem 8
a) yes; b) no