Problem 1 Check, if the signals

a) $x(t) = \mathrm{e}^{-2t} \cdot u(t)$; b) $x(t) = \mathrm{e}^{\mathrm{j}(2t+\pi/4)}$; c) $x(t) = \cos(t)$

are energy or power signals. $u(t)$ is the unit step function. If possible, determine the energy E_∞ and the average power P_∞.

Problem 2 Which of the following signals is periodic? $u(t)$ is the unit step function.

a) $x(t) = 2 \cdot \mathrm{e}^{\mathrm{j}(t+\pi/4)} \cdot u(t)$; b) $x(t) = \mathrm{j} \cdot \mathrm{e}^{\mathrm{j}10t}$; c) $x(t) = \mathrm{e}^{(-1+\mathrm{j})t}$

If a signal is periodic, specify the period T.

Problem 3 Express the real part of each of the following signals in the form

$A \cdot \mathrm{e}^{-at} \cdot \cos(\omega t + \phi)$,

where A, a, ω, and ϕ are real numbers with $A > 0$ and $-\pi < \varphi \le \pi$:

a) $x(t) = -2$ b) $x(t) = \sqrt{2} \cdot \mathrm{e}^{\mathrm{j}\pi/4} \cos(3t + 2\pi)$

c) $x(t) = \mathrm{e}^{-t} \cdot \sin(3t + \pi)$ d) $x(t) = \mathrm{j} \cdot \mathrm{e}^{(-2+\mathrm{j}100)t}$

Problem 4 Compute the convolution of the signals:

a) $x_1(t) = \mathrm{e}^{2t} \cdot u(-t)$; $x_2(t) = u(t-3)$

b) $x_1(t) = \begin{cases} 1+t \ ; & 0 \le t \le 1 \\ 2-t \ ; & 1 < t \le 2 \\ 0 \ ; & \text{elsewhere} \end{cases}$; $x_2(t) = \delta(t+2) + 2 \cdot \delta(t+1)$

c) $x_1(t) = u(t-3) - u(t-5)$; $x_2(t) = \mathrm{e}^{-3t} \cdot u(t)$ d) $x(t) * u(t)$

Problem 5 Derive the following FOURIER transform properties:

a) Time and frequency shifting b) Scaling

c) Differentiation and integration d) Convolution and product

Problem 6 Compute the FOURIER transforms of the signals:

a) $x(t) = \begin{cases} \cos(\omega_0 t) \ ; & t > 0 \\ 0 \ ; & t < 0 \end{cases}$ b) $x(t) = \begin{cases} \sin(\omega_0 t) \ ; & t > 0 \\ 0 \ ; & t < 0 \end{cases}$ c) $x(t) = \begin{cases} 1-t \ ; & 0 < t < 1 \\ 1+t \ ; & -1 < t < 0 \\ 0 \ ; & \text{elsewhere} \end{cases}$

d) $x(t) = 1 + \cos\left(6\pi t + \dfrac{\pi}{8}\right)$ e) $x(t) = t \cdot \left(\dfrac{\sin t}{\pi t}\right)^2$

Problem 7 The medium duration D and the medium bandwidth B of a signal are given by

$$D = \frac{1}{x_{\max}} \int_{-\infty}^{\infty} x(t)\,\mathrm{d}t \ , \quad B_\omega = \frac{1}{X_{\max}} \int_{-\infty}^{\infty} X(\omega)\,\mathrm{d}\omega \ , \quad B_\omega = 2\pi \cdot B$$

Calculate the product $D \cdot B_\omega$ for a rectangular impulse of width T. *Aid:* $\int_0^\infty \mathrm{si}(ax) \cdot \mathrm{d}x = \dfrac{\pi}{2a}$

Answers

Problem 1 a) $P_\infty = 0;\ E_\infty = \frac{1}{4}$; b) $P_\infty = 1;\ E_\infty = \infty$; c) $P_\infty = \frac{1}{2};\ E_\infty = \infty$

Problem 2 a) No ; b) $T = \frac{\pi}{5}$; c) No

Problem 3

a) $A = 2$; $a = 0$; $\omega = 0$; $\varphi = \pi$ b) $A = 1$; $a = 0$; $\omega = 3$; $\phi = 0$

c) $A = 1$; $a = 1$; $\omega = 3$; $\varphi = \frac{\pi}{2}$ d) $A = 1$; $a = 2$; $\omega = 100$; $\varphi = \frac{\pi}{2}$

Problem 4

a) $y(t) = \begin{cases} \frac{1}{2}\cdot e^{2(t-3)} & \text{for } t \le 3 \\ \frac{1}{2} & t \ge 3 \end{cases}$ ... b) $y(t) = \begin{cases} 3+t & -2 < t \le -1 \\ 4+t & -1 < t \le 0 \\ 2-2t & 0 < t \le 1 \\ 0 & \text{elsewhere} \end{cases}$

c) $y(t) = \begin{cases} 0 & -\infty < t \le 3 \\ \frac{1}{3}\left[1 - e^{-3(t-3)}\right] & \text{for } 3 < t \le 5 \\ \frac{1}{3}\left(1 - e^{-6}\right)\cdot e^{-3(t-5)} & 5 < t < \infty \end{cases}$ d) $\int_{-\infty}^{t} x(\tau)\,\mathrm{d}\tau$

Problem 5 See lecture

Problem 6

a) $X(\mathrm{j}\omega) = \frac{\pi}{2}\left(\delta(\omega+\omega_0) + \delta(\omega-\omega_0)\right) - \frac{\mathrm{j}\omega}{\omega^2 - \omega_0^2}, X(f) = \frac{1}{4}\left(\delta(f+f_0) + \delta(f-f_0)\right) - \frac{\mathrm{j}f}{2\pi\cdot\left(f^2 - f_0^2\right)}$

b) $X(\mathrm{j}\omega) = \frac{\mathrm{j}\pi}{2}\left(\delta(\omega+\omega_0) - \delta(\omega-\omega_0)\right) - \frac{\omega_0}{\omega^2 - \omega_0^2}, X(f) = \frac{\mathrm{j}}{4}\left(\delta(f+f_0) - \delta(f-f_0)\right) - \frac{f_0}{2\pi\cdot\left(f^2 - f_0^2\right)}$

c) $X(\mathrm{j}\omega) = \mathrm{si}^2\left(\frac{\omega}{2}\right) = \mathrm{sinc}^2\left(\frac{\omega}{2\pi}\right)$ e) $X(\mathrm{j}\omega) = \begin{cases} \mathrm{j}/2\pi & ;\ -2 \le \omega < 0 \\ -\mathrm{j}/2\pi & ;\ 0 \le \omega < 2 \\ 0 & ;\ \text{elsewhere} \end{cases}$

d) $X(\mathrm{j}\omega) = 2\pi\delta(\omega) + \pi\left[e^{\mathrm{j}\pi/8}\cdot\delta(\omega - 6\pi) + e^{-\mathrm{j}\pi/8}\cdot\delta(\omega + 6\pi)\right]$

Problem 7 $D\cdot B_\omega = 2\pi$, $D\cdot B = 1$