Technische Hochschule OWL **Prof. Dr. Uwe Meier**

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Written Exam: Discrete Signals and Systems (DSS)

Degree Programmes: Information Technology (M. Sc.), Elektrotechnik (M. Sc.)

2022-03-25, 120 min, 100 points available $\rightarrow$ no notebooks, no books

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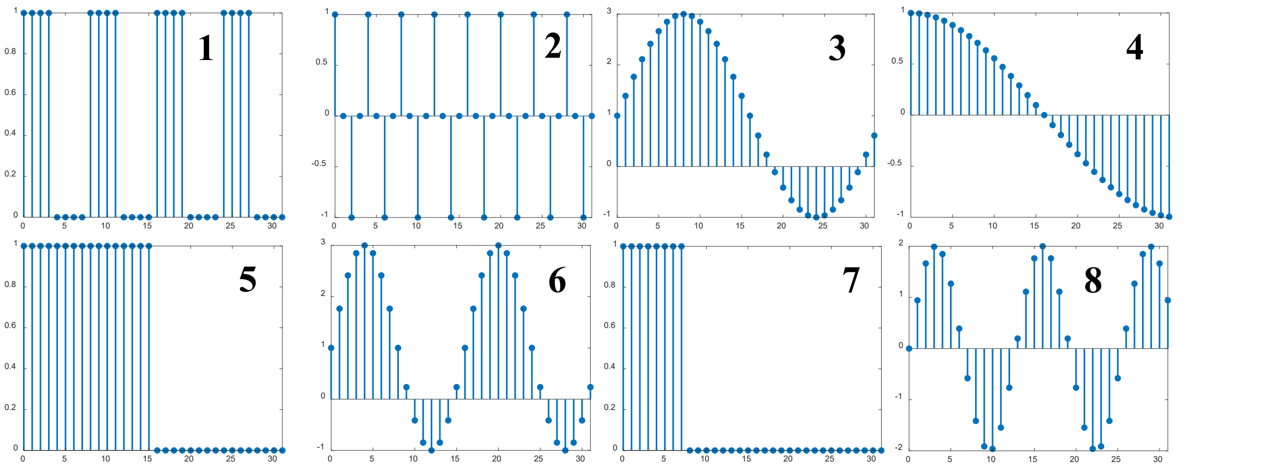
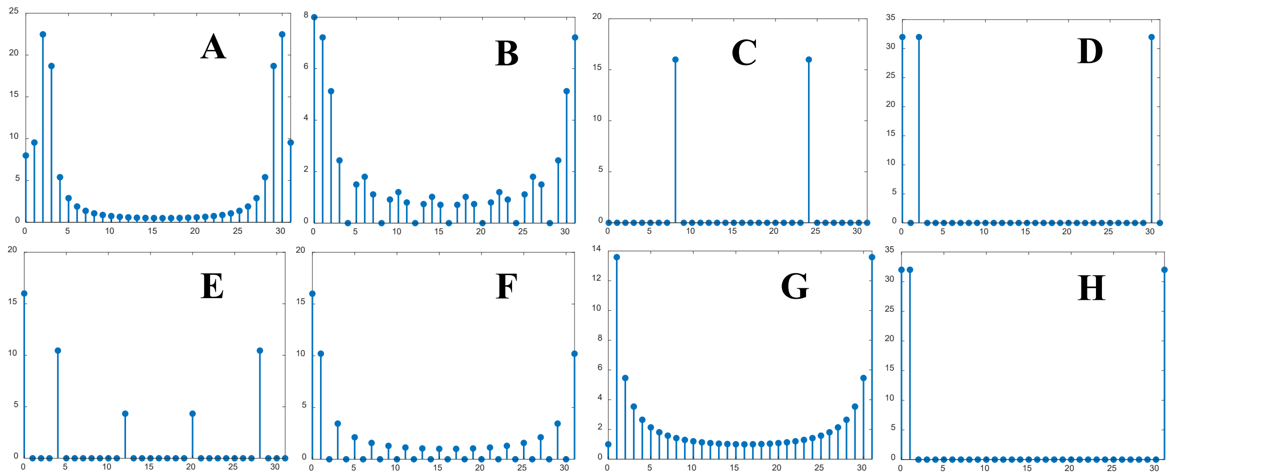
Please: Don't use red ink; start the solution of each problem on a ***new*** *sheet or side of paper; present all solutions thoroughly.*

Problem 1 Mapping task: 8 different discrete-time sequences $x[n]$ and their discrete FOURIER transforms $|X[k]|$ are given. Unfortunately, the relations are missing. Find the correct relations and give the answer in the form (1, C), if you think, that this is correct. *You are allowed to guess without proving your answer. But thinking might increase your success.* ☺

$x[n]$ time domain n



$|X[k]|$ spectrum k



10 points

Problem 2 Time-continuous signals and systems

2.1 Compute the continuous-time FOURIER transform $X(f)$ of the signal

$$x(t) = \frac{t}{1+t^2}$$

by applying the transform pair $\mathrm{j} \cdot \mathrm{sgn}(t) \cdot \mathrm{e}^{-|t|} \circ\!-\!\bullet \frac{4\pi \cdot f}{1+(2\pi \cdot f)^2}$ and useful transform properties.

2.2 Check, if the following systems are linear, time invariant, memoryless and causal. $x(t)$ is the input signal. $y_1(t)$ and $y_2(t)$ are output signals. a and b are real coefficients. *Short proof sufficient for saving time.*

1) $y_1(t) = a \cdot x(t) - b \cdot \frac{\mathrm{d}\, y_1(t)}{\mathrm{d}t}$ 2) $y_2(t) = a \cdot t^2 + x(t+3)$

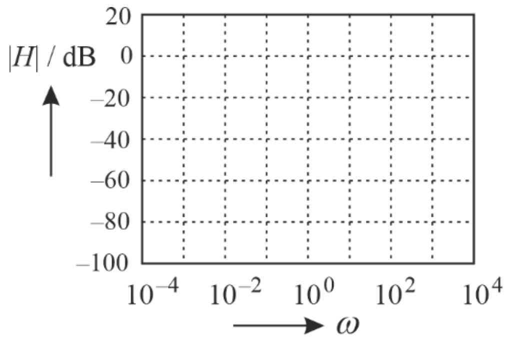
15 points

Problem 3 The system function of a time-continuous LTI system is known

$$H(s) = \frac{s^3}{(s+100)^2 \cdot (s+10)} .$$

3.1 Compute the asymptotic magnitude response for very low and very high frequencies.

3.2 Plot the magnitude response of the BODE diagram as a straight-line approximation.



15 points

Problem 4 A band-limited time-continuous signal with spectral components in the range $25\ \mathrm{Hz} \le |f| \le 35\ \mathrm{Hz}$ shall be sampled with ideal impulse train sampling.

4.1 Show that sampling with 24 Hz avoides aliasing errors.

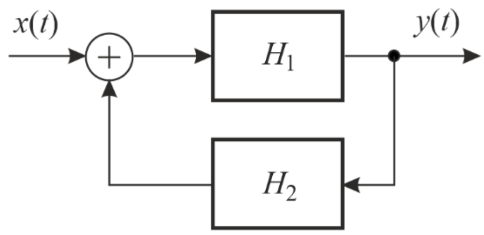
4.2 Determine the range $f_{\mathrm{s,min}} < f_{\mathrm{s}} < f_{\mathrm{s,max}}$ of possible sampling frequencies around 24 Hz for sampling without aliasing errors.

15 points

Problem 5 Two causal time-continuous LTI systems with

$$H_1(s) = \frac{s+1}{s^2 + 2 \cdot s + 5} \quad , \quad H_2(s) = 2$$

are connected as shown in the plot.



5.1 Derive the time-continuous relation between $x(t)$ and $y(t)$.

5.2 Discretize the system with forward EULER and compute the time-discrete relation between $x[n]$ and $y[n]$. Is the discretized system causal?

15 points

Problem 6 Consider the causal time-discrete LTI system function

$$H(z) = \frac{z}{20 \cdot z^2 - 4 \cdot z + 1}.$$

6.1 Compute the zeros and poles.

6.2 Is the system stable? Characterize the system with respect to its phase.

6.3 Determine the impulse response $h[n]$ and verify $h[0]$.

6.4 Plot the ARMA topology of the system.

15 points

Problem 7 The 4-point DFT of the sequence $x[n]$ is given by

$$X[k] = -2\,\mathrm{j} \cdot \sin\left(k \cdot \frac{\pi}{2}\right)$$

7.1 Compute the sequence $x[n]$ in a closed form.

7.2 Assume a measurement system with an observation time of 5 s. The sampling frequency of the AD converter amounts 2 kHz. Compute the resulting frequency resolution and the required number of samples per sequence. How can you improve the frequency resolution?

15 points

Good luck !