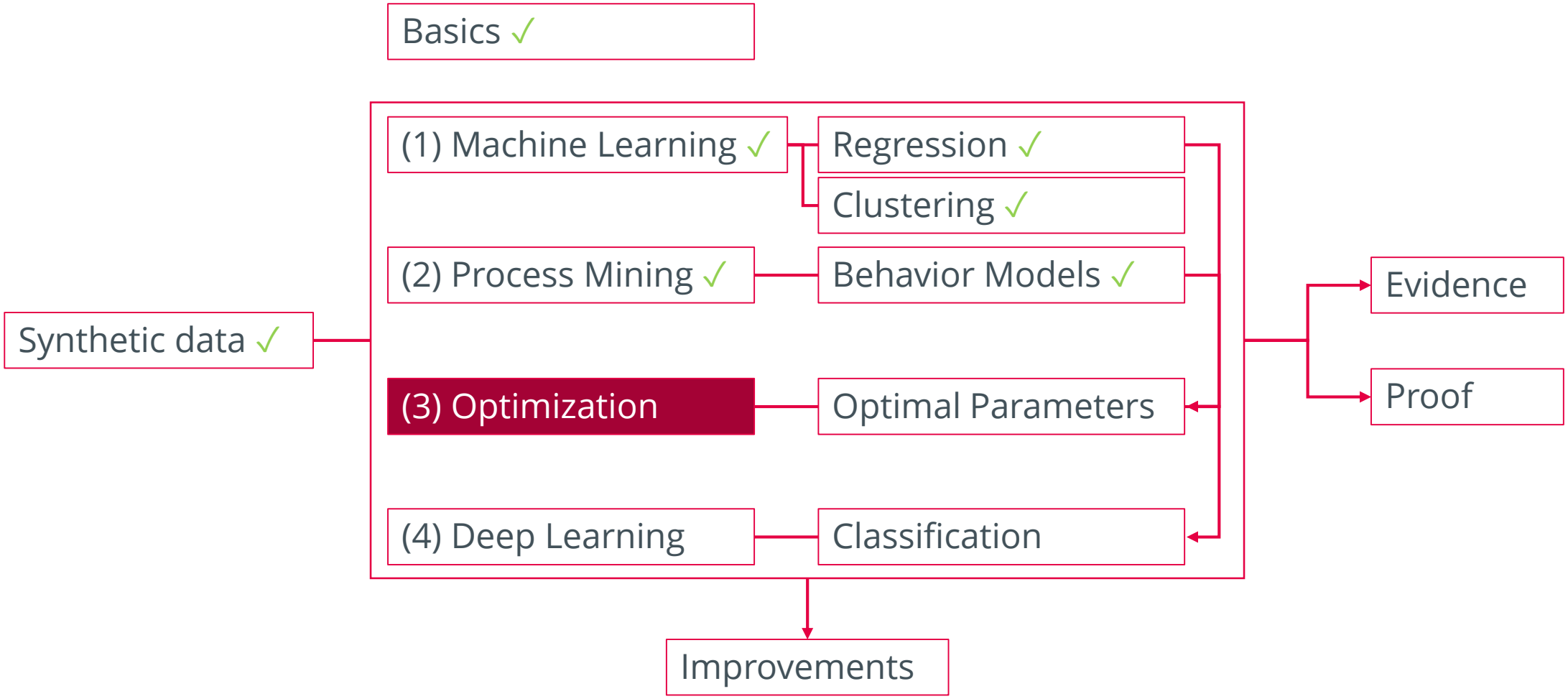
TECHNISCHE HOCHSCHULE
OSTWESTFALEN-LIPPE
UNIVERSITY OF
APPLIED SCIENCES
AND ARTS

Welcome

to Advanced Topics in Algorithms

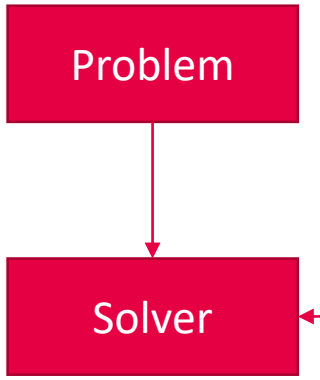
Summary: Advanced Topics in Algorithms



Classification of Optimization Problems

- **Mixed Integer Nonlinear Programming**
- Mixed Integer Linear Programming
- Nonlinear Programming
- Linear Programming
- **Knapsack Problem**
- **Multiple-Choice Knapsack Problem**

Complexity Class	MINLP	MILP	NLP	LP	KP	MCKP
NP-hard	+	+	+	-	-	+
NP-complete	-	-	-	-	+	-
P	-	-	-	+	-	-



Techniques
- Simplex
- Branch-and-bound
- Cutting Planes
- Outer Approximation
- Primal Heuristics
- Preprocessing

- AlphaECP
- ANTIGONE
- BARON
- BONMIN
- DICOPT
- SCIP

Additional information:
https://www.mcs.anl.gov/papers/P3060-1112.pdf

NP-hard vs NP-complete

The set of class **NP-hard** describes problems that are difficult to solve, because no polynomial time algorithm is known, and it is not possible to verify if a solution is a feasible solution in polynomial time.

The set of class **NP-complete** describes problems that are in the set of class NP-hard and in the set of class NP. That means that these problems are difficult to solve, because no polynomial time algorithm is known, but it is possible to verify if a solution is a feasible solution in polynomial time.

Knapsack Problem (1/2)

Which items should be chosen to maximize the amount of money while keeping the overall weight under or equal to 6 kg?

Knapsack Problem (2/2)

Knapsack Problem (KP)

maximize: $\sum_{i=1}^{k} v_i \, b_i$

s.t.: $\sum_{i=1}^{k} w_i \, b_i \leq obj,$

$b_i \in \{0,1\}, i = 1, \ldots, k$

Pyomo is a Python-based, open-source optimization modeling language with a diverse set of optimization capabilities.

http://www.pyomo.org/

```
10
11  model = ConcreteModel()
12  model.elements = Set(initialize = v.keys())
13  model.x = Var(model.elements, within = Binary)
14  model.value = Objective(expr=sum(v[i]*model.x[i] for i in model.elements), sense=maximize)
15  model.weight = Constraint(expr=sum(w[i]*model.x[i] for i in model.elements) <= limit)
16
```

- https://www.gams.com/products/gams/gams-language/
- https://zimpl.zib.de/
- …

Multiple-Choice Knapsack Problem

Multiple-Choice Knapsack Problem (MCKP)

$$\text{maximize:} \quad \sum_{i=1}^{k} \sum_{j \in N_i} v_{ij}\, b_{ij}$$

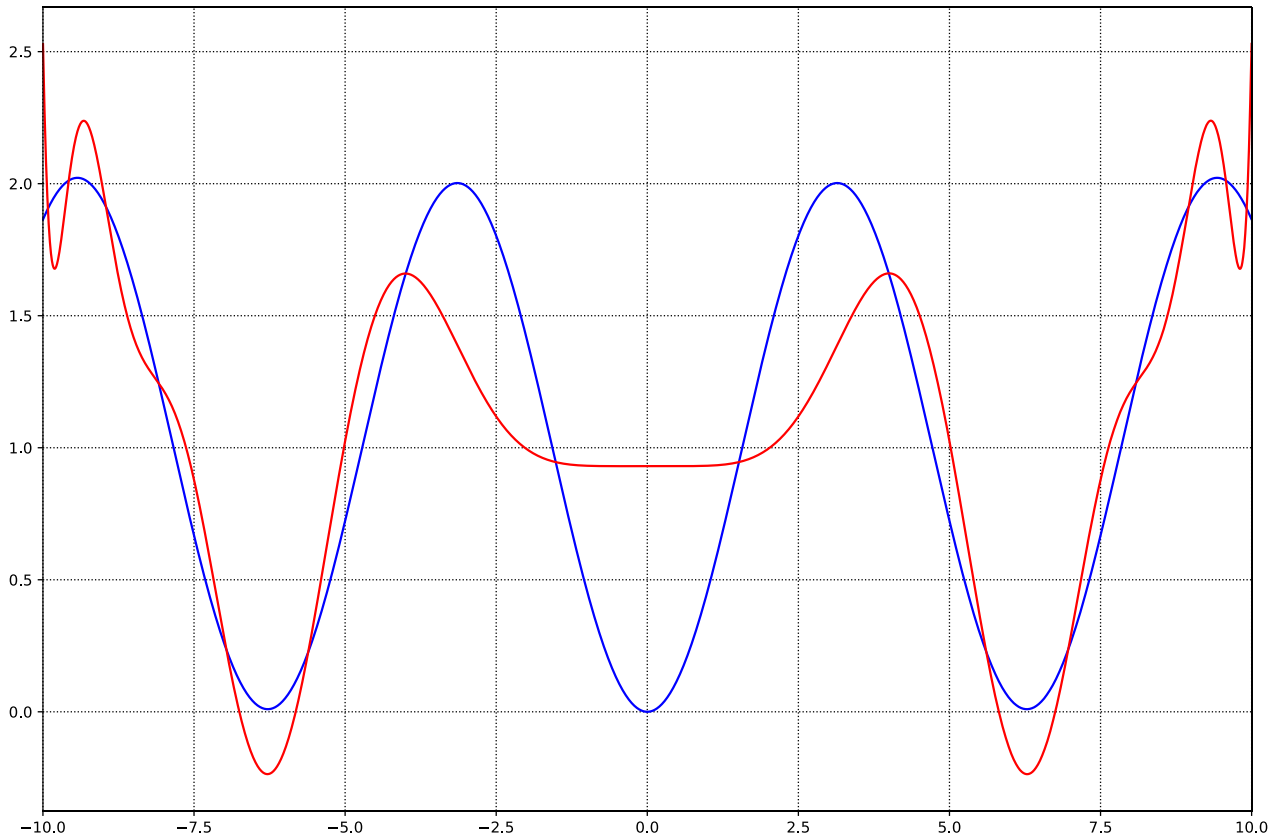
$$\text{s.t.:} \quad \sum_{i=1}^{k} \sum_{j \in N_i} w_{ij}\, b_{ij} \leq obj,$$

$$\sum_{j \in N_i} b_{ij} = 1, i = 1, \ldots, k,$$

$$b_{ij} \in \{0,1\}, i = 1, \ldots, k, j \in N_i$$

Nonlinear Programming

Energy Consumption



Symbolic Representation

$$-1.4 \cdot 10^{-7} x_0^{12} + 6.5 \cdot 10^{-6} x_0^{10} + 2.0 \cdot 10^{-8} x_0^9 - 0.0001587 x_0^8 - 4.2 \cdot 10^{-7} x_0^7 + 0.00150314 x_0^6 + 3.51 \cdot 10^{-6} x_0^5 + 0.00022478 x_0^4 + 6.6 \cdot 10^{-7} x_0^3 + 1.824 \cdot 10^{-5} x_0^2 + 1.589 \cdot 10^{-5} x_0 + 0.9302$$

Mixed Integer Nonlinear Programming

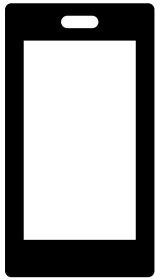
Knapsack?

Goal

Original image (96,615 colors)

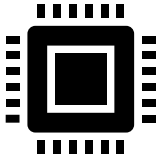
Quantized image (16 colors, K-Means)

Smartphone



?

CPU



Cloud

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Thank you!