OWL University of Applied Sciences **Prof. Dr. Uwe Meier**
Department of Electrical Engineering and Computer Science

Written Exam: Discrete Signals and Systems (DSS)
Degree Programmes: Information Technology (M. Sc.), Elektrotechnik (M. Sc.)
2019-03-29, 120 min, 100 points available → no notebooks, no books

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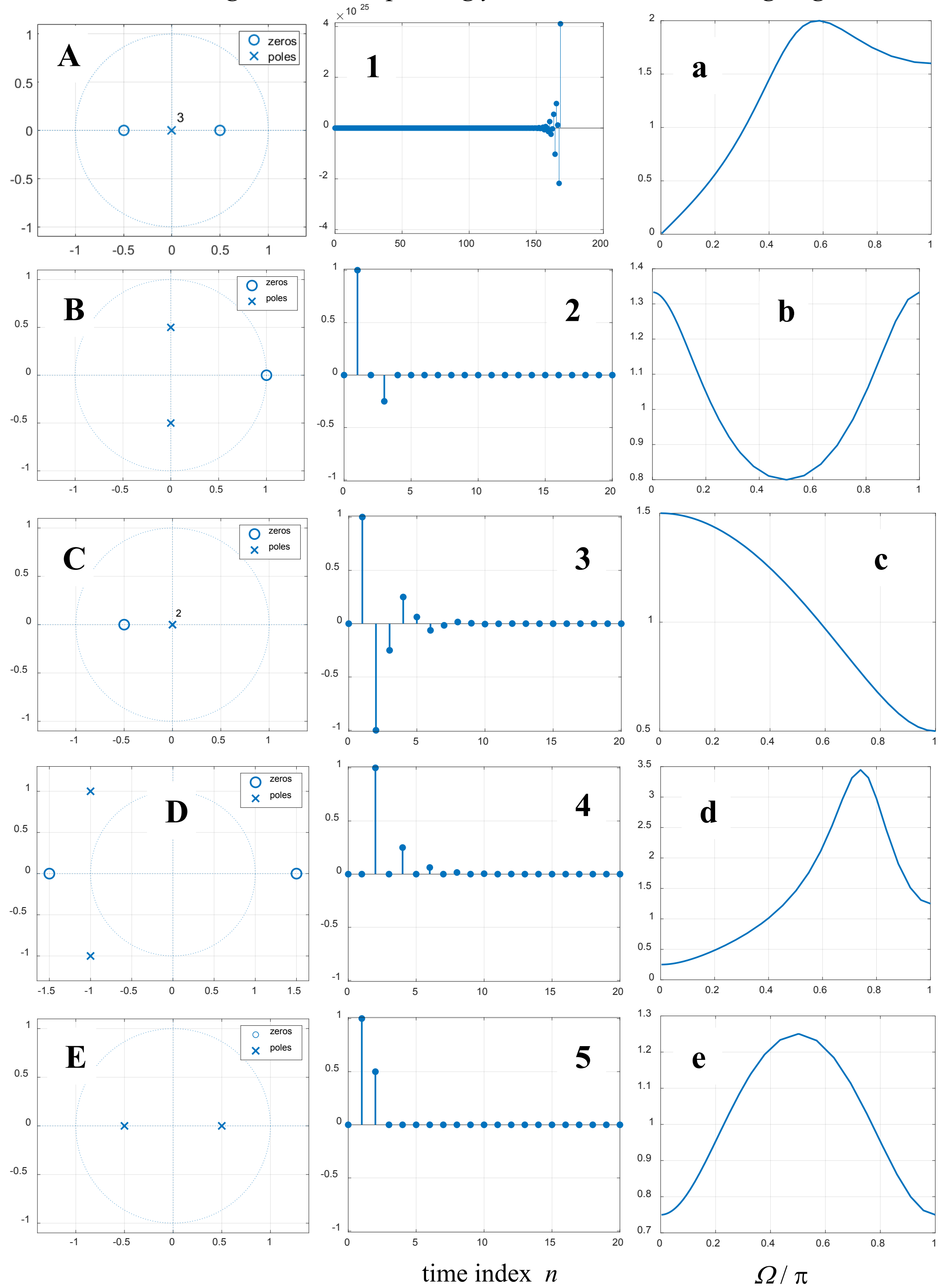
Please: Don't use red ink; start the solution of each problem on a ***new*** *sheet or side of paper; present all solutions thoroughly.*

Problem 1 Mapping task: 5 different discrete-time systems shall be considered. Unfortunately, the given relation of the zero-pole plots to the impulse responses and to the magnitude responses are erroneous. Find the correct relation and give the answer in the form (A, 2, c), if you think, that this is a correct constellation. *You are allowed to guess without proving your answer. But thinking might increase your success.* ☺

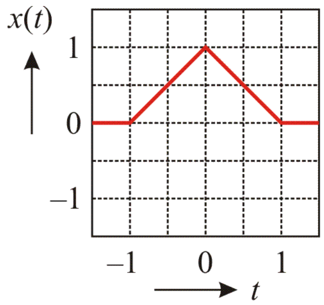


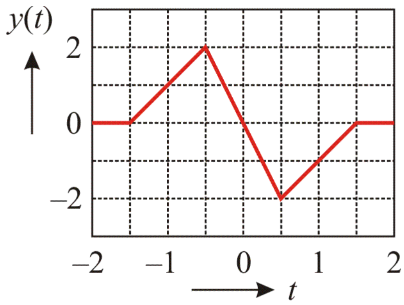
15 points

Problem 2 Time-continous signals

2.1 The spectrum of each of the signals $x_\mathrm{a}(t)$ and $x_\mathrm{b}(t)$ is symmetric and band-limited within the range $-a/2 \le f \le a/2$ and $-b/2 \le f \le b/2$, respectively. Compute the bandwidth B of the signals $y_1(t) = x_\mathrm{a}(t) + x_\mathrm{b}(t)$ and $y_2(t) = x_\mathrm{a}(t) * x_\mathrm{b}(t)$.

2.2 Compute the continuous-time FOURIER transform $X(f)$ of $x(t) = \mathrm{trian}(t)$. Next, write $y(t)$ as a superposition of $\mathrm{trian}(t)$ and compute $Y(f)$. Simplify the result.

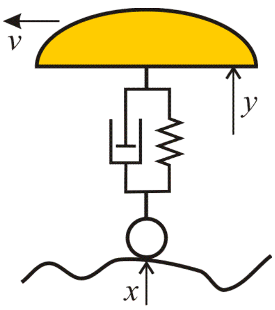




15 points

Problem 3 A car with vertical position y moves with constant velocity v along a street with height profile x. This time-continous system is described by the differential equation

$$\frac{\mathrm{d}^2 y}{\mathrm{d}t^2} = -2\cdot(y-x) - 3\cdot\frac{\mathrm{d}(y-x)}{\mathrm{d}t}\ .$$



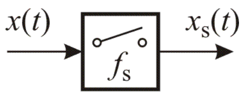
3.1 Compute the system function $H(s) = Y(s)/X(s)$.

3.2 Determine the system response $y(t)$ for $x(t) = 2\cdot u(t)$, i. e. the car moves across a stone. Assume $x = y = 0$ for $t < 0$. Compute the initial value $y(0)$ and the stationary value $y(t \to \infty)$.

3.3 When the car moves across a sinusoidal street profile with amplitude $\hat{x}$ it will oscillate sinusoidally. Determine the amplitude $\hat{y}$.

20 points

Problem 4 The signal $x(t) = \sin(2\pi\cdot 3\ \mathrm{Hz}\cdot t)\cdot\cos(2\pi\cdot 2\ \mathrm{Hz}\cdot t)$ is ideal sampled with $f_\mathrm{s} = 6\ \mathrm{Hz}$. Useful relation: $\sin x\cdot\cos y = 0.5\cdot\sin(x-y) + 0.5\cdot\sin(x+y)$.



4.1 Compute and plot the FOURIER transform $X(f)$ of the unsampled signal $x(t)$.

4.2 Compute and plot the FOURIER transform $X_\mathrm{s}(f)$ of the sampled signal $x_\mathrm{s}(t)$ for $|f| \le 10\ \mathrm{Hz}$.

15 points

Problem 5 The discrete FOURIER transform of a real discrete sequence $x[n]$ of length N is denoted with $X[k]$. Assume that the samples of $X[k]$ are known for $0 \le k \le M < N$.

5.1 Determine M in order to be able to compute the missing samples of $X[k]$ for $(M+1) \le k \le (N-1)$. Consider even and odd N separately.

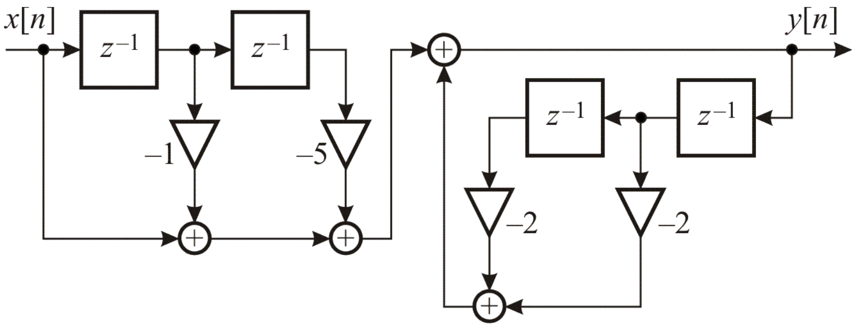
5.2 Derive an algorithm for computing the real and imaginary parts of the missing samples.

5.3 1st example with $N = 6$: $X[0] = 0$, $X[1] = 1 + \mathrm{j}$, $X[2] = 2$, $X[3] = 1$. Compute the missing samples.

5.4 2nd example with $N = 6$: $\{X[k]\} = \{0,\ \mathrm{j},\ 0,\ 1,\ 0,\ -\mathrm{j}\}$. Compute a simplified functional expression for $x[n]$. Don't compute numbers!

15 points

Problem 6 Consider the given time-discrete filter.



6.1 Is it an FIR or an IIR filter?

6.2 Determine the difference equation in the time domain.

6.3 Determine the system function $H(z) = Y(z) / X(z)$.

6.4 Is the system stable?

6.5 Compute the magnitude response for $\Omega = 0$ in dB.

6.6 Plot the magnitude response for $-\pi \le \Omega \le \pi$ approximately.

6.7 Plot a simplified block diagram with a canonic structure.

20 points

Good luck !