Probability and Statistics (PAS)

1.) The following data resulted from independent measurements of the melting point of lead:

$330.0~^{\circ}\mathrm{C}$	$322.0~^{\circ}\mathrm{C}$	$345.0~^{\circ}\mathrm{C}$	$328.6~^{\circ}\mathrm{C}$	$331.0~^{\circ}\mathrm{C}$	$342.0~^{\circ}\mathrm{C}$
$342.4~^{\circ}{\rm C}$	$340.4~^{\circ}{\rm C}$	$329.7~^{\circ}{\rm C}$	$334.0~^{\circ}{\rm C}$	$326.5~^{\circ}\mathrm{C}$	$325.8~^{\circ}{\rm C}$

Assuming that the measurements can be regarded as constituting a normal sample whose mean is the true melting point of lead, determine 95 percent and 99 percent two-sided confidence intervals for the melting point.

- 2.) Suppose the lifetime of LEDs are exponentially distributed with mean  $\mu$ . If the average lifetime of 12 LEDs is 11,200 hours, determine 95 percent two-sided and upper and lower one-sided confidence intervals for  $\mu$ .
- 3.) An airline is interested in determining the proportion of its customers who are flying for reasons of business. How large should the sample be if the airline wants to be 99.0 percent sure that its estimate does not deviate from the actual value by more than 0.01?
- 4.) The capacities (in ampere-hours) of 10 batteries were recorded as follows:

Assuming that a battery's capacity is normally distributed, determine

- (i) a 95% two-sided confidence interval for the mean,
- (ii) a 95% one-sided upper confidence interval for the mean, and
- (ii) a 95% two-sided confidence interval for the standard deviation

of the underlying distribution.

5.) The following table contains 10 data pairs relating the yield of a laboratory experiment  $y_i$  to the temperature  $x_i$  at which the experiment was run.

i	1	2	3	4	5	6	7	8	9	10
$x_i$	100	110	120	130	140	150	160	170	180	190
$y_i$	45	52	54	63	62	68	75	76	92	88

- (i) Determine the estimators for the regression parameters and draw a scatter diagram for the sampled data above and the estimated regression line.
- (ii) Determine a 95% prediction interval for the yield of a single experiment which will be run at the temperature  $x_0 = 155$ .