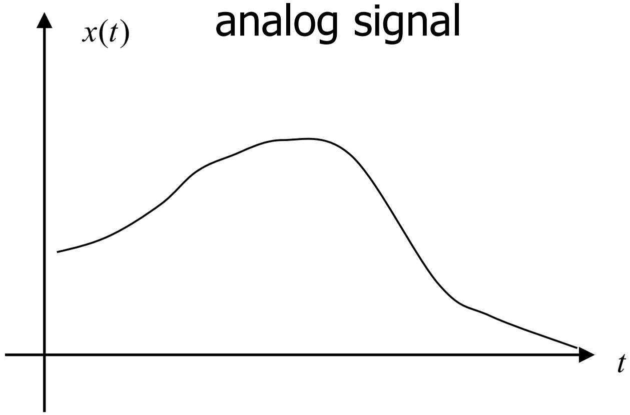
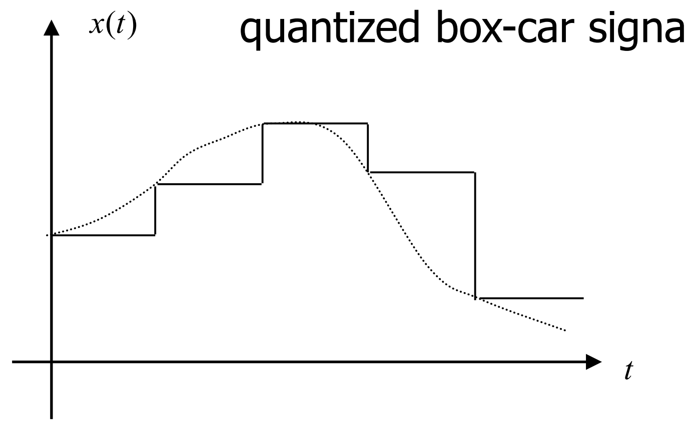
Chapter 4

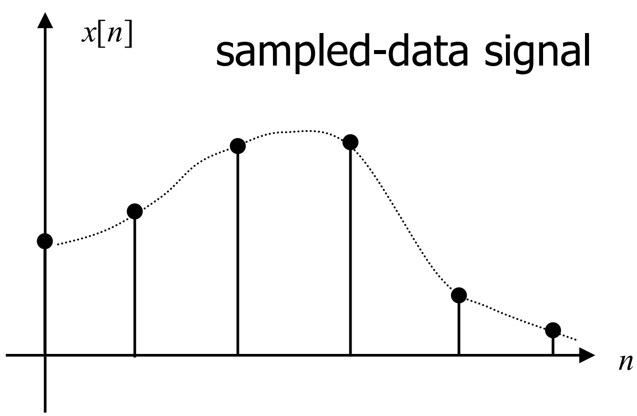
Discrete-Time Signals

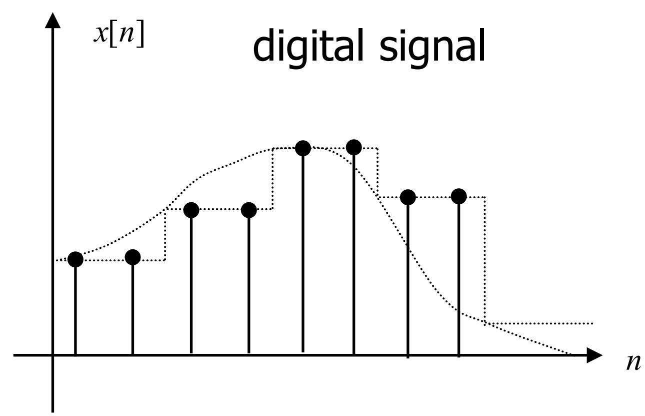
4.1 Overview





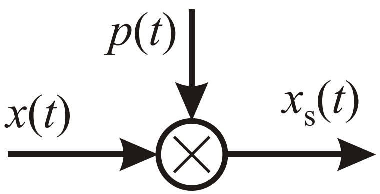
Discrete-Time Signals are suited for computer-based processing





4.2 Sampling and Reconstruction

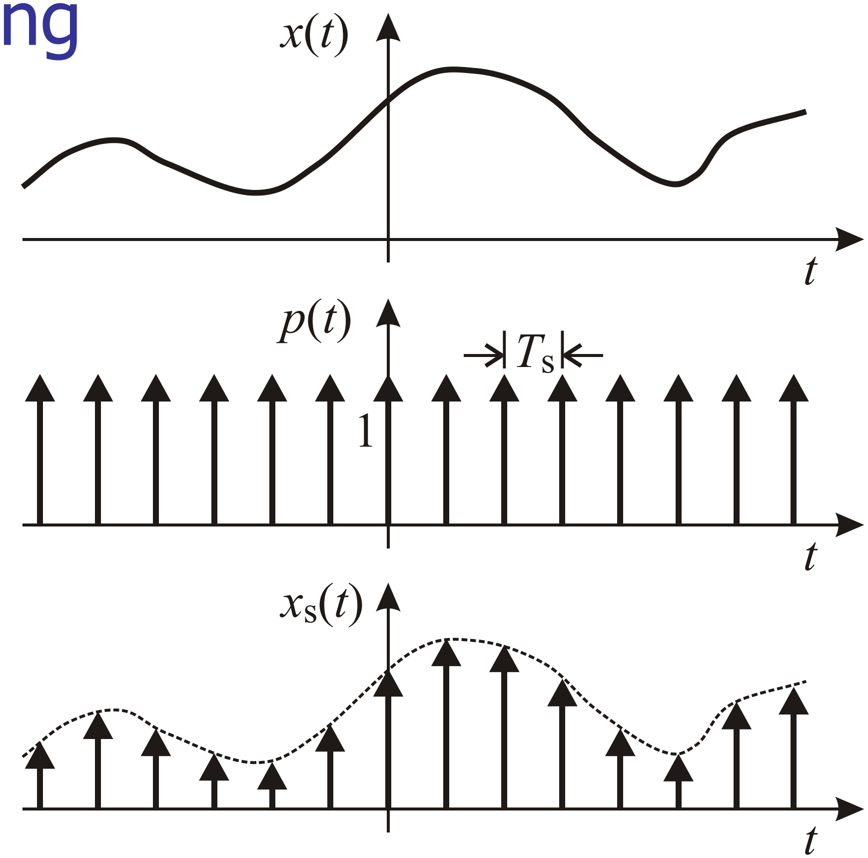
4.2.1 Impulse-Train Sampling



$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

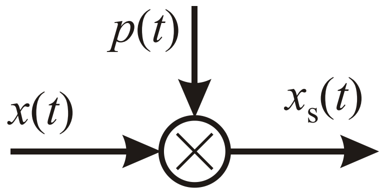
$$x_s(t) = x(t) \cdot p(t)$$

$$= \sum_{n=-\infty}^{\infty} x(nT_s) \cdot \delta(t - nT_s)$$



T_S: sampling period

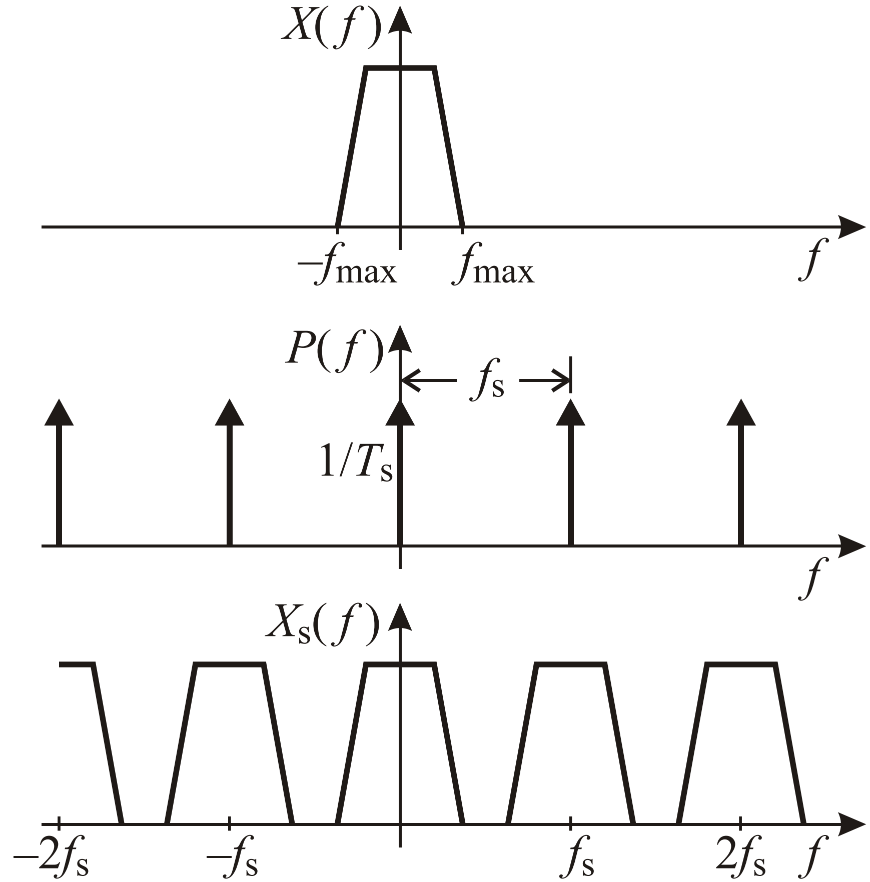
- **Frequency domain**



$$P(f) = \frac{1}{T_s} \sum_{k=-\infty}^{\infty} \delta(f - k \cdot f_s)$$

$$X_s(f) = X(f) * P(f)$$

$$= \frac{1}{T_s} \sum_{k=-\infty}^{\infty} X(f - k \cdot f_s)$$



$f_s = 1/T_s$: sampling frequency

- **Sampling theorem**

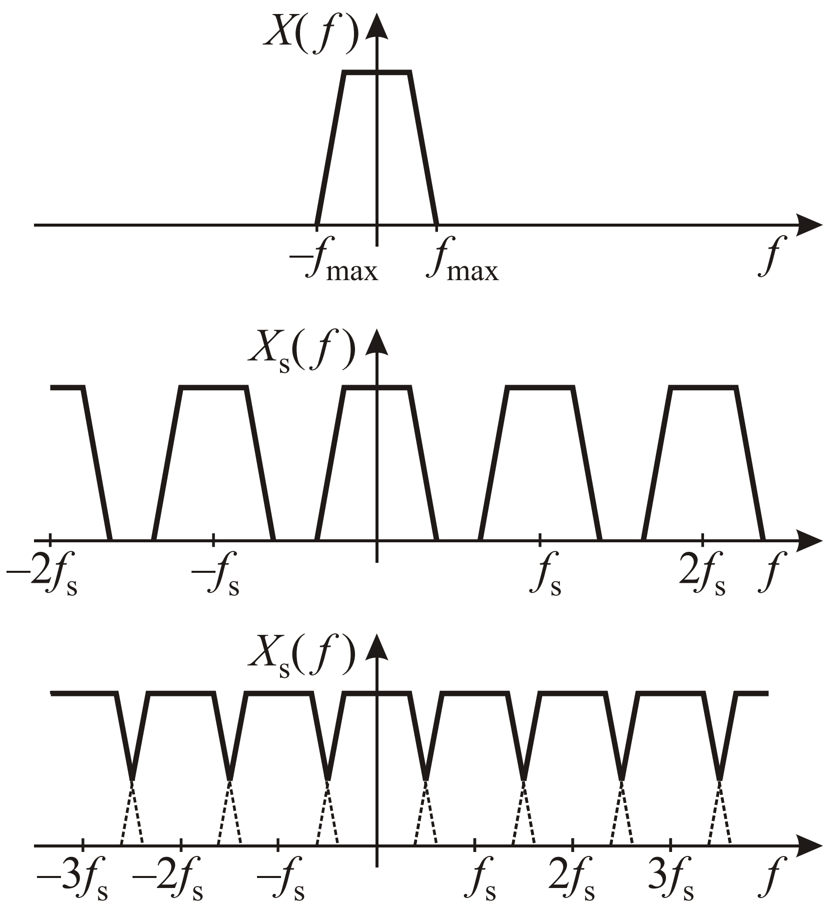
  If $x(t)$ is a band-limited signal with $X(\omega) = 0$ for $|f| > f_{\mathrm{max}}$, then $x(t)$ is uniquely determined by its samples $x(nT_{\mathrm{S}})$, if $f_{\mathrm{S}} > 2f_{\mathrm{max}}$, where $f_{\mathrm{S}} = 1/T_{\mathrm{S}}$.

- **NYQUIST rate**

  The frequency $2f_{\mathrm{max}}$ is referred to as NYQUIST rate. This is the minimal required sampling rate.

HARRY NYQUIST
1889 - 1976

- **Undersampling vs. oversampling**

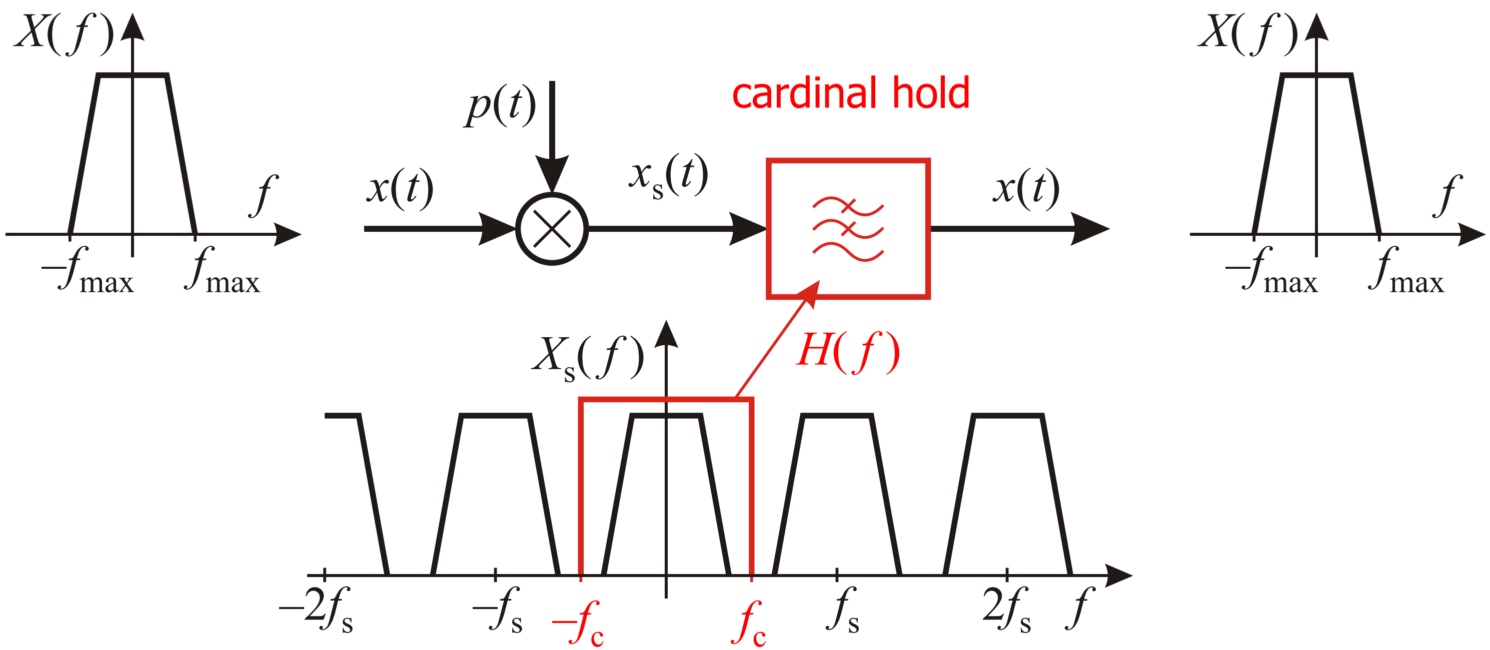


Oversampling: $f_S > 2f_{\max}$

Undersampling: $f_S < 2f_{\max}$
→ **aliasing error**

- **Reconstruction**

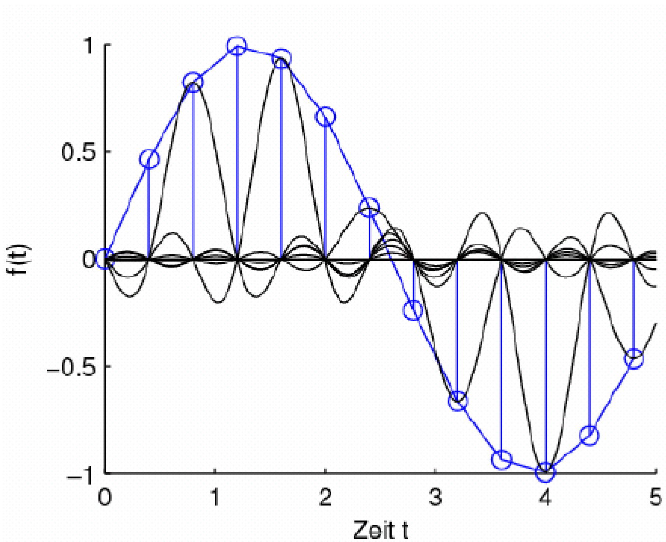
  Given an oversampled signal with $f_S > 2f_{max}$ the original signal $x(t)$ can be reconstructed by an ideal low-pass filter with gain T_S and cutoff frequency greater than f_{max} and less than $f_S - f_{max}$.

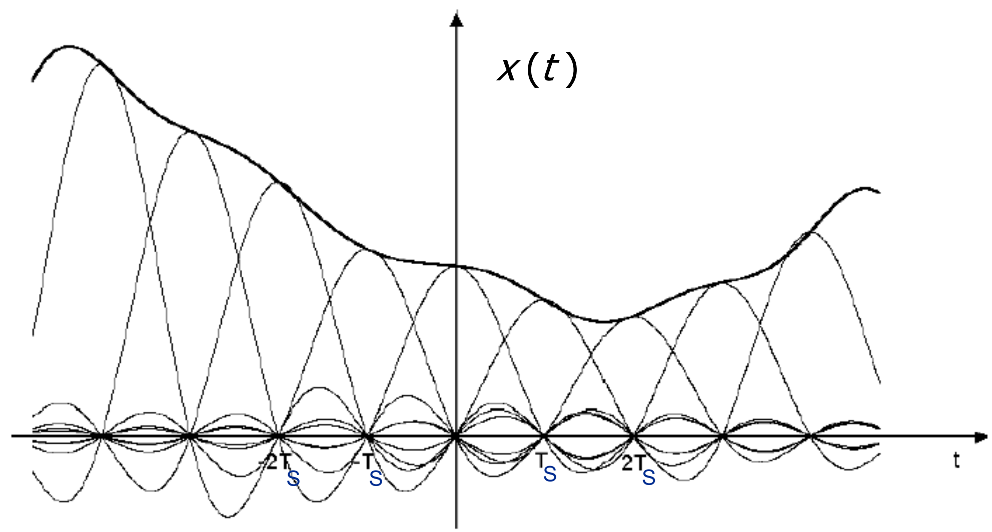


Reconstruction

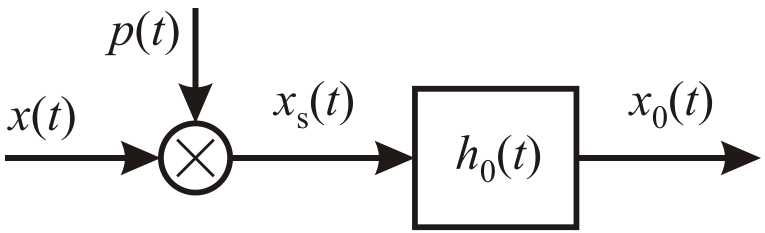
$$X(f) = X_\mathrm{s}(f) \cdot H(f) \qquad\qquad H(f) = T_\mathrm{s} \cdot \mathrm{rect}\left(\frac{f}{f_\mathrm{s}}\right) \quad \bullet\!-\!\circ \quad h(t) = \mathrm{si}\left(\pi f_\mathrm{s} t\right)$$

$$x(t) = x_\mathrm{s}(t) * h(t) = \sum_{n=-\infty}^{\infty} x(nT_\mathrm{s}) \cdot \mathrm{si}\left[\pi f_\mathrm{s}(t - nT_\mathrm{s})\right]$$





4.2.2 Sampling with a Zero-Order Hold

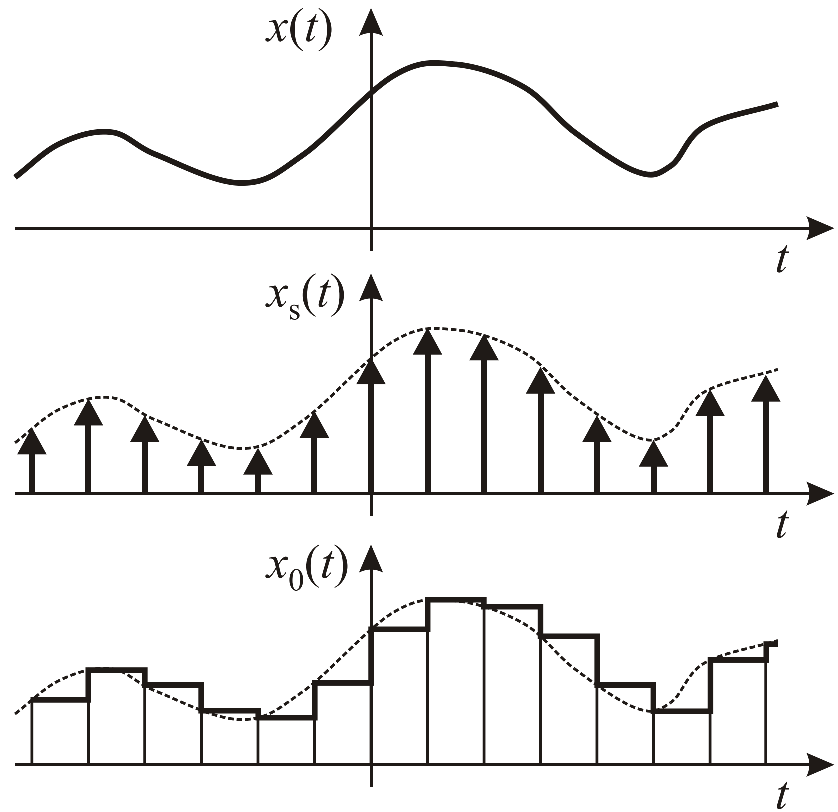


box-car circuit, sample- and-hold circuit, data clamp

$$h_0(t) = \mathrm{rect}\left(\frac{t - T_s/2}{T_s}\right)$$

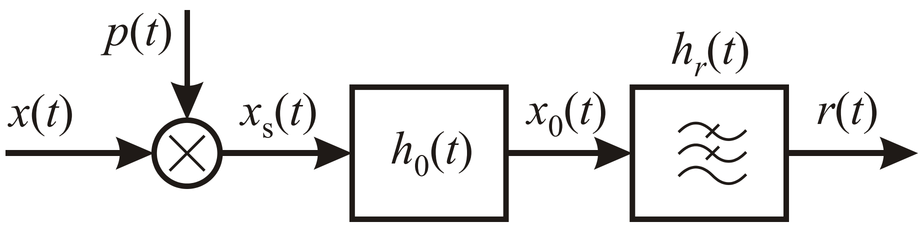
$$x_0(t) = x_s(t) * h_0(t) =$$

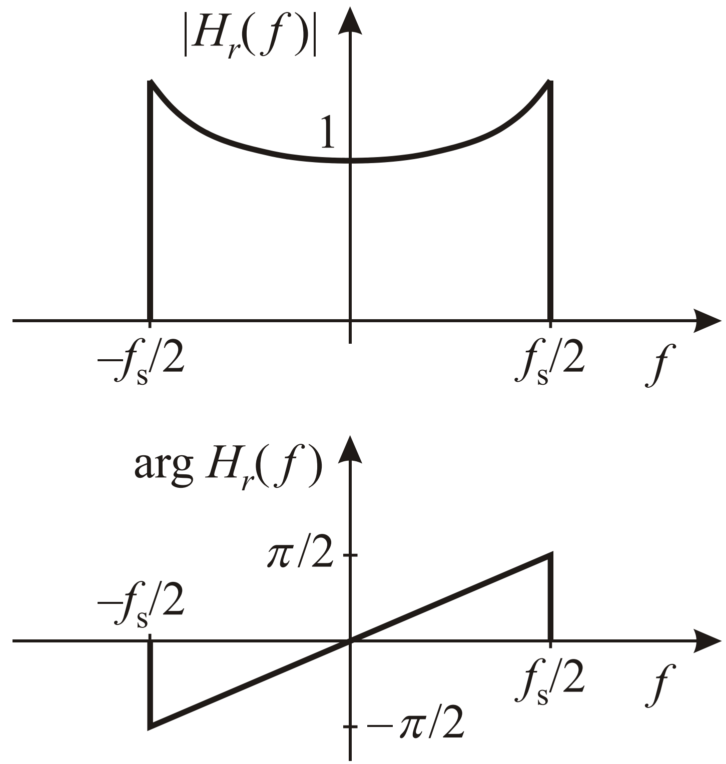
$$= \sum_{n=-\infty}^{\infty} x(nT_s) \cdot h_0(t - nT_s)$$



quantized box-car signal

- **Reconstruction**

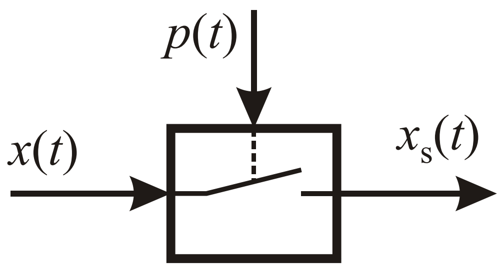




$$H_r(f) = \frac{H(f)\cdot \mathrm{e}^{\mathrm{j}\omega T_s/2}}{T_s \cdot \mathrm{si}\left(\pi f T_s\right)}$$

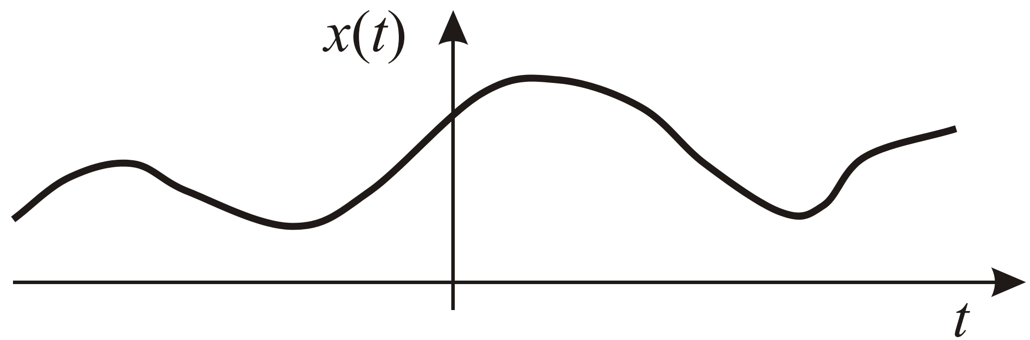
In many situations the output $x_0(t)$ of the zero-order hold can be considered an adequate approximation of the original signal $x(t)$ by itself.

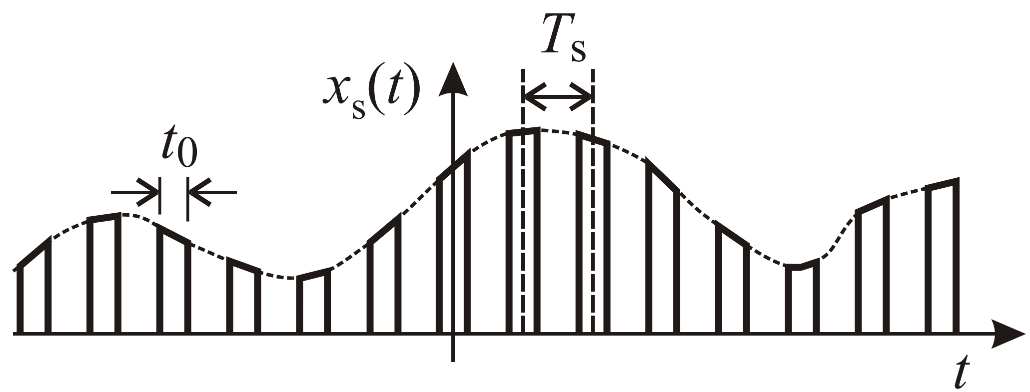
4.2.3 Sampling with a Linear Gate



$$x_s(t) = x(t) \cdot p(t)$$

$$p(t) = \sum_{n=-\infty}^{\infty} \mathrm{rect}\left(\frac{t - nT_s}{t_0}\right)$$





4.3 Signal Notation and Properties

- **Discrete signal**

  $x[n] = x_{\text{cont}}(t = n \cdot T_{\text{s}})$ T_S: sampling period; n: integer

  $$\{x[n]\} = \{\ldots, x[-1], x[0], x[1], x[2], \ldots\} = \{\ldots, 0.9, 0.1, 2.2, 3.7, \ldots\}$$

  - sequence of data
  - MATLAB: $n = 1, \ldots, N$

  $$\vec{x} = \left[x[0], x[1], x[2], \ldots, x[N-1]\right]^{\mathrm{T}}$$

  - vector representation

- **Digital signal**

  $$\{\hat{x}[n]\} = \{\ldots, \hat{x}[-1], \hat{x}[0], \hat{x}[1], \hat{x}[2], \ldots\} = \{\ldots, 1, 0, 2, 4, \ldots\}$$

- **Finite length sequence**

$$n_1 \le n \le n_2 \;\; ; \;\; N = n_2 - n_1 + 1 \;\; ; \;\; N: \text{length, duration}$$

- **Causal signal**

$$x[n] = 0 \;\text{ for }\; n < N_1 \;\text{ with }\; N_1 \ge 0$$

- **Periodic sequence**

$$x[n] = x[n \pm k \cdot N] \;\; ; \;\; k = 1, 2, 3, \ldots \;\; ; \;\; N: \text{period}$$

- **Signal energy**

$$E_\infty = \lim_{N\to\infty} \sum_{n=-N}^{N} |x[n]|^2 = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

- **Signal power**

$$P_\infty = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2$$

Energy signal: $E_\infty < \infty$, $P_\infty = 0$
Power signal: $P_\infty < \infty$, $E_\infty \to \infty$

- **Strength** of a signal: The signal strength is given by its norm.

$$\|x\|_p = \left(\sum_{n=-\infty}^{\infty} |x[n]|^p \right)^{1/p} \quad ; \quad p = 1, 2, \ldots$$

Symmetry Relations

- Conjugate symmetric sequence: $x_{\mathrm{cs}}[n] = x_{\mathrm{cs}}{}^*[-n]$
  - Real sequence → even: $x_{\mathrm{e}}[n] = x_{\mathrm{e}}[-n]$
- Conjugate antisymmetric sequence: $x_{\mathrm{ca}}[n] = -x_{\mathrm{ca}}{}^*[-n]$
  - Real sequence → odd: $x_{\mathrm{o}}[n] = -x_{\mathrm{o}}[-n]$

- Any complex sequence $x[n]$ can be composed as:

$$x[n] = x_{\mathrm{cs}}[n] + x_{\mathrm{ca}}[n]$$

$$x_{\mathrm{cs}}[n] = \frac{x[n] + x^*[-n]}{2} \quad , \quad x_{\mathrm{ca}}[n] = \frac{x[n] - x^*[-n]}{2}$$

$$x_{\mathrm{cs}}[0] = \mathrm{real} \;\; ; \quad x_{\mathrm{ca}}[0] = \mathrm{imaginary} \;\; ; \quad x_{\mathrm{o}}[0] = 0$$

4.4 Test Sequences

- **Unit sample sequence** **Unit step sequence**

$$\delta[n] = \begin{cases} 1 \ , & n = 0 \\ 0 \ , & n \neq 0 \end{cases} \qquad\qquad u[n] = \begin{cases} 1 \ , & n \geq 0 \\ 0 \ , & n < 0 \end{cases}$$

- **Rectangular sequence**

$$\mathrm{rect}_N[n] = \begin{cases} 1 & \text{for } |n| \leq (N-1)/2 \\ 0 & \text{elsewhere} \end{cases} \quad ; \ N \text{ odd}$$

- **Sinusoidal sequence**

$$x[n] = \cos(2\pi f \cdot n \cdot T_\mathrm{s}) = \cos(n \cdot \Omega)$$

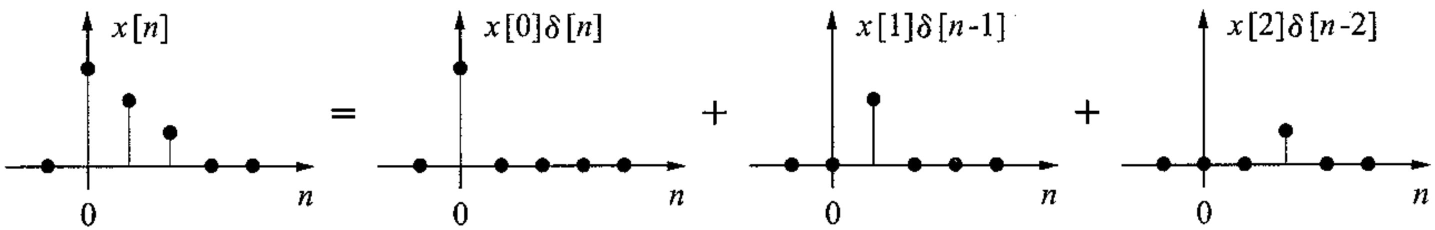
$$\Omega = 2\pi f \cdot T_\mathrm{s} = 2\pi f \,/\, f_\mathrm{s}$$

Ω: normalized angular frequency

4.5 Signal Operations
4.5.1 Elementary Operations

- **Superposition**

$$x[n] = \sum_{i=-\infty}^{\infty} x[i]\delta[n-i]$$



- **Sifting equation**

$$x[n]*\delta[n] = x[n] \;\; ; \quad x[n]*\delta[n-k] = x[n-k]$$

4.5.2 Convolution

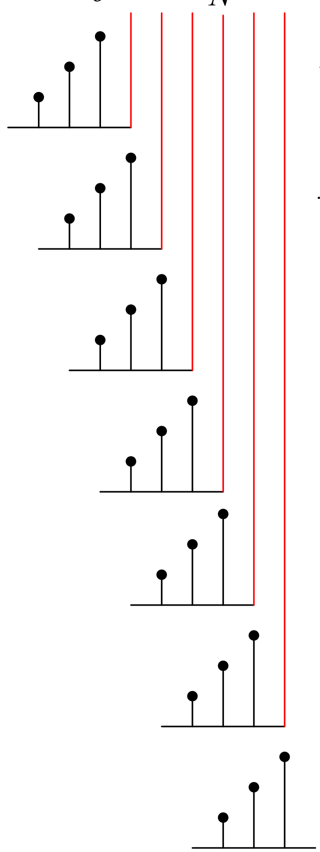
- **Generic**

$$y[n] = \sum_{k=-\infty}^{\infty} x_1[k] x_2[n-k] = x_1[n] * x_2[n]$$

- **Convolution of periodic sequences**

$$y[n] = \frac{1}{N} \sum_{k=0}^{N-1} x_1[k] \cdot x_2[n-k] = \frac{1}{N} \sum_{k=0}^{N-1} x_2[k] \cdot x_1[n-k]$$

$$y[m] = x[m] * h[m]$$



$y[0] = x[0] \cdot h[0] = 1 \cdot 3 = 3$

$y[1] = x[0] \cdot h[1] + x[1] \cdot h[0] = 1 \cdot 2 + 2 \cdot 3 = 8$

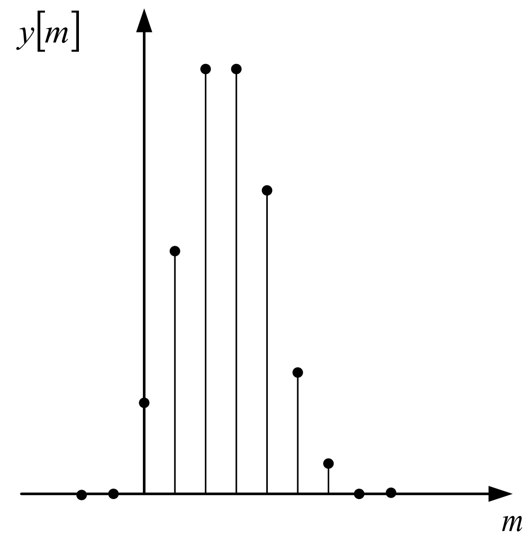
$y[2] = x[0] \cdot h[2] + x[1] \cdot h[1] + x[2] \cdot h[0] = 1 \cdot 1 + 2 \cdot 2 + 3 \cdot 3 = 14$

$y[3] = x[1] \cdot h[2] + x[2] \cdot h[1] + x[3] \cdot h[0] = 2 \cdot 1 + 3 \cdot 2 + 2 \cdot 3 = 14$

$y[4] = x[2] \cdot h[2] + x[3] \cdot h[1] + x[4] \cdot h[0] = 3 \cdot 1 + 2 \cdot 2 + 1 \cdot 3 = 10$

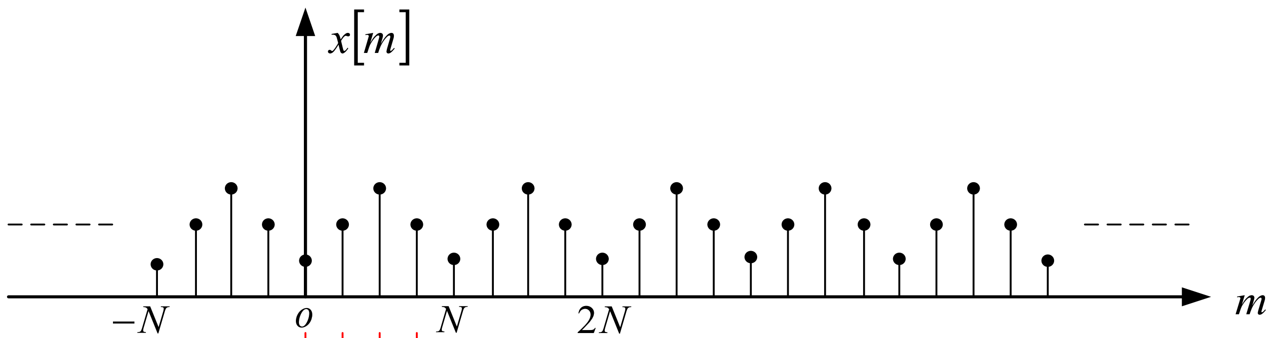
$y[5] = x[3] \cdot h[2] + x[4] \cdot h[1] + x[5] \cdot h[0] = 2 \cdot 1 + 1 \cdot 2 = 4$

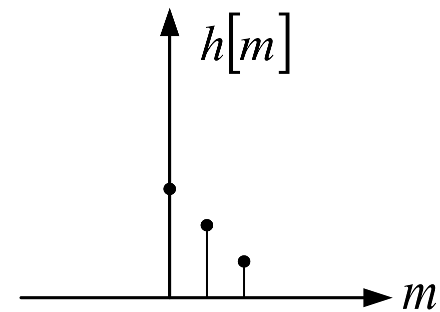
$y[6] = x[4] \cdot h[2] + x[5] \cdot h[1] + x[6] \cdot h[0] = 1 \cdot 1 = 1$

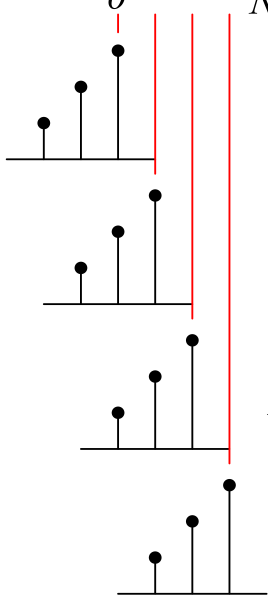


$$\text{Length} = N_1 + N_2 - 1$$

$$y[m] = x[m] * h[m]$$







$$N \cdot y[0] = x[0] \cdot h[0] + x[-1] \cdot h[1] + x[-2] \cdot h[2]$$

$$N \cdot y[1] = x[1] \cdot h[0] + x[0] \cdot h[1] + x[-1] \cdot h[2]$$

$$N \cdot y[2] = x[2] \cdot h[0] + x[1] \cdot h[1] + x[0] \cdot h[2]$$

$$N \cdot y[3] = x[3] \cdot h[0] + x[2] \cdot h[1] + x[2] \cdot h[2]$$

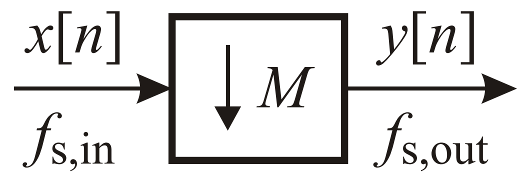
$y[m]$ is periodic with N.

4.5.3 Sampling Rate Alteration

- **Down-sampler**, sampling rate compressor

$$y[n] = x[n \cdot M] \;;\quad M: \text{positive integer}$$

$$f_{\text{s,out}} = f_{\text{s,in}} / M$$

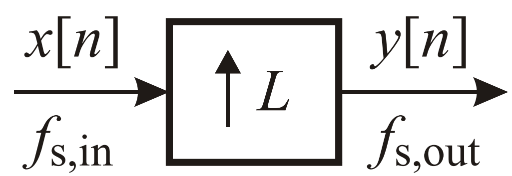


$M - 1$ consecutive samples are removed from the input sequence.

- **Up-sampler**, sampling rate expander

$$y[n] = \begin{cases} x[n/L] & , \quad n = 0, \pm L, \pm 2L, \dots \\ \text{interpolation} & , \quad \text{otherwise} \end{cases} \;;\quad L: \text{positive integer}$$

$$f_{\text{s,out}} = f_{\text{s,in}} \cdot L$$

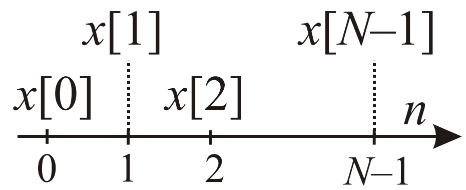


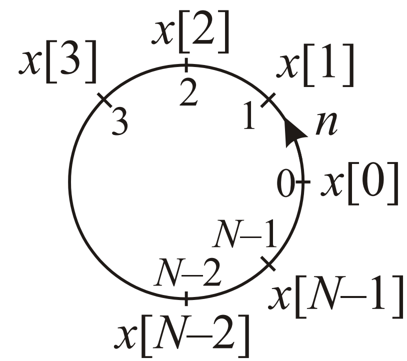
$L - 1$ new samples are inserted between two consecutive samples of the input sequence.

4.5.4 Operations on Finite-Length Sequences

- **Length-*N* sequence**

$$x[n],\ 0 \le n \le N-1$$



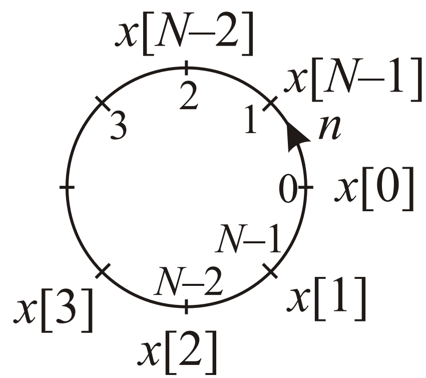


- **Circular time reversal**

$$y[n] = x[\langle -n \rangle_N]$$

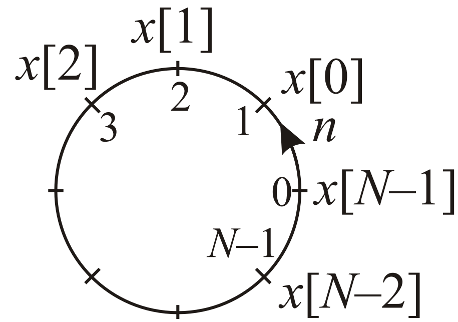
$$\langle m \rangle_N = m \text{ modulo } N = r \ \ : \text{residue}$$

$$r = m + l \cdot N \ \ ; \ \ 0 \le r \le N-1$$



- **Circular shift**

$$x_C[n] = x[\langle n - n_0 \rangle_N]$$



$$x_C[n] = x[\langle n - 1 \rangle_N]$$

Symmetry Relations of Finite-Length Sequences

- Circular conjugate symmetric sequence: $x_{\text{ccs}}[n] = x_{\text{ccs}}{}^*[\langle -n \rangle_N]$
  - Real sequence → circular even: $x_{\text{ce}}[n] = x_{\text{ce}}[\langle -n \rangle_N]$
- Circular conjugate antisymmetric sequence: $x_{\text{cca}}[n] = -x_{\text{cca}}{}^*[\langle -n \rangle_N]$
  - Real sequence → circular odd: $x_{\text{co}}[n] = -x_{\text{co}}[\langle -n \rangle_N]$

- Any finite-length complex sequence $x[n]$ can be composed as:

$$x[n] = x_{\text{ccs}}[n] + x_{\text{cca}}[n] \;\; ; \;\; 0 \le n \le N-1$$

$$x_{\text{ccs}}[n] = \frac{x[n] + x^*[\langle -n \rangle_N]}{2} \;\; , \;\; x_{\text{cca}}[n] = \frac{x[n] - x^*[\langle -n \rangle_N]}{2}$$

$$x_{\text{ccs}}[0] = \text{real} \;\; ; \;\; x_{\text{cca}}[0] = \text{imaginary}$$

$$N \text{ even}: x_{\text{ccs}}[N/2] = \text{real} \;\; ; \;\; x_{\text{cca}}[N/2] = \text{imaginary}$$

Convolution

$$x_1[n], x_2[n],\ 0 \le n \le N-1$$

- **Linear convolution**

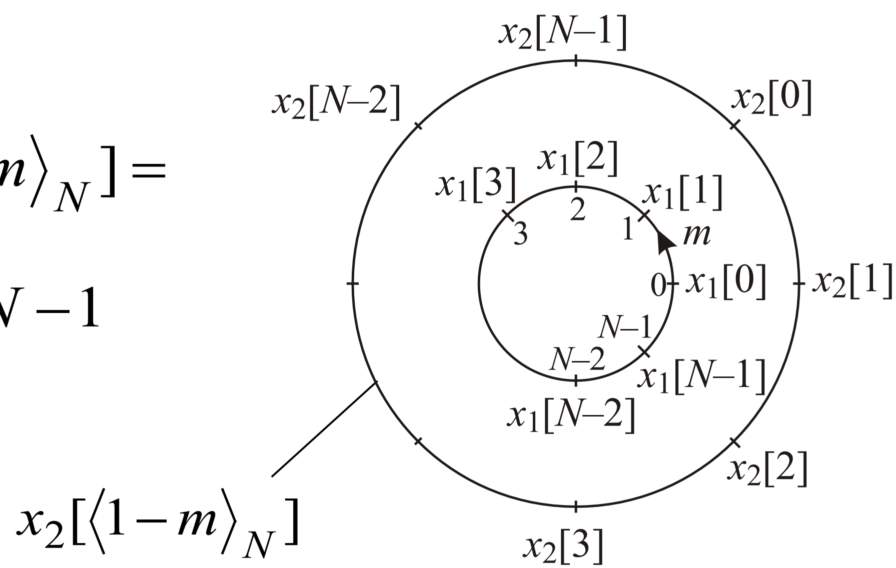
$$y_L[n] = \sum_{m=0}^{N-1} x_1[m] x_2[n-m] = x_1[n] * x_2[n]\ ;\quad 0 \le n \le 2N-2$$

Both length-N sequences are zero-padded to length $2N-1$.

- **Circular convolution**

$$y_C[n] = \sum_{m=0}^{N-1} x_1[m] x_2[\langle n-m \rangle_N] =$$

$$= x_1[n] * x_2[n]\ ;\quad 0 \le n \le N-1$$



4.6 Discrete-Time FOURIER Transform (DTFT)

- **FOURIER transform** of $x[n]$

$$X(\mathrm{e}^{\mathrm{j}\Omega}) = \sum_{n=-\infty}^{+\infty} x[n] \cdot \mathrm{e}^{-\mathrm{j}n\Omega} \quad ; \quad \Omega = \omega \cdot T_\mathrm{s} = 2\pi f / f_\mathrm{s}$$

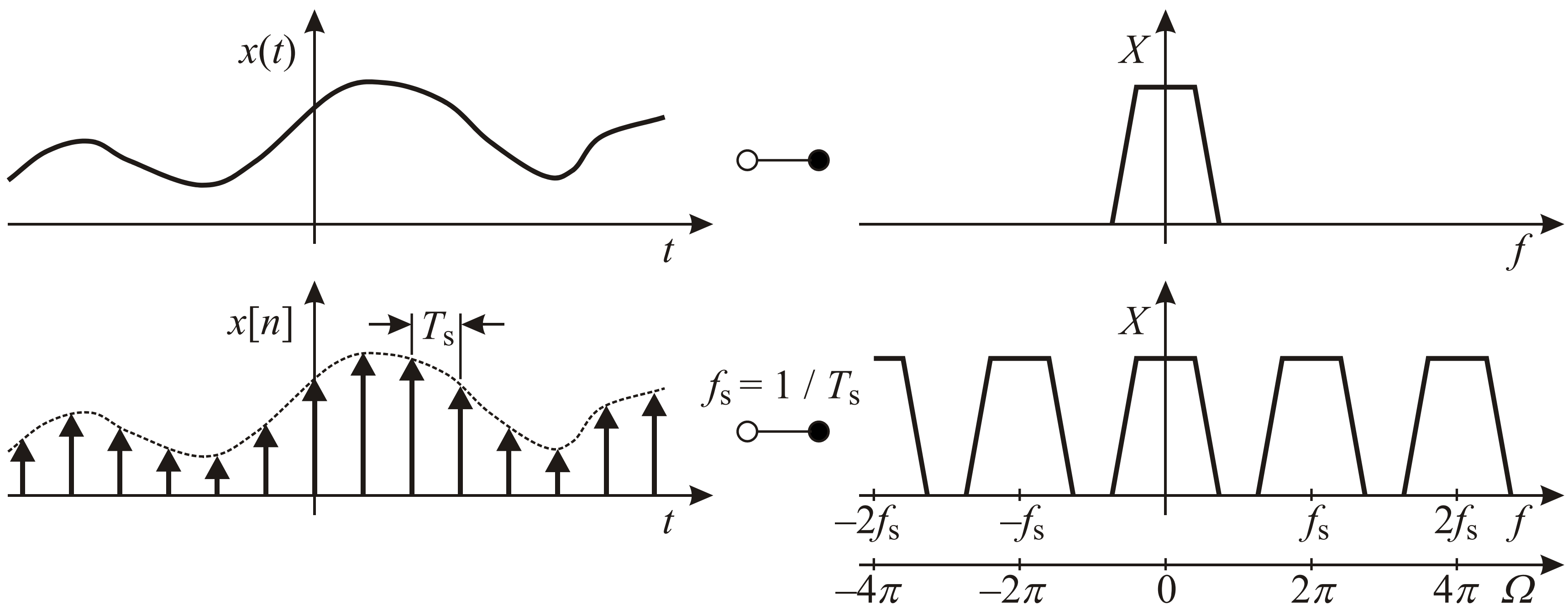
Ω: normalized angular frequency

$$x[n] \circ\!-\!\bullet\ X\left(\mathrm{e}^{\mathrm{j}\Omega}\right) = \mathrm{DTFT}\{x[n]\}$$

- **Inverse FOURIER transform** of $X(\mathrm{e}^{\mathrm{j}\Omega})$

$$x[n] = \frac{1}{2\pi} \cdot \int_{2\pi} X(\mathrm{e}^{\mathrm{j}\Omega}) \cdot \mathrm{e}^{\mathrm{j}n\Omega} \mathrm{d}\,\Omega$$

$$X\left(\mathrm{e}^{\mathrm{j}\Omega}\right) \bullet\!-\!\circ\ x[n] = \mathrm{IDTFT}\left\{X\left(\mathrm{e}^{\mathrm{j}\Omega}\right)\right\}$$



- The FOURIER transform of the discrete signal $x[n]$ is periodic with $\Delta f = f_S$ or $\Delta\Omega = 2\pi$, respectively.
- The FOURIER transform of most practical sequences can be expressed in terms of a sum of a convergent geometric series, which can be summed in a simple closed form.

Some Properties of the FOURIER transform

$$x[n] \circ\!-\!\bullet\; X\left(e^{j\Omega}\right) \qquad y[n] \circ\!-\!\bullet\; Y\left(e^{j\Omega}\right)$$

- **Time shifting** $\quad x[n-n_0] \circ\!-\!\bullet\; e^{-jn_0\Omega} \cdot X\left(e^{j\Omega}\right)$
- **Frequency shifting** $\quad x[n] \cdot e^{jn\Omega_0} \circ\!-\!\bullet\; X\left(e^{j(\Omega-\Omega_0)}\right)$
- **Multiplication** $\quad x[n] \cdot y[n] \circ\!-\!\bullet\; \frac{1}{2\pi} \cdot X\left(e^{j\Omega}\right) * Y\left(e^{j\Omega}\right)$
- **Convolution** $\quad x[n] * y[n] \circ\!-\!\bullet\; X\left(e^{j\Omega}\right) \cdot Y\left(e^{j\Omega}\right)$

4.7 Discrete and Fast FOURIER Transform (DFT, FFT)

- DFT and FFT are **orthogonal finite-length transforms** of length N
- **FOURIER transform** (DFT, FFT) of $x[n]$

$$X[k]=\sum_{n=0}^{N-1} x[n]\cdot \mathrm{e}^{-\mathrm{j}2\pi kn/N}=\sum_{n=0}^{N-1} x[n]\cdot W_N^{kn} \quad ; \quad 0\le k\le N-1 \qquad x[n] \circ\!-\!\bullet\ X[k]$$

- **Inverse FOURIER transform** (IDFT, IFFT) of $X[k]$

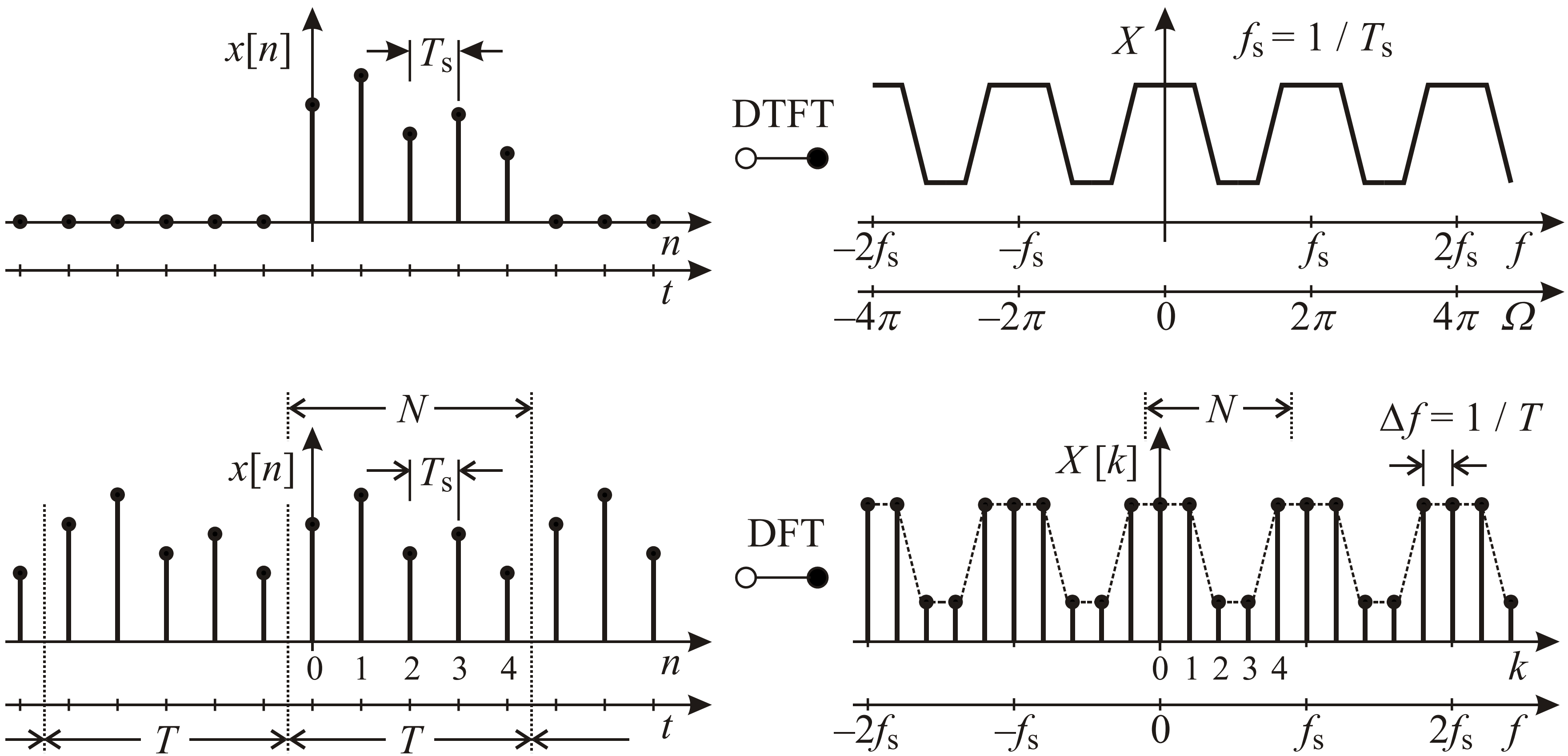
$$x[n]=\frac{1}{N}\cdot\sum_{k=0}^{N-1} X[k]\cdot \mathrm{e}^{\mathrm{j}2\pi kn/N}=\frac{1}{N}\cdot\sum_{k=0}^{N-1} X[k]\cdot W_N^{-kn} \quad ; \quad 0\le n\le N-1$$

$$X[k] \bullet\!-\!\circ\ x[n]$$

- Relation:

$$W_N=\mathrm{e}^{-\mathrm{j}2\pi/N} \quad ; \quad N=\frac{T}{T_\mathrm{s}}=\frac{f_\mathrm{s}}{\Delta f}$$

DTFT versus DFT, FFT



Temporal and Spectral Resolution

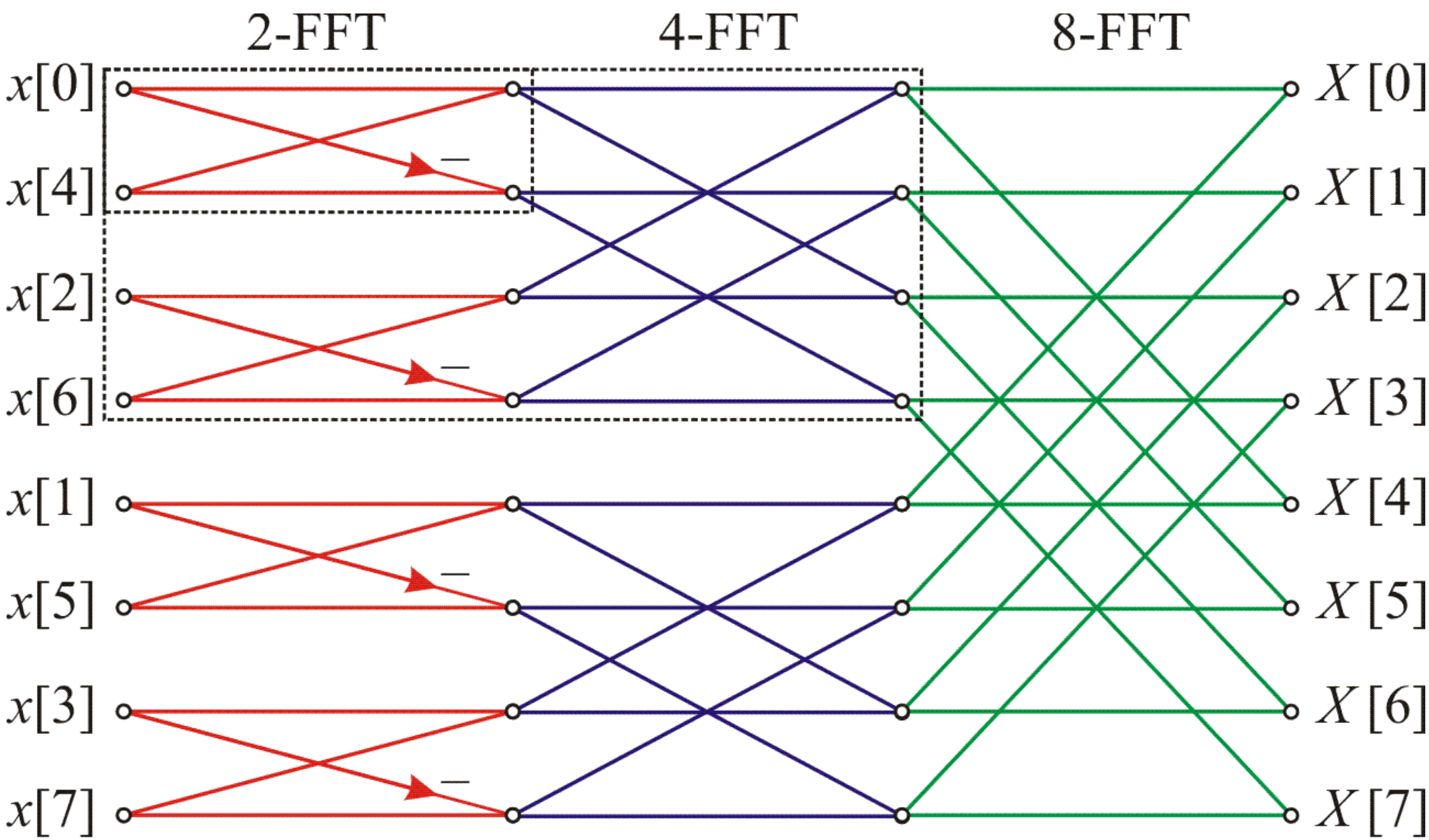
- The sample period T_S and the sampling frequency f_S , respectively, determine the **temporal resolution**.
- The number of samples N together with the sampling frequency f_S determine the observation period T and thus the **spectral resolution** Δf .

Computational Complexity Issues

- DFT requires N^2 complex multiplications and $N(N-1)$ complex additions. This results in $N \cdot (2N-1)$ complex operations.
- FFT requires $N = 2^n$. This results in $N \cdot \mathrm{ld} N = n \cdot 2^n$ complex operations.
- Complex operations:

N	DFT	FFT
4	28	8
8	120	24
16	496	64
32	2,016	160
64	8,128	384
128	32,640	896
256	130,816	2,048

Flow-graph of FFT algorithm



Properties of DFT and FFT

$$x[n],\, y[n],\;\; n = 0,1,\ldots,N-1 \;\circ\!-\!\bullet\; X[k],Y[k],\;\; k = 0,1,\ldots,N-1$$

- **Linearity**

$$a \cdot x[n] + b \cdot y[n] \;\circ\!-\!\bullet\; a \cdot X[k] + b \cdot Y[k]$$

- **Symmetry** properties of a complex sequence $x[n]$

$$x[n] = x_{\mathrm{Re}}[n] + \mathrm{j} \cdot x_{\mathrm{Im}}[n] \;\circ\!-\!\bullet\; X[k] = X_{\mathrm{Re}}[k] + \mathrm{j}\, X_{\mathrm{Im}}[k]$$

$$\mathrm{Re}\{x[n]\} \;\circ\!-\!\bullet\; X_{\mathrm{ccs}}[k] \qquad\qquad \mathrm{j} \cdot \mathrm{Im}\{x[n]\} \;\circ\!-\!\bullet\; X_{\mathrm{cca}}[k]$$

$$x_{\mathrm{ccs}}[n] \;\circ\!-\!\bullet\; \mathrm{Re}\{X[k]\} \qquad\qquad x_{\mathrm{cca}}[n] \;\circ\!-\!\bullet\; \mathrm{j} \cdot \mathrm{Im}\{X[k]\}$$

- **Symmetry** properties of a real sequence $x[n] = x_{\mathrm{ce}}[n] + x_{\mathrm{co}}[n]$

  $x_{\mathrm{ce}}[n] \circ\!-\!\bullet \ \mathrm{Re}\{X[k]\}$ $\quad$ $x_{\mathrm{co}}[n] \circ\!-\!\bullet \ \mathrm{j}\cdot\mathrm{Im}\{X[k]\}$

  ce: circular even
  co: circular odd

  $X[k] = X^*\left[\langle -k\rangle_N\right]$; circular conjugate symmetric

  - $\mathrm{ce}: \mathrm{Re}\{X[k]\} = \mathrm{Re}\left\{X\left[\langle -k\rangle_N\right]\right\}$; $\quad \mathrm{co}: \mathrm{Im}\{X[k]\} = -\mathrm{Im}\left\{X\left[\langle -k\rangle_N\right]\right\}$
  - $\mathrm{ce}: \left|X[k]\right| = \left|X\left[\langle -k\rangle_N\right]\right|$; $\quad \mathrm{co}: \arg\{X[k]\} = -\arg\left\{X\left[\langle -k\rangle_N\right]\right\}$

- **Duality**

  $X[n] \circ\!-\!\bullet \ N\cdot x\left[\langle -k\rangle_N\right]$

- **PARSEVAL's theorem**

$$\sum_{n=0}^{N-1}\left|x[n]\right|^2 = \frac{1}{N}\cdot\sum_{n=0}^{N-1}\left|X[k]\right|^2$$

- **Circular time shifting**

$$x\left[\langle n-n_0\rangle_N\right] \circ\!-\!\bullet\ \mathrm{e}^{-\mathrm{j}2\pi k n_0/N}\cdot X[k]$$

- **Circular frequency shifting**

$$\mathrm{e}^{\mathrm{j}2\pi k_0 n/N}\cdot x[n] \circ\!-\!\bullet\ X\left[\langle k-k_0\rangle_N\right]$$

- ***N*-point circular convolution**

$$\sum_{m=0}^{N-1} x[m]\,y\left[\langle n-m\rangle_N\right] \circ\!-\!\bullet\ X[k]\cdot Y[k]$$

- **Multiplication**

$$x[n]\cdot y[n] \circ\!-\!\bullet\ \frac{1}{N}\cdot\sum_{m=0}^{N-1} X[m]\,Y\left[\langle k-m\rangle_N\right]$$

Leakage Effect

- Periodic signal:

$$x(t) = \mathrm{e}^{\mathrm{j}2\pi f_0 t} \quad \circ\!-\!\bullet \quad X(f) = \delta(f - f_0)$$

- Discrete length-N signal:

$$x[n] = \mathrm{e}^{\mathrm{j}2\pi f_0 n T_\mathrm{s}} \quad ; \quad 0 \le n \le N-1 \quad ; \quad T = N \cdot T_\mathrm{s}$$

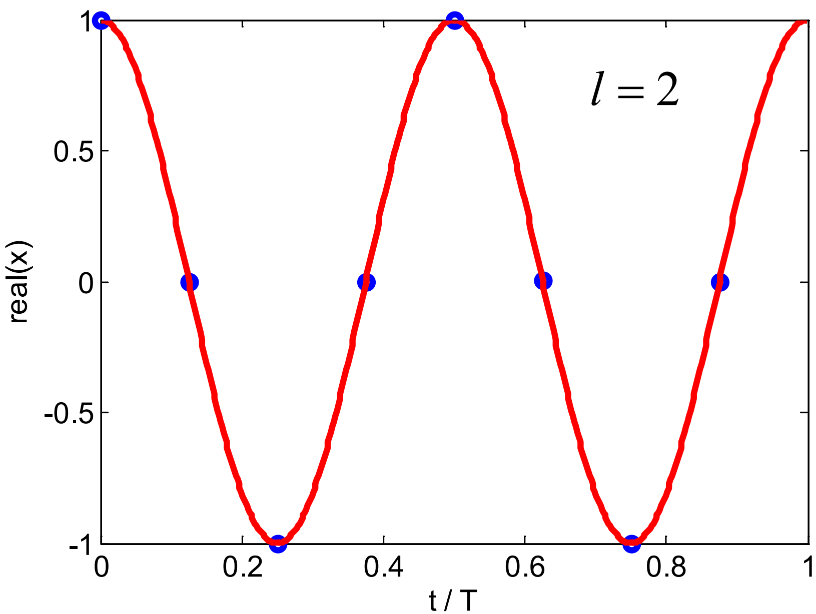
$$X[k] = \mathrm{e}^{\mathrm{j}\pi(f_0 N T_\mathrm{s} - k)\frac{N-1}{N}} \cdot \frac{\sin\left(\pi(f_0 N T_\mathrm{s} - k)\right)}{\sin\left(\frac{\pi}{N}(f_0 N T_\mathrm{s} - k)\right)} \quad ; \quad 0 \le k \le N-1$$

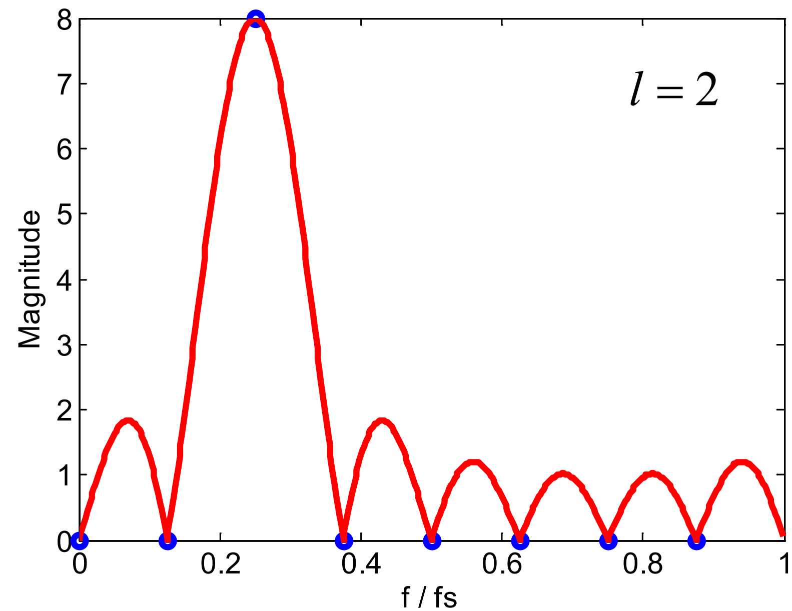
- No leakage, if

$$f_0 N T_s = l = f_0 T = \frac{f_0}{\Delta f} = \frac{T}{1/f_0} \quad ; \; l : \text{positive integer}$$

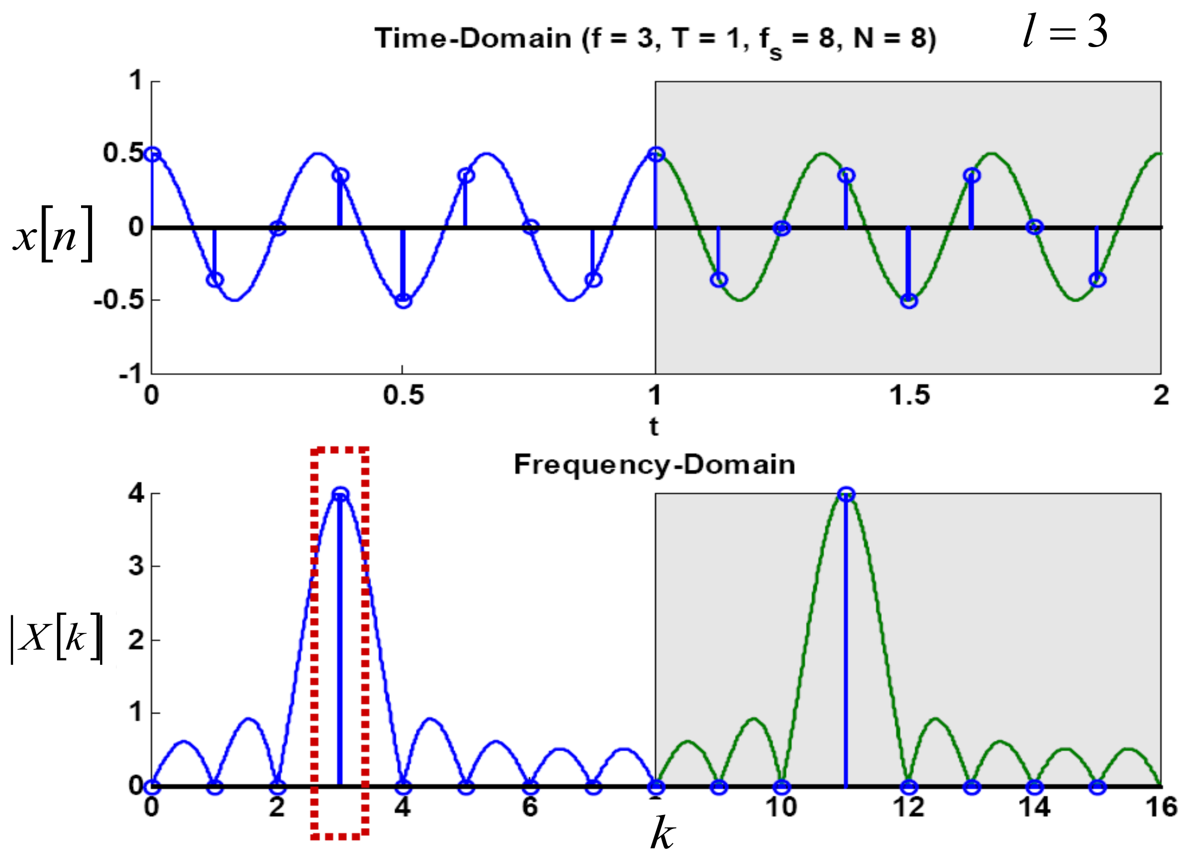
$$X[k] = N \cdot \delta[k - l]$$

$$0 \le k \le N - 1$$





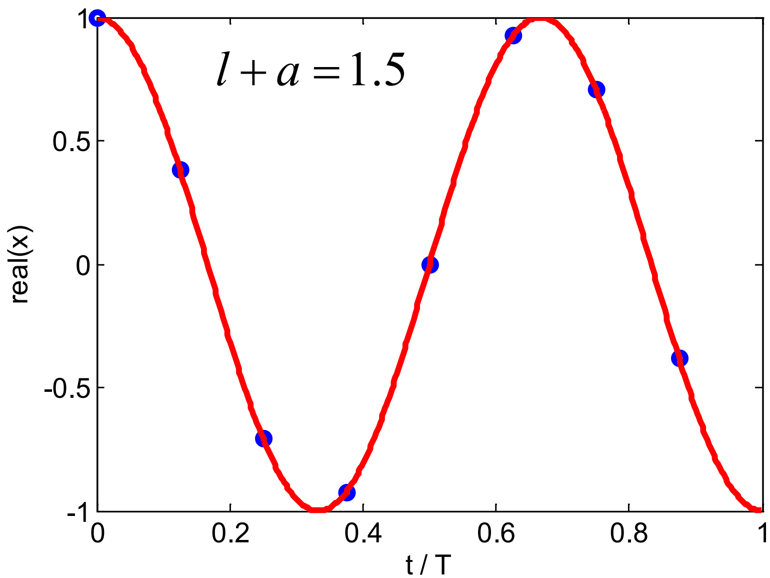
- No leakage:
  - The periodicity of $x(t)$ is conserved.
  - Ideal spectral analysis: one main spectral line

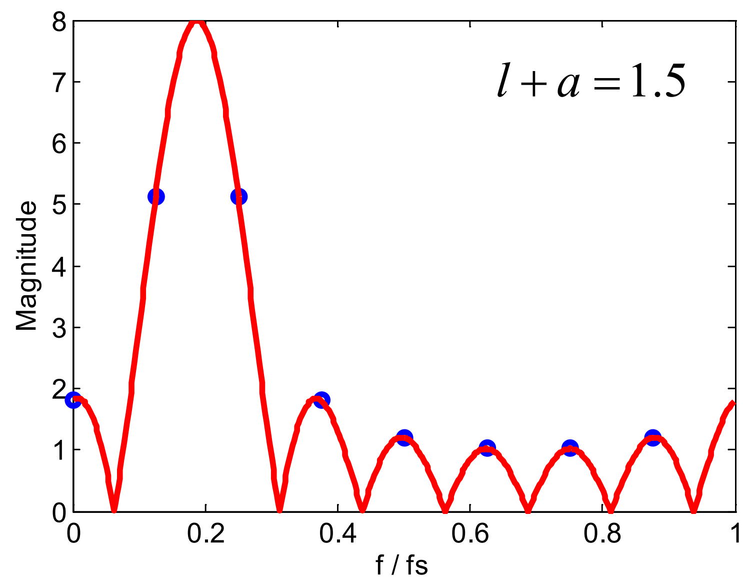


- Leakage effect, if

$$f_0 N T_s = l + a = f_0 T = \frac{f_0}{\Delta f} = \frac{T}{1/f_0} \quad ; \; l : \text{pos. integer} \, ; \; -0.5 < a \le 0.5$$

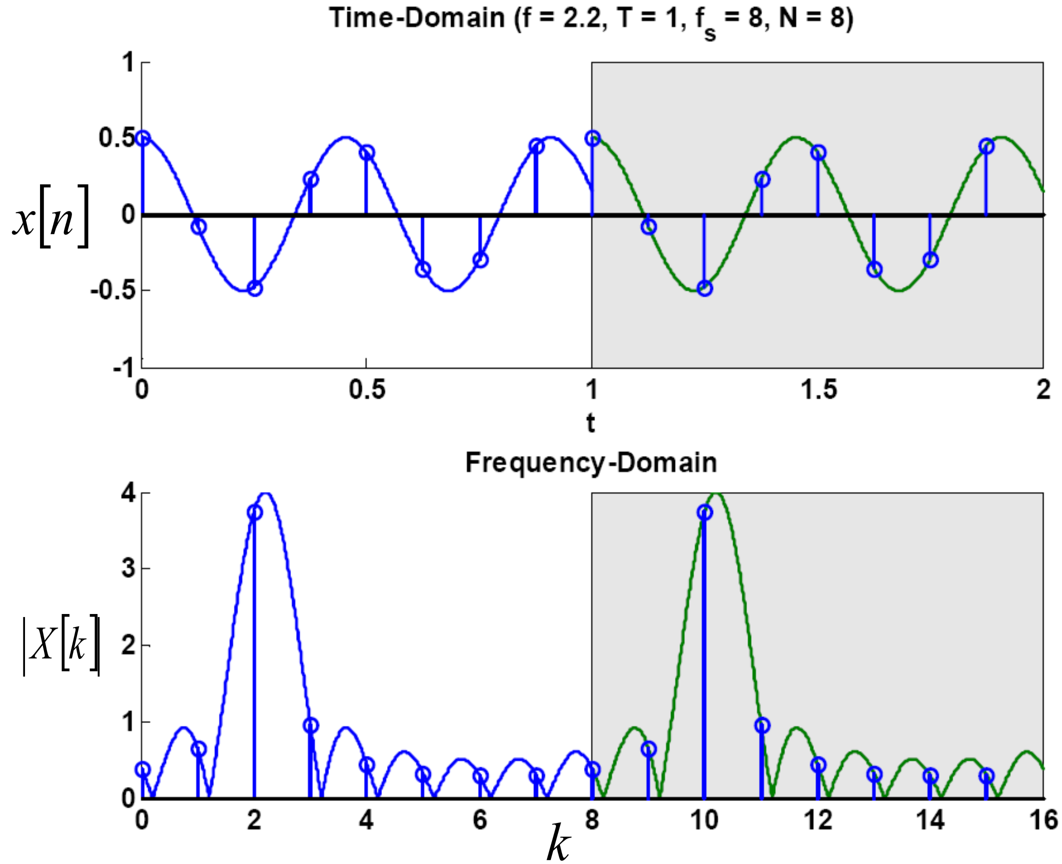
$$X[k] = e^{j\pi(l-k+a)\frac{N-1}{N}} \cdot \frac{\sin\left(\pi(l-k+a)\right)}{\sin\left(\frac{\pi}{N}(l-k+a)\right)} \quad ; \quad 0 \le k \le N-1$$





- The periodicity of $x(t)$ is violated.
- Nonideal spectral analysis: reduced main spectral line + several additional lines
  $\rightarrow$ leakage
  (Lattenzaun-Effekt)

$$l + a = 2.2$$



Zero Padding

- Increasing the observation time $T = NT_S$ by increasing N improves the spectral resolution Δf.