Technische Hochschule OWL
Department of Electrical Engineering and Computer Science

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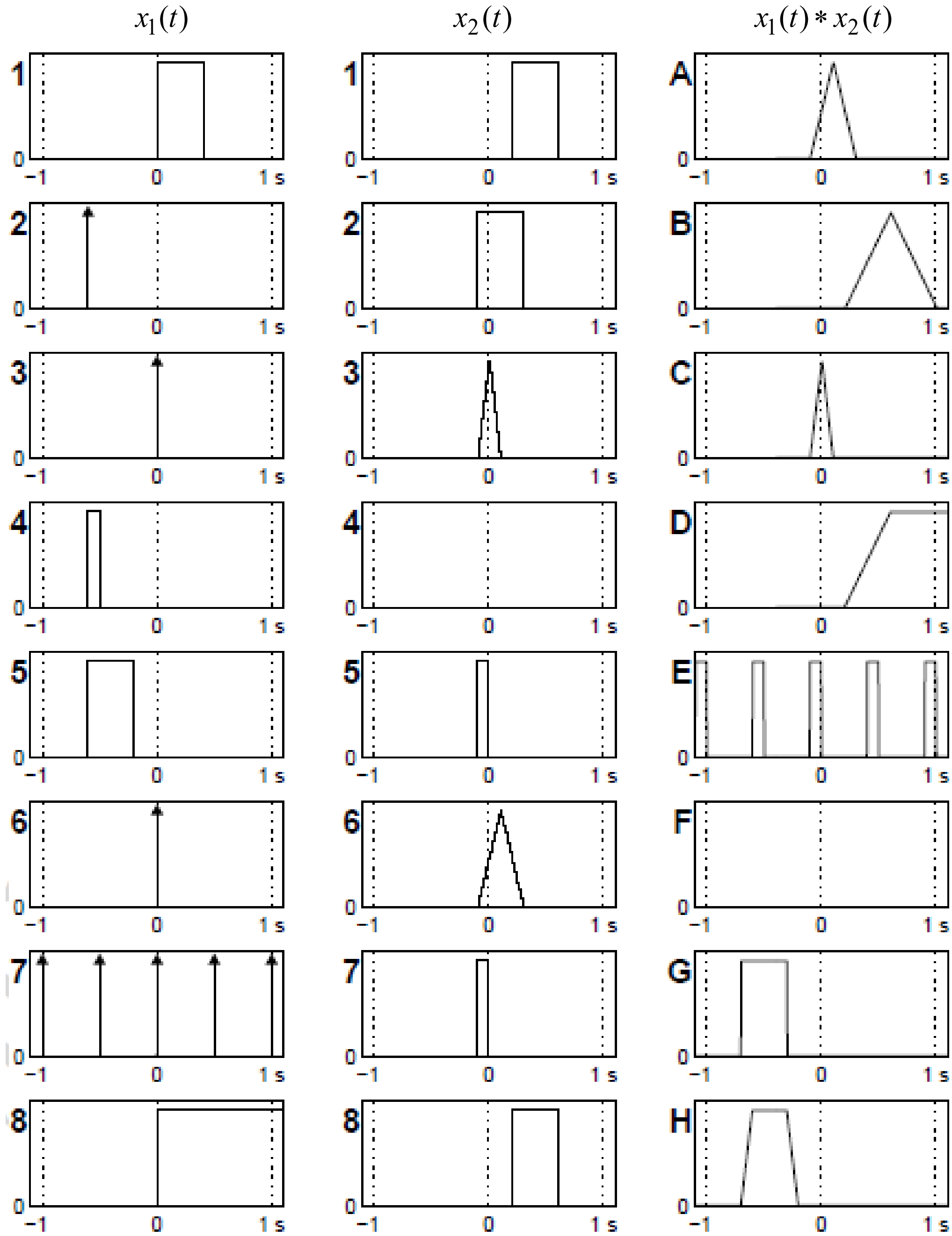
Written Exam: Discrete Signals and Systems (DSS)
Degree Programmes: Information Technology (M. Sc.), Elektrotechnik (M. Sc.)
2021-04-01, 120 min, 100 points available → no notebooks, no books

Please: Don't use red ink; start the solution of each problem on a ***new*** *sheet or side of paper; present all solutions thoroughly.*

Problem 1 Mapping task: 8 different convolutions shall be considered. Unfortunately, the given relation between the signals $x_1(t)$, $x_2(t)$ and the convolution results are erroneous. Find the correct relation and give the answer in the form (1, C), if you think, that this is a correct relation.

You are allowed to guess without proving your answer. But thinking might increase your success. ☺



10 points

Problem 2 Time-continuous signals

2.1 Compute the FOURIER transform $X(f)$ of the signal $x(t)=\begin{cases} e^{-t} & \text{for} \quad -\pi < t \leq \pi \\ 0 & \text{otherwise} \end{cases}$.

2.2 The signal $x(t)$ is periodically extended: $y(t)=\sum_{n=-\infty}^{\infty} x(t-n\cdot 2\pi)$. Compute the FOURIER transform $Y(f)$. *This can be solved separately.*

2.3 Derive the fundamental oscillation of $y(t)$ starting with $Y(f)$. *This can be solved separately.*

15 points

Problem 3 A real, stable, and causal time-continuous LTI system shall be considered. Its system function $H(s)$ has the following features: the number of finite zeros plus finite poles equals five, $H(\infty)=0$, $H(\mathrm{j})=0$, $\left|\mathrm{Re}\{p_i\}\right|=2$ for all finite poles, $\mathrm{Im}\{p_1\}=1$

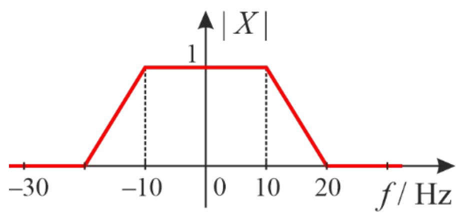
3.1 Plot the pole and zero location and determine $H(s)$.

3.2 Determine the asymptotic magnitude response for very high frequencies, i. e. for $\omega \gg \omega_1$. Compute ω_1.

3.3 Derive the time-domain differential input output relation of the system.

15 points

Problem 4 The magnitude spectrum of a real and even time-continuous signal $x(t)$ is plotted on the right.



4.1 Determine the sampling frequency in order to avoid aliasing errors.

4.2 Plot the spectrum of the sampled signal $x_\mathrm{s}(t)$ with the sampling frequency $f_\mathrm{s}=30\text{ Hz}$.

4.3 Compute the sampled signal $x_\mathrm{s}(t)$ of task 4.2.

4.4 The signal $x(t)$ is impaired by additive noise with spectral components in the range $25\text{ Hz}<|f|<35\text{ Hz}$. Determine the minimum sampling frequency in order to reconstruct the original signal $x(t)$ without errors.

15 points

Problem 5 Consider a causal time-discrete LTI system with system function

$$H(z)=\frac{z^3-\frac{1}{4}z}{\left(z-\frac{1}{2}\right)\cdot\left(z^2-\frac{5}{6}z+\frac{1}{6}\right)}$$

5.1 Is the system stable?

5.2 Simplify the system function and compute the impulse response.

15 points

Problem 6 The system function of a causal time-continuous LTI system is known: $H(s) = \dfrac{s+100}{s+500}$.

6.1 Discretize the system with forward EULER and compute the time-discrete system function.

6.2 Derive a relation for the sample period in order to achieve a stable system.

15 points

Problem 7 The discrete FOURIER transform of a finite-length sequence is given by

$$X[k] = 4 + 4 \cdot \cos\left(k \cdot \frac{\pi}{3}\right) + 2 \cdot \cos\left(k \cdot \frac{2\pi}{3}\right), \text{ with } 0 \le k \le 5.$$

7.1 Compute the sequence $x[n]$.

7.2 Characterize $x[n]$ with respect to possible symmetry features.

15 points

Good luck !