a lower specification limit L so that only 5 percent of the lightbulbs produced will be defective. (That is, determine L so that $P\{X \geq L\} = .95$, where X is the output of a bulb.)

28. A manufacturer produces bolts that are specified to be between 1.19 and 1.21 inches in diameter. If its production process results in a bolt's diameter being normally distributed with mean 1.20 inches and standard deviation .005, what percentage of bolts will not meet specifications?

29. Let $I = \int_{-\infty}^{\infty} e^{-x^2/2}\, dx$.

(a) Show that for any μ and σ

$$\frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{\infty} e^{-(x-\mu)^2/2\sigma^2}\, dx = 1$$

is equivalent to $I = \sqrt{2\pi}$.

(b) Show that $I = \sqrt{2\pi}$ by writing

$$I^2 = \int_{-\infty}^{\infty} e^{-x^2/2}\, dx \int_{-\infty}^{\infty} e^{-y^2/2}\, dy = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-(x^2+y^2)/2}\, dx\, dy$$

and then evaluating the double integral by means of a change of variables to polar coordinates. (That is, let $x = r\cos\theta, y = r\sin\theta, dx\, dy = r\, dr\, d\theta$.)

30. A random variable X is said to have a lognormal distribution if $\log X$ is normally distributed. If X is lognormal with $E[\log X] = \mu$ and $\text{Var}(\log X) = \sigma^2$, determine the distribution function of X. That is, what is $P\{X \leq x\}$?

31. The lifetimes of interactive computer chips produced by a certain semiconductor manufacturer are normally distributed having mean 4.4×10^6 hours with a standard deviation of 3×10^5 hours. If a mainframe manufacturer requires that at least 90 percent of the chips from a large batch will have lifetimes of at least 4.0×10^6 hours, should he contract with the semiconductor firm?

32. In Problem 31, what is the probability that a batch of 100 chips will contain at least 4 whose lifetimes are less than 3.8×10^6 hours?

33. The lifetime of a television picture tube is a normal random variable with mean 8.2 years and standard deviation 1.4 years. What percentage of such tubes lasts

(a) more than 10 years;
(b) less than 5 years;
(c) between 5 and 10 years?

34. The annual rainfall in Cincinnati is normally distributed with mean 40.14 inches and standard deviation 8.7 inches.

(c) $n = 36$;
(d) $n = 64$.

12. An instructor knows from past experience that student exam scores have mean 77 and standard deviation 15. At present the instructor is teaching two separate classes — one of size 25 and the other of size 64.

(a) Approximate the probability that the average test score in the class of size 25 lies between 72 and 82.
(b) Repeat part (a) for a class of size 64.
(c) What is the approximate probability that the average test score in the class of size 25 is higher than that of the class of size 64?
(d) Suppose the average scores in the two classes are 76 and 83. Which class, the one of size 25 or the one of size 64, do you think was more likely to have averaged 83?

13. If X is binomial with parameters $n = 150$, $p = .6$, compute the exact value of $P\{X \leq 80\}$ and compare with its normal approximation both **(a)** making use of and **(b)** not making use of the continuity correction.

14. Each computer chip made in a certain plant will, independently, be defective with probability .25. If a sample of 1,000 chips is tested, what is the approximate probability that fewer than 200 chips will be defective?

15. A club basketball team will play a 60-game season. Thirty-two of these games are against class A teams and 28 are against class B teams. The outcomes of all the games are independent. The team will win each game against a class A opponent with probability .5, and it will win each game against a class B opponent with probability .7. Let X denote its total number of victories in the season.

(a) Is X a binomial random variable?
(b) Let X_A and X_B denote, respectively, the number of victories against class A and class B teams. What are the distributions of X_A and X_B?
(c) What is the relationship between X_A, X_B, and X?
(d) Approximate the probability that the team wins 40 or more games.

16. Argue, based on the central limit theorem, that a Poisson random variable having mean λ will approximately have a normal distribution with mean and variance both equal to λ when λ is large. If X is Poisson with mean 100, compute the exact probability that X is less than or equal to 116 and compare it with its normal approximation both when a continuity correction is utilized and when it is not. The convergence of the Poisson to the normal is indicated in Figure 6.5.

17. Use the text disk to compute $P\{X \leq 10\}$ when X is a binomial random variable with parameters $n = 100, p = .1$. Now compare this with its **(a)** Poisson and

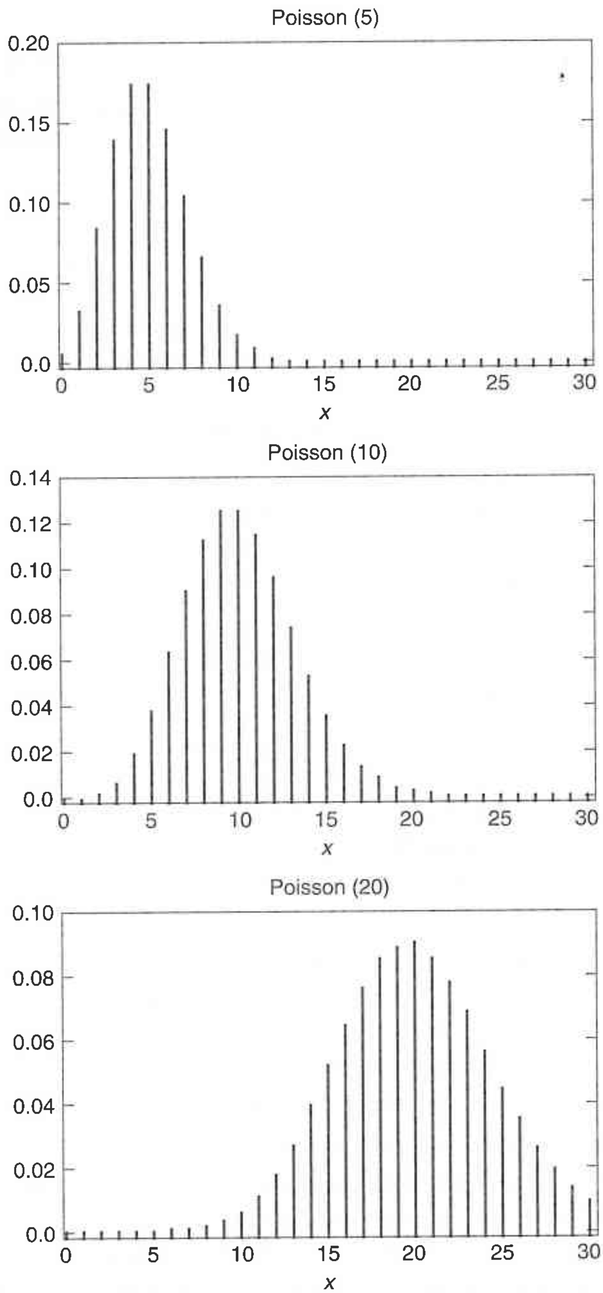

FIGURE 6.5 *Poisson probability mass functions.*