Problem 1 Consider the finite-length sequence of length 7 defined for $-3 \le n \le 3$:

$$\{x[n]\} = \{0,\ 1+\mathrm{j}4,\ -2+\mathrm{j}3,\ 4-\mathrm{j}2,\ -5-\mathrm{j}6,\ -\mathrm{j}2,\ 3\}\ .$$

Determine the conjugate-symmetric and conjugate-antisymmetric sequences.

Problem 2 Consider the causal sequences defined by:

$$\text{a)}\ x[n] = \begin{cases} 3\cdot(-1)^n, & n \ge 0 \\ 0, & \text{otherwise} \end{cases} \qquad \text{b)}\ x[n] = \begin{cases} 1/n, & n \ge 1 \\ 0, & \text{otherwise} \end{cases}$$

Are these sequences energy or power signals? If yes, compute the energy or power, respectively.

Problem 3 Compute the N-point DFT $X[k]$ of the length-N sequences with $0 \le k \le N-1$

a) $\{x[n]\} = \{2, 0, 1, 0, 0, 0, 1, 0\}$; b) $\{x[n]\} = \{1, 2, 3, 4\}$

Check for symmetry relations.

Problem 4 Compute the N-point DFT $X[k]$ of the length-N sequences with $0 \le k \le N-1$

a) $x[n] = \mathrm{e}^{\mathrm{j}2\pi rn/N}$; $0 \le n \le N-1$,

b) $x[n] = \cos\left(2\pi rn/N\right)$; $0 \le n \le N-1$,

where r is an integer in the range $0 < r \le N-1$.

Problem 5 Building resonances between 0.8 Hz and 1.0 Hz must be avoided in skyscrapers, because they cause braking window glasses. Thus, civil engineers need to know exactly the resonance frequencies. The building is equipped with sensors and their signals are being processed digitally. After AD conversion an FFT algorithm is applied.

a) How many FFT points are necessary for the sampling frequency 5.4 Hz in order to achieve a frequency resolution of 0.001 Hz?

b) Determine the observation time.

Problem 6 Use the equation

$$\sum_{n=0}^{N-1} \mathrm{e}^{-\mathrm{j}2\pi kn/N} = \begin{cases} N, & k = l\cdot N,\ l \text{ integer} \\ 0, & \text{otherwise} \end{cases}$$

for the computation of the length-N DFT.

a) Compute the continuous-time FOURIER transform $X(f)$ of $x(t) = \sin(2\pi f_0 t)$ and plot $|X(f)|$.

b) Compute the continuous-time FOURIER transform $X(f)$ of

$$x(t) = \begin{cases} \sin(2\pi f_0 t), & 0 \le t \le 1/f_0 \\ 0, & \text{otherwise} \end{cases}$$

c) Compute $X[k]$ of $x[n] = \sin(2\pi f_0 n T_\mathrm{s})$ with $T_\mathrm{s} = 1/(8 f_0)$ and $N = 8$. Plot $|X[k]|$.

d) Compute $X[k]$ of $x[n] = \sin(2\pi f_0 n T_\mathrm{s})$ with $T_\mathrm{s} = 0.9/(8 f_0)$ and $N = 8$. Plot $|X[k]|$.

e) Perform suitable Matlab simulations and check the results.

Answers

Problem 1

$$\{x_{\mathrm{cs}}[n]\} = \{1.5,\ \ 0.5+\mathrm{j}3,\ \ -3.5+\mathrm{j}4.5,\ \ 4,\ \ -3.5-\mathrm{j}4.5,\ \ 0.5-\mathrm{j}3,\ \ 1.5\}$$

$$\{x_{\mathrm{ca}}[n]\} = \{-1.5,\ \ 0.5+\mathrm{j},\ \ 1.5-\mathrm{j}1.5,\ \ -\mathrm{j}2,\ \ -1.5-\mathrm{j}1.5,\ \ -0.5+\mathrm{j},\ \ 1.5\}$$

Problem 2

a) Power signal: $P_\infty = 4.5$; $E_\infty \to \infty$

b) Energy signal: $E_\infty = \pi^2/6 = 1.645$; $P_\infty = 0$

Problem 3

a) $\{X[k]\} = \{4,\ \ 2,\ \ 0,\ \ 2,\ \ 4,\ \ 2,\ \ 0,\ \ 2\}$

b) $\{X[k]\} = \{10,\ \ -2+\mathrm{j}2,\ \ -2,\ \ -2-\mathrm{j}2\}$

Problem 4

a) $X[k] = \begin{cases} N & \text{for } k = r \\ 0 & \text{otherwise} \end{cases}$; b) $X[k] = \begin{cases} N/2 & \text{for } k = r \\ N/2 & \text{for } k = N-r \\ 0 & \text{otherwise} \end{cases}$

Problem 5

a) 8192 b) 1517 s

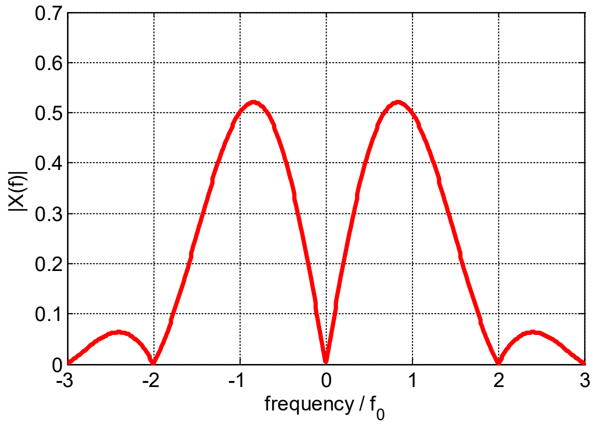
Problem 6

a) $X(f) = \frac{\mathrm{j}}{2}\left(\delta(f+f_0) - \delta(f+f_0)\right)$

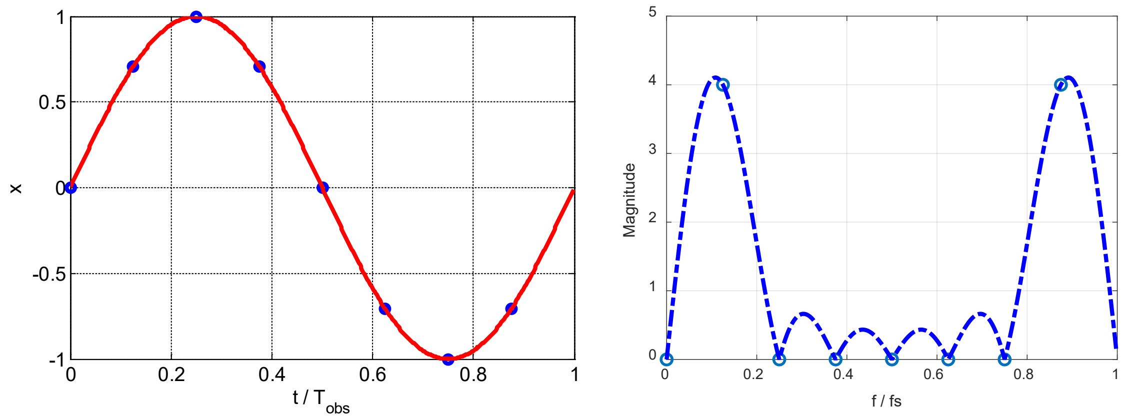
b) $X(f) = \mathrm{j}\frac{T}{2}\left(\mathrm{si}\left(\pi(f+f_0)T\right)\cdot \mathrm{e}^{-\mathrm{j}\pi(f+f_0)T} - \mathrm{si}\left(\pi(f-f_0)T\right)\cdot \mathrm{e}^{-\mathrm{j}\pi(f-f_0)T}\right)$

c) $X[k] = \begin{cases} -4\,\mathrm{j}\ , & k = 1 \\ 4\,\mathrm{j}\ , & k = 7 \\ 0\ , & k = 0,\ 2,\ 3,\ 4,\ 5,\ 6 \end{cases}$

b)



c) $T_s = 1/(8 f_0)$ and $N = 8$



d) $T_s = 0.9/(8 f_0)$ and $N = 8$