1.) Show that for any constant $c \in \mathbb{R}$ and any random variable X, $Var(cX) = c^2\, Var(X)$.

2.) Show by counterexample that being uncorrelated does not imply independence.
Hint: Let X be a random variable with

$$\Pr(X = \pm 1) = \Pr(X = \pm 2) = \frac{1}{4}$$

and $Y := |X|$. Show that X and Y are not independent, but $Cov(X, Y) = 0$.

3.) A fair coin is tossed repeatedly for n times. Determine all values for n, such that with a probability of at least 98% "head" shows up in 49%–51% of all tosses?

4.) Let X, Y be random variables taking values 1, 2 and 3 with joint probabilities $p_{XY}(i, j)$ given by the following matrix:

$$\begin{pmatrix} \frac{1}{24} & \frac{1}{6} & \frac{1}{24} \\ \frac{1}{12} & \frac{1}{3} & \frac{1}{12} \\ \frac{1}{24} & \frac{1}{6} & \frac{1}{24} \end{pmatrix}$$

(i) Determine the marginal pmf's p_X and p_Y.

(ii) Determine $\Pr(X < Y)$.

(iii) Determine whether or not X and Y are independent.

5.) Let X and Y be integer-valued random values with joint pmf

$$p_{XY}(i, j) = \begin{cases} \frac{3^{j-1}e^{-3}}{j!}, & i = 1, j \geq 0, \\ c\frac{6^{j-1}e^{-6}}{j!}, & i = 2, j \geq 0, \\ 0, & \text{otherwise.} \end{cases}$$

(i) Determine c.

(ii) Determine the marginal pmf's p_X and p_Y.

(iii) Determine whether or not X and Y are independent.

6.) Let $p \in (0, 1)$ and $X_1, X_2, ..., X_n$ be independent random variables with $\Pr(X_i = 1) = (1-p)$ and $\Pr(X_i = 2) = p$ for $i = 1, ..., n$. Find the pmf of $X = X_1 + \cdots + X_n$, i.e. determine $\Pr(X = k)$ for all $k \in \mathbb{N}$.

7.) If you buy a lottery ticket in 50 lotteries, in each of which your chance of winning a prize is $\frac{1}{100}$, what is the probability that you will win a prize **(a)** at least once, **(b)** exactly once, and **(c)** at least twice?

8.) In the 1980s, an average of 121.95 workers died on the job each week. Give estimates of the following quantities:

(a) the proportion of weeks having 130 deaths or more;

(b) the proportion of weeks having 100 deaths or less.

Explain your reasoning.

9.) Approximately 80,000 marriages took place in the state of New York last year. Estimate the probability that for at least one of these couples

(a) both partners were born on April 30;

(b) both partners celebrated their birthday on the same day of the year.

State your assumptions.

10.) The probability of error in the transmission of a binary digit over a communication channel is $1/10^3$. Write an expression for the exact probability of more than 3 errors when transmitting a block of 10^3 bits. What is its approximate value? Assume independence.

11.) If X is a Poisson random variable with mean λ, show that $\Pr(X = i)$ first increases and then decreases as i increases, reaching its maximum value when i is the largest integer less than or equal to λ.

12.) A contractor purchases a shipment of 100 transistors. It is his policy to test 10 of these transistors and to keep the shipment only if at least 9 of the 10 are in working condition. If the shipment contains 20 defective transistors, what is the probability it will be kept?

13.) The components of a 6-component system are to be randomly chosen from a bin of 20 used components. The resulting system will be functional if at least 4 of its 6 components are in working condition. If 15 of the 20 components in the bin are in working condition, what is the probability that the resulting system will be functional?

14.) Provide a MatLab script that generates a plot of the hypergeometric(N, D, n) pmf. The plot shall also display the values of the expectation and the standard deviation. These values shall also be depicted in the plot.

15.) If X_1 and X_2 are independent binomial random variables with $X_1 \sim \text{binomial}(n_1, p)$ and $X_2 \sim \text{binomial}(n_2, p)$, then $X_1 + X_2 \sim \text{binomial}(n_1 + n_2, p)$.

16.) If X_1 and X_2 are independent Poisson random variables with $X_1 \sim \text{Poisson}(\lambda_1)$ and $X_2 \sim \text{Poisson}(\lambda_2)$, then $X_1 + X_2 \sim \text{Poisson}(\lambda_1 + \lambda_2)$.