B Transform Tables

Overview

Continuous-Time FOURIER Transform (CTFT)

$$x(t)=\int_{-\infty}^{\infty} X(f)\cdot \mathrm{e}^{\mathrm{j}2\pi ft}\mathrm{d}f \quad \circ\!-\!\bullet \quad X(f)=\int_{-\infty}^{\infty} x(t)\cdot \mathrm{e}^{-\mathrm{j}2\pi ft}\mathrm{d}t$$

$$x(t)=\frac{1}{2\pi}\cdot\int_{-\infty}^{\infty} X(\mathrm{j}\omega)\cdot \mathrm{e}^{\mathrm{j}\omega t}\mathrm{d}\omega \quad \circ\!-\!\bullet \quad X(\mathrm{j}\omega)=\int_{-\infty}^{\infty} x(t)\cdot \mathrm{e}^{-\mathrm{j}\omega t}\mathrm{d}t$$

Discrete-Time FOURIER Transform (DTFT)

$$x[n]=\frac{1}{2\pi}\cdot\int_{2\pi} X(\mathrm{e}^{\mathrm{j}\Omega})\cdot \mathrm{e}^{\mathrm{j}n\Omega}\mathrm{d}\Omega \quad \circ\!-\!\bullet \quad X(\mathrm{e}^{\mathrm{j}\Omega})=\sum_{n=-\infty}^{+\infty} x[n]\cdot \mathrm{e}^{-\mathrm{j}n\Omega}$$

Discrete and fast FOURIER Transform (DFT, FFT)

$$x[n]=\frac{1}{N}\cdot\sum_{k=0}^{N-1} X[k]\cdot \mathrm{e}^{\mathrm{j}2\pi kn/N} \quad \circ\!-\!\bullet \quad X[k]=\sum_{n=0}^{N-1} x[n]\cdot \mathrm{e}^{-\mathrm{j}2\pi kn/N}$$

$0\le n\le N-1$ $\qquad$ $0\le k\le N-1$

LAPLACE Transform (LT) - unilateral

$$x(t)=\frac{1}{2\pi\mathrm{j}}\cdot\int_{\sigma-\mathrm{j}\infty}^{\sigma+\mathrm{j}\infty} X(s)\cdot \mathrm{e}^{st}\mathrm{d}s \quad \circ\!-\!\bullet \quad X(s)=\int_{0_-}^{\infty} x(t)\cdot \mathrm{e}^{-st}\mathrm{d}t;\ s=\sigma+\mathrm{j}\omega$$

***z*-Transform - bilateral**

$$x[n]=\frac{1}{2\pi\mathrm{j}}\cdot\oint_{\mathrm{C}} X(z)\cdot z^{n-1}\,\mathrm{d}z \quad \circ\!-\!\bullet \quad X(z)=\sum_{n=-\infty}^{\infty} x[n]\cdot z^{-n};\ z=r\cdot\mathrm{e}^{\mathrm{j}\Omega}$$

Continuous-Time FOURIER Transform (CTFT)

Definition

$$x(t) = \int_{-\infty}^{\infty} X(f) \cdot e^{j2\pi ft} df \quad \circ\!-\!\bullet \quad X(f) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j2\pi ft} dt$$

$$x(t) = \frac{1}{2\pi} \cdot \int_{-\infty}^{\infty} X(j\omega) \cdot e^{j\omega t} d\omega \quad \circ\!-\!\bullet \quad X(j\omega) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j\omega t} dt$$

Common transform pairs

Time domain $x(t)$	**Transform** $X(f)$	**Transform** $X(j\omega)$
1	$\delta(f)$	$2\pi \cdot \delta(\omega)$
$\delta(t)$	1	1
$\mathrm{rect}\left(\frac{t}{T}\right)$	$T \cdot \frac{\sin(\pi fT)}{\pi fT}$	$T \cdot \frac{\sin(\omega T/2)}{\omega T/2}$
$= \begin{cases} 1 \text{ if } \lvert t\rvert < T/2 \\ 0 \text{ if } \lvert t\rvert > T/2 \end{cases}$	$= T \cdot \mathrm{si}(\pi fT) = T \cdot \mathrm{sinc}(fT)$	$= T \cdot \mathrm{si}\left(\frac{\omega T}{2}\right) = T \cdot \mathrm{sinc}\left(\frac{\omega T}{2\pi}\right)$
$B \cdot \mathrm{si}(\pi Bt) = B \cdot \mathrm{sinc}(Bt)$	$\mathrm{rect}\left(\frac{f}{B}\right)$	$\mathrm{rect}\left(\frac{\omega}{2\pi B}\right)$
$u(t) = \begin{cases} 1 \text{ if } t > 0 \\ 0 \text{ if } t < 0 \end{cases}$	$\frac{1}{j2\pi f} + \frac{1}{2} \cdot \delta(f)$	$\frac{1}{j\omega} + \pi \cdot \delta(\omega)$
$\mathrm{sgn}(t) = \begin{cases} 1 \text{ if } t > 0 \\ -1 \text{ if } t < 0 \end{cases}$	$\frac{1}{j\pi f}$	$\frac{2}{j\omega}$
$e^{j2\pi f_0 t} = e^{j\omega_0 t}$	$\delta(f - f_0)$	$2\pi \cdot \delta(\omega - \omega_0)$
$\cos(2\pi f_0 t) = \cos(\omega_0 t)$	$\frac{1}{2} \cdot [\delta(f + f_0) + \delta(f - f_0)]$	$\pi \cdot [\delta(\omega + \omega_0) + \delta(\omega - \omega_0)]$
$\sin(2\pi f_0 t) = \sin(\omega_0 t)$	$\frac{j}{2} \cdot [\delta(f + f_0) - \delta(f - f_0)]$	$j\pi \cdot [\delta(\omega + \omega_0) - \delta(\omega - \omega_0)]$
$u(t) \cdot e^{-a \cdot t}$; $a > 0$	$\frac{1}{a + j2\pi \cdot f}$	$\frac{1}{a + j\omega}$
$e^{-a \cdot \lvert t\rvert}$; $a > 0$	$\frac{2a}{a^2 + (2\pi \cdot f)^2}$	$\frac{2a}{a^2 + \omega^2}$
e^{-at^2} ; $a > 0$	$\sqrt{\frac{\pi}{a}} e^{-(\pi f)^2/a}$	$\sqrt{\frac{\pi}{a}} e^{-\omega^2/(4a)}$
$\sum_{n=-\infty}^{\infty} \delta(t - nT)$	$\frac{1}{T} \cdot \sum_{k=-\infty}^{\infty} \delta(f - k\frac{1}{T})$	$\frac{2\pi}{T} \cdot \sum_{k=-\infty}^{\infty} \delta(\omega - k\frac{2\pi}{T})$

Properties

Theorem	Time domain	Transform
Symmetry for real time-domain signals	$x(t) = x_{\mathrm{ev}}(t) + x_{\mathrm{od}}(t)$	$X(f) = X_{\mathrm{Re}}(f) + \mathrm{j}\cdot X_{\mathrm{Im}}(f)$
	$x_{\mathrm{ev}}(t)$	$X_{\mathrm{Re}}(f)$; $X_{\mathrm{Re}}(f)$: even
	$x_{\mathrm{od}}(t)$	$\mathrm{j}\cdot X_{\mathrm{Im}}(f)$; $X_{\mathrm{Im}}(f)$: odd
		$X(f) = X^*(-f)$
		$X_{\mathrm{Re}}(f) = X_{\mathrm{Re}}(-f)$; $X_{\mathrm{Im}}(f) = -X_{\mathrm{Im}}(-f)$
		$\lvert X(f)\rvert = \lvert X(-f)\rvert$; $\arg\{X(f)\} = -\arg\{X(-f)\}$
Time reversal	$x(-t)$	$X(-f)$
Conjugation	$x^*(t)$	$X^*(-f)$
Duality	$X(t)$	$x(-f)$
Linearity	$a\cdot x(t) + b\cdot y(t)$	$a\cdot X(f) + b\cdot Y(f)$
Scaling	$x(at)$	$\dfrac{1}{\lvert a\rvert}\cdot X\left(\dfrac{f}{a}\right)$
Time shifting	$x(t-t_0)$	$\mathrm{e}^{-\mathrm{j}2\pi\cdot f\cdot t_0}\cdot X(f)$
Frequency shifting	$\mathrm{e}^{\mathrm{j}2\pi\cdot f_0\cdot t}\cdot x(t)$	$X(f-f_0)$
Differentiation in time domain	$x^{(n)}(t)$	$(\mathrm{j}2\pi\cdot f)^n\cdot X(f)$
Differentiation in frequency domain	$(-\mathrm{j}2\pi\cdot t)^n\cdot x(t)$	$X^{(n)}(f)$
Time integration	$\int_{-\infty}^{t} x(\tau)\cdot \mathrm{d}\tau$	$\dfrac{X(f)}{\mathrm{j}2\pi\cdot f} + \dfrac{1}{2}\cdot X(0)\cdot\delta(f)$; $\dfrac{X(\mathrm{j}\omega)}{\mathrm{j}\omega} + \pi\cdot X(0)\cdot\delta(\omega)$
Time convolution	$x(t) * y(t)$	$X(f)\cdot Y(f)$; $X(\mathrm{j}\omega)\cdot Y(\mathrm{j}\omega)$
Time multiplication	$x(t)\cdot y(t)$	$X(f) * Y(f)$; $\dfrac{1}{2\pi}\left(X(\mathrm{j}\omega) * Y(\mathrm{j}\omega)\right)$
Positive time functions		$x(t) \ge 0 \Rightarrow \lvert X(f)\rvert \le X(0)$
Final value of spectrum		$\int_{-\infty}^{\infty} \lvert x(t)\rvert \cdot \mathrm{d}t < \infty \Rightarrow \lim\limits_{f\to\pm\infty} X(f) = 0$
PARSEVAL's theorem		$\int_{-\infty}^{\infty} x(t)\cdot y^*(t)\cdot \mathrm{d}t = \int_{-\infty}^{\infty} X(f)\cdot Y^*(f)\cdot \mathrm{d}f$
		$\int_{-\infty}^{\infty} \lvert x(t)\rvert^2\cdot \mathrm{d}t = \int_{-\infty}^{\infty} \lvert X(f)\rvert^2\cdot \mathrm{d}f$

Discrete-Time FOURIER Transform (DTFT)

Definition

$$x[n] = \frac{1}{2\pi} \cdot \int_{2\pi} X(\mathrm{e}^{\mathrm{j}\Omega}) \cdot \mathrm{e}^{\mathrm{j}n\Omega} \mathrm{d}\,\Omega \quad \circ\!-\!\bullet \quad X(\mathrm{e}^{\mathrm{j}\Omega}) = \sum_{n=-\infty}^{+\infty} x[n] \cdot \mathrm{e}^{-\mathrm{j}n\Omega}$$

Common transform pairs

Time domain $x[n]$	Transform $X(\mathrm{e}^{\mathrm{j}\Omega})$
$\delta[n]$	1
1 for $-\infty < n < \infty$	$\sum_{k=-\infty}^{\infty} 2\pi \cdot \delta(\Omega + 2\pi \cdot k)$
$u[n]$	$\frac{1}{1-\mathrm{e}^{-\mathrm{j}\Omega}} + \sum_{k=-\infty}^{\infty} \pi \cdot \delta(\Omega + 2\pi \cdot k)$
$\mathrm{e}^{\mathrm{j}n\Omega_0}$	$\sum_{k=-\infty}^{\infty} 2\pi \cdot \delta(\Omega - \Omega_0 + 2\pi \cdot k)$
$\alpha^n \cdot u[n]$ with $\lvert\alpha\rvert < 1$	$\frac{1}{1-\alpha \cdot \mathrm{e}^{-\mathrm{j}\Omega}}$
$(n+1) \cdot \alpha^n \cdot u[n]$ with $\lvert\alpha\rvert < 1$	$\frac{1}{\left(1-\alpha \cdot \mathrm{e}^{-\mathrm{j}\Omega}\right)^2}$
$\mathrm{rect}_N[n] = \begin{cases} 1 \ , & \lvert n\rvert \le N_1 \\ 0 \ , & \lvert n\rvert > N_1 \end{cases}$; $N = 2N_1 + 1$	$\frac{\sin\left((0.5+N_1) \cdot \Omega\right)}{\sin\left(\Omega/2\right)}$; $\Omega \ne k \cdot 2\pi$; $k \in \mathbb{Z}$ $1 + 2 \cdot N_1$; $\Omega = k \cdot 2\pi$; $k \in \mathbb{Z}$
$\frac{\Omega_\mathrm{c}}{\pi} \cdot \frac{\sin\left(n \cdot \Omega_\mathrm{c}\right)}{n \cdot \Omega_\mathrm{c}}$ for $-\infty < n < \infty$	$\begin{cases} 1 \ , & 0 \le \lvert\Omega\rvert \le \Omega_\mathrm{c} \\ 0 \ , & \Omega_\mathrm{c} < \lvert\Omega\rvert \le \pi \end{cases}$

Properties

Theorem	Time domain	Transform
Symmetry for	$x[n]=x_{\mathrm{ev}}[n]+x_{\mathrm{od}}[n]$	$X(\mathrm{e}^{\mathrm{j}\Omega})=X_{\mathrm{Re}}(\mathrm{e}^{\mathrm{j}\Omega})+\mathrm{j}\cdot X_{\mathrm{Im}}(\mathrm{e}^{\mathrm{j}\Omega})$
real	$x_{\mathrm{ev}}[n]$	$X_{\mathrm{Re}}(\mathrm{e}^{\mathrm{j}\Omega})$
sequences	$x_{\mathrm{od}}[n]$	$\mathrm{j}\cdot X_{\mathrm{Im}}(\mathrm{e}^{\mathrm{j}\Omega})$
		$X(\mathrm{e}^{\mathrm{j}\Omega})=X^*(\mathrm{e}^{-\mathrm{j}\Omega})$
		$X_{\mathrm{Re}}(\mathrm{e}^{\mathrm{j}\Omega})=X_{\mathrm{Re}}(\mathrm{e}^{-\mathrm{j}\Omega})$; $X_{\mathrm{Im}}(\mathrm{e}^{\mathrm{j}\Omega})=-X_{\mathrm{Im}}(\mathrm{e}^{-\mathrm{j}\Omega})$
		$\left\|X(\mathrm{e}^{\mathrm{j}\Omega})\right\|=\left\|X(\mathrm{e}^{-\mathrm{j}\Omega})\right\|$; $\arg\left\{X(\mathrm{e}^{\mathrm{j}\Omega})\right\}=-\arg\left\{X(\mathrm{e}^{-\mathrm{j}\Omega})\right\}$
Symmetry for	$x[n]=x_{\mathrm{Re}}[n]+\mathrm{j}\cdot x_{\mathrm{Im}}[n]$	$X(\mathrm{e}^{\mathrm{j}\Omega})=X_{\mathrm{cs}}(\mathrm{e}^{\mathrm{j}\Omega})+X_{\mathrm{ca}}(\mathrm{e}^{\mathrm{j}\Omega})$
complex	$x_{\mathrm{Re}}[n]$	$X_{\mathrm{cs}}(\mathrm{e}^{\mathrm{j}\Omega})=\frac{1}{2}\left\{X(\mathrm{e}^{\mathrm{j}\Omega})+X^*(\mathrm{e}^{-\mathrm{j}\Omega})\right\}$
sequences	$\mathrm{j}\cdot x_{\mathrm{Im}}[n]$	$X_{\mathrm{ca}}(\mathrm{e}^{\mathrm{j}\Omega})=\frac{1}{2}\left\{X(\mathrm{e}^{\mathrm{j}\Omega})-X^*(\mathrm{e}^{-\mathrm{j}\Omega})\right\}$
cs: conjugate symmetric	$x_{\mathrm{cs}}[n]$	$X(\mathrm{e}^{\mathrm{j}\Omega})=X_{\mathrm{Re}}(\mathrm{e}^{\mathrm{j}\Omega})$
ca: conjugate antisymmetric	$x_{\mathrm{ca}}[n]$	$X(\mathrm{e}^{\mathrm{j}\Omega})=\mathrm{j}\cdot X_{\mathrm{Im}}(\mathrm{e}^{\mathrm{j}\Omega})$
Time reversal	$x[-n]$	$X(\mathrm{e}^{-\mathrm{j}\Omega})$
Conjugation	$x^*[n]$	$X^*(\mathrm{e}^{-\mathrm{j}\Omega})$
Conjugation and time reversal	$x^*[-n]$	$X^*(\mathrm{e}^{\mathrm{j}\Omega})$
Linearity	$a\cdot x[n]+b\cdot y[n]$	$a\cdot X(\mathrm{e}^{\mathrm{j}\Omega})+b\cdot Y(\mathrm{e}^{\mathrm{j}\Omega})$
Time shifting	$x[n-n_0]$	$\mathrm{e}^{-\mathrm{j}n_0\Omega}\cdot X(\mathrm{e}^{\mathrm{j}\Omega})$
Frequency shifting	$\mathrm{e}^{\mathrm{j}n\Omega_0}\cdot x[n]$	$X(\mathrm{e}^{\mathrm{j}(\Omega-\Omega_0)})$
Differentiation	$n\cdot x[n]$	$\mathrm{j}\cdot\frac{\mathrm{d}}{\mathrm{d}\,\Omega}X(\mathrm{e}^{\mathrm{j}\Omega})$
Convolution	$x[n]*y[n]$	$X(\mathrm{e}^{\mathrm{j}\Omega})\cdot Y(\mathrm{e}^{\mathrm{j}\Omega})$
Multiplication	$x[n]\cdot y[n]$	$X(\mathrm{e}^{\mathrm{j}\Omega})*Y(\mathrm{e}^{\mathrm{j}\Omega})=\frac{1}{2\pi}\cdot\int_{-\pi}^{\pi}X(\mathrm{e}^{\mathrm{j}\theta})\cdot Y(\mathrm{e}^{\mathrm{j}(\Omega-\theta)})\cdot\mathrm{d}\,\theta$
Summation		$X(\mathrm{e}^{\mathrm{j}0})=\sum_{n=-\infty}^{+\infty}x[n]$
PARSEVAL's theorem		$\sum_{n=-\infty}^{\infty}x[n]\cdot y^*[n]=\frac{1}{2\pi}\cdot\int_{-\pi}^{\pi}X(\mathrm{e}^{\mathrm{j}\Omega})\cdot Y^*(\mathrm{e}^{\mathrm{j}\Omega})\cdot\mathrm{d}\,\Omega$

Discrete and Fast FOURIER Transform (DFT, FFT)

Definition

$$x[n]=\frac{1}{N}\cdot\sum_{k=0}^{N-1}X[k]\cdot e^{j2\pi kn/N} \quad \circ\!-\!\bullet \quad X[k]=\sum_{n=0}^{N-1}x[n]\cdot e^{-j2\pi kn/N}$$

$0\le n\le N-1$ $\qquad$ $0\le k\le N-1$

Example transform pairs

Time domain $x[n]$	Transform $X[k]$
$\delta[n]=\begin{cases}1, & n=0\\ 0, & 1\le n\le N-1\end{cases}$	$1,\ 0\le k\le N-1$
$1,\ 0\le n\le N-1$	$N\cdot\delta[k]=\begin{cases}N, & k=0\\ 0, & 1\le k\le N-1\end{cases}$
$\delta[n-m]=\begin{cases}1, & n=m \text{ for } 0\le m\le N-1\\ 0, & \text{otherwise}\end{cases}$	$e^{-j2\pi km/N}=W_N^{km}$ with $W_N=e^{-j2\pi/N}$
$e^{j2\pi\cdot nl/N}$, $0\le n\le N-1$, $0<l\le N-1$	$\begin{cases}N, & k=l\\ 0, & \text{otherwise}\end{cases}$
$\cos(2\pi\cdot nl/N)$, $0\le n\le N-1$, $0<l\le N-1$	$\begin{cases}N/2, & k=l\\ N/2, & k=N-l\\ 0, & \text{otherwise}\end{cases}$
$\sin(2\pi\cdot nl/N)$, $0\le n\le N-1$, $0<l\le N-1$	$\begin{cases}-j\cdot N/2, & k=l\\ j\cdot N/2, & k=N-l\\ 0, & \text{otherwise}\end{cases}$

Properties

Theorem	Time domain	Transform
Symmetry for	$x[n] = x_{\text{ce}}[n] + x_{\text{co}}[n]$	$X[k] = X_{\text{Re}}[k] + \text{j}\cdot X_{\text{Im}}[k]$
real	$x_{\text{ce}}[n]$	$X_{\text{Re}}[k]$; X_{Re} is circular even
sequences	$x_{\text{co}}[n]$	$\text{j}\cdot X_{\text{Im}}[k]$; X_{Im} is circular odd
ce: circular even		$X[k] = X^*\left[\langle -k\rangle_N\right]$
co: circular odd		$X_{\text{Re}}[k] = X_{\text{Re}}\left[\langle -k\rangle_N\right]$; $X_{\text{Im}}[k] = -X_{\text{Im}}\left[\langle -k\rangle_N\right]$
		$\lvert X[k]\rvert = \left\lvert X\left[\langle -k\rangle_N\right]\right\rvert$; $\arg\{X[k]\} = -\arg\left\{X\left[\langle -k\rangle_N\right]\right\}$
Symmetry for	$x[n] = x_{\text{Re}}[n] + \text{j}\cdot x_{\text{Im}}[n]$	$X[k] = X_{\text{Re}}[k] + \text{j}\cdot X_{\text{Im}}[k]$
complex	$x_{\text{Re}}[n]$	$X_{\text{ccs}}[k] = \frac{1}{2}\left\{X[k] + X^*\left[\langle -k\rangle_N\right]\right\}$
sequences	$\text{j}\cdot x_{\text{Im}}[n]$	$X_{\text{cca}}[k] = \frac{1}{2}\left\{X[k] - X^*\left[\langle -k\rangle_N\right]\right\}$
ccs: circular conjugate symmetric	$x_{\text{ccs}}[n]$	$X[k] = X_{\text{Re}}[k]$
cca: circular conjugate antisymmetric	$x_{\text{cca}}[n]$	$X[k] = \text{j}\cdot X_{\text{Im}}[k]$
Duality	$X[n]$	$N\cdot x\left[\langle -k\rangle_N\right]$
Conjugation	$x^*[n]$	$X^*\left[\langle -k\rangle_N\right]$
Conjugation and time reversal	$x^*\left[\langle -n\rangle_N\right]$	$X^*[k]$
Linearity	$a\cdot x[n] + b\cdot y[n]$	$a\cdot X[k] + b\cdot Y[k]$
Circular time shifting	$x\left[\langle n-n_0\rangle_N\right]$	$W_N^{kn_0}\cdot X[k]$; $W_N = \text{e}^{-\text{j}2\pi/N}$
Circular frequency shifting	$W_N^{-nk_0}\cdot x[n]$	$X\left[\langle k-k_0\rangle_N\right]$
Circular convolution	$\sum_{m=0}^{N-1} x[m]\cdot y\left[\langle n-m\rangle_N\right]$	$X[k]\cdot Y[k]$
Multiplication	$x[n]\cdot y[n]$	$\frac{1}{N}\cdot\sum_{m=0}^{N-1} X[m]\cdot Y\left[\langle k-m\rangle_N\right]$
PARSEVAL's theorem		$\sum_{n=0}^{N-1}\lvert x[n]\rvert^2 = \frac{1}{N}\cdot\sum_{n=0}^{N-1}\lvert X[k]\rvert^2$

LAPLACE Transform (LT) - unilateral

Definition

$$x(t) = \frac{1}{2\pi\,\mathrm{j}} \cdot \int_{\sigma-\mathrm{j}\infty}^{\sigma+\mathrm{j}\infty} X(s) \cdot \mathrm{e}^{st}\,\mathrm{d}s \quad \circ\!-\!\bullet \quad X(s) = \int_{0_-}^{\infty} x(t) \cdot \mathrm{e}^{-st}\,\mathrm{d}t\,;\ s = \sigma + \mathrm{j}\omega$$

Common transform pairs

Time domain $x(t)$	**Transform $X(s)$**	**ROC**
$\delta(t)$	1	All values of s
$u(t)$	$\frac{1}{s}$	$\mathrm{Re}\,\{s\} > 0$
$t \cdot u(t)$	$\frac{1}{s^2}$	
$\frac{1}{n!} \cdot t^n \cdot u(t)$	$\frac{1}{s^{n+1}}$	
$\mathrm{e}^{-at} \cdot u(t)$	$\frac{1}{s+a}$	$\mathrm{Re}\,\{s\} > -a$
$\frac{1}{n!} \cdot t^n \cdot \mathrm{e}^{-at} \cdot u(t)$	$\frac{1}{(s+a)^{n+1}}$	
$\sin(\omega_0 t) \cdot u(t)$	$\frac{\omega_0}{s^2+\omega_0^2}$	$\mathrm{Re}\,\{s\} > 0$
$\cos(\omega_0 t) \cdot u(t)$	$\frac{s}{s^2+\omega_0^2}$	$\mathrm{Re}\,\{s\} > 0$
$\mathrm{e}^{-at} \cdot \sin(\omega_0 t) \cdot u(t)$	$\frac{\omega_0}{(s+a)^2+\omega_0^2}$	$\mathrm{Re}\,\{s\} > -a$
$\mathrm{e}^{-at} \cdot \cos(\omega_0 t) \cdot u(t)$	$\frac{s+a}{(s+a)^2+\omega_0^2}$	$\mathrm{Re}\,\{s\} > -a$

Properties

Theorem	Time domain	Transform	ROC
	$x(t)$	$X(s)$	R_x
	$y(t)$	$Y(s)$	R_y
Conjugation	$x^*(t)$	$X^*(s^*)$	R_x
Linearity	$a \cdot x(t) + b \cdot y(t)$	$a \cdot X(s) + b \cdot X(s)$	$R_x \cap R_y$
Time shifting	$x(t - t_0)\,;\ t_0 > 0$	$\mathrm{e}^{-st_0} \cdot X(s)$	R_x
Frequency shifting	$\mathrm{e}^{s_0 t} \cdot x(t)$	$X(s - s_0)$	$R_x + \mathrm{Re}\ \{s_0\}$
Time scaling	$x(a \cdot t)\,;\ a > 0$	$\frac{1}{a} \cdot X\left(\frac{s}{a}\right)$	$\frac{R_x}{a}$
Frequency scaling	$\frac{1}{a} \cdot x\left(\frac{t}{a}\right)$	$X(a \cdot s)\,;\ a > 0$	
Differentiation in time domain	$\frac{\mathrm{d}^n}{\mathrm{d}t^n} x(t)$	$s^n \cdot X(s) - \sum_{i=1}^{n} s^{n-i} x^{(i-1)}(0-0)$	$R_x \in \mathrm{ROC}$
Integration in time domain	$\int_0^t x(\tau)\,\mathrm{d}\tau$	$\frac{1}{s} \cdot X(s)$	$(R_x \cap \mathrm{Re}\{s\} > 0) \in \mathrm{ROC}$
Differentiation in frequency domain	$(-t)^n \cdot x(t)$	$\frac{\mathrm{d}^n}{\mathrm{d}s^n} X(s)$	
Integration in frequency domain	$\frac{1}{t} \cdot x(t)$	$\int_s^\infty X(u)\,\mathrm{d}u$	
Convolution	$x(t) * y(t)$	$X(s) \cdot Y(s)$	$(R_x \cap R_y) \in \mathrm{ROC}$
Multiplication	$x(t) \cdot y(t)$	$X(s) * Y(s)$	$(R_x \cap R_y) \in \mathrm{ROC}$
Scalar product		$\int_{-\infty}^{\infty} x(t) \cdot y^*(t) \cdot \mathrm{d}t = \int_{-\infty}^{\infty} X(s) \cdot Y^*(s) \cdot \mathrm{d}s$	
Initial-value theorem		$\lim_{t \to 0_+} x(t) = \lim_{s \to \infty} s \cdot X(s)$	
Final-value theorem *Note:* Only single pole at $s = 0$ allowed for $X(s)$. No poles in the right half plane.		$\lim_{t \to \infty} x(t) = \lim_{s \to 0} s \cdot X(s)$	

z-Transform - bilateral

Definition

$$x[n]=\frac{1}{2\pi\,\mathrm{j}}\cdot\oint_{\mathrm{C}} X(z)\cdot z^{n-1}\,\mathrm{d}\,z \quad \circ\!-\!\bullet \quad X(z)=\sum_{n=-\infty}^{\infty} x[n]\cdot z^{-n}\;;\; z=r\cdot \mathrm{e}^{\mathrm{j}\Omega}$$

Common transform pairs

Time domain $x[n]$	Transform $X(z)$	ROC
$\delta[n]$	1	All values of z
$u[n]$	$\dfrac{1}{1-z^{-1}}$	$\|z\|>1$
$\alpha^n\cdot u[n]$	$\dfrac{1}{1-\alpha\cdot z^{-1}}$	$\|z\|>\|\alpha\|$
$n\cdot\alpha^n\cdot u[n]$	$\dfrac{\alpha\cdot z^{-1}}{\left(1-\alpha\cdot z^{-1}\right)^2}$	$\|z\|>\|\alpha\|$
$(n+1)\cdot\alpha^n\cdot u[n]$	$\dfrac{1}{\left(1-\alpha\cdot z^{-1}\right)^2}$	$\|z\|>\|\alpha\|$
$\cos(\Omega_0\cdot n)\cdot u[n]$	$\dfrac{1-\left(\cos\Omega_0\right)\cdot z^{-1}}{1-\left(2\cdot\cos\Omega_0\right)\cdot z^{-1}+z^{-2}}$	$\|z\|>1$
$\sin(\Omega_0\cdot n)\cdot u[n]$	$\dfrac{\left(\sin\Omega_0\right)\cdot z^{-1}}{1-\left(2\cdot\cos\Omega_0\right)\cdot z^{-1}+z^{-2}}$	$\|z\|>1$
$r^n\cdot\cos(\Omega_0\cdot n)\cdot u[n]$	$\dfrac{1-\left(r\cdot\cos\Omega_0\right)\cdot z^{-1}}{1-\left(2r\cdot\cos\Omega_0\right)\cdot z^{-1}+r^2\cdot z^{-2}}$	$\|z\|>\|r\|$
$r^n\cdot\sin(\Omega_0\cdot n)\cdot u[n]$	$\dfrac{\left(r\cdot\sin\Omega_0\right)\cdot z^{-1}}{1-\left(2r\cdot\cos\Omega_0\right)\cdot z^{-1}+r^2\cdot z^{-2}}$	$\|z\|>\|r\|$

Properties

Theorem	Time domain	Transform	ROC
	$x[n]$	$X(z)$	R_x
	$y[n]$	$Y(z)$	R_y
Conjugation	$x^*[n]$	$X^*(z^*)$	R_x
Time reversal	$x[-n]$	$X(1/z)$	$1/R_x$
Linearity	$a \cdot x[n] + b \cdot y[n]$	$a \cdot X(z) + b \cdot X(z)$	$R_x \cap R_y$
Time shifting	$x[n-n_0]$	$z^{-n_0} \cdot X(z)$	R_x, except possibly the point $z = 0$ or $z = \infty$
Multiplication by an exponential sequence	$\alpha^n \cdot x[n]$	$X(z/\alpha)$	$\lvert\alpha\rvert \cdot R_x$
Differentiation	$n \cdot x[n]$	$-z \cdot \frac{\mathrm{d}\,X(z)}{\mathrm{d}\,z}$	R_x, except possibly the point $z = 0$ or $z = \infty$
Convolution	$x[n] * y[n]$	$X(z) \cdot Y(z)$	$\left(R_x \cap R_y\right) \in \mathrm{ROC}$
Multiplication	$x[n] \cdot y^*[n]$	$\frac{1}{2\pi\,\mathrm{j}} \cdot \oint_{\mathrm{C}} X(v) \cdot Y^*\left(z^* / v^*\right) \cdot v^{-1}\,\mathrm{d}\,v$	$\left(R_x \cap R_y\right) \in \mathrm{ROC}$
Summation		$\sum_{n=-\infty}^{\infty} x[n] = X(1)$	
Initial-value theorem		$x[0] = \lim_{z\to\infty} X(z)$ if $x[n] = 0$ for $n < 0$	
Final-value theorem		$\lim_{n\to\infty} x[n] = \lim_{z\to 1}(z-1) \cdot X(z) < \infty$	
PARSEVAL's theorem		$\sum_{n=-\infty}^{\infty} x[n] \cdot y^*[n] = \frac{1}{2\pi\,\mathrm{j}} \cdot \oint_{\mathrm{C}} X(v) \cdot Y^*\left(\frac{1}{v^*}\right) \cdot v^{-1}\,\mathrm{d}\,v$	