PAS

Tuesday, 19. March 2024 10:23

Bonferroni:

① $P(A \cap B) \geq P(A) + P(B) - 1 \longleftarrow$ lower bound

$\min(P(A), P(B)) \longleftarrow$ Higher bound

$P(A \cap B)^c = P(A^c \cup B^c) \leq P(A^c) + P(B^c)$

$\cancel{1} - P(A \cap B) \leq \cancel{1} - P(A) + 1 - P(B)$

$P(A_1 \cap A_2 \cap A_3 \cdots \cap A_n) \geq P(A_1) + P(A_2) + \cdots P(A_n) - (n-1)$

$\min(P(A), P(B), P(C)) = \min(0.73, 0.56, 0.75)$

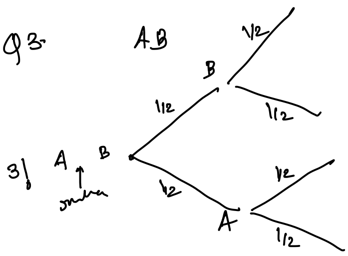
Higher bound. $= 0.56$

Lower bound: $P(A) + P(B) + P(C) - (n-1) \quad n = 3$

$\therefore 0.73 + 0.56 + 0.75 - 2$

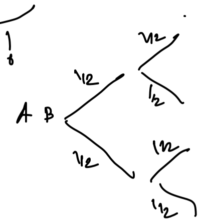
$= 0.04$

Q3. AB



ABC

3!



$\frac{1}{6} + \frac{1}{6} + \frac{1}{6}$

$= \frac{1}{3!} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{24} =$

$\{ 1/2 + 1/2 \} (\cdot 1/2)$

$= \frac{3}{6} = \frac{1}{2} \quad 1/6 \cdot 1/6 \cdot 1/6$

$= \frac{1}{6} \{ 1/2 + \frac{1}{2} \} = \frac{1}{6}$

$\frac{2400}{109000} = \frac{12}{545}$

$\frac{100}{109000} = \frac{1}{109}$

$\left(\frac{1}{100} + \frac{1}{99} \right) \frac{1}{109} \times \frac{12}{454}$

Hyper geometric Variable:

$$P_x(i) = \frac{\binom{D}{i}\binom{N-D}{n-i}}{\binom{N}{n}}$$

$D = 2400$

$N = 109000$

$n = 100$

$i = 2$

$\therefore P_x(2) = \binom{2400}{2}\binom{109000 - 2400}{100 - 2}$

$$P = \frac{\quad}{\binom{109000}{100}}$$

Where, $\binom{n}{k} = \frac{n!}{k!\,(n-k)!}$

$\therefore P_{\mathcal{X}}(2) =$

$$P_{binomial}(i) = \binom{n}{i} P_n^i (1-P_n)^{n-i}$$

$$= \binom{100}{2} (0.02)^2 (1-0.02)^{98}$$

$$= 4950 \times 0.0004 \times 0.1380$$

$$= 0.2734$$

$$\binom{100}{2} = \frac{100!}{2!\,(100-2)!} = \frac{100!}{2!\cdot 98!}$$

$$= \frac{100 \times 99 \times 98!}{2! \times 98!}$$

$$= \frac{100 \times 99}{2!}$$

$P_3 = 1 - q_3(0)$

$P_3 = 1 - 1/3 = 2/3$

$P_4 = 1 - q_4(0)$

$= 1 - \sum$

$$q_n(i) = \sum_{j=0}^{n} (-1)^j \binom{n}{j} (n-j)^{n-j}$$

$$q_3(1) = (-1)^1 \binom{3}{1} (3-1)^{3-1} + (-1)^2 \binom{3}{2} (3-2)^{3-2} + (-1)^3 \binom{3}{3} (3-3)^{3-3}$$

$$= -1 \left(\frac{3!}{2!}\right)(4) + 1\cdot\left(\frac{3!}{2!}\right)(1)^1 + (-1)\left(\frac{3!}{3!\cdot 1}\right)$$

$$= -3 \times 4 + 3 - 1$$

$$= -13 + 3$$

$$= -10$$

$$q_n(i) = (-1)^i \frac{D_{n-i}}{(n-i)!}$$

and $D_n = n! \sum_{k=0}^{n} \frac{(-1)^k}{k!}$

$$\therefore q_3(1) = (-1)^1 \frac{D_2}{2!}$$

and $D_2 = 2! \sum_{k=0}^{3} \frac{(-1)^k}{k!}$

$$= 2! \left\{ 1 - \frac{1}{1!} + \frac{1}{2!} \right\}$$

$$= 2 \left\{ 1 - 1 + \frac{1}{2} \right\} = 2 \cdot \left\{ \frac{1}{2} \right\} = 1$$

$$= 2.5$$

$$\therefore q_3(1) = (-1)^1 \cdot \frac{2 \cdot 1/2}{2!} = (-1)\cdot\frac{1}{2!} = -\frac{1}{2}$$

$$q_4(1) = (-1)^1 \frac{D_3}{3!} \qquad D_3 = 3! \sum_{k=0}^{3} \frac{(-1)^k}{k!}$$

$$= (-1)\cdot\frac{2}{6} \qquad = 3!\left\{ 1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} \right\}$$

$$= -\frac{1}{3} \qquad = 6 \left\{ \frac{1}{2} - \frac{1}{6} \right\} = 2$$

$$q_5(1) = (-1)^1 \frac{D_4}{4!} \qquad D_4 = 4! \left\{ 1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} \right\}$$

$$= (-1)\ \frac{9}{\ } \qquad = 24 \left\{ \frac{1}{2} - \frac{1}{6} + \frac{1}{24} \right\}$$

24 $\quad\quad\quad\quad$ $\left(\frac{1}{2} \quad \frac{}{6} \quad \frac{}{24}\right)$

$= -\frac{3}{8}$ $\quad\quad\quad\quad$ $= 24(3/8)$

$\quad\quad\quad\quad\quad\quad = 9$

$q_5(2) = (-1)^2 \frac{D_3}{3!}$

$= (+1)\frac{2}{6}$

$= \frac{1}{3}$

$q_2(0) = \frac{D_n}{n!}$ $\quad\quad\quad$ $D_2 = 1$

$q_2(1) = \frac{1}{2}$ $\quad\quad$ $\therefore P_2 = 1 - \frac{1}{2} = \frac{1}{2}$

$q_3(0) = \frac{D_3}{3!}$ $\quad\quad\quad$ $P_3 = 1 - \frac{1}{3} = \frac{2}{3}$

$= \frac{2}{6} > \frac{1}{3}$

$\frac{D_{n-1}}{n!}$

$\downarrow$

$\lim_{n\to\infty} q_n(1) = \lim_{n\to\infty} (-1)^1 \left\{1 - \frac{1}{1!} + \frac{1}{2!} - \cdots + \frac{(-1)^n}{n!}\right\} = \frac{\cancel{(n-1)!}/e}{\cancel{(n-1)!}}$

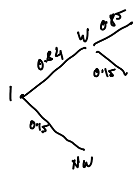
$= \frac{1}{e}$

$\lim_{n\to\infty} q_n(5) = \lim_{n\to\infty} (-1)^5 \left\{\frac{D_{n-5}}{(n-5)!}\right\} = \lim_{n\to\infty} \frac{(-1)}{(n-5)!}\left\{1 - \frac{1}{1!} + \frac{1}{2!} - \cdots + \frac{(-1)^{n-5}}{n-5}\right\}$

$= -\frac{1}{\cancel{(n-5)!}}\left\{\frac{\cancel{(n-5)!}}{e}\right\}$

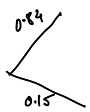
$= -\frac{1}{e}$

T4Q1:



$0.84 \times 0.85 \times 0.15 \times 0.15$

0.016



$0.85 + 0.84 \times 0.15$

0.976

0.84×0.15

$= 0.126$

0.15×0.85

$1 - 0.1275$

0.87

$P(D|\text{no water}) = 0.84$

$P(D|W) = 0.15$

water $= 0.85$

No water $= 0.15$

$A \to$ Alive $\quad\quad$ $A^c \to$ Dead

$B \to W$ $\quad\quad$ $B^c \to$ Forgot to water

Alive $= 1 - 0.15 \times 0.85 + 0.84 \times 0.15$

$= 0.2535$

$P(A|\text{No water}) = 0.18$

$P(A|W) = 0.86$

$P(B) = 0.91$

$= 0.15 + 0.84 \times 0.15$

$= (0.84 + 0.15) \cdot 0.15$

$P(A) = P(A|W) \times P(B) + P(A|\text{No water}) \times P(B^c)$

$= 0.06 \times 0.91 + 0.18 \times 0.09$

$= 0.798$

$$P(B|A^c) = \frac{P(A^c|B) \times P(B)}{P(A^c)}$$

$$= \frac{0.84 \times 0.15}{1 - 0.253}$$

$=$

$$P(B^c|A^c) = \frac{P(A^c|B^c) \times P(B^c)}{P(A^c)}$$

$$= \frac{0.82 \times 0.91}{1 - 0.799}$$

$= 0.2083$

$= 0.983$

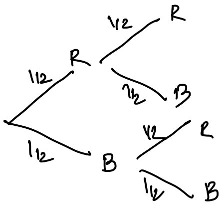
$(0.82 - 0.14) \times 0.09$

$0.09\,(1 - 0.799)$

0.018

R B

$1/2$ $1/2$



$1/2 \times 1/2$

$1/2 \times 1/2 \times 1/2$

$A \to$ cancer

$A^c \to$ Not cacer

$P(A^c) = 0.137$

$P(A) = 0.27$

$P(B) = 0.61$

$P(A) = P(A^c) \times P(B^c) + P(A) \times P(B)$

$= 0.137 \times 0.39 + 0.27 \times 0.61$

$= 0.218$

$= (0.137 - 0.27) \times 0.61$

$P_1 \times P_2 \times P_3 + P_4 \times P_5$

$= 1 \times 2 \times 3 + 4 \times 5$

$= 6 + 20$

$= 26$

$P_1 = P(A \setminus B,C) \cup P(B|A,C) \cup P(C|A,B)$

$= P(A) + P(B) + P(C)$

$= 0.49 + 0.07 + 0.38$

$P = 0.94$

$P_2 = P(A) + P(B) + P(C) - P(A,B) - P(B,C) - P(A,C) + P(A,B,C)$

$= P(A \cup B \cup C)$

$= A + B + C - AB - BC - AC + ABC$

$= 0.94 - 0.0343 - 0.0266 - 0.1862 + 0.013034$

$= 0.7059$

$$P = \binom{4}{4}(0.903)^4 + \binom{4}{3}(0.903)^3(1 - 0.903) + \binom{4}{2}(0.903)^2(1 - 0.903)^2$$

$= 1(0.903)^4 + 4(0.903)^3(0.097) + 12(0.903)^2(0.097)^2$

$= 1-$

$= 0.0426$

$$P(x=2) = \binom{4}{2}(0.903)^2(1-0.903)^2 + \binom{4}{3}(0.903)^3(1-0.903) + \underbrace{\binom{4}{4}}_{1}(0.903)^3\underbrace{(1-0.903)^0}_{1}$$

$= 0.9966$

$P(\overline{X}) = 0.013$

TS Q4:

$\phi_x(t) = ((1-p)+p.e^t)^n$

$\phi_x'(t) = p.e^t.n.((1-p)+p.e^t)^{n-1}$

$\phi_x''(t) = p.e^t.n((1-p)+p.e^t)^{n-1} + n.p.e^t.p.e^t.(n-1)((1-p)+p.e^t)^{n-2}$

$\therefore E(x) = \phi_x'(0) = p.n.((1-\cancel{p})+\cancel{p}.1)^{n-1}$

$= p.n.(1)^{n-1}$

$= p.n$

$\therefore E(x^2) = \phi_x''(0) = p.n. + np.p.(n-1)$

$= pn(1+p(n-1))$

$Var(x) = E(x^2) - (E(x))^2$

$= p.n.(1+p(n-1)) - p^2n^2$

$= p.n.(1+p.(n-1) - p.n)$

$= p.n(1+\cancel{pn} - p - \cancel{pn})$

$= p.n(1-p)$

$=$

TS Q5:

$E(x) = n.p$ $\quad Var(x) = 4.56$

$11.4 = n.p$ $\quad Var(x) = np(1-p) = 4.56$

$\therefore n = \frac{11.4}{0.6}$ $\quad np - np^2 = 4.56$

$\boxed{n=19}$ $\quad 11.4 - 11.4.p = 4.56$

$1-p = 4.56/11.4$

$p = 1-0.4$

$\boxed{p=0.6}$

$\therefore Pr(x=6) = \binom{19}{6}.p^6.(1-p)^{19-6}$

$= 0.0084$

$Pr(x>13) = Pr(14) + Pr(15) + Pr(16) + Pr(17) + Pr(18) + Pr(19)$

$= F(x) = \sum_{i=14}^{19}\binom{19}{i}.(0.6)^{14}.(1-0.6)^{19-14}$

$F(x) = \sum_{i=14}^{19}\binom{19}{i}(0.6)^i(1-0.6)^{19-i}$

$= 0.1629$

① $p=?$

$$P(4) = \binom{20}{4} p^4 (1-p)^{16}$$

$$0.207 = \binom{20}{4} p^4 (1-p)^{16}$$

$$\frac{0.207}{4845} = p^4 (1-p)^{16}$$

$$4.272 \times 10^{-4} = p^4 (1-p)^{16}$$

$$\sqrt[4]{4.272 \times 10^{-4}} = p(1-p)^4$$

$$0.0808 = p(1-p)^4$$

T5Q6:

$$P_r(i+1) = \frac{p}{1-p} \cdot \frac{n-i}{i+1} \cdot P_r(i) \quad \text{where } i = 1, 2, 3 \ldots, n-1$$

let start with $i = 0$:

$$P_r(0+1) = \frac{p}{1-p} \cdot \frac{n}{1} \cdot P_r(0)$$

$$= \frac{p}{1-p} \cdot n \cdot \binom{n}{0} \cdot p^0 \cdot (1-p)^{n-0}$$

$$= \frac{p}{(1-p)} \cdot n \cdot 1 \cdot 1 \cdot (1-p)^n$$

$$= \frac{p \cdot n}{\cancel{(1-p)}} \cdot \cancel{(1-p)^n}$$

$$P_r(0+1) = p \cdot n \cdot (1-p)^{n-1}$$

$$P_r(1) = \binom{n}{1} \cdot p^1 (1-p)^{n-1}$$

Generalysing:

$$P_r(i+1) = \frac{p}{1-p} \cdot \frac{n-i}{i+1} \cdot P_r(i)$$

$$= \frac{p}{1-p} \cdot \frac{n-i}{i+1} \cdot \frac{n!}{i!(n-i)!} \cdot p^i \cdot (1-p)^{n-i}$$

$$= \frac{n-i}{i+1} \cdot \frac{n!}{i!(n-i)!} \, p^{i+1} \cdot (1-p)^{n-i-1}$$

$$= \frac{n!}{(n-i-1)!(i+1)!} \cdot p^{i+1} \cdot (1-p)^{n-(i+1)}$$

$$P_r(i+1) = \binom{n}{i+1} \cdot p^{i+1} \cdot (1-p)^{n-(i+1)}$$

T5Q7:

①

$$\begin{array}{cc}
 & \begin{array}{ccc} y=1 & y=2 & x=3 \end{array} \\
\begin{array}{c} x=1 \\ x=2 \\ \to 3 \end{array} &
\begin{pmatrix} \frac{3}{50} & \frac{21}{100} & \frac{3}{100} \\ \frac{3}{25} & \frac{21}{50} & \frac{3}{50} \\ 1/50 & 7/100 & 1/100 \end{pmatrix}
\begin{array}{l} \to P_x(1) = \sum_y P_{xy}(1,y) \\ \to P_x(2) \\ \to P_x(4) \end{array} \\
 & \begin{array}{ccc} P_y(-1) & P_y(0) & P_y(1) \end{array}
\end{array}$$

$$Z = X + Y$$

②

$$Z = \{1+(-1), 1+0, 1+1, \; 2+(-1), 2+0, 2+1, \; 4+(-1), 4+0, 4+1\}$$

$$Z = \{0, 1, 2, 1, 2, 3, 3, 4, 5\}$$

$$Z = \{0, 1, 2, 3, 4, 5\}$$

③ for $Z = XY$

$$Z = \{1 \cdot (-1), 1(0), 1 \cdot (1), \; 2(-1), 2(0), 2(1), \; 4(-1), 4(0), 4(1)\}$$

$$\therefore Z = \{-1, 0, 1, -2, 0, 2, -4, 0, 4\}$$

$$\therefore Z = \{-4, -2, -1, 0, 1, 2, 4\}$$

Now, $P_Z(0) = (X_1 + Y_2) = 3/50$

$$P_Z(1) = 21/100 + 3/25 = 33/100$$

$$P_Z(-4) = 1/50$$

$$P_Z(-2) = 3/25$$

$$P_Z(-1) = 3/50$$

$P_2(2) = 3/100 + 21/50 = 9/20$

$P_2(3) = 3/50 + 1/50 = 2/25$

$P_2(4) = 7/100$

$P_2(5) = 1/100$

$P_2(0) = 21/100 + 21/50 + 7/100 = 7/10$

$P_2(1) = 3/100$

$P_2(2) = 3/50$

$P_2(4) = 1/100$

(2) $E(X) = P_X(1)\cdot X_1 + P_X(2)\cdot X_2 + P_X(4)\cdot X_3$

$= \frac{3}{10}\cdot 1 + \frac{3}{5}\cdot 2 + \frac{1}{10}\cdot 4$

$= \frac{3}{10} + \frac{6}{5} + \frac{4}{10} = \frac{19}{10}$

$E(Y) = P_Y(1)\cdot Y_1 + P_Y(2)\cdot Y_2 + P_Y(3)\cdot Y_3$

$= \frac{1}{5}\cdot(-1) + \frac{7}{10}\cdot(0) + \frac{1}{10}(1)$

$= -\frac{1}{5} + \frac{1}{10} = \frac{-1}{10}$

$E(Z) = E(X+Y) = E(X) + E(Y) = \frac{19}{10} - \frac{1}{10} = \frac{18}{10}$

$E(X\cdot Y) = E(P) = E(X)\cdot E(Y) = -\frac{19}{100}$

TSQ 8:

$$P_{XY}(i,j) = \begin{cases} \frac{3^{j-1}\cdot e^{-3}}{j!} & \text{for } i=1, j\geq 0 \\ c\cdot\frac{6^{j-1}\cdot e^{-6}}{j!} & \text{for } i=2, j\geq 0 \\ 0 & \text{0therwise} \end{cases}$$

$\sum_{i,j} P_{XY}(i,j) = 1 \quad \forall i,j$

$$\therefore \underbrace{\sum_{j=0}^{\infty} \frac{3^{j-1}\cdot e^{-3}}{j!}}_{j\to\infty} + c\cdot\underbrace{\frac{6^{j-1}\cdot e^{-6}}{j!}}_{j\to\infty} = 1$$

Calculator → Any big number

$$\sum_{j=0}^{15} \frac{3^{j-1}\cdot e^{-3}}{j!} + c\sum_{j=0}^{20} \frac{6^{j-1}\cdot e^{-6}}{j!} = 1$$

$1/3 + c\cdot\frac{1}{6} = 1$

$0.33 + c\cdot 0.166 = 1$

$c = \frac{1-0.33}{0.166}$

$c = 4.03$

$\boxed{c=4}$

$P_X(1) = \sum_{j=0}^{15} \frac{3^{j-1}\cdot e^{-3}}{j!} = 1/3$

$P_X(2) = 4\cdot\sum_{j=0}^{15} \frac{6^{j-1}\cdot e^{-3}}{j!} = 4\cdot 1/6 = 2/3$

$P_Y(j) = \frac{3^{j-1}\cdot e^{-3}}{j!} + 4\cdot\frac{6^{j-1}\cdot e^{-6}}{j!}$

(3) $P_{XY}(1,1) = \frac{3^{(1-1)}\cdot e^{-3}}{1!} = e^{-3}$

$P_X(1)\cdot P_Y(1) = \frac{1}{3}\cdot\left\{ \frac{3^{(1-1)}\cdot e^{-3}}{j!} + 4\,\frac{6^{(1-1)}\cdot e^{-6}}{j!} \right\}$

$= \frac{1}{3}\left\{ \frac{e^{-3}}{j!} + \frac{4\cdot e^{-6}}{j!} \right\}$

T6Q1:

$Pr(X = \pm 1) = Pr(X = \pm 2) = 1/4$

and $Y = |X|$

$\therefore X = \{-2, -1, 1, 2\}$

$Y = \{|-2|, |-1|, |1|, |2|\}$

$Y = \{2, 1, 1, 2\} =$

Now, $Z := X \cdot Y$ for uncorrelated no need to do matrix multiplication.

$= \{(-2)\cdot(1) \quad \cancel{(-2)\cdot(2)}$
$(-1)\cdot\cancel{(1)} \quad (-1)\cdot(2)$
$\cancel{(1)\cdot(1)} \quad (1)(2)$
$\cancel{(2)}\cdot(1) \quad (2)\cdot(2)\} = \{-4, -1, 1, 4\}$

$[z_1, z_2, z_3] = [-4, -1, 1, 4]$

Now, $[P_z(z_1), P_z(z_2) \ldots] = [Pr(x=-2), Pr(x=-1), Pr(x=1), Pr(x=2)]$

$= [1/4, 1/4, 1/4, 1/4]$

$E(X) = \sum_{i=1}^{4} P_X(i) \cdot x_i$

$= P_X(1)\cdot x_1 + P_X(2)\cdot x_2 + P_X(3)\cdot x_3 + P_X(4)\cdot x_4$

$= \frac{1}{4}\cdot(-2) + \frac{1}{4}(-1) + \frac{1}{4}(1) + \frac{1}{4}(-2)$

$E(X) = 0$

$E(Y) = \sum_{i=1}^{2} P_Y(i) \cdot x_i = P_Y(1)\cdot y_1 + P_Y(2) y_2$

$= \frac{1}{2}(2) + \frac{1}{2}(1)$

$E(Y) = 3/2$

$E(Z) = E(X \cdot Y) = E(X) \cdot E(Y) = 0$

$Cov(X, Y) = E(XY) - E(X)\cdot E(Y)$

$= 0 - 0$

$= 0$

$Pr(X=1, Y=1) = 1/4$

$Pr(X=1)\cdot Pr(Y=1) = \frac{1}{4}\cdot \frac{1}{2} = 1/8$

T6Q2:

Because the probability of Bernoulli $= 1/2$, there are two choices.

So, we can write,

$X = \{0, 1\}$

$Y = \{0, 1\}$

and given that, $Z := X + Y$

$\therefore Z = [0+0, 1+0, 0+1, 1+1]$

$= [0, 1, 1, 2]$

$\therefore E(X) = Pr(x_1)\, x_1 + Pr(x_2)\cdot x_2$

$= \frac{1}{2}\cdot 0 + \frac{1}{2}\cdot 1$

$E(X) = \frac{1}{2}$

$E(X^2) = Pr(x_1)\cdot x_1^2 + Pr(x_2) x_2^2$

$= \frac{1}{2}\cdot 0^2 + \frac{1}{2}\cdot 1^2$

$= 1/2$

$E(Y^2) = P_Y(y_1)\cdot y_1^2 + P_Y(y_2)\, y_2^2$

$= \frac{1}{2}\cdot 0^2 + \frac{1}{2}\cdot 1^2$

$= 1/2$

$E(Z) = P_Z(z_1)\cdot z_1 + P_Z(z_2)\cdot z_2 + P_Z(z_3)\cdot z_3$

$= \frac{1}{4}\cdot 0 + \frac{2}{4}\cdot 1 + \frac{1}{4}\cdot 2$

$E(Z) = 1$

$E(Z^2) = \frac{1}{4}\cdot 0^2 + \frac{2}{4}\cdot 1^2 + \frac{1}{4}\cdot 2^2 = 3/2$

$E(Y) = 1/2$

Not for independent

X & Z are not independent !

$\therefore E(X \cdot Z) = E(X)\cdot E(Z) = 1/2 \cdot 1 = 1/2$

$E(X\cdot Z) = E(X \cdot (X+Y)) =$

$$Cov(X,Z) = E(X\cdot Z) - E(X)\cdot E(Z) \qquad\qquad = E(X^2) + E(X\cdot Y)$$

$$= 3/4 - 1/2 \cdot 1 \qquad\qquad = 1/2 + 1/2 \cdot 1/2$$

$$= 1/4 \rightarrow \text{Correlated} \qquad\qquad E(X\cdot Z) = 3/4$$

$$\rho_{XZ} := E\left(\left(\frac{X-m_X}{\sigma_X}\right)\left(\frac{Y-m_Y}{\sigma_Y}\right)\right) = \frac{Cov(X,Y)}{\sigma_X \sigma_Y}$$

where, $\sigma_X = \sqrt{Var}$ $\qquad Var = E(X^2) - E(X)^2$

$\sigma_X = \sqrt{1/4}$ $\qquad = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$

$\sigma_X = 1/2$

$\sigma_Z = \sqrt{Var_Z}$ $\qquad Var_Z \xrightarrow{Z} E(Z^2) - E(Z)^2$

$\sigma_Z = \sqrt{1/2}$ $\qquad = 3/2 - 1 = 1/2$

$$\therefore \rho_{XZ} = \frac{1/4}{\sigma_X \cdot \sigma_Y} = \frac{1/4}{1/2\sqrt{1/2}} = \frac{\sqrt{2}}{2}$$

T6Q3:

$$\phi_X(t) = \frac{pe^t}{1-(1-p)e^t} \qquad \text{for } t < \ln\left(\frac{1}{1-p}\right)$$

Fucking quotient rule. Remember good and day!

$$y = \frac{u}{v}$$

$$y' = \frac{u' \cdot v - v' \cdot u}{v^2}$$

$$\therefore \phi_X'(t) = \frac{p\cdot e^t \cdot (1-(1-p)\cdot e^t) - (0-(1-p)\cdot e^t)\cdot p\cdot e^t}{(1-(1-p)\cdot e^t)^2}$$

$$= \frac{p\cdot e^t \left\{1 - \cancel{(1-p)\cdot e^t} + \cancel{(1-p)\cdot e^t}\right\}}{(1-(1-p)\cdot e^t)^2}$$

$$= \frac{pe^t}{(1-(1-p)\cdot e^t)^2}$$

$$\phi_X'(0) = \frac{p}{(1-(1-p))^2} = \frac{p}{(1-1+p)^2} = \frac{p}{p^2} = 1/p$$

$$\phi_X''(t) = \frac{p\cdot e^t (1-(1-p)\cdot e^t)^2 - 2(1-(1-p)\cdot e^t)\cdot(0-(1-p)\cdot e^t)\cdot p\cdot e^t}{(1-(1-p)\cdot e^t)^4}$$

$$= \frac{p\cdot e^t \left\{(1-(1-p)\cdot e^t) + \cancel{2}(1-p)\cdot e^t\right\} \cancel{(1-(1-p)\cdot e^t)}}{(1-(1-p)\cdot e^t)^{\cancel{4}}}$$

$$= pe^t \frac{(1+(1-p)\cdot e^t)}{(1-(1-p)\cdot e^t)^3}$$

$$\phi_X''(0) = \frac{p\cdot\{1+(1-p)\}}{(1-(1-p))^3} = \frac{p(2-p)}{(-p)^3}$$

$$= \frac{-2+p}{p^2}$$

T6Q4:

Poisson Distribution

①

$$\cancel{e^{-\lambda}} \cdot \frac{\cancel{\lambda^3}}{\cancel{3!}} = \cancel{e^{-\lambda}} \cdot \frac{\cancel{\lambda^4}}{\cancel{4!}\ 4}$$

$$\frac{\lambda}{4} = 1$$

$$\boxed{\lambda = 4}$$

② $\cancel{e^{-\lambda}} \cdot \dfrac{\cancel{\lambda^{i}}}{\cancel{i!}} > \cancel{e^{-\lambda}} \cdot \dfrac{\cancel{\lambda^{i+1}}}{\cancel{(i+1)!}}$

$\dfrac{1}{i+1} > \lambda$

$\dfrac{1}{i+1} > \lambda$

This inequality is only possible if λ is less than 1.

Ex: $\dfrac{1}{i+1} > \lambda$ True if $\lambda = -2$

T7Q5:

A → Prob of drug beneficial ($\lambda = 1.2$)

B → prob. of 0 colds in a year.

P(A|B) → prob of drug is beneficial given 0 cold in a year.

Using byes fune.

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$

P(B|A) → Prob of 0 cold if drug is beneficial

$P(A) \rightarrow 0.78$

$P(B) = P(B|A) \cdot P(A) + P(B|\bar{A}) \cdot P(\bar{A})$

$= e^{-1.2} \cdot 0.78 + e^{-2.2} \cdot 0.22$

$$P(A|B) = \frac{e^{-1.2} \cdot 0.78}{e^{-1.2} \cdot 0.78 + e^{-2.2} \cdot 0.22}$$

$P(A|B) = 0.9059$

T6Q6:

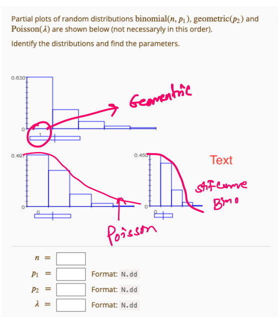
$$P_X(i) = e^{-\lambda} \cdot \frac{\lambda^{i}}{i!}$$

① $P_1 = 1 - \sum_{i=0}^{130} e^{-123.55} \cdot \dfrac{(123.55)^{i}}{i!}$

$= 1 - 16185.05$

$= 16$

T6Q8:


Partial plots of random distributions binomial(n, p_1), geometric(p_2) and Poisson(λ) are shown below (not necessaryly in this order).

Identify the distributions and find the parameters.

n =

p_1 = Format: N.dd

p_2 = Format: N.dd

λ = Format: N.dd

① Bino: $P_X(i) = \binom{n}{i} p^{i} (1-p)^{n-i}$

given, $P_X(0) = 0.450$

$\therefore 0.45 = \binom{n}{0} p^{0} (1-p)^{n}$

$0.45 = 1 \cdot 1 \cdot (1-p)^{n}$

$$n = \frac{\ln(P(x=0))}{\ln(1-P)}$$

$\therefore 0.45 = \left(1 - \dfrac{0.45}{n}\right)^{n}$

$P_X(0) = 1 - P$

$0.45 = 1 - nP$

② Poisson:

$$P_x(i) = e^{-\lambda} \cdot \frac{\lambda^i}{i!}$$

$$0.497 = e^{-\lambda} \cdot \frac{\lambda^i}{i!} \quad i=0$$

$$0.497 = e^{-\lambda} \cdot \frac{1}{1}$$

$$0.497 = e^{-\lambda}$$

$$\ln(0.497) = -\lambda$$

$$\lambda = 0.6991$$

③ Geometric.

$$P_x(i) = (1-p)^{i-1} \cdot p$$

$$P_x(0) = (1-p)^{-1} \cdot p$$

$$0.630 = \frac{p}{1-p}$$

$$0.630 - 0.630p = p$$

$$1.630\,p = 0.630$$

$$p = \frac{0.630}{1.630}$$

$$p = 0.39$$

$$P_X(1) = (1-p)^0 \cdot p = 1 \cdot 0.39 = 0.39$$

$$P_x(2) = (1-p)^1 \cdot p = (1-0.39) \cdot 0.39 = 0.2379$$

T7Q1:

$$P_r(49\% \text{ Mn } 51\%) \geqslant 98\%$$

$$\frac{\sigma^2}{n\,\varepsilon^2} \leq \frac{2}{100}$$

$$\sigma^2 = 1/4, \quad \varepsilon = 1\% = 0.01$$

$$\therefore \frac{1/4}{0.01 \cdot n} \leq \frac{2}{100}$$

$$2n \cdot 0.01 \leq \frac{100}{4}$$

$$n \leq \frac{25}{2 \times 0.01}$$

$$n \leq 12.5 \times 100$$

$$n = 12500$$

T7Q4.

$$\mu = E(x) = \frac{a+b}{2} = 3/2$$

$$\sigma = \sqrt{Var(x)} = \sqrt{\frac{(b-a)^2}{12}}$$

$$\sigma = \frac{b-a}{\sqrt{12}} = \frac{1}{\sqrt{12}} \qquad Var =$$

$$\left\{ \begin{array}{l} \text{put} \\ a=1,\ b=2 \end{array} \right.$$

① $P_r\left(\mu - 0.5\sigma < x < \mu + 0.5\sigma\right) = P_r\left(\frac{a+b}{2} - \frac{1}{2}\left(\frac{a+b}{\sqrt{12}}\right) < x < \frac{a+b}{2} + \frac{1}{2}\left(\frac{a+b}{\sqrt{12}}\right)\right)$

$$= P_r\left(\frac{3}{2} - \frac{1}{2}\cdot\frac{1}{\sqrt{12}} < x < \frac{3}{2} + \frac{1}{2}\cdot\frac{1}{\sqrt{12}}\right) = P_r\left(1.067 < x < 1.933\right)$$

$$\therefore = \int_{3/2 - 1/2 \cdot 1/\sqrt{12}}^{3/2 + 1/2 \cdot 1/\sqrt{12}} f(x)\,dx = \left[\frac{x-1}{1}\right] = \left[\left(\frac{3}{2} + \frac{1}{2}\frac{1}{\sqrt{12}} - 1\right) - \frac{3}{2} + \frac{1}{2}\frac{1}{\sqrt{12}} + 1\right)\right]$$

$$= \sqrt{3}/6$$

$$P_r(x) = \sqrt{3}/6$$

T7Q5:

$$F_x = \begin{cases} 0 & x \leq a \\ \frac{x-a}{b-a} & \text{for } a < x \leq b \\ 1 & \text{for } x > b \end{cases}$$

Given Interval $= [a, b]$

$\therefore F_x(P) = \frac{P - a}{b-a}$

$F_x(P)(b-a) + a = P$

$P(b-a) + a = F_x(P)^{-1}$

quantile:

1st quantile $F_x^{-1}(P) = F_x^{-1}(1/4) = \frac{1}{4}.(b-a)+a$

median $F_x^{-1}(1/2) = \frac{1}{2}(b-a)+a$

3rd quantile $F_x^{-1}(3/4) = \frac{3}{4}(b-a)+a$

T7Q6:

$y := \ln(1/x)$

Using definition of density function:

① $\lim_{x \to -\infty} F_X(x) = 0$

$\therefore$ for $x \leq 0$: $F_Y(x) = 0$

② for $x > 0 \Rightarrow 0 < x < \infty$ $\Pr(a < X \leq b) = F_X(b) - F_X(a)$

$\therefore F_Y(x) := \Pr(Y \leq x)$

$= \Pr(\ln(1/X) \leq x)$

$= \Pr\left(\frac{1}{X} \leq e^x\right)$

$= \Pr(X^{-1} \leq e^x)$

$= \Pr(e^{-x} \leq X)$

$= 1 - \Pr(X \leq e^{-x})$

$= 1 - F_Y(e^{-x})$ uniform $(\underset{a}{0}, \underset{b}{1}]$

$= 1 - \frac{e^{-x} - a}{b-a}$

$= 1 - \frac{e^{-x} - 0}{1-0}$

$F_Y(x) = 1 - e^{-x}$

③ $f_Y(x) = F_Y'(x) = \frac{d}{dx}(0) = 0$; $x<0$

④ $f_Y(x) = F_Y'(x) = e^{-x} = f_Y(x)$; $x > 0$

T7Q7:

$\mu = E(X) = \frac{1}{\lambda}$

$\sigma = \sqrt{Var(X)} = \sqrt{1/\lambda^2}$

$\sigma = \frac{1}{\lambda}$

① $\therefore P(X < \mu) = P(X < 1/\lambda)$

$= F_X(1/\lambda)$

$F_X(x) = (1 - e^{-\lambda \cdot x})$

$\therefore F_X(1/\lambda) = (1 - e^{-\lambda \cdot 1/\lambda}) = 1 - e^{-1}$

$= 0.6321$

② $\Pr(\mu - \sigma < X < \mu + \sigma) = \Pr\left(\frac{1}{\lambda} - \frac{1}{\lambda} < X < \frac{1}{\lambda} + \frac{1}{\lambda}\right)$

$= \Pr(0 < X < 2/\lambda)$

$\Rightarrow F_X(1) - F_X(0)$

$\Rightarrow (1 - e^{-\lambda \cdot 2/\lambda}) - (1 - e^{-\lambda \cdot 0}) = 1 - e^{-2} = 0.8647$

③ $\Pr(\mu - 2\sigma < X < \mu + 2\sigma) =$

$= \Pr\left(\frac{1}{\lambda} - 2\frac{1}{\lambda} < X < \frac{1}{\lambda} + 2\frac{1}{\lambda}\right)$

$= \Pr\left(\frac{-1}{\lambda} < X < \frac{3}{\lambda}\right)$

$\therefore F_X(3/\lambda) - F_X(-1/\lambda) = (1 - e^{-\lambda \cdot 3/\lambda}) - \overbrace{(1 - e^{-\lambda(-1/\lambda)})}^{0}$

$= 1 - e^{-3}$ (zero)

$= 1 - e^{-3}$

$= 0.9502$

④ $P_r(\mu - 3\sigma < X < \mu + 3\sigma)$

$F_X\left(\frac{1}{\lambda} + \frac{3}{\lambda}\right) - F_X\left(\frac{1}{\lambda} - 3 \cdot \frac{1}{\lambda}\right) = F_X\left(\frac{4}{\lambda}\right) - F_X\left(-\frac{2}{\lambda}\right)$

$\uparrow 0$

$= 1 - e^{-\lambda \cdot (4/\lambda)} - 0$

$= 0.9817$

T7Q8:

$E(X) = \frac{1}{\lambda} = 9.5$

$\therefore \lambda = \frac{1}{9.5}$

① $P(X > 12.5) = 1 - P(X \le 12.5)$

$F_X(12.5) = 1 - e^{-\lambda x} = 1 - e^{-\frac{1}{9.5} \cdot 12.5}$

$= 0.7317$

$\therefore P(X > 12.5) = 1 - 0.7317$

$= 0.2683$

② $P(X > (17.5 - 5)) = 1 - P(X < 12.5)$

$=$

$= 0.7317$

T7Q9:

given $r = \frac{1}{382} \cdot s^{-1} = \frac{1}{382 s} = \frac{1}{382/60} = \lambda$

① $P_r(4 < X \le 8) = F_X(8) - F_X(4)$

$= \left(1 - e^{-\frac{1}{382/60} \cdot 8}\right) - \left(1 - e^{-\frac{1}{382/60} \cdot 4}\right)$

$= 0.8181$

② $F_X(1/2) = 1 - e^{-1/382 \cdot 1/2}$ $\quad F_X^{-1}(x) = -\frac{\ln(1-x)}{\lambda}$

$= 0.0755$

$F_X^{-1}(1/2) = 0.9245 = 264.7822$

③ $F_X^{-1}(3/4) - F_X^{-1}(1/4) = 529.5644 - 109.8945$

$= 419.6698$

T7Q10:

$P_r(X > 1000) = 0.717$

$1 - P(X < 1000) = 0.717 \Rightarrow F_X(1000) = 0.717 - 1$

$1 - F_X(1000) = 1 - 0.717$ $\quad 1 - e^{-\lambda \cdot 1000} = 0.717 - 1$

$\cancel{1} - e^{-\lambda \cdot 1000} = \cancel{1} - 0.717$ $\quad - e^{-\lambda \cdot 1000} = -1.283$

$-e^{-\lambda \cdot 1000} = 0.717$ $\quad -\lambda \cdot 1000 = \ln(1.283)$

$-(-\lambda \cdot 1000) = \ln(0.717)$ $\quad \lambda = 0.000249$

$\lambda = \frac{\ln(0.717)}{1000}$

$\lambda = 0.000332$

$x = \frac{\ln(1 - F_X(x))}{-\lambda}$

$\lambda = -\frac{\ln(1 - F_X(x))}{x}$

$= -\frac{\ln(1 - 0.717)}{1000}$

$= 0.00126$ or 0.000332

$F_X(1000) = 0.289$

$1 - e^{-\lambda \cdot 1000} = 0.289$

$\lambda = \frac{\ln(1 - 0.289)}{-1000}$

0.00034

T8Q1:

$$f_{xy}(x,y) = \begin{cases} \frac{c}{(x+y+1)^3} & \text{if } x,y \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

$f(x,y) = f_x(x) \cdot f_y(y)$

for continuous,

$$F_{xy}(x,y) = \int_{-\infty}^{y} \int_{-\infty}^{x} f_{xy}(s,t)\, ds\, dt$$

$$= \int_0^y \int_0^x \frac{c}{(x+y+1)^3}\, ds \cdot dt$$

$x \to s$, $t \to y$

$$= c \int_0^y \int_0^x (s+t+1)^{-3}\, ds \cdot dt$$

$$= c \int_0^y \left\{ \frac{-1}{2} (s+t+1)^{-2} \right\}_0^x$$

$$= c \int_0^y \left\{ -\frac{1}{2} \left((x+t+1)^{-2} - (t+1)^{-2} \right) \right\}$$

$$= -\frac{c}{2} \left\{ \frac{-1}{1} (x+t+1)^{-1} - \frac{(-1)}{1} (t+1)^{-1} \right\}_0^y$$

$$= +\frac{c}{2} \left\{ (x+y+1)^{-1} - (y+1)^{-1} - (x+1)^{-1} + (1)^{-1} \right\}$$

$$F_{xy}(x,y) = \frac{c}{2} \left\{ (x+y+1)^{-1} - (y+1)^{-1} - (x+1)^{-1} + 1 \right\}$$

$$F_x(x) = \lim_{y \to \infty} F_{xy}(x,y) = \frac{c}{2} \left\{ 0 - 0 - \frac{1}{x+1} + 1 \right\}$$

$$\lim_{x \to \infty} F_x(x) = \frac{c}{2}$$

$$\left\{ \begin{array}{l} \lim_{x \to \infty} F_x(x) = 1 \\ \lim_{x \to -\infty} F_x(x) = 0 \end{array} \right.$$

$$1 = \frac{c}{2}$$

$$\boxed{\therefore c = 2}$$

② $F_{xy}(x,y) = (x+y+1)^{-1} - (y+1)^{-1} - (x+1)^{-1} + 1$

$\therefore F_{xy}(2.1, 3.9) = 0.6162$

T8Q2:

① $E(x) = m$

$Var(x) = s^2 \quad > 0$

$x = m$

$Skew(x) := E\left(\left(\frac{x-m}{s}\right)^3 \right) \quad X = \left(\frac{x-m}{s}\right) \quad \therefore \frac{E[x]}{s}$

$E(1/s)$

$Skew(x) - \frac{E(x^3)}{s^3} = ?$

$E\left(\left(\frac{x-m}{s}\right)^3 \right) - E($

② $E(x^3) \Rightarrow E(x^n) = \frac{b^{n+1} - a^{n+1}}{(n+1)(b-a)}$

$$E(x^3) = \frac{b^4 - a^4}{4(b-a)}$$

③ $skew(x) = 0$

④ $E(x^n) = \frac{n!}{\lambda^n}$; $E(x^3) = \frac{3!}{\lambda^3}$

$$= \frac{6}{\lambda^3}$$

$4\ \lambda = c$

T8Q3:

$$f(x) = \frac{c}{2} \cdot e^{-c|x|}$$

① $F_x(x) = \int_{-\infty}^{x} f(s)\, ds \quad ; x \leq 0$

$$\therefore F_X(x) = \int_{-\infty}^{x} \frac{c}{2} \cdot e^{-c|s|} ds$$
$$= \frac{c}{2} \int_{-\infty}^{x} e^{cs} ds$$
$$= \frac{c}{2} \left\{ \frac{1}{c} \cdot e^{cs} \right\}_{-\infty}^{x}$$
$$= \frac{1}{2} \left\{ e^{cx} - \frac{1}{e^{\infty}} \right\}$$
$$F_X(x) = \frac{1}{2} \cdot e^{cx}$$

② For $x > 0$: $F_X(x) = ?$

$$F_X(x) = \int_{-\infty}^{0} f(s)\, ds + \int_{0}^{x} f(s) \cdot ds$$
$$= \int_{-\infty}^{0} \frac{c}{2} \cdot e^{-c|s|} ds + \int_{0}^{x} \frac{c}{2} \cdot e^{-c|s|} \cdot ds$$
$$= \frac{c}{2} \int_{-\infty}^{0} e^{cs} ds + \frac{c}{2} \int_{0}^{x} e^{-cs} ds$$
$$= \frac{c}{2} \left\{ \frac{1}{c} \cdot e^{cs} \right\}_{-\infty}^{0} + \frac{c}{2} \left\{ \frac{-1}{c} \cdot e^{cs} \right\}_{0}^{x}$$
$$= \frac{1}{2} \left\{ e^{0} - e^{-\infty} \right\} - \frac{1}{2} \left\{ e^{cx} - e^{0} \right\}$$
$$= \frac{1}{2} \cdot 1 - \frac{1}{2} \cdot \left\{ e^{cx} - 1 \right\}$$
$$= \frac{1}{2} \left\{ 1 - e^{cx} + 1 \right\}$$
$$= \frac{1}{2} \left\{ 2 - e^{cx} \right\}$$
$$F_X(x) = 1 - \frac{e^{cx}}{2}$$

③
$$E(X) = \int_{-\infty}^{\infty} x \cdot f_X(x)\, dx$$
$$= \int_{-\infty}^{\infty} x \frac{c}{2} \cdot e^{-c|x|} dx$$
$$= \int_{-\infty}^{0} x \cdot \frac{c}{2} e^{+c|x|} dx + \int_{0}^{\infty} x \cdot \frac{c}{2} \cdot e^{-c|x|} dx$$
$$= \frac{c}{2} \left\{ \left[x \cdot \frac{1}{c} \cdot e^{cx} \right] - \int_{-\infty}^{0} \frac{1}{c} \cdot e^{cx} dx \right\} + \frac{c}{2} \left\{ \left[x \cdot \frac{-1}{c} \cdot e^{-cx} \right] - \int_{0}^{\infty} \frac{-1}{c} \cdot e^{-cx} dx \right\}$$
$$= \frac{1}{2} \left\{ \left[x \cdot e^{cx} \right]_{-\infty}^{0} - \left[\frac{1}{c} \cdot e^{cx} \right]_{-\infty}^{0} \right\} - \frac{1}{2} \left\{ \left[x \cdot e^{-cx} \right]_{0}^{\infty} - \left[-\frac{1}{c} \cdot e^{-cx} \right]_{0}^{\infty} \right\}$$
$$= \frac{1}{2} \left\{ [0-0] - \left[\frac{1}{c} - 0 \right] \right\} - \frac{1}{2} \left\{ [0-0] - \left[0 + \frac{1}{c} \right] \right\}$$
$$= \frac{-1}{2c} + \frac{1}{2} \cdot \frac{1}{c}$$
$$= 0$$

④
$$Var(X) = E(X^2) - E(X)^2$$
$$= E(X^2) - 0$$
$$= \int_{-\infty}^{\infty} x^2 \cdot f(x) \cdot dx$$
$$= \frac{2}{c^2}$$

⑤
$$\phi_X(t) = E(e^{tx}) = \int_{-\infty}^{\infty} e^{tx} \cdot f(x) \cdot dx$$
$$=$$

T8Q4:

$$f(x) = \begin{cases} \frac{x}{c} \cdot e^{-\frac{(x/c)^2}{2}} & \text{for } x \geq 0 \\ 0 & \text{for } x < 0 \end{cases}$$

① for $x \leq 0$: $F_X(x) = \int_{-\infty}^{x} f(s)\, ds = 0$

② for $x > 0$: $F_X(x) = \int f(s)\, ds = \int_{}^{x} \ldots e^{-\frac{(s/c)^2}{2}} \ldots$

② for $x > 0$: $F_X(x) = \int_0^x f(s)\,ds = \int_0^x \frac{s}{c^2} \cdot e^{-\ldots}\,ds$

$$= \frac{1}{c^2}\left\{\left[-\frac{c^2}{2}\, s \cdot e^{-(s/c)^2 x}\right]_0^x - \frac{c^2}{2}\left[e^{-(s/c)^2 x}\right]_0^x\right\}$$

$$= -\frac{1}{2}\left[x \cdot e^{-(x/c)/2} - 0\right] - \frac{1}{2}\left[e^{-x/c/2} - 1\right]$$

$$= -\frac{1}{2} \cdot x \cdot e^{-2x/c} - \frac{1}{2} \cdot e^{-2x/c} + 1$$

$$= 1 - \frac{1}{2} \cdot e^{-2x/c}(x+1)$$

T991:

$\mu = 1.7$

$\sigma = 0.05$

$F_X(x) = P(x<1.69) + 1 - P(x<1.71)$

$P = \text{normcdf}(1.69, 1.7, 0.005) + 1 - \text{normcdf}(1.71, 1.7, 0.005)$

$P = 45.5\,\%$

T992:

$\mu = 4.8 \times 10^6\ h$

$\sigma = 3.7 \times 10^5\ h$

$x = 3.9 \times 10^6$

$N = 4$

$P = \text{normcdf}(x, \mu, \sigma)$

$= \text{normcdf}(3.9 \times 10^4, 4.8 \times 10^6, 3.7 \times 10^5)$

T994:

$P_r(X \geq 2n) = ?$

```
%1.
n=158;
p=0.62;
x = 88;
y1 = binocdf(x,n,p)

%2
u=n*p;
sig=sqrt(n*p*(1-p));
pd = makedist('Normal','mu',u,'sigma',sig);
y2=cdf(pd,x)

y3=(y1-y2)/y1

%3
x=x+0.5;
u=n*p;
sig=sqrt(n*p*(1-p));
pd = makedist('Normal','mu',u,'sigma',sig);
y4=cdf(pd,x)

y5=(y1-y4)/y1
```

T995:

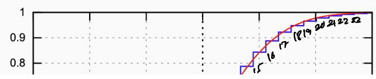
$\mu = \lambda$

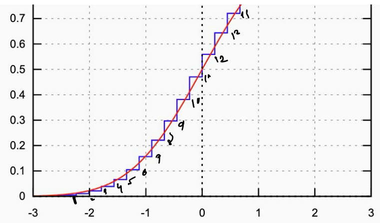
$X_i^\circ = \frac{X_i - \mu}{\sigma}$ $\quad \sigma^2 = \lambda$

$X_i^\circ = \frac{X_i - \lambda}{\sqrt{\lambda}} = \Phi \quad \left\{ 0.5 = \frac{X_i - \lambda}{\sqrt{\lambda}} \right.$

$F_X(\sigma x + \mu) = F_X\left(\sigma\left(x + \frac{\mu}{\sigma}\right)\right) \approx \Phi$

Let X be a random variable with $X \sim \text{Poisson}(\lambda)$ and X^o be the normalized version of X. The following figure shows the cdf F_{X^o} of X^o for some $\lambda \in \mathbb{N}$ and the cdf Φ of the standard normal distribution.





(0) Determine λ: $2^{3}-3=20$

λ =

② $P_x(i) = e^{-\lambda} \cdot \frac{\lambda^i}{i!}$

$P_x(x \le 24) = e^{-20} \cdot \frac{(\lambda)^{20}}{20!}$

T10Q4: $\lambda = \beta = 3.45$

① $m = \frac{1}{\lambda} \times n = \frac{1}{3.45} \times 20 = 5.8 \le$ $\{ m = \frac{n}{\beta} \to$ erlang, $m = \alpha/\beta \to$ Gama

② $s = \sqrt{\frac{n}{\lambda^2}} = \frac{\sqrt{20}}{3.45} = 1.35$

③ $P = \frac{\beta^n}{(n-1)!} x^{n-1} \cdot e^{-\beta x}$

$P_r(x \le 2) = F_x(2)$

gamcdf$(x, \alpha, 1/\beta)$ = gamcdf$(2, 20, 1/3.45)$ = 366e-05

④ $p =$ normcdf$(2, \frac{20}{3.45}, \frac{\sqrt{20}}{3.45})$ = normcdf(x, μ, σ)

$p = 0.0017$

T11Q2:

Bernolli: Appro^n of n

$n \le \frac{(\phi^{-1}(1-\alpha/2))^2}{4 \cdot \delta^2}$

$\alpha = 1 - 99.5$

$\alpha = 0.005$
$\delta = 0.015$

$\therefore$ normInv$(1-\alpha/2)^2 / 4 \cdot \delta^2$

$n \ge$

```
fprintf('%2f', norminv(1-0.005/2)^2/(4*0.015^2))
```

T2Q1:

$\bar{x} = 100.7$
$s^2 = 16$

$I = (\bar{x} - 5.0, \bar{x} + 5.0) = (95.7, 105.7)$

$k \cdot s = 5.0$
$k = 5.0/4$
$k = 1.25$

$\boxed{k = 1.5}$

$\frac{194}{20} \ge 1 - \frac{1}{k^2}$

$Pk \geq \left(1 - \frac{1}{(1.5)^2}\right) \cdot 20$

$Pk \geq \sqrt{60}$

$\therefore \; n_{min} = 8$

Now, $S^2 = \frac{1}{N-1} \sum_{i=1}^{N} (x_i - \bar{x})^2 =$

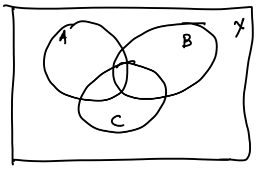
$16 = \frac{1}{19} 12 \cdot a^2$

$a^2 = 16 \times 19 / 12$

$a =$

$\boxed{a = 5.0332}$

T3Q1:



1. $A \setminus (B \cup C)$
2. $(A \cap B) \setminus C$
3. $A \cup B \cup C$
4. $(A \cap B) \cup (A \cap C) \cup (B \cap C)$
5. $A \cap B \cap C$
6. $X \setminus A \cap B \cap C$
8. $(X \setminus (A \cup B)) \cup (X \setminus (B \cup C)) \cup (X \setminus (A \cup C))$
7. $X \setminus (A \cap B) \cup (A \cap C) \cup (B \cap C)$
9. $(A \cap B) \cup (B \cap C) (A \cap C) \setminus (A \cap B \cap C)$
10. X

T3Q2:

$X \leq Pr(A \cap B) \leq Y$

$x = 1 - ((\bar{A}) + (\bar{B})) = 1 - 0.28 - 0.33 = 0.29$

$Y = (\min(Pr(A), Pr B)$

$x = 1 - (\bar{A}) - \bar{B} - \bar{C}$

$= 1 - 0.28 - 0.30 - 0.22$

$x = 0.17$

$Y = \min(A, B, C)$

$A > B > C$

$A > C > B$

$B > C > A$

$B > A > C$

$C > A > B$

$C > B > A$

$2/6 \cdot 1/2$

$= 1/6$

$A < B < C$ ✓

$A < C < B$ ✓ $4/6$

$B < C < A$ ✓

$B < A < C$ ✓

$C < A < B$ ✗

$C < B < A$ ✗

T3Q4:

$p = 1 - Pr(\bar{x})^n$

$= 1 - (5/6)^3$

$p = 0.421$

T3Q5:

$N = 100000$
$D = 1650$
$n = 150$
$i = 1$

$p = \text{hygepdf}(i, N, D, n)$

$p = 0.318$

T4Q1:

$A \rightarrow P(\text{die} \mid \text{without water}) = 0.83$

$P(\text{die} \mid \text{with water}) = 0.07$

$B \rightarrow P(\text{remember to water the plant}) = 0.85$

$P(B^c) = 0.15$

① $P(A^c) = 1 - P(A) \cdot P(B^c) + P(A) \cdot P(B)$

$= 1 - 0.07 \cdot 0.85 + 0.83 \cdot 0.15$

$= 1 - 0.184$

$P(A^c) = 0.816$

② $P(B^c \mid A) = \dfrac{P(A \mid B) \cdot Pr(B)}{Pr(A)}$

$= \dfrac{0.83 \cdot 0.15}{0.184}$

$= 0.677$

T4Q2:

$\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

RR ← 1/4
RB
BR
BB

prob xR
1 Red
1/3

1/3+1/3
= 2/3

1/8

RRR ✓
RRB
RBR
RBB
BRR
BRB
BBR
BBB

1/8

1/4

A → Both balls are colored red (1/4)
B → Next ball red (1/2)

C → First two balls red (1/4)

① $P(A|C) = \frac{P(A|C)\cdot P(A)}{P(C)} = \frac{P(A\cap C)}{P(C)} = \frac{1/4 \cdot 1/2}{1/4} = \frac{1}{2}$

T4Q3:

A → Cancer

B → elevated PSA level

$P(B|A^c) = 0.132$

$P(B|A) = 0.267$

$P(\text{other}) = 0.33$

$P(A|B) = \frac{P(A|B)\cdot P(A)}{P(B)} =$

T4Q4:

① $(P_1 P_2 P_3) + P_4 P_5 - P_1 P_2 P_3 P_4 P_5$ $\quad \{(P_1 \cap P_2 \cap P_3) \cup (P_4 \cap P_5)\}$

$P_1 \cdot P_2 \cdot P_3 + P_4 P_5 - (P_1 P_2 P_3 \cdot P_4 \cdot P_5)$

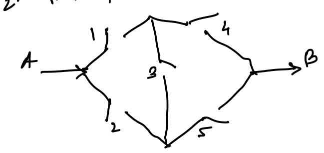
② $((P_1 \cap P_2) \cup (P_3 P_4)) \cap P_5$

$((P_1 \cdot P_2) + (P_3 P_4) - (P_1 P_2 P_3 P_4))\, P_5$

$(P_1 * P_2 + P_3 * P_4 - P_1 * P_2 * P_3 * P_4) * P_5$

③ $(P_1 \cap P_4) \cup (P_1 \cap P_3 \cap P_5) \cup (P_2 \cap P_5) \cup (P_2 \cap P_3 \cap P_4)$

2nd method



$\therefore\ P = \overline{P_3} \cap ((P_1 \cap P_4) \cup (P_2 \cap P_5)) + P_3((P_1 \cup P_2) \cap (P_4 \cup P_5))$

$= \overline{P_3} \cap \left(\overline{\overline{P_1 \cap P_4} \cap \overline{P_2 \cap P_5}}\right) + P_3 \cap \left(\overline{\overline{P_1} \cap \overline{P_2}} \cap \overline{\overline{P_4} \cap \overline{P_5}}\right)$

$= \overline{P_3} \cap \left(\overline{\overline{P_1 \cap P_4} \cap \overline{P_2 \cap P_5}}\right) + P_3 \cap (P_1 \cup P_2 \cap P_4 \cup P_5)$

$= (1 - P_3) * ((P_1 * P_4) + P_2 * P_5) + P_3 * ((P_1 + P_2) * (P_4 + P_5))$

$= (1 - P_3) * (1 - ((1 - P_1 * P_4) * (1 - P_2 * P_5))) + P_3 * ((1 - (1 - P_1) * (1 - P_2)) * (1 - (1 - P_4) * (1 - P_5)))$

T4Q5:

$P = 2/4 = 1/2$

$P_1 = \binom{4}{2} = 6$

$P_1P_2P_3P_4 + (1-P_1)P_2P_3P_4 + (1-P_2)P_1P_3P_4$
$+ (1-P_3)P_1P_2P_4 + (1-P_4)P_1P_2P_3$
$+ (1-P_1)(1-P_2)P_3P_4 + (1-P_2)(1-P_3)P_1P_4$
$+ (1-P_3)(1-P_4)\cdot P_1P_2 + (1-P_1)(1-P_4)\cdot P_2P_3$

T4Q6:

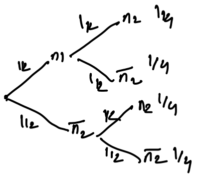
① $P_1 = P_r(A) + P_r(B) + P_r(C)$
$= 0.46 + 0.01 + 0.46$
$P_1 = 0.93$

② $P_2 = P(A \cup B \cup C) = P(A) + P(B) + P(C)$
$- P(A \cap B) - P(A \cap C) - P(B \cap C)$
$- P(A \cap B \cap C)$
$= P_1 - (0.46 \cdot 0.01) - (0.46 \cdot 0.46) - (0.46 \cdot 0.01)$
$+ ABC$

$= 0.93 - 0.2116 - 4.6 \times 10^{-3} - 4.6 \times 10^{-3} + 0.49 \cdot 0.01 \cdot 0.49$
$= 0.7116$

T4Q7:

let say $n=2$
$p = 1/2$



3/4 at least 1 component works.

$\boxed{1 - 3/4}$

T5Q1:

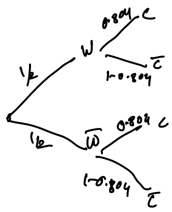
$n = 50$
$p = 0.0098$

① $P(X \geq 1) = 1 - P(X=0)$
$= 1 - \binom{50}{0} p^0 (1-p)^{50-0}$

② $P(X=1) = \binom{50}{1} \cdot p^1 \cdot (1-p)^{50-1}$
$= 0.3024$

③ $P(X \geq 2) = 1 - P(X=0) - P(X=1)$
$= 0.7704$

T5:
Q2



T5Q3: $n = 5$
$P(X \geq 3)$

$$P = 1 - \left(P(x=3) + P(x=4) + P(x=5) \right)$$

$$= 1 - \binom{5}{3} p^3 (1-p)^2 + \binom{5}{4} p^4 (1-p)^1 + \binom{5}{5} p^5 (1-p)^0$$

$$= 1 - 10 \cdot (0.001)^3 (1-0.001)^2 + 5 \cdot (0.001)^4 (1-0.001) + (0.001)^5$$

$$= (0.99999)\ 848$$

TSQ4:

$$\phi_x(t) = ((1-p) + pe^t)^n$$

$$\phi'_x(t) = p \cdot e^t \cdot n \cdot ((1-p) + pe^t)^{n-1}$$

$$\phi''_x(t) = pe^t \cdot n\, ((1-p) + pe^t)^{n-1} + pe^t \cdot n \cdot pe^t \cdot (n-1) \left((1-p) + pe^t\right)^{n-2}$$

$$E(x) = \phi_x(0) = p \cdot n \cdot \left((1-\cancel{p}) + \cancel{p}\right)^{n-1}$$

$$= p \cdot n\, (1)^{n-1}$$

$$= pn$$

$$E(x^2) = \phi''_x(0) = p \cdot n \left((1-\cancel{p}) + \cancel{p}\right)^{n-1} + p \cdot n \cdot p \cdot (n-1) \left((1-\cancel{p}) + \cancel{p}\right)^{n-2}$$

$$= pn + pnp\,(n-1)$$

$$= pn\,(1 + p(n-1))$$

$$Var(x) = E(x^2) - E(x)^2$$

$$= pn\,(1 + p(n-1)) - (pn)^2$$

$$= pn + p^2 n(n-1) - (pn)^2$$

$$= pn + \cancel{p^2n^2} - p^2 n - \cancel{(pn)^2}$$

$$= pn(1-p)$$

TSQ5: $E(x) = 6.4 \quad Var = 3.84$

$$E(x) = np \qquad np(1-p) = 3.84$$

$$np = 6.4 \rightarrow 6.4\,(1-p) = 3.84$$

$$n = \frac{6.4}{0.4} \qquad 6.4 - 6.4p = 3.84$$

$$-6.4p = 3.84 - 6.4$$

$$\boxed{n = 16} \qquad p = \frac{-2.56}{-6.4}$$

$$p = 0.4$$

Now, $P(x=6) = $ binopdf(1, n, p)

TSQ6:

① $P(x=4) = 0.207$

$$P(4) = \binom{20}{4} p^4 (1-p)^{16} \Rightarrow F = @(p)\ \text{nchoosek}(20,4) * p^4 * (1-p)^{16} - 0.207$$

fzero(F, 0.5)

$$p = 0.23$$

$$\frac{0.207}{4845} = p^4 (1-p)^{16}$$

$$\sqrt[4]{4.2724 \times 10^{-5}} = p(1-p)^4$$

$$0.0808 = p(1-p)^4$$

② $S(x) = \max\{z \in Z \mid z \boxed{<} x\}$

for less than → ceiling

$\therefore\ S(x) = \text{ceiling}(x-1)$

③ $p_r(i+1) = \frac{p}{1-p} \cdot \frac{n-i}{i+1} \cdot p_r(i)$

$i = 0$

$p_r(0+1) = \frac{p}{1-p} \cdot n \cdot p_r(0)$

$= \frac{pn}{\cancel{1-p}} 1 \cdot 1 \cdot (1-p)^{n-1}$

$= pn(1-p)^{n-1}$

$\therefore p_r(1) = \binom{n}{1} p^1 (1-p)^{n-1}$

Generalising:

$p_r(i+1) = \frac{p}{1-p} \cdot n \cdot p_r(i)$

$= \frac{p}{1-p} \cdot n \cdot \binom{n}{i+1} \cdot p^{i+1} (1-p)^{n-(i+1)}$

$= \frac{p}{1-p} \cdot n \cdot \frac{n!}{i!\,(n-i)!}$

$\therefore p_r(i+1) = \binom{n}{i+1} \cdot p^{i+1} \cdot (1-p)^{n-(i+1)}$

$\frac{p}{1-p} \cdot \frac{n-i}{i+1} > 1$

$\vdots$

$np - 1 + p > i$

$\therefore$ ceiling$(np - 1 + p)$

④ Floor$(n*p + p)$

TSQ 7:

$x_1 = 1,\ x_2 = 2,\ x_3 = 4$
$y_1 = -1\ \ y_2 = 0\ \ y_3 = 1$

① $P_x(1) = \frac{7}{20} + \frac{1}{10} + \frac{1}{20} = 0.5$

$P_x(2) = \frac{7}{25} + \frac{2}{25} + \frac{1}{25} = 0.4$

$P_x(4) = 0.1$

$P_y(-1) = 0.7$

$P_y(0) = 0.2$

$P_y(1) = 0.1$

7/20	1/10	1/20
7/25	2/25	1/25
7/100	1/50	1/100

(2) $Z = X+Y$

$$Z = \begin{bmatrix} 1+(-1), & 1+0, & 1+1 \\ 2+(-1), & 2+0, & 2+1 \\ 4+(-1), & 4+0, & 4+1 \end{bmatrix} = \begin{bmatrix} 0, & 1, & 2, \\ 1, & 2, & 3 \\ 3, & 4, & 5 \end{bmatrix}$$

$\therefore Z = [0, 1, 2, 3, 4, 5]$

$P_Z(Z=0) = \frac{7}{20}$

$P_Z(Z=1) = \frac{1}{10} + \frac{7}{25} = 19/50$

$P_Z(Z=2) = \frac{1}{20} + 2/25 = 13/100$

$P_Z(Z=3) = \frac{1}{25} + \frac{7}{100} = 11/100$

$P_Z(Z=4) = 1/50$

$P_Z(Z=5) = 1/100$

(3) $Z = X \cdot Y$

$$\therefore Z = \begin{Bmatrix} -1, & 0, & 1 \\ -2, & 0, & 2 \\ -4, & 0, & 4 \end{Bmatrix}$$

$\therefore Z = [-4, -2, -1, 0, 1, 2, 4]$

$P_Z(-4) = 7/100$

$P_Z(-2) = 7/25$

$P_Z(-1) = 7/20$

$P_Z(0) = \frac{1}{10} + \frac{2}{25} + \frac{1}{50} = 1/5$

$P_Z(1) = 1/20$

$P_Z(2) = 1/25$

$P_Z(4) = 1/100$

(4) $E(X) = P_X(1) \cdot x_1 + P_X(2) \cdot x_2 + P_X(4) x_3$

$= 0.5 + 0.4(2) + 0.1 \cdot (4)$

$E(X) = 1.7$

$E(Y) = P_Y(-1) \cdot y_1 + P_Y(0) \cdot y_2 + P_Y(1) y_3$

$= 0.7 \cdot (-1) + 0.2(0) + 0.1(1)$

$E(Y) = -0.6$

TSQ8:

(1) $\sum_{j=0}^{15} \frac{3^{j-1} \cdot e^{-3}}{j!} + c \cdot \frac{6^{j-1} \cdot e^{-6}}{j!} = 1$

$0.333333 + C \cdot 0.166581 = 1$

$C = \frac{1 - 1/3}{0.166581}$

$C = 4.002$

$\boxed{C = 4}$

② $P_X(1) = \sum_{j=0}^{15} \frac{3^{j-1} \cdot e^{-3}}{j!} = 1/3$

$P_X(2) = 4 \cdot \sum_{j=0}^{15} \frac{6^{j-1} \cdot e^{-6}}{j!} = 4 \cdot 1/6$

$P_Y(j) = \frac{3^{j-1} \cdot e^{-3}}{j!} + 4 \frac{6^{j-1} \cdot e^{-6}}{j!}$

③ $P_{XY}(1,1) = e^{-3}$

$P_X(1) \cdot P_Y(1) = \frac{1}{3} \cdot \{ e^{-3} + 4e^{-6} \}$

$= \frac{1}{3}(e^{-3} + 4e^{-6})$

T6Q1:

$X = [-2, -1, 1, 2]$

$Y = |X| = [2, 1, 1, 2] = [1, 2]$

$Z = XY$

① $Z = [-4, -1, 1, 4]$

$P_Z(z) = [1/4, 1/4, 1/4, 1/4]$

② $E(X) = -2 \cdot 1/4 - 1 \cdot 1/4 + 1/4 + 2/4$

$E(X) = 0$

$E(Y) = \frac{1}{2} \cdot 1 + \frac{1}{2} \cdot 2$

$= 3/2$

$E(X \cdot Y) = E(Z) = 0$

$Cov(X, Y) = E(X \cdot Y) - E(X) \cdot E(Y)$

$Cov(X, Y) = 0$

$Pr(X=1, Y=1) = 1/4$

$Pr(X=1) \cdot Pr(Y=1) = \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$

T6Q2:

$E(X) = P$

Bernouli(1/2)

$P = 1/2$

$Z = X + Y$

$X = [0, 1]$

$Y = [0, 1]$

$Z = X + Y$

$Z = [0, 1$
$1, 2]$

$E(X) = \frac{1}{2} \cdot 0 + \frac{1}{2} \cdot 1 = \frac{1}{2}$

$E(Y) = 1/2$

$E(Z) = 1/4 \cdot 0 + \frac{1}{4} + \frac{1}{4} + \frac{2}{4} = 1$

$E(X^2) = 1/2$

$E(Y) = 1/2$

$E(Z^2) = 0 + \frac{1}{4} + \frac{1}{4} + \frac{2^2}{4} = 2/4 + 1 = 6/4 = 3/2$

Now, $E(X \cdot Z) = E(X^2) + E(X \cdot Y) = 1/2 + 1/2 \cdot 1/2 = 3/4$

$Cov(X,Z) = E(XZ) - E(X) \cdot E(Z)$

$= 3/4 - 1/2 \cdot 1$

$= 1/4$

$\rho_{XZ} = \frac{Cov(X,Z)}{\sigma_X \sigma_Z}$

$= \frac{1/4}{1/2 \cdot \sqrt{1/2}}$

$\rho_{XZ} = \frac{\sqrt{2}}{2}$

$Var X = E(X^2) - E(X)^2$
$= 1/2 - 1/4$

$Var(X) = 1/4 \quad \sigma_X = 1/2$

$Var(Z) = E(Z^2) \quad E(Z)^2$
$= 3/2 - 1$
$= 1/2 \quad \sigma = \sqrt{1/2}$

T6Q3:

$\phi_X(t) = \frac{pe^t}{1-(1-p)e^t} \qquad \frac{dv}{du}\left(\frac{u}{v}\right) = \frac{u'v - v'u}{v^2}$

$\therefore \phi_X'(t) = \frac{p \cdot e^t \cdot (1-(1-p) \cdot e^t) - p \cdot e^t \cdot (0-(1-p)e^t)}{(1-(1-p) \cdot e^t)^2}$

$= \frac{p \cdot e^t (1 - \cancel{(1-p) \cdot e^t} + \cancel{(1-p) e^t})}{(1-(1-p) \cdot e^t)^2}$

$= \frac{p \cdot e^t}{(1-(1-p) \cdot e^t)^2}$

$\phi_X''(t) = \frac{(1-(1-p) \cdot e^t)^2 \cdot p \cdot e^t - p \cdot e^t \cdot (2 \cdot (1-(1-p)e^t) \cdot (-(1-p)e^t))}{(1-(1-p) \cdot e^t)^4}$

$= \frac{p \cdot e^t \{ (1-(1-p) \cdot e^t)^2 - 2(1-(1-p) \cdot e^t) \cdot (1-p) \cdot e^t \}}{(1-(1-p) \cdot e^t)^4}$

$= \frac{p \cdot e^t \{ (1 - \cancel{(1-p) e^t}) + \cancel{2}(1-p) \cdot e^t \}}{(1-(1-p) \cdot e^t)^3}$

$\phi_X''(t) = \frac{p \cdot e^t (1 + (1-p) \cdot e^t)}{(1-(1-p) \cdot e^t)^3}$

$\phi_X(0) =$

T6Q4:

1. $P(X=3) = P(X=4)$

$e^{-\lambda} \cdot \frac{\lambda^3}{3!} = e^{-\lambda} \cdot \frac{\lambda^4}{4!}$

$\frac{\lambda}{4} = 1$

$\boxed{\lambda = 4}$

2. $e^{-\lambda} \cdot \frac{\lambda^i}{i!} \geq e^{-\lambda} \cdot \frac{\lambda^{i+1}}{(i+1)!}$

$\frac{1}{i+1} \geq \lambda$

$\therefore \lambda < 1$

T7.Q8:

$E(X) = 1/\lambda = 6.9$

$P(X \geq 11.5) = 1 - P(X < 11.5)$

$= 1 - (1 - e^{-\lambda x})$

$= 1 - (1 - e^{-1/69 \cdot 11.5})$

$= 0.1889$

$P(X > 16.5 - 5) = 1 - P(X < 11.5) = 1 - (1 - e^{-\frac{1}{69} \cdot 11.5})$

$= 1 - 0.1889$

$= 0.811$

T7Q9: $\lambda = \frac{1}{250} s^{-1}$

① $P(2 < x < 6) = P(x = 6) - P(x = 2)$

$= F_x(6) - F_x(2)$

$= (1 - e^{-\frac{1}{250} \cdot 6}) - (1 - e^{-\frac{1}{250} \cdot 2})$

$= -e^{-60/250 \cdot 6} + e^{-60/250 \cdot 2}$

$= 0.3312$

② $F_x^{-1}(1/2) = \frac{-\ln(1-p)}{\lambda} = \frac{-\ln(1 - 1/2)}{1/250}$

$= 173.28$

T7Q9: $P_r(X < 1000) = 0.737$

$\cancel{1-} e^{-\lambda \cdot 1000} \cancel{=} 0.737$

$-e^{-\lambda 1000} = -0.737$

$-\lambda 1000 = \ln(0.737)$

$\lambda = 3.0516 \times 10^{-4}$

② $E(x) = 1/\lambda = 3276.89$

③ $\sigma = 1/\lambda$

T8Q1:

$f_{xy} := \begin{cases} \frac{c}{(x+y+1)^3} & \text{if } x, y \geq 0 \\ 0 & \text{Otherwise} \end{cases}$

$= \int_{-\infty}^{y} \int_{-\infty}^{x} \frac{c}{(x+y+1)^3} \, dx \, dy$

$= c \int_0^y \int_0^x (s+t+1)^{-3} \, ds \, dy$

$= c \int_{-\infty}^{y} \left\{ \frac{1}{-2} \cdot (s+t+1)^{-2} \right\}_{-\infty}^{x}$

$= -\frac{c}{2} \int_0^y \left\{ (x+t+1)^{-2} - (t+1)^{-2} \right\}$

$= -\frac{c}{2} \left\{ \frac{-1}{1} \cdot (x+y+1)^{-1} - (y+1)^{-1} \frac{-1}{1} \right\}_0^y$

$= +\frac{c}{2} \left\{ (x+y+1)^{-1} - (y+1)^{-1} \right\}_0^y$

$= \frac{c}{2} \left\{ (x+y+1)^{-1} - (y+1)^{-1} - (x+1)^{-1} + (1)^{-1} \right\}$

$= \frac{c}{2} \left\{ \cancel{(x+y+1)} - \cancel{(y+1)} - \cancel{(x+1)} \right\}^{-1}$

$= \frac{c}{2} \{1\}$

$F_{xy}(x, y) = c/2$

$\lim_{x, y \to \infty} F_{xy}(x, y) = 1$

$\boxed{c=2}$

② $F_X(3.8)\cdot F_Y(0.6) = \frac{c}{2}\left\{(x+y+1)^{-1}-(y+1)^{-1}-(x+1)^{-1}+1\right\}$

$= \frac{2}{2}\left\{(3.8+0.6+1)^{-1}-(0.6+1)^{-1}-(3.8+1)^{-1}+1\right\}$

$=$

③ $F_X(3.8) = \frac{1}{x+1}+1 = \frac{-1}{3.8+1}+1 = 0.791$

$F_Y(0.6) = \frac{-1}{y+1}+1 = 0.375$

$F_X(3.8)\cdot F_Y(0.6) = 0.296$

$F_{XY}(3.8, 0.6) = 0.3518$

T8Q3:

$f(x) = \frac{c}{2}\cdot e^{-c|x|}$

$\phi_x(t) = \int_{-\infty}^{\infty} e^{tx}\cdot f(x)\cdot dx$

$= \int_{-\infty}^{\infty} x\cdot e^{tx}$

$= \left\{\frac{1}{t}\cdot x\cdot e^{tx} - \int_{-\infty}^{\infty}\frac{1}{t}\cdot e^{tx}dx\right\}$

$= \left\{\frac{x}{t}\cdot e^{tx} - \frac{1}{t}\,t\,e^{tx}\right.$

T9Q1:

$\mu = 1.3$
$\sigma = 0.024$

$P(1.25 < x < 1.35) = P(X<1.35)+1-P(X<1.25)$

$= 3.72\%$

T9Q2:

$\mu = 4.5\times10^{6}$
$\sigma = 3.8\times10^{5}$

$P(X\geq 3.5\times10^{6})$

normcdf $(3.6\times10^{6},\ 4.5\times10^{6},\ 3.8\times10^{5})$