Technische Hochschule OWL **Prof. Dr. Uwe Meier**
Department of Electrical Engineering and Computer Science

Written Exam: Discrete Signals and Systems (DSS)
Degree Programmes: Information Technology (M. Sc.), Elektrotechnik (M. Sc.)
2023-03-24, 120 min, 100 points available → no notebooks, no books

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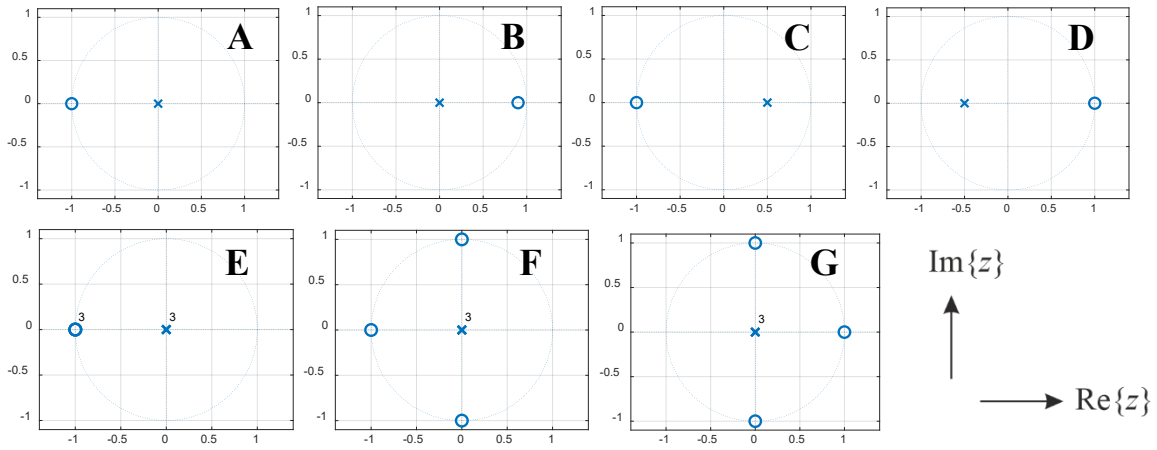
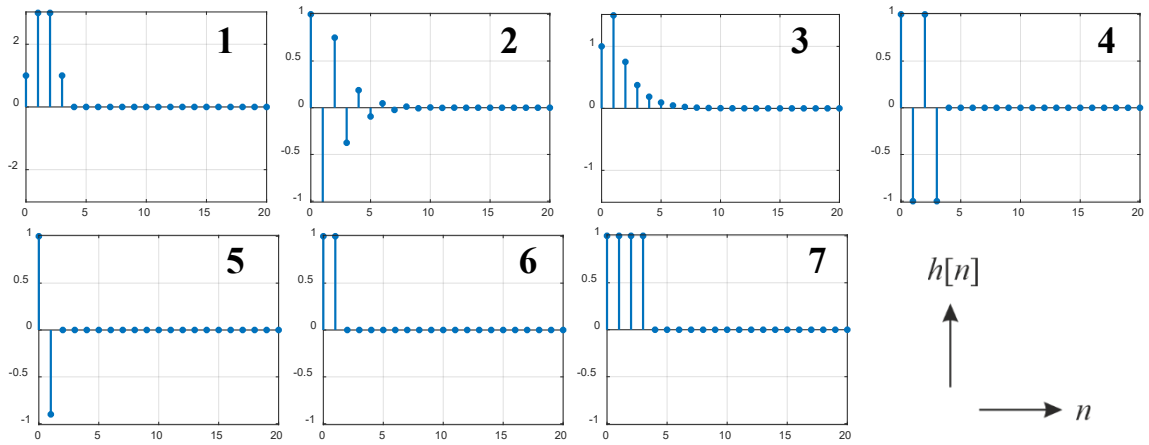
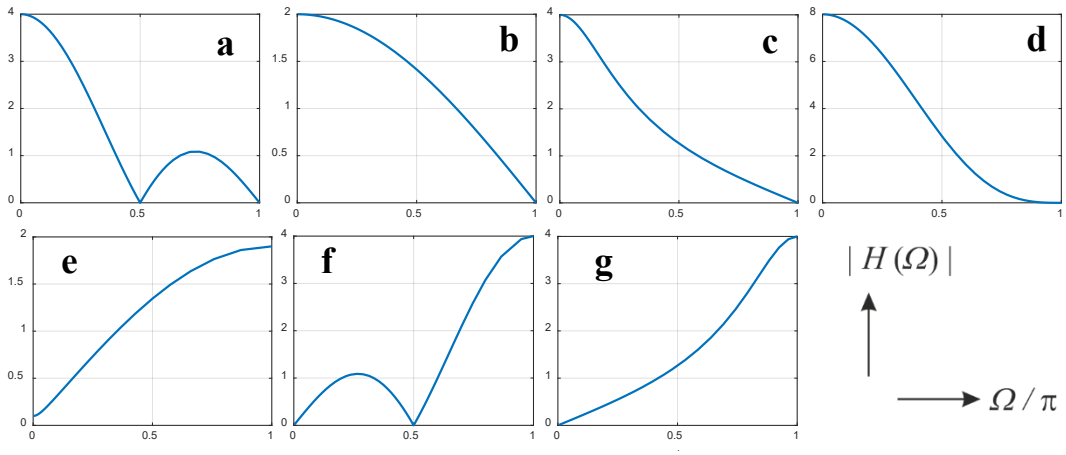
*Please: Don't use red ink; start the solution of each problem on a **new** sheet or side of paper; present all solutions thoroughly.*

Problem 1 Mapping task: 7 different discrete-time systems shall be considered. Find the correct relation of zero-pole plot, impulse response, and magnitude response. Give the answer in the form (A, 2, c), if you think, that this is a correct relation. *You are allowed to guess without proving your answer. But thinking might increase your success.* ☺



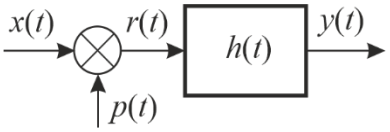




10 points

Problem 2 The plotted time-continuous system with

$$x(t)=\frac{\sin(4\pi\cdot t)}{\pi\cdot t}\ ,\quad p(t)=2\cdot\cos(2\pi\cdot t)\ ,$$
$$h(t)=1+3\cdot\sin(4\pi\cdot t)+2\cdot\cos(8\pi\cdot t)$$



uses a multiplier at the input.

2.1 Compute and plot the spectrum $R(f)$ of the signal $r(t)$.

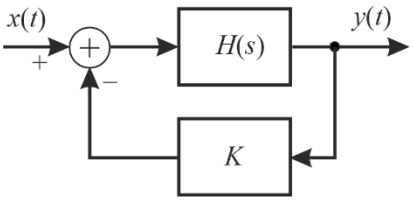
2.2 Determine the output signal $y(t)$.

15 points

Problem 3 The time-continuous closed-loop system with

$$H(s)=\frac{s}{s^2-2s+1}$$

and a proportional feedback path with gain element *K* shall be investigated.

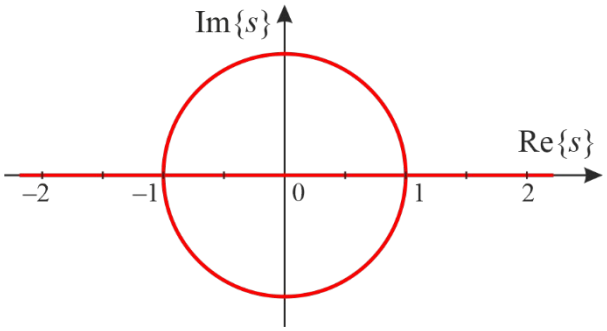


3.1 Plot the root-locus curve as a function of *K*. *You may use the plotted curve if you think it is helpful.*

3.2 Determine *K* for a stable closed-loop system.

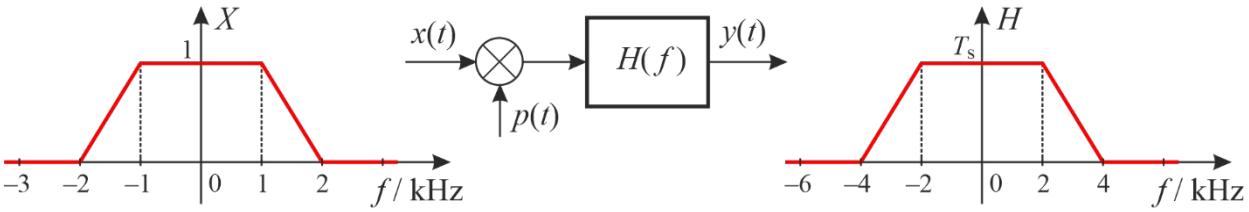
3.3 Determine *K* for a closed-loop system with oscillating impulse response.

3.4 Determine the step response for $K = 4$.



20 points

Problem 4 A time-continuous sensor signal $x(t)$ with given spectrum $X(f)$ is processed with ideal impulse train sampling and a succeeding reconstruction filter with known frequency response $H(f)$. Determine the sampling frequency f_s in order to achieve an aliasing-free reconstruction $y(t)$ of the sensor signal. *An appropriate proof of your result is necessary.*



10 points

Problem 5 The input and output signals of a time-discrete LTI system are given by

$$x[n]=\left(\frac{1}{2}\right)^n\cdot u[n]-\frac{1}{4}\cdot\left(\frac{1}{2}\right)^{n-1}\cdot u[n-1]\ ,\ y[n]=\left(\frac{1}{3}\right)^n\cdot u[n]\ ,$$

where $u[n]$ is the unit step sequence. Derive the system's difference equation relating $y[n]$ as a function of $x[n]$.

15 points

Problem 6 Consider the time-discrete LTI system function

$$H(z) = \frac{(z-2)\cdot(z+1)}{\left(z+\frac{3}{4}\right)\cdot\left(z-\frac{1}{2}\right)}\ ,\quad |z| > \frac{3}{4}\,.$$

6.1 Is the system stable and/or causal?

6.2 Decompose the system into an allpass and a minimum-phase system.

6.3 Determine the impulse response $h[n]$ and verify $h[0]$.

20 points

Problem 7 Given is the periodic time-discrete sequence

$$x[n] = \cos\left(\frac{2\pi\cdot n}{6}\right)\ ,\ n \text{ integer},\ -\infty < n < \infty\,.$$

7.1 Compute the spectrum $X[k]$ of a 12-point DFT.

7.2 Compare the results of $X[k]$ for a 6-point and 16-point DFT. A computation of $X[k]$ is not necessary.

10 points

Good luck !