Information Fusion (IFU)

Summer Semester 2021

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Name: ______________________________

Registration number: ______________________________

Remarks

- You have 120 Minutes to reupload your exam to the eAssessment. Please save enough time for the uploading.
- Lecture slides, exercises, and your personal notes are allowed as aids.
- A written answer is expected in full sentences. Writing down keywords is **not** sufficient.
- **Do not copy text from any source into the answer fields.** Always give your answers in your own words. Copied text leads to a failed exam automatically.
- Total points: 100; passed with at least 40 points.

Points	< 40	$[40, 45.5]$	$[46, 51.5]$	$[52, 57.5]$	$[58, 63.5]$	$[64, 69.5]$
Grade	5.0	4.0	3.7	3.3	3.0	2.7

Points	$[70, 75.5]$	$[76, 81.5]$	$[82, 87.5]$	$[88, 93.5]$	$[94, 100]$
Grade	2.3	2.0	1.7	1.3	1.0

1. Fusion and Conflict in Dempster-Shafer Theory (35 P.)

An autonomous vehicle approaches a traffic light which can either be green (G), yellow (Y), or red (R). The vehicle drives in the USA, so no combination of lights are possible ($\Omega = \{\varnothing, G, Y, R\}$). The traffic light is observed by optical sensors whose signals are fed into an image processing algorithm. The algorithm outputs evidences about the state of the traffic light. If there are no signal outputs, it is assumed that no traffic lights could be detected. To increase the safety of the vehicle three independent camera-based sensors are used: S_1, S_2, and S_3. The measurements are defined via their basic belief assignments $m_i(\circ) \in [0, 1]$, $i \in \{1, 2, 3\}$.

You may use the following symbols for an easier use within this PDF:

- Omega := Ω,
- emptyset := $\varnothing$, and
- you may write indices with an underscore, for example: $m\_1 := m_1$ or $k\_c := k_c$.

1.1 Define the power set of the frame of discernment. /4

1.2 While approaching the traffic lights at a speed of $30\,\mathrm{km\,h^{-1}}$ the sensors output the following basic belief assignments:

	$m_i(Y)$	$m_i(R)$	$m_i(\{Y, R\})$	$m_i(\Omega)$
S_1	0.2	0	0.6	0.2
S_2	0.3	0	0	0.7
S_3	0.6	0	0.2	0.2

All elements of the power set, which are not listed, have a mass of 0.

1.2.1 Interpret the evidential masses $m_1(\{Y, R\}) = 0.6$ and $m_2(\Omega) = 0.7$. /5

1.2.2 Apply DEMPSTER's rule of combination for the sensors S_1 and S_2. Calculate the conflict factor k_c and the joined masses $m_{12}(\circ)$. Interpret your result. /13

1.2.3 Now include the sensor S_3. Calculate the conflict factor k_c and the joined masses $m_{123}(\circ)$. Interpret your result. /13

2. Fusion in Possibility Theory (30 P.)

Dubois and Prade proposed the following fusion equation for possibility values $\pi \in [0,1]$:

$$h(\pi_1, \pi_2) = (1 - k_\mathrm{c}) \cdot \max\left(\frac{\min(\pi_1, \pi_2)}{1 - k_\mathrm{c}}, \min(1 - k_\mathrm{c}, \max(\pi_1, \pi_2))\right),$$
$$k_\mathrm{c} = f(\pi_1, \pi_2) \in [0, 1[\ .$$

2.1 The fusion operator h has an optimistic and a pessimistic mode. In the following we look into the optimistic case in which it is assumed that there is no conflict between sources. Which of the following statements is true in the optimistic case, that is, if $k_\mathrm{c} = 0$? /10

A. $h = 0$.
B. $h = 1$.
C. $h \to \infty$.
D. $h \to -\infty$.
E. $h = \min(\pi_1, \pi_2)$.
F. $h = \max(\pi_1, \pi_2)$.
G. h converts into the arithmetic means operator.
H. h converts into the generalised means operator.
I. h converts into the harmonic means operator.
J. The *orness* of h becomes 1.
K. The *andness* of h becomes 0.
L. The *andness* and *orness* of h becomes 0.5.
M. h becomes undefined.
N. None of the above statements is true.

2.2 Which of the following statements is true regarding the *boundedness* of h? /5

A. h is bounded because $h \leq \max(\pi_1, \pi_2)$.
B. h is bounded because $\min(\pi_1, \pi_2) \leq h \leq \max(\pi_1, \pi_2)$.
C. h is bounded because $h(\pi_{\mathrm{a}_1}, \pi_{\mathrm{a}_2}) \geq h(\pi_{\mathrm{b}_1}, \pi_{\mathrm{b}_2})$ if $\pi_{\mathrm{a}_i} \geq \pi_{\mathrm{b}_i}\ \forall i \in \{1, 2\}$.
D. h is bounded only if $\pi = \pi_i\ \forall i \in \{1, 2\}$.
E. h is not bounded because $\min(\pi_1, \pi_2) \leq h$.
F. h is not bounded because it is possible that $h > 1$.
G. h is not bounded because $h(\pi_{\mathrm{a}_1}, \pi_{\mathrm{a}_2})$ is not always greater than $h(\pi_{\mathrm{b}_1}, \pi_{\mathrm{b}_2})$ if $\pi_{\mathrm{a}_i} \geq \pi_{\mathrm{b}_i}\ \forall i \in \{1, 2, \ldots, n\}$.
H. None of the above statements is true.

2.3 Which of the following statements is true regarding the *idempotency* of h? /5

A. h is idempotent because $h \leq \max(\pi_1, \pi_2)$.

B. h is idempotent because $h(\pi_1, \pi_2) = h\big(\pi_{r(1)}, \pi_{r(2)}, \ldots, \pi_{r(n)}\big)$ for any permutation r on $\mathbb{N}_n$.

C. h is idempotent because $\min(\pi_1, \pi_2) = \max(\pi_1, \pi_2) = \pi$ if $\pi = \pi_i\ \forall i \in \{1, 2\}$.

D. h is idempotent only if $k_\mathrm{c} = 0$.

E. h is idempotent only if $k_\mathrm{c} \to 1$.

F. h is not idempotent if $\pi_1 = \pi_2$.

G. h is not idempotent if $\pi_1 \neq \pi_2$.

H. h is not idempotent because $h \leq \max(\pi_1, \pi_2)$.

I. h is not idempotent because $\min(\pi_1, \pi_2) \leq h$.

J. None of the above statements is true.

2.4 Possibility distributions are closely related to fuzzy membership functions. Would it make sense to apply h to fuzzy memberships ($\mu \in [0, 1]$)? If your answer is yes, what would need to be considered in an application on memberships? If your answer is no, why do you think that it is not sensible to do so? Reason your statements. /10

3. Possibility and Necessity Measures (35 P.)

A sensor outputs an integer-valued variable x. Let information about the true value of x be given in terms of the proposition „x is around 4“ in which the concept „around 4“ is expressed by the fuzzy set A described by a fuzzy membership function $\mu_A(x)$ (cf. Fig. 1). A is defined on $\Theta = \{1, 2, 3, 4, 5, 6, 7\}$. This incomplete information induces a possibility distribution function $\pi_A(x)$ that is numerically identical with the membership function $\mu_A(x)$ under the condition stated by DUBOIS and PRADE: $\pi_{\mathrm{A}}(x = \theta \mid \theta \in A) = \mu_{\mathrm{A}}(\theta \in A \mid x = \theta)$. The nested α-cuts of $\mu_A(x)$ are shown in Fig. 1 also.

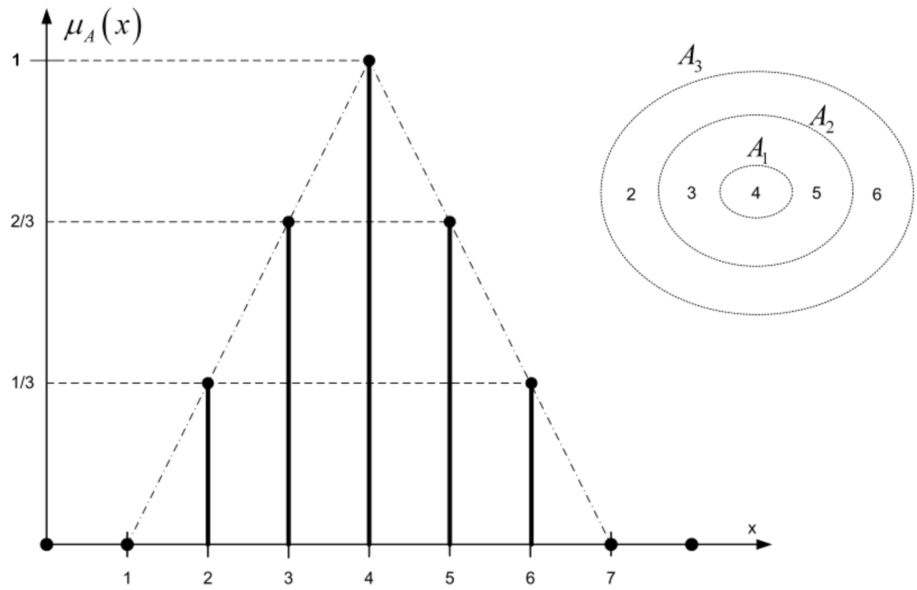

Figure 1: Sketch of a discrete fuzzy set „around 4“.

3.1 State whether the fuzzy set A is normal and whether it is convex. Give reasons for your statements. /4

3.2 Interpreting the fuzzy membership function $\mu_A(x)$ as a possibility distribution $\pi_A(x)$: Is $\pi_A(x)$ an example of total ignorance, partial ignorance, or complete knowledge? Reason your statement. /4

3.3 Calculate the Possibility measure $\Pi(A_i)$ and Necessity measure $N(A_i)$, $i \in \{1, 2, 3\}$. /9

3.4 Calculate the basic belief assignments m_i, $i \in \{1, 2, 3\}$ for the discrete sets A_i. /9

3.5 Transferring a fuzzy set into Dempster-Shafer-Theory is only reasonable if certain condition(s) are met. Define these condition(s). Describe how one could proceed if these requirements are not met. /9