1.) Let X be a binomial random variable:

$$X \sim \text{binomial}(n, p)$$

(i) Use the definition (3.17) of the moment generating function to show that

$$\phi_X(t) = ((1-p) + pe^t)^n$$

for all $t \in \mathbb{R}$.

(ii) Use lemma (3.18) to calculate $E(X)$ and $E(X^2)$.

2.) Provide a MatLab script that generates a plot of the binomial(n, p) pmf. The plot shall also display the values of the expectation and the standard deviation. These values shall also be depicted in the plot.
Use the script to generate plots of binomial(n, p) for $n = 50$ and $p = 0.1, 0.2, 0.3, 0.4, 0.5$ and 0.6.

3.) A satellite system consists of 4 components and can function adequately if at least 2 of the 4 components are in working condition. If each component is, independently, in working condition with probability .6, what is the probability that the system functions adequately?

4.) A communications channel transmits the digits 0 and 1. However, every digit transmitted is incorrectly received with probability 0.01.

In order to reduce the error probability for messages send over the channel, every bit is transmitted five times in a row. I.e., for every message bit 0 or 1 the bit sequence 00000 or 11111 is transmitted respectively. The receiver uses majority decoding: a sequence of five bits is decoded as 0 if there are at least three zeros in the sequence, otherwise it is decoded as 1.

A message consisting of 128 bytes (1024 bits) is encoded and send over the channel. What is the probability that the message contains no error after it is decoded?

5.) Solve problems 3 to 5 from [Ross, p.195].

6.) Let X be a binomial random variable with $E(X) = 7$ and $\text{Var}(X) = 2.1$.

Determine: (a) $\Pr\{X = 4\}$ and (b) $\Pr\{X > 12\}$

7.) The probability of an error in the transmission of a binary digit over a communication channel is 10^{-3}. Assuming independence, write an expression for the exact probability of more than 3 errors when transmitting a block of 10^3 bits.

8.) Let $n \in \mathbb{N}$ and $p \in (0,1)$ and X, Y random variables with $X \sim \text{binomial}(n,p)$ and $Y \sim \text{binomial}(n, 1-p)$. Proof the following identities:

$$
\begin{array}{lrcl}
\text{(a)} & \Pr(X = k) & = & \Pr(Y = n-k) \\
\text{(b)} & \Pr(X \leq i) & = & \Pr(Y \geq n-i)
\end{array}
$$

9.) Let $n \in \mathbb{N}$ and $p \in (0,1)$ and X a random variable with $X \sim \text{binomial}(n,p)$. Proof the following facts about the pmf of X:

(a) $p_X(i+1) = \dfrac{p}{1-p} \cdot \dfrac{n-i}{i+1} \cdot p_X(i)$ for $i = 0, 1, \ldots, n-1$.

(b) For $m = \lfloor (n+1)p \rfloor$ the sequences $p_X(0), p_X(1), p_X(2), ..., p_X(m)$ and $p_X(m), p_X(m+1), p_X(m+2), ..., p_X(n)$ are increasing and decreasing, respectively.

10.) Let $p \in (0,1)$ and X a random variable with $X \sim \text{geometric}(p)$. Use the moment generating function

$$
\phi_X(t) = \frac{p\, e^t}{1 - (1-p)\, e^t} \qquad (t \in \mathbb{R})
$$

to prove:

(i) $E(X) = \frac{1}{p}$ (ii) $\text{Var}(X) = \frac{1-p}{p^2} = \frac{1}{p^2} - \frac{1}{p}$

11.) Provide a MatLab script that generates a plot of the geometric(p) pmf. The plot shall also display the values of the expectation and the standard deviation. These values shall also be depicted in the plot.

12.) The number of times that an individual contracts a cold in a given year is a Poisson random variable with paramerer $\lambda = 3$. Suppose a new wonder drug (based on large quantities of vitamin C) has just been marketed that reduces the Poisson parameter to $\lambda = 2$ for 75 percent of the population. For the other 25 percent of the population, the drug has no appreciable effect on colds.

If an individual tries the drug for a year and has 0 colds in that time, how likely is it that the drug is beneficial for him or her?

13.) Provide a MatLab script that generates a plot of the Poisson(λ) pmf. The plot shall also display the values of the expectation and the standard deviation. These values shall also be depicted in the plot.