Technische Hochschule OWL **Prof. Dr. Uwe Meier**
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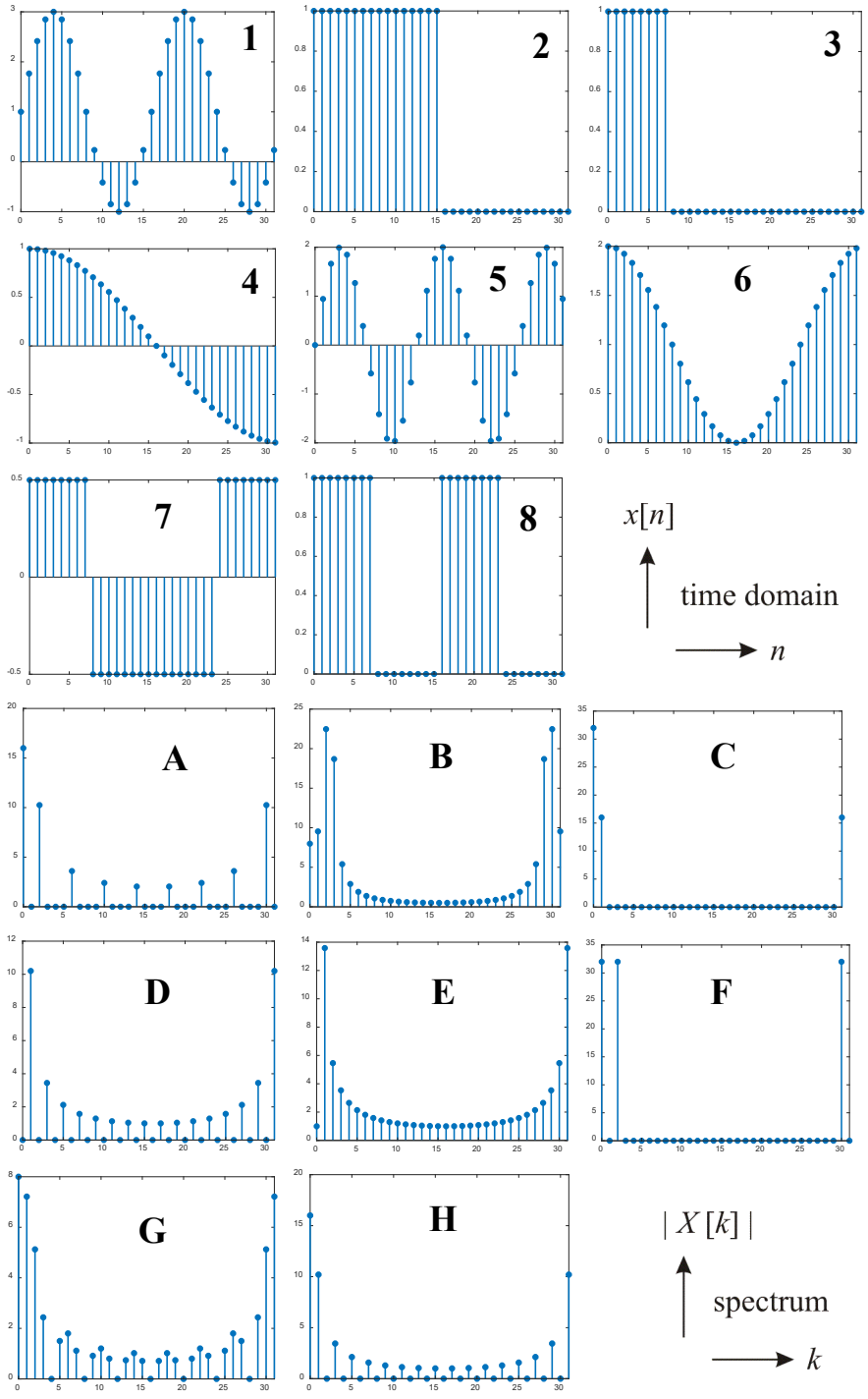
Written Exam: Discrete Signals and Systems (DSS)
Degree Programmes: Information Technology (M. Sc.), Elektrotechnik (M. Sc.)
2020-02-07, 120 min, 100 points available → no notebooks, no books

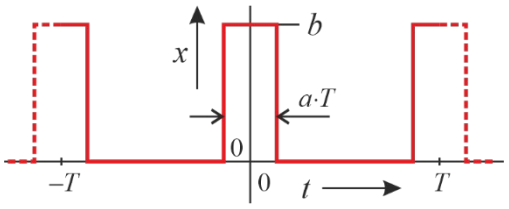
*Please: Don't use red ink; start the solution of each problem on a **new** sheet or side of paper; present all solutions thoroughly.*

Problem 1 Mapping task: 8 different discrete-time sequences $x[n]$ and their discrete FOURIER transforms $|X[k]|$ are given. Unfortunately, the relations are missing. Find the correct relations and give the answer in the form (1, C), if you think, that this is correct. *You are allowed to guess without proving your answer. But thinking might increase your success.* ☺



15 points

Problem 2 Time-continuous signal



2.1 Compute the continuous-time FOURIER transform of the periodic signal $x(t)$ for $0 \le a \le 1$.

2.2 Determine the amplitude of the fundamental oscillation with $a = 0.2$ and $a = 0.8$. Compute the maximal amplitude and the corresponding value of a.

15 points

Problem 3 A time-continuous system responds to the input signal $x(t)$ with the output signal

$$y(t) = -x(t) + 2 \cdot \int_0^{\infty} \mathrm{e}^{-\tau} \cdot x(t-\tau) \cdot \mathrm{d}\tau \ .$$

3.1 Rewrite $y(t)$ by using the convolution operator $*$.

3.2 Determine the impulse response of the system.

3.3 Is the system stable?

15 points

Problem 4 The time-discrete sequence $\{x[n]\} = \{0, -1,\ 2,\ 4,\ 2, -1\}$ with $0 \le n \le 5$ is given.

4.1 Compute an *analytical* expression of the discrete FOURIER transform? *No e-functions should be visible in the final form. No numerical values reqired.*

4.2 Compute a numerical value of the FOURIER transform energy per spectral period.

15 points

Problem 5 The system output of a time-discrete system ist given by the difference equation

$$3 \cdot y[n] - 4 \cdot y[n-1] + y[n-2] = \left(\frac{1}{2}\right)^n \cdot u[n] \ .$$

5.1 Determine the z-transformed output signal $Y(z)$ and its region of convergence.

5.2 Compute the output signal $y[n]$.

5.3 Is $y[n]$ absolutely summable?

15 points

Problem 6 Check, if the following systems are linear, time invariant and causal. $x(t)$, $x[n]$ and $y(t)$, $y[n]$ are the input and output signals, respectively. *Short proof sufficient for saving time.*

1) $y[n] = \left| x[n + n_0] \right|$, n_0 real

2) $y(t) = \dfrac{\mathrm{d}}{\mathrm{d}t}\{x(t) + t\}$

3) $y(t) = t^2 \cdot x(t)$

4) $y[n-1] = n^2 \cdot y[n-2] + n \cdot y[n] + x[n]$

15 points

Problem 7 The time-continuous signal $y(t) = \cos(2\pi \cdot 20\ \mathrm{Hz} \cdot t) + 0.1 \cdot \sin(2\pi \cdot 24\ \mathrm{Hz} \cdot (t - \pi/2))$ can only be observed for a short duration. Choose the minimal observation time T_o for a continuous-time FOURIER transform in order to avoid loss of important spectral content.

10 points

Good luck !