

20. Suppose that distinct integer values are written on each of 3 cards. These cards are then randomly given the designations A, B, and C. The values on cards A and B are then compared. If the smaller of these values is then compared with the value on card C, what is the probability that it is also smaller than the value on card C?
21. There is a 60 percent chance that the event  $A$  will occur. If  $A$  does not occur, then there is a 10 percent chance that  $B$  will occur.
- What is the probability that at least one of the events  $A$  or  $B$  occurs?
  - If  $A$  is the event that the democratic candidate wins the presidential election in 2012 and  $B$  is the event that there is a 6.2 or higher earthquake in Los Angeles sometime in 2013, what would you take as the probability that both  $A$  and  $B$  occur? What assumption are you making?
22. The sample mean of the annual salaries of a group of 100 accountants who work at a large accounting firm is \$130,000 with a sample standard deviation of \$20,000. If a member of this group is randomly chosen, what can we say about
- the probability that his or her salary is between \$90,000 and \$170,000?
  - the probability that his or her salary exceeds \$150,000?
- Hint:* Use the Chebyshev inequality.
23. Of three cards, one is painted red on both sides; one is painted black on both sides; and one is painted red on one side and black on the other. A card is randomly chosen and placed on a table. If the side facing up is red, what is the probability that the other side is also red?
24. A couple has 2 children. What is the probability that both are girls if the eldest is a girl?
25. Fifty-two percent of the students at a certain college are females. Five percent of the students in this college are majoring in computer science. Two percent of the students are women majoring in computer science. If a student is selected at random, find the conditional probability that
- this student is female, given that the student is majoring in computer science;
  - this student is majoring in computer science, given that the student is female.
26. A total of 500 married working couples were polled about their annual salaries, with the following information resulting.

Wife	Husband	
	Less than \$25,000	More than \$25,000
Less than \$25,000	212	198
More than \$25,000	36	54

Thus, for instance, in 36 of the couples the wife earned more and the husband earned less than \$25,000. If one of the couples is randomly chosen, what is

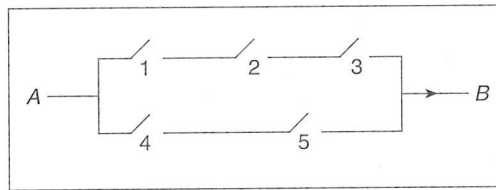
- (a) the probability that the husband earns less than \$25,000;
  - (b) the conditional probability that the wife earns more than \$25,000 given that the husband earns more than this amount;
  - (c) the conditional probability that the wife earns more than \$25,000 given that the husband earns less than this amount?
27. There are two local factories that produce radios. Each radio produced at factory  $A$  is defective with probability .05, whereas each one produced at factory  $B$  is defective with probability .01. Suppose you purchase two radios that were produced at the same factory, which is equally likely to have been either factory  $A$  or factory  $B$ . If the first radio that you check is defective, what is the conditional probability that the other one is also defective?
28. A red die, a blue die, and a yellow die (all six-sided) are rolled. We are interested in the probability that the number appearing on the blue die is less than that appearing on the yellow die which is less than that appearing on the red die. (That is, if  $B$  ( $R$ ) [ $Y$ ] is the number appearing on the blue (red) [yellow] die, then we are interested in  $P(B < Y < R)$ .)
- (a) What is the probability that no two of the dice land on the same number?
  - (b) Given that no two of the dice land on the same number, what is the conditional probability that  $B < Y < R$ ?
  - (c) What is  $P(B < Y < R)$ ?
  - (d) If we regard the outcome of the experiment as the vector  $B, R, Y$ , how many outcomes are there in the sample space?
  - (e) Without using the answer to (c), determine the number of outcomes that result in  $B < Y < R$ .
  - (f) Use the results of parts (d) and (e) to verify your answer to part (c).
29. You ask your neighbor to water a sickly plant while you are on vacation. Without water it will die with probability .8; with water it will die with probability .15. You are 90 percent certain that your neighbor will remember to water the plant.
- (a) What is the probability that the plant will be alive when you return?
  - (b) If it is dead, what is the probability your neighbor forgot to water it?
30. Two balls, each equally likely to be colored either red or blue, are put in an urn. At each stage one of the balls is randomly chosen, its color is noted, and it is then returned to the urn. If the first two balls chosen are colored red, what is the probability that
- (a) both balls in the urn are colored red;
  - (b) the next ball chosen will be red?

31. A total of 600 of the 1,000 people in a retirement community classify themselves as Republicans, while the others classify themselves as Democrats. In a local election in which everyone voted, 60 Republicans voted for the Democratic candidate, and 50 Democrats voted for the Republican candidate. If a randomly chosen community member voted for the Republican, what is the probability that she or he is a Democrat?
32. Each of 2 balls is painted black or gold and then placed in an urn. Suppose that each ball is colored black with probability  $\frac{1}{2}$ , and that these events are independent.
- (a) Suppose that you obtain information that the gold paint has been used (and thus at least one of the balls is painted gold). Compute the conditional probability that both balls are painted gold.
  - (b) Suppose, now, that the urn tips over and 1 ball falls out. It is painted gold. What is the probability that both balls are gold in this case? Explain.
33. Each of 2 cabinets identical in appearance has 2 drawers. Cabinet *A* contains a silver coin in each drawer, and cabinet *B* contains a silver coin in one of its drawers and a gold coin in the other. A cabinet is randomly selected, one of its drawers is opened, and a silver coin is found. What is the probability that there is a silver coin in the other drawer?
34. Prostate cancer is the most common type of cancer found in males. As an indicator of whether a male has prostate cancer, doctors often perform a test that measures the level of the PSA protein (prostate specific antigen) that is produced only by the prostate gland. Although higher PSA levels are indicative of cancer, the test is notoriously unreliable. Indeed, the probability that a noncancerous man will have an elevated PSA level is approximately .135, with this probability increasing to approximately .268 if the man does have cancer. If, based on other factors, a physician is 70 percent certain that a male has prostate cancer, what is the conditional probability that he has the cancer given that
- (a) the test indicates an elevated PSA level;
  - (b) the test does not indicate an elevated PSA level?

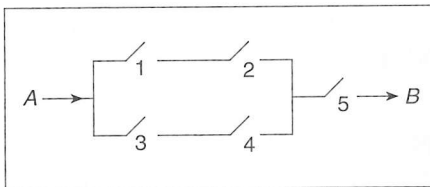
Repeat the preceding, this time assuming that the physician initially believes there is a 30 percent chance the man has prostate cancer.

35. Suppose that an insurance company classifies people into one of three classes — good risks, average risks, and bad risks. Their records indicate that the probabilities that good, average, and bad risk persons will be involved in an accident over a 1-year span are, respectively, .05, .15, and .30. If 20 percent of the population are “good risks,” 50 percent are “average risks,” and 30 percent are “bad risks,” what proportion of people have accidents in a fixed year? If policy holder *A* had no accidents in 1987, what is the probability that he or she is a good (average) risk?

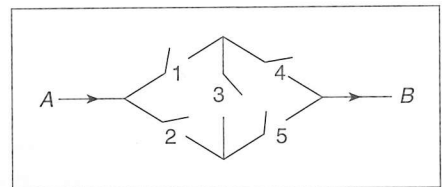
36. A pair of fair dice is rolled. Let  $E$  denote the event that the sum of the dice is equal to 7.
- (a) Show that  $E$  is independent of the event that the first die lands on 4.
- (b) Show that  $E$  is independent of the event that the second die lands on 3.
37. The probability of the closing of the  $i$ th relay in the circuits shown is given by  $p_i, i = 1, 2, 3, 4, 5$ . If all relays function independently, what is the probability that a current flows between  $A$  and  $B$  for the respective circuits?



(a)



(b)



(c)

- (a) If the  $i$ th component functions with probability  $P_i, i = 1, 2, 3, 4$ , compute the probability that a 2-out-of-4 system functions.
- (b) Repeat (a) for a 3-out-of-5 system.
38. An engineering system consisting of  $n$  components is said to be a  $k$ -out-of- $n$  system ( $k \leq n$ ) if the system functions if and only if at least  $k$  of the  $n$  components function. Suppose that all components function independently of each other.
39. Five independent flips of a fair coin are made. Find the probability that
- (a) the first three flips are the same;
- (b) either the first three flips are the same, or the last three flips are the same;
- (c) there are at least two heads among the first three flips, and at least two tails among the last three flips.
40. Suppose that  $n$  independent trials, each of which results in any of the outcomes 0, 1, or 2, with respective probabilities .3, .5, and .2, are performed. Find the probability that both outcome 1 and outcome 2 occur at least once. (Hint: Consider the complementary probability.)

41. A parallel system functions whenever at least one of its components works. Consider a parallel system of  $n$  components, and suppose that each component independently works with probability  $\frac{1}{2}$ . Find the conditional probability that component 1 works, given that the system is functioning.
42. A certain organism possesses a pair of each of 5 different genes (which we will designate by the first 5 letters of the English alphabet). Each gene appears in 2 forms (which we designate by lowercase and capital letters). The capital letter will be assumed to be the dominant gene in the sense that if an organism possesses the gene pair  $xX$ , then it will outwardly have the appearance of the  $X$  gene. For instance, if  $X$  stands for brown eyes and  $x$  for blue eyes, then an individual having either gene pair  $XX$  or  $xX$  will have brown eyes, whereas one having gene pair  $xx$  will be blue-eyed. The characteristic appearance of an organism is called its *phenotype*, whereas its genetic constitution is called its *genotype*. (Thus 2 organisms with respective genotypes  $aA, bB, cC, dD, eE$  and  $AA, BB, CC, DD, EE$  would have different genotypes but the same phenotype.) In a mating between 2 organisms each one contributes, at random, one of its gene pairs of each type. The 5 contributions of an organism (one of each of the 5 types) are assumed to be independent and are also independent of the contributions of its mate. In a mating between organisms having genotypes  $aA, bB, cC, dD, eE$ , and  $aa, bB, cc, Dd, ee$ , what is the probability that the progeny will (1) phenotypically, (2) genotypically resemble
- the first parent;
  - the second parent;
  - either parent;
  - neither parent?
43. Three prisoners are informed by their jailer that one of them has been chosen at random to be executed, and the other two are to be freed. Prisoner  $A$  asks the jailer to tell him privately which of his fellow prisoners will be set free, claiming that there would be no harm in divulging this information because he already knows that at least one of the two will go free. The jailer refuses to answer this question, pointing out that if  $A$  knew which of his fellow prisoners were to be set free, then his own probability of being executed would rise from  $\frac{1}{3}$  to  $\frac{1}{2}$  because he would then be one of two prisoners. What do you think of the jailer's reasoning?
44. Although both my parents have brown eyes, I have blue eyes. What is the probability that my sister has blue eyes?
45. A set of  $k$  coupons, each of which is independently a type  $j$  coupon with probability  $p_j$ ,  $\sum_{j=1}^n p_j = 1$ , is collected. Find the probability that the set contains either a type  $i$  or a type  $j$  coupon.
46. Suppose that distinct integer values are written on each of 3 cards. Suppose you are to be offered these cards in a random order. When you are offered a card you

must immediately either accept it or reject it. If you accept a card, the process ends. If you reject a card, then the next card (if a card remains) is offered. If you reject the first two cards offered, then you must accept the final card.

- (a) If you plan to accept the first card offered, what is the probability that you will accept the highest valued card?
- (b) If you plan to reject the first card offered, and to then accept the second card if and only if its value is greater than the value of the first card, what is the probability that you will accept the highest valued card?

47. Let  $A, B, C$  be events such that  $P(A) = .2, P(B) = .3, P(C) = .4$ . Find the probability that at least one of the events  $A$  and  $B$  occurs if

- (a)  $A$  and  $B$  are mutually exclusive;
- (b)  $A$  and  $B$  are independent.

Find the probability that all of the events  $A, B, C$  occur if

- (c)  $A, B, C$  are independent;
- (d)  $A, B, C$  are mutually exclusive.

48. Two percent of woman of age 45 who participate in routine screening have breast cancer. Ninety percent of those with breast cancer have positive mammographies. Ten percent of the women, who do not have breast cancer will also have positive mammographies. Given a woman has a positive mammography, what is the probability she has breast cancer?