Problem 1 Compute the LAPLACE transforms, determine the ROC and the zero-pole plots for each of the following functions.

a) $x(t) = \mathrm{rect}\left(\frac{t - T/2}{T}\right)$

b) $x(t) = u(t) \cdot \left(\frac{2}{3} - \frac{1}{5} \cdot \mathrm{e}^{2t} - \frac{7}{15} \cdot \mathrm{e}^{-3t}\right)$

c) $x(t) = u(t) \cdot \mathrm{e}^{-2t} + u(t) \cdot \mathrm{e}^{-t} \cdot \cos(3t)$

Problem 2 Compute the causal time functions of the given LAPLACE transforms.

a) $X(s) = \frac{s-4}{s^3 + s^2 - 6s}$ b) $X(s) = \frac{1}{s^2 + s + 1}$ c) $X(s) = \frac{s}{s^2 + s + 1}$

Plot and compare the zero-pole characteristics. Check the initial- and final-value theorems.

Problem 3 The input to and the output of an LTI system are given by

$$x(t) = u(t) \cdot \mathrm{e}^{-3t} \quad ; \quad y(t) = u(t) \cdot \left[\mathrm{e}^{-t} - \mathrm{e}^{-2t}\right]$$

Determine the systems differential equation under the condition of initial rest.

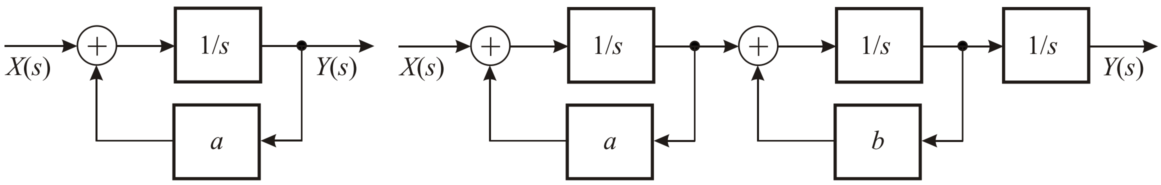
Problem 4 Determine the system function $H(s)$ of the system with the impulse response

$$h(t) = u(t) \cdot (2 - t) \cdot 4 \cdot t \cdot \mathrm{e}^{-2t}.$$

Plot the zero-pole diagram.

Problem 5 Consider a stable system with a real impulse response. Its system function has two poles and a) no, b) one, c) two zeros. Discuss and plot possible impulse responses.

Problem 6 The block diagrams of two systems are given. a, b are real.



System 1 **System 2**

Determine for each of the systems:

a) system function, b) zero-pole plot, c) stability, d) impulse response, e) ARMA topology.

Problem 7 The differential equation of an LTI system is given by

$$\tau \cdot \frac{\mathrm{d}\,y(t)}{\mathrm{d}t} + y(t) = x(t)$$

where τ denotes the time constant. Determine $y(t)$ for $x(t) = 8 \cdot u(t)$ under the condition of initial rest. Solve the differential equation in the time domain and apply LAPLACE transform as well.

Answers

Problem 1

a) $X(s)=\frac{1}{s}\cdot\left(1-\mathrm{e}^{-sT}\right)$; ROC : entire s - plane ; $X(s)$ not rational

b) $X(s)=\dfrac{(s-z_1)}{(s-p_1)\cdot(s-p_2)\cdot(s-p_3)}$; $\sigma>2$; $z_1=4$; $p_1=0$; $p_2=2$; $p_3=-3$

c) $X(s)=\dfrac{2\cdot(s-z_1)\cdot(s-z_2)}{(s-p_1)\cdot(s-p_2)\cdot(s-p_3)}$; $\sigma>-1$; $z_{1/2}=-1{,}25\pm\mathrm{j}2{,}11$; $p_1=-2$; $p_{2/3}=-1\pm\mathrm{j}3$

Problem 2

a) $x(t)=u(t)\cdot\left(\frac{2}{3}-\frac{1}{5}\cdot\mathrm{e}^{2t}-\frac{7}{15}\cdot\mathrm{e}^{-3t}\right)$

b) $x(t)=u(t)\cdot\frac{2}{\sqrt{3}}\cdot\mathrm{e}^{-t/2}\cdot\sin\left(\frac{\sqrt{3}}{2}t\right)$

c) $x(t)=u(t)\cdot\mathrm{e}^{-t/2}\cdot\left(\cos\left(\frac{\sqrt{3}}{2}t\right)-\frac{1}{\sqrt{3}}\sin\left(\frac{\sqrt{3}}{2}t\right)\right)$

Problem 3

$$2\cdot y(t)+3\cdot\frac{\mathrm{d}y}{\mathrm{d}t}+\frac{\mathrm{d}^2y}{\mathrm{d}t^2}=3\cdot x(t)+\frac{\mathrm{d}x}{\mathrm{d}t}$$

Problem 4

$H(s)=\dfrac{8\cdot(s-z)}{(s-p)^3}$; $z=-1$; $p=-2$

Problem 6

$H_1(s)=\dfrac{1}{s-a}$; $H_2(s)=\dfrac{1}{s-a}\cdot\dfrac{1}{s-b}\cdot\dfrac{1}{s}$

System 1: Stable for $a<0$; System 2: Not stable

Problem 7

$y(t)=8\cdot\left(1-\mathrm{e}^{-t/\tau}\right)\cdot u(t)$