Problem 1 The impulse response of a moving average filter is given by

$$h[n] = \sum_{i=0}^{M-1} \frac{1}{M} \cdot \delta[n-i].$$

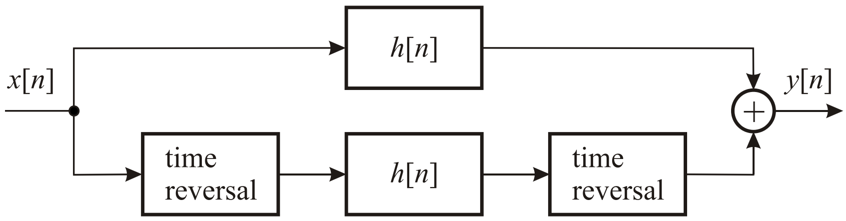
Compute the transfer function and the pole-zero location for $M > 1$. Is the system stable?

Problem 2 A causal, finite-dimensional, linear, time-invariant IIR filter is characterized by the constant coefficient difference equation

$$y[n] = x[n-1] - 1.2 \cdot x[n-2] + x[n-3] + 1.3 \cdot y[n-1] - 1.04 \cdot y[n-2] + 0.222 \cdot y[n-3].$$

Determine the transfer function and the pole-zero plot. Is the system stable?

Problem 3 Let a causal LTI discrete-time system be characterized by a real impulse response $h[n]$ with the DTFT $H\left(e^{j\Omega}\right)$. Consider the given system with a complex finite-length sequence $x[n]$. Determine the frequency response $G\left(e^{j\Omega}\right)$ of the overall system and characterize the system.



Problem 4 The frequency response of an LTI discrete-time system is given:

$$H\left(e^{j\Omega}\right) = \begin{cases} e^{-j6\Omega} & , \ 0 \le |\Omega| \le \frac{\pi}{4} \\ 0 & , \ \frac{\pi}{4} < |\Omega| \le \pi \end{cases}$$

a) Plot the magnitude and phase response and characterize the system.

b) Determine the impulse response.

c) Is the system realizable?

d) Truncate the impulse response to $0 \le n \le 12$ and discuss the realizability and the truncation effect.

Problem 5 Consider the following causal IIR transfer function:

$$H(z) = \frac{3 \cdot z^3 + 2 \cdot z^2 + 5}{(0.5 \cdot z + 1) \cdot (z^2 + z + 0.6)}$$

a) Is $H(z)$ a stable transfer function?

b) If it is not stable, find a stable transfer function $G(z)$ such that $\left|G\left(e^{j\Omega}\right)\right| = \left|H\left(e^{j\Omega}\right)\right|$.

c) Is there any other transfer function having the same magnitude response as that of $H(z)$?

Problem 6 Consider the problem 7 from ex3-2 again: Discretize the differential equation with $T_s = \tau / 10$ and compute $y[n]$ in the time domain and additionally by applying the z-transform.

Answers

Problem 1

$$H(z) = \frac{1}{M}\cdot\frac{1}{z^{M-1}}\cdot\frac{z^M-1}{z-1} = \frac{1}{M}\cdot\frac{1}{z^{M-1}}\cdot(z-z_1)\cdot(z-z_2)\cdots(z-z_{M-1})$$

$M-1$ finite zeros at $z_k = \mathrm{e}^{\mathrm{j}2\pi k/M}$ with $k = 1, 2, \ldots, M-1$

$M-1$ poles at $z = 0$. Stable.

Problem 2

$$H(z) = \frac{z^2 - 1.2\cdot z + 1}{z^3 - 1.3\cdot z^2 + 1.04\cdot z - 0.222}$$

$z_{1,2} = 0.6 \pm \mathrm{j}0.8$; $p_1 = 0.3$; $p_{2,3} = 0.5 \pm \mathrm{j}0.7$; stable

Problem 3

$G\left(\mathrm{e}^{\mathrm{j}\Omega}\right) = 2\cdot\mathrm{Re}\left\{H\left(\mathrm{e}^{\mathrm{j}\Omega}\right)\right\}$; zero-phase transfer function

Problem 4

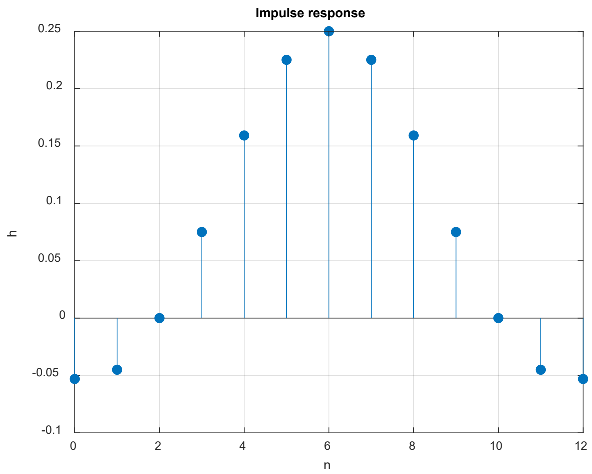
a) Linear-phase lowpass filter

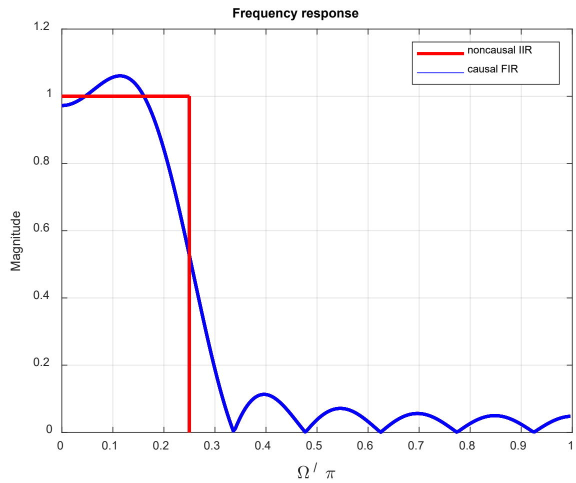
b) $h[n] = \frac{1}{4}\cdot\frac{\sin(\pi\cdot(n-6)/4)}{\pi\cdot(n-6)/4}$

c) Noncausal with $-\infty < n < \infty$. Not realizable.

d) Causal FIR filter with length 13. Rectangular magnitude response is modified.

$$H_\mathrm{T}\left(\mathrm{e}^{\mathrm{j}\Omega}\right) = \left\{\mathrm{rect}\left(\frac{\Omega}{\pi/2}\right) * \frac{\sin(6.5\cdot\Omega)}{\sin(\Omega/2)}\right\}\cdot\mathrm{e}^{-\mathrm{j}6\Omega}$$





Problem 5

a) No

b) $G(z) = \dfrac{3 \cdot z^3 + 2 \cdot z^2 + 5}{(z+0.5)\cdot(z^2 + z + 0.6)}$

c) Yes, infinitely many

Problem 6

$$y[n] = 8 \cdot \left(1 - \left(\frac{9}{10}\right)^n\right) \cdot u[n]$$