Information Fusion – Inference and Fusion Strategies

Combination Techniques for Uncertain Information in Measurement and Signal Processing

4.1 Evidence Theory Fusion

Information Fusion Prof. Dr.-Ing. Volker Lohweg

Lectures – Contents

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4.1.0 True positive / negative rate and all that …

Binary confusion matrix

- For assigments of data, features, decisions, the terms *true positives (tp), true negatives (tn), false positives (fp)*, and *false negatives (fn)* compare the results of a decision unit with trusted external judgments (supervised learning).
- The terms *positive* and *negative* refer to the *expectation*, and the terms *true* and *false* refer to whether that prediction corresponds to the external judgment (sometimes known as the *observation*).

		Expectation
Prediction	tp	fp*
	fn**	tn

* Type I error; ** Type II error

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4.1.0 True positive / negative rate and all that …

Binary confusion matrix

- True positive = correctly identified
- False positive = incorrectly identified
- True negative = correctly rejected
- False negative = incorrectly rejected

$$TP = \sum_i tp_i \qquad TN = \sum_i tn_i$$

$$FP = \sum_i fp_i \qquad FN = \sum_i fn_i$$

- Some formulae
  - True Positive Rate (TPR) (**Recall, Sensitivity**): $TPR = \frac{TP}{TP + FN} = 1 - FNR$
  - False Negative Rate (FNR) (**Miss Rate**): $FNR = \frac{FN}{FN + TP} = 1 - TPR$

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4.1.0 True positive / negative rate and all that …

Binary confusion matrix

- Some formulae, cont'd
  - True Negative Rate (TNR) (**Specitivity**): $TNR = \frac{TN}{TN+FP} = 1 - FPR$
  - Positive Prediction Value (PPV) (**Precision**): $PPV = \frac{TP}{TP+FP} = 1 - FDR$
  - False Positive Rate (FPR) (**Fall-out**): $FPR = \frac{FP}{FP+TN} = 1 - TNR$
  - False Discovery Rate (FDR): $FDR = \frac{FP}{FP+TP} = 1 - PPV$
  - Accuracy: $ACC = \frac{TP+TN}{TP+TN+FP+FN}$
  - F-measure (**F1 score**): $F_1 = 2 \cdot \frac{PPV \cdot TPR}{PPV + TPR}$

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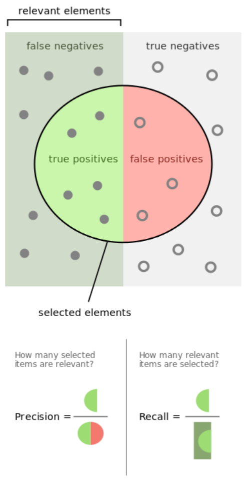
4.1.0 True positive / negative rate and all that …



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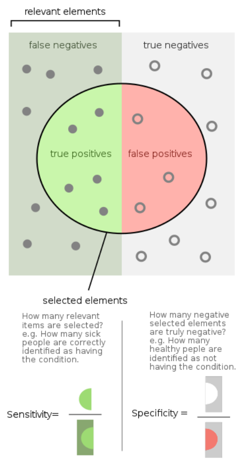

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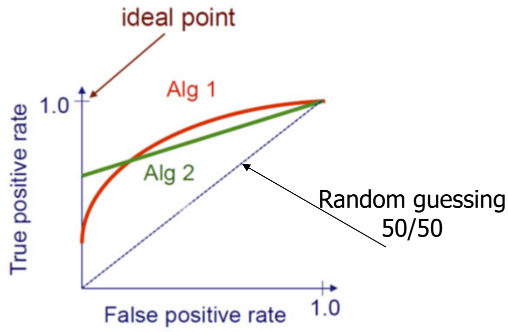
4.1.0 True positive / negative rate and all that …

Receiver Operating Curve (ROC)

- A *Receiver Operating Curve* (*ROC*) plots the TP-rate vs. the FP-rate as a threshold on the confidence of an instance.



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4.1.1 Production streams

- Application of Poisson Distribution in Production Processes

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4.1.1 Production streams

- It is well known, that production processes can be described with a binomial probability distribution in the case of decorrelated occurrences of true and false decisions in a process.
- This fact is in general true for high volume production with low waste rates (e.g. banknotes, electronic devices, screws, etc.). It can also be assumed, that the process is
  - stationary – the occurrence is only dependent on the measurement and analysis time,
  - memoryless – the number of events is not dependent on the statistic of the occurrences itself and
  - ordinal – two events with different results can only occur consecutively.

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4.1.1 Production streams

- In this case the statistical behaviour can be described with a Poisson stream (Poisson distribution):

$$p(x,\lambda) = \frac{\lambda^x}{x!} \cdot e^{-\lambda} \Rightarrow W_P(\lambda, M) = \sum_{x=0}^{M} \frac{\lambda^x}{x!} \cdot e^{-\lambda},$$

- were M is the number of occurrences and $\lambda = N \cdot p = const.$ is the expected value of the process.

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4.1.1 Production streams

- Example 4.1-1 "Banknote production I"

  - For example, in a lot of 20,000 produced sheets are 1,000 sheets classified as bad (*wwwc* = 5%) and 19,000 classified as good.
  - A) The number of **good sheets** in a bad sheet pile must be checked. It is assumed that the a-priori probability of such a pseudo error (False rejection rate FRR) is *FRR* = 0.005 (0.5%) and the number of sheets in the bad pile is *N* = 1,000.
  - B) The number of **bad sheets** in a good sheet pile must be checked. It is assumed that the a-priori probability of such a pseudo error (False acceptance rate FAR) is *FAR* = 0.005 (0.5%) and the number of sheets in the good pile is *N* = 19,000.

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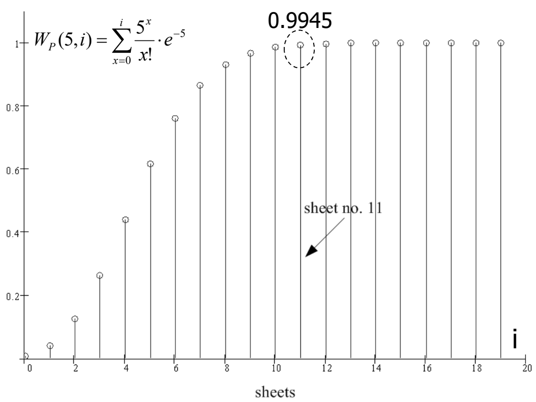
4.1.1 Production streams

- Example 4.1-1 "Banknote production – FRR"



We assume a statistical acceptance level of approx. 99.5%:

Applying the parameters to a/m equation results in the discrete distribution curve. Here, M = 11 for p = 0.9945. That is: **With a probability of 99.45 % there are not more than 11 good sheets in the bad sheet pile.**

$$W_P(\lambda, M)=\sum_{x=0}^{M}\frac{\lambda^x}{x!}\cdot e^{-\lambda}, \lambda = N\cdot p$$

with $\lambda = 1{,}000\cdot 0.005 = 5$ and $M = 11 \rightarrow$

$$W_P(5,11)=\sum_{x=0}^{11}\frac{5^x}{x!}\cdot e^{-5} = 0.9945$$

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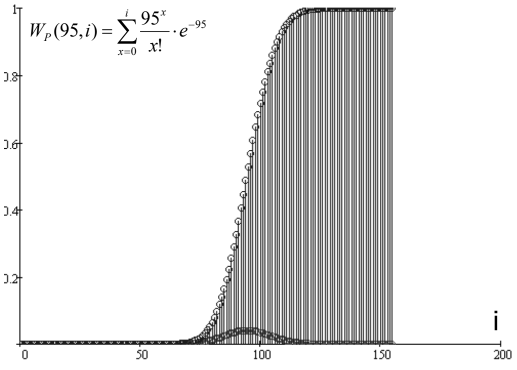
4.1.1 Production streams

- Example 4.1-1 "Banknote production – FAR"



We assume a statistical acceptance level of approx. 99.5%:

Applying the parameters to a/m equation results in the discrete distribution curve. Here, M = 120 for p = 0.9943. That is: **With a probability of 99.43 % there are not more than 120 bad sheets in the good sheet pile**.

$$W_P(\lambda, M) = \sum_{x=0}^{M} \frac{\lambda^x}{x!} \cdot e^{-\lambda}, \lambda = N \cdot p$$

with $\lambda = 19{,}000 \cdot 0.005 = 95$ and $M = 120 \rightarrow$

$$W_P(95,120) = \sum_{x=0}^{120} \frac{95^x}{x!} \cdot e^{-95} = 0.9943$$

Is a FAR of 0.5% ok in a production process?

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4.1.1 Production streams

- The mentioned procedure applies to monomodal (mono-sensor) systems in general. In the next step it is assumed that different channels of a multimodal system operate on a specific detection or inspection procedure.
- The signal sources may not be of the same type – they can be different (image sensors, acoustical sensors, etc.). However, a signal decorrelation in a broader sense is necessary.

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4.1.1 Production streams

- Now we examine the multimodal sensor case, supposing decorrelated signals. Therefore, the probability of a statistical event (signal) is assumed as independent of all other events.
- We consider a system with four decorrelated inspection channels which can be described via their distribution functions $W(E_k)$ of the events E_k, $k \in \{1,2,3,4\}$. The complementary probability of the mentioned events is

$$W\left(E_k^c\right) = 1 - W\left(E_k\right)$$

- If now e.g. n = 4 inspection channels are operating parallel, then the probability that at least one specific channels will detect an error, is assumed to be high.

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4.1.1 Production streams

- The process validation operation V reflects the process description on a **logical level**. We assume that at least one channel will deliver the correct inspection result (error, no error) with a specific probability.
- Therefore, the function V has to be interpreted in a sense of disjunctive probability (union) interconnection.
- The table shows the connection in form of a disjunction matrix.

E_4	E_3	E_2	E_1	F
0	0	0	0	0
0	0	0	D	D
0	0	D	0	D
0	0	D	D	D
0	D	0	0	D
.	.	.	.	.
D	D	D	D	D

D: detection

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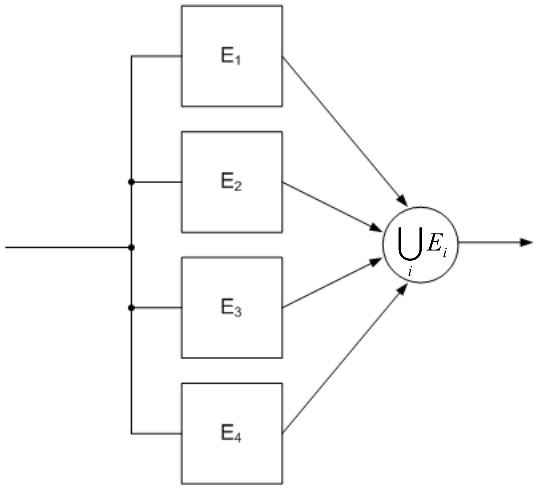
4.1.1 Production streams

- Coupling sketch



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4.1.1 Production streams

- As the probability distribution functions of the channels are usually equipped with different λ_i the valuation function has to be defined in the **canonical** disjunctive form. For example, the connection of two sensor probability functions is as follows:

$$F = \left(E_2^c \wedge E_1\right) \vee \left(E_2 \wedge E_1^c\right) \vee \left(E_2 \wedge E_1\right),$$

$$F^c = E_2^c \wedge E_1^c \; .$$

E_2	E_1	F
0	0	0
0	D	D
D	0	D
D	D	D

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4.1.1 Production streams

- The resulting distribution function which is coupled via the valuation function is therefore (two channels):

$$
\begin{aligned}
W(E_1+E_2) &= \overbrace{(1-W(E_2))\cdot W(E_1)}^{W\left(E_2^c \wedge E_1\right)} + (1-W(E_1))\cdot W(E_2) + W(E_2)\cdot W(E_1) = \cdots \\
\cdots &= W(E_1) - W(E_2)\cdot W(E_1) + W(E_2) - W(E_2)\cdot W(E_1) + W(E_2)\cdot W(E_1) = \cdots \\
\cdots &= W(E_1) + W(E_2) - W(E_2)\cdot W(E_1) = 1 - \left((1-W(E_1))\cdot(1-W(E_2))\right)
\end{aligned}
$$

- The general case:

$$W\left(E_1+\cdots+E_n\right) = 1-\prod_{k=1}^{n}\left(1-W\left(E_k\right)\right)$$

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4.1.1 Production streams

- In particular, for the Poisson distribution it follows from the above:

$$W_P\left(E_1+\cdots+E_n\right) = 1-\prod_{k=1}^{n}\left(1-\sum_{i=0}^{M}\frac{\lambda_k{}^i}{i!}\cdot e^{-\lambda_k}\right)$$

$$\lambda_k = N\cdot p_k$$

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4.1.1 Production streams

- Example 4.1-2 "Banknote production II – FAR, revisited"
  - As a second example, we refer to the first example in this section.
  - As an assumption we consider a four channel system (n = 4) with M = 100, N = 19,000, FAR = 0.005.
  - The probability that there are not more than 100 bad sheets in the good pile of 19,000 sheets is approx. 72% in a monomodal system. Whereas the probability for a four channel multimodal system results in 99.4%.

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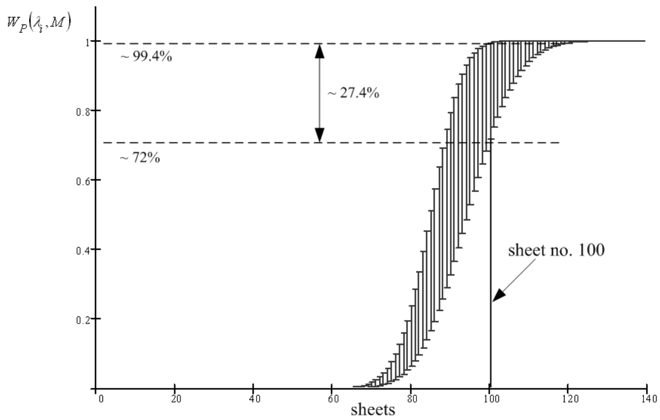
4.1.1 Production streams

- Example 4.1-2 "Banknote production II – FAR"



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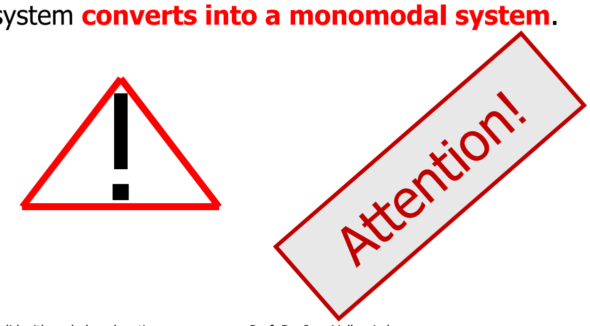
4.1.1 Production streams

- However, it must be accentuated that the above mentioned holds in cases where the detection result is a contribution of more than one channel.
- In some cases only one signal source is able to detect a specific signal class. Then a multimodal system **converts into a monomodal system**.



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4.1.2 Bayesian Inference

- Application of Bayesian Inference (Fusion) in an medical example

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4.1.2 Bayesian Inference

- COVID-19 Antibody test
  - **Sensitivity** (true positive rate, TPR) measures the proportion of actual positives that are correctly identified as such (e.g., the percentage of sick people who are correctly identified as having the condition).
  - **Specificity** (true negative rate, TNR) measures the proportion of actual negatives that are correctly identified as such (e.g., the percentage of healthy people who are correctly identified as not having the condition).

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4.1.2 Bayesian Inference

- Example 4.1-3 "Medical Test, COVID-19"
  - This is the simplest example of reasoning by combining sources of information.
  - Test for rare disease is positive with 90% sensitivity.
  - What's the probability that you have the disease?
  - What if the test is repeated?
    - Event D = "Disease did occur" → P[D].
    - Event D^c = "Disease did not occur" → P[D^c].
    - Event T = "Test is positive" → P[T].
    - Event T^c = "Test is not positive" → P[T^c].

    - We know $P[T|D] = 0.9$ ← The probability that the test is positive under the condition that the disease occurred is 90%.

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4.1.2 Bayesian Inference

- Example 4.1-3 "Medical Test", cont'd
  - We want P[D|T] = "The probability of the having the disease given a positive test".
  - What's the prior P[D]?
    - Disease is rare, so let's assume P[D] = 0.001.
  - What about P[T]?
    - What's the interpretation of that?

$$P[D|T] = \frac{P[T|D] \cdot P[D]}{P[T]}$$

$$P[T] = P[T \cap D] + P[T \cap D^C] = P[T|D] \cdot P[D] + P[T|D^c] \cdot P[D^c]$$

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4.1.2 Bayesian Inference

- Example 4.1-3 "Medical Test", cont'd

$$P[D|T] = \frac{P[T|D] \cdot P[D]}{P[T|D] \cdot P[D] + P[T|D^c] \cdot P[D^c]}$$

  - Some calculations
    - P[D] = 0.001, P[T|D] = 0.9
    - P[T|D^c] = 0.1 (FP), P[D^c] = 0.999
    - P[T] = P[T|D] P[D]+ P[T|D^c] P[D^c] = 0.9 · 0.001 + 0.1 · 0.999 = 0.101
      - (Total Probability / Evidence)
    - P[D|T] = 0.0089.
    - The probability of the having the disease given a positive test is 0.89%.
    - Homework: What is P[D|T] if the test is 100% reliable (TPR)?
    - Roche* claims TPR = 100% and TNR = 99,8%

*https://www.roche.com/media/releases/med-cor-2020-05-03.htm

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4.1.2 Bayesian Inference

- Combining Information
  - We will follow the last example for clarification.
  - Let us assume we would like to process a second test.
  - We have to calculate

$$P[D|T_1,T_2] = \text{the probability of two combined tests.}$$

  - Let us assume the tests will be processed temporally independent t_1: $T_1 \rightarrow t_2$: T_2 ($t_1 < t_2$).
  - This fact results in (independent events):

$$P[T_1,T_2] = P[T_1]\cdot P[T_2].$$

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4.1.2 Bayesian Inference

- Combining Information
  - We get $$P[D|T_1,T_2] = \frac{P[T_1,T_2|D]\cdot P[D]}{P[T_1,T_2]}$$

$$P[D|T_1,T_2] = \frac{P[T_1|D]\cdot P[T_2|D]\cdot P[D]}{P[T_1]\cdot P[T_2]}$$

the old posterior

  - Now we rearrange the equation slightly.

$$P[D|T_1,T_2] = \frac{P[T_2|D]\cdot P[T_1|D]\cdot P[D]}{P[T_2]\cdot P[T_1]} = \frac{P[T_2|D]}{P[T_2]}\cdot\frac{P[T_1|D]\cdot P[D]}{P[T_1]}$$

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4.1.2 Bayesian Inference

- Combining Information
  - Now

$$P[D|T_1,T_2]=\frac{P[T_2|D]}{P[T_2]}\cdot\frac{P[T_1|D]\cdot P[D]}{P[T_1]}=\frac{P[T_2|D]}{P[T_2]}\cdot P[D|T_1]=\cdots$$

$$\cdots=\frac{P[T_2|D]}{P[T_2|D]\cdot P[D\,|\,T_1]+P[T_2|D^c]\cdot P[(D\,|\,T_1)^c]}\cdot P[D|T_1]$$

This is how Bayesian reasoning combines old information with new information to update our belief states.

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4.1.2 Bayesian Inference

- Example 4.1-4 "Medical Test II"

$$P[D|T_1,T_2]=\frac{P[T_2|D]}{P[T_2|D]\cdot \underbrace{P[D\,|\,T_1]}_{0.0089}+P[T_2|D^c]\cdot \underbrace{P[(D\,|\,T_1)^c]}_{0.9911}}\cdot P[D|T_1]$$

- Some calculations
  - $P[D|T_1] = 0.0089$, $P[T_2|D] = 0.9$
  - $P[T_2|D^c] = 0.1$ (FP), $P[D^c] = 0.9911$
  - $P[T] = P[T_2|D]\,P[D|T_1] + P[T_2|D^c]\,P[(D|T_1)^c] = 0.9 \cdot 0.0089 + 0.1 \cdot 0.9911 = 0.107$
    - (Total Probability / Evidence)
  - $P[D|T] = 0.075$.
  - The probability of the having the disease given a positive test is 7.5%.

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4.1.3 Decision Trees

- Application of Bayesian Decision Trees in Production Processes

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4.1.4 DST – Inference

- Application of DST inference / fusion in a measurement process

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4.1.4 DST – Inference

- Dempster-Shafer inference
  - The problem we now face is how to combine two independent sets of mass assignments. The original combination rule, known as **Dempster's rule of combination**, is a generalization of Bayes Rule.
  - This rule strongly emphasizes the agreement between multiple sources and ignores *all* the conflicting evidence through a normalization factor. This can be considered a strict AND-operation.
  - Use of that rule has come under serious criticism when significant conflict in the information is encountered. We will discuss this later on → action in research.

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4.1.4 DST – Inference

- Dempster-Shafer inference
  - These multiple sources provide different assessments for the same frame of discernment and Dempster-Shafer theory is based on the assumption that these sources are **independent**. The requirement for establishing the independence of sources is an important philosophical question.
  - Specifically, the combination (called the **joint mass**) is calculated from the two sets of masses m_1 and m_2 in the following manner:

$$m_{12}(X) = m_1(X) \oplus m_2(X) = \begin{cases} \dfrac{\sum\limits_{A_i \cap B_j = X} m_1(A_i) \cdot m_2(B_j)}{1 - k_C} & \text{for } X \neq \varnothing,\ A_i \in 2^{\Omega},\ B_j \in 2^{\Omega} \\ 0 & \text{for } X = \varnothing \end{cases}$$

$$k_C = \sum_{A_i \cap B_j = \varnothing} m_1(A_i) \cdot m_2(B_j)$$

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4.1.4 DST – Inference

- Dempster-Shafer inference
  - The *variable* k_C represents basic belief mass associated with conflict. This is determined by the summing the products of the bba's of all sets where the intersection is null. This rule is commutative, associative, but not idempotent or continuous.
  - The denominator in Dempster's rule, 1- k_c is a normalization factor. This has the effect of *completely* ignoring conflict and attributing any probability mass associated with conflict to the null set [Yager, 1987].
  - Problem

$$\lim_{k_c \to 1} m_{12}(X) = \lim_{k_c \to 1} \frac{\sum_{A_i \cap B_j = X} m_1(A_i) \cdot m_2(B_j)}{1 - k_C} \Rightarrow m_{12}(X) \to \infty$$

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4.1.4 DST – Inference

- Dempster-Shafer inference
  - Highly reliable and conflicting sensing sources should be avoided.
  - Example 3.2-3 "Weather report, revisited II"
    - We assume two weather reports (WR) are existing.
      - WR1 forecasts for the next day: 60% rainy (0.6); 30% alternating cloudy and sunny (0.3).
      - WR2 forecasts for the next day: 80% sunny (0.8).
    - We will now use the DST rule of combination. We define the numerator of the DST rule as

$$N(X) = \sum_{A_i \cap B_j = X} m_1(A_i) \cdot m_2(B_j) .$$

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4.1.4 DST – Inference

rainy (0.6)
cloudy/sunny (0.3)
sunny (0.8)

- Dempster-Shafer inference
  - Example 4.1-6 "Weather report, revisited II"

$N_{ij}(X) \underset{A_i \cap B_j = X}{=} m_1(A_i)\cdot m_2(B_j)$	$m_1(\{C\}) = 0.6$	$m_1(\{A,B\}) = 0.3$	$m_1(\{\Omega\}) = 0.1$
$m_2(\{A\}) = 0.8$	$C(\{\varnothing\}) = 0.48$	$C(\{A\}) = 0.24$	$C(\{A\}) = 0.08$
$m_2(\{\Omega\}) = 0.2$	$C(\{C\}) = 0.12$	$C(\{A,B\}) = 0.06$	$C(\{\Omega\}) = 0.02$

$$k_C = \sum_{A_i \cap B_j = \varnothing} m_1(A_i)\cdot m_2(B_j) = C(\{\varnothing\}) = 0.48$$

$$1 - k_C = 1 - 0.48 = 0.52$$

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4.1.4 DST – Inference

- Dempster-Shafer inference
  - Example 4.1-6 "Weather report, revisited II, cont'd"
    - Supporting evidences

$m_{12}(\{\varnothing\})$	= 0
$m_{12}(\{A\})$	$= \frac{0.24 + 0.08}{0.52} \approx 0.62$
$m_{12}(\{C\})$	$= \frac{0.12}{0.52} \approx 0.23$
$m_{12}(\{A,B\})$	$= \frac{0.06}{0.52} \approx 0.11$
$m_{12}(\{\Omega\})$	$= \frac{0.02}{0.52} \approx 0.04$

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4.1.4 DST – Inference

- Dempster-Shafer inference
  - Example 4.1-6 “Weather report, revisited II, cont’d”
    - Belief, Plausibility, Ignorance, support intervals

	Bel	Pl	Igr = Pl-Bel
$\{\varnothing\}$	0	0	0
$\{A\}$	0.62	0.62 + 0.11 + 0.04 = 0.77	0.15
$\{B\}$	0	0.11 + 0.04 = 0.15	0.15
$\{C\}$	0.23	0.23 + 0.04 = 0.27	0.04
$\{A,B\}$	0.62+0.11 = 0.73	0.62 + 0.11 + 0.04 = 0.77	0.04
$\{A,C\}$	0.62 + 0.23 = 0.85	0.62 + 0.23 + 0.11 + 0.04 = 1	0.15
$\{B,C\}$	0.23	0.23 + 0.11 + 0.04 = 0.38	0.15
$\{\Omega\}$	0.62 + 0.23 + 0.11 + 0.04 = 1	0.62 + 0.23 + 0.11 + 0.04 = 1	0

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4.1.4 DST – Inference

- Dempster’s Combination Matrix
  - Suppose there are 2 sources that both provide some support, but also some doubt regarding the hypothesis H.
  - Let Bel_1(H) and Bel_2(H) denotes the belief, Igr_1(H) and Igr_2 (H) denotes the Ignorance (uncertainty) and Pl_1(H) and Pl_2(H) denotes the plausibility.
  - Hence, Pl(H) = 1 – D(H); Igr(H) = Pl(H) – Bel(H)
  - Remember that the following equation always are true:
    - Bel(H) + Igr(H) + D(H) = 1
  - Hence,
    - Bel 1(H) + Igr 1(H) + D 1(H) = 1
    - Bel 2(H) + Igr 2(H) + D 2(H) = 1
    - Bel i(H) + Igr i(H) + D i(H) = 1 etc. are always true

4.1.4 DST – Inference

	$Bel_1(H)$	$Igr_1(H)$	$D_1(H)$
$Bel_2(H)$	Belief	Belief	Conflict
$Igr_2(H)$	Belief	Ignorance	Doubt
$D_2(H)$	Conflict	Doubt	Doubt

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4.1.4 DST – Inference

- Combination of two sources
  - Two frames of discernments

$$\Omega_1 = \{A_1, B_1\} \rightarrow 2^{\Omega_1} = \{\{\varnothing\}, \{A_1\}, \{B_1\}, \{\Omega_1\}\}$$

$$\Omega_2 = \{A_2, B_2\} \rightarrow 2^{\Omega_2} = \{\{\varnothing\}, \{A_2\}, \{B_2\}, \{\Omega_2\}\}$$

  - Masses

m	$\{\varnothing\}$	$\{A_1\}$	$\{B_1\}$	$\{A_1, B_1\}$
$\{\varnothing\}$	$\{\varnothing\}$	$\{\varnothing\}$	$\{\varnothing\}$	$\{\varnothing\}$
$\{A_2\}$	$\{\varnothing\}$	$\{A_1\}\{A_2\}$	$\{B_1\}\{A_2\}$	$\{A_1, B_1\}\{A_2\}$
$\{B_2\}$	$\{\varnothing\}$	$\{A_1\}\{B_2\}$	$\{B_1\}\{B_2\}$	$\{A_1, B_1\}\{B_2\}$
$\{A_2, B_2\}$	$\{\varnothing\}$	$\{A_1\}\{A_2, B_2\}$	$\{B_1\}\{A_2, B_2\}$	$\{A_1, B_1\}\{A_2, B_2\}$

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4.1.4 DST – Inference

- Example 4.1-7 "Two Sources"
  - Two frames of discernments

$$\Omega_1 = \{A_1, B_1\} \rightarrow 2^{\Omega_1} = \{\{\varnothing\}, \{A_1\}, \{B_1\}, \{\Omega_1\}\}$$

$$\Omega_2 = \{A_2, B_2\} \rightarrow 2^{\Omega_2} = \{\{\varnothing\}, \{A_2\}, \{B_2\}, \{\Omega_2\}\}$$

  - Let us assume sensor 1 (infrared) delivers the following events $\{A_1\}$ = {object detected in < 100 mm distance}, $\{B_1\}$ = {object detected in 100 mm … 500 mm distance}, $\{\varnothing\}$ = {no object detected}, $\{A_1, B_1\}$ = {result uncertain}
  - Let us assume sensor 2 (ultra sonic) delivers the following events $\{A_2\}$ = {object detected in < 100 mm distance}, $\{B_2\}$ = {object detected in 100 mm or greater distance}, $\{\varnothing\}$ = {no object detected}, $\{A_2, B_2\}$ = {result uncertain}

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4.1.4 DST – Inference

- Example 4.1-7 "Two Sources", cont'd
  - Combinations: example for $\{A_1\}$ = {object detected in < 100 mm distance}

Cut out

m_{12}	$\{A_1\}$
$\{\varnothing\}$	$\{\varnothing\}$
$\{A_2\}$	$\{A_1\}\ \{A_2\}$
$\{B_2\}$	$\{A_1\}\ \{B_2\}$
$\{A_2, B_2\}$	$\{A_1\}\ \{A_2, B_2\}$

→ $\{\varnothing\}$= {no object detected}

→ $\{A_1\}$ = {object detected in < 100 mm distance}
$\{A_2\}$ = {object detected in < 100 mm distance}

→ $\{A_1\}$ = {object detected in < 100 mm distance}
$\{B_2\}$ = {object detected in 100 mm or greater distance}

→ $\{A_1\}$ = {object detected in < 100 mm distance}
$\{A_2, B_2\}$ = {result uncertain}

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4.1.4 DST – Inference

- Example 4.1-7 "Two Sources", cont'd
  - Combinations: example for $\{A_2\}$ = {object detected in < 100 mm distance}

Cut out

m_{12}	$\{\varnothing\}$	$\{A_1\}$	$\{B_1\}$	$\{A_1, B_1\}$
$\{A_2\}$	$\{\varnothing\}$	$\{A_1\}\ \{A_2\}$	$\{B_1\}\ \{A_2\}$	$\{A_1, B_1\}\ \{A_2\}$

$\{\varnothing\}$ = {no object detected}

$\{A_1\}$ = {object detected in < 100 mm distance}
$\{A_2\}$ = {object detected in < 100 mm distance}

$\{A_2\}$ = {object detected in < 100 mm distance}
$\{B_1\}$ = {object detected in 100 mm or greater distance}

$\{A_2\}$ = {object detected in < 100 mm distance}
$\{A_1, B_1\}$ = {result uncertain}

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4.1.4 DST – Inference

- Example 4.1-7 "Two Sources", cont'd
  - Combinations: example for $\{A_1\}$ = {object detected in < 100 mm distance}

$\{\varnothing\}$ = {no object detected} → $m_1(A_1) \cdot m_2(\varnothing) = 0$

$\{A_1\}$ = {object detected in < 100 mm distance}
$\{A_2\}$ = {object detected in < 100 mm distance} → $m_1(A_1) \cdot m_2(A_2)$

$\{A_1\}$ = {object detected in < 100 mm distance}
$\{B_2\}$ = {object detected in 100 mm or greater distance} → $m_1(A_1) \cdot m_2(B_2)$ conflict

$\{A_1\}$ = {object detected in < 100 mm distance}
$\{A_2, B_2\}$ = {result uncertain} → $m_1(A_1) \cdot m_2(\Omega_2)$

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4.1.4 DST – Inference

- Example 4.1-7 "Two Sources", cont'd
  - Combinations: example for $\{A_2\}$ = {object detected in < 100 mm distance}

{∅} = {no object detected} → $m_1(A_1)\cdot m_2(\varnothing)=0$

$\{A_1\}$ = {object detected in < 100 mm distance}
$\{A_2\}$ = {object detected in < 100 mm distance} → $m_1(A_1)\cdot m_2(A_2)$

$\{A_2\}$ = {object detected in < 100 mm distance}
$\{B_1\}$ = {object detected in 100 mm or greater distance} → $m_2(A_2)\cdot m_1(B_1)$ conflict

$\{A_2\}$ = {object detected in < 100 mm distance}
$\{A_1, B_1\}$ = {result uncertain} → $m_2(A_2)\cdot m_1(\Omega_1)$

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4.1.4 DST – Inference

- Combination of two sources
  - Combinations, example for {A}

$$N(X)=\sum_{A_i\cap B_j=X} m_1(A_i)\cdot m_2(B_j) \qquad k_C=\sum_{A_i\cap B_j=\varnothing} m_1(A_i)\cdot m_2(B_j)$$

$$m_{12}(\{A\})=\frac{m_1(\{A_1\})m_2(\{A_2\})+m_1(\{A_1\})m_2(\{A_2,B_2\})+m_2(\{A_2\})m_2(\{A_1,B_1\})}{1-[m_1(\{A_1\})m_2(\{B_2\}+m_2(\{A_2\})m_1(\{B_1\}]}$$

  - More sources are connected accordingly (first two sources → result and the next source an so forth).

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4.1.4 DST – Inference

- Example 4.1-7 "Two Sources", cont'd
  - Combinations: example for {A} = {object detected in < 100 mm distance}
  - Let us assume:

$$m_1(A_1) = 0.6,\ m_1(B_1) = 0.35,\ m_1(\Omega_1) = 0.05$$
$$m_2(A_2) = 0.8,\ m_2(B_2) = 0.15,\ m_2(\Omega_2) = 0.05$$

$$m_{12}(\{A\}) = \frac{m_1(\{A_1\})m_2(\{A_2\}) + m_1(\{A_1\})m_2(\{A_2,B_2\}) + m_2(\{A_2\})m_1(\{A_1,B_1\})}{1-[m_1(\{A_1\})m_2(\{B_2\} + m_2(\{A_2\})m_1(\{B_1\}]}$$

$$m_{12}(\{A\}) = \frac{0.6\cdot 0.8 + 0.6\cdot 0.05 + 0.8\cdot 0.05}{1-[0.6\cdot 0.15 + 0.8\cdot 0.35]} = 0.873$$

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4.1.4 DST – Inference

- Example 4.1-7 "Two Sources", cont'd
  - Combinations: example for {A} = {object detected in < 100 mm distance}
  - Let us assume:

$$m_1(A_1) = 0.6,\ m_1(B_1) = 0.35,\ m_1(\Omega_1) = 0.05$$
$$m_2(A_2) = 0.8,\ m_2(B_2) = 0.15,\ m_2(\Omega_2) = 0.05$$

A ≈ 87.3%
B ≈ 12.3%
Ω ≈ 0.4%

m	{∅}	$\{A_1\}$	$\{B_1\}$	$\{A_1, B_1\}$
{∅}	{∅}	{∅}	{∅}	{∅}
$\{A_2\}$	{∅}	0.6•0.8 = 0.48	0.35•0.8 = 0.28	0.8•0.05 = 0.04
$\{B_2\}$	{∅}	0.6•0.15 = 0.09	0.35•0.15 = 0.0525	0.15•0.05 = 0.0075
$\{A_2, B_2\}$	{∅}	0.6•0.05 = 0.03	0.35•0.05 = 0.0175	0.05•0.05 = 0.0025

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4.1.4 DST – Inference

- Mass determination
  - The sources generate hypotheses *H* with a specific **certainty** *cer*(S,H). Sometimes this is also called discount factor δ ($0 < \delta < 1$) [Shafer 76].
  - The certainty is based on expert's know-how, measurements, etc. It is a **subjective** certainty.
  - A second measure ist the so called **reliability** *rel* (S) of a sensing source.
  - The reliability is the **objective** measure how certain a source is.
  - Therefore, a mass (BPA) of a source can be defined as follows:

$$m(H) = rel(S) \cdot cer(S,H)$$

  - The reliability can only be generated by measurements. Usually the value is predicted.

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4.1.4 DST – Inference

- Mass determination
  - 1. It will be checked how often the source generates a **true** measurement result based on a hypothesis (true positive → TP).
  - 2. It will be checked how often the source generates a **false** measurement result based on a hypothesis (false negative → FN).
  - The relative occurrence is then

$$occ(H) = TPR(H) = \frac{TP}{TP + FN}$$

  - As we have no further information, we are not able to distinguish between an FN (2.) or an FP (1.).
  - Therefore, we define as follows:

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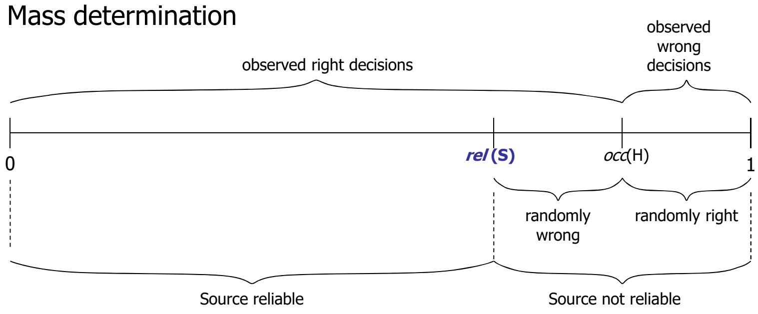
4.1.4 DST – Inference

- Mass determination



$$rel(S) = 1 - 2\cdot(1 - occ(H)) = \cdots$$
$$rel(S) = 1 - 2 + 2occ(H) = 2occ(H) - 1.$$

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4.1.4 DST – Inference

- Example 4.1-7 "Two Sources", cont'd
  - Homework
    - Write an Matlab m-file which …
    - calculates the joined masses for all others supporting evidences
    - calculates Bel(), Pl(), and Igr()
    - Comment the results

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4.1.4 DST – Inference

- Limitations of Dempster-Shafer Theory [Zadeh 84]
  - Normalization

    …may lead to counter-intuitive results, as it ignores belief that objects considered may not exist (e.g. diagnosis)
  - e.g. two doctors:

$$
\begin{aligned}
m_A(meningitis) &= 0.99\\
m_A(brain\ tumor) &= 0.01\\
m_B(concussion) &= 0.99\\
m_B(brain\ tumor) &= 0.01
\end{aligned}
$$

the result is brain tumor with which both doctors disagree.

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4.1.4 DST – Inference

- Combination alternatives
  - Murphy's Rule (trade-off, [Mu00])
  - Yager's Rule (conflict moved to universal set, [Ya83], [Ya85], [Ya87])
  - Campos' Rule (based on DST, [Ca06])
  - …

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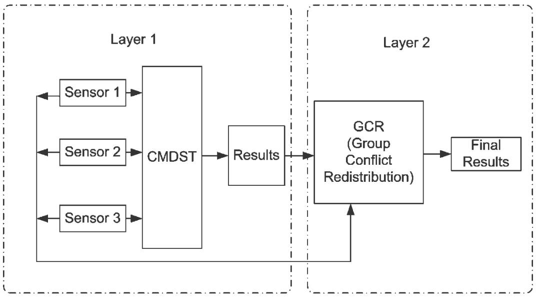
4.1.4 DST – Inference

- Research at Discrete Systems Group, financed by the German Ministry of Education and Research (BMBF)
  - Psychological Evidence: Decision making has been traditionally studied at three levels: *individual*, *group* and *organizational*.



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4.1.5 Fuzzy – Inference

- Application of Fuzzy inference / fusion in a measurement process

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4.1.5 Fuzzy – Inference

- The most commonly used fuzzy inference technique is the so-called Mamdani method.
- In 1975, Ebrahim Mamdani of London University built one of the first fuzzy systems to control a steam engine and boiler combination.
- He applied a set of fuzzy rules supplied by experienced human operators.

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4.1.5 Fuzzy – Inference

- The Mamdani-style fuzzy inference process is performed in four steps:
  - fuzzification of the input variables,
  - rule evaluation;
  - aggregation of the rule outputs, and finally
  - defuzzification.

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4.1.5 Fuzzy – Inference – Mamdani System

- We examine a simple two-input one-output problem that includes three rules:
- Rule 1:

  IF <optical sensor is on "green"> OR <acoustical sensor signal is low>

  THEN (alarm) risk is low A3 B1

- Rule 2:

  IF <optical sensor is on "yellow"> AND <acoustical sensor signal is medium>

  THEN risk is normal A2 B2

- Rule 3:

  IF <optical sensor is on "red"> AND <acoustical sensor signal is high>

  THEN risk is high A1

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4.1.5 Fuzzy – Inference – Mamdani System

- We examine a simple two-input one-output problem that includes three rules:

Rule 1:		Rule 1:	
IF	x is A3	IF	optical sensor: "green"
OR	y is B1	OR	acoustical sensor signal is low
THEN	z is C1	THEN	(alarm) risk is low
Rule 2:		Rule 2:	
IF	x is A2	IF	optical sensor: "yellow"
AND	y is B2	AND	acoustical sensor signal is medium
THEN	z is C2	THEN	risk is normal
Rule 3:		Rule 3:	
IF	x is A1	IF	optical sensor: "red"
AND	y is B2	AND	acoustical sensor signal is high
THEN	z is C3	THEN	risk is high

A1 A2 A3 B1 B2

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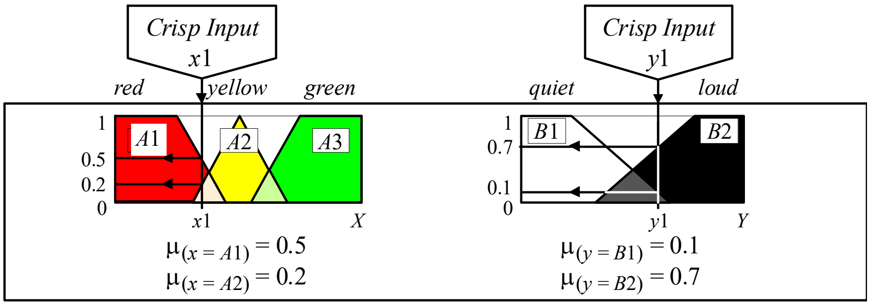
4.1.5 Fuzzy – Inference – Mamdani System

- Step 1: Fuzzification (general)
  - The first step is to take the crisp inputs, x1 and y1 (optical sensor signal and acoustical sensor signal), and determine the degree to which these inputs belong to each of the appropriate fuzzy sets.



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4.1.5 Fuzzy – Inference – Mamdani System

- Step 2: Rule Evaluation
  - The second step is to take the fuzzified inputs, e.g.
    - μ(x=A1) = 0.5,
    - μ(x=A2) = 0.2,
    - μ(y=B1) = 0.1 and
    - μ(y=B2) = 0.7,

    and apply them to the premises of the fuzzy rules.
  - If a given fuzzy rule has multiple premises (antecedents), the fuzzy operator (AND or OR) is used to obtain a single number that represents the result of the premise (antecedent) evaluation.
  - This number (the degree of membership value) is then applied to the conclusion (consequent) membership function.

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4.1.5 Fuzzy – Inference – Mamdani System

- Step 2: Rule Evaluation
  - To evaluate the disjunction of the rule premises, we use the **fuzzy OR operation**. Typically, fuzzy expert systems make use of the classical fuzzy operation **union**:

$$\mu(A \cup B)(x) = \max\,[\mu A(x),\ \mu B(x)]$$

  - Similarly, in order to evaluate the conjunction of the rule premises, we apply the **fuzzy AND operation intersection**:

$$\mu(A \cap B)(x) = \min\,[\mu A(x),\ \mu B(x)]$$

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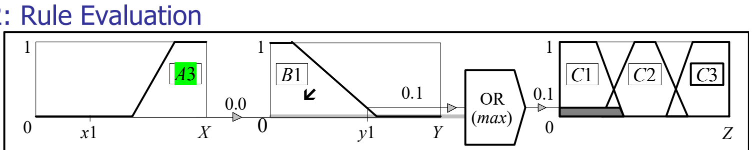
4.1.5 Fuzzy – Inference – Mamdani System

- Step 2: Rule Evaluation



Rule 1: IF *x* is *A*3 (0.0) OR *y* is *B*1 (0.1) THEN *z* is *C*1 (0.1)

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4.1.5 Fuzzy – Inference – Mamdani System

- Step 3: Aggregation
  - Aggregation is the process of unification of the outputs of all rules.
  - We take the membership functions of all rule conclusions previously clipped or scaled and combine them into a single fuzzy set.
  - The input of the aggregation process is the list of clipped or scaled conclusion membership functions, and the output is one fuzzy set for each output variable.

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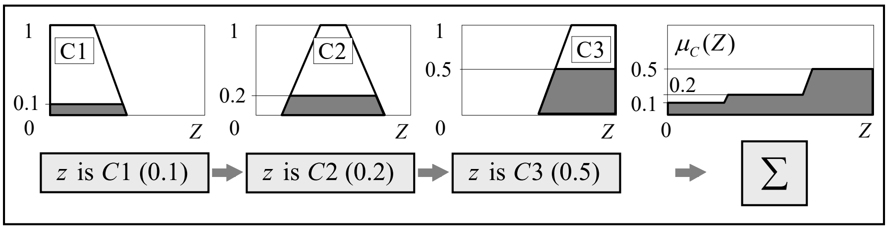
4.1.5 Fuzzy – Inference – Mamdani System

- Step 3: Aggregation



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4.1.5 Fuzzy – Inference – Mamdani System

- Step 4: Defuzzification
  - The last step in the fuzzy inference process is defuzzification.
  - Fuzziness helps us to evaluate the rules, but the final output of a fuzzy system has to be a crisp number.
  - The input for the defuzzification process is the aggregate output fuzzy set and the output is a single number.
  - There are several defuzzification methods, but probably the most popular one is the centroid technique.
  - It finds the point where a vertical line would slice the aggregate set into two equal masses. Mathematically this centre of gravity (CoG) is expressed e.g. as:

$$CoG_x = \frac{\int_a^b \mu_A(x)\cdot x\,dx}{\int_a^b \mu_A(x)\,dx}$$

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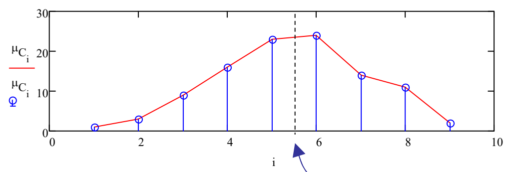
4.1.5 Fuzzy – Inference – Mamdani System

- Step 4: Defuzzification
  - Centroid defuzzification method finds a point representing the centre of gravity of the fuzzy set A, on the interval, [ab].
  - A reasonable estimate can be obtained by calculating the CoG over sample points.
  - Example:



$\mu_C =$

	0
0	0
1	1
2	3
3	9
4	16
5	23
6	24
7	14
8	11
9	2

$$CoG = \frac{\sum_{i=1}^{9} \mu_C(i)\cdot i}{\sum_{i=1}^{9} \mu_C(i)} = \frac{1\cdot 1 + 3\cdot 2 + 9\cdot 3 + 16\cdot 4 + 23\cdot 5 + 24\cdot 6 + 14\cdot 7 + 11\cdot 8 + 2\cdot 9}{1+3+9+16+23+24+14+11+2} \approx 5.45$$

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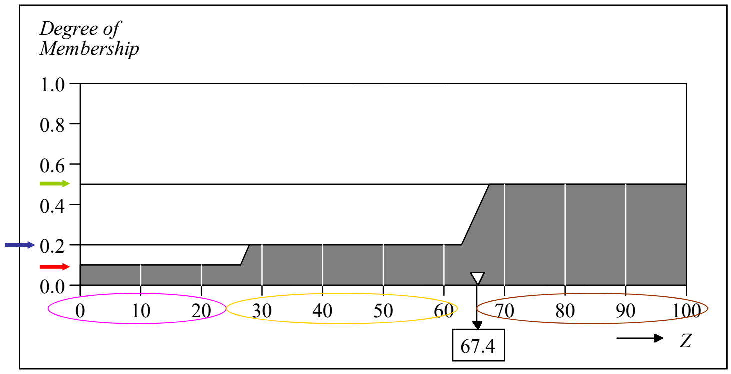
4.1.5 Fuzzy – Inference – Mamdani System

- Step 4: Defuzzification

$$COG = \frac{(0+10+20)\cdot 0.1 + (30+40+50+60)\cdot 0.2 + (70+80+90+100)\cdot 0.5}{0.1+0.1+0.1+0.2+0.2+0.2+0.2+0.5+0.5+0.5+0.5} = 67.4$$



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4.1.5 Fuzzy – Inference – Sugeno System

- Mamdani-style inference, as we have just seen, requires us to find the centroid of a two-dimensional shape by integrating across a continuously varying function.
  - In general, this process is not computationally efficient.
- **Michio Sugeno** suggested to use a single Delta impulse (Dirac impulse), a *singleton*, as the membership function of the rule consequent.
  - A **singleton**, or more precisely a **fuzzy singleton**, is a fuzzy set with a membership function that is unity at a single particular point on the universe of discourse (frame of discernment) and zero everywhere else.

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4.1.5 Fuzzy – Inference – Sugeno System

- Sugeno-style fuzzy inference is very similar to the Mamdani method. Sugeno changed only a rule consequent. Instead of a fuzzy set, he used a mathematical function of the input variable.
- The format of the **Sugeno-style fuzzy rule** is

  IF < *x* is *A* > AND < *y* is *B* > THEN < *z* is *f* (*x*, *y*) >

  - where *x*, *y* and *z* are linguistic variables; *A* and *B* are fuzzy sets on universe of discourses *X* and *Y*, respectively; and *f* (*x*, *y*) is a mathematical function.

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4.1.5 Fuzzy – Inference – Sugeno System

- The most commonly used **zero-order Sugeno fuzzy model** applies fuzzy rules in the following form:

  IF < *x* is *A* > AND < *y* is *B* > THEN < *z* is *k* >

  where *k* is a constant.
- In this case, the output of each fuzzy rule is constant. All consequent membership functions are represented by singletons.

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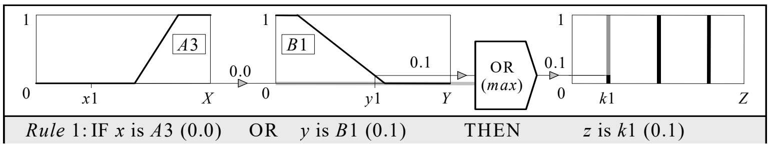
4.1.5 Fuzzy – Inference – Sugeno System

- Step 3: Sugeno Aggregation

k1=20



k2=50

k3=80

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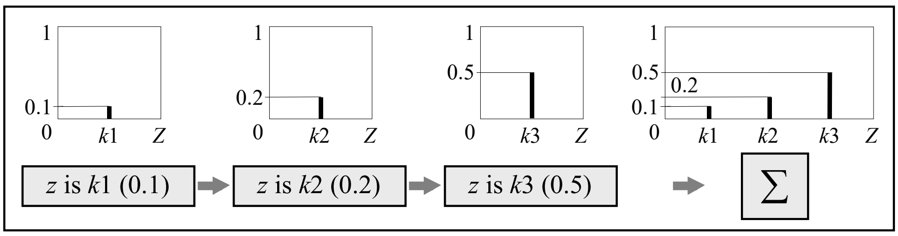
4.1.5 Fuzzy – Inference – Sugeno System

- Step 3: Sugeno Aggregation



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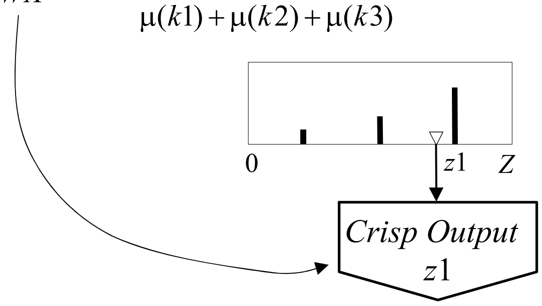
4.1.5 Fuzzy – Inference – Sugeno System

- Step 4: Sugeno Defuzzification
  - Weighted Average (WA)

$$WA = \frac{\mu(k1)\cdot k1 + \mu(k2)\cdot k2 + \mu(k3)\cdot k3}{\mu(k1)+\mu(k2)+\mu(k3)} = \frac{0.1\cdot 20 + 0.2\cdot 50 + 0.5\cdot 80}{0.1+0.2+0.5} = 65$$



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4.1.5 Fuzzy – Inference – Sugeno System

- How to make a decision on which method to apply: Mamdani or Sugeno?
  - Mamdani method is widely accepted for capturing expert knowledge.
  - It allows us to describe the expertise in more intuitive, more human-like manner.
  - On the other hand, Sugeno method is computationally effective and works well with optimisation and adaptive techniques, which makes it very attractive in control problems, particularly for dynamic nonlinear systems.

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