Information Fusion (IFU), Summer Semester 2023

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# Excercise 7: Conflict in Dempster-Shafer Theory

The handling of conflict by Dempster's Rule of Combination (DRC) has been under heavy criticism over the years. The aim of this exercise is to understand why and in which situations the DRC can be problematic. In the second part of the exercise several alternative fusion rules are presented. These rules address the drawback of DRC with different strategies.

The exercise requires to have read and understood lectures L_IFU_32, L_IFU_41 and L_IFU_5.

## 7.1 Conflict Handling in Dempster's Rule of Combination

In this task we revisit Zadeh's example of two highly conflicting sources. Two doctors ($s_1$ and $s_2$) examine a patient suffering from a headache. They agree that the patient either suffers from meningitis (M), concussion (C), or a brain tumour (B):

| | $\varnothing$ | $M$ | $B$ | $C$ | $\{M, C\}$ | $\{M, B\}$ | $\{C, B\}$ | $\Omega$ |
|---|---|---|---|---|---|---|---|---|
| $s_1$ | 0 | 0.99 | 0.01 | 0 | 0 | 0 | 0 | 0 |
| $s_2$ | 0 | 0 | 0.01 | 0.99 | 0 | 0 | 0 | 0 |

1. We have already seen that the fusion results using the DRC are not intuitive in this example. Why is this the case? Do the conflicting masses influence the result at all?
2. Fusion rules aside: What do you think should a reasonable fusion result look like? Should mass be assigned to the frame of discernment? Or rather to $M$ and $C$? Can you think of other possible solutions?

### 7.2 Alternative Fusion Rules

Unsatisfied with the diagnoses of the two doctors the patient seeks out two additional experts:

| | $\varnothing$ | $M$ | $B$ | $C$ | $\{M, C\}$ | $\{M, B\}$ | $\{C, B\}$ | $\Omega$ |
|---|---|---|---|---|---|---|---|---|
| $s_1$ | 0 | 0.99 | 0.01 | 0 | 0 | 0 | 0 | 0 |
| $s_2$ | 0 | 0 | 0.01 | 0.99 | 0 | 0 | 0 | 0 |
| $s_3$ | 0 | 0.99 | 0.01 | 0 | 0 | 0 | 0 | 0 |
| $s_4$ | 0 | 0.70 | 0.20 | 0.10 | 0 | 0 | 0 | 0 |

Since the DRC gave unsatisfactory results, the patient looks also for alternative fusion rules. These are given in Table 1. Having new expert's opinions and alternative rules, how likely are meningitis, brain tumour, and concussion to be the cause of the patient's sufferings? Proceed with the following steps:

1. Calculate the fusion results of the sources $\{s_1, s_2\}$, $\{s_1, s_2, s_3\}$, $\{s_1, s_2, s_3, s_4\}$ using the fusion rules given in Table 1. Use the *fusion world model*, i. e. , fuse the sources serially one after another.
2. Describe the fusion strategies applied by these rules in your own words (e. g. , DRC enhances the majority opinion regardless of how strongly or weakly this opinion is supported).
3. Which of the rules result in intuitive, which result in unintuitive results given this example?
4. Could you imagine other disadvantages of these alternative rules?

## References


[1] Arthur P. Dempster. Upper and lower probabilities induced by a multivalued mapping. *Annals of Mathematical Statistics*, 38:325–339, 1967.

[2] Roland R. Yager. On the Dempster-Shafer framework and new combination rules. *Information Sciences*, 41(2):93–137, 1987.

[3] Catherine K. Murphy. Combining belief functions when evidence conflicts. *Decision Support Systems*, 29(1):1 – 9, 2000.

[4] Uwe Mönks. *Information Fusion Under Consideration of Conflicting Input Signals*. Technologies for Intelligent Automation. Springer, Berlin, Heidelberg, 2017. Doctoral dissertation.

[5] F. Campos and S. Cavalcante. An extended approach for Dempster-Shafer theory. In *Proceedings Fifth IEEE Workshop on Mobile Computing Systems and Applications*, pages 338–344, 2003.

| Fusion rule | Equations |
|---|---|
| DRC [1] | $m(A) = \frac{1}{1-\mathrm{k_c}} \cdot \sum_{B\cap C=A} m_1(B)\cdot m_2(C)$ <br> $\mathrm{k_c} = \sum_{B\cap C=\varnothing} m_1(B)\cdot m_2(C)$ |
| Yager [2] | $m(A) = \sum_{B\cap C=A} m_1(B)\cdot m_2(C),\ A\neq\Omega$ <br> $m(\Omega) = m_1(\Omega)\cdot m_2(\Omega) + \mathrm{k_c}$ <br> $\mathrm{k_c} = \sum_{B\cap C=\varnothing} m_1(B)\cdot m_2(C)$ |
| Murphy [3] | $m^{(n)}(A) = \frac{1}{n}\sum_{i=1}^{n} m_i(A)$, with $n$ being the amount of fused sources. |
| BalTLCS [4] L_IFU_5 | $m(A) = \frac{\mathrm{k_c}}{\mathrm{B_C(n)}} \cdot \frac{1}{n}\sum_{i=1}^{n} m_i(A) + \frac{1}{\mathrm{B_C(n)}}\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} m_i(A)\cdot m_j(A),$ <br> $\mathrm{k_c} = \sum_{i=1}^{n}\sum_{\substack{j=1,\\ j\neq i}}^{n}\sum_{\substack{A_k\cap A_l=\varnothing,\\ k<l}} m_i(A_k)\cdot m_j(A_l)$ <br> $B_{\mathrm{C}}(n) = \frac{n(n-1)}{2}$ |
| Campos [5] [Optional] | $m^{(f)}(A) = \frac{1}{1+\log(K_f)} \cdot \frac{1}{1-\mathrm{k_c}_f} \cdot \sum_{B\cap C=A} m_1(B)\cdot m_2(C),\ A\neq\Omega,$ <br> with $f$ counting the number of fusions (e. g. , $f=2$ would mean in our example that $s_1$ and $s_2$ have already been fused and the result is now fused with $s_3$). <br> $m^{(f)}(\Omega) = \frac{1}{1-\mathrm{k_c}_f}\cdot m_1(\Omega)\cdot m_2(\Omega) + \left(1 - \sum_{\substack{A\in\mathcal{P}(\Omega),\\ A\neq\Omega}} m^{(f)}(A)\right)$ <br> $\mathrm{k_c}_f = \sum_{B\cap C=\varnothing} m_1(B)\cdot m_2(C)$, i. e. the conflict of the current fusion. <br> $K_f = \frac{1}{f-\sum_{i=1}^{f}\mathrm{k_c}_i}$, i. e. an expression based on the sum of conflicts of all fusions steps so far. |

Table 1: Different fusion rules in Dempster-Shafer theory. For all rules $m(\varnothing) = 0$.