Information Fusion (IFU), Summer Semester 2023
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Excercise 4: Dempster-Shafer Theory

This exercise focuses on the Dempster-Shafer theory (DST) which can be interpreted as a generalisation of Bayesian probability. DST is specifically designed to handle some types of epistemic uncertainty and to fuse evidences from multiple sources.

To be able to complete this exercise, you are required to have worked through the lectures up to and including L_IFU_32.

4.1 Ignorance

In the last exercise we discussed that probability theory is unable to represent ignorance properly.

1. Again, let us assume the example given in [1, p. 91]. But this time we will change it a little bit as follows: Are there carbon-based living beings near the star Sirius (proposition A_1: yes; proposition A_2: no)? Assuming total ignorance (we have no clue), how would a basic belief assignment look like?
2. What would happen (both in probability theory and in DST) if we added a third proposition A_3 which corresponds to the existence of silicon-based life? What would happen after adding A_4 being the proposition that sulfur-based life exists?
3. Now assume the coin-example from last exercise (well-balanced new coin, never tossed before). Tossing the coin, the frame of discernment is $\Omega = \{B_1, B_2\}$, B_1 is „head side up“ and B_2 is „tail side up“. Assign basic beliefs to B_1 and B_2.

4.2 Belief and Plausibility

DST sums up all evidence supporting a proposition into a degree of belief. All evidence which does not contradict a proposition is condensed into a degree of plausibility. In DST the belief function is defined by

$$\mathrm{Bel}(A) = \sum_{X \subseteq A} m(X)$$

and the plausibility function by

$$\mathrm{Pl}(A) = \sum_{X \cap A \neq \varnothing} m(X).$$

A helpful relation between belief and plausibility is

$$\mathrm{Pl}(A) = 1 - \mathrm{Bel}(A^{\mathrm{c}}).$$

(with A^{c} being the complement of A). In this task a condition monitoring example is provided for which beliefs and plausibilities are to be determined.

Before shipping electrical motors to customers, the motors must pass a quality assurance. In the quality check each motor is observed by sensors. Data obtained by these sensors are fed into a pre-trained machine learning model (a neural net) which determines if the motor is healthy (proposition H), has a bearing damage (F_1), or shows signs of eccentricity (F_2).

1. Define the frame of discernment Ω.
2. Define the power set $\mathcal{P}(\Omega)$.

The basic belief assignments for this example are:

- $m(H) = 0.3$,
- $m(F_1) = 0.1$,
- $m(F_2) = 0$,
- $m(\{F_1, F_2\}) = 0.4$, and
- $m(\Omega) = 0.2$.

The evidential masses of all other elements in $\mathcal{P}(\Omega)$ are 0. This basic belief assignment adheres to the basic axioms of DST which are (i) $m(\varnothing) = 0$ and (ii) $\sum_{X \in \mathcal{P}(\Omega)} m(X) = 1$.

3. Interpret the evidential mass $m(\{F_1, F_2\}) = 0.4$. What does the evidence mean?
4. Calculate the belief and plausibility of each element in $\mathcal{P}(\Omega)$ (including the empty set and Ω).
5. After having completed this part of the exercise, which categories in the taxonomy of ignorance (see Figure 1) do you think DST is well suited for?

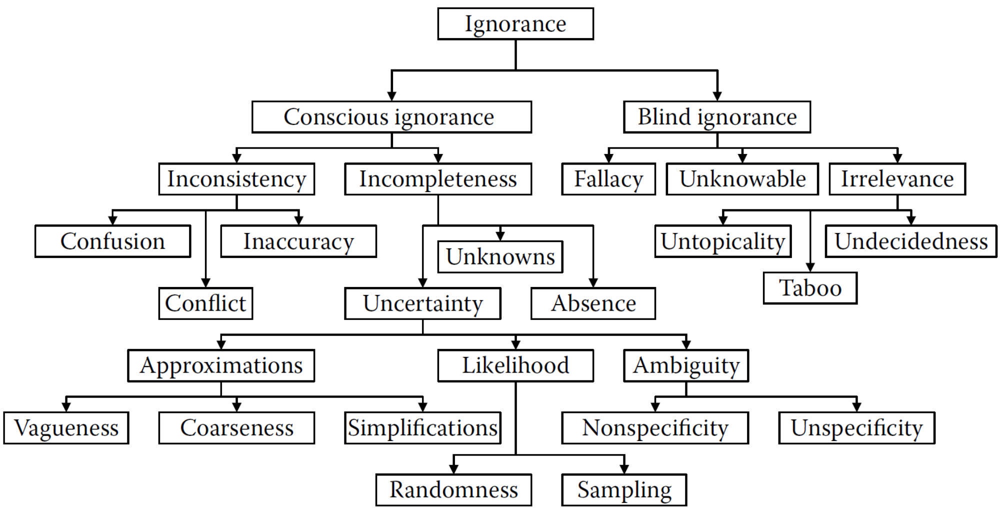

Figure 1: A taxonomy of ignorance given in [2, p.53].

4.3 Fusion

Many approaches have been published for the fusion of evidential masses. Most famous is DEMPSTER*'s rule of combination* (DRC) [3]. Other rules are proposed by, e. g., YAGER [4] CAMPOS [5], or MÖNKS [6].

1. The DRC is used in the lecture's example (L_IFU_32 (pp. 9-13)) to fuse information of weather reports. Do you have an idea why the forecasts' evidences are not simply averaged (using the arithmetic mean)?

The DRC is formally defined as follows[1]:

$$m_{12}(A) = \begin{cases} \dfrac{\sum\limits_{B\cap C=A} m_1(B)\cdot m_2(C)}{1-k_{\mathrm{c}}} & \text{if } A \neq \varnothing, \\ 0 & \text{if } A = \varnothing, \end{cases}$$

in which $B \in \mathcal{P}(\Omega)$, $C \in \mathcal{P}(\Omega)$, and

$$k_{\mathrm{c}} = \sum_{B\cap C=\varnothing} m_1(B)\cdot m_2(C).$$

2. Understand the given equations.

In the following, we extend the condition monitoring example and apply the DRC. In addition to the neural net (source s_1), the quality of the motor is now also checked by a support vector machine (source s_2) and a fuzzy pattern classifier (source s_3). To keep the task manageable, the propositions are reduced to healthy H and faulty F. The basic belief assignments for these sources are:

$$\begin{aligned} &m_1(H) = 0.7, \quad &&m_1(F) = 0.2, \quad &&m_1(\Omega) = 0.1, \\ &m_2(H) = 0.2, \quad &&m_2(F) = 0.4, \quad &&m_2(\Omega) = 0.4, \\ &m_3(H) = 0.9, \quad &&m_3(F) = 0, \quad &&m_3(\Omega) = 0.1. \end{aligned}$$

3. Fuse the evidential masses of s_1 and s_2 using DEMPSTER's rule of combination. What are the values for the resulting masses $m_{12}(H)$, $m_{12}(F)$, and $m_{12}(\Omega)$?
4. Fuse the combined masses $m_{12}(\circ)$ with s_3.
5. What have we achieved by fusion in this example?
6. Does the order, in which the sources are fused, have an impact on the final fusion result (remember the *fusion world model*)?

[1] The formal definition of the DRC and the conflict factor k_{c} are given in lecture 4.1. Nonetheless, both are already applied in the weather report example and the formal definitions are not too hard to understand.

References

[1] Marco Prioli and Simona Salicone. *Measurement Uncertainty within the Theory of Evidence*. Springer Series in Measurement Science and Technology. Springer International Publishing, 2018.

[2] Bilal M. Ayyub and George Jiri Klir. *Uncertainty Modeling and Analysis in Engineering and the Sciences*. Chapman & Hall/CRC, Boca Raton, FL, 2006.

[3] Arthur P. Dempster. Upper and lower probabilities induced by a multivalued mapping. *Annals of Mathematical Statistics*, 38:325–339, 1967.

[4] Roland R. Yager. On the Dempster-Shafer framework and new combination rules. *Information Sciences*, 41(2):93–137, 1987.

[5] Fabio Campos. *Decision Making in Uncertain Situations: An Extension to the Mathematical Theory of Evidence*. Dissertation.Com., 2006. Doctoral dissertation.

[6] Uwe Mönks. *Information Fusion Under Consideration of Conflicting Input Signals*. Technologies for Intelligent Automation. Springer, Berlin, Heidelberg, 2017. Doctoral dissertation.