Day 2:Euler's Number

Akshay Gadre

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1 What is it?

Euler's number(e) is one of the most famous mathematical constant. Unlike other famous constants like π or Pythagoras' constant $\sqrt{2}$, which are defined by geometry, e is actually defined by rate of change. It is thus used as the natural base of log in differential and integral calculus.

It is the most optimum rate of change which could be received from compound interest given a starting amount. It was first discovered thus by Swiss mathematician Jacob Bernoulli while studying compound interest.

2 Origin:

Let's say you start with some principal amount and a rate of interest. Now with P, in a year you get 100% interest. So by the end of the first year you will have 2P.

Now let's compound the interest every semester i.e 50% interest every 6 months. So by the end of the first semester you get 1.5P and at the end of the first year 2.25P which is better.

Let's reduce the time interval even more let's say $\frac{100}{52}$% interest every week. With correct calculation we end up with 2.69259695444P.

What Bernoulli wanted to find out was what would be the best possible case of output after the first year. He knew that it would be when the rate would be calculated infinitesimally.

$$\lim_{N\to\infty}(1+\frac{1}{N})^N$$

However Bernoulli didn't actually find the exact value of e which is irrational. Euler found the value of e as 2.71828182845904523536028747135266249775724709369995 and showed it is irrational using the following infinite series:

$$e = 2 + \cfrac{1}{1 + \cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{4+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{6+\cfrac{1}{1+\dots}}}}}}}}}$$

3 Other formulae and significance

$$e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \frac{1}{5!} + ...$$

The significance of e is that it is the natural language of growth. This property comes out of all three, value, gradient and area of its curve being the same value at every point on the curve. Thus, when we use this as the base for exponential numbers the mathematics becomes much easier as we avoid unnecessary constants.

Also it is part of the golden equation of mathematics:

$$e^{i\pi} + 1^{\infty} = 0$$