QUANT FORNULAS

(1) Number System

Natural numbers : 1, 2, - - - -

Whole numbers : 0, 1, 2, - - - -

Composite numbers : Having more than 2 factors.

Prime numbers : Only two factors i.e. 1 & the no. itself.

☆ Remainder when $p \geq 5$ is divided by 6 is $\boxed{1 \text{ or } 5}$

☆ Remainder when p^2 ($p \geq 5$) is divided by 12 or 24 is $\boxed{1}$

Integers : - - - - -4, -3, -2, -1, 0, 1, 2, 3, - - - -

Real Numbers : No. which can be represented on number line

Rational Numbers & Irrational numbers.

Let us take two numbers n_1 & n_2.

HCF : Highest of all common divisors of n_1 & n_2

LCM : Smallest no. exactly divisible by n_1 & n_2.

HCF × LCM = product of 2 numbers.

$$\text{HCF of fraction} = \frac{\text{HCF of numerator}}{\text{LCM of denominator}} \qquad \text{LCM of fraction} = \frac{\text{LCM of numerator}}{\text{HCF of denominator}}$$

☆ Any no. which when divided by p, q, r leaving the same remainder R in each case will be of the form $N = LCM(p, q, r) + R$

Scanned by CamScanner

☆ Any no. which when divided by p, q or r leaving respective remainders of s, t & u where $p-s = q-t = r-u = R$ will be of the form: $N = LCM(p, q, r) - R$

☆ Largest no with which numbers p, q, or r are divided giving remainders s, t & u will be $N = HCF[(p-s), (q-t), (r-u)]$

☆ Largest no. with which p, q, r are divided giving same remainders is $N = HCF[(p-q), (p-r)]$.

<u>Divisibility</u>.

2: Even no. 3: sum of digits multiple of 3

4: last two digits divisible by 4 5: last digit either 0 or 5.

6: both divisible by 2 or 3. 8: last three digits /8.

9: sum of digits multiple of 9.

11: sum of alternate no. is same or differ by 11.

12: by 3 & 4 15 by 3 & 5 16: last 4 digits /16.

18: by 2 & 9 25: last two digits either 00, 25, 50 or 75.

☆ Any digit repeated 6 times is divisible by 3, 7, 11, 13, 37

<u>Euler Number</u>.

$E(p) = p\left[1 - \frac{1}{a}\right]\left[1 - \frac{1}{b}\right] \cdots$ $E(p)$ = No. of co-primes less than p.

$a, b, \ldots$ are prime factors of p.

☆ $\frac{a^{E(b)}}{b}$ $\boxed{R \text{ is always } 1}$

☆ a & b are co-prime

Sum of co-primes less than p

$S = \frac{p}{2} \times E(p)$.

Scanned by CamScanner

★ last digits of any powers follow a cyclic pattern.

When base ends with 2, 3, 7, 8 Cyclicity : 4

" " " " 4, 9 Cyclicity : 2

" " " " 0, 1, 5, 6, Cyclicity : 1

★ Remainders of a power also follow cyclic pattern.

Chinese Remainder Theorem.

$\frac{p}{q}$ $q = a \times b$ a, b are co-primes.

Let $\frac{p}{a} = N = at_1 + r$, t: quotient r: remainder.

Let $\frac{p}{b} = M = at_2 + r$. We need to make $N = M$

★ R of $\frac{a^{p-1}}{p}$ is 1 a, p are co-prime.

★ If p is prime $\frac{(p-1)!}{p}$ $\boxed{R = p-1}$ and $\frac{(p-2)!}{p}$ $\boxed{R = 1}$

Let $N = a^p b^q c^r \ldots$ a, b, c : prime no. p, q, r : +ve integers.

- No. of factors $= (p+1)(q+1)(r+1)\ldots$
- No. of ways of expressing N as a product of two $= \frac{1}{2}\left[(p+1)(q+1)(r+1)\ldots\right]$
- Sum of all factors $= \left[a^0 + a^1 + \ldots a^p\right]\left[b^0 + b^1 + \ldots b^q\right]\left[c^0 + c^1 + \ldots c^r\right]$
- Product of all factors $= (N)^{(\text{total factors})/2}$

★ $N! = N \times (N-1) \times (N-2) \times \ldots 1$

Highest power of prime p in N is given as

$$\left[\frac{N}{p}\right] + \left[\frac{N}{p^2}\right] + \left[\frac{N}{p^3}\right] + \ldots$$

Scanned by CamScanner

☆ No. of zeros in N! is given as

$$\left[\frac{N}{5}\right] + \left[\frac{N}{25}\right] + \cdots$$

☆ $(M!)_{10} = (N)_b$. No. of zeros N will end is given as

$$\left[\frac{M}{b}\right] + \left[\frac{M}{b^2}\right] + \left[\frac{M}{b^3}\right] + \cdots \text{ if } b \text{ is prime.}$$

b : composite then use other method.

☆ $a^n - b^n$ → always divisible by $a-b$

→ n : even divisible by $a+b$

☆ $a^n + b^n$ → never divisible by $a-b$

→ n : odd divisible by $a+b$.

☆ Sums of consecutive integers

In how many ways N can be expressed as a sum of two or more consecutive integers

→ $N = (2)^p (3)^q (p_1)^r (p_2)^s \cdots$

Consider only odd factors.

So, Ans $= (q+1)(r_1+1)(r_2+1) \cdots - 1$

Unit's digit.

☆ To find unit digit of a^b, then use cyclicity method.

☆ " " " " " a^{b^c}, then find remainder of

$R = \frac{b^c}{\text{Cyclicity}}$, then find unit digit of $\underline{a^R}$.

Last two digits Some shortcuts.

☆ If $N = 2^{10 \times \text{Even}}$ = ___(76). eg. $2^{40} = 2^{10\times 4}$ ∴ ___(76)

☆ If $N = 2^{10 \times \text{odd}}$ = ___(24). eg $2^{30} = 2^{10\times 3}$ ∴ ___(24)

☆ If $N = (___A1)^{__B}$ = ___(A1)1 eg. $(161)^{29}$ ∴ (41).

☆ If $N = (___\text{odd5})^{\text{odd}}$ = ___(75) eg. $(35)^{11}$ ___(75)

else ___(25) $(45)^{19}$ ___(25)

Scanned by CamScanner

(2) <u>Logarithms</u>

☆ $x = \log_a m \quad a>0,\ m>0.$

Properties.

1: $\log_a 1 = 0 \ \forall\, a>0\ a\neq 1$ 2: $\log_a a = 1 \ \forall\, a>0.$

☆ $\log(a_1 a_2 a_3 \ldots a_k) = \log a_1 + \log a_2 + \log a_3 + \ldots \log a_k$

☆ $\log\left(\frac{b_1}{b_2}\right) = \log b_1 - \log b_2.$

☆ $\log m^n = n \log m$

☆ $\log_c b = \dfrac{\log_c b}{\log_c a}$ ☆ $\log_a b = \dfrac{1}{\log_b a}.$

☆ If $a>1$ & $\log_a x_1 > \log_a x_2 \Rightarrow x_1 > x_2$

☆ If $a<1$ & " " $\Rightarrow x_1 < x_2$

☆ If the log to any base a gives (n...), then no. of integers possible is given by $\boxed{a^{n+1} - a^n}$

☆ $\log_{a^q} m^p = \dfrac{p}{q} \log_a m$

☆ $a^{\log_a N} = N$

☆ $a^{\log_b c} = c^{\log_b a}.$

Scanned by CamScanner

(3) Sequences & Series.

Arithmetic Progressions

☆ $T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}[2a + (n-1)d]$ or $\frac{n}{2}[a + l]$ (last term)

$$AM = \frac{a_1 + a_2 + \cdots a_n}{n}$$

Geometric Progressions

☆ $T_n = ar^{n-1}$ $\quad S_n = \frac{a(r^n - 1)}{r-1}$ $\quad S_\infty = \frac{a}{r-1}$ [if $r<1$]

$$GM = \sqrt[n]{a_1 a_2 \cdots a_n}$$

Harmonic Progression

If a, b, c, d are in AP, then $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}, \frac{1}{d}$ are in HP

☆ $HM = \frac{2ab}{a+b}$.

☆ $AM \geq GM$ [for positive numbers] & $GM \geq HM$

☆ Sum of 1st n natural numbers

$S = 1 + 2 + 3 + \cdots n = \frac{n(n+1)}{2}$

☆ Sum of squares of 1st n natural numbers $S = \frac{n(n+1)(2n+1)}{6}$.

☆ Sum of the cubes of 1st n " " $S = \left[\frac{n(n+1)}{2}\right]^2$

☆ Sum of 1st n odd natural numbers $S = 1 + 3 + 5 + \cdots (2n-1) = n^2$

☆ " " " " even " " $S = 2 + 4 + 6 + \cdots 2n \rightarrow n(n+1)$

☆ $\frac{1}{6} + \frac{1}{12} = \frac{1}{2.3} + \frac{1}{3.4} = \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right)$

Special Series

☆ $S_1 = \frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \cdots$

$S_1 = \frac{1}{1} - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \cdots$

☆ $S_2 = \frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \cdots$

$S_2 = \frac{1}{2}\left[\frac{2}{1.3} + \frac{2}{3.5} + \cdots\right]$

$= \frac{1}{2}\left[\frac{1}{1} - \frac{1}{3} + \frac{1}{3} - \frac{1}{5} + \cdots\right]$

☆ For an ∞ AGP

$S_\infty = \frac{a}{1-r} + \frac{rd}{(1-r)^2}$

Scanned by CamScanner

☆ $\frac{1}{a(a+d)(a+2d)} = \frac{1}{2d^2}\left[\frac{1}{a} - \frac{2}{a+d} + \frac{1}{a+2d}\right]$

$S = \frac{2}{5} + \frac{6}{5^2} + \frac{12}{5^3} + \ldots$

$\frac{5S}{5} = \frac{2}{5} + \frac{6}{25} + \ldots$

$\frac{4S}{5} = \frac{2}{5} + \frac{4}{25} + \frac{6}{125} + \ldots$

Subtracting $\frac{4S}{25} = \frac{2}{25} + \frac{4}{125} + \ldots$

$xS = \frac{2}{5} + \frac{2}{25} + \frac{2}{125} + \ldots$

Infinite GP.

(4) Simple & Compound Interest

☆ $S.I = \frac{P \times R \times T}{100}$ P: Principle R: Rate T: Time.

$A = P + S.I$

☆ $C.A = P\left[1 + \frac{R}{100}\right]^T$ $C.I = CA - P$.

☆ If compound interest in half-yearly then $R' = \frac{R}{2}$ $T' = 2T$

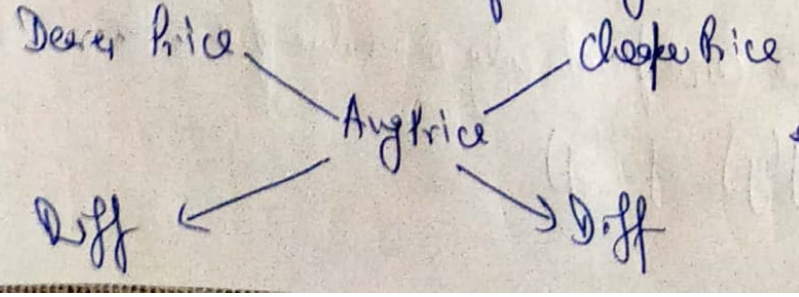
☆ Difference between compound & simple interest over two years is given by $P\left[\frac{r}{100}\right]^2$

(5) Averages, Mixtures & Allegations

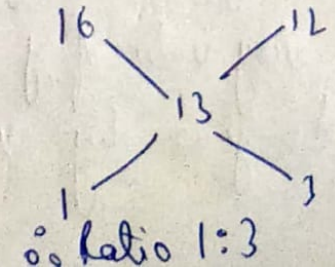
☆ Average of n numbers $A_n = \frac{(x_1 + x_2 + \ldots x_n)}{n}$

☆ Weighted average $A_w = \frac{n_1A_1 + n_2A_2 + n_3A_3 + \ldots n_kA_k}{n_1 + n_2 + n_3 + \ldots n_k}$.

Graphical representation of Alligation



eg.



∴ Ratio 1:3

Scanned by CamScanner

☆ If there is P volume of pure liquid initially & in each operation Q volume is taken out & replaced by Q volume of water, then at the end of n such operations,

Concentration of liquid in solution $= \left(\frac{P-Q}{P}\right)^n \times P$

(6) Algebra

Functions

A function is a rule that attributes to every number x from D one definite number y belongs to the set of real numbers.

e.g. $y = f(x)$ $\quad y = 4x^2$

☆ Even function : $f(x) = f(-x)$. e.g. $y = x^2$ $\quad y = |x|$

- Sum, diff., product, quotient of an even function is even
- Graph is symmetrical about y-axis

☆ Odd function : $f(x) = -f(-x)$ e.g. $y = x^3$ $\quad y = x^5 + x^3$

- Sum, difference of an odd function is an odd function
- Product & quotient " " " " " " even "
- Graph is symmetrical about origin

☆ Inverse function : $y = f(x)$.

For every value of y, there corresponds only one value of x, then inverse exists given by $x = g(y) = f^{-1}(y)$

Scanned by CamScanner

Shifting of Graphs.

☆ $y=f(x)$ & $y=f(x)+c$ → Graph shifted up by c units

☆ $y=f(x)$ & $y=f(x)-c$ → " " down by c units

☆ $y=f(x)$ & $y=f(x+c)$ → " " left " " "

☆ $y=f(x)$ & $y=f(x-c)$ → " " right " " "

For any real function $f(x)$

☆ $\frac{f(x)+f(-x)}{2}$ = Even ☆ $\frac{f(x)-f(-x)}{2}$ = Odd.

☆ $f(x+y)=f(x)+f(y)$ $f(x)=kx$

☆ $f(x+y)=f(x).f(y)$ $f(x)=a^x$

☆ $f(xy)=f(x)+f(y)$ $f(x)=k\log x$

<u>Inequalities</u>.

a, b are real numbers.

If ☆ $a>b$, $\forall c$ $a+c>b+c$ ☆ $a>b$, $c>0 \Rightarrow ac>bc$

☆ $a>b$, $c<0 \Rightarrow ac<bc$ ☆ $a>b$, $c>d \Rightarrow a+c>b+d$ & $ac>bd$.

☆ $a^2+b^2 \geq 2ab$ [Equality for $a=b$]

☆ $a^3+b^3 \geq ab(a+b)$ $a,b>0$

☆ $|a_1+a_2+\cdots a_n| \leq |a_1|+|a_2|+\cdots|a_n|$

☆ $|a-b| \geq |a|-|b|$

☆ $(a+b)(b+c)(c+a) \geq 8abc$ $a,b,c \geq 0$ & equality for $a=b=c$

☆ $\frac{a}{b}+\frac{b}{a} \geq 2$ if $a,b>0$ or $a,b<0$.

☆ $a+b=2 \Rightarrow a^4+b^4 \geq 2$

☆ If $a \geq 1$ then $2 \leq \left[1+\frac{1}{a}\right]^a < 2.8$.

☆ $2^n > n^2$ for $n \geq 5$

Scanned by CamScanner

a, b are +ve nos.

☆ If $a+b$ = constant ; max (ab) when $a=b$

☆ If ab = constant ; min $(a+b)$ when $a=b$

Quadratic & Other Equations

☆ ax^2+bx+c $a, b, c \in R$ & $a \neq 0$.

D = Discriminant $= b^2-4ac$.

☆ If $D<0$, equation has imaginary roots & conjugate pairs.

☆ If $D>0$, " " two distinct "

☆ If $D=0$, " " equal roots.

☆ If D = perfect square, roots are rational else irrational & conjugate.

Roots $= \alpha, \beta$ Sum $= \frac{-b}{a}$ Product $= \frac{c}{a}$.

☆ If roots $\frac{1}{\alpha}, \frac{1}{\beta}$, replace x by $\frac{1}{x}$ eg: $a\left(\frac{1}{x}\right)^2+b\left(\frac{1}{x}\right)+c=0$.

☆ " " $(\alpha+k), (\beta+k)$ replace x by $x-k$ eg $a(x-k)^2+b(x-k)+c=0$

☆ " " $\alpha k, \beta k$ replace x by $\frac{x}{k}$ eg $a\left(\frac{x}{k}\right)^2+b\left(\frac{x}{k}\right)+c=0$.

If $a>0$, min value $= \frac{(4ac-b^2)}{4a}$ at $x = \frac{-b}{2a}$

If $a<0$, max value = " " " "

If $p(x)$ is divided by $(ax+b)$, Remainder $= p\left(\frac{-b}{a}\right)$ $ax+b=0$, $x=\frac{-b}{a}$.

Degree of the remainder is always less than divisor.

☆ $a>0, D<0$ $f(x)>0 \ \forall x \in R$ ☆ $a<0, D<0$ $f(x)<0 \ \forall x \in R$.

☆ $a>0, D=0$ $f(x) \geq 0$ $f(x)=0$ at vertex ☆ $a<0, D=0$ $f(x) \leq 0$ $f(x)=0$ at vertex

☆ $a>0, D>0$ $f(x)>0 \ \forall x \in (-\infty, \alpha) \cup (\beta, \infty)$
$f(x)<0 \ \forall x \in (\alpha, \beta)$

☆ $a<0$ $D>0$.
$f(x)<0 \ \forall x \in (-\infty, \alpha) \cup (\beta, \infty)$
$f(x)>0 \ \forall x \in (\alpha, \beta)$.

Scanned by CamScanner

☆ Some Imp. Formulas.

$(a-b)^3 = a^3 - b^3 - 3ab(a-b)$

$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

$(a^2+b^2) = (a+b)^2 - 2ab$

$(a^2-b^2) = (a-b)(a+b)$

$(a^3+b^3) = (a+b)(a^2-ab+b^2)$

$(a^3-b^3) = (a-b)(a^2+ab+b^2)$

$(a+b+c)^2 = a^2+b^2+c^2+2(ab+bc+ca)$

If $a+b+c=0 \Rightarrow a^3+b^3+c^3 = 3abc$

$a^n - b^n = (a-b)[a^{n-1} + a^{n-2}b + a^{n-3}b^2 + \cdots b^{n-1}]$

$(a+b+c)^3 = a^3+b^3+c^3+3(a+b+c)(ab+bc+ca) - 3abc$

$a^3+b^3+c^3 = (a+b+c)(a^2+b^2+c^2-ab-bc-ca) + 3abc$

Scanned by CamScanner

let $f(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \cdots + a_1 x + a_0$

☆ Sum of roots $S_1 = \frac{(-1)^1 a_{n-1}}{a_n}$.

☆ Sum of product of roots taken two at a time $S_2 = \frac{(-1)^2 a_{n-2}}{a_n}$.

☆ Sum of product of roots taken m $(m \le n)$ at a time

$$S_m = \sum (-1)^m \frac{a_{n-m}}{a_n}.$$

☆ Any equation of an odd degree must have at least <u>one real root</u>.

☆ $a_1 x^2 + b_1 x + c_1 = 0 \qquad a_2 x^2 + b_2 x + c_2 = 0$.

Common root condition $\frac{(c_1 a_2 - c_2 a_1)^2}{1} = (b_1 c_2 - b_2 c_1)(a_1 b_2 - a_2 b_1)$

Descartes' Rule of Signs

$f(x) = 4x^7 + 3x^6 + x^5 + 2x^4 - x^3 + 9x^2 + x + 1 = 0$.

(1) Positive root case

$f(x) = 4x^7 + 3x^6 + x^5 + 2x^4 - x^3 + 9x^2 + x + 1 = 0$

2 sign changes ∴ 0 or 2 positive root

(2) Negative root case

$f(-x) = -4x^7 + 3x^6 - x^5 + 2x^4 + x^3 + 9x^2 - x + 1$

5 sign changes ∴ 1 or 3 or 5 negative root.

Scanned by CamScanner

(7) Profit, loss & Discounts

CP : Amount paid to purchase SP : Price at which product is sold

MP : price at which article is marked.

☆ Profit = SP − CP ☆ loss = CP − SP

☆ % Profit = $\frac{\text{Profit} \times 100}{CP}$ ☆ % loss = $\frac{\text{loss} \times 100}{CP}$

Discount = MP − SP ☆ % Discount = $\frac{\text{Discount} \times 100}{MP}$

☆ If two items are sold at same price x, one at profit p % & other at loss of p %, then Overall loss = $\frac{p^2}{100}$ %.

Absolute value of loss = $\frac{2p^2 x}{100^2 - p^2}$

☆ If CP of two items are same, one is sold at profit p % & other at loss p %, then no loss & no gain

☆ If a trader professes to sell at CP but uses false weight

$$\text{Gain \%} = \frac{\text{Error}}{\text{True value} - \text{Error}} \times 100.$$

☆ Buy x get y free, % discount = $\frac{y}{x+y} \times 100$. [CP of x articles = SP of $(x+y)$ articles.

☆ When, there are two successive discounts

$$\text{Desired} = \left(a + b - \frac{ab}{100}\right)$$

☆ If CP of x = SP of y resultant profit / loss % = $\left(\frac{y-x}{y} \times 100\right)$

Scanned by CamScanner

(8) Ratio, Proportion & Variation

Ratio $\frac{a}{b}$ $a \to$ antecedent, $b \to$ consequent.

☆ $\frac{a}{b} = \frac{ma}{mb}$ ☆ $\frac{a}{b} = \frac{a/d}{b/d}$ $m, d \neq 0$.

☆ $\frac{a}{b} = \frac{p}{q} \Rightarrow aq = bp$ ☆ $\frac{a}{b} > \frac{p}{q} \Rightarrow aq > bp$ ☆ $\frac{a}{b} < \frac{p}{q} \Rightarrow aq < bp$

☆ If $a, b, x > 0$, then

- If $a > b$, $\Rightarrow \frac{a+x}{b+x} < \frac{a}{b}$
- If $a < b \Rightarrow \frac{a+x}{b+x} > \frac{a}{b}$
- " " " $\Rightarrow \frac{a-x}{b-x} > \frac{a}{b}$
- " " " $\Rightarrow \frac{a-x}{b-x} < \frac{a}{b}$.

☆ If $\frac{a}{p} = \frac{b}{q} = \frac{c}{r} = \frac{d}{s} = \ldots$ then $a:b:c:d \ldots = p:q:r:s \ldots$

and it is equal to $\frac{a+b+c+d+\ldots}{p+q+r+s+\ldots}$

☆ Duplicate & Sub-duplicate ratio of $a:b$ is $a^2:b^2$ & $\sqrt{a}:\sqrt{b}$.

☆ $a_1x + b_1y + c_1z = 0$ $a_2x + b_2y + c_2z = 0$.

$$\begin{matrix} b_1 & c_1 & a_1 & b_1 \\ b_2 & c_2 & a_2 & b_2 \end{matrix}$$

$b_1c_2 - b_2c_1 : c_1a_2 - c_2a_1 : a_1b_2 - a_2b_1$

☆ $a:b :: b:c$ [a, b, c are in continued proportion].

$b \to$ mean proportion of a & c

$b = \sqrt{ac}$

Scanned by CamScanner

☆ If four quantities are in proportion, then

$a : b :: c : d$ means.

$\frac{a}{b} = \frac{c}{d}$ $ad = bc$

Extremes

☆ If $a : b :: b : c$.

then $a : c = a^2 : b^2$

Operations on proportions

(1) Inverendo : If $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{b}{a} = \frac{d}{c}$

(2) Alternando : If $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a}{c} = \frac{b}{d}$

(3) Componendo : If $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a+b}{b} = \frac{c+d}{d}$

(4) Dividendo : If $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a-b}{b} = \frac{c-d}{d}$.

(5) Componendo & Dividendo : If $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a+b}{a-b} = \frac{c+d}{c-d}$.

☆ Direct proportion: When A varies directly with B $A \propto B$ $\frac{A}{B} = \text{const}$

☆ Inverse proportion: When A varies indirectly with B $A \propto \frac{1}{B}$ $AB = \text{const}$

(9) Partnerships

☆ When two or more than two persons run a business jointly, they are called partners & the deal is known as partnership.

☆ When investments of all partners are of same time.

Let A invest ₹x B invest ₹y A's profit : B's profit = $x : y$.

☆ When investment are of different time period A → p months, B → q months.

A's profit : B's profit = $xp : yq$

☆ A partner who manages the business is called working partner.

☆ A partner who simply invests money is called sleeping partner.

Scanned by CamScanner

Ratio & Proportion (Contd..).

☆ Given the sum of two unknown variables, the product of the two variables say (a & b) is highest when they are equal.

eg. $a+b=10$

$ab = \max$ when $a=b=5$ $\therefore ab=25$.

☆ Given the product of two unknown variables, the sum of the two variables will be the least when they are equal.

eg. $ab=100$

$a+b = \min$ when $a=b=10$ $\therefore a+b=20$.

☆ $a^x \times b^y \times c^z$ is maximum when $\frac{a}{x}=\frac{b}{y}=\frac{c}{z}$ if $a+b+c=\text{const}$.

eg. max value of $a^3 \times b^2$ where $a+b=10$. — ①

Now, product is max when $\frac{a}{3}=\frac{b}{2} \Rightarrow 2a=3b$ — ②

Solving ① & ② we get $a=6$ $b=4$.

Max value $= 6^3 \times 4^2 = 3456$.

Scanned by CamScanner

(10) Set Theory & Venn Diagrams

☆ Cardinality of a set : No. of distinct elements

☆ Subset : $A \subseteq B$ Proper subset $A \subset B$

☆ If cardinality or order is n, total subsets $= 2^n$

☆ Disjoint sets : $A \cap B = \phi$ → empty set

☆ $A - B$: elements of A not belonging to B ☆ A' or $A^c = U - A$ → Universal set

☆ Symmetric difference of 2 sets $A \Delta B = (A-B) \cup (B-A)$

☆ $A \cup A = A$, $A \cap A = A$

$A \cup B = B \cup A$

$A \cup (B \cup C) = (A \cup B) \cup C$

$A \cap (B \cap C) = (A \cap B) \cap C$

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$C - (C - A) = C \cap A$

$C - (A \cup B) = (C - A) \cap (C - B)$

$C - (A \cap B) = (C - A) \cup (C - B)$

$(A^c)^c = A$

$(A \cup B)^c = A^c \cap B^c$

$(A \cap B)^c = A^c \cup B^c$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

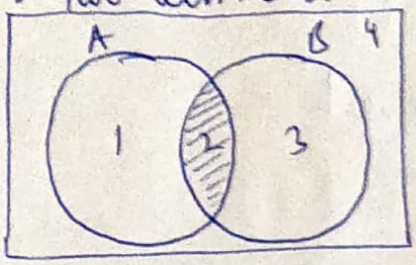
☆ If $O(A) = m$ $O(B) = n$ $O(A \times B) = mn$

$A \times B \neq B \times A$

$n(A \times B) = n(B \times A)$

Scanned by CamScanner

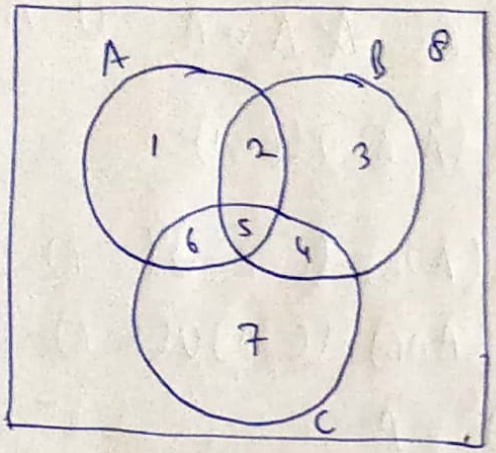
Fig 1: Two attributes A & B



1 : Only A
2 : Both A & B ($A \cap B$)
3 : Only B
4 : Neither A nor B.

So, $A \cup B = A + B - A \cap B$

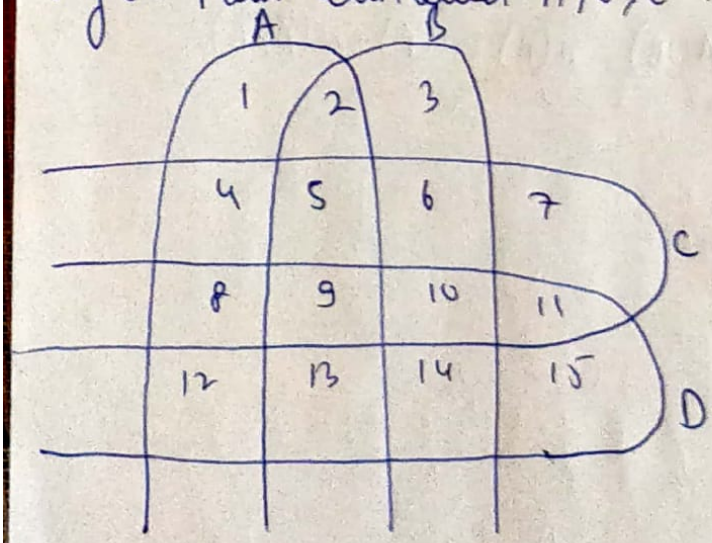
Fig 2: Three attributes A, B & C.



1 : Only A
2 : Only A & B not C
3 : Only B
4 : Only B & C not A
5 : Both A, B & C ($A \cap B \cap C$)
6 : Only A & C not B
7 : Only C
8 : Neither A nor B nor C.

So, $(A \cup B \cup C) = A + B + C - (A \cap B) - (B \cap C) - (A \cap C) + (A \cap B \cap C)$.

Fig 3: Four attributes A, B, C & D.



1 : Only A
2 : Only A & B
3 : Only B
4 : Only A & C
5 : Only A, B & C
6 : Only B & C
7 : Only C
8 : Only A & D & C
9 : All [$A \cap B \cap C \cap D$]
10 : Only B, C & D
11 : Only C & D
12 : Only A & D
13 : Only A, C & D
14 : Only B & D
15 : Only D.

Scanned by CamScanner

(11) Time, Speed & Distance

$$\text{Speed} = \left(\frac{\text{Distance}}{\text{Time}}\right)$$

☆ kmph → msec × $\frac{5}{18}$ ☆ msec → kmph × $\frac{18}{5}$

☆ when s = const $d \propto t$ ☆ when d = const $s \propto \frac{1}{\text{time}}$. $\frac{s_1}{s_2} = \frac{t_2}{t_1}$

☆ when t = const $s \propto d$

☆ When a man covers certain distance at x kmph & same at y kmph.

$$\text{Avg Speed} = \left(\frac{2xy}{x+y}\right) \text{ kmph.}$$

☆ Two bodies moving in opposite direction Rel Speed = $S_1 + S_2$

☆ " " " " same " Rel Speed = $|S_1 - S_2|$

TRAINS

Let S_1: Speed of train 1 L_1: length of train 1 S_0: Speed of object

S_2: Speed of train 2 L_2: length of train 2 L_0: length of object t = time

Case I: Time taken by train 1 to pass a pole or stationary object

$$t = \frac{L_1}{S_1}$$

Case II: Time taken by train 1 to pass an stationary object of length L_0

$$t = \frac{L_1 + L_0}{S_1}$$

Case III: Time taken by train 1 to pass moving object (like man)

$$t = \frac{L_1}{S_1 \pm S_0}$$

+ → opposite direction

− → same "

Case IV: If object has length & moving

$$t = \frac{L_1 + L_0}{S_1 \pm S_0}$$

+ → opposite direction

− → same "

Scanned by CamScanner

Case V: Two trains 1 & 2 are moving, Time taken to cross each other.

$$t = \frac{L_1 + L_2}{S_1 \pm S_2}$$

$+ \rightarrow$ opposite direction

$- \rightarrow$ same "

☆ If two trains start at same point time from A & B towards each other & after crossing take a & b sec to reach B & A, then.

$$\frac{\text{A's speed}}{\text{B's speed}} = \frac{\sqrt{b}}{\sqrt{a}}$$

Boat & Streams

Let u: speed of boat in still water

v: speed of stream.

☆ Speed downstream $= (u+v)$ ☆ Speed upstream $= (u-v)$

Let a: speed downstream b: speed upstream.

☆ Speed in still water $= \frac{(a+b)}{2}$ ☆ Speed of stream $= \frac{(a-b)}{2}$

Circular Tracks

Let L: length of circular track a: speed of A b: speed of B c: speed of C

	Same Direction	Opposite Direction
Time taken to meet first time	$\left(\frac{L}{a-b}\right)$	$\left(\frac{L}{a+b}\right)$
Time taken to meet 1st time at starting point	$LCM\left(\frac{L}{a}, \frac{L}{b}\right)$	$LCM\left(\frac{L}{a}, \frac{L}{b}\right)$

3 people → same direction

Time taken to meet 1st time $= LCM\left(\frac{L}{a-b}, \frac{L}{a-c}\right)$

↳ fastest runner.

Time taken to meet 1st time at starting point $= LCM\left(\frac{L}{a}, \frac{L}{b}, \frac{L}{c}\right)$

Scanned by CamScanner

Circular Tracks (Contd)..

★ If two bodies are moving in the same direction, their relative speed = $(a-b)$

★ If two bodies are moving in opposite direction, their relative speed = $(a+b)$.

★ No. of distinct points at which body meets on a track

→ If they are moving in same direction, points = $(a-b)$ a & b coprime

→ " " " " " opposite " " = $(a+b)$

→ All these points are equidistant from each other.

★ No. of distinct points 3 bodies will meet on a circular track

① Find out pairwise distinct meeting points

② The overall answer will be HCF of pairwise value.

Eg A, B, C are running with speeds 7, 9, 13 on a circular track A & B clockwise, C anti-clockwise.

So, A & B → $9-7=2$ distinct points

A & C → $7+13=20$ " "

B & C → $9+13=22$ " "

∴ A, B, C will meet in HCF $(2, 20, 22)$ i.e. 2 distinct points.

★ To meet at diametrically opposite point of the track, the distinct points should be even, else they'll never meet.

Scanned by CamScanner

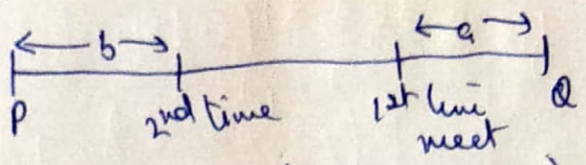
★ When 3 people are running on a track of L meter at speed a, b, c m/sec respectively. then they meet for the first time

$$LCM\left(\frac{L}{a \pm b}, \frac{L}{b \pm c}, \frac{L}{a \pm c}\right) sec$$

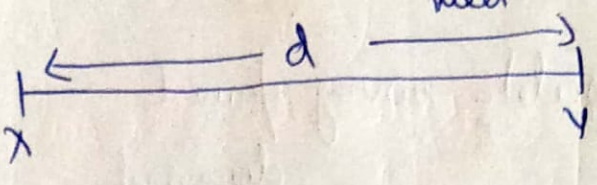
+ → opposite direction.

− → same direction

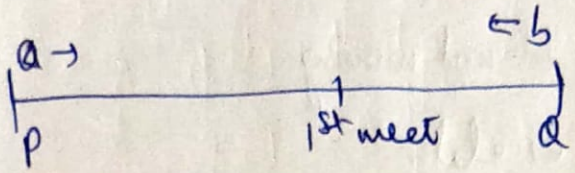
Scanned by CamScanner

☆ 

$PQ = (3a - b)$

☆ 

1st meet distance covered = d
2nd " " " = 2d
& so, on.

☆ 

After 1st meet a took t_1 to reach Q
b took t_2 " " P

∴ 1st meet $t = \sqrt{t_1 t_2}$

(12) Clocks

A clock has 360° angles. Angle traced by minute hand in 60 min = 360°
" " " hour " " " = 30°

Now, speed of minute hand $= \frac{360^\circ}{60\,min} = 6^\circ/min$

Speed of hour hand $= \frac{30^\circ}{60\,min} = \left(\frac{1}{2}\right)^\circ/min$

Relative speed of minute hand & hour hand $= 6 - \frac{1}{2} = \left(\frac{11}{2}\right)^\circ/min$

☆ To solve questions, consider hour hand stationary & find angle traced by minute hand & use relative speed.

☆ In 12 hours, 2 hands of a clock

$0^\circ \rightarrow 11$ times = Every $\frac{12}{11}$ hours $= 1\frac{1}{11}$ hrs $= 65\frac{5}{11}$ min

$180^\circ \rightarrow 11$ times

90° or any other $\angle \rightarrow 22$ times.

Scanned by CamScanner

TSD (Contd..) <u>Escalators</u>.

★ Type 1: When 1 person is moving.

Let t_1: time to walk up on an escalator moving upwards

t_2: " " " " " " " " downwards.

t = time to walk up if escalator is not moving.

Let a: speed of A & x: speed of escalator.

When escalator is moving up $S_{eff} = (a+x)$

" " " " down " $= (a-x)$.

" " " not moving " $= a$

$\therefore (a+x), (a), (a-x)$ are in AP

$\therefore$ time will be in HP.

Hence $t = \dfrac{2t_1 t_2}{t_1 + t_2}$.

Let t_1: time to walk down on an escalator moving downwards Steps $= l_1$

t_2: " " run " " " " " " Steps covered $= l_2$

L: total steps of the escalator.

Let x: speed of escalator.

D_1: distance (walking) $= l_1 + t_1 x$ D_2: distance (running) $= l_2 + t_2 x$

Now $D_1 = D_2$

$l_1 + t_1 x = l_2 + t_2 x$

$x = ?$

Total steps $= l_1 + t_1$

Scanned by CamScanner

☆ Type 2: When two people are moving

Let t_1: time taken by A_1 to reach top of upward escalator $t_1 > t_2$

t_2: " " " A_2 " " " " " " "

s_1: steps taken per second by A_1 $s_2 > s_1$

s_2: steps " " " " A_2

S = total steps.

Let x: speed of escalator or x steps / second.

D_1 (distance by A_1) $= t_1(s_1 + x)$ D_2 (distance by A_2) $= t_2(s_2 + x)$

$D_1 = D_2$.

$x = ?$ $S = t_1(s_1 + x)$

II If l_1: steps taken by A_1 to reach top then $t_1 = \frac{l_1}{s_1}$ $t_2 = \frac{l_2}{s_2}$

& l_2: " " " " " " " & approach previous

III Let l_1: time taken by A_1 to reach top of upward moving escalator

$l_2 > l_1$ l_2: " " " A_2 " " " " " " "

Ratio of speeds $A_2 : A_1$ assume 4:3 For every 4 steps by A_2, 3 steps by A_1

Ratio of distance $l_2 : l_1$ let assume (32:30)

Ratio of times $t_2 : t_1 = 4:5$

Let n: no. of steps in escalator.

$$\frac{(n-32)}{(n-30)} = \frac{4}{5} \qquad n = 40 \text{ steps.}$$

Scanned by CamScanner

IV. Ravi is climbing on a moving escalator going up & takes 30 steps. Rakesh is coming down. For every 5 steps of Rakesh, 3 steps of Ravi. Both take same time to reach other end.

(a) What is total steps in escalator?

Let their speeds by 5s & 3s & x: speed of escalator.

$5s - x = 3s + x \quad \therefore x = s$

Speed of Ravi : speed of escalator $= 3s : s = 3:1$

$\therefore$ When Ravi takes 30 steps, escalator takes 10 steps

Total steps $= 30 + 10 = 40$.

(b) Difference in no. of steps both of them had taken when they crossed each other

Both of them would have covered 20 steps.

Ravi (Since 3:1) $\therefore$ 15 steps + 5 escalator.

Rakesh (Since 5:1) $\therefore$ 25 steps − 5 escalator.

Difference $= 25 - 15 = 10$ steps.

Scanned by CamScanner

Circular Tracks (contd..).

☆ If two people are running on a circular track with speeds in ratio a:b where a & b are co-primes. Let l be the length

⊙ They will meet at (a+b) distinct points, if they are running in opposite direction

⊙ They will meet at |a-b| distinct points, if they are running in the same direction

⊙ The time for their first meeting $= \frac{l}{(a+b)}$ [Opposite]

⊙ " " " " " " $= \frac{l}{|a-b|}$ [Same]

Some Special Series

☆ Method of Differences : When the differences b/w two consecutive terms is in AP.

$S = 1 + 4x + 9x^2 + 16x^3 + \ldots \infty$ or $S = 1^2x^0 + 2^2x^1 + 3^2x^2 + 4^2x^3 + \ldots \infty$ $|x| < 1$

$Sx = x + 4x^2 + 9x^3 + 16x^4 + \ldots \infty$

$(1-x)S = 1 + 3x + 5x^2 + 7x^3 + \ldots \infty$ Hence, it is infinite AGP $a = 1$ $d = 2$ $r = x$.

$(1-x)S = \frac{1+x}{(1-x)^2}$ $\therefore S = \frac{1+x}{(1-x)^3}$ Using infinite AGP formula.

☆ Interpolation

$S = \sqrt{1+\frac{1}{1^2}+\frac{1}{2^2}} + \sqrt{1+\frac{1}{2^2}+\frac{1}{3^2}} + \ldots \sqrt{1+\frac{1}{2007^2}+\frac{1}{2008^2}}$ Find $S_{2007} = ?$

$S_1 = T_1 = \sqrt{1+\frac{1}{1^2}+\frac{1}{2^2}} = \frac{3}{2} = 2 - \frac{1}{2}$ $S_2 = T_1 + T_2 = \frac{3}{2} + \sqrt{1+\frac{1}{2^2}+\frac{1}{3^2}} = \frac{8}{3} = 3 - \frac{1}{3}$

On interpolating we can say $S_3 = 4 - \frac{1}{4}$ $S_4 = 5 - \frac{1}{5}$ and so on

$\therefore S_{2007} = 2008 - \frac{1}{2008}$

Scanned by CamScanner

(13) Time & Work

☆ If A does a work in 'a' days & B does a work in 'b' days, then A & B does '$\frac{1}{a}$', '$\frac{1}{b}$' of the work respectively & their combined work is $\left[\frac{1}{a}+\frac{1}{b}\right]$. No. of days $= \frac{ab}{a+b}$.

☆ If C tries to negative their work & C works 'c' days, so, combined work of A, B & C is $\left(\frac{1}{a}+\frac{1}{b}-\frac{1}{c}\right)$.

☆ Work Rate × Time = Work Done.

☆ If M_1 men work for H_1 hours/day & worked for D_1 days & completed W_1 work and if M_2 men work for H_2 hours/day & worked for D_2 days & completed W_2 work, then.

$$\frac{M_1 H_1 D_1}{W_1} = \frac{M_2 H_2 D_2}{W_2}$$

☆ If A is twice efficient as B ⇒ A does twice the work as B in the same time or A will require half the time required by B to do same work.

☆ A pipe can fill a tank in 'x' hours ⇒ part filled in 1 hr $= \frac{1}{x}$

☆ " " " empty " " " 'y' " ⇒ " emptied in 1 hr $= \frac{1}{y}$

☆ If $y > x$, then on opening both pipes, net part filled in 1 hr $= \left(\frac{1}{x}-\frac{1}{y}\right)$

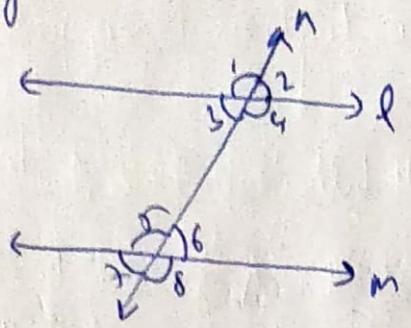
☆ If $x > y$, " " " " " , net part emptied in 1 hr $= \left(\frac{1}{y}-\frac{1}{x}\right)$.

Scanned by CamScanner

(14) Geometry

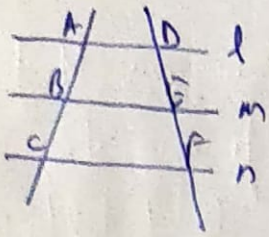
☆ Angle $< 90^\circ \rightarrow$ Acute | $90^\circ \le$ Angle $< 180^\circ \rightarrow$ Obtuse | 90° = Right angle

☆ Angle $> 180^\circ$ & $< 360^\circ \rightarrow$ Reflex Angle.



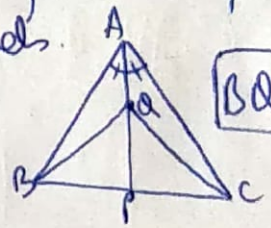
1. Corresponding angles are equal eg $1=5$, $2=6$
2. Alternate interior angles are equal eg $4=5$, $3=6$
3. " Exterior angles are equal eg $2=7$, $1=8$
4. Interior angles on same side of transversal are supplementary eg $3+5=4+6=180^\circ$

☆ Here $l \parallel m$ & n is transversal.

☆ 

If $l \parallel m \parallel n$ then $\frac{AB}{BC} = \frac{DE}{EF}$

☆ Any point on ∠ bisector is equidistant from both ends.



$\boxed{BQ = CQ}$

Polygons

☆ Concave polygon: in which all diagonals lie inside the polygon.

☆ Each side of a regular polygon subtends an angle of $\frac{360}{n}$ at centre. ($n \rightarrow$ sides)

☆ Sum of all angles of a polygon with n sides $= (n-2)180^\circ$

☆ Sum of all exterior angles $= 360^\circ$

☆ Area of a polygon $= \frac{ns^2}{4} \times \cot\left(\frac{180}{n}\right)$ $\quad n$ = no. of sides, s = length of side.

☆ Perimeter $= n \times s$.

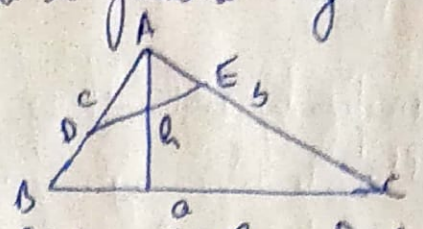
☆ No. of diagonals $= {}^nC_2 - n$

Scanned by CamScanner

Triangles.

☆ Sum of two sides is greater than the third eg $a+b>c$, $b+c>a$, $c+a>b$

☆ Side opposite to greatest angle will be the greatest & vice-versa



☆ Sine Rule : $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

☆ Cosine Rule : $\cos A = \frac{b^2+c^2-a^2}{2bc}$, $\cos B = \frac{a^2+c^2-b^2}{2ac}$, $\cos C = \frac{b^2+a^2-c^2}{2ab}$

☆ Area of Triangle $= \frac{1}{2}ah = \frac{1}{2}bc\sin A = \frac{1}{2}ab\sin C = \frac{1}{2}ac\sin B$

$= \sqrt{s(s-a)(s-b)(s-c)}$ $\quad s =$ semi-perimeter.

$= \frac{abc}{4R} = rs$ $\quad r \to$ in radius, $R \to$ circumradius.

☆ $\frac{AD}{DB} = \frac{x}{y}$ $\quad \frac{AE}{EC} = \frac{l}{m}$ $\quad \frac{Ar(\Delta ADE)}{Ar(\Delta ABC)} = \frac{lx}{(x+y)(l+m)}$

Congruency condition

☆ SAS, ASA, AAS, SSS, SSA

☆ Similarity condition : AAA, SSS, SAS

In Similar Δ's

Ratio of sides = Ratio of altitudes = Ratio of medians = Ratio of angle bisec =

Ratio of In radius = Ratio of circumradius = (Ratio of area)$^{1/2}$

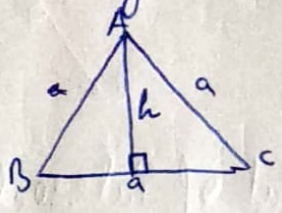
Scanned by CamScanner

☆ **Scalene Triangle** : No side equal, All general properties of Δ's applies

☆ **Equilateral Triangle** : All three sides are equal & angles

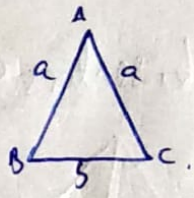
Side $= a$ Altitude $= \frac{\sqrt{3}}{2}a$ Area $= \frac{\sqrt{3}}{4}a^2$

In radius $= \frac{a}{2\sqrt{3}}$ Circumradius $= \frac{a}{\sqrt{3}}$



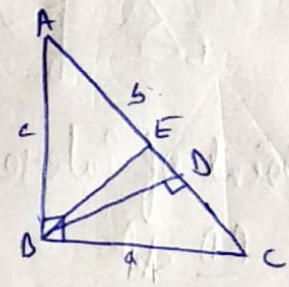
☆ **Isosceles Triangle** : Two sides are equal.

- Angles opposite to equal sides are equal
- Area $= \frac{b}{4}\sqrt{4a^2-b^2}$



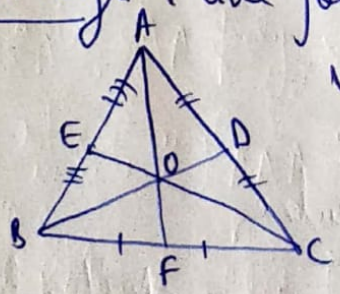
☆ **Right Triangle** : One of the angles is 90°

- Area $= \frac{1}{2} \times AB \times BC$ $R = \frac{\text{Hypotenuse}}{2}$
- $BD = \frac{AB \times BC}{AC}$ $r = \frac{ac}{a+b+c}$
- $EA = EC = EB$ $AB^2 + BC^2 = AC^2$



(3,4,5), (5,12,13), (7,24,25), (8,15,17), (9,40,41), (11,60,61)

Medians of a triangle : line joining mid-point of a side to the opposite vertex.



AF, BD, CE are medians

☆ Median divides the triangle into two equal areas

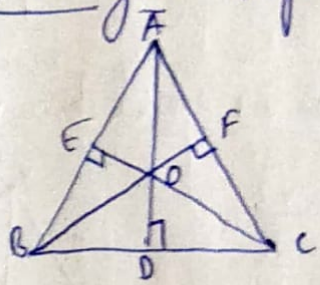
☆ Intersection of three medians is called centroid.

☆ Centroid divides median in the ratio 2:1 $\frac{AO}{OF} = \frac{BO}{OD} = \frac{CO}{OE} = 2$.

☆ $AB^2 + AC^2 = 2(AF^2 + BF^2)$ Apollonius Theorem ☆☆

Scanned by CamScanner

Altitudes of a Triangle: Perpendicular drawn from vertex to the opposite side

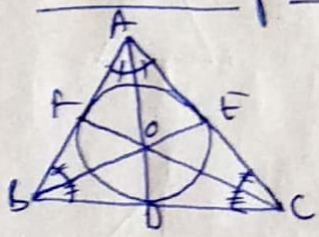


AD, BF, CE are altitudes

☆ Point of intersection is called orthocentre.

☆ $\angle A + \angle BOC = \angle B + \angle AOC + \angle C + \angle BOA = 180^\circ$

Internal angle bisectors of a Triangle: lines bisecting the interior angles of a triangle

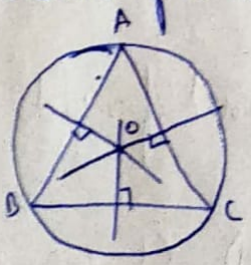


AD, BE, CF are angle bisector

☆ The point of intersection is the inradius.

☆ $\frac{AB}{AC} = \frac{BD}{CD}$ ☆

Perpendicular bisectors of a Triangle: line that is ⊥ side & bisect it.



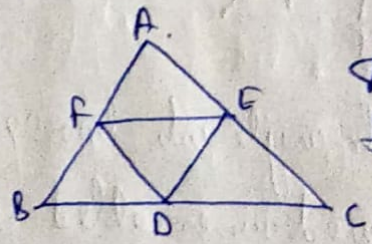
☆ Point of intersection is the circumcentre

Line Joining the Mid point

D, E, F are mid points

FE ∥ BC, DE ∥ AB, DF ∥ AC

☆ $FE = \frac{BC}{2}$ $DE = \frac{AB}{2}$ $DF = \frac{AC}{2}$



☆ If a line ∥ base & passes through mid-point of one side it will pass through mid-point of other side.

☆ $ar(\Delta DEF) = ar(\Delta AFE) = ar(\Delta BFD) = ar(DEC) = \frac{1}{4} ar(\Delta ABC)$

Scanned by CamScanner

	Circumcentre	Orthocentre
Acute	Inside	Inside
Right	Hypotenuse	Vertex
Obtuse	Outside	Obtuse

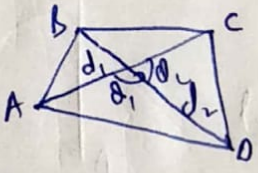
☆ In a right Δ, length of median drawn to hypotenuse is equal to half of hypotenuse
∴ This median = circumradius

☆ In equilateral Δ, centroid, orthocentre, circumcentre, incentre all coincide

☆ In isosceles Δ, centroid, " " " all lie on median.

☆ In equilateral Δ $\frac{R}{r} = \frac{2}{1}$ Area $= \frac{4}{1}$

Quadrilaterals

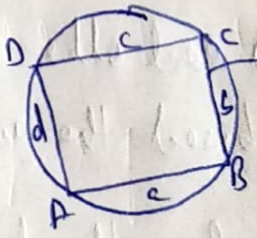


☆ Area $= \frac{1}{2} d_1 d_2 \sin\theta_1 = \frac{1}{2} d_1 d_2 \sin\theta_2$

☆ Area $= \frac{1}{2} \times$ diagonal $\times$ Sum of perpendicular drawn from opposite vertices
$= \frac{1}{2} d (h_1 + h_2)$

☆ Area of circumscribed quadrilateral.

$A = \sqrt{(s-a)(s-b)(s-c)(s-d)}$ $s = \frac{a+b+c+d}{2}$



→ cyclic quadrilateral

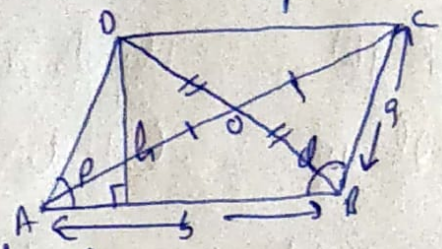
☆ $(a \times c) + (b \times d) = AC \times BD$ Sum of all angles $= 360^\circ$

Types

1) Parallelogram
2) Rectangle
3) Square
4) Rhombus
5) Trapezium
6) Hexagon.

Scanned by CamScanner

Parallelogram : Quadlateral whose opposite sides are parallel



☆ Area $= bh = ab \sin P = ab \sin Q$

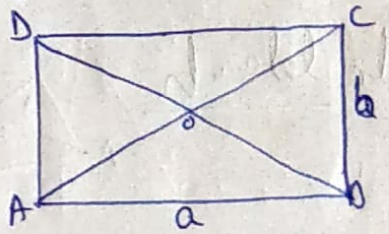
☆ Diagonals bisect each other

☆ Parallelogram inscribed in a circle is a rectangle

☆ " circumscribed about " " " rhombus

☆ $AC^2 + BD^2 = 2(AB^2 + BC^2)$ $\angle P + \angle Q = 180^\circ$

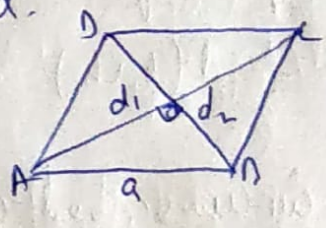
Rectangle : A parallelogram with all angles 90°.



☆ Area $= ab$ Perimeter $= 2(a+b)$

☆ Diagonals are equal $= \sqrt{a^2+b^2}$ & bisect each other.

Rhombus : A ||gm having all sides equal.

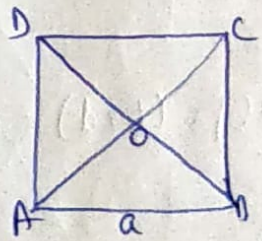


☆ Area $= \frac{1}{2} d_1 d_2$

☆ Diagonals bisect each other at 90°

☆ Diagonals also bisect the angles $\angle BCA = \angle DCA$ & $\angle ADB = \angle CDB$

Square : Rectangle with adjacent sides equal



☆ Area $= a^2$ Perimeter $= 4a$

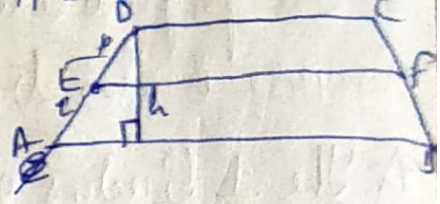
$= \frac{1}{2}(\text{diagonal})^2$ Diagonal $= \sqrt{2}a$

☆ ~~Diagonal = $\frac{a}{2}$~~ ☆ Diagonals are equal & bisect at 90°

☆ $r = \frac{a}{2}$ $R = \frac{a}{\sqrt{2}}$

Scanned by CamScanner

Trapezium : Quadlateral having only two sides || to each other



☆ Area $= \frac{1}{2}(AB+CD)\,h$

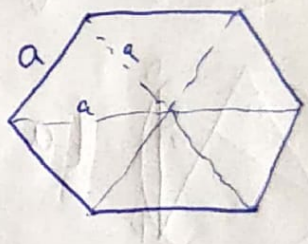
☆ Median $= \frac{1}{2}(AB+CD)$

☆ For any line $EF \parallel AB$ $\quad EF = \frac{p(AB) + q(DC)}{AD}$

☆ $AC^2 + BD^2 = AD^2 + BC^2 + 2 \cdot AD \cdot CD$

☆ If non-parallel sides are equal, then diagonals are equal too & it is called isosceles trapezium.

Regular Hexagon : A closed figure with six equal sides.

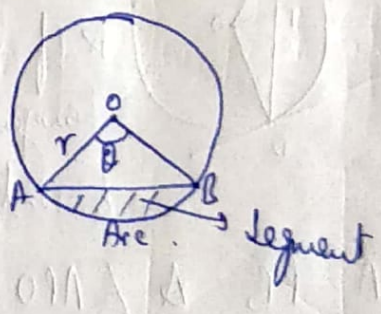


☆ Area $= 6 \times \left(\frac{\sqrt{3}}{4}a^2\right) = \frac{3\sqrt{3}}{2}a^2$

☆ It is combination of 6 equilateral Δ's $\boxed{R = a}$

Circles

☆ A circle is set of all points in a plane that lie at a constant distance from a fixed point.



☆ Area $= \pi r^2$ $\quad$ Circumference $= 2\pi r$

☆ Length (arc AB) $= \frac{\theta}{360} \times 2\pi r$

☆ Area of sector $= \frac{\theta}{360} \times \pi r^2$

☆ Area of segment $= \frac{\theta}{360} \times \pi r^2 - \frac{1}{2} r^2 \sin\theta$

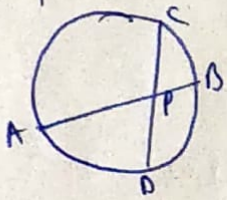
☆ Length of chord AB $= 2r \sin\left(\frac{\theta}{2}\right)$

Scanned by CamScanner

☆ The perpendicular from the centre of a circle to a chord bisects it. Converse is also true.

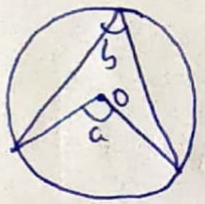
☆ The ⊥ bisectors of two chords intersect at the centre.

☆ Equal chords are equidistant from the centre. Converse is also true.

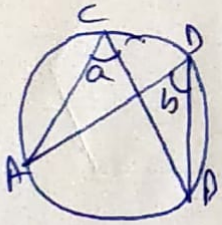
☆ 

$\boxed{AP \times PB = CP \times DP}$

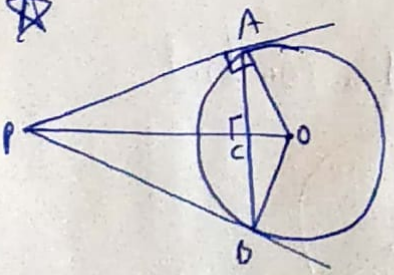
☆ Equal chords subtend equal angle at the center.

☆ 

Angle subtended by an arc of a circle is double the angle subtended by it at any point $\boxed{a = 2b}$

☆ 

$\boxed{a = b}$ ☆ Angle inscribed in a semicircle is 90°

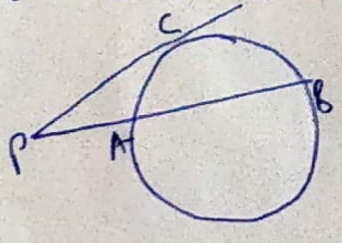
☆ 

☆ PA is tangent to the circle & PA ⊥ OA

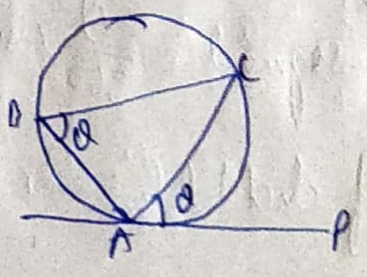
☆ If two tangents are drawn to circle & length of two tangent are equal & line joining external point to circle bisects the angle.

☆ PA = PB ☆ ∠APO = ∠BPO AB ⊥ OP

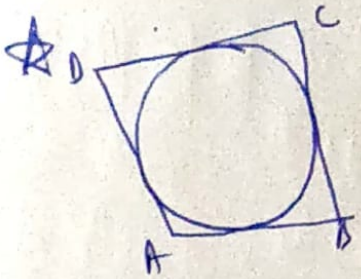
☆ From an external point P, a secant P-A-B & tangent PC



$\boxed{PA \times PB = PC^2}$

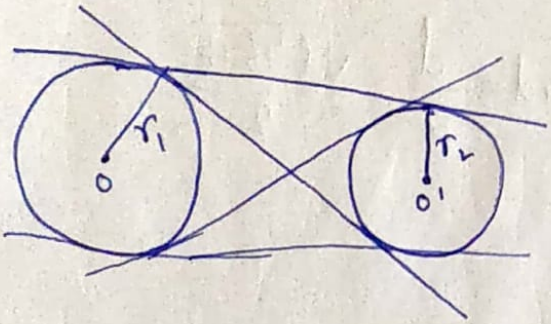
☆ 

Scanned by CamScanner



$\boxed{AB + DC = AD + BC}$

☆ When two circles intersect each other, the line joining the centres bisects the common chord & is ⊥ chord.



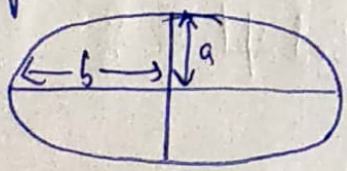
☆ length of direct common tangent

$$\sqrt{(OO')^2 - (r_1 - r_2)^2}$$

☆ length of transverse common tangent

$$\sqrt{(OO')^2 - (r_1 + r_2)^2}$$

Ellipse



$P = \pi(a+b)$

$A = \pi ab$

Star

☆ Sum of all angles $= (n-4)\pi$

☆ Largest rectangle that can be inscribed in a circle is a square.

☆ Perimeter of any inscribed polygon < circumference of the circle

☆ " " " circumscribed " > " " " "

Scanned by CamScanner

(15) Mensuration

☆ **Cuboid**. l: length b: breadth h: height.

length of body diagnol $= \sqrt{l^2+b^2+h^2}$

$C.S.A = 2(l+b)h$ $\quad$ $T.S.A = 2(lb+bh+lh)$

Volume $= lbh$

Radius of sphere circumscribing the cuboid $= \frac{d}{2} = \frac{\sqrt{l^2+b^2+h^2}}{2}$

☆ **Cube** s: side.

$C.S.A = 4s^2$ $\quad$ $TSA = 6s^2$

Volume $= s^3$

face diagnol $= \sqrt{2}s$ $\quad$ Body diagnol $= \sqrt{3}s$

Radius of circumscribed sphere $= \frac{\sqrt{3}a}{2}$ $\quad$ R of inscribed sphere $= \frac{a}{2}$

☆ **Prisms**. right prism : if sides faces are rectangle.

C.S.A = Perimeter of base X height

T.S.A. = CSA + 2X area of base.

Volume = Area of base X height

☆ **Cylinder** r: radius h: height.

$C.S.A = 2\pi rh$

$T.S.A = 2\pi rh + \pi r^2$

Volume $= \pi r^2 h$

Scanned by CamScanner

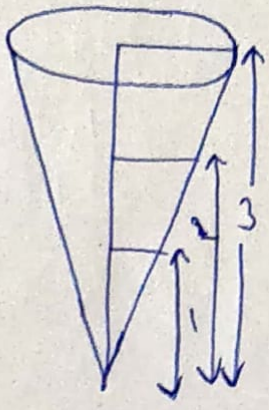
☆ Cone

r : radius h : height l : slant height

$l = \sqrt{h^2 + r^2}$

$CSA = \pi r l$ $TSA = \pi r l + \pi r^2$

$\text{Volume} = \frac{1}{3}\pi r^2 h$



☆ Ratio of volumes of cones up to height 1m, 2m, 3m is $1^3 : 2^3 : 3^3 = 1 : 8 : 27$.

☆ Ratio of volume above 2m, b/w 2 & 1 & below 1m
$(27-8) : (8-1) : 1 = 19 : 7 : 1$

☆ Pyramid

$C.S.A = \frac{\text{perimeter}}{2} \times \text{slant height}$

$T.S.A = C.S.A + \text{area of base}$

$\text{Volume} = \frac{\text{base area}}{3} \times \text{height}$

☆ Tetrahedron : A pyramid having 4 equilateral Δ's as four faces.

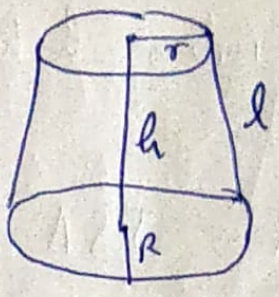
a : side

$C.S.A. = \frac{3\sqrt{3}a^2}{4}$ $\text{Height} = \frac{\sqrt{6}a}{3}$

$T.S.A = \sqrt{3}a^2$ $\text{Volume} = \frac{\sqrt{2}a^3}{12}$

Scanned by CamScanner

☆ Frustum of a cone



Slant height $l = \sqrt{h^2 + (R-r)^2}$

$C.S.A = \pi(r+R)l$

$T.S.A = C.S.A + \pi(r^2+R^2)$ Volume $= \frac{1}{3}\pi h(r^2+R^2-rR)$

☆ Sphere r: radius

Surface Area $= 4\pi r^2$ Volume $= \frac{4}{3}\pi r^3$

☆ Hemisphere r: radius

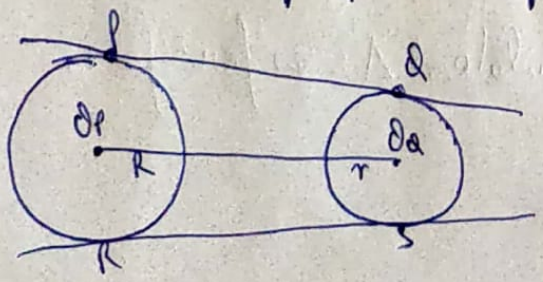
$C.S.A = 2\pi r^2$

$T.S.A = 3\pi r^2$ Volume $= \frac{2}{3}\pi r^3$

☆ Shell: R: outer radius r: inner radius

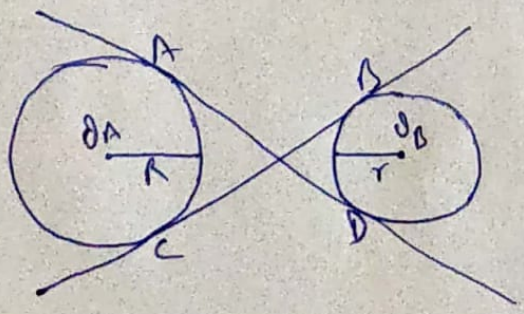
Volume $= \frac{4}{3}\pi(R^3 - r^3)$

☆ Some more properties of Circles.



length of direct common tangent

$PQ = \sqrt{(O_P O_Q)^2 - (R-r)^2}$



Length of transverse common tangent

$AD = \sqrt{(O_A O_B)^2 - (R+r)^2}$

Scanned by CamScanner

(16) Coordinate Geometry

☆ Distance Formula : let $P(x_1, y_1)$ & $Q(x_2, y_2)$

$$PQ = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

☆ Section Formula : let $P(x, y)$, $A(x_1, y_1)$ $B(x_2, y_2)$ & divide ratio $m:n$

⊙ Internal division $x = \dfrac{mx_2 + nx_1}{m+n}$ $y = \dfrac{my_2 + ny_1}{m+n}$

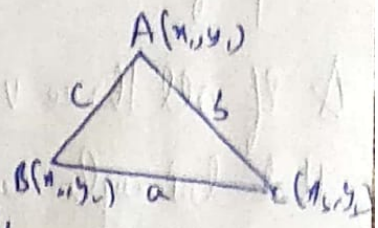
⊙ External division $x = \dfrac{mx_2 - nx_1}{m-n}$ $y = \dfrac{my_2 - ny_1}{m-n}$

☆ Area of a triangle $= \dfrac{1}{2}\left[\dfrac{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)}{1}\right]$

☆ If 3 points are collinear ∴ area = 0.

☆ Area of quadrilateral $= \dfrac{1}{2}\begin{vmatrix} x_1 - x_3 & y_1 - y_3 \\ x_2 - x_4 & y_2 - y_4 \end{vmatrix}$

☆ Centriod of a △ $x = \dfrac{x_1 + x_2 + x_3}{3}$ $y = \dfrac{y_1 + y_2 + y_3}{3}$



☆ Incentre of a △ $x = \dfrac{ax_1 + bx_2 + cx_3}{a+b+c}$ $y = \dfrac{ay_1 + by_2 + cy_3}{a+b+c}$

☆ St. line equation : $\dfrac{y - y_1}{x - x_1} = \dfrac{y_2 - y_1}{x_2 - x_1} = m$, $y = mx + c$,

$\dfrac{x}{a} + \dfrac{y}{b} = 1$, $ax + by + c = 0$

☆ If $m_1 = m_2 \Rightarrow \parallel$ lines , if $m_1 m_2 = -1 \Rightarrow \perp$ lines.

☆ $l_1 : a_1x + b_1y + c_1 = 0$ $l_2 = a_2x + b_2y + c_2 = 0$.

$$\cos\theta = \left|\frac{a_1a_2 + b_1b_2}{\sqrt{a_1^2 + b_1^2}\sqrt{a_2^2 + b_2^2}}\right| \qquad \tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

Scanned by CamScanner

☆ length of perpendicular or distance of a point from a line.

$$L = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}} \qquad l: ax+by+c=0, \quad P:(x_1, y_1).$$

☆ Distance b/w two ∥ lines $= \left|\frac{c_1 - c_2}{\sqrt{a^2+b^2}}\right|$

☆ Change of axes: If origin (0,0) is shifted to (h,k), then coordinates of point (x,y) will be (X,Y) where.

$$x = X + h \qquad y = Y + k$$

☆ Rotation of Axes: $x = X\cos\theta - Y\sin\theta \qquad y = X\sin\theta + Y\cos\theta$

$$X = x\cos\theta + y\sin\theta \qquad Y = -x\sin\theta + y\cos\theta.$$

☆ Eq. of circle: $(x-a)^2 + (y-b)^2 + r^2$

☆ Eq. of common chord is given by $C_1 = C_2$ [Solve].

☆ If all three vertices of a triangle have integral coordinates, then that triangle can't be equilateral triangle.

Scanned by CamScanner

(17) Permutations & Combinations

☆ Selection of 2 men out of 3 is given by ${}^3C_2$ $\boxed{☆\ 0! = 1}$

☆ Arrangement of " " " " " " " " ${}^3P_2$

$${}^nC_r = \frac{n!}{r!(n-r)!} = {}^nC_{n-r} \quad ☆\ {}^nP_r = \frac{n!}{(n-r)!}, \quad {}^nP_n = n!$$

☆ ${}^nP_r = r! \times {}^nC_r$

☆ MNP rule : M ways of doing 1st things, N ways for 2nd & P ways of 3rd

- If works are mutually inclusive ⇒ Ways = $M \times N \times P$
- " " " " exclusive ⇒ Ways = $M + N + P$

☆ No. of permutation of n different things taken all at a time = $n!$

☆ " " " " " " " in which P_1 alike of one type, P_2 of 2nd type, P_3 of third $= \frac{n!}{P_1!\,P_2!\,P_3!}$

☆ No. of permutation of n different things taken r at a time & repetition is allowed $= n^r$

☆ No. of selection of r things out of n identical things = 1

☆ Selection of zero or more things out of n different things $= {}^nC_0 + {}^nC_1 + \dots 2^n$.

☆ " " one " " " " " " " " " $= {}^nC_1 + {}^nC_2 + \dots {}^nC_n = 2^n - 1$

☆ No. of ways of distributing n identical things among r persons in which each person can get any no. of things $= {}^{n+r-1}C_{r-1}$

Scanned by CamScanner

★ No. of ways of dividing $(m+n)$ different things in two groups containing m & n things respectively $= {}^{m+n}C_m \times {}^{n}C_n = \frac{(m+n)!}{m!\,n!}$

★ Ways of dividing $2n$ different items in two groups containing n things $= \frac{(2n)!}{2!\,(n!)^2}$

★ No. of ways of distributing n identical things among r people such that ~~each~~ each get atleast $1 = {}^{n-1}C_{r-1}$

★ Some Formulas

⊙ ${}^{n-1}P_r = (n-r)\,{}^{n-1}P_{r-1}$

⊙ ${}^{n}P_r = {}^{n-1}P_r + r\,{}^{n-1}P_{r-1}$

⊙ $1\times1! + 2\times2! + 3\times3! + \ldots + n\times n! = {}^{n+1}P_{n+1} - 1! = (n+1)! - 1$

⊙ ${}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r$

⊙ ${}^{n}C_x = {}^{n}C_y \Rightarrow x = y$ or $x+y = n$

⊙ ${}^{n}C_r = \frac{n}{r}\,{}^{n-1}C_{r-1}$

⊙ $\frac{{}^{n}C_r}{(r+1)} = \frac{{}^{n+1}C_{r+1}}{(n+1)}$

⊙ $\frac{{}^{n}C_r}{{}^{n}C_{r-1}} = \frac{n-r+1}{n}$

⊙ ${}^{n}C_0 + {}^{n}C_1 + {}^{n}C_2 + \ldots {}^{n}C_n = 2^n$

⊙ ${}^{2n+1}C_0 + {}^{2n+1}C_1 + \ldots {}^{2n+1}C_n = 2^{2n}$

⊙ ${}^{n}C_0 + {}^{n}C_2 + {}^{n}C_4 + \ldots = {}^{n}C_1 + {}^{n}C_3 + \ldots = 2^{n-1}$

⊙ ${}^{n}C_n + {}^{n+1}C_n + {}^{n+2}C_n + \ldots {}^{2n-1}C_n = {}^{2n}C_{n+1}$

⊙ For ${}^{n}C_r$ to be greatest

n even $\Rightarrow r = n/2$ ${}^{n}C_{n/2}$ $\quad$ n odd $\Rightarrow r = \frac{(n+1)}{2}$ or $\frac{(n-1)}{2}$

★ No. of ways of distributing n distinct things among r distinct groups $= r^n$ [where ^ what].

q. 20 distinct chocolates into 4 distinct boxes.

where → box $\quad$ what → chocolate $\therefore$ Ans $= 4^{20}$.

Scanned by CamScanner

Circular Permutations

☆ Ways of arranging n people on a circular track $= (n-1)!$

☆ When clockwise & anti-clockwise are not different ways $= \frac{(n-1)!}{2}$

Some More Results

☆ No. of terms in $(a_1 + a_2 + \cdots a_n)^m = {}^{m+n-1}C_{n-1}$

☆ If all possible n-digit numbers using n distinct digits are formed

sum of all numbers $= (n-1)! \times (\text{Sum of } n \text{ digits}) \times (111\ldots1)_{n \text{ times}}$

☆ No. of diagonals of n-sided regular polygon $= {}^nC_2 - n$

☆ If m points in a plane & n are co-linear

Δ's $= {}^mC_3 - {}^nC_3$ $\quad$ St line $= {}^mC_2 - {}^nC_2 + 1$

☆ $x_1 + x_2 + x_3 + \cdots x_n = y \quad ; y \geq 0$

⊙ No. of +ve integral solution $= {}^{y-1}C_{n-1}$

⊙ No. of non-ve " " $= {}^{y+n-1}C_{n-1}$

⊙ No. of integral solution or real or real +ve or real non-ve solution $= \infty$

☆ $A + B + C = 12 \; ; A, B, C \geq 0$ Find no. of soln

OR

ways of distributing 12 identical chocolates to 3 persons

OR

How many no. less than 1000 will have digit sum 12 $[A, B, C \leq 9]$

OR

No. of terms in the expansion $(A+B+C)^{12}$

Scanned by CamScanner

Case I : <u>No limit</u>.

$A+B+C=12$ ways $= {}^{12+2-1}C_{3-1} = {}^{14}C_2$

Case II : <u>Lower limit</u>

$A+B+C=12 \quad A, B, C \geq 1$

Now $A+B+C=9$ ways $= {}^{11}C_2$

Case III : <u>Upper limit</u>

Convert UL to LL

$A+B+C=12 \quad A \leq 6, B \leq 7, C \leq 8$

Violate the rule by giving $(6+1)=7$ to A

$A=7$

$A+B+C=5 \rightarrow {}^{7}C_2$

Now give $(7+1)=8$ to B $\Rightarrow A+B+C=4 \rightarrow {}^{6}C_2$

" " $(8+1)=9$ to C $\Rightarrow A+B+C=3 \rightarrow {}^{5}C_2$

Total $= \boxed{{}^{14}C_2 - {}^{7}C_2 - {}^{6}C_2 - {}^{5}C_2}$

★ If UL is less than half of total no eg $A+B+C=12 \quad A, B, C \leq 5$

<u>Leave</u>

Case IV : <u>Inequality</u>

$A+B+C \geq 12$ (Not determined)

$A+B+C \leq 10$

Add another variable

$A+B+C+D=10 \rightarrow {}^{13}C_3$

Scanned by CamScanner

~~Some Cases~~

S → D [Special permutation]
12 similar chocolate — 3 persons

S → S [Combination]
12 similar balls — 3 similar boxes

D → S [Std. Combination]
12 diff. letters — 5 similar letter box

D → D [Std. Permutation]
12 diff letters — 5 diff. letter box

☆ No. of squares in a square of $n \times n = 1^2 + 2^2 + 3^2 + \dots n^2$

☆ " " rectangles " " " " " $= 1^3 + 2^3 + 3^3 + \dots n^3$

A rectangle having m rows & n columns.

☆ Squares $= mn + (m-1)(n-1) + (m-2)(n-2) \dots$

☆ Rectangles $= (1+2+\dots m)(1+2+\dots+n)$

☆ Derrangement: No. of ways of placing letters in envelopes such that no letter is placed in corresponding envelope is

$$D_n = n!\left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \dots (-1)^n \frac{1}{n!}\right]$$

☆ Out of n things, r goes to wrong place, (n-r) goes to right place

Ways $= {}^nC_{n-r} D_r$

Scanned by CamScanner

☆ Distribution (distinct into identical)

Eg. No. of ways of distributing 5 distinct balls into 3 identical boxes.

$(5,0,0) \rightarrow {}^5C_5 = 1$

$(4,1,0) \rightarrow {}^5C_4 \times {}^1C_1 = 5$

$(3,2,0) \rightarrow {}^5C_3 \times {}^2C_2 = 10$

$(3,1,1) \rightarrow [{}^5C_3 \times {}^2C_1 \times {}^1C_1]/2! = 10$

$(2,2,1) \rightarrow [{}^5C_2 \times {}^3C_2 \times {}^1C_1]/2! = 15$

Total cases $= 1+5+10+10+15 = 41$

☆ $xy = N$

1) Natural solution = Factors of N
2) Integral solution = $2 \times$ Factors of N
3) Real solution $= \infty$

Scanned by CamScanner

(18) Probability

- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- If A & B are independent $P(A \cap B) = P(A) \times P(B)$
- Conditional probability $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$
- Baye's Theorem $= \frac{p_i q_i}{p_1 q_1 + p_2 q_2 + \cdots p_n q_n}$
- Odds in favour $= \frac{\text{No. of favorable outcomes}}{\text{No. of non-favorable outcomes}}$
- Odds in against $= \frac{1}{\text{Odds in favour}}$
- Expectation = Probability of winning × Reward of winning
- If the probability of an event occuring is P, then probability of that event occuring 'r' times in 'n' trials is $= {}^nC_r P^r (1-P)^{n-r}$

Scanned by CamScanner

☆ No. of ways a number can be written as sum of two or more consecutive natural numbers is = [No. of odd factors − 1].

☆ Similarly, as a sum of two or more consecutive integers

$$= 2^{\alpha} \text{ No. of odd factors} - 1$$

☆ If x, y, z, w are natural numbers & consecutive terms of an AP with common diff 'd' then $xyzw + d^4$ is always a perfect square

☆ If $N = a^x \times b^y \times c^z$, then set of pairs (a, b) such that $LCM(a, b) = N$ is

$(2x+1)(2y+1)(2z+1)$ ordered pairs.

$$\frac{[(2x+1)(2y+1)(2z+1) - 1]}{2} + 1 \text{ unordered pairs.}$$

☆ If $N = 5a + b$

then last non zero digit of $N!$ is calculated as $2^a a! b!$

& last non zero two digits of $N!$ " " as $12^a a! (5a+1) \ldots (5a+5)$

Scanned by CamScanner

☆☆ $x^2+y^2=N$

→ It will have integer solutions only if N contains primes of the form $(4k+1)$ type.

→ If all primes are $(4k+1)$ type then positive soln = (Factors of N)
[In case of perfect squares take 0 also].

→ If it contains some $(4k+3)$ type primes with odd power then '0' solns.

→ If it contains even primes of $(4k+1)$ along with $(4k+1)$, then
+ve solution = (No. of factors of $(4k+1)$ form only).

→ Integral soln = 4 × Positive.

<u>Examples</u>

1) $x^2+y^2 = 5^4 \times 13^5$

Since all factors are of $(4k+1)$ type ∴ +ve solution = $5\times6=30$ & 120 = integral

2) $x^2+y^2 = 7^3 \times 37^5$

Since $7=(4k+3)$ with odd power ∴ 0 solution

3) $x^2+y^2 = 19^4 \times 37^7$

Since $19=(4k+1)$ has even power, so ignore this term

Factors of $37^7 = 8$ ∴ +ve = 8 integral = $8\times4=32$

Scanned by CamScanner

Divisibility (Contd.)

☆ 7/11/13 : Alternate sum & subtraction of three digits taken at a time should be divisible

eg. $\underline{949}\,\overline{531}\,\underline{254} \Rightarrow (949+254) - 531 = 672 \bmod 7 = 0$.

☆ 9 : Sum of digits taken one digit at a time [$\because 10 \bmod 9 = 1$]

☆ 99 : " " " " two digits in " "

☆ 999/27/37 : Sum of digits taken 3 digits at a time

eg. $\underbrace{368}\,\underbrace{330}\,\underbrace{301} \Rightarrow 368+330+301 = 999 \bmod 27/37/999 = 0$.

☆ Now $100 \bmod 101 = -1$ ($\therefore$ ~~Sum of~~ Alternate sum & subtraction of two digits).

In basic terms

Let $abcdef \bmod k = ?$

☆ If $1000 \bmod k = 1$ then $abc + def$

☆ If $100 \bmod k = 1$ then $ab + cd + ef$

☆ If $1000 \bmod k = -1$ then $def - abc$

☆ If $100 \bmod k = -1$ then $ef - cd + ab$.

☆ $\underline{32}\,\underline{32}\,\underline{32}\,\underline{3} \bmod 99$

$32 \times 3 = 96$

$963 \bmod 99 = \underline{72}$

☆ $\underline{345}\,\underline{345}\,\underline{345}\,\underline{34} \bmod 999$

$345 \times 3 = 1037$

$1034 \bmod 999 = \underline{637}$

Euler concepts

$12^{107} \bmod 37 = ?$

Now $E(37) = 36$

So, $12^{108} \bmod 37 = 1$

$12 \times 12^{107} \bmod 37 = -36$

$12^{107} \bmod 37 = -3$ or $\underline{34}$

☆ No. of numbers co prime to 20 from 101–300

$\Rightarrow$ Range [Euler of (20)] $E(20) =$

$\Rightarrow 200 \times \left[\frac{1}{2} \times \frac{4}{5}\right] = \underline{80}$

Scanned by CamScanner

Factors Sum

Let $N = 360 = 2^3 \times 3^2 \times 5^1$

☆ Sum of all factors $= (2^0 + \dots 2^3)(3^0 + \dots 3^2)(5^0 + 5^1) = 1170$

☆ Sum of even factors $= (2^1 + \dots 2^3)$ " " $= 1092$ [Just exclude 2^0]

☆ Sum of odd factors $= 2^0(3^0 + \dots 3^2)(5^0 + 5^1) = 78$ [Exclude all power of 2]

☆ How to calculate squares & cubes

☆ No. of perfect squares in $N = 2^5 \times 3^7$

→ Squares = even powers.

So, find even powers of each prime

$2^5 \Rightarrow 2^0, 2^2, 2^4 \Rightarrow 3$

$3^7 \Rightarrow 3^0, 3^2, 3^4, 3^6 \Rightarrow 4$.

∴ Total perfect squares $= 3 \times 4 = 12$.

→ Cubes = 3k form.

So, find 3k form powers of each prime

$2^5 \Rightarrow 2^0, 2^3 \Rightarrow 2$

$3^7 \Rightarrow 3^0, 3^3, 3^6 \Rightarrow 3$

∴ Total perfect cubes $= 2 \times 3 = 6$

Scanned by CamScanner

☆☆ Integral Solutions

→ $ax + by = n$

Non-negative integral solutions $= \left[\frac{n}{\text{lcm}(a,b)}\right] + 1$; if n is divisible by any of a & b.

☆ For positive soln remove those including $x=0$ or $y=0$ from answer.

eg. $7x + 13y = 1001$

$$\text{Soln} = \left[\frac{1001}{\text{lcm}(7,13)}\right] + 1 = \left[\frac{1001}{91}\right] + 1 = \underline{12.}$$

For positive, we'll remove cases for $(143,0)$ & $(0,77)$ ∴ $\underline{12-2=10}$

☆ $|x|+|y|=n$ has $4n$ integral soln. $n \in N$

☆ $|x|+|y|+|z|=n$ has $4n^2+2$ integral soln when $n=0$ only 1 soln

☆ $|x|+|y|+|z|+|w|=n$ has $\frac{8n}{3}[n^2+2]$ integral solns.

eg. Integral soln for $|a|+|b|+|c|=10 \Rightarrow 4(10)^2+2 = 402$.

☆ $xy = N$ [N is a natural number].

→ Positive soln = Factors of N

→ Integral " = 2 × Factors of N

→ Ways in which N is represented as a product of 2 no $= \left(\frac{\text{Factors}}{2}\right)$

Scanned by CamScanner

☆ $x+y=N$ [N is a natural number].

→ No. of +ve solns such that x & y are co-prime $= E(N)$

For unordered [i.e. variables are not mentioned] $= \frac{E(N)}{2}$

☆☆ $\frac{a}{x}+\frac{b}{y}=\frac{1}{n}$

→ Positive integral solution = Factors of $[ab n^2]$

→ Integral solution = $2\times$ Factors of $[a\times b\times n^2]-1$

[eg. $\frac{1}{x}+\frac{1}{y}=\frac{1}{12}$
Factors of $[1\times1\times12^2]=144$
$F(144)=15$
+ve=15 integ $=2\times15-1=29$]

☆☆ $\frac{a}{x}-\frac{b}{y}=\frac{1}{n}$

→ Positive integral solution = Factors of $[a\times b\times n^2]$ below a^n

→ Integral solution = $2\times$ Factors of $[a\times b\times n^2]-1$

☆ No. of ways a number can be written as difference of squares

① If N = odd then $\frac{\text{No. of factors}}{2}$

② If N = even then $\frac{\text{Factors of}\left(\frac{N}{4}\right)}{2}$

③ If $N=4k+2$ then no pairs.

☆ for integral = 4 × Positive

eg. Soln for $x^2-y^2=600$.

N = even $\frac{600}{4}=150$

Factors of $150\,[2^1\times3^1\times5^2]=12$ so $\frac{12}{2}=6$ +ve

& $6\times4=24$ integral

④ N = perfect square.

$x^2-y^2=900$.

N = even. $\frac{N}{4}=225$

$225=3^2\times5^2=(3\times3)=9$ factors.

So, Positive soln $=\left[\frac{9}{2}\right]=4$.

Integer $=4\times4=16$

BUT, there will be one including non-negative for 30^2-0^2

So, Integral $=16+2=18$

Scanned by CamScanner

☆ Fibonacci & its Applications

$F(n+1) = F(n) + F(n-1)$

1, 1, 2, 3, 5, 8, 13, ...

(Q) A fair coin is tossed 10 times, what is the probability that no two consecutive tosses show head.

Toss 1 times → 2 ways (H, T)

Toss 2 times → 3 ways (HT, TT, TH)

Hence fibonacci starts with 2 & 10th term will be favorable

2, 3, 5, 8, 13, 21, 34, 55, 89, 144

Favorable = 144 Total = 2^{10} Probability = $\frac{144}{2^{10}}$.

(Q) In how many ways you can step 8 stair using only 1 or 2 at a time

General way

11111111 : 1 way

2111111 : $\frac{7!}{6!}$ = 7 ways

221111 : $\frac{6!}{4!2!}$ = 15 ways

22211 : $\frac{5!}{3!2!}$ = 10 ways

2222 : 1 way

Total = 34.

$\boxed{\text{Shortcut : } 8^{th} \text{ term of Fibonacci} = 34}$

Scanned by CamScanner

☆ But in case we would be using 1 or 2 or 3 steps, then we can't use fibonacci, but similar type series.

So, if question is: ways to step up 7 stairs using 1 or 2 or 3 at a time then upto 3 terms we could use $2^0, 2^1, 2^2$ i.e. 1, 2, 4 ways

Now from here next term will be sum of previous three terms & 7th term is answer.

1, 2, 4, 7, 13, 24, 44 ∴ 44 ways.

☆☆ Similarly, it can be applied to n number of steps using similar method.

Scanned by CamScanner

☆ Selection of Non-Consecutive Things ☆

→ No. of ways of selecting r non consecutive things from n distinct things in a row $= {}^{n-r+1}C_r$

eg. Select 4 non consecutive books from 10 distinct books on a shelf?

$n=10 \quad r=4 \quad \therefore {}^{10-4+1}C_4 = {}^{7}C_4 = 35.$

→ No. of ways of selecting r non-consecutive things from n distinct things in a circle $= {}^{n-r+1}C_r - {}^{n-r-1}C_{r-1}$

Eg. Select 4 non-consecutive books from 10 distinct books arranged in circle.

$\Rightarrow {}^{10-4+1}C_4 - {}^{10-4-1}C_{4-1} \Rightarrow {}^{7}C_4 - {}^{5}C_1 = 25.$

☆ Concept behind non consecutive

Let's select 3 non-consecutive out of 10.

a | b | c Total cases $= {}^{10}C_3$.

but we want non-consecutive ∴ at least 1 gap should be there.

a+1 | b+1 | c = 10

a, b, c to be selected out of 8 ways

∴ ${}^{8}C_3$ or by formula ${}^{10-3+1}C_3 = {}^{8}C_3$

Now for at least two instead of +1 we will put +2

a+2 | b+2 | c = 10

a, b, c to be selected out of 6 ways

∴ ${}^{6}C_3$

Scanned by CamScanner

☆ Expressing as a Product of 3 Numbers.

→ No. of ways a number N can be written as product of three numbers

Let $N = 360 = 2^3 \times 3^2 \times 5 = a^x b^y c^z$

Now, the primes 2, 3, 5 can be anyway arranged among a, b, c such that power of each primes remain same.

∴ For 2^3, $x+y+z=3 \rightarrow {}^5C_2$

For 3^2, $x+y+z=2 \rightarrow {}^4C_2$

For 5^1, $x+y+z=1 \rightarrow {}^3C_2$

∴ total ordered solution $= {}^5C_2 \times {}^3C_2 \times {}^4C_2 = 180$

Unordered: there are three possible triplets abc i.e. all distinct, which is arranged in 3! ways; aab is arranged in 3!/2 ways & aaa in only 1 way.

So, from total, remove aab & aaa type. We are left with ordered distinct triplets. Now divide it by 3! & add aab & aaa cases.

So, for 180 aab type (1, 1, 180), (2, 2, 45), (3, 3, 20), (6, 6, 5)

So, total 4 which can be arranged in $4 \times 3 = 12$ ways

aaa type : Not possible

∴ Unordered $= \dfrac{(180 - 3 \times 4)}{3!} + 4 = 32$

Scanned by CamScanner

★★ Very Very Important ★★

→ In ques like, find no. of ways for $a+b+c=100$ $[a,b,c \in N]$ we simply apply the formula ${}^{n-1}C_{r-1}$ [or ${}^{n+r-1}C_{r-1}$ for $a,b,c \geq 0$] because, we want to find ordered solution.

BUT, when variables are not mentioned, we need to find unordered solutions. Let's see how to calculate that:

(a) How many ways 100 can be represented as sum of 3 natural numbers?

$x+y+z=100 \Rightarrow {}^{100-1}C_{3-1} = {}^{99}C_2$, we get all ordered solutions

Now in ${}^{99}C_2$, we have three types:

① (x,y,z) each arranged in $3!$ ways → in order $(1,2,3)$, $(1,3,2)$, $(2,1,3)$ etc are different but in unordered these are same $(1,2,3)$.

② (x,x,y) each arranged in $3!/2$ ways

③ (x,x,x) in this case $=0$.

First, we'll calculate (x,x,y) type.

Now $2x+y=100$ x ranges from 1 to 49 & each is arranged in $\frac{3!}{2}=3$ ways

So, all distinct $= {}^{99}C_2 - 3\times 49$

So, distinct unordered $= \dfrac{{}^{99}C_2 - 3\times 49}{3!}$

And, we already calculated 49 unordered.

So, (x,x,y) type $= 3\times 49$

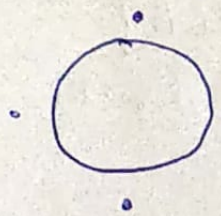
(x,y,z) type $= {}^{99}C_2 - 3\times 49$

(x,x,x) type $=0$

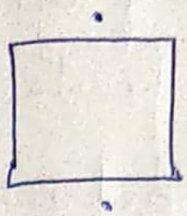
∴ Unordered solution $=$ $(x,y,z \text{ type})/3! + (x,x,y)/3 + (x,x,x)$ ①

$= [{}^{99}C_2 - 3\times 49]/3! + 49 =$ ans ∴

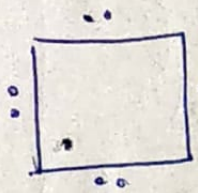
★ Concept of arranging around a polygon



Distinct pts = 1



Dt pt = 1



Distinct points = 2.

So, arranging n people around a polygon with r on each side

$r \times (n-1)!$

eg. How many ways to arrange 15 people around a pentagon with 3 on each side

Ans- $r = 3$ $n = 15$ $\Rightarrow 3 \times 14!$

★★ If sides are odd, then no symmetry unless polygon is regular

eg. Arrange 12 people around an isosceles triangle with 4 on each side.

Ans → 12! (linear arrangement)

So, basically funda for unordered done in previous page is

1. Remove all aab & aaa type from total
2. left with all distinct
3. Divide by 3! to get unordered
4. Add aab & aaa cases seperately

$\therefore$ Ans $= \dfrac{(\text{Total} - 3 \times aab - aaa)}{3!} + aab + aaa$

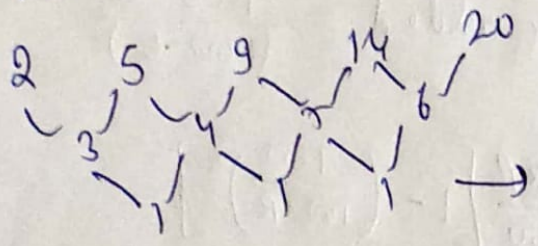
Scanned by CamScanner

Number System Contd.

☆ If x & y are two integers such that
$x, y \geq 3$ & $x < y$ then $x^y > y^x$ eg $4^5 > 5^4$

☆ A concept of sequence & series

If we have series like



→ Since difference is constant in 2nd time.

$\therefore \boxed{T_n = an^2 + bn + c}$

Substitute $n = 1, 2, 3$ in the given question & find a, b, c

☆ Number of primes

→ 1 - 30 ⇒ 10 primes

→ 1 → 50 ⇒ 15 primes

→ 1 → 100 ⇒ 25 primes

→ 1 → 200 ⇒ 46 primes

→ 1 → 1000 ⇒ 168 primes.

☆ To calculate even factors

$N = 2^x \times 5^y \times c^z$, take a 2 common

$N = 2(2^{x-1} \times 5^y \times c^z)$ → find factors of remaining

Even factors $= x(y+1)(z+1)$

☆ To calculate odd factors,

$N = 2^x \times b^y \times c^z$, exclude 2^x

Odd factors $= (y+1)(z+1)$.

Scanned by CamScanner

Allegation & Mixtures

$Q_c \rightarrow$ quantity of cheaper. $Q_d \rightarrow$ quantity of costlier

$P_c \rightarrow$ price of " $P_d \rightarrow$ price of costlier

$P_m \rightarrow$ price of mean $Q_m \rightarrow$ quantity of mea.

$$\therefore \frac{Q_c}{Q_d} = \frac{P_d - P_m}{P_m - P_c}.$$

☆ P gram of ingredient sol has a% ingredient. To increase it to b%

$$\text{quantity added} = \frac{P(b-a)}{(100-b)}.$$

Simple & Compound Interest

$$\text{Equal installments under CI} = \frac{P \times R}{100}\left[1 - \left(\frac{100}{100+R}\right)^2\right]$$

eg. $P = 25000$ $R = 10\%$ $n = 2$.

$$\text{Annual installment } \frac{25000 \times 10}{100}\left[1 - \left(\frac{100}{100+10}\right)^2\right] = 14404.$$

☆ No. of years taken for an amount to double at CI at r% per annum is $\left[\frac{72}{r}\right]$ or more accurate $\left[\frac{69}{r} + 0.35\right]$.

☆ No. of years to triple at CI $\Rightarrow \left[\frac{114}{r}\right]$.

☆ If sum A becomes B in T_1 years at CI, then after T_2 years

$$\text{Sum} = \frac{(B)^{T_2/T_1}}{(A)^{(T_2/T_1 - 1)}}$$

Scanned by CamScanner