CBSE MATH

Made Simple

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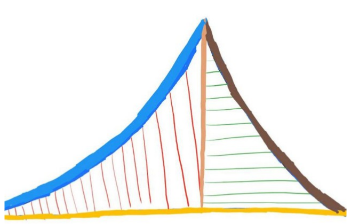

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Introduction

This book links high school coordinate geometry to linear algebra and matrix analysis through solved problems.

Chapter 1

Vectors

1.1. 2024

1.1.1. 12

1. For any two vectors $\overrightarrow{a}$ and $\overrightarrow{b}$, which of the following statements is always true?

   (a) $\overrightarrow{a} \cdot \overrightarrow{b} \geq |\overrightarrow{a}||\overrightarrow{b}|$

   (b) $\overrightarrow{a} \cdot \overrightarrow{b} = |\overrightarrow{a}||\overrightarrow{b}|$

   (c) $\overrightarrow{a} \cdot \overrightarrow{b} \leq |\overrightarrow{a}||\overrightarrow{b}|$

   (d) $\overrightarrow{a} \cdot \overrightarrow{b} < |\overrightarrow{a}||\overrightarrow{b}|$

2. The unit vector perpendicular to both vectors $\hat{i} + \hat{k}$ and $\hat{i} - \hat{k}$ is:

   (a) $2\hat{j}$

   (b) $\hat{j}$

   (c) $\frac{\hat{i}-\hat{k}}{\sqrt{2}}$

   (d) $\frac{\hat{i}+\hat{k}}{\sqrt{2}}$

3. Direction ratios of a vector parallel to line $\frac{x-1}{2} = -y = \frac{2z+1}{6}$ are :

   (a) $2, -1, 6$

   (b) $2, 1, 6$

   (c) $2, 1, 3$

   (d) $2, -1, 3$

4. Assertion (A): For two non-zero vectors $\overrightarrow{a}$ and $\overrightarrow{b}$, $\overrightarrow{a} \cdot \overrightarrow{b} = \overrightarrow{b} \cdot \overrightarrow{a}$

   Reason (R): For two non-zero vectors $\overrightarrow{a}$ and $\overrightarrow{b}$, $\overrightarrow{a} \times \overrightarrow{b} = \overrightarrow{b} \times \overrightarrow{a}$

   (a) Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explaination of Assertion (A).

   (b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explaination of Assertion (A)

   (c) Assertion (A) is true, but Reason (R) is false

   (d) Assertion (A) is false, but Reason (R) is true

5. The position vectors of vertices of $\triangle\ ABC$ are $A(2\hat{\imath} - \hat{\jmath} + \hat{k})$, $B(\hat{\imath} - 3\hat{\jmath} - 5\hat{k})$ and $C(3\hat{\imath} - 4\hat{\jmath} - 4\hat{k})$. Find all the angles of $\triangle\ ABC$.

Chapter 2

Linear Forms

Chapter 3

Circles

Chapter 4

Intersection of Conics

Chapter 5

Probability

5.1. 2024

5.1.1. 12

1. Let E be an event of a sample space S of an experiment, then $P(S|E) =$

   (a) $P(S \cap E)$

   (b) $P(E)$

   (c) 1

   (d) 0

2. A pair of dice is thrown simultaneously. If X denotes the absolute difference of the numbers appearing on top of the dice, then find the probability distribution of X.

3. Airplanes are by far the safest mode of transportation when the number of transported passengers are measured against personal injuries and fatality tools. Previous records state that the probability of an airplane crash is 0.00001%. Further, there are 95% chances that there

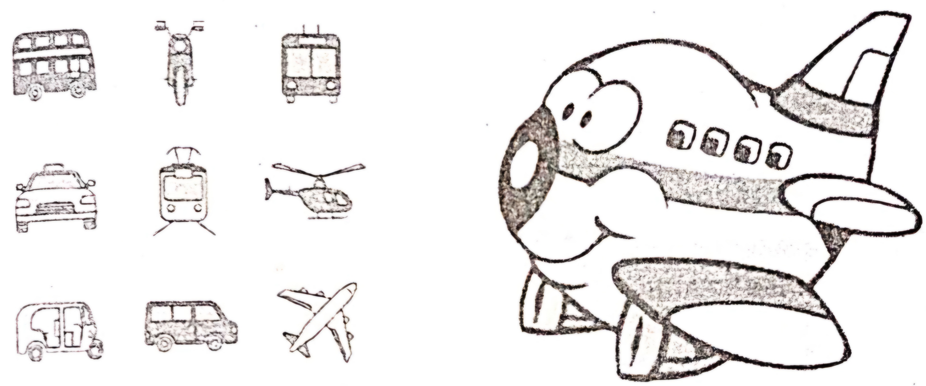

Figure 5.1: 1

will be survivors after a plane crash. Assume that in case of no crash, all travellers survive.

Let E_1 be the event that there is a plane crash and E_2 be the event that there is no crash. Let A be the event that passengers survive after the journey.

On the basis of the above information, answer the following questions:

(i) Find the probability that the airplane will not crash.

(ii) Find $P(A|E_1) + P(A|E_2)$.

(iii) Find $P(A)$

(iv) Find $P(E_2|A)$.

Chapter 6

Construction

Chapter 7

Optimization

Chapter 8

Algebra

8.1. 2024

8.1.1. 12

1. let $f : R_+ \to [-5, \infty)$ be defined as $f(x) = 9x^2 + 6x - 5$ where R_+ is the set of all non-negative real numbers, then f is:

   (a) one-one

   (b) onto

   (c) bijective

   (d) neither one-one nor onto

2. The number of points of discontinuity of $f(x) = \begin{cases} |x| + 3, & if x \leq -3 \\ -2x, & if -3 < x < 3 \\ 6x + 2, & if x \geq 3 \end{cases}$ is:

   (a) 0

   (b) 1

(c) 2

(d) infinite

3. The function $f(x) = x^3 - 3x^2 + 12x - 18$ is:

   (a) strictly decreasing on R

   (b) strictly increasing on R

   (c) neither strictly increasing nor strictly decreasing on R

   (d) strictly decreasing on $(-\infty, 0)$

4. Find the domain of the function $f(x) = \sin^{-1}(x^2 - 4$. Also, find its range.

5. If $f(x) = |\tan 2x|$, then find the value of $f'(x)$ at $x = \frac{\pi}{3}$.

6. If M and m denote the local maximum and local minimum values of the function $f(x) = x + \frac{1}{x}(x \neq 0)$ respectively, find the value of $(M - m)$.

7. Show that $f(x) = e^x - e^{-x} + x - \tan^{-1} x$ is strictly increasing in its domain.

8. Show that a function $f : R \to R$ defined bt $f(x) = \frac{2x}{1+x^2}$ is neither one-one nor onto. Further, find set A so that the given function $f : R \to A$ becomes an onto function.

9. A relation R is defined on $N \times N$ (where N is the set f natural numbers) as:

$$(a, b)R(c, d) \leftrightarrow a - c = b - d$$

   Show that R is an equivalence relation.

10. The month of September is celebrtaed as the Rashtriya Poshan Maah across the country. Following a healthy and well-balanced diet is crucial in order to supply the body with the proper nutrients it needs. A balanced diet also keeps us mentally fit and promotes improved level of energy.

A dietician wishes to minimize the cost of a diet involving two types

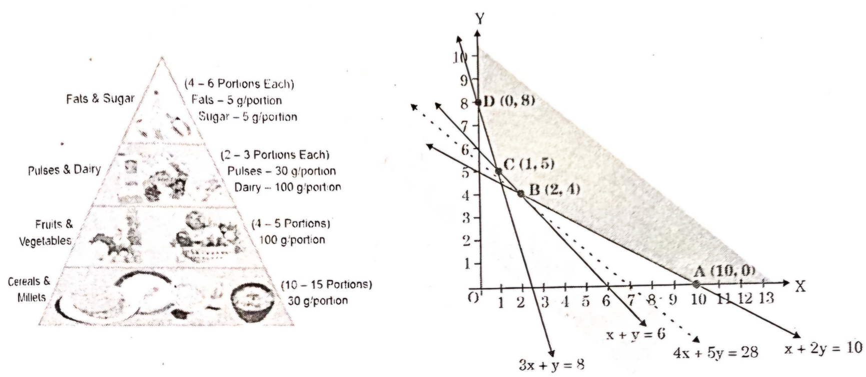

Figure 8.1: 1

of foods,food $X(xkg)$ and fodd $Y(ykg)$ which are available at the rate of $|16/kg$ and $|20/kg$ respectively. The feasible region satisfying the constraints is shown in the graph.

On the basis of the above information, answer the following questions:

(i) Identify and write all the constraints which determine the given feasible region in the above graph.

(ii) If the objective is to minimize cost $Z = 16x + 20y$, find the values of x and y at which cost is minimum. Also, find minimum cost assuming that minimum cost is possiblr for the given unbounded region.

Chapter 9

Geometry

9.1. 2024

9.1.1. 12

1. The coordinates of the foot of the perpendicular drawn from the point $(0, 1, 2)$ on th x-axis are given by:

   (a) $(1, 0, 0)$

   (b) $(2, 0, 0)$

   (c) $(\sqrt{5}, 0, 0)$

   (d) $(0, 0, 0)$

2. If a line makes an angle of 30° with the positive direction of x-axis, 120° with the positive direction of y-axis, then the angle which it makes with the positive direction of z-axis is:

   (a) 90°

   (b) 120°

   (c) 60°

(d) 0°

3. Find the equation of the line which bisects the line segment joining points $A(2,3,4)$ and $B(4,5,8)$ and is perpendicular to the lines $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$

4. If A_1 denotes the area of region bounded by $y^2 = 4x$ and x-axis in the first quadrant and A_2 denotes the area of region bounded by $y^2 = 4x$, $x = 4$, find $A_1 : A_2$.

Chapter 10

Discrete

Chapter 11

Number Systems

Chapter 12

Differentiation

12.1. 2024

12.1.1. 12

1. The derivative of $\tan^{-1}(x^2)$ w.r.t. x is:

   (a) $\frac{x}{1+x^4}$

   (b) $\frac{2x}{1+x^4}$

   (c) $-\frac{2x}{1+x^4}$

   (d) $\frac{1}{1+x^4}$

2. Overspeeding increases fuel consumption and decreases fuel economy as a result of tyre rolling friction and air resistance. While vehicles reach optimal fuel economy at different speeds, fuel mileage usualy decreases rapidly at speeds above 80km/h.

   The relation btween fuel consumption F(l/100 km) and speed V(km/h) under some constraints is given as $F = \frac{V^2}{500} - \frac{V}{4} + 14$.

   On the basis of the above information, answer the following questions:

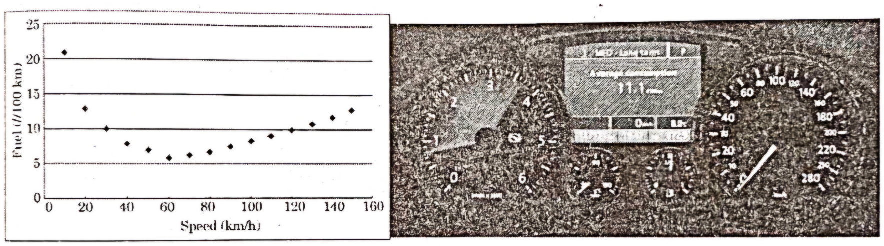

Figure 12.1: 1

(i) Find F, when $V = 40km/h$.

(ii) Find $\frac{dF}{dV}$.

(iii) Find he speed V for which fuel consumption F is minimum.

(iv) Find the quantity of fuel required to travel 600 km at the speed V at which $\frac{dF}{dV} = -0.01$.

1. The differential equation $\frac{dy}{dx} = F(x, y)$ will not be a homogeneous differential equation, if $F(x, y)$ is:

   (a) $\cos x - \sin(\frac{y}{x})$

   (b) $\frac{y}{x}$

   (c) $\frac{x^2+y^2}{xy}$

   (d) $\cos^2(\frac{x}{y})$

2. The degree of the differential equation $(y'')^2 + (y')^3 = x \sin(y')^3$ is:

   (a) 1

   (b) 2

   (c) 3

(d) not defined

3. If $y =$ cosec $(\cot^{-1} x)$, then prove that $\sqrt{1+x^2}\frac{dy}{dx} - x = 0$.

4. If $x = e^{\cos 3t}$ and $y = e^{\sin 3t}$, prove that $\frac{dy}{dx} = -\frac{ylogx}{xlogy}$.

5. Show that: $\frac{d}{dx}(|x|) = \frac{x}{|x|}, x \neq 0$.

6. Find the particular solution of the differetial equation given by $2xy + y^2 - 2x^2\frac{dy}{dx} = 0; y = 2$, when x=1.

7. Find the general solution of the differential equation:

$$ydx = (x + 2y^2)\text{dy}$$

Chapter 13

Integration

13.1. 2024

13.1.1. 12

1. $\int\limits_0^{\frac{\pi}{2}} \frac{\sin x - \cos x}{1+\sin x \cos x} dx$ is equal to:

   (a) π

   (b) $Zero(0)$

   (c) $\int\limits_0^{\frac{\pi}{2}} \frac{2\sin x}{1+\sin x \cos x} dx$

   (d) $\frac{\pi^2}{4}$

2. Find : $\int \frac{e^{4x}-1}{e^{4x}+1} dx$

3. Evaluate:

   $\int\limits_2 -2\sqrt{\frac{2-x}{z+x}} dx$

4. Find:

   $\int \frac{1}{x[(logx)^2 - 3logx - 4]} dx$

5. Find: $\int x^2 \cdot \sin^{-1}(x^{\frac{3}{2}}) dx$

Chapter 14

Functions

Chapter 15

Matrices

15.1. 2024

15.1.1. 12

1. If the sum of all the elements of 3×3 scalar matrix is 9, then the product of all elements is:

   (a) 0

   (b) 9

   (c) 27

   (d) 729

2. If $\begin{vmatrix} -a & b & c \\ a & -b & c \\ a & b & -c \end{vmatrix} = kabc$, then the value of k is:

   (a) 0

   (b) 1

   (c) 2

(d) 4

3. If $A = [a_{ij}]$ be a 3×3 where $a_{ij} = i - 3j$,then which of the following is false?

   (a) $a_{11} < 0$

   (b) $a_{12} + a_{21} = -6$

   (c) $a_{13} > a_{31}$

   (d) $a_{31} = 0$

4. If $F(x) = \begin{pmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 0 \end{pmatrix}$ and $[F(x)]^2$=$F(kx)$, then the value of k is:

   (a) 1

   (b) 2

   (c) 0

   (d) -2

5. Assertion (A): For any symmetric matrix A, $B'AB$ is a skew-symmetric matrix.

   Reason (R): A square matrix P is kew-symmetric if $P' = -P$

   (a) Both Assertion and Reason are true, and Reason is the correct explaination of Assertion.

   (b) Both Assertion and Reason are true, but Reason is not the correct explaination of Assertion.

(c) Assertion is true, but Reason is false.

(d) Assertion is false, but Reason is true.

6. Solve the following system of equations,using matrices:

$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4,\ \frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1,\ \frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$ where $x, y, z \neq 0$

7. If $A = \begin{pmatrix} 1 & \cot x \\ -\cot x & 1 \end{pmatrix}$,then show that $A'A^{-}1 = \begin{pmatrix} -\cos 2x & -\sin 2x \\ \sin 2x & -\cos 2x \end{pmatrix}$

Chapter 16

Trignometry

16.1. 2024

16.1.1. 12

1. Find the value of $\tan^{-1}(-\frac{1}{\sqrt{3}}) + \cot^{-1}(\frac{1}{\sqrt{3}}) + \tan^{-1}[\sin(-\frac{\pi}{2})]$