CBSE MATH

Made Simple

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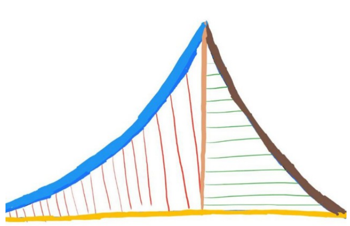

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Introduction

This book links high school coordinate geometry to linear algebra and matrix analysis through solved problems.

Chapter 1

Vectors

1.1. 2024

1.1.1. 12

1. For any two vectors $\overrightarrow{a}$ and $\overrightarrow{b}$, which of the following statements is always true?

   (a) $\overrightarrow{a} \cdot \overrightarrow{b} \geq |\overrightarrow{a}||\overrightarrow{b}|$

   (b) $\overrightarrow{a} \cdot \overrightarrow{b} = |\overrightarrow{a}||\overrightarrow{b}|$

   (c) $\overrightarrow{a} \cdot \overrightarrow{b} \leq |\overrightarrow{a}||\overrightarrow{b}|$

   (d) $\overrightarrow{a} \cdot \overrightarrow{b} < |\overrightarrow{a}||\overrightarrow{b}|$

2. The unit vector perpendicular to both vectors $\hat{i} + \hat{k}$ and $\hat{i} - \hat{k}$ is:

   (a) $2\hat{j}$

   (b) $\hat{j}$

   (c) $\frac{\hat{i}-\hat{k}}{\sqrt{2}}$

   (d) $\frac{\hat{i}+\hat{k}}{\sqrt{2}}$

3. Direction ratios of a vector parallel to line $\frac{x-1}{2} = -y = \frac{2z+1}{6}$ are :

   (a) $2, -1, 6$

   (b) $2, 1, 6$

   (c) $2, 1, 3$

   (d) $2, -1, 3$

4. Assertion (A): For two non-zero vectors $\overrightarrow{a}$ and $\overrightarrow{b}$, $\overrightarrow{a} \cdot \overrightarrow{b} = \overrightarrow{b} \cdot \overrightarrow{a}$

   Reason (R): For two non-zero vectors $\overrightarrow{a}$ and $\overrightarrow{b}$, $\overrightarrow{a} \times \overrightarrow{b} = \overrightarrow{b} \times \overrightarrow{a}$

   (a) Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explaination of Assertion (A).

   (b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explaination of Assertion (A)

   (c) Assertion (A) is true, but Reason (R) is false

   (d) Assertion (A) is false, but Reason (R) is true

5. The position vectors of vertices of $\triangle\ ABC$ are $A(2\hat{i} - \hat{j} + \hat{k})$, $B(\hat{i} - 3\hat{j} - 5\hat{k})$ and $C(3\hat{i} - 4\hat{j} - 4\hat{k})$. Find all the angles of $\triangle\ ABC$.

Chapter 2

Linear Forms

Chapter 3

Circles

Chapter 4

Intersection of Conics

Chapter 5

Probability

5.1. 2024

5.1.1. 12

1. Let E be an event of a sample space S of an experiment, then $P(S|E) =$

   (a) $P(S \cap E)$

   (b) $P(E)$

   (c) 1

   (d) 0

2. A pair of dice is thrown simultaneously. If X denotes the absolute difference of the numbers appearing on top of the dice, then find the probability distribution of X.

3. Airplanes are by far the safest mode of transportation when the number of transported passengers are measured against personal injuries and fatality tools. Previous records state that the probability of an airplane crash is 0.00001%. Further, there are 95% chances that there

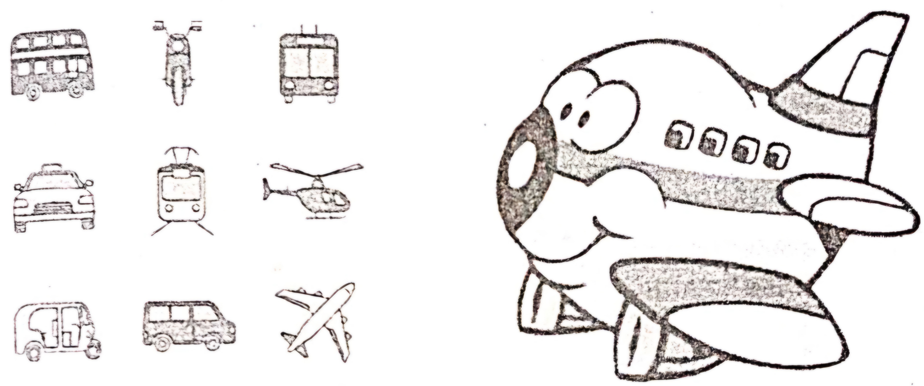

Figure 5.1: 1

will be survivors after a plane crash. Assume that in case of no crash, all travellers survive.

Let E_1 be the event that there is a plane crash and E_2 be the event that there is no crash. Let A be the event that passengers survive after the journey.

On the basis of the above information, answer the following questions:

(i) Find the probability that the airplane will not crash.

(ii) Find $P(A|E_1) + P(A|E_2)$.

(iii) Find $P(A)$

(iv) Find $P(E_2|A)$.

5.2. 2006

5.2.1. 10

1. A card is drawn at random from a well-shuffled deck of playing cards.

   Find the probability that the card drawn is

   (a) a card of spades or an ace

   (b) a red king

   (c) neither a king nor a queen

   (d) either a king or a queen.

Chapter 6

Construction

Chapter 7

Optimization

Chapter 8

Algebra

8.1. 2024

8.1.1. 12

1. let $f : R_+ \to [-5, \infty)$ be defined as $f(x) = 9x^2 + 6x - 5$ where R_+ is the set of all non-negative real numbers, then f is:

   (a) one-one

   (b) onto

   (c) bijective

   (d) neither one-one nor onto

2. The number of points of discontinuity of $f(x) = \begin{cases} |x| + 3, & if x \leq -3 \\ -2x, & if -3 < x < 3 \\ 6x + 2, & if x \geq 3 \end{cases}$ is:

   (a) 0

   (b) 1

(c) 2

(d) infinite

3. The function $f(x) = x^3 - 3x^2 + 12x - 18$ is:

   (a) strictly decreasing on R

   (b) strictly increasing on R

   (c) neither strictly increasing nor strictly decreasing on R

   (d) strictly decreasing on $(-\infty, 0)$

4. Find the domain of the function $f(x) = \sin^{-1}(x^2 - 4$. Also, find its range.

5. If $f(x) = |\tan 2x|$, then find the value of $f'(x)$ at $x = \frac{\pi}{3}$.

6. If M and m denote the local maximum and local minimum values of the function $f(x) = x + \frac{1}{x}(x \neq 0)$ respectively, find the value of $(M - m)$.

7. Show that $f(x) = e^x - e^{-x} + x - \tan^{-1} x$ is strictly increasing in its domain.

8. Show that a function $f : R \to R$ defined bt $f(x) = \frac{2x}{1+x^2}$ is neither one-one nor onto. Further, find set A so that the given function $f : R \to A$ becomes an onto function.

9. A relation R is defined on $N \times N$ (where N is the set f natural numbers) as:

   $$(a, b)R(c, d) \leftrightarrow a - c = b - d$$

   Show that R is an equivalence relation.

10. The month of September is celebrtaed as the Rashtriya Poshan Maah across the country. Following a healthy and well-balanced diet is crucial in order to supply the body with the proper nutrients it needs. A balanced diet also keeps us mentally fit and promotes improved level of energy.

A dietician wishes to minimize the cost of a diet involving two types

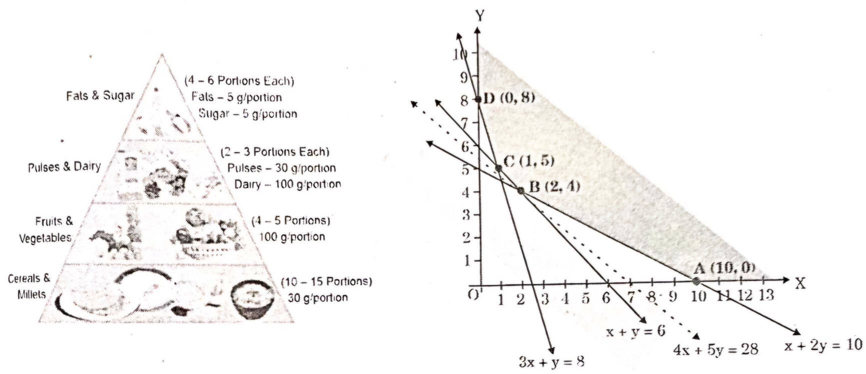

Figure 8.1: 1

of foods,food $X(xkg)$ and fodd $Y(ykg)$ which are available at the rate of $|16/kg$ and $|20/kg$ respectively. The feasible region satisfying the constraints is shown in the graph.

On the basis of the above information, answer the following questions:

(i) Identify and write all the constraints which determine the given feasible region in the above graph.

(ii) If the objective is to minimize cost $Z = 16x + 20y$, find the values of x and y at which cost is minimum. Also, find minimum cost assuming that minimum cost is possiblr for the given unbounded region.

8.2. 2006

8.2.1. 10

1. Solve the system of equations:

   $\frac{bx}{a} - \frac{ay}{b} + a + b = 0$ and $bx - ay + 2ab = 0.$

2. Given that:

   $P = \frac{x+2y}{x+y} + \frac{x}{y}$,Q= $\frac{x+y}{x-y} - \frac{x-y}{x+y}$ and R=$\frac{x+2y}{x+y}$ - $\frac{x}{x+y}$

3. If $(x+2)(x-3)$ is the HCF of the polynomials $p(x) = (x^2+x-2)(3x^2-8x+c)$ and $q(x) = (x^2+x-12)(2x^2+x+b)$,

   find the values of c and b.

4. Using the quadratic formula, solve the equation: $A^2b^2x^2-(4b^4-3a^4)x-12a^2b^2 = 0.$

5. A household article is available for ₹970 cash or ₹210 cash down payment followed by three equal monthly installments. If the rate of interest charged under the installment plan is 16% per annum, find the amount of each installment

6. A man borrows ₹25,200 from a finance company and has to repay it in two equal annual installments. If the interest is charged at the rate of 10% per annum compounded annually, calculate the amount of each

installment.

7. The speed of a boat in still water is 11 km/hr. It can go 12 km upstream and return downstream to the original point in 2 hours 45 minutes. Find the speed of the stream.

8. A bucket made up of a metal sheet is in the form of a frustum of a cone. Its depth is 24 cm and the diameters of the top and bottom are 30 cm and 10 cm respectively. Find the cost of milk which can completely fill the bucket at the rate of ₹20 per liter and the cost of the metal sheet used, if it costs ₹10 per 100 cm^2. ($Use \pi = 3.14$)

9. Mrs. Ruchi's salary is Rs.$32,250$ per month exclusive of HRA. She donates ₹12,000 to Prime Minister's Relief Fund (100% exemption). She also donates ₹6,000 to a school and gets a relief of 50% on this donation. She contributes ₹5,000 per month towards her Provident Fund. She pays a quarterly premium of Rs.$2,500$ towards her LIC policy and invests ₹25,000 in NSCs. If ₹2,700 is the tax deducted each month from her salary for 11 months, find the tax deducted from her salary in the last month of the year.

Income Tax Rates:

Slab	Tax Rate
Up to Rs.$1,35,000$	No tax
From Rs.$1,35,001$ to Rs.$1,50,000$	10% of the taxable income above Rs.$1,35,000$
From Rs.$1,50,001$ to Rs.$2,50,000$	Rs.$1,500 + 20\%$ of the income exceeding Rs.$1,50,000$
Rs.$2,50,001$ and above	Rs.$21,500 + 30\%$ of the amount exceeding Rs.$2,50,000$

Education Cess: 2% of the income tax

Chapter 9

Geometry

9.1. 2024

9.1.1. 12

1. The coordinates of the foot of the perpendicular drawn from the point $(0, 1, 2)$ on th x-axis are given by:

   (a) $(1, 0, 0)$

   (b) $(2, 0, 0)$

   (c) $(\sqrt{5}, 0, 0)$

   (d) $(0, 0, 0)$

2. If a line makes an angle of 30° with the positive direction of x-axis, 120° with the positive direction of y-axis, then the angle which it makes with the positive direction of z-axis is:

   (a) 90°

   (b) 120°

   (c) 60°

(d) $0°$

3. Find the equation of the line which bisects the line segment joining points $A(2,3,4)$ and $B(4,5,8)$ and is perpendicular to the lines $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$

4. If A_1 denotes the area of region bounded by $y^2 = 4x$ and x-axis in the first quadrant and A_2 denotes the area of region bounded by $y^2 = 4x$, $x = 4$, find $A_1 : A_2$.

9.2. 2006

9.2.1. 10

1. In Figure 1, $\angle BAC = 90°$. $AD \parallel BC$. Prove that $AB^2 + CD^2 = BD^2 + AC^2$.

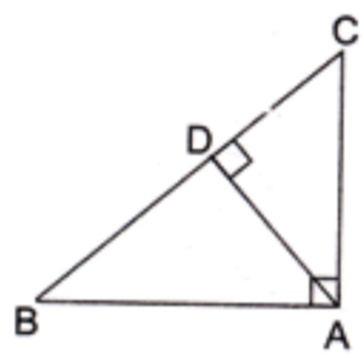

Figure 9.1: 1

2. In Figure 2, $PT = 6$ cm, $AR = 5$ cm. Find the length of PA.

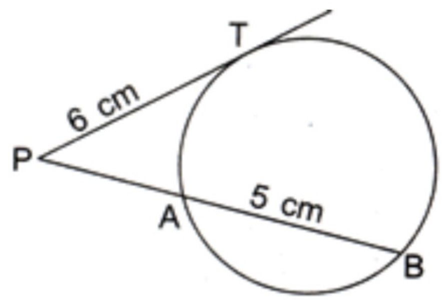

Figure 9.2: 2

3. Draw the graphs of the following equations: $3x-4y+6=0, 3x+y-9=0$ Also, determine the co-ordinates of the vertices of the triangle formed by these lines and the x axis.

4. A solid is in the form of a right circular cylinder with hemispherical ends. The total height of the solid is 58 cm and the diameter of the cylinder is 28cm. Find the total surface area of the solid $\pi \approx \frac{22}{7}$

5. . Construct a triangle ABC in which $BC = 7$ cm, and median $AD = 5$ cm, $\angle A = 60°$ Write the steps of construction also.

   .

6. Show that the points A(6, 2), B(2, 1), C(1, 5) and D(5, 6) are the vertices of a square

7. Find the value of p for which the points (- 5, 1), (1, p) and $4, -2$ are collinear.

8. . Prove that in a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

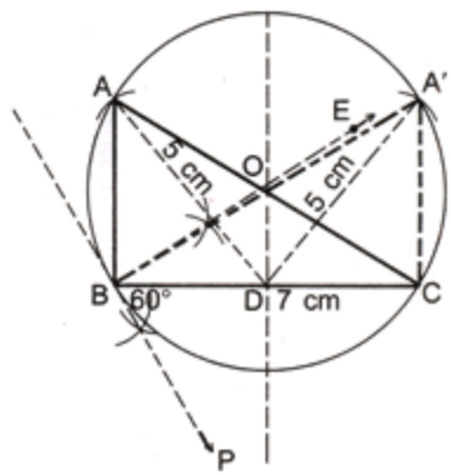

Figure 9.3: 3

Makeing ue of the above, prove the following:

in fig:4, $ABCD$ is a fig:4 rhombus. prove that $4AB^2 = AC^2 + BD^2$.

9. Prove that I a line touch a circle and from the point of contact a chord is drawn, the angles which this chord makes with the given line are equal respectively to the angles formed in the corresponding alternate segments. Using the above, do the following:

   AB is a diameter and AC is a chord of a circle such that $\angle BAC = 30°$. The tangent at C intersects AR produced in a point I Prove that $BC = RD$.

10. A man standing on the deck of a ship, which is 10 m above the water level, observes the angle of elevation of the top of a hill as $60°$ and the angle of depression of the base of the hill as 300. Calculate the distance of the hill from the ship and the height of the hill.

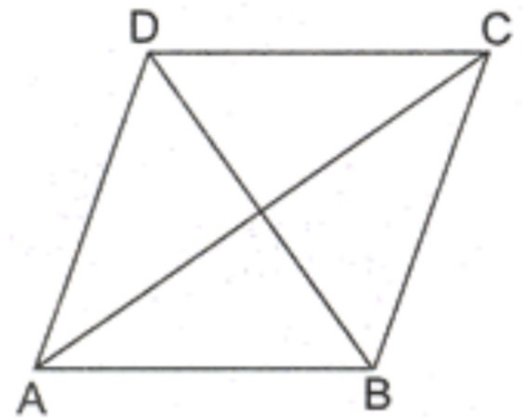

Figure 9.4: 4

11. From a window x meters high above the ground in a street, the angles of elevation and depression of the top and foot of the other house on the opposite side of the street are α and β respectively. Show that the height of the opposite house is $x(1 + \tan\alpha \cot\beta)$ meters.

Chapter 10

sequences

10.1. 2006

10.1.1. 10

1. The 5^{th} term of an Arithmetic Progression (A.P.) is 26 and the 10th term is 51. Determine the 15^{th} term of the A.P.

2. Find the sum of all the natural numbers less than 100 which are divisible by 6.

Chapter 11

Datahandling

11.1. 2006

11.1.1. 10

1. The following table shows the monthly expenditure of company. Draw apie chart for the data.

	Amount (in Rs.)
Wages	4800
Materials	3200
Taxation	2400
Adm. Expenditure	3000
Miscellaneous	1000

2. The Arithmetic Mean of the following frequency distribution is 47.

Determine the value of p.

Classes	Frequency
0 - 20	8
20 - 40	15
40 - 60	20
60 - 80	p
80 - 100	5

Chapter 12

Discrete

Chapter 13

Number Systems

Chapter 14

Differentiation

14.1. 2024

14.1.1. 12

1. The derivative of $\tan^{-1}(x^2)$ w.r.t. x is:

   (a) $\frac{x}{1+x^4}$

   (b) $\frac{2x}{1+x^4}$

   (c) $-\frac{2x}{1+x^4}$

   (d) $\frac{1}{1+x^4}$

2. Overspeeding increases fuel consumption and decreases fuel economy as a result of tyre rolling friction and air resistance. While vehicles reach optimal fuel economy at different speeds, fuel mileage usualy decreases rapidly at speeds above 80km/h.

   The relation btween fuel consumption F(l/100 km) and speed V(km/h) under some constraints is given as $F = \frac{V^2}{500} - \frac{V}{4} + 14$.

   On the basis of the above information, answer the following questions:

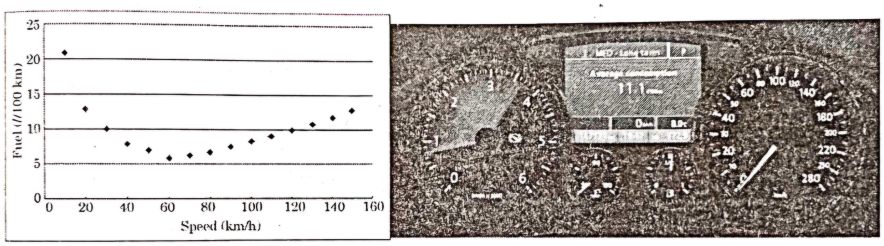

Figure 14.1: 1

(i) Find F, when $V = 40km/h$.

(ii) Find $\frac{dF}{dV}$.

(iii) Find he speed V for which fuel consumption F is minimum.

(iv) Find the quantity of fuel required to travel 600 km at the speed V at which $\frac{dF}{dV} = -0.01$.

1. The differential equation $\frac{dy}{dx} = F(x, y)$ will not be a homogeneous differential equation, if $F(x, y)$ is:

   (a) $\cos x - \sin(\frac{y}{x})$

   (b) $\frac{y}{x}$

   (c) $\frac{x^2+y^2}{xy}$

   (d) $\cos^2(\frac{x}{y})$

2. The degree of the differential equation $(y'')^2 + (y')^3 = x \sin(y')^3$ is:

   (a) 1

   (b) 2

   (c) 3

(d) not defined

3. If $y = \operatorname{cosec}(\cot^{-1} x)$, then prove that $\sqrt{1+x^2}\frac{dy}{dx} - x = 0$.

4. If $x = e^{\cos 3t}$ and $y = e^{\sin 3t}$, prove that $\frac{dy}{dx} = -\frac{ylogx}{xlogy}$.

5. Show that: $\frac{d}{dx}(|x|) = \frac{x}{|x|}, x \neq 0$.

6. Find the particular solution of the differetial equation given by $2xy + y^2 - 2x^2\frac{dy}{dx} = 0; y = 2$, when x=1.

7. Find the general solution of the differential equation:

$$ydx = (x + 2y^2)\text{dy}$$

Chapter 15

Integration

15.1. 2024

15.1.1. 12

1. $\int\limits_{0}^{\frac{\pi}{2}} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx$ is equal to:

   (a) π

   (b) $Zero(0)$

   (c) $\int\limits_{0}^{\frac{\pi}{2}} \frac{2 \sin x}{1 + \sin x \cos x} dx$

   (d) $\frac{\pi^2}{4}$

2. Find : $\int \frac{e^{4x} - 1}{e^{4x} + 1} dx$

3. Evaluate:

   $$\int\limits_{2} -2\sqrt{\frac{2-x}{z+x}} dx$$

4. Find:

   $$\int \frac{1}{x[(logx)^2 - 3logx - 4]} dx$$

5. Find: $\int x^2 \cdot \sin^{-1}(x^{\frac{3}{2}}) dx$

Chapter 16

Functions

Chapter 17

Matrices

17.1. 2024

17.1.1. 12

1. If the sum of all the elements of 3×3 scalar matrix is 9, then the product of all elements is:

   (a) 0

   (b) 9

   (c) 27

   (d) 729

2. If $\begin{vmatrix} -a & b & c \\ a & -b & c \\ a & b & -c \end{vmatrix} = kabc$, then the value of k is:

   (a) 0

   (b) 1

   (c) 2

(d) 4

3. If $A = [a_{ij}]$ be a 3×3 where $a_{ij} = i - 3j$,then which of the following is false?

   (a) $a_{11} < 0$

   (b) $a_{12} + a_{21} = -6$

   (c) $a_{13} > a_{31}$

   (d) $a_{31} = 0$

4. If $F(x) = \begin{pmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 0 \end{pmatrix}$ and $[F(x)]^2$=$F(kx)$, then the value of k is:

   (a) 1

   (b) 2

   (c) 0

   (d) -2

5. Assertion (A): For any symmetric matrix A, $B'AB$ is a skew-symmetric matrix.

   Reason (R): A square matrix P is kew-symmetric if $P' = -P$

   (a) Both Assertion and Reason are true, and Reason is the correct explaination of Assertion.

   (b) Both Assertion and Reason are true, but Reason is not the correct explaination of Assertion.

(c) Assertion is true, but Reason is false.

(d) Assertion is false, but Reason is true.

6. Solve the following system of equations,using matrices:

$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4,\ \frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1,\ \frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$ where $x, y, z \neq 0$

7. If $A = \begin{pmatrix} 1 & \cot x \\ -\cot x & 1 \end{pmatrix}$,then show that $A'A^{-}1 = \begin{pmatrix} -\cos 2x & -\sin 2x \\ \sin 2x & -\cos 2x \end{pmatrix}$

Chapter 18

Trignometry

18.1. 2024

18.1.1. 12

1. Find the value of $\tan^{-1}(-\frac{1}{\sqrt{3}}) + \cot^{-1}(\frac{1}{\sqrt{3}}) + \tan^{-1}[\sin(-\frac{\pi}{2})]$