SC107- Calculus

Home Work 11

Week 12: November 23, 2017

Tutorial Discussion Week: November 23, 2017

Tutors: Krishna Gopal Benerjee and Dixita Limbachiya

(1) Verify that the following functions (explicit or implicit) are solution of the corresponding differential equations.

(a). $y^2 = e^{2x} + c \qquad yy' = e^{2x}$

(b). $y = ce^{kx} \qquad y' = ky$

(c). $y = c_1 \sin 2x + c_2 \cos 2x \qquad y'' = -4y$

(d). $y = c_1 e^{2x} + c_2 e^{-2x} \qquad y'' = 4y$

(2) Find the general solution.

(a). $xy' = 1$

(b). $y' = xe^{x^2}$

(c). $(1 + x^2)dy + (1 + y^2)dx = 0$

(d). $y \log y dx - x dy = 0$

(3) Show that

$$y = xe^{x^2} \int_0^x e^{-t^2} dt$$

is a solution of

$$y' = 2xy + 1$$

(4)Verify that the following equations are homogeneous and solve them.

(a). $xy' = y + 2xe^{-\frac{y}{x}}$

(b). $xy' = \sqrt{x^2 + y^2}$

(5) Find the value of n for which each of the following equations is exact and solve the equation for that values of n.

(a). $(xy^2 + nx^2y)dx + (x^3 + x^2y)dy = 0$

(b). $(x + ye^{2xy})dx + (nxe^{2xy})dy = 0$

(6) Show that if $\frac{\left(\frac{\partial M}{\partial y} - \frac{\partial n}{\partial x}\right)}{N}$ is a function of $g(x)$ then the integrating factor

$$\mu = e^{\int g(x)dx}$$

(7) Solve each of the following equations by finding an integrating factor.

(a). $e^x dx + (e^x \cot y + 2y \csc y)dy = 0$

(b). $ydx + (x - 2x^2y^3)dy = 0$

(c). $(x + 3y^2)dx + 2xydy = 0$

(8) The equation $\frac{dy}{dx} + p(x)y = Q(x)y^n$ which is known as Bernoullis equation is linear when $n = 0$ *or* 1. Show that it can be reduced to a linear equation for any other value of n by the change of variable $z = y^{1-n}$ and apply this method to solve the following equation.

$$xy' + y = x^4y^3$$

(9) Solve the following equations.

(a). $x^2y'' = 2xy' + (y')^2$

(b). $yy'' - (y')^2 = 0$

.....