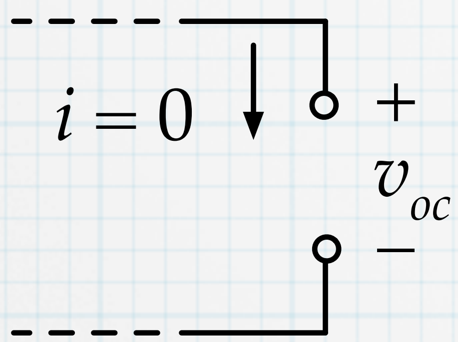
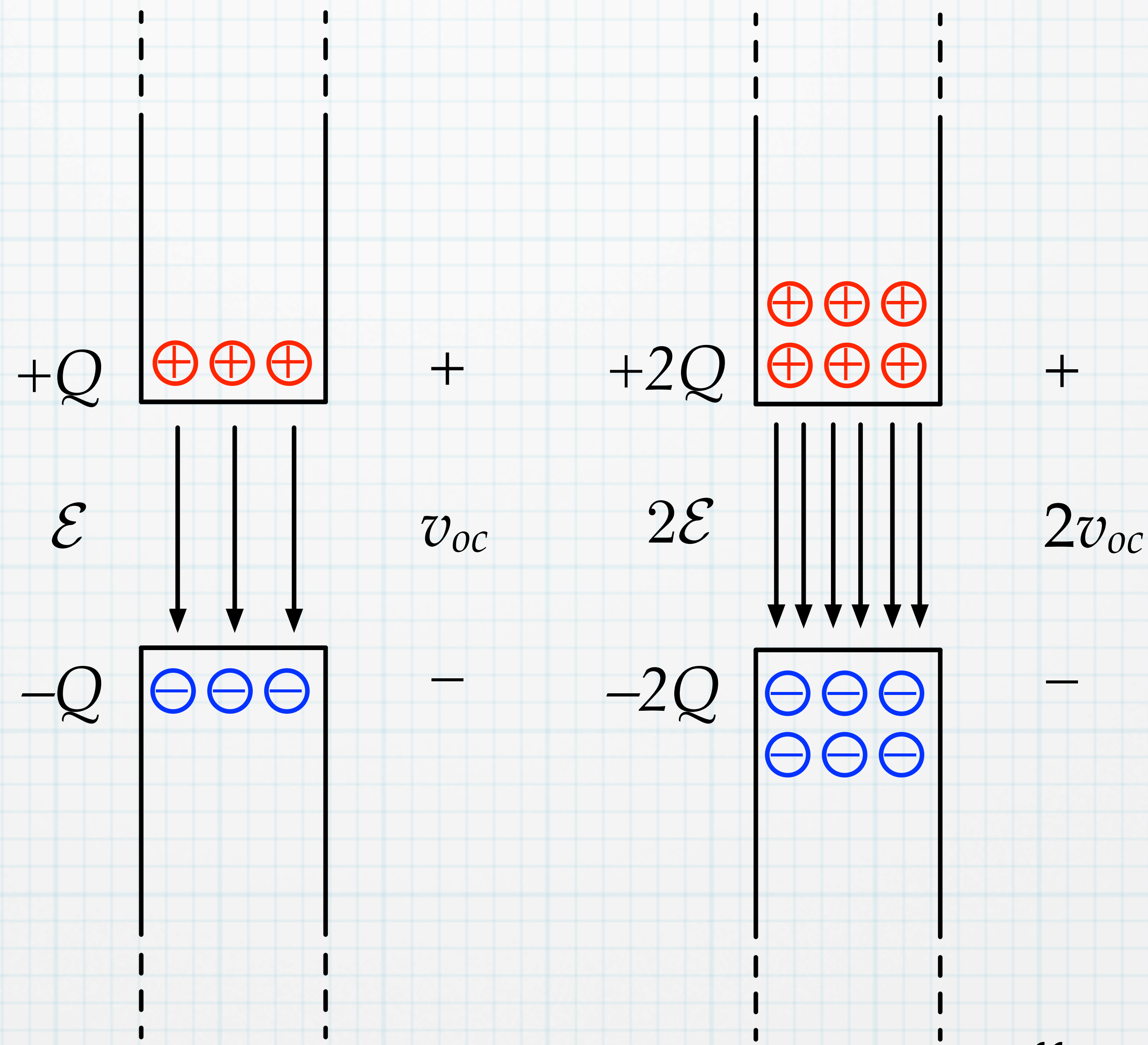
Capacitors

Consider an open circuit:



double the voltage



$$Q \propto v_{oc}$$

$$\mathcal{E} \propto v_{oc}$$

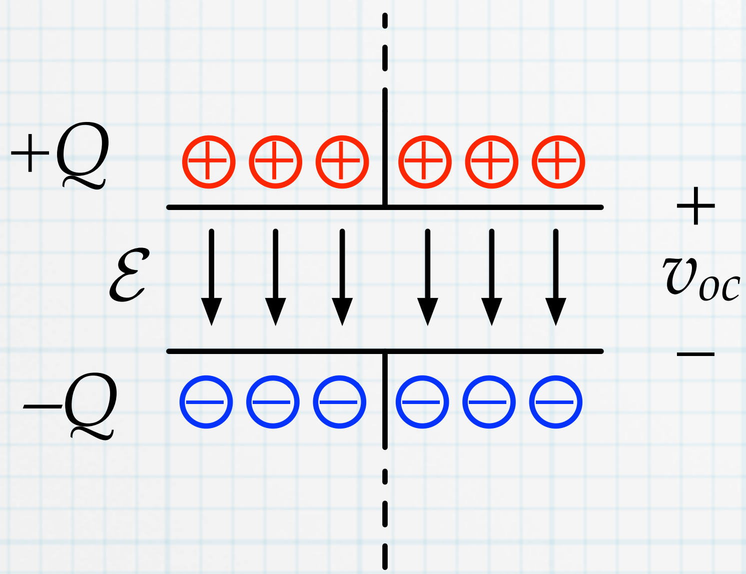
There is some energy "stored".

There was a bit of current flow when the voltage changed.

Effect is weak for dangling wires. (But not zero!)

Capacitance

Change the geometry – have parallel plates with area A



same voltage

- much more charge
- much more electric field
- much more energy stored

$$Q = CV \qquad Q = \epsilon \mathcal{E} A \qquad \mathcal{E} = \frac{V}{d}$$

$C \rightarrow$ capacitance

farads (F) = C / V

increase charge with better dielectric material and more area.

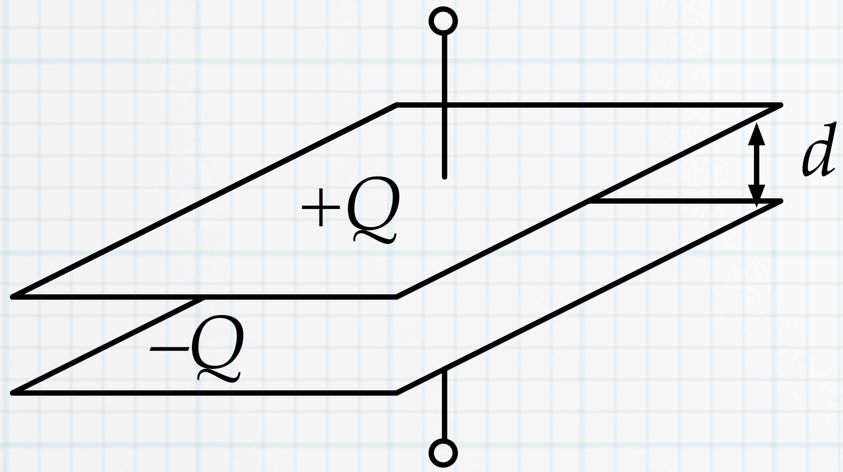
increase field (and hence Q) by moving plates closer together

air: $\varepsilon_o = 8.85\text{x}10^{-12}$ F/m

other materials: $\varepsilon = \varepsilon_r \varepsilon_o$

relative dielectric: ε_r = constant

Parallel-plate capacitor



2 plates, each with area A.

$$C = \frac{\epsilon A}{d}$$

Example: A = 1 cm^2, d = 0.001 cm, air dielectric

$$C = \frac{\left(8.8 \times 10^{-14}\mathrm{F/cm}\right)\left(1\mathrm{cm}^2\right)}{0.001\mathrm{cm}} = 8.85 \times 10^{-11}\mathrm{F} = 88.5\mathrm{pF}$$

Wow. Very small value, and it is already a fairly large area plate.

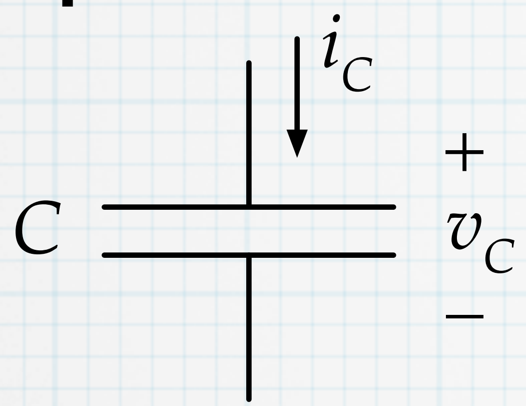
Higher values?

- higher dielectric material between the electrodes
- thinner dielectric
- winding or stacking to get larger surface area into a smaller volume.

Other configurations are possible, but parallel-plate is most common.

Values range from 10 pF to 100 μF, (and higher). A 1-F capacitor is huge and quite rare.

Capacitor current



Note that passive sign convention.

At DC, $i_c = 0$. (It's just a fancy open circuit.)

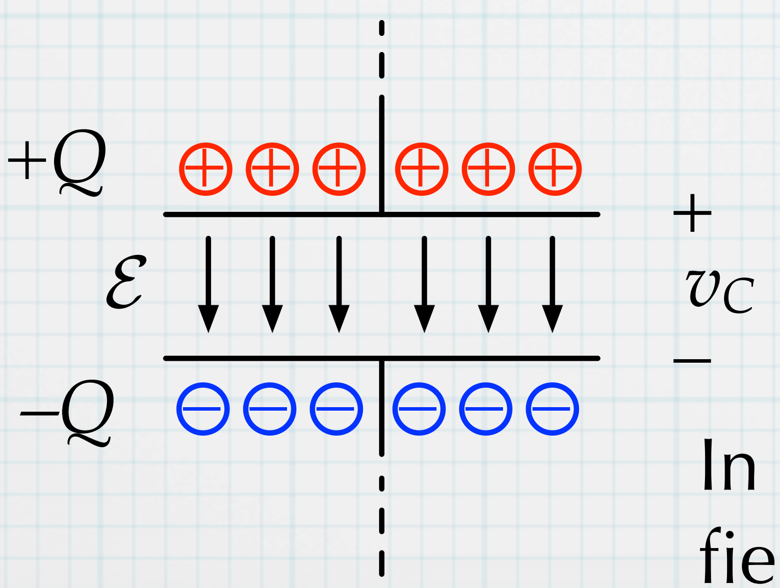
However, some current must flow when voltage is *changing*. Otherwise, the charge would not change.

$$Q = Cv_C$$

$$\boxed{\frac{dQ}{dt} = i_C = C\frac{dv_c}{dt}}$$

Current only flows when voltage is changing.

As current flows, the capacitor charge increases or decreases.

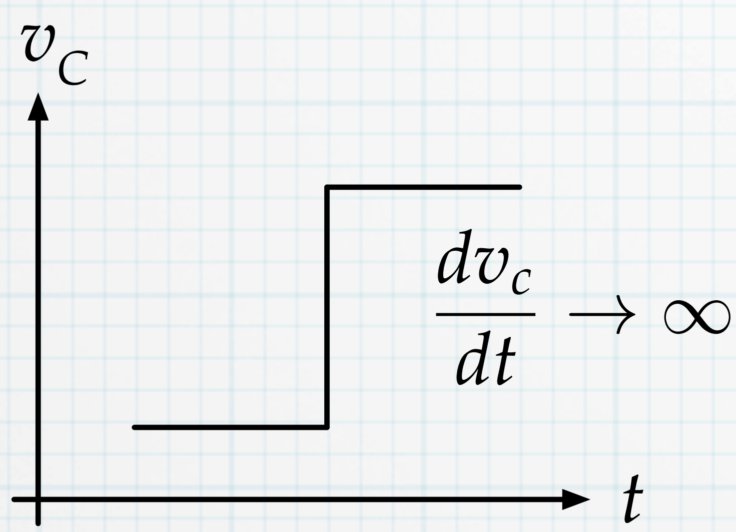


$$\frac{dv_C}{dt} \rightarrow \frac{d\mathcal{E}}{dt} \qquad \frac{d\mathcal{E}}{dt} \rightarrow \frac{dQ}{dt}$$

In Maxwell's equations, the current due to changing field is called *displacement* current.

$$i_C = C\frac{dv_c}{dt}$$

Capacitor voltage cannot change instantaneously.



infinite current!

can't happen.

Note, though, that current can change instantaneously.

also $$v_C\left(t\right) = \frac{1}{C}\int_0^t i_C\left(t'\right)dt' + v_C\left(0\right)$$

Capacitor energy

An energy storage device

- Charge the cap to some voltage. Charge (and energy) stays. Remove it later.
- Ideal capacitor dissipates no energy – no heat generated.
- Real capacitors do show some leakage. (Large resistor in parallel.) Usually negligible.

When charging a capacitor, the power being delivered is given by:

$$P_C(t) = v_C(t)\, i_C(t) = C v_C \frac{dv_C}{dt}$$

The energy delivered by the source, and hence the energy stored in the capacitor is (assuming $v_C = 0$ at $t = 0$ and $v_C(t_f) = V_C$.)

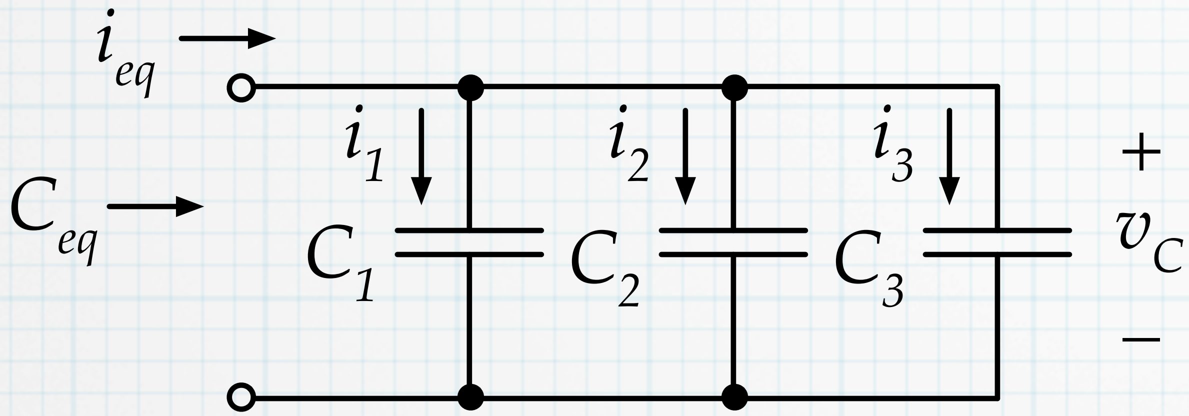
$$P_C(t)\, dt = C v_C dv_C$$

$$E = \int_0^{t_f} P_C(t)\, dt = C \int_0^{V_C} v_C dv_C$$

$$\boxed{E = \frac{1}{2} C V_C^2}$$

Combinations of capacitors

Parallel

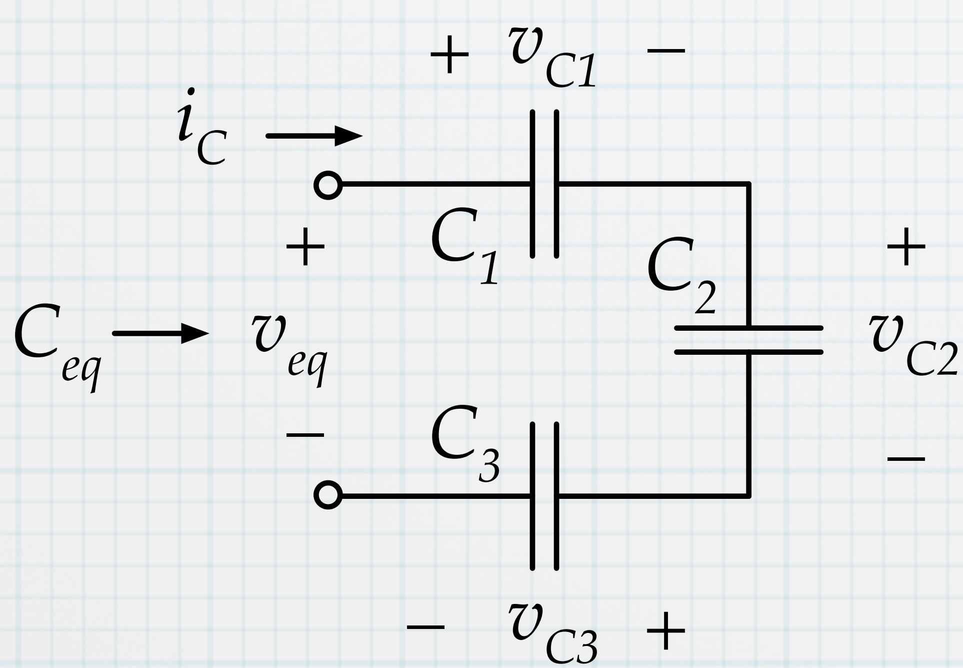


$$i_{eq} = i_1 + i_2 + i_3$$

$$C_{eq}\frac{dv_C}{dt} = C_1\frac{dv_C}{dt} + C_2\frac{dv_C}{dt} + C_3\frac{dv_C}{dt}$$

$$C_{eq} = C_1 + C_2 + C_3$$

Series



$$v_{eq} = v_{c1} + v_{c2} + v_{c3}$$

$$\frac{dv_{eq}}{dt} = \frac{dv_{c1}}{dt} + \frac{dv_{c2}}{dt} + \frac{dv_{c3}}{dt}$$

$$\frac{i_C}{C_{eq}} = \frac{i_C}{C_1} + \frac{i_C}{C_2} + \frac{i_C}{C_3}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Capacitors combinations are exactly opposite those of resistors.