SC107- Calculus

Home Work 12

Week 13: October 30, 2017

Tutorial Discussion Week: October 30, 2017

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Assume for problems 1 to 4, that if $y_1(x)$ and $y_2(x)$ are two solutions and neither is a constant multiple of the other, then $c_1y_1(x) + c_2y_2(x)$ is a general solution.

Q.(1) Answer the following.

(a). Verify that $y_1 = 1$ and $y_2 = x^2$ are solution of the reduced equation.

$$xy^{''} - y^{'} = 0,$$

and write down the general solution.

(b). Determine the value of a for which $y_p = ax^3$ is a particular solution of the complete equation

$$xy^{''} - y^{'} = 3x^2.$$

Use this solution and the result of part a to write down the general solution of this equation.

(c). Can you discover y_1,y_2 and y_p by inspection?

Q.(2) Verify that $y_1 = 1$ and $y_2 = \log x$ are solution of the equation $xy^{''} - y^{'} = 0$, and write down the general solution. Can you discover y_1 and y_2 by inspection?

Q.(3) Use inspection or experiment to find a particular solution for each of the following equations:

(a). $x^2y^{''} + x^2y^{'} + xy = 1$

(b). $y^{''} - 2y = \sin x$

(c). $y^{''} - 2y^{'} = 6$

Q.(4) If $y_1(x)$ and $y_2(x)$ are two solutions of equation $y^{''} + P(x)y^{'} + Q(x)y = 0$ on an interval [a,b] and have a common zero in this interval, show that one is a constant multiple of the other.

Q.(5) Find two linearly independent solution of $x^2y^{''} - 2y = 0$ on the interval [1,2] and determine the particular solution satisfying the initial conditions .

Q.(6) Answer the following.

(a). Use one (or both) of the methods (reduction of orders) to find all solutions of

$$y^{''} - (y^{'})^2 = 0.$$

(b). Verify that $y_1 = 1$ and $y_2 = \log x$ are linearly independent solution of the equation in part a on any interval to the right of the origin. Is $y = c_1 + c_2 \log x$ the general solution? If not, why not ?

Q.(7) Use the Wronskian to prove that two solutions of a homogeneous equation on an interval [a,b] are linearly dependent if

(a). they have a common zero x_0 in the interval (problem 4).

(b). they have maxima and minima at the same point x_0 in the interval.

Q.(8) Consider the two functions $f(x) = x^3$ and $g(x) = x^2 |x|$ on the interval [-1,1].

(a). Show that their Wronskian $W(f, g)$ vanishes identically.

(b). Show that f and g are not linearly dependent.

(c). Do (a) and (b) contradict Lemma 2 (See CCV Notes)? If not, why not?

Q.(9) The equation $xy^{''} + 3y' = 0$ has the obvious solution $y_1 = 1$. Find y_2 and the general solution

Q.(10) Find the solution of $y^{''} - xf(x)y' - f(x)y = 0$.

Q.(11) Find the general solution of each of the following equations:

(a). $y^{''} - 4y^{'} + 4y = 0$

(b). $4y^{''} - 12y^{'} + 9y = 0$

(c). $y'' - 9y' + 20y = 0$

Q.(12) In this problem we present another way of discovering the second linearly independent solution of $y'' + py' + qy = 0$ when the roots of the auxiliary equations are real and equal. If $m_1 \neq m_2$, verify that the differential equation

$$y'' - (m_1 + m_2)y' + m_1 m_2 y = 0 \text{ has}$$

$$y = \frac{e^{m_1 x} - e^{m_2 x}}{m_1 - m_2} \text{ as a solution.}$$

Q.(13) Use the principle of superposition to find the general solution of

$$y'' + 4y = 4\cos 2x + 6\cos x + 8x^2 - 4x.$$

Q.(14) Answer the following.

(a). Show that the method of variation of parameters applied to the equation

$$y'' + y = f(x). \text{ leads to a particular solution}$$

$$y_p(x) = \int_0^x f(t)\sin(x - t)dt.$$

(b). Find the similar formula for a particular solution of the equation $y'' + k^2 y = f(x)$ where k is a positive constant.

Q.(15) Find the particular solution of $y'' - 2y' + y = 2x$ first by inspection and then by variation of parameters

Q.(16) Find the general solution of $y'' - y = 0$.

Q.(17)Find a particular solution by successive integration of operator method.

$$y'' - y = e^{-x}.$$

Q.(18) Find a particular solution by series expansion of operator method.

$$4y'' + y = x^4.$$