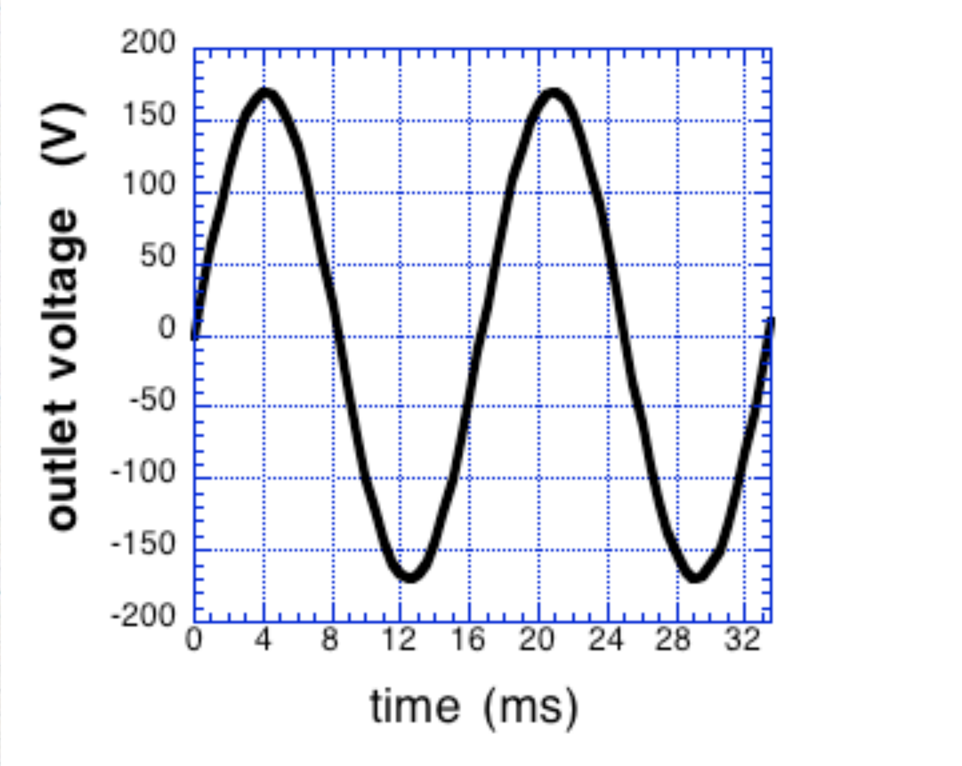
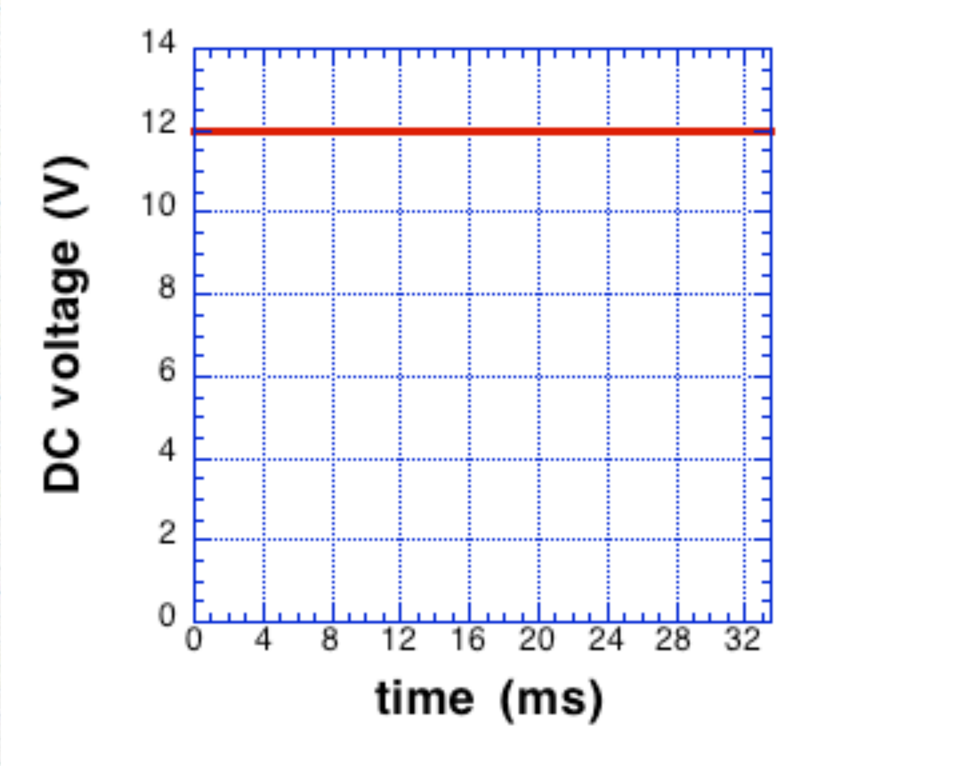
Rectifier circuits & DC power supplies

Goal: Generate the DC voltages – needed for most electronics – starting with the AC power that comes through the power line.



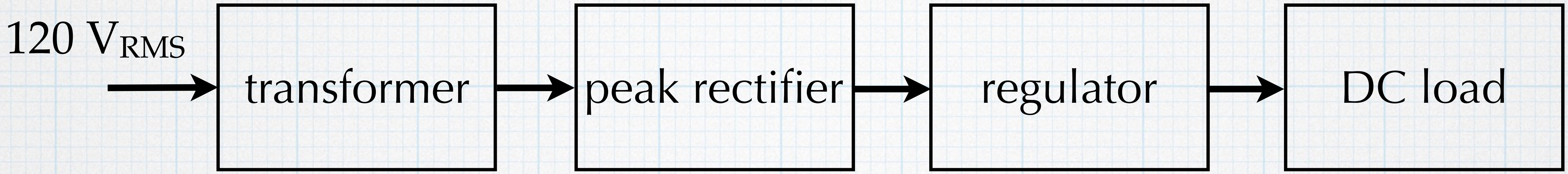
?



120 V_{RMS} $f = 60$ Hz ($T = 16.67$ ms)

$$V_{ac} = (170\text{V})\sin\left(\frac{2\pi}{T}t\right)$$

How to take time-varying voltage with an average value of 0 and turn it into a DC voltage?



transformer : reduces AC amplitude to something safe and manageable. V_{peak} from the transformer will be a few volts bigger than the desired DC voltage.

peak rectifier : breaks up the AC waveform and produces a $V_{DC} \approx V_{peak}$.

regulator : Refines the output of the rectifier. (optional)

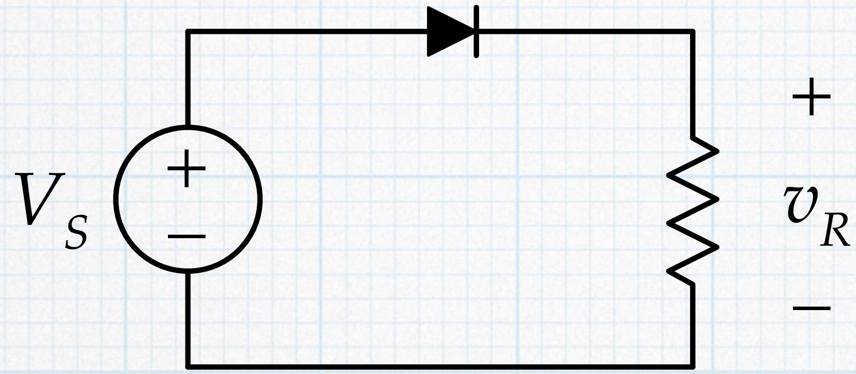
Issues:

- Total power
- Efficiency
- Cost
- Load regulation (Does V_{DC} change as the load draws different amounts of current?)
- Line regulation (Does V_{DC} change if the input AC amplitude changes?)

Half-wave rectifier

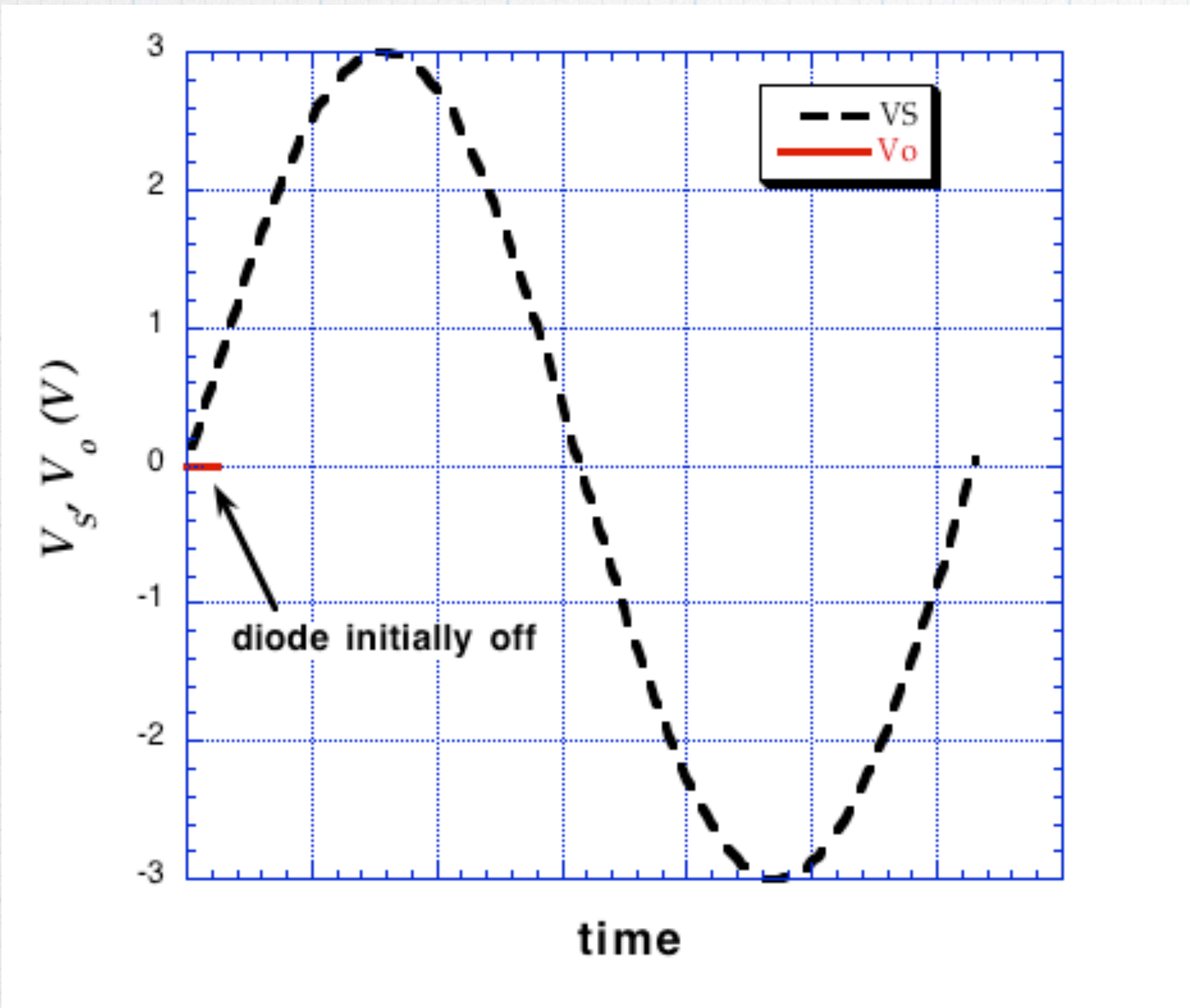
$$V_S(t) = V_p \sin\left(\frac{2\pi}{T}t\right)$$

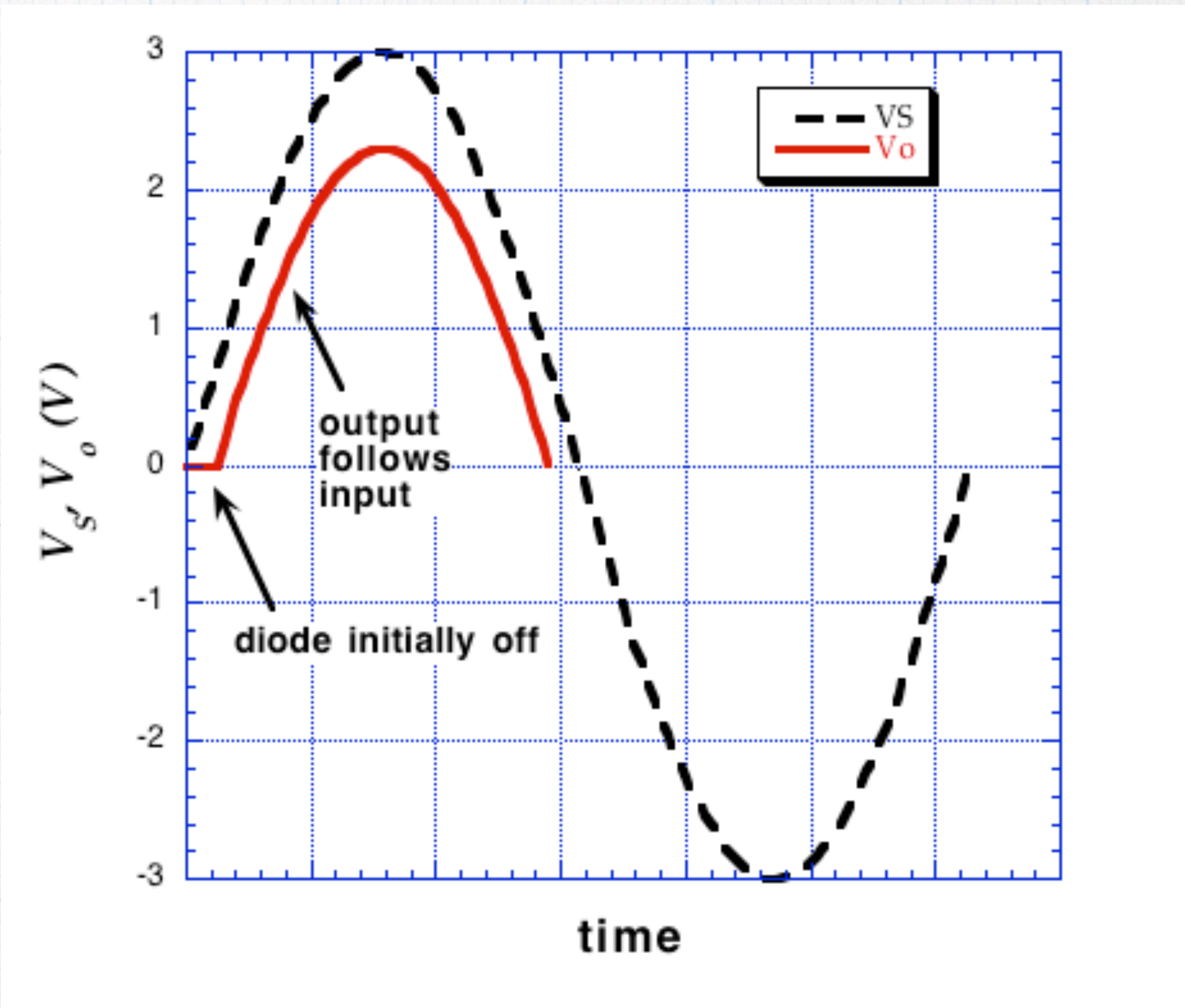
$V_p = 3$ V.



Resistor represent a load.

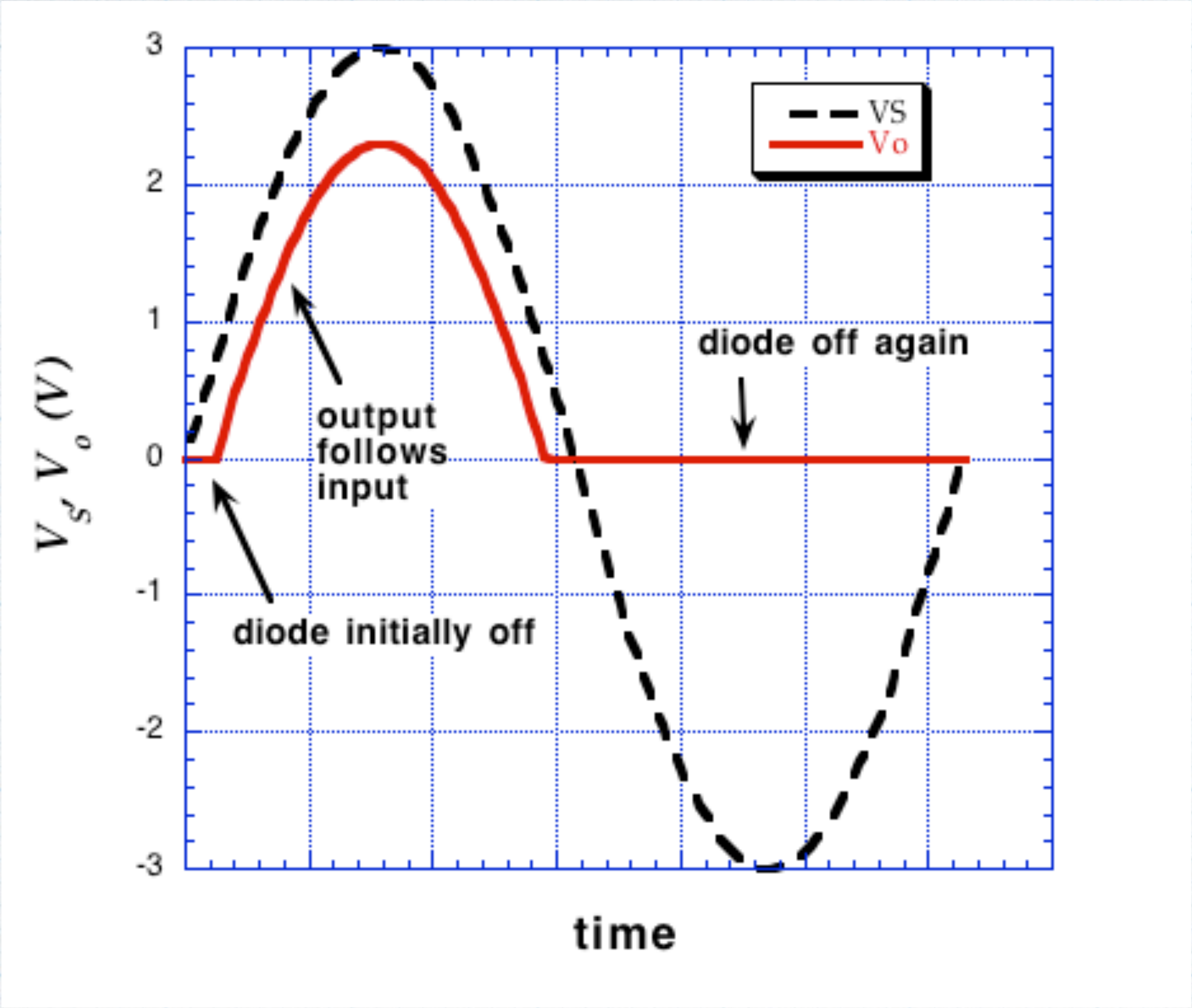
We are trying to deliver DC power to the load.





Diode is off until $V_S > 0.7$ V.

Current flows when diode is in forward conduction. The output tracks the input during positive half cycle.



The diode turns off when $V_S < 0.7$ V. It stays off during the negative half cycle of the sinusoid.

$$V_S > 0: \quad v_R(t) \approx V_p \sin\left(\frac{2\pi}{T}t\right) - 0.7\text{V}$$

$$V_S < 0: \quad v_R(t) = 0:$$

$$V_o(avg) \approx \frac{V_P}{\pi} - \frac{0.7\text{V}}{2}$$

$$\neq 0\ !$$

To get the negative half of the cycle, turn the diode around.

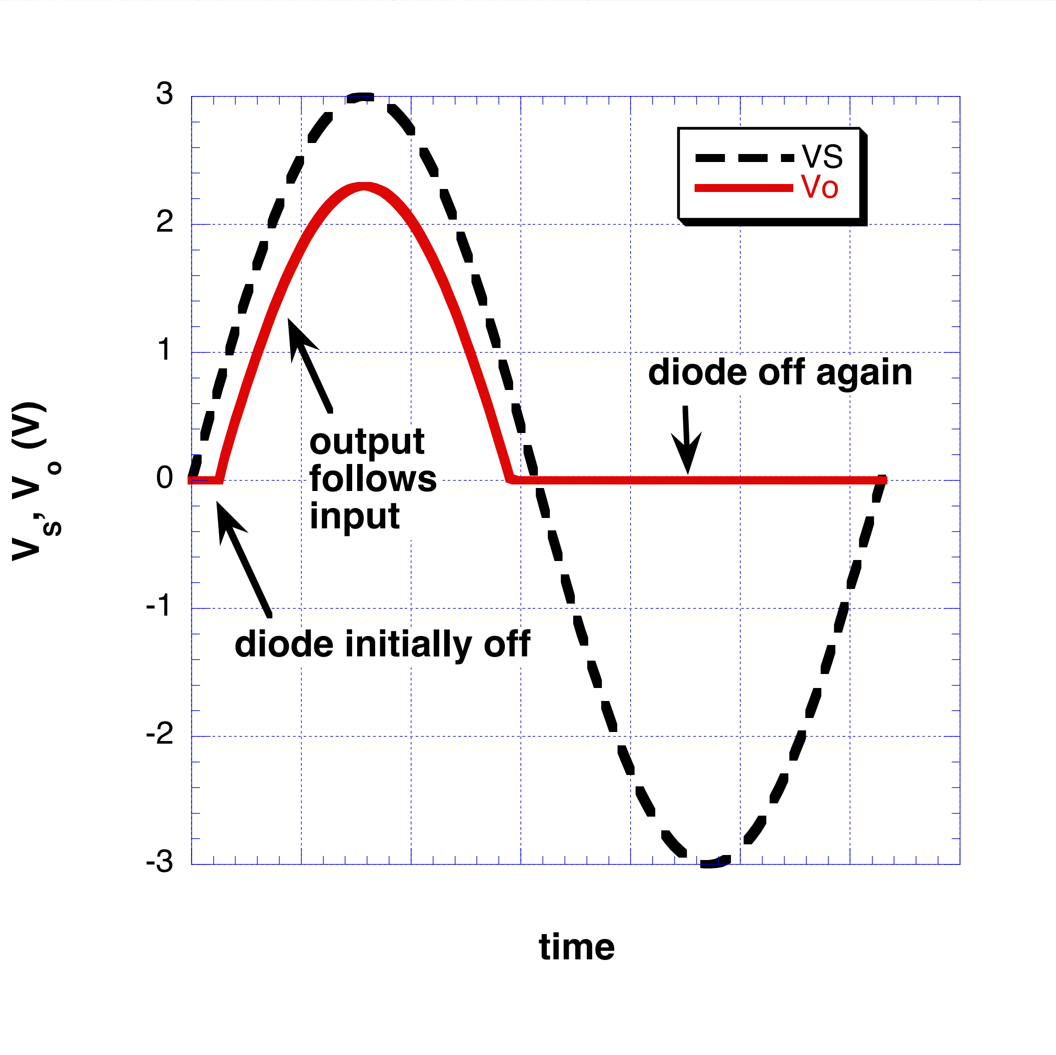
Time delay

Note that since the diode will not turn on until the sinusoid goes above ≈ 0.7 V, there is time delay before the rectifier "turns on". It is a simple matter to determine the delay time, using the "on-off" diode model:

$$0.7\text{V} = V_p \sin\left(\frac{2\pi}{T} t'\right)$$

$$t' = \frac{T}{2\pi} \arcsin\left(\frac{0.7\text{V}}{V_p}\right)$$

If f = 60 Hz (T = 16.67 ms) and V_p = 3 V, t' = 0.62 ms.

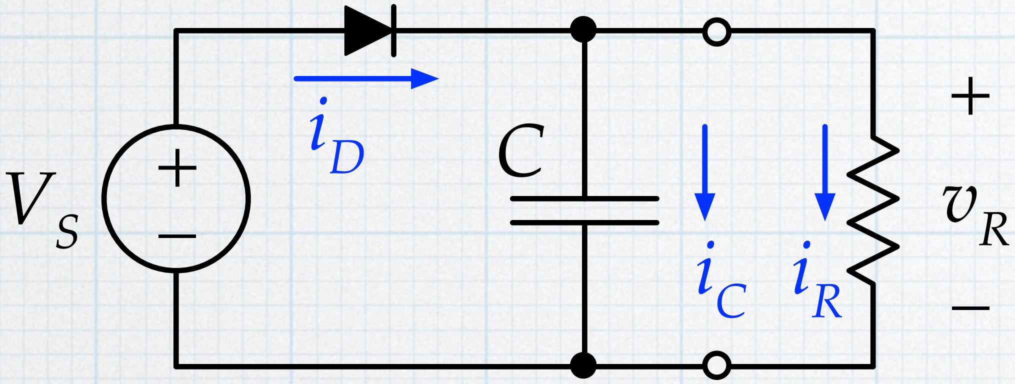


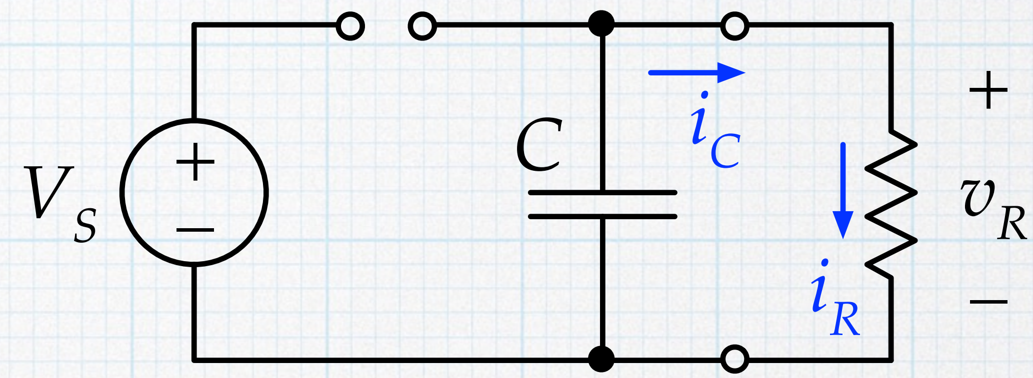
There is a similar time offset at the other end of the positive half cycle.

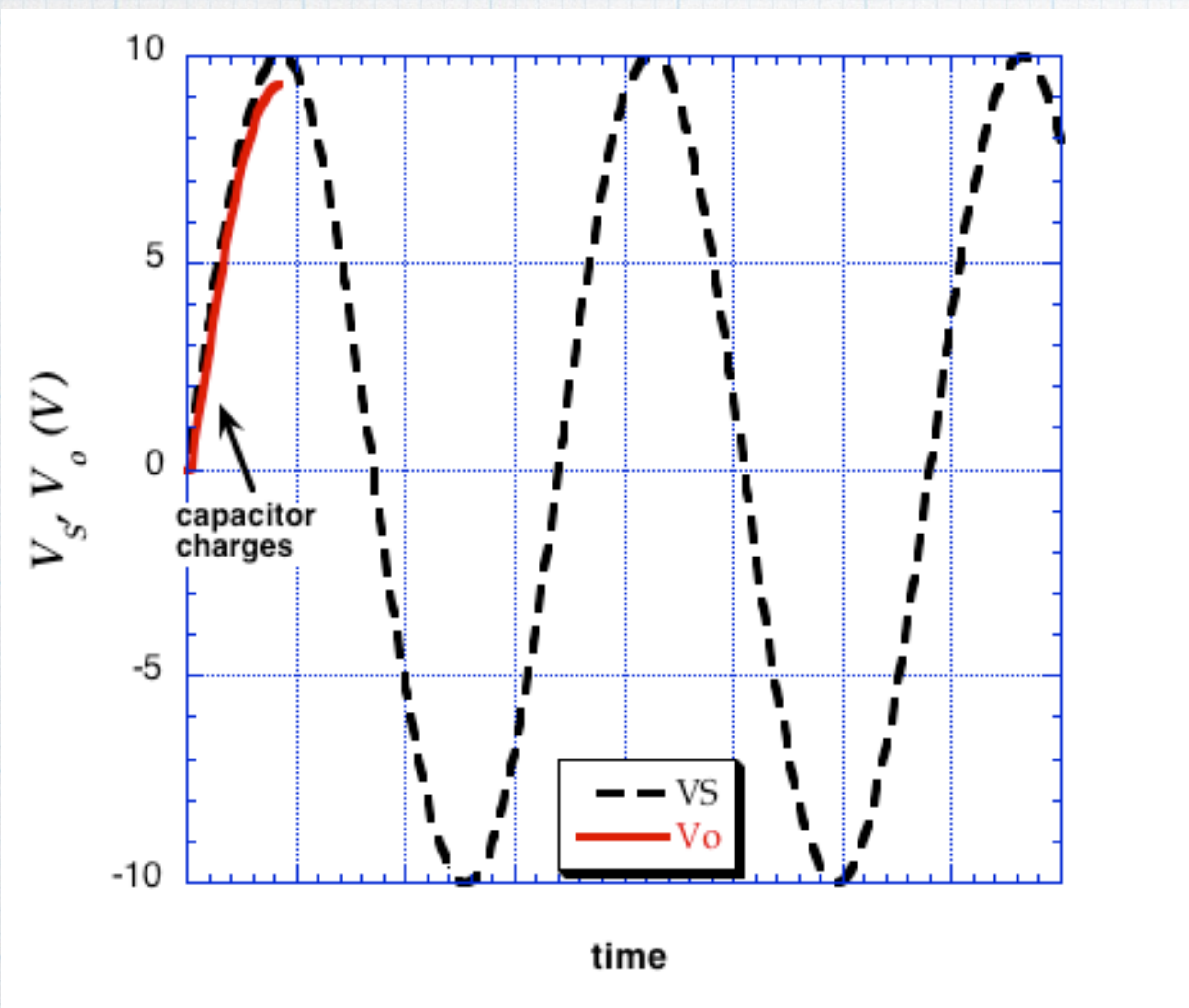
The effect of the time offset become negligible if $V_P >> 0.7$ V.

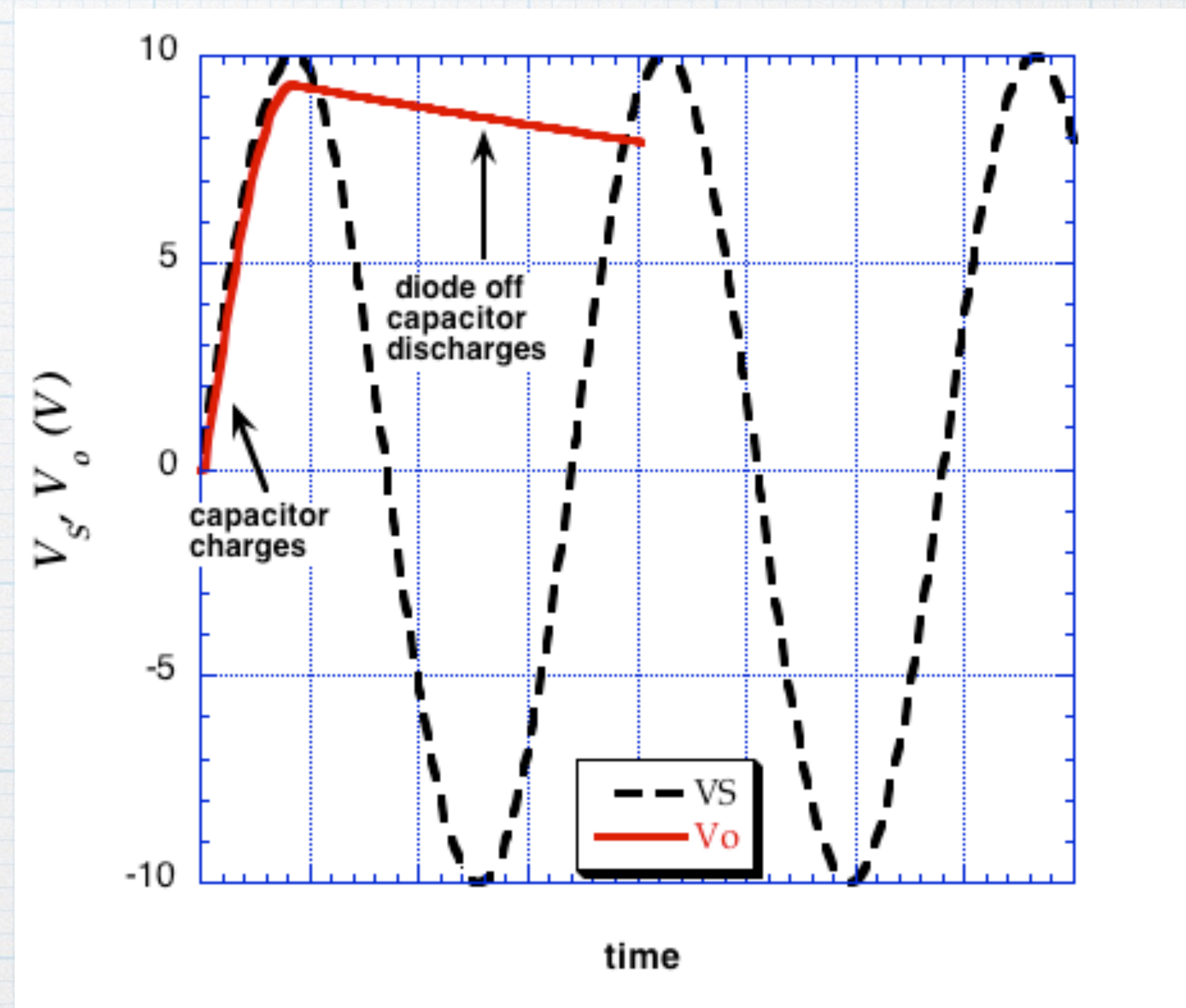
Peak rectifier

Add a largish capacitor after the diode, in parallel with the load.



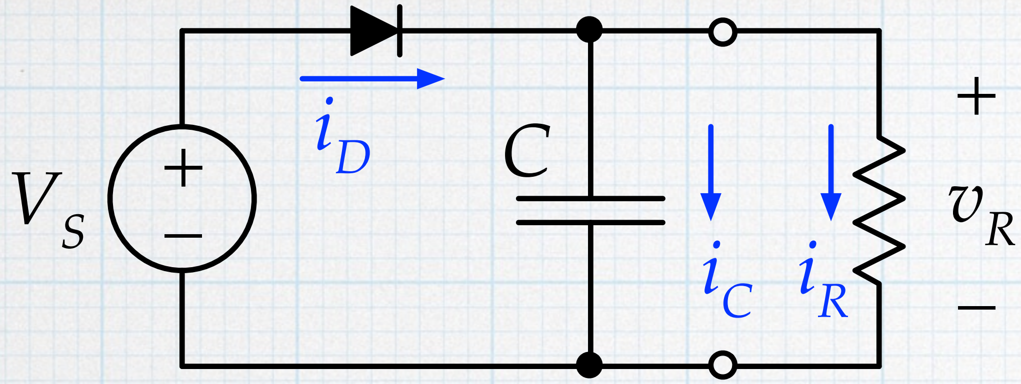


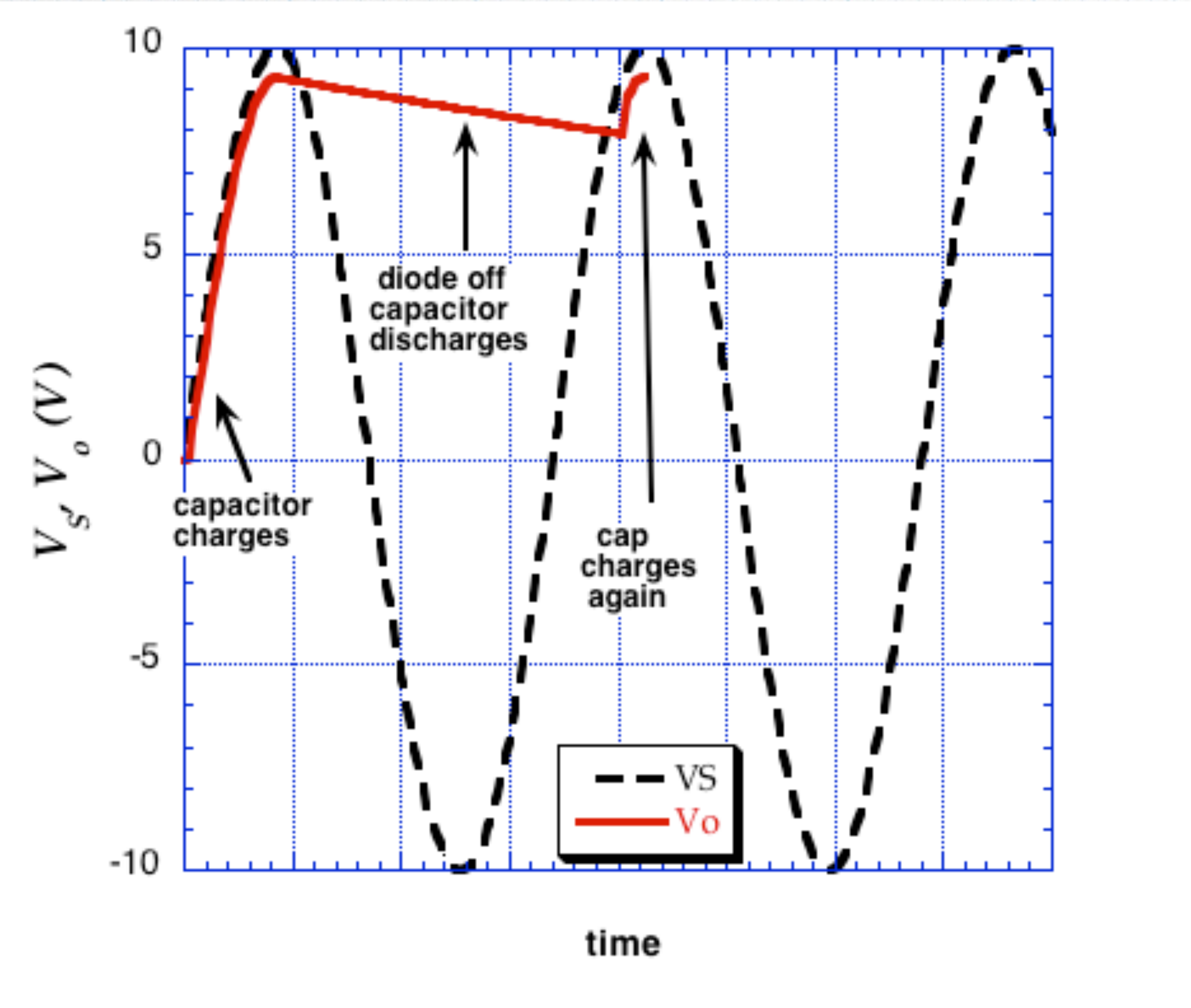




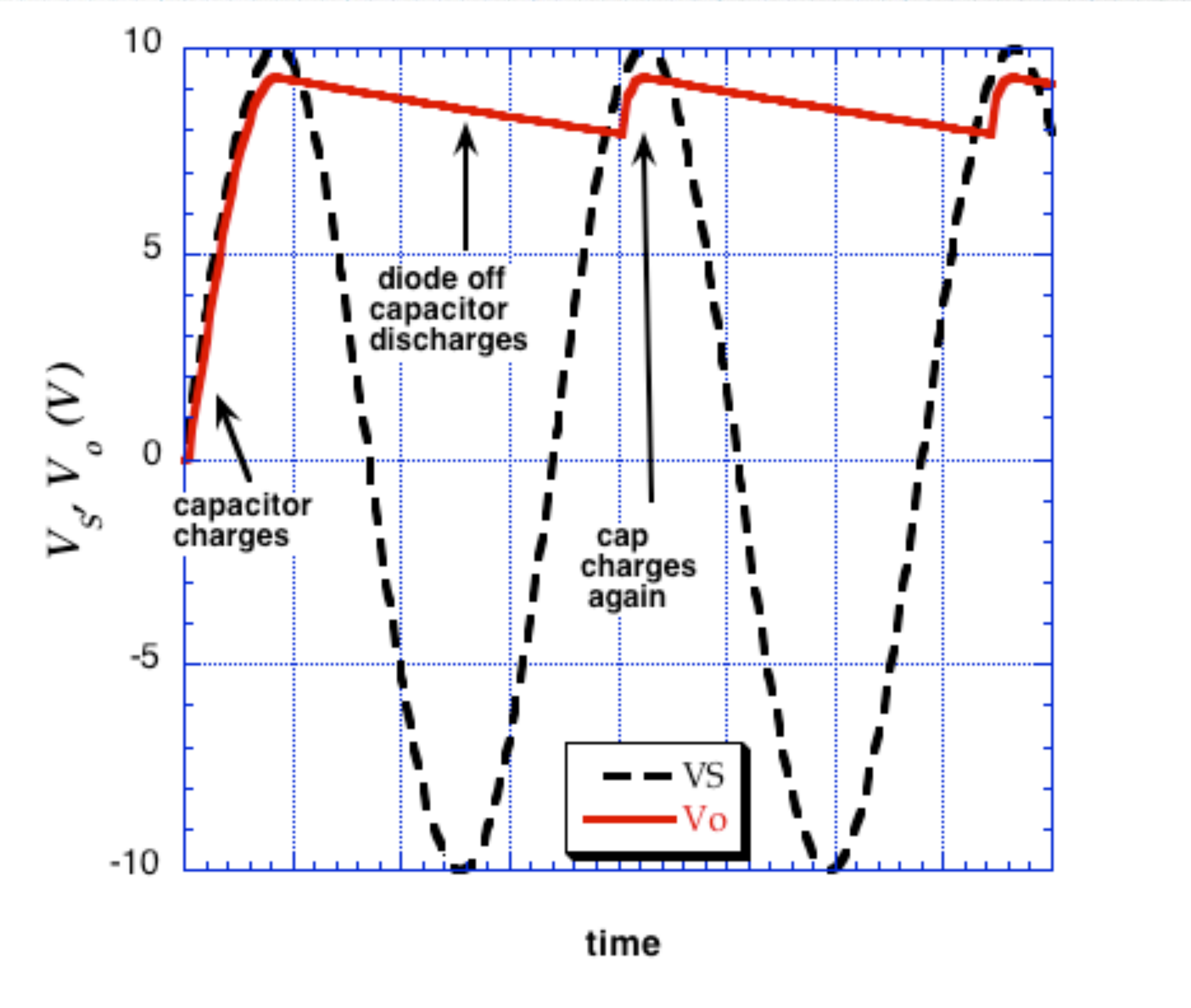
Initially, diode is on & cap charges to V_P - 0.7 V.

While $V_S < v_C$, diode is off! Cap discharges through load.

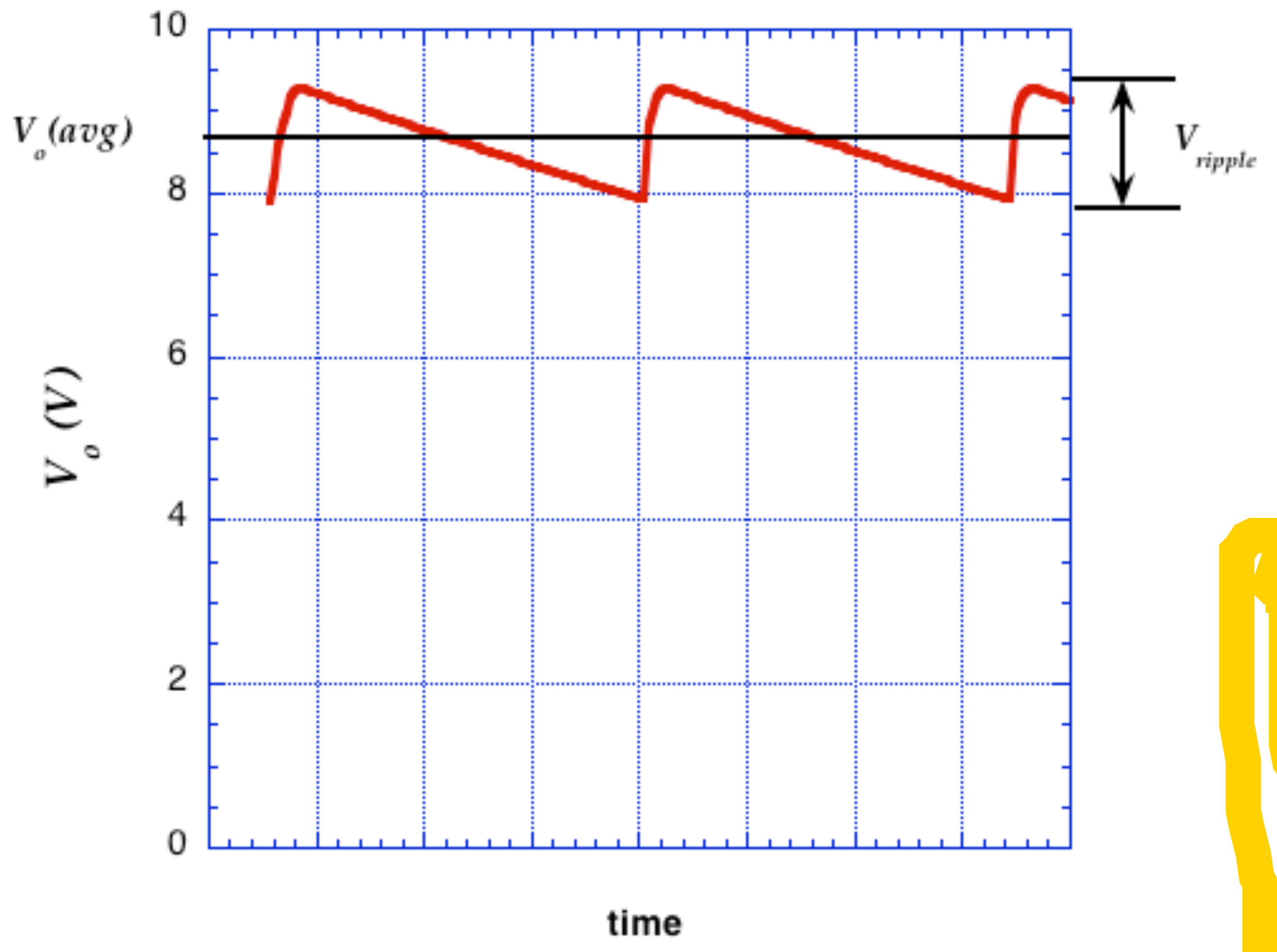


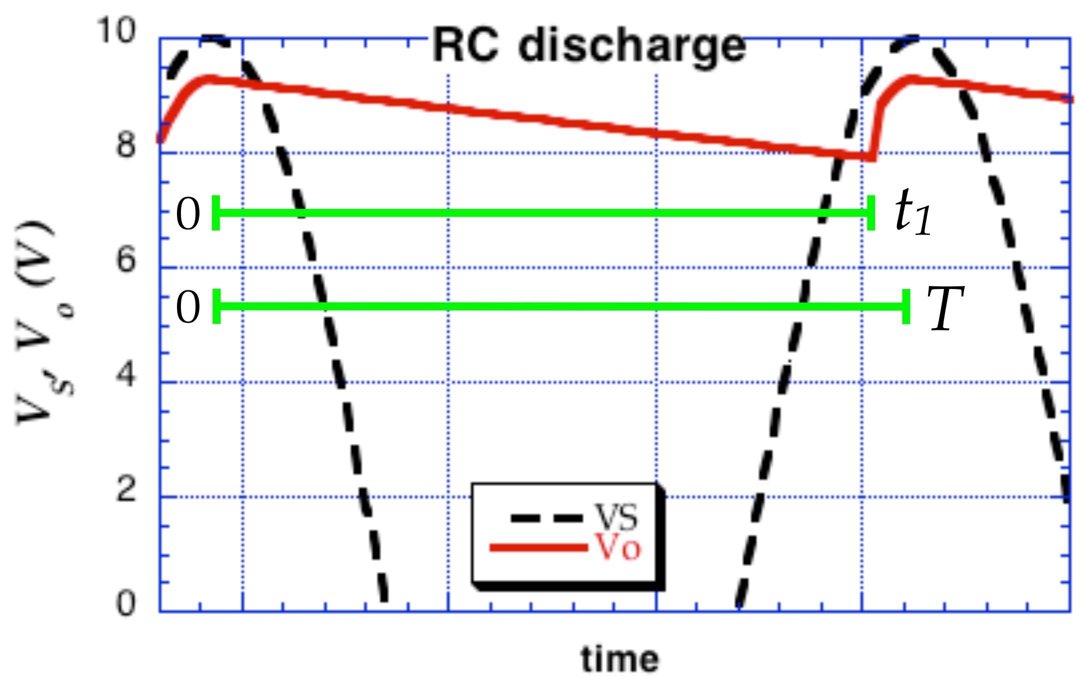


Diode stay off until V_S comes back around and becomes bigger than v_C. Then diode comes on again and re-charges the capacitor.



When V_S falls to less than v_C, the diode turn off again, and the cycle continues.





Not a a perfect DC voltage at output. There is some variation (ripple) around an average value.

$$V_o\,(max) = V_P - 0.7\mathrm{V}$$

$$V_o\,(min) = [V_P - 0.7\mathrm{V}]\exp\left(-\frac{t_1}{RC}\right)$$

$$\approx [V_P - 0.7\mathrm{V}]\exp\left(-\frac{T}{RC}\right)$$

$$V_{ripple} = V_o\,(max) - V_o\,(min)$$

$$= [V_P - 0.7\mathrm{V}]\left[1 - \exp\left(-\frac{T}{RC}\right)\right]$$

$$V_o\,(avg) \approx V_o\,(max) - \frac{V_{ripple}}{2}$$

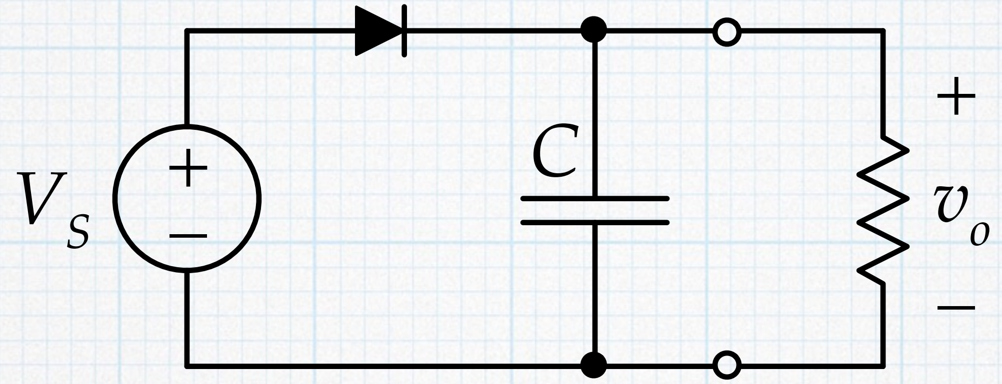
t_1 = time when diode conducts again.

$t_1 \approx T$

Example 1

$$V_S = (15\text{V})\sin\left(\frac{2\pi}{T}t\right)$$

$T = 16.67$ ms



$C = 100\ \mu$F

$R = 5000\ \Omega$

Find the average value of v_o and the ripple voltage. Repeat for R = 1000 Ω and 200 Ω.

$$V_{ripple} = [V_P - 0.7\text{V}]\left[1 - \exp\left(-\frac{T}{RC}\right)\right]$$

$$= [15\text{V} - 0.7\text{V}]\left[1 - \exp\left(-\frac{16.67\text{ms}}{(5000\Omega)(100\mu\text{F})}\right)\right]$$

$$= 0.47\text{ V}$$

$$V_o(avg) = V_o(max) - \frac{V_{ripple}}{2} = 14.3\text{V} - \frac{0.47\text{V}}{2} = 14.1\text{V}$$

$R = 1\ \text{k}\Omega$

$V_{ripple} = 2.19$ V

$V_o\,(\text{avg}) = 13.2$ V

$R = 200\ \Omega$

$V_{ripple} = 8.09$ V

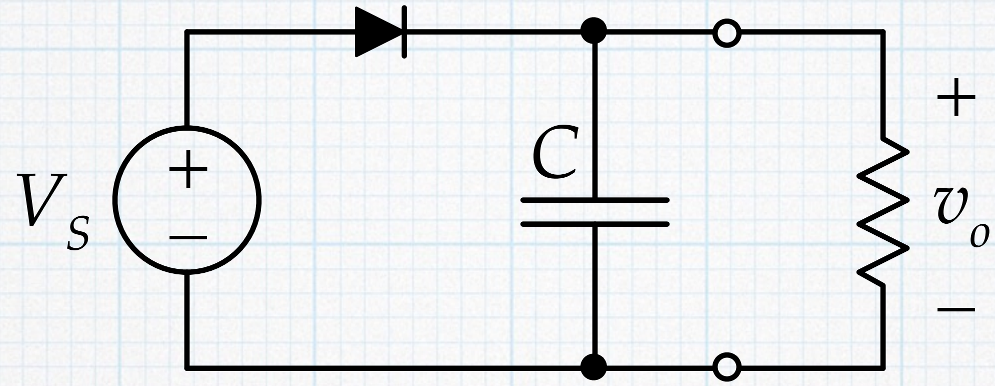
$V_o\,(\text{avg}) = 10.2$ V

Drawing more current causes the ripple to increase and V_{DC} to droop. Can fight this with more capacitance.

Example 2

$$V_S = (25\text{V})\sin\left(\frac{2\pi}{T}t\right)$$

$T = 16.67$ ms



$R = 1000\ \Omega$

Find the capacitance so that the ripple will be no bigger than 1 V.

What is the DC voltage?

$$V_{ripple} = [V_P - 0.7\text{V}]\left[1 - \exp\left(-\frac{T}{RC}\right)\right]$$

$$C = -\frac{T}{R}\left[\ln\left(1 - \frac{V_{ripple}}{V_P - 0.7\text{V}}\right)\right]^{-1} = -\frac{16.67\text{ms}}{1000\Omega}\left[\ln\left(1 - \frac{1\text{V}}{24.3\text{V}}\right)\right]^{-1} = 397\ \mu\text{F}$$

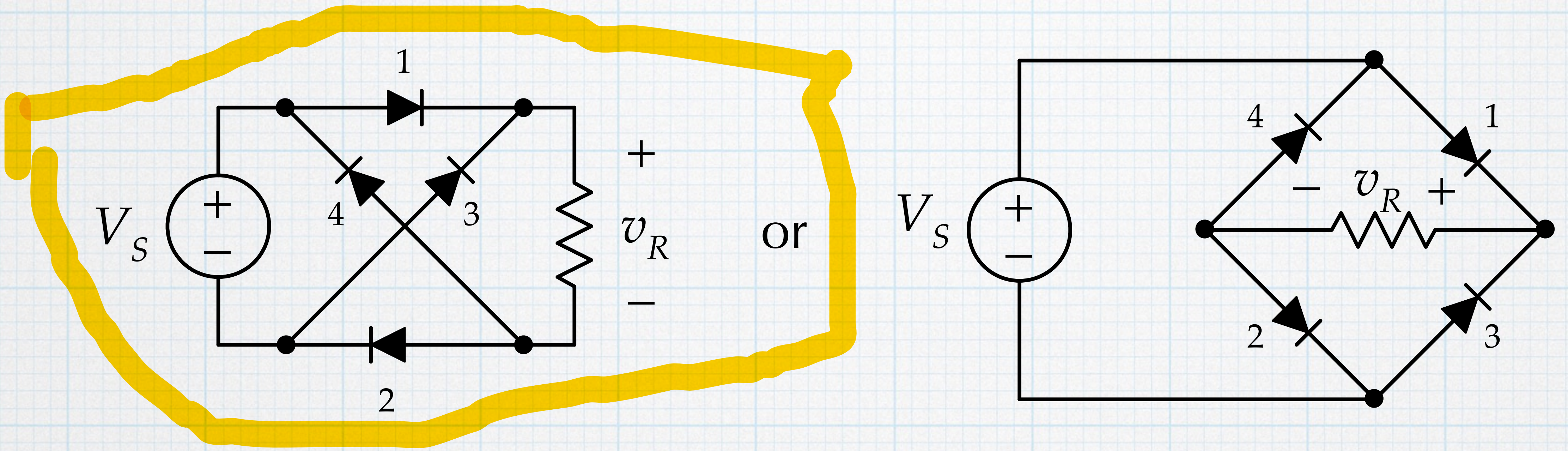
$$V_o\,(avg) = V_o\,(max) - \frac{V_{ripple}}{2} = 24.3\text{V} - \frac{1\text{V}}{2} = 23.8\text{V}$$

What capacitance is needed to limit the ripple to 0.1 V?

$C = 4000\ \mu$F !!!

Full-wave rectifier

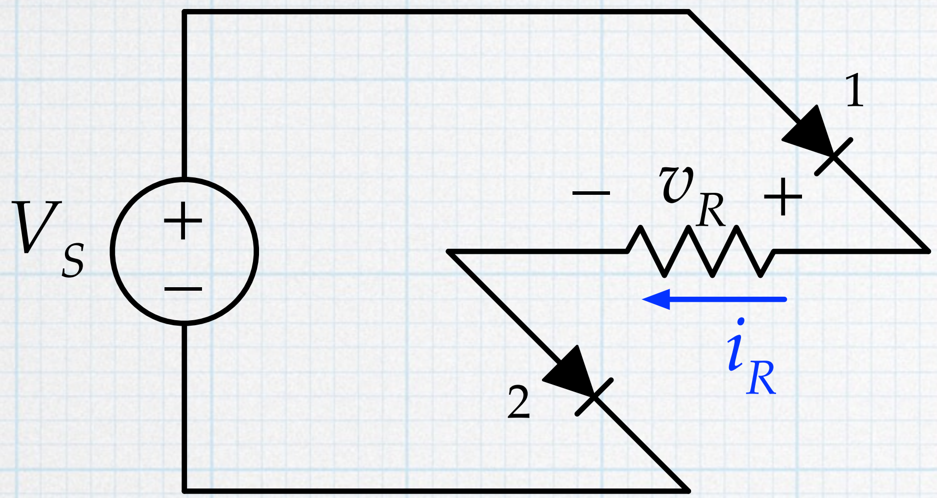
With a few more diodes, we can rectify the entire sinusoidal input.

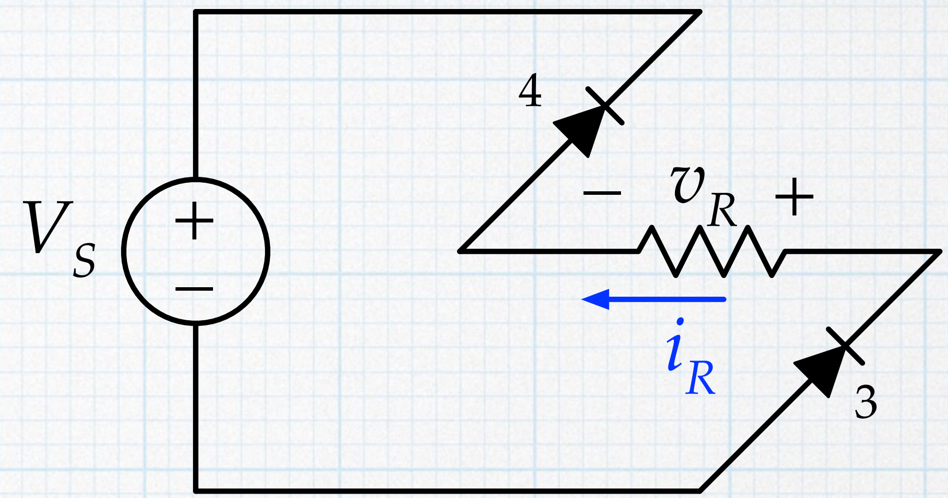


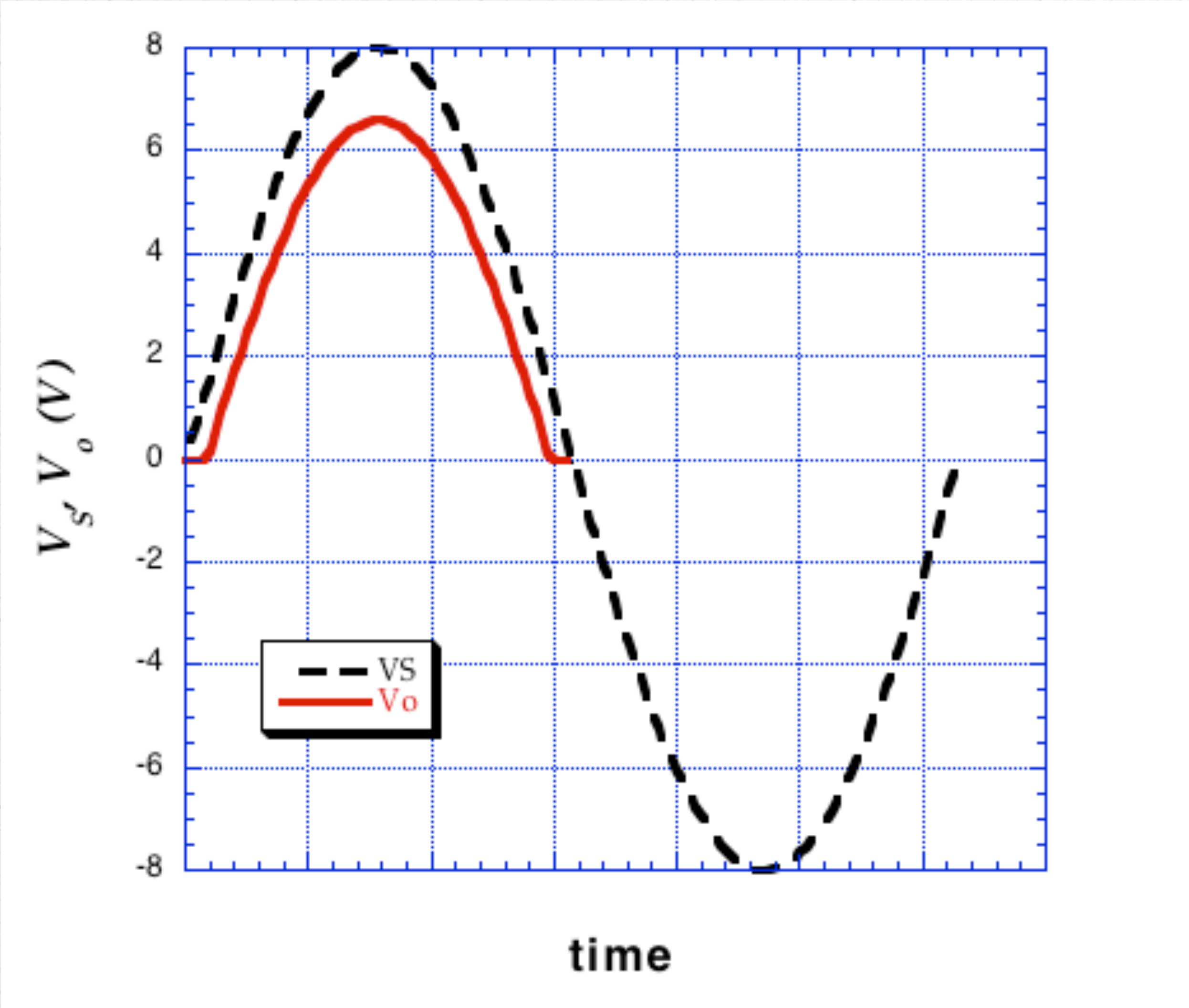
The diodes are in a bridge configuration.

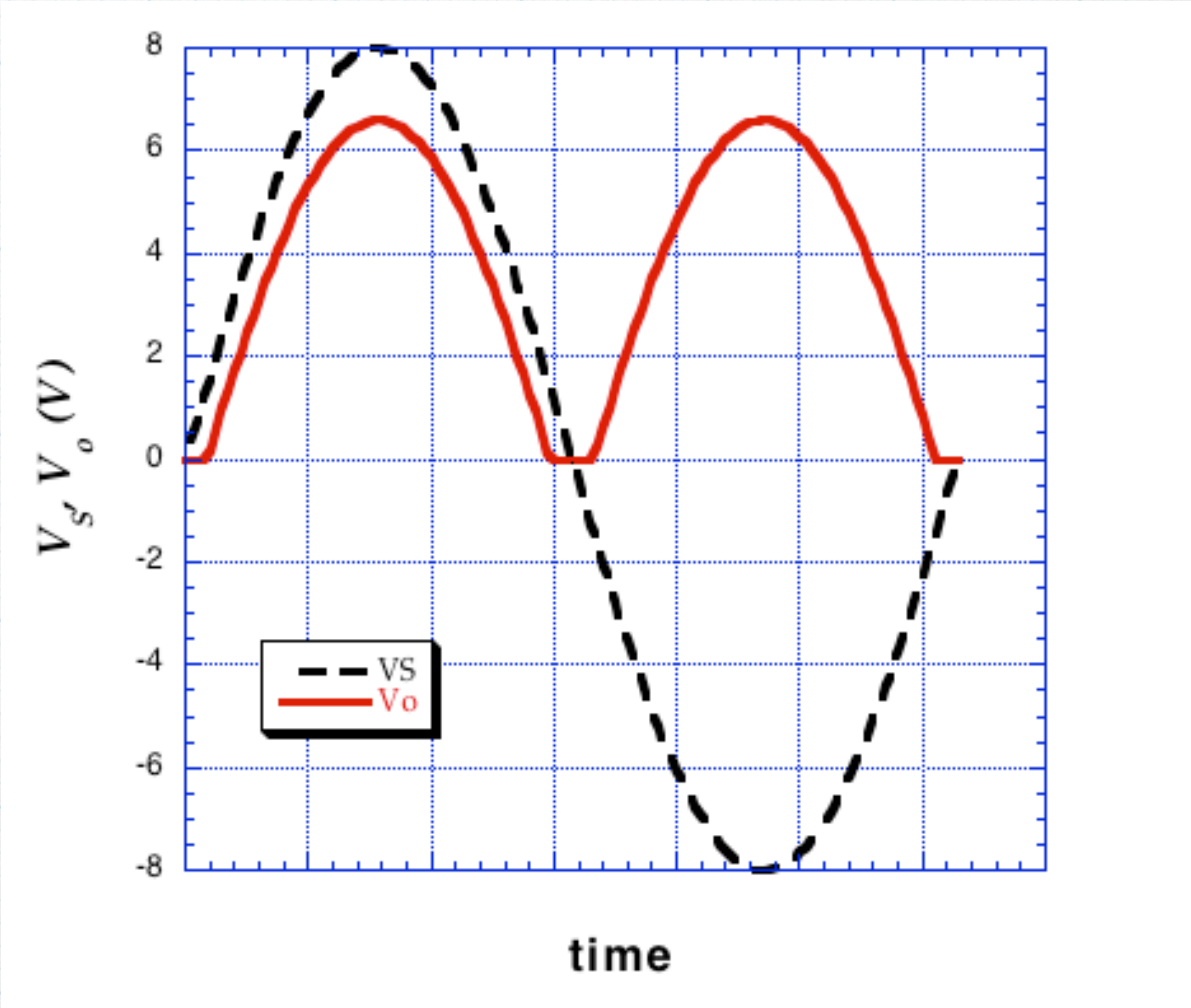
During the positive half cycle of the input, diodes 1 and 2 will be forward biased. Current will flow from the positive source through those diodes and the resistor to generate a positive voltage across the resistor.

During the negative half cycle of the input, diodes 3 and 4 will be forward biased. Current will flow from the negative source through those diodes and the resistor to generate a positive voltage across the resistor, again.









Note that there are no two diode drops in the conduction path(s).

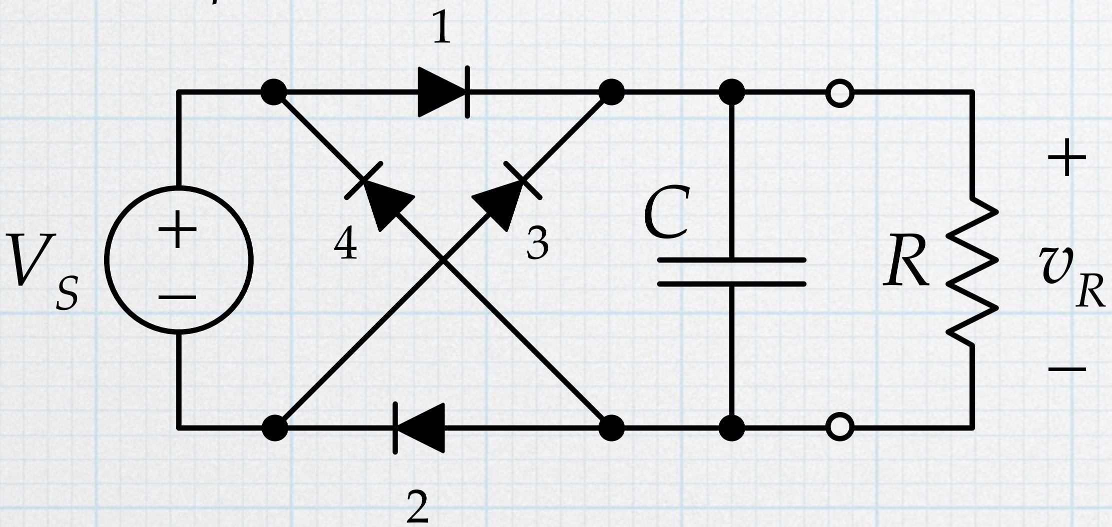
Also, the frequency is effectively doubled.

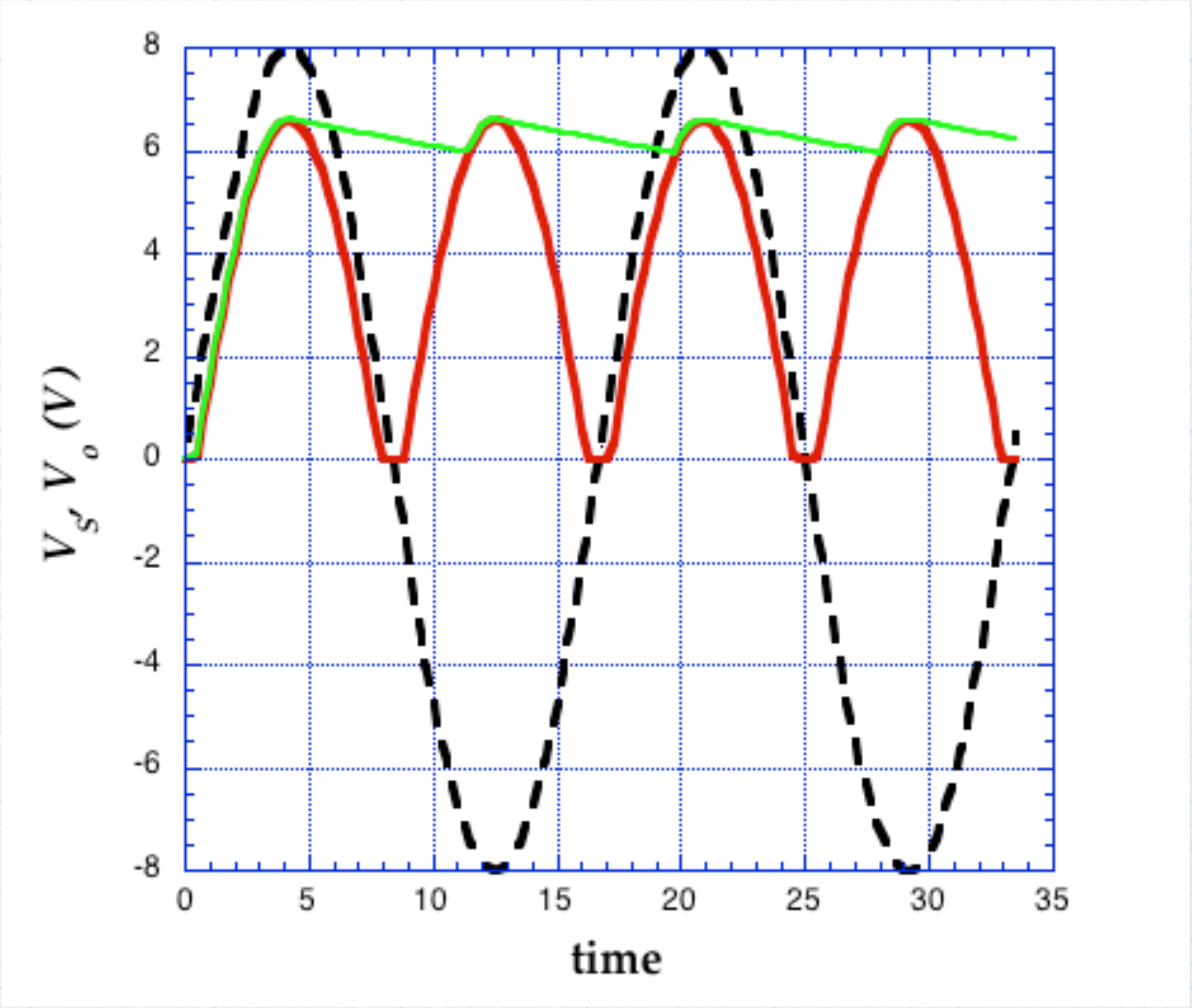
Full-wave peak rectifier

Placing a capacitor in parallel with the load, turns the circuit into a full-wave peak rectifier. It behaves essentially the same as the half-wave peak rectifier except with twice the frequency (half the period).

$$V_S(t) = V_p \sin\left(\frac{2\pi}{T}t\right)$$

$V_p = 8$ V.





The ripple voltage is calculated in exactly the same way, except that the period is cut in half (frequency doubled).

$$V_{ripple} = [V_P - 1.4\text{V}]\left[1 - \exp\left(-\frac{T}{2RC}\right)\right]$$

Same as doubling capacitance!

Example 3

You want to use a wall transformer that produces 10-V_{RMS} at the secondary to generate a DC voltage. The desired voltage DC should be greater than 12 V and it should be able to supply at least 50 mA while keeping the voltage ripple to less than 5%. Design the rectifier to meet these goals. (Note: f = 60 Hz.)

10 V_{RMS} → 14.1 V amplitude

effective $R_L \approx V_o \ / \ I_o = 12.0 \text{ V} \ / \ (50 \text{ mA}) = 240 \ \Omega$

Note: This would be the minimum value of effective resistance. If we choose C to meet the ripple requirement, then we will still be safe if we use a slightly higher V_o.

Two options: half-wave or full-wave rectifier. Try both.

Half-wave:

$V_o(\text{max}) = V_p - 0.7 \text{ V} = 13.4 \text{ V} \rightarrow V_{ripple} \leq 0.67 \text{ V}.$

$$C = -\frac{T}{R}\left[\ln\left(1 - \frac{V_{ripple}}{V_P - 0.7\text{V}}\right)\right]^{-1} = 1350 \ \mu\text{F}$$

$V_o(\text{avg}) = V_o(\text{max}) - V_{ripple} \ / \ 2 = 13.06 \text{ V}.$

Full-wave:

$$V_o(\max) = V_p - 2(0.7\text{ V}) = 12.74\text{ V} \rightarrow V_{ripple} \le 0.64\text{ V}.$$

$$C = -\frac{T}{2R}\left[\ln\left(1 - \frac{V_{ripple}}{V_o\,(\max)}\right)\right]^{-1} = 673\mu\text{F}$$

$$V_o(\text{avg}) = V_o(\max) - V_{ripple}\ /\ 2\ = 12.42\text{ V}.$$

Either approach will work and meet the requirements. The full-wave version uses extra diodes, but only half the capacitance. Since diodes are nearly free (pennies per piece), but big capacitors are relatively expensive, the full-wave circuit will actually cost less than the half-wave.

This is why full-wave rectifiers are used more commonly than half-wave rectifiers.

Component manufactures supply full-wave bridge rectifiers packaged as single unit with the transformer sinusoid as input the rectified waveform as the output.