

SC107- Calculus

Home Work 13

Week 14: November 06, 2017

Tutorial Discussion Week: November 06, 2017

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Q.(1) If p is an integer greater than or equal to zero. Show that the series

$$(1) \quad \sum_{n=1}^{\infty} \frac{p(p-1)(p-2)\dots(p-n+1)}{n!} x^n$$

converges for $|x| < 1$ and diverges for $|x| > 1$

Q.(2) Chebyshev equation is

$$(2) \quad (1-x)^2 y'' - xy' + p^2 y = 0.$$

Where p is a constant.

(a). Find the two linearly independent solutions valid for $|x| < 1$.

(b). Show that if $p = n$ where $n \geq 0, n$ is an integer, then there is a polynomial solution of degree n . When these are multiplied by suitable constants, they are called Chebyshev's polynomial

Q.(3) When $p > 0$ Bessels equation becomes $x^2 y'' + xy' + x^2 y = 0$. Show that its indicial equation has only one root, and use the method of this section to deduce that

$$(3) \quad y = \sum_{n=0}^{\infty} \frac{(-1)^n}{2^{2n}(n!)^2} x^{2n}$$

Q.(4) Bessel equation of order $p = 1$ is $x^2 y'' + xy' + (x^2 - 1)y = 0$. Show that $m_1 - m_2 = 2$ and that the equation has only one Frobenius series solution. Then find it.