SC105- Calculus with Complex Variables

Home Work on Application of Partial Differential Equations

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1. Show that the equation $u_t = c^2(u_{xx} + u_{yy} + u_{zz})$ with $u = 0$ on the boundary S of a closed and bounded region D and $u(x, y, z, 0) = 0$, has only the trivial solution *i.e.* $u = 0\ \forall x, y, z \in D$ and $\forall\ t$.
2. Find the temperature in a bar of length 10 cm, and of homogeneous material with $c^2 = 1$, which is perfectly insulated laterally, whose ends are kept at temperature $0^o c$ and whose initial temperature (in $^o c$) is $f(x)$, where (i) $f(x) = \begin{cases} x & \text{if } 0 \leq x \leq 5, \\ 0 & \text{if } 5 < x \leq 10 \end{cases}$, (ii) $f(x) = x(10 - x)$.
3. Find the temperature in a bar of length L which is perfectly insulated also at the ends at $x = 0$ and $x = L$, assuming $u(x, 0) = f(x)$. [Note: flux of heat through the face at the end is proportional to $\frac{\partial u}{\partial x}$].
4. Find the temperature $u_L(x)$ after a long time $t \to \infty$ in a bar of length L, which is partially insulated laterally and whose ends are kept in different temperatures, $u(0, t) = T_1$ and $u(L, t) = T_2$.
5. Find the temperature in a bar of length L and of homogeneous material, which is perfectly insulated laterally, whose ends are kept in different temperatures, $u(0, t) = T_1$, is given by $u(L, t) = T_2$ and whose initial temperature is proportional to $f(x)$.
6. Show that the temperature in an infinite bar (with extends to infinity both side and is laterally perfectly insulated) with initial temperature given by $u(x, 0) = f(x)$,

$$u(x, t) = \frac{1}{2a\sqrt{\pi t}} \int_{-\infty}^{\infty} f(v) e^{\left(-\frac{(x-v)^2}{4a^2 t}\right)} dv.$$

7. Find the deflection $u(x, t)$ of the vibrating string (length $L = \pi$, ends fixed and $c^2 = \frac{T}{\rho} = 1$) corresponding to zero initial velocity and initial deflection $f(x)$ prescribed as
   (i) $f(x) = k(\sin x + \sin 3x)$,
   (ii) $f(x) = 0.01x(\pi - x)$,
8. The forced vibrations of an elastic strings (of length L) under an external force $P = A\rho \sin \omega t$ per unit length and acting normal to the string are given by $u_{tt} = c^2 u_{xx} + P/\rho$. If the two ends are kept fixed and initial velocity is zero, initial deflection is prescribed as f(x), determine $u(x, t)$. [Hint: $u(x, t) = \sum_0^\infty G_n(t) \sin \frac{n\pi x}{L}$ satisfies the boundary conditions; write Fourier sine series of P/ρ in term of $\sin \frac{n\pi x}{L}$; substituting the two in the equation for forced vibrations obtain the equations for $G_n(t)$].
9. Find the potential in the interior as well as exterior of the surface $R = 1$ if the potential on the surface is prescribed as (i) $f(\rho) = 1$, (ii) $f(\rho) = \cos^2 \rho$.