SC 105

Calculus and Complex Variables

Dhirubhai Ambani Institute of Information and Communication Technology (DA-IICT)

Version 3 (Fall 2009)

INSTRUCTIONS:

- There are 3 double sided pages (6 printed pages). Ensure that you have all the pages.
- Answer **all questions**, writing clearly in the space provided.
- Show all your work and explain how you arrived at your answers, unless explicitly told to do otherwise.
- Write your name and student number **clearly** at the top of each page. If you do not follow this you will get zero right away.
- You have **two hours** to complete the test
- Marks for each question are indicated in brackets at right. You may use point form for your answers, but make sure the points are clear and unambiguous. I am more interested in your thought process.
- You may use last page (page number 6) for rough work. If you need more paper for rough work please ask for supplementary sheets. Do not ask for full answer sheets. Your answers should be on the question paper in the space provided.

FOR MARKER'S USE ONLY

Question	Possible	Received
1	5	
2	5	
3	5	
4	5	
TOTAL	20	

1. Linear Differential Equation

   (a) If $y_1(x)$ and $y_2(x)$ are two solutions of $y'' + P(x)y' + Q(x)y = 0$ on $[a, b]$ then show that they are linearly dependent if and only if their Wronskian $W(y_1, y_2) = 0$. Comment what will happen to this result if $y_1(x), y_2(x)$ are not the solutions of the equation. Explain. (5)

2. Chebyshev Equation

   (a) Solve for series solution about $x = 0$ of the following equation.

$$(1 - x^2)y'' - xy' + p^2y = 0,$$

   where p is a real positive integer. (5)

3. Initial Value Problem (IVP)

   (a) Find the solution of the following IVP.

$$y'' + 4y' + 13y = e^{-t}, y(0) = 0, y'(0) = 2.$$

(5)

4. Heat Equation -Initial-Boundary Value Problem (IBVP)

   (a) Find the Fourier series solution of the following one dimensional heat equation:

   $$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}, 0 < x < l, t > 0,$$

   with the initial conditions: $u(x,0) = f(x), 0 < x < l$ and the boundary conditions: $u(0,t) = u(l,t) = 0, t > 0$. Where c^2 is the thermal diffusivity. (5)

Rough