

RC transients

Circuits having capacitors:

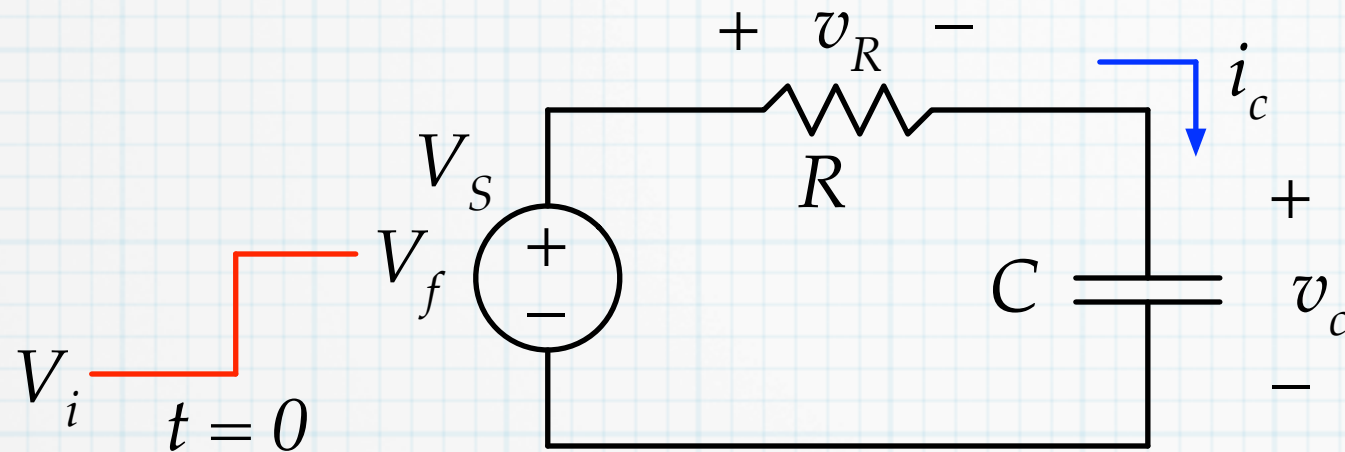
- At DC – capacitor is an open circuit, like it's not there.
- Transient – a circuit changes from one DC configuration to another DC configuration (a source value changes or a switch flips). Determine the DC state (current, voltages, etc.) before the change. Then determine what happens after the change. Over time, the circuit will settle into a new DC state, where the capacitors are again open-circuits. In between will be an interval during which currents and voltages are changing as the capacitors charge or discharge. Since this lasts for only a “short” time, this is known as a *transient* effect.
- AC – currents and voltages are changing continuously, so capacitors are charging and discharging continuously. This requires special techniques and is the next topic for EE 201.

Solving a circuit with transient changes

1. Determine the DC voltages on the capacitors before the change occurs. These may be given, or you may have to solve for them from the original configuration.
2. Let the change occur instantaneously at time $t = 0$. The capacitors will maintain their voltages into the “instant” just after the change. (Recall: capacitor voltage cannot change instantaneously.)
3. Analyze the circuit using standard methods (node-voltage, mesh-current, etc.) Since capacitor currents depend on dv_c/dt , the result will be a differential equation.
4. Solve the differential equation, using the capacitor voltages from before the change as the initial conditions.
5. The resulting equation will describe the charging (or discharging) of the capacitor voltage during the transient and give the final DC value once the capacitor is fully charged (or discharged).

In practice, we will solve only one circuit and try to understand it completely. Then, when we encounter other circuit configurations, we will make those fit the prototype.

Simple RC circuit transient (physics)



In the circuit, V_S abruptly changes value from V_i to V_f at $t = 0$. Assume that the source was at V_i for a “very long time” before $t = 0$.

1. For $t < 0$, $v_c = V_i$ and $i_c = 0$.

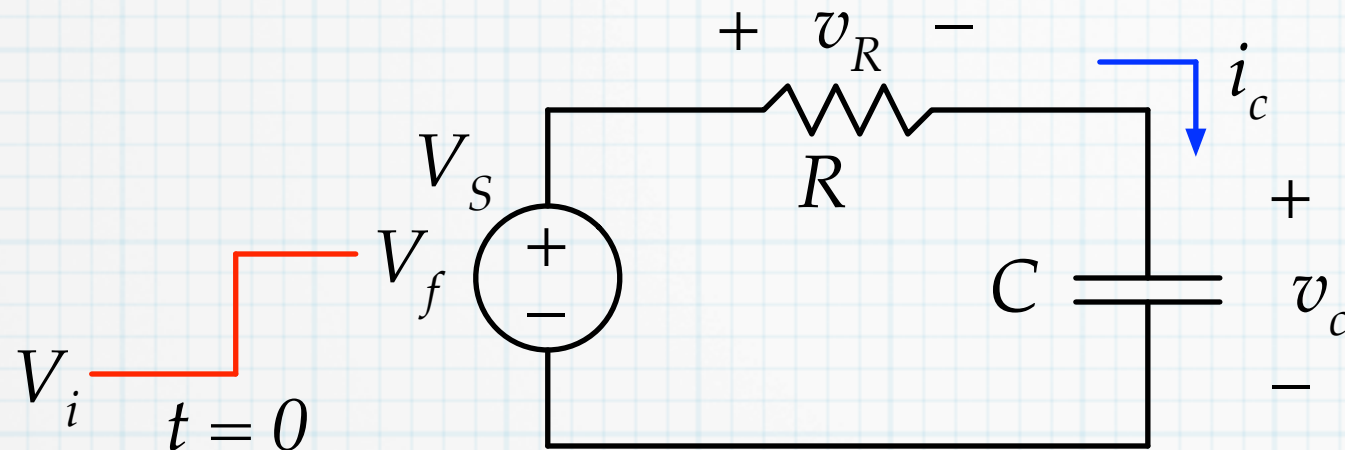
2. At $t = 0$, V_S changes. $v_c = V_i$ (still), but i_c jumps abruptly:

$$i_c(0) = \frac{V_S - v_c(0)}{R} = \frac{V_f - V_i}{R} \quad \text{Capacitor begins to charge.}$$

3. For $t > 0$, as the capacitor charges, v_c will increase. As v_c increases, i_c will decrease.

4. After a “sufficiently long time”, v_c will charge to V_f . The current drops to zero. The transient is complete.

Simple RC circuit transient (math)



In the circuit, V_S abruptly changes value from V_i to V_f at $t = 0$. Assume that the source was at V_i for a “very long time” before $t = 0$.

For $t < 0$, $v_c = V_i$ and $i_c = 0$.

$$i_c = i_R$$

$$C \frac{dv_c(t)}{dt} = \frac{V_f - v_c(t)}{R}$$

$$\frac{dv_c}{V_f - v_c} = \frac{1}{RC} dt$$

$$\int \frac{dv_c}{V_f - v_c} = \frac{1}{RC} \int dt$$

$$\int_{V_i}^{v_c(t)} \frac{dv_c}{V_f - v_c} = \frac{1}{RC} \int_0^t dt'$$

$$-\ln(V_f - v_c) \Big|_{V_i}^{v_c(t)} = \frac{t}{RC}$$

$$\ln \left[\frac{V_f - V_i}{V_f - v_c(t)} \right] = \frac{t}{RC}$$

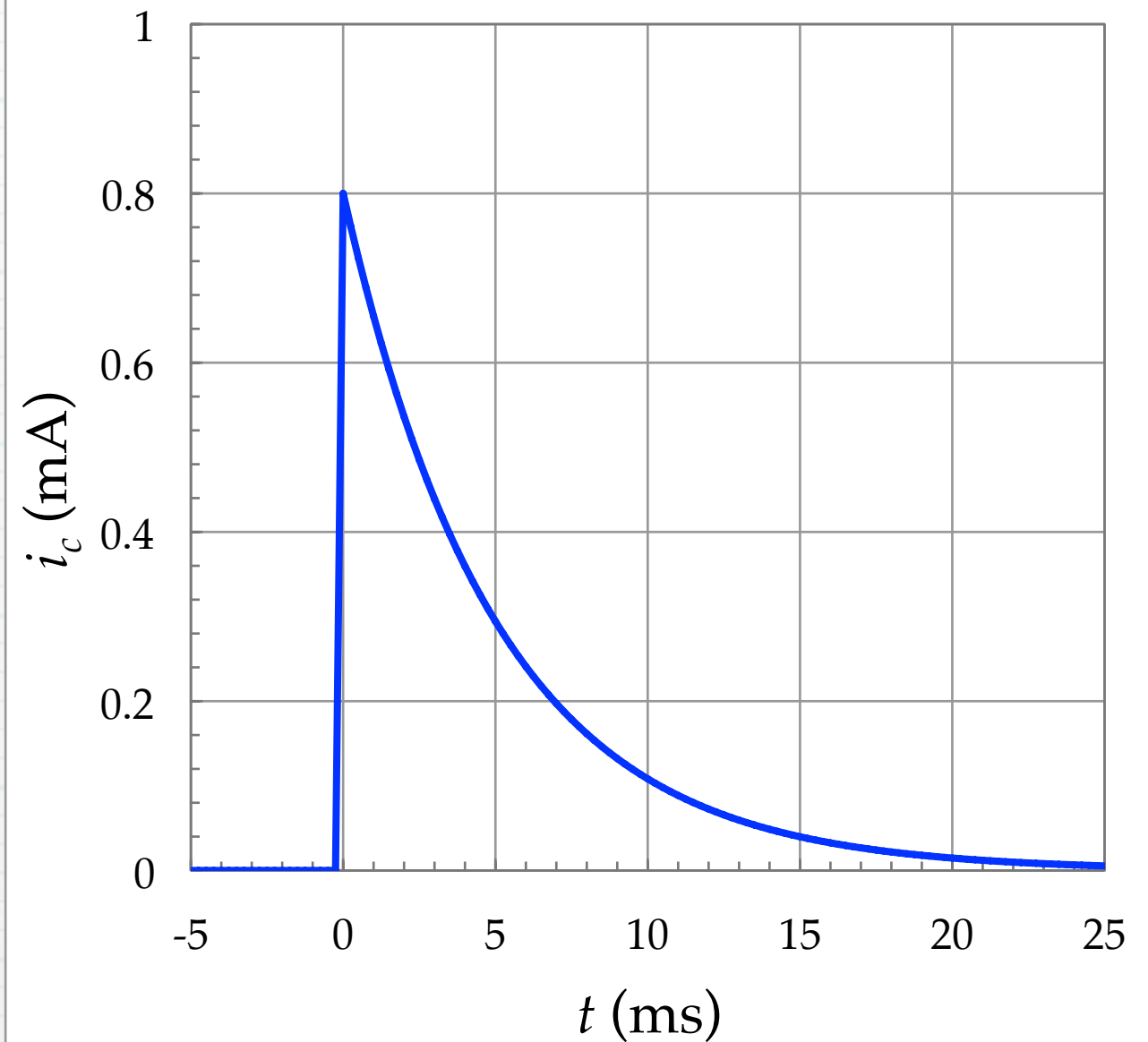
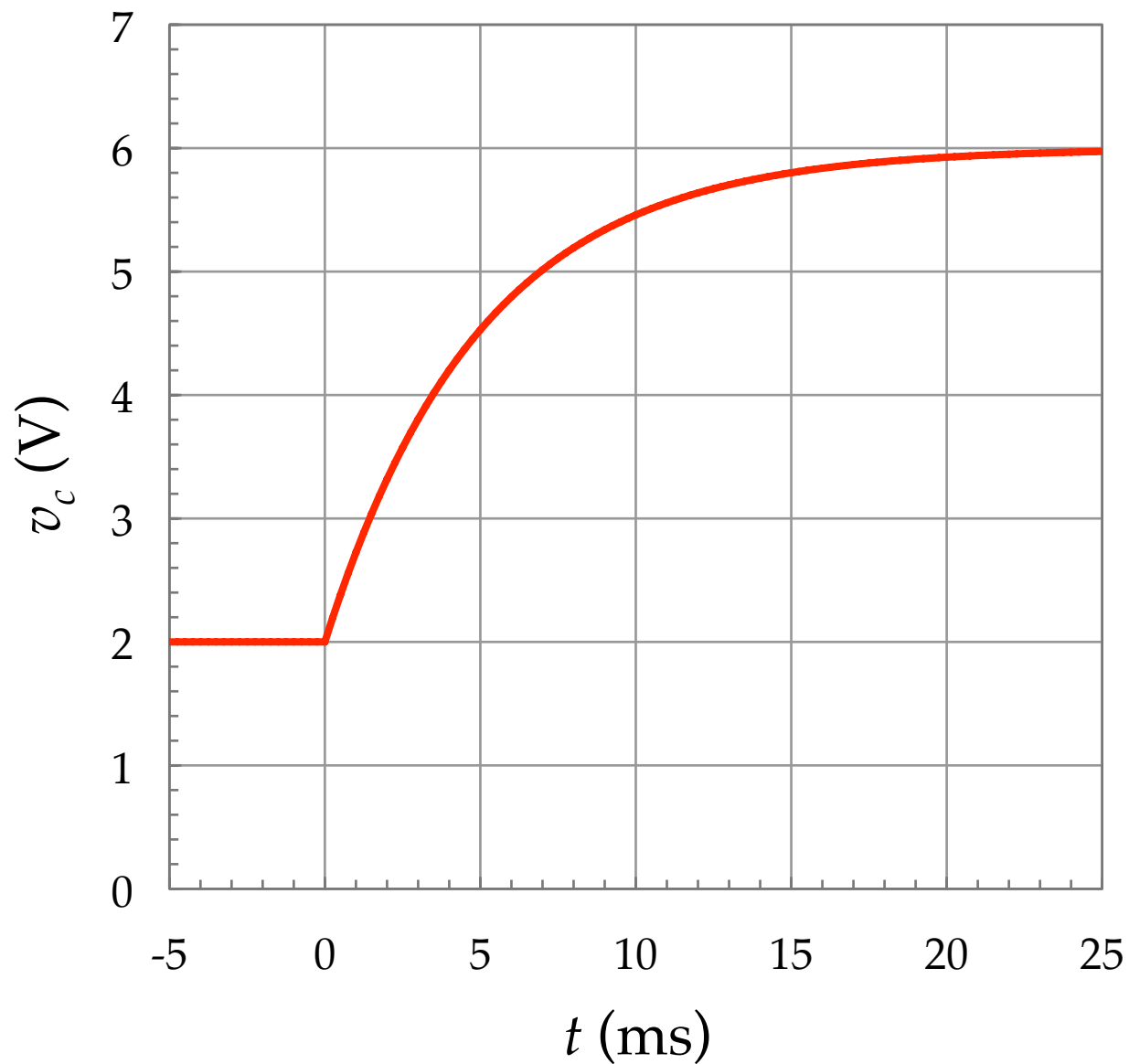
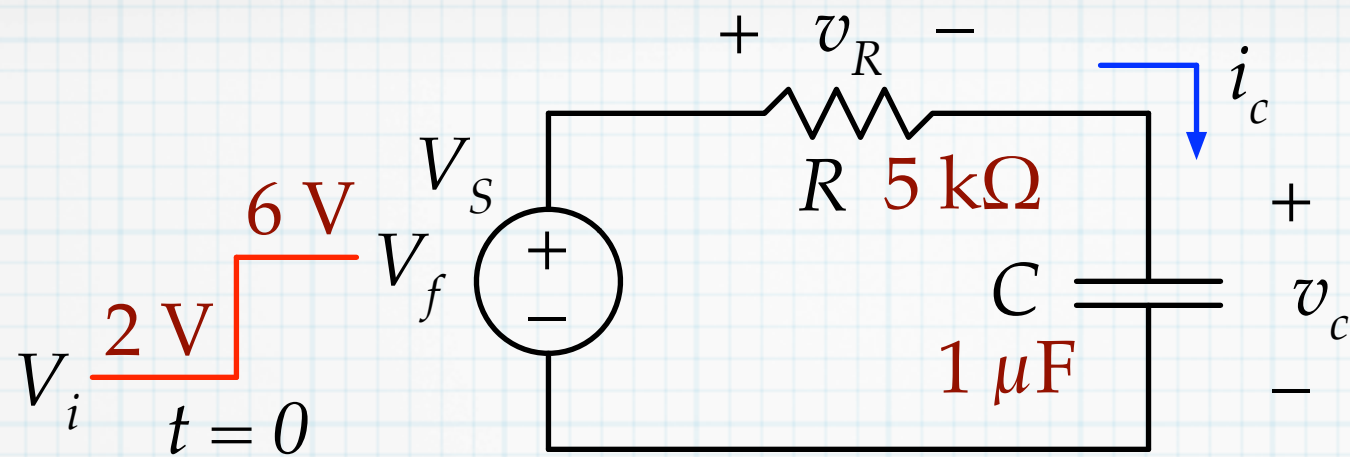
$$v_c(t) = V_f - [V_f - V_i] \exp \left(-\frac{t}{RC} \right)$$

$$v_c(t) = V_f - [V_f - V_i] \exp\left(-\frac{t}{RC}\right)$$

- At $t = 0$, $v_c = V_i$, as expected.
- As $t \rightarrow \infty$, $v_c \rightarrow V_f$, also as expected.
- In the between the capacitor voltage changes according to a decaying exponential, given above.
- At $t = 0$, i_c jumps to up to a maximum value.
- For $t > 0$, i_c decays away exponentially, as the voltage approaches is final value.

$$i_c(t) = \frac{V_f - v_c(t)}{R} \quad i_c(t) = C \frac{dv_c(t)}{dt} = \frac{V_f - V_i}{R} \exp\left(-\frac{t}{RC}\right)$$

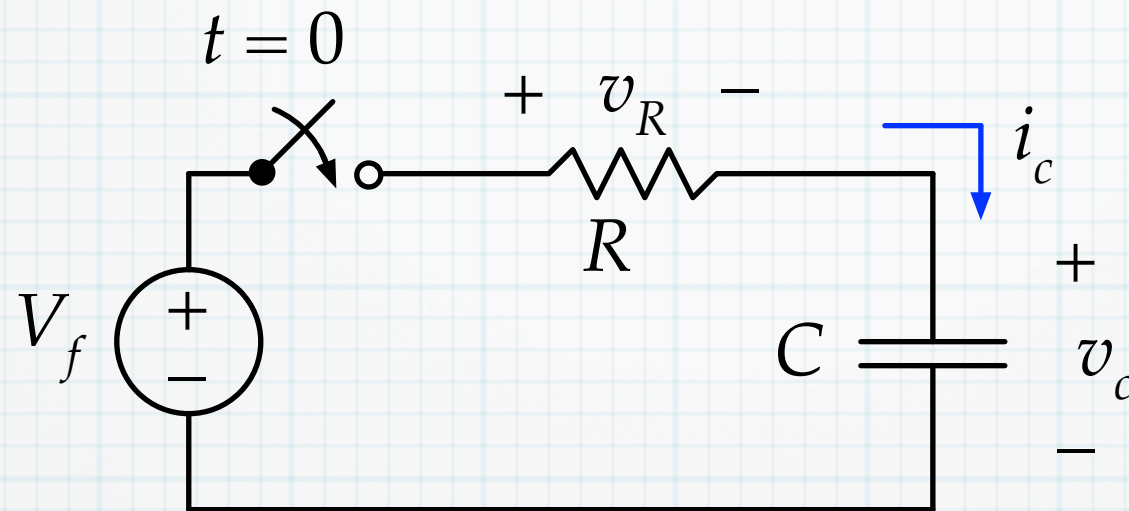
Finally, note that the equation works just as well for discharging as it does for charging. In the example, we used $V_f > V_i$, but that was never a requirement in the derivation of the equation.



Plots of capacitor voltage and current for a simple RC circuit with $V_f = 6$ V, $V_i = 2$ V, $R = 5$ kΩ, $C = 1$ μF ($RC = 5$ ms).

Using a switch

The same transient phenomena occurs when using a switch to change a circuit.



1. For $t < 0$, if the capacitor has some charge, it will have an initial voltage, $v_c(t < 0) = V_i$. Also, $i_c = 0$.
2. At $t = 0$, the switch closes. The capacitor maintains its value, $v_c(0) = V_i$. The capacitor current abruptly jumps to

$$i_c(t) = \frac{V_f - V_i}{R}$$

3. For $t > 0$, the capacitor voltage rises as the capacitor charges. The current drops as the voltage rises.

$$v_c(t) = V_f - [V_f - V_i] \exp\left(-\frac{t}{RC}\right)$$

RC time constant

- The decay is characterized by a time constant, $\tau = RC$.
(Check the units.)

- RC determines the time scale for the transient.

If $R = 1\text{k}\Omega$ and $C = 1\text{ }\mu\text{F}$, $\tau = RC = 1\text{ ms}$.

If $R = 1\text{k}\Omega$ and $C = 1\text{ nF}$, $\tau = RC = 1\text{ }\mu\text{s}$.

If $R = 100\text{ }\Omega$ and $C = 10\text{ pF}$, $\tau = RC = 1\text{ ns}$.

Changing R changes charging (or discharging) current. Changing C changes the amount of charge required.

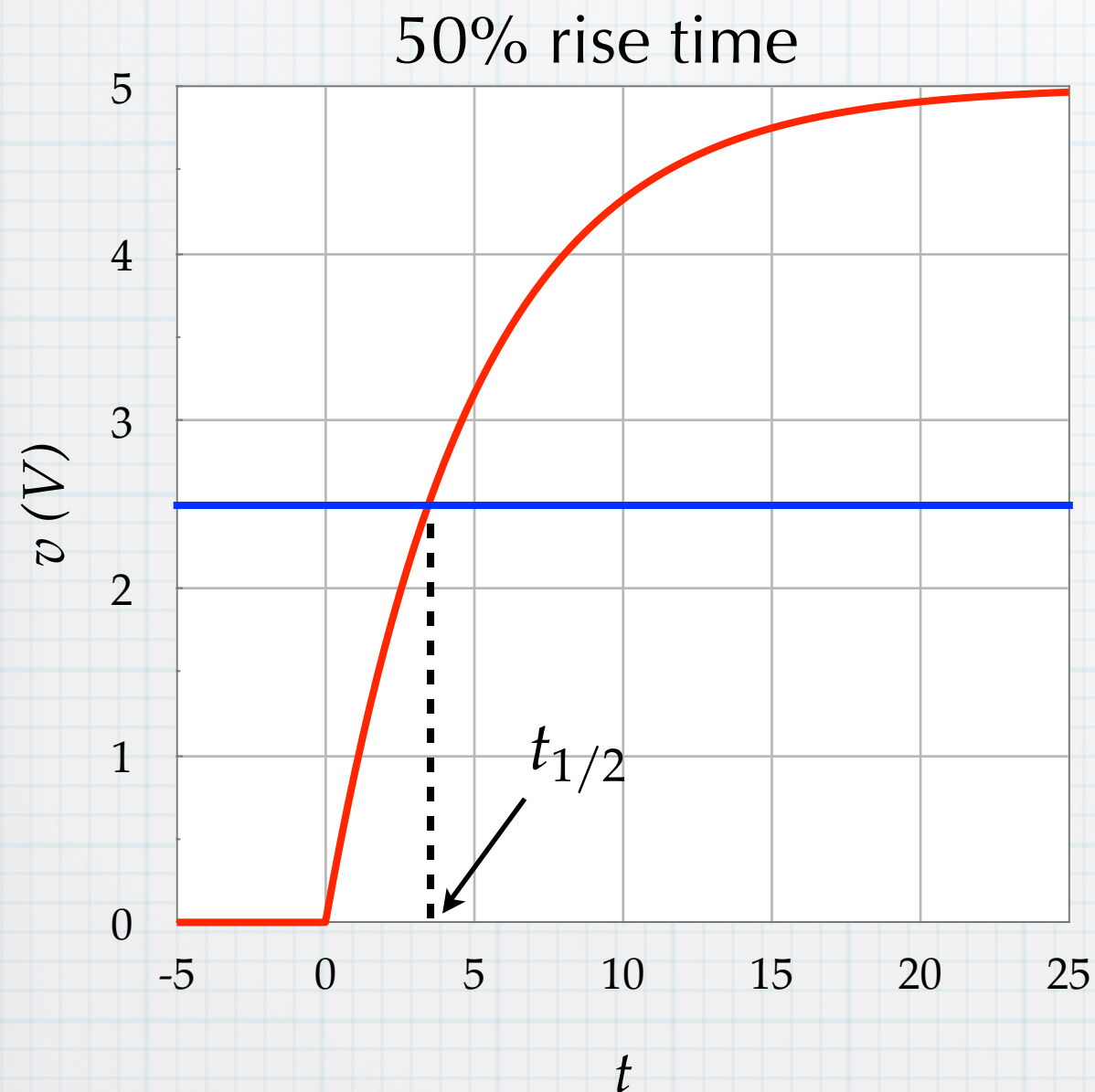
- After about 5 time constants, most of the change has occurred.

If $t = 5RC \rightarrow \exp(-5) = 0.0067 \rightarrow 99.3\%$ of the transition is done.

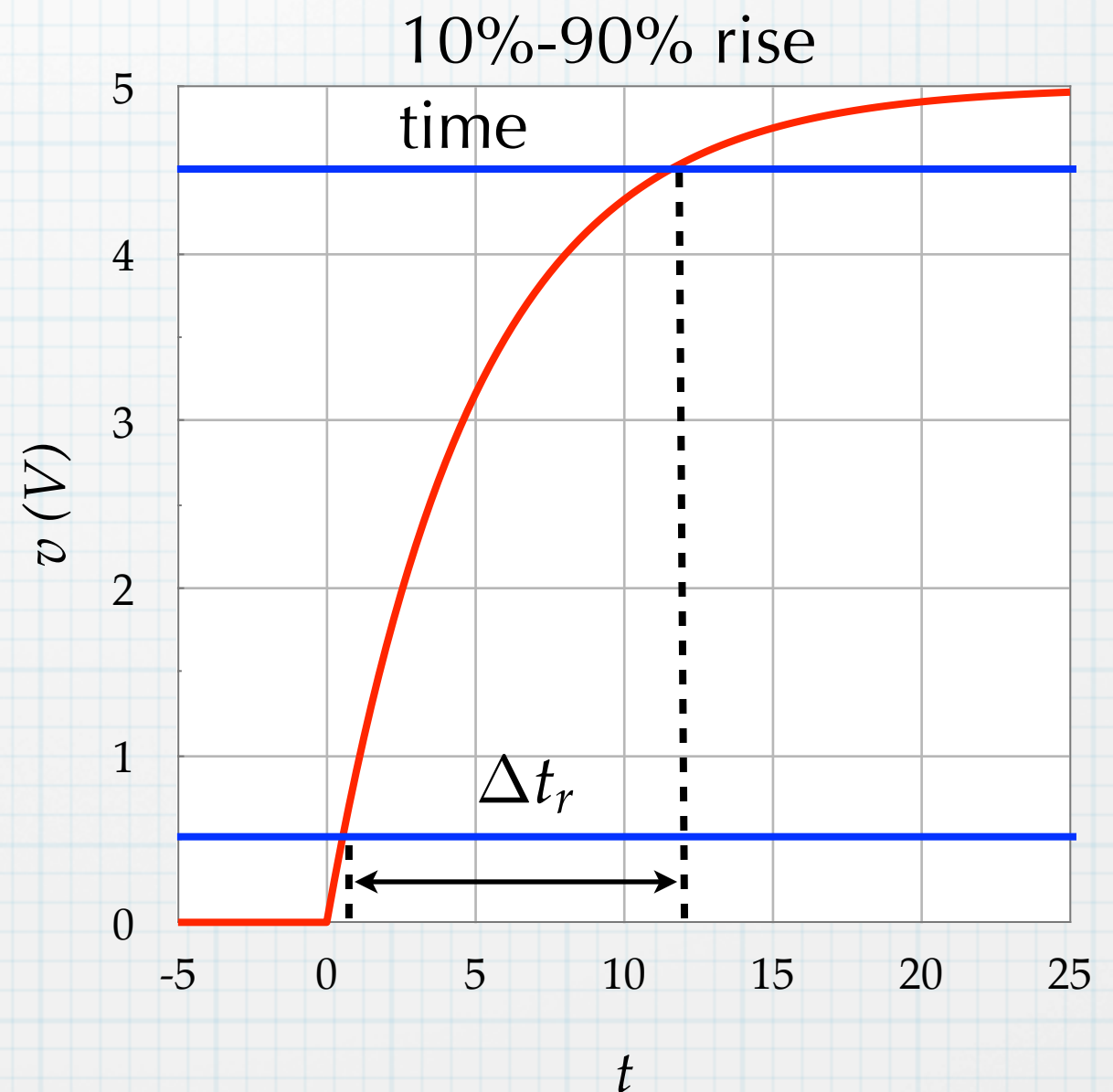
(Theoretically, v_c never really gets to V_f . But as engineers, we have to be practical.)

Rise/fall times

Non-zero transition times for voltages or currents are typical in circuits. There isn't necessarily a resistor, but there is always some capacitance or inductance present somewhere. There are a couple of commonly accepted definitions for rise/fall times.

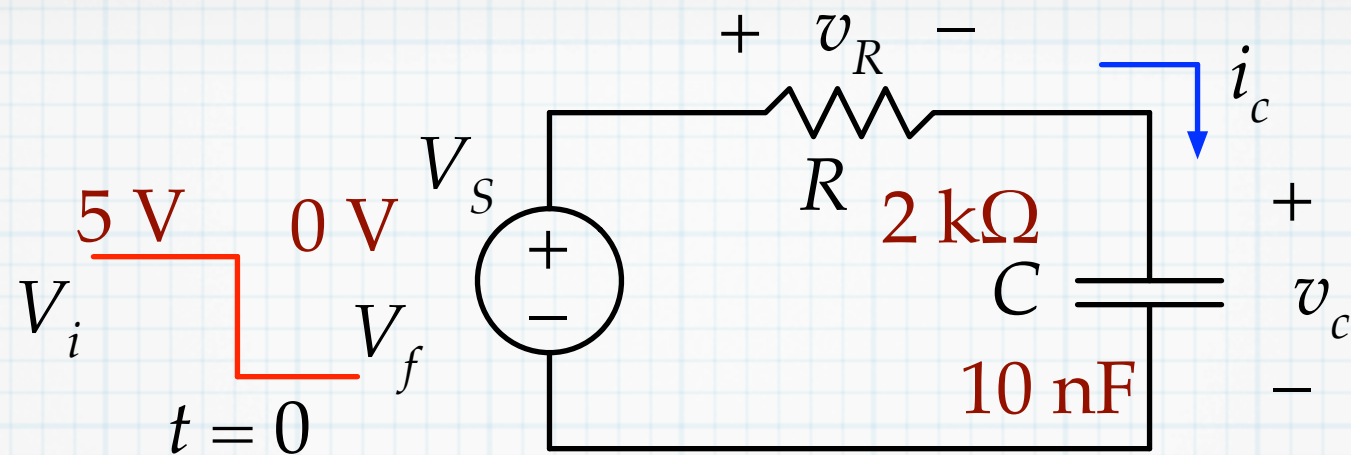


If RC circuit $t_{1/2} = 0.693RC$



$\Delta t_r = 2.20RC$

Example



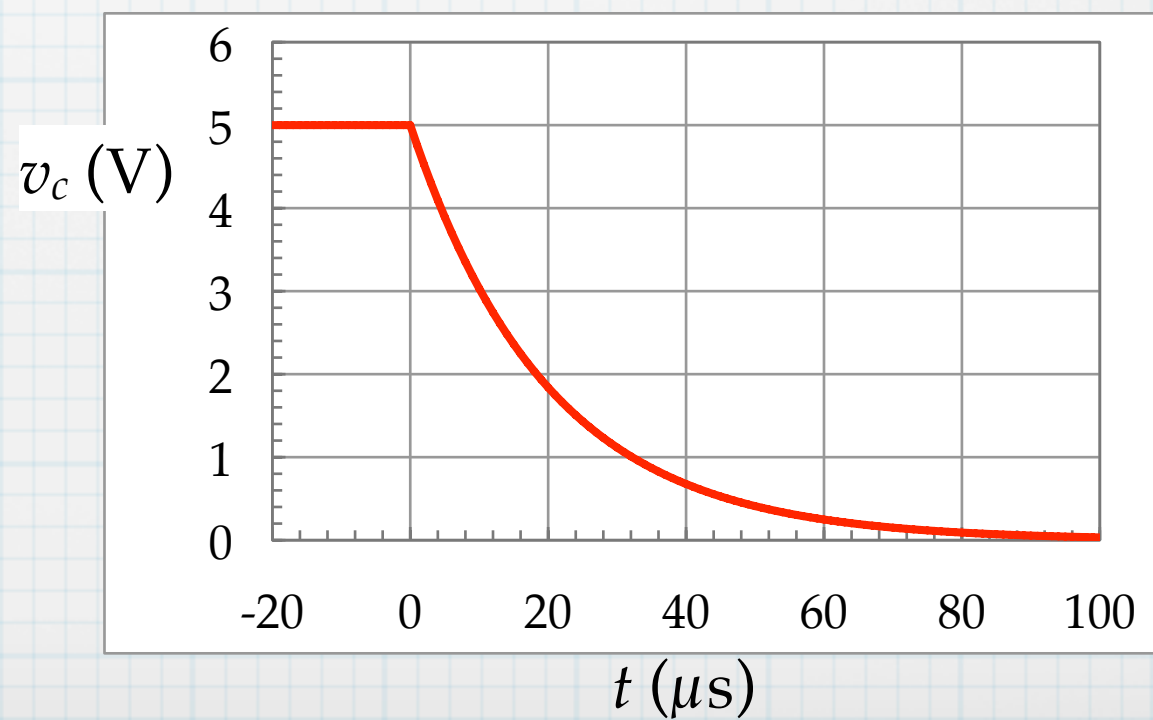
In the RC circuit above, V_s abruptly changes value from $V_i = 5\text{ V}$ to $V_f = 0$ (essentially shorted out) at $t = 0$. Assume that the source was at V_i for a “very long time” before $t = 0$. Find the expression for the capacitor voltage as a function of time. Find the time at which $v_c = 1\text{ V}$.

Apply the transient function directly, with $V_f = 0$, $V_i = 5\text{ V}$, and $RC = 20\text{ }\mu\text{s}$.

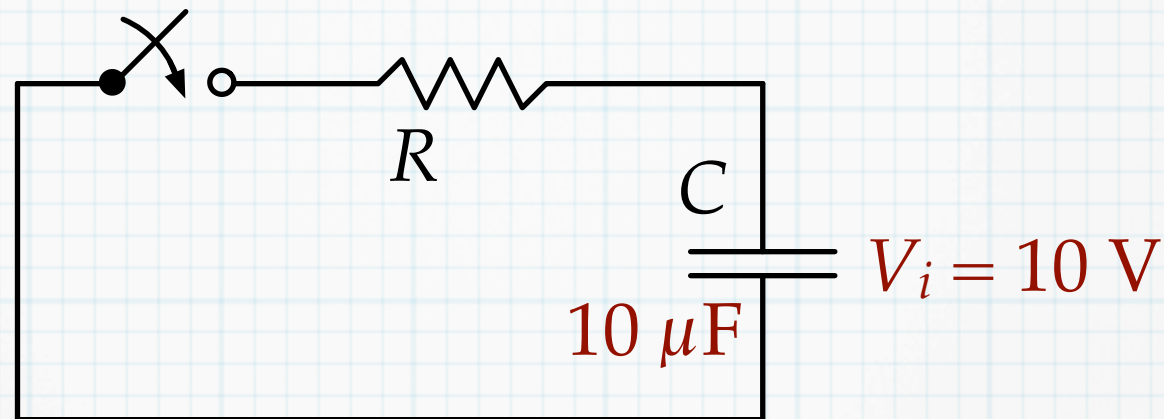
$$v_c(t) = 0 - [0 - 5\text{V}] \exp\left(-\frac{t}{20\mu\text{s}}\right) = [5\text{V}] \exp\left(-\frac{t}{20\mu\text{s}}\right)$$

$$1\text{V} = [5\text{V}] \exp\left(-\frac{t'}{20\mu\text{s}}\right)$$

$$t' = (20\mu\text{s}) \ln\left[\frac{5\text{V}}{1\text{V}}\right] = 32.2\mu\text{s}$$



Example 2



The capacitor above is charged to $V_i = 10\ \text{V}$. It is discharged by connecting a resistor in parallel (using a switch). Find the value of the resistor so that the capacitor is 90% discharged in 5 ms.

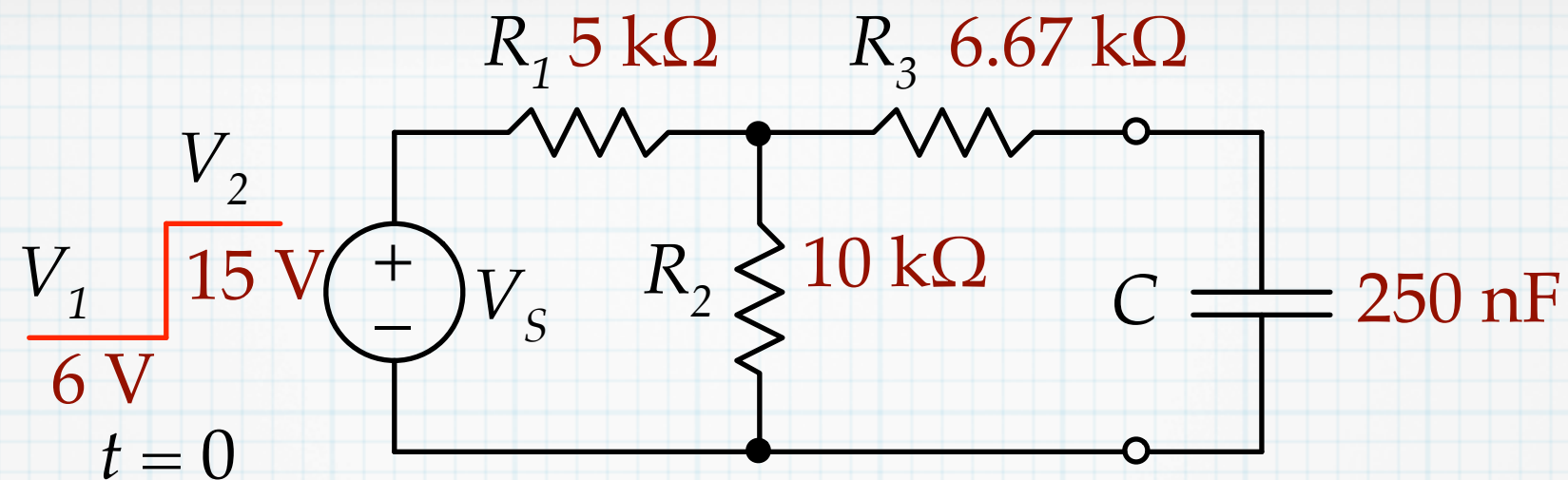
Note that this is exactly the same situation as the previous example – $V_f = 0$, so the capacitor simply discharges through the resistor. We can write the capacitor voltage equation directly.

$$v_c(t) = V_i \exp\left(-\frac{t}{RC}\right)$$

To meet the discharge requirement: (Note: at 90% discharge, $v_c = 1\ \text{V}$.)

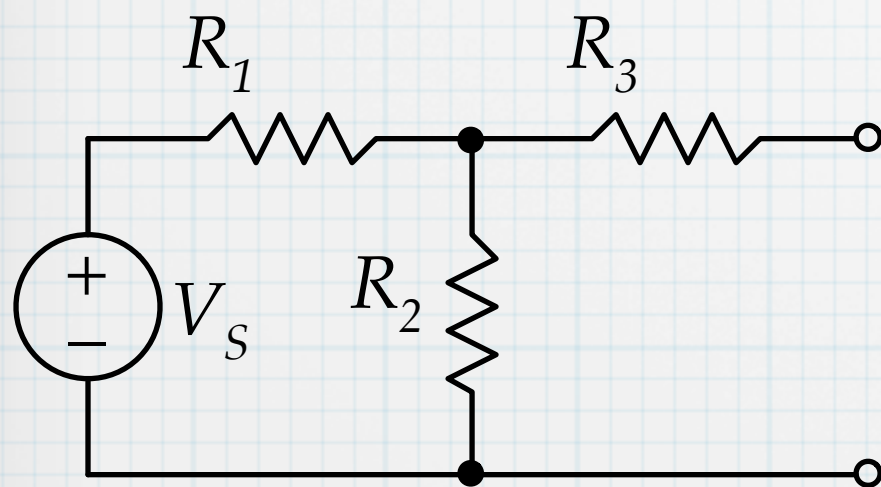
$$R = \frac{t}{C \ln\left(\frac{V_i}{v_c}\right)} = \frac{5\text{ms}}{(10\mu\text{F}) \ln\left(\frac{10\text{V}}{1\text{V}}\right)} = 217\Omega$$

Example 3



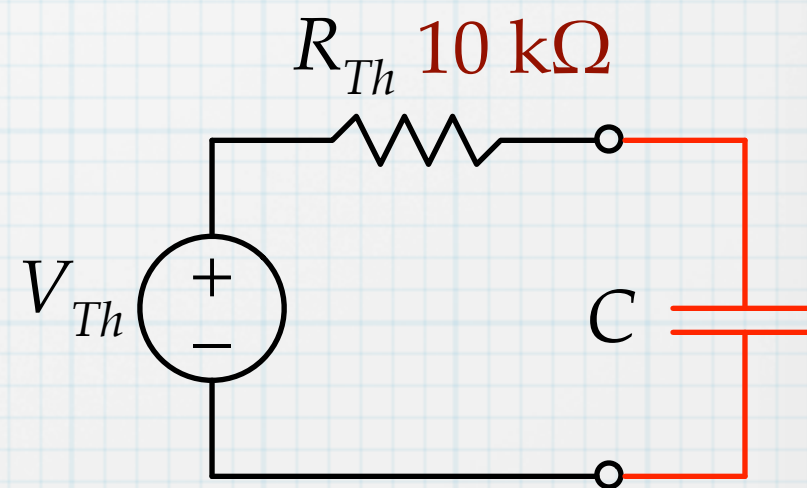
In the RC circuit above, V_s abruptly changes value from V_1 to V_2 at $t = 0$. Find the expression for the capacitor voltage as a function of time.

We could write a couple of loop or node equations, slash through some of algebra, and then solve the resulting differential equation. But instead we will use the Thevenin equivalent to make the circuit fit our equation.



$$V_{Th} = \frac{R_2}{R_1 + R_2} V_s = \frac{2}{3} V_s$$

$$R_{Th} = R_3 + R_1 || R_2 = 10 \text{ k}\Omega$$



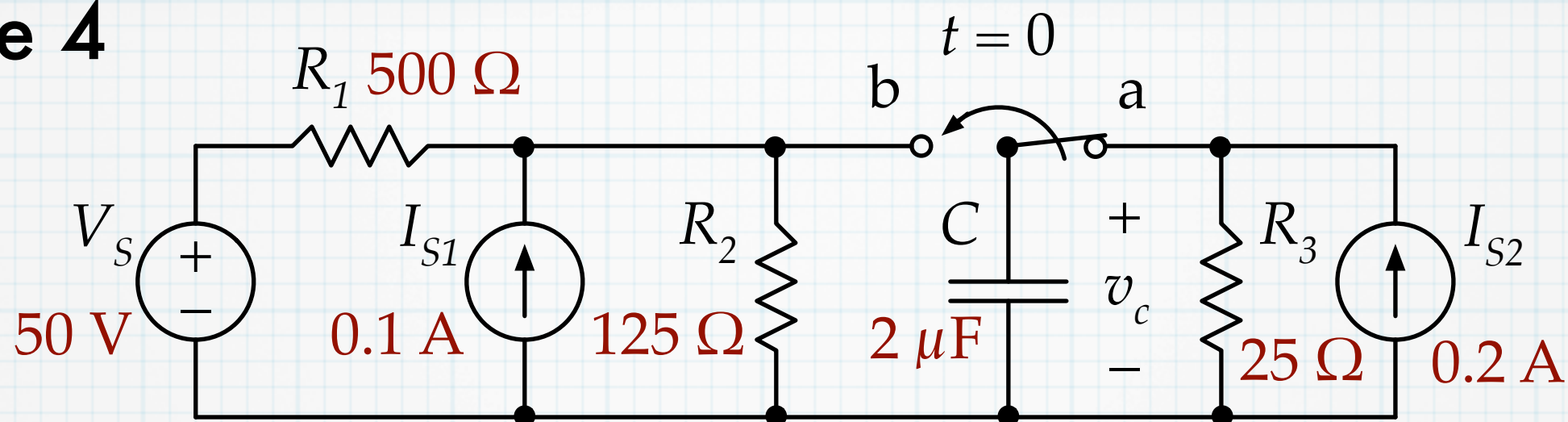
$$t < 0, V_i = 0.667 V_1 = 4 \text{ V}.$$

$$t \geq 0, V_f = 0.667 V_2 = 10 \text{ V}.$$

$$RC = (10 \text{ k}\Omega)(250 \text{ nF}) = 2.5 \text{ ms}.$$

$$v_c(t) = 10 \text{ V} - [6 \text{ V}] \exp\left(-\frac{t}{2.5 \text{ ms}}\right)$$

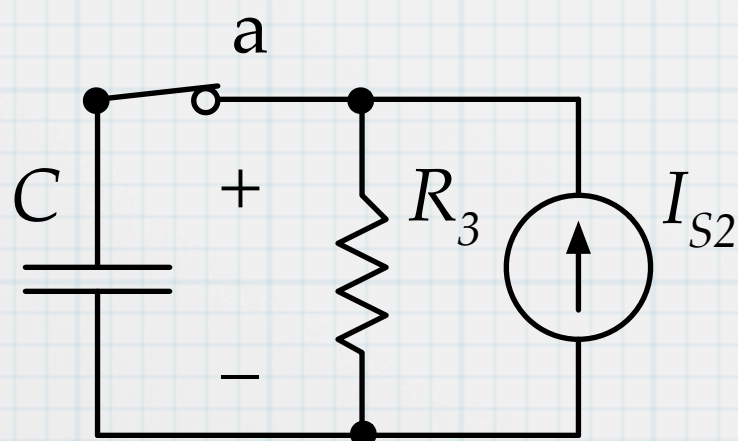
Example 4



In the circuit above, the switch has been in position a “for a very long time”. At $t = 0$, it abruptly switches to position b. Find the expression for the capacitor voltage as a function of time.

What is going on here !?!

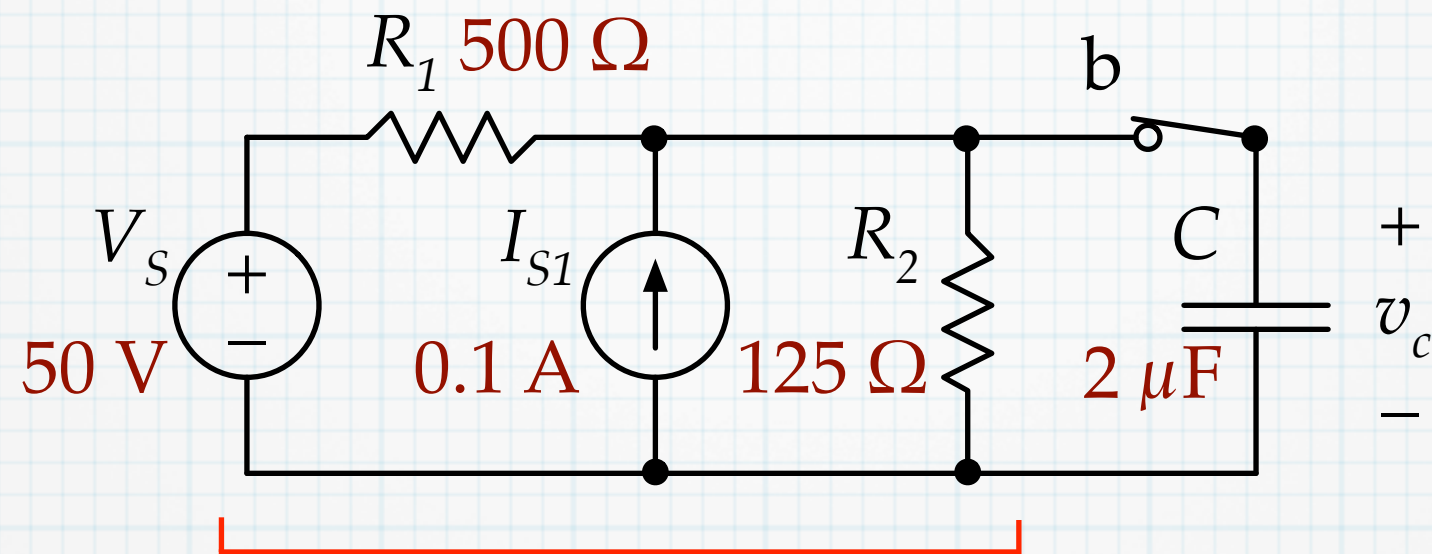
First, for $t < 0$, the portion of the circuit to the left of the switch is irrelevant – it is not connected to the capacitor. The circuit to the right of the switch determines V_i for the capacitor.



Since it has been this way for a long time (all transients have passed), $i_c = 0$.

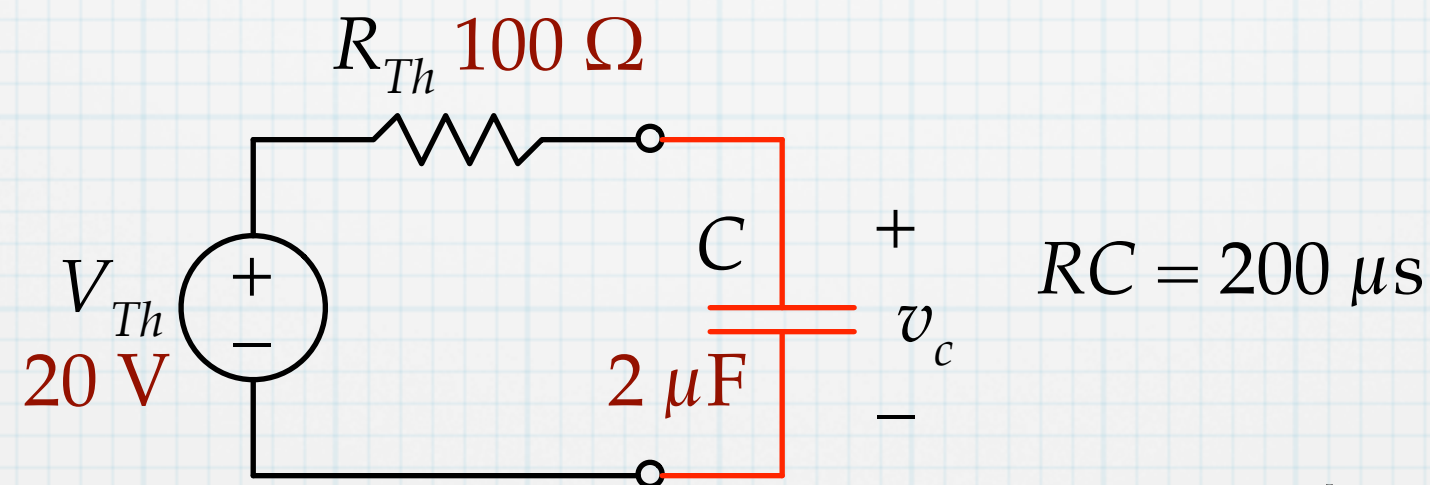
$$v_c = V_i = v_{R3} = I_{S2}R_3 = 5 \text{ V}$$

After the flip, the circuit to the right of the switch is irrelevant. The circuit to the left will determine the charging (or discharging) of the capacitor.



Find Thevenin as seen by capacitor.

Use your favorite method. Note that $V_{Th} = V_f$.



$$v_c(t) = V_f - [V_f - V_i] \exp\left(-\frac{t}{RC}\right)$$

$$= 20V - [15V] \exp\left(-\frac{t}{200\mu s}\right)$$

The capacitor starts at 5 V and charges to 20 V with a time constant of 200 μ s.

To study:

1. Work through the solution to the differential equation (slide 4) and make sure that you understand it thoroughly.
2. Try to find the solution to Example 3 *without* using the Thevenin equivalent. It can be done.
3. For the example shown on slide 6, calculate the capacitor charge before and after the transient. Then integrate the current equation and show that everything balances.
4. For the example shown on slide 6, calculate the capacitor energy before and after the transient. Calculate the total energy delivered by the source during the transient. Show that everything balances. (Don't forget about the power consumed in the resistor.)
5. For example 3, show that all of the energy stored in the capacitor is dissipated in the resistor during the discharge transient.
6. In example 4, determine the capacitor voltage function if the switch stays in position b “for a long time” and flips to position a.