SC105- Calculus with Complex Variables

Home Work on Partial Differential Equations

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1. Determine characteristic curves and reduce the following partial differential equations into one of the three canonical forms

   1. $u_{xx} - 2u_{xy} - 3u_{yy} + u_x = 0,$
   2. $x^2 u_{xx} + y^2 u_{yy} = 0,$
   3. $u_{xx} - (5 + 2x^2)u_{xy} - (1 + x^2)(4 + x^4)u_{yy} = 0,$
   4. $y^2 u_{xx} - 2xyu_{xy} - x^2 u_{yy} = \frac{y^2}{x} u_x + \frac{x^2}{y} u_y,$
   5. $y^2 u_{xx} + u_{yy} = 0$

2. Find where the following partial differential equations are elliptic, parabolic or hyperbolic

   1. $u_{xx} + xu_{xy} + yu_{yy} = u_x + u_y,$
   2. $yu_{xx} + 2u_{xy} + xu_{yy} = u + \sin(x + y),$
   3. $u_{xx} + 2xu_{xy} + (1 - y^2)u_{yy} = 0,$
   4. $yu_{xx} + 2u_{xy} + u_{yy} = uu_x + u_y^2.$

3. Reduce the following partial differential equation to the canonical form and find their solutions.

   1. $u_{xx} + xu_{yy} = \frac{1}{2x} u_x \ (x > 0),$
   2. $4y^2 u_{xx} + e^{2x} u_{yy} = e^{-2x} u_x - \frac{1}{4y^3} u_y,$
   3. $u_{xx} + 2u_{xy} + u_{yy} = 4xy,$
   4. $u_{xx} + y^4 u_{yy} = 2y^3 u_y.$

4. By use the transformation $x = r\cos\theta$, $y = \sin\theta$, show that $u_{xx} + u_{yy} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}.$
5. If u is independent of θ, solve Laplace equation $\nabla^2 u = 0$.
6. By use of cylindrical coordinates $x = r\cos\theta$, $y = r\sin\theta$, $z = z$, show that $u_{xx} + u_{yy} + u_{zz} = u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} + u_{zz}.$
7. By use of spherical poler coordinates $x = r\cos\theta\sin\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\phi$, show that $u_{xx} + u_{yy} + u_{zz} = \frac{1}{r^2}\left[\frac{\partial}{\partial r}\left(r^2 u_r\right) + \frac{1}{\sin\phi}\frac{\partial}{\partial \phi}\left(\sin\phi\ u_\phi\right) + \frac{1}{\sin^2\phi}u_{\theta\theta}\right].$
8. If u is independent of θ and ϕ, solve Laplace equation $\nabla^2 u = 0$.
9. Reduce two dimensional wave equation $\frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$ to canonical form and find solution. Interpret obtained solution as wave motion.
10. Find all possible solutions with spherical symmetry of the following equation $\nabla^2 u + a^2 u = 0$.