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UNIT - 1

Transmisson Lines and Waveguides

★ **Introduction:**

conventional two conductor transmission lines are commonly used for transmitting microwave energy. If a line is properly matched to its characteristic impedence at each terminal, its efficiency can reach maximum.

usually all impedances elements are assumed as lumped constants but it isn't true for a long transmission line over a wide range of frequencies. Frequencies of operation are so high that inductances of short lengths of conductors and capacitances between short conductors and their surroundings cannot be neglected. these inductances and capacitances are distributed along the length of a conductor and their effects combine at each point of the conductor. since the wavelength is short in comparison to the physical length of the line, distributed parameters cannot be represented accurately by means of a lumped-parameter equivalent circuit.

Thus microwave transmission lines can be analyzed in terms of voltage, current and impedance only by the distributed-circuit theory. If the spacing between the lines is smaller than the wavelength of the transmitted signal, the transmission line must be analyzed as a waveguide.

★ **Transmission - line Equations and Solutions:**

A transmission line can be analyzed by the solution of:

- Maxwell's field equations
- methods of distributed-circuit theory.

The solution of Maxwell's equations involve three space variables in addition to the time variable. The distributed circuit method involves only one space variable in addition to time variable.

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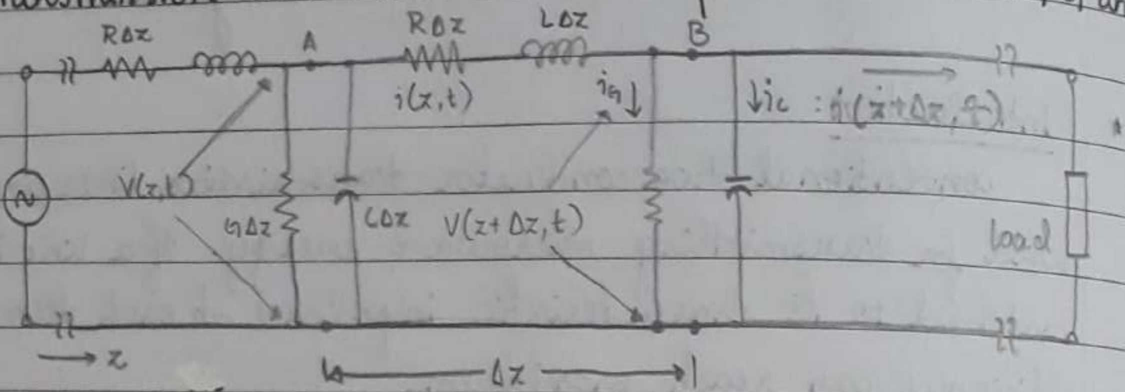
- **Distributed Circuit Theory:**

The schematic circuit of a conventional two-conductor transmission line with constant parameters R, L, G and C



The parameters are expressed per unit length and the wave propagation is assumed in the positive z direction

ie. R : Ω/m ; L : H/m ; C : F/m ; G : ℧/m

Applying KVL to loop 2

$$v(z,t) = i(z,t)R\Delta z + L\Delta z \frac{\partial i(z,t)}{\partial t} + v(z+\Delta z,t)$$

$$v(z,t) - v(z+\Delta z,t) = i(z,t)R\Delta z + L\Delta z \frac{\partial i(z,t)}{\partial t}$$

$$\frac{v(z,t) - v(z+\Delta z,t)}{\Delta z} = i(z,t)R + L\frac{\partial i(z,t)}{\partial t}$$

applying limit $\Delta z \to 0$

$$\frac{-\partial v}{\partial z} = Ri + L\frac{\partial i}{\partial t} \quad \text{——} (1)$$

here $-\partial v = -[v(z+\Delta z,t) - v(z,t)]$

Applying KCL at point B.

$$i(z,t) = i(z+\Delta z,t) + v(z+\Delta z,t)G\Delta z + C\Delta z \frac{\partial v(z+\Delta z,t)}{\partial t}$$

$$i(z,t) - i(z+\Delta z,t) = v(z+\Delta z,t)G\Delta z + C\Delta z \frac{\partial v(z+\Delta z,t)}{\partial t}$$

$$\frac{i(z,t) - i(z+\Delta z,t)}{\Delta z} = v(z+\Delta z,t)G + C\frac{\partial v(z+\Delta z,t)}{\partial t}$$

applying limit $\Delta z \to 0$

$$\frac{-\partial i}{\partial z} = Gv + C\frac{\partial v}{\partial t} \quad \text{——} (2)$$

here $-\partial i = -[i(z+\Delta z,t) - i(z,t)]$

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Differentiating eq ① wrt z, we get

$$-\frac{\partial^2 v}{\partial z^2} = R\frac{\partial i}{\partial z} + L\frac{\partial}{\partial z}\frac{\partial i}{\partial t} \quad \text{——— ③}$$

Substituting $\partial i/\partial z$ from eq ② in eq ③, we get

$$-\frac{\partial^2 v}{\partial z^2} = -R\left[Gv + C\frac{\partial v}{\partial t}\right] - L\frac{\partial}{\partial t}\left[Gv + C\frac{\partial v}{\partial t}\right]$$

$$\frac{\partial^2 v}{\partial z^2} = RGv + RC\frac{\partial v}{\partial t} + LG\frac{\partial v}{\partial t} + LC\frac{\partial^2 v}{\partial t^2}$$

$$\boxed{\frac{\partial^2 v}{\partial z^2} = RGv + \frac{\partial v}{\partial t}(RC + LG) + LC\frac{\partial^2 v}{\partial t^2}} \quad \text{——— ④}$$

The final transmission line equation in voltage form

Similarly

Differentiating eq ② wrt z, we get

$$-\frac{\partial^2 i}{\partial z^2} = G\frac{\partial v}{\partial z} + C\frac{\partial}{\partial z}\frac{\partial v}{\partial t} \quad \text{——— ⑤}$$

Substituting $\partial v/\partial z$ from eq ① in eq ⑤, we get

$$-\frac{\partial^2 i}{\partial z^2} = -G\left[Ri + L\frac{\partial i}{\partial t}\right] - C\frac{\partial}{\partial t}\left[Ri + L\frac{\partial i}{\partial t}\right]$$

$$\frac{\partial^2 i}{\partial z^2} = GRi + GL\frac{\partial i}{\partial t} + RC\frac{\partial i}{\partial t} + LC\frac{\partial^2 i}{\partial t^2}$$

$$\boxed{\frac{\partial^2 i}{\partial z^2} = RGi + \frac{\partial i}{\partial t}\left[GL + RC\right] + LC\frac{\partial^2 i}{\partial t^2}} \quad \text{——— ⑥}$$

The final transmission line equation in current form

All these transmission line equations are also applicable to general transient solution. The voltage and current on the line are the functions of both position z and time t.

The instantaneous line voltage and current can be expressed as:

$$v(z,t) = \text{Re}\, V(z)e^{j\omega t} \quad \text{——— ⑦}$$

$$i(z,t) = \text{Re}\, I(z)e^{j\omega t} \quad \text{——— ⑧}$$

where Re stands for real part of.

$V(z)$ and $I(z)$ are complex quantities of the sinusoidal functions of position z on the line and is known as phasors.

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The phasors give the magnitudes and phases of the sinusoidal function at each position of z, and they can be expressed as :

$$V(z) = V_+ e^{-\gamma z} + V_- e^{\gamma z} \quad \text{— (9)}$$

$$I(z) = I_+ e^{-\gamma z} + I_- e^{\gamma z} \quad \text{— (10)}$$

$$\gamma = \alpha + j\beta \quad \text{— (11)}$$

(propagation constant)

where : V_+ and I_+ are complex amplitudes in positive z direction

V_- and I_- are complex amplitudes in negative z direction

α : attenuation constant (nepers per unit length)

β : phase constant (radians per unit length)

Substituting $j\omega$ for $\partial/\partial t$

For eq (1)

$$\frac{-\partial v}{\partial z} = Ri + Lj\omega i \quad \text{— (12)}$$

differentiating wrt z

$$\frac{\partial^2 v}{\partial z^2} = -(R + j\omega L)\frac{\partial i}{\partial z}$$

From eq (2)

$$\frac{\partial^2 v}{\partial z^2} = [-(R + j\omega L)][-(Gv + Cj\omega v)]$$

$$\frac{\partial^2 v}{\partial z^2} = (R + j\omega L)(G + j\omega C)v$$

$$\frac{\partial^2 v}{\partial z^2} = \gamma^2 v \quad \text{— (13)}$$

because: $\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$

For eq (2)

$$-\frac{\partial i}{\partial z} = Gv + Cj\omega v \quad \text{— (14)}$$

differentiating wrt z

$$\frac{\partial^2 i}{\partial z^2} = \frac{-\partial v}{\partial z}(G + j\omega C)$$

From eq (1)

$$\frac{\partial^2 i}{\partial z^2} = [-(G+j\omega C)][-(Ri+Lj\omega i)]$$

$$\frac{\partial^2 i}{\partial z^2} = (R+Lj\omega)(G+j\omega C)\, i$$

$$\frac{\partial^2 i}{\partial z^2} = \gamma^2 i \quad \text{— (15)}$$

because $\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$

From eq (12), (13), (14) and (15)

$$\frac{dV}{dz} = -Z I \quad \text{— (16)}$$

because $R+j\omega L = Z$

$$\frac{dI}{dz} = -Y V \quad \text{— (17)}$$

because $G+j\omega C = Y$

$$\frac{d^2V}{dz^2} = \gamma^2 V \quad \text{— (18)}$$

$$\frac{d^2I}{dz^2} = \gamma^2 I \quad \text{— (19)}$$

These are the transmission line equations in phasor form of the frequency domain.

the following is substituted in above equations:

$Z = R + j\omega L$ (Ω/m)

$Y = G + j\omega C$ $(\mho/m)$

$\gamma = \sqrt{ZY} = \alpha + j\beta$ (propagation constant)

For a lossless line

$R = G = 0$

Therefore

$$\frac{dV}{dz} = -j\omega L I$$

$$\frac{dI}{dz} = -j\omega C V$$

$$\frac{d^2V}{dz^2} = -\omega^2 LCV$$

$$\frac{d^2I}{dz^2} = -\omega^2 LCI$$

Eq (18) and (19) are similar to equations of the electric and magnetic waves respectively but these transmission-line equations are one-dimensional.

- **Solutions of Transmission line Equations:**

The equation (18) is given as:

$$\frac{d^2 V}{dz^2} = \gamma^2 V$$

The possible solution for the above equation is

$$V = V_+ e^{-\gamma z} + V_- e^{\gamma z}$$

$$V = V_+ e^{-\alpha z} e^{-j\beta z} + V_- e^{\alpha z} e^{j\beta z} \quad \text{——} \quad (20)$$

The term involving $e^{-j\beta z}$ shows a wave travelling in positive z direction and term involving $e^{j\beta z}$ shows a wave travelling in negative z direction.

βz is called the electrical length of the line and is measured in radians.

The equation (19) is given as:

$$\frac{d^2 I}{dz^2} = \gamma^2 I$$

The possible solution for the above equation is:

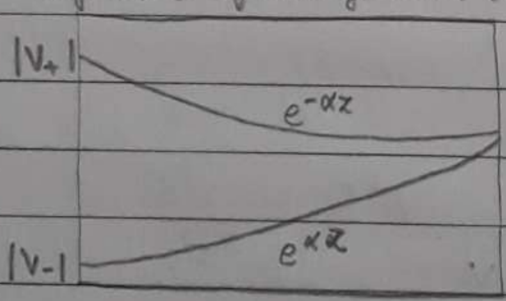
$$I = Y_0 (V_+ e^{-\gamma z} - V_- e^{\gamma z})$$

$$I = Y_0 (V_+ e^{-\alpha z} e^{-j\beta z} - V_- e^{\alpha z} e^{j\beta z}) \quad \text{——} \quad (21)$$

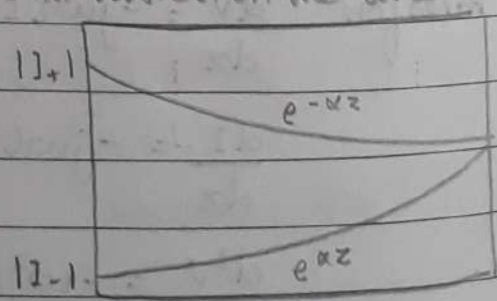
From the above equation, the characteristic impedance of the line is defined as:

$$Z_0 = \frac{1}{Y_0} = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = R_0 \pm j X_0 \quad \text{——} \quad (22)$$

Magnitude of voltage and current waves on the line:



voltage wave



current wave

At microwave frequencies:

$$R \ll \omega L \quad \text{and} \quad G \ll \omega C$$

By using the binomial expansion, the propagation constant can be expressed as:

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$\gamma = \sqrt{(j\omega)^2 LC}\sqrt{\left(1+\frac{R}{j\omega L}\right)\left(1+\frac{G}{j\omega C}\right)}$$

$$\gamma \approx j\omega\sqrt{LC}\left[\left(1+\frac{R}{2j\omega L}\right)\left(1+\frac{G}{2j\omega C}\right)\right]$$

$$\gamma \approx j\omega\sqrt{LC}\left[1+\frac{1}{2}\left(\frac{R}{j\omega L}+\frac{G}{j\omega C}\right)\right]$$

$$\gamma = \frac{1}{2}\left[R\sqrt{\frac{C}{L}}+G\sqrt{\frac{L}{C}}\right]+j\omega\sqrt{LC} \quad \text{—— (23)}$$

Therefore, the attenuation constant is given by

$$\boxed{\alpha = \frac{1}{2}\left[R\sqrt{\frac{C}{L}}+G\sqrt{\frac{L}{C}}\right]} \quad \text{—— (24)}$$

and the phase constant is given by

$$\boxed{\beta = \omega\sqrt{LC}} \quad \text{—— (25)}$$

Similarly, the character impedance is found to be

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

$$Z_0 = \sqrt{\frac{L}{C}}\left(1+\frac{R}{j\omega L}\right)^{1/2}\left(1+\frac{G}{j\omega C}\right)^{-1/2}$$

$$Z_0 \approx \sqrt{\frac{L}{C}}\left(1+\frac{R}{2j\omega L}\right)\left(1-\frac{G}{2j\omega C}\right)$$

$$Z_0 \approx \sqrt{\frac{L}{C}}\left[1+\frac{1}{2}\left(\frac{R}{j\omega L}-\frac{G}{j\omega C}\right)\right]$$

$$\boxed{Z_0 \approx \sqrt{\frac{L}{C}}} \quad \text{—— (26)}$$

From eq (25), the phase velocity is

$$v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = c = 3\times10^8\ m/s \quad \text{—— (27)}$$

when the dielectric of a lossy microwave transmission line is not air, the phase velocity is smaller than the velocity of light in vacuum and is given by:

$$v_e = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{c}{\sqrt{\mu_r \varepsilon_r}} \quad \text{—— (28)}$$

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In general, the relative phase velocity factor can be defined as:

$$\text{velocity factor} = \frac{\text{actual phase velocity}}{\text{velocity of light in vacuum}}$$

$$\boxed{v_r = \frac{v_e}{c} = \frac{1}{\sqrt{\mu_r \varepsilon_r}}} \quad \text{—— (29)}$$

A loss-loss transmission line filled only with dielectric medium, such as a coaxial line with solid dielectric between conductors, has a velocity factor on the order of about 0.65.

Q1: A transmission line has the following parameters:

$R = 2\Omega/m \qquad G = 0.5\,m℧/m \qquad f = 1\,GHz$

$L = 8\,nH/m \qquad C = 0.23\,pF$

calculate: a) the characteristic impedance (Z_0)

b) the propagation constant.

a. the characteristic impedence

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$$Z_0 = \sqrt{\frac{2 + j\,2\pi(10^9)\,8(10^{-9})}{0.5(10^{-3}) + j\,2\pi(10^9)\,0.23(10^{-12})}}$$

$$Z_0 = \sqrt{\frac{2 + 16\pi j}{0.5(10^{-3}) + 0.46\pi j(10^{-3})}}$$

$$Z_0 = \sqrt{\frac{50.31\,\angle 87.72^\circ}{1.53(10^{-3})\,\angle 70.91^\circ}} = 181.33\,\angle 8.4^\circ = 179.38 + 26.58j$$

b. The propagation constant

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$\gamma = \sqrt{(2 + j2\pi(10^9)8(10^{-9}))(0.5(10^{-3}) + j2\pi(10^9)0.23(10^{-12}))}$$

$$\gamma = \sqrt{(2 + 50.26j)(0.5\times10^{-3} + 1.45\times10^{-3}j)}$$

$$\gamma = \sqrt{(50.31\,\angle 87.72^\circ)(1.53\times10^{-3}\,\angle 70.97^\circ)}$$

$$\gamma = \sqrt{76.97\times10^{-3}\,\angle 158.69^\circ}$$

$$\gamma = 0.2774\,\angle 79.35^\circ = 0.051 + 0.273j$$

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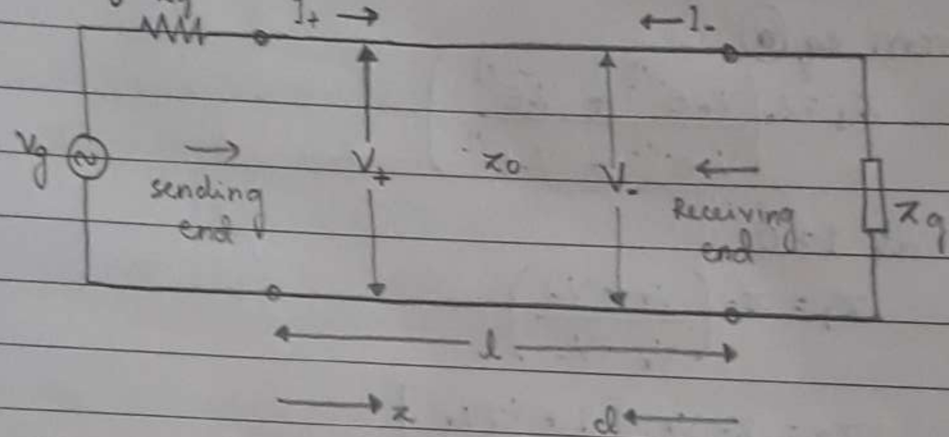
★ Reflection Coefficient and Transmission coefficient:

- **Reflection coefficient:**

The travelling wave along the line contains two components: one travelling in the positive z direction and the other travelling the negative x direction. If the load impedance is equal to the line characteristic impedance, the reflected travelling wave does not exist.



Transmission line terminated in a load impedance

The incident voltage and current waves travelling along the transmission line are given by:

$$V = V_+ e^{-\gamma z} + V_- e^{+\gamma z} \quad \text{---} (1)$$

$$I = I_+ e^{-\gamma z} - I_- e^{+\gamma z} \quad \text{---} (2)$$

in which the current wave can be expressed in terms of the voltage by:

$$I = \frac{V_+}{Z_0} e^{-\gamma z} - \frac{V_-}{Z_0} e^{\gamma z} \quad \text{---} (3)$$

$Z_0 = V_+ / I_+ \; ; -Z_0 = V_- / I_-$

If the line has a length of l, the voltage and current at the receiving end become

$$V_l = V_+ e^{-\gamma l} + V_- e^{\gamma l} \quad \text{---} (4)$$

$$I_l = \frac{1}{Z_0}\left(V_+ e^{-\gamma l} - V_- e^{\gamma l} \right) \quad \text{---} (5)$$

The ratio of voltage to current at the receiving end is the load impedance

$$Z_l = \frac{V_l}{I_l} = Z_0 \left(\frac{V_+ e^{-\gamma l} + V_- e^{\gamma l}}{V_+ e^{-\gamma l} - V_- e^{\gamma l}} \right) \quad \text{---} (6)$$

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The reflection coefficient is defined as:

Reflection coefficient = $\dfrac{\text{Reflected voltage or current}}{\text{Incident voltage or current.}}$

$$\Gamma = \frac{V_{ref}}{V_{inc}} = \frac{I_{ref}}{I_{inc}} \quad \text{——— (7)}$$

The reflection coefficient at the receiving end

$$\Gamma_l = \frac{V_- e^{\gamma l}}{V_+ e^{-\gamma l}} \quad \text{——— (8)}$$

From eq (6)

$$Z_l = Z_0 \left(\frac{V_+ e^{-\gamma l} + V_- e^{\gamma l}}{V_+ e^{-\gamma L} - V_- e^{\gamma L}} \right)$$

at $l = 0$

$$Z_l = Z_0 \left(\frac{V_+ + V_-}{V_+ - V_-} \right)$$

$$Z_l = Z_0 \frac{V_+}{V_+} \left[\frac{1 + V_-/V_+}{1 - V_-/V_+} \right]$$

$$Z_l = Z_0 \left[\frac{1+\Gamma}{1-\Gamma} \right]$$

$$Z_l - Z_l\Gamma = Z_0 + Z_0\Gamma$$

$$\Gamma(Z_0 + Z_l) = Z_l - Z_0$$

$$\boxed{\Gamma_l = \frac{Z_l - Z_0}{Z_l + Z_0}} \quad \text{——— (9)}$$

The reflection coefficient is a complex quantity that can be expressed as:

$$\Gamma_l = |\Gamma_l|\, e^{j\theta_l} \quad \text{——— (10)}$$

where: $|\Gamma_l|$: magnitude $\quad |\Gamma_l| \le 1$

θ_l : phase angle between the incident and reflected voltages at receiving end (phase angle of the reflection coefficient)

The general solution of the reflection coefficient at any point on the line corresponds to the incident and reflected waves at that point, each attenuated in the direction of its own progress along the line.

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The generalized reflection coefficient is defined as

$$\Gamma = \frac{V_- e^{\gamma z}}{V_+ e^{-\gamma z}}$$

From the figure, let $z = l - d$

Then the reflection coefficient at some point located at a distance d from the receiving end is.

$$\Gamma_d = \frac{V_- e^{\gamma(l-d)}}{V_+ e^{-\gamma(l-d)}} = \frac{V_- e^{\gamma l}}{V_+ e^{-\gamma L}} e^{-2\gamma d}$$

$$\therefore \Gamma_d = \Gamma_l e^{-2\gamma d} \quad \text{———} \quad (11)$$

but wkt $\gamma = \alpha + j\beta$

Therefore

$$\boxed{\begin{aligned} \Gamma_d &= \Gamma_l e^{-2\alpha d} e^{-2j\beta d} \\ \Rightarrow \Gamma_d &= |\Gamma_l| e^{-2\alpha d} e^{j(\theta_l - 2\beta d)} \end{aligned}} \quad \text{———} \quad (12)$$

For a lossy line, both the magnitude and phase of the reflection coefficient are changing in an inward spiral.

For a loss-less line, $\alpha = 0$, the magnitude of the reflection coefficient remains constant and only the phase of Γ is changing circularly toward the generator with an angle of $-2\beta d$.

Γ_l will be zero and there will be no reflection from the receiving end when the terminating impedance is equal to the characteristic impedance of the line.

Thus a terminating impedance that differs from the characteristic impedance will create a reflected wave travelling toward the source from the termination. The reflection, upon reaching the sending end, will itself be reflected if the source impedance is different from the line characteristic impedance at the sending end.

- Transmission coefficient

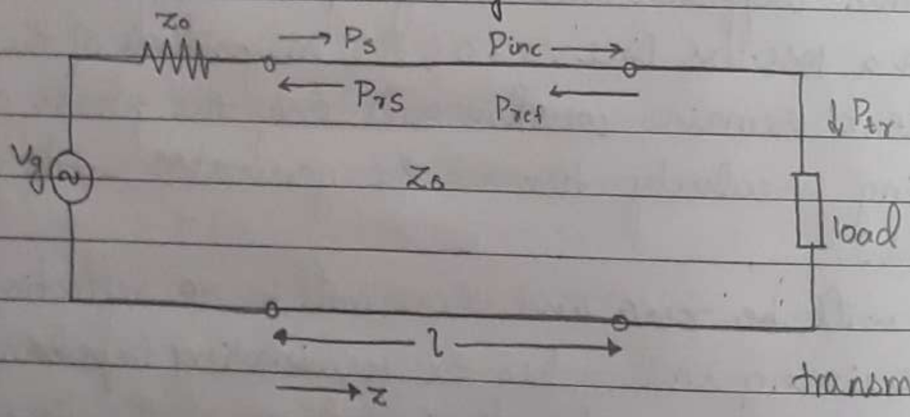
A transmission line terminated in its characteristic impedance Z_0 is called a properly terminated line. Otherwise it is called an improperly terminated line. There is a reflection coefficient Γ at any point along an improperly terminated line.

According to the principle of conservation of energy, the incident power minus the reflected power must be equal to the power transmitted to the load. This can be expressed as:

$$1 - \Gamma_l^2 = \frac{Z_0}{Z_l} T^2 \quad \text{——— (13)}$$

where T is the transmission coefficient and it is defined as:

$$T = \frac{\text{transmitted voltage or current}}{\text{incident voltage or current}} = \frac{V_{tr}}{V_{inc}} = \frac{I_{tr}}{I_{inc}} \quad \text{——— (14)}$$



Power transmission on a line

Let the traveling waves at the receiving end be

$$V_+ e^{-\gamma l} + V_- e^{\gamma l} = V_{tr} e^{-\gamma l}$$

$$\frac{V_+}{Z_0} e^{-\gamma l} - \frac{V_- e^{\gamma l}}{Z_0} = \frac{V_{tr}}{Z_l} e^{-\gamma l}$$

wkt $\Gamma_l = \dfrac{V_- e^{\gamma l}}{V_+ e^{-\gamma l}} = \dfrac{Z_l - Z_0}{Z_l + Z_0}$

$$T = \frac{V_{tr}}{V_+} = \frac{V_+ e^{-\gamma l} + V_- e^{\gamma l}}{V_+ e^{-\gamma l}} = 1 + \frac{V_- e^{\gamma l}}{V_+ e^{-\gamma l}}$$

$$\boxed{T = 1 + \Gamma} = 1 + \frac{Z_l - Z_0}{Z_l + Z_0}$$

$$T = \frac{2Z_l}{Z_l + Z_0} \quad \text{——— (15)}$$

The power carried by the two waves in the side of the incident and reflected wave is

$$P_{inr} = P_{inc} - P_{ref} = \frac{(V_+ e^{-\alpha l})^2}{2Z_0} - \frac{(V_- e^{\alpha l})^2}{2Z_0} \quad \text{(16)}$$

The power carried to the load by the transmitted waves is:

$$P_{tr} = \frac{(V_{tr} e^{-\alpha l})^2}{2Z_L} \quad \text{(17)}$$

considering eq (13)

LHS:
$$1 - \Gamma_l^2 = 1 - \left(\frac{Z_L - Z_0}{Z_L + Z_0}\right)^2$$

$$1 - \Gamma_l^2 = \frac{(Z_L + Z_0)^2 - (Z_L - Z_0)^2}{(Z_L + Z_0)^2}$$

$$1 - \Gamma_l^2 = \frac{(Z_L + \cancel{Z_0} + Z_L - \cancel{Z_0})(\cancel{Z_L} + Z_0 - \cancel{Z_L} + Z_0)}{(Z_L + Z_0)^2}$$

$$1 - \Gamma_l^2 = \frac{(2Z_L)(2Z_0)}{(Z_L + Z_0)^2} \times \frac{Z_L}{Z_L}$$

$$1 - \Gamma_l^2 = \frac{Z_0}{Z_L} \frac{4Z_L^2}{(Z_L + Z_0)^2}$$

$$1 - \Gamma_l^2 = \frac{Z_0}{Z_L}\left(\frac{2Z_L}{Z_L + Z_0}\right)^2$$

$$\boxed{1 - \Gamma_l^2 = \frac{Z_0}{Z_L} T^2}$$

Q2: A certain transmission line has a characteristic impedance of $75 + j0.01\,\Omega$ and is terminated in a load impedance of $70 + j50\,\Omega$. compute:

a) the reflection coefficient

b) the transmission coefficient

c) the relationship shown in the above equation

d) the transmission coefficient equals the algebraic sum of 1 plus the reflection coefficient.

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Given:

$$Z_0 = 75 + 0.01j \ \Omega$$

$$Z_L = 70 + 50j \ \Omega$$

a. the Reflection coefficient

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$\Gamma = \frac{70 + 50j - 75 - 0.01j}{70 + 50j + 75 + 0.01j}$$

$$\Gamma = \frac{-5 + 49.99j}{145 + 50.01j} = \frac{50.24 \angle 95.71^\circ}{153.38 \angle 19.03^\circ}$$

$$\Gamma = 0.33 \angle 76.68^\circ = 0.076 + 0.32j$$

b. the transmission coefficient

$$T = \frac{2Z_L}{Z_L + Z_0} = \frac{2(70 + 50j)}{70 + 50j + 75 + 0.01j}$$

$$T = \frac{140 + 100j}{145 + 50.01j}$$

$$T = \frac{172.05 \angle 35.54^\circ}{153.38 \angle 19.03^\circ}$$

$$T = 1.12 \angle 16.51^\circ = 1.07 + 0.32j$$

c. $$1 - \Gamma^2 = \frac{Z_0}{Z_L} T^2$$

$$\frac{Z_L}{Z_0}(1 - \Gamma^2) = T^2$$

$$T^2 = (1.12 \angle 16.51)^2 = 1.25 \angle 33.02^\circ$$

$$\frac{Z_L}{Z_0}(1 - \Gamma^2) = \frac{70 + 50j}{75 + 0.01j}\left(1 - (0.33 \angle 76.68^\circ)^2\right)$$

$$= \frac{86.02 \angle 35.54^\circ}{75 \angle 0^\circ}\left(1 - 0.11 \angle 153.36^\circ\right)$$

$$= 1.15 \angle 35.54^\circ \,(1.1 \angle -2.55^\circ)$$

$$= 1.26 \angle 33^\circ$$

d. $T = 1.07 + 0.32j$

$1+\Gamma = 1 + 0.07 + 0.32j = 1.07 + 0.32j$

★ Standing Wave and Standing Wave Ratio (SWR):

- **Standing Wave:**

The general solutions of the transmission-line equation consists of two waves travelling in opposite directions with equal amplitude is given as:

$$V = V_+ e^{-\gamma z} + V_- e^{\gamma z}$$

$$V = V_+ e^{-\alpha z} e^{-j\beta z} + V_- e^{\alpha z} e^{j\beta z}$$

$$V = V_+ e^{-\alpha z}(\cos\beta z - j\sin\beta z) + V_- e^{\alpha z}(\cos\beta z + j\sin\beta z)$$

$$V = (V_+ e^{-\alpha z} + V_- e^{\alpha z})\cos\beta z - j(V_+ e^{-\alpha z} - V_- e^{\alpha z})\sin\beta z \quad \text{—— (1)}$$

With no loss it can be assumed that $V_+ e^{-\alpha z}$ and $V_- e^{\alpha z}$ are real. Then the voltage wave equation is

$$V_s = V_0 e^{-j\phi} \quad \text{—— (2)}$$

This is called the Equation of the voltage standing wave where

$$V_0 = [(V_+ e^{-\alpha z} + V_- e^{\alpha z})^2 \cos^2\beta z + (V_+ e^{-\alpha z} - V_- e^{\alpha z})^2 \sin^2\beta z]^{1/2} \quad \text{—— (3)}$$

which is called the standing wave pattern of the voltage wave or the amplitude of the standing wave.

$$\phi = \arctan\left(\frac{V_+ e^{-\alpha z} - V_- e^{\alpha z}}{V_+ e^{-\alpha z} + V_- e^{\alpha z}} \tan\beta z\right) \quad \text{—— (4)}$$

which is called the Phase Pattern of the Standing Wave. The maximum and minimum values of eq (3) can be found by differentiating the equation with respect to βz. and equating the result to zero.

1. The maximum amplitude is

$$V_{max} = V_+ e^{-\alpha z} + V_- e^{\alpha z} = V_+ e^{-\alpha z}(1 + |\Gamma|) \quad \text{—— (5)}$$

this occurs at $\beta z = n\pi$, where $n = 0, \pm 1, \pm 2 \ldots$

2. The minimum amplitude is

$$V_{min} = V_+ e^{-\alpha z} - V_- e^{\alpha z} = V_+ e^{-\alpha z}(1 - |\Gamma|) \quad \text{—— (6)}$$

this occurs at $\beta z = \frac{(2n-1)\pi}{2}$, where $n = 0, \pm 1, \pm 2 \ldots$

3. The distance between any two successive maxima or minina is one-half wavelength.

$$\beta z = n\pi \quad z = \frac{n\pi}{\beta} = \frac{n\pi}{2\pi/\lambda} = \frac{n\lambda}{2} \quad (n = 0, \pm 1, \pm 2 \dots)$$

Similarly

$$I_{max} = I_+ e^{-\alpha z} + I_- e^{\alpha z} = I_+ e^{-\alpha z}(1 + |\Gamma|) \quad \text{—— (7)}$$

$$I_{min} = I_+ e^{-\alpha z} - I_- e^{\alpha z} = I_+ e^{-\alpha z}(1 - |\Gamma|) \quad \text{—— (8)}$$

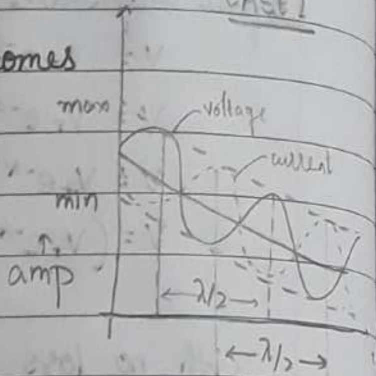
From eq (3)

CASE 1: when $V_+ \neq 0$ and $V_- = 0$ the standing wave pattern becomes

$$V_0 = V_+ e^{-\alpha z}$$

CASE 2: When $V_+ = 0$ and $V_- \neq 0$ the standing wave pattern becomes: $V_0 = V_- e^{\alpha z}$

CASE 1

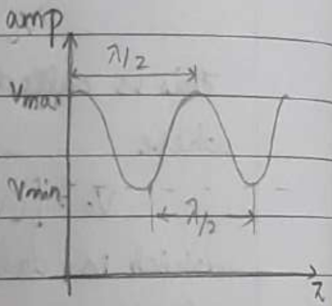


a. lossy-line

CASE 3: When the positive wave and negative wave have equal amplitudes or magnitude of the reflection coefficient is unity, the standing wave pattern with a zero phase is

$$V_s = 2V_+ e^{-\alpha z} \cos \beta z \quad \text{—— (9)}$$

which is called as a Pure Standing Wave.

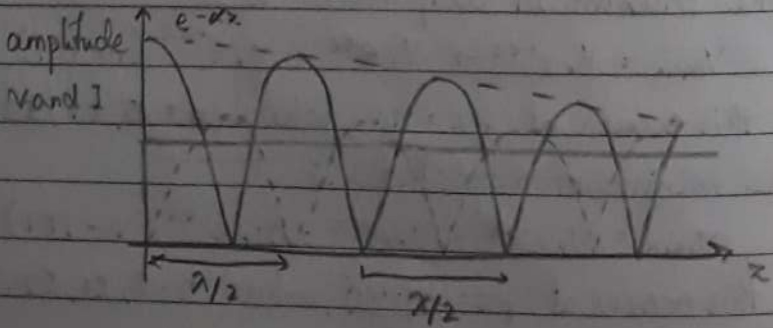


b. lossless line

Similarly pure standing wave for current is

$$I_s = -j 2 Y_0 V_+ e^{-\alpha z} \sin \beta z \quad \text{—— (10)}$$

Eq (9) and (10) shows that the voltage and current standing waves are 90° out of phase along the line. The points of zero current is called the current Nodes.



Pure standing waves of voltage and current

- Standing Wave Ratio:

Standing waves result from the simultaneous presence of waves travelling in opposite directions on a transmission line. The ratio of maximum of the standing wave pattern to the minimum is defined as the standing wave ratio.

$$\text{Standing wave Ratio} = \frac{\text{maximum voltage or current}}{\text{minimum voltage or current}}$$

$$S = \frac{|V_{max}|}{|V_{min}|} = \frac{|I_{max}|}{|I_{min}|}$$

- The standing wave ratio results from the fact that the two traveling wave components add in phase at some points and subtract at other points.
- The distance between successive maxima or minima is $\lambda/2$
- The standing wave ratio of a pure traveling wave is unity and of a pure standing wave is infinite.
- When the standing wave ratio is unity, there is no reflected wave and the line is called a flat line.
- The standing wave ratio:
  - cannot be defined on a lossy line as the standing wave pattern changes from one position to another.
  - is fairly constant over a low-loss line and it may be defined over some region.
  - it is same through out the line for a lossless line.

Also

$$\boxed{S = \frac{1+|\Gamma|}{1-|\Gamma|}} \quad \text{and} \quad \boxed{|\Gamma| = \frac{S-1}{S+1}}$$

Q3: A transmission line has a characteristic impedance of $50 + 0.01j\ \Omega$ and is terminated in a load impedance of $73 - 42.5j\ \Omega$. Calculate:

a. the reflection coefficient

b. the standing wave ratio

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Given

$Z_0 = 50 + 0.01j$

$Z_L = 73 - 42.5j$

a. the reflection coefficient

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$\Gamma = \frac{73 - 42.5j - 50 - 0.01j}{73 - 42.5j + 50 + 0.01j}$$

$$\Gamma = \frac{23 - 42.51j}{123 - 42.49j} = \frac{48.33 \angle -61.58^\circ}{130.13 \angle -19.06^\circ}$$

$$\Gamma = 0.377 \angle -42.58^\circ$$

b. Standing wave ratio

$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

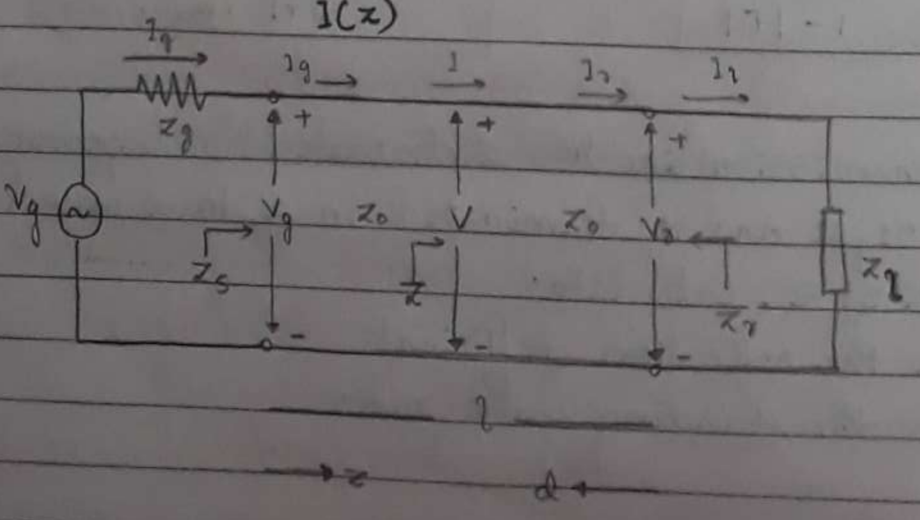
$$S = \frac{1 + 0.377}{1 - 0.377} = 2.21$$

★ Line Impedance and Line Admittance:

• **Line Impedance:**

The line impedance of a transmission line is the complex ratio of the voltage phasor at any point to the current phasor at that point.

$$Z = \frac{V(z)}{I(z)} \quad \text{—— (1)}$$



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The current or voltage along a line is the sum of the respective incident and reflected wave.

$$V = V_{inc} + V_{ref} = V_+ e^{-\gamma z} + V_- e^{\gamma z} \quad \text{—— (2)}$$

$$I = I_{inc} + I_{ref} = Y_0 (V_+ e^{-\gamma z} - V_- e^{\gamma z}) \quad \text{—— (3)}$$

At the sending end, $z = 0$

From eq (2) and (3)

$$V_s = V_+ + V_-$$

$$I_s Z_s = V_+ + V_- \quad \text{—— (4)}$$

$$I_s Z_0 = V_+ - V_- \quad \text{—— (5)}$$

From eq (4) and (5)

$$2V_+ = I_s Z_s + I_s Z_0 \qquad 2V_- = I_s Z_s - I_s Z_0$$

$$V_+ = \frac{I_s}{2}(Z_s + Z_0) \quad \text{—— (6)} \qquad V_- = \frac{I_s}{2}(Z_s - Z_0) \quad \text{—— (7)}$$

Substituting eq (6) and (7) in eq (2)

$$V = \frac{I_s}{2}(Z_s + Z_0)e^{-\gamma z} + \frac{I_s}{2}(Z_s - Z_0)e^{\gamma z}$$

$$V = \frac{I_s}{2}\left[(Z_s + Z_0)e^{-\gamma z} + (Z_s - Z_0)e^{\gamma z}\right] \quad \text{—— (8)}$$

Substituting eq (6) and (7) in eq (3)

$$I = \frac{1}{Z_0}\left[\frac{I_s}{2}(Z_s + Z_0)e^{-\gamma z} - \frac{I_s}{2}(Z_s - Z_0)e^{\gamma z}\right]$$

$$I = \frac{I_s}{2Z_0}\left[(Z_s + Z_0)e^{-\gamma z} - (Z_s - Z_0)e^{\gamma z}\right] \quad \text{—— (9)}$$

The line impedance at any point z from the sending end in terms of Z_s and Z_0 is given by:

$$Z = Z_0 \frac{(Z_s + Z_0)e^{-\gamma z} + (Z_s - Z_0)e^{\gamma z}}{(Z_s + Z_0)e^{-\gamma z} - (Z_s - Z_0)e^{\gamma z}} \quad \text{—— (10)}$$

At $z = l$ the line impedance at the receiving end in terms of Z_s and Z_0 is given by:

$$Z_r = Z_0 \frac{(Z_s + Z_0)e^{-\gamma l} + (Z_s - Z_0)e^{\gamma l}}{(Z_s + Z_0)e^{-\gamma l} - (Z_s - Z_0)e^{\gamma l}} \quad \text{—— (11)}$$

Alternatively the line impedance can be expressed in terms of Z_l and Z_0. At $z = l$; $V_r = I_l Z_l$, then

$$I_l Z_l = V_+ e^{-\gamma l} + V_- e^{\gamma l}$$

$$I_l Z_0 = V_+ e^{-\gamma l} - V_- e^{\gamma l}$$

i.e. $$V_+ = \frac{I_l}{2}(Z_l + Z_0)e^{\gamma l} \quad \text{—— (12)}$$

$$V_- = \frac{I_l}{2}(Z_l - Z_0)e^{-\gamma l} \quad \text{—— (13)}$$

Substituting the above equations in eq (2) and (3) let $(z = l-d)$

$$V = \frac{I_l}{2}\left[(Z_l + Z_0)e^{\gamma d} + (Z_l - Z_0)e^{-\gamma d}\right] \quad \text{—— (14)}$$

$$I = \frac{I_l}{2Z_0}\left[(Z_l + Z_0)e^{\gamma d} - (Z_l - Z_0)e^{-\gamma d}\right] \quad \text{—— (15)}$$

The line impedance at any point from the receiving end in terms of Z_l and Z_0 is

$$Z = Z_0 \frac{(Z_l + Z_0)e^{\gamma d} + (Z_l - Z_0)e^{-\gamma d}}{(Z_l + Z_0)e^{\gamma d} - (Z_l - Z_0)e^{-\gamma d}} \quad \text{—— (16)}$$

The line impedance at the sending end can be found by letting $d = l$

$$Z_s = Z_0 \frac{(Z_l + Z_0)e^{\gamma l} + (Z_l - Z_0)e^{-\gamma l}}{(Z_l + Z_0)e^{\gamma l} - (Z_l - Z_0)e^{-\gamma l}} \quad \text{—— (17)}$$

Equations (10), (11), (16), (17) can be simplified by replacing the exponential functions with either hyperbolic functions or circular functions. The hyperbolic functions are obtained from : $$e^{\pm\gamma z} = \cosh(\gamma z) \pm \sinh(\gamma z)$$

Substituting in eq (10)

$$Z = Z_0 \frac{Z_s \cosh(\gamma z) - Z_0 \sinh(\gamma z)}{Z_0 \cosh(\gamma z) - Z_s \sinh(\gamma z)} = Z_0 \frac{Z_s - Z_0 \tanh(\gamma z)}{Z_0 - Z_s \tanh(\gamma z)} \quad \text{—— (18)}$$

Substituting in eq (16)

$$Z = Z_0 \frac{Z_L \cosh(\gamma d) + Z_0 \sinh(\gamma d)}{Z_0 \cosh(\gamma d) + Z_L \sinh(\gamma d)} = Z_0 \frac{Z_l + Z_0 \tanh(\gamma d)}{Z_0 + Z_L \tanh(\gamma d)} \quad \text{—— (19)}$$

For lossless line $\gamma = j\beta$

$$\sinh(j\beta z) = j\sin(\beta z)$$

$$\cosh(j\beta z) = j\,\cancel{\cos}(\beta z)$$

$$\therefore Z_0 = R_0$$

Therefore,

$$Z = R_0 \frac{Z_s \cos\beta z - jR_0 \sin\beta z}{R_0 \cos\beta z - jZ_s \sin\beta z} = R_0 \frac{Z_s - jR_0 \tan\beta z}{R_0 - jZ_s \tan\beta z} \quad (20)$$

and

$$Z = R_0 \frac{Z_L \cos\beta d + jR_0 \sin\beta d}{R_0 \cos\beta d + jZ_L \sin\beta d} = R_0 \frac{Z_L + jR_0 \tan\beta d}{R_0 + jZ_L \tan\beta d} \quad (21)$$

- Impedance interms of reflection coefficient or standing-wave ratio

Line impedance at receiving end

$$Z = Z_0 \frac{(Z_L + Z_0)e^{\gamma d} + (Z_L - Z_0)e^{-\gamma d}}{(Z_L + Z_0)e^{\gamma d} - (Z_L - Z_0)e^{-\gamma d}}$$

$$Z = Z_0 \frac{\left[1 + \left(\frac{Z_L - Z_0}{Z_L + Z_0}\right)e^{-2\gamma d}\right]}{\left[1 - \left(\frac{Z_L - Z_0}{Z_L + Z_0}\right)e^{-2\gamma d}\right]}$$

$$Z = Z_0 \left[\frac{1 + \Gamma_L e^{-2\gamma d}}{1 - \Gamma_L e^{-2\gamma d}}\right] \quad (22) \qquad \text{because } \frac{Z_L - Z_0}{Z_L + Z_0} = \Gamma_L$$

wkt,

$\gamma = \alpha + j\beta$

$\Gamma = \Gamma_L e^{-2\gamma d}$

$\Gamma = \Gamma_L e^{-2\alpha d} e^{j(\theta_L - j2\beta d)} = \Gamma_L e^{-2\alpha d} e^{j\phi} \quad (\phi = \theta_L - j2\beta d)$

The simple equation for the line impedance at a distance d from the load is expressed by:

$$Z = Z_0 \frac{1 + \Gamma}{1 - \Gamma} \qquad \text{here } \Gamma = |\Gamma| e^{j\phi}$$

The impedance variation along a lossless line is:

$$\boxed{Z(d) = Z_0 \frac{1 + |\Gamma| e^{j\phi}}{1 - |\Gamma| e^{j\phi}} = R_0 \frac{1 + |\Gamma|(\cos\phi + j\sin\phi)}{1 - |\Gamma|(\cos\phi + j\sin\phi)}} \quad (23)$$

Line impedance in terms of reflection coefficient

wkt,

$$\Gamma = \frac{S - 1}{S + 1}$$

Therefore

$$\boxed{Z = R_0 \frac{(S+1) + (S-1)e^{j\phi}}{(S+1) - (S-1)e^{j\phi}}} \quad (24)$$

Line impedance in terms of standing-wave ratio (SWR)

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- Determination of characteristic Impedance:

1. Measure the sending end impedance with the receiving end short-circuited.

$$Z_{sc} = Z_0 \tanh(\gamma l)$$

2. Measure the sending end impedance with the receiving end open-circuited.

$$Z_{oc} = Z_0 \coth(\gamma l)$$

Then the characteristic impedance of the measured transmission line is given by

$$\boxed{Z_0 = \sqrt{Z_{sc} Z_{oc}}}$$

- Normalized Impedance:

The normalized impedance of a transmission line is defined as:

$$z = \frac{Z}{Z_0} = \frac{1+\Gamma}{1-\Gamma} = r \pm jx$$

wkt $Z = R_0 \dfrac{(S+1)+(S-1)e^{j\phi}}{(S+1)-(S-1)e^{j\phi}}$ lossless ($Z_0 = R_0$)

and $Z_{sc} = Z_0 \tanh(\gamma l)$

1. The maximum normalized impedance

$$z_{max} = \frac{Z_{max}}{R_0} = \frac{|V_{max}|}{R_0|I_{min}|} = \frac{1+|\Gamma|}{1-|\Gamma|} = S$$

2. The minimum normalized impedance

$$z_{min} = \frac{Z_{min}}{R_0} = \frac{|V_{min}|}{R_0|I_{max}|} = \frac{1-|\Gamma|}{1+|\Gamma|} = \frac{1}{S}$$

- Line Admittance:

The characteristic admittance is defined as

$$Y_0 = \frac{1}{Z_0} = G_0 \pm jB_0$$

The generalized admittance is defined as

$$Y = \frac{1}{Z} = G \pm jB$$

The normalized admittance is

$$y = \frac{Y}{Y_0} = \frac{Z_0}{Z} = \frac{1}{z} = g \pm jb$$

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Q4: A lossless line has a characteristic impedance of 50Ω and is terminated in a load resistance of 75Ω. The line is energized by a generator which has an output impedance of 50Ω and an open circuit output voltage of 30V (rms). The line is assumed to be 2.25 wavelengths long. Determine:

a. the input impedance

b. the magnitude of the instantaneous load voltage

c. the instantaneous power delivered to the load.

Given:

$Z_0 = R_0 = 50\Omega$ (lossless) $V_+ = 30V$

$R_L = 75\,\Omega$

a. the input impedance

$$Z_{in} = \frac{R_0^2}{R_L} = \frac{(50)^2}{75} = \underline{\underline{33.33\Omega}}$$

b. The magnitude of instantaneous load voltage

Reflection coefficient

$$\Gamma_L = \frac{R_L - R_0}{R_L + R_0} = \frac{75-50}{75+50} = \frac{25}{125} = 0.2$$

The line that is 2.25 wavelengths long looks like a quarter wave line

$$\therefore \beta d = \frac{2\pi}{\lambda}\frac{\lambda}{4} = \frac{\pi}{2}$$

$$V_L = V_+ e^{-j\beta l}(1 + \Gamma_L)$$

$$V_L = 30(1+0.2) = 36V$$

c. The instantaneous power delivered to the load.

$$P_L = \frac{V_L^2}{R_L} = \frac{(36)^2}{75} = 17.28W$$

★ Applications and Limitations of Transmission lines

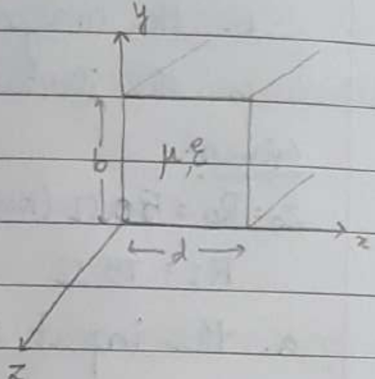
• Applications:

1. They are used to transmit signal, ie EM waves from one point to another.
2. They can be used for impedance matching purposes.

3. They can be used as circuit elements like inductors, capacitors.
4. They can be used as stubs by properly adjusting their lengths.

★ Rectangular Waveguides:

A rectangular wave guide is a hollow metallic tube with a rectangular cross section. The conducting walls of the guide confine the electromagnetic fields and thereby guide the electromagnetic wave. They have dielectric medium. There are three different modes of propagation



- TE : Transverse electric
- TM : Traversverse magnetic
- TEM : Transverse electric and magnetic

When the waves travel longitudinally down the guide, the plane waves are reflected from walls to wall. This process results in a component of either electric or magnetic field in the direction of propagation of the resultant wave.

Therefore the wave is no longer a transverse electromagnetic (TEM) wave. Thus any uniform plane wave in a lossless guide may be resolved into TE and TM waves.

For TE mode of propagation

$$\Delta^2 H_z = -\omega^2 \mu \epsilon H_z \quad \text{--- (1)}$$

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + \frac{\partial^2 H_z}{\partial x^2} + \omega^2 \mu \varepsilon H_z = 0$$

Here $\partial^2/\partial z^2 = \gamma^2$ (propagation constant)

$$\therefore \frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + \gamma^2 H_z + \omega^2 \mu \varepsilon H_z = 0$$

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + (\gamma^2 + \omega^2 \mu \varepsilon) H_z = 0$$

Date:

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considering $\gamma^2 + \omega^2\mu\varepsilon = h^2$

we get,

$$\boxed{\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + h^2 H_z = 0} \quad \text{—— (2)}$$

For TM mode of propagation

$$\nabla^2 E_z = -\omega^2\mu\varepsilon E_z \quad \text{—— (3)}$$

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \frac{\partial^2 E_z}{\partial z^2} + \omega^2\mu\varepsilon E_z = 0$$

here $\partial^2/\partial z^2 = \gamma^2$ (propagation constant)

$$\therefore \frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \gamma^2 E_z + \omega^2\mu\varepsilon E_z = 0$$

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + (\gamma^2 + \omega^2\mu\varepsilon) E_z = 0$$

considering $\gamma^2 + \omega^2\mu\varepsilon = h^2$, we get

$$\boxed{\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + h^2 E_z = 0} \quad \text{—— (4)}$$

UNIT - 4

Strip Lines

* **Introduction:**

A transmission line is a connector which transmits energy from one to another.

There are four types of transmission lines:

- Two wire parallel transmission lines
- coaxial lines
- strip type substrate transmission lines
- waveguides.

If a uniform lossless transmission line is considered, for a wave travelling in one direction, the ratio of the amplitudes of voltage and current along a line which has no reflections is called as characteristic Impedance.

- **Classification:**

Transmission Lines.

- → Multi Conductor lines
  - → Coaxial Lines
  - → strip line
    - → Micro strip line
    - → coplanar line
    - → Parallel strip line
    - → shielded strip line
- → Single conductor line (wave Guides)
  - → Rectangular wave guide
  - → circular wave guide
  - → Elliptical wave guide
  - → Single ridged wave guide
- → Open Boundary structured

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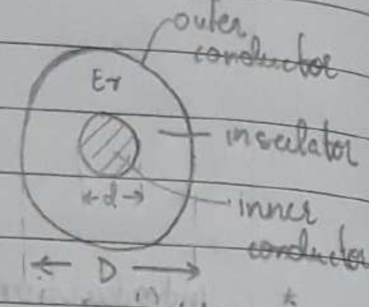
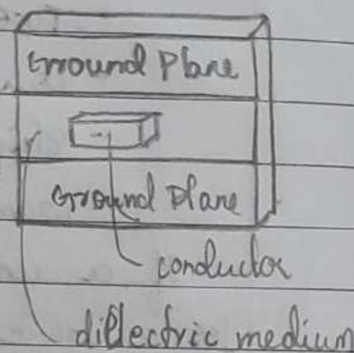
- coaxial lines:
  - used for great frequency applications.
  - inner electrode : diameter 'd' then concentric cylindrical insulating material around it which is then enclosed by an outer conductor with a diameter 'D'.



- strip lines:
  - also known a "sandwith line."
  - evolved from flatened coaxial transmission line.
  - consists of
    - top ground plane
    - bottom ground plane
    - a centre conductor
  - W: width of thin conducting strip
    B: distance between ground planes separation
  - The region between the ground planes is filled with dielectric medium.

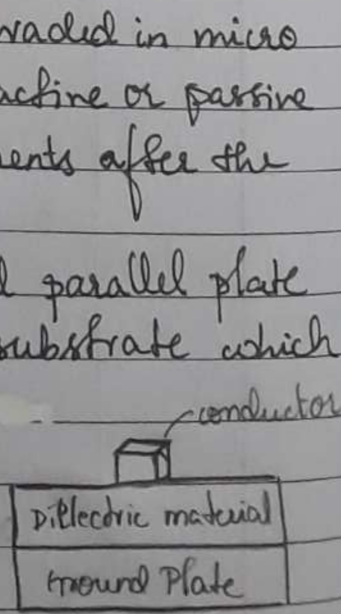


Disadvantage of strip lines.

Microstrip lines:

The strip line has a difficulty that is not reachable for adjustment and tuning. This is evaded in micro strip lines which permits mounting of active or passive devices and also makes minor adjustments after the circuit is fabricated.

A microstrip line is a asymmetrical parallel plate transmission line having di-electric substrate which has a metallixed ground on the bottom and a thin conducting strip on top with thickness 't' and width 'w'.



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- Transverse Electro-Magnetic (TEM) mode:
  - It is mode of propagation where electric and magnetic field lines are all restricted to directions normal (transverse) to the direction of propagation.
  - Benefits: media is non-dispersive i.e., phase velocity and characteristic impedance are constant over a wide band.
  - TEM mode is preferred in coax.
    - E-field lines run radially
    - M-field lines run in circles around the centre conductor.
  - In TEM mode all the field lines exist within a homogeneous medium. (TEM - strip lines and parallel plate wave guide)
  - Quasi-TEM mode

    It exists in microstrip lines. The term quasi refers that this wave resembles a TEM wave.

    In microstrip, wave propagates through the air above the top pattern and through the dielectric substrate. Due to different mediums having different resistivities wave propagates with different speeds in both the regions.

- Comparison

	Strip lines	Micro strip lines
Mode	TEM mode	Quasi TEM mode
Bandwidth	High	High
Losses	High	High
Dispersion	nil as the field lines are confined in dielectric substrate	high as the field lines are partly in dielectric substrate and partly in air
Power Handling capacity	Low	Lower
construction	complex	simpler
Radiation	better	Greater tendency to radiate
Mounting components	Fair	Easy

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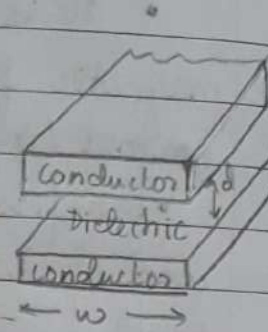
- Parallel and coplanar strip lines:

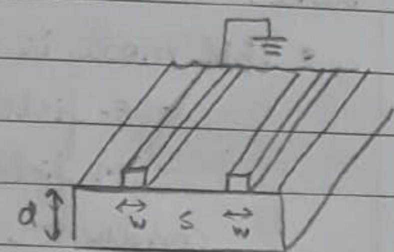
Parallel strip lines.

- It is similar to a two conductor transmission line
- It supports quasi-TEM mode

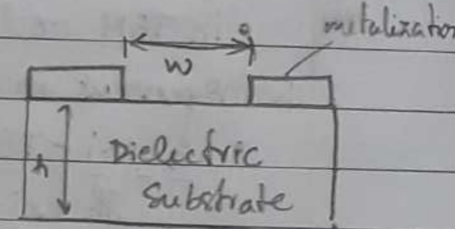


Coplanar Strip lines

- It is shaped by two leading strips with one of them grounded. Both are placed on the same substrate surface.



- A slot line transmission line consist of a slot or gap in a conducting coating on a dielectric substrate.



slot line

- Wave Guides:

A hallow metallic tube of uniform cross section for transmission of electromagnetic waves by successive reflections from the inner walls of the tube is called as wave guide. It has no centre conductor.

Disadvantages: bandwidth is limited
wave gets dispersed

- Open Boundary Structures:
  - Also known as open electromagnetic waveguides
  - a wave guide that is not completely surrounded in a metal shielding can be measured as an open waveguide.
  - Free space is also measured as a king of open waveguide
  - It is a physical device with longitudinal axial symmetry and unbounded cross-section, capable of guiding electromagnetic waves.
  - They own a spectrum which is no longer separate

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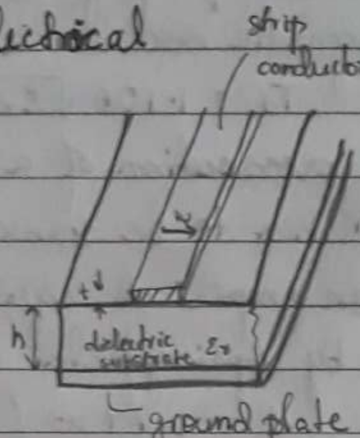
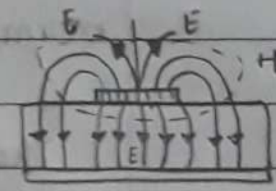
* Microstrip Lines:

Microstrip is a type of electrical transmission line which can be fabricated with any technology where a conductor is separated from a ground plate by a dielectric layer known as the substrate. Microstrip lines are used to convey microwave-frequency signals.





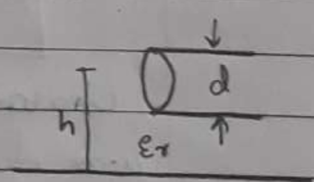
* Characteristic Impedance of Microstrip lines:

Characteristic Impedance of micro strip line is a function of:

- strip line width
- strip line thickness
- distance between line and the ground plane
- the homogenous dielectric constant.

Comparative method to determine the characteristic impedance of a wire-over ground transmission line is given by:

$$Z_0 = \frac{60}{\sqrt{\varepsilon_r}} \ln \frac{4h}{d} \quad \text{for } h \gg d \qquad (1)$$



ε_r: dielectric constant

h: height from the centre of the wire to the ground plane

d: diameter of the wire

Effective dielectric constant:

$$\varepsilon_{re} = 0.475\,\varepsilon_r + 0.67 \qquad (2)$$ ε_r: relative dielectric const.

Propagation delay time per unit length:

$$T_d = \sqrt{\mu\varepsilon} \qquad (3)$$

μ: permeability of the medium

ε: permittivity of the medium

In free space

$$T_d = \sqrt{\mu_0\varepsilon_0} = 3.33\ \text{ns/m or } 1.016\ \text{ns/ft}$$

$\mu_0 = 4\pi \times 10^{-7}$ H/m or 3.83×10^{-7} H/ft

$\varepsilon_0 = 8.854 \times 10^{-12}$ F/m or 2.69×10^{-12} F/ft

Thus the propagation delay time for a line in a nonmagnetic medium is :

$$T_d = 1.106\sqrt{\varepsilon_r} \quad ns/ft \qquad (4)$$

The crossection of a microstrip is rectangular which has to be transformed into a circular conductor. Thus an emperical equation for transformation is found by springfield which is given by an equation :

$$d = 0.67\,w\left(0.8 + \frac{t}{w}\right) \qquad (5)$$

where d : diameter of the wire over the ground
w : width of the microstrip line
t : thickness of the microstrip line

Substituting eq (5) and (2) in eq (1)

$$Z_0 = \frac{60}{\sqrt{0.475\varepsilon_r + 0.67}} \ln \frac{4h}{0.67w\left(0.8+\frac{t}{w}\right)}$$

$$Z =$$

$$Z_0 = \frac{60\sqrt{0.475}}{\sqrt{0.475\varepsilon_r+0.67}} \ln \frac{4h}{0.536w+0.67t}$$

$$\boxed{Z_0 = \frac{87}{\sqrt{\varepsilon_r+1.41}} \ln\left[\frac{5.98h}{0.8w+t}\right]}$$

characteristics impedance for a wide microstrip line was derived by Assadourian is expressed by

$$Z_0 = \frac{h}{w}\sqrt{\frac{\mu}{\varepsilon}} = \frac{377h}{\sqrt{\varepsilon_r}\,w} \quad \text{for } (w \gg h)$$

Q1: A certain microstrip line has the following parameters:
$\varepsilon_r = 5.23$; $h = 7$ mils , $t = 2.8$ mils , $w = 10$ mils
calculate the characteristic impedance Z_0 of the line:

$$Z_0 = \frac{87}{\sqrt{\varepsilon_r+1.41}} \ln\left[\frac{5.98h}{0.8w+t}\right]$$

$$Z_0 = \frac{87}{\sqrt{5.23+1.41}} \ln\left[\frac{5.98(7)}{0.8(10)+2.8}\right]$$

$$Z_0 = 45.75\,\Omega$$

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★ **Losses in Micro Strip Lines:**

Attenuation factor (α): Losses per unit length along the microstrip line can be expressed as the sum of these two losses:

- Dielectric losses in the substrate (α_d)
- Ohmic ~~skin~~ loss in the strip conductor and the ground plate (α_c).

Attenuation factor or constant can be expressed as:

$$\alpha = \alpha_d + \alpha_c \quad \text{—— (6)}$$

The power carried by a wave travelling in the positive direction in a quasi-TEM mode is:

$$P = \frac{1}{2} V I^*$$

$$P = \frac{1}{2}\left(V_+ e^{-\alpha z} I_+ e^{-\alpha z}\right)$$

$$P = \frac{1}{2}\left(V_+ e^{-\alpha z} \frac{V_+}{Z_0} e^{-\alpha z}\right)$$

$$P = \frac{1}{2} \frac{|V_+|^2}{Z_0} e^{-2\alpha z}$$

$$P = P_0 e^{-2\alpha z} \quad \text{—— (7)}$$ where $P_0 = \frac{|V_+|^2}{2Z_0}$ is power at $z = 0$

Since $P = P_0 e^{-2\alpha z}$, α can be expressed as:

$$\frac{dP}{dz} = -2P(z)\alpha \quad \text{—— (8)}$$

$$\alpha = \frac{dP/dz}{-2P(z)} \quad \text{—— (9)}$$

The gradient of power in the z direction can be further expressed as:

$$-\frac{dP(z)}{dz} = \frac{d}{dz}\left(\frac{1}{2} V I^*\right) \quad \text{—— (10)}$$

$$= \frac{1}{2}\left(-\frac{dV}{dz}\right) I^* + \frac{1}{2}\left(-\frac{dI^*}{dz}\right) V$$

$$= \frac{1}{2}(RI) I^* + \frac{1}{2} \sigma V^* V$$

$$-\frac{dP(z)}{dz} = \frac{1}{2}|I|^2 R + \frac{1}{2}|V|^2 \sigma$$

$$= P_c + P_d \quad \text{—— (11)}$$

Substituting eq (11) in (9)

$$\alpha = \frac{P_d + P_c}{2P(z)} = \frac{P_d}{2P(z)} + \frac{P_c}{2P(z)}.$$

Therefore

$$\alpha_d = \frac{P_d}{2P(z)} \quad \text{and} \quad \alpha_c = \frac{P_c}{2P(z)}$$

- Dielectric Losses:

When the conductivity of the dielectric is not negligible, the electric field and magnetic field in the dielectric are not in time phase which leads to dielectric attenuation given by the dielectric attenuation constant : $\alpha_d = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}$ Np/cm

where σ : conductivity of the dielectric substrate board. The dielectric constant which can be expressed in terms of dielectric loss tangent is given by :

$$\tan\theta = \frac{\sigma}{\omega\varepsilon} \Rightarrow \sigma = \omega\varepsilon\tan\theta$$

$$\therefore \alpha_d = \frac{\omega\varepsilon\tan\theta}{2}\sqrt{\frac{\mu}{\varepsilon}}$$

$$\boxed{\alpha_d = \frac{\omega}{2}\sqrt{\mu\varepsilon}\tan\theta \ \text{Np/cm}}$$

1 Np = 8.686 dB

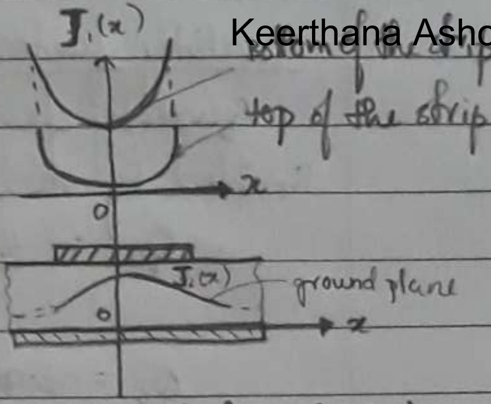
$$\alpha_d = 1.634 \times 10^3 \frac{q\sigma}{\sqrt{\varepsilon_{re}}} \ \text{dB/cm}$$

ε_{re} : relative dielectric constant

q : dielectric filling factor

$$q = \frac{\varepsilon_{re} - 1}{\varepsilon_r - 1}$$

- **Ohmic Losses:**



current density (J)

  - It is dependent on the type of metal used for the transmission lines and their dimensions.
  - It is due to resistance in path mainly due to irregularies in conductors (nonperfect conductors)
  - Microstrip conductor contributes the major part of ohmic losses.

Assuming current distribution is uniform and confined to the region $|x| < w/2$, the conducting attenuation constant of a wide ~~micro~~strip line is given by:

$$\alpha_c = \frac{8.686 R_s}{Z_0 w} \ dB/m$$

where $R_s = \sqrt{\frac{\pi f \mu}{\sigma}}$ is the surface skin resistance

$R_s = \frac{1}{\delta \sigma}$ $\quad \delta = \sqrt{\frac{1}{\pi f \mu \sigma}}$ is the skin depth in cm.

- **Radiation Losses:**

It depends on : - substrate's thickness
- Dielectric constant
- Geometry

The radiation losses are calculated for several discontinuities using following approximations:

1. TEM transmission
2. Uniform dielectric in the neighbourhood of the strip
3. Neglecting the radiation from TE field component parallel to the strip
4. Substrate thickness much less than the free space length.

Radiation loss and characteristic impedance Z_0 are inversely proportional.

$$\frac{P_{rad}}{P_t} = 240\pi^2 \left(\frac{h^2}{\lambda_0}\right) \frac{F(\varepsilon_{re})}{Z_0}$$

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* Quality factor Q of microstrip lines:

Quality factor of the striplines is very high but limited by radiation losses of the substrates.

$$Q_c = \frac{27.3}{\alpha_c}$$

$$Q_c = 3.95 \times 10^{-6} \left(\frac{h}{R_s}\right) f = 15.14\, h \sqrt{f}$$

Similarly

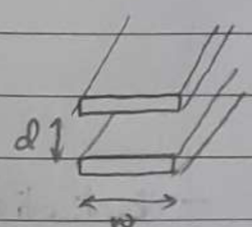
$$Q_d = \frac{27.3}{\alpha_d}$$

* Parallel strip lines:

- Distributed lines:

Two perfectly parallel strips separated by a perfect dielectric slab of uniform thickness.

considering $w \gg d$



Inductance : $L = \frac{\mu_c d}{w}$ H/m

Capacitance : $C = \frac{\varepsilon_d w}{d}$ F/m

Resistance : $R = \frac{2R_s}{w} = \frac{2}{w}\sqrt{\frac{\pi f \mu_c}{\sigma_c}}$ Ω/m

Conductance : $G = \frac{\sigma_d w}{d}$

- Characteristic Impedance:

$$Z_0 = \sqrt{\frac{L}{C}} = \frac{377}{\sqrt{\varepsilon_{rd}}}\frac{d}{w} \quad \text{for } w \gg d$$

phase velocity along the strip line

$$v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}} = \frac{c}{\sqrt{\varepsilon_{rd}}} \text{ m/s}$$

- Attenuation losses:

The propagation constant of a parallel strip is:

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} \quad \text{for } R \ll \omega L,\ G \ll \omega C$$

$$\gamma = \frac{1}{2}\left[R\sqrt{\frac{C}{L}} + G\sqrt{\frac{L}{C}}\right] + j\omega\sqrt{LC}$$

Attenuation constant will be

$$\alpha = \frac{1}{2}\left(R\sqrt{\frac{C}{L}} + G\sqrt{\frac{L}{C}}\right) \text{ Np/m}$$

$$\beta = \omega\sqrt{LC}$$

$$\alpha_c = \frac{1}{2} R\sqrt{\frac{C}{L}} = \frac{1}{d}\sqrt{\frac{\pi f \varepsilon_d}{\sigma_c}} \quad \text{Np/m}$$

$$\alpha_d = \frac{1}{2} G\sqrt{\frac{L}{C}} = \frac{188\,\sigma_d}{\sqrt{\varepsilon_{rd}}} \quad \text{Np/m}$$

* Coplanar strip lines:

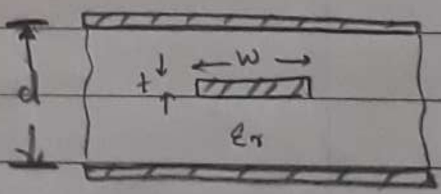
$$Z_0 = \frac{2P_{avg}}{I_0^2}$$

$$P_{avg} = \frac{1}{2}\,\mathrm{Re}\iint (E \times H^*)\cdot u_z \, dx\,dy$$

where E is Electric field intensity in +ve x direction
H is magnetic field in +ve y direction.

* Shielded strip lines:

$$Z_0 = \frac{94.15}{\sqrt{\varepsilon_r}}\left(\frac{w}{d}k + \frac{C_f}{8.854\varepsilon_r}\right)^{-1}$$



where $k = \dfrac{1}{1 - t/d}$

t = the strip thickness
d = distance between the two ground planes

$$C_f = \frac{8.854\varepsilon_r}{\pi}\left[2k\ln(k+1) - (k-1)\ln(k^2-1)\right]$$

fringe capacitance in pF/m

UNIT - 6

An Introduction to Radar

★ **Basic Radar:**

- **Introduction**

RADAR : Radio Detection and Ranging

RADAR is an electromagnetic based detection system that works by radiating electromagnetic waves and then studying the echo or the reflected back waves.

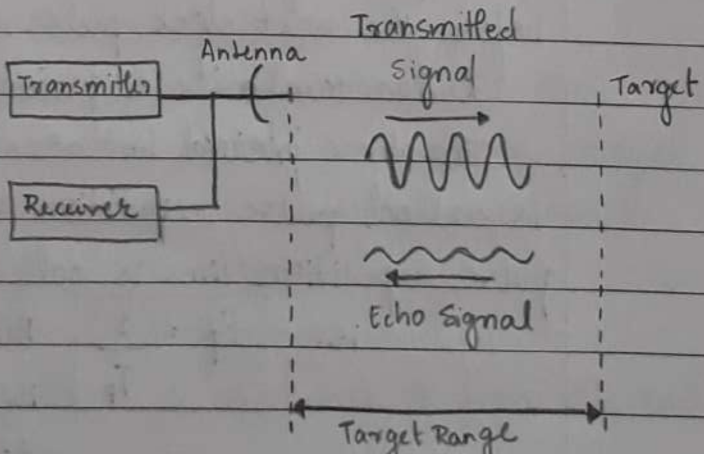
Detection is about whether the target is present or not and also if it is stationary or non-stationary (movable).

The nature of the echo signal provides information about the target:

- Range / Distance : using the time taken by the radiated energy to travel to the target and return back.
- Location: using directive antenna to sense the angle of arrival of the echo signal.
- Moving Target: with the help of shifting frequency based on the doppler effect.

- **Basic Principle of Radar:**

Radar is used for detecting the objects and finding their location. Radar mainly consists of a transmitter and a receiver. It uses the same antenna for both transmitting and receiving the signals.



BASIC PRINCIPLE OF RADAR

The function of the transmitter is to transmit the Radar signal in the direction of the target present. Target reflects this received signal in various directions. The signal, which is reflected back towards the antenna gets received by receiver.

- Terminology of Radar Systems:

- **Range:**

The distance between Radar and target is called Range of the target (R) Radar transmits a signal to the target and accordingly the target sends an echo signal to the Radar with the speed of light c.

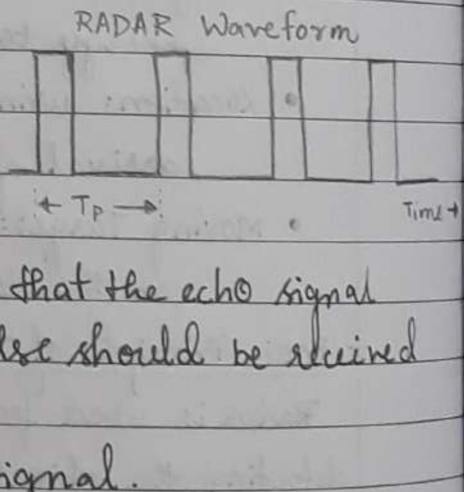
Let the time taken for the signal to travel from Radar to target and back to Radar be 'T'.

The two way distance between the Radar and target will be '2R' where R is the range.

wkt : $\text{speed} = \dfrac{\text{distance}}{\text{time}}$

$$c = \frac{2R}{T} \qquad \therefore \text{Range}: \boxed{R = \frac{cT}{2}}$$

- **Pulse Repetition Frequency:**



Radar signal should be transmitted at every clock pulse.

The duration between the two clock pulses should be properly chosen in such a way that the echo signal corresponding to present clock pulse should be recieved before the next clock pulse.

Radar transmits a periodic signal.

The time interval between the successive clock pulses is called pulse repetition time, T_p. The reciprocal of pulse repetition time is called pulse repetition frequency f_p

ie. $\boxed{f_p = \dfrac{1}{T_p}}$ Therefore pulse repetition frequency is the frequency at which Radar transmits the signal.

- **Maximum Unambiguous Range:**

Radar signals should be transmitted at every clock pulse. If we select a shorter duration between the two clock pulses, then the echo signal corresponding to present clock pulse will be received after the next clock pulse

Due to this, the range of the target seems to be smaller than the actual range.

Thus, we have to select the duration between two clock pulses in such a way that the echo signal corresponding to the present clock pulse will be received before the next clock pulse starts. Then we will get the true range of the target which is also called as maximum unambiguous range.

$\therefore$ Here $R = R_{un}$ and $T = T_p$

Therefore,

$$\boxed{R_{un} = \frac{CT_P}{2}}$$

- Minimum Range:

Minimum Range of the target is the time required for the echo signal to receive at Radar after the signal being transmitted from the Radar as pulse width. It is also called the shortest range of the target.

Here $R = R_{min}$ and $T = \tau$

Therefore,

$$\boxed{R_{min} = \frac{C\tau}{2}}$$

* The Simple Form of the Radar Equation:

Radar equation relates the range of a radar to the characteristics of the transmitter, receiver, antenna, target and environment. Where as the Radar range equation is useful to know the range of the target theoretically.

Power density is the ratio of power and area. Therefore power density P_{di} at a distance R from the radar can be represented as:

$$P_{di} = \frac{P_t}{4\pi R^2} \quad \text{——— (1)}$$

where P_t is the power transmitted by the transmitter

This equation is valid for isotropic antenna, but Radars use directional antenna. Therefore the power density P_{dd} due to directional antenna is given by:

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$$P_{dd} = \frac{P_t G}{4\pi R^2} \quad \text{--- (2)}$$ where G is antenna gain

Target radiates the power in different directions from the received input power. The amount of power which is reflected back towards the Radar depends on its cross section. So the power density P_{de} of the echo signal at Radar can be represented as:

$$P_{de} = P_{dd} \frac{\sigma}{4\pi R^2} \quad \text{--- (3)}$$ where σ is the Radar cross section.

Substituting eq (2) in eq (3).

$$P_{de} = \frac{P_t G}{4\pi R^2} \frac{\sigma}{4\pi R^2} \quad \text{--- (4)}$$

The amount of power Received P_r by the Radar depends on the effective aperture A_e of the receiving antenna.

$$P_r = P_{de} A_e \quad \text{--- (5)}$$

Substituting eq (4) in eq (5)

$$P_r = \frac{P_t G}{4\pi R^2} \frac{\sigma}{4\pi R^2} A_e$$

$$P_r = \frac{P_t G \sigma A_e}{(4\pi)^2 R^4}$$

Therefore

$$R^4 = \frac{P_t G \sigma A_e}{(4\pi)^2 P_r}$$

$$\boxed{R = \left[\frac{P_t G \sigma A_e}{(4\pi)^2 P_r}\right]^{1/4}} \quad \text{--- (6)}$$

- Standard Form of Radar Range Equation

If the echo signal is having the power less than the power of maximum detectable signal, then Radar cannot detect the target since it is beyond the maximum limit of the Radar's range. Therefore, the range of the target is said to be maximum range when the received echo signal is having the power equal to that of minimum detectable signal.

Hence $R = R_{max}$ and $P_r = S_{min}$

Therefore, to find maximum range of the target

$$\boxed{R_{max} = \left[\frac{P_t G \sigma A_e}{(4\pi)^2 S_{min}}\right]^{1/4}} \quad \text{—— (7)}$$

Standard Radar Range Equation

- Modified Forms of Radar Range Equation

Antenna gain is given by

$$G = \frac{4\pi A_e}{\lambda^2} \quad \text{—— (8)}$$

λ is the operating wavelength $\lambda = c/f$

substituting eq (8) in eq (7)

$$R_{max} = \left[\frac{P_t \sigma A_e}{(4\pi)^2 S_{min}}\left(\frac{4\pi A_e}{\lambda^2}\right)\right]^{1/4}$$

$$\boxed{R_{max} = \left[\frac{P_t \sigma A_e^2}{4\pi \lambda^2 S_{min}}\right]^{1/4}} \quad \text{—— (9)}$$

Modified Form of Radar Range Equation

Also from eq (8)

$$A_e = \frac{G\lambda^2}{4\pi} \quad \text{—— (10)}$$

Substituting eq (10) in eq (7)

$$R_{max} = \left[\frac{P_t G \sigma}{(4\pi)^2 S_{min}}\left(\frac{G\lambda^2}{4\pi}\right)\right]^{1/4}$$

$$\boxed{R_{max} = \left[\frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 S_{min}}\right]^{1/4}} \quad \text{—— (11)}$$

Modified Form of Radar Range Equation

Q1: Calculate the maximum range of Radar for the following specifications:

Peak power transmitted by the Radar : $P_t = 250\,kW$

Gain of transmitting antenna : $G = 4000$

Effective Aperture of receiving antenna : $A_e = 4\,m^2$

Radar cross section of the target : $\sigma = 25\,m^2$

Power of minimum detectable signal : $S_{min} = 10^{-12}\,W$

sol: Using the standard Radar Range Equation

$$R_{max} = \left[\frac{P_t G A_e \sigma}{(4\pi)^2 S_{min}}\right]^{1/4}$$

$$R_{max} = \left[\frac{250\times10^3 \times 4000 \times 4 \times 25}{(4\pi)^2 \times 10^{-12}}\right]^{1/4} = \underline{\underline{158\,km}}$$ is the max range of Radar

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* ## Performance Factors:

The factors that affect the performance of Radar are known as Radar performance factors.

From the standard form of Radar range equation:

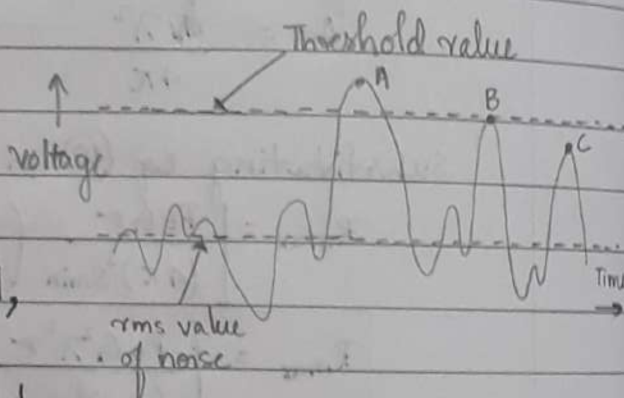
$$R_{max} = \left[\frac{P_t G \sigma A_e}{(4\pi)^2 S_{min}} \right]^{1/4}$$

In order to get the range of the Radar as maximum:

- Peak power transmitted by Radar P_t should be high
- Gain of the transmitting antenna G should be high
- Radar cross section of the target σ should be high
- Effective aperture of the receiving antenna A_e should be high.
- Power of minimum detectable signal S_{min} should be low.

- **Minimum Detectable Signal**

The echo signal of minimum power that can be detected by the Radar is called the minimum detectable signal, below which the Radar cannot detect the echo signal.



Echo signal is received by the Radar along with noise. If the threshold value is used for detection of target then it is called threshold detection.

The proper threshold value is selected based on the strength of the signal to be detected.

In the above figure, A and B are valid detection whereas C is a missing detection.

- **Receiver Noise.**

It is the noise generated by the receiver. One such receiver noise is thermal noise which occurs due to thermal motion of conduction electrons.

The thermal noise power, N_i produced at the receivers is given by:

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$$N_i = kT_oB_n$$

where k is Boltzmann's constant : 1.38×10^{-23} J/deg
T_o is the absolute temperature : 290°K
B_n is the bandwidth of the receiver.

- Figure of Merit:

It is the ratio of input SNR and output SNR which is given by:

$$F = \frac{(SNR)_i}{(SNR)_o} = \frac{S_i/N_i}{S_o/N_o} = \frac{N_oS_i}{N_iS_o}$$

$$\therefore \; S_i = \frac{FN_iS_o}{N_o}$$

Substituting $N_i = kT_oB_n$

$$S_i = F\,kT_oB_n\left(\frac{S_o}{N_o}\right)$$

Input signal power is minimum when output SNR is also minimum.

Therefore, $S_{min} = FkT_oB_n\left(\frac{S_o}{N_o}\right)_{min}$

Substituting S_{min} in standard form of Radar range equation

$$R_{max} = \left[\frac{P_t G \sigma A_e}{(4\pi)^2 FkT_oB_n\left(\frac{S_o}{N_o}\right)_{min}}\right]^{1/4}$$

Hence to have maximum Radar range
- Figure of merit F should be low
- Bandwith of receiver B_n should be low

- NOTE: Unit of Range

Nautical Mile : a unit used in measuring distances at sea, equal to 1852 meters (2025 yards approx)
The nautical mile is one minute of lattitude
It is used when travelling or navigating long distances

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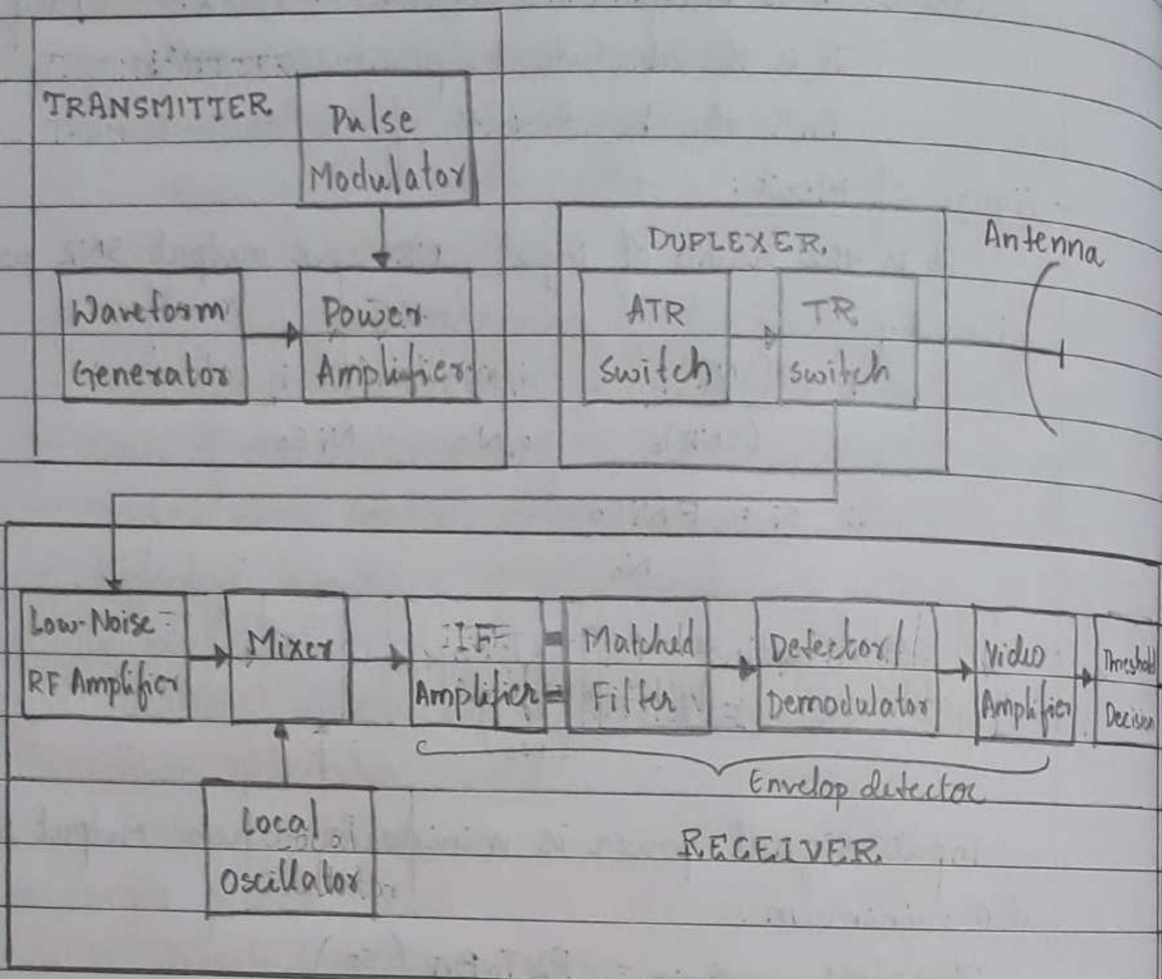
★ Radar Block Diagram:



- Transmitter:
  - Waveform generator: Generates low power radar signal or electromagnetic signal.
  - Pulse Modulator: It turns the transmitter on and off in synchronous with the input pulse to generate pulse waveform.
  - Power Amplifier: Ex: klystron, Travelling wave tube or transistor amplifier, Magnetron oscillator.

  The output of the transmitter is delivered to the antenna by a waveguide or other form of transmission lines, where it is radiated into space.
- Duplexer:

  Duplexer allows a single antenna to be used on a time shared basis for transmission and receiving. It consists of gas discharge device known as TR switch (Transmit-Receive) and other ATR switch (Anti-Transmit-Receive).

  TR directs during transmission and ATR directs the echo signal during reception.

During transmission, the receiver is cut off so that high power flows to the antenna and not to the receiver. Similarly during reception, the duplexer directs the echo signal to the receiver.

Next part is the superheterodyne receiver.

- Receiver:
  - Low-Noise RF amplifier: The first stage can be a low-noise RF amplifier, a parametric amplifier or low-noise transistor.
  - Mixer - Local Oscillator: converts the RF signal to an intermediate frequency where it is amplified by IF amplifier.
  - IF Amplifier - Matched Filter: Amplifies the intermediate frequency generated by the mixer. Signal bandwidth of the superheterodyne receiver is determined by the frequency of IF stage. The matched filter is designed such that its frequency response function H(f) maximizes the SNR at the output.
  - Detector / Demodulator: crystal diode is used as a detector which helps to extract the original signal from the carrier. The combination of IF amplifier, detector and video amplifier acts as an envelop detector which passes the envelop and rejects carrier frequency.

  In Radar that detects the doppler shift of the echo signal the envelop detector is replaced by phase detector.

- Threshold Detection:

  The threshold decision is based on the magnitude of receiver output and the threshold level. If output signal is above threshold the target is found and if output signal is below threshold only noise is present.

  The threshold is set such that the false alarm rate of receiver is limited.

- Integrator: adds echo pulses together to increase the SNR. It is found in video amplifier portion.
- Signal Processor: It is found before the detection decision, It passes the desired echo signal and rejects unwanted signal, noise and clutter. Ex: Matched Filter.

Date:

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* **Radar Frequencies:**
  - Operational Radar : Frequency ranging from 100MHz to 36GHz

HF	3-30MHz
VHF	30-300MHz
UHF	300-1000 MHz
L	1000-2000MHz
S	2000-4000 MHz
C	4000-8000 MHz
X	8000-12,000 MHz
K_u	12-18 GHz
K	18-27GHz
K_a	27-40 GHz
mm	40-300 GHz

- HF Over the Horizon Radars (OHR) operates at frequencies as low as few MHz.
- Experimental millimeter wave radars operates at frequencies higher than 240 GHz.

* **Application of Radar:**
  - **Military:**
    - Surveillance system includes target detection, target recognition and target tracking
    - Missile system: used to track target, direct weapon and assess battle damage.
    - Used for detecting fixed or moving target on the battle field.
  - **Remote sensing:**
    - weather observation
    - Planetary Observation
    - Mapping of sea ice to route shipping in efficient manner
  - **Air Traffic Control (ATC):**
    - To control air traffic in airport vicinity
    - Enroute from one airport to another (Air Route Surveillance Radar)
    - Maps region of rain

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- Ground vehicular traffic (Air Surface Detection Equipment)
- To observe weather in vicinity of airport (Terminal Doppler Weather Radar - TDWR).

• <u>Law Enforcement and Highway Safety:</u>
  - Radar speed Meter: Used by police to observe speed limits.

• <u>Aircraft Safety and Navigation:</u>
  - Airborne weather avoidance radar - dangerous wind
  - Terrain avoidance and terrain following radars.
  - Ground mapping Radar: used by military to image a scene.
  - Radar Altimeter: height of aircraft above the terrain

• <u>Ship safety:</u>
  - collision avoidance during poor visibility
  - For surveillance of harbors and river traffic

• <u>space:</u>
  - Space vehicle radar: landing on moon
  - Ground based radar: detection and tracking satellites
  - Radio ~~Astronomy~~: nature of meteors, moon and planets.

★ <u>Doppler Effect:</u>

Change in frequency of electromagnetic signal that propagates from radar to the moving target and back to the radar is the <u>Doppler Frequency Shift</u>.

- The two possible cases according to Doppler effect are:
  i. If the frequency of the received signal increases, then the target is moving towards the Radar.
  ii. If the frequency of the received signal decreases, then the target is moving away from the Radar.

- <u>Derivation of Doppler Frequency:</u>

The two way communication path is 2R, where R is the range of the target.

Total phase change in two way propagation path is:

$$\phi = 2\pi \frac{2R}{\lambda} \quad \text{——— (1)}$$

because Path difference = $\frac{\text{wavelength}}{2\pi} \times$ Phase difference.

Hence the total angle of excursion made by the electromagnetic wave during two way communication path is:

$$\phi = \frac{4\pi R}{\lambda} \quad \text{—— (2)}$$

Since target is moving, R changes and also the phase changes.

Therefore differentiating eq (2) w.r.t time gives the rate of change of phase which is angular frequency.

$$\omega_d = \frac{d\phi}{dt} = \frac{4\pi}{\lambda}\frac{dR}{dt} = \frac{4\pi}{\lambda} V_r$$

but $\omega_d = 2\pi f_d$

Therefore $f_d = \frac{2}{\lambda} V_r$

NOTE: The knot is a unit of speed equal to one nautical mile per hour

wkt: $\lambda = \frac{c}{f_t}$

$$\therefore \boxed{f_d = \frac{2 f_t}{c} V_r}$$

$$\boxed{f_d = \frac{1.03 V_r}{\lambda} \ kt/m \simeq \frac{V_r}{\lambda} \ kt/m}$$

Q2: If the Radar operates at a frequency of 5GHz, then find the Doppler frequency of an aircraft moving with a speed of 100 kMph.

Sol: Given: Speed of aircraft: $V_r = 100$ kMph

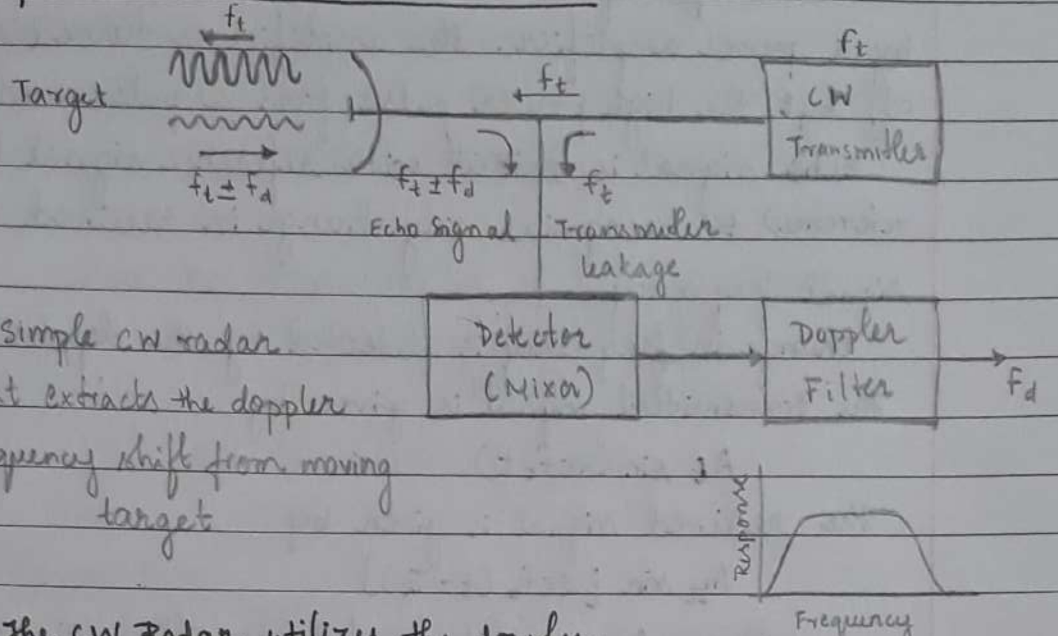
$$V_r = \frac{100 \times 10^3}{60 \times 60} = 27.78 \ m/s$$

Doppler frequency is given by

$$f_d = \frac{2 f_t V_r}{c}$$

$$f_d = \frac{2(5 \times 10^9)}{3 \times 10^8} 27.78 = 926 \ Hz$$ is the value of Doppler frequency

★ **Simple Continuous-Wave (CW) Radar**



Simple CW radar that extracts the doppler frequency shift from moving target

The CW Radar utilizes the doppler frequency shift for detection of the target.

The transmitter generates continuous sinusoidal oscillations of frequency f_t which is radiated by the antenna.

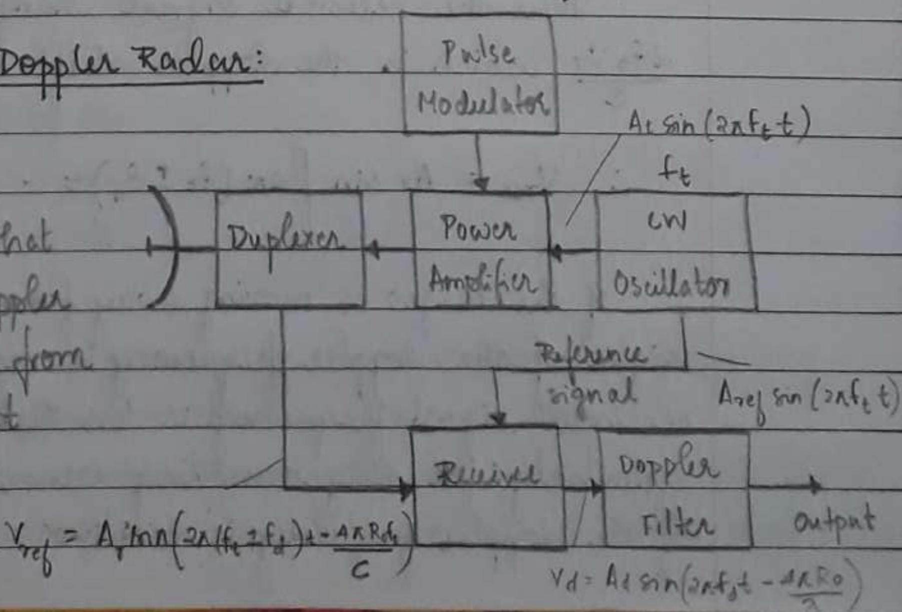
The CW Radar receives the echo signal which will be shifted in frequency by an amount of $\pm f_d$ when the target is moving.

The transmitter leakage signal acts as reference signal to determine the change in frequency.

The detector/mixer mixes heterodyne the echo signal with frequency $f_t \pm f_d$ with the transmitter leakage signal f_t.

The Doppler filter allows the difference frequency from the detector to pass and rejects the higher frequencies.

★ **Simple Pulse Doppler Radar:**



Simple Pulse Doppler Radar that extracts the doppler frequency shift from moving target

The output of the stable cw oscillator is amplified by a power amplifier. The amplifier is turned on and off by the high power pulse from a pulse modulator. Echo signal is mixed with reference signal (coherent reference) to recognize any change in received echo signal frequency.

change in frequency is detected by the doppler filter.

The transmitted signal is given by

$$A_t \sin(2\pi f_t t)$$

The received signal is given by

$$A_r \sin[2\pi f_t (t - T_R)]$$

where A_t : amplitude of transmitted signal
A_r : amplitude of receiver signal
T_R : round trip time $= \frac{2R}{c}$
$R = R_0 - v_r t$

Therefore the received signal is

$$V_{rec} = A_r \sin\left[2\pi f_t \left(t - \frac{2R}{c}\right)\right]$$

$$= A_r \sin\left[2\pi f_t \left(t - \frac{2}{c}(R_0 - v_r t)\right)\right]$$

$$= A_r \sin\left[2\pi f_t \left(t - \frac{2R_0}{c} + \frac{2v_r t}{c}\right)\right]$$

$$= A_r \sin\left[2\pi f_t \left(1 + \frac{2v_r}{c}\right)t - \frac{4\pi f_t R_0}{c}\right]$$

Thus the received signal changes by a factor of $\frac{2f_t v_r}{c}$ which is the doppler shift f_d.

$$\therefore V_{rec} = A_r \sin\left[2\pi (f_t \pm f_d)t - \frac{4\pi f_t R_0}{c}\right]$$

If the target is moving away from the radar, the sign of the doppler frequency is negative, thus the received signal frequency is less than that of transmitted. Similarly for target moving towards the radar, the sign of the doppler frequency is positive.

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Then the received signal is heterodyne with the reference signal $A_{ref} \sin(2\pi f_t t)$

Therefore the output of the receiver is

$$V_d = A_d \sin\left(2\pi f_d t - \frac{4\pi R_0}{\lambda}\right)$$

where A_d: Amplitude of difference signal.

f_d: doppler frequency $= \frac{2 f_t V_r}{c}$

f_t: Transmitter frequency $= \frac{c}{\lambda}$

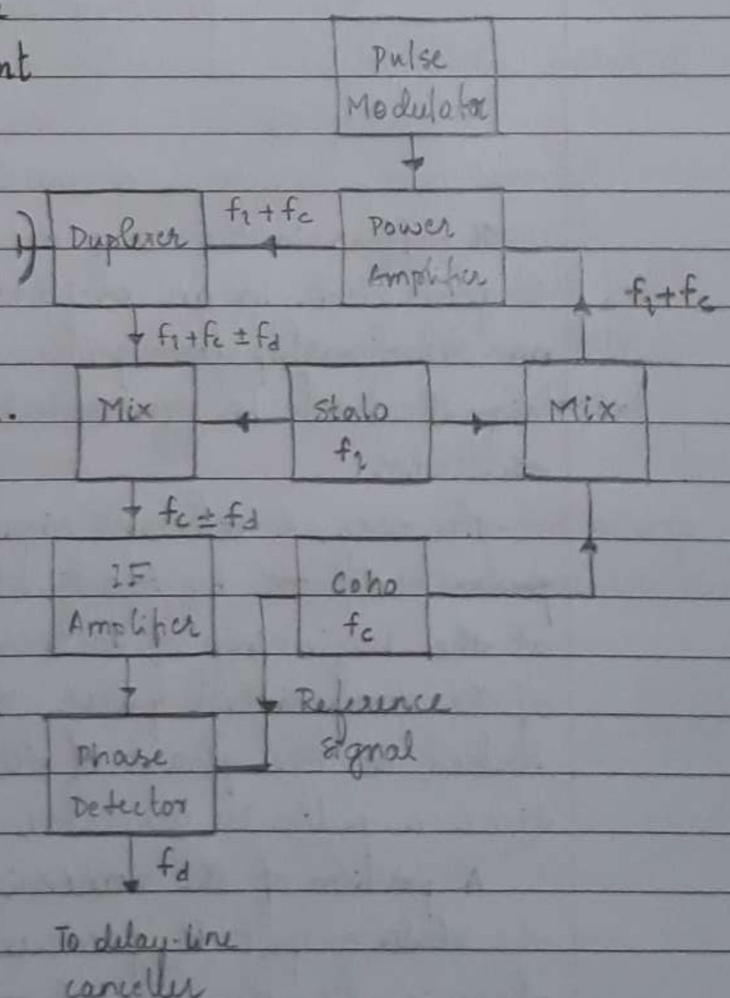
★ **MTI Radar:**

– **MTI Radar with Power Amplifier Transmitter:**

The coherent reference is supplied by an coherent oscillator called coho.

The output of coho f_c is mixed with the local oscillator frequency f_1 and is called stalo i.e., stable local oscillator.

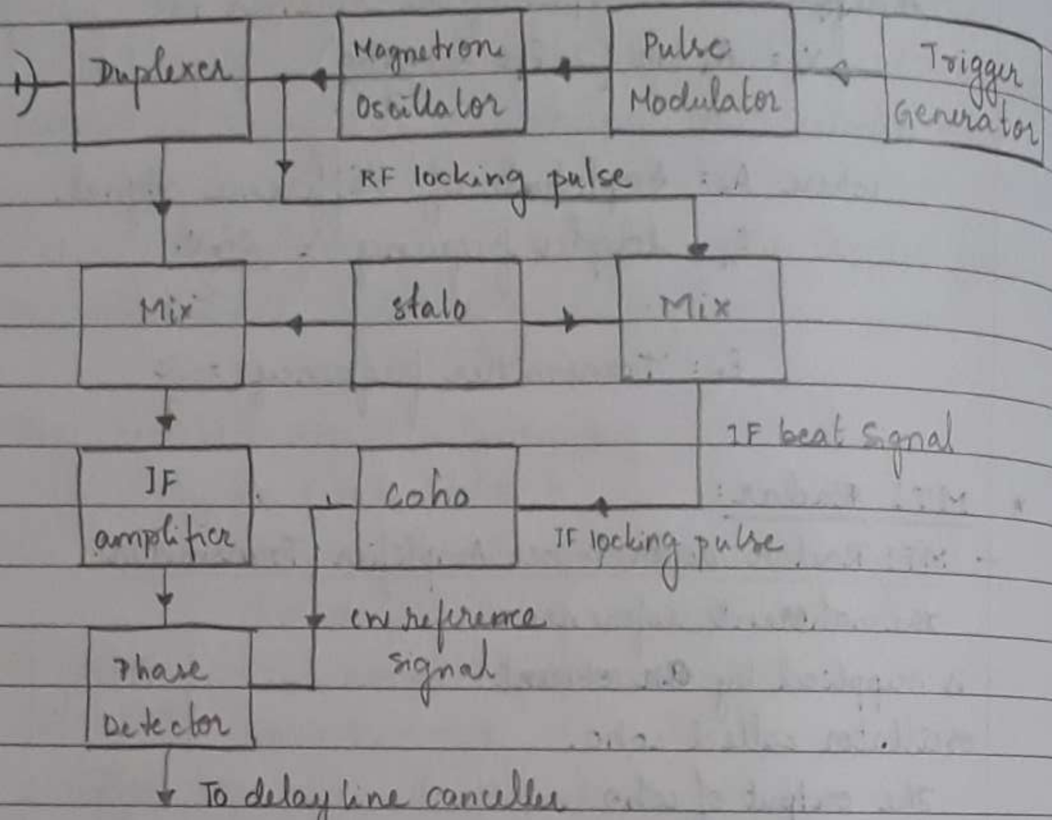
The characteristic feature of MTI radar is that the transmitted radar signal must be coherent with the reference signal in the receiver, this is achieved by the coho signal.



The function of the stalo is to provide necessary frequency translation IF to transmitted frequency. Any stalo shift is cancelled on the reception.

The reference signal from coho and IF echo signal is fed to phase detector (mixer) whose output is proportional to the phase difference between the two input signals.

- MTI Radar with power oscillator transmitter:



It differs in the way in which the reference signal is generated. In an oscillator the phase of the RF bears no relationship from pulse to pulse. Thus the reference signal cannot be generated by a continuously running oscillator.

However, a coherent signal may be obtained by the power oscillator by readjusting the phase of the coho at the beginning of each sweep according to the phase of the transmitted pulse. The phase of the coho is locked to the phase of the transmitted pulse each time a pulse is generated.

A portion of the transmitted signal is mixed with the stalo output to produce an IF beat signal whose phase is directly related to the phase to be transmitted.

IF phase is applied to the coho and causes the phase of the coho CW oscillations to "lock" in step with the phase of the IF reference pulse.

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The phase of the coho is then related to the phase of the transmitted pulse and may be used as the reference signal for the echos received from that particular pulse.
Upon the next transmission another IF locking pulse is generated to relock the phase of the CW coho until the next locking pulse comes along.

* **Delay Line cancellers:**

Delay line canceller is a filter which eliminates the DC components of echo signals received from stationary targets i.e., it allows AC components of echo signals received from non-stationary targets (moving targets).

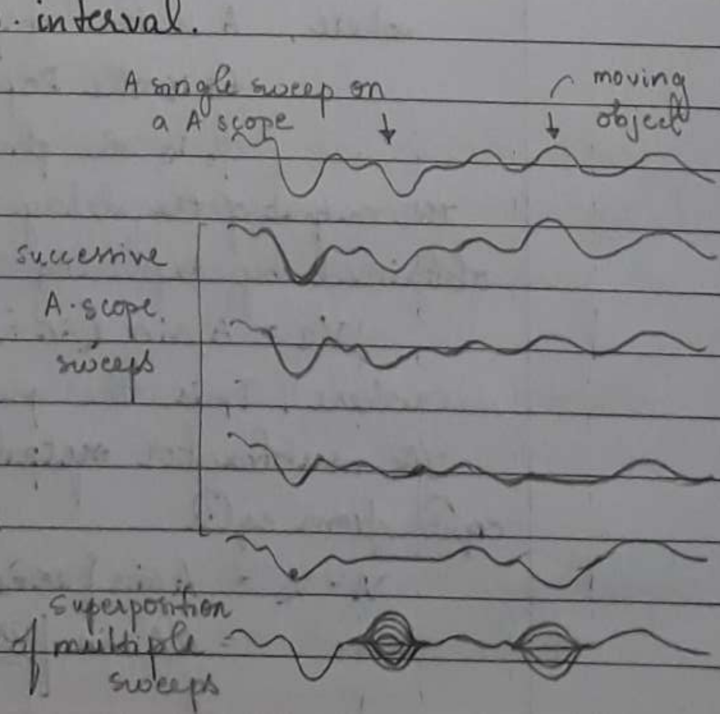
- **Need for delay line cancellers:**

To detect the doppler frequency shift on the basis of the frequency change within a single pulse:
- the radar pulse width should be long enough.
- the doppler frequency should be large enough.

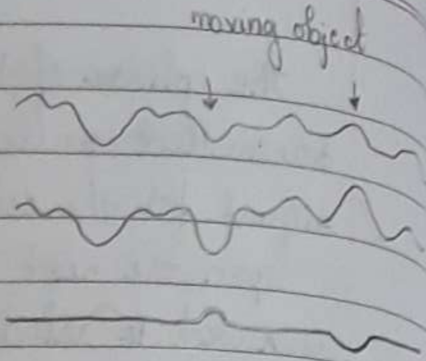
This condition is usually not met when detecting an aircraft because doppler frequency f_d is much smaller than $1/\tau$. Thus the doppler effect cannot be utilized with a single short pulse.

The fixed clutter echos remain the same from sweep to sweep, where sweep is the time between two transmitted pulse or pulse repetition interval.

Although the butterfly effect is suitable for recognizing moving targets on an A scope, it is not appropriate for display on the PPI.
One method employed to extract doppler information from display on PPI scope is delay-line canceller.



If one sweep is subtracted from the previous sweep, the fixed clutter echos will cancel and will not be detected or displayed.



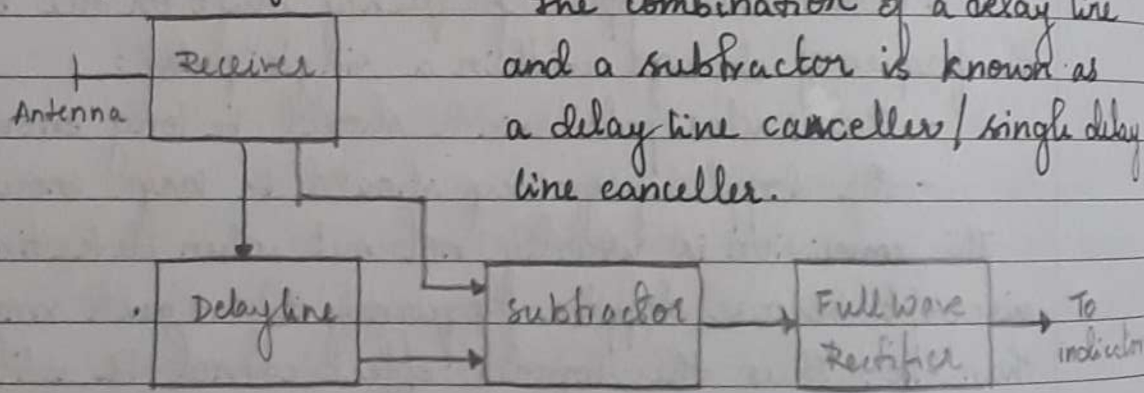
Due to the presence of doppler frequency shift the moving target changes in amplitude from sweep to sweep. Hence on subtracting the result will be uncancelled residue.

- Types of Delay Line Cancellers:

Delay lines cancellers are classified based on the number of delay lines present.

- Single Delay Line Cancellers:

The combination of a delay line and a subtractor is known as a delay line canceller / single delay line canceller.



The received echo signal after the Doppler effect is given by:

$$V_1 = A \sin(2\pi f_d t - \phi_0) \quad \text{--- (1)}$$

where, A is the amplitude of video signal

f_d is the Doppler frequency

ϕ_0 is the phase shift: $\phi_0 = \frac{4\pi f_t R_0}{c}$

The output of the delay line canceller is obtained by replacing t by $t - T_P$ in eq (1)

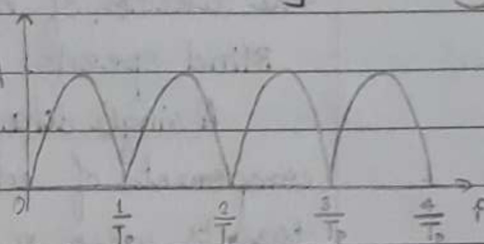
$$V_2 = A \sin[2\pi f_d(t - T_P) - \phi_0] \quad \text{--- (2)}$$

where, T_P is the pulse repetition time

The subtractor output is obtained by subtracting eq (2) from eq (1).

$$V_1 - V_2 = A \sin[2\pi f_d t - \phi_0] - A \sin[2\pi f_d (t - T_P) - \phi_0]$$

$$V_1 - V_2 = 2A \sin\left[\frac{2\pi f_d t - \phi_0 - (2\pi f_d t - 2\pi f_d T_P - \phi_0)}{2}\right]$$
$$\cos\left[\frac{2\pi f_d t - \phi_0 + (2\pi f_d t - 2\pi f_d T_P - \phi_0)}{2}\right]$$
$$= 2A \sin\left[\frac{2\pi f_d T_P}{2}\right] \cos\left[\frac{2\pi f_d (2t - T_P) - 2\phi_0}{2}\right]$$
$$= 2A \sin(2\pi f_d T_P) \cos\left[2\pi f_d\left(t - \frac{T_P}{2}\right) - \phi_0\right] \quad (3)$$



The frequency response function of the single delay-line canceller is:

$$H(f) = 2 \sin(\pi f_d T_P)$$

From eq (3), the frequency response of the single delay line canceller is zero when $\pi f_P T_P$ is equal to integral multiples of π.

i.e: $\pi f_d T_P = n\pi$

$$\therefore f_d = \frac{n}{T_P} \quad (4)$$

From eq (4)

Therefore the frequency response of the single delay line canceller is zero when Doppler frequency f_d is equal to the integral multiples of pulse repetition time T_P.

wkt $f_P = \frac{1}{T_P}$ (5)

substituting eq (5) in eq (4)

$$f_d = n f_P \quad (6)$$

Therefore the frequency response of the single delay line canceller is zero when Doppler frequency f_d is equal to the integral multiples of pulse repetition frequency f_P.

Advantages:

- It eliminates fixed clutter which has zero Doppler frequency.

Disadvantages:

- The frequency response is zero when moving targets have doppler frequencies at f_P and its harmonics.

- The clutter spectrum at zero frequency is a delta function with finite width, that clutter will appear in the pass band of the delay line canceller. (i.e, the finite width of the clutter spectrum)

This results in target speeds called blind speed where the target will not be detected. There will be uncancelled clutter residue that can interfere with the detection of moving target.

Blind speeds:

A single delay line canceller eliminates the DC components of echo signals received from stationary targets when n is equal to zero.

It also eliminates the AC components of echo signals received from non-stationary targets, when the Doppler frequency f_d is equal to integer multiples of pulse repetition frequency f_p.

Hence the relative velocities for which the frequency response of the single delay line canceller becomes zero are called blind speeds.

The mathematical expression for blind speed is given by:

$$v_n = \frac{n\lambda}{2T_p} \quad \text{——— (7)}$$

$$v_n = \frac{n\lambda f_p}{2} \quad \text{——— (8)}$$

where n is an integer (1, 2, 3 . . .)

λ is the operating wavelength

Q3: An MTI Radar operates at a frequency of 6GHz with a pulse repetition frequency of 1kHz. Find the first, second and third blind speeds of this radar.

Sol: Given: Operating frequency : $f = 6\,GHz$

Pulse repetition frequency : $f_p = 1\,kHz$

Operating wavelength

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{6\times10^9} = 0.05\,m$$

Blind speed

$$v_n = \frac{n\lambda f_P}{2}$$

$$v_n = n\,\frac{0.05\times1\times10^3}{2} = 25n$$

Substituting $n = 1, 2$ and 3 in the above equation we can find the first, second and third blind speeds of this Radar.

First blind speed

$$v_1 = 25(1) = \underline{25\,m/s}$$

Second blind speed

$$v_2 = 25(2) = \underline{50\,m/s}$$

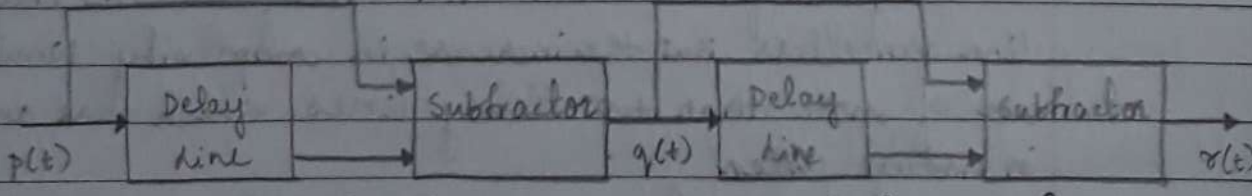
Third blind speed

$$v_3 = 25(3) = \underline{75\,m/s}$$

Blind speed can be serious limitation in MTI radar since they cause some desired moving targets to be cancelled along with the undesired clutter at zero frequency.
The ways in which the effect of blind speed can be reduced are:

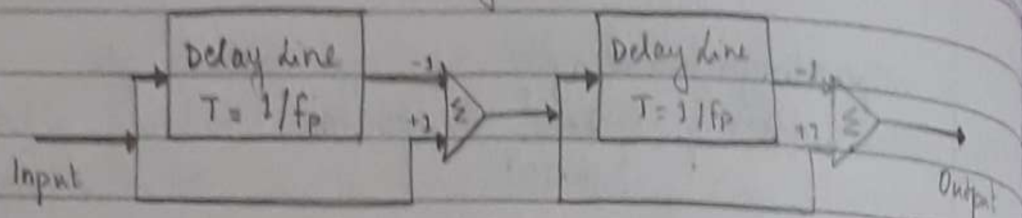
1. Operate the radar at long wavelength
2. Operate at high pulse repetition frequency
3. Operate with more than one pulse repetition frequency
4. Operate with more than one RF frequency.

- Double Delay Line Canceller:



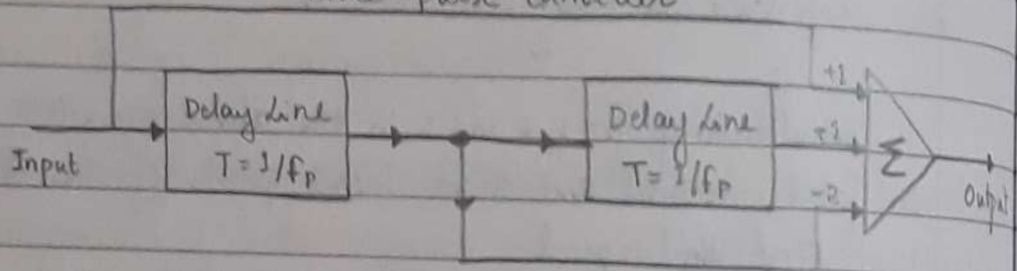
When two single delay line canceller which consists of a delay line and a subtractor are cascaded together, then that combination is called Double Delay Line Canceller.

Double Delay line canceller



The output of the double delay line canceller is given by:
$s(t) - s(t+T_p) - [s(t+T_p) - s(t+2T_p)]$

Three pulse canceller

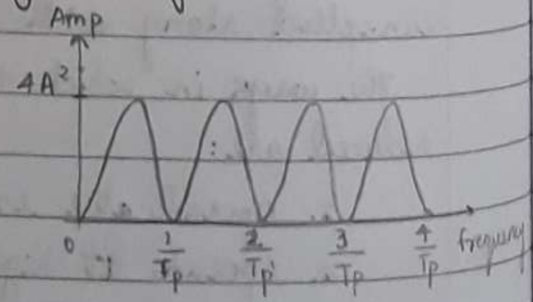


The output of the three-pulse canceller is given by:
$s(t) - 2s(t+T_p) + s(t+2T_p)$

Thus the double delay line canceller and the three pulse canceller have the same frequency response function.

The advantage of double delay line canceller is that it rejects the clutter broadly. The output of two delay line cancellers which are cascaded will be equal to the square of the output of single delay line canceller.

So the magnitude of output of double delay line canceller, which is present at MTI Radar receiver will be equal to
$4A^2(\sin(\pi f_d T_p))^2$



★ <u>Digital MTI Processing:</u>

Early MTI filters were with analog acoustic delay line cancellers. But the increase in complexity limits the use of more than two delay lines which are analog in nature.

Digital technology allowed the delays to be obtained by storing words in memory. Thus with digital filters more sophisticated filters can be readily obtained when a large number of pulses are available for processing

- Advantages:
  - Greater dynamic range can be obtained.
  - Eliminate unwanted changes in the delay times of acoustic delay lines due to temperature change.
  - Easy to change the delay time in synchronous with the radar's pulse repetition frequency.
  - Digital filters are readily available in the market with many different filter characteristics.
  - Digital MTI is more stable and reliable are requires less adjustment during operation.
  - I and Q channel compensates the blind phases which causes a loss due to the diffrence in phase between the echo signal and MTI reference signal.

- Blind Phase

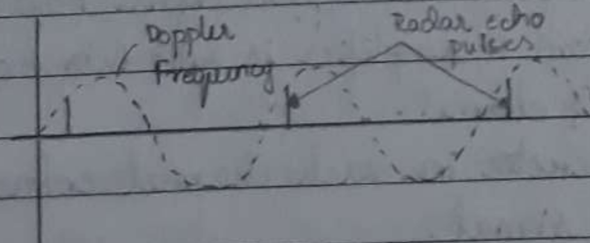
Refer pg: 1-15 – MTI radar with power amplifier

As there is only one phase detector and single channel there is loss when the Doppler shifted signal is not sampled at + and - peak of the sine wave. When the phase between the Doppler signal and sampling at pulse repetition frequency results in a loss it is called blind phase.

Blind speed: When sampling pulse appears at the same point in the doppler cycle at each sampling time.

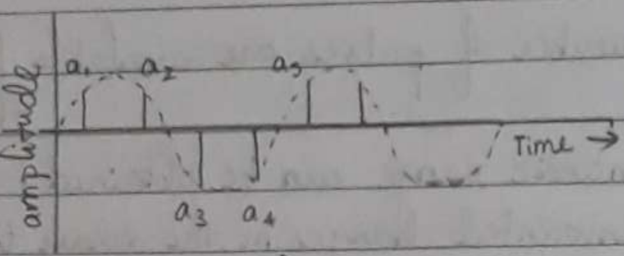
I channel : Inphase channel : Reference Coho signal : $\sin 2\pi f_d t$

Q channel : Quadrature channel : Reference Coho signal : $\cos 2\pi f_d t$

a. 

Example of Blind speed
The doppler frequency is equal to pulse repetition frequency

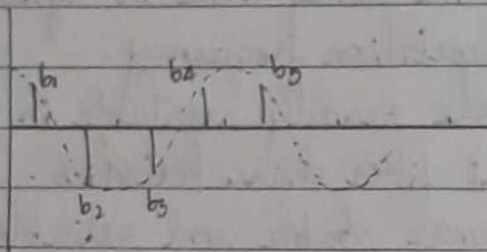
b.



Example of blind phase in I channel

After processing through two pulse MTI, half of the signal energy is lost if only the I channel is used.

c.

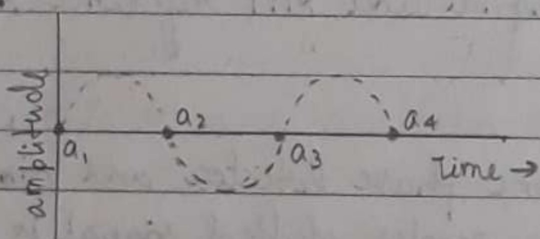


Example of Blind phase in Q channel

there rate of sampling is same but the signal is shifted by 90°.

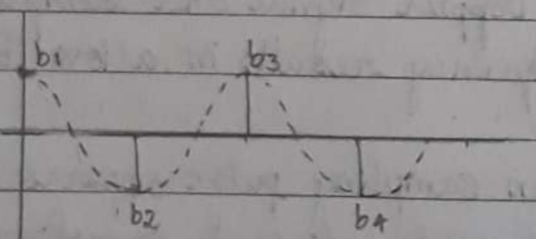
Thus both I channel and Q channel is used to solve the problem.

d.



Pulse Repetition Frequency is twice the doppler frequency. Phase of the sampling is such that there is no response at all since the sampling is at zeros of the Doppler frequency.

e.



The sampling is at the positive and negative peaks of the Doppler frequency so that there is complete recovery of the signal.

- Block Diagram of Digital MTI Doppler Signal Processor:
  - The signal from IF amplifier is split into two channels: I channel and Q channel.
  - The phase detector in each channel extracts the doppler-shifted signal.
  - In the I channel, the doppler signal is represented as:

$$A_d \cos(2\pi f_d t + \phi_0)$$

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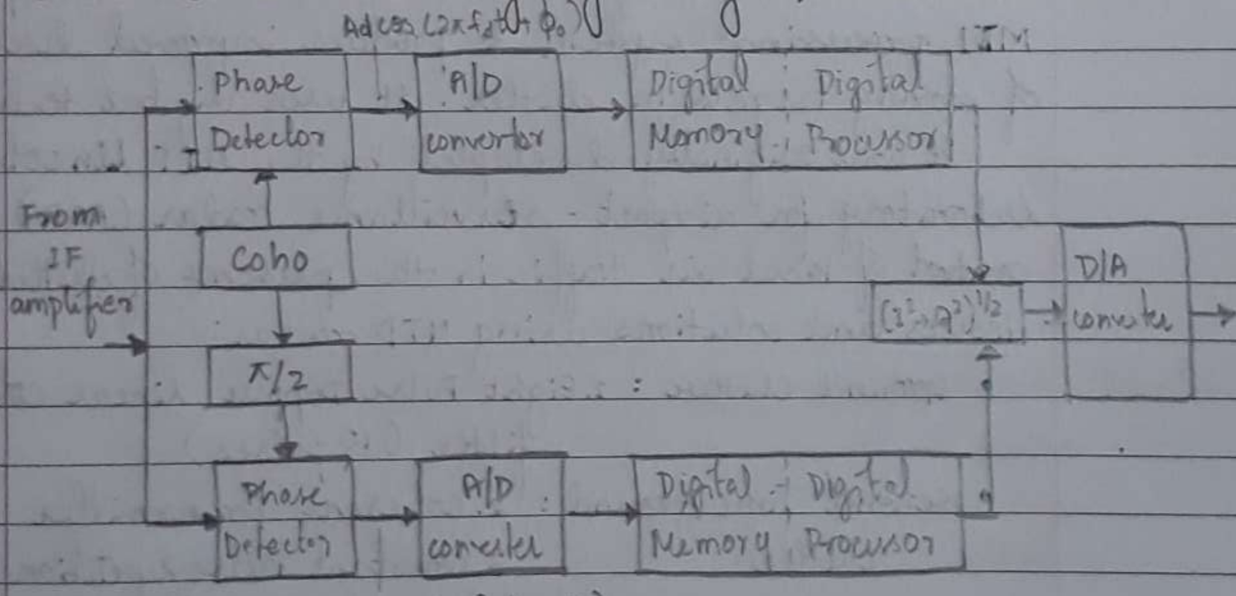
- In Q channel the doppler signal is represented as:

$$A_d \sin(2\pi f_d t + \phi_0)$$

- The signals are then digitized by the A/D converter. A sample and hold circuit is needed ahead of the ADC for more effective digitizing.
- The digital words are stored in a digital memory for the required delay times and are processed with suitable algorithm to provide the desired doppler filtering. Magnitude of the Doppler signal is:

$$\sqrt{I^2 + Q^2}$$

- Then the combined unipolar output can be converted to an analog signal by D/A converter.



- Limitations on improvement factor A/D converter:
  - Quantization Noise: It is the quantization of analog signal resulting in noise or uncertainity.
  - Improvement factor due to quantization noise is given by:

$$I_q = 20\log[(2^N - 1)\sqrt{0.75}]\ dB$$

where N - number of bits.

For 1 bit : $I_q = 6\,dB$

For 10 bit : $I_q = 10\,dB$

In general A/D converter generally requires one or more additional bits to achieve the desired performance.

- Dynamic Range.

The maximum signal to noise ratio that can be handled by an A/D converter without saturation is given by

$$\text{dynamic range} = \frac{2^{2N-3}}{k^2}$$

where: N - number of bits in A/D converter
K - rms noise level divided by the quantization level.

★ **Moving Target Detector:**

The moving target detector (MTD) is an example of MTI processing system to produce improved detection of moving targets in clutter. It uses digital technique. It was originally developed by the MIT Lincoln Laboratory for airport-surveillance radar (ASR) for control of local air traffic in the presence of clutter.

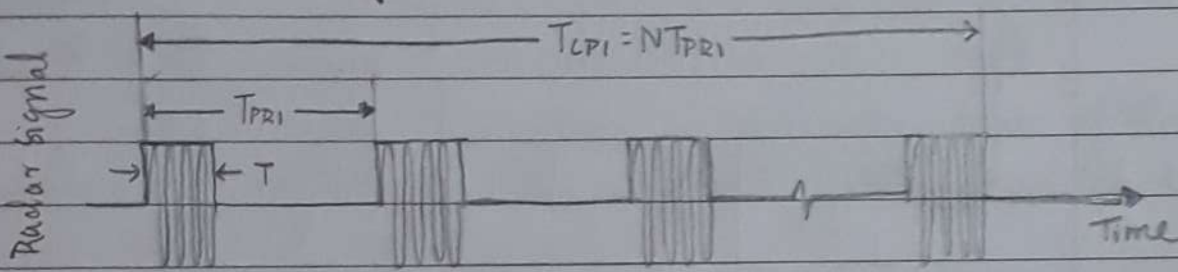
- Issues and solutions using MTD radar:
  - Ground Clutter : 1. Eight Pulse Doppler, Linear coherent filter (10 pulses)
  - Second Time Around Clutter : 2. Coherent Transmitter
    3. Constant Pulse Repetition frequency with coherent processing interval.
  - Rain: 4. Doppler Filter Bank
    5. Adaptive Thresholding
  - Tangential Target, Blind Speeds : 6. Fine Grained Clutter Map
    7. Multiple Pulse Repetition Frequencies.

- Characteristics of original MTD:

It is similar to ASR-8 radar:
- Operating band S-band (2.7 to 2.9 GHz)
- Pulse width 0.6 μsec
- Azimuth beam width - 1.35 degree

- Antenna rotation rate - 12.8 rpm
- Average Pulse Repetition Frequency - 1040 Hz
- Average Power - 875 watts
- Range coverage - 60 nmi

- <u>Coherent Processing Interval:</u>



T : Pulse length

$B = 1/T$: Band Width

T_{PRI} : Pulse Repetition Interval

$f_p = 1/T_{PRI}$: Pulse Repetition Frequency

$\delta = T/T_{PRI}$: Duty Cycle (%)

$T_{CPI} = NT_{PRI}$: Coherent Processing Interval

N : Number of pulses in the CPI

N = 2, 3, 4 for MTI

N usually much greater (8 to 1000) for Pulse Doppler

- The pulses are transmitted at a constant pulse repetition frequency.
- On receiving these 10 pulse, they were called coherent Processing Interval (CPI)
- Then Pulse Repetition Frequency is changed to avoid blind speed and to unmask moving targets from moving clutter. Also to eliminate second time around clutter echoes.

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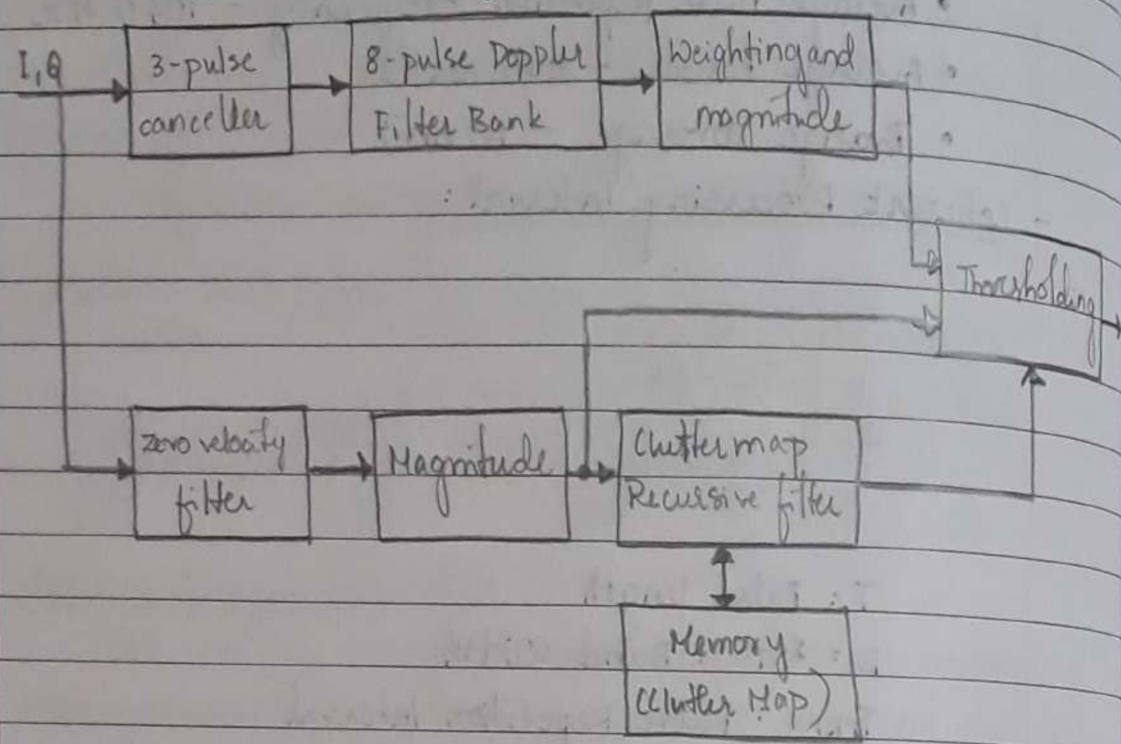
- Block Diagram of Moving Target Detector (MTD)



- <u>3-pulse canceller:</u>

The filter bank is implmented by a FFT. There are 10 pulses in the CPI but only 8 doppler filters are there since the three pulse canceller requires all three pulses before it can cancel clutter.

The first two pulses are discared and are called as filled pulses as the are needed to fill the canceller before cancellation of the clutter can begin.

The 3-pulse canceller reduces the dynamic range of the signals which the doppler filter bank has to handle. It compensates for the lack of adequate cancellation of stationary clutter in the Doppler filters. It acts like a double delay canceller.

- <u>8-pulse Doppler Filter Bank:</u>

The doppler filter bank separates moving targets from moving weather clutter.

When the moving clutter (ex: rain, birds with non zero doppler shift) and the target echo signal power appears in different doppler filters, the clutter echo will not interfere with the detection of the desired moving target. The moving clutter echo can be easily eliminated

Doppler Filter Bank (linear scale) 8 Filter - DFT - 13dB side lobes will give poor suppression of rain clutter.

- Frequency-domain weighting

After the filter bank the Doppler filter outputs are weighted to reduce the effects of the side lobes in the filter. The output of the filter are modified by subracting 25% of the outputs of the filters on either side.

If A, B, C represents the unweighted outputs of three continuous filters, then the weighted output applied to filter B is $B - (A/4 + C/4)$.

This is used to reduce the Doppler-filter side lobes for the better clutter attenuation. The magnitude of this operation is $(I^2 + Q^2)^2$

- Zero velocity Filter:

The 3-pulse canceller removes all echoes with zero velocity, the zero velocity filter had to be restablished in order to produce the clutter map.

- Clutter Map:

A conventional MTI processor eliminates stationary clutter, but it also eliminates aircraft moving on a crossing trajectory which causes the aircraft's radial velocity to be zero.

MTD makes use of larger crossection ~~of~~ the target to detect it. It uses clutter map that stores the magnitude of clutter echoes in digital memory. It then establishes the thresholds used for detecting those targets which produces zero velocity. About 10 to 20 scans are required to establish steady state values.

- Adaptive Thresholding:

It is used to allow detection of moving targets in stationary and moving clutter. The thresholding is continuously adapted to the local environment.

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* Performance metric of MTI filter:

1. **Clutter Attenuation:**

Delay line canceller: to cancel stationary clutter with zero doppler shift

Drawback: insufficient attenuation of clutter that results from finite width of clutter spectrum.

Finite width of clutter spectrum is due to:

a. internal motion of clutter
b. Instabilities of stalo and coho oscillators
c. Other imperfection of the radar and its signal processors.
d. Finite signal duration.

considering MTI filter.

Clutter Attenuation

$$CA = \frac{\text{MTI filter i/p clutter power}}{\text{MTI filter o/p clutter power}} = \frac{C_i}{C_o}$$

The ~~clutter~~ power spectral density is represented by a gaussian function given by

$$S(f) = \frac{P_c}{\sqrt{2\pi}\,\sigma_f} e^{-f^2/2\sigma_f^2}$$

P_c : constant clutter mean power.

Thus input power is

$$C_i = \int_{-\infty}^{\infty} S(f)\,df = P_c$$

Output power is

$$C_o = \int_{-\infty}^{\infty} |H(f)^2|\, S(f)\,df$$

Frequency response of single delay line canceller is given by

$$H(f) = 2\sin(\pi f T_r) = 2\sin\left(\pi \frac{f}{f_r}\right)$$

Therefore

$$C_o = \int_{-\infty}^{\infty} |H(f)|^2 S(f)\, df = \frac{4 P_c \pi^2 \sigma_f^2}{f_r}$$

Hence

$$\boxed{CA = \frac{C_i}{C_o} = \frac{P_c f_r^2}{4 P_c \pi^2 \sigma_f^2} = \left(\frac{f_r}{2\pi \sigma_f}\right)^2}$$ clutter Attenuation.

2. MTI Improvement Factor:

MTI improvement factor includes the signal gain as well as the clutter attenuation. It is defined as the signal to clutter ratio at the input of the filter, averaged over all target radial velocities of interest.

Improvement factor

$$I = \frac{\text{SCR at the output}}{\text{SCR at the input}}$$

$$I = \frac{S_o/C_o}{S_i/C_i} = \frac{S_o}{S_i}\frac{C_i}{C_o} = \frac{S_o}{S_i}(CA)$$

$$I = \begin{pmatrix}\text{Average power gain} \\ \text{of MTI filter}\end{pmatrix}(CA)$$

wkt, transfer function of MTI filter is $H(f)$, then

$$I = |H(f)|^2_{av}\,(CA)$$

since ~~for all~~ the frequency it will not be the same, the average is considered.

wkt, pulsed MTI radar is periodic with period f_r, then

$$|H(f)|^2_{av} = \frac{1}{f_r}\int_{-f_r/2}^{f_r/2} A \sin\left(\frac{\pi f}{f_r}\right) df = 2$$

Therefore

MTI improvement factor: $\boxed{I = 2(CA)}$

$$\boxed{I = 2\left(\frac{f_r}{2\pi \sigma_f}\right)^2}$$ for single Delay line canceller.

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3. <u>Sub Clutter Visibility</u> : (SCV)

It describes the Radar stability to detect non-stationary targets embedded in a strong clutter background for a given probability of detection and false alarm.

$$\boxed{SCV = \frac{I}{(SCR)_0 / P_d}}$$

$$\boxed{SCV = \frac{\text{MTI improvement factor}}{\text{min MTI output SCR for given } P_d}}$$