ANTENNAS AND WAVE PROPAGATION

UNIT - 01

Antenna Basics

* **Basic principle of radiation:**

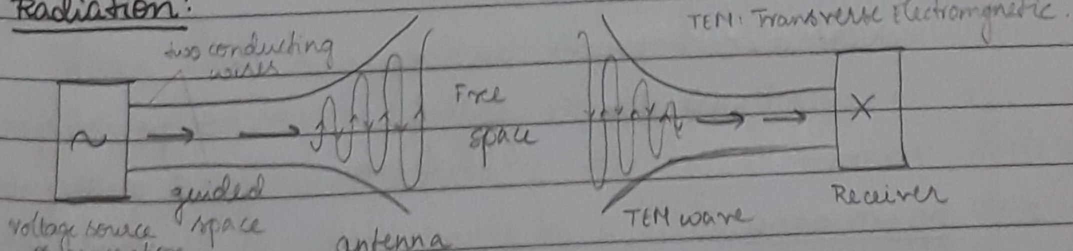
Radiation is produced by acceleration or deceleration of a charge. Basic equation of radiation is:

$$\frac{di}{dt} L = Q \frac{dv}{dt} \Rightarrow \dot{i} L = Q \dot{v}$$

i: time changing current
L: length of current element
Q: charge
$\dot{v}$: time changing velocity (acceleration of charge)

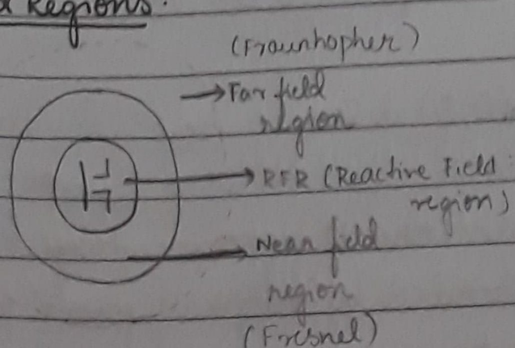
Radiation occurs only when
- there is acceleration or deceleration of charges
- the two wire line / cable is bent.

* **Radiation:**



The transmitter converts electric signals into radio waves and then the receiver converts radio waves into electrical signal.

* **Field Regions:**



In the far field region, the field components are traverse to the radiation direction and all power flow is directed radially outward. The shape of the field pattern is independent of the distance.

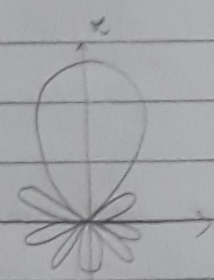
In the near field region, the longitudinal component of the electric field may be significant and power flow is not entirely radial. The shape of the field pattern depends on the distance.

The region near the poles of the sphere acts as a reflector.

★ Basic antenna parameters:

1. Radiation pattern:

The pattern has its main lobe (maximum radiation) in the z-direction with minor lobes (side and back) in the other directions.



Normalized field pattern:

$$E_\theta(\theta,\phi)_n = \frac{E_\theta(\theta,\phi)}{E_\theta(\theta,\phi)_{max}}$$

Normalized power pattern:

$$P_n(\theta,\phi)_n = \frac{S(\theta,\phi)}{S(\theta,\phi)_{max}}$$

S: power per unit area or Poynting vector

2. Beam area:

The beam area or beam solid angle Ω_A of an antenna is given by the integral of the normalized power pattern over a sphere.

$$\Omega_A = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} P_n(\theta,\phi)\sin\theta \, d\theta \, d\phi$$

$$\Omega_A = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} P_n(\theta,\phi)\, d\Omega$$

Beam area of an antenna is approximately equal to the product of angles subtended by the half power points of main lobe

$$\Omega_A \simeq \theta_{HP}\,\phi_{HP}$$

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3. **Radiation intensity:**

The power radiated from an antenna per unit solid angle is called radiation intensity U.

$$P_n(\theta,\phi) = \frac{U(\theta,\phi)}{U(\theta,\phi)_{max}} = \frac{S(\theta,\phi)}{S(\theta,\phi)_{max}}$$

4. **Beam efficiency:**

The total beam area Ω_A consists of the main beam area Ω_M and the minor lobe area Ω_m.

$$\therefore \Omega_A = \Omega_M + \Omega_m$$

The ratio of the main beam area to the total beam area is called the beam efficiency.

$$\varepsilon_M = \frac{\Omega_M}{\Omega_A}$$

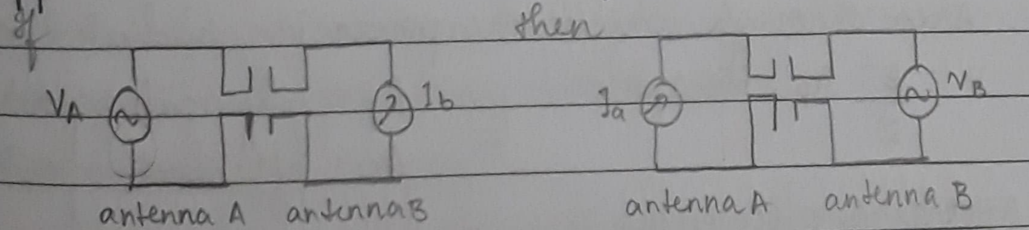
The ratio of the minor beam area to the total beam area is called the stray factor.

$$\varepsilon_m = \frac{\Omega_m}{\Omega_A}$$

$$\therefore \varepsilon_M + \varepsilon_m = 1$$

5. **Reciprocity**

If an emf is applied to the terminals of an antenna A and the current is measured at the terminals of another antenna B, then the equal current will be obtained at the terminals of A if the same emf is applied to the terminals of antenna B.



6. **Directivity and gain:**

The directivity of an antenna is equal to the ratio of the maximum power density $P(\theta,\phi_{max})$ to the average value over a sphere as observed in the far field of an antenna.

$$D = \frac{P(\theta,\phi)_{max}}{P(\theta,\phi)_{av}}$$

The average power density over a sphere

$$P(\theta,\phi)_{av} = \frac{1}{4\pi}\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} P(\theta,\phi)\sin\theta\, d\theta\, d\phi$$

$$D = \frac{P(\theta,\phi)_{max}}{\frac{1}{4\pi}\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} P(\theta,\phi)\sin\theta\, d\theta\, d\phi}$$

$$D = \frac{4\pi}{\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi}\underbrace{\left(P(\theta,\phi)/P(\theta,\phi)_{max}\right)}_{\text{Normalized power pattern}} d\Omega}$$

$$D = \frac{4\pi}{\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} P_n(\theta,\phi)\, d\Omega}$$

$$D = \frac{4\pi}{\Omega_A}$$

The gain G of an antenna is an actual or realised quantity which is less than the directivity due to ohmic loses. The ratio of the gain to the directivity is the antenna efficiency factor (k)

$$G = kD$$

$$D = \frac{41253^{\circ}}{\theta^{\circ}_{HP}\,\phi^{\circ}_{HP}}$$ approximate directivity.

7. <u>Antenna Apertures</u>

Area through which the power is radiated or received.

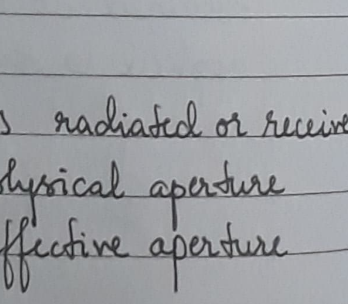
$$P = \frac{E^2}{Z} A_p = S A_p$$

A_p: physical aperture
A_e: effective aperture

Aperture efficiency

$$\varepsilon_{ap} = \frac{A_e}{A_p}$$

$A_e < A_p$.



consider an antenna with an effective aperture Ae, which radiates all of its power in a conical pattern of beam area Ω_A. Assuming a uniform field E_a over the aperture. The power radiated is

$$P = \frac{E_a^2}{Z_0} A_e$$

Assuming a uniform field E_r in the far field at a distance r, the power radiated is

$$P = \frac{E_r^2}{Z_0} r^2 \Omega_A$$

$$\therefore \frac{E_a^2}{\cancel{Z_0}} A_e = \frac{E_r^2}{\cancel{Z_0}} r^2 \Omega_A$$

wkt $\quad E_r = \frac{E_a A_e}{r\lambda}$

$$\therefore \cancel{E_a^2} \cancel{A_e} = \frac{\cancel{E_a^2} A_e^{\cancel{2}} \cancel{r^2} \Omega_A}{\cancel{r^2} \lambda^2}$$

$$\lambda^2 = A_e \Omega_A$$

$$\therefore \text{Directivity} = 4\pi \frac{A_e}{\lambda^2}$$

8. Effective height:

$$V = hE$$

$$h = \frac{V}{E}$$

h: effective height
E: incident field
V: voltage induced.

9. Bandwidth:

The bandwidth of an antenna expresses its ability to operate over a wide frequency range. The radiation pattern of an antenna may change dramatically outside its specified operating bandwidth.

10. Radiation efficiency:

Radiation efficiency of an antenna is the ratio of power radiated to the power accepted by antenna.

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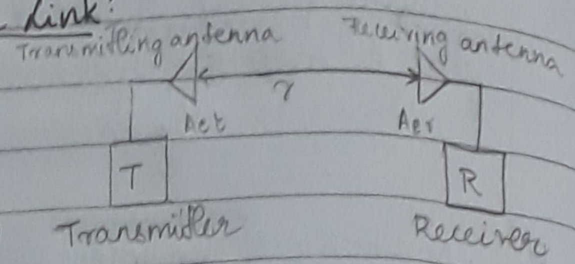
★ Radio communication link:



Let the transmitter feed a power P_t to a transmitting antenna of effective aperture A_{et}.
At a distance r a receiving antenna of effective aperture A_{er} intercepts some of the power radiated by the transmitting antenna and delivers to the receiver.
The power per unit area available at the receiving antenna is

$$S_r = \frac{P_t}{4\pi r^2}$$

If the antenna has gain G_t, the power per unit area available at the receiving antenna will be increased

$$S_r = \frac{P_t G_t}{4\pi r^2}$$

The power at receiving antenna of effective aperture A_{er}

$$P_r = S_r A_{er} = \frac{P_t G_t A_{er}}{4\pi r^2}$$

The gain of transmitting antenna is

$$G_t \simeq D_t = \frac{4\pi A_{et}}{\lambda^2}$$

$$\therefore \frac{P_r}{P_t} = \frac{\cancel{4\pi} A_{et}}{\lambda^2} \frac{A_{er}}{\cancel{4\pi} r^2}$$

$$\frac{P_r}{P_t} = \frac{A_{et} A_{er}}{\lambda^2 r^2}$$ Friis Transmission Formula

11. Antenna Temperature:

There is a continuous background of noise-like electromagnetic radiations in the atmosphere. These radiations arrive from outer space and are termed as cosmic waves. The antenna observes the cosmic noise at one temperature and an intervening absorbing atmosphere noise at another.

The combined temperature of the cosmic noise and atmospheric noise is called the space temperature, the brightness temperature or the antenna temperature of an ideal antenna.

NOTE:

- Isotropic : non directional (in all directions)
  Anisotropic : directional (single direction)
- An antenna is a transition device, or transducer between a guided wave and a free-space wave or vice-versa. It is a device which interfaces a circuit and space.

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UNIT - 01

Antenna Basics

- **Isotropic Antennas**
  Isotropic antennas radiate signals in all directions.

- **Anisotropic Antennas**
  Anisotropic antennas radiate signals only in one direction

★ **Basic Radiation Principle:**

$$\boxed{\dot{i}L = Q\dot{v}}$$

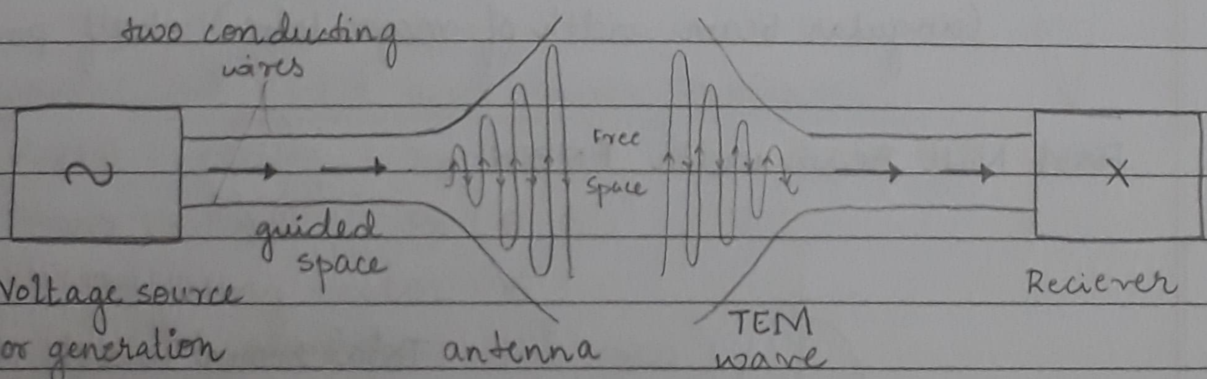
where

L : length of current element
Q : charge
$\dot{v}$: acceleration of charges

Radiation occurs only when
- there is accelleration of charges
- the two wire line/cable is bent.

★ **Radiation:**

The block diagram is given by



where TEM is transverse electromagnetic waves

The transmitter converts electric signals into radio waves and then the reciever converts radio waves into electrical signals.

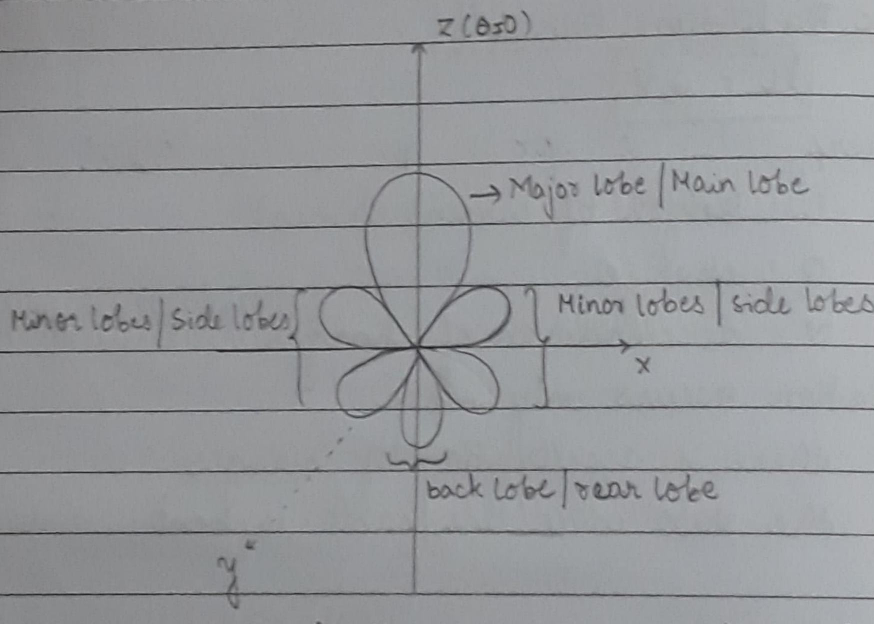
* **Basic Parameters:**

The basic parameters of antennas are

a. radiation pattern

b. beam area.

* **Antenna Pattern:**

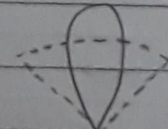


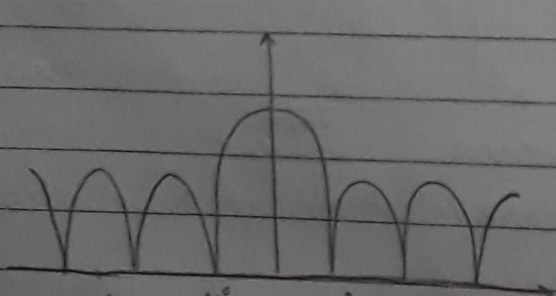
The maximum radiation occurs in major lobe.

Half power beam width HPBW = $\frac{1}{\sqrt{2}}$ max

(angular beam width of major lobe): half power

First Null Beam width FNBW





Total beam width of major axis | lobe

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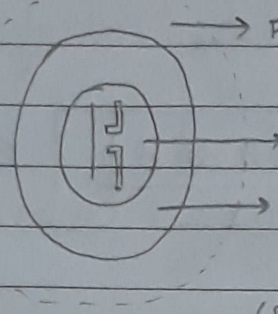
Date

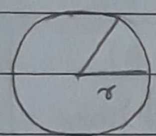
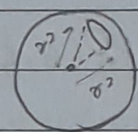
Page

★ Field Regions:



1. Reactive Field Region
2. Radiating near field region
3. Radiating far field region

★

Radian : r	Steradian
Plane Angle	solid angle
 $C = 2\pi r$ $\Rightarrow$ 2π radians	 $A = 4\pi r^2$ $\Rightarrow$ 4π steradian

$$1\ sr = (1\ \text{radian})^2$$

$$1\ sr = \left(\frac{180}{\pi}\right)^2 \deg$$

$$\boxed{1\ sr = 3282.8064\ \deg}$$

4π steradian

$4\pi \times 3282.8064 = 41253^{\square}$

★ Normalised Field and Power Pattern:

- Field Pattern : $E_\theta(\theta, \phi)$ $\quad P_n = E_n^2$
- Power Pattern : $P_\theta(\theta, \phi)$

Poynting vector : S

(power per unit area)

* Antenna Parameters:

1. **Beam Area / Beam solid Angle** : Ω_A

$$\boxed{\Omega_A = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} P_\theta(\theta,\phi)\underbrace{\sin\theta\, d\theta\, d\phi}_{d\Omega}} \quad Sr$$

Approximation

$$\boxed{\Omega_A = \phi_{HP}\,\theta_{HP}}$$

HP: Half power

Integral of normalised power factor.

2. **Radiation Intensity** : U

Power radiated per unit solid angle (Sr)

$$\boxed{P_n(\theta,\phi) = \frac{U(\theta,\phi)}{U(\theta,\phi)_{max}} = \frac{S(\theta,\phi)}{S(\theta,\phi)_{max}}}$$

3. **Beam Efficiency** : E_M

$$\boxed{\Omega_A = \Omega_M + \Omega_m}$$

where Ω_M : major lobe area

Ω_m : minor lobe area

$$\boxed{E_M = \frac{\Omega_M}{\Omega_A}}$$

Stray factor: ratio of minor lobe area to beam area

$$\boxed{E_m = \frac{\Omega_m}{\Omega_A}}$$

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4. <u>Directivity</u> : D

It is the ratio of maximum power factor to average power factor.

$$\boxed{D = \frac{P(\theta,\phi)_{max}}{P(\theta,\phi)_{av}}}$$

$$P_{\theta}(\theta,\phi) = \frac{1}{4\pi \sin\theta \, d\theta \, d\phi}$$

$$\boxed{D = \frac{4\pi}{\Omega_A}} \qquad \boxed{D = \frac{41253^{\circ}}{\theta^{\circ}_{HP} \, \phi^{\circ}_{HP}}}$$

approximate directivity.

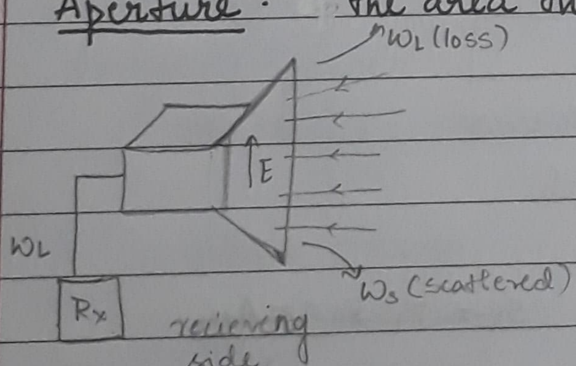
5. <u>Gain</u> : G

$$\boxed{G = KD}$$

$$\boxed{G = 4\pi \frac{\text{Radiation Intensity}}{\text{Power Incident}}}$$ dimensionless

The gain is always less than directivity due to ohmic losses during transmission.

6. <u>Aperture</u> : the area through which signal is recieved / transmitted

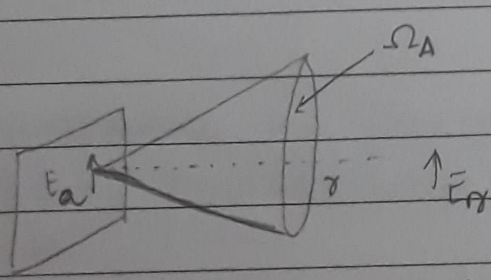


HORN ANTENNA

$$\boxed{P = S A_p}$$

$$\boxed{P = \frac{E^2}{Z} A_p}$$

$$\boxed{\varepsilon_p = \frac{A_e}{A_p}}$$



$$P = \frac{E_a^2 A_e}{Z_0} \qquad P = \frac{E_r^2 r^2 \Omega_A}{Z_0}$$

Equating both the equations

$$\frac{E_a^2 A_e}{Z_0} = \frac{E_r^2 r^2 \Omega_A}{Z_0}$$

$$\boxed{E_r = \frac{E_a A_e}{r\lambda}}$$ substituting

$$E_a^2 A_e = \left(\frac{E_a A_e}{r\lambda}\right)^2 r^2 \Omega_A$$

$$1 = \frac{A_e \Omega_A}{\lambda^2} \Rightarrow \boxed{\Omega_A = \frac{\lambda^2}{A_e}}$$

wkt $D = \frac{4\pi}{\Omega_A}$

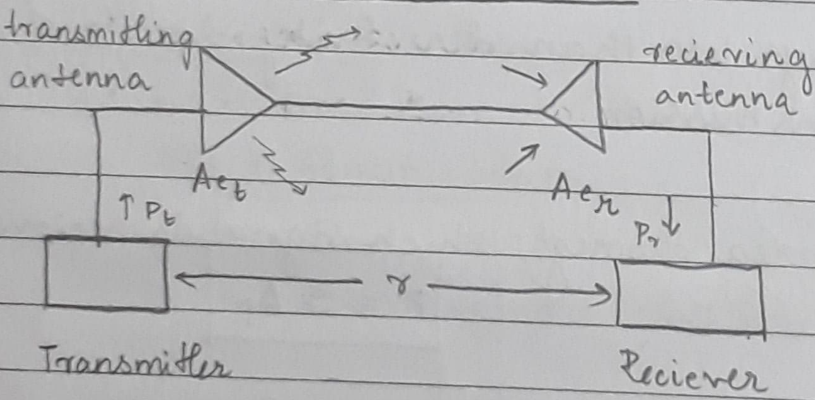
$$\boxed{D = \frac{4\pi}{\lambda^2} A_e}$$

7. <u>Effective Height</u> : h_e

$$\boxed{h_e = \frac{V}{E}}$$

where
V - Voltage
E - Electric field

★ <u>Friis Transmission Formula</u>:



Radio communication Link

Power of transmitter

$$P_t = S_r \times 4\pi r^2$$

~~$S_r = P_t \times \frac{4\pi A_{et}}{\lambda^2}$~~

$$\boxed{S_r = \frac{P_t}{4\pi r^2}}$$

$$\boxed{S_r = \frac{P_t G_t}{4\pi r^2}}$$

$G = kD$ but $G_t = \frac{4\pi A_{et}}{\lambda^2}$

$k = 1 \Rightarrow G = D$

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$$S_r = \frac{P_t}{\cancel{4\pi} r^2} \times \frac{\cancel{4\pi} A_{et}}{\lambda^2}$$

$$P_r = S_r A_{er}$$

$$P_r = \frac{P_t A_{et} \cdot A_{er}}{\lambda^2 r^2}$$

$$\boxed{\frac{P_r}{P_t} = \frac{1}{\lambda^2 r^2} A_{et} A_{er}}$$ Friis Transmission Formula

Q: A antenna has field pattern given as $E(\theta) = \cos^2\theta$ for $0 \le \theta \le 90°$. Find the half power beam width.

Formula

$$HPBW = 2 \,|\, \theta_{max} - \theta_{hp} \,|$$

Given

$$E(\theta) = \cos^2\theta$$

Equating to 1 to find θ_{max}

$\cos^2\theta = 1$ — Power is maximum

$\cos\theta = 1$ — at angle which gives 1.

$\theta = \cos^{-1} 1$ — Hence equate to 1.

$\underline{\theta_{max} = 0}$

For Half power

$$\cos^2\theta = \frac{1}{\sqrt{2}}$$

$$\cos\theta = 0.8408$$

$$\theta = \cos^{-1}(0.8408)$$

$$\therefore \underline{\theta_{hp} = 32.77°}$$

$HPBW = 2 \,|\, \theta_{max} - \theta_{hp} \,|$

$= 2 \,|\, 0 - 32.77 \,|$

$\underline{\theta_{HP} = 66°}$

Q: An antenna has field pattern given as $E(\theta) = \cos^2\theta$ for $0 \le \theta \le 90°$. Find the beam area.

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Date
Page

$$\Omega_A = \int_{\phi=0}^{\phi=2\pi} \int_{\theta=0}^{\theta=\pi/2} P_n \sin\theta \, d\theta \, d\phi$$

wkt $P_n = E_n^2$

$$\Omega_A = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} E_n^2 \sin\theta \, d\theta \, d\phi$$

$$= \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} \cos^4\theta \sin\theta \, d\theta \, d\phi$$

$$= \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} \cos^3\theta \cos\theta \sin\theta \, d\theta \, d\phi$$

Let $\cos\theta = x$ as $\theta \to 0$ $x \to 1$

$-\sin\theta = dx$ $\theta \to \pi/2$ $x \to 0$

$$\therefore \Omega_A = \int_{\phi=0}^{2\pi} \int_{x=0}^{1} x^3 x \, dx \, d\phi$$

$$= \int_{\phi=0}^{2\pi} \int_{x=0}^{1} x^4 \, dx \, d\phi$$

$$= \int_{\phi=0}^{2\pi} \frac{x^5}{5}\Big|_0^1 \, d\phi$$

$$= \int_{\phi=0}^{2\pi} \left[\frac{1}{5}\right] d\phi$$

$$= \frac{1}{5} \phi \Big|_0^{2\pi}$$

$$\Omega_A = \frac{2\pi}{5} \text{ sr}$$

Approximation

$\Omega_A = \theta_{HP} \phi_{HP}$

$\Omega_A^{approx} = 66(66)$

$\Omega_A^{approx} = 4356^\square$

ϕ_{HP} is not given

$\phi_{HP} = \theta_{HP}$

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Date
Page

Q: Estimate the directivity of an antenna with $\theta_{HP} = 2^\circ$ and $\phi_{HP} = 1^\circ$.

Directivity

$$D = \frac{41253^\circ}{\theta^\circ_{HP}\,\phi^\circ_{HP}}$$

$$D = \frac{41253^\circ}{2^\circ \cdot 1^\circ} = \underline{20626.5}$$

Q: The radiation intensity of an antenna is given by $U = B_0 \cos\theta$ only in the upper hemisphere. Find the

- exact directivity
- approximate directivity
- decibel difference

given

$U = B_0 \cos\theta$ $\qquad B_0 = 1$

Exact directivity

$$D = \frac{4\pi}{\Omega_A} = \frac{4\pi}{\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi/2} U_n \sin\theta \, d\theta \, d\phi}$$

$$D = \frac{4\pi}{\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi/2} B_0 \cos\theta \sin\theta \, d\theta \, d\phi}$$

let $x = \sin\theta$ $\qquad \theta \to 0 \quad x \to 0$

$dx = \cos\theta \, d\theta$ $\qquad \theta \to \pi/2 \quad x \to 1$

$$D = \frac{4\pi}{\int_{\phi=0}^{2\pi}\int_{0}^{1} B_0 \, x \, dx \, d\phi}$$

$$D = \frac{4\pi}{B_0 \int_{\phi=0}^{2\pi} \frac{x^2}{2}\Big|_0^1 d\phi} = \frac{\cancel{4\pi}}{B_0 \left(\frac{1}{\cancel{2}}\right)(\cancel{2\pi})}$$

$$D_{exact} = \frac{4}{B_0} \quad \Rightarrow \quad \underline{D = 4}$$

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Approximation Directivity

wkt $P_n = E_n^2 = U$

$\Rightarrow E_n = \sqrt{\cos\theta}$

For θ_{max}

$\sqrt{\cos\theta} = 1$

$\cos\theta = 1$

$\theta_{max} = 0^\circ$

For θ_{hp}

$\sqrt{\cos\theta} = 1/\sqrt{2}$

$\cos\theta = 1/2$

$\theta_{hp} = 60^\circ$

$HPBW = 2\,|\theta_{max} - \theta_{hp}|$

$= 2\,|0 - 60^\circ|$

$\theta_{HP} = 120^\circ$

$$D = \frac{41253}{\theta_{HP}\,\phi_{HP}}$$

$\phi_{HP} = \theta_{HP}$ (not given)

$$D = \frac{41253}{120(120)}$$

$D_{approx} = 2.86$

Decibel Difference

$$\text{Decibel difference} = 10 \log \left| \frac{D_{exact}}{D_{approx}} \right|$$

$$= 10 \log \left| \frac{4}{2.86} \right|$$

$= 10(0.145)$

$= 1.45$

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Date ____
Page ____

Q: A normalised field pattern of an antenna is given by $E_n = \sin\theta \sin\phi$ where $0 \le \theta \le \pi$ and $0 \le \phi \le \pi$. Find

- Exact directivity
- Approximate directivity
- Decimel directivity

given:

$E_n = \sin\theta \sin\phi$

Exact directivity

$$D = \frac{4\pi}{\Omega A}$$

$$D = \frac{4\pi}{\int_{\phi=0}^{\pi}\int_{\theta=0}^{\pi} E_n^2 \sin\theta\, d\theta\, d\phi}$$

$$D = \frac{4\pi}{\int_{\phi=0}^{\pi}\int_{\theta=0}^{\pi} \sin^3\theta \sin^2\phi\, d\theta\, d\phi}$$

$$D = \frac{4\pi}{\int_{\phi=0}^{\pi}\int_{\theta=0}^{\pi} \left[\frac{-\sin 3\theta + 3\sin\theta}{4}\right]\left[\frac{1-\cos 2\phi}{2}\right] d\theta\, d\phi}$$

$$D = \frac{4\pi}{1/8 \int_{\phi=0}^{\pi} (1-\cos 2\phi)\, d\theta \int_{\theta=0}^{\pi} (-\sin 3\theta + 3\sin\theta)\, d\theta}$$

$$D = \frac{4\pi}{\frac{1}{8}\left[\phi - \frac{\sin 2\phi}{2}\right]_0^{\pi}\left[\frac{\cos 3\theta}{3} - 3\cos\theta\right]_0^{\pi}}$$

$$D = \frac{4\pi}{\frac{1}{8}\left[\pi - 0 - 0 + 0\right]\left[-\frac{1}{3} + 3 - \frac{1}{3} + 3\right]}$$

$$D = \frac{\cancel{4}^{2}\pi}{\frac{\pi}{\cancel{8}}\left[\frac{\cancel{16}}{3}\right]} \Rightarrow \underline{D_{exat} = 6}$$

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Approximate Directivity:

given $E_M = \sin\theta \sin\phi$

since ϕ is not given $\theta = \phi$

$\therefore \sin^2\theta = 1$ $\quad\quad$ $\sin^2\phi = 1$

$\sin\theta = 1$ $\quad\quad$ $\sin\phi = 1$

$\theta_{max} = 90^\circ$ $\quad\quad$ $\phi = 90^\circ$

$\sin\theta = 1/\sqrt{2}$ $\quad\quad$ $\sin\phi = 1/\sqrt{2}$

$\theta_{hp} = 45^\circ$ $\quad\quad$ $\phi_{hp} = 45^\circ$

$HPBW = 2|\theta_{max} - \theta_{hp}|$

$HPBW = 2|90 - 45|$

$HPBW = 90^\circ$

wkt $D = \dfrac{41253}{\theta_{HP}\,\phi_{HP}}$

$D = \dfrac{41253}{90^\circ \cdot 90^\circ}$

$D = 5.09$

Decibel Difference

$D_{dB} = 10 \log \left| \dfrac{D_{exact}}{D_{approx}} \right|$

$D_{dB} = 10 \log \left| \dfrac{6}{5.09} \right|$

$D_{dB} = 10\,(0.0714)$

$D_{dB} = 0.714$

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Date ______
Page ______

Q: Calculate the approximate directivity from the half power beam width of an unidirectional antenna if the normalised power pattern is given by. ~~$P_n = \cos\theta$~~

- $P_n = \cos\theta$
- $P_n = \cos^2\theta$
- $P_n = \cos^3\theta$
- $P_n = \cos^n\theta$

In all the cases these patterns are unidirectional with P_n having a value only for $0 < \theta < 90^\circ$ and $0 \leq \phi < \pi$.

$P_n = \cos\theta$

wkt $P_n = E_n^2 = \cos\theta$

$\Rightarrow E_n = \sqrt{\cos\theta}$

$\sqrt{\cos\theta} = 1$ $\qquad$ $\sqrt{\cos\theta} = 1/\sqrt{2}$

$\cos\theta = 1$ $\qquad$ $\cos\theta = 1/2$

$\theta_{max} = 0^\circ$ $\qquad$ $\theta_{HP} = 60^\circ$

$HPBW = 2|\theta_{max} - \theta_{HP}|$

$= 2|0 - 60^\circ|$

$HPBW = 120^\circ = \theta_{HP}$

$$D_{approx} = \frac{41253}{\theta_{HP}\,\phi_{HP}}$$

$$D_{approx} = \frac{41253}{120^\circ \times 120^\circ}$$

$D_{approx} = 2.86$

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$\underline{P_n = \cos^2\theta}$

$P_n = E_n^2 = \cos^2\theta$

$\Rightarrow E_n = \cos\theta$

$\cos\theta = 1$ $\qquad\qquad$ $\cos\theta = 1/\sqrt{2}$

$\theta_{max} = 0^\circ$ $\qquad\qquad$ $\theta_{hp} = 45^\circ$

$HPBW = \theta_{HP} = 2|\theta_{max} - \theta_{hp}|$

$\theta_{HP} = 2|0-45^\circ|$

$\theta_{HP} = 90^\circ$

$$D_{approx} = \frac{41253}{\theta_{HP}\,\phi_{HP}}$$

$$D_{approx} = \frac{41253}{90^\circ\,90^\circ} = 5.092$$

$\underline{P_n = \cos^3\theta}$

$P_n = E_n^2 = \cos^3\theta$

$\Rightarrow E_n = \cos^{3/2}\theta$

$\cos^{3/2}\theta = 1$ $\qquad\qquad$ $\cos^{3/2}\theta = 1/\sqrt{2}$

$\cos\theta = 1$ $\qquad\qquad$ $\cos\theta = 0.7936$

$\theta_{max} = 0^\circ$ $\qquad\qquad$ $\theta_{hp} = 37.47^\circ$

$HPBW = \theta_{HP} = 2|\theta_{max} - \theta_{hp}|$

$\theta_{HP} = 2|0 - 37.47^\circ|$

$\theta_{HP} = 74.94^\circ$

$$D_{approx} = \frac{41253}{\theta_{HP}\,\phi_{HP}}$$

$$D_{approx} = \frac{41253}{74.94^\circ \times 74.94^\circ} = 7.34$$

$P_n = \cos^n\theta$

$P_n = E_n^2 = \cos^n\theta$

$\Rightarrow E_n = \cos^{n/2}\theta$

$\cos^{n/2}\theta = 1$

$\cos\theta = 1$

$\theta_{max} = 0^\circ$

$\cos^{n/2}\theta = 1/\sqrt{2}$

$\cos^{n/2}\theta = (1/2)^{1/2}$

$\cos^n\theta = 1/2$

θ varies: $0 < \theta < 90^\circ$

$\cos\theta = (1/2)^{1/n}$

$\theta_{hp} = \cos^{-1}\sqrt[n]{1/2}$

general formula

$HPBW = \theta_{HP} = 2|\theta_{max} - \theta_{hp}|$

$\theta_{HP} = 2|0^\circ - \cos^{-1}\sqrt[n]{1/2}|$

$\theta_{HP} = 2\cos^{-1}\sqrt[n]{1/2}$

$$D_{approx} = \frac{41253}{\phi_{HP}\ \theta_{HP}}$$

$$D_{approx} = \frac{41253}{2\cos^{-1}\sqrt[n]{1/2} \cdot 2\cos^{-1}\sqrt[n]{1/2}}$$

$$D_{approx} = \frac{10313.25}{[\cos^{-1}\sqrt[n]{1/2}]^2}$$

Q: What is the maximum effective aperture of a microwave antenna with the directivity of 900?

given: $D = 900$

$$D = \frac{4\pi A_e}{\lambda^2}$$

$$\Rightarrow A_e = \frac{D\lambda^2}{4\pi}$$

$$A_e = \frac{900\lambda^2}{4\pi}$$

$$A_e = 71.62\lambda^2$$

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Q: What is the maximum effective aperture for a beam antenna having a HPBW of $30°$ and $35°$, in perpendicular planes intersecting in the beam axis at 900 MHz?

- Given:

$\theta_{HP} = 30°$ $\phi_{HP} = 35°$

$f = 900\,MHz$

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{900\times10^6} = 0.33\,m$$

$$A_e = \frac{\lambda^2}{\Omega_A}$$

$\Omega_A = \theta_{HP}\phi_{HP}$

$\Omega_A = 30 \times 35$

$\Omega_A = 1050°$

$$A_e = \frac{(0.33)^2}{1050°}$$

$1050° = 1050 / 3282.8064$

$= 0.3198^{\square}$

$$A_e = \frac{(0.33)^2}{0.3198^{\square}}$$

$A_e = 0.34\,m$

Q. What is the maximum power received at a distance of 0.5 km over freespace of 1 GHz circuit consisting of a transmitting antenna with 25 dB gain and a receiving antenna with 20 dB? The transmitting antenna input is 150 W.

- ~~Transmission gain~~

$$A_e = \frac{D\lambda^2}{4\pi}$$

$f = 1\,GHz$

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{1\times10^9} = 0.3\,m$$

$$\frac{P_r}{P_t} = \frac{\frac{D_t\lambda^2}{4\pi} \times \frac{D_r\lambda^2}{4\pi}}{(r\lambda)^2} \quad \text{--- (1)}$$

$$\left(\because \frac{P_r}{P_t} = \frac{A_{et}A_{er}}{r^2\lambda^2}\right)$$

$D_{dB} = 10 \log D_t$

$25 = 10 \log D_t$

$\Rightarrow D_t = 316.22$

$D_{dB} = 10 \log D_r$

$20 = 10 \log D_r$

$D_r = 100$

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Substituting in eq (1)

$$\frac{P_r}{P_t} = \frac{(316.22)(100)(0.3)^2}{16\pi^2 (500)^2}$$

$$P_r = 150 \times 7.2 \times 10^{-5}$$

$$P_r = 10.8\ mW$$

Q: What is the maximum power received at a distance of 0.6km over a free space of 2GHz circuit consisting of transmitting antenna with 25dB gain and receiving antenna with 20dB? The transmitting input is 150W.

$f = 2GHz$

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{2\times10^9} = 0.15\ m$$

$$\frac{P_r}{P_t} = \frac{\frac{D_t\lambda^2}{4\pi} \times \frac{D_r\lambda^2}{4\pi}}{(r\lambda)^2}$$

$D_{dB} = 10 \log D_t$

$25 = 10 \log D_t$

$D_t = 316.22$

$D_{dB} = 10 \log D_r$

$20 = 10 \log D_r$

$D_r = 100$

$$\therefore \frac{P_r}{150} = \frac{(316.22)(100)(0.15)^2}{16\pi^2 (500)^2}$$

$$P_r = 150 \times 1.25 \times 10^{-5}$$

$$P_r = 1.877\ mW$$

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Q: Calculate the approximate directivity from the half power beam width of unidirectional antenna if the field pattern is given by $E(\theta,\phi) = \cos^2\theta \sin^{3/2}\phi$ where $0 \le \theta < \pi$ and $0 \le \phi \le \pi$. Also calculate the exact directivity and decibel difference.

— given

$E = \cos^2\theta \sin^{3/2}\phi$

Approximate directivity

$\cos^2\theta = 1$

$\cos\theta = 1$

$\theta_{max} = 0^\circ$

$\cos^2\theta = 1/\sqrt{2}$

$\cos\theta = (1/2)^{1/4}$

$\theta_{HP} = 32.76^\circ$

$\theta_{HP} = 2|\theta_{max} - \theta_{hp}|$

$= 2|-32.76^\circ|$

$\theta_{HP} = 65.53^\circ$

$\sin^{3/2}\phi = 1$

$\sin\phi = 1$

$\phi_{max} = 90^\circ$

$\sin^{3/2}\phi = 1/\sqrt{2}$

$\sin\phi = (1/2)^{1/3}$

$\phi_{hp} = 52.53^\circ$

$\phi_{HP} = 2|\phi_{max} - \phi_{hp}|$

$= 2|90 - 52.53|$

$\phi_{HP} = 74.93^\circ$

$$D_{approx} = \frac{41253}{\theta_{HP}\,\phi_{HP}}$$

$$D_{approx} = \frac{41253}{65.53 \times 74.93}$$

$D_{approx} = 8.4$

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Date ______
Page ______

$\sin^3 x = \frac{3\sin x - \sin 3x}{4}$

$\cos^3 x = \frac{3\cos x + \cos 3x}{4}$

Exact Directivity

$$D_{exact} = \frac{4\pi}{\Omega_A}$$

$$D_{exact} = \frac{4\pi}{\int_{\phi=0}^{\pi}\int_{\theta=0}^{\pi} En^2 \sin\theta \, d\theta \, d\phi}$$

$$D_{exact} = \frac{4\pi}{\int_{\phi=0}^{\pi}\int_{\theta=0}^{\pi} \cos^4\theta \sin^3\phi \, \cancel{\sin\theta} \, d\theta \, d\phi}$$

$$D_{exact} = \frac{4\pi}{\int_{\phi=0}^{\pi} \sin^3\phi \, d\phi \int_{\theta=0}^{\pi} \cos^4\theta \sin\theta \, d\theta}$$

$$D_{exact} = \frac{4\pi}{\int_{\phi=0}^{\pi} \sin^3\phi \, d\phi \; 2\int_{\theta=0}^{\pi/2} \cos^4\theta \sin\theta \, d\theta}$$

Formula: $\int_0^{\pi/2} \cos^n\theta \sin\theta \, d\theta = \frac{1}{n+1} \Rightarrow \int_0^{\pi/2} \cos^4\theta \sin\theta \, d\theta = \frac{1}{5}$

$$D_{exact} = \frac{{}^{2}4\pi}{2\left(\frac{1}{5}\right)\int_{\phi=0}^{\pi} \frac{3\sin\phi - \sin 3\phi}{4} \, d\phi}$$

$$D_{exact} = \frac{40\pi}{\left[-3\cos\phi + \frac{\cos 3\phi}{3}\right]_0^{\pi}}$$

$$D_{exact} = \frac{40\pi}{\left[3 - \frac{1}{3} + 3 - \frac{1}{3}\right]}$$

$D_{exact} = 23.56$

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Decibel difference

$$D_{dB} = 10 \log \left| \frac{D_{exact}}{D_{approx}} \right|$$

$$D_{dB} = 10 \log \left| \frac{8.4}{23.56} \right|$$

$$D_{dB} = -4.478$$

Q: 1 GHz satellite antenna has an e-plate beam width of 12° and h-plate beam width of 10°. The antenna conductivity and mismatch total loss -3dB. Estimate the gain of antenna.

$G = KD$

but $K = 1 - \text{losses}$

$-3dB$ losses

$\Rightarrow -3dB = 10 \log(\text{loss})$

$\log(\text{losses}) = -0.3$

$\text{losses} = 0.5012$

$\therefore k = 1 - \text{losses}$

$k = 1 - 0.5012$

$k = 0.4988 \approx 0.5$

$$D = \frac{41253^\circ}{10^\circ \times 12^\circ}$$

$D = 343.625$

$D_{dB} = 10 \log(343.625)$

$D_{dB} = 25.36$

$\therefore G = 0.5 \times 25.36$

$G = 12.68$

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Date
Page

Q: Two spacecrafts are separated by 100 Mm. Each has an antenna with directivity of 1000 operating at 2.5GHz. If craft A is receiver and requires 20 dB over 1pW, what transmitter power is required on craft B to achieve this signal level.

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{2.5\times10^9}$$

given $r = 100\times10^6$ m

$D = 1000$

$P_r(dB) = 20 dB$

$$\lambda = 0.12 \text{ m}$$

$P_r = 10 \log P_r$

$20 dB = 10 \log P_r$

$\log P_r = 2$

$P_r = 100$

20 dB over 1pW

$\therefore P_r = 100\times10^{-12}$

$P_r = 10^{-10}$

$$\frac{P_r}{P_t} = \frac{\frac{D_t\lambda^2}{4\pi}\times\frac{D_r\lambda^2}{4\pi}}{\lambda^2 r^2}$$

$$\frac{10^{-10}}{P_t} = \frac{1000\times1000\times(0.12)^2}{16\pi^2(10^8)^2}$$

$$P_t = \frac{10^{-10}\times16\pi^2\times10^{16}}{(0.12)^2\times10^6}$$

$$P_t = 10.966 \text{ kW}$$

Q: Two spacecrafts are separated by 3 Mm. Each has an antenna with directivity 200 operating at 2GHz. If craft A receiver is receiver and requires 20dB over 1pW, what transmitter power is required on craft B to achieve this signal level.

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{2\times10^9} = 0.15m$$

$r = 3\times10^6 m$

$D = 200$

$P_r(dB) = 20dB$ over $1pW$

$P_r = 10\log P_r$ — P_r over $1pW$

$20 = 10\log P_r$ — $\therefore P_r = 10^2\times10^{-12}$

$\log P_r = 2$ — $P_r = 10^{-10}\ W$

$P_r = 10^2 W$

$$\frac{P_r}{P_t} = \frac{\frac{D_t\lambda^2}{4\pi}\times\frac{D_r\lambda^2}{4\pi}}{\lambda^2 r^2}$$

$$\frac{10^{-10}}{P_t} = \frac{200\times200\times(0.15)^2}{16\pi^2(3\times10^6)^2}$$

$$P_t = \frac{16\pi^2(9)10^{12}\times10^{-10}}{4\times10^4(0.15)^2}$$

$P_t = 157.92\ W$

Q: Obtain the actual and approximate directivity for given pattern. $P_n = \cos^5\theta\cos\phi$ where $0<\theta<\pi/2$ and $0\le\phi\le2\pi$.

Given: $P_n = \cos^5\theta$

Exact directivity

$$D = \frac{4\pi}{\Omega_A}$$

$$D = \frac{4\pi}{\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi/2} P_n \sin\theta\, d\theta\, d\phi}$$

$$D = \frac{4\pi}{\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi/2} \cos^5\theta\sin\theta\, d\theta\, d\phi\cos\phi}$$

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let $\cos\theta = x$ as $\theta \to 0$ $x \to 1$

$\Rightarrow dx = -\sin\theta \, d\theta$ $\theta \to \pi/2$ $x \to 0$

$$\therefore D = \frac{4\pi}{\int_{\phi=0}^{2\pi} \int_{x=0}^{1} x^5 \, dx \, d\phi \cos\phi}$$

$$D = \frac{4\pi}{\frac{x^6}{6}\Big|_0^1 \sin\phi\Big|_0^{2\pi}}$$

$$D_{exact} = \frac{4\pi}{\frac{1}{6}[0]}$$

$$D_{exact} = \infty$$

Approximate Directivity:

$P_n = E_n^2 = \cos^5\theta \cos\phi$

$\Rightarrow E_n = \cos^{5/2}\theta \cos^{1/2}\phi$

$\cos^{5/2}\theta = 1$

$\cos\theta = 1$

$\theta_{max} = 0^\circ$

$\cos^{5/2}\theta = 1/\sqrt{2}$

$\cos^5\theta = 1/2$

$\cos\theta = (1/2)^{1/5}$

$\theta_{hp} = 29.47^\circ$

$\theta_{HP} = 2|\theta_{max} - \theta_{hp}|$

$= 2|0 - 29.47^\circ|$

$\theta_{HP} = 58.95^\circ$

$\cos^{1/2}\phi = 1$

$\cos\phi = 1$

$\phi_{max} = 0^\circ$

$\cos^{1/2}\phi = 1/\sqrt{2}$

$\cos\phi = 1/2$

$\phi_{hp} = 60^\circ$

$\phi_{HP} = 2|\phi_{max} - \phi_{hp}|$

$= 2|0 - 60^\circ|$

$\phi_{HP} = 120^\circ$

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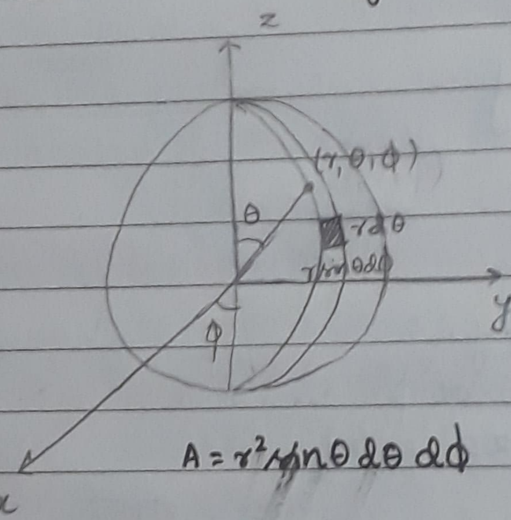
$$D_{approx} = \frac{41253}{\theta_{HP}\,\phi_{HP}}$$

$$D_{approx} = \frac{41253}{58.95 \times 120}$$

$$\underline{\underline{D_{approx} = 5.83}}$$

* **Spherical coordinate system**

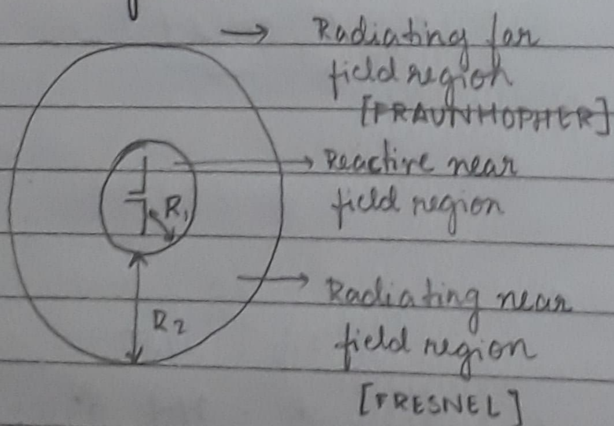


$A = r^2 \sin\theta \, d\theta \, d\phi$

r : radial distance
θ : polar angle
φ : Azimuth angle

r : 0 to R
θ : 0 to π
φ : 0 to 2π

* **Field Regions:**



$R_1 = 0.62\sqrt{\dfrac{D^3}{\lambda}}$
$R_2 = \dfrac{2D^2}{\lambda}$

Far field region : only radiation.
Reactive near field region : inductive coupling

UNIT - 02

Point Sources and Arrays

★ **Power Theorem:**

"The total power radiated by the source is the integral over the surface of the sphere of the radial component S_r, of the average Poynting vector"

$$P = \oiint S \cdot ds = \oiint S_r \cdot ds$$

Here $ds = r^2 \sin\theta \, d\theta \, d\phi$

For an isotropic source, S_r is independent of θ and ϕ

$$P = S_r \oiint ds$$

$$P = S_r \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} r^2 \sin\theta \, d\theta \, d\phi$$

$$P = S_r r^2 \left[-\cos\theta\right]_0^{\pi} \left[\phi\right]_0^{2\pi}$$

$$P = S_r r^2 [2][2\pi]$$

Therefore

$$S_r = \frac{P}{4\pi r^2}$$

★ **Radiation Intensity:**

$$P = \oiint U \, d\Omega$$

For isotropic point source

$$P = \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} U_0 \sin\theta \, d\theta \, d\phi$$

$$P = U_0 \left[-\cos\theta\right]_0^{\pi} \left[\phi\right]_0^{2\pi}$$

$$P = U_0 (2)(2\pi)$$

Therefore

$$U_0 = \frac{P}{4\pi}$$

wkt $S_r = \dfrac{P}{4\pi r^2}$

Hence $U_0 = S_r r^2$ $\quad$ $P = U_m 4\pi$ $\quad$ $\dfrac{U_m}{U_0}$ = directivity

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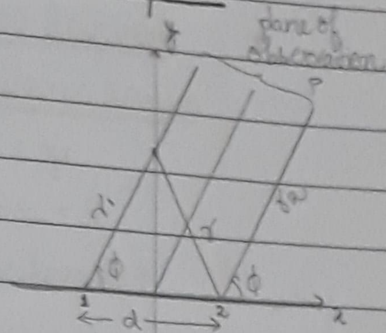
* A pattern showing the variation of the electric field intensity at a constant radius r as a function of angle (θ, ϕ) is called a **field pattern**.

* **Two isotropic sources of same magnitude and phase:**

Consider two isotropic point sources having equal amplitudes and oscillating in same phase.

Let the two sources be separated by a distance d.



If the phase angle between the two fields is $\beta d \cos\phi = \psi$

then far field component at P due to 1 is $E_1 = E_0 e^{-j\frac{\psi}{2}}$

far field component at P due to 2 is $E_2 = E_0 e^{j\frac{\psi}{2}}$

where E_0 is the amplitude of field components at distance r.

The total field at a distance r in the direction ϕ is

$$E = E_1 + E_2$$

$$E = E_0 e^{-j\psi/2} + E_0 e^{j\psi/2}$$

$$E = 2E_0 \cos\frac{\psi}{2} \quad \Rightarrow \quad E_{max} = 2E_0$$

Array factor

$$AF = \frac{|E|}{|E_{max}|} = \frac{2E_0 \cos\psi/2}{2E_0}$$

Normalized field

$$E = \cos\frac{\psi}{2}$$

$$AF = \cos\left[\frac{\beta d \cos\phi}{2}\right]$$

* **Two isotropic source of same magnitude and opposite phase:**

Consider two isotropic point sources having equal amplitudes and oscillating in opposite phase.

Let the two sources be separated by a distance d.

If the phase angle between the two fields is $\psi = \beta d \cos\phi$

then far field component at P due to 1 is $E_1 = -E_0 e^{-j\frac{\psi}{2}}$

far field component at P due to 2 is $E_2 = E_0 e^{j\frac{\psi}{2}}$

The total field at a distance r in the direction ϕ is

$$E = E_1 + E_2$$

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$$E = -E_0 e^{-j\frac{\psi}{2}} + E_0 e^{j\frac{\psi}{2}}$$

$$E = 2j E_0 \sin\frac{\psi}{2}$$

Normalised field.

$$E = \sin\frac{\psi}{2}$$

* **general case of two isotropic point sources of equal amplitude and any phase difference:**

$$\psi = \beta d \cos\phi + \delta$$

$$E = 2E_0 \cos\frac{\psi}{2}$$

Normalised

$$E = \cos\frac{\psi}{2} = \cos\left[\frac{\beta d}{2}\cos\phi + \frac{\delta}{2}\right]$$

NOTE: path difference $= \frac{d\cos\phi}{\lambda}$

phase difference $= 2\pi$ path difference

$$= 2\pi \frac{d\cos\phi}{\lambda} = \beta d \cos\phi$$

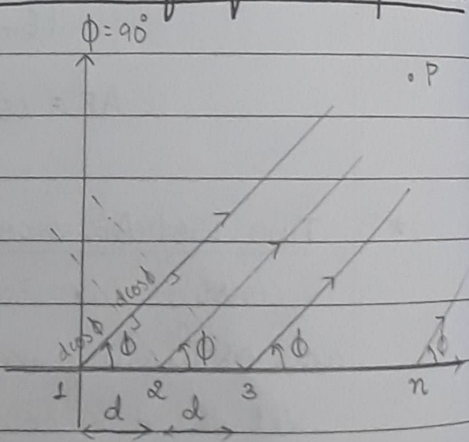
with initial phase difference

$$\psi = \beta d \cos\phi + \delta.$$

* **Linear arrays of n isotropic sources of equal amplitude and spacing:**

consider a linear uniform array of n-isotropic antennas along the axis $\phi = 0°$.

consider a point P at a distance r from the origin.



Total field E at P is

$$E = E_1 + E_2 + E_3 + \cdots + E_n$$

$$E = E_0 e^{j0} + E_0 e^{j\psi} + E_0 e^{2j\psi} + \cdots + E_0 e^{(n-1)j\psi}$$

$$E e^{j\psi} = E_0 e^{j\psi} + E_0 e^{2j\psi} + E_0 e^{3j\psi} + \cdots + E_0 e^{nj\psi}$$

Therefore

$$E e^{j\psi} - E = E_0 e^{nj\psi} - E_0$$

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$$E\left[e^{j\psi}-1\right] = E_0\left[e^{nj\psi}-1\right]$$

$$E = E_0\left[\frac{e^{nj\psi}-1}{e^{j\psi}-1}\right]$$

$$E = E_0\left[\frac{e^{nj\frac{\psi}{2}}\left[e^{nj\frac{\psi}{2}}-e^{-nj\frac{\psi}{2}}\right]}{e^{j\psi/2}\left[e^{j\frac{\psi}{2}}-e^{-j\frac{\psi}{2}}\right]}\right]$$

$$E = E_0\left[\frac{e^{nj\frac{\psi}{2}}\,2j\sin n\psi/2}{e^{j\frac{\psi}{2}}\,2j\sin\psi/2}\right]$$

$$E = E_0\,\underbrace{e^{(n-1)j\frac{\psi}{2}}}_{\text{phase angle}}\underbrace{\left[\frac{\sin n\psi/2}{\sin\psi/2}\right]}_{\text{magnitude}}$$

The peak value is obtained when $\psi = 0$

$$|E|_{max} = E_0\left.\frac{d/d\psi\,(\sin n\psi/2)}{d/d\psi\,(\sin\psi/2)}\right|_{\psi=0}$$

$$E_{max} = E_0\left.\frac{n/2\,\cos n\psi/2}{1/2\,\cos\psi/2}\right|_{\psi=0}$$

$$E_{max} = nE_0$$

★ <u>Uniform linear array</u>

maximum occurs at $\psi = 0$: principle maximum

minimum occurs at $\frac{N\psi}{2} = \pm k\pi$: nulls.

first secondary maxima is called side lobe

the secondary maxima occurs at $\frac{N\psi}{2} = \pm(2k+1)\frac{\pi}{2}$

<u>Broadside Array</u>

$\delta = 0 \quad \phi = 90^\circ$

<u>End fire Array</u>

$\delta = -\beta d \quad \phi = 0^\circ$

<u>Extended end fire Array</u>

$\delta = -\beta d - \frac{\pi}{n}$

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* BWFN, HPBW, Directivity:

1. **Broadside Array:**

$$\phi_0 = 2\sin^{-1}\left(\pm \frac{\lambda}{nd}\right) \quad : \text{Exact BWFN}$$

$$\phi_0 = \frac{2\lambda}{nd} \quad : \text{Approx BWFN}$$

$$HPBW = \frac{\phi_0}{2} = \sin^{-1}\left(\pm \frac{\lambda}{nd}\right) \quad : \text{Exact HPBW}$$

$$HPBW = \frac{\lambda}{nd} \quad : \text{Approx HPBW}$$

$$D = \frac{2nd}{\lambda} = 2L_\lambda \quad : \text{directivity.}$$

2. **End Fire Array:**

$$\phi_0 = 4\sin^{-1}\left(\pm \sqrt{\frac{\lambda}{2nd}}\right) \quad : \text{Exact BWFN}$$

$$\phi_0 = 2\sqrt{\frac{2\lambda}{nd}} \quad : \text{Approx BWFN}$$

$$HPBW = \frac{\phi_0}{2} = 2\sin^{-1}\left(\pm \sqrt{\frac{\lambda}{2nd}}\right) \quad : \text{Exact HPBW}$$

$$HPBW = \frac{\phi_0}{2} = \sqrt{\frac{2\lambda}{nd}} \quad : \text{Approx HPBW}$$

$$D = \sqrt{\frac{2nd}{\lambda}} = \sqrt{2L_\lambda} \quad : \text{directivity.}$$

3. **Extended Endfire Array:**

$$\phi_0 = 4\sin^{-1}\left(\mp \sqrt{\frac{\lambda}{4nd}}\right) \quad : \text{Exact BWFN}$$

$$\phi_0 = 2\sqrt{\frac{\lambda}{nd}} \quad : \text{Approx BWFN}$$

$$HPBW = \frac{\phi_0}{2} = 2\sin^{-1}\left(\pm \sqrt{\frac{\lambda}{4nd}}\right) \quad : \text{Exact HPBW}$$

$$HPBW = \frac{\phi_0}{2} = \sqrt{\frac{\lambda}{nd}} \quad : \text{Approx HPBW}$$

$$D = \sqrt{\frac{4nd}{\lambda}} = 2\sqrt{L_\lambda} \quad : \text{Directivity.}$$

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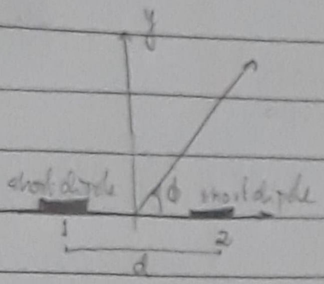
* Nonisotropic but similar point sources:

consider two non isotropic sources at a distance d from each other.



Let the two sources be short dipoles which are parallel to x axis and both the sources are identical.

The field pattern of an isolated short dipole is

$$E_0 = E_0' \sin\phi$$

wkt the field pattern of a two element array is

$$E = 2E_0 \cos\frac{\psi}{2}$$

$$\therefore E = 2E_0' \sin\phi \cos\frac{\psi}{2}$$

Normalized

$$E = \sin\phi \cos\frac{\psi}{2} \quad \text{where } \psi = \beta d\cos\phi + \delta$$

* Principle of pattern multiplication:

The total field pattern of an array of nonisotropic but similar sources is the product of the individual source pattern and the pattern of an array of isotropic point sources each located at phase center of the individual source and having the same relative amplitude and phase while the total phase pattern is the sum of the phase patterns of the individual sources and the array of isotropic point sources.

$$E = \underbrace{f(\theta,\phi)\,F(\theta,\phi)}_{\text{Field pattern}} \underbrace{\angle f_p(\theta,\phi) + F_p(\theta,\phi)}_{\text{phase pattern}}$$

where $f(\theta,\phi)$ and $f_p(\theta,\phi)$ represents the field and phase pattern of individual sources and $F(\theta,\phi)$ and $F_p(\theta,\phi)$ represents the field and phase pattern of array of isotropic sources.

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UNIT - 02

Point Source and Arrays

* **Point source:**

The factors deciding if the antenna is a point source are:

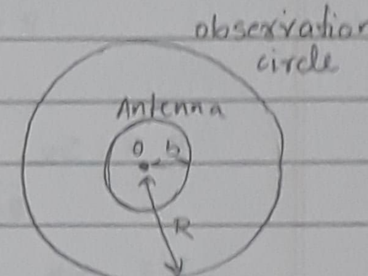
$$\frac{\text{Distance between antenna and point of observation}}{\text{Physical size of antenna}} > 1$$

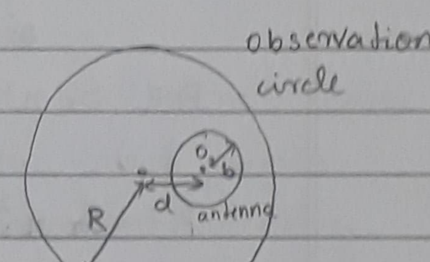
On moving the antenna such that the circles do not coincide then there is a variation in field measurements. This is negligible when :

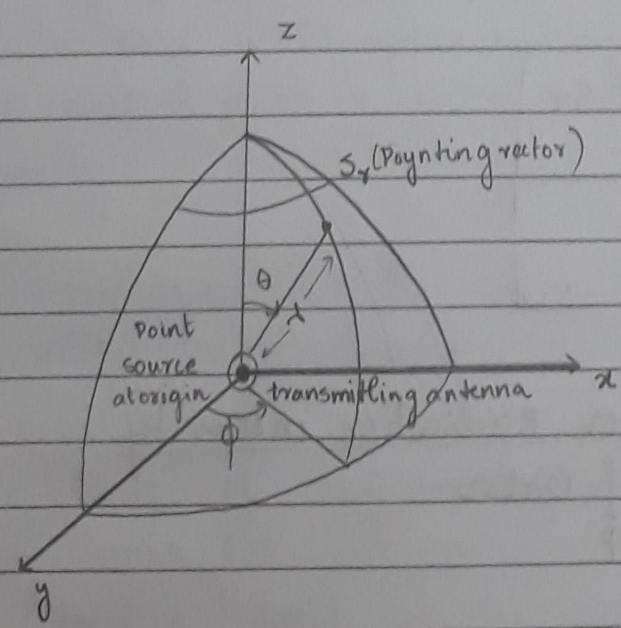
$R \gg d$

$R \gg b$

$R \gg \lambda$

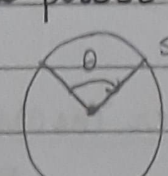






For isotropic source (radiating in all direction)

The power pattern in 2d



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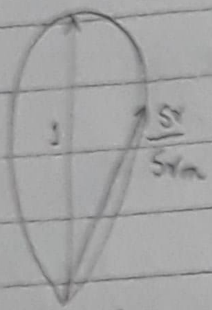
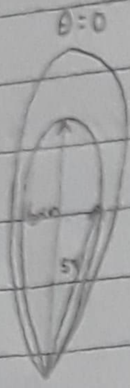
For anisotropic source



Normalised power pattern



Absolute power pattern
$S_r = W/m^2$

S_r : relative power pattern

Normalised power

$$\frac{S_r}{S_{rm}}$$

★ Power Theorem :

$$P = \oint\!\!\oint S_r \, ds$$

" The total power radiated is the integral over the surface of the radial component of the poynting vector"

$$P = S_r \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} ds$$

$$P = S_r \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} r^2 \sin\theta \, d\theta \, d\phi$$

$$P = S_r r^2 \left[-\cos\theta \Big|_0^{\pi} \right] \left[\phi \Big|_0^{2\pi} \right]$$

$$P = S_r r^2 [1+1][2\pi]$$

$P = 4\pi r^2 S_r$
$S_r = \dfrac{P}{4\pi r^2}$

★ Power Theorem for Radiation intensity :

$$P = \int_{\theta}\int_{\phi} U \, d\Omega$$

For isotropic point source

$$P = \int_\theta \int_\phi U_0 \, d\Omega$$

$$P = \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} U_0 \sin\theta \, d\theta \, d\phi$$

$$P = U_0 \left[-\cos\theta \right]_0^{\pi} \left[\phi \right]_0^{2\pi}$$

$$P = U_0 (2)(2\pi)$$

$$P = 4\pi U_0 \Rightarrow \boxed{U_0 = \frac{P}{4\pi}}$$

wkt $S_r = \dfrac{P}{4\pi r^2}$

$$S_r = \frac{U_0}{r^2} \Rightarrow \boxed{U_0 = S_r r^2}$$

Q: A source has cosine radiation pattern $U = U_m \cos\theta$ where U_m maximum radiation intensity. Find the total power radiated by the cosine source. The radiation intensity has a value only in the upper hemisphere i.e., $0 < \theta < \pi/2$ and $0 < \phi < 2\pi$ and is zero in lower hemisphere. Also calculate the directivity.

given:

$$U = U_m \cos\theta$$

$$P = \int_{\theta=0}^{\pi/2} \int_{\phi=0}^{2\pi} U_m \cos\theta \, d\Omega$$

$$P = U_m \int_{\theta=0}^{\pi/2} \int_{\phi=0}^{2\pi} \cos\theta \sin\theta \, d\theta \, d\phi$$

$$P = U_m \left[\frac{1}{2} \right] \left[2\pi \right]$$

$$\underline{\underline{P = U_m \pi}}$$

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wkt $P = 4\pi U_0$

$\Rightarrow 4\pi U_0 = U_m \pi$

$\therefore \frac{U_m}{U_0} = 4$ Directivity : $\frac{U_m}{U_0}$

Q: Find total power radiated and directivity of unidirectional sine source $U = U_m \sin\theta$

$$P = \int_{\theta=0}^{\pi/2} \int_{\phi=0}^{2\pi} U_m \sin\theta \, d\Omega$$

$$P = \int_{\theta=0}^{\pi/2} \int_{\phi=0}^{2\pi} U_m \sin\theta \sin\theta \, d\theta \, d\phi$$

$$P = \int_{\theta=0}^{\pi/2} \int_{\phi=0}^{2\pi} U_m \left[\frac{1-\cos 2\theta}{2} \right] d\theta \, d\phi$$

$$P = \frac{U_m}{2} [2\pi] \int_{\theta=0}^{\pi/2} (1-\cos 2\theta) \, d\theta$$

$$P = U_m \pi \left[\theta - \frac{\sin 2\theta}{2} \right]_0^{\pi/2}$$

$$P = U_m \pi \left[\frac{\pi}{2} - \cancel{\frac{\sin \pi}{2}}^{0} - 0 + \cancel{\frac{\sin 0}{2}}^{0} \right]$$

$$P = \frac{U_m \pi^2}{2}$$

wkt $P = 4\pi U_0$

$\therefore \frac{U_m \pi^2}{2} = 4\pi U_0$

$$\text{Directivity} = \frac{U_m}{U_0} = \frac{8}{\pi}$$

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Date ______
Page ______

Q: Find the total power radiated and directivity of bidirectional and unidirectional source $U = U_m \cos^2\theta$

$$P = \int_{\theta=0}^{\pi/2} \int_{\phi=0}^{2\pi} U_m \cos^2\theta \, d\Omega$$

$$P = \int_{\theta=0}^{\pi/2} \int_{\phi=0}^{2\pi} U_m \cos^2\theta \sin\theta \, d\theta \, d\phi$$

$$P = U_m \int_{\theta=0}^{\pi/2} \cos^2\theta \sin\theta \, d\theta \int_{\phi=0}^{2\pi} d\phi$$

$$P = U_m \left[\frac{1}{3}\right] \left[2\pi\right]$$

$$P = \frac{2\pi U_m}{3} \,//$$

wkt $P = 4\pi U_0$

$$\therefore \frac{2\pi U_m}{3} = 4\pi U_0$$

$$\therefore \text{Directivity}: D = \frac{U_m}{U_0} = 6 \,//$$ Unidirectional

$$P = \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} U_m \cos^2\theta \, d\Omega$$

$$P = \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} U_m \cos^2\theta \sin\theta \, d\theta \, d\phi$$

$$P = U_m \int_{\theta=0}^{\pi} \cos^2\theta \sin\theta \, d\theta \int_{\phi=0}^{2\pi} d\phi$$

$$P = U_m \, 2\left[\frac{1}{3}\right] \left[2\pi\right]$$

$$P = \frac{4\pi U_m}{3} \,//$$

wkt $P = 4\pi U_0$

$$\therefore \frac{4\pi U_m}{3} = 4\pi U_0$$

$$\therefore \text{Directivity} = \frac{U_m}{U_0} = 3 \,//$$ Bidirectional

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Q: Find the total power radiated and directivity of both unidirectional and bidirectional source $U = U_m \sin^2\theta$

$$P = \int_{\theta=0}^{\pi/2} \int_{\phi=0}^{2\pi} U_m \sin^2\theta \, d\Omega \quad : \text{Unidirectional}$$

$$P = \int_{\theta=0}^{\pi/2} \int_{\phi=0}^{2\pi} U_m \sin^2\theta \sin\theta \, d\theta \, d\phi$$

$$P = U_m \int_{\theta=0}^{\pi/2} \sin^3\theta \, d\theta \int_{\phi=0}^{2\pi} d\phi$$

$$P = U_m \int_{\theta=0}^{\pi/2} \left[\frac{3\sin\theta - \sin 3\theta}{4}\right] d\theta \left[\phi\right]_0^{2\pi}$$

$$P = \frac{U_m \cancel{2}\pi}{\cancel{4}_2} \int_{\theta=0}^{\pi/2} (3\sin\theta - \sin 3\theta) \, d\theta$$

$$P = \frac{U_m \pi}{2} \left[-3\cos\theta + \frac{\cos 3\theta}{3}\right]_0^{\pi/2}$$

$$P = \frac{U_m\pi}{2}\left[\cancel{-3\cos\frac{\pi}{2}} \cancel{+\frac{1}{3}\cos\frac{3\pi}{2}} + 3\cos 0 - \frac{\cos 0}{3}\right]$$

$$P = \frac{U_m\pi}{2}\left[3 - \frac{1}{3}\right]$$

$$P = \frac{U_m\pi}{2}\left[\frac{8}{3}\right]$$

$$P = \frac{4\pi U_m}{3} \quad //$$

wkt $P = 4\pi U_0$

$$\therefore \frac{4\pi U_m}{3} = 4\pi U_0$$

$$\Rightarrow \text{directivity} = \frac{U_m}{U_0} = 3 \quad //$$

$$P = \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} U_m \sin^2\theta \, d\Omega \quad : \text{bidirectional}$$

$$P = \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} U_m \sin^2\theta \sin\theta \, d\theta \, d\phi$$

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$$P = U_m \int_{\theta=0}^{\pi} \sin^3\theta \, d\theta \int_{\phi=0}^{2\pi} d\phi$$

$$P = U_m \int_{\theta=0}^{\pi} \left[\frac{3\sin\theta - \sin 3\theta}{4} \right] d\theta \, \left[\phi \right]_0^{2\pi}$$

$$P = \frac{\cancel{2}\pi U_m}{\cancel{4}_2} \int_{\theta=0}^{\pi} (3\sin\theta - \sin 3\theta) \, d\theta$$

$$P = \frac{U_m \pi}{2} \left[-3\cos\theta + \frac{\cos 3\theta}{3} \right]_0^{\pi}$$

$$P = \frac{U_m \pi}{2} \left[-3\cos\pi + \frac{1}{3}\cos 3\pi + 3\cos 0 - \frac{1}{3}\cos 0 \right]$$

$$P = \frac{U_m \pi}{2} \left[-(-3) + \frac{1}{3}(-1) + 3 - \frac{1}{3}(1) \right]$$

$$P = \frac{U_m \pi}{2} \left[3 - \frac{1}{3} + 3 - \frac{1}{3} \right]$$

$$P = \frac{U_m \pi}{\cancel{2}} \left[\frac{\cancel{16}^{8}}{3} \right]$$

$$P = \frac{8\pi U_m}{3}$$

wkt $P = 4\pi U_0$

$$\therefore \frac{8\pi U_m}{3} = 4\pi U_0$$

$$\Rightarrow \text{directivity} = \frac{U_m}{U_0} = \frac{3}{2} = 1.5$$

★ **Arrays:**

It is a combination of two or more antennas.

It improves the gain and the directivity.

→ 1. Two Istropic source of same magnitude and phase.

consider two point sources equidistant from the origin. Plane of observation is present in the far field.

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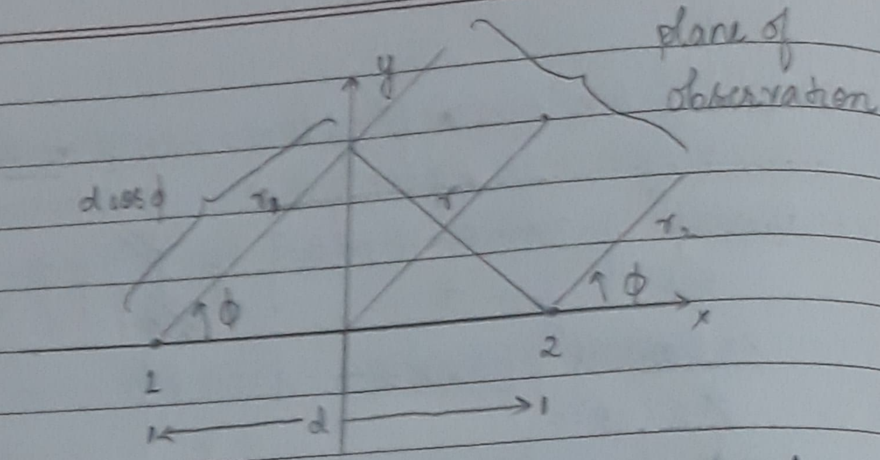

Let the distance between the sources be d.

path difference = $d\cos\phi$

path difference (λ) = $\frac{d\cos\phi}{\lambda}$

phase difference = $2\pi \times$ path difference

$= \frac{2\pi}{\lambda} d\cos\phi$

$$\boxed{\frac{2\pi}{\lambda} = \beta}$$

$$\boxed{\psi = \beta d\cos\phi}$$

$$\psi = d_r\cos\phi + \delta$$
$$= \beta d\cos\phi + \delta$$

d_r : distance in radians

$$E_1 = E_0 e^{-j\frac{\psi}{2}}$$

$$E_2 = E_0 e^{j\frac{\psi}{2}}$$

$E = E_1 + E_2$

$E = E_0 e^{-j\frac{\psi}{2}} + E_0 e^{j\frac{\psi}{2}}$

$E = 2E_0 \left[\frac{e^{-j\frac{\psi}{2}} + e^{j\frac{\psi}{2}}}{2}\right]$

$E = 2E_0 \cos\frac{\psi}{2}$

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Date
Page

$$\boxed{E_{total} = 2E_0 \beta d \cos\phi = 2E_0 d_r \cos\phi} \quad \underset{I \to unity}{= 2E_0 \cos}\left[\frac{\beta d \cos\phi}{2}\right]$$

$$\text{Array factor} = \frac{E_{total}}{E_{total\,max}} = \frac{2E_0 \cos\psi/2}{2E_0}$$

$$\boxed{\text{Array factor} = \cos\frac{\psi}{2}}$$

Q: Obtain the radiation pattern of two isotropic arrays field with same magnitude for both the sources and they are inphase. The distance between the elements is $\lambda/2$.

given: $d = \lambda/2$

$\delta = 0$

$\psi = \beta d \cos\phi$

$\psi = \frac{2\pi}{\lambda}\frac{\lambda}{2}\cos\phi$

$\psi = \pi\cos\phi$

$E = 2E_0 \cos\left[\frac{\beta d\cos\phi}{2}\right]$

$E = \cos\left[\frac{\pi\cos\phi}{2}\right]$

$E = \cos\left[\frac{\pi}{2}\cos\phi\right]$

a. Maximum direction or peak

$\cos\left[\frac{\pi}{2}\cos\phi\right] = 1$

$\frac{\pi}{2}\cos\phi = 0$

$\cos\phi = 0$

$\phi = \pm 90^\circ$

b. Nulls

$\cos\left[\frac{\pi}{2}\cos\phi\right] = 0$

$\frac{\pi}{2}\cos\phi = \pm\frac{\pi}{2}$

$\cos\phi = \pm 1$

$\phi = 0^\circ, 180^\circ$

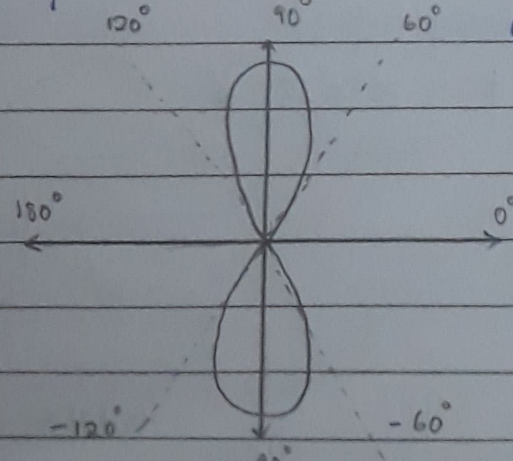
c. Half power level

$\cos\left[\frac{\pi}{2}\cos\phi\right] = \frac{1}{\sqrt{2}}$

$\frac{\pi}{2}\cos\phi = \pm\frac{\pi}{4}$

$\cos\phi = \pm\frac{1}{2}$

$\phi = 60^\circ, 120^\circ$



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Q: Obtain the radiation pattern of two isotropic array field with same magnitude for both the sources and they are out of phase. The distance between the elements is $\lambda/2$

given: $d = \lambda/2$

$\delta = \pi$

$\psi = \beta d \cos\phi + \delta$

$\psi = \frac{2\pi}{\lambda}\frac{\lambda}{2}\cos\phi + \pi$

$\psi = \pi(1+\cos\phi)$

$E = 2E_0 \cos\frac{\psi}{2}$

$E = \cos\frac{\psi}{2}$

$E = \cos\left[\frac{\pi}{2}\cos\phi + \frac{\pi}{2}\right]$

a. Maximum direction or peak

$\cos\left[\frac{\pi}{2}\cos\phi + \frac{\pi}{2}\right] = 1$

$\frac{\pi}{2}\cos\phi + \frac{\pi}{2} = 0$

$\frac{\pi}{2}\cos\phi = -\frac{\pi}{2}$

$\cos\phi = \pm|-1|$

$\phi = \pi = 0^\circ, 180^\circ$

b. Nulls

$\cos\left[\frac{\pi}{2}\cos\phi + \frac{\pi}{2}\right] = 0$

$\frac{\pi}{2}\cos\phi + \frac{\pi}{2} = \pm\frac{\pi}{2}$

$\cos\phi + 1 = \pm 1$

$\cos\phi = 0$

$\phi = \pm 90^\circ$

c. Half power level

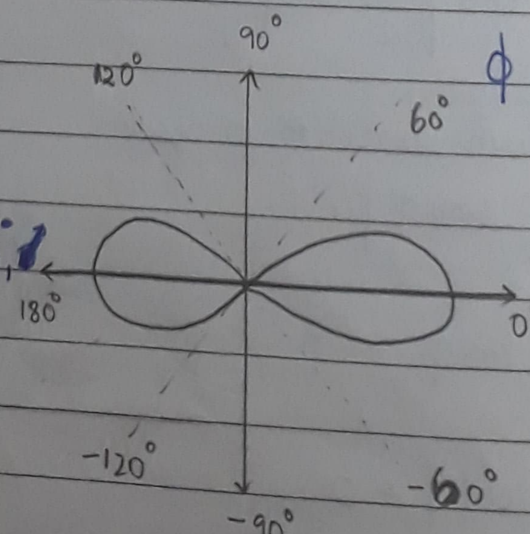
$\cos\left[\frac{\pi}{2}\cos\phi + \frac{\pi}{2}\right] = \frac{1}{\sqrt{2}}$

$\frac{\pi}{2}\left[\cos\phi + 1\right] = \frac{\pi}{4}$

$\cos\phi \pm 1 = \frac{1}{2}$

$\cos\phi = \pm\left|\frac{-1}{2}\right|$

$\phi = \pm 120^\circ, \pm 60^\circ$



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Date ____
Page ____

Q: Obtain the radiation pattern of two isotropic array field with same magnitude for both the sources and they are out of phase $\delta = \pi/2$. The distance between the elements is $\lambda/2$.

given: $d = \lambda/2$

$\delta = \pi/2$

$\psi = \beta d \cos\phi + \delta$

$\psi = \frac{2\pi}{\lambda} \frac{\lambda}{2} \cos\phi + \frac{\pi}{2}$

$\psi = \pi\left[\cos\phi + \frac{1}{2}\right]$

$E = 2E_0 \cos\frac{\psi}{2}$

$E = \cos\frac{\psi}{2}$

$E = \cos\left[\frac{\pi}{2}\cos\phi + \frac{\pi}{4}\right]$

a. Maximum direction or peak

$\cos\left[\frac{\pi}{2}\cos\phi + \frac{\pi}{4}\right] = 1$

$\frac{\pi}{2}\cos\phi + \frac{\pi}{4} = 0$

$\frac{\pi}{2}\cos\phi = -\frac{\pi}{4}$

$\cos\phi = \pm\left|-1/2\right|$

$\phi = \pm 60^\circ, \pm 120^\circ$

b. Nulls

$\cos\left[\frac{\pi}{2}\cos\phi + \frac{\pi}{4}\right] = 0$

$\frac{\pi}{2}\cos\phi + \frac{\pi}{4} = \pm\frac{\pi}{2}$

$\cos\phi + \frac{1}{2} = \pm 1$

$\cos\phi = 1/2$

$\phi = \pm 60^\circ, \pm 120^\circ$

c. Half power level

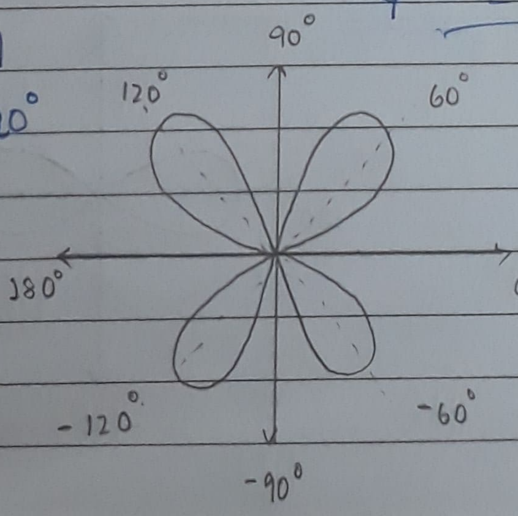
$\cos\left[\frac{\pi}{2}\cos\phi + \frac{\pi}{4}\right] = \frac{1}{\sqrt{2}}$

$\frac{\pi}{2}\cos\phi + \frac{\pi}{4} = \frac{\pi}{4}$

$\cos\phi + \frac{1}{2} = \frac{1}{2}$

$\cos\phi = 0$

$\phi = \pm 90^\circ$



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Q: Obtain the radiation pattern of two isotropic array field with same magnitude for both the sources and they are out of phase $\delta = \pi/2$. The distance between the elements is $\lambda/4$.

— given $d = \lambda/4$

$\delta = \pi/2$

$\psi = \beta d \cos\phi + \delta$

$\psi = \frac{2\pi}{\lambda} \frac{\lambda}{4_2} \cos\phi + \frac{\pi}{2}$

$\psi = \frac{\pi}{2} \cos\phi + \frac{\pi}{2}$

$E = 2E_0 \cos\frac{\psi}{2}$

$E = \cos\left[\frac{\pi}{4}\cos\phi + \frac{\pi}{4}\right]$

a. Maximum direction or peak

$\cos\left[\frac{\pi}{4}\cos\phi + \frac{\pi}{4}\right] = 1$

$\frac{\pi}{4}\cos\phi + \frac{\pi}{4} = 0$

$\cos\phi + 1 = 0$

$\cos\phi = -1$

$\phi = 0^\circ, 180^\circ$

b. Nulls

$\cos\left[\frac{\pi}{4}\cos\phi + \frac{\pi}{4}\right] = 0$

$\frac{\pi}{4}\cos\phi + \frac{\pi}{4} = \frac{\pi}{2}$

$\frac{1}{2}\cos\phi + \frac{1}{2} = 1$

$\cos\phi \pm 1 = 2$

$\cos\phi = 1$

$\phi = 0^\circ, 180^\circ$

c. Half power level

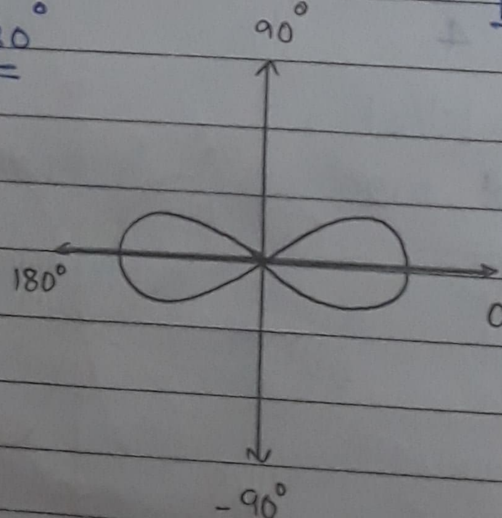
$\cos\left[\frac{\pi}{4}\cos\phi + \frac{\pi}{4}\right] = \frac{1}{\sqrt{2}}$

$\frac{\pi}{4}\cos\phi + \frac{\pi}{4} = \frac{\pi}{4}$

$\cos\phi + 1 = 1$

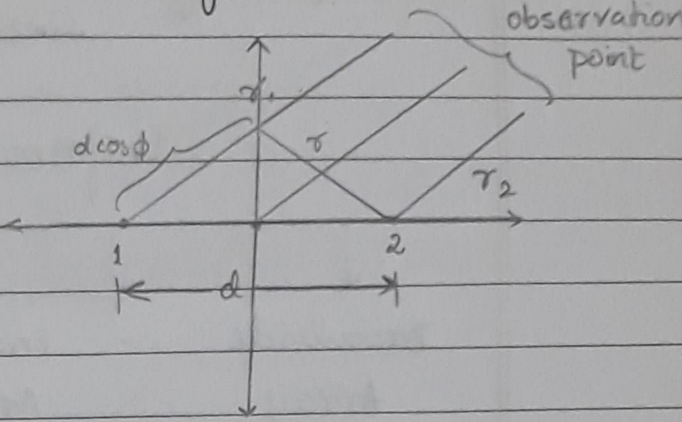
$\cos\phi = 0$

$\phi = \pm 90^\circ$



→ 2. Two isotropic point source of same magnitude and opposite phase

path difference $= \frac{d\cos\phi}{\lambda}$



total phase difference

$= 2\pi \times$ path difference

$= \frac{2\pi}{\lambda} d\cos\phi$

$$\boxed{\psi = \beta d\cos\phi + \delta}$$

$$E_{total} = E_1 + E_2$$

$$= -E_1 + E_2 \quad \text{(opposite phase)}$$

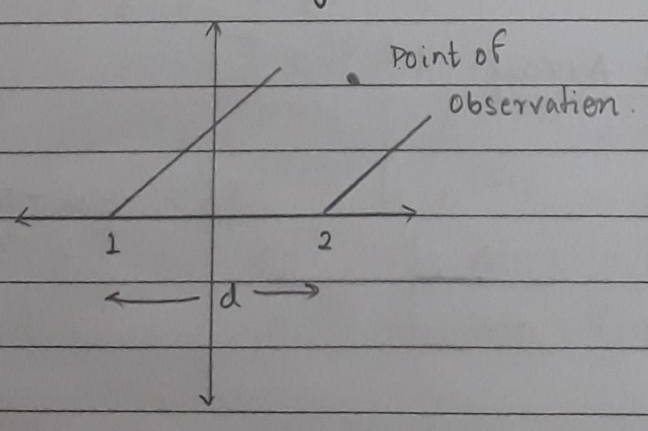
$$= -E_0 e^{-j\frac{\psi}{2}} + E_0 e^{j\frac{\psi}{2}}$$

$$= E_0 2j \left[\frac{e^{j\frac{\psi}{2}} - e^{-j\frac{\psi}{2}}}{2j} \right]$$

$$\boxed{E_{total} = \underbrace{E_0}_{\text{unity}} 2j \sin\frac{\psi}{2}}$$

$$\boxed{|E| = \sin\frac{\psi}{2}}$$

→ 3. General case of two isotropic point sources of same amplitude and any phase difference :



$$\psi = \beta d\cos\phi + \delta$$

$$E = \cos\frac{\psi}{2}$$

* **Linear Array**: when antennas are spaced equally.

Linear Array

↓

Uniform Linear Array

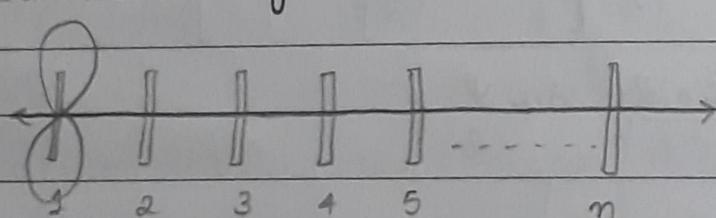
↓

- Broadside Array
- End-fire Array
- Extended End Fire Array (Hansen-Woodyard array)

When antennas are spaced equally then they are called as linear array.

When antennas have same properties and are spaced equally then they are called as uniform linear array.

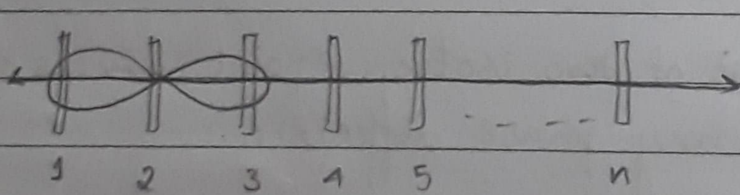
Broadside Array



Axis of array

$\delta = 0 \quad \phi = 90^\circ$

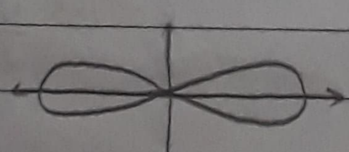
End fire Array



Axis of array

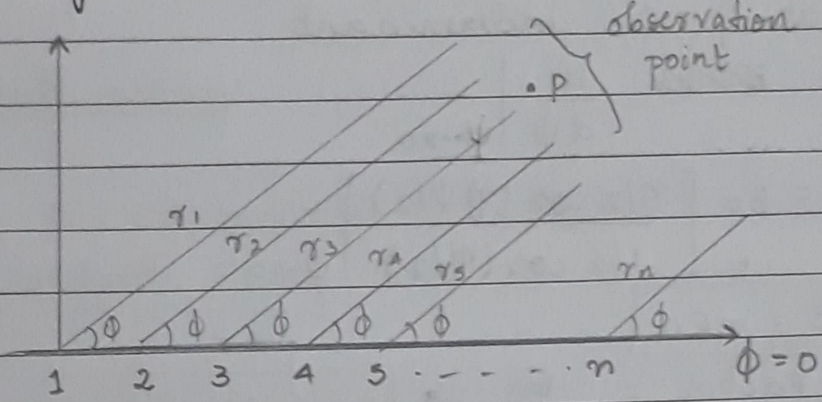
$\phi = 0 \quad \delta = -\beta d$

Extended End Fire Array



$\delta = -\beta d - \frac{\pi}{n}$

Linear Arrays of n isotropic sources of equal amplitude and spacing:



Path difference $= d\cos\phi$

Phase difference $= 2\pi \dfrac{d}{\lambda}\cos\phi$

$E_1 = E_0 e^0 = E_0$

$E_2 = E_0 e^{j\psi}$

$E_3 = E_0 e^{j2\psi}$

$\vdots$

$E_n = E_0 e^{j(n-1)\psi}$

Total path difference

$\psi = \beta d \cos\phi + d$

$E = E_1 + E_2 + E_3 + \dots + E_n$

$$E = E_0 + E_0 e^{j\psi} + E_0 e^{2j\psi} + E_0 e^{3j\psi} + \dots + E_0 e^{j(n-1)\psi} \quad \text{—— (1)}$$

$$E e^{j\psi} = E_0 e^{j\psi} + E_0 e^{j2\psi} + E_0 e^{3j\psi} + \dots + E_0 e^{jn\psi} \quad \text{—— (2)}$$

Subtracting eq (1) from eq (2)

$$E e^{j\psi} - E = -E_0 + E_0 e^{jn\psi}$$

$$E(e^{j\psi} - 1) = E_0 (e^{jn\psi} - 1)$$

$$E = E_0 \left[\frac{e^{jn\psi} - 1}{e^{j\psi} - 1}\right]$$

$$E = E_0 \frac{e^{jn\psi/2}}{e^{j\psi/2}} \left[\frac{e^{jn\psi/2} - e^{-jn\psi/2}}{e^{j\psi/2} - e^{-j\psi/2}}\right]$$

$$\boxed{E = E_0 \frac{e^{jn\psi/2}}{e^{j\psi/2}} \left[\frac{\sin(n\psi/2)}{\sin(\psi/2)}\right]}$$

$$\boxed{E = E_0 \underbrace{e^{j(n-1)\psi/2}}_{\text{phase angle}} \underbrace{\left[\frac{\sin(n\psi/2)}{\sin(\psi/2)}\right]}_{\text{magnitude}}}$$

$$E = E_0 \left[\frac{\sin(n\psi/2)}{\sin(\psi/2)} \right]$$

at $\psi = 0$: inderminant

$\therefore$ ~~E = E0~~ $\frac{d}{d\psi}\Big|_{\psi \to 0}$

$$E = E_0 \left[\frac{n/2 \cos(n\psi/2)}{1/2 \cos(\psi/2)} \right]_{\psi \to 0}$$

$$\boxed{E_{max} = nE_0}$$

$$\text{Array Factor} = \frac{E}{E_{max}}$$

$$\text{Array Factor} = \frac{\cancel{E_0} \left[\frac{\sin(n\psi/2)}{\sin(\psi/2)} \right]}{n\cancel{E_0}}$$

$$\boxed{\text{Array Factor} = \frac{1}{n} \left[\frac{\sin(n\psi/2)}{\sin(\psi/2)} \right]}$$

- Salient Features of uniform linear array.
  - The maximum value occurs at $\psi = 0$. The maximum value at $\psi = 0$ is called principle maximum of array.
  - The minimum value occurs at $\frac{N\psi}{2} = \pm k\pi$ where $k = 0, \pm 1, \pm 2 \cdots$ These minimas are called nulls.
  - A secondary maximum occurs at

$$\frac{N\psi}{2} = \pm (2k+1)\frac{\pi}{2}$$

The first secondary maximum is called side lobe. The ratio of first secondary maximum to the principle maximum is called side lobe ratio.

- The angular difference between the first null on either side of the main beam is called null-to-null beam width.

- The angular width between 3dB point on the main beam is called half power beam width.

★ **Broadside array:**

1. Peak:

wkt $\psi = \beta d \cos\phi \mp \delta$

For broadside array $\delta = 0$

$\therefore \boxed{\psi = \beta d \cos\phi}$

Maximum value occurs at $\psi = 0$

$\therefore \beta d \cos\phi = 0$

$\cos\phi = 0$

$\boxed{\phi = \pm 90^\circ}$

2. Side Lobe:

$$\frac{N\psi}{2} = \pm (2k+1)\frac{\pi}{2}$$

For broadside array

$\psi = \beta d \cos\phi$

$$\therefore \frac{N}{2}[\beta d \cos\phi] = \pm(2k+1)\frac{\pi}{2}$$

$$\cos\phi = \pm \frac{(2k+1)\pi}{N\beta d}$$

$$\boxed{\phi = \cos^{-1}\left[\frac{\pm(2k+1)\pi}{N\beta d}\right]}$$

$\beta d = d_r$

3. Nulls:

$$\frac{N\psi}{2} = \pm k\pi$$

$$\frac{N}{2}[\beta d \cos\phi] = \pm k\pi$$

$$\cos\phi = \pm \frac{2k\pi}{N\beta d}$$

$$\boxed{\phi = \cos^{-1}\left[\frac{\pm 2k\pi}{N\beta d}\right]}$$

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* End Fire Array:

1. Peak:

wkt $\psi = \beta d \cos\phi + \delta$

For end fire array $\delta = -\beta d$

$\therefore \boxed{\psi = \beta d \cos\phi - \beta d}$

Maximum value occurs at $\psi = 0$

$\therefore \beta d \cos\phi - \beta d = 0$

$\cos\phi - 1 = 0$

$\cos\phi = 1$

$\boxed{\phi = 0^\circ}$

2. Side lobes:

$$\frac{N\psi}{\cancel{2}} = \pm(2k+1)\frac{\pi}{\cancel{2}}$$

$$N[\beta d(\cos\phi - 1)] = \pm(2k+1)\pi$$

$$\cos\phi - 1 = \pm\frac{(2k+1)\pi}{N\beta d}$$

$$\boxed{\phi = \cos^{-1}\left[\pm\frac{(2k+1)\pi}{N\beta d} + 1\right]}$$

3. Nulls:

$$\frac{N\psi}{2} = \pm k\pi$$

$$\frac{N}{2}[\beta d(\cos\phi - 1)] = \pm k\pi$$

$$\cos\phi - 1 = \pm\frac{2k\pi}{N}$$

$$\boxed{\phi = \cos^{-1}\left[\pm\frac{2k\pi}{N} + 1\right]}$$

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Q: A linear array of isotropic antennas satify the following parameters

$n = 4 ; \delta = 0 ; d = \lambda/2.$

Obtain the radiation pattern and find the beam widths between the first nulls and half power beam width

→ Given $\delta = 0$

Hence it is broad side array

Peaks:

Maximum radiation occurs at $\psi = 0$

$\therefore \psi = \beta d \cos\phi + \cancel{\delta}^{0}$

$\psi = \beta d \cos\phi$

where $\beta d \cos\phi = 0$

$\Rightarrow \underline{\underline{\phi = \pm 90^\circ}}$

Side lobe:

$$\phi = \cos^{-1}\left[\pm \frac{(2k+1)\pi}{n\beta d}\right]$$

$$\phi = \cos^{-1}\left[\pm \frac{(2k+1)\pi}{4\left(\frac{2\pi}{\cancel{\lambda}}\right)\frac{\cancel{\lambda}}{2}}\right]$$

$$\phi = \cos^{-1}\left[\pm \frac{(2k+1)}{4}\right]$$

at $k = 0$

$$\phi = \cos^{-1}\left[\pm \frac{1}{4}\right] = \underline{\underline{\pm 75.52^\circ}}, \underline{\underline{\pm 104.47^\circ}}$$

at $k = 1$

$$\phi = \cos^{-1}\left[\pm \frac{3}{4}\right] = \underline{\underline{\pm 41.40^\circ}}, \underline{\underline{\pm 138.59^\circ}}$$

$$\cancel{\phi = \cos^{-1}\left[\frac{-1}{4}\right] = \pm 75.52^\circ}$$

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at $k = 2$

$$\phi = \cos^{-1}\left[+\frac{5}{4}\right] = \text{not defined } (\because > 1)$$

Nulls

$$\phi = \cos^{-1}\left[\pm\frac{2k\pi}{n\beta d}\right]$$

$$\phi = \cos^{-1}\left[\frac{\pm \cancel{2} k \cancel{\pi}}{{}_2\cancel{4}\left(\frac{2\cancel{\pi}}{\cancel{\lambda}}\right)\frac{\cancel{\lambda}}{\cancel{2}}}\right]$$

$$\phi = \cos^{-1}\left[\pm\frac{k}{2}\right]$$

at $k = 0$

$$\phi = \cos^{-1}[\pm 0] = \underline{\underline{\pm 90^\circ}}$$

at $k = 1$

$$\phi = \cos^{-1}\left[\pm\frac{1}{2}\right] = \underline{\underline{\pm 60^\circ}}, \underline{\underline{\pm 120^\circ}}$$

$$\cancel{\phi = \cos^{-1}\left[\frac{1}{2}\right] = \pm 60^\circ}$$

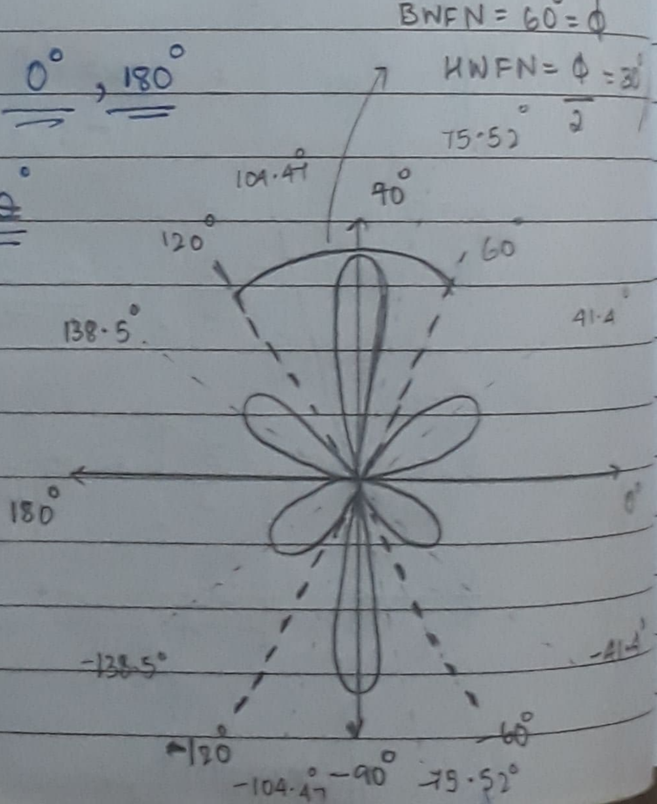
at $k = 2$

$$\phi = \cos^{-1}[\pm 1] = \underline{\underline{0^\circ}}, \underline{\underline{180^\circ}}$$

$$\cancel{\phi = \cos^{-1}[1] = 0^\circ}$$

BWFN $= 60^\circ = \phi$

HWFN $= \frac{\phi}{2} = 30^\circ$



sidelobes at $\pm 75.52^\circ$ and $\pm 104.47^\circ$ are not present as side lobes are not present next to the major lobes.

NOTE:

Out of the possible nulls, the nulls in the direction of peaks are eliminated. Based on the remaining nulls side lobes are considered.

Nulls for k = 0 are to be eliminated since they appear in the direction of peaks.

Nulls for k = 1 are always adjacent to the main lobes known as first nulls.

Q: A linear array consists of four isotropic sources. The distance between the adjacent antenna is $\lambda/2$. The power is applied with equal magnitude and a phase difference of $-d_r$. Obtain the field pattern and find beam width between the first null and HPBW.

– Maximum value occurs at $\psi = 0$

Peaks:

$$\beta d \cos\phi + \delta = 0$$

$$\beta d \cos\phi + \beta d = 0$$

$$\beta d(\cos\phi + 1) = 0$$

$$\cos\phi = -1$$

$$\phi = 0^\circ, 180^\circ$$

Side lobes:

$$\frac{N\psi}{2} = \pm\frac{(2k+1)\pi}{2}$$

$$N[\beta d(\cos\phi - 1)] = \pm(2k+1)\pi$$

$$\cos\phi - 1 = \pm\frac{(2k+1)\pi}{N\beta d}$$

$$\therefore \phi = \cos^{-1}\left[\pm\frac{(2k+1)\pi}{N\beta d} + 1\right]$$

therefore

$$\phi = \cos^{-1}\left[\pm\frac{\pi(2k+1)}{4\left(\frac{2\pi}{\lambda}\right)\left(\frac{\lambda}{2}\right)} + 1\right] = \cos^{-1}\left[\pm\frac{(2k+1)}{4} + 1\right]$$

at $k = 0$

$$\phi = \cos^{-1}\left[\pm \frac{1}{4} + 1\right] = \text{Not defined}$$
$\pm 41.4^\circ$

at $k = 1$

$$\phi = \cos^{-1}\left[\pm \frac{3}{4} + 1\right] = \text{Not defined}$$
$\pm 75.5^\circ$

at $k = 2$

$$\phi = \cos^{-1}\left[\pm \frac{5}{4} + 1\right] = \text{Not defined}$$
$\pm 104.47^\circ$

at $k = 3$

$$\phi = \cos^{-1}\left[\pm \frac{7}{4} + 1\right] = \text{Not defined}$$
$\pm 138.5^\circ$

at $k = 4$

$$\phi = \cos^{-1}\left[\pm \frac{9}{4} + 1\right] = \text{Not defined}$$

<u>Nulls:</u>

$$\frac{N\psi}{2} = \pm k\pi$$

$$\frac{N}{2}\left[\beta d(\cos\phi - 1)\right] = \pm k\pi$$

$$\cos\phi - 1 = \pm \frac{2k\pi}{N\beta d}$$

$$\phi = \cos^{-1}\left[\pm \frac{2k\pi}{N\beta d} + 1\right]$$

$$\phi = \cos^{-1}\left[\pm \frac{\cancel{2}k\cancel{\pi}}{2\,\cancel{4}\left(\frac{\cancel{2\pi}}{\cancel{\lambda}}\right)\left(\frac{\cancel{\lambda}}{2}\right)} + 1\right]$$

$$\phi = \cos^{-1}\left[\pm \frac{k}{2} + 1\right]$$

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at $k = 0$

$$\phi = \cos^{-1}[1] = \underline{\underline{0^\circ, 180^\circ}}$$

at $k = 1$

$$\phi = \cos^{-1}\left[\pm \frac{1}{2} + 1\right] = \text{Not defined}$$

$$\underline{\underline{\pm 60^\circ}}$$

at $k = 2$

$$\phi = \cos^{-1}[\pm 1 + 1] = \text{Not defined}$$

$$\underline{\underline{\pm 90^\circ}}$$

at $k = 3$

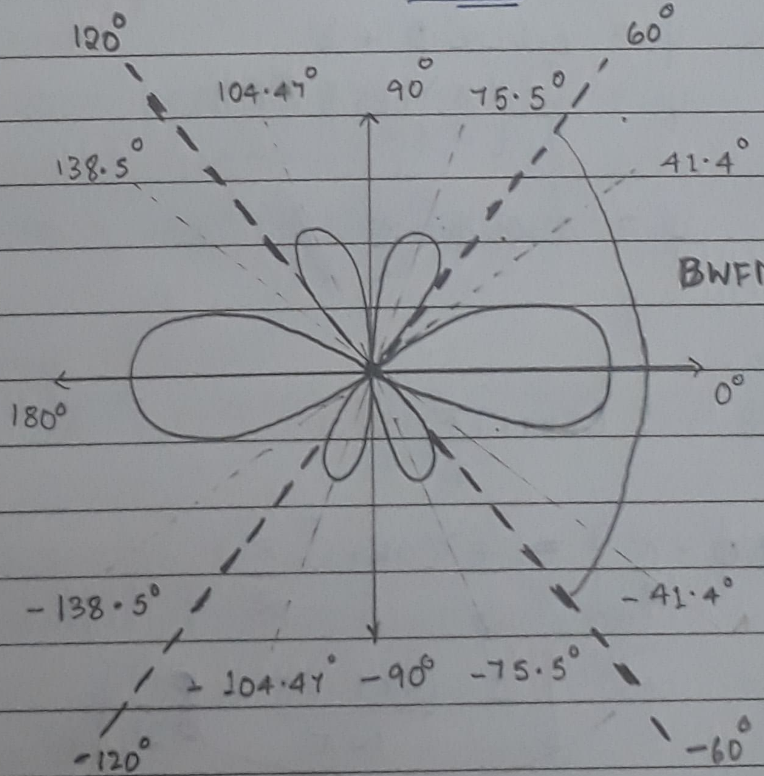
$$\phi = \cos^{-1}\left[\pm \frac{3}{2} + 1\right] = \text{Not defined}$$

$$\underline{\pm 120^\circ}$$

at $k = 4$

$$\phi = \cos^{-1}[\pm 2 + 1] = \text{Not defined}$$

$$\underline{\underline{0^\circ, 180^\circ}}$$



BWFN = 120°

HWFN = 60°

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Q: An array of 4 isotropic antenna are placed along the axis with distance between the adjacent elements $\lambda/2$. The peak is ~~to~~ be obtained in the direction $60°$ from the axis of the array. What should be the phase difference between the adjacent elements? complete the pattern and find the beam width and half power beam width.

Given: $N = 4$

$d = \lambda/2$

Peak: $\psi = \beta d \cos\phi + \delta$

$$0 = \beta d \cos\frac{\pi}{3} + \delta$$

$$\therefore \delta = -\frac{2\pi}{\lambda}\left(\frac{\lambda}{2}\right)\left(\frac{1}{2}\right)$$

$$\delta = -\frac{\pi}{2}$$

Therefore $\psi = \beta d \cos\phi + \delta$

$$\psi = \frac{2\pi}{\lambda}\left(\frac{\lambda}{2}\right)\cos\phi - \frac{\pi}{2}$$

$$\psi = \pi\cos\phi - \frac{\pi}{2}$$

sidelobe:

$$\frac{N\psi}{2} = \pm(2k+1)\frac{\pi}{2}$$

$$\frac{N}{\cancel{2}}\left[\pi\cos\phi - \frac{\pi}{2}\right] = \pm(2k+1)\frac{\pi}{\cancel{2}}$$

$$\phi = \cos^{-1}\left[\pm\frac{(2k+1)\pi}{N\pi} + \frac{1}{2}\right]$$

$$\phi = \cos^{-1}\left[\pm\frac{(2k+1)\cancel{\pi}}{4\cancel{\pi}} + \frac{1}{2}\right]$$

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at $k=0$

$$\phi = \cos^{-1}\left[\pm\frac{1}{4}+\frac{1}{2}\right] = \pm 41.4^\circ,\ \pm 75.52^\circ$$

at $k=1$

$$\phi = \cos^{-1}\left[\pm\frac{3}{4}+\frac{1}{2}\right] = \text{Not defined}$$

$\pm 104.47^\circ$

at $k=2$

$$\phi = \cos^{-1}\left[\pm\frac{5}{4}+\frac{1}{2}\right] = \text{Not defined}$$

$\pm 138.59^\circ$

at $k=3$

$$\phi = \cos^{-1}\left[\pm\frac{7}{4}+\frac{1}{2}\right] = \text{Not defined}$$

Nulls :

$$\frac{N\psi}{2} = \pm k\pi$$

$$\frac{N}{2}\left[\pi\cos\phi - \frac{\pi}{2}\right] = \pm k\pi$$

$$\phi = \cos^{-1}\left[\pm\frac{2k\cancel{\pi}}{N\cancel{\pi}}+\frac{1}{2}\right]$$

$$\phi = \cos^{-1}\left[\pm\frac{k}{2}+\frac{1}{2}\right]$$

at $k=0$

$$\phi = \cos^{-1}\left[\frac{1}{2}\right] = \pm 60^\circ$$

at $k=1$

$$\phi = \cos^{-1}\left[\pm\frac{1}{2}+\frac{1}{2}\right] = 0^\circ,\ \pm 90^\circ,$$

at $k=2$

$$\phi = \cos^{-1}\left[\pm 1+\frac{1}{2}\right] = \text{not defined}$$

$\pm 120^\circ$

at $k=3$

$$\phi = \cos^{-1}\left[\pm\frac{3}{2}+\frac{1}{2}\right] = \text{not defined}$$

180°

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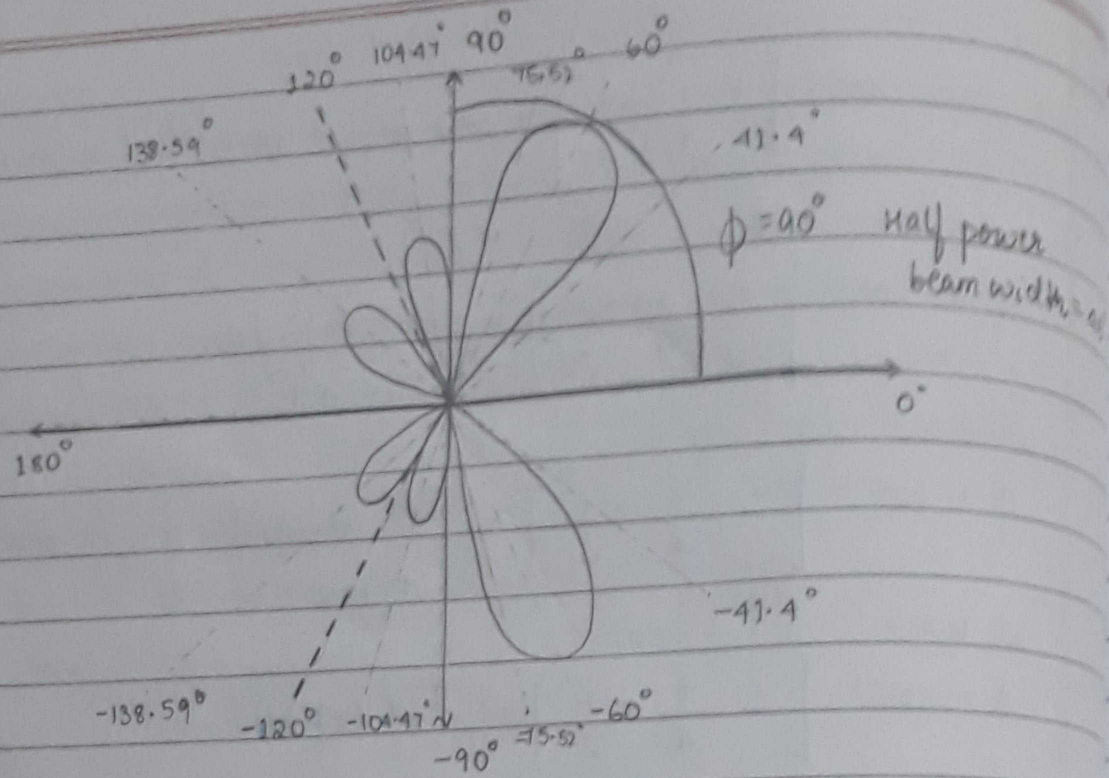

Q: 4 isotropic sources are placed at a distance $\lambda/6$ apart. They have a phase difference of $\pi/3$ between the adjacent elements. Find beam width between the first nulls and HPBW.

given: $d = \lambda/6 \quad \delta = \pi/3$.

$\psi = \beta d \cos\phi + \delta$

peaks:

$\beta d \cos\phi + \delta = 0$

$\beta d \cos\phi + \frac{\pi}{3} = 0$

$\frac{2\pi}{\lambda}\left(\frac{\lambda}{6}\right)\cos\phi = \frac{-\pi}{3}$

$\cos\phi = -1$

$\phi = 0°, 180°$

side lobes:

$\frac{N\psi}{2} = \pm(2k+1)\frac{\pi}{2}$

$N\left[\beta d \cos\phi + \frac{\pi}{3}\right] = \pm(2k+1)\pi$

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Date
Page

$$\phi = \cos^{-1}\left[\pm\frac{(2k+1)\pi}{N\beta d} - \frac{\pi}{3\beta d}\right]$$

$$\phi = \cos^{-1}\left[\pm\frac{(2k+1)\pi}{4\left(\frac{2\pi}{\lambda}\right)\left(\frac{\lambda}{6}\right)} - \frac{\pi}{3\left(\frac{2\pi}{\lambda}\right)\left(\frac{\lambda}{6}\right)}\right]$$

$$\phi = \cos^{-1}\left[\frac{\pm 3(2k+1) - 1}{4}\right]$$

at $k = 0$

$$\phi = \cos^{-1}\left[\frac{\pm 3 - 1}{4}\right] = \pm 104.47^\circ$$

Not defined

at $k = 1$

$$\phi = \cos^{-1}\left[\frac{\pm 9 - 1}{4}\right] = \text{Not defined}$$

Nulls :

$$\frac{N\psi}{2} = \pm k\pi$$

$$N\left[\beta d\cos\phi + \frac{\pi}{3}\right] = \pm 2k\pi$$

$$\frac{2\pi}{\lambda}\left(\frac{\lambda}{6}\right)\cos\phi + \frac{\pi}{3} = \pm\frac{k\pi}{2}$$

$$\frac{\pi}{3}\cos\phi = \pm\frac{k\pi}{2} - \frac{\pi}{3}$$

$$\phi = \cos^{-1}\left[\pm\frac{3k}{2} - 1\right]$$

at $k = 0$

$$\phi = \cos^{-1}[-1] = \underline{180^\circ}$$

at $k = 1$

$$\phi = \cos^{-1}\left[\pm\frac{3}{2} - 1\right] = \underline{\pm 60^\circ}$$

at $k = 2$

$$\phi = \cos^{-1}[\pm 3 - 1] = \text{Not defined}$$

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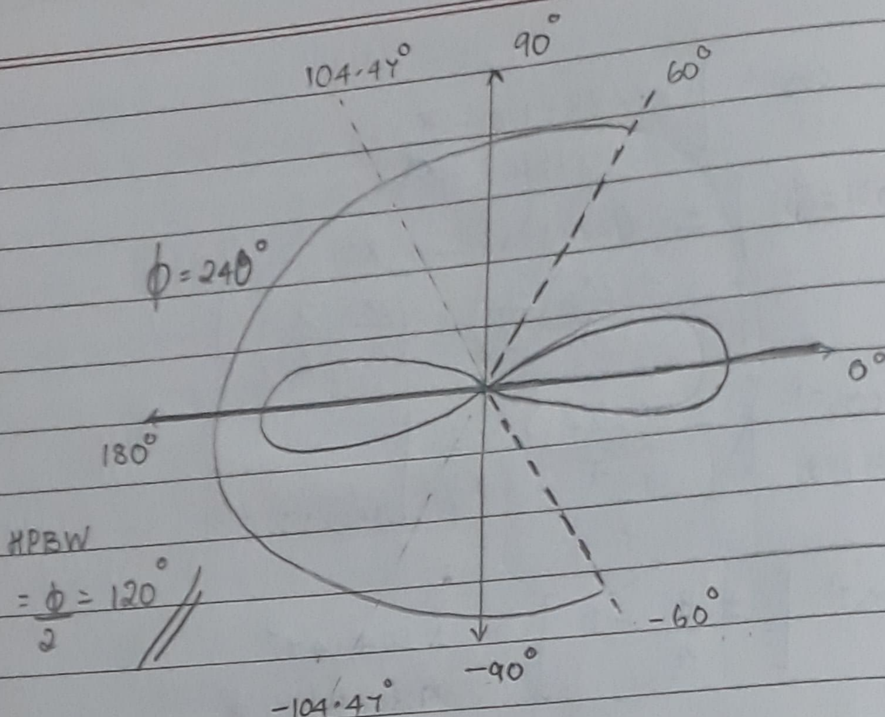

★ General expression for beam width between the first nulls approximate half power beam width and approximate directivity:

1. Broadside Array:

Beam width between the first nulls

nulls: $\frac{n\psi}{2} = \pm k\pi$

$$\psi = \beta d \cos\phi$$

$$\frac{n}{2}\left[\beta d \cos\phi\right] = \pm k\pi$$

$$\frac{2\pi}{\lambda} d\cos\phi = \pm\frac{2\pi k}{n}$$

$$\phi = \cos^{-1}\left[\pm\frac{k\lambda}{dn}\right]$$

First null is at ~~k=~~0 but as it is not consider hence the first null is present at $k=1$.

$$\therefore \phi_0' = \cos^{-1}\left[\pm\frac{\lambda}{nd}\right]$$

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$BWFN = 2(90° - \phi_0')$

$BWFN = 2\sin^{-1}\left[\pm\frac{\lambda}{nd}\right]$

Beam width between first nulls.

$\boxed{\cancel{BWFN = 2\phi'}}$

Exact BWFN

$$\boxed{\phi_0 = 2\sin^{-1}\left(\pm\frac{\lambda}{nd}\right)}$$ exact BWFN

when $nd >> k\lambda$: approximate

$$\boxed{\phi_0 = \frac{2\lambda}{nd}}$$ approximate BWFN

Half power beam width

$$\boxed{HPBW = \frac{\phi_0}{2} = \frac{\lambda}{nd}}$$ approximate

Directivity:

wkt $D = \frac{4\pi}{\theta_{HP} \times \phi_{HP}}$ | $\theta_{HP} = 2\pi$

$D = \frac{4\pi}{2\pi \times \lambda/nd}$

$D = \frac{2nd}{\lambda}$

$$\boxed{D = 2L_\lambda}$$ $L_\lambda = nd/\lambda$

2. End Fire Array:

Beam width between the first nulls:

nulls: $\frac{n}{2}\psi = \pm k\pi$

$\frac{n}{2}[\beta d\cos\phi - \beta d] = \pm k\pi$

$\frac{n}{2}\left(\frac{2\pi}{\lambda}\right)d(\cos\phi - 1) = \pm k\pi$

$$\cos\phi - 1 = \pm\frac{k\lambda}{nd}$$

$$\cancel{1} - 2\sin^2\left(\frac{\phi}{2}\right) - \cancel{1} = \pm\frac{k\lambda}{nd}$$

$$\sin\left(\frac{\phi}{2}\right) = \mp\sqrt{\frac{k\lambda}{2nd}}$$

at $k = 1$ $\boxed{\phi' = 2\sin^{-1}\left[\mp\sqrt{\frac{\lambda}{2nd}}\right]}$ BWFN $= 2\phi'$

$$\boxed{\phi_0 = 4\sin^{-1}\left[\pm\sqrt{\frac{\lambda}{2nd}}\right]}$$

exact BWFN

First nulls are present at $k = 1$

approximate BWFN

$$\phi_0 = 4\sqrt{\frac{\lambda}{2nd}} = 2\sqrt{\frac{2\lambda}{nd}}$$

$$\boxed{\phi_0 = 2\sqrt{\frac{2\lambda}{nd}}}$$

$$\boxed{\text{BWFN} = \phi_0 = 2\sqrt{\frac{2\lambda}{nd}}}$$ approximate

- Half Power Beam width

$$\boxed{\text{HPBM} = \sqrt{\frac{2\lambda}{nd}}}$$

- Directivity:

$$D = \frac{4\pi}{\theta_{HP} \times \phi_{HP}}$$

$$D = \frac{\cancel{4\pi}^{2}}{\cancel{2\pi}\sqrt{\frac{2\lambda}{nd}}}$$

$$D = \sqrt{\frac{\cancel{4}^{2}nd}{2\lambda}}$$

$$\boxed{D = \sqrt{2L_\lambda}}$$

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Date
Page

3. Extended End Fire Array:

- Beam width between first nulls:

nulls : $\frac{n\psi}{2} = \pm k\pi$

but $\psi = \beta d \cos\phi + \delta$

$\psi = \beta d \cos\phi - \beta d - \frac{\pi}{n}$

$$\frac{n}{2}\left[\beta d(\cos\phi - 1) - \frac{\pi}{n}\right] = \pm k\pi$$

$$\cancel{\frac{n}{2}}\left[\frac{2\pi}{\lambda} d(\cos\phi - 1) - \frac{\pi}{n}\right] = \pm \frac{2k\pi}{n}$$

$$\frac{2}{\lambda} d(\cos\phi - 1) - \frac{1}{n} = \pm \frac{2k}{n}$$

$$\frac{2}{\lambda} d(\cos\phi - 1) = \pm \frac{2k}{n} + \frac{1}{n}$$

$$\cos\phi - 1 = \pm \frac{\cancel{2}k\lambda}{\cancel{2}nd} + \frac{\lambda}{2nd}$$

$$\cancel{1} - 2\sin^2\frac{\phi}{2} \cancel{- 1} = \pm \frac{k\lambda}{nd} + \frac{\lambda}{2nd}$$

at $k = 1$

$$\sin\frac{\phi}{2} = \sqrt{\mp \frac{k\lambda}{2nd} - \frac{\lambda}{4nd}}$$

$$\phi' = 2\sin^{-1}\sqrt{\pm\frac{\lambda}{2nd} - \frac{\lambda}{4nd}}$$

$$\boxed{\phi_0 = 2\phi' = 4\sin^{-1}\left[\mp\sqrt{\frac{\lambda}{4nd}}\right]}$$

$$\phi' = 2\sin^{-1}\sqrt{\pm\frac{\lambda}{4nd}}$$

Exact BWFN

First nulls are present at $k = 1$

approximate BWFN

$$2\phi = 4\sqrt{\frac{\lambda}{4nd}}$$

$$\boxed{2\phi = 2\sqrt{\frac{\lambda}{nd}}}$$

Approximate BWEN

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- Half Power beam width

$$\boxed{HPBW = \frac{1}{8}\sqrt{\frac{\lambda}{nd}}}$$

- Directivity

$$D = \frac{4\pi}{\theta_{HP} \times \phi_{HP}}$$

$$D = \frac{4\pi}{2\pi \times \sqrt{\frac{\lambda}{nd}}}$$

$$D = \sqrt{\frac{4nd}{\lambda}}$$

$$\boxed{D = 2\sqrt{L\lambda}}$$

Q: If an array of isotropic radiators is operated at a frequency of 6GHz and is required to produce a broadside beam then find the beamwidth between the first nulls if the array length is 10m. Also find the directivity.

- Given: $f = 6GHz = 6 \times 10^9$ Broadside array

$l = 10m$

wkt $\lambda = \frac{c}{f} = \frac{3 \times 10^8}{6 \times 10^9} = 0.05m$

Beamwidth between the first nulls.

$$BWFN = \frac{2\lambda}{nd} = \frac{2\lambda}{l}$$

$$\therefore BWFN = \frac{2(0.05)}{10} = 0.01 \text{ radians} = 0.572^\circ$$

Directivity: $D = \frac{2nd}{\lambda} = \frac{2(10)}{0.05} = 400$

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Date ______
Page ______

Q: Find the directivity of a broadside array of 10 isotropic elements with a separation of $\lambda/4$ between the elements.

- Given: Broadside array.

$n = 10 \quad d = \lambda/4$

Directivity: $D = \dfrac{2nd}{\lambda} = \dfrac{2(10)\lambda/4}{\lambda} = 5$ //

Q: A uniform linear array is required to produce an end-fire beam when it is operated at a frequency of 10 GHz. It contains 50 radiators and are spaced at a distance of 0.5λ. Find the phase shift required to produce an end-fire array and also find the array length.

- Given: End fire array

$f = 10\,GHz = 10 \times 10^9\,Hz$

$n = 50 \qquad \lambda = \dfrac{c}{f} = \dfrac{3\times10^8}{10\times10^9} = \underline{0.03m}$

$d = 0.5\lambda$

For an end fire array

$\delta = -\beta d = -\dfrac{2\pi}{\lambda} d$

$\delta = \dfrac{-2\pi}{\lambda}(0.5\lambda) = -\pi$ // radians.

Arraylength

$l = nd = 50(0.5\lambda) = 50(0.5)(0.03) = 0.75m$ //

Q: An array contains 100 isotropic radiators with an interelement spacing of 0.5λ. It is required to produce a broadside array and end-fire array beam. Find the beam width between the first nulls, Half power beam width and directivity for both broad side and end-fire array.

Given: $n = 100$

$d = 0.5\lambda$

a. Broadside array

Beam width between the first nulls.

$$BWFN = \frac{2\lambda}{nd} = \frac{2\lambda}{100(0.5\lambda)} = 0.04 \text{ radians}$$

Half power beam width

$$HPBW = \frac{\lambda}{nd} = \frac{\lambda}{100(0.5\lambda)} = 0.02 \text{ radians}$$

Directivity

$$D = \frac{2nd}{\lambda} = \frac{2(100)(0.5\lambda)}{\lambda} = 100$$

b. End-fire array

Beam width between the first nulls.

$$BWFN = 2\sqrt{\frac{2\lambda}{nd}} = 2\sqrt{\frac{2\lambda}{100(0.5\lambda)}} = 0.4 \text{ radians}$$

Half power beam width

$$HPBW = \sqrt{\frac{2\lambda}{nd}} = \sqrt{\frac{2\lambda}{100(0.5\lambda)}} = 0.2 \text{ radians}$$

Directivity

$$D = \sqrt{\frac{2nd}{\lambda}} = \sqrt{\frac{2(100)(0.5\lambda)}{\lambda}} = 10$$

Q: Find the beam width between the first nulls of a broadside, end-fire and extended end fire array when

$l = 10\lambda, 50\lambda, 20\lambda$

$n = 20, 100, 50$ respectively.

Also find directivity

a. Broad side Array:

Beam width between the first nulls.

$$BWFN = \frac{2\lambda}{nd} = \frac{2\lambda}{10\lambda} = 0.2 \text{ radians}$$

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Directivity : $D = \frac{2nd}{\lambda} = \frac{2(10\lambda)}{\lambda} = 20$

b. End-Fire array

Beam width between first nulls :

$BWFN = 2\sqrt{\frac{2\lambda}{nd}} = 2\sqrt{\frac{2\lambda}{50\lambda}} = 0.4$

Directivity : $D = \sqrt{\frac{2nd}{\lambda}} = \sqrt{\frac{2(50\lambda)}{\lambda}} = 10$

c. Extended End fire array

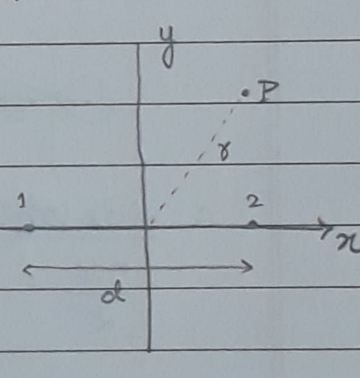
Beam width between first nulls

$BWFN = \sqrt{\frac{\lambda}{nd}} = \sqrt{\frac{\lambda}{20\lambda}} = 0.22$

Directivity : $D = \sqrt{\frac{4nd}{\lambda}} = \sqrt{\frac{4(20\lambda)}{\lambda}} = 8.94$

* Non-Isotropic point sources but similar point :

Let us consider two non-isotropic point sources separated by a distance 'd'



$E_0 = E_0' \sin\phi$

but $E = 2E_0 \cos\frac{\psi}{2}$

Therefore $E = 2E_0' \sin\phi \cos\frac{\psi}{2}$

Field Pattern : $\boxed{E = \sin\phi \cos\frac{\psi}{2}}$

* Principle of pattern multiplication :

It is applicable only when the non isotropic point sources are similar and identical.

" The total field pattern of an array of non-isotropic but similar sources is the product of individual source pattern and the pattern of an array of isotropic point sources each located at the phase center of the individual sou… and having the same relative amplitude and phase. While the total phase pattern is the sum of the phase patterns of the individual sources and the array of the isotropic point sources.

$$E = f(\theta,\phi)\, F_p(\theta,\phi) \times \angle f(\theta,\phi) + F_p(\theta,\phi)$$

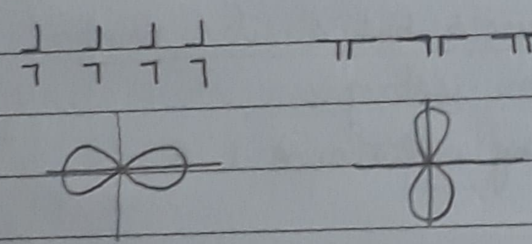
Q: Six vertical radiators are spaced $\lambda/2$ distance apart. The power is applied with equal amplitude and in phase. Find the field pattern and HPBW and Beam width between the first nulls.

$n = 6$ $\quad \delta = 0$

$d = \lambda/2$ $\quad$ given: 6 vertical radiators.

NOTE:

vertical $\qquad$ Horizontal



a. side lobes:

$$\frac{n\psi}{2} = \pm(2k+1)\frac{\pi}{2}$$

$$\psi = \beta d \cos\phi + \delta$$

$$\psi = \frac{2\pi}{\lambda}\left(\frac{\lambda}{2}\right)\cos\phi + 0$$

$$\psi = \pi\cos\phi$$

$$6(\pi\cos\phi) = \pm(2k+1)\pi$$

$$\cos\phi = \pm\frac{(2k+1)}{6}$$

$$\phi = \cos^{-1}\left[\pm\frac{(2k+1)}{6}\right]$$

at $k = 0$

$$\phi = \cos^{-1}\left[\pm\frac{1}{6}\right] = \pm 80.4^\circ, \pm 99.59^\circ$$

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at $k = 1$

$$\phi = \cos^{-1}\left[\pm \frac{1}{2}\right] = \pm 60^\circ, \pm 120^\circ$$

at $k = 2$

$$\phi = \cos^{-1}\left[\pm \frac{5}{6}\right] = \pm 33.56^\circ, \pm 146.4^\circ$$

b. nulls :

$$\frac{n\psi}{2} = \pm k\pi$$

$$\frac{6}{2}\left[\pi \cos\phi\right] = \pm k\pi$$

$$\cos\phi = \pm \frac{k}{3}$$

$$\phi = \cos^{-1}\left[\pm \frac{k}{3}\right]$$

at $k = 0$

$$\phi = \cos^{-1}(0) = \pm 90^\circ$$

at $k = 1$

$$\phi = \cos^{-1}\left[\pm \frac{1}{3}\right] = \pm 70.52^\circ, \pm 109.47^\circ$$

at $k = 2$

$$\phi = \cos^{-1}\left[\pm \frac{2}{3}\right] = \pm 48.19^\circ, \pm 131.81^\circ$$

at $k = 3$

$$\phi = \cos^{-1}\left[\pm 1\right] = 0^\circ, 180^\circ$$

c. Peak :

$$\psi = \pi \cos\phi = 0$$

$$\cos\phi = 0$$

$$\phi = \pm 90^\circ$$

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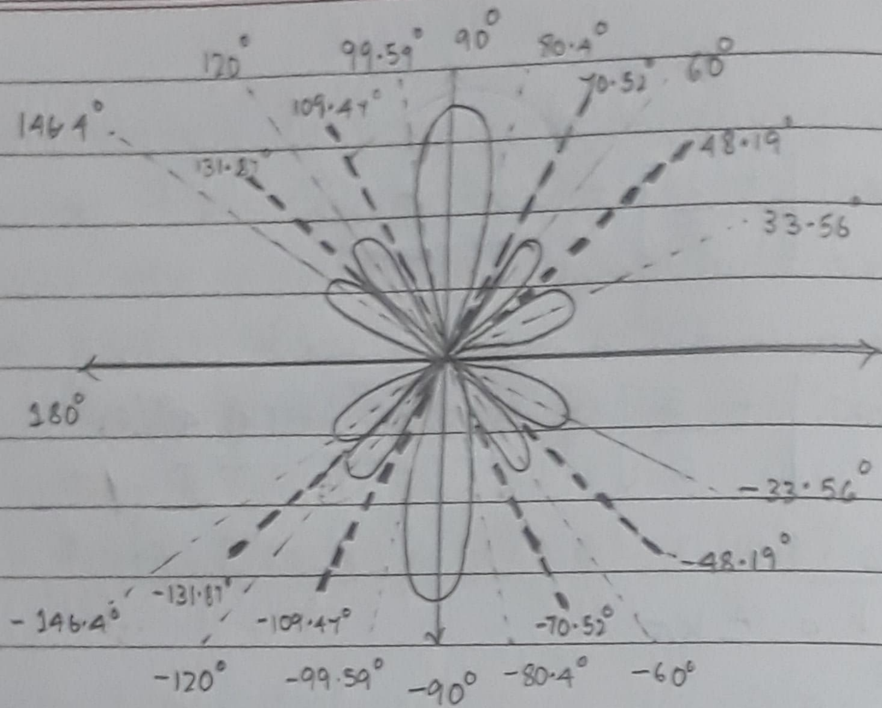

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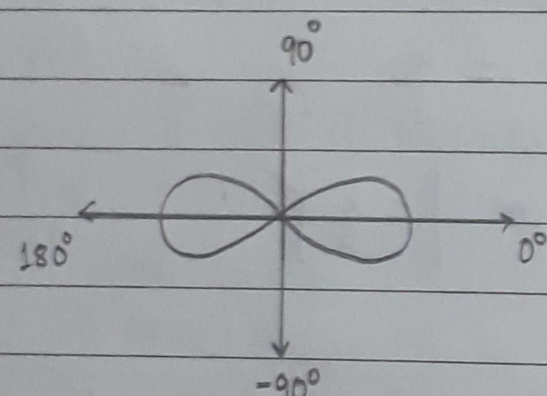
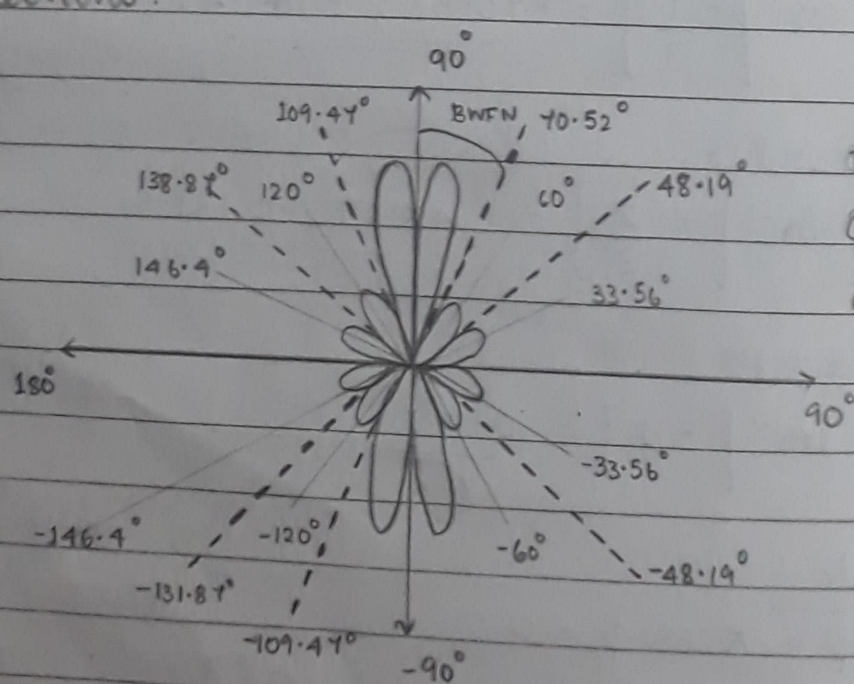


There will be no radiation in the null directions.

⇓



BWFN

$\phi = 90° - 70.52°$

$\phi = 19.48°$

HPBW

$\frac{\phi}{2} = 9.74°$

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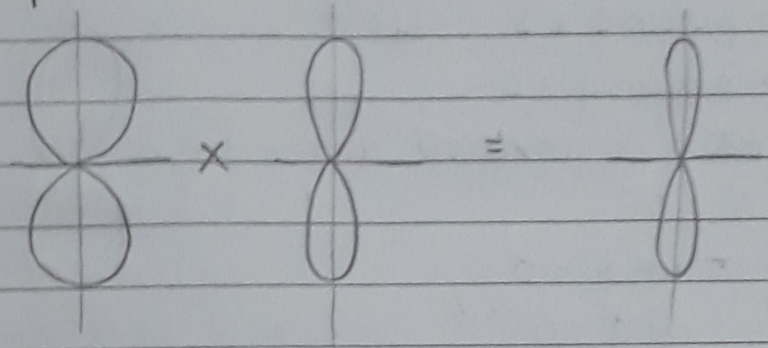
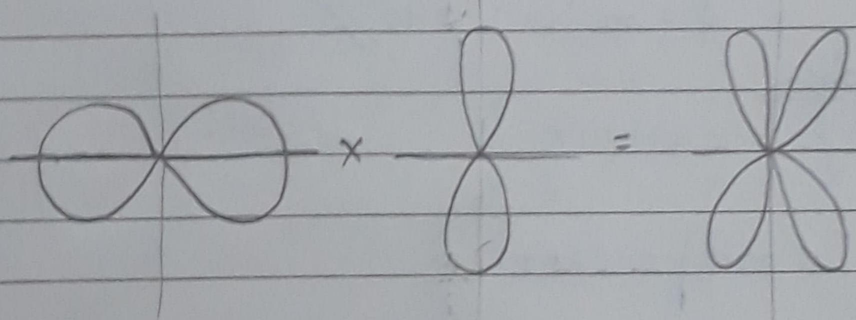
Date ______
Page ______

NOTE:

Examples of pattern multiplication.

Two nonisotropic but identical point sources of the same amplitude and phase.





Similar sources indicates that the variation with absolute angle ϕ of both the amplitude and phase of the field is the same. (The patterns not only must be of the same shape but also must be oriented in the same direction to be called similar). The maximum amplitudes of the individual sources may be unequal, if they are also equal then the sources are not only similar but are identical.

There will be no radiation in the direction of nulls after pattern multiplication.

Q: Four vertical radiators spaced $\lambda/2$ distance apart are placed along a straight line. The power is applied with same amplitude and in phase. Find the field pattern, Beam width between the first nulls and HPBW.

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- given: $n = 4$ $\quad \delta = 0$

$d = \lambda/2$

Peak:

$\psi = \beta d \cos\phi + \delta$

$\psi = \beta d \cos\phi = 0$

$\left(\frac{2\pi}{\cancel{\lambda}}\right)\left(\frac{\cancel{\lambda}}{\cancel{2}}\right)\cos\phi = 0$

$\pi \cos\phi = 0$

$\cos\phi = 0$

$\phi = \cos^{-1} 0 = \pm 90^\circ$ //

Side lobes:

$\frac{n\psi}{2} = \pm (2k+1)\frac{\pi}{2}$

$\frac{\cancel{4}^{2}}{\cancel{2}}\left[\cancel{\pi}\cos\phi\right] = \pm (2k+1)\frac{\cancel{\pi}}{2}$

$\cos\phi = \pm \frac{(2k+1)}{4}$

$\phi = \cos^{-1}\left[\pm \frac{(2k+1)}{4}\right]$ //

at $k = 0$

$\phi = \cos^{-1}\left[\pm \frac{1}{4}\right] = \pm 75.52^\circ, \pm 104.47^\circ$ //

at $k = 1$

$\phi = \cos^{-1}\left[\pm \frac{3}{4}\right] = \pm 41.4^\circ, \pm 138.59^\circ$ //

Nulls:

$\frac{n\psi}{2} = \pm k\pi$

$\frac{\cancel{4}^{2}}{\cancel{2}}\left[\cancel{\pi}\cos\phi\right] = \pm k \cancel{\pi}$

$\cos\phi = \pm \frac{k}{2}$

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$$\phi = \cos^{-1}\left[\pm \frac{k}{2}\right]$$

at $k=0$

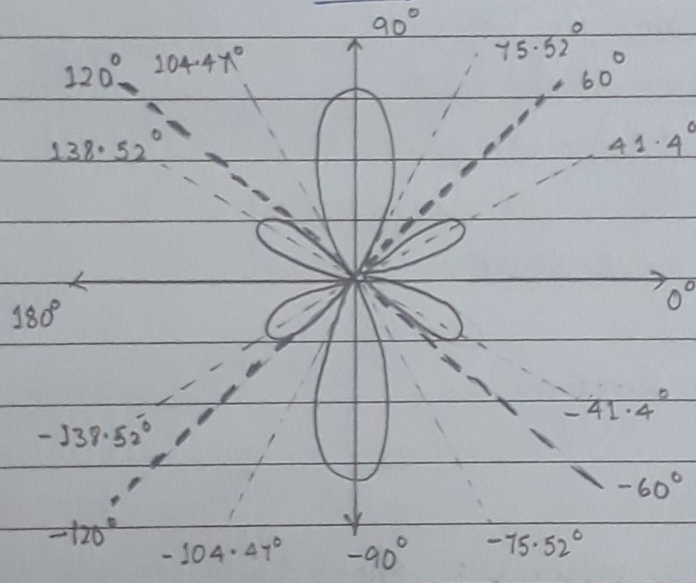
$$\phi = \cos^{-1}(0) = \pm 90^\circ$$

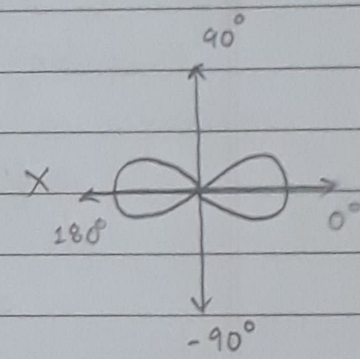
at $k=1$

$$\phi = \cos^{-1}\left[\pm \frac{1}{2}\right] = \underline{\pm 60^\circ, \pm 120^\circ}$$

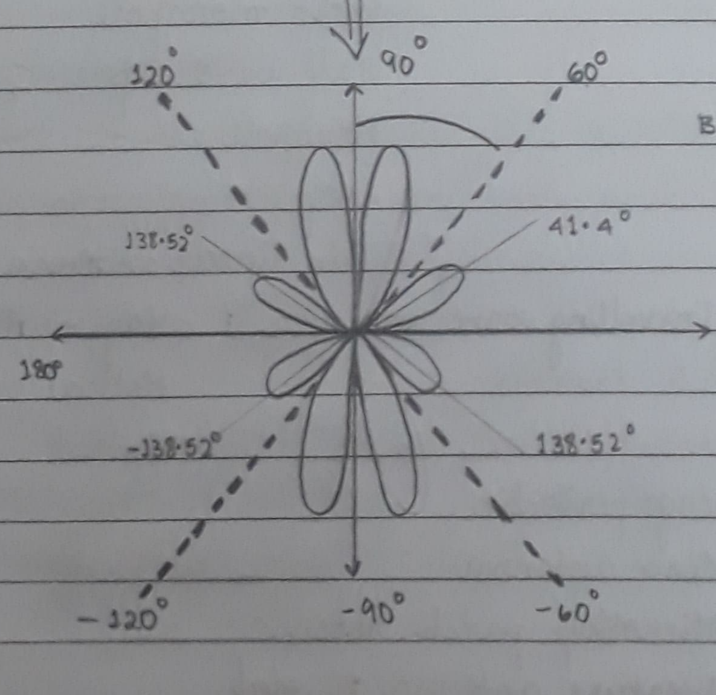
at $k=2$

$$\phi = \cos^{-1}[\pm 1] = \underline{0^\circ, 180^\circ}$$





$\Downarrow$



BWFN

$\phi = 30^\circ$

HPBW

$\frac{\phi}{2} = 15^\circ$

Date
Page

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Q: The lower lobe of an 8 element uniform broadside array was observed to be 45° with a frequency 40 MHz. Estimate the distance using exact method.

Given:

Broadside array

$n = 8$ $f = 40\,MHz$ $BWFN = 45°$

$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{40 \times 10^6} = 7.5$

Exact BWFN

$$\phi_0 = 2 \sin^{-1}\left(\frac{\lambda}{nd}\right)$$

$$45° = 2 \sin^{-1}\left(\frac{7.5}{8(d)}\right)$$

$$\sin^{-1}\left(\frac{7.5}{8d}\right) = 22.5°$$

$$\frac{7.5}{8d} = 0.3827$$

$$\underline{\underline{d = 2.45\,m}}$$

NOTE: Antennas:

- Wire antenna : short dipole : narrow band applications
  - short monopole :
  - Half wave dipole : GSM applications
  - Monopole : radio broad casting
  - Loop : aircrafts
  - Thin linear antenna.
- Travelling wave antenna : Yagi-uda : field of RADAR
  - Helical Transmission and receiving VHF signals through ionosphere
  - Spiral Defence application
- Log periodic
- Lens antenna
- Microstrip patch antenna
- Aperture antenna : slot
  - horn

UNIT - 03

Electric Dipoles and Thin Linear Antenna

A short linear conductor is often called a short dipole. ($L \ll \lambda$)

* current distribution

$$I = I_0 e^{j\omega t}$$

$$[I] = I_0 e^{j\omega[t - r/c]}$$

$[I]$: retarded current $\qquad [q] = \frac{[I]}{j\omega}$

r/c : retardation time

$\omega r/c$: phase retardation

Vector Retarded Potential

$$A = \frac{\mu_0}{4\pi}[I]L$$

Polar coordinates

$$A_r = A\cos\theta = \frac{\mu_0}{4\pi}[I]L\cos\theta$$

$$A_\theta = -A\sin\theta = -\frac{\mu_0}{4\pi}[I]L\sin\theta$$

$$A_\phi = 0$$

Scalar Retarded Potential

$$V = \frac{1}{4\pi\varepsilon_0}[I]L\cos\theta\left[\frac{1}{j\omega r^2} + \frac{1}{rc}\right]$$

* Fields of a short dipole:

$$E = -j\omega A - \nabla V$$

$$E = a_r E_r + a_\theta E_\theta + a_\phi E_\phi$$

$$A = a_r A_r + a_\theta A_\theta + a_\phi A_\phi$$

$$\nabla V = a_r \frac{\partial V}{\partial x} - \frac{a_\theta}{r}\frac{\partial V}{\partial \theta} + \frac{a_\phi}{r\sin\theta}\frac{\partial V}{\partial \phi}$$

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General case:

$$E_r = \frac{1}{2\pi\varepsilon_0}[I]L\cos\theta\left[\frac{1}{j\omega r^3}+\frac{1}{cr^2}\right]$$

$$E_\theta = \frac{1}{4\pi\varepsilon_0}[I]L\sin\theta\left[\frac{1}{j\omega r^3}+\frac{1}{cr^2}+\frac{j\omega}{rc^2}\right]$$

Magnetic field

$$H = \frac{1}{\mu_0}(\nabla\times A)$$

$$\nabla\times A = \begin{vmatrix} 1/r^2\sin\theta & 1/r\sin\theta & 1/r \\ \partial/\partial r & \partial/\partial\theta & \partial/\partial\phi \\ A_r & rA_\theta & r\sin A_\phi \end{vmatrix}$$

$$H = a_r H_r + a_\theta H_\theta + a_\phi H_\phi$$

$$\nabla\times A = \frac{a_r}{r^2\sin\theta}\left[\frac{\partial}{\partial\theta}(r\sin A_\phi)-\frac{\partial}{\partial\phi}(rA_\theta)\right]$$
$$+\frac{a_\theta}{r\sin\theta}\left[\frac{\partial}{\partial\phi}(A_r)-\frac{\partial}{\partial r}(r\sin A_\phi)\right]$$
$$+\frac{a_\phi}{r}\left[\frac{\partial}{\partial r}(rA_\theta)-\frac{\partial}{\partial\theta}(A_r)\right]$$

$$H_\phi = \frac{1}{4\pi}[I]L\sin\theta\left[\frac{j\omega}{rc}+\frac{1}{r^2}\right]$$

Far field: $(1/r)$

$$E_\theta = \frac{1}{4\pi\varepsilon_0}[I]L\sin\theta\left[\frac{j\omega}{rc^2}\right]$$

$$H_\phi = \frac{1}{4\pi}[I]L\sin\theta\left[\frac{j\omega}{rc}\right]$$

Impedance of space (Intrinsic Impedance)

$$\frac{E_\theta}{H_\phi} = \frac{1}{\varepsilon_0 c} = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 376.7\,\Omega$$

also:
$$E_\theta = j\frac{\beta}{4\pi\varepsilon_0 c r}[I]L\sin\theta = j\frac{[I]L\sin\theta}{r}\left[\frac{60\pi}{\lambda}\right]$$

$$H_\phi = j\frac{\beta}{4\pi r}[I]L\sin\theta = j\frac{[I]L\sin\theta}{r}\left[\frac{1}{2\lambda}\right]$$

Quasi-stationary:

$$E_r = \frac{q_0 L\cos\theta}{2\pi\varepsilon_0 r^3} \qquad E_\theta = \frac{q_0 L\sin\theta}{4\pi\varepsilon_0 r^3}$$

$$H_\phi = \frac{I_0 L\sin\theta}{4\pi r^2}$$

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★ Radiation resistance of short dipole

$$S_r = \frac{1}{2} R_e (E_\theta \times H_\phi^*)$$

$$R_r = 20L^2\beta^2 = \frac{80L^2\pi^2}{\lambda^2} = 790\frac{L^2}{\lambda^2}$$

$$\int e^{ax}\sin(bx+c) = \frac{e^{ax}}{a^2+b^2}\left[a\sin(bx+c) - b\cos(bx+c)\right]$$

★ Thin Linear Antenna:

Current distribution

$$I = I_0 \sin \omega t$$

$$[I] = I_0 e^{j\omega(t - r/c)} \sin\left[\beta\left(\frac{L}{2} \pm x\right)\right]$$

Far fields

$$H_\phi = j\frac{[I_0]}{2\pi r}\left[\frac{\cos[(\beta L\cos\theta)/2] - \cos(\beta L/2)}{\sin\theta}\right]$$

$$E_\theta = \frac{j60[I_0]}{r}\left[\frac{\cos[(\beta L\cos\theta)/2] - \cos(\beta L/2)}{\sin\theta}\right]$$

because $E_\theta = 120\pi H_\phi$

Radiation resistance of $\lambda/2$ dipole

$$S_r = \frac{1}{2} E_\theta H_\phi^*$$

$$R_r = 73\Omega$$

★ Folded Dipole:

$$Z_{in} = 4Z_D$$

$$R_r' = 292\ \Omega \ ; \ R'_r = n^2 R_r$$

$$Z_T = 2j\tan\left(\frac{\beta L}{2}\right) = \infty \text{ when } L = \lambda/2$$

NOTE:

$$\%\text{efficiency} = \frac{R_r}{R_r + R_L} \times 100$$

Effective aperture

$$A_e = \frac{D\lambda^2}{4\pi} \quad \text{or} \quad A_{em} = \frac{V^2}{4SR_r}$$

where $V = EL$: short dipole $\qquad S = EH.$

$V = \frac{E\lambda}{\pi}$: Halfwave dipole

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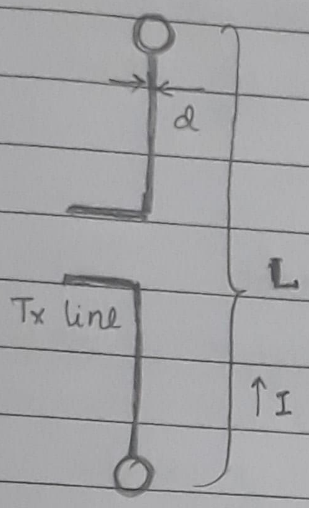
UNIT - 03

Electric Dipole and Thin Linear Antenna

* **Wire Antenna:**



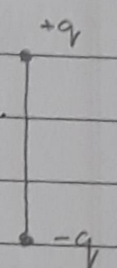
short dipole : $L << \lambda/4$ (smallest)
short monopole : $L << \lambda/8$
Half-wave dipole : $L = \lambda/2$
Monopole : $L = \lambda/4$
loop
Thin linear antenna : $L >> \lambda/4$

It has a uniform flow of current throughout the length due to capacitor plates and the small length

* – **Short dipole :**

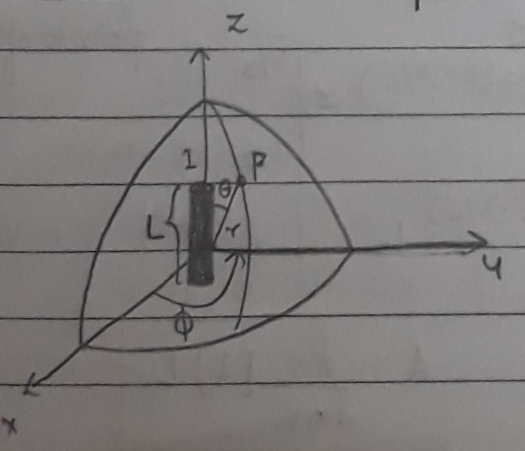
Assuming

1. The diameter of the short dipole satisfies $d << L$.
2. The transmission line does not radiate
3. The radiation through the capacitors are neglected.



Then the short dipole becomes a conductor of length L with +q and -q charge on each end.

– **Fields of a short dipole:**



current distribution
Vector Retarded potential
Scalar Retarded potential

$$E = -j\omega A - \nabla V$$

$$H = \frac{1}{\mu_0}(\nabla \times A)$$

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1. <u>current distribution</u>

$$\boxed{I = I_0 e^{j\omega t}}$$

where ωt = phase time

$t = t' = r/c$: retardation time

I_0 : maximum current.

$\omega t = \omega (t - r/c)$

Therefore

$$\boxed{[I] = I_0 e^{j\omega (t - r/c)}}$$

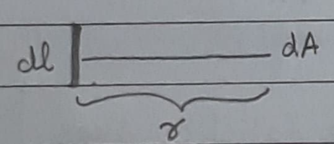
Here

$[I]$: denoted retarded current

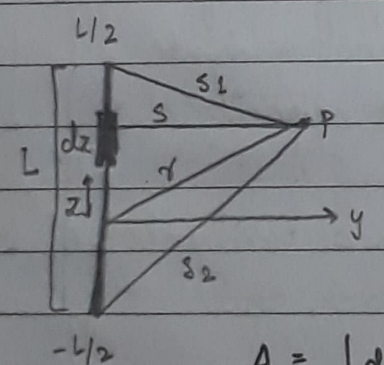
$\frac{\omega r}{c}$: retardation phase

r/c : retardation time.

2. <u>Vector Retarded Potential:</u>



$$\boxed{dA = \frac{\mu_0}{4\pi} \frac{[I]\, dl}{r}}$$



$$\boxed{dA = \frac{\mu_0}{4\pi} \frac{[I]\, dz}{s}}$$

Here $r = s$

because the length of $dz = s$ from the point of observation

$$A = \int dA$$

$$A = \int_{-L/2}^{L/2} \frac{\mu_0}{4\pi} \frac{I_0 e^{j\omega(t - s/c)}}{s} dz$$

$$A = \frac{\mu_0}{4\pi} \frac{I_0}{s} e^{j\omega(t - s/c)} z \Big|_{-L/2}^{L/2}$$

$$A = \frac{\mu_0}{4\pi} \frac{I_0 e^{j\omega(t-s/c)}}{s} L$$

$$A = \frac{\mu_0}{4\pi} \frac{[I] L}{s}$$

$$\boxed{A = \frac{\mu_0}{4\pi r} [I] L}$$

Spherical coordinate system

$$A_r = A_x \cos\theta$$
$$A_\theta = -A_x \sin\theta$$
$$A_\phi = 0$$

Therefore

$$A_r = A_z \cos\theta \qquad A_\theta = -A_z \sin\theta$$

$$\boxed{A_r = \frac{\mu_0}{4\pi r}[I]L\cos\theta} \qquad \boxed{A_\theta = -\frac{\mu_0}{4\pi r}[I]L\sin\theta}$$

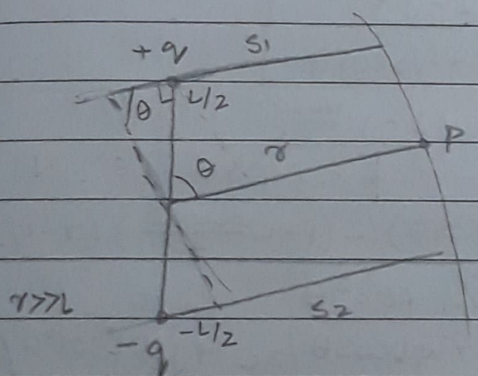
$$\boxed{A_\phi = 0}$$

3. Scalar Retarded Potential:

Potential due to a charge

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$$

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{[q]}{S_1} - \frac{[q]}{S_2}\right]$$



but $\frac{dq}{dt} = I$

$\therefore \quad q = \int I\,dt$

Hence

$$[q] = \int [I]\,dt$$
$$= \int I_0 e^{j\omega(t-r/c)}\,dt$$
$$= \frac{I_0 e^{j\omega(t-r/c)}}{j\omega}$$

$$\therefore \quad \boxed{[q] = \frac{[I]}{j\omega}}$$

$$S_1 = r - L/2\cos\theta$$
$$S_2 = r + L/2\cos\theta$$

Therefore

$$V = \frac{I_0}{4\pi\varepsilon_0 j\omega}\left[\frac{e^{j\omega(t-S_1/c)}}{S_1} - \frac{e^{j\omega(t-S_2/c)}}{S_2}\right]$$

$$V = \frac{I_0}{4\pi\varepsilon_0 j\omega}\left[\frac{e^{j\omega\left[t-\left(\frac{r-L/2\cos\theta}{c}\right)\right]}}{r - L/2\cos\theta} - \frac{e^{j\omega\left[t-\left(\frac{r+L/2\cos\theta}{c}\right)\right]}}{r + L/2\cos\theta}\right]$$

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$$V = \frac{I_0 e^{j\omega(t-r/c)}}{j\omega 4\pi \varepsilon_0}\left[\frac{e^{\frac{j\omega L/2\cos\theta}{c}}}{r-L/2\cos\theta} - \frac{e^{\frac{-j\omega L/2\cos\theta}{c}}}{r+L/2\cos\theta}\right]$$

$$V = \frac{[I]}{j\omega 4\pi\varepsilon_0}\left[\frac{e^{jx}}{r-L/2\cos\theta} - \frac{e^{-jx}}{r+L/2\cos\theta}\right]$$

where $x = \frac{\omega L\cos\theta}{2c}$

$$x = \frac{2\pi f L\cos\theta}{2c}$$

$$x = \frac{2\pi c L\cos\theta}{2c\lambda}$$

$$x = \frac{\pi L\cos\theta}{\lambda} \qquad \frac{L}{\lambda} << 1 \therefore x << 1$$

but $e^{jx} = 1+jx$ and $e^{-jx} = 1-jx$

Therefore

$$V = \frac{[I]}{j\omega 4\pi\varepsilon_0}\left[\frac{1+jx}{r-L/2\cos\theta} - \frac{1-jx}{r+L/2\cos\theta}\right]$$

$$V = \frac{[I]}{j\omega 4\pi\varepsilon_0}\left[\frac{1+j\frac{\omega L\cos\theta}{2c}}{r-L/2\cos\theta} - \frac{1-j\frac{\omega L\cos\theta}{2c}}{r+L/2\cos\theta}\right]$$

$$V = \frac{[I]}{j\omega 4\pi\varepsilon_0}\left[\frac{(1+j\frac{\omega L\cos\theta}{2c})(r+L/2\cos\theta) - (1-j\frac{\omega L\cos\theta}{2c})(r-L/2\cos\theta)}{r^2 - L^2/4\cos^2\theta}\right]$$

$$V = \frac{[I]}{j\omega 4\pi\varepsilon_0}\left[\frac{r + L/2\cos\theta + jr\omega L\cos\theta/2c + j\omega L^2\cos^2\theta/4c - r + L/2\cos\theta + jr\omega L\cos\theta/2c - j\omega L^2\cos^2\theta/4c}{r^2 - L^2/4\cos^2\theta}\right]$$

Since $L << \lambda$ Hence the square terms are neglected

$$V = \frac{[I]}{j\omega 4\pi\varepsilon_0}\left[\frac{L\cos\theta + jr\omega L\cos\theta/c}{r^2}\right]$$

$$V = \frac{[I]L\cos\theta}{j\omega 4\pi\varepsilon_0 r^2}\left[1 + \frac{j\omega r}{c}\right]$$

$$\boxed{V = \frac{[I]L\cos\theta}{4\pi\varepsilon_0}\left[\frac{1}{j\omega r^2} + \frac{1}{rc}\right]}$$

Keerthana Ashok

Date
Page

General case :

wkt

$$E = -j\omega A - \nabla V$$

in spherical coordinate system

$$E = a_r E_r + a_\theta E_\theta + a_\phi E_\phi$$

$$A = a_r A_r + a_\theta A_\theta + a_\phi A_\phi$$

$$\nabla V = a_r \frac{\partial V}{\partial r} - \frac{a_\theta}{r}\frac{\partial V}{\partial \theta} + \frac{a_\phi}{r\sin\phi}\frac{\partial V}{\partial \phi}$$

Therefore

$$E_r = -j\omega(A_r) - \frac{\partial V}{\partial r}$$

$$E_\theta = -j\omega(A_\theta) - \frac{1}{r}\frac{\partial V}{\partial \theta}$$

$$E_\phi = -j\omega(A_\phi) - \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}$$

wkt

$$A_r = \frac{\mu_0}{4\pi r}[I]L\cos\theta = \frac{\mu_0 L}{4\pi r} I_0 e^{j\omega(t-r/c)}\cos\theta$$

$$A_\theta = \frac{-\mu_0}{4\pi r}[I]L\cos\theta = \frac{-\mu_0 L}{4\pi r} I_0 e^{j\omega(t-r/c)}\cos\theta$$

$$A_\phi = 0.$$

Here

$$V = \frac{[I]L\cos\theta}{4\pi\varepsilon_0}\left[\frac{1}{j\omega r^2} + \frac{1}{rc}\right]$$

$$V = \frac{I_0 e^{j\omega(t-r/c)} L\cos\theta}{4\pi\varepsilon_0}\left[\frac{1}{j\omega r^2} + \frac{1}{rc}\right]$$

differentiating partially wrt r.

$$\frac{\partial V}{\partial r} = \frac{I_0 L\cos\theta}{4\pi\varepsilon_0}\left[e^{j\omega(t-r/c)}\left(\frac{-2}{j\omega r^3} - \frac{1}{r^2 c}\right) + \left(\frac{1}{j\omega r^2} + \frac{1}{rc}\right)e^{j\omega(t-r/c)}\left(\frac{j\omega}{c}\right)\right]$$

Keerthana Ashok

$$\frac{\partial V}{\partial r} = \frac{[I]L\cos\theta}{4\pi\varepsilon_0}\left[\frac{-2}{j\omega r^3} - \frac{1}{r^2c} - \frac{1}{r^2c} - \frac{j\omega}{rc^2}\right]$$

$$\frac{\partial V}{\partial r} = \frac{[I]L\cos\theta}{4\pi\varepsilon_0}\left[\frac{-2}{j\omega r^3} - \frac{2}{r^2c} - \frac{j\omega}{rc^2}\right]$$

Substituting for E_r.

$$E_r = -j\omega\left[\frac{\mu_0}{4\pi r}[I]L\cos\theta\right] - \frac{[I]L\cos\theta}{4\pi\varepsilon_0}\left[\frac{-2}{j\omega r^3} - \frac{2}{r^2c} - \frac{j\omega}{rc^2}\right]$$

$$E_r = \frac{[I]L\cos\theta}{4\pi\varepsilon_0}\left[\frac{-j\omega\mu_0\varepsilon_0}{r} + \frac{2}{j\omega r^3} + \frac{2}{r^2c} + \frac{j\omega}{rc^2}\right]$$

$$E_r = \frac{[I]L\cos\theta}{4\pi\varepsilon_0}\left[\cancel{\frac{-j\omega}{rc^2}} + \frac{2}{j\omega r^3} + \frac{2}{r^2c} + \cancel{\frac{j\omega}{rc^2}}\right]$$

$$\boxed{E_r = \frac{[I]L\cos\theta}{2\pi\varepsilon_0}\left[\frac{1}{j\omega r^3} + \frac{1}{r^2c}\right]}$$

Similarly differentiating V partially wrt θ

$$\frac{\partial V}{\partial \theta} = \frac{-[I]L\sin\theta}{4\pi\varepsilon_0}\left[\frac{1}{j\omega r^2} + \frac{1}{rc}\right]$$

Substituting for E_θ

$$E_\theta = -j\omega\left[-\frac{\mu_0}{4\pi r}[I]L\sin\theta\right] - \frac{1}{r}\left[-\frac{[I]L\sin\theta}{4\pi\varepsilon_0}\left(\frac{1}{j\omega r^2} + \frac{1}{rc}\right)\right]$$

$$E_\theta = \frac{[I]L\sin\theta}{4\pi\varepsilon_0}\left[\frac{j\omega\mu_0\varepsilon_0}{r} + \frac{1}{j\omega r^3} + \frac{1}{r^2c}\right]$$

$$\boxed{E_\theta = \frac{[I]L\sin\theta}{4\pi\varepsilon_0}\left[\frac{j\omega}{rc^2} + \frac{1}{j\omega r^3} + \frac{1}{r^2c}\right]}$$

Similarly differentiating V partially wrt ϕ

$$\frac{\partial V}{\partial \phi} = 0.$$

Substituting for E_ϕ

$$E_\phi = -j\omega\{0\} - \frac{1}{r\sin\theta}(0)$$

$$\boxed{E_\phi = 0}$$

Keerthana Ashok

Date
Page

similarly

wkt $H = \frac{1}{\mu_0}(\nabla \times A)$

where $H = a_r H_r + a_\theta H_\theta + a_\phi H_\phi$

$$\nabla \times A = \frac{a_r}{r^2 \sin\theta}\left[\frac{\partial}{\partial\theta}\cancel{(r \sin A_\phi)}^{0} - \frac{\partial}{\partial\phi}\cancel{(r A_\theta)}^{0}\right]$$

$$+ \frac{a_\theta}{r\sin\theta}\left[\frac{\partial}{\partial\phi}\cancel{(A_r)}^{0} - \frac{\partial}{\partial r}\cancel{(r \sin A_\phi)}^{0}\right]$$

$$+ \frac{a_\phi}{r}\left[\frac{\partial}{\partial r} r A_\theta - \frac{\partial}{\partial\theta}(A_r)\right]$$

as the ϕ components are zero

$$\nabla \times A = \frac{a_\phi}{r}\left[\frac{\partial}{\partial r} r A_\theta - \frac{\partial}{\partial\theta}(A_r)\right]$$

where $A_r = \frac{\mu_0}{4\pi r}[I]L\cos\theta$

and $A_\theta = \frac{-\mu_0}{4\pi r}[I]L\sin\theta$

$$\nabla \times A = \frac{a_\phi}{r}\left[\frac{\partial}{\partial r}\left(-\cancel{r}\frac{\mu_0}{4\pi \cancel{r}}[I]L\sin\theta\right) - \frac{\partial}{\partial\theta}\left(\frac{\mu_0}{4\pi r}[I]L\cos\theta\right)\right]$$

$$\nabla \times A = \frac{a_\phi}{r}\left[\frac{\partial}{\partial r}\left(\frac{-\mu_0}{4\pi}L\sin\theta\, I_0 e^{j\omega(t-r/c)}\right) - \frac{\partial}{\partial\theta}\left(\frac{\mu_0}{4\pi r}[I]L\cos\theta\right)\right]$$

$$\nabla \times A = \frac{a_\phi}{r}\left[\frac{-\mu_0 L\sin\theta}{4\pi} I_0 e^{j\omega(t-r/c)}\left(\frac{-j\omega}{c}\right) + \frac{\mu_0[I]L\sin\theta}{4\pi r}\right]$$

$$\nabla \times A = \frac{a_\phi [I]L\sin\theta\,\mu_0}{4\pi r}\left[\frac{j\omega}{c} + \frac{1}{r}\right]$$

Therefore

$$H_\phi = \frac{1}{\mu_0}(\nabla \times A)$$

$$H\phi = \frac{1}{\cancel{\mu_0}}\left[\frac{\cancel{\mu_0}}{4\pi r}[I]L\sin\theta\left(\frac{j\omega}{c} + \frac{1}{r}\right)\right]$$

$$\boxed{H\phi = \frac{[I]L\sin\theta}{4\pi}\left(\frac{j\omega}{rc} + \frac{1}{r^2}\right)}$$

Therefore

$$E_r = \frac{[I]L\cos\theta}{2\pi\varepsilon_0}\left[\frac{1}{j\omega r^3} + \frac{1}{r^2 c}\right]$$

$$E_\theta = \frac{[I]L\sin\theta}{4\pi\varepsilon_0}\left[\frac{j\omega}{rc^2} + \frac{1}{j\omega r^3} + \frac{1}{r^2 c}\right]$$

$$H\phi = \frac{[I]L\sin\theta}{4\pi}\left[\frac{j\omega}{rc} + \frac{1}{r^2}\right]$$

$\frac{1}{r^3}$: Electrostatic Field

$\frac{1}{r^2}$: Induction Field / Near Field

$\frac{1}{r}$: Radiation Field / Far Field.

→ For Far Field :

$$E_\theta = \frac{[I]L\sin\theta}{4\pi\varepsilon_0}\left[\frac{j\omega}{rc^2}\right]$$

$$H\phi = \frac{[I]L\sin\theta}{4\pi}\left[\frac{j\omega}{rc}\right]$$

Intrinsic Impedence

$$\frac{E_\theta}{H\phi} = \frac{1}{\varepsilon_0 c}$$

$$\frac{E_\theta}{H\phi} = \frac{\sqrt{\varepsilon_0\mu_0}}{\varepsilon_0}$$

$\mu_0 = 4\pi \times 10^{-7}$

$\varepsilon_0 = 8.854 \times 10^{-12}$

$$\boxed{\frac{E_\theta}{H\phi} = \sqrt{\frac{\mu_0}{\varepsilon_0}} = \eta} = 376\,\Omega = 120\pi$$

Keerthana Ashok

Date ______
Page ______

wkt $\omega = 2\pi f = \frac{2\pi c}{\lambda} = \beta c$

therefore

$$E_\theta = \frac{[I] L \sin\theta}{4\pi \varepsilon_0} \left[\frac{j\beta \cancel{c}}{r c^{\cancel{2}}} \right]$$

$$\therefore \boxed{E_\theta = j \frac{[I] L \sin\theta \, \beta}{4\pi \varepsilon_0 c r}}$$

$$H_\phi = \frac{[I] L \sin\theta}{4\pi} \left[\frac{j\beta \cancel{c}}{r \cancel{c}} \right]$$

$$\therefore \boxed{H_\phi = \frac{j [I] L \sin\theta \, \beta}{4\pi r}}$$

similarly

taking $\frac{\omega}{4\pi \varepsilon_0 c^2} = \frac{2\cancel{\pi}\cancel{c}}{{}_2\cancel{4\pi} \varepsilon_0 c^{\cancel{2}} \lambda} = \frac{1}{2\varepsilon_0 c \lambda} = \frac{\sqrt{\mu_0 \varepsilon_0}}{2\varepsilon_0 \lambda}$

$$\therefore \frac{\omega}{4\pi \varepsilon_0 c^2} = \sqrt{\frac{\mu_0}{\varepsilon_0}} \frac{1}{2\lambda} = \frac{120\pi}{2\lambda} = \frac{60\pi}{\lambda}$$

substituting:

$$\boxed{E_\theta = \frac{[I] L \sin\theta}{r} j \left[\frac{60\pi}{\lambda} \right]}$$

and $\frac{\omega}{4\pi c} = \frac{2\cancel{\pi}\cancel{c}}{{}_2\cancel{4\pi}\cancel{c}\lambda} = \frac{1}{2\lambda}$

$$\boxed{H_\phi = \frac{[I] L \sin\theta}{r} j \left[\frac{1}{2\lambda} \right]}$$

* <u>Quasi-Stationary / dc-current condition:</u>

This condition is for low frequencies.

$$\omega^{\to 0} = 2\pi f^{\to 0}$$

Retarded current is replaced by

$$[I] = [q] j\omega$$

Therefore

$$E_r = \frac{[q] L \cos\theta}{2\pi \varepsilon_0} j\omega \left[\frac{1}{j\omega r^3} + \frac{1}{r^2 c} \right]$$

$$\therefore E_r = \frac{[q]L\cos\theta}{2\pi\varepsilon_0}\left[\frac{1}{r^3} + \frac{j\omega}{r^2 c}\right]$$

$$\boxed{E_r = \frac{[q]L\cos\theta}{2\pi\varepsilon_0 r^3} = \frac{q_0 L\cos\theta}{2\pi\varepsilon_0 r^3}}$$

similarly.

$$E_\theta = \frac{[q]L\sin\theta}{4\pi\varepsilon_0} j\omega\left[\frac{j\omega}{rc^2} + \frac{1}{j\omega r^3} + \frac{1}{r^2 c}\right]$$

$$E_\theta = \frac{[q]L\sin\theta}{4\pi\varepsilon_0}\left[-\frac{\omega^2}{rc^2} + \frac{1}{r^3} + \frac{j\omega}{r^2 c}\right]$$

$$\boxed{E_\theta = \frac{[q]L\sin\theta}{4\pi\varepsilon_0 r^3} = \frac{q_0 L\sin\theta}{4\pi\varepsilon_0 r^3}}$$

and

$$H_\phi = \frac{[I]L\sin\theta}{4\pi}\left[\frac{j\omega}{rc} + \frac{1}{r^2}\right]$$

$$\boxed{H_\phi = \frac{I_0 L\sin\theta}{4\pi r^2}}$$

★ **Radiation Resistance of Short Dipole : R_r**

- The Poynting vector of the far field is integrated over a large sphere to obtain the total power radiated.
- The total power obtained is equated to I^2R where I is the rms current on the dipole and R is called as the radiation resistance of the dipole.

Poynting vector is given by

$$S = \frac{1}{2}\operatorname{Re}(E_\theta \times H_\phi^*)$$

$$S_r = \frac{1}{2}\operatorname{Re}(E_\theta \times H_\phi^*)$$

H_ϕ^* : complex conjugate of H_ϕ

By power theorem wkt

Keerthana Ashok

Date
Page

$$P = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} S_r \, ds$$

For sphere $ds = r^2 \sin\theta \, d\theta \, d\phi$

Therefore

$$P = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} \frac{1}{2} Re\left(E_\theta H_\phi^*\right) r^2 \sin\theta \, d\theta \, d\phi$$

$$\frac{E_\theta}{H_\phi} = Z$$

$$E_\theta = Z H_\phi = \sqrt{\frac{\mu_0}{\varepsilon_0}} H_\phi$$

$$P = \frac{1}{2}\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} Re\left(\sqrt{\frac{\mu_0}{\varepsilon_0}} |H_\phi|^2\right) r^2 \sin\theta \, d\theta \, d\phi$$

$$P = \frac{1}{2}\int_{\phi=0}^{2\pi} d\phi \; 120\pi \int_{\theta=0}^{\pi} \left| j \frac{I_0 e^{j\omega(t - r/c)} L \sin\theta \, \omega}{4\pi r c} \right|^2 r^2 \sin\theta \, d\theta$$

$$P = \frac{1}{\cancel{2}} (\cancel{2}\pi) \, 1\cancel{20}\pi \int_{\theta=0}^{\pi} \frac{I_0^2 L^2 \sin^2\theta \, \omega^2}{16\pi^2 \cancel{r^2} c^2} \cancel{r^2} \sin\theta \, d\theta$$

$$P = \frac{15 \, \omega^2 I_0^2 L^2}{2c^2} \int_{\theta=0}^{\pi} \sin^3\theta \, d\theta$$

$$P = \frac{15 (4\pi^2 c^2) I_0^2 L^2}{2c^2 \lambda^2} \int_{\theta=0}^{\pi} \left[\frac{3\sin\theta - \sin 3\theta}{4}\right] d\theta$$

$$P = \frac{15 I_0^2 L^2 \pi^2}{2\lambda^2} \left[-3\cos\theta + \frac{\cos 3\theta}{3} \right]_0^{\pi}$$

$$P = \frac{15 I_0^2 L^2 \pi^2}{2\lambda^2} \left[-3(-1) + \left(\frac{-1}{3}\right) + 3 - \frac{1}{3} \right]$$

$$P = \frac{15 I_0^2 L^2 \pi^2}{2\lambda^2} \left[6 - \frac{2}{3} \right]$$

$$P = \frac{15 I_0^2 L^2 \pi^2}{2\lambda^2} \left[\frac{16}{3} \right]$$

$$\boxed{P = \frac{40 I_0^2 L^2 \pi^2}{\lambda^2}}$$

wkt $\beta = \frac{2\pi}{\lambda}$

$$\boxed{\therefore P = 10 I_0^2 L^2 \beta^2}$$

$$P = I^2 R$$

$$\therefore 10 I_0^2 L^2 \beta^2 = I^2 R$$

$$10 \cancel{I_0^2} L^2 \beta^2 = \left[\frac{I_0}{\sqrt{2}}\right]^2 R_r$$

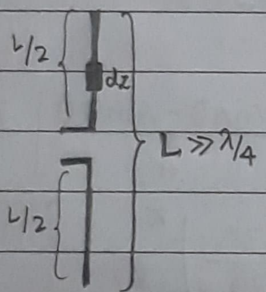
$$\therefore \boxed{R_r = 20 L^2 \beta^2}$$

wkt $\beta = \dfrac{2\pi}{\lambda}$

$$\boxed{R_r = \frac{80 L^2 \pi^2}{\lambda^2}}$$

$$\boxed{R_r = \frac{789.56 L^2}{\lambda^2}}$$

★ **Fields for thin Linear Antenna: $(L \gg \lambda/4)$**



Current distribution
Vector Retarded Potential
Scalar Retarded Potential.

1. **Current Distribution**

$$\boxed{i = I_0 \sin \omega t}$$

$$i = I_0 \sin 2\pi f t$$

$$i = I_0 \sin \frac{2\pi c}{\lambda} t$$

$$\boxed{i = I_0 \sin \beta L}$$

$$\boxed{i = I_0 \sin \beta\left(\frac{L}{2} \pm z\right)}$$

$L/2 + z \quad : z < 0$

$L/2 - z \quad : z > 0$

$$\boxed{[I] = I_0 \sin\beta\left(\frac{L}{2} \pm z\right) e^{j\omega(t - s/c)}}$$

Far field component for short dipole

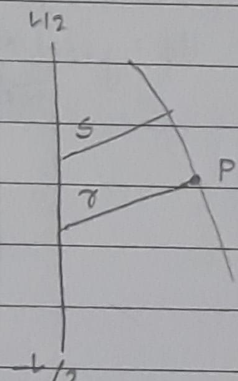
$$E_\theta = j\,\frac{[I] L \sin\theta}{r}\left[\frac{60\pi}{\lambda}\right]$$

$$dE_\theta = j\,\frac{[I] \sin\theta}{r}\left[\frac{60\pi}{\lambda}\right] dz$$

$$H_\phi = j\,\frac{[I] L \sin\theta}{r}\left[\frac{1}{2\lambda}\right]$$

$$dH_\phi = j\,\frac{[I] \sin\theta}{r}\left[\frac{1}{2\lambda}\right] dz$$

wkt $Z = \frac{E_\theta}{H_\phi} \Rightarrow H_\phi = \frac{E_\theta}{Z}$



$$H_\phi = \int_{-L/2}^{L/2} j\,\frac{[I] \sin\theta}{s}\left[\frac{1}{2\lambda}\right] dz$$

$$H_\phi = \int_{-L/2}^{L/2} j\, I_0 \sin\beta\left(\frac{L}{2} \pm z\right) e^{j\omega(t - s/c)}\,\frac{\sin\theta\, dz}{2s\lambda}$$

$$H_\phi = \frac{j I_0 \sin\theta}{2s\lambda} \int_{-L/2}^{L/2} e^{j\omega(t - s/c)} \sin\beta\left(\frac{L}{2} \pm z\right) dz$$

$$H_\phi = \frac{j I_0 \sin\theta}{2s\lambda}\left[\int_{-L/2}^{0} \sin\beta\left(\frac{L}{2} + z\right) e^{j\omega(t - s/c)} dz + \int_{0}^{L/2} \sin\beta\left(\frac{L}{2} - z\right) e^{j\omega(t - s/c)} dz\right]$$

$$H_\phi = \frac{j I_0 \sin\theta}{2r\lambda}\left[\int_{-L/2}^{0} \sin\left(\beta\frac{L}{2} + \beta z\right) e^{j\omega(t - (r - z\cos\theta)/c)} dz + \int_{0}^{L/2} \sin\left(\beta\frac{L}{2} - \beta z\right) e^{j\omega(t - (r - z\cos\theta)/c)} dz\right]$$

$$H_\phi = \frac{j I_0 \sin\theta}{2r\lambda} e^{j\omega(t - r/c)}\left[\int_{-L/2}^{0} \sin\left(\beta\frac{L}{2} + \beta z\right) e^{j\omega z\cos\theta/c} dz + \int_{0}^{L/2} \sin\left(\beta\frac{L}{2} - \beta z\right) e^{j\omega z\cos\theta/c} dz\right]$$

$$H_\phi = j\frac{[I_0] \sin\theta}{2r\lambda}\left[\int_{-L/2}^{0} \sin\left(\beta\frac{L}{2} + \beta z\right) e^{j\omega z\cos\theta/c} dz + \int_{0}^{L/2} \sin\left(\beta\frac{L}{2} - \beta z\right) e^{j\omega z\cos\theta/c} dz\right]$$

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$$\int e^{ax}\sin(bx+c) = \frac{e^{ax}}{a^2+b^2}\left[a\sin(bx+c) - b\cos(bx+c)\right]$$

$\therefore$ Here $a = \dfrac{j\omega\cos\theta}{c} = \dfrac{j2\pi c\cos\theta}{\lambda c} = j\beta\cos\theta$

$b = \beta$ and $c = \beta\dfrac{L}{2}$

$$\therefore H\phi = j\frac{[I_0]\sin\theta}{2r\lambda}\left\{\left[\frac{e^{j\beta\cos\theta z}}{\beta^2-\beta^2\cos^2\theta}\left[j\beta\cos\theta\sin\left(\beta z+\beta\frac{L}{2}\right) + \beta\cos\left(\beta z+\beta\frac{L}{2}\right)\right]\right]\Bigg|_{-L/2}^{0} + \int_0^{L/2}\sin\left(\beta\frac{L}{2}-\beta x\right)e^{j\omega z\cos\theta/c}\,dz\right\}$$

$$H\phi = j\frac{[I_0]\sin\theta}{2r\lambda}\left\{\left[\frac{1}{\beta^2\sin^2\theta}\left(j\beta\cos\theta\sin\beta\frac{L}{2} + \beta\cos\beta\frac{L}{2}\right) \frac{e^{-j\beta\cos\theta L/2}}{\beta^2\sin^2\theta}\left(-j\beta\cos\theta\,\cancel{\sin(0)}^{0} + \beta\,\cancel{\cos(0)}^{1}\right)\right]\right] + \int_0^{L/2}\sin\left(\beta\frac{L}{2}-\beta z\right)e^{j\omega z\cos\theta/c}\,dz\right\}$$

$$H\phi = \frac{j[I_0]\sin\theta}{2r\lambda}\left\{\left[\frac{1}{\beta\sin^2\theta}\left(\cancel{j\cos\theta\sin\beta\frac{L}{2}} + \beta\cos\beta\frac{L}{2}\right.\right.\right. \cos\left(\frac{L\beta\cos\theta}{2}\right) - j\sin\left(\frac{L\beta}{2}\cos\theta\right)\Bigg]$$

$$+\left[\frac{1}{\beta\sin^2\theta}\left(\cos\left(\frac{\beta L}{2}\cos\theta\right) + \cancel{j\sin\left(\frac{\beta L}{2}\cos\theta\right)}\right)\right.$$

$$\left[\cancel{-j\cos\theta\sin\left(\frac{\beta L}{2}\right)} - \cos\left(\frac{\beta L}{2}\right)\right]$$

$$H\phi = \frac{j[I_0]}{\cancel{2}r\cancel{\lambda}}\left[\frac{\cancel{2}}{\beta\sin\theta}\left(\cos\left(\frac{\beta L}{2}\cos\theta\right) - \cos\left(\frac{\beta L}{2}\right)\right)\right]$$

$\dfrac{2\pi}{\cancel{\lambda}}$

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$$\boxed{H_\phi = \frac{j[I_0]}{2\pi r}\left[\frac{\cos\left(\frac{\beta L}{2}\cos\theta\right) - \cos(\beta L/2)}{\sin\theta}\right]}$$

wkt

$$\frac{E_\theta}{H_\phi} = 120\pi$$

$$E_\theta = 120\pi H_\phi$$

$$E_\theta = \cancel{120}^{60}\cancel{\pi}\,\frac{j[I_0]}{\cancel{2\pi}r}\left[\frac{\cos(\beta L/2\cos\theta) - \cos(\beta L/2)}{\sin\theta}\right]$$

$$\boxed{E_\theta = \frac{60j[I_0]}{r}\left[\frac{\cos(\beta L/2\cos\theta) - \cos(\beta L/2)}{\sin\theta}\right]}$$

CASE 1 : $L = \lambda/2$

$$E_\theta = \frac{60j[I_0]}{r}\left[\frac{\cos[(2\pi/\lambda)(\lambda/4)\cos\theta] - \cos[(2\pi/\lambda)(\lambda/4)]}{\sin\theta}\right]$$

$$E_\theta = \frac{60j[I_0]}{r}\left[\frac{\cos(\pi/2\cos\theta) - \cos(\pi/2)}{\sin\theta}\right]$$

$$\boxed{E_\theta = \frac{60j[I_0]}{r}\,\frac{\cos(\pi/2\cos\theta)}{\sin\theta}}$$

$$\boxed{H_\phi = \frac{j[I_0]}{2\pi r}\,\frac{\cos(\pi/2\cos\theta)}{\sin\theta}}$$

CASE 2 : $L = \lambda$

$$E_\theta = \frac{60j[I_0]}{r}\left[\frac{\cos[(2\pi/\lambda)(\lambda/2)\cos\theta] - \cos[(2\pi/\lambda)(\lambda/2)]}{\sin\theta}\right]$$

$$E_\theta = \frac{60j[I_0]}{r}\left[\frac{\cos(\pi\cos\theta) - \cos(\pi)}{\sin\theta}\right]$$

$$\boxed{E_\theta = \frac{60j[I_0]}{r}\left[\frac{\cos(\pi\cos\theta) + 1}{\sin\theta}\right]}$$

$$\boxed{H_\phi = \frac{j[I_0]}{2\pi r}\left[\frac{\cos(\pi\cos\theta) + 1}{\sin\theta}\right]}$$

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CASE 3: $L = 3\lambda/2$

$$E_\theta = \frac{60j[I_0]}{r}\left[\frac{\cos[(2\pi/\lambda)(3\lambda/4)\cos\theta] - \cos[(2\pi/\lambda)(3\lambda/4)]}{\sin\theta}\right]$$

$$E_\theta = \frac{60j[I_0]}{r}\left[\frac{\cos(3\pi/2\cos\theta) - \cos(3\pi/2)}{\sin\theta}\right]$$

$$\boxed{E_\theta = \frac{60j[I_0]}{r}\frac{\cos(3\pi/2\cos\theta)}{\sin\theta}}$$

$$\boxed{H_\phi = \frac{j[I_0]}{2\pi r}\frac{\cos(3\pi/2\cos\theta)}{\sin\theta}}$$

★ Radiation resistance of $\lambda/2$ dipole:

- The poynting vector of the field is integrated over a large sphere to obtain the total power radiated.

$$S_r = \frac{1}{2}E_\theta H_\phi^*$$

$$S_r = \frac{1}{2}120\pi|H_\phi|^2 = 60\pi|H_\phi|^2$$

By power theorem, wkt

$$P = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} S_r\, ds$$

$$P = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} 60\pi|H_\phi|^2 r^2\sin\theta\, d\theta\, d\phi$$

$$P = \overset{15}{60\cancel{\pi}}\cancel{r^2}\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi}\frac{I_0^2}{\cancel{4\pi^2 r^2}}\frac{\cos^2(\pi/2\cos\theta)}{\sin^2\theta}\cancel{\sin\theta}\, d\theta\, d\phi$$

$$P = 15I_0^2(2\cancel{\pi})\int_{\theta=0}^{\pi}\frac{\cos^2(\pi/2\cos\theta)}{\cancel{\pi}\sin\theta}\, d\theta$$

$$P = 30I_0^2\int_{\theta=0}^{\pi}\frac{\cos^2(\pi/2\cos\theta)}{\sin\theta}\, d\theta$$

- The total power radiated that is obtained is equated to I^2R where I is the rms current on the dipole and R is called as the radiation resistance of the dipole

$$P = 30 \cancel{I_0^2} \int_{\theta=0}^{\pi} \frac{\cos^2(\pi/2\cos\theta)}{\sin\theta} d\theta = \frac{\cancel{I_0^2}}{2} R_r$$

$$\therefore R_r = 60 \int_{\theta=0}^{\pi} \frac{\cos^2(\pi/2\cos\theta)}{\sin\theta} d\theta$$

Let $\cos\theta = u$

$\therefore -\sin\theta\, d\theta = du \Rightarrow d\theta = -du/\sin\theta$

as $\theta \to \pi \quad u \to -1$

$\theta \to 0 \quad u \to 1$

$$R_r = -60 \int_{1}^{-1} \frac{\cos^2(\pi/2\, u)}{\sin^2\theta} du$$

$$R_r = 60 \int_{-1}^{1} \frac{\cos^2(\pi/2\, u)}{1-u^2} du$$

$$\frac{1}{1-u^2} = \frac{1}{2}\left[\frac{1}{1-u} + \frac{1}{1+u}\right]$$

$$\therefore R_r = 30 \left[\int_{-1}^{1} \frac{\cos^2(\pi/2\, u)}{1-u} du + \int_{-1}^{1} \frac{\cos^2(\pi/2\, u)}{1+u} du\right]$$

Let $1-u = v/\pi$ | Let $1+u = v'/\pi$

$u = 1 - v/\pi$ | $u = v'/\pi - 1$

$du = -dv/\pi$ | $du = dv'/\pi$

as $u \to 1 \quad v \to 0$ | as $u \to 1 \quad v' \to 2\pi$

$u \to -1 \quad v \to 2\pi$ | as $u \to -1 \quad v' \to 0$

$$\therefore R_r = 30 \left[\int_{2\pi}^{0} \frac{-\cos^2(\cancel{\pi}/2\,(\frac{\pi - v}{\cancel{\pi}}))}{v/\cancel{\pi}} \frac{dv}{\cancel{\pi}} + \int_{0}^{2\pi} \frac{\cos^2(\cancel{\pi}/2\,(\frac{v'-\pi}{\cancel{\pi}}))}{v'/\cancel{\pi}} \frac{dv'}{\cancel{\pi}}\right]$$

$$R_r = 30 \left[\int_{0}^{2\pi} \frac{\cos^2(\frac{\pi - v}{2})}{v} dv + \int_{0}^{2\pi} \frac{\cos^2(\frac{v'-\pi}{2})}{v'} dv'\right]$$

Since $\int_a^b x\,dx = \int_a^b y\,dy$

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$$\therefore R_r = 30(2)\int_0^{2\pi} \frac{\cos^2\left(\frac{v-\pi}{2}\right)}{v}\, dv$$

$$R_r = \frac{60}{2}\int_0^{2\pi} \frac{1+\cos(v-\pi)}{v}\, dv$$

$$R_r = 30\left[\int_0^{2\pi} \frac{1}{v}\, dv - \int_0^{2\pi} \frac{\cos v}{v}\, dv\right]$$

$$R_r = 30\left[\cancel{\log v}\Big|_0^{2\pi} - \left(\cancel{\log v} - \frac{v^2}{2\cdot 2!} + \frac{v^4}{4\cdot 4!} - \frac{v^6}{6\cdot 6!} + \cdots\right)\Big|_0^{2\pi}\right]$$

$$R_r = 30\left[\frac{v^2}{2\cdot 2!} - \frac{v^4}{4\cdot 4!} + \frac{v^6}{6\cdot 6!} - \cdots\right]_0^{2\pi}$$

$$R_r = 30(2.41)$$

$$\boxed{R_r = 73\,\Omega}$$

Q: A half wave dipole radiating in free space is driven by a current of 0.5 A at the terminals. Calculate the electric and magnetic fields from the antenna at a distance of 1 km at angles 45°, 90° and 120°.

$I_0 = 0.5\,A$

$r = 1\,km$

Electric field

$$E_\theta = \frac{60 j [I_0]}{r}\left[\frac{\cos(\pi/2 \cos\theta)}{\sin\theta}\right]$$

$$E_\theta = \frac{60(0.5)}{1000}\left[\frac{\cos(\pi/2 \cos\theta)}{\sin\theta}\right]$$

at $\theta = 45^\circ$

$$E_\theta = 0.03\left[\frac{\cos(\pi/2 \cos 45^\circ)}{\sin 45^\circ}\right] = 0.042\ V/m$$

at $\theta = 90^\circ$

$$E_\theta = 0.03\left[\frac{\cos(\pi/2 \cos 90^\circ)}{\sin 90^\circ}\right] = 0.03\ V/m$$

Keerthana Ashok

Date
Page

at $\theta = 120°$

$$E_\theta = 0.03\left[\frac{\cos(\pi/2 \cos 120°)}{\sin 120°}\right] = 0.034 \text{ V/m}$$

Magnetic field

$$H_\phi = \frac{E_\theta}{120\pi}$$

at $\theta = 45°$

$$H_\phi = \frac{0.042}{120\pi} = 0.11\text{m Wb/m}$$

at $\theta = 90°$

$$H_\phi = \frac{0.03}{120\pi} = 0.079\text{m Wb/m}$$

at $\theta = 120°$

$$H_\phi = \frac{0.034}{120\pi} = 0.09\text{m Wb/m}$$

Q: A isotropic radiator has a field strength given by $\frac{10I}{r}$ V/m where I is the terminal current and r is the distance in meters. Find the radiation resistance.

$$E_\theta = \frac{10I}{r} \text{ V/m}$$

$$S_r = \frac{1}{2} E_\theta H_\phi$$

$$S_r = \frac{1}{2} E_\theta \frac{E_\theta}{120\pi}$$

$$S_r = \frac{1}{240\pi} E_\theta^2$$

$$S_r = \frac{1}{240\pi} \frac{100I^2}{r^2}$$

$$S_r = \frac{5I^2}{12\pi r^2}$$

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Power

$$P = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} S_r \, ds$$

$$P = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} \frac{5 I^2}{120\pi \cancel{r^2}} \cancel{r^2} \sin\theta \, d\theta \, d\phi$$

$$P = \frac{5 I^2}{12\pi}\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} \sin\theta \, d\theta \, d\phi$$

$$P = \frac{5 I^2}{\cancel{12\pi}_{6}} [\cancel{2\pi}] \left[-\cos\theta\right]_0^{\pi}$$

$$P = \frac{5 I^2}{6}\left[-\cos\pi + \cos 0\right]$$

$$P = \frac{5 I^2}{6}[2]$$

$$P = \frac{5 I^2}{3} = \frac{I^2}{2} R_r$$

$$\therefore R_r = \frac{10}{3}$$

$$R_r = 3.33\,\Omega$$

Q: A 2m long vertical wire carries a current of 5A at 1MHz. Find the strength of the radiated field at 30km in a direction at right angles to the axis of the wire assuming wire is situated in free space.

$f = 1\,MHz$ $\qquad \lambda = \frac{c}{f} = \frac{3\times 10^8}{1\times 10^6} = 300\,m$

$I = 5A$ $\qquad r = 30km$

$L = 2m$ $\qquad L < \lambda/4$

$\theta = 90^\circ$ $\qquad$ short dipole

Radiated field - far field

$$E_\theta = \frac{[I] L \sin\theta}{r}\frac{60\pi}{\lambda} = \frac{5(2)\sin 90}{3\cancel{0}\times 10^3}\frac{\cancel{60}\pi}{\cancel{300}_{15}}$$

$$E_\theta = 0.21\,mV/m$$

Keerthana Ashok

Date
Page

$$H\phi = \frac{[I]L\sin\theta}{r}\left[\frac{1}{2\lambda}\right] = \frac{5(2)\sin 90^\circ}{30\times10^3}\ \frac{1}{2(300)}$$

$$\underline{H\phi = 0.55\mu\ Wb/m}$$

Q: Find the effective height of an ariel antenna given the field strength of 1.5 mV/m is produced at a distance 50 km by a transmitter operating at 150 kHz with a radial current of 25A.

— $E_\theta = 1.5\,mV/m$

$\lambda = \frac{c}{f} = \frac{3\times10^8}{150\times10^3}$

$r = 50\,km$

$f = 150\,kHz$

$\underline{\lambda = 2\,km}$

$I = 25\,A$

Assuming $L < \lambda/4$ — short dipole

$$E_\theta = \frac{[I]L\sin\theta}{r}\ \frac{60\pi}{\lambda}$$

$$1.5\times10^{-3} = \frac{25\,L\sin 90^\circ}{50\times10^3}\ \frac{60\pi}{2\times10^3}$$

$$\underline{L = 100\,m}$$

Q: A dipole antenna of length 5 cm is operated at a frequency of 100 MHz with $I_0 = 120$ mA at 45° and $r = 1$ m. Find E_r, E_θ and $H\phi$ using quasi stationary phase expression.

— $L = 5\,cm$ $\theta = 45^\circ$

$f = 100\,MHz$ $r = 1\,m$ $[q] = [I]/j\omega$

$I_0 = 120\,m$

$$E_r = \frac{[q]L\cos\theta}{2\pi\varepsilon_0 r^3} = \frac{120/2\pi(100\times10^6)}{2\pi(8.854\times10^{-12})}\ \frac{5\times10^{-2}\cos 45^\circ}{1^3}$$

Keerthana Ashok

Date
Page

Imp also

Q: Demonstrate mathematically the maximum effective aperture and directivity of a short dipole antenna is ~~given that~~: $A_{em} = 0.119\lambda^2$ and $D = 1.5$.

— wkt

$$A_e = \frac{D\lambda^2}{4\pi} \qquad A_{em} = \frac{V^2}{4SR_r}$$

$$A_{em} = \frac{\cancel{E^2}\cancel{L^2}}{4\frac{\cancel{E^2}}{\cancel{120}\pi}\,\frac{\cancel{80}\cancel{L^2}\cancel{\pi^2}}{\lambda^2}} \qquad \text{where } \boxed{V = EL} \qquad R_r = \frac{80L^2\pi^2}{\lambda^2}$$

$$S = E \times H$$

$$S = \frac{E^2}{120\pi}$$

$$A_{em} = \frac{3\lambda^2}{8\pi} = 0.119\lambda^2\ m^2$$

$$A_e = \frac{D\lambda^2}{4\pi}$$

$$0.119\cancel{\lambda^2} = \frac{D\cancel{\lambda^2}}{4\pi} \Rightarrow D = 1.495$$

Q: Demonstrate mathematically the effective aperture and the directivity of a half wave dipole antenna is $A_{em} = 0.13\lambda^2$ and $D = 1.64$.

—

$$A_{em} = \frac{V^2}{4SR_r} \qquad \boxed{V = \frac{E\lambda}{\pi}}$$

$$S = \frac{E^2}{120\pi} \text{ and } R_r = 73$$

$$A_{em} = \frac{\cancel{E^2}\lambda^2}{\frac{\cancel{E^2}}{120\cancel{\pi}}\,\pi^2\, 73} = \frac{120\lambda^2}{73\pi} = 0.13\lambda^2\ m^2$$

$$A_e = \frac{D\lambda^2}{4\pi}$$

$$0.13\cancel{\lambda^2} = \frac{D\cancel{\lambda^2}}{4\pi}$$

$$D = 0.13\,(4\pi) = 1.64$$

Q: For a short dipole $\lambda/15$ long, find the efficiency, radiation resistance if the loss resistance is 1Ω. Also find the directivity and effective aperture.

Given: $L = \lambda/15$

Radiation resistance

$$R_r = \frac{80\pi^2 L^2}{\lambda^2} = \frac{80\pi^2\lambda^2}{\lambda^2 15^2} = \underline{3.51\Omega}$$

Percentage efficiency $= \frac{R_r}{R_r + R_L} \times 100$

$$= \frac{3.51}{3.51+1} \times 100 = \underline{77.82\%}$$

Effective aperture

$$A_{em} = \frac{v^2}{4 S R_r}$$

short dipole

$v = EL$

$$A_{em} = \frac{E^2 L^2}{4\left(\frac{E^2}{120\pi}\right) 3.51}$$

30

$$A_{em} = \frac{\lambda^2 30\pi}{3.51 (15)^2} = 0.119\lambda^2 \ m^2$$

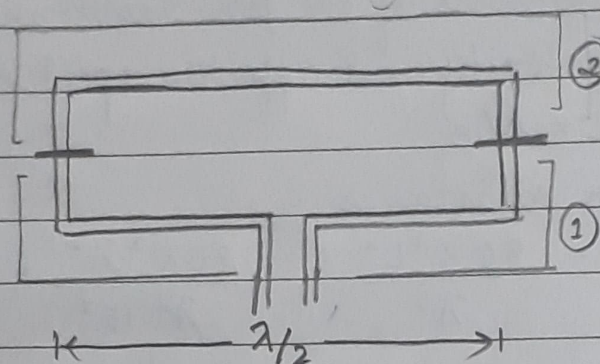
Directivity

$$D = \frac{A_e 4\pi}{\lambda^2}$$

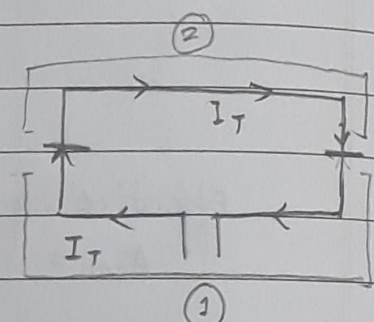
$$D = \frac{0.119\lambda^2 (4\pi)}{\lambda^2}$$

$$\underline{D = 1.49}$$

* Folded Dipole:

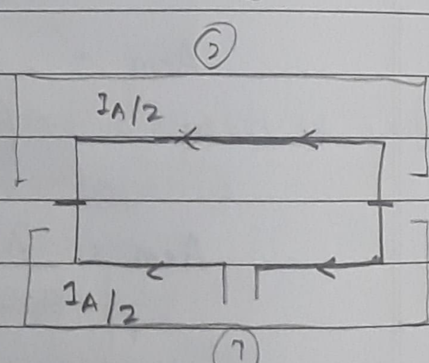


1. Transmission mode



The electromagnetic fields gets cancelled as the currents in the dipoles are opposite in direction. Hence there is no radiation.

2. Antenna mode



The currents in the two dipoles are in the same direction. The current is equally divided. The radiation occurs only in this mode.

Total current

$$I = I_T + I_A/2$$

where $I_T = \frac{V/2}{Z_T} = \frac{V}{2Z_T}$

$$\frac{I_A}{2} = \frac{V}{4Z_D}$$

$$\therefore I = \frac{V}{2Z_T} + \frac{V}{4Z_D} = V\left[\frac{1}{2Z_T} + \frac{1}{4Z_D}\right]$$

$$I = V\left[\frac{2Z_D + Z_T}{4Z_T Z_D}\right]$$

classmate Keerthana Ashok

Date
Page

$$\frac{V}{I} = \frac{4Z_T Z_D}{2Z_D + Z_T}$$

$$Z_{in} = \frac{4Z_T Z_D}{2Z_D + Z_T}$$

$$Z_{in} = \frac{\cancel{Z_T}}{\cancel{Z_T}}\left[\frac{4Z_D}{2Z_D/Z_T + 1}\right]$$

as $Z_T = \infty$

$$\boxed{Z_{in} = 4Z_D}$$

wkt $R_r = 73\,\Omega$

$\therefore R_r' = 4(73)$

$$\boxed{R_r' = 292\,\Omega}$$

$$Z_T = 2j\tan\left(\frac{\beta L}{2}\right)$$

$$Z_T = 2j\tan\left(\frac{2\pi}{\lambda}\,\frac{\lambda/2}{2}\right)$$

$$Z_T = \infty$$

In general the radiation resistance of a folded dipole

$$[R_r' = n^2 R_r]$$

where n: number of elements.

UNIT - 04

Loop, Helix, Yagi-uda and Parabolic

* **Yagi-uda antenna:** (parasitic antenna)

$L_a = 0.46\lambda$: active / driven element
$L_r = 0.475\lambda$: reflector
$L_{d_1} = 0.44\lambda$
$L_{d_2} = 0.44\lambda$
$L_{d_3} = 0.43\lambda$
$L_{d_4} = 0.40\lambda$
} directors

$S_L = 0.25\lambda$: b/w reflector and driven element
$S_d = 0.31\lambda$: b/w director and driven element
$d = 0.01\lambda$: diameter
$L = 1.5\lambda$: length of array (Boom)

For 3 elements

$$L_a = \frac{478}{f_{MHz}} \text{ feet}$$

$$L_r = \frac{492}{f_{MHz}} \text{ feet}$$

$$L_d = \frac{461.5}{f_{MHz}} \text{ feet}$$

$$S = \frac{142}{f_{MHz}} \text{ feet}$$

* **Helical antenna:**

$C = \pi D$: circumference — D: diameter

$\alpha = \tan^{-1}\left(\frac{S}{C}\right) = \tan^{-1}\left(\frac{S}{\pi D}\right)$: pitch angle

S: spacing b/w turns
N: number of turns

$L_a = NS$: Axial length

$L = \sqrt{S^2 + C^2}$: Length of one turn

d: diameter of helix

Normal mode: (Resonant stub Helix Antenna)

The field is maximum in a plane normal to the helix and minimum along its axis.

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$D << \lambda$, $L << \lambda$, $S << \lambda$

Helix to loop : $\alpha = 0$ because $S = 0$.

The far field radiated by a short dipole of length S and constant I_0 is E_θ and is given by:

$$E_\theta = j\,\frac{60\pi [I]\, S \sin\theta}{r\lambda}$$

The far field radiated by a small loop of diameter D/2 is E_ϕ and is given by

$$E_\phi = \frac{120\pi^2 [I] \sin\theta}{r}\left(\frac{A}{\lambda^2}\right) \qquad A = \pi r^2 = \pi\left(\frac{D}{2}\right)^2$$

Axial Ratio

$$AR = \frac{|E_\theta|}{|E_\phi|} = \frac{\dfrac{60\pi [I]\, S \sin\theta}{r\lambda}}{\dfrac{120\pi^2 [I] \sin\theta\, A}{r\lambda^2}}$$

$$AR = \frac{S\lambda}{2\pi A}$$

$$AR = \frac{S\lambda \cancel{4}^{2}}{\cancel{2}\pi^2 D^2}$$

$$AR = \frac{2S\lambda}{\pi^2 D^2} = \frac{2S\lambda}{C^2}$$

CASE 1 : $AR = 0$ when $S = 0$ and $E_\theta = 0$

Linearly polarized wave of horizontal polarization

Helix is a loop.

CASE 2 : $AR = \infty$ when $D = 0$ and $E_\phi = 0$

Linearly polarized wave of vertical polarization

Helix is a vertical dipole

CASE 3 : $AR = 1$ when E_θ and E_ϕ exists, $|E_\theta| = |E_\phi|$

$$AR = 1 = \frac{2S\lambda}{C^2} \qquad C^2 = 2S\lambda \qquad C = \sqrt{2S\lambda}$$

circular polarization

$$\alpha = \tan^{-1}\left(\frac{S}{C}\right) = \tan^{-1}\left(\frac{S}{\pi D}\right)$$

$$\alpha = \tan^{-1}\left(\frac{S}{C}\right) = \tan^{-1}\left(\frac{C^2}{2\lambda C}\right) = \tan^{-1}\left(\frac{\pi D}{2\lambda}\right)$$

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Radiation resistance

$$R_r = 640\left(\frac{L_a}{\lambda}\right)^2$$

Total length of wire

$$L_n = NL = N\sqrt{s^2+c^2}$$

Total length of antenna

$$L_a = NS$$

Axial mode:

Only one major lobe and its maximum radiation intensity is along the axis of the helix.

$$3/4 < C/\lambda < 4/3$$

$$S \simeq \lambda_4 \qquad 12^\circ < \alpha < 14^\circ$$

Radiation resistance $= 140\, C_\lambda$

$$HPBW = \frac{52^\circ}{C_\lambda\sqrt{N S_\lambda}}$$

$$BWFN = \frac{115^\circ}{C_\lambda\sqrt{N S_\lambda}}$$

Directivity $= 12 N C_\lambda^2 S_\lambda$

Axial Ratio $AR = \frac{2N+1}{2N}$

* Loop Antenna

$$E_\phi = \frac{120\pi^2 [I] \sin\theta}{r}\left[\frac{A}{\lambda^2}\right]$$: Far field : small loop

Far field : general case

$$E_\phi = \frac{60\pi\beta [I] a}{r} J_1(\beta a \sin\theta)$$

$$H_\theta = \frac{[I]\beta a}{r} J_1(\beta a \sin\theta)$$

$$R_r = 20\pi^2(\beta a)^4$$

small loop: $R_r = 31.2\left(\frac{A}{\lambda^2}\right)^2 k\Omega$ $\qquad D = 1.5$

Large loop: $R_r = 3720\left(\frac{a}{\lambda}\right)$ $\qquad D = 4.25\left(\frac{a}{\lambda}\right)$

Keerthana Ashok

Date ______
Page ______

UNIT - 04

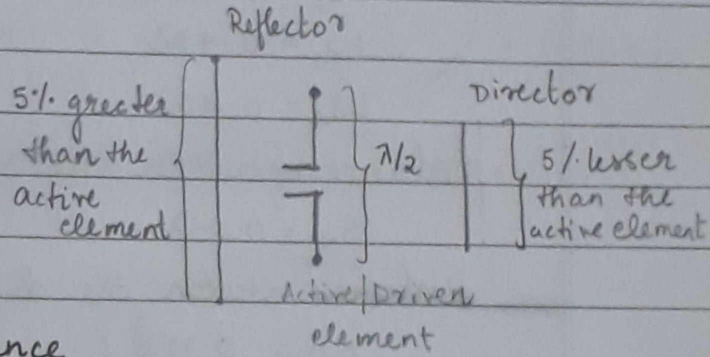
Loop, Helix, Yagi-uda and Parabolic

* **Yagi-uda Antenna:**

It is also called as Parasitic Antenna.

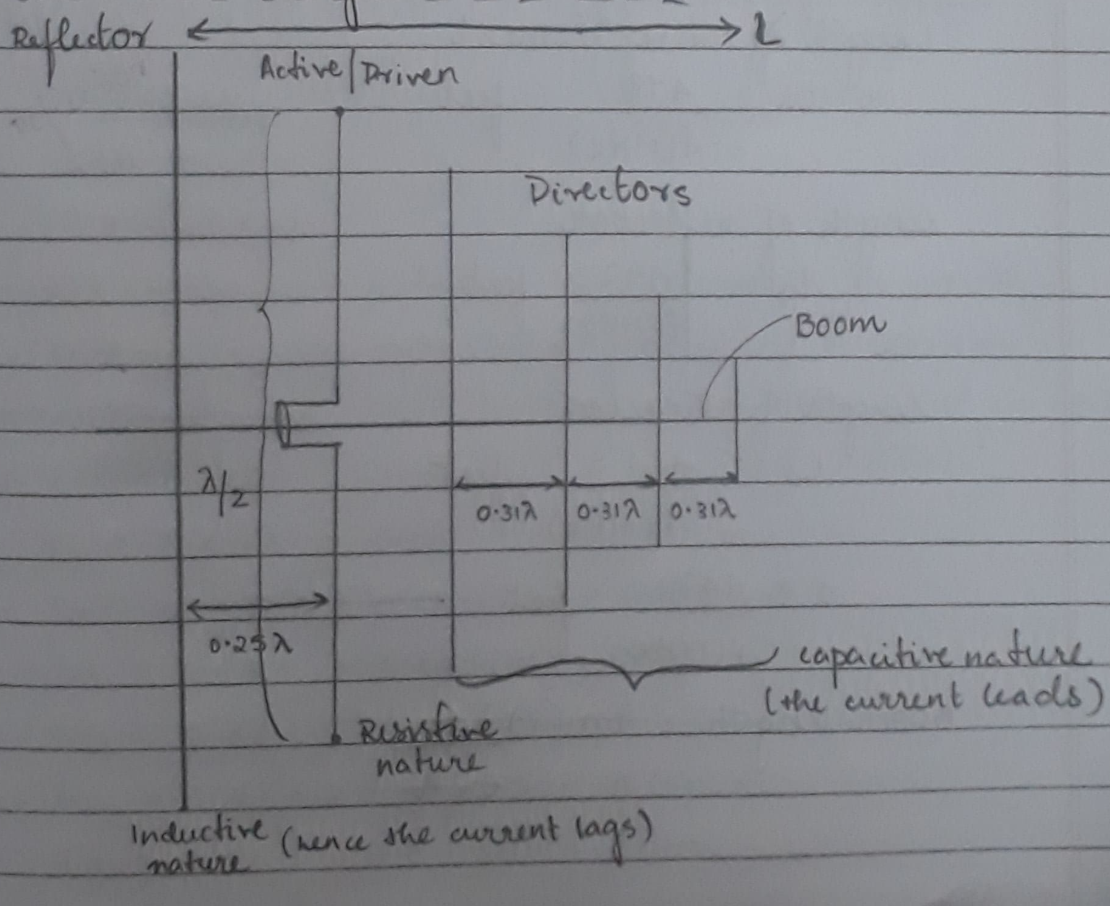
It consists of three elements: one active element and two parasitic elements.

The field or the voltage is applied to the active element where as the parasitic elements are energised by the active element.



It is used for long distance communication i.e., for larger gain or directivity.

Six element Yagi-Uda Antenna



- Design Equations (for 6 elements)

Length of active element

$L_a = 0.46\lambda$

Length of reflector

$L_r = 0.475\lambda$

Length of the directors

$L_{d_1} = 0.44\lambda$

$L_{d_2} = 0.44\lambda$

$L_{d_3} = 0.43\lambda$

$L_{d_4} = 0.40\lambda$

Distance between the reflector and active element

$S_R = 0.25\lambda$

Distance between the directors

$S_d = 0.31\lambda$

Diameter $d = 0.01\lambda$ Gain = 12dB

$L = 1.5\lambda$

- Design equations (for 3 elements)

Length of active element

$L_a = \frac{478}{f(MHz)}$ feet

Length of reflector

$L_r = \frac{492}{f(MHz)}$ feet

Length of director

$L_d = \frac{461.5}{f(MHz)}$ feet

$S = \frac{142}{f(MHz)}$ feet

general:

$L_a = \lambda/2$

$L_r = L_a + 5\% L_a$

$L_{d_1} = L_a - 5\% L_a$

$L_{d_2} = L_{d_1} - 5\% L_{d_1}$

Boom length $= (n-1)d$

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Q: Design a 3 element yagi-uda antenna to operate at a frequency of 175 MHz.

— Given a 3 element yagi uda antenna

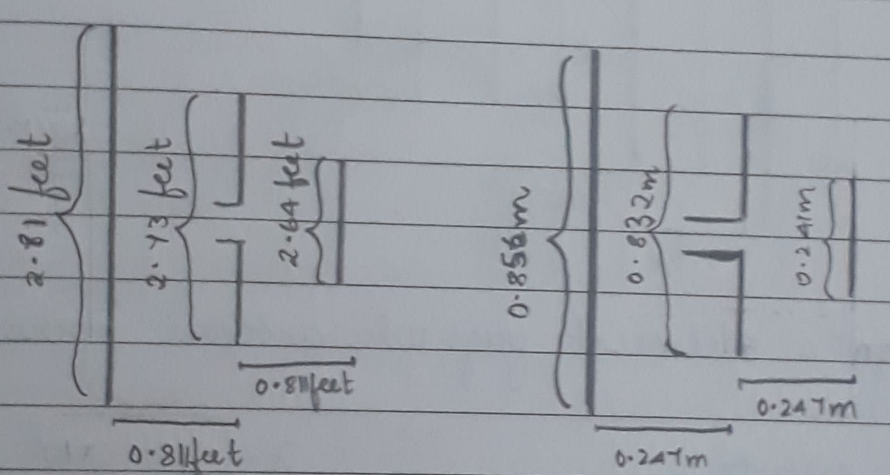
$f = 175\,MHz$

$L_a = \dfrac{478}{f(MHz)}$ feet $= \dfrac{478}{175} = 2.73$ feet $= 0.832\,m$

$L_r = \dfrac{492}{f(MHz)}$ feet $= \dfrac{492}{175} = 2.81$ feet $= 0.856\,m$

$L_d = \dfrac{461.5}{f(MHz)}$ feet $= \dfrac{461.5}{175} = 2.64$ feet $= 0.804\,m$

$S = \dfrac{142}{f(MHz)}$ feet $= \dfrac{142}{175} = 0.811$ feet $= 0.247\,m$



Q: Design a yagi uda antenna of 6 elements to provide a gain of 12dB if the operating frequency is 200 MHz.

— $f = 200\,MHz$ $\qquad \lambda = \dfrac{c}{f} = \dfrac{3\times10^8}{200\times10^6} = 1.5\,m$

$G = 12\,dB$

$L_a = 0.46\lambda = 0.46(1.5) = 0.69\,m$

$L_r = 0.475\lambda = 0.475(1.5) = 0.7125\,m$

$L_{d_1} = 0.44\lambda = 0.44(1.5) = 0.66\,m$

$L_{d_2} = 0.44\lambda = 0.44(1.5) = 0.66\,m$

$L_{d_3} = 0.43\lambda = 0.43(1.5) = 0.645\,m$

$L_{d_4} = 0.40\lambda = 0.40(1.5) = 0.6\,m$

Keerthana Ashok

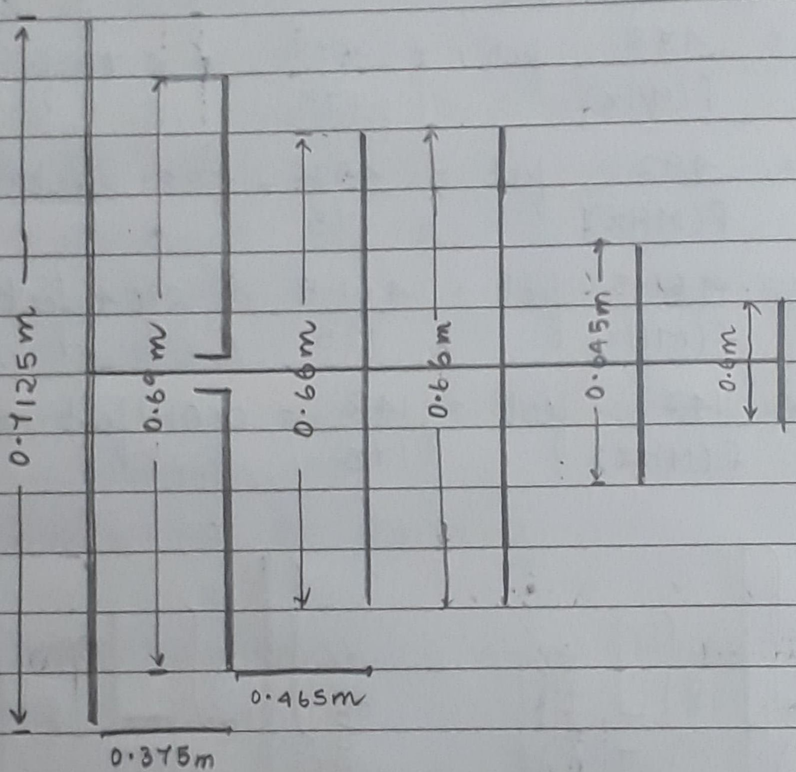
Date

Page

$S_r = 0.25\lambda = 0.25(1.5) = 0.375\ m$

$S_d = 0.31\lambda = 0.31(1.5) = 0.465\ m$

$\cancel{d = 0.01\lambda = 0.01(1.5) = 0.015\ m}$



Q: Design a 5 element yagi uda antenna operating at 4.2 GHz.

$f = 4.2\ GHz$

$\lambda = \frac{c}{f} = \frac{3\times10^8}{4.2\times10^9} = 0.07\ m$

$L_a = \lambda/2 = 0.035\ m$

$L_r = L_a + 5\%\ L_a$

$= 0.035 + 0.05(0.35) = 0.03675\ m$

$L_{d_1} = L_a - 5\%\ L_a$

$= 0.035 - 0.05(0.35) = 0.03325\ m$

$L_{d_2} = L_{d_1} - 5\%\ L_{d_1}$

$= 0.03325 - 0.05(0.03325) = 0.03158\ m$

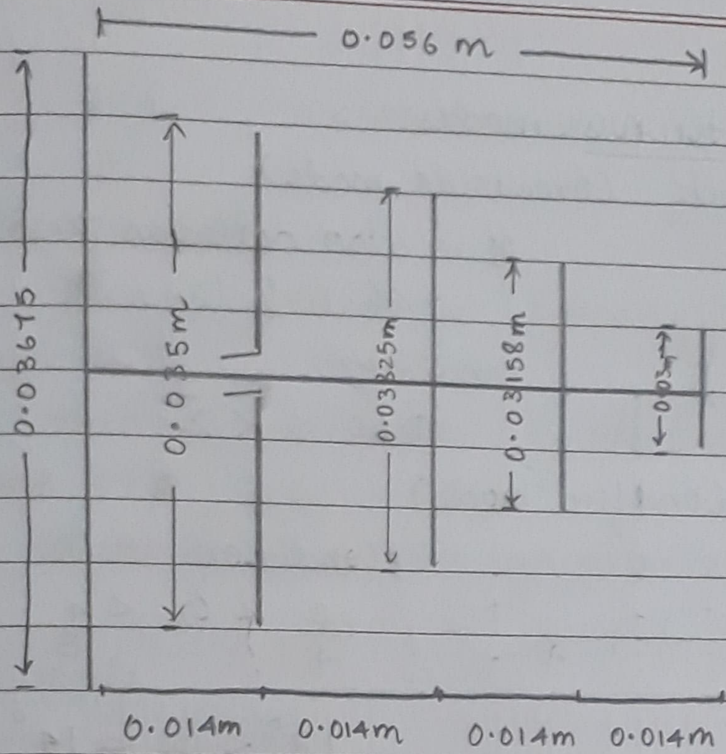
$L_{d_3} = L_{d_2} - 5\%\ L_{d_2}$

$= 0.03158 - 0.05(0.03158) = 0.03000\ m$

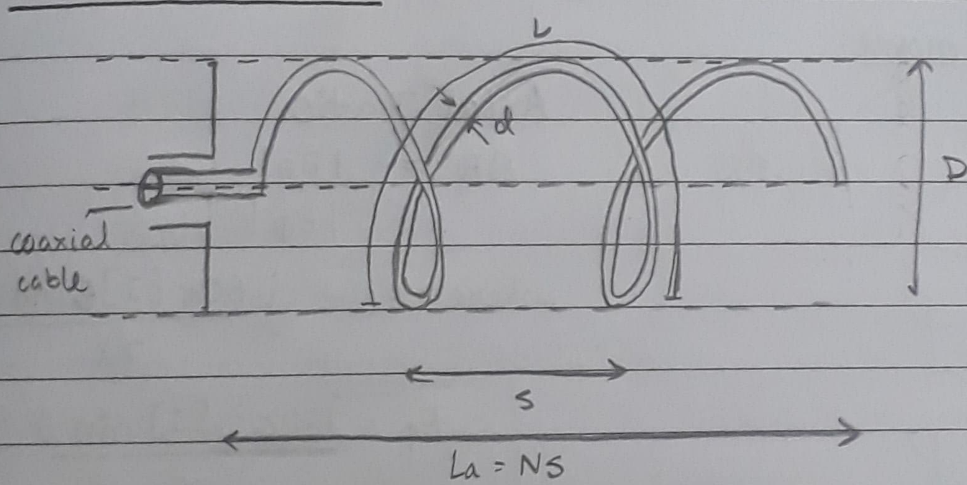
$d = 0.2\lambda = 0.2(0.07) = 0.014\ m$

Boom length $= (n-1)d = (5-1)(0.014) = 0.056\ m$

Date
Page

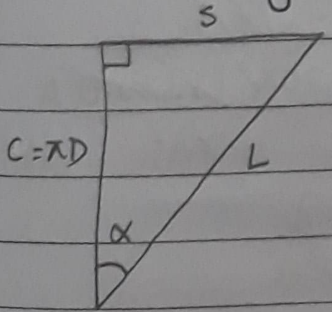


* Helical Antenna:



$D_{min} = \frac{3\lambda}{4}$

Helical triangle



D: Diameter of helix
C: circumference
S: distance between two turns.

$$L^2 = S^2 + C^2$$

$$L = \sqrt{S^2 + C^2}$$

Pitch angle

$$\boxed{\alpha = \tan^{-1}\left(\frac{S}{C}\right)}$$

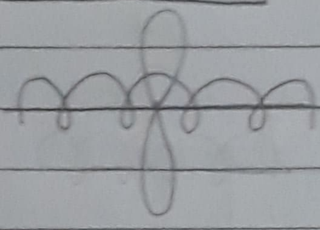
when $S = 0$ $\alpha = 0°$: Helix loop

when $D = 0$ $\alpha = 90°$: Helix linear conductor

Date
Page

It operates in two modes.

— **Normal mode** (Broadside mode)



It is also called as Resonant stab helix

condition

$D, C, S < \lambda$

— **Axial mode** (Endfire mode)

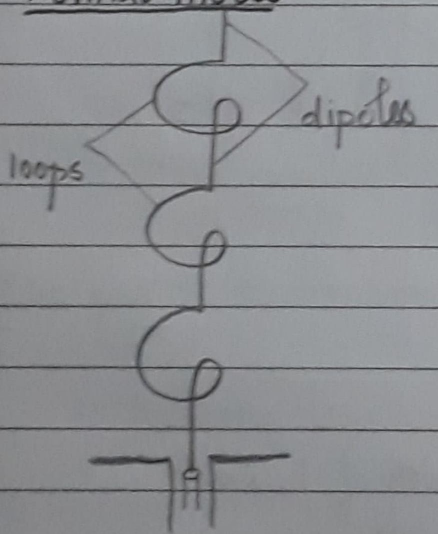
Condition

$\frac{3}{4} < C_\lambda < \frac{4}{3}$

$12^\circ < \alpha < 14^\circ$

$N > 3$

Normal mode:



Axial Ratio

$$AR = \frac{|E_\theta|}{|E_\phi|}$$

where $E_\theta = j\,\frac{60\pi [I] S \sin\theta}{r\lambda}$

$$E_\phi = \frac{120\pi^2 [I] \sin\theta}{r} \frac{A}{\lambda^2}$$

$$\therefore AR = \frac{\frac{60\pi [I] S \sin\theta}{r\lambda}}{\frac{2\,120\pi^2 [I] \sin\theta\, A}{r\lambda^2}}$$

$$AR = \frac{S\lambda}{2\pi A}$$

Area: $A = \pi D^2/4$

$$AR = \frac{S\lambda\, 4}{2\pi(\pi D^2)}$$

$$\boxed{AR = \frac{2S\lambda}{\pi^2 D^2} = \frac{2S\lambda}{C^2}}$$

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$AR = \frac{2S\lambda}{C^2}$ $\quad 0 < AR < \infty$

Spacing: $S = 0$

then $AR = 0$: loop. (Linear Horizontal Polarization)

Diameter, $D = 0 \Rightarrow C = 0$

then $AR = \infty$: Helix, dipole (Linear vertical polarization)

~~Where~~ $S = D$; $|E_\theta| = |E_\phi|$

~~where~~ $AR = 1$: (Circular polarization)

Axial mode:

Design equations.

$$AR = \frac{2N+1}{2N}$$

$$HPBW = \frac{52^\circ}{C_\lambda \sqrt{N S_\lambda}}$$

$$BWFN = \frac{115^\circ}{C_\lambda \sqrt{N S_\lambda}}$$

$$\text{Directivity} = 12 N C_\lambda^2 S_\lambda$$

$$\text{Radiation resistance} = 140 C_\lambda \, \Omega$$

$$\alpha = \tan^{-1}\left(\frac{S}{C}\right)$$

L_a : actual length

L : length of single turn

L_n : length of the wire

$$L_a = NS$$

$$L = \sqrt{S^2 + C^2}$$

$$L_n = NL = N\sqrt{S^2+C^2}$$

$$C = \pi D$$

For circular polarization.

$$AR = \frac{2S\lambda}{C^2} = 1$$

$$\boxed{C = \sqrt{2S\lambda}}$$

$$\alpha = \tan^{-1}\left(\frac{S}{C}\right)$$

$$\alpha = \tan^{-1}\left(\frac{C^2}{2\lambda C}\right)$$

$$\boxed{\alpha = \tan^{-1}\left(\frac{C}{2\lambda}\right) = \tan^{-1}\left(\frac{\pi D}{2\lambda}\right)}$$

Q: A helical antenna has 10 turns, 100mm diameter and 70mm turn spacing. The frequency is 1GHz. Calculate Half power beam width and directivity.

$N = 10$

$D = 100\text{mm}$

$S = 70\text{mm}$

$f = 1\text{MHz}$

$C = \pi D = 100\pi\text{mm}$

$S_\lambda = \frac{S}{\lambda} = \frac{70\text{m}}{0.3} = 0.23\text{m}$

$\lambda = \frac{c}{f} = \frac{3\times10^8}{10^9} = 0.3\text{m}$

$C_\lambda = \frac{C}{\lambda} = \frac{100\pi\text{m}}{0.3} = 1.05\text{m}$

Half power beam width

$$HPBW = \frac{52^\circ}{C_\lambda\sqrt{NS_\lambda}}$$

$$= \frac{52^\circ}{1.05\sqrt{10(0.23)}} = 32.65^\circ$$

Directivity

$$D = 12NC_\lambda^2S_\lambda$$

$$D = 12(10)(1.05)^2(0.23)$$

$$D = 30.43$$

$$D_{dB} = 10\log D$$

$$D_{dB} = 10\log(30.43)$$

$$D_{dB} = 14.83$$

Q: A mono filar axial mode helical antenna has 30 turns, $\lambda/3$ diameter, $\lambda/5$ turn spacing. Find HPBW, directivity and radiation resistance and BWFN.

→ Given: $d = \lambda/3$ $\qquad C = \pi d = \pi\lambda/3$

$s = \lambda/5$ $\qquad C_\lambda = \frac{C}{\lambda} = \frac{\pi}{3}$; $S_\lambda = \frac{s}{\lambda} = \frac{1}{5}$

$N = 30$

Half power beam width

$$HPBW = \frac{52^\circ}{C_\lambda\sqrt{N S_\lambda}} = \frac{52^\circ}{\pi/3\sqrt{30\times0.2}} = 20.27^\circ$$

Directivity

$$D = 12 N C_\lambda^2 S_\lambda = 12(30)(\pi/3)^2(1/5) = 78.95$$

$$D_{dB} = 10\log(78.95) = 18.97\,dB$$

Beam width between first nulls

$$BWFN = \frac{115^\circ}{C_\lambda\sqrt{N S_\lambda}} = \frac{115^\circ}{\pi/3\sqrt{30\times0.2}} = 44.85^\circ$$

Radiation resistance

$$R_r = 140\,C_\lambda = 140(\pi/3) = 146.6\Omega$$

Q: An helix antenna in axial mode with 10 turns operates at $f = 8GHz$ with $c = 0.92\lambda$ and pitch angle $= 13^\circ$. Find HPBW, axial ratio, radiation resistance and directivity.

→ Given: $c = 0.92\lambda$ $\qquad f = 8GHz$

$\alpha = 13^\circ$

$$\alpha = \tan^{-1}\left(\frac{s}{c}\right) \Rightarrow s = c\tan\alpha$$

$$s = 0.92\lambda(\tan 13^\circ) = 0.212\lambda$$

$$S_\lambda = 0.212m$$

$$HPBW = \frac{52^\circ}{C_\lambda\sqrt{N S_\lambda}} = \frac{52^\circ}{0.92\sqrt{10\times0.212}} = 38.82^\circ$$

$$AR = \frac{2N+1}{2N} = \frac{21}{20} = 1.05$$

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$R_r = 140 C_\lambda = 140 \times 0.92 = 128.8\,\Omega$

$D = 12 N C_\lambda^2 S_\lambda = 12(10)(0.92)^2(0.212)$

$= 21.532$

$D_{dB} = 10\log(21.532) = 13.33\,dB$

Q: An helix antenna operating in axial mode with 20 turns operating at a frequency of 1 GHz, diameter 100mm and pitch angle is 13°. Calculate HPBW, AR, directivity, turning spacing axial length, length of the wire and radiation resistance and BWFN.

Given: $N = 20$ $f = 1GHz$

$d = 100mm$ $\alpha = 13°$

$\lambda = \frac{c}{f} = \frac{3\times10^8}{1\times10^9} = 0.3m$

$c = \pi d = \pi(0.1) = 0.314\,m$

$\alpha = \tan^{-1}(S/C)$

$S = C\tan\alpha = 0.314\tan 13$

$S = 0.0725\,m$

$C_\lambda = \frac{0.314}{0.3} = 1.047$

$S_\lambda = \frac{0.0725}{0.3} = 0.2417$

$HPBW = \frac{52°}{C_\lambda\sqrt{N S_\lambda}}$

$= \frac{52°}{1.047\sqrt{20(0.2417)}}$

$= 22.59°$

$BWFN = \frac{115°}{C_\lambda\sqrt{N S_\lambda}}$

$= \frac{115°}{1.047\sqrt{20(0.2417)}}$

$= 50°$

$AR = \frac{2N+1}{2N} = \frac{2(20)+1}{2(20)} = 1.025$

$D = 12 N C_\lambda^2 S_\lambda = 12(20)(1.047)^2(0.2417) = 63.58$

$D_{dB} = 10\log(63.58) = 18.03\,dB$

$L_a = SN = 20(0.0725) = 1.45m$

$L_n = N\sqrt{S^2 + C^2} = 20\sqrt{0.0725^2 + 0.314^2} = 6.41m$

$R_r = 140 C_\lambda = 140(1.047) = 146.6\,\Omega$

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Q: A stub helix antenna operating in cellular telephone band at 883 MHz is designed with 4 ~~turn~~ helix and has a axial length of 2.25 inches and 0.2 inch in diameter. Find axial ratio, turn spacing, radiation resistance, length of helix.

- Stub helix : Normal mode

$f = 883 MHz$

$N = 4$

$La = 2.25$ inches $= 0.054m$

$d = 0.2$ inches $= 0.005m$

$c = \pi d = 0.005\pi$ m

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{883\times10^6} = 0.339m$$

$$AR = \frac{2S\lambda}{c^2}$$

$La = NS$

$$S = \frac{La}{N} = \frac{0.54}{4} = 0.01425m$$

$$AR = \frac{2(0.0142)(0.339)}{(0.005\pi)^2} = 39.05$$

$$R_r = 640\left(\frac{La}{\lambda}\right)^2 = 640(0.168)^2 = 18.09\Omega$$

$$L_n = N\sqrt{s^2+c^2} = 0.0848$$

Q: Design 5 turn helical antenna which operates at 100MHz in normal mode. The spacing between the turns is $\lambda/50$. It is designed such that the antenna posses the circular polarization. Determine circumference, length of single turn, overall length of helix and pitch angle.

- Given : $N = 5$

$s = \lambda/50$

$f = 400 MHz$

$$\lambda = \frac{3\times10^8}{400\times10^6} = 0.75m$$

circumference $c = \sqrt{2 s \lambda}$

$$c = \sqrt{2 \frac{\lambda}{50} \lambda} = \sqrt{\frac{2\lambda^2}{50}} = \sqrt{\frac{2(0.75)^2}{50}} = 0.15m$$

length of single turn

$$L = \sqrt{c^2 + s^2} = \sqrt{(0.15)^2 \left(\frac{0.75}{50}\right)^2} = 0.1507m$$

overall length of helix

$$L_n = N\sqrt{c^2 + s^2} = 5(0.1507) = 0.753m$$

Pitch angle:

$$\alpha = \tan^{-1}\left(\frac{c}{2\lambda}\right) = \tan^{-1}\left(\frac{0.15}{2 \times 0.75}\right) = 5.71^\circ$$

Q: Design an end fire circular beam polarized helix having HPBW of 45°, pitch angle of 13° and the circumference in 60cm at a frequency of 500 MHz. Determine the number of turns needed, directivity in dB, axial ratio, radiation resistance and BWFN.

Given: HPBW = 45° $f = 500MHz$

$\alpha = 13^\circ$ $\lambda = \frac{c}{f} = \frac{3 \times 10^8}{500 \times 10^6} = 0.6m$

$c = 60cm = 0.6m$

$$\alpha = \tan^{-1}\left(\frac{s}{c}\right)$$

$$\therefore s = c \tan\alpha = 0.6 \tan(13) = 0.1385 m$$

$$HPBW = \frac{52^\circ}{C_\lambda \sqrt{N S_\lambda}}$$

$$45 = \frac{52}{\frac{0.6}{0.6}\sqrt{N\left(\frac{0.1385}{0.6}\right)}}$$

$$N = 5.78 \approx 6$$

$$D = 12 N C_\lambda^2 S_\lambda = 12(6)(1)^2(0.23) = 16.56$$

$$D_{dB} = 10\log(16.56) = 12.19$$

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$$AR = \frac{2N+1}{2N} = \frac{2(6)+1}{2(6)} = 1.083$$

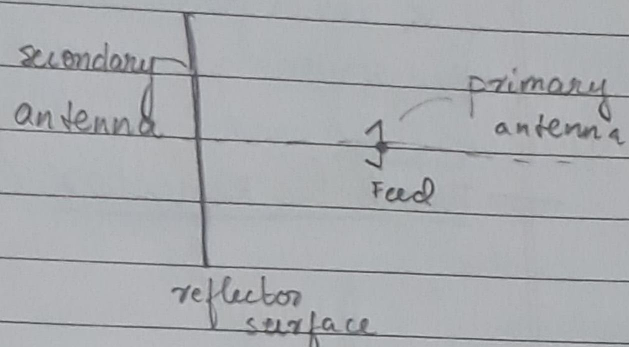
$$R_r = 140\, C_\lambda = 140(1) = 140\,\Omega$$

$$BWFN = \frac{115}{C_\lambda \sqrt{N S_\lambda}} = \frac{115}{1\sqrt{6(0.23)}} = 97.89^\circ$$

* **Reflector Antenna:**

Gain: 20 dB to 70 dB

There are two types of reflector antennas:



a. Active reflector

The field is applied.

i. Rod reflector
ii. Planar reflector
iii. corner reflector
iv. Parabolic reflector

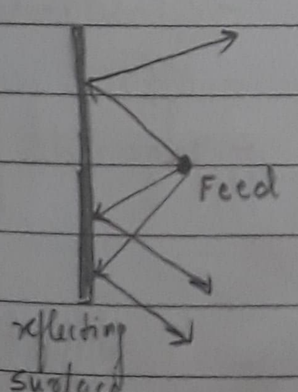
b. Passive reflector

No field is applied or energised.

i. Retro reflector.

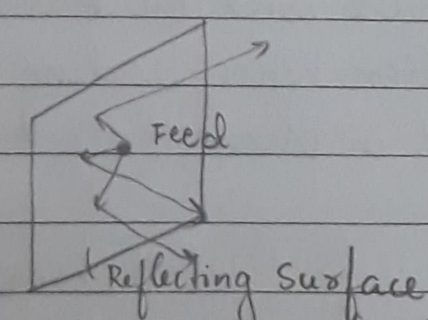
The reflector reflects all the back radiations into the forward direction.

- **Rod Reflector:**



diameter is small thus back lobe reduces the gain

- **Planar Reflector:**



Here there are no back lobes hence there is more gain. The directional property of reflector antenna is decreased.

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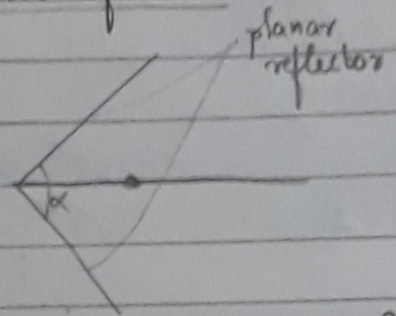
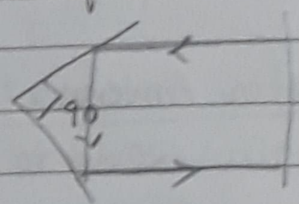
— Corner Reflector:

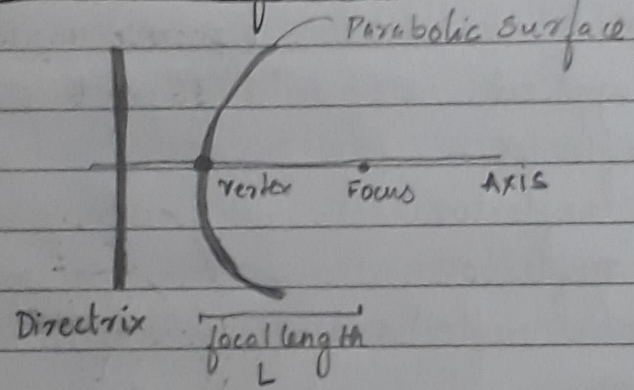


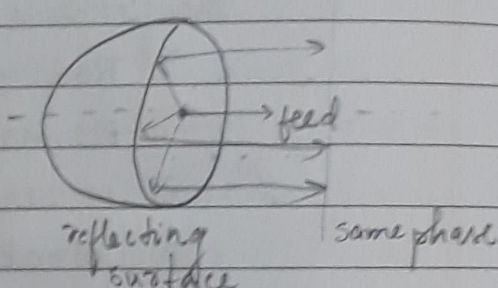
If $\alpha = 90^\circ$ no feed antenna is required. This type of antenna is called as **Retroreflector** (passive reflector)



α varies from 0 to 180°
when $\alpha = 180^\circ$: planar reflector

— **Parabolic Reflector** (Paraboloid / Dish Antenna)





pencil beam

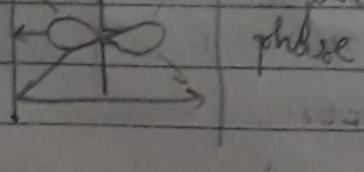
more gain (upto 70dB)
operates above 1GHz

They are of two types:
1. cylindrical Parabolic reflector
2. Paraboloid

Types of feeds used
- dipole (half wave)
- hornfeed
- cassegrain feed
- offset feed
- Yagi uda feed

Half wave dipole feed

The reflected ray gets interfered with the back radiation hence leading to change in phase



Yagi uda feed

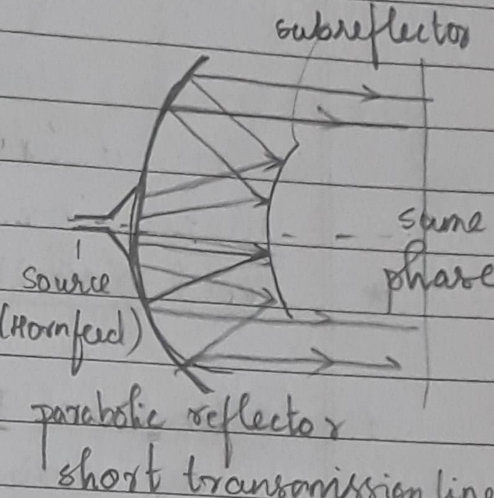
since Yagi uda is large in size we need a large reflector and it increase the cost.

Horn feed

The size is smaller than Yagi uda. Hence this feed can be preferred.

Cassegrain feed

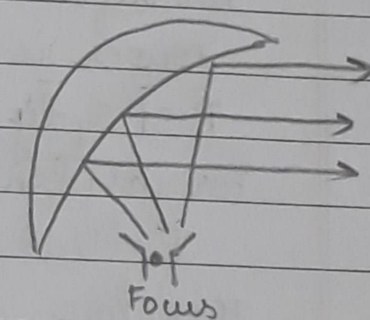
- parabolic reflector (main)
- Horn feed
- Subreflector (hyperboloid)



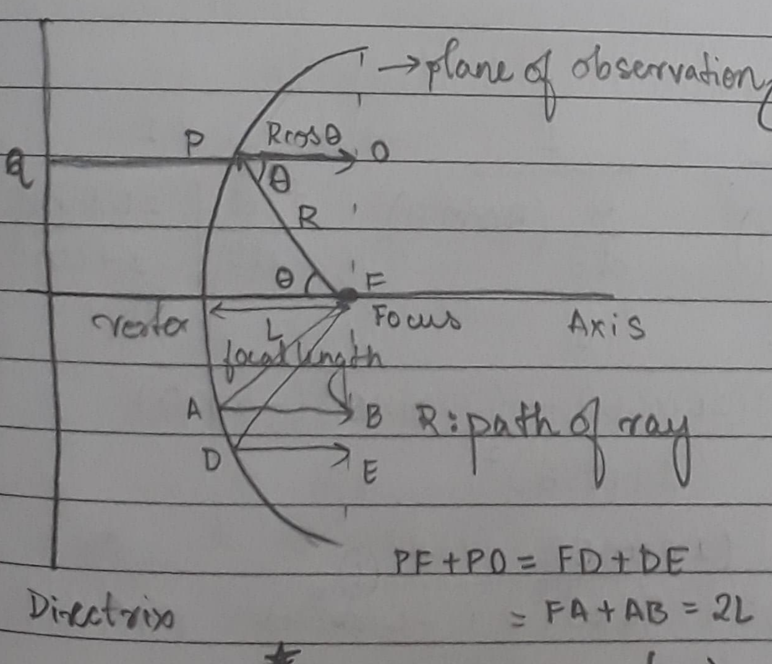
Some of the electromagnetic waves are lost as the rays reflect back to the subreflector from the parabolic reflector.

Offset feed

The reflected and the incident waves do not collide. Horn feed is placed on the focus.



★ Geometry :



PF + PO = FD + DE

= FA + AB = 2L

① ★ PF + PO = 2L (property)

R + R cos θ = 2L

R (1 + cos θ) = 2L

$$\boxed{R = \frac{2L}{1+\cos\theta}}$$

polar coordinate system

$$\boxed{(y-y_0) = 4(x-x_0)}$$

cartesian coordinate system

② ★ PF = PQ (property.)

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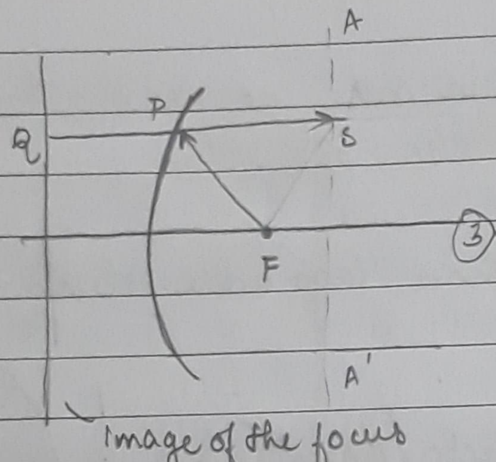


$PS = QS - PQ$

$PS = QS - PF$ $\quad (\because PQ = PF)$

(3) ★ $PS + PF = QS$

★ **Field Distribution / Power Density Ratio:**

1. **Cylindrical Parabola** (Line source half wave dipole)

dy : strip length

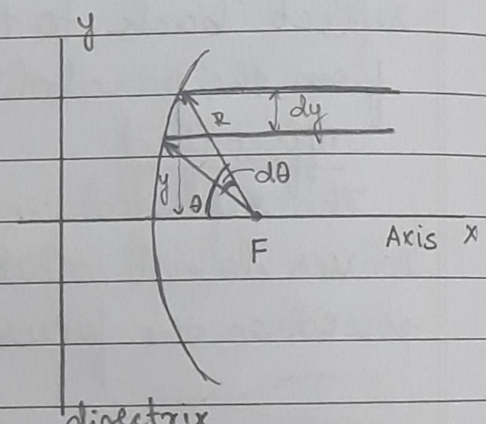
For reflected rays

Power is

$P = S_y\,dy$

For incident ray

$P = U\,d\theta$



Therefore

$$S_y\,dy = U\,d\theta$$

$$\frac{S_y}{U} = \frac{d\theta}{dy}$$

$$\frac{S_y}{U} = \frac{1}{\frac{d}{d\theta}(y)} = \frac{1}{\frac{d}{d\theta}(R\sin\theta)} = \frac{1}{\frac{d}{d\theta}\left[\frac{2L\sin\theta}{1+\cos\theta}\right]}$$

$$\frac{S_y}{U} = \frac{1}{\frac{(1+\cos\theta)(2L\cos\theta) - (2L\sin\theta)(-\sin\theta)}{(1+\cos\theta)^2}}$$

$$\frac{S_y}{U} = \frac{(1+\cos\theta)^2}{2L\cos\theta + 2L\cos^2\theta + 2L\sin^2\theta}$$

$$\frac{S_y}{U} = \frac{(1+\cos\theta)^{\cancel{2}}}{2L\,\cancel{(1+\cos\theta)}}$$

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$$\boxed{S_\psi = \left[\frac{1+\cos\theta}{2L}\right] U}$$

Power density ~~ratio~~ at $\psi = \theta$ and $\psi = 0$ at $\theta = 0$

$$S_0 = \frac{U}{L}$$

$$\frac{S_\theta}{S_0} = \frac{\left[\frac{1+\cos\theta}{2L}\right]\cancel{U}}{\cancel{U}/\cancel{L}}$$

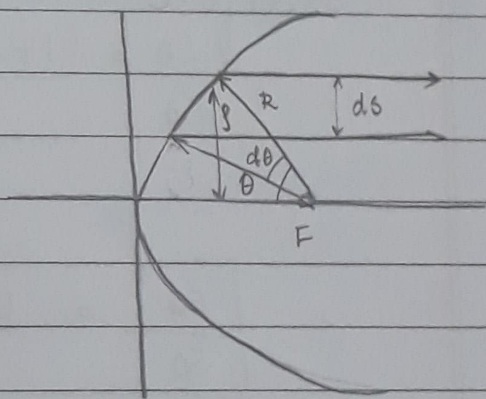
$$\boxed{\frac{S_\theta}{S_0} = \frac{1+\cos\theta}{2}}$$ Power density ratio

Field Distribution

wkt $P = E^2$

$$\boxed{\frac{E_\theta}{E_0} = \sqrt{\frac{1+\cos\theta}{2}}}$$

2. <u>Paraboloid</u>:



For reflected rays

$P = 2\pi \rho \, d\rho \, S_\rho$

For incident rays

$P = 2\pi \sin\theta \, d\theta \, U$

Equating the powers

$$\cancel{2\pi}\rho \, d\rho \, S_\rho = \cancel{2\pi}\sin\theta \, d\theta \, U$$

$$\frac{S_\rho}{U} = \frac{\sin\theta}{\rho}\frac{d\theta}{d\rho}$$

$$\frac{S_\rho}{U} = \frac{\sin\theta}{\rho\left(\frac{d\rho}{d\theta}\right)} = \frac{\cancel{\sin\theta}}{R\cancel{\sin\theta}\left(\frac{d(R\sin\theta)}{d\theta}\right)}$$

$$\frac{S_\rho}{U} = \frac{1}{\frac{2L}{1+\cos\theta}\frac{d}{d\theta}\left[\frac{2L\sin\theta}{1+\cos\theta}\right]}$$

$$\frac{S_\rho}{U} = \frac{1}{\frac{2L}{1+\cos\theta}\left[\frac{(1+\cos\theta)(2L\cos\theta) - (2L\sin\theta)(-\sin\theta)}{(1+\cos\theta)^2}\right]}$$

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Date

Page

$$\frac{S_\theta}{U} = \frac{1}{\frac{2L}{1+\cos\theta}\left[\frac{2L\cos\theta + 2L\cos^2\theta + 2L\sin^2\theta}{(1+\cos\theta)^2}\right]}$$

$$\frac{S_\theta}{U} = \frac{1}{\frac{2L}{1+\cos\theta}\left[\frac{2L\,\cancel{(1+\cos\theta)}}{(1+\cos\theta)^{\cancel{2}}}\right]}$$

$$\frac{S_\theta}{U} = \frac{(1+\cos\theta)^2}{4L^2}$$

$$\boxed{S_\theta = \frac{(1+\cos\theta)^2}{4L^2}U}$$

Power density ~~ratio~~ at $\theta = \theta$ and $\theta = 0$

$$S_0 = \frac{U}{L}$$

therefore

$$\frac{S_\theta}{S_0} = \frac{\frac{(1+\cos\theta)^2\,\cancel{U}}{4L^{\cancel{2}}}}{\frac{\cancel{U}}{\cancel{L}}}$$

$$\boxed{\frac{S_\theta}{S_0} = \frac{(1+\cos\theta)^2}{4}}$$ Power density ratio

Field distribution

wkt $P = E^2$

$$\boxed{\frac{E_\theta}{E_0} = \sqrt{\frac{S_\theta}{S_0}} = \frac{1+\cos\theta}{2}}$$

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* **Design Equations :**

L : length
D : diameter

	CIRCULAR APERTURE	RECTANGULAR APERTURE
HPBW	$\frac{58^\circ}{D_\lambda}$	$\frac{51^\circ}{L_\lambda}$
BWFN	$\frac{140^\circ}{D_\lambda}$	$\frac{115^\circ}{L_\lambda}$
Directivity	$9.87 D_\lambda^2 \simeq 9.9 D_\lambda^2$	$12.56 L_\lambda L_\lambda'$ (Rectangular) $12.56 L_\lambda^2$ (square aperture)
Power gain over $\lambda/2$ dipole	$6(D_\lambda)^2$	$7.7 L_\lambda L_\lambda'$ (rectangular) $7.7 L_\lambda^2$ (square aperture)

Q: Estimate the power gain of a paraboloid reflector of open mouth aperture of 10λ.

– given : $D_\lambda = 10\lambda$

$$\text{Gain} = 6 D_\lambda^2$$

$$= 6\left(\frac{D}{\lambda}\right)^2$$

$$= 6\left(\frac{10\lambda}{\lambda}\right)^2$$

$$= 600 = 27.78\,dB$$

Q: Calculate BWFN and power gain of a 2m paraboloidal reflector operating at 6GHz.

– given : $D = 2m$

$$f = 6GHz = 6\times10^9\,Hz$$

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{6\times10^9} = 0.05\,m$$

$$D_\lambda = \frac{D}{\lambda} = \frac{2}{0.05} = 40$$

$$BWFN = \frac{140^\circ}{D_\lambda} = \frac{140^\circ}{40} = 3.5^\circ$$

Power gain $= 6D_\lambda^2 = 6(40^2) = 9600$
$= 39.82\ dB$

Q: A 64 m diameter dish antenna operating at a frequency of 1.43 GHz is fed by a non directional antenna. Calculate HPBW, BWFN and the power gain with respect to half wave dipole assuming even illumination.

→ Given: $D = 64 m$
$f = 1.43 GHz$

$\lambda = \frac{c}{f} = \frac{3\times10^8}{1.43\times10^9} = 0.21 m$

$D_\lambda = \frac{D}{\lambda} = \frac{64}{0.21} = 304.7$

$HPBW = \frac{58^\circ}{D_\lambda}$

$= \frac{58}{304.7} = 0.19^\circ$

$BWFN = \frac{140^\circ}{D_\lambda} = \frac{140}{304.5} = 0.459^\circ$

Power gain $= 6D_\lambda^2$
$= 6(304.7)^2 = 557052.54$
$= 57.45\ dB$

Q: Specify the diameter of the parabolic reflector if the gain is 70 dB at 15 GHz.

→ given: $G = 70 dB$
$f = 15 GHz$

$\lambda = \frac{c}{f} = \frac{3\times10^8}{15\times10^9} = 0.02 m$

$70 = 10\log(6D_\lambda^2)$
$6D_\lambda^2 = 10^7$
$D_\lambda = \sqrt{\frac{10^7}{6}} = 1290.99$

$D = D_\lambda(\lambda) = 1290.99(0.02)$
$D = 25.82 m$

Keerthana Ashok

Date ______
Page ______

Q: A paraboloid operating at 5GHz has a radiation pattern with a BWFN of 10°. Find the mouth diameter of paraboloid, HPBW, power gain and directivity.

- Given: $f = 5GHz$ $\qquad \lambda = \frac{c}{f} = \frac{3\times10^8}{5\times10^9} = 0.06\,m$

$BWFN = 10^\circ$

$$BWFN = \frac{140^\circ}{D\lambda}$$

$$10^\circ = \frac{140^\circ}{D\lambda} \Rightarrow D\lambda = 14$$

$$\therefore D = D\lambda\,\lambda = 14(0.06) = 0.84\,m$$

$$HPBW = \frac{58^\circ}{D\lambda}$$

$$= \frac{58^\circ}{14} = 4.14^\circ$$

$$\text{Power gain} = 6D\lambda^2 = 6(14)^2 = 1176$$
$$= 30.7\,dB$$

Directivity

$$= 9.9D\lambda^2 = 9.9(14)^2 = 1940.4$$
$$= 32.87\,dB$$

Q: For a paraboloid reflector of diameter 6m the illumination efficiency B is 0.65. The frequency of operation is 10GHz. Find BWFN, HPBW, power gain, directivity and capture area.

- Given: $B = 0.65$ $\qquad \lambda = \frac{c}{f} = \frac{3\times10^8}{10\times10^9} = 0.03\,m$

$f = 10GHz$

$D = 6m$ $\qquad D\lambda = \frac{D}{\lambda} = \frac{6}{0.03} = 200$

$$BWFN = \frac{140^\circ}{D\lambda} = \frac{140^\circ}{200} = 0.7^\circ$$

$$HPBW = \frac{58^\circ}{D\lambda} = \frac{58^\circ}{200} = 0.29^\circ$$

Power gain $= 6D_\lambda^2 = 6(200)^2 = 240000$
$= 53.8dB$

Directivity $= 9.9D_\lambda^2 = 9.9(200)^2 = 396000$
$= 55.97dB$

capture area

$A_c = BA$

A: actual area

$A = \pi r^2 = \frac{\pi D^2}{4}$

$A_c = 0.65(28.27)$
$= 18.37\ m^2$

$A = \frac{\pi(6)^2}{4} = 28.27\ m^2$

For half wave dipole
the illumination efficiency $B = 0.65$

Q: A paraboloid reflector operates at a frequency of 10GHz and power gain of 75dB is provided. Find the capture area, BWFN, HPBW, directivity.

Given: $G = 75dB$
$f = 10GHz$

$\lambda = \frac{c}{f} = \frac{3\times10^8}{10\times10^9} = 0.03m$

$75 = 10\log(6D_\lambda^2)$
$6D_\lambda^2 = 10^{7.5}$
$D_\lambda = \sqrt{\frac{10^{7.5}}{6}} = 2295.7$

$D = D_\lambda(\lambda) = 2295.7(0.03) = 68.87m$

capture area

$A_c = BA$

$A = \frac{\pi D^2}{4} = \frac{\pi(68.87)^2}{4}$

$A_c = 0.65(3725.2)$
$A_c = 2421.38\ m^2$

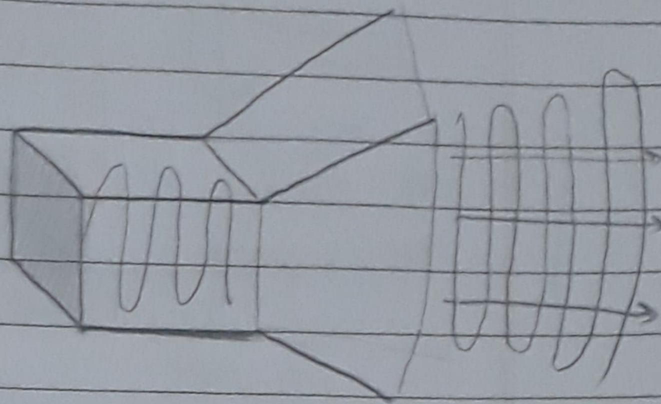
$A = 3725.2\ m^2$

$BWFN = \frac{140^\circ}{D_\lambda} = \frac{140^\circ}{2295.7} = 0.061^\circ$

$HPBW = \frac{58^\circ}{D_\lambda} = \frac{58^\circ}{2295.7} = 0.025^\circ$

Directivity $= 9.9D_\lambda^2 = 9.9(2295.7)^2 = 77.17dB$

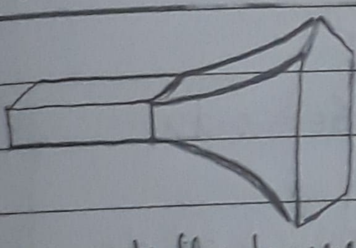
* Horn Antenna:



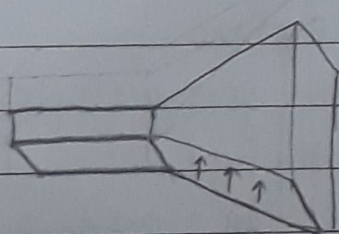
Transition region

$377\Omega = 120\pi\Omega$

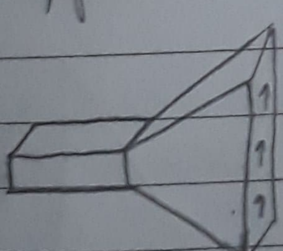
Rectangular Horn	Conical Horn
1. Exponentially tapered pyramidal horn	Exponentially tapered conical / circular horn
2. Sectoral E plane horn	TEM biconical horn
3. Sectoral H-plane horn	TE_{10} biconical horn
4. Pyramidal horn	



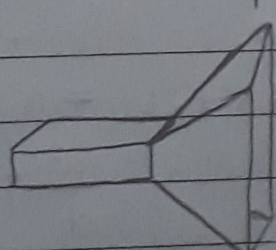
Exponentially tapered pyramidal horn



Sectoral H-plane horn

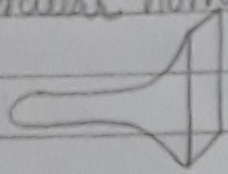


Sectoral E plane horn



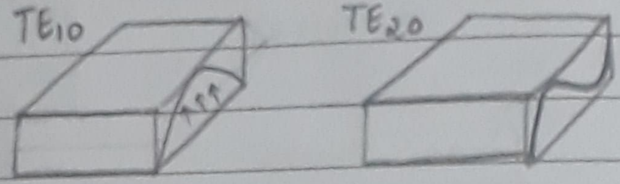
Pyramidal Horn

circular horn



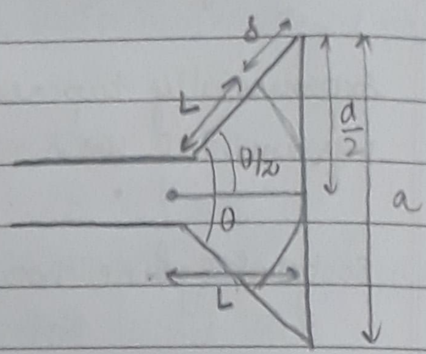
TE_{10} 1: indicates number of half wave along the breadth
0: indicates number of half wave along the length

TE : H wave
TM : E wave



★ **Design Equations:**

δ : path length difference
a : aperture of horn antenna
L : Axial length / horn length
θ : flare angle.



$$\cos\frac{\theta}{2} = \frac{L}{L+\delta}$$

$$\sin\frac{\theta}{2} = \frac{a/2}{L+\delta}$$

$$\tan\frac{\theta}{2} = \frac{a/2}{L}$$

$$\boxed{\tan\frac{\theta}{2} = \frac{a}{2L}}$$

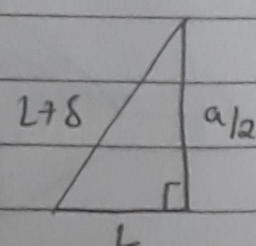
In E plane
$\delta_E = 0.1\lambda$ to 0.25λ : $\delta_E = 0.2\lambda$

In H plane
$\delta_H = 0.1\lambda$ to 0.4λ : $\delta_H = 0.375\lambda$

$$(L+\delta)^2 = (a/2)^2 + L^2$$

$$\cancel{L^2} + \delta^2 + 2L\delta = a^2/4 + \cancel{L^2}$$

$$2L\delta = \frac{a^2}{4} - \cancel{\delta^2} \text{ neglected}$$



$$\boxed{L = \frac{a^2}{8\delta}}$$

Approximate directivity

$$\boxed{D = \frac{41253^0}{\theta^{\circ}_{HP(E)} \times \theta^{\circ}_{HP(H)}} = 4\pi}$$

$\theta_{HP}(E) = \frac{56^\circ}{a_{E\lambda}}$	$\theta_{HP}(H) = \frac{67^\circ}{a_{H\lambda}}$

Directivity through aperture (actual directivity)

$$D = \frac{4\pi A_e}{\lambda^2}$$

$$D = \frac{4\pi \varepsilon_{ap} A_p}{\lambda^2}$$

Effective aperture $= \varepsilon_{ap} = 0.6$

Aperture efficience : $\varepsilon_{ap} = \frac{A_e}{A_p}$

$$D = \frac{4\pi(0.6)A_p}{\lambda^2}$$

$$\boxed{D = \frac{7.5 A_p}{\lambda^2}}$$

$$\boxed{\text{Gain} = 0.6\, D_{actual}}$$

For rectangular antenna

$$A_p = a_E \times a_H$$

For circular antenna

$$A_p = \pi r^2$$

Q. The aperture dimensions of a pyramidal horn are 12x16cm. It is operating at a frequency of 6GHz. Find the HPBW in E plane and H plane, directivity, approximate directivity and power gain.

Given: $a_E = 12cm$ $\quad f = 6GHz$

$a_H = 6cm$ $\quad \lambda = \frac{c}{f} = \frac{3\times10^8}{6\times10^9} = 0.05m$

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Date
Page

HPBW in E plane and H plane

$\theta_{HP}(E) = \frac{56^\circ}{a_{E\lambda}}$ $\qquad a_{E\lambda} = \frac{a_E}{\lambda} = \frac{0.12}{0.05} = 2.4$

$\theta_{HP}(E) = \frac{56^\circ}{2.4} = 23.33^\circ$

$\theta_{HP}(H) = \frac{67^\circ}{a_{H\lambda}}$ $\qquad a_{H\lambda} = \frac{a_H}{\lambda} = \frac{0.06}{0.05} = 1.2$

$\theta_{HP}(H) = \frac{67^\circ}{1.2} = 55.83^\circ$

Actual directivity

$D = \frac{7.5 A_P}{\lambda^2}$ $\qquad A_P = a_E \times a_H$

$= 0.12 \times 0.06$

$= 0.0072\, m^2$

$D = \frac{7.5(0.0072)}{(0.05)^2}$

$D_{act} = 21.6 = 13.34\, dB$

Approximate directivity

$D = \frac{41253}{\theta^\circ_{HP}(E) \times \theta^\circ_{HP}(H)}$

$D_{approx} = \frac{41253}{(23.33)(55.83)} = 31.67$

Power gain $= 0.6\, D_{actual}$

$= 0.6(21.6)$

$= 12.96\, W$

$= 11.12\, dB$

Q: Find the power gain of square horn antenna whose aperture size is 8λ.

Given: $a_E = a_H = 8\lambda$

$D_{act} = \frac{7.5 A_P}{\lambda^2} = \frac{7.5\, a_E a_H}{\lambda^2} = \frac{7.5(8\lambda)^2}{\lambda^2} = 480$

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Q: Design a pyramidal horn antenna. Calculate the horn length, flare angle in both E plane and H plane and aperture in H plane for the given E plane aperture 18λ. Find directivity through aperture, approximate directivity and power gain.

Given: $a_E = 18\lambda$

Horn length

$\delta_E = 0.2\lambda$

$\delta_H = 0.375\lambda$

$$L = \frac{a_E^2}{8\delta_E} = \frac{(18\lambda)^2}{8(0.2\lambda)} = 202.5\lambda$$

$$a_H^2 = 8\delta_H L = 8(0.375\lambda)(202.5\lambda) = 607.5\lambda^2$$

$$a_H = 24.65\lambda$$

Flare angle

$$\tan\frac{\theta_E}{2} = \frac{a_E}{2L} \quad \text{(in E plane)}$$

$$\theta_E = 2\tan^{-1}\left(\frac{18\lambda}{2(202.5\lambda)}\right)$$

$$\theta_E = 5.09^\circ$$

$$\tan\frac{\theta_H}{2} = \frac{a_H}{2L} \quad \text{(in H plane)}$$

$$\theta_H = 2\tan^{-1}\left(\frac{24.65\lambda}{2(202.5\lambda)}\right)$$

$$\theta_H = 6.96^\circ$$

Directivity

$$D_{act} = \frac{7.5 A_P}{\lambda^2} = \frac{7.5(a_E a_H)}{\lambda^2}$$

$$D_{act} = \frac{7.5(18\lambda)(24.65\lambda)}{\lambda^2} = 3327.75 = 35.22\,\text{dB}$$

Approximate directivity

$$D_{approx} = \frac{41253}{\theta^\circ_{HP(E)} \times \theta^\circ_{HP(H)}}$$

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$$\theta_{HP}(E) = \frac{56^\circ}{a_{E\lambda}}$$

$$a_{E\lambda} = \frac{a_E}{\lambda} = 18$$

$$= \frac{56^\circ}{18} = 3.11^\circ$$

$$a_{H\lambda} = \frac{a_H}{\lambda} = 24.65$$

$$\theta_{HP}(H) = \frac{67^\circ}{a_{H\lambda}} = \frac{67^\circ}{24.65} = 2.718^\circ$$

$$D_{approx} = \frac{41253}{(3.11)(2.718)}$$

$$D_{appox} = 4880.29 = 36.88\ dB$$

$$\text{Power gain} = 0.6\ D_{act}$$
$$= 0.6(3327.75)$$
$$= 1996.65\ W = 33.0\ dB$$

Q: Horn antenna is required to have the HPBW of 19° in both the planes, determine the dimensions of the horn antenna, calculate the flare angle θ_E and θ_H. Also calculate directivity through aperture and approximate directivity.

Given: $\theta_{HP}(E) = \theta_{HP}(H) = 19^\circ$

$$\theta_{HP}(E) = \frac{56^\circ}{a_{E\lambda}}$$

$$a_{E\lambda} = \frac{56^\circ}{19} = 2.95$$

$$L = \frac{a_E^2}{8\delta_E} \qquad \delta_E = 0.2\lambda$$

$$a_E = 2.95\lambda$$

$$L = \frac{(2.95\lambda)^2}{8(0.2\lambda)} = 5.4\lambda$$

$$\theta_{HP}(H) = \frac{67^\circ}{a_{H\lambda}}$$

$$a_{H\lambda} = \frac{67}{19} = 3.52$$

$$a_H = 3.52\lambda$$

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Flare angle

$$\tan\frac{\theta_E}{2} = \frac{a_E}{2L}$$

$$\theta_E = 2\tan^{-1}\left(\frac{2.95\lambda}{2(5.4\lambda)}\right)$$

$$\theta_E = 30.55^\circ$$

$$\tan\frac{\theta_H}{2} = \frac{a_H}{2L}$$

$$\theta_H = 2\tan^{-1}\left(\frac{3.52\lambda}{2(5.4\lambda)}\right)$$

$$\theta_H = 36.1^\circ$$

Directivity

$$D_{act} = \frac{7.5A_P}{\lambda^2} = \frac{7.5a_E a_H}{\lambda^2}$$

$$D_{act} = \frac{7.5(2.95\lambda)(3.52\lambda)}{\lambda^2}$$

$$D_{act} = 77.88 = 18.91\,dB$$

Approximate directivity

$$D_{approx} = \frac{41253}{\theta_{HP}(E)\,\theta_{HP}(H)}$$

$$D_{approx} = \frac{41253}{(19)(19)} = 114.27 = 20.58\,dB$$

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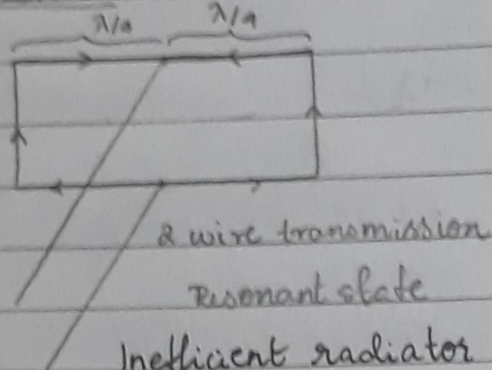
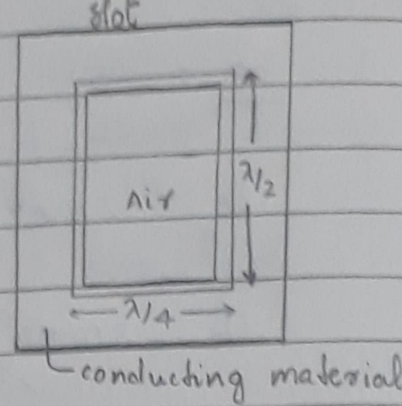
classmate
Date
Page

★ **Slot Antenna:**

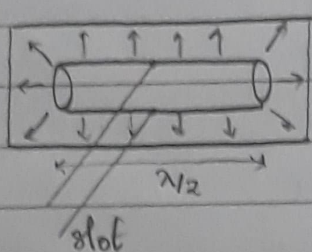
complement of dipole antenna

- Structures





Inefficient radiator. The current is confined only to the edges

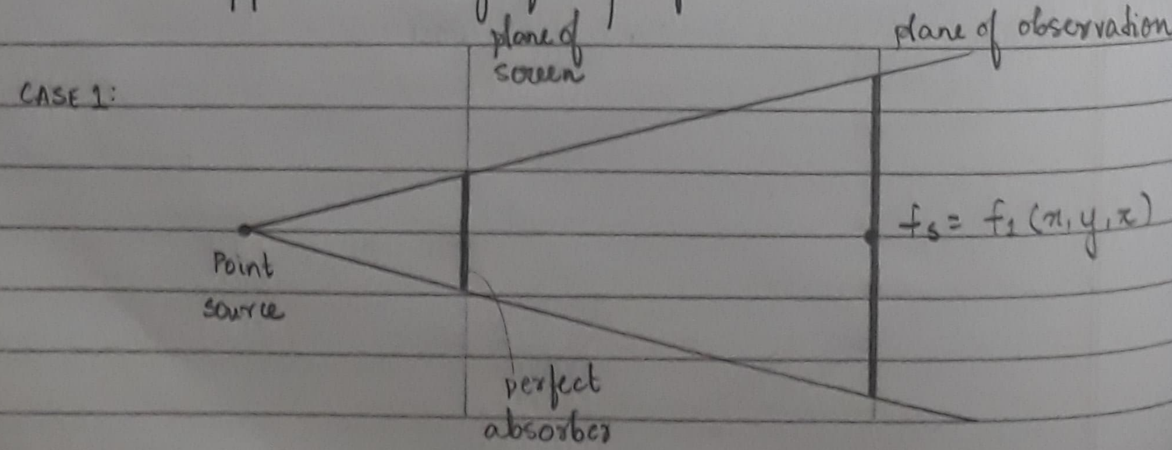


Efficient radiator. The current runs through out the metal conductor

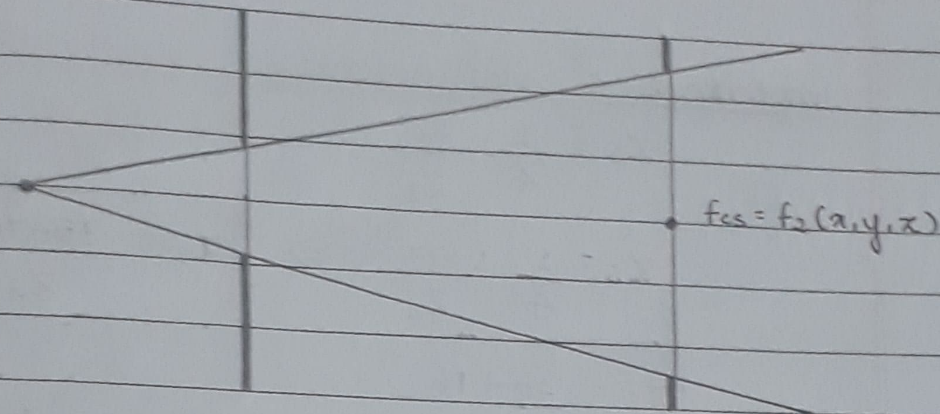
- **Babinet's Principle**

"The field at any point behind a plane having a screen if added to the field at the same point when the complimentary screen is substituted is equal to the field when no screen is present".

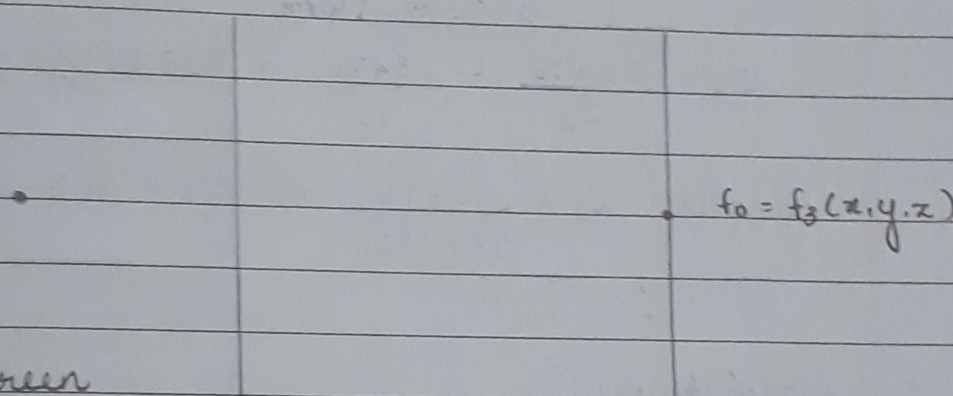
- slot antenna works on Babinet's principle
- It works only in optics
- Polarization will ~~not~~ be considered
- It is applicable only for perfect absorbers.



CASE 2:



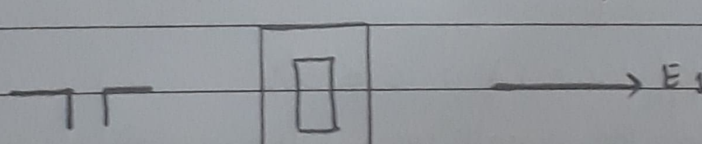
CASE 3:

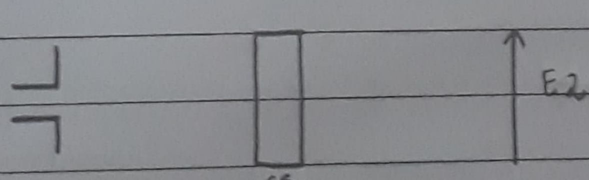


No screen

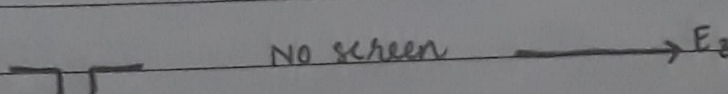
$f_0 = f_s + f_{cs}$

- <u>Booker's principle</u> (Extension of Babinet's principle)
  - EM wave
  - Polarization is considered
  - conducting sheet is thin and infinitismally large.

1. 

2. 

Complementary screen

3. 

$E_3 = E_1 + E_2$

$\frac{E_1}{E_3} + \frac{E_2}{E_3} = 1$

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- Impedance of slot antennas:

$$Z_s = \frac{1}{4}\frac{Z_0^2}{Z_d}$$

$$Z_s = \frac{1}{4}\frac{(377)^2}{Z_d} \Rightarrow \boxed{Z_s = \frac{35476}{Z_d}\ \Omega}$$

$$\boxed{Z_s = \frac{35476}{R_d + jX_d}}$$ complex form

$$Z_s = \frac{35476}{Z_d} = \frac{35476}{73} = 363\ \Omega$$

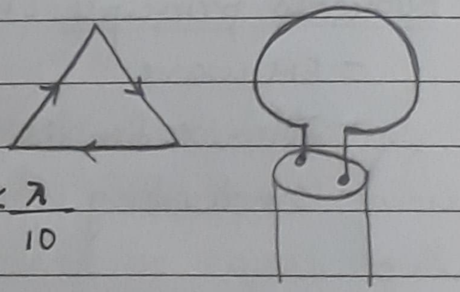
complex : $Z_s = \dfrac{35476}{73+425j} \Rightarrow \boxed{Z_s = 363 - 211j\ \Omega}$

* Loop Antenna:

A simple conductor which is in the shape of a loop.

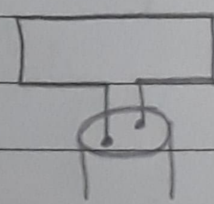
Loop antenna can be considered to be a small magnetic dipole.

I is uniform for small dimensions.

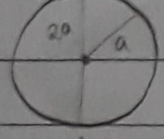


small loop : $A < \frac{\lambda^2}{100}$; $C = 2\pi r < \frac{\lambda}{10}$

Large loop : $C = 1\lambda$; $C \leq 5\lambda$

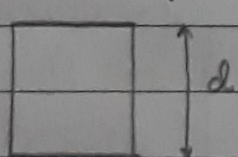


- Far field for small loop (E_ϕ and H_θ)



circular loop

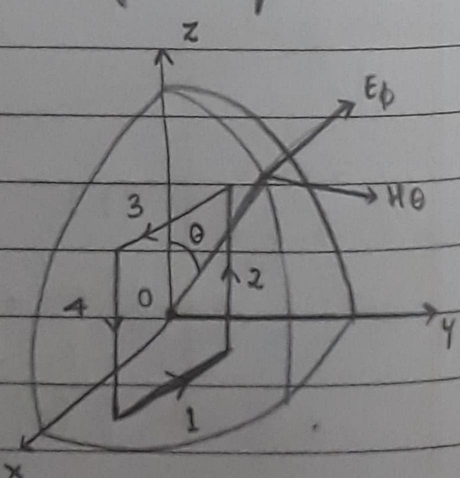
$A = \pi a^2 = d^2$



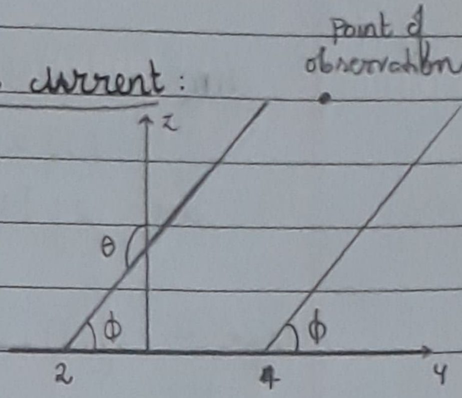
square loop

$A = d^2$

Geometry of square



* **Array of two isotropic source - opposite current:**



$$E_\phi = -\frac{E_{\phi_0} e^{j\psi/2}}{\cancel{2}} + \frac{E_{\phi_0} e^{-j\psi/2}}{\cancel{2}}$$

$$E_\phi = -2j\left[\frac{E_{\phi_0} e^{j\psi/2} - E_{\phi_0} e^{-j\psi/2}}{2j}\right]$$

$$\boxed{E_\phi = -2j\, E_{\phi_0} \sin \psi/2}$$

Electric field component of short dipole

$$E_{\phi_0} = j\frac{60\pi [I] L \sin\theta}{r\lambda}$$

$$E_{\phi_0} = j\frac{60\pi [I] d}{r\lambda}$$

For $\sin\frac{\psi}{2}$; $\psi = \beta d \cos\phi$

$$\psi = \frac{2\pi}{\lambda} d \sin\theta$$

$$\sin\frac{\psi}{2} = \sin\left[\frac{2\pi}{\lambda}\frac{d}{\cancel{2}}\sin\theta\right]$$

$$\Rightarrow \sin\frac{\psi}{2} = \frac{\pi d}{\lambda}\sin\theta$$

$$E_\phi = \cancel{-}2\cancel{j}\left[\cancel{j}\frac{60\pi [I] L \sin\theta}{r\lambda}\right]\frac{\pi d}{\lambda}\sin\theta$$

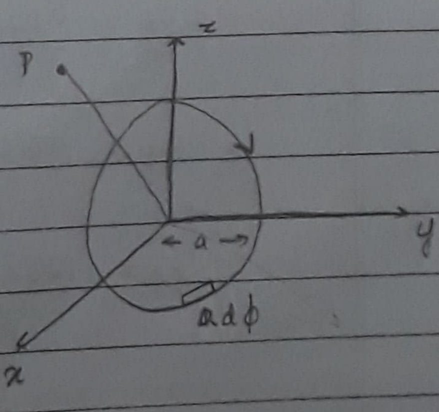
$$E_\phi = 120\pi^2 \frac{[I] \sin\theta}{r}\frac{d^2}{\lambda^2}$$

at $\theta = 90°$

$L = d$.

$$\boxed{E_\phi = \frac{120\pi^2 [I] \sin\theta}{r}\left[\frac{A}{\lambda^2}\right]}$$

- **Far fields for Loop - General case:**



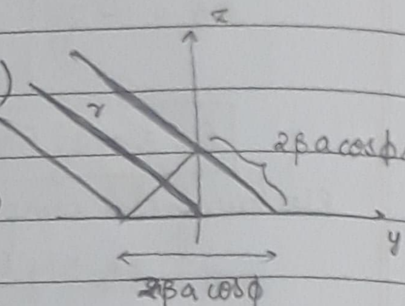
$$dA_\phi = \frac{\mu dM}{4\pi r}$$

dM : current moment

single short dipole

$= [I]\, a\, d\phi \cos\phi \sin \psi/2$

$dM = 2j\,[I]\, a\, d\phi \cos\phi \sin(\beta a \cos\phi \sin\theta)$



$dA_\phi = \frac{\mu}{4\pi r} 2j\,[I]\, a\, d\phi \cos\phi \sin(\beta a \cos\phi \sin\theta)$

$dA_\phi = \frac{\mu}{2\pi r} j\,[I]\, a\, d\phi \cos\phi \sin(\beta a \cos\phi \sin\theta)$

Total vector potential $= \int_0^\pi dA_\phi = A_\phi$

$$A_\phi = \int_0^\pi \frac{\mu}{2\pi r} j\,[I]\, a \cos\phi \sin(\beta a \cos\phi \sin\theta)\, d\phi$$

$$A_\phi = \frac{j\mu [I] a}{2\pi r} \int_0^\pi \cos\phi \sin(\beta a \cos\phi \sin\theta)\, d\phi$$

Bessel function of order 1.

$$J_0(x) = \frac{1}{\pi} \int_0^\pi \sin(x\cos\theta)\cos\theta\, d\theta$$

$$A_\phi = \frac{j\mu [I] a}{2r} \left[\frac{1}{\pi} \int_0^\pi \sin(\beta a \sin\theta \cos\phi) \cos\phi\, d\phi \right]$$

$$A_\phi = \frac{j\mu [I] a}{2r} J_0(\beta a \sin\theta) \quad \text{vector potential}$$

$$E_\phi = -j\omega A_\phi$$

$$E_\phi = -j\omega \frac{j\mu [I] a}{2r} J_1(\beta a \sin\theta)$$

$$E_\phi = \frac{\mu\omega [I] a}{2r} J_1(\beta a \sin\theta)$$

$\omega = \beta c$

$$E_\phi = \frac{\mu \beta c [I] a}{2r} J_1(\beta a \sin\theta)$$

$\mu c = 120\pi$

$$E_\phi = \frac{120\pi \beta [I] a}{2r} J_1(\beta a \sin\theta)$$

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Date
Page

therefore

$$\boxed{E_\phi = \frac{60\pi\beta[I]a}{r} J_1(\beta a \sin\theta)}$$

similarly

$$H = \frac{E}{120\pi} = \boxed{H_\theta = \frac{[I]\beta a}{2r} J_1(\beta a \sin\theta)}$$

- Radiation resistance of loop:

$$S_r = 1/2 \, Re \, \cancel{z} \, E_\phi H_\theta^*$$

$$S_r = 1/2 \, Re \, z \, |H_\theta|^2$$

$$S_r = \frac{1}{2} 120\pi \frac{I_0^2\beta^2 a^2}{(2r)^2} J_1^2(\beta a \sin\theta)$$

$$P = \iint S_r \, ds$$

$$P = \int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} \frac{1}{\cancel{2}} \cancel{120}^{\cancel{60}\,15}\pi \frac{I_0^2(\beta a)^2}{\cancel{4r^2}} J_1^2(\beta a \sin\theta) \cancel{r^2} \sin\theta \, d\theta \, d\phi$$

$$P = 15\pi I_0^2 \beta^2 a^2 (2\pi) \int_{\theta=0}^{\pi} J_1^2(\beta a \sin\theta) \sin\theta \, d\theta$$

$$J_1(x) = x/2$$

$$\therefore J_1^2(\beta a \sin\theta) = \left(\frac{\beta a \sin\theta}{2}\right)^2 = \beta^2 a^2 \frac{\sin^2\theta}{4}$$

$$P = 30\pi^2 \beta^2 a^2 I_0^2 \frac{\beta^2 a^2}{4} \int_{\theta=0}^{\pi} \sin^2\theta \sin\theta \, d\theta$$

$$P = \frac{15}{2} \pi^2 I_0^2 (\beta a)^4 \int_{\theta=0}^{\pi} \frac{3\sin\theta - \sin 3\theta}{4} \, d\theta$$

$$P = \frac{15}{8} I_0^2 \pi^2 (\beta a)^4 \left[-3\cos\theta + \frac{\cos 3\theta}{3}\right|_0^\pi$$

$$P = \frac{15}{8} I_0^2 \pi^2 (\beta a)^4 \left[3 - \frac{1}{3} + 3 - \frac{1}{3}\right]$$

$$P = \frac{\cancel{15}^{5}}{\cancel{8}} I_0^2 \pi^2 (\beta a)^4 \left(\frac{\cancel{16}^{2}}{\cancel{3}}\right)$$

$$P = 10 I_0^2 \pi^2 (\beta a)^4$$

Therefore

$$10\pi^2 \cancel{I_0^2} (\beta a)^4 = \frac{\cancel{I_0^2}}{2} R_r$$

$$\boxed{R_r = 20\pi^2 (\beta a)^4}$$

Area $= \pi a^2$

$$A^2 = (\pi a^2)^2$$

$$R_r = 20 A^2 \beta^4 = 20 A^2 \left(\frac{2\pi}{\lambda}\right)^4 = 31170.9 \frac{A^2}{\lambda^4}$$

small loop $\boxed{R_r = 31.2 \left(\frac{A}{\lambda^2}\right)^2 k\Omega}$

A : area; $C < \lambda/10$ and $A < \lambda^2/100$

large loop $\boxed{R_r = 3720 \left(\frac{a}{\lambda}\right)}$ a : radius

Directivity

$$\boxed{\begin{array}{l} D = 3/2 = 1.5 \\ D = 4.25(a/\lambda) \end{array}}$$

$D = 3/2 = 1.5$: Small loop

$D = 4.25(a/\lambda)$: Large loop

Q: A circular loop antenna has a diameter of 1.8λ. Find the directivity and radiation resistance.

Given: $d = 1.8\lambda \Rightarrow a = 0.9\lambda$

$$C = 2\pi a = 2\pi(0.9\lambda) = 5.65\lambda$$

Since : $C \geq 5\lambda$: Large loop.

$$\text{Directivity} = 4.25\left(\frac{a}{\lambda}\right) = 4.25\left(\frac{0.9\lambda}{\lambda}\right)$$

$$= 3.825 = 5.826\,dB$$

Radiation resistance

$$R_r = 3720\left(\frac{a}{\lambda}\right) = 3720\left(\frac{0.9\lambda}{\lambda}\right) = 3348\,\Omega$$

UNIT - 05

Antenna Types

* **Slot Antenna:**

- Babinet's principle

$F_S = f_1(x,y,z)$

$F_{CS} = f_2(x,y,z)$ $\quad F_0 = F_S + F_{CS}$

$F_0 = f_3(x,y,z)$

- Booker's principle

$E_0 = E_1 + E_2$

$\frac{E_1}{E_0} + \frac{E_2}{E_0} = 1$

- Impedance of slot antenna

The terminal impedance Z_s of a slot antenna is equal to the 1/4 of the square of the intrinsic impedance of the surrounding medium divided by the terminal impedance Z_d of the complementary dipole antenna.

$$Z_s = \frac{Z_0^2}{4Z_d} = \frac{(377)^2}{4Z_d} = \frac{35476}{Z_d}$$

$$Z_s = \frac{35476}{R_d + jX_d} = \frac{35476}{R_d^2 + X_d^2}(R_d - jX_d)$$

The terminal impedance of infinitely thin slot is

$$Z_s = \frac{35476}{73 + 42.5j} = 363 - j211\ \Omega$$

* **Horn Antenna:**

Path length difference

$\delta_E = 0.2\lambda \quad \delta_H = 0.375\lambda$

Flare angle

$\theta_E = 2\tan^{-1}\left(\frac{a_E}{2L}\right) \quad \theta_H = 2\tan^{-1}\left(\frac{a_H}{2L}\right)$

Horn length

$L_E = \frac{a_E^2}{8\delta_E} \quad L_H = \frac{a_H^2}{8\delta_E}$

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Directivity (actual)

$$D = \frac{4\pi A_e}{\lambda^2} = \frac{4\pi \varepsilon_{ap} A_p}{\lambda^2} = \frac{4\pi (0.6) A_p}{\lambda^2} = \frac{7.5 A_p}{\lambda^2}$$

For rectangular Horn $A_p = a_E a_H$

For circular Horn $A_p = \pi r^2$

Directivity (approx)

$$D = \frac{41253}{\theta^\circ_{HP} \phi^\circ_{HP}} = \frac{41253}{\theta_{HP_E} \times \theta_{HP_H}}$$

where $\theta_{HP(E)} = \frac{56^\circ}{a_{E\lambda}}$ $\theta_{HP(H)} = \frac{67^\circ}{a_{H\lambda}}$

Gain $G = kD = 0.6D$

★ <u>Parabolic Reflectors:</u>

$$R = \frac{2L}{1+\cos\theta}$$

- <u>Field distribution and Power density ratio</u>

cylindrical Parabola

$$\frac{S_\theta}{S_0} = \frac{1+\cos\theta}{2} \qquad \frac{E_\theta}{E_0} = \sqrt{\frac{1+\cos\theta}{2}}$$

Paraboloid

$$\frac{S_s}{S_0} = \frac{(1+\cos\theta)^2}{4} \qquad \frac{E_s}{E_0} = \frac{1+\cos\theta}{2}$$

$HPBW = \frac{58^\circ}{D_\lambda}$ Directivity $= 9.9 D_\lambda^2$

$BWFN = \frac{140^\circ}{D_\lambda}$ Power gain $= 6 D_\lambda^2$

capture area

$A_c = BA$ A : actual area $= \pi r^2 = \pi D^2/4$

B : illumination efficiency $= 0.65$

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* **Embedded antenna:**
metallic conductor embedded in a dielectric material with dielectric constant greater than 1. Antenna size will be reduced.

* **Plasma Antenna:**
Plasma antenna employs ionized gas enclosed in a tube as the conducting elements instead of metal conductors.

* **Antennas for ground penetrating radar:** GPR
For the detection of underground anomalities both natural and man made. (metallic and non metallic)

* **Antenna measurement technique:**

- **Measurement of gain**

$$G = \frac{U_m}{U_0} = \frac{k U_m}{U_0}$$

a. absolute method

$$P_r = P_T G_T G_R \left(\frac{\lambda}{4\pi R}\right)^2$$

b. comparison method

$$G_{AUT} = \frac{P_{AUT}}{P_{ref}} G_{ref}$$

c. celestial Radio sources.

$$G_{AUT} = \frac{8\pi k \Delta T_A}{LS\lambda^2}$$ $k \Delta T_A B$: noise power

d. Radar techniques.

$$G_{AUT} = \frac{8\pi R^2}{a\lambda}\sqrt{\frac{P_r}{P_t}}$$

- **Measurement of phase**
a. direct method
b. reference antenna method
c. differential method

- **Measurement of Directional pattern**
a. copolar pattern
b. cross polar pattern

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UNIT - 06

Radio Wave Propagation

* Electromagnetic waves

- General classification:

0-16km: Troposphere
16-80km: Stratosphere
80-400km: Ionosphere

1. plane wave:
2. uniform plane wave
3. non uniform plane wave
4. slow wave
5. forward wave
6. backward wave
7. travelling wave
8. standing wave
9. surface wave

- classification based on the presence of field components:
1. Transverse Electric or H wave
2. Transverse Magnetic or E wave
3. Transverse Electromagnetic (TEM) or EH or HE wave

- classification based on modes of propagation:
1. ground waves
2. Space waves
3. sky waves.

* Ground wave propagation:

$$Z_S = \frac{\omega\mu}{\sqrt{\sigma^2 + \omega^2\varepsilon_r}}$$ surface wave impedance of earth

* Space wave propagation:

R_1: distance travelled by direct wave

R_2: distance travelled by reflected wave.

$$R_1 = d\sqrt{1 + \left(\frac{h_t - h_r}{d}\right)^2} \qquad R_2 = d\sqrt{1 + \left(\frac{h_t + h_r}{d}\right)^2}$$

For larger distances

$$R_1 = d + \frac{(h_t - h_r)^2}{2d} \qquad R_2 = d + \frac{(h_t + h_r)^2}{2d}$$

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Path difference $= R_2 - R_1 = \frac{2h_t h_r}{d}$

Phase difference $= \frac{2\pi}{\lambda}\left(\frac{2h_t h_r}{d}\right) = \frac{4\pi h_t h_r}{\lambda d}$

Resultant field E_R at the receiver

$$E_R = E_d + E_r e^{-j\psi}$$

The total phase : $\psi = 180^\circ + \alpha$

α : phase difference due to path difference.

$$E_R = 2E_s \sin(\alpha/2)$$

$$E_R = \frac{2E_0}{d} \sin \frac{2\pi h_t h_r}{\lambda d}$$

* **Sky wave Propagation:**

Critical frequency

$f_c = 9\sqrt{N}$ N: electron density

Maximum usable frequency

$f_{MUF} = f_c \sec \phi_i$: secant law

$$f_{MUF} = f_c \sqrt{1 + \left(\frac{d}{2h}\right)^2}$$

Skip distance

$$d = 2h \sqrt{\frac{f_{MUF}^2}{f_c^2} - 1}$$