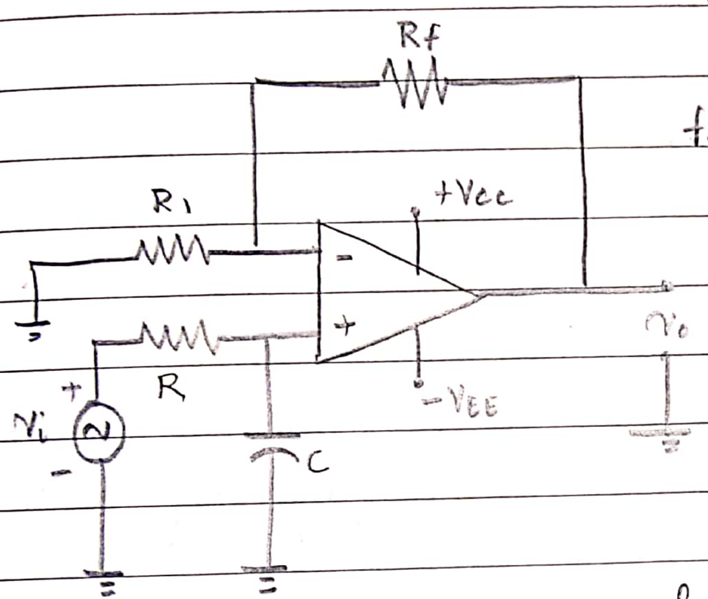
Keerthana Ashok

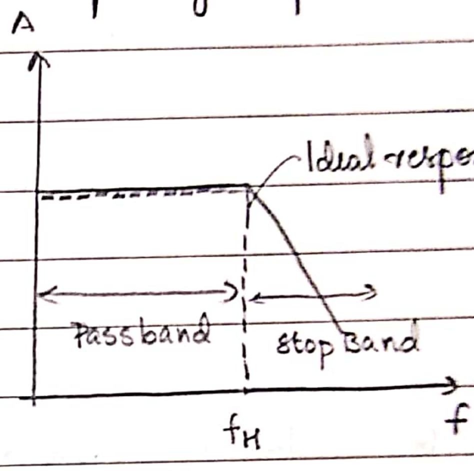
Low Pass Filter :



$$A_V = 1 + \frac{R_f}{R_1}$$

$$f_H = \frac{1}{2\pi f C}$$

Frequency response



f_H : Higher cut off frequency.

A low pass filter has a constant gain from 0Hz to a high cut off frequency f_H. A low pass filter allows frequency upto f_H and then attenuates the higher frequency. After f_H the gain is decreasing with the increase in input frequency. Bandwidth is equal to f_H.

Frequencies between 0Hz to f_H : pass band frequencies

Frequencies which are above f_H are attenuated : stop band frequencies.

Q1: Design a low pass filter at a cut off frequency $f_H = 10kHz$ and pass band gain of 2.

Sol: given: $A_f = 2$ $\quad f_H = 10kHz$

$$f_H = \frac{1}{2\pi RC} \qquad A_f = 1 + \frac{R_f}{R_1}$$

$$RC = \frac{1}{2\pi(10k)} \qquad 2 = 1 + \frac{R_f}{R_1}$$

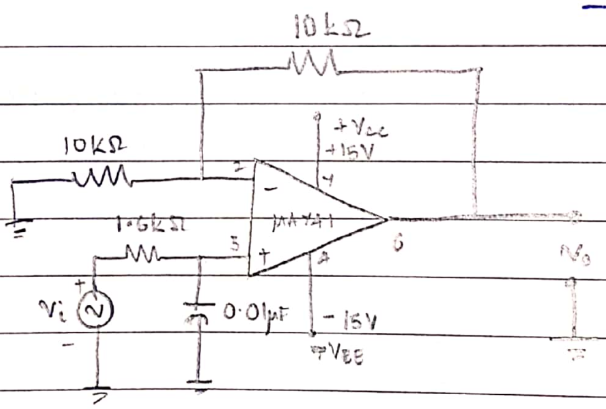
Assuming $C = 0.01\mu F$ $\qquad \frac{R_f}{R_1} = 1$

$$R = \frac{1}{2\pi(10k)(0.01\mu)} \qquad \Rightarrow R_f = R_1$$

$$\underline{R = 1.6k\Omega}$$

Assuming $R_f = 10k\Omega$

$\therefore \underline{R_1 = 10k\Omega}$



Q2: Design a low pass filter at a cut off frequency of 15.9 kHz with a pass band gain of 1.5.

Sol: $f_H = \frac{1}{2\pi RC}$ $\qquad A = 1.5 = 1 + \frac{R_f}{R_1}$

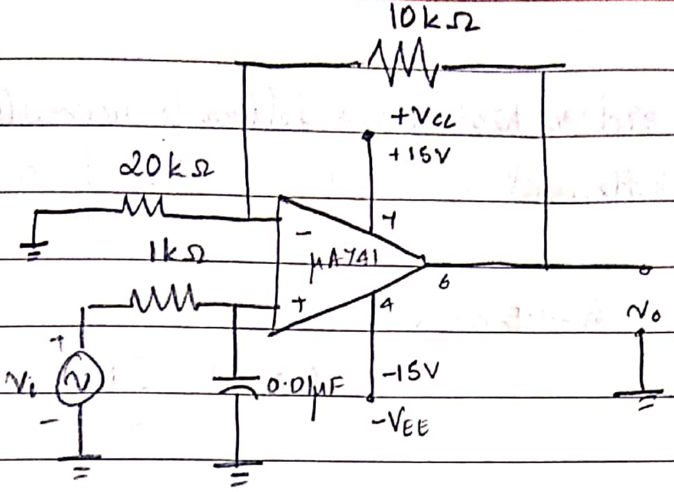
Assuming $C = 0.01\mu F$ $\qquad \Rightarrow R_f = 0.5R_1$

$$R = \frac{1}{2\pi(15.9k)(0.01\mu)}$$

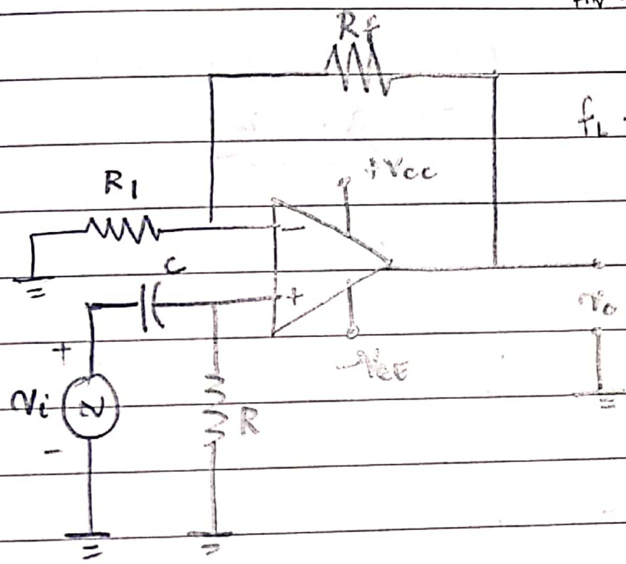
Assuming $R_f = 10k\Omega$

$\underline{R_1 = 20k\Omega}$

$\underline{R = 1k\Omega}$



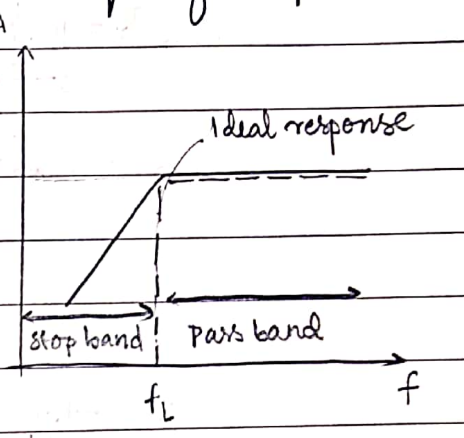
* High Pass Filter:



$A_V = 1 + \frac{R_f}{R_1}$

$f_L = \frac{1}{2\pi RC}$

Frequency response



f_L: low cutoff frequency

A high pass filter has a constant gain from lower cutoff frequency and above. A high pass filter attenuates frequencies upto f_L and then allows frequency above f_L.

From 0Hz to f_L the gain increases with increase in input frequency.

Frequencies between 0Hz to f_L : stop band frequencies as they are attenuated and frequencies above f_L : pass band frequencies.

Q1: Design a first order high pass filter to have lower cut off frequency of 1.5kHz and a pass band gain of 1.5.

Sol: Given: $f_L = 1.5kHz$ $A = 1.5$

$$f_L = \frac{1}{2\pi RC}$$

$$RC = \frac{1}{2\pi(1.5k)}$$

Assuming $C = 0.01\mu F$

$R = 10.6k\Omega$

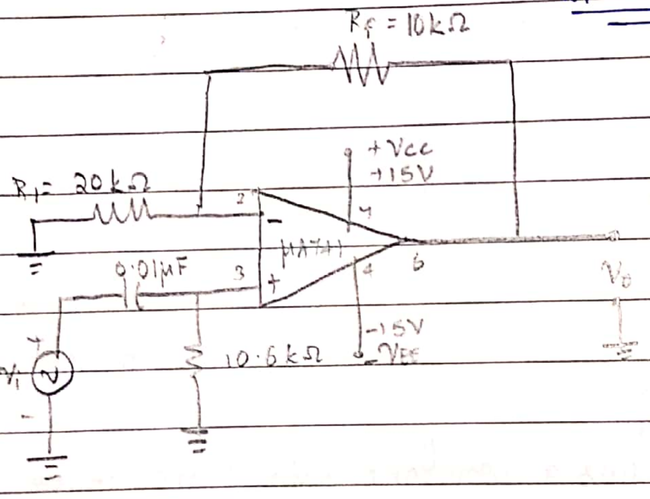
$$A = 1 + \frac{R_f}{R_1}$$

$$1.5 = 1 + \frac{R_f}{R_1}$$

$R_f = 0.5R_1$

Assuming $R_f = 10k\Omega$

$R_1 = 20k\Omega$



* Differentiator and Integrator:

→ Design steps for differentiator and integrator:

Select f_a equal to highest frequency of the input signal to be differentiated. Then assuming the value of $C_1 < 1\mu F$. Calculate the value of R_f

$$f_a = \frac{1}{2\pi R_f C_f} \qquad f_b = \frac{1}{2\pi R_1 C_1} \qquad R_1C_1 = R_fC_f$$

Choose $f_b = 10f_a$ or $f_b = 20f_a$ and calculate R_1 and C_f.

(In most of the wave shaping circuits and FM modulator we use integrator and differentiator.)

Differentiator:

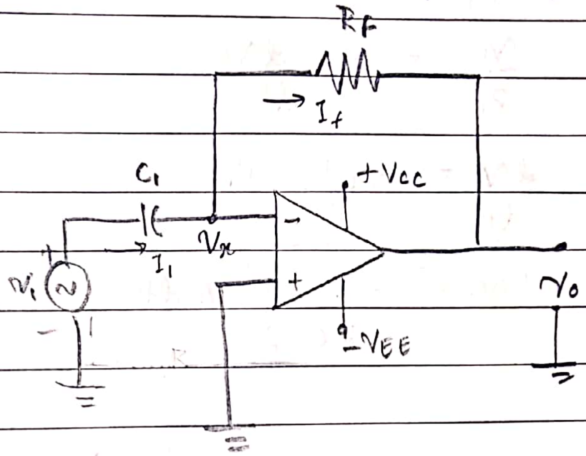
The circuit performs the mathematical operation of differentiation i.e., the output waveform is the derivative of the input waveform. The differentiator is constructed by replacing the input resistance R_i of inverting amplifier by a capacitor C_1.

Due to virtual ground

$$V_x = 0$$

Applying KCL

$$I_1 = I_f$$



$$C_1 \frac{d[V_i - V_x]}{dt} = \frac{V_x - V_o}{R_f}$$

$$C_1 \frac{dV_i}{dt} = \frac{-V_o}{R_f}$$

$$\boxed{V_o = -R_f C_1 \frac{dV_i}{dt}}$$

Differentiator

Input:	Output:
cosine	sine
triangular	square wave

Integrator:

A circuit in which the output voltage waveform is the integral of the input voltage waveform is the integrator or the integration amplifier. Such a circuit is obtained by using a basic inverting amplifier configuration if the feedback resistor R_f is replaced by a feedback capacitor C_f.

Due to virtual ground

$v_x = 0$

Applying KVL

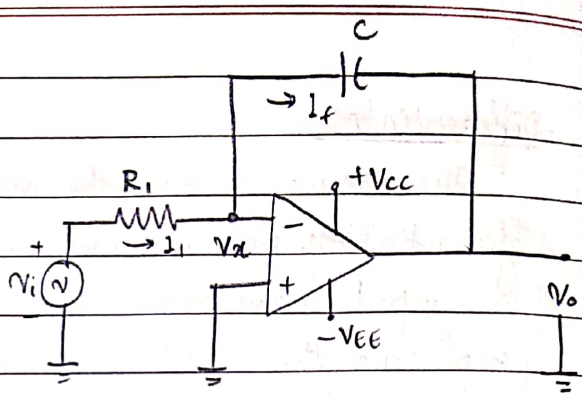
$I_1 = I_f$

$$\frac{V_i - \cancel{V_x}^{\approx 0}}{R_1} = C \frac{d}{dt}[\cancel{V_x}^{0} - V_o]$$

$$\frac{V_i}{R_1} = -C\frac{dV_o}{dt}$$

$$\frac{dV_o}{dt} = -\frac{1}{RC} V_i$$

$$\boxed{V_o = -\frac{1}{RC}\int_0^t V_i \, dt}$$



Integrator:

Input	Output
sine	cosine
square	triangular.

Q1: a. Design a differentiator to differentiate an input signal that varies in frequency from 10Hz to about 1kHz.
b. If a sine wave of 1V peak at 1000Hz is applied to the differentiator of part a, then draw the output waveform.

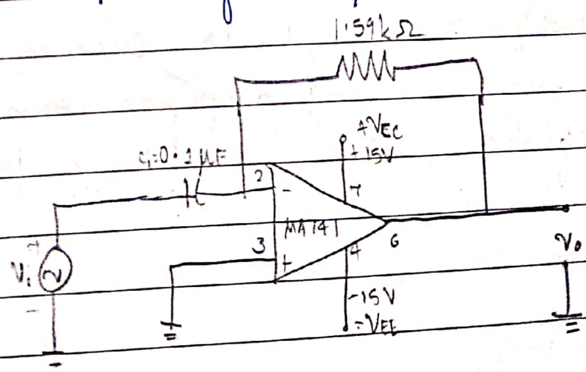
Sol: Given: $f_a = 1kHz$ ~~$f_b = 10Hz$~~

$$f_a = \frac{1}{2\pi R_f C_f}$$

Assuming $C_f < 1\mu F$

$$\Rightarrow R_f = \frac{1}{2\pi(1k)(0.01\mu)}$$

$$R_f = 1.59k\Omega$$



$f_b = 20 f_a$

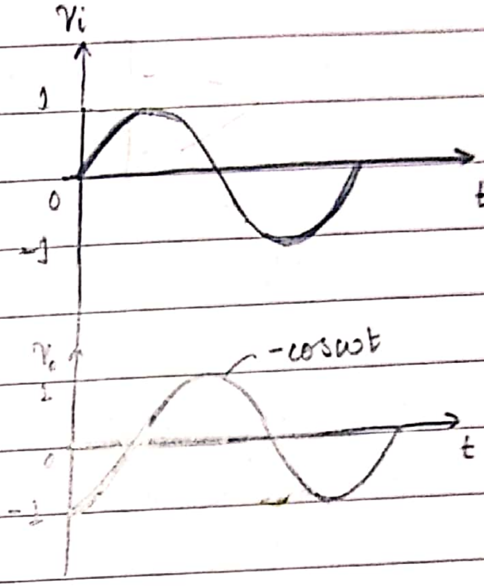
$f_b = 20(1k)$

$f_b = 20kHz$

$$f_b = \frac{1}{2\pi R_1 C_1}$$

$$20k = \frac{1}{2\pi R_1 (0.1\mu F)}$$ assuming $C_1 < 1\mu F$

$$\underline{R_1 = 79.57\Omega}$$



b. $V_o = -R_f C_1 \frac{dv_i}{dt}$

$$V_o = -(1.59k)(0.1\mu)\frac{d}{dt}(\sin\omega t)$$

$$V_o = -159 \times 10^{-6}\ \omega \cos\omega t$$

$$V_o = -159 \times 10^{-6}(2\pi(1000))\cos\omega t$$

$$\underline{V_o = -0.999 \cos\omega t}$$

Q2: Design a differentiator using op-amp to differentiate an input signal with $f_{max} = 200Hz$. Also draw the output waveform for a sine wave input of $1V_p$ at $200Hz$.

sol: $f_a = 200Hz$

Assume: $C_f = 0.1\mu F$

$$f_a = \frac{1}{2\pi R_f C_f} \Rightarrow R_f = \frac{1}{2\pi(0.1\mu F)(200)}$$

$$\therefore \underline{R_f = 7.95k\Omega}$$

$$f_b = 10f_a = 2000Hz$$

$$f_b = \frac{1}{2\pi R_1 C_1} \Rightarrow R_1 = \frac{1}{2\pi(2000)(0.1\mu)}$$

$$\therefore \underline{R_1 = 0.795k\Omega}$$

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$v_i = 1 \sin(2\pi \times 200)t$

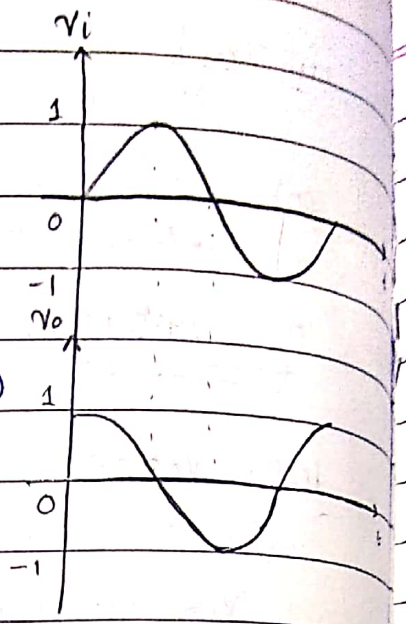
$v_o = -R_f C_1 \dfrac{dv_i}{dt}$

$v_o = -(7.95k)(0.1\mu) \dfrac{d}{dt} \sin(2\pi \times 200)t$

$v_o = -7.95 \times 10^{-4} \cos(2\pi 200t) \times 2\pi(200)$

$v_o = -0.999 \cos(2\pi 200t)$

$v_o \approx -1 \cos(2\pi 200t)$



Q3: Assuming $R_f = 100k\Omega$ and $R_1 = 10k\Omega$ and $C_f = 10nF$ in a practical integrator circuit, determine the lower frequency limit of integrator and output response for the sine wave input.

Sol: $R_1 = 10k\Omega$ $R_f = 100k\Omega$ $C_f = 10nF$

$f_a = \dfrac{1}{2\pi R_f C_f}$

$\Rightarrow f_a = \dfrac{1}{2\pi(100k)(10n)} = 159\,Hz$

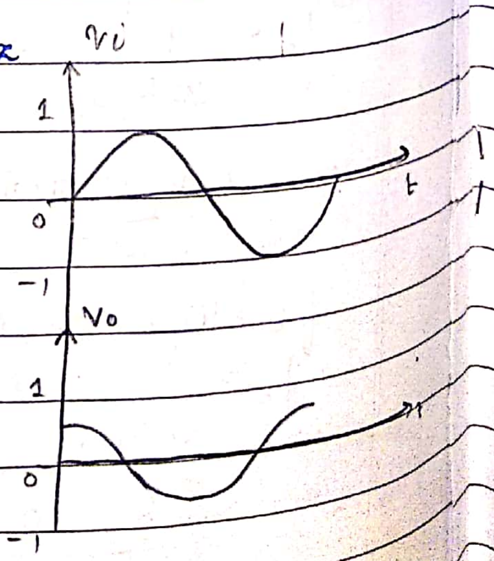
For an input voltage of $1V_p$ and $2.5kHz$

$v_o = \dfrac{-1}{R_1 C_f} \int v_i(t)\, dt$

$v_o = \dfrac{-1}{(10k)(10n)} \int \sin 2\pi(2.5k)t\, dt$

$v_o = -10000 \dfrac{[-\cos 2\pi(2.5k)t]}{2\pi(2.5k)}$

$v_o = 0.636 \cos 2\pi(2.5k)t$



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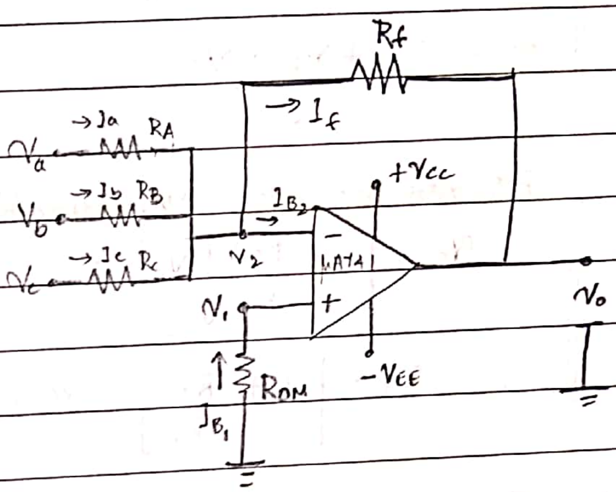
* Summing, Scaling and averaging amplifiers using op-amp:

Different configurations.

- Inverting configuration
- Noninverting configuration
- Differential configuration.

* Inverting configuration:

Due to virtual ground



$V_1 = V_2 = 0$

$\Rightarrow I_{B1} = I_{B2} = 0$

$I_{B1} = I_{B2} = I_B$

Applying KCL at V_2

$I_a + I_b + I_c = I_f + \cancel{I_B}^{0}$

$$\frac{V_a}{R_a} + \frac{V_b}{R_b} + \frac{V_c}{R_c} = \frac{-V_o}{R_f}$$

$$\Rightarrow V_o = -\left[\frac{R_f}{R_a}V_a + \frac{R_f}{R_b}V_b + \frac{R_f}{R_c}V_c\right]$$

a. Summing amplifier:

If $R_a = R_b = R_c = R$, then

$$\boxed{V_o = \frac{-R_f}{R}\left[V_a + V_b + V_c\right]}$$

If gain is 1 then $V_o = -[V_a + V_b + V_c]$

b. Scaling or Weighted Amplifier:

$$\frac{R_f}{R_a} \neq \frac{R_f}{R_b} \neq \frac{R_f}{R_c}$$

$$\boxed{V_o = -\left[\frac{R_f}{R_a}V_a + \frac{R_f}{R_b}V_b + \frac{R_f}{R_c}V_c\right]}$$

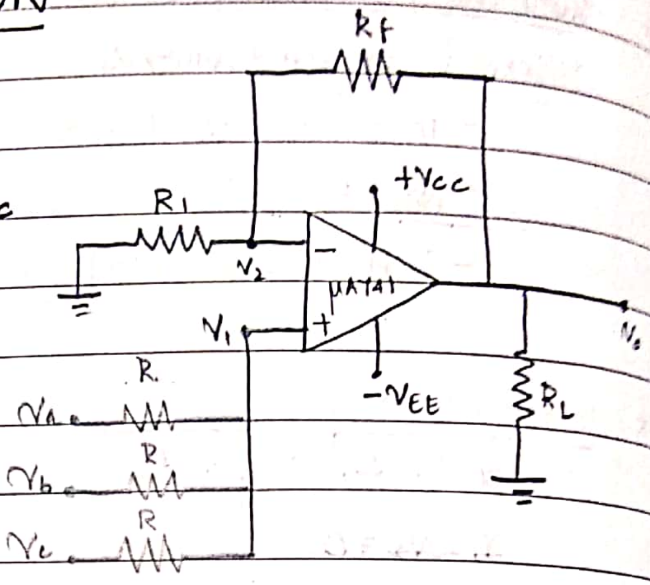
c. Averaging amplifier:

If $R_a = R_b = R_c = R$ and gain $\frac{R_f}{R} = \frac{1}{n}$ where n is number of inputs.

$$\boxed{V_o = \frac{-1}{n}\left[V_a + V_b + V_c\right]}$$

* NON INVERTING CONFIGURATION:

Here $R_a = R_b = R_c = R$



By superposition principle

$$V_1 = \frac{R/2}{R+R/2} V_a + \frac{R/2}{R+R/2} V_b + \frac{R/2}{R+R/2} V_c$$

$$V_1 = \frac{V_a}{3} + \frac{V_b}{3} + \frac{V_c}{3}$$

$$V_1 = \frac{V_a + V_b + V_c}{3}$$

For noninverting configuration

$$V_0 = \left[1 + \frac{R_f}{R_1}\right] V_1$$

$$\therefore \quad V_0 = \left[1 + \frac{R_f}{R_1}\right]\left[\frac{V_a + V_b + V_c}{3}\right]$$

a. **Summing Amplifier**

If $\left[1 + \frac{R_f}{R_1}\right]$ is number of inputs.

$$\boxed{V_0 = [V_a + V_b + V_c]}$$

b. **Averaging Amplifier**

— No sign change or phase reversal occurs between the average of input and output.

— In non-inverting configuration V_1 is the average of all the three inputs whereas in inverting configuration V_0 is the average of all the three inputs.

$$\boxed{V_0 = -\left[\frac{V_a + V_b + V_c}{3}\right]}$$

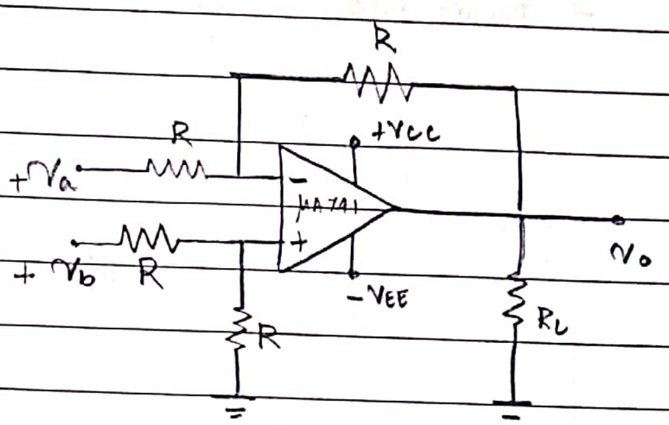
* DIFFERENTIAL CONFIGURATION:

By using differential configuration we can construct summing amplifier and subtractor.

a. Subtractor:

For inverting

$$V_o = -\frac{R_f}{R_1} V_i$$



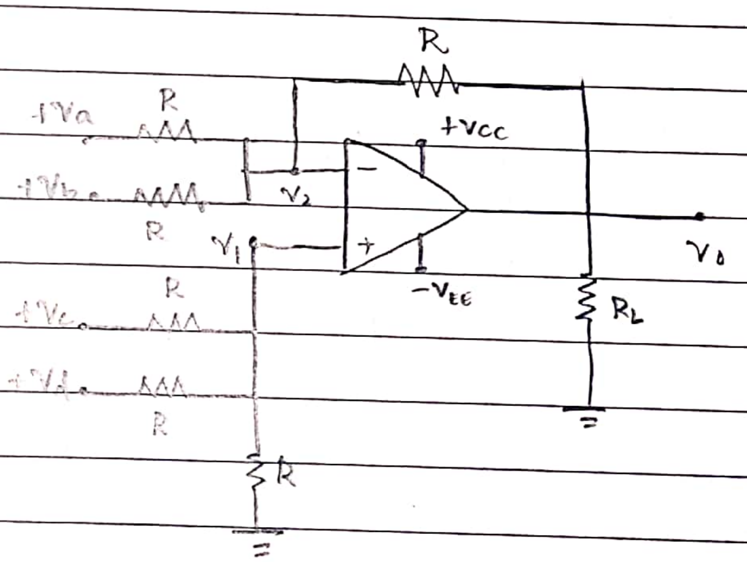
$$\therefore V_o = \frac{-R}{R}[V_a - V_b]$$

$$\boxed{V_o = V_b - V_a}$$

b. Summing amplifier:

By superposition principle

$$V_{oa} = \frac{-R}{R} V_a = -V_a$$



$$V_{ob} = \frac{-R}{R} V_b = -V_b$$

$$V_{oc} = \left[1 + \frac{R}{R/2}\right] V_1 = V_c$$

$$V_{od} = \left[1 + \frac{R}{R/2}\right] V_1 = V_d$$

Output voltage

$$V_o = V_{oa} + V_{ob} + V_{oc} + V_{od}$$

$$\boxed{V_o = -V_a - V_b + V_c + V_d}$$

$$V_1 = \frac{R/2}{R + R/2} V_c = \frac{V_c}{3}$$

$$V_1 = \frac{R/2}{R + R/2} V_d = \frac{V_d}{3}$$

Q1: In the circuit shown in the figure (above)

$R = 1k\Omega$

$V_a = 2V$, $V_b = 3V$ and $V_c = 4V$, $V_d = 5V$

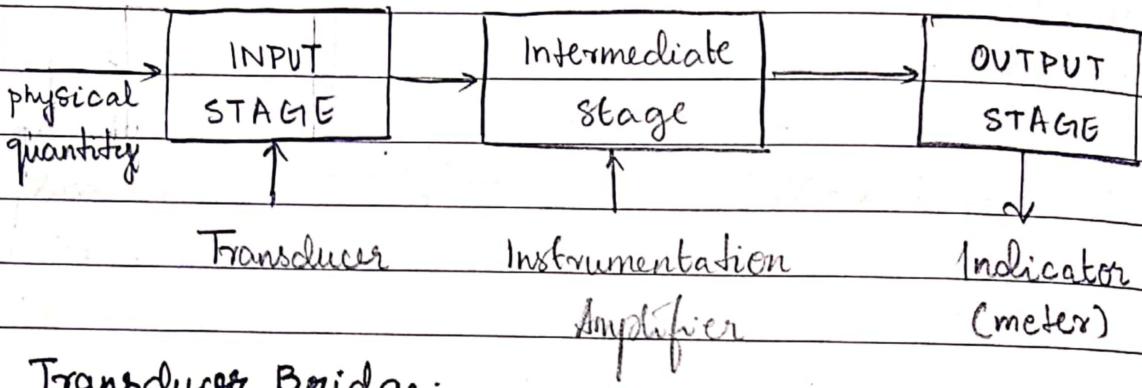
and the supply voltage is ±15V. Determine the output voltage V_o.

sol: $V_o = -2 - 3 + 4 + 5 = 4V$

★ Instrumentation Amplifier:

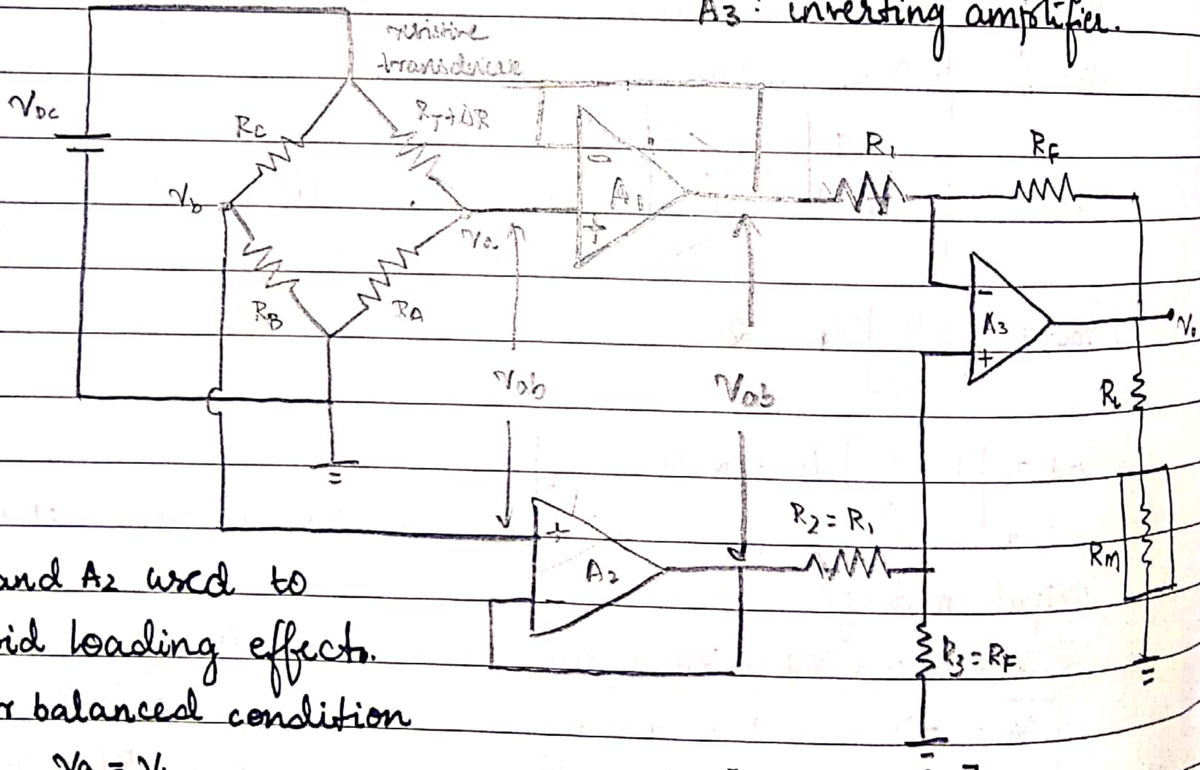
Application: used to measure physical quantities such as temperature, flow rate, pressure.

– Basic Block Diagram of Instrumentation Amplifier



– Transducer Bridge:

A_1 and A_2 : voltage follower
A_3 : inverting amplifier.



A_1 and A_2 used to avoid loading effect.

For balanced condition

$$V_a = V_b$$

$$R_a = R_b = R_c = R_T$$

$$\frac{R_C}{R_B} = \frac{R_T}{R_A}$$

$$V_{ab} = V_a - V_b$$

$$V_a = V_{DC}\left[\frac{R_A}{R_A + (R_T + \Delta R)}\right]$$

$$V_b = V_{DC}\left[\frac{R_B}{R_B + R_C}\right]$$

$$\therefore V_a - V_b = V_{DC}\left[\frac{R_A}{R_A + (R_T + \Delta R)} - \frac{R_B}{R_B + R_C}\right]$$

Let $R_A = R_B = R_C = R_T = R$

$$V_{ab} = V_{DC}\left[\frac{R}{2R+\Delta R} - \frac{\cancel{R}}{2\cancel{R}}\right]$$

$$V_{ab} = V_{DC}\left[\frac{\cancel{2R} - \cancel{2R} - \Delta R}{2(2R+\Delta R)}\right]$$

$$V_{ab} = -V_{DC}\left[\frac{\Delta R}{2(2R+\Delta R)}\right]$$

For A_3 amplifier

$$V_o = \frac{-R_f}{R_1} V_{ab}$$

$$V_o = \frac{-R_f}{R_1}\left[-V_{DC}\left(\frac{\Delta R}{2(2R+\Delta R)}\right)\right] \qquad 2R+\Delta R \approx 2R$$

$$\boxed{V_o = \frac{R_f}{R_1} V_{DC} \frac{\Delta R}{4R}} \Rightarrow V_o \propto \Delta R.$$

Q1: In an instrumentation amplifier $R_1 = 1k\Omega$, $R_f = 4.7k\Omega$, then $R_a = R_b = R_c = 100k\Omega$, then $V_{DC} = 5V$. The opamp supply voltage is $\pm 15V$. The transducer is a thermistor with the following specifications: $R_T = 100k\Omega$. At the reference temperature of $25°C$, the temperature coefficient of resistance is equal to $1k\Omega/°C$ or $1\%/°C$. Determine the output voltage at $0°C$ and at $100°C$.

Sol: Given: $R_1 = 1k\Omega$ $\quad R_f = 4.7k\Omega$ $\quad V_{DC} = 5V$

$R_a = R_b = R_c = 100k\Omega$ $\quad +V_{cc}$ & $-V_{EE} = \pm 15V$

Reference temp: $25°C$. $\quad R_T = 100k\Omega$

$= 1k\Omega/°C$.

as $R_a = R_b = R_c = R_T = 100k\Omega$ at $25°C$

it is balanced

hence $V_a = V_b$ and $V_o = 0$

Temp (0°c)

CASE 1 : $\Delta R = -1k\ (0-25)$

$$\Delta R = 25k\Omega$$

$$V_o = \frac{R_f}{R_1} V_{DC} \left[\frac{\Delta R}{4R} \right]$$

$$V_o = \frac{4.7k}{1k} (5) \frac{25k}{4(100k)}$$

$$V_o = 1.468\ V$$

CASE 2 : $\Delta R = -1k\ (100-25)$

$$\Delta R = -75k\Omega$$

$$V_o = \frac{R_f}{R_1} V_{DC} \left[\frac{\Delta R}{4R} \right]$$

$$V_o = -\frac{4.7k}{1k} (5) \frac{75k}{4(100k)}$$

$$V_o = -4.4V$$

* Voltage to current converter (Transconductance Amplifier)

Applications:

- low voltage ac or dc voltmeter
- zener diode tester
- LED

The conversion can be done by two methods

a. V - I converter using floating load.

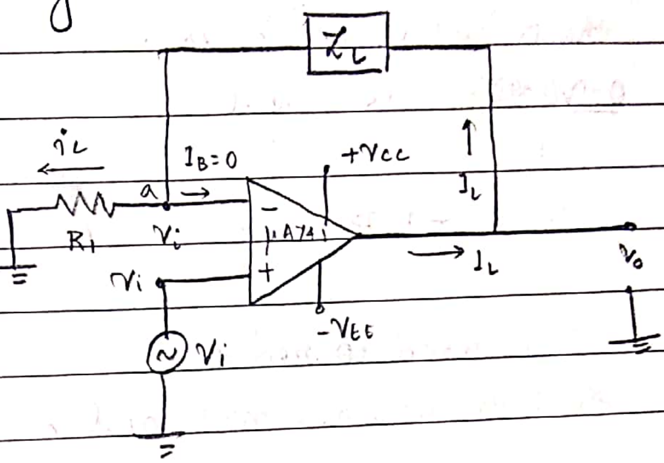
b. V - I converter using grounded load.

1. V-I converter using floating load

at 'a' due to virtual ground potential is V_i

Since $I_B = 0$

I_L is current across R_1



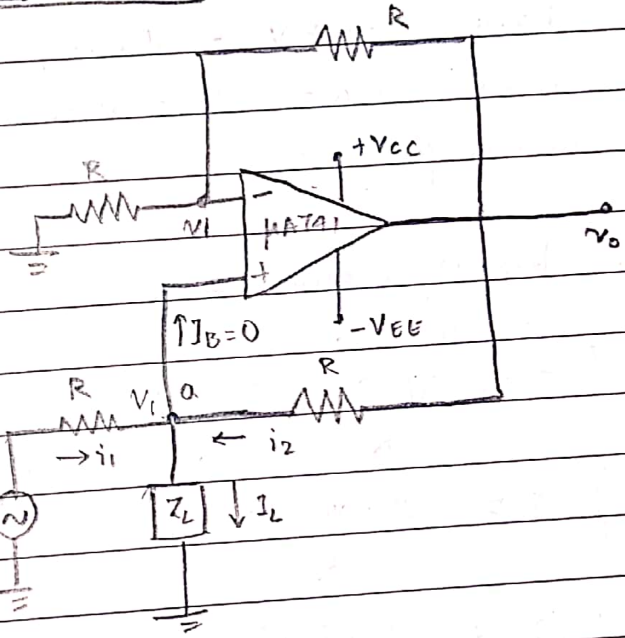
$\therefore V_i = I_L R_1$ input

$\therefore I_L = \frac{V_i}{R_1}$ output

2. V-I converter using grounded load

Due to virtual ground at inverting terminal potential is V_1.

At node a



$I_L = i_1 + i_2$

$I_L = \frac{V_i - V_1}{R} + \frac{V_o - V_1}{R}$

$I_L = \frac{V_i - 2V_1 + V_o}{R}$

$I_L R = V_i - 2V_1 + V_o$

$V_1 = \frac{V_i + V_o - I_L R}{2}$ input

but $V_i = I_L R$

$I_L = \frac{V_i}{R}$

* Current to Voltage converter (Transresistance Amplifier)

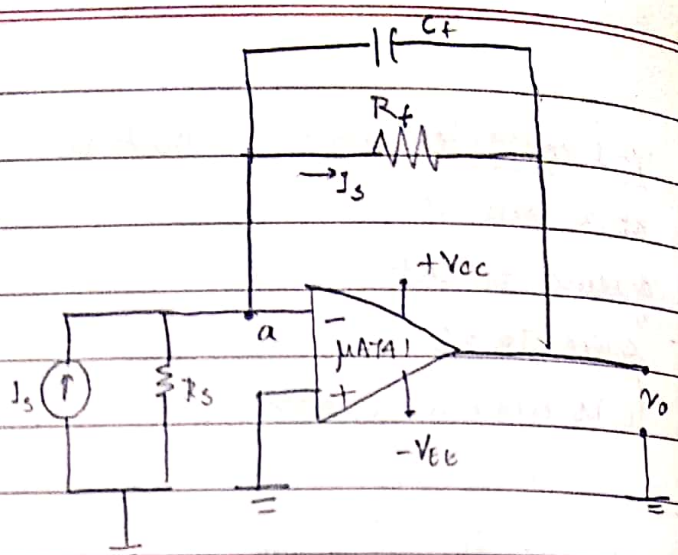
Applications

- Photo diode
- Photo multiplier
- Photo FET Detector

Due to virtual ground the potential at 'a' is 0.

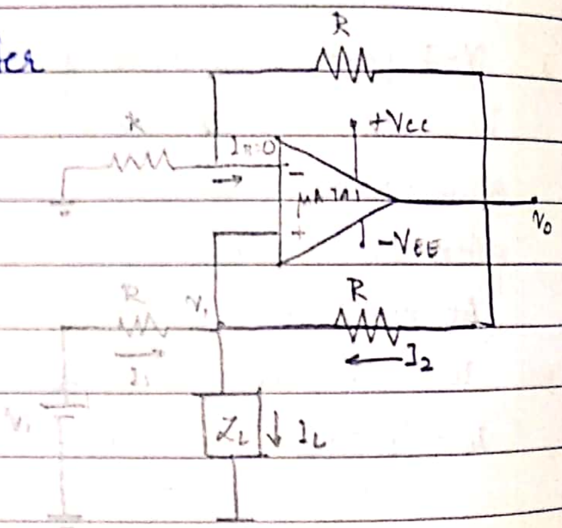
$$\frac{0 - V_o}{R_f} = I_s \quad \text{Input}$$

$$V_o = -I_s R_f \quad \text{output}$$



C_f is used to reduce the high frequency noise and oscillations.

Q: For a voltage to current converter shown, $V_i = 5V$, $R = 10k\Omega$, $V_1 = 1V$. Find the load current and output voltage.



Sol: Initially assume $I_B = 0$

$$I_L = I_1 + I_2$$

$$I_L = \frac{V_i - V_1}{R} + \frac{V_o - V_1}{R}$$

$$I_L = \frac{V_i + V_o - 2V_1}{R}$$

$$I_L = \frac{5 + 2 - 2(1)}{10k}$$

$$I_L = 0.5mA$$

For non inverting amplifier

$$V_o = \left[1 + \frac{R_F}{R_1}\right] V_i$$

Here $R_F = R_1 = R$

$$V_o = 2V_1$$

$$V_o = 2V$$

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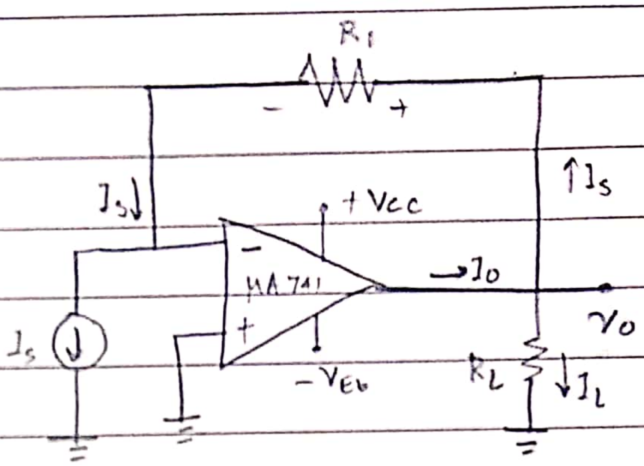
classmate

Date

Page

Current Amplifier:



At output applying KVL

$$I_o = I_s + I_L$$

$$I_o = \frac{V_o}{R_1} + \frac{V_o}{R_L}$$

$$I_o = V_o\left[\frac{1}{R_1} + \frac{1}{R_L}\right]$$

$$\therefore \boxed{V_o = \frac{R_1R_2}{R_1+R_2} I_o}$$

but $V_o = I_L R_L$

$$\frac{R_1R_2}{R_1+R_2} I_o = I_L R_L$$

$$\frac{R_1R_2}{R_1+R_2}\left[I_s + I_L\right] = I_L R_L$$

$$\boxed{I_L = I_s \frac{R_1}{R_L}}$$

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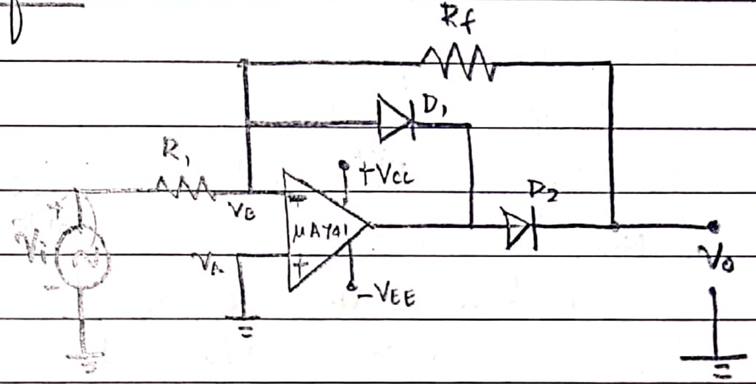
Unit - 3

LINEAR APPLICATIONS OF AN OP AMP - II

* **Precision Rectifier :**

The precision rectifier is obtained with an operational amplifier in order to have a circuit behave like an ideal diode and rectifier. It is useful for high precision signal processing.

— **Precision Half Wave Rectifier:**



Due to virtual ground

$V_A = V_B = 0$

During positive half cycle

D_1 - forward biased

D_2 - reverse biased

$\therefore V_o = 0$

During negative half cycle

D_1 - reverse biased

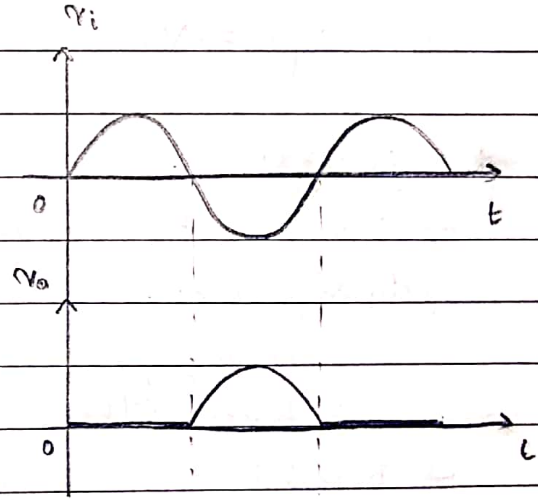
D_2 - forward biased

then it is like an inverting amplifier

$\therefore V_o = \frac{-R_f}{R_1} V_i$

If $R_f = R_1$ then $\boxed{V_o = -V_i}$

(Positive)



— **Precision Full Wave Rectifier:**

During positive half cycle

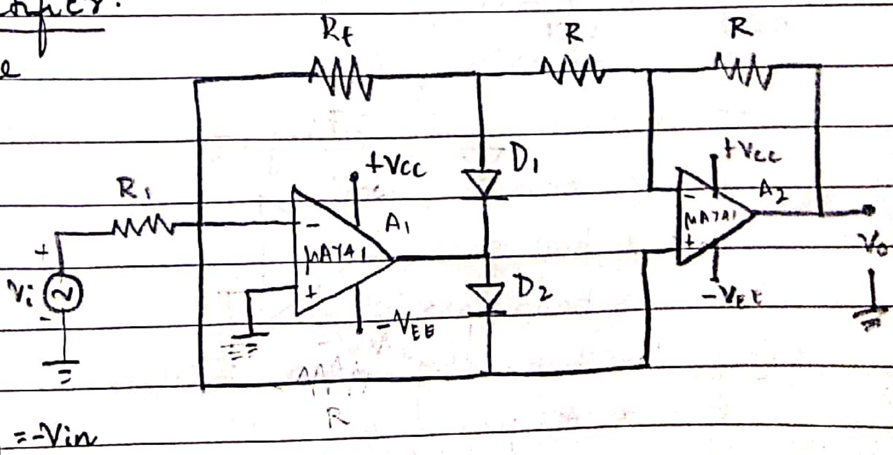
A_1 is negative

$\therefore D_1$ is forward biased

D_2 is reverse biased

Here op-amp 1 acts as inverting amplifier

$\therefore V_{o_1} = \frac{-R_f}{R_1} V_{in} \Rightarrow V_{o_1} = -V_{in}$



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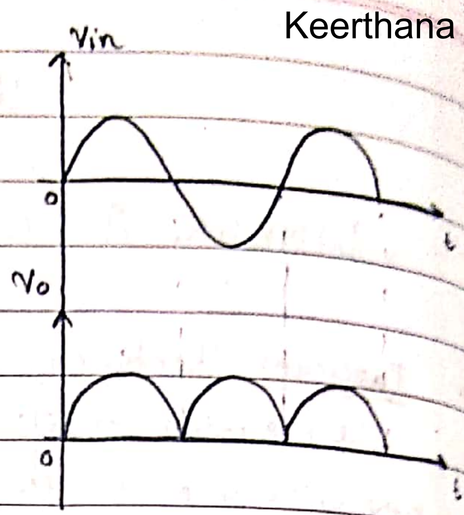


If $R_1 = R_f$ then $V_{o_1} = -V_{in}$

Now op amp 2 acts as an inverting adder

$$\therefore V_o = -\left[\frac{R}{R/2} V_{o_1} + \frac{R}{R} V_{in}\right]$$

but $V_{o_1} = -V_{in}$

$$\therefore V_o = -\left[-2V_{in} + V_{in}\right]$$

$$\therefore \boxed{V_o = V_{in}}$$

During negative half cycle

A_1 is positive

$\therefore$ D_1 is reverse biased and D_2 is forward biased

But due to virtual ground output of op amp 1, $V_{o_1} = 0$.

Now op amp 2 has an output

$$V_o = -\left[\frac{R}{R} V_{in} + \frac{R}{R/2} V_{o_1}\right]$$

$$\therefore \boxed{V_o = -V_{in}}$$

★ **Filters:**

– Based on the nature of signal processing, filters are classified as:

a. Analog filter

b. Digital filter.

– Based on the components used, filters are classified as:

a. Active filters

b. Passive filters.

– Based on the frequency, filters are classified as:

a. lowpass filter.

b. high pass filter.

3. bandpass filter.

4. band stop / band elimination / band reject filter.

5. All pass filter.

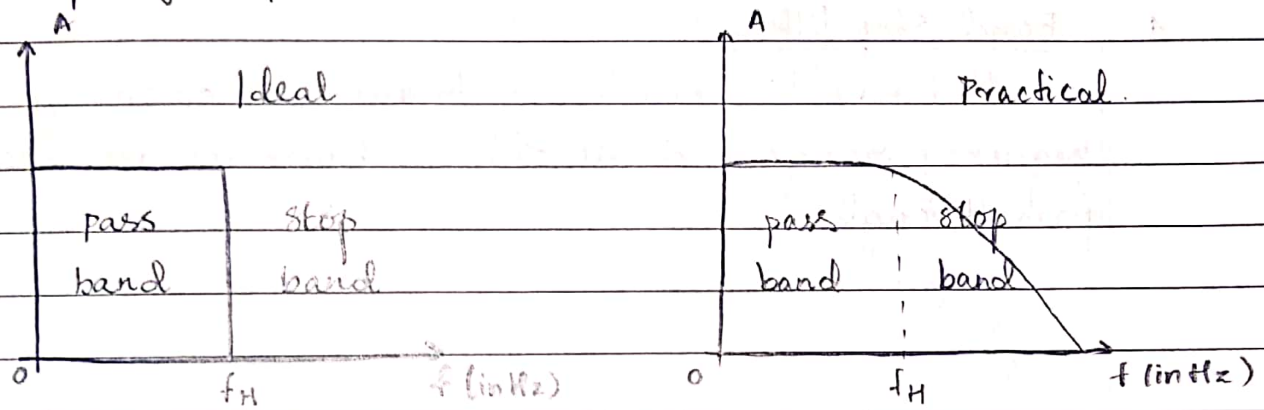
* Active filters:

Passive filters (R, L, C) along with op-amp / transistor forms an active filter.

1. Low pass filter:

It allows the frequency range upto the cut off frequency f_H, beyond which it attenuates.

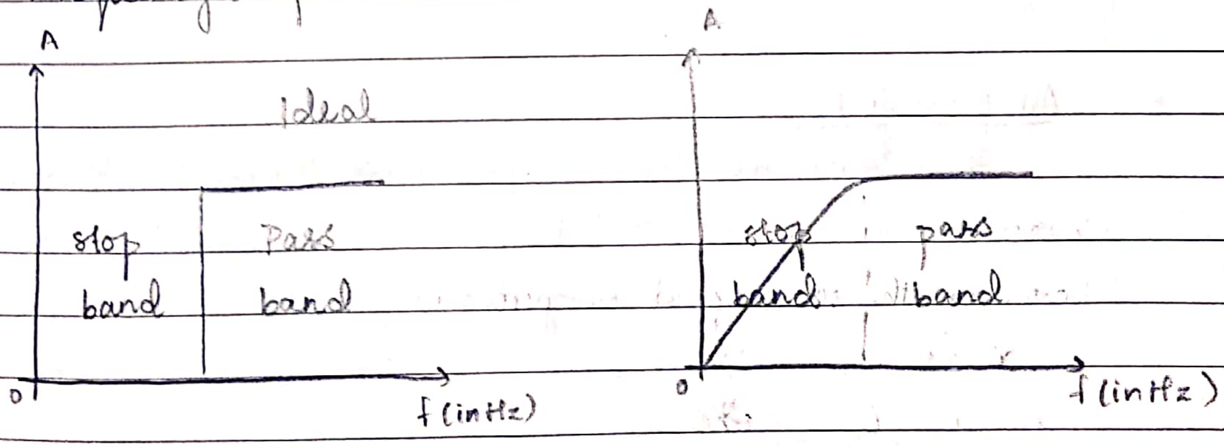
Frequency response



2. High pass filter:

It allows the frequency range beyond the cut off frequency, f_L, until f_L it is attenuated.

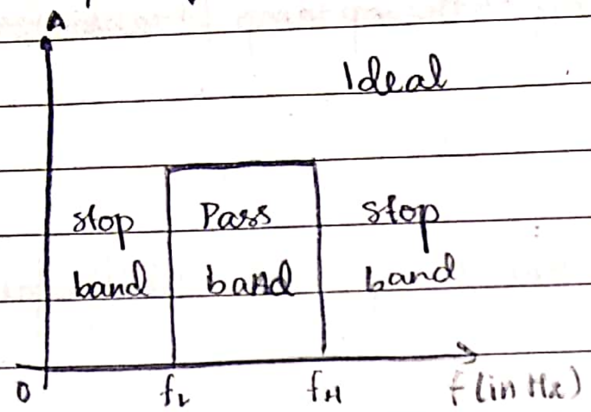
Frequency response

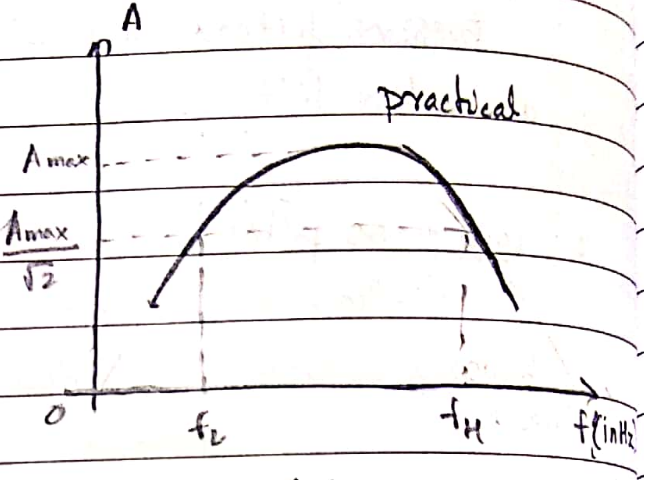


3. Band Pass filter:

It allows the band of frequency of the range f_L to f_H; other frequencies are attenuated.
(combination of high pass and low pass filter)

Frequency response

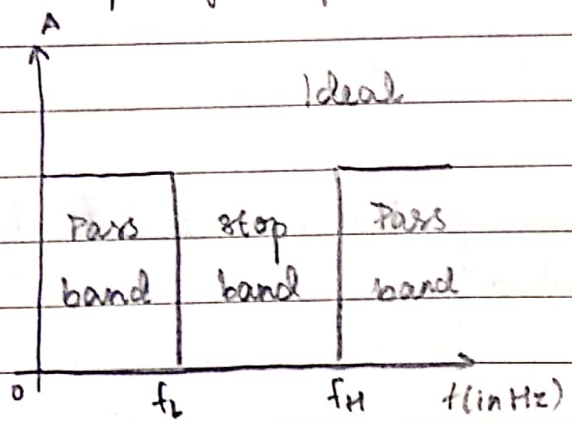


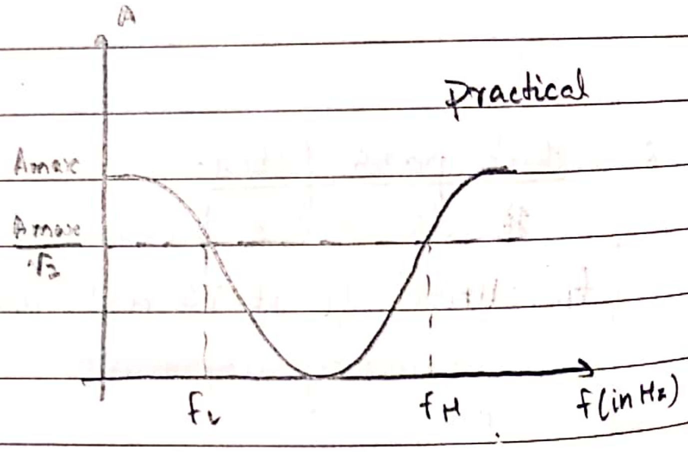


4. **Band stop filter** (notch filter : based on shape of frequency response)

It blocks / attenuates the frequencies from f_L to f_H frequency range and all other frequencies are allowed to pass through.

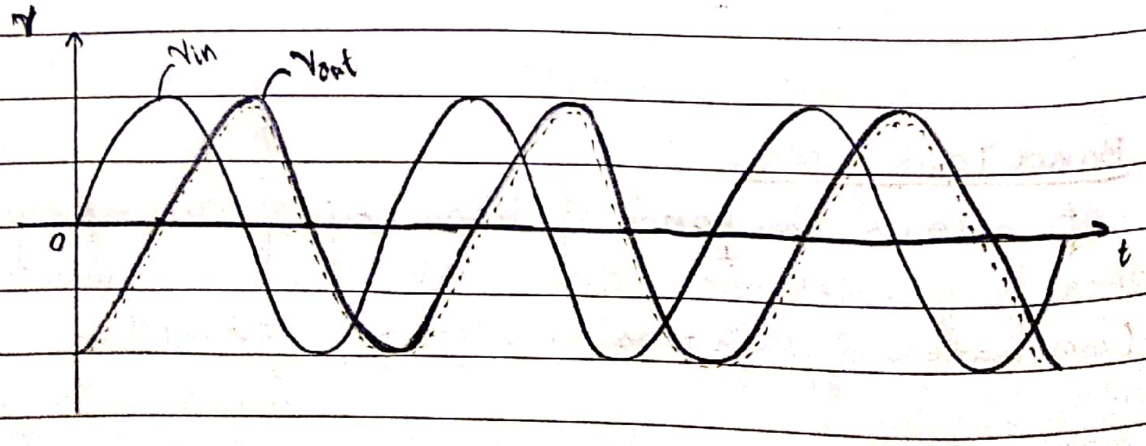
Frequency response





5. **All pass filter**

It allows all frequency to pass through, that is from zero to maximum. But there is around 90° phase shift between the input and output.



* **Order of filter:**

Depending upon the gain roll off the filters may be classified as first order, second order, third order, etc.

Order of filter	Rate of roll off of the gain
I order	−20 dB / decade
II order	−40 dB / decade
III order	−60 dB / decade
IV order	−80 dB / decade

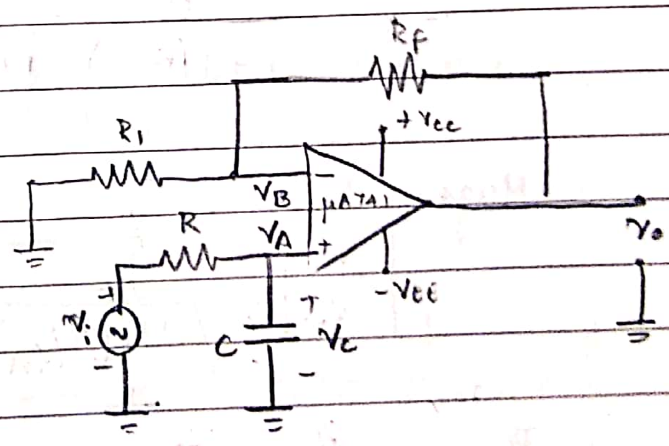
Note: Number of RC network is equal to the order of the filter.

* Filters are also classified as:
  1. Butterworth filter
  2. Chebyshev filter
  3. Elliptic filter
  4. Bessel filter

* **I order Low Pass Butterworth filter:**

For a non inverting amplifier

$$A = 1 + \frac{R_f}{R_1}$$



$$\therefore V_o = \left[1 + \frac{R_f}{R_1}\right] V_A \quad \text{—— (1)}$$

By superposition,

$$V_A = V_c = \frac{-jX_c V_i}{R - jX_c}$$

$$V_A = V_c = \frac{\frac{-j}{2\pi f c} V_i}{R - \frac{j}{2\pi f c}} = \frac{\frac{-j V_i}{2\pi f c}}{\frac{2\pi f CR - j}{2\pi f c}}$$

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$$V_A = V_C = \frac{-jV_i}{2\pi fRC - j}$$

$$V_A = V_C = \frac{V_i / j}{2\pi fRC + \frac{1}{j}}$$

$$V_A = V_C = \frac{V_i / \cancel{j}}{\frac{j2\pi fRC + 1}{\cancel{j}}}$$

$$\therefore V_A = V_C = \frac{V_i}{j2\pi fRC + 1}$$

Substituting in eq 1

$$V_o = \left[1 + \frac{R_f}{R_1}\right]\left[\frac{V_i}{1 + j2\pi fRC}\right]$$

Here $A_f = 1 + \frac{R_f}{R_1}$ and $\frac{1}{2\pi RC} = f_H$

$$\therefore V_o = A_f \left[\frac{V_i}{1 + jf/f_H}\right]$$

$$\boxed{\frac{V_o}{V_i} = \frac{A_f}{1 + j\left(\frac{f}{f_H}\right)}}$$

Magnitude

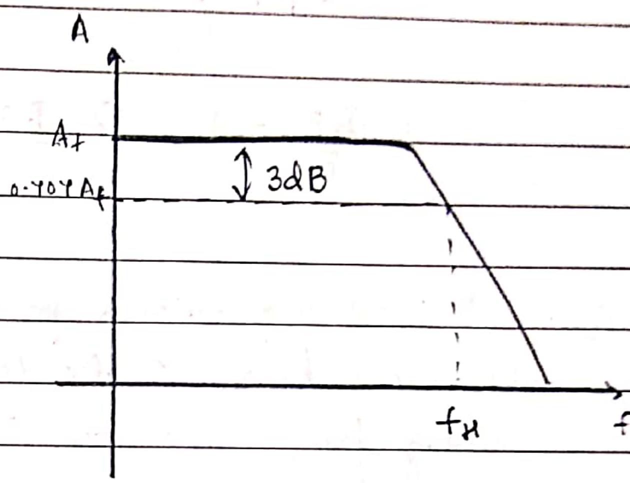
$$\boxed{\left|\frac{V_o}{V_i}\right| = \frac{A_f}{\sqrt{1 + \left(f/f_H\right)^2}}}$$

Phase angle

$$\boxed{\phi = \tan^{-1}\left(\frac{f}{f_H}\right)}$$

Frequency response of Low pass Butterworth filter:

$$\frac{v_o}{v_i} = \frac{A_f}{1+j(f/f_H)}$$



CASE 1: For frequency $f = 0$.

$A = A_f$

CASE 2: For frequencies $f < f_H$

$A = A_f$

CASE 3: For frequency $f = f_H$

$$A = \frac{A_f}{\sqrt{2}} = 0.707A_f$$

CASE 4: For frequencies $f > f_H$

$A < A_f$

Design rule for lowpass Butterworth filter:

Assume $C < 1\mu F$.

$$f_H = \frac{1}{2\pi RC}$$

Gain: $A_f = 1 + \frac{R_f}{R_1}$

Frequency scaling technique to find new resistance

Ex: $f_H = 10kHz$ $f_H' = 12kHz$

$$R_{New} = R\left(\frac{f_H}{f_H'}\right)$$

Q: Design a low pass filter at a cut off frequency of 10kHz with a pass band gain of 2.

Sol: Given: $f_H = 10kHz$ $A_f = 2$

Assuming $C = 0.01\mu F$

$$f_H = \frac{1}{2\pi RC} \Rightarrow R = \frac{1}{2\pi(10k)(0.01\mu)} = 1.59k\Omega$$

$$A_f = 1 + \frac{R_f}{R_1}$$

$$2 = 1 + \frac{R_f}{R_1}$$

$$\frac{R_F}{R_1} = 1 \Rightarrow R_f = R_1$$

For $R_f = 10k\Omega$

$R_1 = 10k\Omega$

Q2: If in the previous problem the cut off frequency is changed from 10kHz to 12kHz, find the new value of R using frequency scaling techique.

Sol: Given: $f_H = 10kHz$ $f_H' = 12kHz$

$$R_{New} = R\left(\frac{f_H}{f_H'}\right)$$

$$R_{New} = 1.59k\left(\frac{10k}{12k}\right)$$

$$R_{New} = 1.326k\Omega$$

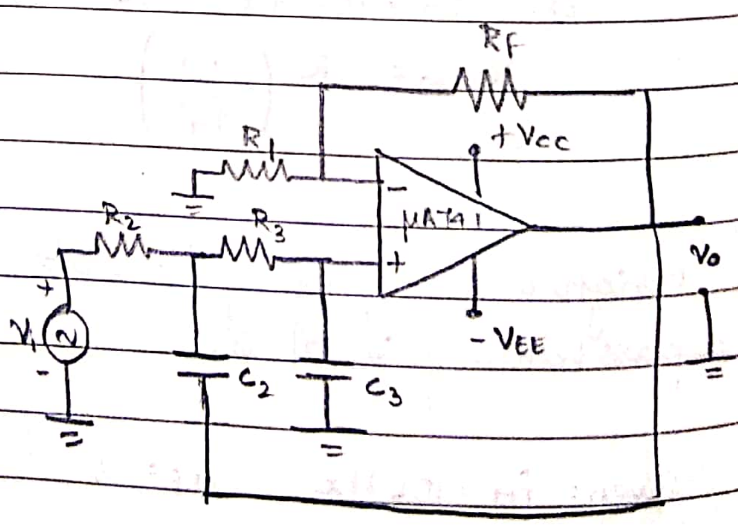
★ II Order Low Pass Butterworth Filter:

Let $R_2 = R_3 = R$

and $C_2 = C_3 = C$

$$f_H = \frac{1}{2\pi\sqrt{R_2R_3C_2C_3}}$$

$$\therefore f_H = \frac{1}{2\pi RC}$$



Magnitude

$$\boxed{\left|\frac{V_O}{V_i}\right| = \frac{A_f}{\sqrt{1+\left(\frac{f}{f_H}\right)^4}}}$$

Designing of II order Butterworth low pass filter :

Assume $C < 1\mu F$

$$A_f = 1 + \frac{R_F}{R_1}$$

$$f_H = \frac{1}{2\pi\sqrt{R_2R_3C_2C_3}}$$ If $R_2 = R_3$ & $C_2 = C_3$ then $f_H = \frac{1}{2\pi RC}$

$R_1 \leq 100\,k\Omega$

If the gain value is not given then assume that

$$A_f = 1 + \frac{R_f}{R_1} = 1.586$$

where $\frac{R_f}{R_1} = 0.586 \Rightarrow R_f = 0.586R_1$

Q1: Design a II order low pass Butterworth filter to have a higher cutoff frequency of 1.5 kHz.

Sol: Given: $f_H = 1.5\,kHz$

Assuming $C = 0.05\mu F$

$$f_H = \frac{1}{2\pi RC}$$

$$\Rightarrow R = \frac{1}{2\pi f_H C} = \frac{1}{2\pi(1.5k)(0.05\mu)} = \underline{2.12\,k\Omega}$$

Since A_f is not given, assuming $A_f = 1.586$

$$\therefore 1 + \frac{R_f}{R_1} = 1.586$$

$$\frac{R_f}{R_1} = 0.586$$

$$R_f = 0.586R_1$$

Assuming $R_1 = 27\,k\Omega$

$$\underline{R_f = 15.822\,k\Omega}$$

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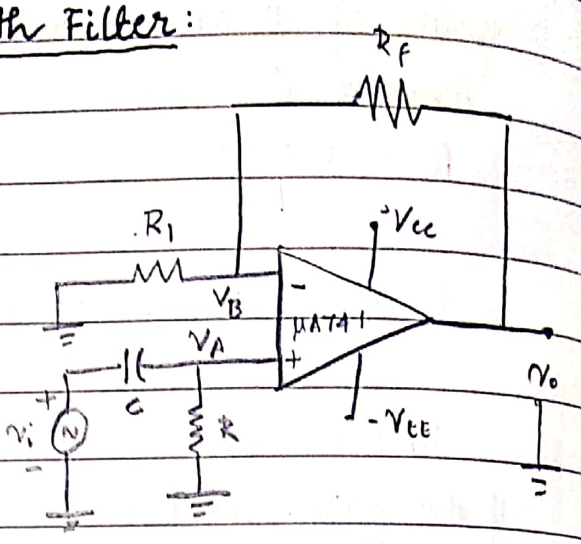
* I Order high pass Butterworth Filter:

for non inverting amplifier



$$A = 1 + \frac{R_f}{R_1}$$

$$\therefore \quad V_o = \left[1 + \frac{R_f}{R_1}\right] V_A \quad \text{——} \; (1)$$

By superposition

$$V_A = \frac{R V_i}{R - jX_c}$$

$$V_A = \frac{R V_i}{jX_c\left(\frac{R}{jX_c} - 1\right)} \qquad \text{where } X_C = \frac{1}{2\pi f C}$$

$$V_A = \frac{R V_i}{\frac{j}{2\pi f C}\left[\frac{2\pi f RC}{j} - 1\right]} = \frac{R V_i}{\frac{j}{2\pi f C}\left[\frac{2\pi f RC - j}{j}\right]}$$

$$V_A = \frac{2\pi f RC\, V_i}{2\pi f RC - j} = \frac{2\pi f RC\, V_i}{2\pi f RC + 1/j}$$

$$V_A = \frac{j\, 2\pi f RC V_i}{j\, 2\pi f RC + 1}$$

Substituting in eq (1)

$$V_o = \left[1 + \frac{R_f}{R_1}\right]\left[\frac{j\, 2\pi f RC\, V_i}{j\, 2\pi f RC + 1}\right]$$

$$V_o = \left[1 + \frac{R_f}{R_1}\right]\left[\frac{jf/f_L}{jf/f_L + 1}\right] V_i \qquad \frac{1}{2\pi RC} = f_L$$

Here $\left[1 + \frac{R_f}{R_1}\right] = A_F$

$$\boxed{\frac{V_o}{V_i} = \frac{A_F\, j(f/f_L)}{j(f/f_L) + 1}}$$

Magnitude

$$\boxed{\left|\frac{V_o}{V_i}\right| = \frac{A_F (f/f_L)}{\sqrt{1 + (f/f_L)^2}}}$$

Frequency response of High Pass Butterworth filter:

$$\frac{v_o}{v_i} = \frac{A_f\, j(f/f_L)}{1 + j(f/f_L)}$$

CASE 1: $f = 0$

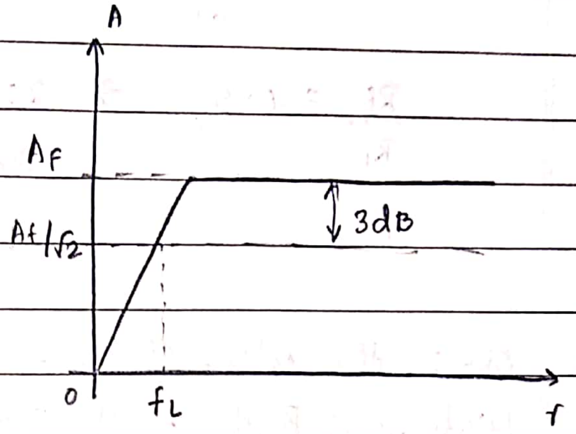
$A = 0$

CASE 2: $f < f_L$

$A < A_f$

CASE 3: $f \simeq f_L$

$A = \frac{A_f}{\sqrt{2}} = 0.707 A_f$



CASE 4: $f > f_L$

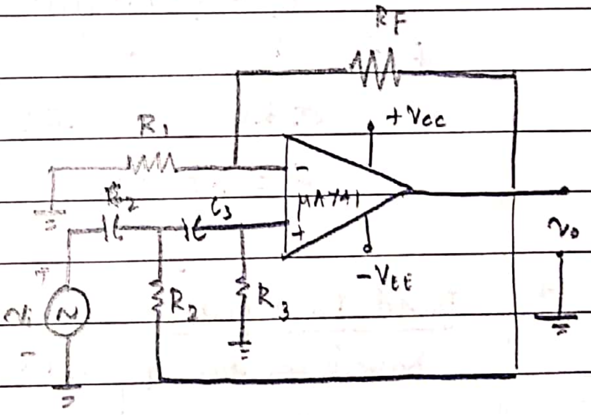
$A = A_f$

* **II order high pass Butterworth Filter:**

Let $R_2 = R_3 = R$ and $C_2 = C_3 = C$

$$f_L = \frac{1}{2\pi\sqrt{R_2 R_3 C_2 C_3}}$$

$$\therefore f_L = \frac{1}{2\pi RC}$$



Magnitude

$$\boxed{\left|\frac{v_o}{v_i}\right| = \frac{A_f (f/f_L)}{\sqrt{1 + \left(\frac{f}{f_4}\right)^4}}}$$

Q1: Design a first order high pass filter to have a lower cut off frequency of 1.5kHz and the pass band gain of 1.5.

Sol: Given: $f_L = 1.5\,kHz$ $A = 1.5$

Assuming $C = 0.01\mu F$

$$f_L = \frac{1}{2\pi RC} \Rightarrow R = \frac{1}{2\pi f C} = \frac{1}{2\pi(1.5k)(0.01\mu)} = \underline{10.61\,k\Omega}$$

$$A_f = 1 + \frac{R_f}{R_1}$$

$$1.5 = 1 + \frac{R_f}{R_1}$$

$$\frac{R_f}{R_1} = 0.5 \Rightarrow R_f = 0.5R_1$$

Assuming $R_1 = 10k\Omega$

$\therefore R_f = 5k\Omega$

Q2: Obtain the value of lower cut off frequency of II order High pass Butterworth filter. Given: $R_1 = 47k\Omega$ and $R_2 = R_3 = 10k\Omega$, $C_2 = C_3 = 0.01\mu F$.

Sol: Given: $R_1 = 47k\Omega$

$R_2 = R_3 = 10k\Omega$

$C_2 = C_3 = 0.01\mu F$

$$f = \frac{1}{2\pi RC} = \frac{1}{2\pi(10k)(0.01\mu)}$$

$$f = 1.59\,kHz$$

* **Band Pass Filter:**

Based on the Q factor it is classified as
1. Wide band pass filter
2. Narrow band pass filter

$$Q\text{-factor} = \frac{f_c}{BW} \Rightarrow Q \propto \frac{1}{BW} \qquad Q = \frac{f_c}{f_H - f_L}$$

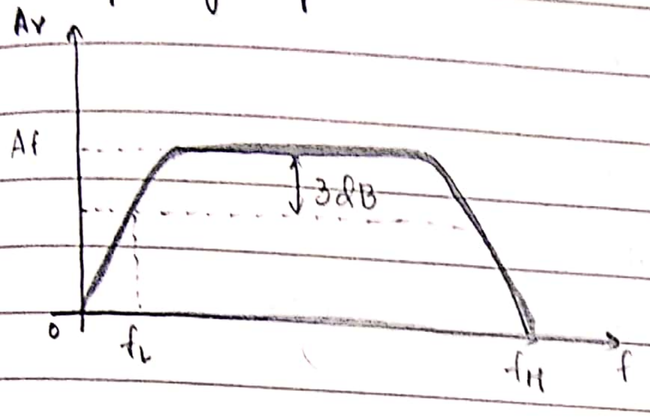
If $Q < 10$: wide band pass filter

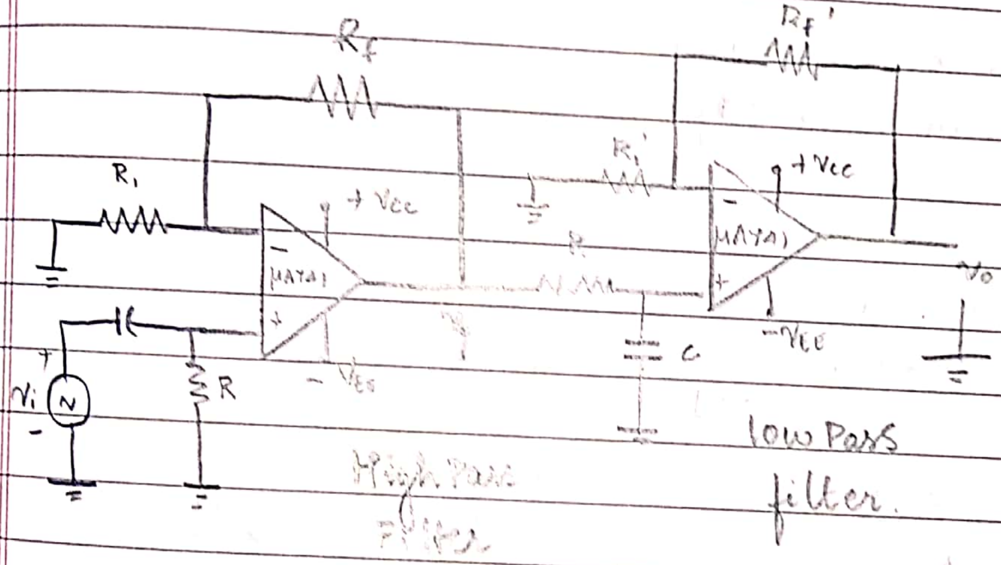
If $Q > 10$: Narrow band pass filter.

Band pass filter allows all the frequencies in the range from lower cut off frequency f_L to higher cut off frequency f_H. All other frequencies are attenuated.

- **Wide Bandpass Filter :** (high pass filter + low pass filter)

Frequency responce





Gain is given by

$$A_f = A_1 \cdot A_2$$

High pass filter: $A_1 = \dfrac{A_{f_1}(f/f_L)}{\sqrt{1+(f/f_L)^2}}$

Low pass filter: $A_2 = \dfrac{A_{f_2}}{\sqrt{1+(f/f_H)^2}}$

$$\therefore \boxed{A_f = \frac{A_{f_1}(f/f_L)}{\sqrt{1+(f/f_L)^2}} \cdot \frac{A_{f_2}}{\sqrt{1+(f/f_H)^2}}}$$

Q1: Design a wide band pass filter to have lower and higher cut off frequency of 500Hz and 1.5kHz respectively and a pass band gain of 4. Also calculate Q-factor of the filter.

Sol: Given: $f_L = 500Hz$ $f_H = 1.5kHz$

$A_f = 4$.

To design high pass filter

assuming $C_1 = 0.01\mu F$.

$$f_L = \frac{1}{2\pi RC}$$

$$\Rightarrow R = \frac{1}{2\pi f_L C_1} = \frac{1}{2\pi(500)(0.01\mu)} = \underline{\underline{31.8k\Omega}}$$

To design low pass filter

assuming $C_2 = 0.01\mu F$.

$$f_H = \frac{1}{2\pi R_2 C_2}$$

$$\Rightarrow R = \frac{1}{2\pi f_H C_2} = \frac{1}{2\pi(1.5k)(0.01\mu)} = \underline{\underline{10.61k\Omega}}$$

$A_f = 4$

$A_1 = A_2 = \sqrt{4} = 2$

$(\because A_1 A_2 = 4)$

$$2 = 1 + \frac{R_F}{R_1}$$

$\therefore$ $\underline{\underline{R_F = R_1 = 10k\Omega}}$

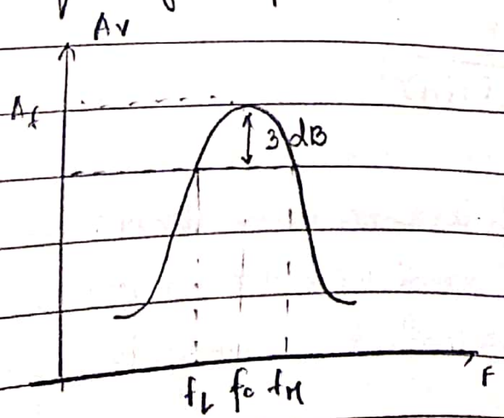
$$Q\text{-factor} = \frac{f_c}{f_H - f_L}$$

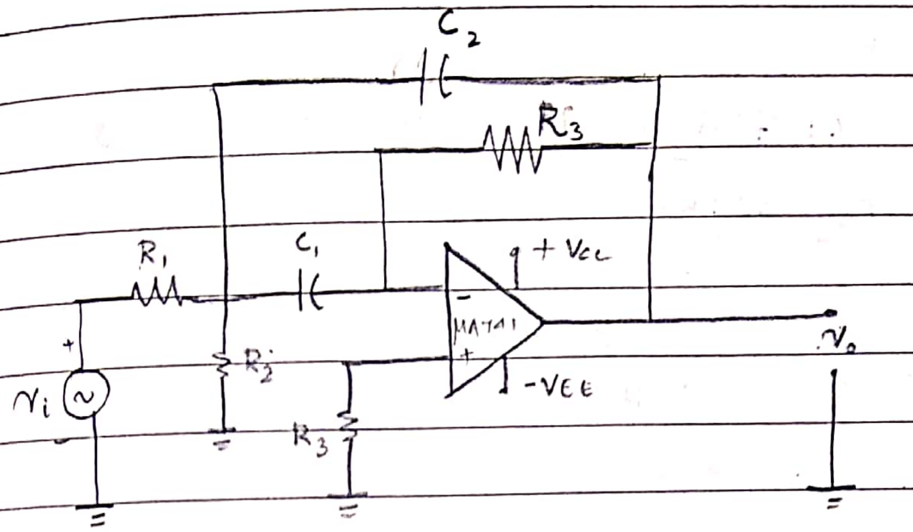
$$Q = \frac{\sqrt{f_L f_H}}{f_H - f_L}$$

$$Q = \frac{\sqrt{500(1.5k)}}{1500 - 500} = \underline{\underline{0.866}}$$

★ **Narrow Band Pass Filter:**

Frequency response





Design rule:

Take $C_1 = C_2 = C$ and $C < 1\mu F$

calculate all resistors based on the relationship given below:

$$R_1 = \frac{Q}{2\pi f_c C A_f} \qquad A_f = \frac{R_3}{2R_1}$$

$$R_2 = \frac{Q}{2\pi f_c C (2Q^2 - A_f)} \qquad A_f < 2Q^2$$

$$R_3 = \frac{Q}{\pi f_c C}$$

If the center frequency changes from f_c to f_c'

$$R_2' = R_2 \left(\frac{f_c}{f_c'}\right)^2$$

Q1: Design a narrow pass band filter for which $f_c = 1kHz$, $Q = 3$ and $A_f = 10$. How do you change the centre frequency to 1.2 kHz keeping A_f and BW constant.

Sol: Given: $f_c = 1kHz$; $Q = 3$; $A_f = 10$

$f_c' = 1.2 kHz$

$C_1 = C_2 = C = 0.01\mu F$

$$R_1 = \frac{Q}{2\pi f_c C A_f} = \frac{3}{2\pi (1k)(0.01\mu F)(10)}$$

$R_1 = 4.77k\Omega$

$$R_2 = \frac{Q}{2\pi f_c C(2Q^2 - A_f)} = \frac{3}{2\pi(1k)(0.01\mu)(2(9)-10)}$$

$$R_2 = 5.9\,k\Omega$$

$$R_3 = \frac{Q}{\pi f_c C} = \frac{3}{\pi(1k)(0.01\mu)}$$

$$R_3 = 95.49\,k\Omega$$

$$R_2' = R_2\left(\frac{f}{f'}\right)^2 = 5.9k\left[\frac{1k}{1.2k}\right]^2$$

$$R_2' = 4.14\,k\Omega$$

* **Band Stop Filter:**

It is classified as:

1. Wide band stop filter
2. Narrow band stop filter.

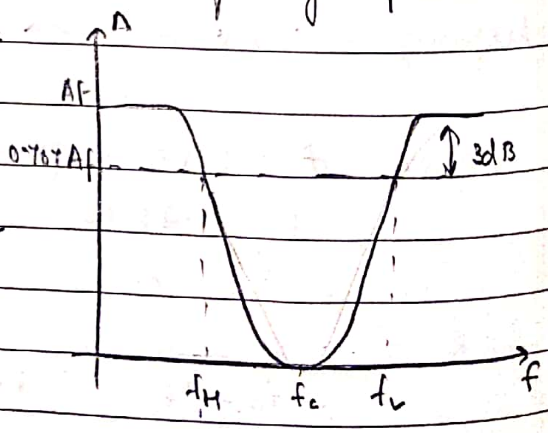
– **Wide band stop filter:**

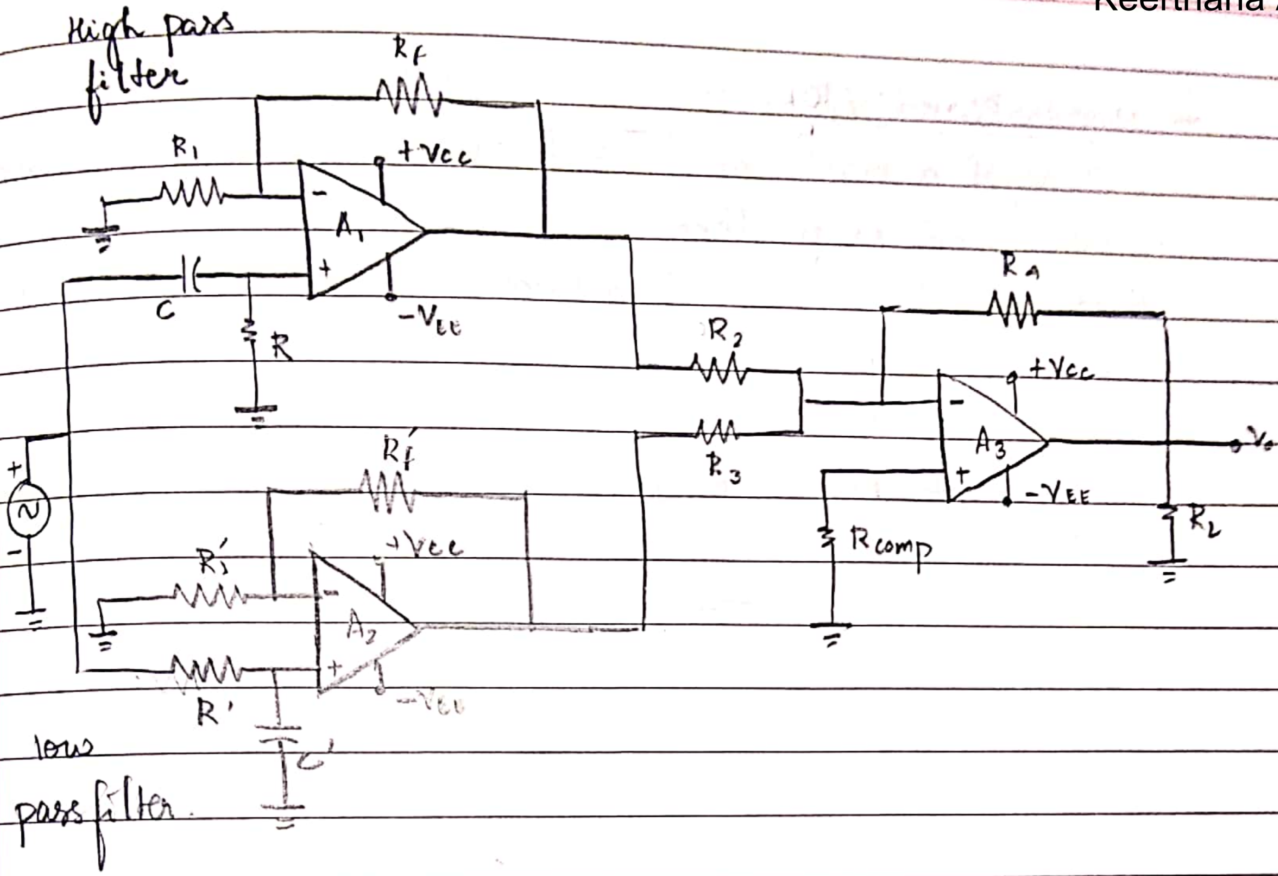
(Lowpass filter + high pass filter + summing amplifier)

– The lowpass and the high pass filter has equal pass band gain. This condition has to satisfied for a wide band stop filter.

– The lower cut off frequency f_L of high pass filter should be greater than higher cut off frequency f_H of low pass filter.

Frequency response





Q: Design a wide band stop filter to have $f_H = 500Hz$ and $f_L = 1.5kHz$.

Sol: Given: $f_L = 1.5kHz$; $f_H = 500Hz$

Assuming $C = 0.05\mu F$ and $C' = 0.01\mu F$

$$f_H = \frac{1}{2\pi RC}$$

$$\Rightarrow R = \frac{1}{2\pi f_H C} = \frac{1}{2\pi(500)(0.05\mu)} = \underline{6.36k\Omega}$$

$$f_L = \frac{1}{2\pi R'C'}$$

$$\Rightarrow R' = \frac{1}{2\pi f_L C'} = \frac{1}{2\pi(1.5k)(0.01\mu)} = \underline{10.61k\Omega}$$

$R_1' = R_f' = 10k\Omega$

$R_1 = R_f = 10k\Omega$

If the gain of the summing amplifier is assumed as 1,

$R_2 = R_3 = R_4 = 10k\Omega \Rightarrow R_{comp} = \frac{10k}{3} = \underline{3.33k\Omega}$

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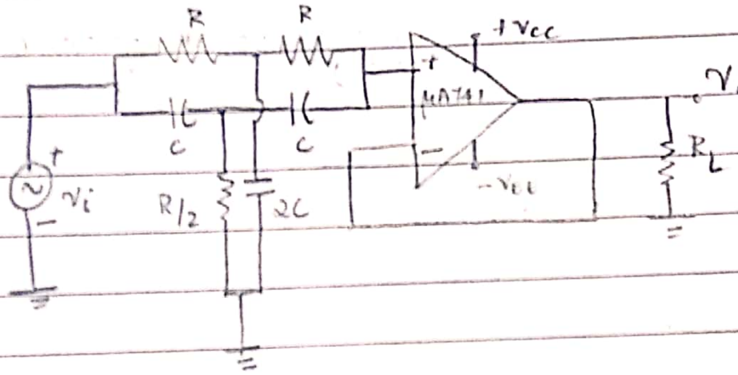
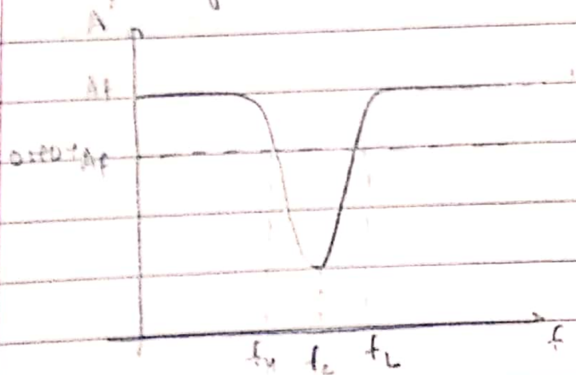
- Narrow Band stop Filter: (Notch filter)

To reject a signal at a particular frequency we used narrow band stop filter.

It is used in a twin-T network



Frequency response



Notch filter frequency

$$f_N = \frac{1}{2\pi RC}$$

Q: Design an active notch filter to eliminate 100Hz frequency.

Sol: $f_N = 100\,Hz$

Assuming $C = 0.01\mu F$

$$f_N = \frac{1}{2\pi RC}$$

$$\Rightarrow R = \frac{1}{2\pi f_N C} = \frac{1}{2\pi(100)(0.01\mu)} = \underline{\underline{159.1k\Omega}}$$

$\underline{\underline{R/2 = 79.57k\Omega}}$ $\quad$ $\underline{\underline{2C = 0.02\mu F}}$

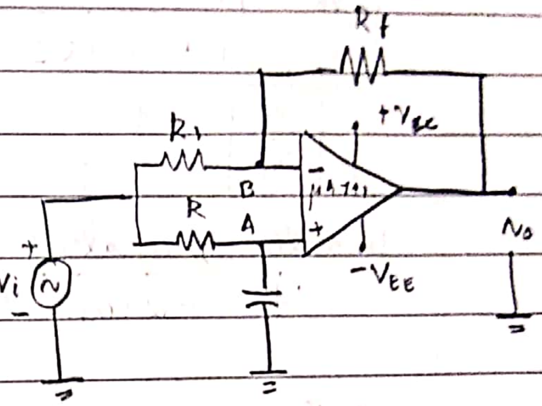
* All Pass Filter:

It allows all the frequencies with certain phase shift.

Let us consider $R_1 = R_f$

Using superposition

R_1 is grounded hence acts as non inverting amplifier.



$$\therefore V_{o_1} = \left[1 + \frac{R_f}{R_1}\right] V_A$$

$$V_{o_1} = 2V_A \qquad (\because R_f = R_1)$$

R is grounded hence acts as inverting amplifier.

$$V_{o_2} = \frac{-R_f}{R_1} V_i$$

$$V_{o_2} = -V_i \qquad (\because R_f = R_1)$$

By Voltage divider rule

$$V_A = \left[\frac{-jX_c}{R - jX_c}\right] V_i$$

$$\text{but } X_c = \frac{1}{2\pi f C}$$

$$V_A = \frac{V_i}{1 + j2\pi f RC}$$

Therefore

$$V_o = V_{o_1} + V_{o_2}$$

$$V_o = 2V_A - V_i$$

$$V_o = \frac{2V_i}{1 + j2\pi f RC} - V_i$$

$$V_o = V_i\left[\frac{2}{1 + j2\pi f RC} - 1\right]$$

$$v_o = v_i \left[\frac{1 - j2\pi fRC}{1 + j2\pi fRC} \right]$$

$$\left| \frac{v_o}{v_i} \right| = \frac{\sqrt{1 + (2\pi fRC)^2}}{\sqrt{1 + (2\pi fRC)^2}}$$

$$\boxed{\left| \frac{v_o}{v_i} \right| = 1}$$

In order to analyse phase shift ϕ, we have :

$$\boxed{\phi = -2\tan^{-1}(2\pi fRC)}$$

If the phase shift is negative, then the output lags behind the input by an angle ϕ.

If the phase shift is positive, then the output leads the input by an angle ϕ.

Q: Find the phase shift in an all pass filter given a signal frequency 2kHz and R = 20 kΩ and C = 0.01μF.

Sol: Given: $f = 2\,kHz$

$R = 20\,k\Omega$

$C = 0.01\,\mu F$

$$\phi = -2\tan^{-1}(2\pi fRC)$$

$$\phi = -2\tan^{-1}(2\pi(2k)(20k)(0.01\mu))$$

$$\phi = -2\tan^{-1}(2.5132)$$

$$\phi = -136.6^\circ$$

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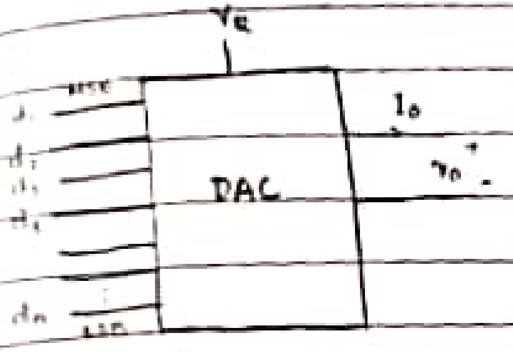
Data converters:

Digital to Analog Data converters:

It can be done by:

1. Binary Weighted Resistor
2. R-2R ladder Network.



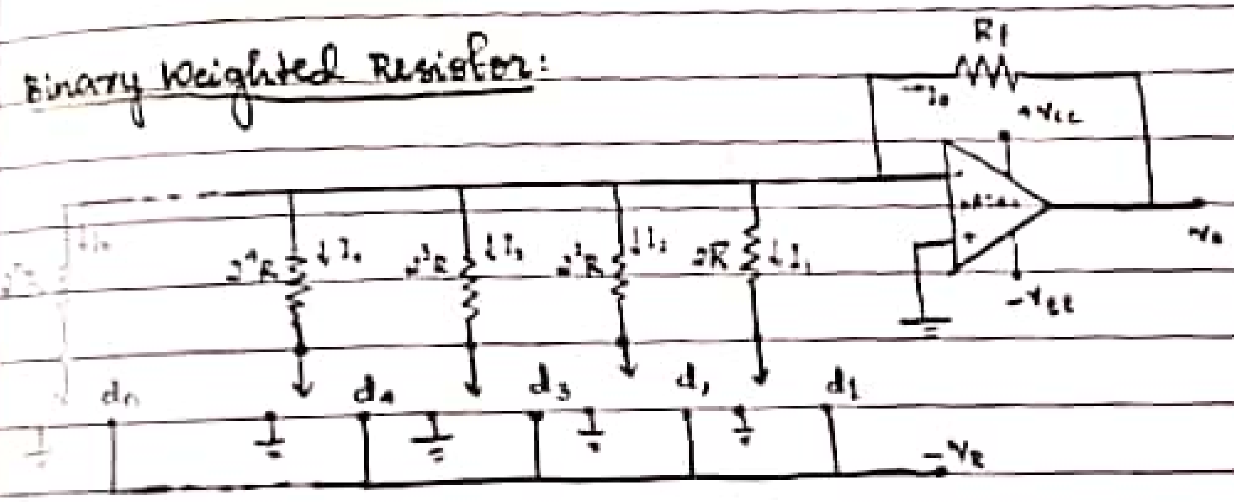
Output voltage

$V_0 = k V_{FS} (d_1 2^{-1} + d_2 2^{-2} + \ldots d_n 2^{-n})$

where k : scaling factor = 1.

V_{FS} : Full scale output voltage V_{FS}.

1. Binary Weighted Resistor:



$I_0 = I_1 + I_2 + I_3 + \cdots I_n$

$$I_0 = \frac{V_R}{2R} d_1 + \frac{V_R}{2^2 R} d_2 + \frac{V_R}{2^3 R} d_3 + \frac{V_R}{2^4 R} d_4 + \cdots + \frac{V_R}{2^n R} d_n$$

$$I_0 = \frac{V_R}{R} \left[2^{-1} d_1 + 2^{-2} d_2 + 2^{-3} d_3 + 2^{-4} d_4 + \cdots + 2^{-n} d_n \right]$$

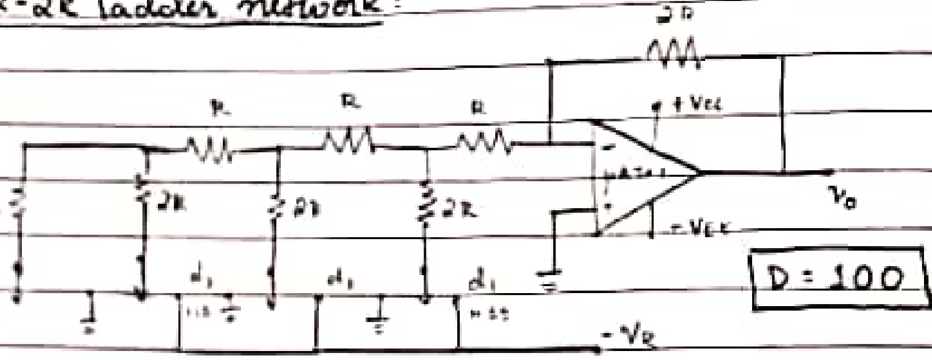
but, $V_0 = I_0 R_f$

$$\therefore V_0 = \frac{V_R}{\cancel{R}} \left[2^{-1} d_1 + 2^{-2} d_2 + 2^{-3} d_3 + \cdots + 2^{-n} d_n \right] \cancel{R_f}$$

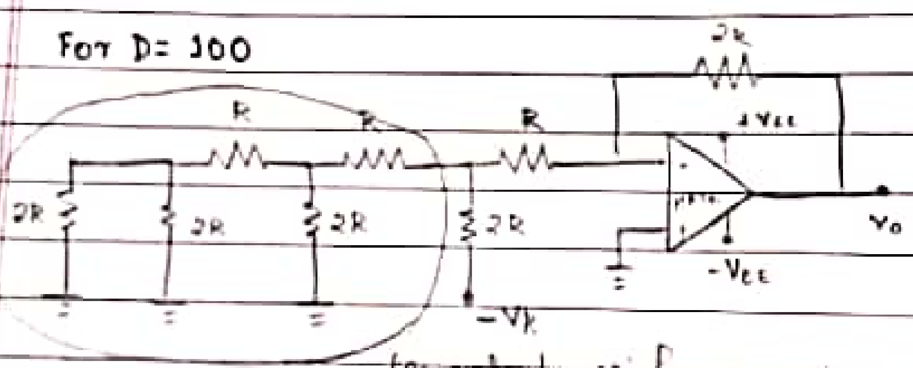
$$\therefore \boxed{V_0 = V_R \left[2^{-1} d_1 + 2^{-2} d_2 + 2^{-3} d_3 + \cdots + 2^{-n} d_n \right]}$$

But there will be a voltage drop across each resistor hence to overcome this drawback by R-2R ladder network

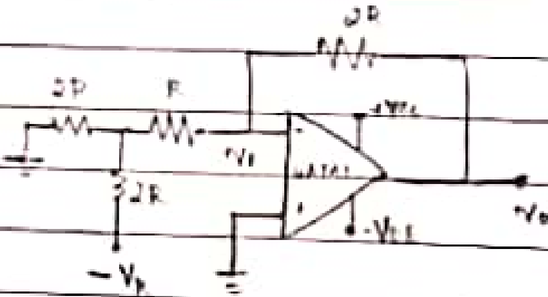
2. R-2R ladder network:



For D = 100



Equivalent resistance = 2R

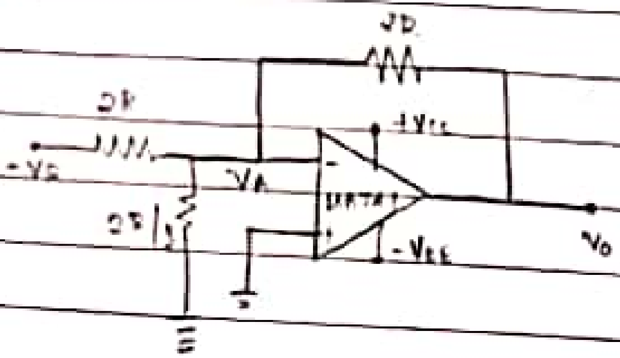


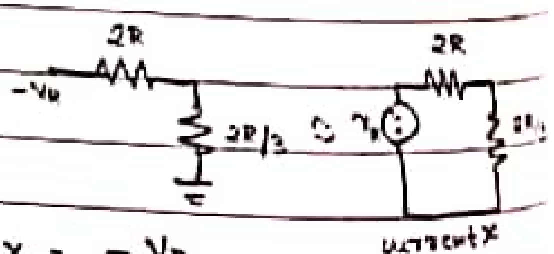
Due to virtual ground

$V_A = 0$

$\therefore$ resistance R is grounded at A.

$\therefore R \| 2R = 2R/3$





$$X = \frac{-V_R}{(2R + 2R/3)}$$

$$\therefore X = \frac{-3V_R}{8R}$$

Voltage at node A

$$V_A = IR$$

$$V_A = x \frac{2R}{3}$$

$$V_A = \frac{-3V_R}{8R}\left(\frac{2R}{3}\right)$$

$$V_A = \frac{-V_R}{4}$$

For inverting amplifier

$$V_0 = \frac{-R_f}{R_1} V_i$$

$$V_0 = \frac{-2R}{R}\left[\frac{-V_R}{4}\right]$$

$$V_0 = \frac{V_R}{2}$$