Keerthana Ashok

classmate
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INTRODUCTION:

Circuit Analysis is the fundation for electrical technology. The function of the circuit in terms of two basic building blocks - charge and energy.

The Ohm's law, $V = IR$ can be successfully applied to simple circuits, but in practice, the circuits may consist of one or more sources and number of electrical parameters connected in different ways.

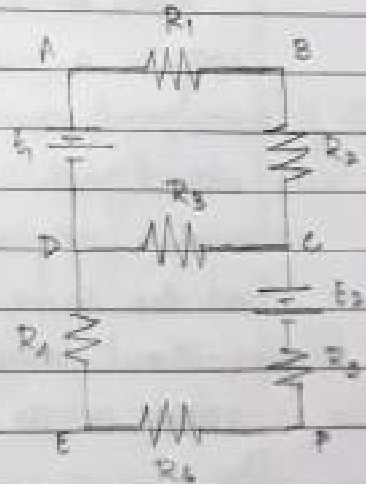
The different electrical parameters or elements are resistors, capacitors and inductors. The combination of such elements along with various sources of energy give rise to complicated electrical circuits referred as network.

The network analysis means to find the current through or voltage across any branch of the network by using fundamental laws and various simplification techniques

NETWORK TOPOLOGY:

1. Network:

Any arrangement of various energy sources along with different circuit elements is called an electrical network. Such a network is shown in the figure.



2. Network elements:

Any individual circuit element with two terminals which can be connected to other circuit element is called network elements. Network elements can either be passive elements or active elements. Active elements are the elements which supply power or energy to the network (voltage source and current source). Passive elements are the elements which store energy or dissipate energy in the form of heat. Resistors, capacitors and inductors are passive elements.

3. <u>Branch:</u>

A part of the network which connects various parts of the network with one another is called a branch. In the figure: AB, BC, CD, DA, DE, EF, CF are the various branches.

4. <u>Junction Point:</u>

A point where three or more branches meet is called a junction point. Point D and C are the junction points of the given network.

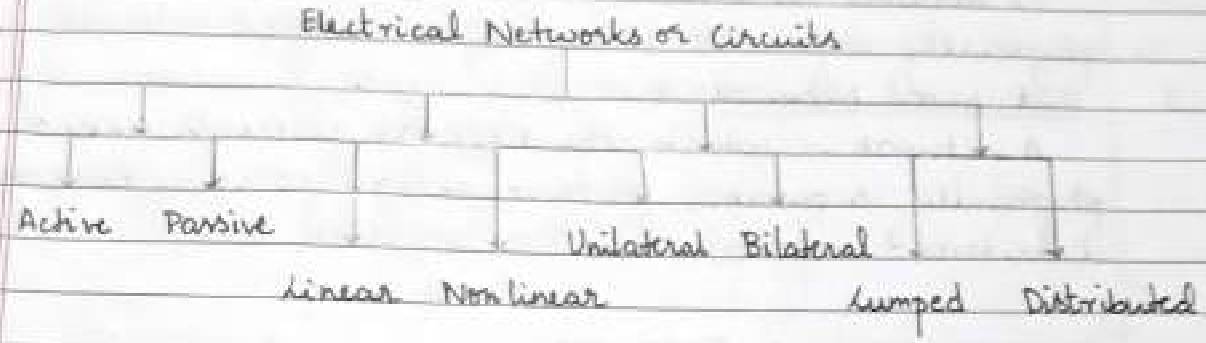
5. <u>Node:</u>

A point at which two or more elements are joined together is called node. The junction points are also the nodes of the network. In the network shown A, B, C, D, E, F are the nodes of the network.

6. <u>Mesh / loop:</u>

Mesh ~~or loop~~ is a set of branches forming a closed loop in a network in such a way that if one branch is removed then remaining branches do not form a closed path. A loop also can be defined as a closed path which originates from a particular node, terminating at the same node travelling through various ~~other~~ nodes without travelling through any node twice. In the figure ABCDA, DCFED and ABCFEDA are the loops.

CLASSIFICATION OF NETWORKS:



1. **Linear Network:**
A circuit or network whose parameters or elements like resistors, capacitors and inductors are always constant irrespective of the change in time, voltage and temperature.

2. **Non-linear Network:**
A circuit or network whose parameters or elements like resistors, capacitors and inductors are varied due to the change in time, voltage and temperature.

3. **Bilateral Network:**
A circuit whose characteristics is same irrespective of the direction of current through various elements of it is called bilateral network. Ex: Network consisting only resistors.

4. **Unilateral Network:**
A circuit whose characteristics is different with respect to the direction of current through various elements of it is called unilateral network. Ex: Network consisting only diodes.

5. **Active Network:**
A network which contains atleast one source of energy is called active network.

6. **Passive Network:**
A circuit which contains no source of energy is called passive network.

7. ~~Lumped Network~~: Distributed network:
A network in which all the network elements are physically separable is known as lumped network.

8. ~~Distributed Network~~: Lumped network:
A network in which the network elements cannot be physically separable for analysis purposes is known as distributed network. Ex: Transmission lines.

★ PASSIVE CIRCUIT ELEMENTS:

1. Resistors:
Power absorbed by the resistor is given by $P = VI$
The equation for energy absorbed or delivered to a resistor is $Pt = VIt = \frac{V^2t}{R} = I^2Rt$

2. Inductors:
Total voltage V across N turns of an inductor is $V = N\frac{d\phi}{dt}$
The power in an inductor is given by $d/dt[1/2\,Li^2]$
The energy stored in an inductor is $\frac{1}{2}LI^2$.

3. Capacitor:
Power in a capacitor is given by $d/dt[1/2\,CV^2]$
Voltage across the capacitor is given by q/C
The energy stored in a capacitor is $\frac{1}{2}CV^2$

★ ACTIVE CIRCUIT ELEMENTS:
There are two types of energy sources: Voltage source and current source. These are classified as
i) Ideal source
ii) Practical source.

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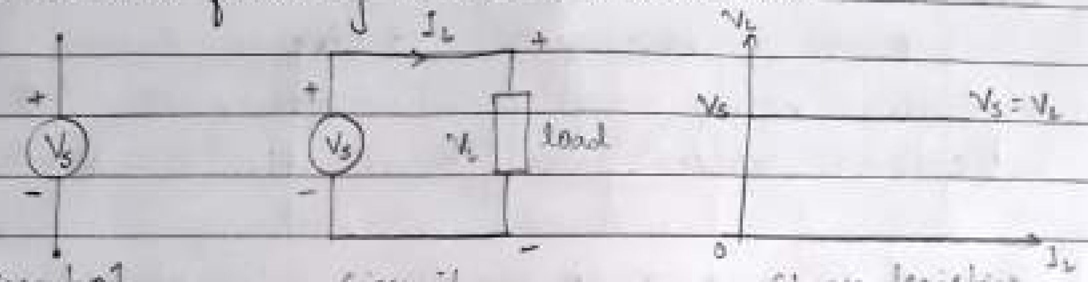
1. <u>Ideal Voltage Source:</u>

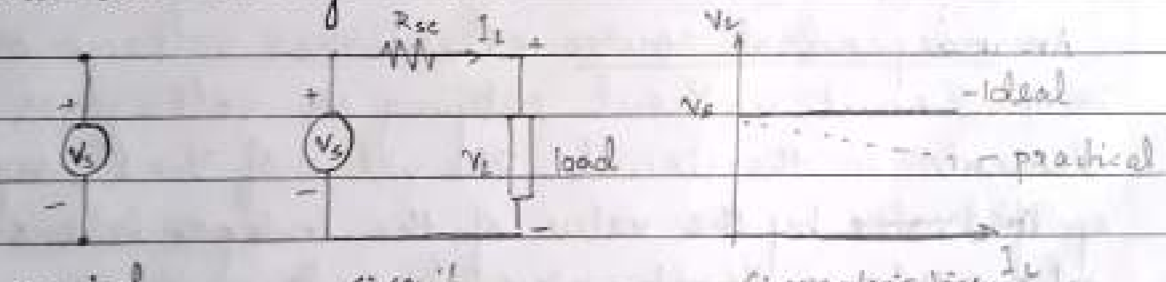
An ideal voltage source is a circuit element that maintains the prescribed voltage across the terminals regardless of the current flowing in those terminals.



symbol   circuit   Characteristics

2. <u>Practical Voltage Source:</u>

Practical voltage source has a small internal resistance R_{se} with voltage source



symbol   circuit   Characteristics

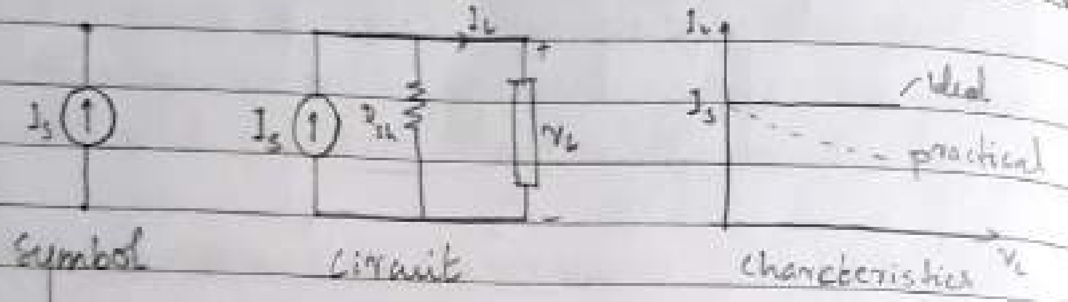
3. <u>Ideal Current Source:</u>

An ideal current source is a circuit element that maintains a prescribed current through its terminals regardless of the voltage across the terminals.

Symbol   circuit   Characteristics

4. <u>Practical current source:</u>
Practical current source has high internal resistance shown in parallel with current source and is denoted as R_{sh}.

$I_s = I_L + I_{sh}$ $\quad I_{sh} \uparrow \quad I_L \downarrow$



– Ideal voltage and current sources can be further divided into: independent sources or dependent sources.

An <u>independent source</u> establishes voltage or current in the circuit without relying on voltage or current elements in the circuit. The value of V or I supply is specified ~~value~~ by the value of the independent source ~~alone In contrast, value depends~~

In contrast, a <u>dependent source</u> establishes a voltage or current whose value depends on voltage or current elsewhere in the circuit. We cannot specify the value of a dependent source, unless we know the value of voltage or current on which it depends.

<u>NOTE:</u>
Note that a circle is used to represent an independent sources. Diamond symbol is used to represent the dependent source.

<u>Dependent sources</u> are the values depend on voltage or current in the circuit and further classified as

1. <u>Voltage dependent voltage source</u> (VDVS)
It produces the voltage as a function of voltages

else where in the given circuit.

ii. Current dependent current source (CDCS)

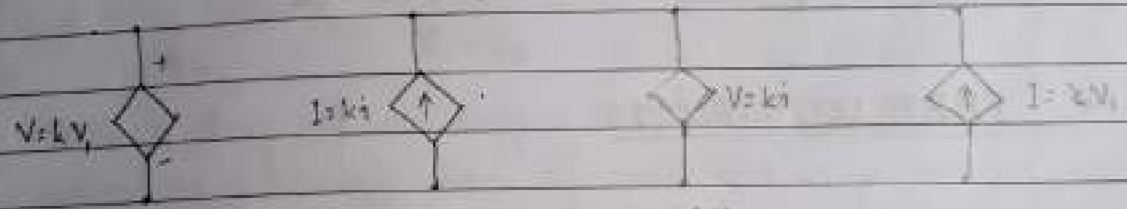
It produces ~~current~~ current as a function of current else where in the given circuit.

iii. Current dependent voltage source (CDVS)

It produces a voltage as a function of current

iv. Voltage dependent current source (VDCS)

It produces a current as a function of voltage where k is a constant V_1 and I_1 are voltage and current respectively present else where in the circuit.



VDVS CDCS CDVS VDCS

★ Ohm's Law:

It states that "The current flowing through the conductor is directly proportional to the potential difference across the two terminals, provided physical conditions are constant."

★ SERIES CIRCUIT:

1. Resistors in series:

Equivalent resistance of the series circuit is arithmetic sum of resistances connected in series. For n resistances in series.

$$R_{eq} = R_1 + R_2 + R_3 + \cdots + R_n$$

2. <u>Inductors in series:</u>

Equivalent inductance of series circuit is the arithmetic sum of inductors connected in series.

For n inductors in series

$$L_{eq} = L_1 + L_2 + L_3 + \cdots + L_n$$

3. <u>Capacitors in series:</u>

Equivalent capacitance of series circuit is the arithmetic sum of the reciprocals of capacitances.

For n capacitors in series

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_n}$$

★ <u>PARALLEL CIRCUITS:</u>

1. <u>Resistors in parallel:</u>

Equivalent resistance of the parallel circuits is the arithmetic sum of the reciprocals of resistances

For n resistances in parallel

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}$$

2. <u>Inductors in parallel:</u>

Equivalent inductance of the parallel circuits is the arithmetic sum of the reciprocals of inductances.

For n inductors in parallel

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \cdots + \frac{1}{L_n}$$

3. <u>Capacitors in parallel</u>

Equivalent capacitance of parallel circuit is the arithmetic sum of capacitors connected in parallel.

For n capacitors in parallel

$$C_{eq} = C_1 + C_2 + \cdots + C_n$$

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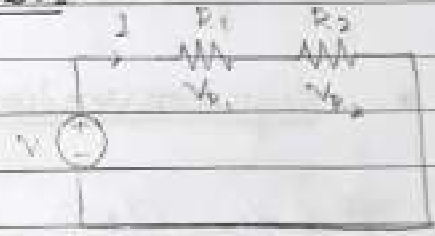
★ SHORT AND OPEN CIRCUITS:

The resistance of short circuit is zero, thus the voltage across the short circuit is zero though current flows through the short circuit path.

The resistance of open circuit is infinity, thus the current through open circuit is zero though there exists voltage across the open circuit terminals.

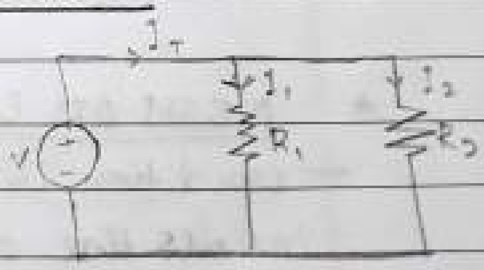
★ VOLTAGE DIVISION IN SERIES CIRCUIT:



$$V_{R_1} = \frac{VR_1}{R_1+R_2} \qquad V_{R_2} = \frac{VR_2}{R_1+R_2}$$

Voltage drop across any resistors or a combination of resistors in a series circuit is equal to the ratio of that resistance to the total resistance multiplied by the source voltage.

★ CURRENT DIVISION IN PARALLEL CIRCUIT:



$$I_1 = \frac{I_T R_2}{R_1+R_2} \qquad I_2 = \frac{I_T R_1}{R_1+R_2}$$

The current in any branch is equal to ratio of opposite branch resistance to the total resistance value multiplied by the total current in the circuit.

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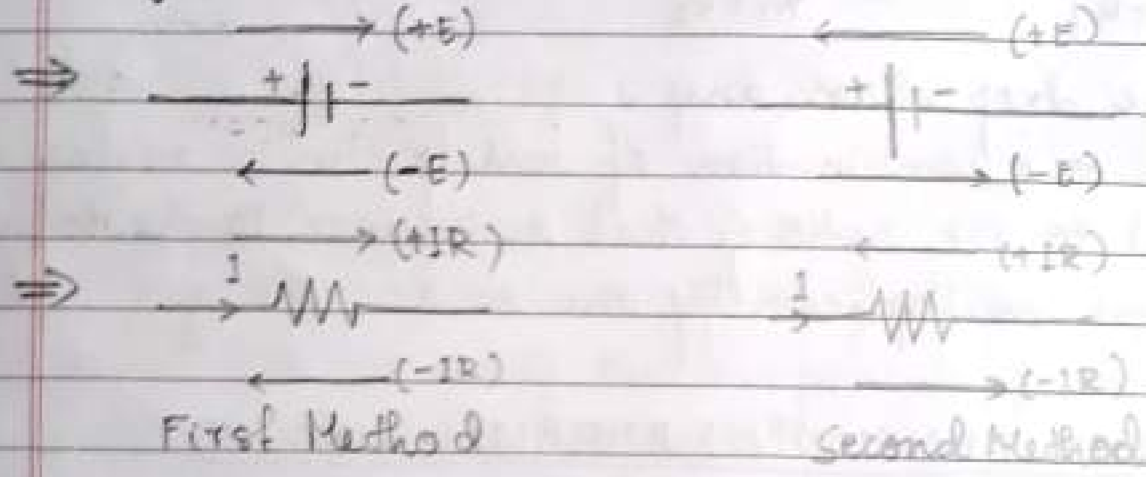
★ **KIRCHOFF'S LAWS:**

1. **Kirchoff's Voltage Law:**
In any closed electrical circuit algebraic sum of product of current and resistance and the sum of emf connected in it is equal to ~~the~~ zero.

2. **Kirchoff's Current Law:**
The algebraic sum of currents meeting at a point is equal to zero. i.e., $\Sigma I = 0$.

★ **Sign Convention:**



★ **MESH OR LOOP ANALYSIS:**

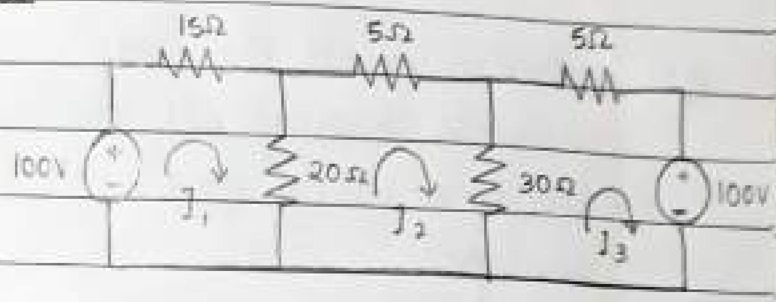
— Condition is that it should not have any other loop inside the circuit.

— Number of mesh currents will be equal to mesh loops

— Assume the current flowing in clockwise direction

Mesh analysis with independent voltage and current sources:

Q1: Perform mesh analysis and determine current through 30Ω resistor.



Sol: Applying KVL to loop 1

$100 = 15I_1 + 20(I_1 - I_2)$

$35I_1 - 20I_2 - 100 = 0$ —— ①

Applying KVL to loop 2

$5I_2 + 30(I_2 - I_3) + 20(I_2 - I_1) = 0$

$-20I_1 + 55I_2 - 30I_3 = 0$ —— ②

Applying KVL to loop 3

$100 + 30(I_3 - I_2) + 5I_3 = 0$

$-30I_2 + 35I_3 + 100 = 0$ —— ③

$I_1 = 1.943A$

$I_2 = -1.6A$

$I_3 = -4.23A$

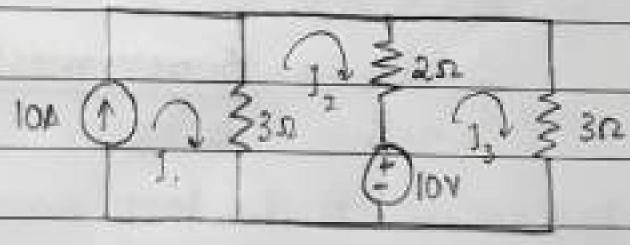
current across 30Ω

$I_{30\Omega} = I_3 - I_2$

$I_{30\Omega} = -4.23 - (-1.6)$

$I_{30\Omega} = -2.63A$

Q2: Determine loop currents for the circuit shown using mesh analysis.



Sol: From loop 1

$I_1 = 10A$

Applying KVL to loop 2

$10 + 3(I_2 - I_1) + 2(I_2 - I_3) = 0$

$5I_2 - 2I_3 - 20 = 0$ —— ①

$I_2 = 5.714\,A$

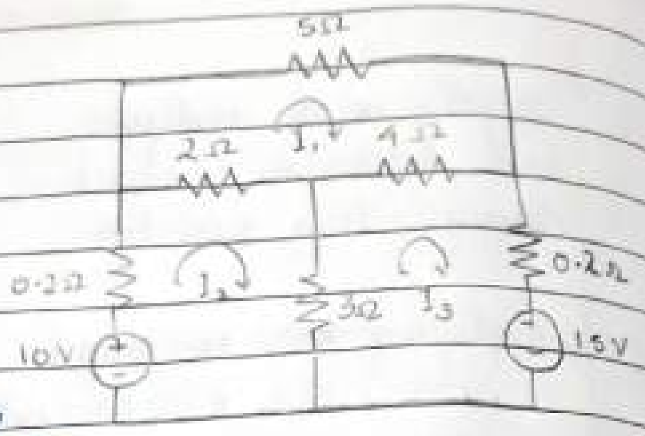
$I_3 = 4.286\,A$

Applying KVL to loop 3

$10 = 2(I_3 - I_2) + 3I_3$

$-2I_2 + 5I_3 - 10 = 0$ —— (2)

Q3: Determine all the loop currents through all the resistors using mesh analysis.



Sol: Applying KVL to loop 1

$5I_1 + 4(I_1 - I_3) + 2(I_1 - I_2) = 0$

$11I_1 - 2I_2 - 4I_3 = 0$ —— (1)

Applying KVL to loop 2

$-10 + 0.2I_2 + 2(I_2 - I_1) + 3(I_2 - I_3) = 0$

$-2I_1 + 5.2I_2 - 3I_3 - 10 = 0$ —— (2)

Applying KVL to loop 3

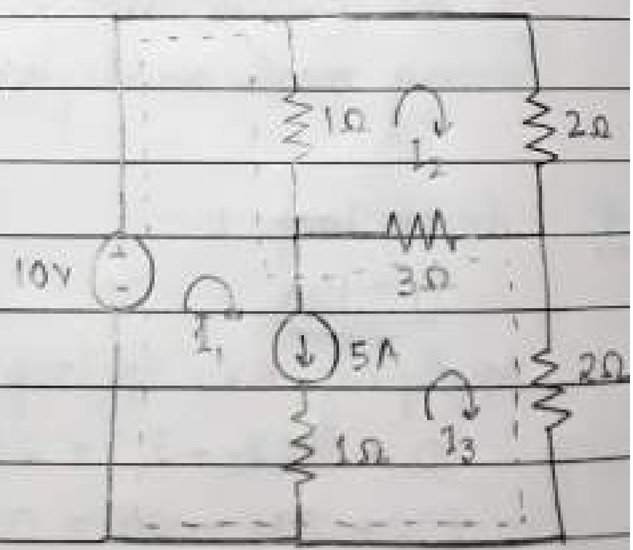
$15 + 3(I_3 - I_2) + 4(I_3 - I_1) + 0.2I_3 = 0$

$-4I_1 - 3I_2 + 7.2I_3 + 15 = 0$ —— (3)

$I_1 = -0.903\,A$ $\quad I_2 = 0.111\,A$ $\quad I_3 = -2.539\,A$

Super mesh

Q4: Determine all the loop currents.



Sol: Constraint equation

$I_1 - I_3 = 5A$ —— (1)

Applying KVL to loop 2

$2I_2 + 3(I_2 - I_3) + 1(I_2 - I_1) = 0$

$-I_1 + 6I_2 - 3I_3 = 0$ —— (2)

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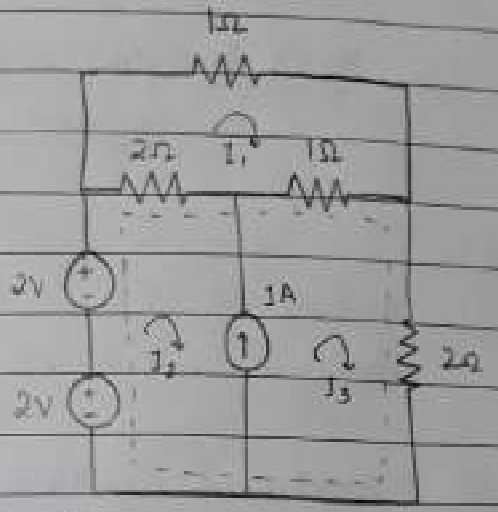
Super mesh

$-10 + 1(I_1 - I_2) + 3(I_3 - I_2) + 2I_3 = 0$

$I_1 - 4I_2 + 5I_3 - 10 = 0$ —— ③

$I_1 = 7.5A$ $I_2 = 2.5A$ $I_3 = 2.5A$

Q5 Determine all the loop currents



Sol: constraint equation

$I_3 - I_2 = 1A$ —— ①

Applying KVL to loop 1

$1I_1 + 1(I_1 - I_3) + 2(I_1 - I_2) = 0$

$4I_1 - 2I_2 - I_3 = 0$ —— ②

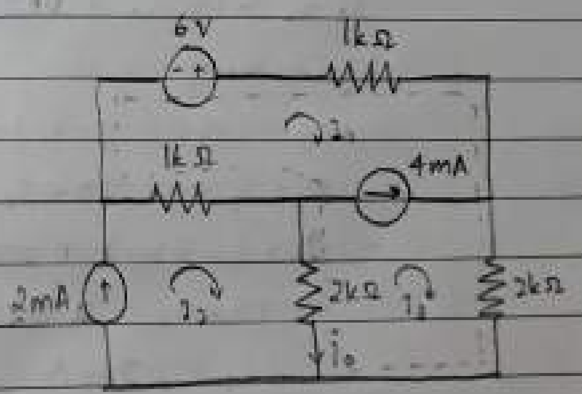
Super mesh

$-2 - 2 + 2(I_2 - I_1) + 1(I_3 - I_1) + 2I_3 = 0$

$-3I_1 + 2I_2 + 3I_3 - 4 = 0$ —— ③

$I_1 = 0.727A$ $I_2 = 0.636A$ $I_3 = 1.636A$

Q6: Find the current i_o in the circuit shown.



Sol: constraint equation

$I_3 - I_1 = 4mA$ —— ①

From loop 2

$I_2 = 2mA$

Super mesh

$-6 + 1k\,I_1 + 2k\,I_3 + 2k(I_3 - I_2) + 1k(I_1 - I_2) = 0$

$2k\,I_1 - 3k\,I_2 + 4k\,I_3 - 6 = 0$

$2k\,I_1 - 3k(2m) + 4k\,I_3 - 6 = 0$

$2k\,I_1 + 4k\,I_3 - 12 = 0$ —— ②

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$I_1 = -0.67mA$ $I_3 = 3.33mA$

$i_o = I_2 - I_3 = 2m - 3.33m$

$i_o = -1.33mA$

★ Super Mesh:

A more general technique for mesh analysis method, when a current source is common to two meshes involves a concept of super mesh. A super mesh is created from two meshes that has a current source as a common element. The current source is in the interior of super mesh. Thus we reduce the number of mesh by one for each current source present.

★ Mesh Analysis with Dependent Voltage and current Sources:

The presence of one or more dependent sources nearly requires each of these source quantities and the variable on which it depends to be expressed in terms of assigned meshcurrents. That is to begin with, ~~the~~ we treat the dependent source as though it is an independent source while writting the KVL equation.

We write the constraint equation for the dependent source.

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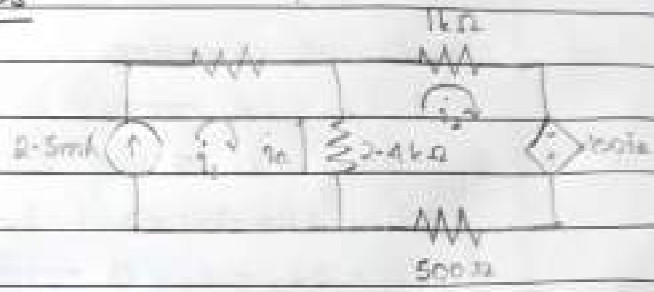
Mesh Analysis with dependent voltage and current sources

Q 7: Use the mesh current method to solve for i_a shown in the current.



Sol: Constraint equation

$$i_a = i_2 - i_1 \quad \text{——} \ (1)$$

From loop 1

$$\underline{i_1 = 2.5mA}$$

Applying KVL to loop 2

$$1k\,i_2 - 150 i_a + 0.5k\,i_2 + 2.4k(i_2 - i_1) = 0$$

Substituting eq (1)

$$1k\,i_2 - 150(i_2 - i_1) + 0.5k\,i_2 + 2.4k\,i_2 - 2.4k\,i_1 = 0$$

$$-2.25k\,i_1 + 3.75k\,i_2 = 0$$

$$-2.25k(2.5m) + 3.75k\,i_2 = 0$$

$$3.75k\,i_2 = 5.625$$

$$\underline{\underline{i_2 = 1.5mA}}$$

$$\therefore i_a = i_2 - i_1 = 1.5m - 2.5m$$

$$\underline{\underline{i_a = 1mA}}$$

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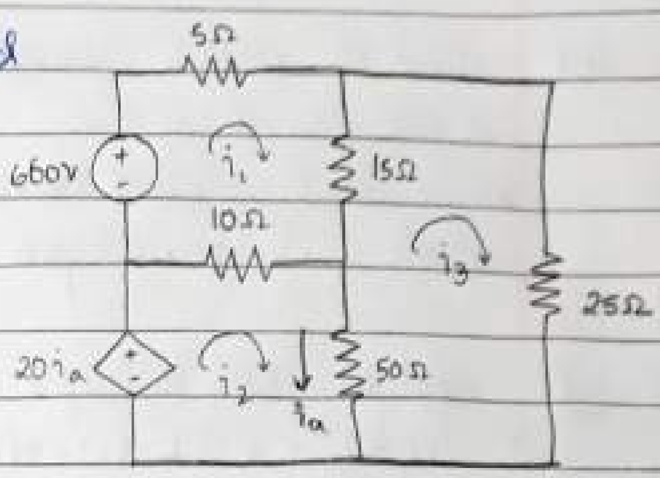
Q8: use the mesh analysis method to find the power delivered by the dependent voltage source in the circuit shown.



Sol: Constraint equation

$i_a = i_2 - i_3$ —— ①

Applying KVL to loop 1

$-660 + 5i_1 + 15(i_1 - i_3) + 10(i_1 - i_2) = 0$

$30i_1 - 10i_2 - 15i_3 - 660 = 0$ —— ②

Applying KVL to loop 2

$-20i_a + 10(i_2 - i_1) + 50i_a = 0$

$10(i_2 - i_1) + 30i_a = 0$

From eq ① $10(i_2 - i_1) + 30(i_2 - i_3) = 0$

$-10i_1 + 40i_2 - 30i_3 = 0$ —— ③

Applying KVL to loop 3

$25i_3 + 50(i_3 - i_2) + 15(i_3 - i_1) = 0$

$-15i_1 - 50i_2 + 90i_3 = 0$ —— ④

$i_1 = 42A$ $i_2 = 27A$ $i_3 = 22A$

Substituting in eq ①

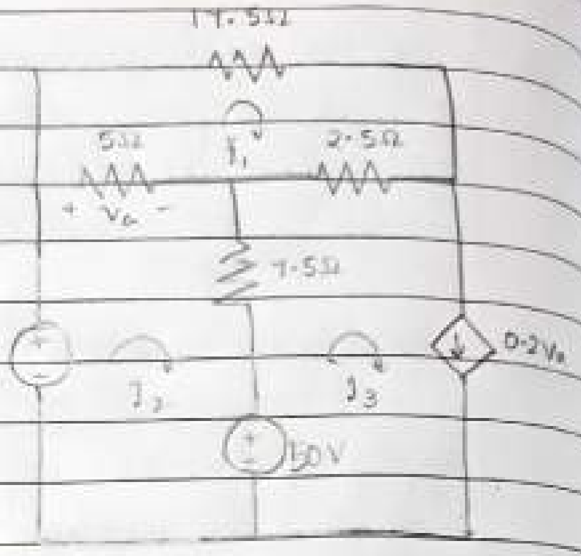
$i_a = i_2 - i_3 = 27 - 22 = 5A$

Power delivered by the dependent voltage source is

$P = (20i_a)\, i_2$

$P = (20 \times 5)\, 27$

$P = 2700W$



Q9: Find the power delivered in the circuit by the independent sources using mesh current analysis.

Sol: Applying KVL to loop 1

$17.5I_1 + 2.5(I_1 - I_3) + 5(I_1 - I_2) = 0$

$25I_1 - 5I_2 - 2.5I_3 = 0$ —— (1)

Applying KVL to loop 2

$50 - 125 + 5(I_2 - I_1) + 7.5(I_2 - I_3) = 0$

$-5I_1 + 12.5I_2 - 7.5I_3 - 75 = 0$ —— (2)

Applying KVL to loop 3

$-50 + 7.5(I_3 - I_2) + 2.5(I_3 - I_1) = 0$

$-2.5I_1 - 7.5I_2 + 10I_3 - 50 = 0$ —— (3)

$I_1 = 7.143\,A$ $\quad I_2 = 23.506\,A$ $\quad I_3 = 24.415\,A$

~~Kerch~~ From loop 3

$I_3 = 0.2V_a$

$V_a = \dfrac{I_3}{0.2} = \dfrac{24.415}{0.2} = 122.075V$

~~Power delivered by dependent source is~~

~~$P = 0.2V_a(I_3)$~~

~~$= 0.2(122.075)$~~

Power delivered by 125V voltage source

$P = 125(I_2) = 125(23.506)$

$P = 2.938\,kW$

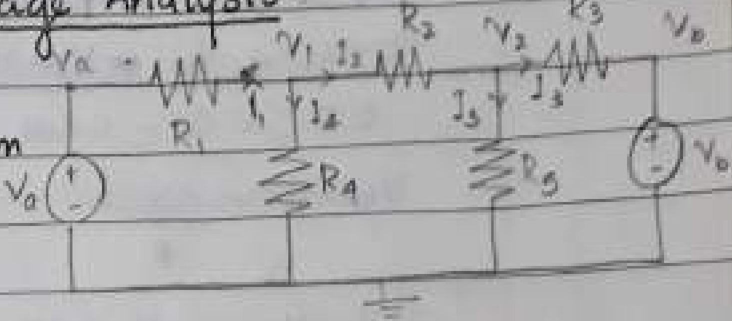
Power delivered by 50V voltage source

$P = 50(I_3 - I_2) = 50(24.415 - 23.506)$

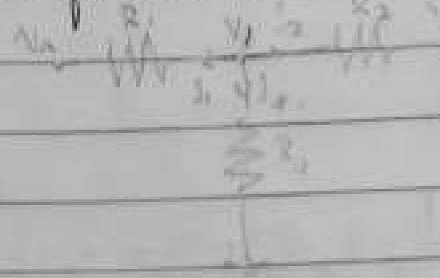
$P = 50(0.909)$

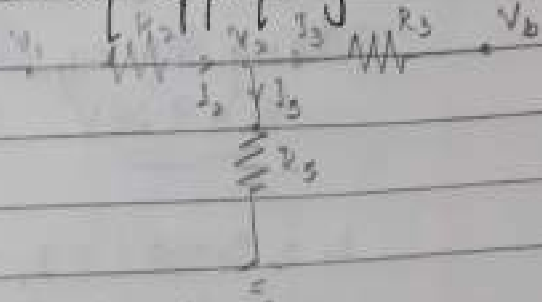
$P = 45.45\,W$

★ <u>Node Analysis or Node Voltage Analysis:</u>



In node analysis, KVL is used to write the equilibrium equation.
A node is defined as a junction of two or more branches. If we define one node of the network as a reference node as zero potenial or ground, the remaining nodes of the network will have fixed potential relative to this reference Equations relating to all nodes except for the reference node can be written by applying KCL.





$$I_1 + I_2 + I_4 = 0$$

$$\frac{V_1 - V_a}{R_1} + \frac{V_1 - V_2}{R_2} + \frac{V_1 - 0}{R_4}$$

$$I_2 = I_3 + I_5$$

$$\frac{V_1 - V_2}{R_2} = \frac{V_2 - V_b}{R_3} + \frac{V_2 - 0}{R_5}$$

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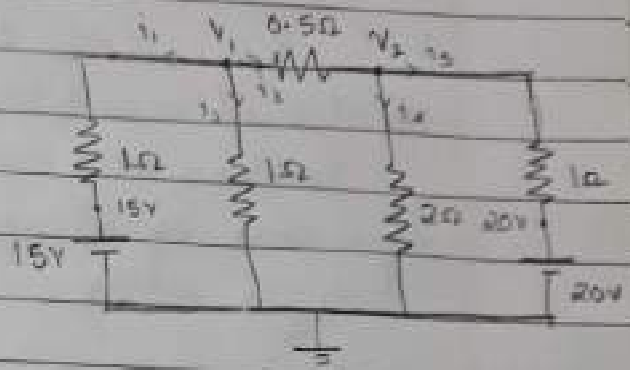
Nodal Analysis

Q10: Determine all the node voltages and branch currents using nodal analysis.



Sol: Node V_1:

$$\frac{V_1-15}{1}+\frac{V_1-0}{1}+\frac{V_1-V_2}{0.5}=0$$

$$V_1-15+V_1+2V_1-2V_2=0$$

$$4V_1-2V_2-15=0 \quad \text{—— (1)}$$

Node V_2:

$$\frac{V_1-V_2}{0.5}=\frac{V_2-0}{2}+\frac{V_2-20}{1}$$

$$4V_1-4V_2=V_2+2V_2-40$$

$$4V_1-7V_2+40=0 \quad \text{—— (2)}$$

$$V_1=9.25V \qquad V_2=11V$$

$$i_1=\frac{V_1-15}{1}=9.25-15=-5.75A$$

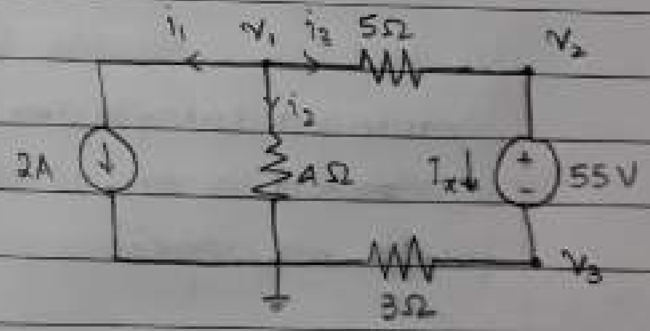
$$i_2=\frac{V_1}{1}=9.25A$$

$$i_3=\frac{V_1-V_2}{0.5}=\frac{9.25-11}{0.5}=-3.5A$$

$$i_4=\frac{V_2}{2}=\frac{11}{2}=5.5A$$

$$i_5=\frac{V_2-20}{1}=11-20=-9A$$

Q11: Use the node voltage method to find how much power the 2A source extracts from the circuit given.



Sol: Node V_1:

$$2+\frac{V_1-0}{4}+\frac{V_1-V_2}{5}=0$$

$$40+5V_1+4V_1-4V_2=0$$

$$9V_1-4V_2+40=0 \quad \text{—— (1)}$$

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Node V_2

$$\frac{V_1 - V_2}{5} = I_x \quad \text{--- (2)}$$

Node V_3

$$\frac{V_3 - 0}{3} = I_x \quad \text{--- (3)}$$

Constraint equation

$$V_2 - V_3 = 55 \quad \text{--- (4)}$$

Equating eq (2) and eq (3)

$$\frac{V_1 - V_2}{5} = \frac{V_3}{3}$$

$$3V_1 - 3V_2 - 5V_3 = 0 \quad \text{--- (5)}$$

Solving eq (1), eq (4) and eq (5)

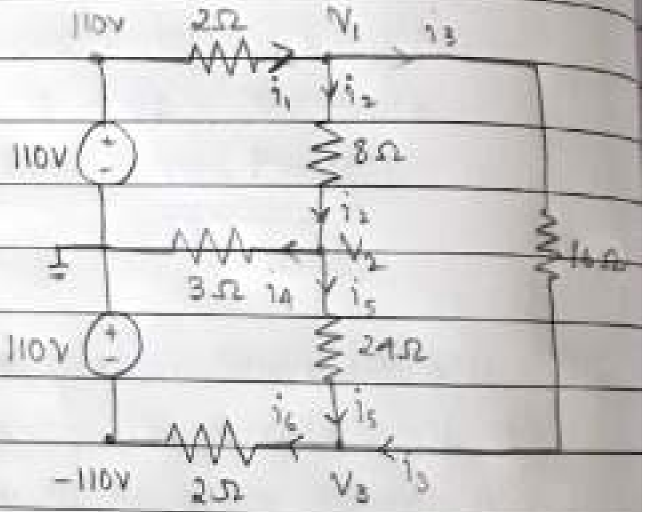
$\underline{V_1 = 13V}$ $\quad$ $\underline{V_2 = 39.25V}$ $\quad$ $\underline{V_3 = -15.75V}$

Power the 2A source extracts.

$$P = V_1 I_1 = 13(2) = \underline{26W}$$

Q12: Use the node voltage method to find the branch currents.

Test your solution for branch currents by showing total power dissipated is equal to the power developed.



Sol: Node V_1

$$\frac{-V_1 + 110}{2} = \frac{V_1 - V_2}{8} + \frac{V_1 - V_3}{16}$$

$$-8V_1 + 880 + 2V_1 - 2V_2 + V_1 - V_3 = 0$$

$$11V_1 - 2V_2 - V_3 - 880 = 0 \quad \text{--- (1)}$$

Scanned by CamScanner

Node V_2

$$\frac{V_1 - V_2}{8} = \frac{V_2 - 0}{3} + \frac{V_2 - V_3}{24}$$

$$3V_1 - 3V_2 = 8V_2 + V_2 - V_3$$

$$3V_1 - 12V_2 + V_3 = 0 \quad \text{—— (2)}$$

Node V_3

$$\frac{V_2 - V_3}{24} + \frac{V_1 - V_3}{16} = \frac{V_3 + 110}{2}$$

$$2V_2 - 2V_3 + 3V_1 - 3V_3 = 24V_3 + 2640$$

$$3V_1 + 2V_2 - 29V_3 - 2640 = 0 \quad \text{—— (3)}$$

$V_1 = 74.643V$ $V_2 = 11.786V$ $V_3 = -82.5V$

$$i_1 = \frac{110 - V_1}{2} = \frac{110 - 74.643}{2} = 17.678A$$

$$i_2 = \frac{V_1 - V_2}{8} = \frac{74.643 - 11.786}{8} = 7.857A$$

$$i_3 = \frac{V_1 - V_3}{16} = \frac{74.643 + 82.5}{16} = 9.821A$$

$$i_4 = \frac{V_2}{3} = \frac{11.786}{3} = 3.928A$$

$$i_5 = \frac{V_2 - V_3}{24} = \frac{11.786 + 82.5}{24} = 3.928A$$

$$i_6 = \frac{V_3 + 110}{2} = \frac{-82.5 + 110}{2} = 13.75A$$

Total power developed

$$P = 110 i_1 + 110 i_6$$

$$P = 110(17.678) + 110(13.75)$$

$$P = 3457.08 W$$

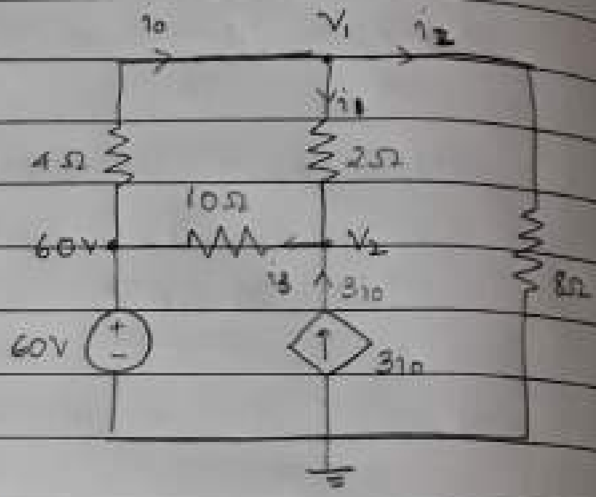
Total power dissipated

$$P = i_1^2(2) + i_2^2(8) + i_3^2(16) + i_4^2(3) + i_5^2(24) + i_6^2(2)$$

$$P = (17.678)^2(2) + (7.857)^2(8) + (9.821)^2(16) + (3.928)^2(3) + (3.928)^2(24) + (13.75)^2(2)$$

$$P = \underline{\underline{3456.23\,W}}$$

Q13: Determine i_o using nodal analysis.



Sol: Node V_1

$$\frac{-V_1+60}{4} = \frac{V_1-0}{8} + \frac{V_1-V_2}{2}$$

$$2V_1 - 120 + V_1 + 4V_1 - 4V_2 = 0$$

$$7V_1 - 4V_2 - 120 = 0 \quad \text{—— (1)}$$

Node V_2

$$\frac{V_1-V_2}{2} + 3i_o = \frac{V_2-60}{10}$$

$$\frac{V_1-V_2}{2} + 3\left(\frac{60-V_1}{4}\right) = \frac{V_2-60}{10}$$

$$10V_1 - 10V_2 + 900 - 15V_1 = 2V_2 - 120$$

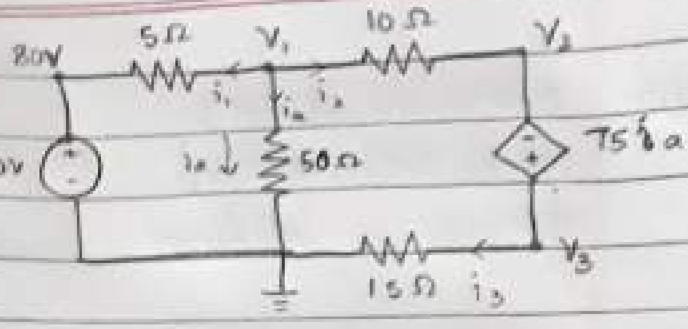
$$-5V_1 - 12V_2 + 1020 = 0 \quad \text{—— (2)}$$

$$\underline{\underline{V_1 = 53.077\,V}} \qquad \underline{\underline{V_2 = 62.885\,V}}$$

$$i_o = \frac{60-V_1}{4}$$

$$i_o = \frac{60-53.077}{4} = \frac{6.923}{4} = \underline{\underline{1.73\,A}}$$

Q14: Refer the circuit shown and find the power delivered by the dependent source in the network.



Sol: Constraint equation

$V_3 - V_2 = 75 i_a$ —— ① $\Rightarrow V_3 - V_2 = \frac{75 V_1}{50}$ $\Rightarrow 3V_1 + 2V_2 - 2V_3 = 0$ —— ①

Node V_1

$$\frac{V_1 - 80}{5} + \frac{V_1 - 0}{50} + \frac{V_1 - V_2}{10} = 0$$

$$10V_1 - 800 + V_1 + 5V_1 - 5V_2 = 0$$

$$16V_1 - 5V_2 - 800 = 0 \quad \text{—— ②}$$

Node V_2

$$i_2 = \frac{V_1 - V_2}{10} = i_3$$

$$\frac{V_1 - V_2}{10} = \frac{V_3 - 0}{15}$$

$$3V_1 - 3V_2 - 2V_3 = 0 \quad \text{—— ③}$$

$V_1 = 50\,V$ $\quad V_2 = 0\,V$ $\quad V_3 = 75\,V$

$i_a = \frac{V_1}{50} = \frac{50}{50} = 1\,A$ $\qquad i_3 = \frac{V_3}{15} = \frac{75}{15} = 5\,A$

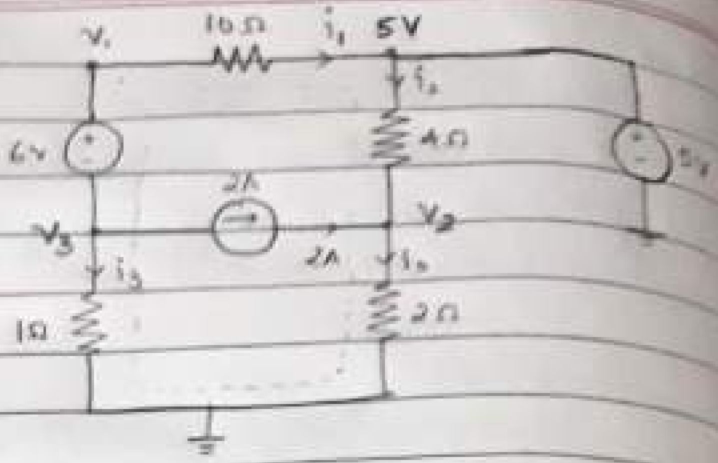
Power delivered by the dependent source is

$$P = 75 i_a i_3$$

$$P = 75(1)(5)$$

$$P = 375\,W$$

Q15: Determine all the node voltages for the circuit shown



Sol: constraint equation

$V_1 - V_3 = 6$ —— ①

Node V_2

$$\frac{5-V_2}{4} + 2 = \frac{V_2 - 0}{2}$$

$5 - V_2 + 8 = 2V_2$

$3V_2 = 13$

$\underline{V_2 = 4.33V}$

Super Node

$i_1 + 2A + i_3 = 0$

$$\frac{V_1 - 5}{10} + 2 + \frac{V_3 - 0}{1} = 0$$

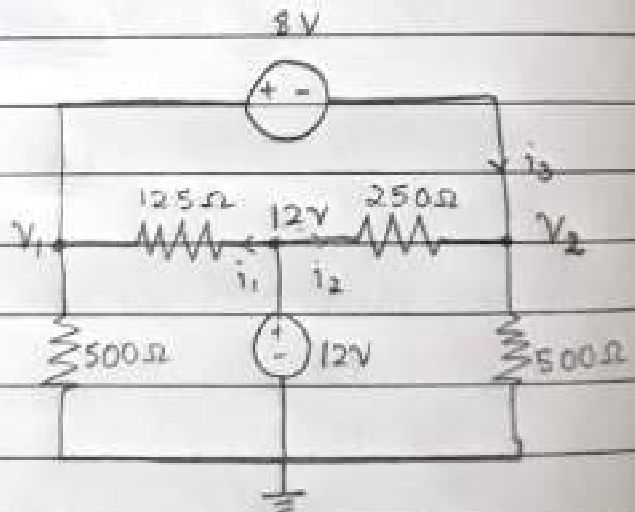
$V_1 - 5 + 20 + 10V_3 = 0$

$V_1 + 10V_3 + 15 = 0$ —— ①

$\underline{V_1 = 4.091V}$

$\underline{V_3 = -1.909V}$

Q16: Find All the nodal voltages.



Sol: constraint equation

$V_1 - V_2 = 8V$ —— ①

Node (12V)

$i_1 = i_2$

$$\frac{12 - V_1}{125} = \frac{12 - V_2}{250}$$

$24 - 2V_1 = 12 - V_2$

$2V_1 - V_2 - 12 = 0$ —— ②

$\underline{V_1 = 4V}$

$\underline{V_3 = -4V}$

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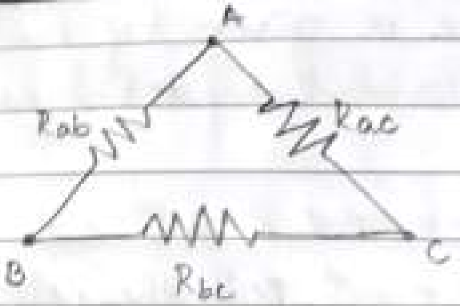
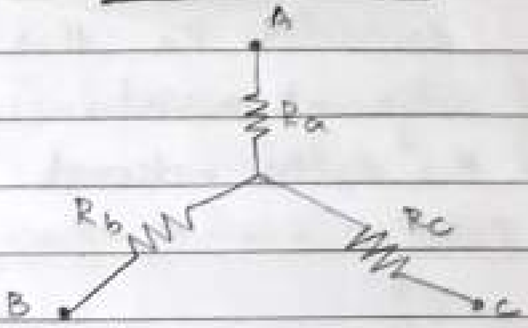
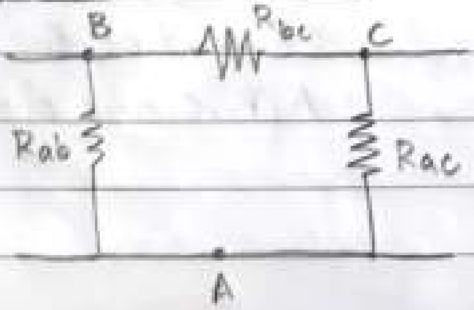
★ **Delta Network**



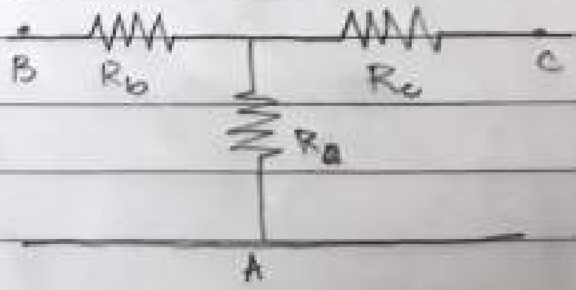
★ **Star Network** (Y network)



★ **Pi / π Network**



★ **T / Tee Network**



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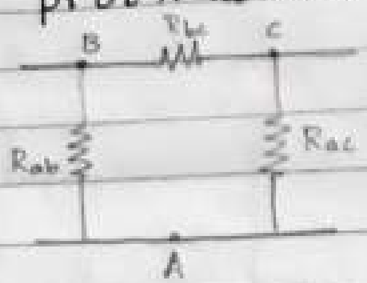
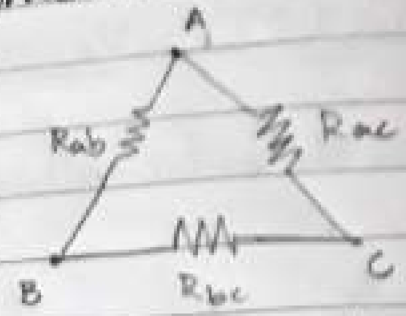
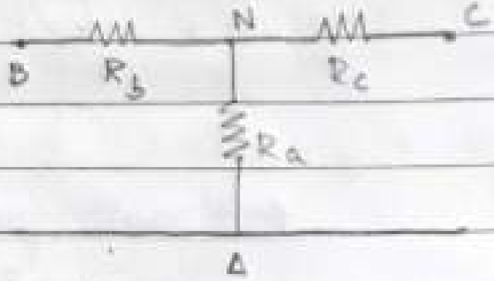
pi or π network can be converted into delta network.



$\equiv$



T or Tee network can be converted into star network



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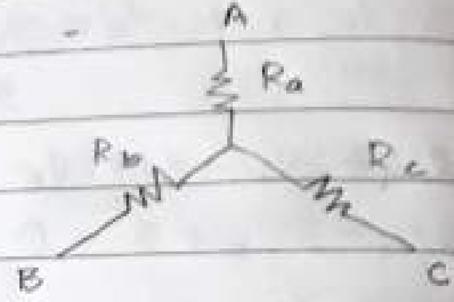
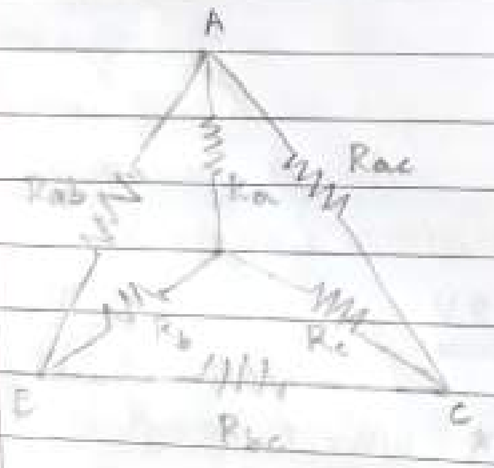


* <u>Delta to Star Conversion</u>:



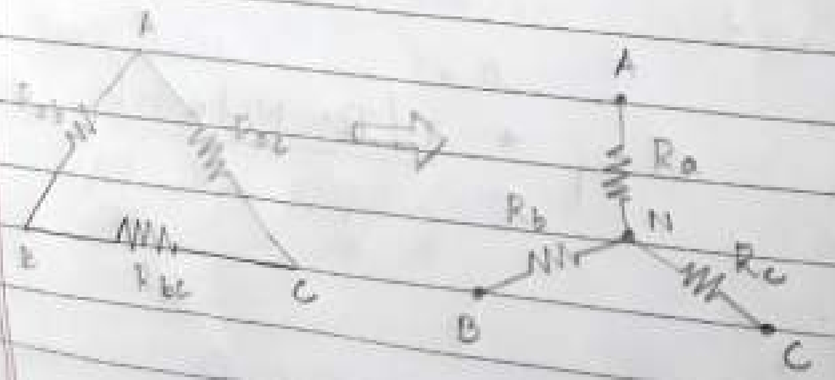
$$R_a = \frac{R_{ab} \times R_{ac}}{R_{ab} + R_{bc} + R_{ac}}$$

$$R_b = \frac{R_{ab} \times R_{bc}}{R_{ab} + R_{bc} + R_{ac}}$$

$$R_c = \frac{R_{ac} \times R_{bc}}{R_{ab} + R_{bc} + R_{ac}}$$

Consider R_a, this resistance is given by multiplication of adjacent resistances of R_a divided by total resistance in the delta network.

<u>Proof</u>:



Both the network are not connected to any other, i.e., the nodes A, B, C are open circuit

Resistance between node A and B

$$R_{ab} \parallel (R_{ac} + R_{bc}) \Rightarrow \frac{R_{ab}(R_{ac}+R_{bc})}{R_{ab}+R_{bc}+R_{ac}} = R_a + R_b \quad \text{--- } (1)$$

Resistance between node B and C

$$R_{bc} \parallel (R_{ab} + R_{ac}) \Rightarrow \frac{R_{bc}(R_{ab}+R_{ac})}{R_{ab}+R_{bc}+R_{ac}} = R_b + R_c \quad \text{--- } (2)$$

Resistance between node C and A

$$R_{ac} \parallel (R_{ab} + R_{bc}) \Rightarrow \frac{R_{ac}(R_{ab}+R_{bc})}{R_{ab}+R_{bc}+R_{ac}} = R_a + R_c \quad \text{--- } (3)$$

Adding equations (1), (2) and (3)

$$\frac{R_{ab}(R_{ac}+R_{bc}) + R_{bc}(R_{ab}+R_{ac}) + R_{ac}(R_{ab}+R_{bc})}{R_{ab}+R_{bc}+R_{ac}} = 2R_a + 2R_b + 2R_c$$

$$\frac{R_{ab}R_{ac} + R_{ab}R_{bc} + R_{bc}R_{ab} + R_{bc}R_{ac} + R_{ac}R_{ab} + R_{ac}R_{bc}}{R_{ab}+R_{bc}+R_{ac}}$$

$$= 2(R_a + R_b + R_c)$$

$$\frac{\cancel{2}(R_{ab}R_{ac} + R_{ab}R_{bc} + R_{bc}R_{ac})}{R_{ab}+R_{bc}+R_{ac}} = \cancel{2}(R_a + R_b + R_c)$$

$$\frac{R_{ab}R_{ac} + R_{ab}R_{bc} + R_{bc}R_{ac}}{R_{ab}+R_{bc}+R_{ac}} = R_a + R_b + R_c \quad \text{--- } (4)$$

Subtracting eq (4) and eq (1)

$$\frac{\cancel{R_{ab}R_{ac}} + \cancel{R_{ab}R_{bc}} + R_{bc}R_{ac} - \cancel{R_{ab}R_{ac}} - \cancel{R_{ab}R_{bc}}}{R_{ab}+R_{bc}+R_{ac}} = \cancel{R_a} + \cancel{R_b} + R_c - \cancel{R_a} - \cancel{R_b}$$

$$\boxed{R_c = \frac{R_{bc}R_{ac}}{R_{ab}+R_{bc}+R_{ac}}}$$

Subtracting eq ④ and eq ②

$$\frac{R_{ab}R_{ac} + \cancel{R_{ab}R_{bc}} + \cancel{R_{bc}R_{ac}} - R_{ab}R_{bc} - R_{bc}R_{ac}}{R_{ac} + R_{ab} + R_{bc}} = R_a + \cancel{R_b} + \cancel{R_c} - R_b - R_c$$

$$\boxed{R_a = \frac{R_{ab}R_{ac}}{R_{ac}+R_{ab}+R_{bc}}}$$

Subtracting eq ④ and eq ③

$$\frac{R_{ab}R_{ac} + R_{ab}R_{bc} + R_{bc}R_{ac} - R_{ab}R_{ac} - R_{ac}R_{bc}}{R_{ac} + R_{ab} + R_{bc}} = R_a + R_b + \cancel{R_c} - R_a - R_c$$

$$\boxed{R_b = \frac{R_{ab}R_{bc}}{R_{ac}+R_{ab}+R_{bc}}}$$

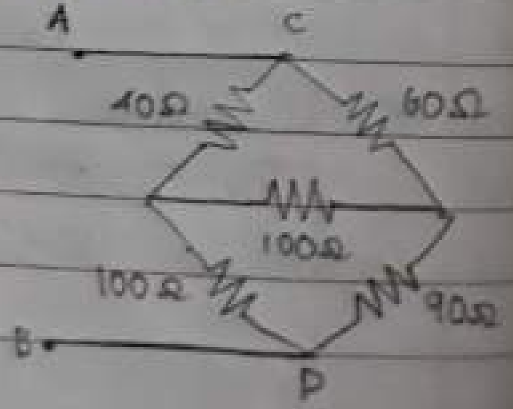
SPECIAL CASE:

If suppose in delta network all resistances are of same value then

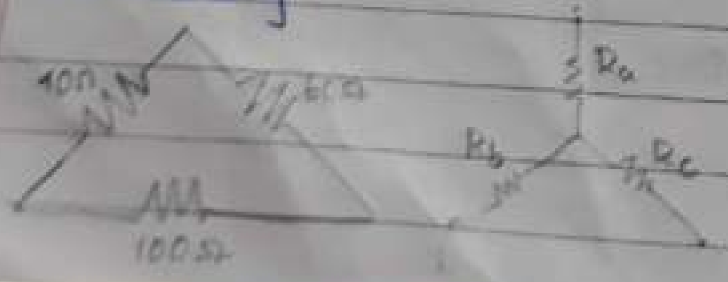
$$R_a = \frac{R \times R}{R+R+R} = \frac{R^2}{3R}$$

$$\boxed{R_a = R_b = R_c = \frac{R}{3}}$$

Q1: Find equivalent resistance between A and B using delta to star conversion.

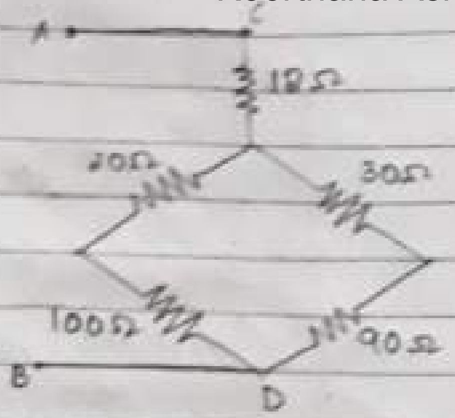


Sol: Considering



$$R_a = \frac{60 \times 40}{60+40+100} = \frac{2400}{200}$$

$R_a = 12\,\Omega$

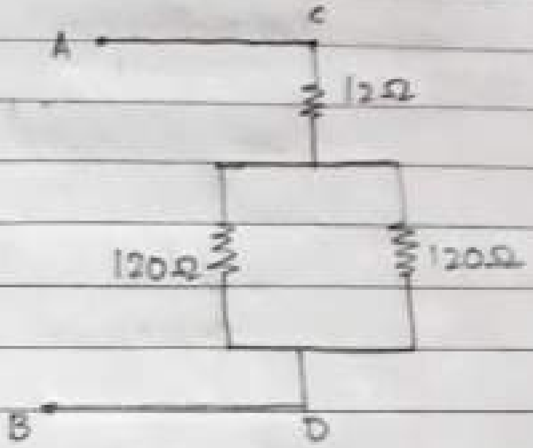


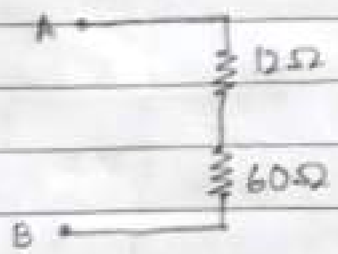
$$R_b = \frac{40 \times 100}{60+40+100} = \frac{4000}{200}$$

$R_b = 20\,\Omega$

$$R_c = \frac{60 \times 100}{60+40+100} = \frac{6000}{200}$$

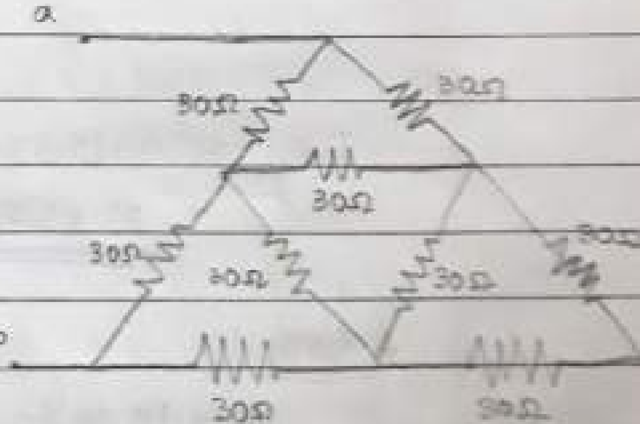
$R_c = 30\,\Omega$



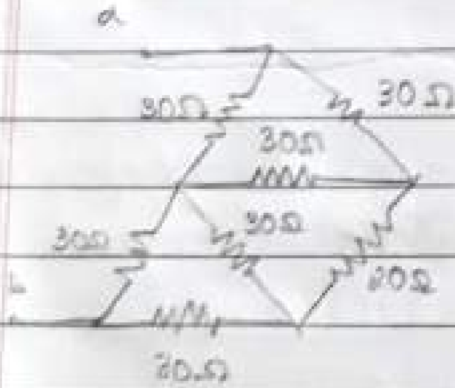


$R_{eq} = 72\,\Omega$

Q2: Obtain equivalent resistance R_{ab} for the circuit shown.

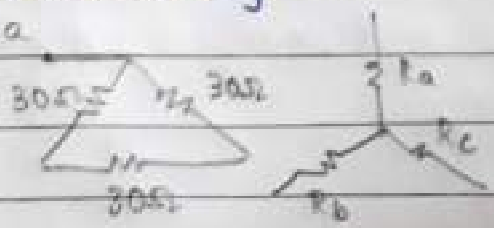


sol:



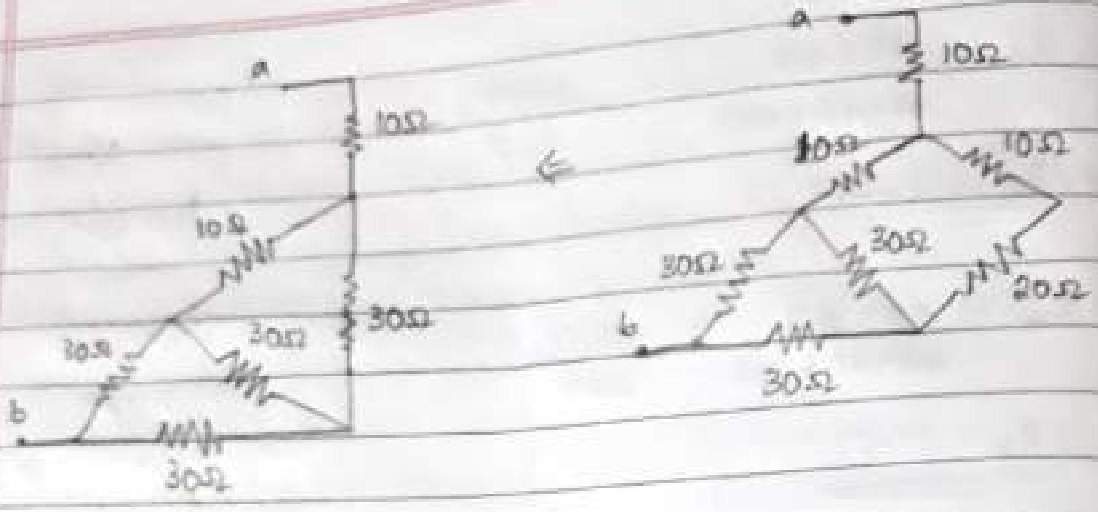
⇐

Considering

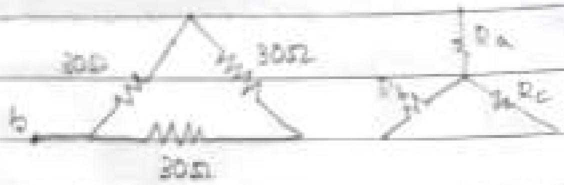


$$R_{ab} = R_{bc} = R_{ac} = \frac{R}{3} = \frac{30}{3}$$

$R_a = R_b = R_c = 10\,\Omega$

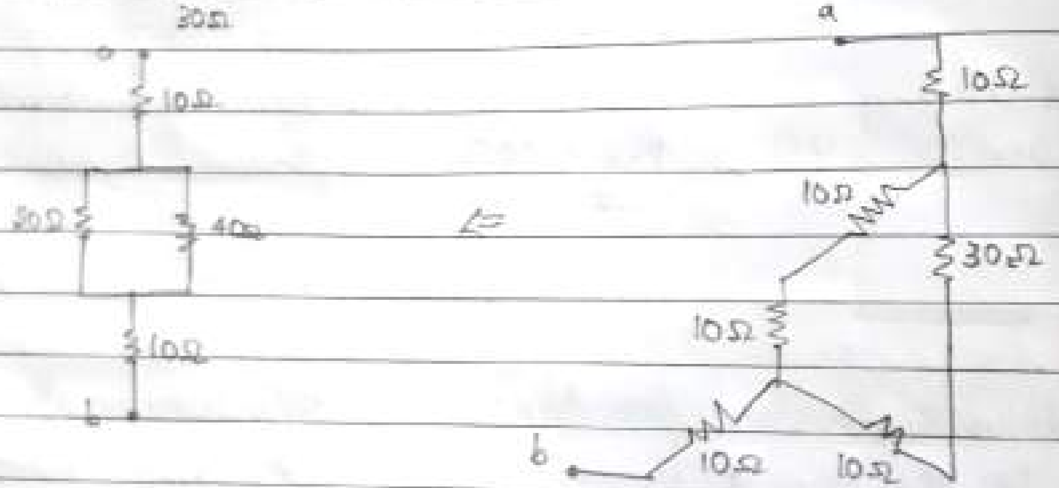


Considering



$R_{ab} = R_{bc} = R_{ac} = 30\,\Omega$

$R_a = R_b = R_c = \frac{30}{3} = \underline{10\,\Omega}$



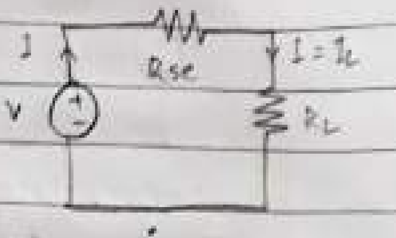
$R_{eq} = 10 + 13.33 + 10$
$= \underline{33.33\,\Omega}$

★ NOTE :

star to delta conversion - Tutorial

* Source Transformation:

Source transformation is a procedure which transforms one source into another while retaining the terminal characteristics of the original source.



i

Source Transformation is based on the concept of equivalence. An equivalent circuit is one whose terminal characteristics remain identical to that of original.

ii

The term equivalence as applied to circuit means an identical effect at the terminals, but not within the equivalent circuit themselves.

i. $I_L = I = \dfrac{V}{R_{se}+R_L}$

ii. $I_L = I\left(\dfrac{R_{sh}}{R_{sh}+R_L}\right)$

Equating i and ii

$$\frac{V}{R_{se}+R_L} = I\left(\frac{R_{sh}}{R_{sh}+R_L}\right)$$

$$R_{se} = R_{sh}$$

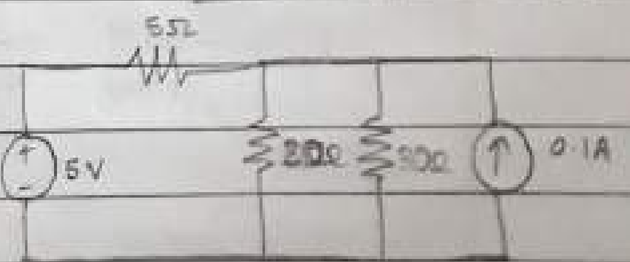
$$V = I R_{sh} = I R_{se}$$

$$\therefore \boxed{I = \frac{V}{R_{se}}} \quad \text{or} \quad \boxed{I = \frac{V}{R_{sh}}}$$

Q1: A circuit shown, find the current i by reducing the circuit using source transformation.

sol: 3V voltage source into current source

$I = \dfrac{V}{R_{se}} = \dfrac{3}{30} = \underline{0.1A}$



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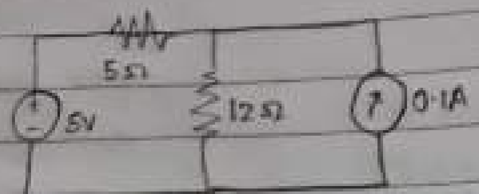
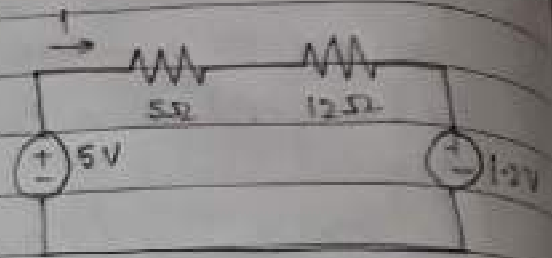


converting 0.1A current source into voltage source

$V = I R_{sh}$

$V = 0.1(12)$

$V = 1.2V$

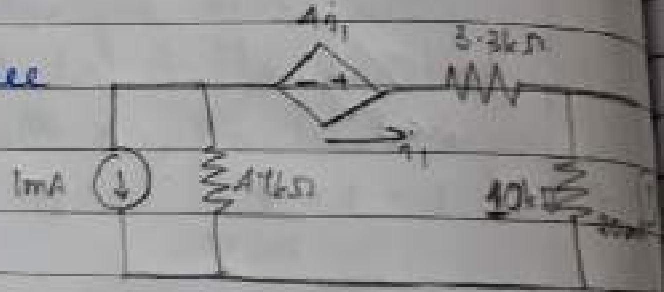


Applying KVL

$-5 + i(5+12) + 1.2 = 0$

$17i = 3.8 \qquad \underline{i = 0.22A}$

Q2: Find the current i_1 using source transformation



sol: converting 20mA current source into voltage source

$V_1 = I_1 R_{sh_1}$

$V_1 = 20m(10k)$

$\underline{V_1 = 200V}$

converting 1mA current source into voltage source

$V_2 = I_2 R_{sh_2}$

$V_2 = 1m(47k\Omega)$

$\underline{V_2 = 47V}$

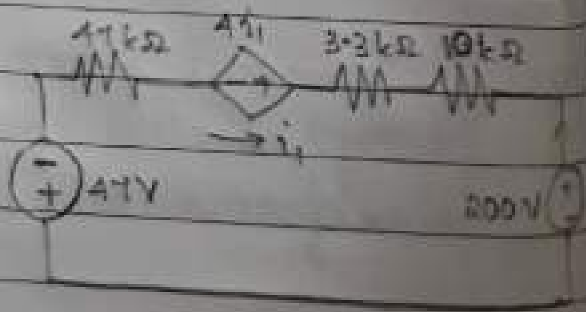
Applying KVL

$47 + (47k)i_1 - 4i_1 + (3.3k+10k)i_1 + 200 = 0$

$60.3k i_1 - 4i_1 + 247 = 0$

$60296 i_1 + 247 = 0$

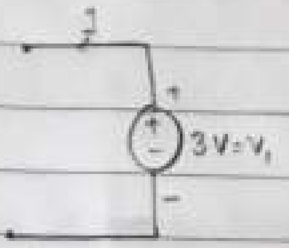
$i_1 = \frac{-247}{60296} = \underline{-4.096mA}$



★ Power :

$P = VI = I^2R$

If power is positive then it is absorbing power.



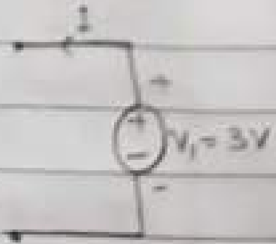
$I = -5A$

$P = V_1(-I)$

$P = 3(-(-5))$

$\underline{P = 15W}$

Power is absorbed



$I = 4A$

$P = V_1(-I)$

$P = 3(-4)$

$\underline{P = -12W}$

Power is delivered by 3V source.

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UNIT - 2

Network Theorems

The aim of the theorems is to reduce complexity and make the design easy. If you want to concentrate on certain portion of the circuit we cannot apply KVL or KCL. Here portion of the circuit is taken and it is solved to find current and voltage.
We have different theorems to solve this namely:-

1. Superposition theorem
2. Thevinins theorem
3. Reciprocity theorem
4. Maximum Power Transfer theorem
5. Millman's theorem

★ The principle of superposition is applicable only for linear systems. The concept of superposition can be explained mathematically by

$$\begin{matrix} i_1 \rightarrow V_1 \\ i_2 \rightarrow V_2 \end{matrix} \Rightarrow i_1 + i_2 \longrightarrow V_1 + V_2$$

We can state that a device if excited by a current i_1 will produce a response of V_1, similarly excitation of current i_2 will cause a response of V_2. Then if we use an excitation i_1+i_2, we will find a response V_1+V_2.
The theorem has the ability to reduce a complicated problem into several easier problems each containing only a single independent source.
"In any linear circuit containing multiple independent sources, the current or voltage at any point in the network may be calculated as algebraic sum of the individual contributions of each source acting alone."

The three steps involved are:

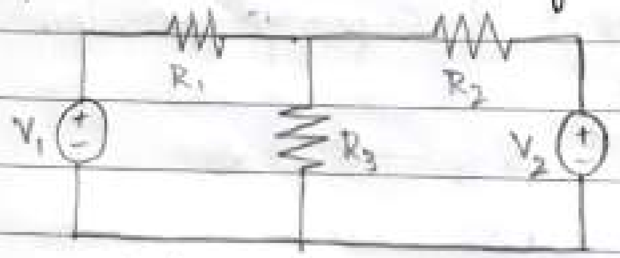
1. In a circuit comprising of many independent sources, only one source is allowed to be active in the circuit the rest are deactivated

2. To deactivate a voltage source replace it with a short circuit and to deactivate a current source replace it with a open circuit.

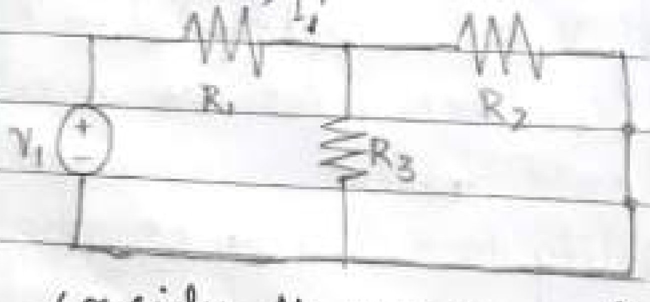
3. The response obtained by applying each source one at a time are then added algebraically to obtain a solution.

Limitations

- Only applicable for linear circuits.
- It applies only to the current and voltage in a linear circuit, but cannot be used to determine power because power is a non linear function.



consider V_1 source and short circuit V_2 source.



consider V_2 source and short circuit V_1 source

Total response is given by

$I_1 = I_1' + I_1''$

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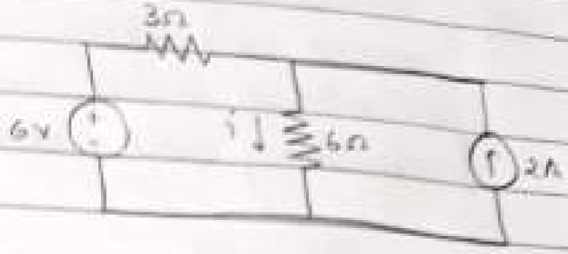
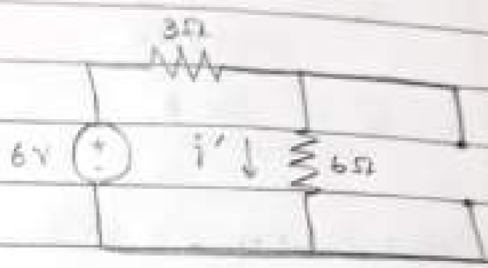
Super position

Q1: Find the current in 6Ω resistor using superposition theorem for the given circuit.



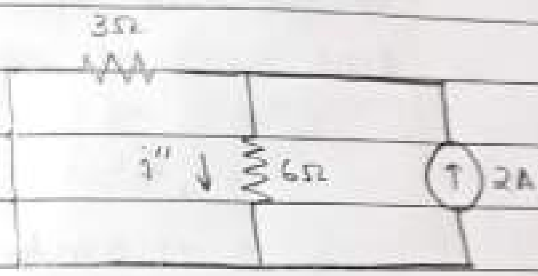
Sol: considering 6V voltage source and open circuiting 2A current source



$$i' = \frac{6}{3+6} = \frac{6}{9}$$

$$\underline{\underline{i' = 0.67A}}$$

considering 2A current source and short circuiting 6V voltage source



current divider rule

$$i'' = \frac{2(3)}{3+6} = \frac{6}{9}$$

$$\underline{\underline{i'' = 0.67A}}$$

The current in 6Ω resistor is

$$i = i' + i''$$

$$i = 0.67 + 0.67 = \underline{\underline{1.34\ A}}$$

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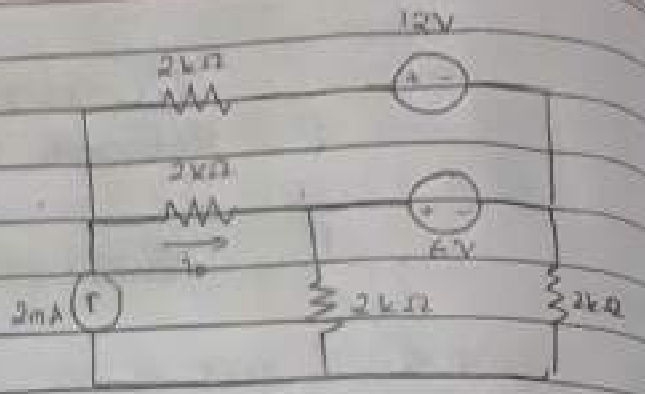
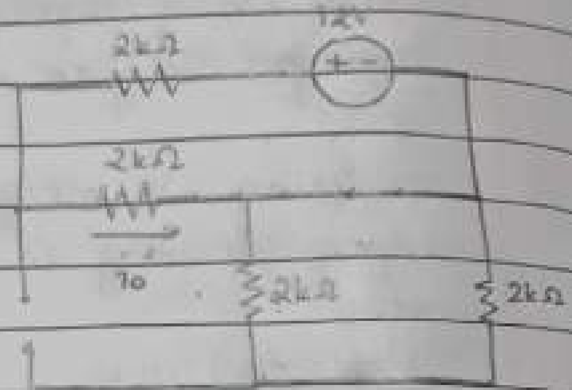
Q2: use superposition to find i_o in the circuit shown



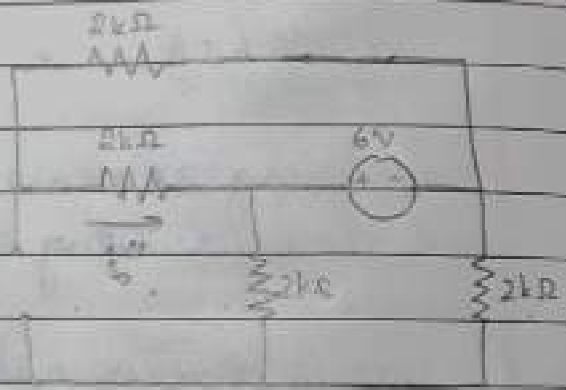
Sol: considering 12V voltage source and short circuiting 6V voltage source and open circuiting 2mA current source.



$$i_o' = \frac{12}{2k+2k}$$

$$i_o' = \underline{3mA}$$

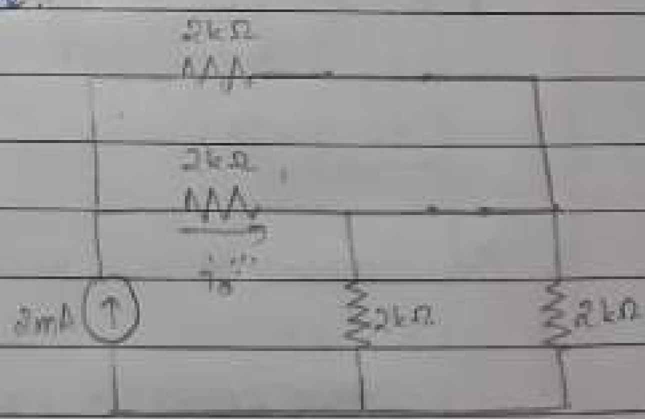
considering 6V voltage source. Short circuiting 12V voltage source and open circuiting 2mA current source



$$i_o'' = \frac{-6V}{4k\Omega}$$

$$i_o'' = \underline{-1.5mA}$$

considering 2mA current source. Short circuiting 12V and 6V voltage source.



$$i_o''' = \frac{2m(2k)}{2k+2k}$$

$$i_o''' = \frac{4}{4k} = \underline{1mA}$$

$$\therefore i_o = i_o' + i_o'' + i_o'''$$

$$= 3m - 1.5m + 1m$$

$$i_o = \underline{2.5mA}$$

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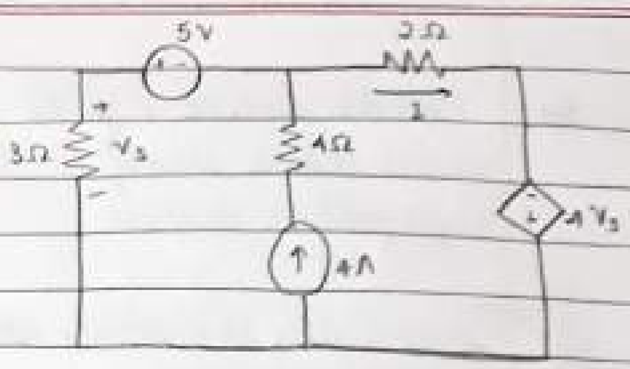
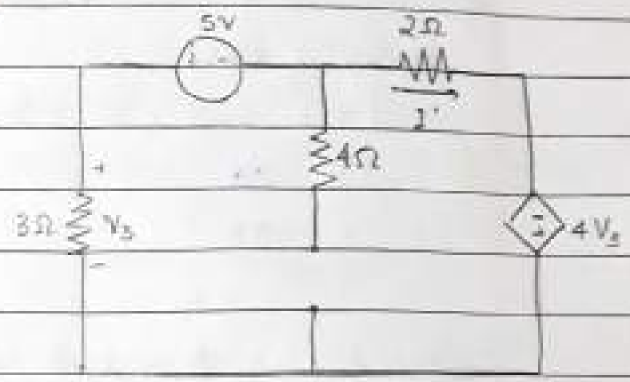
Q3. Determine the current through 2Ω resistor of the network shown using super position theorem.



Sol: considering 5V voltage source. 4A current source is open circuited.



Applying KVL

$5 + 2I' - 4V_3 - V_3 = 0$

$2I' + 5 - 5V_3 = 0$

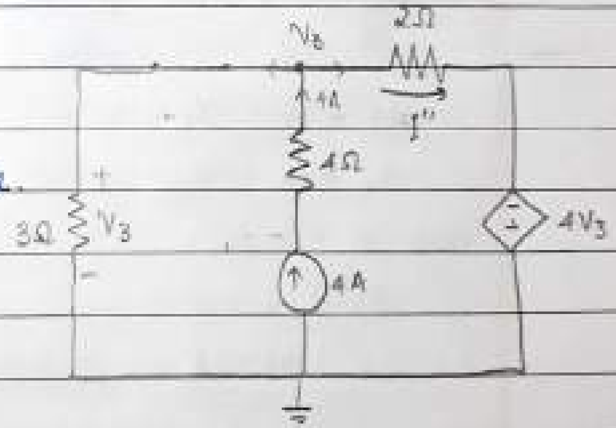
but $V_3 = -3I'$

$2I' + 5 + 15I' = 0$

$17I' + 5 = 0$

$\Rightarrow \underline{I' = -0.294\,A}$

considering 4A current source. Short circuiting 5V voltage source.



$\frac{V_3 - 0}{3} + \frac{V_3 + 4V_3}{2} = 4$

$2V_3 + 3V_3 + 12V_3 = 24$

$17V_3 = 24$

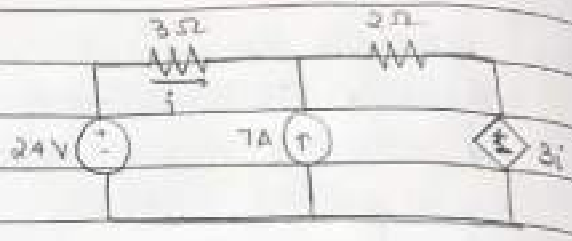
$\Rightarrow V_3 = \frac{24}{17}$ $\quad \therefore I'' = \frac{V_3 + 4V_3}{2} = \frac{5V_3}{2} = \underline{3.529\,A}$

$\underline{V_3 = 1.412\,V}$

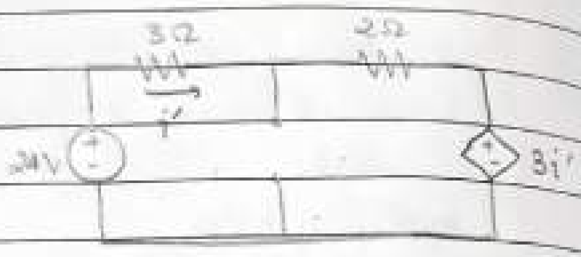
$\therefore$ Total current $I = I' + I'' = -0.294 + 3.529$

$\underline{I = 3.235\,A}$

Q4: Find the current i for the circuit shown using superposition theorem.



Sol: considering 24V voltage source open circuiting 7A current source.

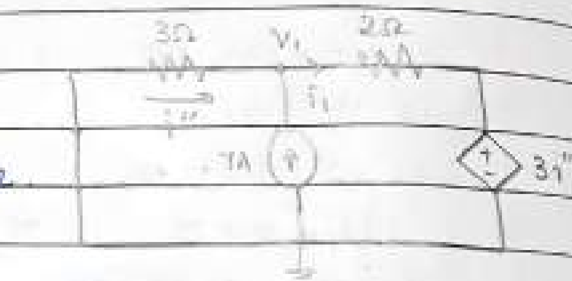


Applying KVL.

$-24 + 3i' + 2i' + 3i' = 0$

$8i' = 24$

$\underline{i' = 3A}$



Considering 7A current source. Short circuiting 24V voltage source.

At Node V_1

$\frac{0 - V_1}{3} \quad i'' + 7 = \frac{V_1 - 3i''}{2}$

$2i'' + 14 = V_1 - 3i''$

$5i'' + 14 = V_1$

but $V_1 = -3i''$

$8i'' + 14 = 0$

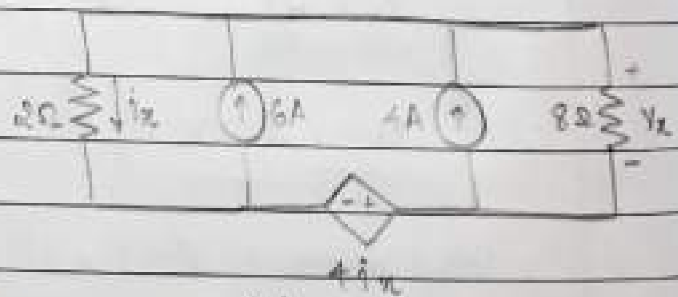
$\underline{i'' = -1.75A}$

$\therefore i = i' + i''$

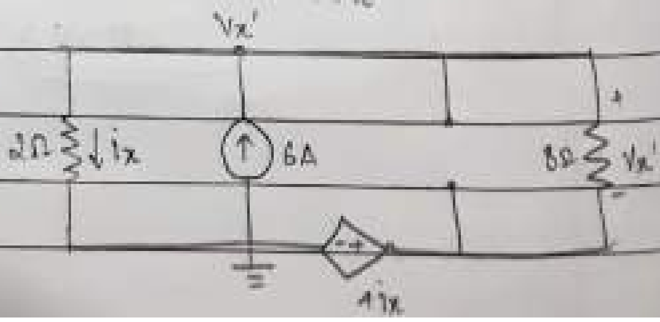
$i = 3 - 1.75$

$\underline{i = 1.25A}$

Q5: Find the potential V_x for the circuit shown using superposition theorem.



Sol: considering 6A current source Open circuiting 4A current source



At node V_x'

$\frac{V_x' - 0}{2} + \frac{V_x' - 4i_x}{8} = 6$

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but $i_x = V_x'/2$

$$\frac{V_x'}{2} + \frac{V_x' - 2V_x'}{8} = 6$$

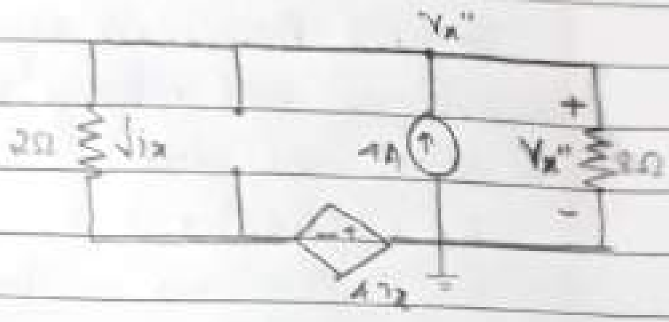
$$4V_x' - V_x = 48$$

$$3V_x' = 48$$

$$\underline{\underline{V_x' = 16V}}$$

Considering 4A current source.

Open circuiting 6A current source.



At node V_x''

$$\frac{V_x'' + 4i_x}{2} + \frac{V_x'' - 0}{8} = 4$$

but $i_x = \frac{V_x'' + 4i_x}{2} \Rightarrow 2i_x = V_x'' + 4i_x \Rightarrow i_x = \frac{-V_x''}{2}$

$$\frac{V_x'' - 2V_x''}{2} + \frac{V_x''}{8} = 4$$

$$-4V_x'' + V_x'' = 32$$

$$-3V_x'' = 32$$

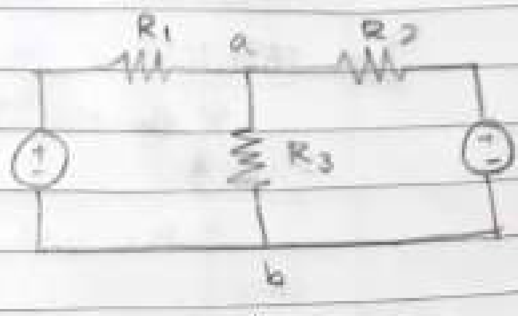
$$\underline{\underline{V_x'' = -10.67V}}$$

$V_x = V_x' + V_x''$

$V_x = 16V - 10.67V$

$\underline{\underline{V_x = 5.33V}}$

★ Thevenins Theorem :

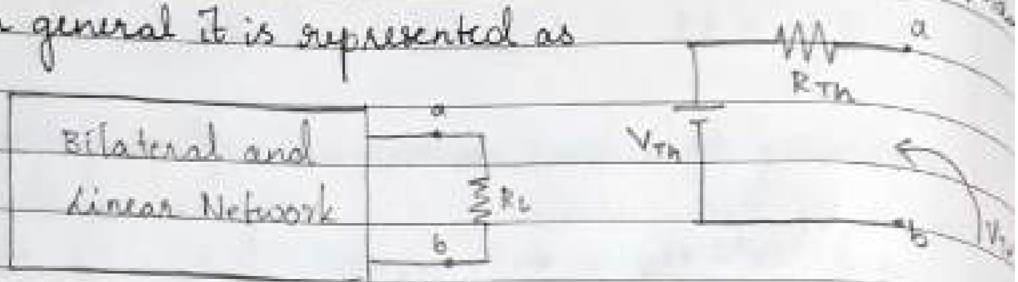


consider the circuit shown,
If we want to find voltage or current across the element we can use KVL, KCL or any analysis.
·But we want to calculate voltage and current for 20 different values of R_3, i.e., sometimes we have to see the behaviour of the circuit for different load. KVL and KCL is

time consuming, at that case Thevenin's theorem is mo useful technique. We can replace entire circuit with these two terminals 'a' and 'b' with equivalent voltag and series resistance.

Any linear bilateral network containing voltage and current sources can be replaced by single equivalent voltage source in series with one equivalent resistance across its terminal (maybe dependent or independ

In general it is represented as



V_{Th} is the voltage defined across the network after removing the load. R_{Th} is the resistance that defined across the two terminals after removing independent sources and replacing by equivalent resistance.

Voltage source is replaced by short circuit and current source is replaced by open circuit.

Since voltage source has zero series internal resistanc and current source has infinite parallel internal resistance.

CASE 1:

If the circuit consists of all independent sources

CASE 2:

If the circuit contains independent and dependent sour

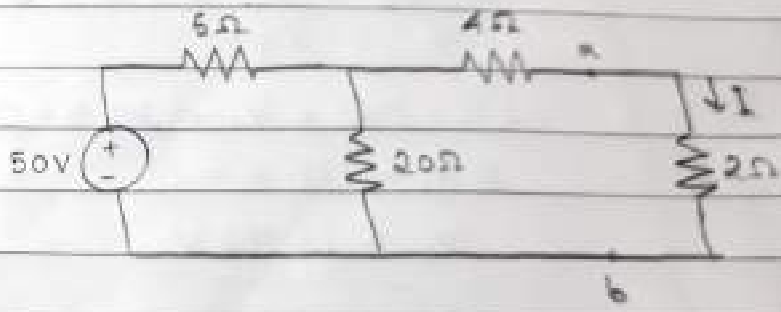
CASE 3:

If the circuit contains only dependent sources.

Thevenin's Theorem :

case 1 If the circuit consists only of independent sources.

Q. Find the current I through 2Ω resistor.



Sol Load resistance 2Ω is removed.

To find thevenin's equivalent voltage.

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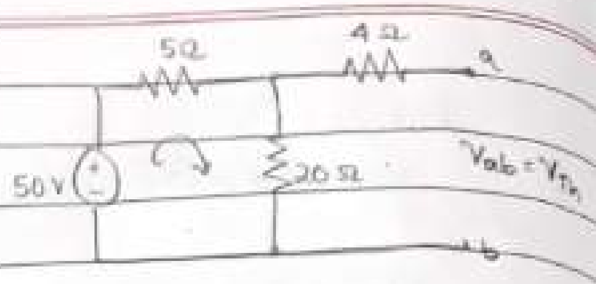
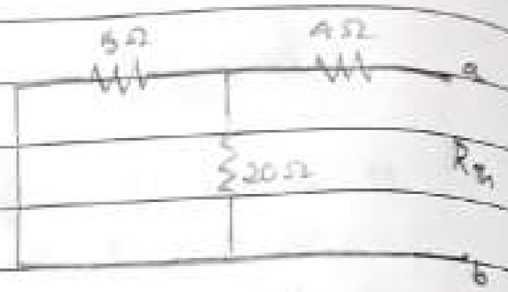


Applying KVL

$50 = (5+20)i$

$i = \frac{50}{25} = 2A$

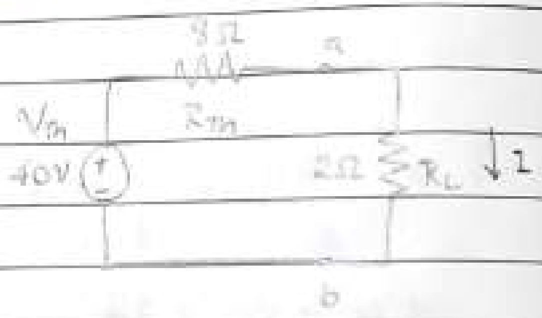
$V_{Th} = 20i = 20(2) = 40V$



To find Thevenin's equivalent resistance 50V voltage source is short circuited

$R_{Th} = (5 \| 20) + 4$

$R_{Th} = \frac{5(20)}{25} + 4$ $\quad R_{Th} = 8\Omega$



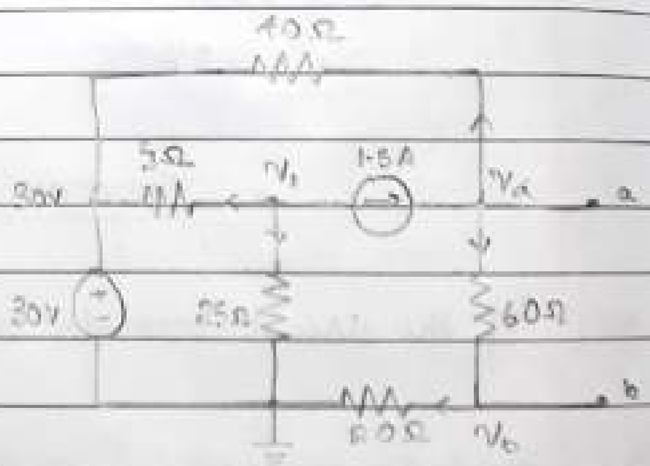
Applying KVL

$40 = 8I + 2I$

$40 = 10I$

$I = 4A$

Q7: For the circuit, get the equivalent circuit using Thevenin's theorem.



Sol: At node V_1

$\frac{V_1 - 30}{5} + \frac{V_1}{25} + 1.5 = 0$

$5V_1 - 150 + V_1 + 37.5 = 0$

$6V_1 - 112.5 = 0$

$V_1 = 18.75V$

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At node V_a

$$\frac{V_a - 30}{40} + \frac{V_a - V_b}{60} = 1.5$$

$$3V_a - 90 + 2V_a - 2V_b = 180$$

$$5V_a - 2V_b - 270 = 0 \quad \text{—— (1)}$$

At node V_b

$$\frac{V_a - V_b}{60} = \frac{V_b - 0}{20}$$

$$V_a - V_b = 3V_b$$

$$V_a = 4V_b$$

Substituting in eq (1)

$$20V_b - 2V_b - 270 = 0$$

$$18V_b = 270$$

$$\underline{\underline{V_b = 15V}}$$

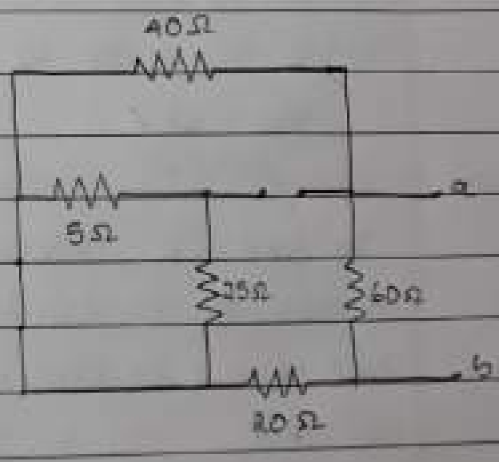
$$V_a = 4V_b$$

$$V_a = 4(15)$$

$$\underline{\underline{V_a = 60V}}$$

$$V_{Th} = V_{ab} = V_a - V_b = 60 - 15$$

$$\underline{\underline{V_{Th} = 45V}}$$

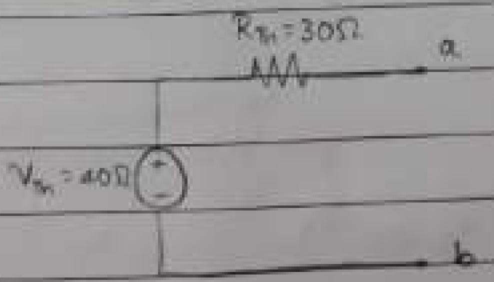


To find R_{Th}, short circuiting 30V voltage source and open circuiting 1.5A current source

$$R_{Th} = (40 + 20) \parallel 60$$

$$R_{Th} = 60 \parallel 60$$

$$\underline{\underline{R_{Th} = 30\Omega}}$$



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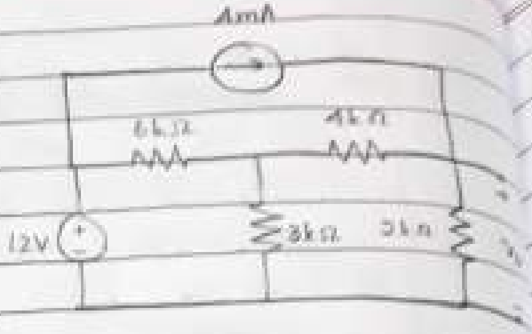
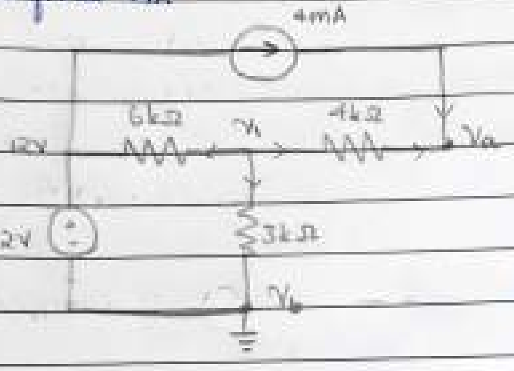
Q8: Find V_o in the circuit using Thevenin's theorem.



sol: Removing the load resistance 2kΩ.

To find V_{Th}



At node V_1

$$\frac{V_1-12}{6k}+\frac{V_1-V_a}{4k}+\frac{V_1-0}{3k}=0$$

$$2V_1-24+3V_1-3V_a+4V_1=0$$

$$9V_1-3V_a-24=0 \quad \text{---} (1)$$

At node V_a

$$4m+\frac{V_1-V_a}{4k}=0$$

$$V_1-V_a+16=0 \quad \text{---} (1)$$

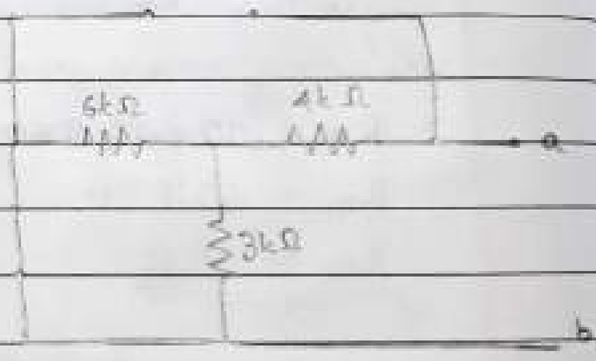
$V_1 = 12V$

$V_a = 28\,V$

$$V_{Th}=V_{ab}=V_a-V_b=28-0$$

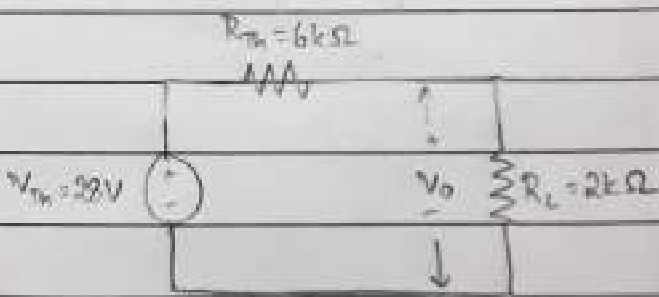
$$V_{Th}=28\,V$$

To find R_{Th}, 12V voltage source is short circuited and 4mA current source is open circuited.



$$R_{Th}=(6k \parallel 3k)+4k$$

$$R_{Th}=2k+4k=6k\Omega$$



$$V_o=\frac{V_{Th}R_L}{R_{Th}+R_L}=\frac{28(2k)}{6k+2k}$$

$$V_o=\frac{56k}{8k}=7V$$

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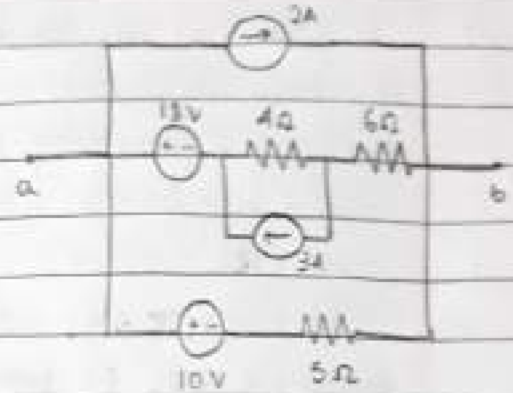
Q9: Find the thevenin's equivalent circuit at the terminals a-b.



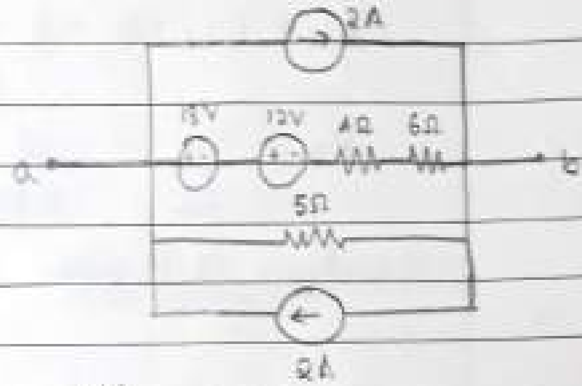
Sol: Using source transformation

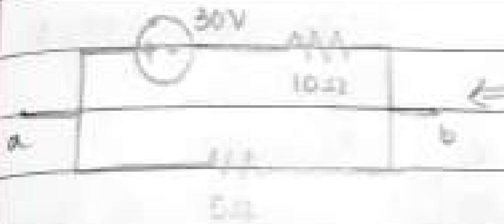
$V = I \; R_{sh}$

$V = 3(4) = 12V$

Similarly

$I = \frac{V}{R_{se}} = \frac{10}{5} = 2A$



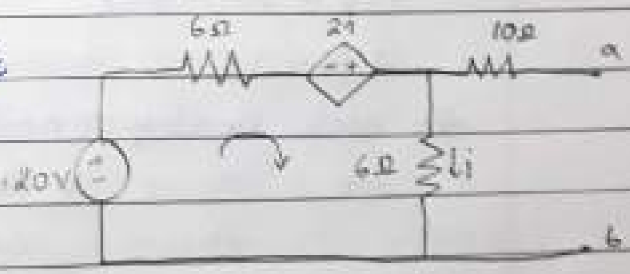


$V_{Th} = 30V$

$R_{Th} = 10 \| 5 = \underline{\underline{3.33\Omega}}$

CASE 2: If the circuit contains independent and dependent sources

Q10: Find the Thevenin's equivalent for the circuit shown.



Sol: Applying KVL

$-20 + 6i - 2i + 6i = 0$

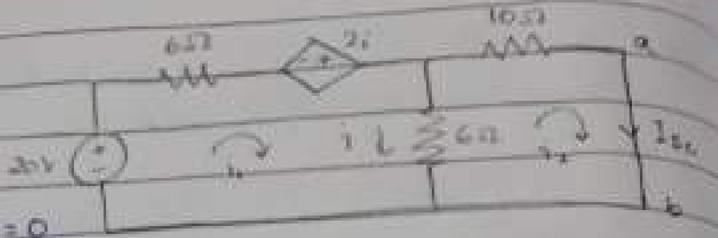
$10i = 20$

$\underline{\underline{i = 2A}}$

$V_{Th} = 6i = 6(2)$

$\underline{\underline{V_{Th} = 12V}}$

To find R_{Th}, the load is short circuited



Applying KVL to loop 1

$-20 + 6i_1 - 2i + 6(i_1 - i_2) = 0$

$-20 + 12i_1 - 2(i_1 - i_2) - 6i_2 = 0$

$-20 + 10i_1 - 4i_2 = 0$ —— ①

Applying KVL to loop 2

$10i_2 + 6(i_2 - i_1) = 0$

$-6i_1 + 16i_2 = 0$ —— ②

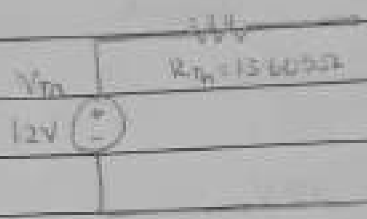
$i_1 = 2.353A$

$i_2 = 0.882A$

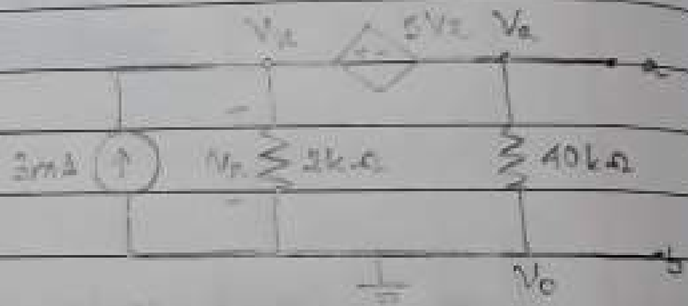
$I_{sc} = i_2 = 0.882A$

$$R_{Th} = \frac{V_{Th}}{I_{sc}} = \frac{12}{0.882}$$

$R_{Th} = 13.605\,\Omega$



Q11: Find the Thevenin's equivalent circuit across a and b.



Sol: Constraint equation

$V_x - V_a = 5V_x$

$4V_x + V_a = 0$ —— ①

At node V_x (Super node)

$$3m = \frac{V_x}{2k} + \frac{V_a - V_b}{40k}$$

$20V_x + V_a - 120 = 0$ —— ②

$V_x = -4.5V$

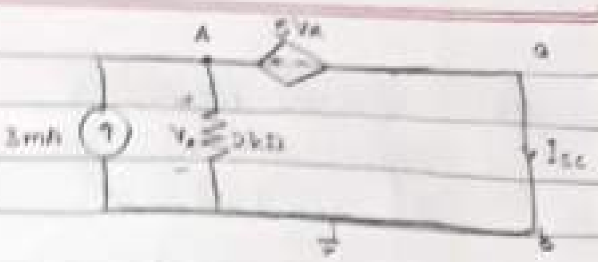
$V_a = 30V$

$V_{Th} = V_{ab} = V_a = 30V$

To find R_{Th}, the load is shorted.

~~Ap but $V_x = I_{sc}(2k)$~~

~~$I_{sc}(2k) + 5I_{sc}(2k) = 0$~~

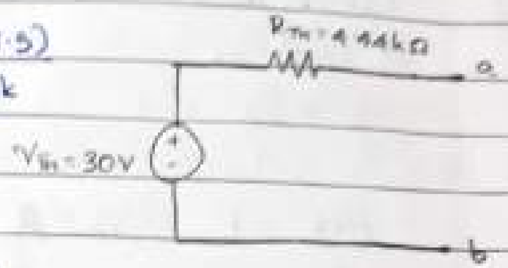


At node A

$$3m = \frac{V_x}{2k} + I_{sc}$$

$$I_{sc} = 3m - \frac{V_x}{2k} = 3m - \frac{(-7.5)}{2k}$$

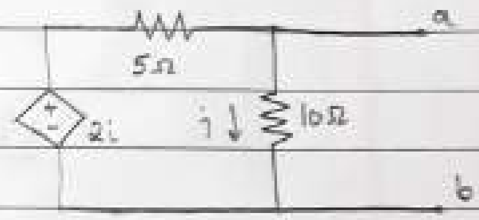
$$\underline{I_{sc} = 6.75\,mA}$$



$$R_{Th} = \frac{V_{Th}}{I_{sc}} = \frac{30}{6.75} = \underline{4.44\,k\Omega}$$

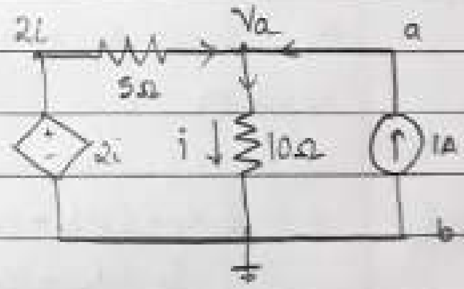
CASE 3: If the circuit contains only dependent sources.

Q12: Find the Thevenin's equivalent circuit as seen from the terminal a and b from the circuit.



Sol: $V_{Th} = 0$, as no independent source is present and current is zero.

To find R_{Th}, a test current or test voltage should be applied across the terminals.



$$R_{Th} = \frac{V_{Th}}{\text{Test current}}$$

Nodal analysis

$$\frac{2i - V_a}{5} + 1 = \frac{V_a}{10}$$

$$\frac{V_a - 5V_a}{25} + 1 = \frac{V_a}{10}$$

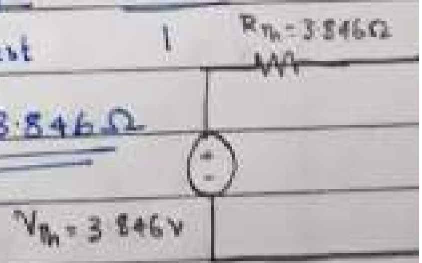
$$-8V_a + 50 = 5V_a$$

$$13V_a = 50$$

$$V_{Th} = V_{ab} = V_a = \underline{3.846\,V}$$

$$R_{Th} = \frac{V_{Th}}{I_{Test}} = \frac{3.846}{1}$$

$$\underline{R_{Th} = 3.846\,\Omega}$$



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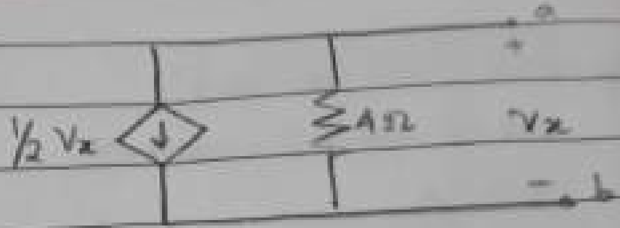
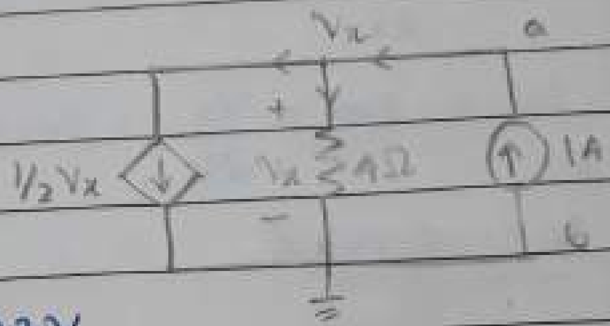
Q13: Find the thevenin's equivalent circuit for the circuit shown.



Sol: Taking test current = 1A

At node V_x

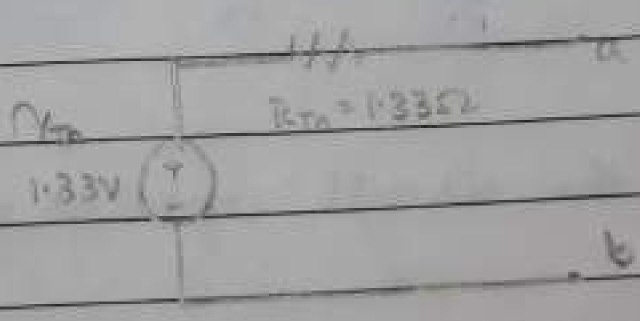


$$\frac{V_x}{2} + \frac{V_x}{4} = 1$$

$$\frac{3V_x}{4} = 1 \qquad V_x = \frac{4}{3} = \underline{\underline{1.33V}}$$

$$\underline{\underline{V_{Th} = 1.33V}}$$

$$R_{Th} = \frac{V_{Th}}{I_{test}}$$



$$R_{Th} = \frac{1.33}{1A} = \underline{\underline{1.33\Omega}}$$

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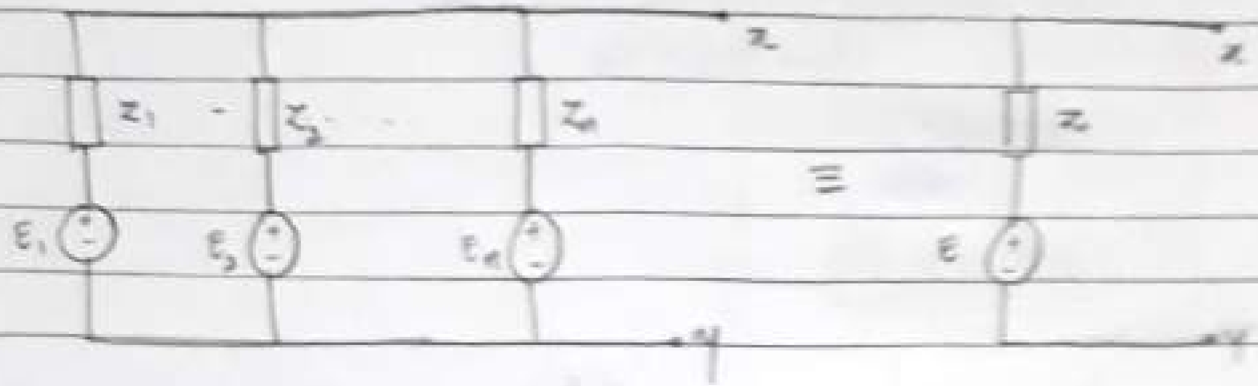
* ## Millman's Theorem :



$$E = \frac{E_1Y_1 + E_2Y_2 + \cdots + E_nY_n}{Y}$$

where $Y = Y_1 + Y_2 + Y_3 \cdots Y_n$

$Y_1 = \frac{1}{Z_1}$; $Y_2 = \frac{1}{Z_2}$; $Y_3 = \frac{1}{Z_3}$ ..... $Y_n = \frac{1}{Z_n}$

Z - Impedence

Y - Admitance

combining number of voltage sources or current sources into a single equivalent voltage or current source using the theorem.

" If n number of generators having generated emf's $E_1, E_2 \cdots E_n$ and internal impedences $Z_1, Z_2 \cdots Z_n$, then the emfs and inpedences can be combined to give a single equivalent emf E with internal impedence of equivalent value Z"

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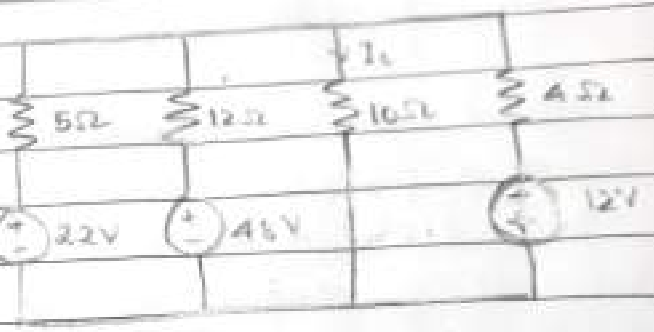
Millman's Theorem

Q14: Refer the circuit, find the current through 10 Ω resistor



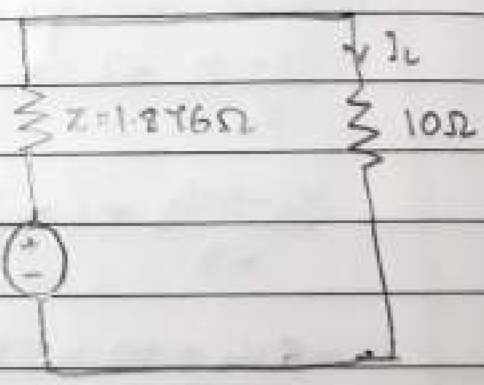
Sol:

$$E = \frac{E_1Y_1 + E_2Y_2 + E_3Y_3}{Y_1 + Y_2 + Y_3}$$

$$E = \frac{22/5 + 48/12 - 12/4}{1/5 + 1/12 + 1/4} = \frac{5.4}{0.533} = \underline{\underline{10.13V}}$$

$$Z = \frac{1}{Y_1 + Y_2 + Y_3}$$

$$Z = \frac{1}{1/5 + 1/2 + 1/4} = \frac{1}{0.533} = \underline{\underline{1.876\Omega}}$$



$$I_L = \frac{E}{Z + R_L} = \frac{10.13}{1.876 + 10} = \underline{\underline{0.853A}}$$

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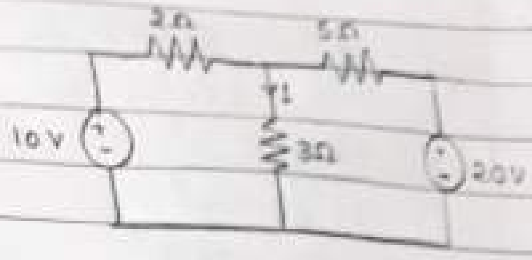
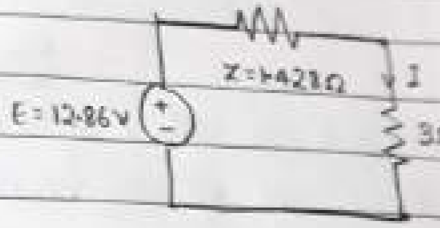
Q15: Calculate the current shown for the given circuit



Sol:
$$E = \frac{E_1Y_1 + E_2Y_2}{Y_1 + Y_2}$$

$$E = \frac{10/2 + 20/5}{1/2 + 1/5} = \frac{9}{0.7} = \underline{\underline{12.86V}}$$

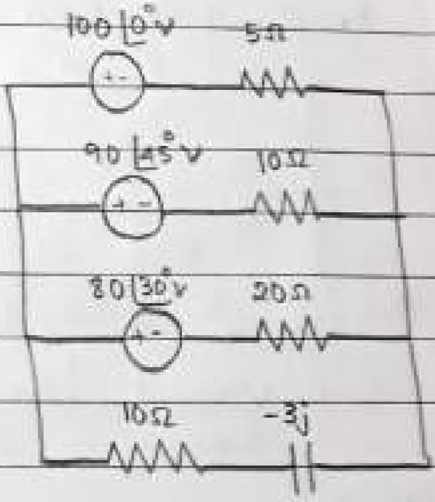


$$Z = \frac{1}{Y_1 + Y_2}$$

$$Z = \frac{1}{1/2 + 1/5} = \frac{1}{0.7} = \underline{\underline{1.428\Omega}}$$

$$I = \frac{E}{Z + R_L} = \frac{12.86}{1.428 + 3} = \underline{\underline{2.9A}}$$

Q16: Find the current through $10 - 3j$ using Millman's theorem.



Sol:
$100\angle 0^\circ = 100 + 0j$

$90\angle 45^\circ = 63.64 + 63.64j$

$80\angle 30^\circ = 69.28 + 40j$

$$E = \frac{E_1Y_1 + E_2Y_2 + E_3Y_3}{Y_1 + Y_2 + Y_3}$$

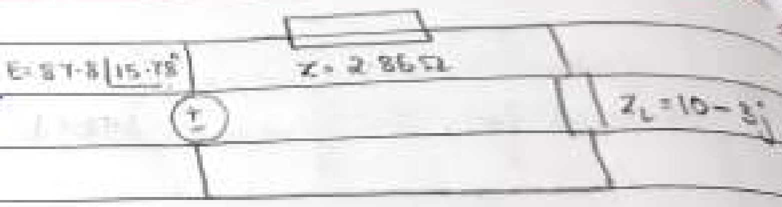
$$E = \frac{(100+0j)/5 + (63.64+63.64j)/10 + (69.28+40j)/20}{1/5 + 1/10 + 1/20}$$

$$E = \frac{20 + 6.364 + 6.364j + 3.214 + 2j}{0.35} = \frac{29.578 + 8.364j}{0.35}$$

$$E = 84.5 + 23.89j = 87.8\angle 15.78^\circ$$

$$Z = \frac{1}{Y_1 + Y_2 + Y_3} = \frac{1}{1/5 + 1/10 + 1/20} = \frac{1}{0.35} = \underline{\underline{2.86\Omega}}$$

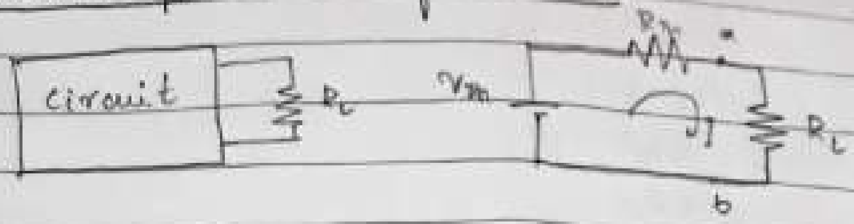
$$I = \frac{E}{Z+Z_L} = \frac{84.5+23.89j}{2.86+10-3j}$$



$$I = \frac{87.8\angle 15.78^\circ}{13.2\angle -13.13^\circ}$$

$$I = 6.65\angle 28.91^\circ$$

Maximum power transfer theorem:



In the circuit analysis we can determine the maximum power using maximum power transfer theorem. It states that " Maximum power delivered by a source represented by its Thevenin's equivalent circuit is attained when the load resistance R_L is equal to Thevenin's resistance R_{th}."

Some of the applications are audio system and radio transmission system.

$$P = I^2 R_L$$

$$P = \left(\frac{V_{th}}{R_{th}+R_L}\right)^2 R_L$$

$$P = \frac{V_{th}^2}{(R_{th}+R_L)^2} \cdot R_L \quad \text{—— (1)}$$

diff P wrt to R_L

$$\frac{dP}{dR_L} = \frac{(R_{th}+R_L)^2 V_{th}^2 - 2(R_{th}+R_L) V_{th}^2 R_L}{(R_{th}+R_L)^4} = 0$$

$$\frac{dP}{dR_L} = V_{th}^2 \left[\frac{(R_{th}+R_L) - 2R_L}{(R_{th}+R_L)^3}\right] = 0$$

$$\frac{dP}{dR_L} = V_{th}^2 \left[\frac{R_{th}-R_L}{(R_{th}+R_L)^3}\right] = 0$$

wkt $V_{th} \neq 0$

$$\therefore \frac{R_{th}-R_L}{(R_{th}+R_L)^3} = 0$$

$$R_{th} - R_L = 0 \Rightarrow \boxed{R_{th} = R_L}$$

Subtituting in eq ①

$$P_{max} = \frac{V_{th}^2}{(R_L + R_L)^2} R_L$$

$$= \frac{V_{th}^2}{4 R_L^2} R_L$$

$$\boxed{P_{max} = \frac{V_{th}^2}{4 R_L}}$$

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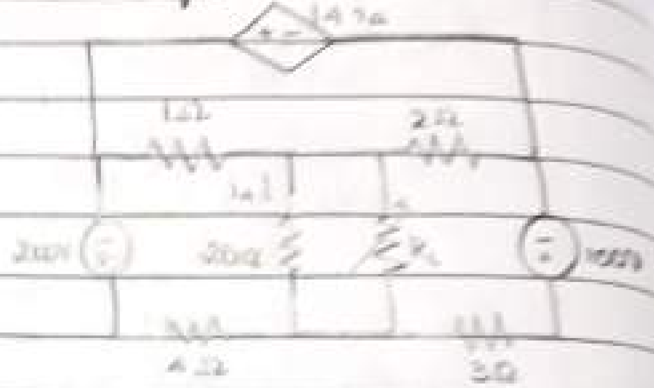
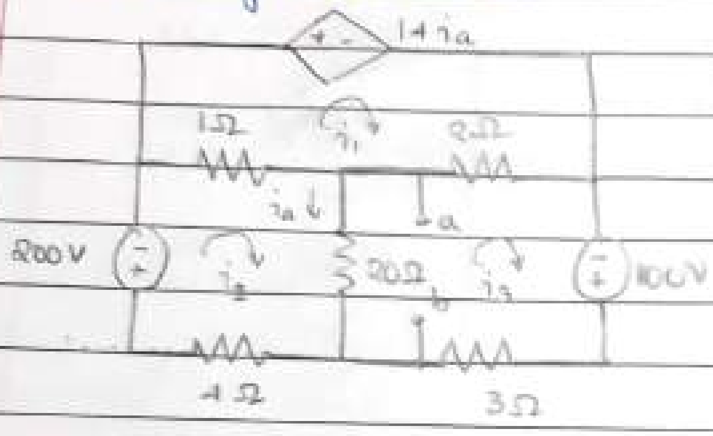
Maximum Power Transfer

Q17: Find the value of R_L for maximum power transfer and also find maximum power transferred to the load.



Sol: Removing the load R_L



Applying KVL to loop 1

$14 i_a + 2(i_1 - i_3) + 1(i_1 - i_2) = 0$

$14(i_2 - i_3) + 2(i_1 - i_3) + (i_1 - i_2) = 0$

$3i_1 + 13i_2 - 16i_3 = 0$ —— (1)

Applying KVL to loop 2

$200 + 1(i_2 - i_1) + 20(i_2 - i_3) + 4i_2 = 0$

$-i_1 + 25i_2 - 20i_3 + 200 = 0$ —— (2)

Applying KVL to loop 3

$-100 + 3i_3 + 20(i_3 - i_2) + 2(i_3 - i_1) = 0$

$-2i_1 - 20i_2 + 25i_3 - 100 = 0$ —— (3)

$i_1 = 37.5A$

$i_2 = -2.5A$

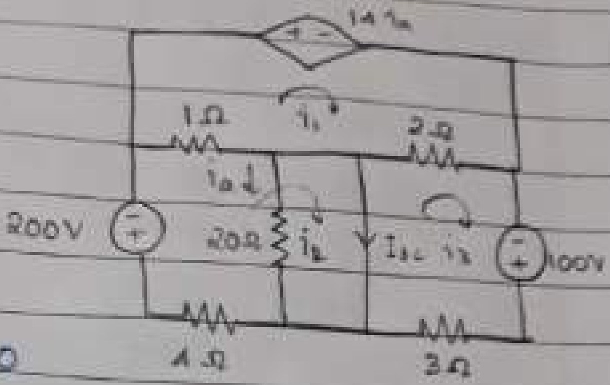
$i_3 = 5A$

$V_{Th} = 20 i_a = 20(i_2 - i_3) = 20(-2.5 - 5)$

$V_{Th} = -150V$

To find R_{Th}, short circuit the load.

$i_a = 0$, as the current flows through the short circuited path.



Applying KVL to loop 1

$14 i_a + 2(i_1 - i_3) + 1(i_1 - i_2) = 0$

$3i_1 - i_2 - 2i_3 = 0$ —— (1)

Applying KVL to loop 2

$200 + 1(i_2 - i_1) + 4(i_2) = 0$

$-i_1 + 5i_2 + 200 = 0$ —— (2)

Applying KVL to loop 3

$-100 + 3i_3 + 2(i_3 - i_1) = 0$

$-2i_1 + 5i_3 - 100 = 0$ —— (3)

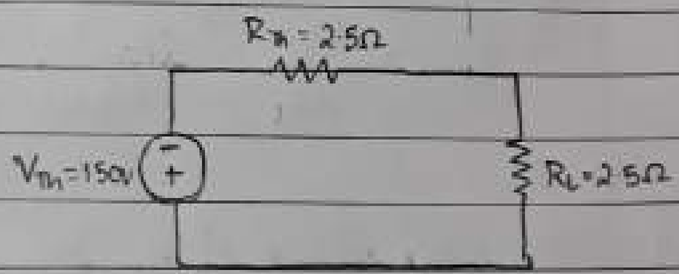
$i_1 = 0A$

$i_2 = -40A$

$i_3 = 20A$

$$R_{Th} = \frac{V_{Th}}{I_{sc}} = \frac{-150}{i_2 - i_3}$$

$$R_{Th} = \frac{-150}{-40-20} = 2.5\Omega$$

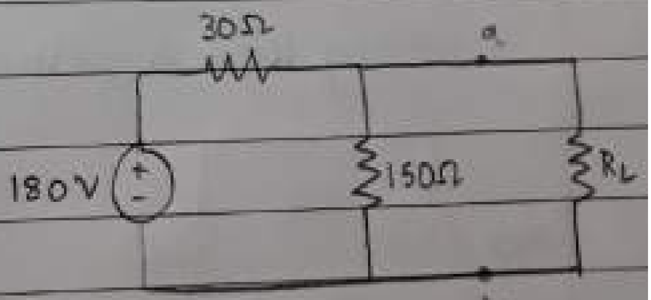


For max power transfer

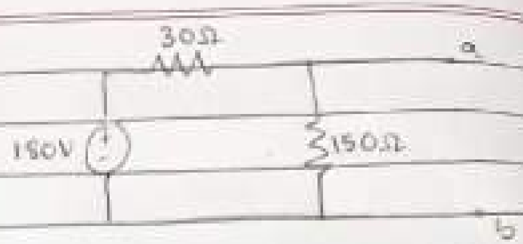
$R_{Th} = R_L = 2.5\Omega$

$$P_{max} = \frac{V_{Th}^2}{4R_L} = \frac{(150)^2}{4(2.5)} = 2.25kW$$

18: Find the load R_L that will result in maximum power delivered to the load for the circuit shown. Also determine maximum power.



Sol: Removing the load R_L

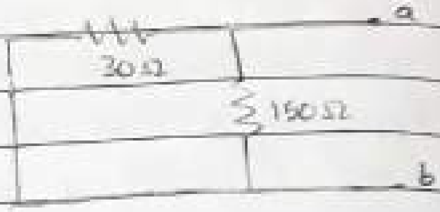


$$I = \frac{V}{30+150}$$

$$I = \frac{180}{180} = 1A$$

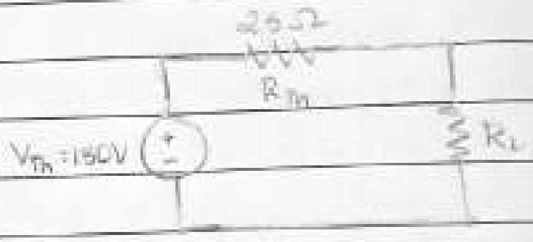
$$V_{Th} = 150(1) = 150V$$

To find R_{Th}, short circuit 180V voltage source.



$$R_{Th} = 30 \| 150$$

$$R_{Th} = 25\Omega$$



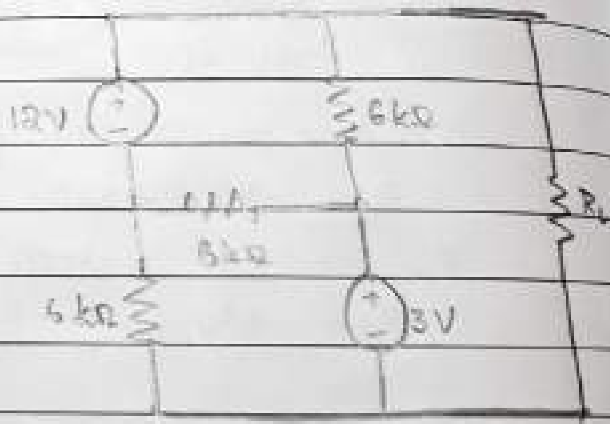
For max power transfer

$$R_L = R_{Th} = 25\Omega$$

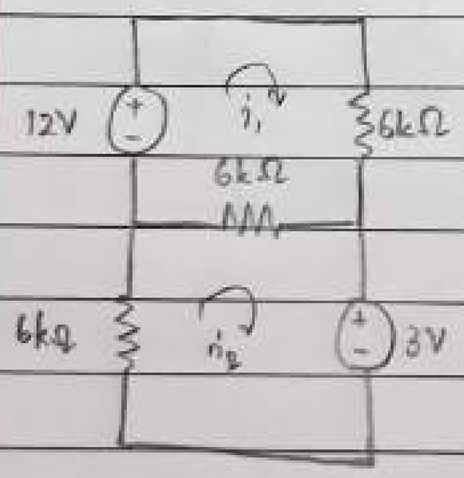
Max power transfered

$$P_{max} = \frac{V_{Th}^2}{4R_L} = \frac{(150)^2}{4(25)} = 225W$$

Q19: Find the value of R_L for maximum power transfer and also find maximum power transfered to the load.



sol: Removing load resistance R_L



Applying KVL to loop 1

$$-12 + 6k\,i_1 + 6k(i_1 - i_2) = 0$$

$$12k\,i_1 - 6k\,i_2 - 12 = 0 \quad \text{—} (1)$$

Applying KVL to loop 2

$$3 + 6k\,i_2 + 6k(i_2 - i_1) = 0$$

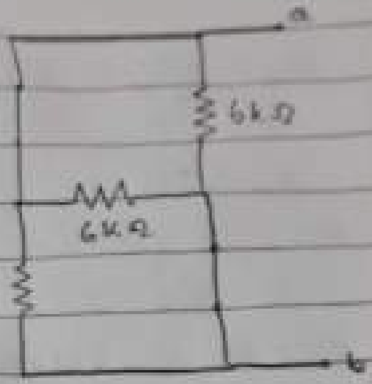
$$-6k\,i_1 + 12k\,i_2 + 3 = 0 \quad \text{—} (2)$$

$$i_1 = 1.167mA \qquad i_2 = 0.33mA$$

$V_{Th} = 6k(i_1) + 3$

$V_{Th} = 6k(1.167m) + 3 = 10V$

To find R_{Th}, 12V and 3V voltage sources are short circuited.

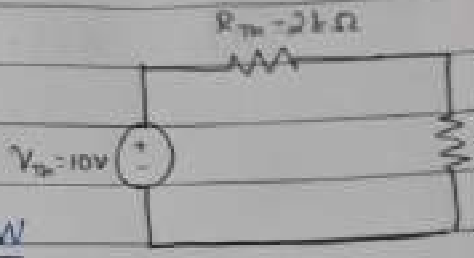


$R_{Th} = (6k \| 6k \| 6k)$

$R_{Th} = 2k\Omega$

For max power transfer

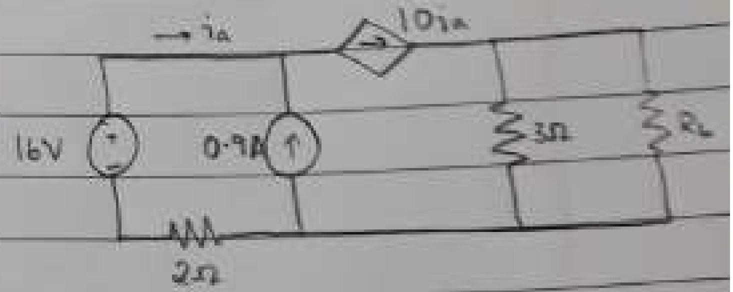
$R_L = R_{Th} = 2k\Omega$



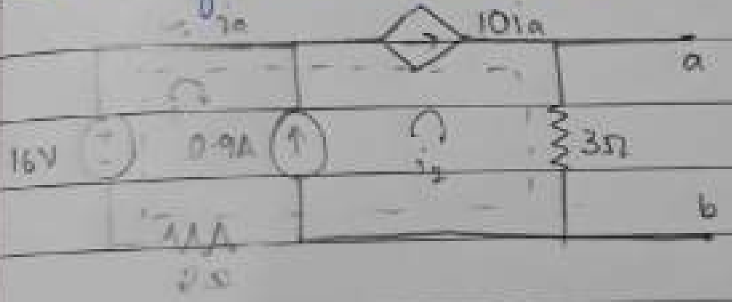
Max power transfered

$$P_{max} = \frac{V_{Th}^2}{4R_L} = \frac{(10)^2}{4(2k)} = 12.5mW$$

20: Find R_L and the maximum power transfered to the load



A: Removing load resistance R_L



~~Apply supermesh~~

~~[illegible]~~

To find R_{Th}, short circuit the load.

constraint equation

$i_2 - i_1 = 0.9A$ —— (1)

but $i_2 = 10i_a$ and $i_a = i_1$

$i_2 = 10i_1$

From eq (1)

$10i_1 - i_1 = 0.9$

$i_1 = 0.1A$ $\quad i_2 = 1A$

at node A by KCL

$i_a + 0.9 = 10i_a$

$0.9 = 9i_a$

$i_a = 0.1$

$I_{sc} = 10i_a = 1A$

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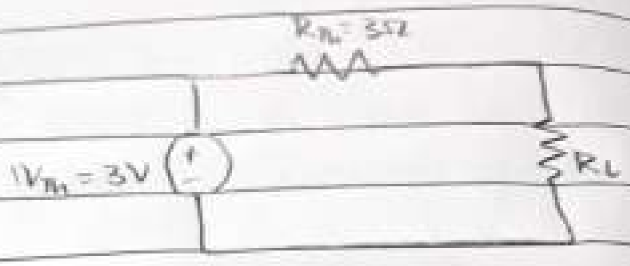
Date
Page

$$R_{Th} = \frac{V_{Th}}{I_{sc}} = \frac{3i_2}{10i_a} = \frac{3(1)}{1}$$

$R_{Th} = 3\Omega$

For max power transfer

$R_L = R_{Th} = 3\Omega$



Max power transfered

$$P_{max} = \frac{V_{Th}^2}{4R_L} = \frac{3^2}{4(3)} = 0.75W$$

UNIT - 3

Resonant Circuits

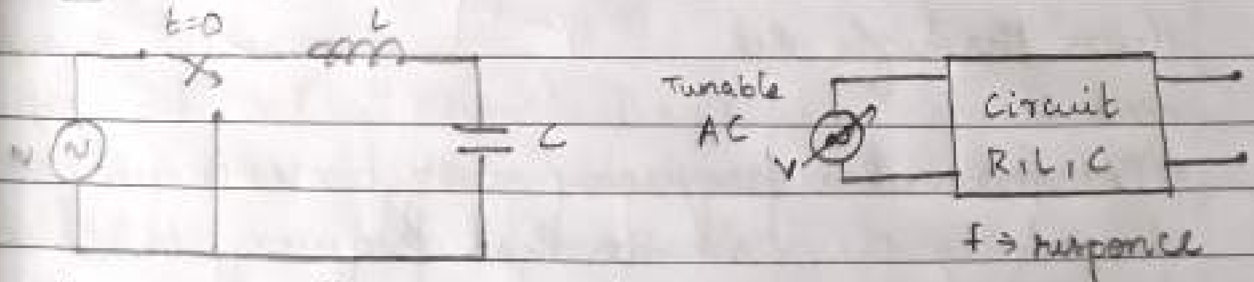
Resonance occurs in an electrical circuit due to the presence of energy storing elements like inductor and capacitors. There are two types of resonance namely ~~Resonance~~ series resonance and parallel resonancy. These are classified based on the network elements that are connected in series or parallel.

* Resonance in Electrical Circuits:

Resonance is nothing but the phenomenon at which the response of the circuit is maximum for the given particular frequency. That particular frequency when tuned will get a maximum response at the output is called the resonance in electrical circuit and that particular frequency where we get maximum output is known as Resonance frequency.

Inductor and capacitor deals with resonance because both have ability to store energy.

Ex: LC circuit



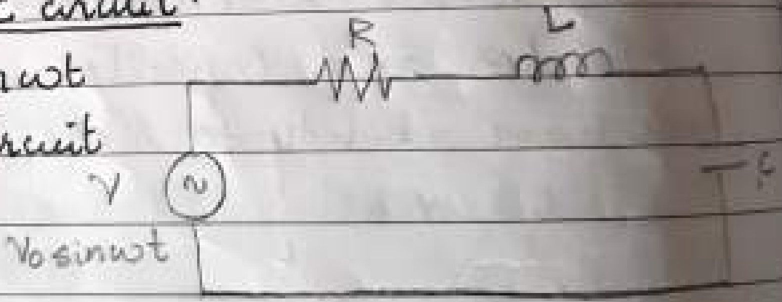
When we apply same voltage source V to the circuit at time t=0, we are removing voltage source and replace with short circuit. As we know that L and C stores energy even if voltage source is removed, L will supply the current, later capacitor acts as voltage source and again supply energy back to inductor. In this way energy gets transfered between capacitor and inductor.

Rate of energy transfer depends on L and C, because of transfer of energy we see the oscillations in the circuit, since the circuit is ideal and no resistance is involved then this oscillations will continue forever. But all the circuits have some resistance in the circuit because of which the energy will get dissipated across the resistor and eventually oscillations will drain out. Hence to maintain the oscillation an external source with same frequency of oscillation applied to LC circuit so that the oscillations will continue forever.

L and C have tendency to produce oscillations, R has tendency to reduce these oscillations.

★ **Series Resonance in RLC circuit:**

A voltage of $V = V_0 \sin \omega t$ is applied across RLC circuit hence some current flows through the circuit



At particular frequency we get maximum current flowing in the circuit, that frequency is called resonant frequency.

At lower frequency, there will not be current flowing through the circuit and at higher frequency, there will not be current flowing through the circuit.

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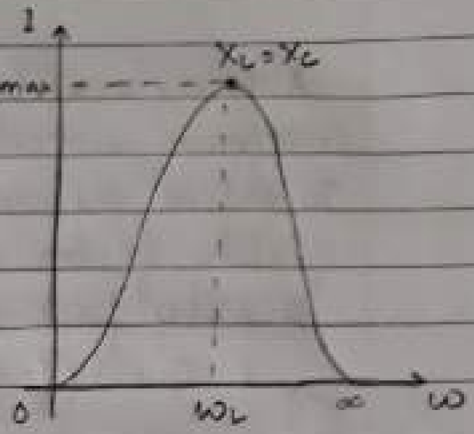


As we move towards resonant frequency the current suddenly increases and as we move away from resonant frequency current decreases eventually.

wkt $X_L = 2\pi f L = \omega L$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{\omega C}$$

At $\omega = 0$
$X_L = 0$
$X_C = \infty$

At this condition capacitor acts as open circuit.

At $\omega = \infty$
$X_L = \infty$
$X_C = 0$

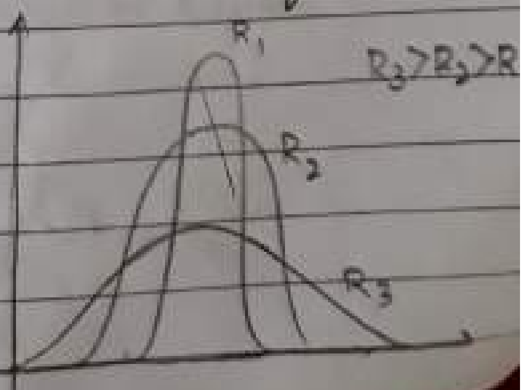
At this condition inductor acts as open circuit

Hence at lower and higher frequencies the current flowing is minimum and at resonant frequency $X_L = X_C$. The impedence of the circuit is purely resistive where we find maximum current flowing in the circuit.

RLC circuits provide some kind of selectivity for some partiular frequency and this principle is used in radio recivers.

For good selectivity the resonance curve should be more narrower, to acheive this the value of resistanc should be as low as possible.

Three different resonance curve for three different resistance values. For good selectivety the resistance should be as low as possible.



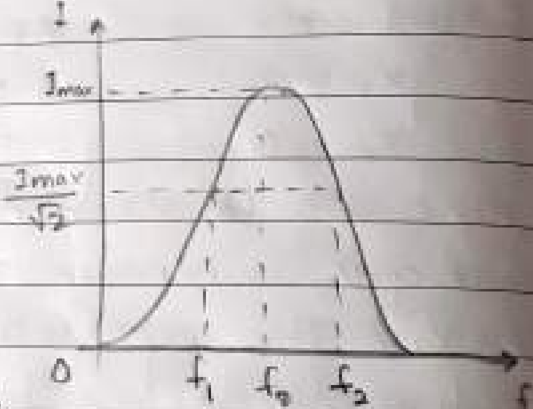
A low resistance value, the peak current (maximum current) is shown in the response curve. It depends on to define the selectivity of curve. quality factor and band width.

Quality factor is defined as

$$\text{Quality factor} = \frac{2\pi \times \text{energy stored in L or C}}{\text{Energy dissipated across R}}$$

As the value of R increases, the circuit has less tendency to oscillate because the resonant curve gets wider. Basically quality factor defines sharpness of the resonant curve.

Band width



The difference between half power frequencies.

Half power frequency is a frequency at which power of the circuit is reduced to half. In terms of current it is given by $I_{max}/\sqrt{2}$, where f_1 is lower cut off frequency and f_2 is upper cut off frequency. $BW = f_2 - f_1$

And the diffrence between f_2 and f_1 is Band width.

In terms of angular frequency: $BW = \omega_2 - \omega_1$

For good selectivity, the bandwidth of the circuit should be as narrow as possible.

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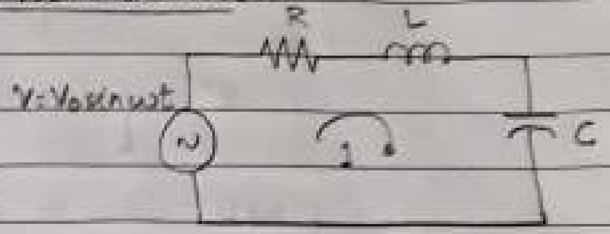
Mathematical expression for series RLC circuit

1. Resonant Frequency

$$I = \frac{V}{Z}$$



where $Z = R + X_L + X_C$

$$Z = R + j\omega L - \frac{j}{\omega C}$$

$$Z = R + j\left[\omega L - \frac{1}{\omega C}\right]$$

$$I_{max} = \frac{V}{Z_{min}} \Rightarrow \omega L - \frac{1}{\omega C} = 0 \Rightarrow Z_{min} = R$$

Resonance condition

$$X_L = X_C$$

$$\omega L = \frac{1}{\omega C} \Rightarrow Z_{min} = R$$

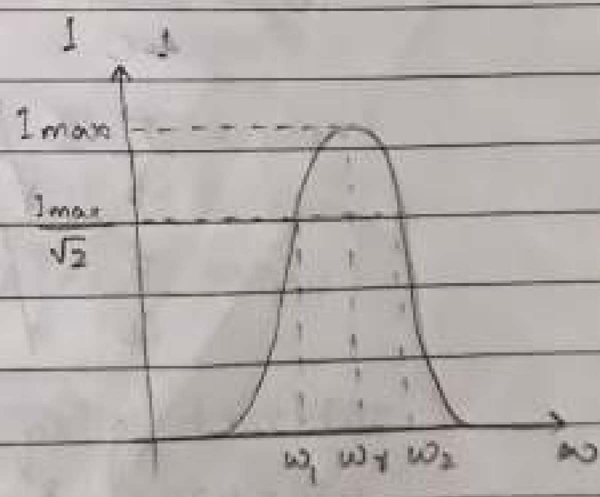
$$I_{max} = \frac{V}{R}$$

$$\omega L = \frac{1}{\omega C} \Rightarrow \omega^2 = \frac{1}{LC}$$

$$\therefore \boxed{\omega_r = \frac{1}{\sqrt{LC}}} \quad \boxed{f_r = \frac{1}{2\pi\sqrt{LC}}}$$

2. Bandwidth:

at $\omega_1 \Rightarrow I_1 = \frac{I_{max}}{\sqrt{2}}$



$$I_1 = \frac{V}{\sqrt{R^2 + \left(\omega_1 L - \frac{1}{\omega_1 C}\right)^2}}$$

squaring on both sides

$$I_1^2 = \frac{V^2}{R^2 + \left(\omega_1 L - \frac{1}{\omega_1 C}\right)^2} = \frac{I_{max}^2}{2} = \frac{V^2}{2R^2}$$

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$$\therefore 2R^2 = R^2 + \left(\omega_1 L - \frac{1}{\omega_1 C}\right)^2$$

$$R^2 = \left(\omega_1 L - \frac{1}{\omega_1 C}\right)^2$$

$$\omega_1 L - \frac{1}{\omega_1 C} = \pm R$$

$|||^{ly}$ $\omega_2 L - \frac{1}{\omega_2 C} = \pm R$

$[\omega_1 < \omega_2]$ since X_C is dominant

$[\omega_2 > \omega_1]$ since X_L is dominant

$$\omega_2 L - \frac{1}{\omega_2 C} = R$$

$$\omega_1 L - \frac{1}{\omega_1 C} = -R$$

$$\omega_1^2 LC - 1 = -R\omega_1 C$$

$$\omega_1^2 - \frac{1}{LC} + \frac{R\omega_1}{L} = 0$$

$$\omega_1 = \frac{-R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$\boxed{\omega_1 = \frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}} \quad \text{—— (1)}$$

3.

$$\omega_2 L - \frac{1}{\omega_2 C} = R$$

$$\omega_2^2 LC - 1 = R\omega_2 C$$

$$\omega_2^2 - \frac{1}{LC} - \frac{R\omega_2}{L} = 0$$

$$\omega_2 = \frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$\boxed{\omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}} \quad \text{—— (2)}$$

4.

subtracting eq (1) by eq (2)

$$\omega_2 - \omega_1 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} - \left(\frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right)$$

$$\boxed{\text{Bandwidth} = \omega_2 - \omega_1 = \frac{R}{L}}$$

$$\boxed{\text{Bandwidth} = f_2 - f_1 = \frac{R}{2\pi L}}$$

3. <u>Relation between upper cut off frequency, lower cut off frequency and resonant frequency</u>

Multiply ω_1 and ω_2

$$\omega_1\omega_2 = \left(\frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right)\left(\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right)$$

$$\omega_1\omega_2 = \left[\cancel{\left(\frac{R}{2L}\right)^2} + \frac{1}{LC} - \cancel{\left(\frac{R}{2L}\right)^2}\right]$$

$$\omega_1\omega_2 = \frac{1}{LC}$$

wkt $\frac{1}{LC} = \omega_r^2$

$\therefore \quad \omega_r^2 = \omega_1\omega_2$

$$\Rightarrow \boxed{\omega_r = \sqrt{\omega_1\omega_2}}$$

4. <u>Quality Factor</u>

At resonant frequency

$$Q = \frac{X_C}{R} = \frac{X_L}{R}$$

$$Q = \frac{\omega L}{R} = \frac{1}{\omega C R}$$

$$\boxed{Q = \frac{1}{R}\sqrt{\frac{L}{C}}} \qquad \boxed{Q = \frac{\omega_r}{BW}}$$

Summary

Impedence	$Z_{min} = R$
Band width	R/L
Quality factor	$\frac{\omega L}{R} = \frac{1}{\omega CR} \Rightarrow \frac{1}{R}\sqrt{\frac{L}{C}}$ and $\frac{\omega_r}{BW}$
ω_0	$\frac{1}{\sqrt{LC}}$
current	$I_{max} = \frac{V}{R}$

NOTE:

Impedence of series resonant circuits in terms of quality factor and frequency deviation or fractional variation.

Q - Quality factor

δ - Fractional variation / Frequency deviation.

δ is given by

$$\delta = \frac{f - f_r}{f_r}$$

where f - actual frequency

f_r - resonant frequency

In terms of angular frequency

$$\frac{\omega}{\omega_r} = 1 + \delta$$

Impedence of series resonant circuit in terms of R, Q and δ is given by

$$Z = R[1 + jQ\delta(2-\delta)]$$ Derivation (Tutorial)

Derive an expression for the impedence of a series RLC circuit in terms of resistance, quality factor and frequency deviation

wkt $Z = R + j\left(\omega L - \frac{1}{\omega C}\right)$

$$Z = R\left[1 + j\left(\frac{\omega_r L}{R}\frac{\omega}{\omega_r} - \frac{1}{\omega_r RC}\frac{\omega_r}{\omega}\right)\right]$$

$$Z = R\left[1 + j\left(Q\frac{\omega}{\omega_r} - Q\frac{\omega_r}{\omega}\right)\right]$$

$$Z = R\left[1 + jQ\left(\frac{\omega}{\omega_r} - \frac{\omega_r}{\omega}\right)\right]$$

wkt $\delta = \frac{\omega - \omega_r}{\omega_r} \Rightarrow \delta = \frac{\omega}{\omega_r} - 1 \Rightarrow \frac{\omega}{\omega_r} = \delta + 1$

similarly $\cancel{\delta = \frac{\omega - \omega_r}{\omega_r}}$ $\frac{\omega_r}{\omega} = \frac{1}{\delta + 1}$

$$Z = R\left[1 + jQ\left(\delta + 1 - \frac{1}{\delta + 1}\right)\right] = R\left[1 + jQ\left[(1+\delta) - (1+\delta)^{-1}\right]\right]$$

$$\cancel{Z = R\left[1 + jQ\left(\delta^2 + 2\delta + 1 - 1\right)\right]}$$

$$\cancel{Z = R\left[1 + j\delta Q(\delta + 2)\right]}$$

By binomial series $(1+\delta)^{-1} = 1 - \delta + \delta^2 - \delta^3 \dots$

$$Z = R\left[1 + jQ(2\delta - \delta^2)\right]$$

$$Z = R\left[1 + jQ\delta(2 - \delta)\right]$$

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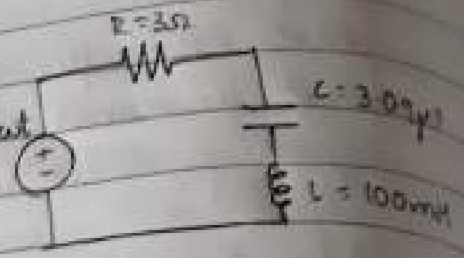

Q1: Find:

a. Resonant frequency and half power frequency

b. Calculate quality factor and band width

c. the amplitude of the current at ω_0, ω_{c_1} and ω_{c_2}

Sol: a. $\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{100\times10^{-3}\times3.09\times10^{-6}}}$

$\omega_0 = 1.798$ k rad/sec. 1800 rad/sec

$$\omega_1 = \frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$= \frac{-3}{2(100\times10^{-3})} + \sqrt{\left[\frac{3}{2(100\times10^{-3})}\right]^2 + \frac{1}{100\times10^{-3}\times3.09\times10^{-6}}}$$

$\omega_1 = 1784$ rad/sec

$$\omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$= \frac{3}{2(100\times10^{-3})} + \sqrt{\left(\frac{3}{2(100\times10^{-3})}\right)^2 + \frac{1}{100\times10^{-3}\times3.09\times10^{-6}}}$$

$\omega_2 = 1814$ rad/sec

b. Quality Factor

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{3}\sqrt{\frac{100\times10^{-3}}{3.09\times10^{-6}}}$$

$Q = 59.96$

Band width : $BW = \omega_2 - \omega_1 = 1814 - 1784$

$BW = 30$ rad/sec

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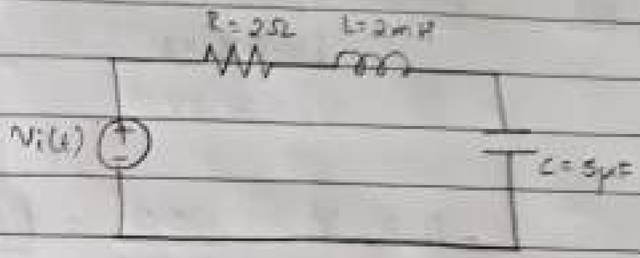
c. At resonant frequency (ω_0)

$$I = \frac{V}{R} = \frac{10}{3} = \underline{3.33A}$$

At ω_{c_1} and ω_{c_2}

$$\boxed{I = \frac{V}{\sqrt{2}R}} = \frac{10}{\sqrt{2}(3)} = \underline{2.36A}$$



Q2. Find:

a. Resonant frequency

b. Quality factor

c. Band width

d. Determine change in Q and the band width if R is changed to 0.4Ω.

sol: a. $$\omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{2\times10^{-3}\times5\times10^{-6}}}$$

$$\underline{\omega_r = 10k \text{ rad/sec}}$$

b. $$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{2}\sqrt{\frac{2\times10^{-3}}{5\times10^{-6}}} = \underline{10}$$

c. $$BW = \frac{R}{L} = \frac{2}{2\times10^{-3}} = \underline{1000 \text{ rad/sec}}$$

d. $R = 0.4\Omega$

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{0.4}\sqrt{\frac{2\times10^{-3}}{5\times10^{-6}}} = \underline{50}$$

$$BW = \frac{R}{L} = \frac{0.4}{2\times10^{-3}} = \underline{200 \text{ rad/sec}}$$

Q3: A resistor and capacitor are connected in series with a variable inductance. When the circuit is energised for a 200V / 50Hz supply, the maximum current delivered by varying the inductance is 0.314 A. The voltage across the capacitor is 300V. Find the circuit parameters R, L, C.

Sol: Given: $V = 200V$ $\quad f = 50Hz$

$I = 0.314A$

$V_c = 300V$

$$R = \frac{V}{I} = \frac{200}{0.314} = 636.94\,\Omega$$

$$I = \frac{V_c}{X_c} \Rightarrow X_c = \frac{V_c}{I} = \frac{300}{0.314} = 955.4\,\Omega$$

$$X_c = \frac{1}{\omega C} \Rightarrow C = \frac{1}{X_c \omega} = \frac{1}{X_c 2\pi f}$$

$$C = \frac{1}{955.4 \times 2 \times 3.14 \times 50} = 3.33\,\mu F$$

At resonant frequency

$$X_c = X_L$$

$$\frac{1}{\omega C} = \omega L$$

$$L = \frac{1}{\omega^2 C} = \frac{1}{(2 \times 3.14 \times 50)^2 \times 3.33 \times 10^{-6}}$$

$$L = 3.04\,H$$

4: A series resonant circuit resonating at 1000kHz with effective value Q=100 is having a total value of resistance in the circuit of 50Ω. The applied voltage is 10V. Calculate the magnitude and phase angle of current and impedence of the circuit below 10kHz.

Sol: Given: $f_r = 1000\,kHz$ $\quad V = 10V$

$Q = 100$ $\quad f - f_r = -10\,kHz$

$R = 50\Omega$

$$\delta = \frac{f - f_r}{f_r} = \frac{-10k}{1000k} = -0.01$$

$$Z = R\left[1 + j\delta Q(2-\delta)\right]$$

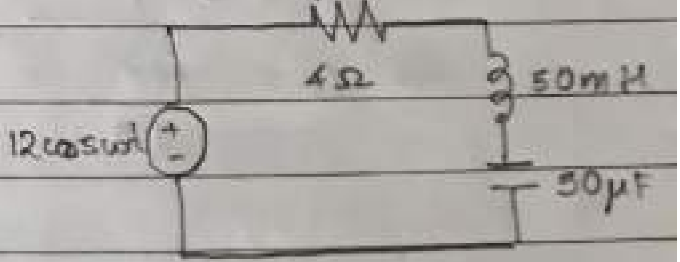
$$Z = 50\left[1 + j(-0.01)(100)(2+0.01)\right]$$

$$Z = 50 - 100j\ \Omega$$

$$Z = 112.25\angle -63.54^\circ\ \Omega$$

$$I = \frac{V}{Z} = \frac{10\angle 0}{112.25\angle -63.54} = 0.089\angle 63.54^\circ$$

5: A variable frequency voltage source drives the network shown determine the resonant frequency, quality factor band width and average power dissipated by the network at resonance.



Sol: $$\omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{50\times10^{-3}\times 50\times10^{-6}}}$$

$$\omega_r = 632.45\ rad/sec$$

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Quality factor

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{4}\sqrt{\frac{50\times10^{-3}}{50\times10^{-6}}} = 7.9$$

Band width

$$BW = \frac{R}{L} = \frac{4}{50\times10^{-3}} = 80 \text{ rad/sec}$$

Average power dissipated

$$= \frac{V^2}{R} = \frac{12^2}{4} = 36W$$

Q6: The quality factor of a series RLC network is 100. The maximum amplitude of the current is 10A. When the applied voltage is 100V. If L = 0.1H, find the value of C.

Sol: Given: $Q = 100$ $\quad V = 100V$

$I_{(max)} = 10A$ $\quad L = 0.1H$

$$I_{(max)} = \frac{V}{R}$$

$$R = \frac{V}{I_{(max)}} = \frac{100}{10} = 10\Omega$$

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$C = \frac{1}{\frac{Q^2R^2}{L}} = \frac{1}{\frac{100^2\times10^2}{(0.1)}} = 0.1\mu F$$

* Power Equations:

wkt $I_{max} = \frac{V}{R}$

Power in the circuit is maximum and given by

$$P_{max} = I_{max}^2 R$$

Half of maximum power is given by

$$\frac{P_{max}}{2} = \left(\frac{I_{max}}{\sqrt{2}}\right)^2 R$$

At resonant frequency

$$P_{max} = I_{max}^2 R$$

At frequency f_1, power in the circuit is half

$$P' = \frac{1}{2} I_{max}^2 R$$

At frequency f_2, power in the circuit is half

$$P' = \frac{1}{2} I_{max}^2 R$$

Q1: Design a series resonant circuit to have $\omega_r = 2500$ rad/s, $Z = 100\Omega$, BW = 500 rad/sec.

Sol: Given: $\omega_r = 2500$ rad/sec

$Z = 100\Omega$

BW = 500 rad/sec

At resonance $Z = R = 100\Omega$

$$BW = \frac{R}{L} \Rightarrow L = \frac{R}{BW} = \frac{100}{500 \text{ rad/sec}} = 0.2H$$

$$Q = \frac{\omega_r}{BW} = \frac{2500}{500}$$

$Q = 5$

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} \Rightarrow \frac{1}{C} = \frac{Q^2R^2}{L} = \frac{5^2(100)^2}{0.2} \Rightarrow C = 10.8\mu F$$

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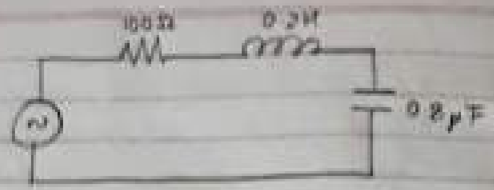
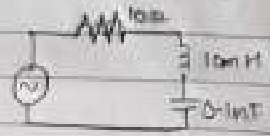


Q2: A series resonant circuit has L = 10 mH, select C and R so that $\omega_0 = 10^4$ rad/sec and the band width is 10^3 rad/sec.

Sol: $\omega_r = \frac{1}{\sqrt{LC}} \Rightarrow C = \frac{1}{\omega_r^2 L} = \frac{1}{(10^4)^2(10\times10^{-3})} = \underline{0.1\,mF}$

$BW = \frac{R}{L} \Rightarrow R = (BW)L = 10^3(10\times10^{-3}) = \underline{10\Omega}$



Q3: A coil of inductance 9H and resistance 50Ω in series with a capacitor is supplied at constant voltage from a variable frequency source. If the maximum current is 1A at 75Hz, Find the frequency when the current is 0.5A

Sol: given $L = 9H$, $R = 50\Omega$, $I_{max} = 1A$ at $f_r = 75Hz$

$f_r = \frac{1}{2\pi\sqrt{LC}}$

$75 = \frac{1}{2\pi\sqrt{9\times C}} \Rightarrow \underline{C = 0.5\mu F}$

$V = I_{max}R$

$V = 1\times50$

$\underline{V = 50V}$

$Z = \sqrt{R^2 + (X_L - X_C)^2}$

$Z = \frac{V}{I} = \frac{50}{0.5} = \underline{100\Omega}$

$Z^2 = (R^2 + (X_L - X_C)^2)$

$100^2 = 50^2 + \left(\omega(9) - \frac{1}{\omega(0.5\mu)}\right)^2$

$7500 = \left(9\omega - \frac{2\times10^6}{\omega}\right)^2$

$7500\omega^2 = (9\omega^2 - 2\times10^6)^2$

$86.6\omega = 9\omega^2 - 2\times10^6$

$9\omega^2 - 86.6\omega - 2\times10^6 = 0$

$\omega_2 = \underline{476.24\text{ rad/sec}}$ $\omega_1 = \underline{466.61\text{ rad/sec}}$

$f_2 = \frac{\omega_2}{2\pi} = \frac{476.24}{2\pi} = \underline{75.79\,Hz}$

$f_1 = \frac{\omega_1}{2\pi} = \frac{466.61}{2\pi} = \underline{74.26\,Hz}$

Q4: A series resonant LCR circuit has a resonant frequency of 80k rad/sec and quality factor of 8. Find the bandwidth, upper cut off frequency and lower cut off frequency.

Sol: $\omega_r = 80k$ rad/sec $\quad \omega_1 = 80k - 5k \quad \omega_2 = 80 + 5k$

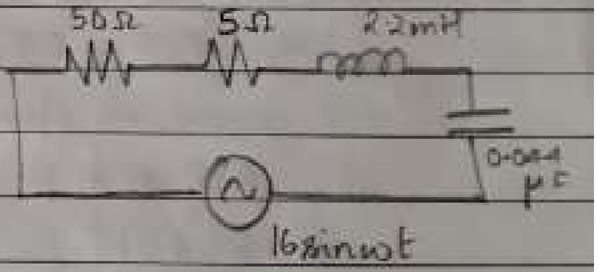
$Q = 8 \quad = \underline{75k \text{ rad/sec}} \quad = \underline{85k \text{ rad/sec}}$

$$Q = \frac{\omega_r}{BW} \Rightarrow BW = \frac{\omega_r}{Q} = \frac{80k}{8} = \underline{10k \text{ rad/sec}}$$

$$BW = \omega_2 - \omega_1 = 2\Delta\omega = \frac{R}{L} \Rightarrow \Delta\omega = \frac{R}{2L} = \frac{BW}{2} = \frac{10k}{2} = \underline{5k \text{ rad/sec}}$$

~~$\Rightarrow \omega_1 = BW - \Delta\omega = 10k - 5k = 5k$ rad/sec $\quad \omega_2 = BW + \Delta\omega = 10k + 5k = 15k$ rad/sec~~

Q5: ~~A variable frequency~~ For the circuit shown determine maximum power dissipated by the circuit, bandwidth and approximate value of cut off frequencies and actual frequencies. Also find the power dissipated at these frequencies.



Sol: Given: $R = 55\Omega$

$L = 2.2mH$

$C = 0.09\mu F$

$$I_{max} = \frac{V}{R} = \frac{16}{55} = \underline{0.29A}$$

$$P_{max} = I_{max}^2 R = (0.29)^2 55 = \underline{4.6255W}$$

$$BW = \frac{R}{L} = \frac{55}{2.2\times10^{-3}} = \underline{25k \text{ rad/sec}} \quad \underline{BW = 3.97 kHz}$$

$$BW = \omega_2 - \omega_1 = 2\Delta\omega$$

$$BW = 2\Delta f = \frac{R}{2\pi L} \Rightarrow \Delta f = \frac{R}{4\pi L} = \frac{55}{4\pi(2.2\times10^{-3})} = \underline{1.98kHz}$$

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$\omega_r = \frac{1}{\sqrt{LC}}$

$f_r = \frac{1}{2\pi\sqrt{LC}}$

$= \frac{1}{2\pi\sqrt{2.2\times10^{-3}\times0.044\times10^{-6}}}$

$f_r = 16.176\,kHz$

upper cut off frequency

$f_2 = f_r + \Delta f$

$f_2 = 16.176 + 1.98$

$f_2 = 18.156\,kHz$

Lower cut off frequency

$f_1 = f_r - \Delta f$

$f_1 = 16.176 - 1.98$

$f_1 = 14.196\,kHz$

P'_{max} (at cut off frequency)

$\frac{P_{max}}{2} = \frac{4.6225}{2} = 2.311\,W$

Q6: A series RLC circuit consists of resistance $1k\Omega$ and an inductance of 100mH in series with capacitance of 10pF. If 100V is applied as input across the combination then determine.

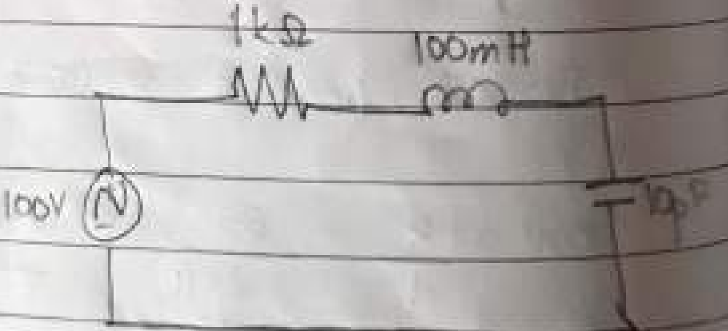
i. Resonant frequency
ii. Maximum current in the circuit
iii. Quality factor
iv. Half power frequencies

Sol: Given: $R = 1k\Omega$

$L = 100mH$

$C = 10pF$

$V = 100V$



i. Resonant frequency

$\omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{100\times10^{-3}\times10\times10^{-12}}} = 1M\,rad/sec$

$$f_r = \frac{\omega_r}{2\pi} = \underline{159.15\,kHz}$$

ii. Maximum current

$$I_{max} = \frac{V}{R} = \frac{100}{10^3} = \underline{0.1\,A}$$

iii. Quality factor

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{10^3}\sqrt{\frac{100\times10^{-3}}{10\times10^{-12}}} = \underline{100}$$

iv. Half power frequencies

$$BW = \frac{R}{2\pi L} = \frac{10^3}{2\pi\, 100\times10^{-3}} = \underline{1591\,Hz}$$

k

$$BW = 2\Delta f \Rightarrow \Delta f = \frac{BW}{2} = \underline{0.796\,kHz}$$

∴ Upper cut of frequency

$$f_2 = f_r + \Delta f = \underline{159.9\,kHz}$$

∴ Lower cut off frequency

$$f_1 = f_r - \Delta f = 159.15k - 0.796k = \underline{158.3\,kHz}$$

Q7: In an RLC series network inductance L = 8mH, the capacitance C = 0.3μF, Resistance R = 15Ω. Determine the current flowing through the circuit when the input voltage is 7.5V.

Find i. resonant frequency

ii. frequency 3% above the resonant frequency

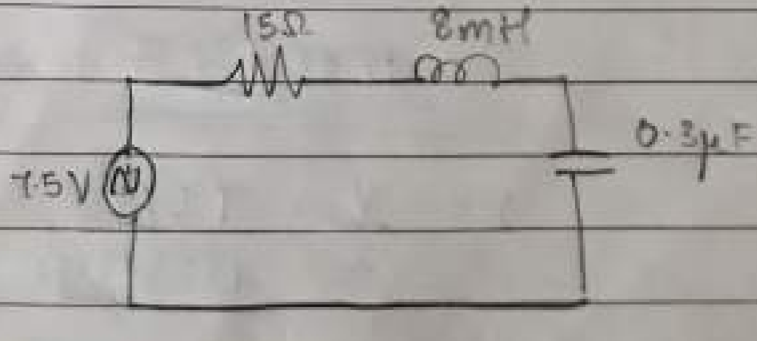
iii. impedance of the circuit when the frequency is 3% above the resonant frequency

Sol: Given: $L = 8\,mH$

$C = 0.3\,\mu F$

$R = 15\,\Omega$

$V = 7.5\,V$



i. Resonant frequency

$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{8\times10^{-3}\times0.3\times10^{-6}}} = \underline{3.25\,kHz}$$

ii. $\delta = 3\% = 0.03$

$$\delta = \frac{f - f_r}{f_r}$$

$$0.03 = \frac{f - 3.25k}{3.25k}$$

$$f = (0.03)3.25k + 3.25k$$

$$f = \underline{3.344\,kHz}$$

$$\cancel{X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(3.344\times10^{3})0.3\times10^{-6}} = 158.6\,\Omega}$$

$$\cancel{X_L = 2\pi fL = 2\pi(3.344\times10^{3})8\times10^{-3} = 168.24\,\Omega}$$

Impedence :

$$\cancel{Z = \sqrt{R^2 + (X_L - X_C)^2}}$$

$$\cancel{Z = \sqrt{15^2 + (168.24 - 158.5)^2} = 11.4\,\Omega}$$

iii $Z = R\left[1 + jQ\delta(2-\delta)\right]$

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$Z = 15\left[1 + j(10.88)(0.03)(2-0.03)\right]$$

$$Q = \frac{1}{15}\sqrt{\frac{8\times10^{-3}}{0.3\times10^{-6}}}$$

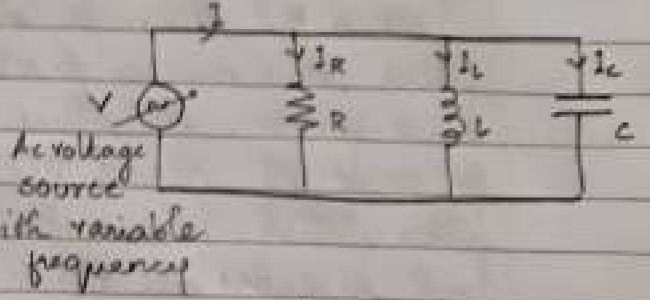
$$Z = 15 + 9.64j$$

$$\underline{Q = 10.88}$$

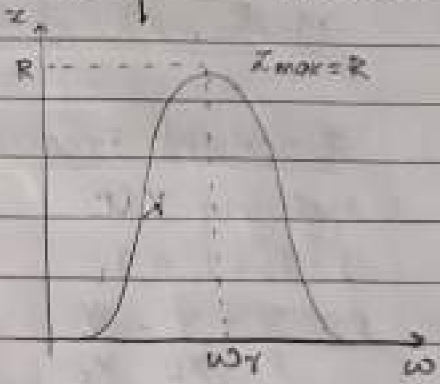
$$Z = \underline{17.83\angle 32.72^\circ}$$

$$I = \frac{V}{Z} = \frac{7.5\angle 0^\circ}{17.83\angle 32.72^\circ} = \underline{0.42\angle -32.72^\circ}$$

Resonance in Parallel Circuits:



In series ~~or~~ parallel resonance, the resonance condition remains same i.e., $X_L = X_C$. During the resonance the impedence of the circuit is purely resistive, that is voltage and current are in phase. At resonance condition impedence is maximum and the current supplied by the source is minimum.



consider an impedence versus frequency curve for RLC circuit at resonant frequency, impedence of the circuit is maximum and when moved away from the resonant condition the impedence is reduced.

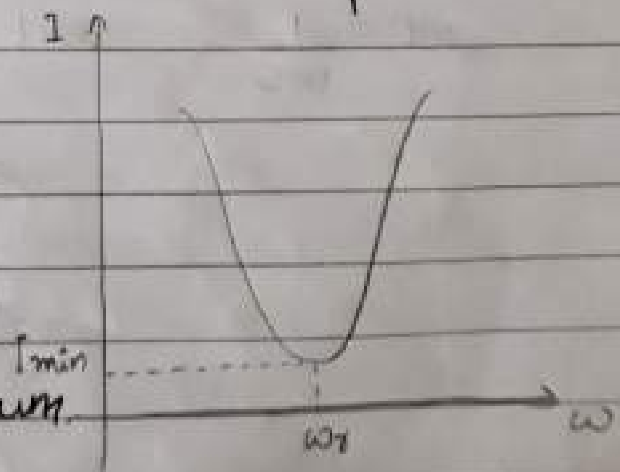
At $\omega = 0$, $X_C = 1/\omega C = \infty$

$X_L = \omega L = 0$

At $\omega = \infty$, $X_C = 1/\omega C = 0$

$X_L = \omega L = \infty$

Hence at lower and higher frequencies impedence is minimum and at resonance condition impedence is maximum.



Consider the current versus frequency curve.

At resonant condition the current that is flowing through the circuit is minimum. As moved away from the resonant frequency current increases.

When the curve is observed it acts as ~~a~~ band stop filter, this means that it rejects some frequencies and passes all other frequencies.

In case of series RLC circuit, it acts as band pass filter, this means that it passes few frequencies and rejects all other frequencies.

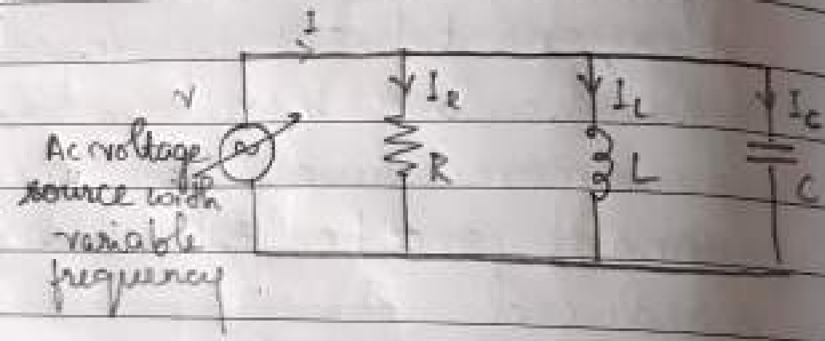
Parallel RLC circuit → Rejector circuit

Series RLC circuit – Acceptor circuit

★ **Mathematical Expressions for Parallel RLC circuit:**

1. Resonant Frequency

Applying KCL:



$$I = I_R + I_L + I_C$$

$$I = \frac{V}{R} + \frac{V}{X_L} + \frac{V}{X_C}$$

$$I = V\left[\frac{1}{R} + \frac{1}{j\omega L} + j\omega C\right]$$

$$I = V\left[\underbrace{\frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)}_{\text{Admittance}}\right]$$

$$I = VY$$

$$I_{min} = VY_{min}$$

Resonant condition

$$\omega L = \frac{1}{\omega C} \qquad \text{wkt } Y = \frac{1}{R}$$

$$\Rightarrow I = \frac{V}{R}$$

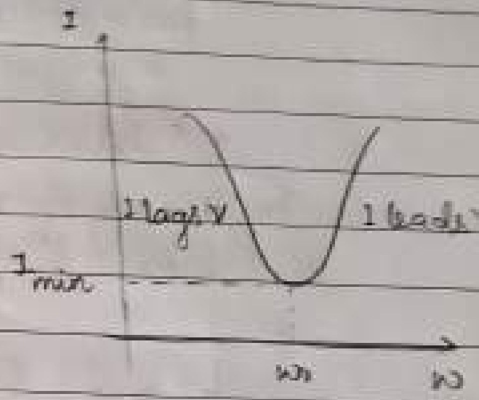
$$\Rightarrow Z_{max} = R$$

To find resonant frequency

$$\omega^2 = \frac{1}{LC}$$

$$\boxed{\omega_r = \frac{1}{\sqrt{LC}}}$$

$$\boxed{f_r = \frac{1}{2\pi\sqrt{LC}}}$$



At resonant frequency the current flowing in the circuit $I = V/R$, that is circuit will be purely resistive.

At $\omega = 0$, $X_L = 0$, that means inductor acts as short circuit and entire current flows through the inductor. Hence $X_C \gtrless X_L$. For the frequency less that ω_r, $X_C > X_L$, the circuit will behave as inductive circuit. For the frequency greater that ω_r, $X_C < X_L$, the circuit will behaves as capacitive circuit.

At higher frequency $\omega = \infty$, $X_C = 0$, that means capacitor acts as short circuit and entire current flows though the capacitor. Hence the capacitive reactance is minimum. Therefore current leads voltage.

To define the sharpness of the resonant curve, two parameters are used.

1. <u>Quality factor</u>: As the value of quality factor increases, sharpness of the curve increases and the resonance curve gets narrower. As the value of quality factor decreases the curve gets wider, i.e., instead of rejecting one single frequency it rejects all other frequencies

Equation for quality factor.

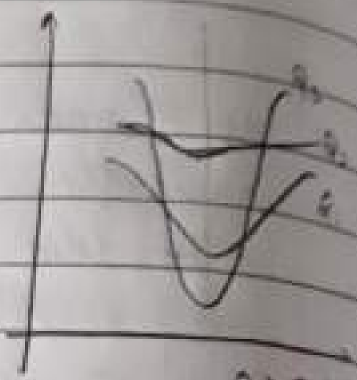
$$Q = 2\pi \times \frac{\text{Energy stored in } L \text{ (or) } C}{\text{Energy dissipated a/c } R}$$

$$Q = \omega \times \frac{\text{Energy stored in } L \text{ (or) } C}{\text{Energy dissipated a/c } R}$$

wkt

$$\frac{1}{2} L I^2 = \frac{1}{2} C V^2$$

Energy stored in inductor = Energy stored in capacitor



$Q_1 > Q_2 > Q$

$R_3 > R_2 > R$

$I = I_{max}$

In terms of rms

$$I_{rms} = \frac{I_{max}}{\sqrt{2}} \qquad V_{rms} = \frac{V_{max}}{\sqrt{2}}$$

$$L I^2_{rms} = C V^2_{rms}$$

$$\therefore \quad Q = \omega \frac{C V^2_{rms}}{I^2_{rms} R} = \omega \frac{C V^2_{rms}}{V^2_{rms} / R}$$

$$Q = \omega C R$$

@resonance $X_L = X_C$

$$\boxed{Q = \frac{R}{X_C} = \frac{R}{X_L}} \qquad Q \propto R$$

$$\cancel{Q = \frac{\omega L I^2_{rms}}{I^2_{rms} R} = \frac{\omega L}{R}}$$

Therefore as R increases Q increases, hence the sharpness of the curve increases & narrower.

3\. b. Band width

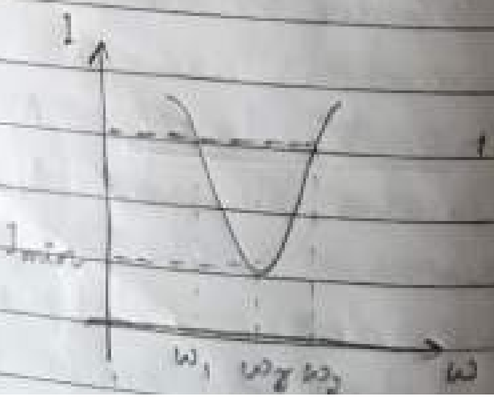
$$I_1 = I_2 = \sqrt{2}\, I_{min}$$

$$\Delta\omega = \omega_2 - \omega_1$$

considering at ω_1

$$I_1 = \sqrt{2}\, I_{min}$$

when I_{min} then $Z_{max} = R$



$$I_1 = V\sqrt{\frac{1}{R^2} + \left(\omega_1 C - \frac{1}{\omega_1 L}\right)^2} \quad \text{(Admittance)}$$

Squaring on both sides

$$I_1^2 = V^2\left(\frac{1}{R^2} + \left(\omega_1 C - \frac{1}{\omega_1 L}\right)^2\right) = 2I_{min}^2 = \frac{2V^2}{R^2}$$

$$\therefore \frac{2V^2}{R^2} = V^2\left[\frac{1}{R^2} + \left(\omega_1 C - \frac{1}{\omega_1 L}\right)^2\right]$$

$$\frac{1}{R^2} = \left(\omega_1 C - \frac{1}{\omega_1 L}\right)^2$$

$$\omega_1 C - \frac{1}{\omega_1 L} = \pm\frac{1}{R}$$ [$\omega_1 < \omega_r$] since X_L is dominant (Inductive)

|||ly $$\omega_2 C - \frac{1}{\omega_2 L} = \pm\frac{1}{R}$$ [$\omega_2 > \omega_r$] since X_L is dominant (capacitive)

$$\omega_1 C - \frac{1}{\omega_1 L} = -\frac{1}{R} \qquad \omega_2 C - \frac{1}{\omega_2 L} = +\frac{1}{R}$$

$$\omega_1^2 LC - 1 + \frac{\omega_1 L}{R} = 0 \qquad \omega_2^2 LC - 1 - \frac{\omega_1 L}{R} = 0$$

$$\boxed{\omega_1 = \frac{-1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}} \quad \text{—(1)}$$

$$\boxed{\omega_2 = \frac{+1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}} \quad \text{—(2)}$$

Subtracting eq (1) from eq (2)

$$\omega_2 - \omega_1 = \frac{1}{2RC} + \cancel{\sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}} + \frac{1}{2RC} - \cancel{\sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}}$$

$$\omega_2 - \omega_1 = 2\left(\frac{1}{2RC}\right)$$

$$\boxed{\text{Bandwidth} = \omega_2 - \omega_1 = \frac{1}{RC}}$$

$$\boxed{\text{Bandwidth} = f_2 - f_1 = \frac{1}{2\pi RC}}$$

4. Lower and upper cut off frequency in terms of resonant frequency.

$$\omega_1 \omega_2 = \left(\frac{-1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}\right)\left(\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}\right)$$

$$= \left(\frac{1}{2RC}\right)^2 + \frac{1}{LC} - \left(\frac{1}{2RC}\right)^2$$

$$\omega_1 \omega_2 = \frac{1}{LC} = \omega_r^2$$

$$\Rightarrow \boxed{\omega_r = \sqrt{\omega_1 \omega_2}}$$

5. Quality Factor in terms of R, L, C

wkt

$$Q = \frac{R}{X_C} = \omega_r CR = \frac{\omega_r}{1/RC}$$

wkt $\frac{1}{RC} = BW = \Delta\omega$

$$\Rightarrow \boxed{Q = \frac{\omega_r}{\Delta\omega} = \frac{\omega_r}{BW} = R\sqrt{\frac{C}{L}}}$$

i. $I_L = \frac{V}{X_L} = \frac{IR}{X_L} = QI$

ii. $I_C = \frac{V}{X_C} = \frac{IR}{X_C} = QI$

6. ω_1 and ω_2 in terms of band width

wkt

$$\omega_1 = \left(\frac{-1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}\right)$$

$$\omega_2 = \left(\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}\right)$$

substituting

$$\omega_r^2 = \frac{1}{LC} \quad \text{and} \quad BW = \frac{1}{RC}$$

$$\therefore \boxed{\omega_1 = -\frac{BW}{2} + \sqrt{\left(\frac{BW}{2}\right)^2 + \omega_r^2}}$$

$$\boxed{\omega_2 = \frac{BW}{2} + \sqrt{\left(\frac{BW}{2}\right)^2 + \omega_r^2}}$$

ω_1 and ω_2 in terms of Quality Factor:

substituting $BW = \frac{\omega_r}{Q}$

$$\boxed{\omega_1 = \omega_r\left[-\frac{1}{2Q} + \sqrt{\left(\frac{1}{2Q}\right)^2 + 1}\right]}$$

$$\boxed{\omega_2 = \omega_r\left[\frac{1}{2Q} + \sqrt{\left(\frac{1}{2Q}\right)^2 + 1}\right]}$$

Summary:

Impedance	$Z_{max} = R$
Band width	$BW = \frac{1}{RC}$
Quality factor	$Q = R\sqrt{\frac{C}{L}} = \omega_r RC$
Resonant frequency	$\omega_r = \frac{1}{\sqrt{LC}}$
current	$I_{min} = \frac{V}{R}$

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Q1. Find ω_r, Quality factor, Bandwidth, ω_1 and ω_2, Maximum power at ω_r.

Sol: Given: $R = 8k\Omega$ $\quad C = 8\mu F$

$L = 0.2mH$ $\quad V = 100\angle 0^\circ V$

$$Q = R\sqrt{\frac{C}{L}} = 8\times10^3\sqrt{\frac{8\times10^{-6}}{0.2\times10^{-3}}} = \underline{1600}$$

$$\omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.2\times10^{-3}\times 8\times10^{-6}}} = \underline{25k\ rad/sec}$$

$$BW = \frac{1}{RC} = \frac{1}{8\times10^3\times 8\times10^{-6}} = \underline{15625\ rad/sec}$$

$$\omega_1 = \omega_r\left[\frac{-1}{2Q} + \sqrt{\left(\frac{1}{2Q}\right)^2+1}\right]$$

$$\omega_1 = 25\times10^3\left[\frac{-1}{2(1600)} + \sqrt{\left(\frac{1}{2(1600)}\right)^2+1}\right]$$

$$\underline{\omega_1 = 24992.18\ rad/sec}$$

$$\omega_2 = \omega_r\left[\frac{1}{2Q} + \sqrt{\left(\frac{1}{2Q}\right)^2+1}\right]$$

$$\omega_2 = 25\times10^3\left[\frac{1}{2(1600)} + \sqrt{\left(\frac{1}{2(1600)}\right)^2+1}\right]$$

$$\underline{\omega_2 = 25007.81\ rad/sec}$$

At $\omega = \omega_r$

$Z = R = 8k\Omega$

$$I_{min} = \frac{V}{Z} = \frac{100}{8\times10^3} = \underline{12.5\angle 0^\circ\ mA}$$

Power: $P_{max} = I_{min}^2 R = (12.5)^2\, 8k = \underline{1.25\ W}$

Q: A parallel RLC circuit has a quality factor of 100, Q=100, Determine the remaining component values for

a. $R = 1k\Omega$, $C = 1\mu F$ $\Rightarrow L = ?$

b. $L = 12mH$, $C = 2.4nF$ $\Rightarrow R = ?$

c. $R = 122k\Omega$, $L = 100pH$ $\Rightarrow C = ?$

case a: $R = 1k\Omega$ $C = 1\mu F$ $Q = 100$

$$Q = R\sqrt{\frac{C}{L}} \Rightarrow \cancel{1\times10^{3}\sqrt{\frac{1\times10^{-6}}{L}} = 100}$$

$$\Rightarrow L = \frac{R^2C}{Q^2} = \frac{(10^3)^2(1\times10^{-6})}{(100)^2}$$

$$\underline{L = 0.1mH}$$

case b: $L = 12mH$ $C = 2.4nF$ $Q = 100$

$$Q = R\sqrt{\frac{C}{L}}$$

$$R = Q\sqrt{\frac{L}{C}} = 100\sqrt{\frac{12\times10^{-3}}{2.4\times10^{-9}}}$$

$$\underline{R = 223.6k\Omega}$$

case c: $R = 122k\Omega$ $L = 100pH$ $Q = 100$

$$Q = R\sqrt{\frac{C}{L}}$$

$$\Rightarrow C = \frac{Q^2L}{R^2} = \frac{(100)^2(100\times10^{-12})}{(122\times10^3)^2}$$

$$\underline{C = 6.71\times10^{-14}}$$

Parallel resonance in AC parallel circuits:

Resonant Frequency:

practical coil

Branch 1

$$Z_1 = R + jX_L$$

$$Y_1 = \frac{1}{Z_1} = \frac{1}{R+jX_L}$$

$$Y_1 = \frac{1}{R+jX_L} \times \frac{R-jX_L}{R-jX_L}$$

$$Y_1 = \frac{R-jX_L}{R^2+X_L^2}$$

$$Y_1 = \frac{R}{R^2+X_L^2} - j\frac{X_L}{R^2+X_L^2} \quad \text{——— (1)}$$

Branch 2

$$Z_2 = -jX_C$$

$$Y_2 = \frac{1}{Z_2} = \frac{-1}{jX_C} = \frac{j}{X_C} \quad \text{——— (2)}$$

Total admittance

$$Y_T = Y_1 + Y_2$$

$$Y_T = \frac{R}{R^2+X_L^2} - j\frac{X_L}{R^2+X_L^2} + \frac{j}{X_C}$$

$$Y_T = \frac{R}{R^2+X_L^2} + j\left(\frac{1}{X_C} - \frac{X_L}{R^2+X_L^2}\right)$$

At resonance

$$\frac{1}{X_C} - \frac{X_L}{R^2+X_L^2} = 0$$

$$\frac{X_L}{R^2+X_L^2} = \frac{1}{X_C}$$

$$X_L X_C = R^2 + X_L^2$$

$$2\pi f_r L \cdot \frac{1}{2\pi f_r C} = R^2 + (2\pi f_r L)^2$$

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$$* \frac{L}{C} = R^2 + 4\pi^2 f_r^2 L^2$$

$$\frac{L}{C} - R^2 = 4\pi^2 f_r^2 L^2$$

$$\sqrt{\frac{L}{C} - R^2} = 2\pi f_r L$$

$$\Rightarrow f_r = \frac{1}{2\pi L}\sqrt{\frac{L}{C} - R^2}$$

$$f_r = \frac{1}{2\pi}\sqrt{\frac{L}{CL^2} - \frac{R^2}{L^2}}$$

$$\boxed{f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}}$$

- Dynamic Resistance:

$$Y_T = \frac{R}{R^2 + X_L^2} \quad \text{(at resonance)}$$

$$\boxed{R_D = \frac{1}{Y_T} = \frac{R^2 + X_L^2}{R}}$$

- Impedence:

$$Z = \frac{R^2 + X_L^2}{R} \quad \text{(at resonance)}$$

$$Z = \frac{R^2 + \omega^2 L^2}{R} \qquad * \text{ since } R^2 + 4\pi^2 f_r^2 L^2 = \frac{L}{C}$$

$$\boxed{Z = \frac{L}{CR}}$$

Q1: A coil of inductance 31.8mH with resistance of 12 Ω is connected in parallel with a capacitor across 250V, 50Hz supply. Determine the value of capacitance, if no reactive current is taken from the supply.

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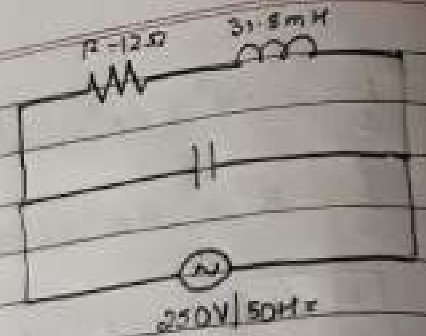


Sol: $f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$

$50 = \frac{1}{2\pi}\sqrt{\frac{1}{(31.8\times10^{-3})C} - \frac{12^2}{(31.8\times10^{-3})^2}}$

$50(2\pi) = \frac{1}{(31.8\times10^{-3})C} - \frac{12^2}{(31.8\times10^{-3})^2}$

$C = 1.30 \times 10^{-4} F$

Q: A practical parallel resonance circuit consists a coil of 0.1H inductance with 10Ω leakage resistance 10μF capacitor in parallel with it. Find frequency at which the current in the circuit is purely resistive. Also find impedence under resonance.

Sol: $f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$

$F_r = \frac{1}{2\pi}\sqrt{\frac{1}{0.1(10\times10^{-6})} - \frac{10^2}{(0.1)^2}} = 158.35Hz$

$z = \frac{L}{CR} = \frac{0.1}{10\times10^{-6}\times10} = 1000\Omega$

Q: In a parallel network inductance $L = 40mH$ and capacitance $C = 5\mu F$. Determine the resonant frequency of the circuit if $R_L = 0$ and $R_L = 40\Omega$.

Sol: a. $\omega_r = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$ $R = 0$

$\omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{40\times10^{-3}\times5\times10^{-6}}} = 2236.06$ rad/sec

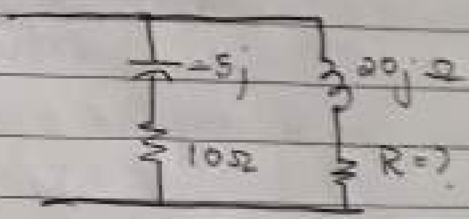
355.8Hz

b. $\omega_r = \sqrt{\dfrac{1}{LC} - \dfrac{R^2}{L^2}}$

$\omega_r = \sqrt{\dfrac{1}{40m \, 5\mu} - \dfrac{40^2}{(40m)^2}} = 2000 \text{ rad/sec}$

$\underline{318.3 \text{ Hz}}$

4. A capacitor of reactance 5Ω is connected in series with a 10Ω resistor. The whole circuit is then connected in parallel with a coil of inductive reactance 20Ω and a variable resistor. Determine the value of this resistance for which the parallel network is resonant.



Sol: $Y_T = \dfrac{1}{10-5j} + \dfrac{1}{R+20j}$

$= \dfrac{1}{10-5j} \times \dfrac{10+5j}{10+5j} + \dfrac{1}{R+20j} \times \dfrac{R-20j}{R-20j}$

$= \dfrac{10+5j}{10^2+5^2} + \dfrac{R-20j}{R^2+20^2}$

$= \dfrac{10}{125} + \dfrac{5j}{125} + \dfrac{R}{R^2+20^2} - \dfrac{20j}{R^2+20^2}$

$Y_T = \dfrac{10}{125} + \dfrac{R}{R^2+20^2} + j\left[\dfrac{5}{125} - \dfrac{20}{R^2+20^2}\right]$

At resonance

$\dfrac{5}{125} - \dfrac{20}{R^2+20^2} = 0$

$\dfrac{\cancel{5}}{125} = \dfrac{\cancel{20}^{\,4}}{R^2+20^2}$

$R^2 + 400 = 500$

$R^2 = 100$

$\underline{R = 10\Omega}$

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Q: A coil of resistance 300Ω, an inductance 100mH and 400pF capacitor are connected:

i. in series

ii. in parallel

Find for each connection.

a. resonant frequency

b. impedance at resonance

Sol: i. in series:

a. resonant frequency

$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{100m \times 400p}} = \underline{25 \cdot 16Hz}$$

b. Impedance at resonance

$$Z = R = \underline{300\Omega}$$

ii. in parallel:

a. resonant frequency

$$f_r = \frac{1}{2\pi\sqrt{LC}} = \underline{25 \cdot 16Hz}$$

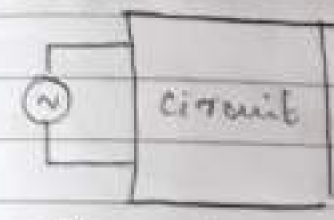
UNIT - 4

Transient Analysis and Initial Conditions.

* Steady State:

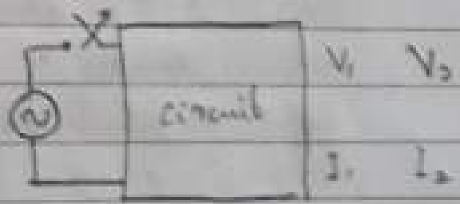
A state or condition of the system or process that does not change in time. In a network the branch current and node voltage are not changing wrt time is said to be in steady state. Also if V and I are having constant amplitude and frequency through out the time interval, then it is said to be in steady state.

* Transient:



consider a circuit and assume voltage source is connected since long time, that means the circuit has attained the steady state condition, i.e., voltage and current in the circuit has attained the steady state condition and with V_1 and I_1.

Now if the voltage source is suddenly ~~suddenly~~ removed from the circuit, then the circuit will attain voltage V_2 and current I_2, to reach this new value, the circuit will take sometime or transition time which is known as transient.



* Importance of transient and why the study of transient analysis in circuit:

Let us consider a circuit, if some spike enters into the circuit because of such component the circuit might fail, it is necessary to see the behaviour of the

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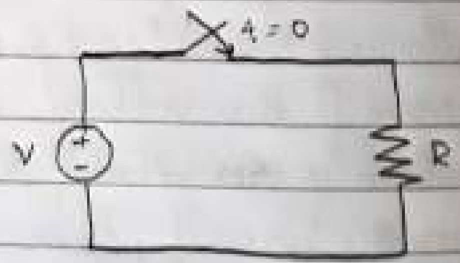
circuit when something abruptly changes in the circuit and how the circuit behaves to this abrupt changes. To see the behaviour of the circuit it is necessary to see voltage and current during abrupt changes or during transient changes. If we analyse voltage and current during transient, we get to know of the circuit behaves to these abrupt changes whether the circuit will be able to sustain such transient.

Transient is importance particularly in switching applications such as transistor as switch.

It is important to understand the behaviour of transistor that how fast is goes from one to off state.

★ Behaviour of basic components:

1. RESISTOR:



consider resistor R at time $t=0$, R is connected to voltage source V. Let time before the switching is closed

$\Rightarrow$ $t=0^-$: At this time I through R is 0.

When switch is closed at $t=0$

current I flows through the circuit $I = V/R$.

When just after switch is closed

At $t=0^+$, $I_R = V/R$

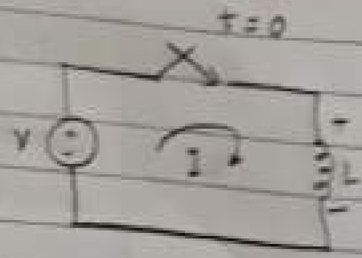
"Resistor responds immediately to the rate of change of current and voltage."

$t=0^-$	$I=0$
$t=0$	$I=V/R$
$t=0^+$	$I=V/R$

2. INDUCTOR:

case 1

Before time at $t=0$, no current flows in the circuit. Let $t=0$, switch is closed.



wkt voltage across the inductor is

$$V_L = L \cdot \frac{di}{dt}$$

Voltage across the inductor is proportional to rate of change of current.

When switch is closed, suddenly I current flows through L

$$t=0 \quad dt \to 0 \quad \frac{di}{dt} \to \infty$$

$$V_L \to \infty \text{ (practically not possible)}$$

"Inductor opposes instantenous change of current"

The I flows through L,

$$t=0^- \Rightarrow t=0^+$$

same current flows when $t=0^+$

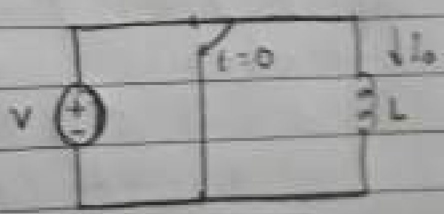
Let at $t=0^-$ current i_L is

$$i_L(0^-)=0 \quad i_L(0^+)=0$$

At $t=0^+ \to$ inductor will act as open circuit.

case 2

Similarly voltage V is applied continuously across circuits since long time. Hence current I_0 is flowing through the inductor L.



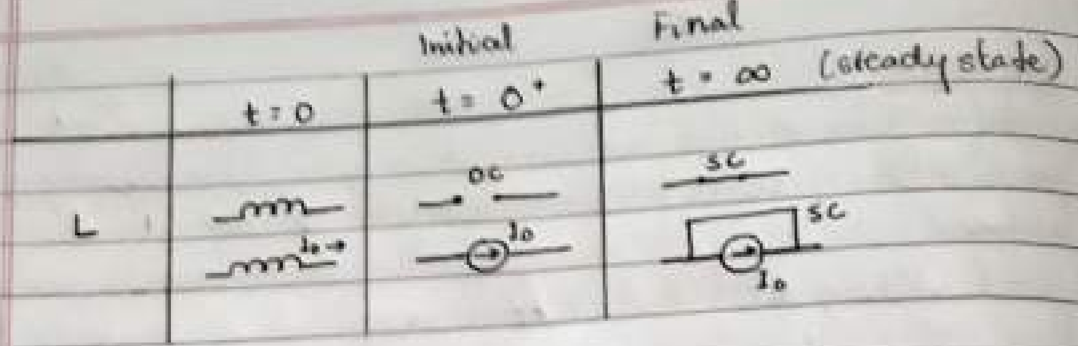
$$t=0^- \longrightarrow I_0$$

$$t=0^+ \longrightarrow I_0$$

$$I_L(0^+) = I_L(0^-) = I_0 \quad \text{(current source)}$$

Once transidtion period is over, there is no change in the current. At $t=\infty$, then $V_L=0$

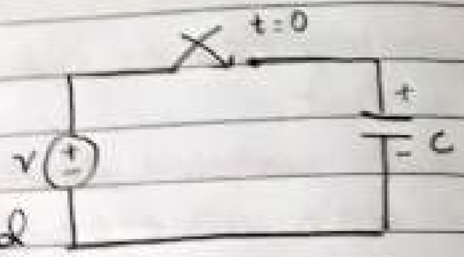
"Inductor acts as short circuit".



3. **CAPACITOR:**

CASE I

Before the time $t=0$, voltage across across the capacitor is zero. $t=0^-$ $v_c=0$



At $t=0$ voltage is appeared across the capacitor. the charge across the capacitor is given by

$$q = CV$$

$$\frac{dq}{dt} = C\frac{dv}{dt}$$

current through the capacitor is proportional to rate of change of voltage

$$i_c = C\frac{dv}{dt}$$

As soon as the switch is closed at $t=0$, voltage appears across the capacitor

$t=0$ $dt \to 0$ $\frac{dv}{dt} \to \infty$

$i_c \to \infty$ (practically not possible)

"capacitor opposes instantenous change of voltage"

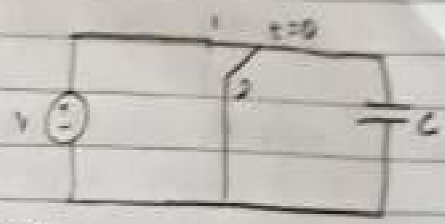
Whatever voltage appears across the capacitor when $t=0^-$, same voltage appears across $t=0^+$.

$t=0^- \Rightarrow t=0^+$

$v_c(0^-)=0$ $v_c(0^+)=0$

CASE 2

At $t=0$, switch is moved from position 1 to 2. wkt capacitor opposes instanteneous change in voltage.



same voltage V will appear across capacitor at $t=0^+$.

$$t=0^- \Rightarrow V_0$$
$$t=0^+ \Rightarrow V_0$$
$$V_c(0^-) = V_c(0^+) = V_0$$

Hence capacitor acts as voltage source during this transient. Once when transient period is over, there will be no change in voltage, i.e., $I_c = 0$

at $t=\infty$ no change in voltage takes place and $i_c = 0 \Rightarrow$ acts as open circuit.

	$t=0$	Initial $t=0^+$	Final $t=\infty$ (steady state)
C	—||—	SC	OC
	—||— V_0	V_0	V_0 OC

- **Prodeve for evaluating initial condition:**

It is solved by taking the value of currents and voltages and then it is solved for derivatives for finding initial values of current and voltages.

An equivalent network of the original network at $t=0^+$ is constructed according to the following rules:

- 1. Replace all inductors with open circuit or current sources having the value of current at $t=0^+$.
- 2. Replace all capacitors with short circuit or voltage source having the value V_0 if there is an initial charge

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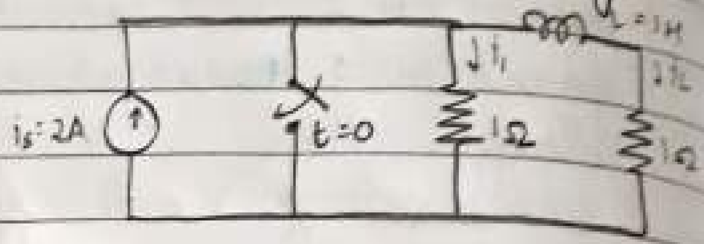
3. Resistors are left in the network without change.

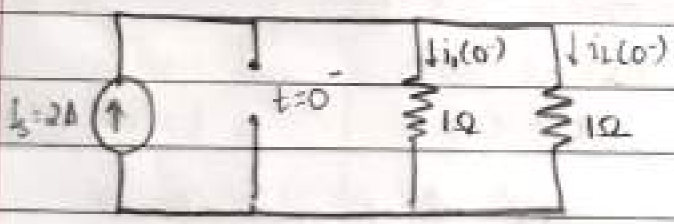
Q1: Refer the circuit shown.
Find $i_1(0^+)$, $i_2(0^+)$
the circuit is in steady state for $t < 0$.



Sol: ~~[illegible]~~ at $t=0^-$: open (steady state)
at $t=0$: closed
at $t=0^+$: closed.

To find:
$i_1(0^+)$ and $i_2(0^+)$

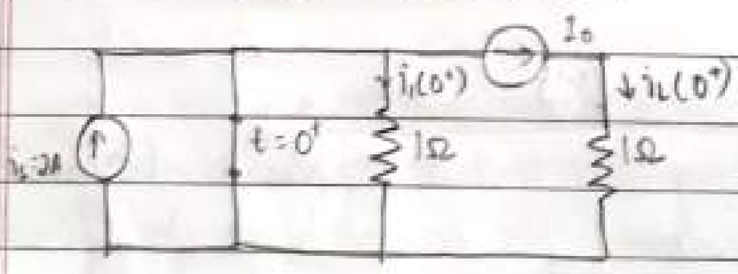
Equivalent circuit:



$i_1(0^-) = 1A$
$i_2(0^-) = 1A$

at $t=0^-$

wkt for inductor
$i_2(0^-) = i_2(0^+) = 1A$

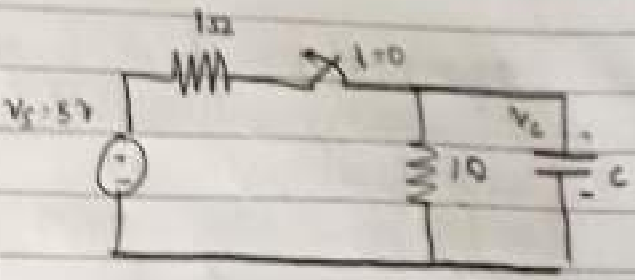


$i_1(0^+) = 0A$
$i_2(0^+) = 0A$

at $t=0^+$

At $t=0^+$ switch is just closed, the voltage across resistor will be zero because of the switch being short circuited hence $i_1(0^+) = 0A$. Hence current through the resistor changes from 1A to 0A.

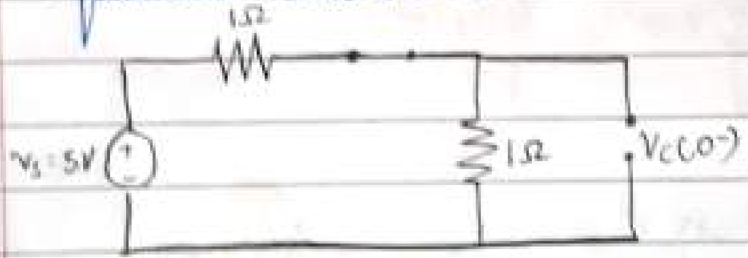
Q2: Refer the circuit.
Find $v_c(0^+)$.
Assume that the switch was in closed state for a long time.



Sol: at $t=0^-$: ~~open~~ ~~open~~ closed (steady state)
at $t=0$: ~~closed~~ open
at $t=0^+$: ~~closed~~ open

To find $v_c(0^+)$

Equivalent circuit at $t=0^-$

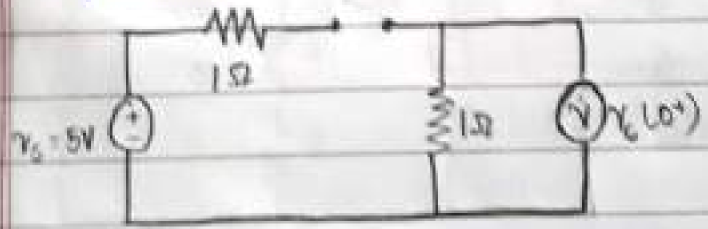


$I = \frac{5}{2} = 2.5A$

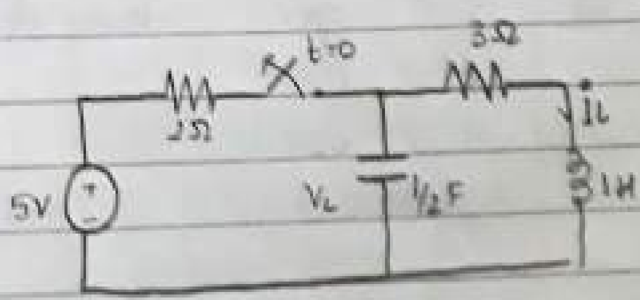
$\underline{V_c(0^-) = 2.5V}$

wkt $V_c(0^-) = V_c(0^+) = 2.5V$

Equivalent circuit at $t=0^+$



Q3: Refer the circuit, find $i_L(0^+)$, $V_c(0^+)$. The circuit is in steady state with switch in the closed condition



Sol: at $t=0^-$: closed (steady state)
at $t=0$: open
at $t=0^+$: open

To find: $i_L(0^+)$, $V_c(0^+)$

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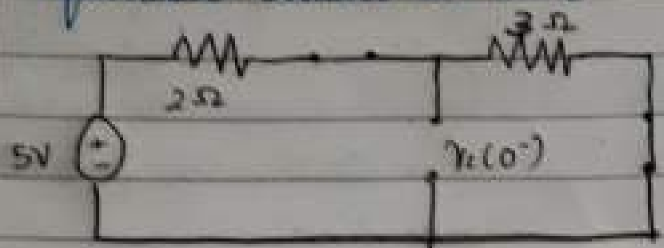
Equivalent circuit at $t = 0^-$



$i_L(0^-) = \frac{5}{5} = 1A$

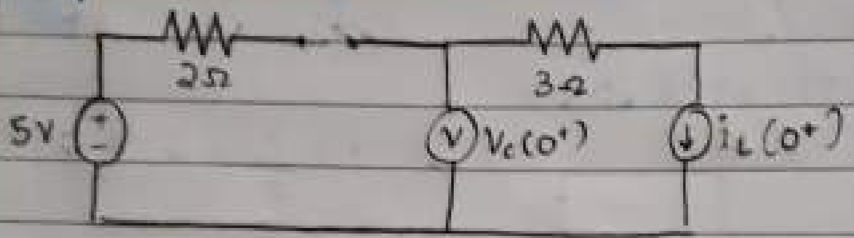
$V_c(0^-) = 1(3) = 3V$

~~Equivalent circuit at~~

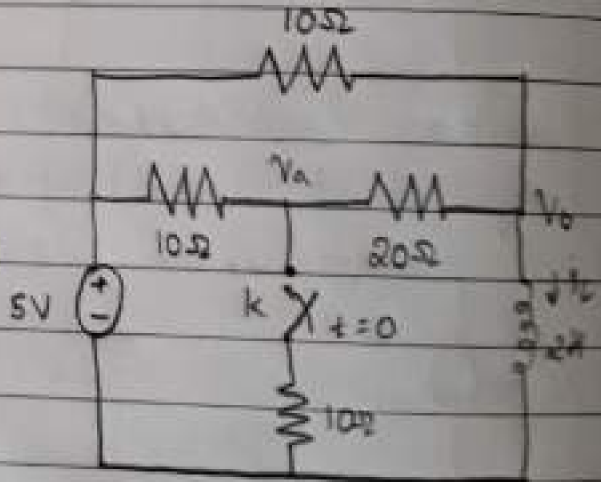
w.k.t $i_L(0^-) = i_L(0^+) = 1A$

$V_c(0^-) = V_c(0^+) = 3V$

Equivalent circuit at $t = 0^+$



Q4: In the circuit shown, a steady state is reached with switch k open. At $t = 0$, the switch is closed, for the element values given determine the values of $V_a(0^-)$ and $V_a(0^+)$.



Sol: $t = 0^-$ open (steady state)

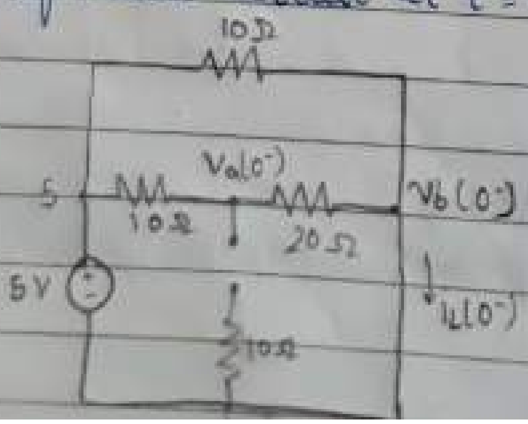
$t = 0$ closed

$t = 0^+$ closed

To find:

$V_a(0^-)$ and $V_a(0^+)$

Equivalent circuit at $t = 0^-$



$5 - V_a(0^-) = \frac{5(10)}{10+20} = \frac{5}{3}$

$V_a(0^-) = 5 - 5/3 = 10/3 = 3.33V$

$V_b(0^-) = 0$

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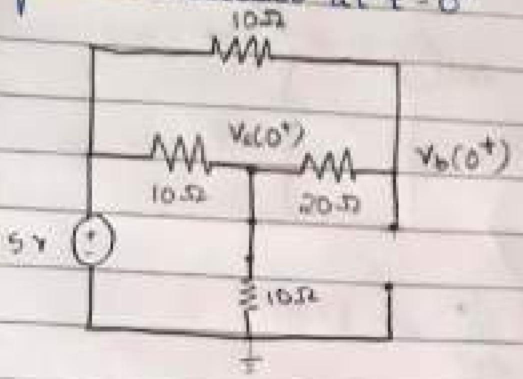
Equivalent circuit at $t=0^+$



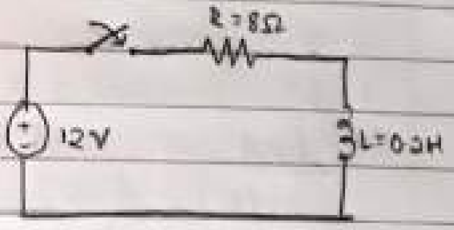
$$\frac{5-V_a(0^+)}{10} = \frac{V_a(0^+)}{10} + \frac{V_a(0^+)-0}{20}$$

$$5-V_a(0^+) = 3V_a(0^+)$$

$$4V_a(0^+) = 5$$

$$V_a(0^+) = 1.25V$$

Q.5: Switch k is closed at $t=0$, with zero current in the inductor. Find the values of $i, \frac{di}{dt}, \frac{d^2i}{dt^2}$ at $t=0^+$, if $R=8\Omega$ and $L=0.2H$.



Sol: $t=0^-$ open $\qquad i(0^-)=0$

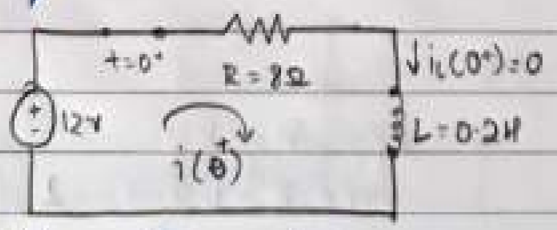
$t=0$ closed $\qquad \therefore i(0^-)=i(0^+)=0$.

$t=0^+$ closed

To find:

$$i(0^+),\ \frac{di(0^+)}{dt},\ \frac{d^2i(0^+)}{dt^2}$$

Equivalent circuit at $t=0^+$



Applying KVL

$$12 = i(0^+)8 + L\frac{di(0^+)}{dt} \quad -①$$

$$12 = 0.2\frac{di(0^+)}{dt}$$

$$\therefore \frac{di(0^+)}{dt} = 60\ A/sec$$

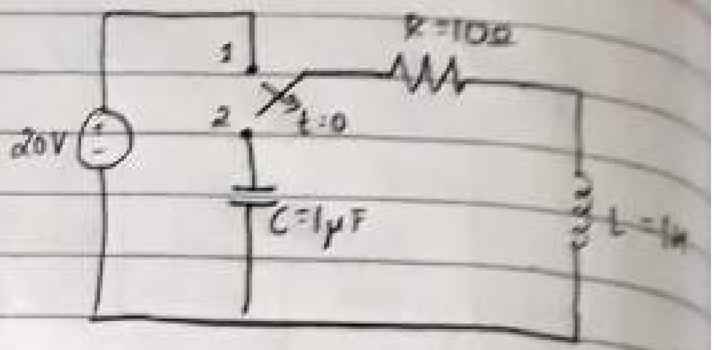
diff eq ① wrt t

$$8\frac{di(0^+)}{dt} + L\frac{d^2i(0^+)}{dt^2} = 0$$

$$8(60) + 0.2\frac{d^2i(0^+)}{dt^2} = 0$$

$$\therefore \frac{d^2i(0^+)}{dt^2} = -2400\ A/sec^2$$

Q6: Refer the circuit shown, the switch k is changed from position 1 to position 2 at $t=0$ steady state condition having been reached at position 1.



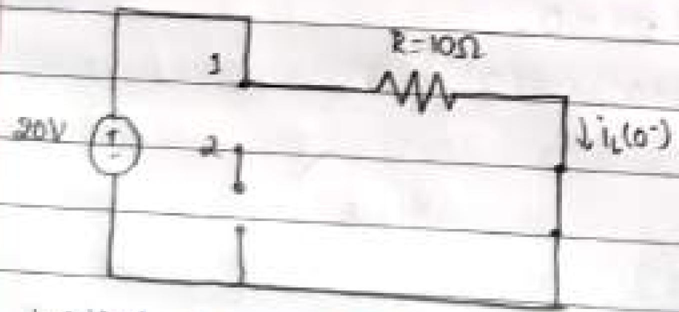
Find the value of i, $\frac{di}{dt}$, $\frac{d^2i}{dt^2}$ at $t=0^+$

Sol: $t=0^-$ position 1 (steady state)
$t=0$ position 2
$t=0^+$ position 2

To find:

$$i(0^+),\ \frac{di(0^+)}{dt},\ \frac{d^2i(0^+)}{dt^2}$$

Equivalent circuit at $t=0^-$



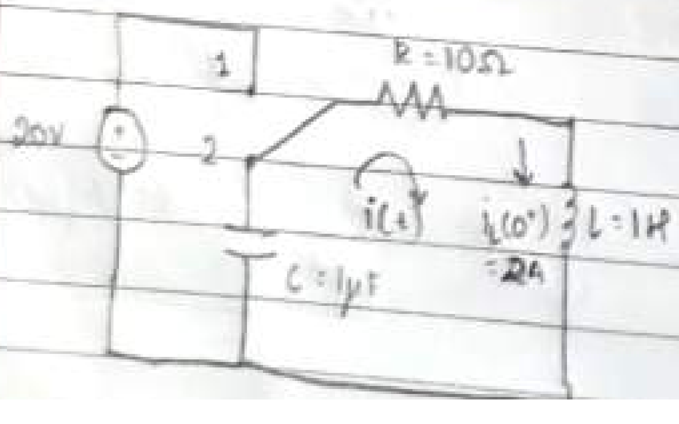
$$i(0^-) = \frac{20}{10} = 2A$$

$$i(0^-) = i(0^+) = 2A$$

Initial charge on capacitor is $V_c(0^-) = 0V$

$\therefore V_c(0^-) = V_c(0^+) = 0V$

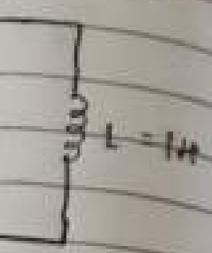
Equivalent circuit at $t=0^+$



Applying KVL

$$i(t)R + L\frac{di(t)}{dt} + \frac{1}{C}\int_{0^+}^{t} i(t)\,dt = 0$$

$$i(t)R + L\frac{di(t)}{dt} + V_c(t) = 0 \quad \text{--- (1)}$$



at $t = 0^+$

$$i(0^+)R + L\frac{di(0^+)}{dt} + V_c(0^+) = 0$$

$$2(10) + 1\frac{di(0^+)}{dt} + 0 = 0$$

$$\frac{di(0^+)}{dt} = -20\,A/sec$$

diff eq ① wrt t

$$\frac{di(t)}{dt}R + L\frac{d^2i(t)}{dt^2} + \frac{dV_c(t)}{dt} = 0$$

$$R\frac{di(t)}{dt} + L\frac{d^2i(t)}{dt^2} + \frac{i(t)}{C} = 0$$

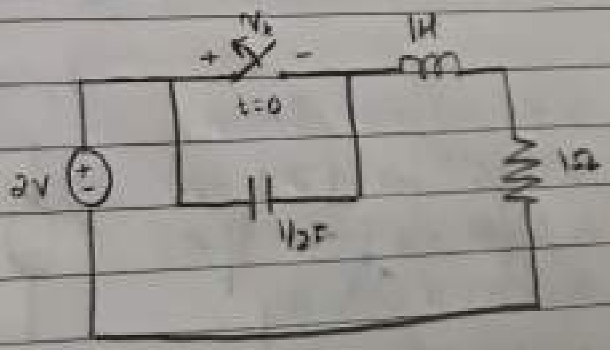
at $t = 0^+$

$$R\frac{di(0^+)}{dt} + L\frac{d^2i(0^+)}{dt^2} + \frac{i(0^+)}{C} = 0$$

$$10(-20) + 1\frac{d^2i(0^+)}{dt^2} + \frac{(2)}{10^{-6}} = 0$$

$$\frac{d^2i(0^+)}{dt^2} = 200 - 2\times10^{6}$$

$$\frac{d^2i(0^+)}{dt^2} = \underline{199.8\times10^3\,A/sec^2}$$



Q7: In the network shown the circuit is in steady state with switch k is closed. At t=0, the switch is open, determine the voltage across the switch V_k, $\frac{dV_k}{dt}$ at $t=0$.

$i(z)dz = 0$

$= 0$ ①

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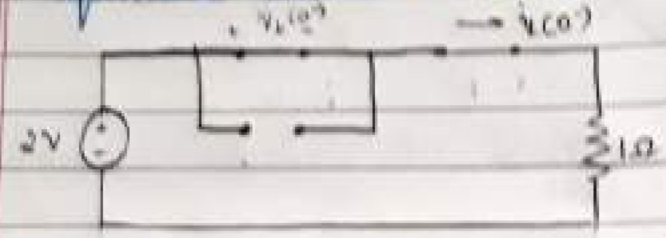
Sol: $t = 0^-$ closed (steady state)
$t = 0$ open
$t = 0^+$ open

To find: $V_k(0^+)$, $\frac{dV_k(0^+)}{dt}$

Equivalent circuit at $t = 0^-$



$V_k(0^-) = 0$

$\therefore V_k(0^-) = V_k(0^+) = 0$

$i_L(0^-) = 2A$

$\therefore i_L(0^-) = i_L(0^+) = 2A$

Equivalent circuit at $t = 0^+$

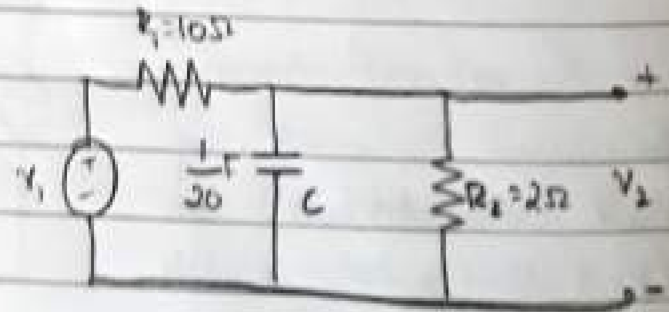
$i(t) = C\frac{dV_k(t)}{dt}$

at $t = 0^+$

$i(0^+) = C\frac{dV_k(0^+)}{dt}$

$2 = 0.5\frac{dV_k(0^+)}{dt}$ $\therefore \frac{dV_k(0^+)}{dt} = 4V/sec$

Q8: In the network shown, $v_1(t) = e^{-t}$ for $t \geq 0$ and is 0 for all $t < 0$. If the capacitor is initially uncharged, determine the value of $\frac{d^2v_2}{dt^2}$ and $\frac{d^3v_2}{dt^3}$ at $t = 0^+$.



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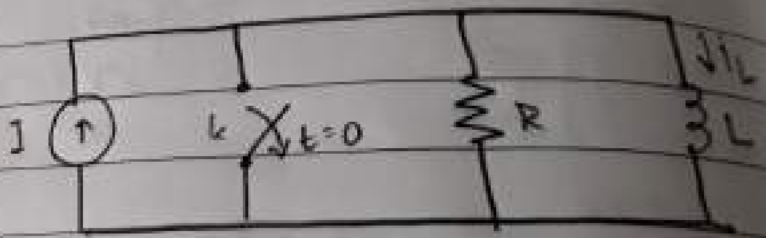
Q7: In a given network, the switch k is open at $t = 0$. At $t = 0^+$, solve for the values of v, $\frac{dv}{dt}$ and $\frac{d^2v}{dt^2}$



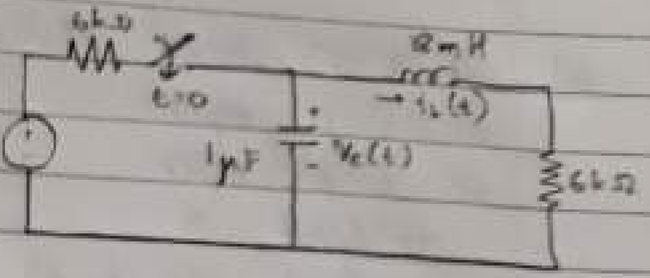
If $I = 2A$, $R = 200\,\Omega$, $L = 1H$

$t = 0^-$: closed
$t = 0$: open
$t = 0^+$: open

To find:

$v(0^+)$, $\frac{dv(0^+)}{dt}$, $\frac{d^2v(0^+)}{dt^2}$

Q. The switch shown has been open for long time before closing at $t=0$. Find $v_c(0^+)$, $i_L(0^+)$, $v_c(\infty)$, $i_L(\infty)$



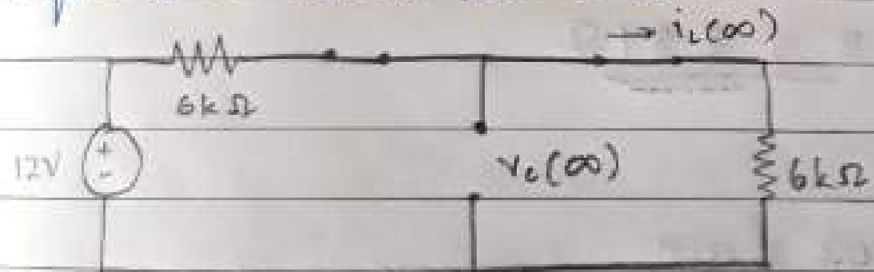
at $t=0^-$: open (steady state)
at $t=0$: closed
at $t=0^+$: closed

To find:
$v_c(0^+)$, $i_L(0^+)$, $v_c(\infty)$, $i_L(\infty)$

Equivalent circuit at $t=0^-$

$i_L(0^-)=0$
$\therefore i_L(0^-)=i_L(0^+)=0$
$v_c(0^-)=0$
$\therefore v_c(0^-)=v_c(0^+)=0$

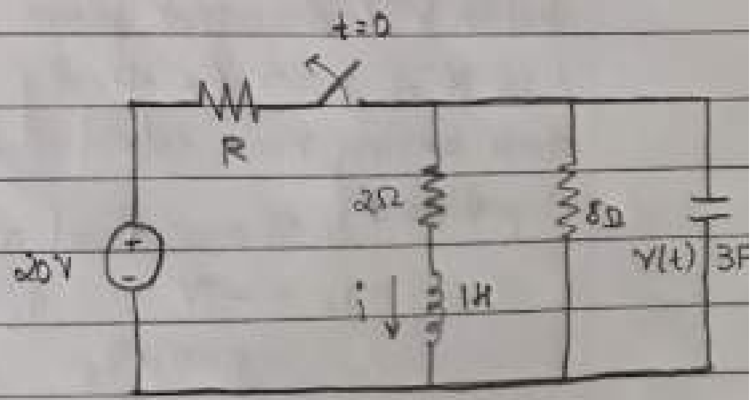
Equivalent circuit at $t=\infty$



$v_c(\infty)=6V$

$i_L(\infty)=1mA$

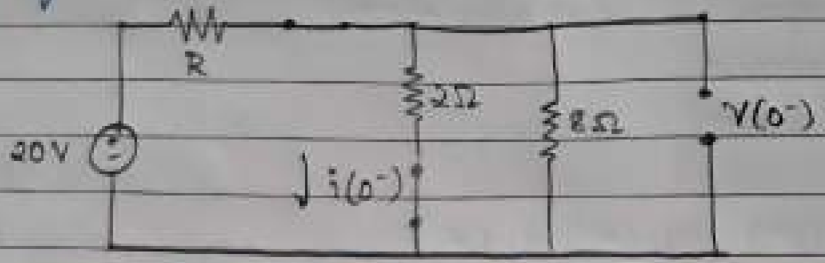
Refer the circuit shown. It is required that $V(0)=4V$ and $i(0)=3.5A$, determine the required resistance for it. Assume that the switch has been closed for a long time before it is open at $t=0$.



sol: at $t=0^-$ closed (steady state)
at $t=0$ open
at $t=0^-$ open

To find: R

equivalent circuit at $t=0^-$



Given $V(0^-) = 7V$ and $i(0^-) = 3.5A$

$$2 \| 8 = 1.6\Omega$$

$$V(0^-) = \frac{1.6(20)}{1.6+R}$$

$$7 = \frac{32}{1.6+R}$$

$$1.6 + R = 4.57$$

$$R = 2.87\Omega$$

Q12: Refer the network shown switch k is changed from A to B at $t=0$ (a steady state having been established at position A). Show that at $t=0$: $i_1 = i_2 = \frac{-V}{R_1+R_2+R_3}$, $i_3 = 0$

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at $t=0^-$ position A (steady state)

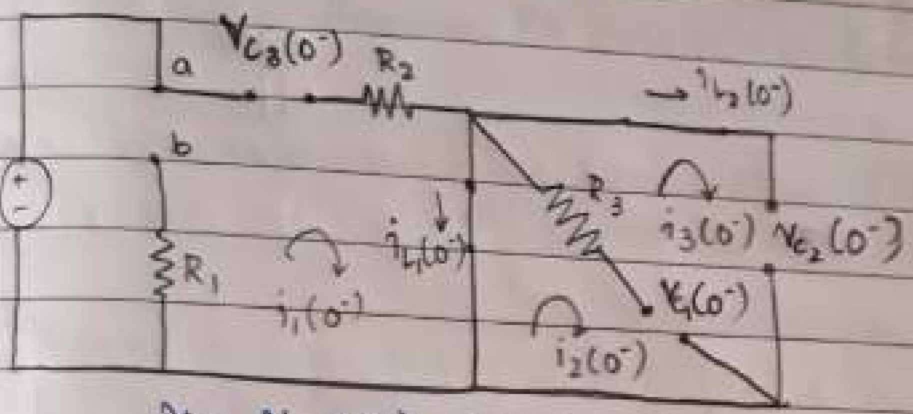
at $t=0$ position B

at $t=0^+$ position B

To prove:

$$i_1 = i_2 = \frac{-V}{R_1+R_2+R_3}, \quad i_3 = 0 \text{ at } t=0$$

Equivalent circuit at $t=0^-$



$i_{L_1}(0^-) = 0$

$i_{L_2}(0^-) = 0$

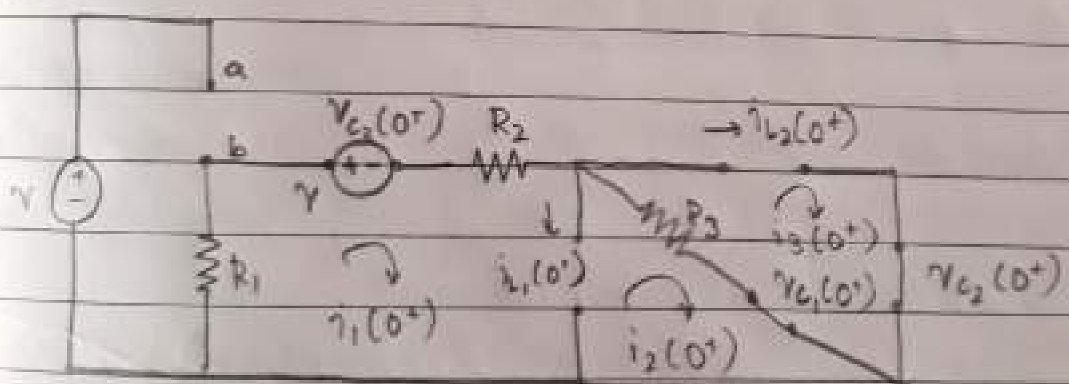
$V_{C_1}(0^-) = 0$

$V_{C_2}(0^-) = 0$

$$V = V_{C_3}(0^-) + \cancel{i_1(0^-)R_2}^{\,0}$$

$$V = V_{C_3}(0^-)$$

Equivalent circuit at $t=0^+$



$i_1(0^+) = i_2(0^+)$ because $i_{L_1}(0^+) = 0$

$i_3(0^+) = 0$ because $i_{L_2}(0^+) = 0$

Applying KVL to the loop

$$V + i_1(0^+)R_2 + i_2(0^+)R_3 + i_1(0^+)R_1 = 0$$

$$V + i_1(0^+)[R_1+R_2+R_3] = 0 \qquad \because i_1(0^+) = i_2(0^+)$$

$$i_2(0^+) = i_1(0^+) = \frac{-V}{R_1+R_2+R_3}$$

UNIT - 5

Laplace Transformations and Applications

Laplace transformation is an important technique which converts differential equation and integral equation into an algebraic equation and makes calculation easy.

It converts time domain to frequency domain.

Defination:

Laplace transform of function $f(t)$ is defined as

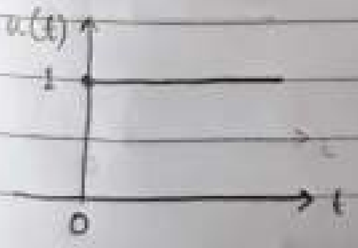
$$L[f(t)] = F(s) = \int_0^\infty f(t)\, e^{-st}\, dt$$

where $f(t)$ is time domain and $f(s)$ is frequency domain. Once the integral has been evaluated $f(t)$, a time domain function is transfered to $F(s)$, a frequency domain function.

* Three important singularity functions are

1. Unit step Function
2. Delta Function
3. Ramp Function

— **Unit Step Function:**



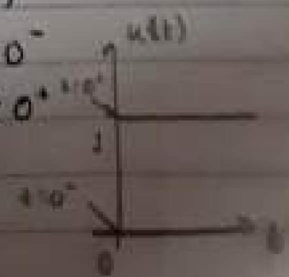
Step function is not defined at $t=0$

$u(t)$ is 0 for negative values of t

$u(t)$ is 1 for positive values of t

Also, we can define,

$$u(t) = \begin{cases} 0 & t \le 0^- \\ 1 & t \ge 0^+ \end{cases}$$



Mathematical defination

$$\boxed{u(t) = \begin{cases} 0, & t < 0 \\ 1, & t > 0 \end{cases}}$$

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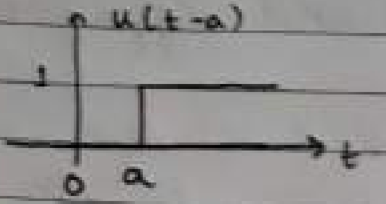
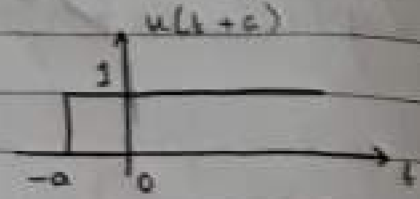
Step function occuring at $t = a$ is expressed as $u(t-a)$



$$u(t-a) = \begin{cases} 0 & t-a<0 \text{ or } t<a \\ 1 & t-a>0 \text{ or } t>a \end{cases}$$

Step function occuring at $t = -a$ is expressed as $u(t+a)$

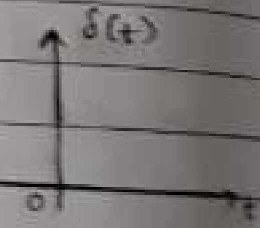


$$u(t+a) = \begin{cases} 0 & t+a<0 \text{ or } t<-a \\ 1 & t+a>0 \text{ or } t>-a \end{cases}$$

– <u>Delta Function</u>: (unit impulse function)

Derivative of unit step function.

$$\boxed{\delta(t) = \frac{d}{dt}u(t) = \begin{cases} 0 & t<0 \\ 0 & t>0 \\ 1 & t=0 \end{cases}}$$



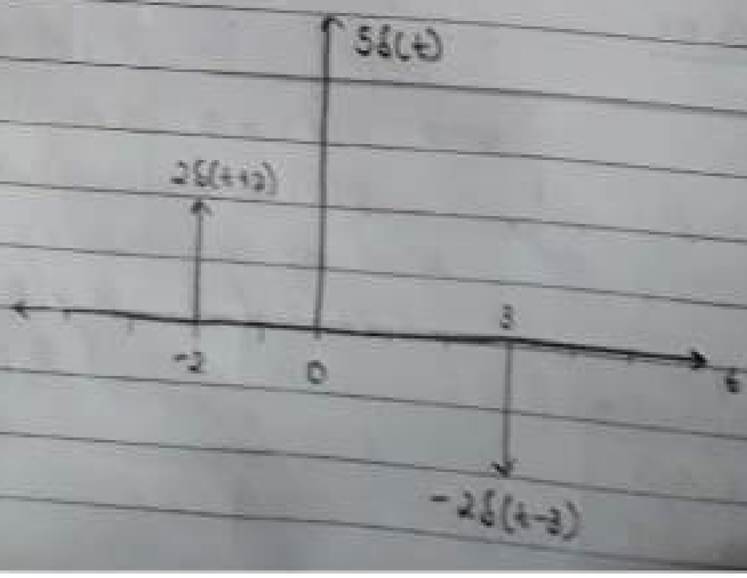
It is a very short duration pulse of unit area

$$\int_{0^-}^{0^+} \delta(t)\,dt = 1$$

$t = 0^-$ time just before $t = 0$

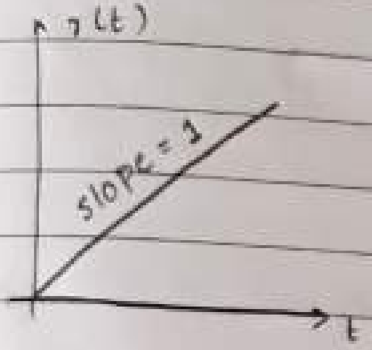
$t = 0^+$ time just after $t = 0$

<u>Ex</u>: $2\delta(t+2)$, $5\delta(t)$, $-2\delta(t-3)$



- Unit Ramp Function:

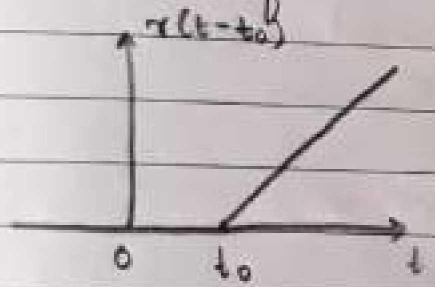
Integrating the unit step function results in unit ramp function $r(t)$.



$$r(t) = \int_{-\infty}^{t} u(\tau)\,d\tau = t\,u(t)$$

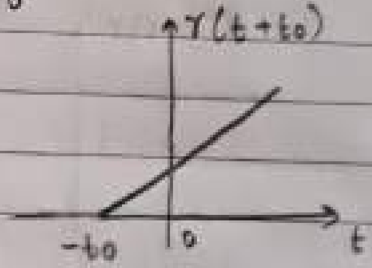
$$\boxed{r(t) = \begin{cases} 0 & t \le 0 \\ t & t \ge 0 \end{cases}}$$

A ramp is a function that changes at a constant rate.



Unit ramp function delayed by to.

$$r(t-t_0) = \begin{cases} 0 & t \le t_0 \\ t-t_0 & t \ge t_0 \end{cases}$$



Unit ramp function advanced by to.

$$r(t+t_0) = \begin{cases} 0 & t \le -t_0 \\ t+t_0 & t \ge -t_0 \end{cases}$$

* Laplace Transform Pairs:

$f(t)$ $t \ge 0$	$F(s)$
$\delta(t)$	1
$u(t)$	$1/s$
t	$1/s^2$
t^n	$\frac{n!}{s^{n+1}}$
e^{-at}	$\frac{1}{s+a}$

te^{-at}	$\frac{1}{(s+a)^2}$
$t^n e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$
$t \sin \omega t$	$\frac{2\omega s}{(s^2+\omega^2)^n}$
$t \cos \omega t$	$\frac{s^2-\omega^2}{(s^2+\omega^2)^2}$

* Properties of Laplace Transforms:

1. Linearity Property

- If $L[f_1(t)] = F_1(s)$ and $L[f_2(t)] = F_2(s)$ then

$$\boxed{L[a_1 f_1(t) + a_2 f_2(t)] = aF_1(s) + a_2 F_2(s)}$$

Q: Find the laplace transform of $f(t) = (A + Be^{-bt})u(t)$

sol: By applying Laplace transform

$L[f(t)] = L[(A + Be^{-bt})u(t)]$

by linearity property

$L[f(t)] = A\,L[u(t)] + B\,L[e^{-bt}u(t)]$

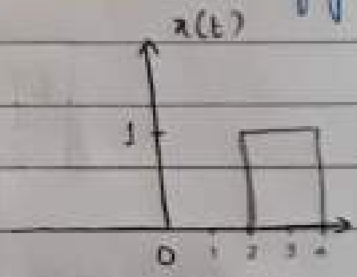
$$L[f(t)] = \frac{A}{s} + \frac{B}{s+b} = \frac{A(s+b)+Bs}{s(s+b)} = \frac{s(A+B)+Ab}{(s+b)s}$$

2. Time Shifting

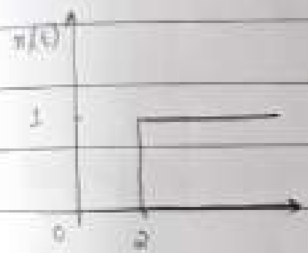
- If $L[x(t)] = X(s)$

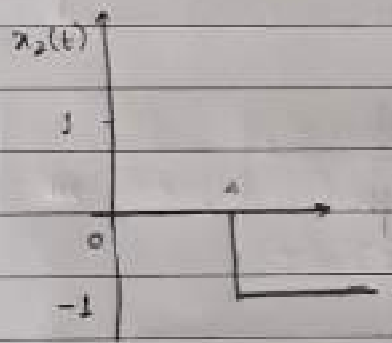
$$\boxed{L[x(t-t_0)u(t-t_0)] = e^{-t_0 s}X(s)}$$

Q: Find the laplace transform of $x(t)$ as shown in the fig.



Sol: $x(t) = x_1(t) + x_2(t)$





$x_1(t) = u(t-2)$

$x_2(t) = -u(t-4)$

$\therefore x(t) = u(t-2) - u(t-4)$

Using time shifting property

$$L[x(t)] = X(s) = \frac{1}{s}e^{-2s} - \frac{1}{s}e^{-4s}$$

$$L[x(t)] = X(s) = \frac{1}{s}\left(e^{-2s} - e^{-4s}\right)$$

3. Frequency Shifting

If $L[x(t)] = X(s)$ then $\boxed{L[e^{s_0 t}x(t)] = X(s-s_0)}$

4. $\boxed{L[x(at)] = \frac{1}{a}X\left(\frac{s}{a}\right)}$ Time Scaling

Q: Find the laplace transform of
$x(t) = \sin(2w_0 t)\, u(t)$

Sol: $L[x(t)] = L[\sin(2w_0 t)\, u(t)] = \frac{2w_0}{s^2 + 4w_0^2}$ //

5. Time Differentiation:
If $L[x(t)] = X(s)$ then

$$\boxed{L\left[\frac{dx(t)}{dt}\right] = sX(s) - x(0)}$$

6. Integration in time domain
For a causal sign (o/p depends on previous and present i/p)
$y(t) = \int_0^t x(\tau)\, d\tau$

$$\boxed{L[y(t)] = Y(s) = \frac{X(s)}{s}}$$

7. Differentiation in frequency domain
For a sign $x(t)$, $t \geq 0$

$$\boxed{L[-t x(t)] = \frac{dX(s)}{dt}}$$

Q: Find the laplace transform of
$x_1(t) = te^{-3t} u(t)$

Sol: $L[x_1(t)] = L[te^{-3t} u(t)]$
$= \frac{d}{dt} \frac{1}{(s+3)} = \frac{-1}{(s+3)^2}$ //

1. Convolution theorem : SLE

Initial Value Theorem:

2. Initial value theorem allows us to find $x(0)$ directly from its laplace transform $X(s)$

If $x(t)$ is a causal signal

$$x(0) = \lim_{s \to \infty} sX(s)$$

Q. Find the initial value of

$$X(s) = \frac{s+1}{(s+1)^2 + 3^2}$$

$$x(0) = \lim_{s \to \infty} s\left[\frac{s+1}{(s+1)^2 + 3^2}\right]$$

$$\cancel{x(0) = s\left[\frac{s/s+1}{1 + 3^2/(s+1)^2}\right]}$$

$$x(0) = \lim_{s \to \infty} \left(\frac{1 + 1/s}{1 + 2/s + 1/s^2 + 3^2/s^2}\right)$$

$$\underline{\underline{x(0) = 1}}$$

Verification:

Taking inverse laplace of $X(s)$

$$L^{-1}[X(s)] = x(t)$$

$$\underline{\underline{x(t) = e^{-t}\cos 3t}}$$

at $t = 0$

$$\underline{\underline{x(0) = 1}}$$

10. Final Value Theorem

Final value theorem allows us to find $x(\infty)$ directly from its laplace transform $X(s)$.

If $x(t)$ is causal signal

$$\lim_{t\to\infty} x(t) = \lim_{s\to 0} sX(s)$$

Q: Find the final value of

$$X(s) = \frac{10}{(s+1)^2+10^2}$$

$$\lim_{t\to\infty} x(t) = \lim_{s\to 0} sX(s)$$

$$x(\infty) = \lim_{s\to 0} s\left[\frac{10}{(s+1)^2+10^2}\right]$$

$$x(\infty) = 0$$

Verification

Taking inverse laplace transform

$$x(t) = e^{-t}\sin 10t$$

$t \to \infty$

$$x(\infty) = 0$$

★ Waveform Synthesis:

Laplace transform's of nonperiodic and periodic function

NON PERIODIC FUNCTION:

Q 1. Obtain the Laplace Transform of the Gate function given

NOTE:

- Unit step function $L[u(t)] = \frac{1}{s}$ When delayed by T $L[u(t-T)] = e^{-Ts}/s$

- Ramp function

$L[t\,u(t)] = \frac{1}{s^2}$

when shifted by T instant

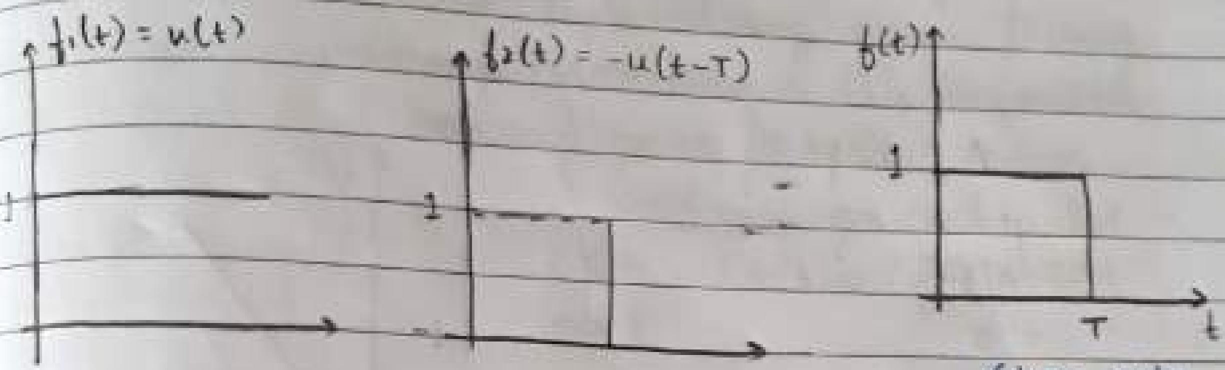
$L[r(t-T)] = \frac{e^{-Ts}}{s^2}$

- impulse function

$L[\delta(t)] = 1$

when delayed by $\delta(t-T)$

$L[\delta(t-T)] = e^{-Ts}$



Given gate function.

$f(t) = f_1(t) + f_2(t)$

$f(t) = u(t) - u(t-T)$

Taking laplace transform

$F(s) = \frac{1}{s} - \frac{e^{-Ts}}{s}$

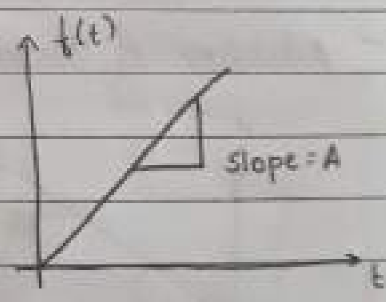
$F(s) = \frac{1-e^{-Ts}}{s}$

NOTE:

- Ramp function

$f(t) = At \quad t \geq 0$

$f(t) = 0 \quad t < 0$

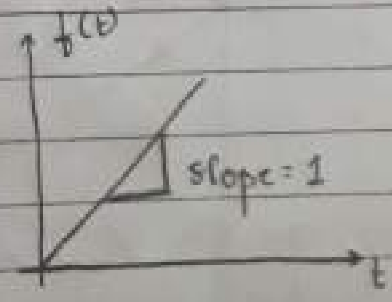


- Unit ramp

slope = 1

$f(t) = t \quad t \geq 0$

$f(t) = 0 \quad t < 0$



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- Any waveform if multiplied by unit step function doesnot change its value.

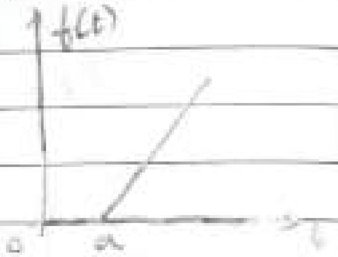
$$f(t) = At\,u(t) \quad t \geq 0$$
$$= 0 \quad t < 0$$

- Ramp of magnitude A can be expressed in the form of ramp function f(t) where $t\,u(t)$ is unit ramp function that is why it is unit ramp function.

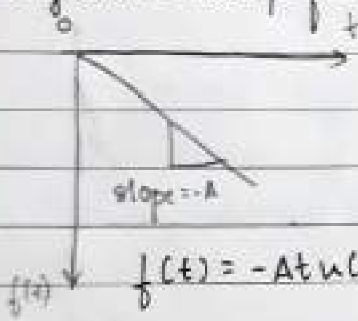
A = slope of ramp function

- Delayed ramp function

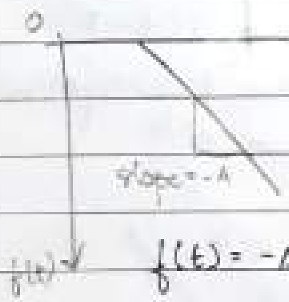
$$f(t) = A(t-a)\,u(t-a) \;,\; t \geq a$$
$$= 0 \;,\; t < a$$



- Negative ramp function



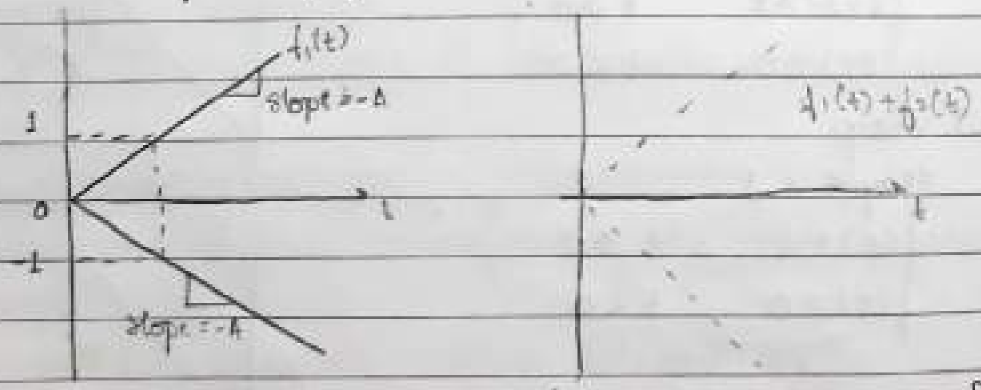
$f(t) = -At\,u(t)$



$f(t) = -A(t-a)\,u(t-a)$

Q2:

- Addition of ramp function



At $t > 0$ they cancel each other to produce constant valued function.

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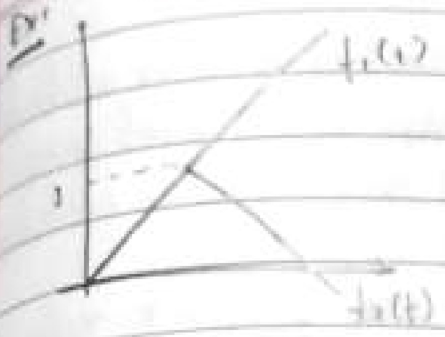
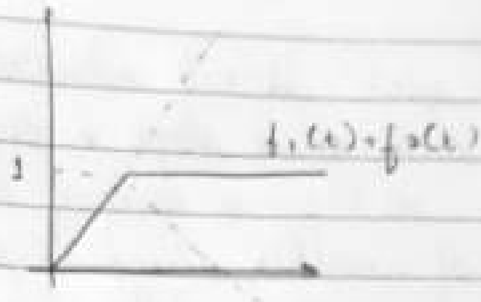
Ex:

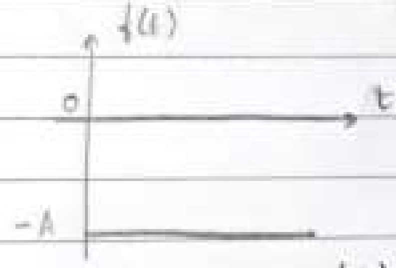




The addition of two ramps having same slope with opposite sign produces a constant valued function for all $t \geq$ at that instant where the two ramps are added.

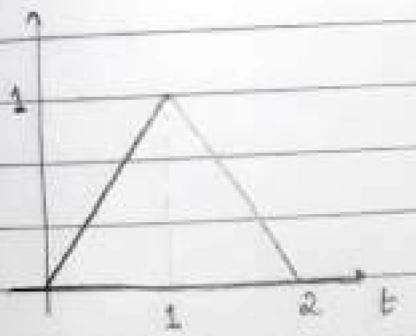
– Unit step function

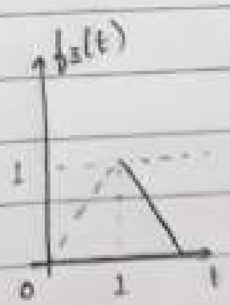
$f(t) = A\,u(t)$

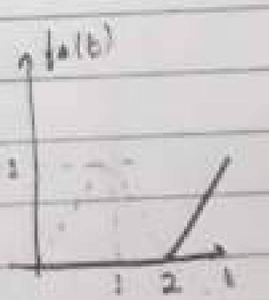


$f(t) = -A\,u(t)$

Express the waveform shown in terms of standard functions







$f_1(t) = t\,u(t)$

$f_2(t) = -(t-1)u(t-1)$

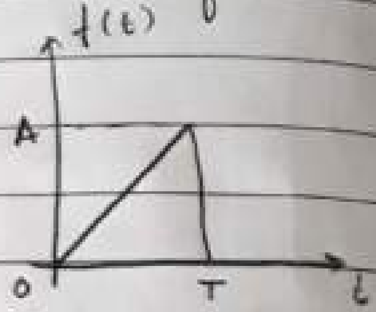
$f_3(t) = -(t-1)u(t-1)$

$f_4(t) = (t-2)u(t-2)$

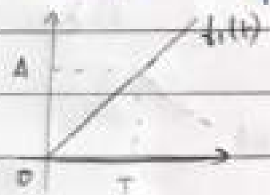
- $f_1(t)$: at $t=0$ indicates that another ramp is added with same slope in opposite sign to end first ramp
  at $t=1$: a ramp of slope -1 exists, hence addition of another ramp of slope -1 at $t=0$
  from $t=2$: exists a function with constant value of 0, hence addition of ramp with equal slope but opposite sign.

$$f(t) = f_1(t) + f_2(t) + f_3(t) + f_4(t)$$

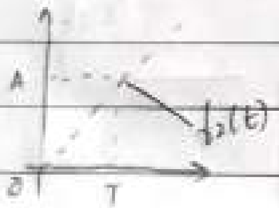
Q3: obtain Laplace transform of sawtooth waveform shown



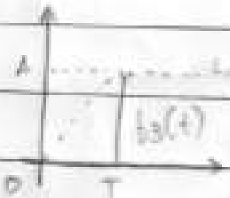
sol: slope $= \frac{A}{T}$



$$f_1(t) = \frac{A}{T} r(t)$$



$$f_2(t) = -\frac{A}{T} r(t-T)$$



$$f_3(t) = -A\,u(t-T)$$

$$\therefore f(t) = \frac{A}{T} r(t) - \frac{A}{T} r(t-T) - A u(t-T)$$

$$f(t) = \frac{A}{T}\left[t u(t) - (t-T) u(t-T)\right] - A u(t-T)$$

$$L[f(t)] = \frac{A}{T}\frac{1}{s^2} - \frac{A}{T}\frac{e^{-Ts}}{s^2} - A\frac{e^{-Ts}}{s} \cancel{\frac{A}{s}}$$

$$= \frac{A}{Ts^2}\left[1 - e^{-Ts}\right] - Ae^{-Ts}$$

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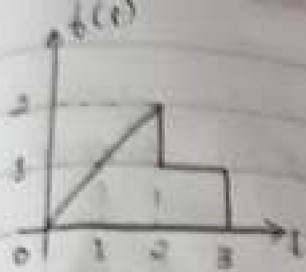
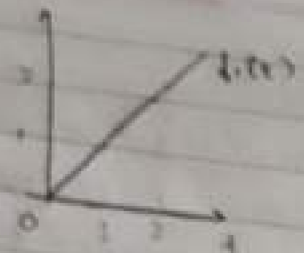
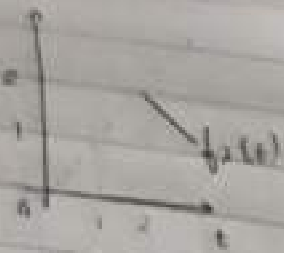
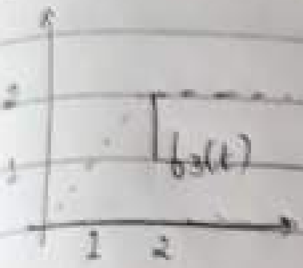


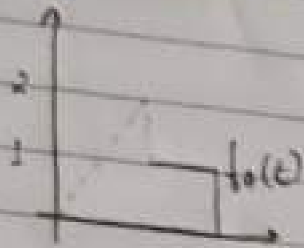




$f_1(t) = r(t)$

$f_2(t) = -r(t-2)$





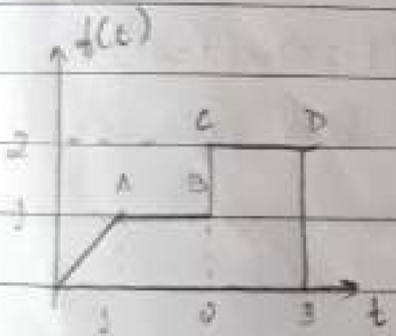
$f_3(t) = -u(t-2)$ $\quad f_4(t) = -u(t-3)$

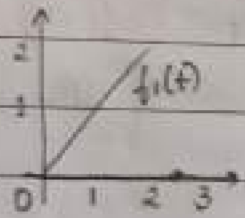
$$\therefore f(t) = r(t) - r(t-2) - u(t-2) - u(t-3)$$

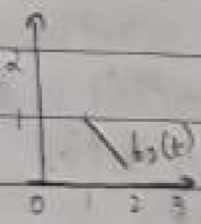
$$= t\,u(t) - (t-2)\,u(t-2) - u(t-2) - u(t-3)$$

$$L[f(t)] = \frac{1}{s^2} - \frac{e^{-2s}}{s^2} - \frac{e^{-2s}}{s} - \frac{e^{-3s}}{s}$$

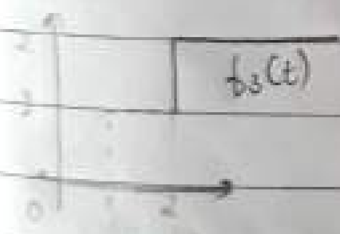
$$= \frac{1}{s^2}\left[1 - e^{-2s}\right] - \frac{1}{s}\left[e^{-2s} + e^{-3s}\right]$$

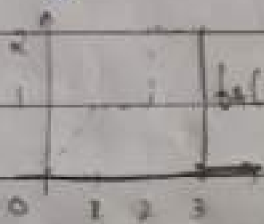






$f_1(t) = r(t)$ $\quad f_2(t) = -r(t-1)$



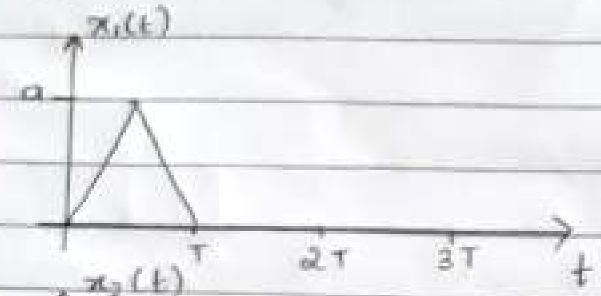


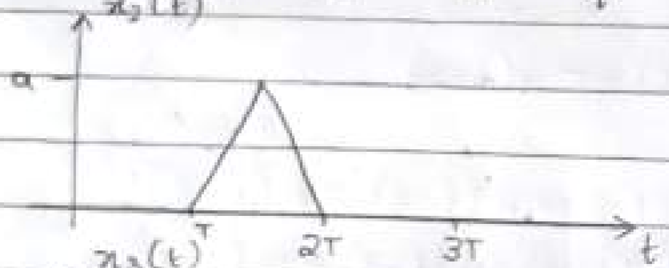
$f_3(t) = u(t-2)$ $\quad f_4(t) = -2u(t-3)$

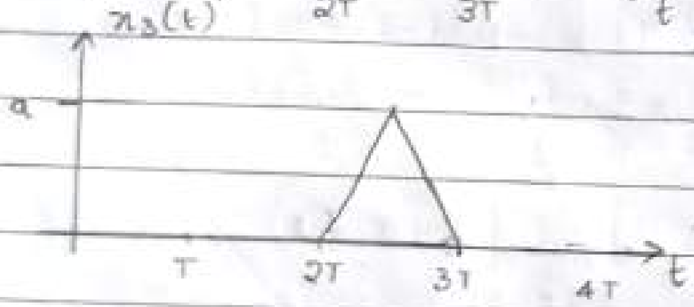
$$\therefore f(t) = r(t) - r(t-1) + u(t-2) - 2u(t-3)$$

★ **Periodic function**:

Consider a function $x(t)$ that is periodic shown. The function $x(t)$ can be represented as the sum of time shifted functions $x_1(t)$, $x_2(t)$, $x_3(t)$ and so on.



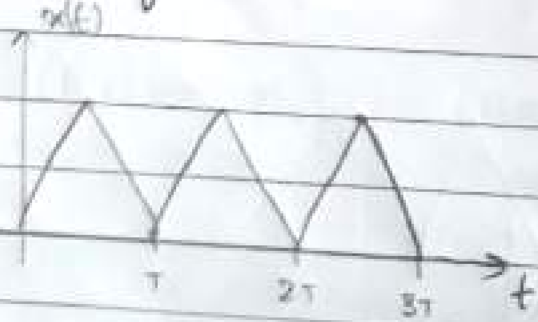




$$x(t) = x_1(t) + x_2(t) + x_3(t) + \dots$$

$$= x_1(t) + x_1(t-T)\,u(t-T) + x_1(t-2T)\,u(t-2T) + \dots$$

where $x_1(t)$ is the waveform described over first period of $x(t)$.



Laplace transform and Time shifting property

$$X(s) = X_1(s) + X_1(s)\,e^{-Ts} + X_1(s)\,e^{-2Ts} + \dots$$

$$X(s) = X_1(s)\,(1 + e^{-Ts} + e^{-2Ts} + \dots)$$

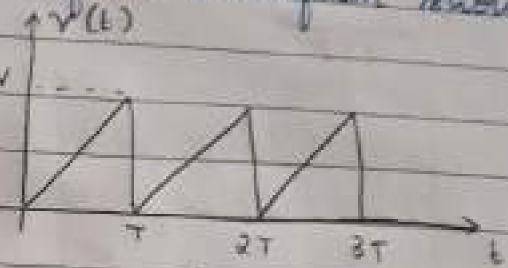
wkt $1 + a + a^2 + \dots = \frac{1}{1-a}$, $|a|<1$

$$\boxed{X(s) = X_1(s)\left(\frac{1}{1-e^{-Ts}}\right)}$$

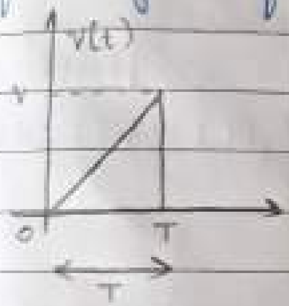
T - overall time period.

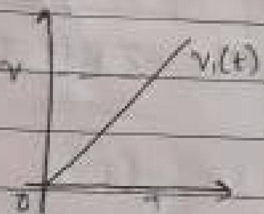
$X_1(s)$ is Laplace transform of $x(t)$ defined over first period only

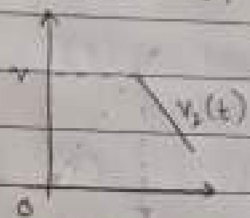
Q. Find the Laplace transform of the waveform shown:



consider the first cycle of the given function

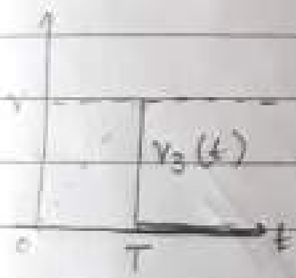






$v_1(t) = \frac{V}{T} r(t)$ $\quad$ $v_2(t) = \frac{-V}{T} r(t-T)$

$v_1(t) = \frac{Vt}{T} u(t)$ $\quad$ $v_2(t) = -\frac{V}{T}(t-T)u(t-T)$



$v_3(t) = -Vu(t-T)$

$$v(t) = v_1(t) + v_2(t) + v_3(t)$$

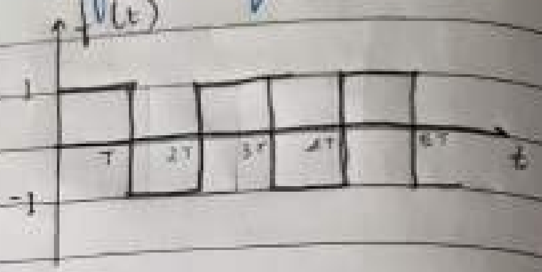
$$v(t) = \frac{Vt}{T}u(t) - \frac{V}{T}(t-T)u(t-T) - Vu(t-T)$$

$$V_1(s) = \frac{V}{T}\frac{1}{s^2} - \frac{V}{T}\frac{e^{-Ts}}{s^2} + \cancel{\frac{V}{T}\frac{Te^{-Ts}}{s}} - V\frac{e^{-Ts}}{s}$$

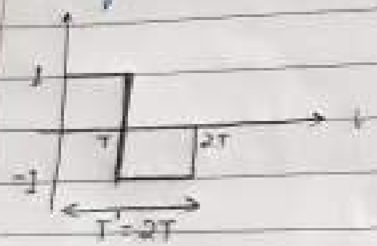
$$V_1(s) = \frac{V}{T}\left[\frac{1}{s^2} - \frac{e^{-Ts}}{s^2} - \frac{T}{s}\right]$$

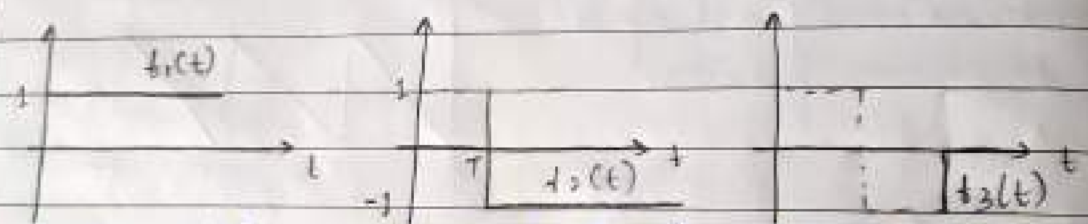
$$V(s) = \frac{V_1(s)}{1-e^{-Ts}} = \frac{V}{T(1-e^{-Ts})}\left[\frac{1}{s^2} - \frac{e^{-Ts}}{s^2} - \frac{T}{s}\right]$$

Q: Obtain the laplace transform of the square wave shown below.



Sol: consider the first cycle of the function $f(t)$





$f_1(t) = u(t)$ $\quad$ $f_2(t) = -2u(t-T)$ $\quad$ $f_3(t) = u(t-2T)$

$$f(t) = f_1(t) + f_2(t) + f_3(t)$$
$$f(t) = u(t) - 2u(t-T) + u(t-2T)$$

$$F_1(s) = \frac{1}{s} - \frac{2e^{-Ts}}{s} + \frac{e^{-2Ts}}{s}$$

$$F(s) = \frac{F_1(s)}{1-e^{-2Ts}} = \frac{1-2e^{-Ts}+e^{-2Ts}}{s(1-e^{-2Ts})}$$

$$= \frac{(1-e^{Ts})^2}{s(1-e^{-Ts})(1+e^{-Ts})}$$

$$= \frac{1}{s}\frac{(1-e^{-Ts})}{(1+e^{-Ts})}$$

$$F(s) = \frac{1}{s}\tanh\left(\frac{Ts}{2}\right)$$

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NOTE: Differentiation in time domain

Let $F(s)$ be the Laplace transform of $f(t)$.

$$L\left[\frac{d}{dt}f(t)\right] = sF(s) - f(0)$$

For n^{th} order derivative

$$L\left[\frac{d^n f(t)}{dt^n}\right] = s^n F(s) - s^{n-1}f(0) - s^{n-2}f'(0) - \cdots - f^{(n-1)}(0)$$

For $n = 2$

$$L\left[\frac{d^2 f(t)}{dt^2}\right] = s^2F(s) - sf(0) - f'(0)$$

For $n = 3$

$$L\left[\frac{d^3 f(t)}{dt^2}\right] = s^3F(s) - s^2f(0) - sf'(0) - f''(0)$$

Integration:

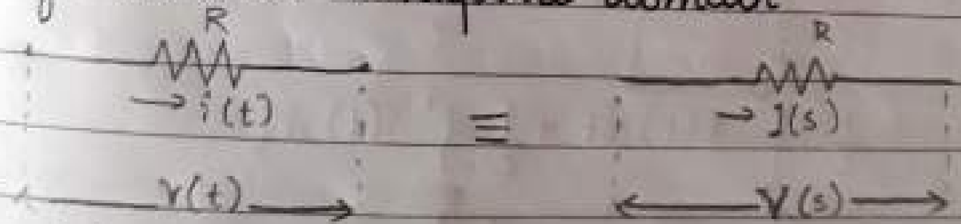
$$L\left[\int_0^t f(t)\,dt\right] = \frac{F(s)}{s}$$

For n^{th} integral

$$L\left[\int_0^{t_1}\int_0^{t_2}\int_0^{t_3}\cdots\int_0^{t_n} f(t)\,dt_1, dt_2, dt_3 \ldots dt_n\right] = \frac{F(s)}{s^n}$$

* RLC behaviour in Laplace domain:

1. Single resistor in Laplace domain



$v(t) = i(t)R$

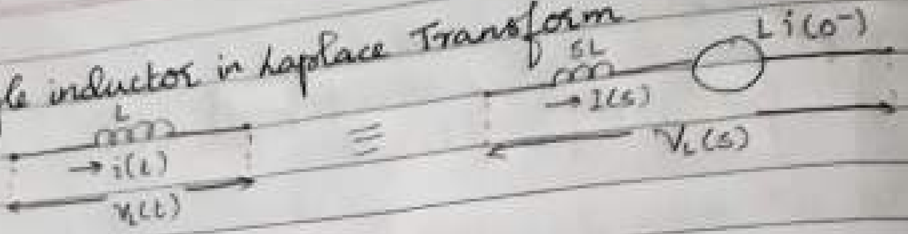
Laplace transform

$V(s) = I(s)R$

$Z(s) = \frac{V(s)}{I(s)} = R$ (transform impedance)

$Y(s) = \frac{1}{Z(s)} = \frac{I(s)}{V(s)} = \frac{1}{R} = G$ (transform admittance)

2. Single inductor in Laplace Transform



$$v_L(t) = L\frac{di(t)}{dt}$$

Laplace transform

$$v_L(s) = L[sI(s) - i(0^-)]$$
$$v_L(s) = sL\,I(s) - Li(0^-)$$

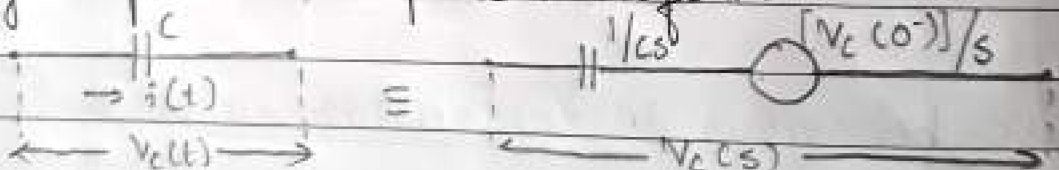
The product of $Li(0^-)$ represents the voltage source which takes into account the effect of initial current through inductor.

To find transform impedance of inductor, assuming zero initial condition.

$$i(0^-) = 0$$
$$V_L(s) = sL\,I(s)$$
$$Z(s) = \frac{V_L(s)}{I(s)} = SL$$

$$Y(s) = \frac{1}{Z(s)} = \frac{I(s)}{V(s)}$$
$$Y(s) = \frac{1}{SL}$$

3. Single capacitor in Laplace Transform



$$V_c(t) = \frac{1}{C}\int_{-\infty}^{t} i(t)\,dt$$
$$= \frac{1}{C}\int_{-\infty}^{0^-} i(t)\,dt + \frac{1}{C}\int_{0}^{t} i(t)\,dt$$
$$V_c(t) = V_c(0^-) + \frac{1}{C}\int_{0}^{t} i(t)\,dt$$

Laplace transform

$$V_c(s) = \frac{V_c(0^-)}{s} + \frac{1}{C}\left[\frac{I(s)}{s}\right]$$

$V_c(0^-)$ is initial charge that is initial voltage in the capacitor, to take into account the effect of initial voltage the limits $-\infty$ to t is split as $-\infty$ to 0^- and 0 to t.

The initial voltage term represents voltage source $\frac{V_c(0^-)}{s}$ in laplace domain.

To find transform impedance of capacitor, assuming zero initial voltage

$V_c(0^-) = 0$

$$V_c(s) = \frac{1}{C}\frac{I(s)}{s}$$

$$Y(s) = \frac{1}{Z(s)} = \frac{I(s)}{V_c(s)} = Cs$$

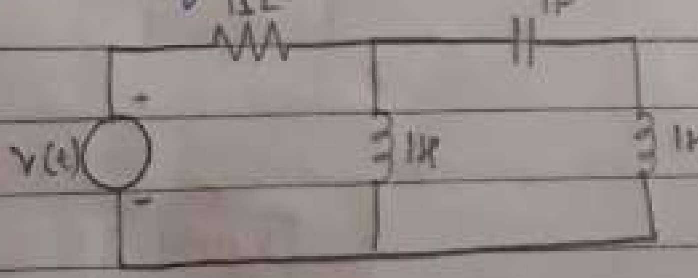
$$Z(s) = \frac{V_c(s)}{I(s)} = \frac{1}{sC}$$

The entire time domain can be converted to s domain network assuming zero initial condition.

	Z(s)	Y(s)
R	R	1/R = G
L	SL	1/SL
C	1/SC	SC

Q: Find the equivalent impedance for the circuit shown

Q: Convert the given circuit into s domain.



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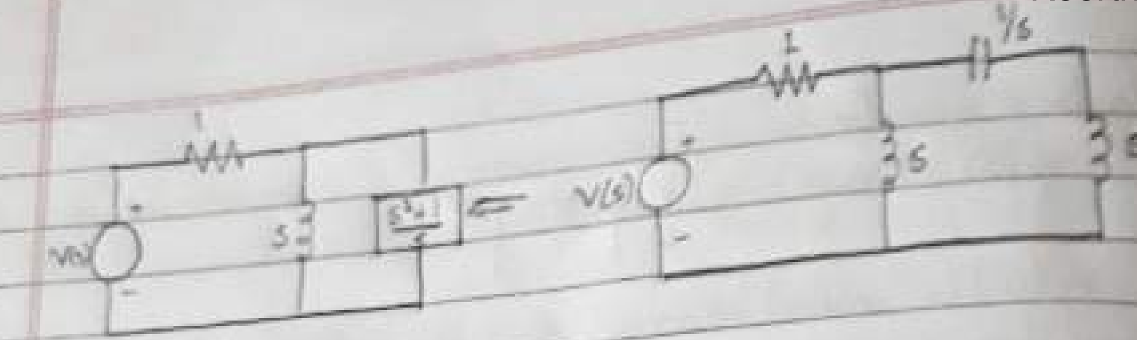
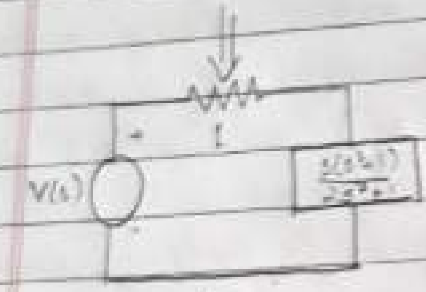


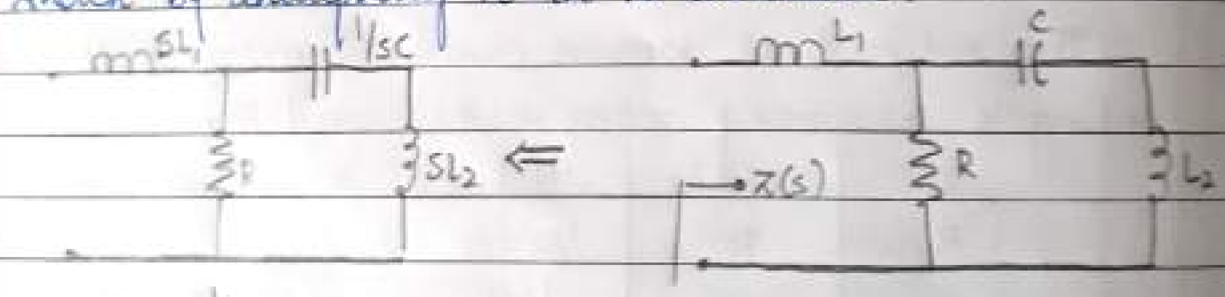


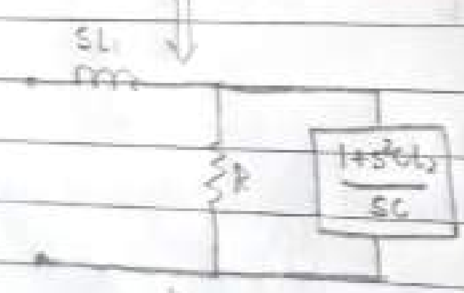
$$\frac{s\left(\frac{s^2+1}{s}\right)}{s+\frac{s^2+1}{s}} = \frac{(s^2+1)s}{2s^2+1}$$

$$1+\frac{(s^2+1)s}{2s^2+1} = \frac{2s^2+1+s^3+s}{2s^2+1}$$

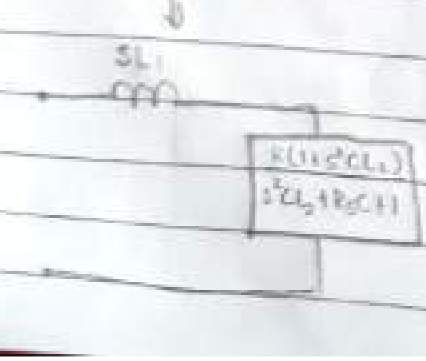
$$Z(s) = \frac{s^3+2s^2+s+1}{2s^2+1}$$

Q: Determine the input impedance $Z(s)$ of the circuit shown by analysing it in s domain.





$$\frac{1}{sC}+sL_2 = \frac{1+s^2CL_2}{sC}$$



$$\frac{R\left(\frac{1+s^2CL_2}{sC}\right)}{R+\underline{1+s^2CL_2}} = \frac{R(1+s^2CL_2)}{s^2CL_2+RsC+1}$$

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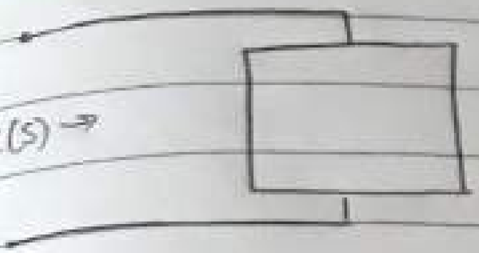
$Z(s) \rightarrow$



$$sL_1 + \frac{R(1+s^2CL_2)}{s^2CL_2 + RsC + 1} = Z(s)$$

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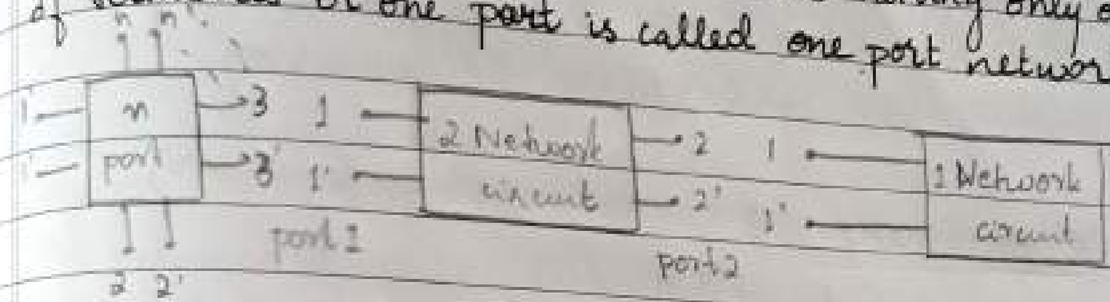
UNIT - 6

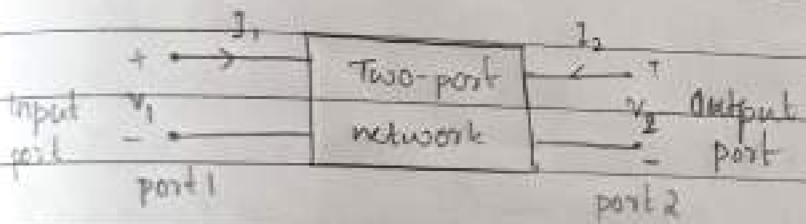
Two Port Network

* **Port:**

A pair of terminals at which electrical signal may enter or leave is called a port. Mininimum number of terminals required is two. A network having only one pair of terminals or one port is called one port network.



* **2 Port network:**



I_1, I_2 are entering. I_1, I_2, V_1, V_2 are four variables. This network can be treated as a black box for the analysis without knowing network details ~~without~~ these parameters can be determined.

In order to describe the relationship between voltage and currents of the port, one requires the linear equations equal to the number of ports.

We require two linear equation for two port in terms of 4 above mentioned variables. we can obtain these equations by considering two variables as dependent variables and other two as independent variables.

As the network consists only linear components the linear relationship can be obtained by writing two variables in terms of other two variables.

There are 6 possible ways of considering two independent variables out of 4 variables. Thus there are 6 different pair of equations defining their own sets of parameters such as impedance, admittance, hybrid, inverse hybrid, transmission and inverse transmission.

* **Z parameter** (open circuit impedance parameters).

I_1 and I_2 independent

V_1 and V_2 dependent

$V_1 = f_1(I_1, I_2)$

$V_2 = f_2(I_1, I_2)$

In equation form

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad \text{—} (1)$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad \text{—} (2)$$

In matrix form

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$[V] = [Z][I]$$

Let $I_2 = 0$; port 2 is open circuited

From eq (1) $V_1 = Z_{11} I_1$

$$\boxed{Z_{11} = \left. \frac{V_1}{I_1} \right|_{I_2=0} \quad \Omega}$$

From eq (2) $V_2 = Z_{21} I_1$

$$\boxed{Z_{21} = \left. \frac{V_2}{I_1} \right|_{I_2=0} \quad \Omega}$$

Let $I_1 = 0$, port 1 is open circuited

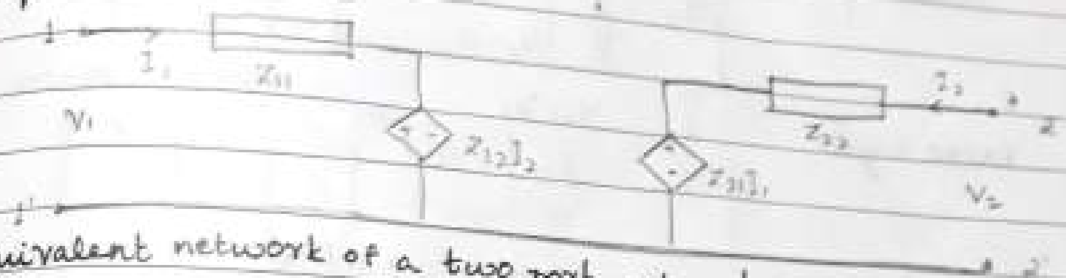
From eq ① $V_1 = Z_{12} I_2$

$$\boxed{Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} \Omega}$$

From eq ② $V_2 = Z_{22} I_2$

$$\boxed{Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} \Omega}$$

Equivalent circuit :



Equivalent network of a two port network in terms of z parameter

Z parameters can be obtained by assigning value of the independent variables as zero.

Z_{11} - open circuit driving point input impedance

Z_{21} - open circuit forward transfer impedance

Z_{12} - open circuit reverse transfer impedance

Z_{22} - open circuit driving point output impedance

Z parameter is defined only when the current in one port is zero or one of the port is open circuited. Hence the name open circuit impedance parameter

* Y parameter (short circuit admittance parameter)

I_1 and I_2 dependent

V_1 and V_2 independent

$I_1 = f_1(V_1, V_2)$

$I_2 = f_2(V_1, V_2)$

In equation form

$I_1 = Y_{11} V_1 + Y_{12} V_2$ —— ①

$I_2 = Y_{21} V_1 + Y_{22} V_2$ —— ②

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In matrix form

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$[I] = [Y][V]$$

Let $V_2 = 0$; port 2 is short circuited

From eq ① $\quad I_1 = Y_{11} V_1$

$$\boxed{Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \quad \mho}$$

From eq ② $\quad I_2 = Y_{21} V_1$

$$\boxed{Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} \quad \mho}$$

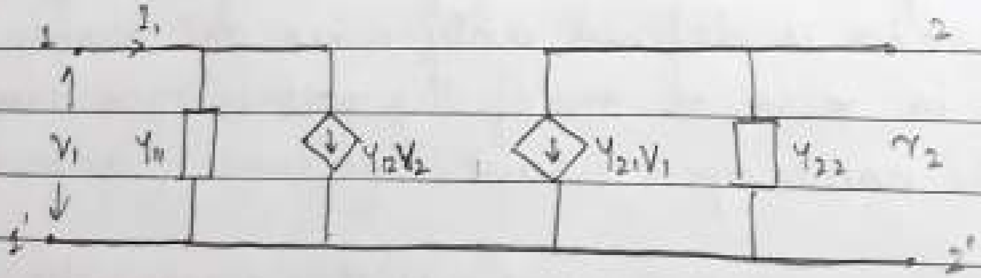
Let $V_1 = 0$; port 1 is short circuited

From eq ① $\quad I_1 = Y_{12} V_2$

$$\boxed{Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} \quad \mho}$$

From eq ② $\quad I_2 = Y_{22} V_2$

$$\boxed{Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} \quad \mho}$$



Y_{11} - short circuit driving point input admittance

Y_{22} - short circuit driving point output admittance

Y_{21} - short circuit forward transfer admittance

Y_{12} - short circuit reverse transfer admittance

Y parameters are defined only when the voltage in any port is zero or one of the port is short circuited. Hence the name short circuit admittance parameter.

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h-parameter | hybrid parameter

I_1, V_2 — independent

I_2, V_1 dependent

$V_1 = f_1(I_1, V_2)$

$I_2 = f_2(I_1, V_2)$

In equation form

$$V_1 = h_{11} I_1 + h_{12} V_2 \quad \text{— (1)}$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \quad \text{— (2)}$$

In matrix form

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} V_2 \\ I_1 \end{bmatrix}$$

let $V_2 = 0$; port 2 is short circuited

From eq (1) $V_1 = h_{11} I_1$

$$\boxed{h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} \quad \Omega}$$

From eq (2) $I_2 = h_{21} I_1$

$$\boxed{h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}}$$

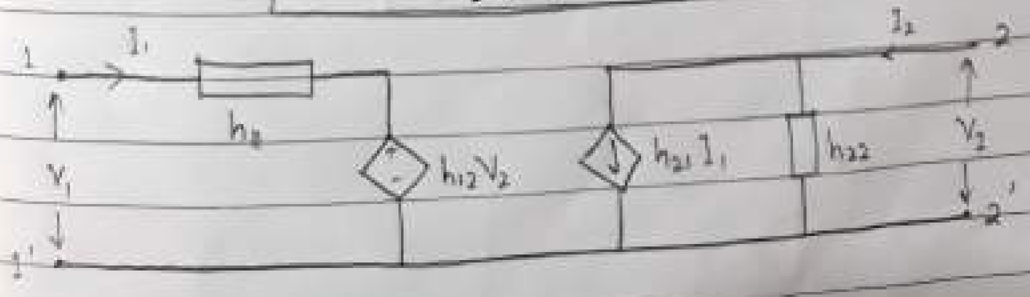
let $I_1 = 0$; port 1 is open circuited

From eq (1) $V_1 = h_{12} V_2$

$$\boxed{h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}}$$

From eq (2) $I_2 = h_{22} V_2$

$$\boxed{h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} \quad \mho}$$



★ ABCD parameter | transmission parameter.

V_1, I_1 dependent

V_2, I_2 independent

$V_1 = f_1(V_2, I_2)$

$I_1 = f_2(V_2, I_2)$

In equation form

$V_1 = AV_2 - BI_2$ —— ①

$I_1 = CV_2 - DI_2$ —— ②

In matrix form

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

Let $I_2 = 0$; port 2 is open circuited

From eq ① $V_1 = AV_2$

$$\boxed{A = \left.\frac{V_1}{V_2}\right|_{I_2=0}}$$

From eq ② $I_1 = CV_2$

$$\boxed{C = \left.\frac{I_1}{V_2}\right|_{I_2=0} \quad \mho}$$

Let $V_2 = 0$; port 2 is short circuited

From eq ① $V_1 = -BI_2$

$$\boxed{-B = \left.\frac{V_1}{I_2}\right|_{V_2=0} \quad \Omega}$$

From eq ② $I_1 = -DI_2$

$$\boxed{-D = \left.\frac{I_1}{I_2}\right|_{V_2=0}}$$

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Conversions

* **Z parameter in terms of other parameters**

1. Y parameter

wkt $I_1 = Y_{11}V_1 + Y_{12}V_2$ —— ①

$I_2 = Y_{21}V_1 + Y_{22}V_2$ —— ②

In matrix form

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Solving using Cramer's rule for V_1 and V_2

$$V_1 = \frac{\begin{vmatrix} I_1 & Y_{12} \\ I_2 & Y_{22} \end{vmatrix}}{\begin{vmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{vmatrix}} = \frac{Y_{22}I_1 - Y_{12}I_2}{Y_{11}Y_{22} - Y_{12}Y_{21}}$$

$$V_1 = \left(\frac{Y_{22}}{\Delta Y}\right)I_1 + \left(\frac{-Y_{12}}{\Delta Y}\right)I_2 \quad —— ③$$

where $Y_{11}Y_{22} - Y_{12}Y_{21} = \Delta Y$

similarly

$$V_2 = \frac{\begin{vmatrix} Y_{11} & I_1 \\ Y_{21} & I_2 \end{vmatrix}}{\begin{vmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{vmatrix}} = \frac{Y_{11}I_2 - Y_{21}I_1}{Y_{11}Y_{22} - Y_{12}Y_{21}}$$

$$V_2 = \left(\frac{-Y_{21}}{\Delta Y}\right)I_1 + \left(\frac{Y_{11}}{\Delta Y}\right)I_2 \quad —— ④$$

comparing z parameters equations

$V_1 = Z_{11}I_1 + Z_{12}I_2$

$V_2 = Z_{21}I_1 + Z_{22}I_2$

with eq ③ and eq ④

$Z_{11} = \dfrac{Y_{22}}{\Delta Y}$ $\quad Z_{12} = \dfrac{-Y_{12}}{\Delta Y}$

$Z_{21} = \dfrac{-Y_{21}}{\Delta Y}$ $\quad Z_{22} = \dfrac{Y_{11}}{\Delta Y}$

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$$\therefore [z] = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \begin{bmatrix} \frac{y_{22}}{\Delta y} & \frac{-y_{12}}{\Delta y} \\ \frac{-y_{21}}{\Delta y} & \frac{y_{11}}{\Delta y} \end{bmatrix}$$

2. <u>h parameter</u> :

wkt $V_1 = h_{11} I_1 + h_{12} V_2$ —— (1)

$I_2 = h_{21} I_1 + h_{22} V_2$ —— (2)

From eq (2)

$$h_{22} V_2 = I_2 - h_{21} I_1$$

$$V_2 = \left(\frac{1}{h_{22}}\right) I_2 + \left(\frac{-h_{21}}{h_{22}}\right) I_1 \quad \text{—— (3)}$$

~~From~~ substituting eq (3) in eq (1)

$$V_1 = h_{11} I_1 + h_{12} \left[\left(\frac{-h_{21}}{h_{22}}\right) I_1 + \left(\frac{1}{h_{22}}\right) I_2\right]$$

$$V_1 = h_{11} I_1 - \frac{h_{12} h_{21}}{h_{22}} I_1 + \frac{h_{12}}{h_{22}} I_2$$

$$V_1 = \frac{h_{11} h_{22} - h_{12} h_{21}}{h_{22}} I_1 + \frac{h_{12}}{h_{22}} I_2$$

where $h_{11} h_{22} - h_{12} h_{21} = \Delta h$

$$V_1 = \left(\frac{\Delta h}{h_{22}}\right) I_1 + \left(\frac{h_{12}}{h_{22}}\right) I_2 \quad \text{—— (4)}$$

comparing z parameters equation

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

with eq (3) and eq (4)

$$z_{11} = \frac{\Delta h}{h_{22}} \qquad z_{12} = \frac{h_{12}}{h_{22}}$$

$$z_{21} = \frac{-h_{21}}{h_{22}} \qquad z_{22} = \frac{1}{h_{22}}$$

$$\therefore [z] = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \begin{bmatrix} \frac{\Delta h}{h_{22}} & \frac{h_{12}}{h_{22}} \\ \frac{-h_{21}}{h_{22}} & \frac{1}{h_{22}} \end{bmatrix}$$

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3. ABCD parameter:

wkt $V_1 = AV_2 - BI_2$ —— (1)

$I_1 = CV_2 - DI_2$ —— (2)

Z parameter equations

$V_1 = Z_{11}I_1 + Z_{12}I_2$ —— (3)

$V_2 = Z_{21}I_1 + Z_{22}I_2$ —— (4)

From eq (2)

$CV_2 = I_1 + DI_2$

$V_2 = \frac{1}{C}I_1 + \frac{D}{C}I_2$ —— (5)

Comparing with eq (4)

$Z_{21} = \frac{1}{C}$ and $Z_{22} = \frac{D}{C}$

Substituting eq (5) in eq (1)

$V_1 = A\left(\frac{1}{C}I_1 + \frac{D}{C}I_2\right) - BI_2$

$V_1 = \frac{A}{C}I_1 + \frac{AD-BC}{C}I_2$ —— (6)

comparing eq (6) With eq (3)

$Z_{11} = \frac{A}{C}$ $Z_{12} = \frac{AD-BC}{C}$ where $AD-BC = \Delta T$

$\therefore Z_{12} = \frac{\Delta T}{C}$

$$\therefore [Z] = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} \frac{A}{C} & \frac{\Delta T}{C} \\ \frac{1}{C} & \frac{D}{C} \end{bmatrix}$$

★ Y parameter in terms of other parameters :

1. Z parameter

wkt $V_1 = Z_{11} I_1 + Z_{12} I_2$ —— ①

$V_2 = Z_{21} I_1 + Z_{22} I_2$ —— ②

In matrix form

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

solving using Cramer's rule for I_1 and I_2

$$I_1 = \frac{\begin{vmatrix} V_1 & Z_{12} \\ V_2 & Z_{22} \end{vmatrix}}{\begin{vmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{vmatrix}} = \frac{Z_{22} V_1 - Z_{12} V_2}{Z_{11} Z_{22} - Z_{21} Z_{12}}$$

$$I_1 = \left(\frac{Z_{22}}{\Delta Z}\right) V_1 + \left(\frac{-Z_{12}}{\Delta Z}\right) V_2 \quad \text{—— ③}$$

where $Z_{11} Z_{22} - Z_{21} Z_{12} = \Delta Z$

Similarly

$$I_2 = \frac{\begin{vmatrix} Z_{11} & V_1 \\ Z_{21} & V_2 \end{vmatrix}}{\begin{vmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{vmatrix}} = \frac{Z_{11} V_2 - Z_{21} V_1}{Z_{11} Z_{22} - Z_{12} Z_{21}}$$

$$I_2 = \left(\frac{-Z_{21}}{\Delta Z}\right) V_1 + \left(\frac{Z_{11}}{\Delta Z}\right) V_2 \quad \text{—— ④}$$

Comparing Y parameter equation

$I_1 = Y_{11} V_1 + Y_{12} V_2$

$I_2 = Y_{21} V_1 + Y_{22} V_2$

with eq ③ and eq ④

$Y_{11} = \dfrac{Z_{22}}{\Delta Z}$ $\quad Y_{12} = \dfrac{-Z_{12}}{\Delta Z}$

$Y_{21} = \dfrac{-Z_{21}}{\Delta Z}$ $\quad Y_{22} = \dfrac{Z_{11}}{\Delta Z}$

$$\therefore \quad [Y] = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \frac{Z_{22}}{\Delta Z} & \frac{-Z_{12}}{\Delta Z} \\ \frac{-Z_{21}}{\Delta Z} & \frac{Z_{11}}{\Delta Z} \end{bmatrix}$$

2. h parameter:

wkt $V_1 = h_{11} I_1 + h_{12} V_2$ —— (1)

$I_2 = h_{21} I_1 + h_{22} V_2$ —— (2)

From eq (1)

$$h_{11} I_1 = V_1 - h_{12} V_2$$

$$I_1 = \left(\frac{1}{h_{11}}\right) V_1 + \left(\frac{-h_{12}}{h_{11}}\right) V_2 \quad \text{—— (3)}$$

Substituting eq (3) in eq (2)

$$I_2 = h_{21} \left[\left(\frac{1}{h_{11}}\right) V_1 + \left(\frac{-h_{12}}{h_{11}}\right) V_2 \right] + h_{22} V_2$$

$$I_2 = \frac{h_{21}}{h_{11}} V_1 + \left[\frac{-h_{12} h_{21} + h_{22} h_{11}}{h_{11}} \right] V_2$$

$$I_2 = \left(\frac{h_{21}}{h_{11}}\right) V_1 + \left(\frac{\Delta h}{h_{11}}\right) V_2 \quad \text{—— (4)}$$

where $\Delta h = h_{11} h_{22} - h_{12} h_{21}$

comparing y parameters equation

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

with eq (3) and eq (4)

$$Y_{11} = \frac{1}{h_{11}} \qquad Y_{12} = \frac{-h_{12}}{h_{11}}$$

$$Y_{21} = \frac{h_{21}}{h_{11}} \qquad Y_{22} = \frac{\Delta h}{h_{11}}$$

$$\therefore \quad [Y] = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \frac{1}{h_{11}} & \frac{-h_{12}}{h_{11}} \\ \frac{h_{21}}{h_{11}} & \frac{\Delta h}{h_{11}} \end{bmatrix}$$

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3. ABCD parameter:

wkt $V_1 = AV_2 - BI_2$ — ①

$I_1 = CV_2 - DI_2$ — ②

From eq ①

$$BI_2 = -V_1 + AV_2$$

$$I_2 = \left(\frac{-1}{B}\right)V_1 + \left(\frac{A}{B}\right)V_2 \quad \text{— ③}$$

From eq ②

$$I_1 = -DI_2 + CV_2$$

Substituting eq ③

$$I_1 = \frac{D}{B}V_1 - \frac{AD}{B}V_2 + CV_2$$

$$I_1 = \frac{D}{B}V_1 + \frac{(AD+BC)}{B}V_2$$

$$I_1 = \left(\frac{D}{B}\right)V_1 + \left(\frac{-\Delta T}{B}\right)V_2 \quad \text{— ④}$$

Comparing ~~with~~ y parameter equation

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

with eq ③ and eq ④

$$Y_{11} = \frac{D}{B} \qquad Y_{12} = \frac{-\Delta T}{B}$$

$$Y_{21} = \frac{-1}{B} \qquad Y_{22} = \frac{A}{B}$$

$$\therefore [Y] = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} \frac{D}{B} & \frac{-\Delta T}{B} \\ \frac{-1}{B} & \frac{A}{B} \end{bmatrix}$$

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* **h parameter in terms of other parameters:**

1. z parameter

wkt $V_1 = z_{11} I_1 + z_{12} I_2$ —— ①

$V_2 = z_{21} I_1 + z_{22} I_2$ —— ②

From eq ②,

$z_{22} I_2 = -z_{21} I_1 + V_2$

$I_2 = \left(\frac{-z_{21}}{z_{22}}\right) I_1 + \left(\frac{1}{z_{22}}\right) V_2$ —— ③

Substituting eq ③ in eq ①

$V_1 = z_{11} I_1 + z_{12} \left[\left(\frac{-z_{21}}{z_{22}}\right) I_1 + \left(\frac{1}{z_{22}}\right) V_2 \right]$

$V_1 = \left(z_{11} - \frac{z_{12} z_{21}}{z_{22}} \right) I_1 + \left(\frac{z_{12}}{z_{22}}\right) V_2$

$V_1 = \frac{z_{11} z_{22} - z_{12} z_{21}}{z_{22}} I_1 + \left(\frac{z_{12}}{z_{22}}\right) V_2$

$V_1 = \left(\frac{\Delta z}{z_{22}}\right) I_1 + \left(\frac{z_{12}}{z_{22}}\right) V_2$ —— ④

comparing h parameter equation

$V_1 = h_{11} I_1 + h_{12} V_2$

$I_2 = h_{21} I_1 + h_{22} V_2$

with eq ③ and eq ④

$h_{11} = \frac{\Delta z}{z_{22}}$ $\quad h_{12} = \frac{z_{12}}{z_{22}}$

$h_{21} = \frac{-z_{21}}{z_{22}}$ $\quad h_{22} = \frac{1}{z_{22}}$

$\therefore [h] = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} \frac{\Delta z}{z_{22}} & \frac{z_{12}}{z_{22}} \\ \frac{-z_{21}}{z_{22}} & \frac{1}{z_{22}} \end{bmatrix}$

2. <u>Y parameter</u>

wkt $I_1 = Y_{11}V_1 + Y_{12}V_2$ —— ①

$I_2 = Y_{21}V_1 + Y_{22}V_2$ —— ②

From eq ①

$Y_{11}V_1 = I_1 - Y_{12}V_2$

$V_1 = \left(\frac{1}{Y_{11}}\right)I_1 + \left(\frac{-Y_{12}}{Y_{11}}\right)V_2$ —— ③

Substituting eq ③ in eq ②

$I_2 = Y_{21}\left[\left(\frac{1}{Y_{11}}\right)I_1 + \left(\frac{-Y_{12}}{Y_{11}}\right)V_2\right] + Y_{22}V_2$

$I_2 = \left(\frac{Y_{21}}{Y_{11}}\right)I_1 + \left[\frac{-Y_{12}Y_{21} + Y_{11}Y_{22}}{Y_{11}}\right]V_2$

$I_2 = \left(\frac{Y_{21}}{Y_{11}}\right)I_1 + \left(\frac{\Delta Y}{Y_{11}}\right)V_2$ —— ④

comparing h parameter equations

$V_1 = h_{11}I_1 + h_{12}V_2$

$I_2 = h_{21}I_1 + h_{22}V_2$

with eq ③ and eq ④

$h_{11} = \frac{1}{Y_{11}}$ $h_{12} = \frac{-Y_{12}}{Y_{11}}$

$h_{21} = \frac{Y_{21}}{Y_{11}}$ $h_{22} = \frac{\Delta Y}{Y_{11}}$

$\therefore [h] = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} \frac{1}{Y_{11}} & \frac{-Y_{12}}{Y_{11}} \\ \frac{Y_{21}}{Y_{11}} & \frac{\Delta Y}{Y_{11}} \end{bmatrix}$

3. <u>ABCD parameter</u> :

wkt $V_1 = AV_2 - BI_2$ —— ①

$I_1 = CV_2 - DI_2$ —— ②

From eq ②

$$DI_2 = -I_1 + CV_2$$

$$I_2 = \left(\frac{-1}{D}\right)I_1 + \left(\frac{C}{D}\right)V_2 \quad \text{——} ③$$

Substituting eq ③ in eq ①

$$V_1 = AV_2 - B\left[\left(\frac{-1}{D}\right)I_1 + \left(\frac{C}{D}\right)V_2\right]$$

$$V_1 = \left(\frac{B}{D}\right)I_1 + \left(\frac{AD-BC}{D}\right)V_2$$

$$V_1 = \left(\frac{B}{D}\right)I_1 + \left(\frac{\Delta T}{D}\right)V_2 \quad \text{——} ④$$

comparing h parameter equations

$$V_1 = h_{11}I_1 + h_{12}V_2$$

$$I_2 = h_{21}I_1 + h_{22}V_2$$

with eq ③ and eq ④

$$h_{11} = \frac{B}{D} \qquad h_{12} = \frac{\Delta T}{D}$$

$$h_{21} = \frac{-1}{D} \qquad h_{22} = \frac{C}{D}$$

$$\therefore [h] = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} \frac{B}{D} & \frac{\Delta T}{D} \\ \frac{-1}{D} & \frac{C}{D} \end{bmatrix}$$

★ ABCD parameter interms of other parameters:

1. Z parameter:

$$V_1 = Z_{11}I_1 + Z_{12}I_2 \quad \text{——} ①$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2 \quad \text{——} ②$$

From eq ②

$$Z_{21}I_1 = V_2 - Z_{22}I_2$$

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$$I_1 = \left(\frac{1}{Z_{21}}\right)V_2 - \left(\frac{Z_{22}}{Z_{21}}\right)I_2 \quad \text{--- (3)}$$

Substituting eq (3) in eq (1)

$$V_1 = Z_{11}\left[\left(\frac{1}{Z_{21}}\right)V_2 - \left(\frac{Z_{22}}{Z_{21}}\right)I_2\right] + Z_{12}I_2$$

$$V_1 = \frac{Z_{11}}{Z_{21}}V_2 - \left[\frac{+Z_{11}Z_{22}}{Z_{21}} - Z_{12}\right]I_2$$

$$V_1 = \left(\frac{Z_{11}}{Z_{21}}\right)V_2 - \left(\frac{+Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{21}}\right)I_2$$

$$V_1 = \left(\frac{Z_{11}}{Z_{21}}\right)V_2 - \left(\frac{\Delta Z}{Z_{21}}\right)I_2 \quad \text{--- (4)}$$

comparing ABCD parameters equations

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

with eq (3) and eq (4)

$$A = \frac{Z_{11}}{Z_{21}} \qquad B = \frac{\Delta Z}{Z_{21}}$$

$$C = \frac{1}{Z_{21}} \qquad D = \frac{Z_{22}}{Z_{21}}$$

$$\therefore [T] = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \frac{Z_{11}}{Z_{21}} & \frac{\Delta Z}{Z_{21}} \\ \frac{1}{Z_{21}} & \frac{Z_{22}}{Z_{21}} \end{bmatrix}$$

2. Y parameter :

$$I_1 = Y_{11}V_1 + Y_{12}V_2 \quad \text{--- (1)}$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2 \quad \text{--- (2)}$$

From eq (2)

$$Y_{21}V_1 = -Y_{22}V_2 + I_2$$

$$V_1 = \left(\frac{-Y_{22}}{Y_{21}}\right)V_2 - \left(\frac{-1}{Y_{21}}\right)I_2 \quad \text{--- (3)}$$

substituting eq ③ in eq ①

$$I_1 = y_{11}\left[\left(\frac{-y_{22}}{y_{21}}\right)V_2 - \left(\frac{-1}{y_{21}}\right)I_2\right] + y_{12}V_2$$

$$I_1 = \left(-\frac{y_{11}y_{22}}{y_{21}} + y_{12}\right)V_2 - \left(\frac{-y_{11}}{y_{21}}\right)I_2$$

$$I_1 = \left(\frac{-y_{11}y_{22} + y_{12}y_{21}}{y_{21}}\right)V_2 - \left(\frac{-y_{11}}{y_{21}}\right)I_2$$

$$I_1 = \left(\frac{-\Delta y}{y_{21}}\right)V_2 - \left(\frac{-y_{11}}{y_{21}}\right)I_2 \quad \text{--- ④}$$

comparing ABCD parameter equation

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

with eq ③ and eq ④

$$A = \frac{-y_{22}}{y_{21}} \qquad B = \frac{-1}{y_{21}}$$

$$C = \frac{-\Delta y}{y_{21}} \qquad D = \frac{-y_{11}}{y_{21}}$$

$$\therefore [T] = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \frac{-y_{22}}{y_{21}} & \frac{-1}{y_{21}} \\ \frac{-\Delta y}{y_{21}} & \frac{-y_{11}}{y_{21}} \end{bmatrix}$$

3. <u>h parameter</u>:

$$V_1 = h_{11}I_1 + h_{12}V_2 \quad \text{--- ①}$$

$$I_2 = h_{21}I_1 + h_{22}V_2 \quad \text{--- ②}$$

From eq ②

$$h_{21}I_1 = -h_{22}V_2 + I_2$$

$$I_1 = \left(\frac{-h_{22}}{h_{21}}\right)V_2 - \left(\frac{-1}{h_{21}}\right)I_2 \quad \text{--- ③}$$

substituting eq ③ in eq ①

$$V_1 = h_{11}\left[\left(\frac{-h_{22}}{h_{21}}\right)V_2 - \left(\frac{-1}{h_{21}}\right)I_2\right] + h_{12}V_2$$

$$V_1 = \left(\frac{-h_{11}h_{22}}{h_{21}} + h_{12}\right)V_2 - \left(\frac{-h_{11}}{h_{21}}\right)I_2$$

$$V_1 = \left(\frac{-h_{11}h_{22} + h_{12}h_{21}}{h_{21}}\right)V_2 - \left(\frac{-h_{11}}{h_{21}}\right)I_2$$

$$V_1 = \left(\frac{-\Delta h}{h_{21}}\right)V_2 - \left(\frac{-h_{11}}{h_{21}}\right)I_2 \quad \text{—— (4)}$$

Comparing ABCD parameter equations

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

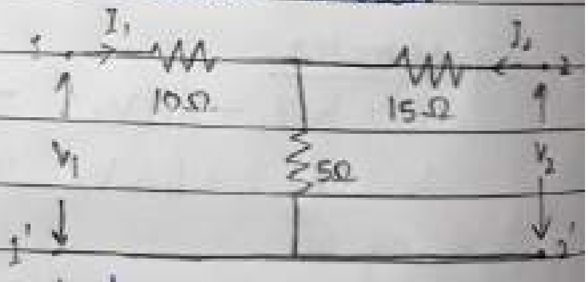
with eq (3) and eq (4)

$$A = \frac{-\Delta h}{h_{21}} \qquad B = \frac{-h_{11}}{h_{21}}$$

$$C = \frac{-h_{22}}{h_{21}} \qquad D = \frac{-1}{h_{21}}$$

$$\therefore [T] = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \frac{-\Delta h}{h_{21}} & \frac{-h_{11}}{h_{21}} \\ \frac{-h_{22}}{h_{21}} & \frac{-1}{h_{21}} \end{bmatrix}$$

Q: Find the z parameter for the network shown.



Sol: z parameter equation

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

– Let $I_2 = 0$; port 2 is open circuited

Applying KVL to input loop

$$V_1 = I_1(10) + I_1(5)$$

$$V_1 = 15\,I_1$$

$$Z_{11} = \frac{V_1}{I_1} = 15\,\Omega$$

$$Z_{21} = \frac{V_2}{I_1} = 5\,\Omega$$

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- let $I_1 = 0$; port 1 is open circuited
Applying KVL to output loop

$$V_2 = 15(I_2) + 5(I_2)$$

$$V_2 = 20 I_2$$

$$Z_{22} = \frac{V_2}{I_2} = 20\,\Omega$$

$$Z_{12} = \frac{V_1}{I_2} = 5\,\Omega$$

$$\therefore [Z] = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} 15 & 5 \\ 5 & 20 \end{bmatrix}$$

Q: A two port network has the following Z parameters $Z_{11} = 10\,\Omega$, $Z_{22} = 12\,\Omega$, $Z_{12} = Z_{21} = 5\,\Omega$ compute the Y parameter for this network.

sol: Given: $[Z] = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} 10 & 5 \\ 5 & 12 \end{bmatrix}$

$$\Delta Z = Z_{11} Z_{22} - Z_{12} Z_{21} = (10 \times 12) - (5 \times 5)$$

$$\Delta Z = 120 - 25 = 95$$

Y parameter in terms of Z parameter

$$Y_{11} = \frac{Z_{22}}{\Delta Z} = \frac{12}{95} = 0.126$$

$$Y_{12} = \frac{-Z_{12}}{\Delta Z} = \frac{-5}{95} = -0.05$$

$$Y_{21} = \frac{-Z_{21}}{\Delta Z} = \frac{-5}{95} = -0.05$$

$$Y_{22} = \frac{Z_{11}}{\Delta Z} = \frac{10}{95} = 0.105$$

$$\therefore [Y] = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} 0.126 & -0.05 \\ -0.05 & 0.105 \end{bmatrix}$$

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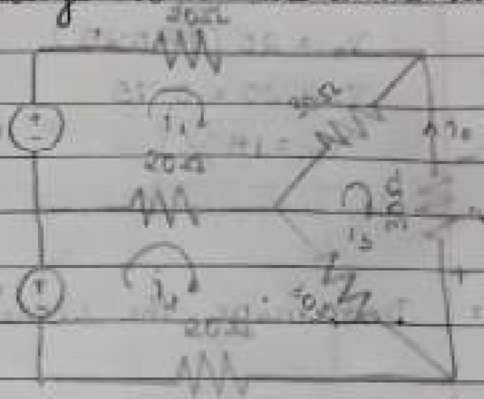
Q1: Determine the current i_o and voltage V_o in the circuit shown:



sol: Loop 1:

$-80+20i_1+30(i_1-i_3)+20(i_1-i_2)=0$

$-80+70i_1-20i_2-30i_3=0$ —— (1)

Loop 2:

$-80+20(i_2-i_1)+30(i_2-i_3)+20i_2=0$

$-80-20i_1+70i_2-30i_3=0$ —— (2)

Loop 3:

$30i_3+30(i_3-i_2)+30(i_3-i_1)=0$

$-30i_1-30i_2+90i_3=0$ —— (3)

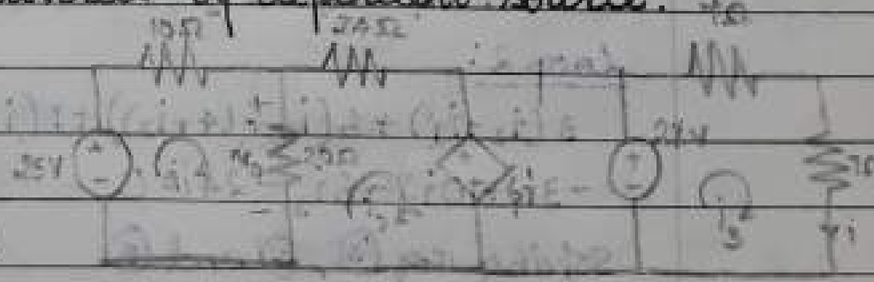
solving eq (1), (2) and (3)

$i_1=2.67A$ $\quad i_2=2.67A$ $\quad i_3=1.78A$ $\Rightarrow$ $\boxed{i_o=-1.78A}$

~~Applying KCL~~ $V_o=30i_o=30(1.78)$

~~$i_1+i_o=i_3$~~ $\boxed{V_o=53.4V}$

Q2: Find V_o and power delivered by dependent source.



sol: Loop 1:

$-25+10(i_1)+20(i_1-i_2)=0$

$-25+30i_1-20i_2=0$ — (1)

Loop 2:

$20(i_2-i_1)+24i_2+6i_3=0$ $\quad (i=i_3)$

$-20i_1+44i_2+6i_3=0$ — (2)

Loop 3:

$-21+7i_3+7i_3=0$

$14i_3=21$

$i_3=1.5A$

Substituting i_3 in (2)

$-20i_1+44i_2+9=0$ — (3)

solving eq (1) and (3)

$i_1=1A$ $\quad i_2=0.25A$

$v_o = 20(i_1 - i_2)$

$v_o = 20(1 - 0.25)$

$v_o = 20 \times 0.75$

$\boxed{v_o = 15\,V}$

Power delivered by dependent source

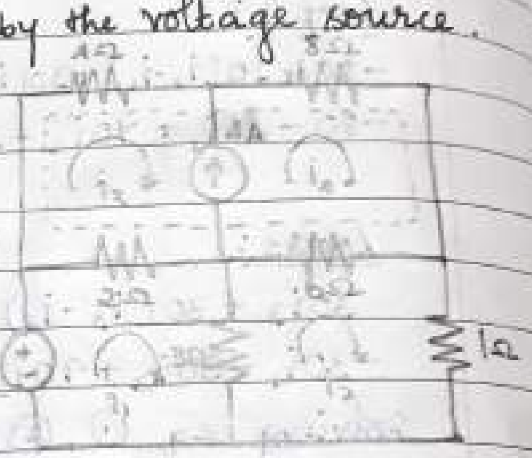
$P = VI$

$P = (6i)(i_2)$

$P = 6 \times 1.5 \times 0.25$

$\boxed{P = 2.25\,W}$

Q3: Determine the current supplied by the voltage source.



sol: Super mesh:

$4i_3 + 8i_4 + 6(i_4 - i_2) + 2(i_3 - i_1) = 0$

$-2i_1 - 6i_2 + 6i_3 + 14i_4 = 0$ — (1)

Constraint Equation

$4 = i_4 - i_3$ — (2) $\Rightarrow i_4 = 4 + i_3$

Substituting i_4 in (1)

$-2i_1 - 6i_2 + 6i_3 + 14(4 + i_3) = 0$

$-2i_1 - 6i_2 + 20i_3 + 56 = 0$ — (3)

Loop 1:

$-30 + 2(i_1 - i_3) + 3(i_1 - i_2) = 0$

$-30 + 5i_1 - 3i_2 - 2i_3 = 0$ — (4)

Loop 2:

$3(i_2 - i_1) + 6(i_2 - (4 + i_3)) + 1(i_2) = 0$

$-3i_1 + 10i_2 - 6i_3 - 24 = 0$ — (5)

Solving eq (3), (4) and (5)

$i_1 = 8.56\,A$ $\quad i_2 = 4.636\,A$ $\quad i_3 = -0.553\,A$ $\quad i_4 = 3.447\,A$

Current supplied by the voltage source

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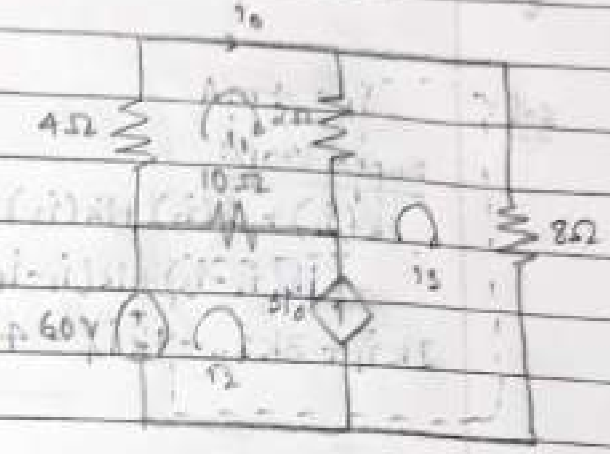
Q4: Determine i_o in the circuit shown:



sol: Loop 1:

$4i_1 + 2(i_1 - i_3) + 10(i_1 - i_2) = 0$

$16i_1 - 10i_2 - 2i_3 = 0$ — ①

Super mesh:

$-60 + 10(i_2 - i_1) + 2(i_3 - i_1) + 8i_3 = 0$

$-60 - 12i_1 + 10i_2 + 10i_3 = 0$ — ②

Constraint Equation

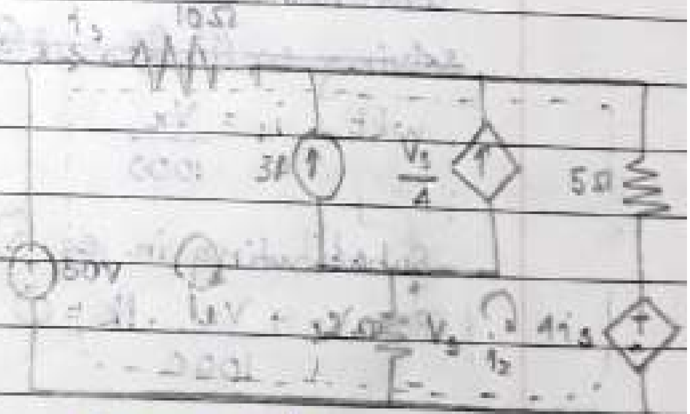
$3i_o = i_3 - i_2$

$\Rightarrow 3i_1 = i_3 - i_2$ — ③

solving eq ①, ② and ③

$i_1 = 1.73A$ $i_2 = 1.44A$ $i_3 = 6.63A$ $\Rightarrow$ $\boxed{i_o = 1.73A}$

Q5: Determine V_s and i_s



sol: $i_s = i_1$

~~Super mesh 1:~~

$\cancel{-50 + 10i_1 + 2(i_1 - i_2) = 0}$

$\cancel{-50 + 12i_1 - 2i_2 = 0}$ — ①

~~Super mesh 2:~~

$\cancel{2(i_2 - i_1) + 5i_2 + 4i_1 = 0}$

$\cancel{2i_1 + 7i_2 = 0}$ — ②

~~Solving eq ① and ②~~

$\cancel{i_1 = 3.98A}$ $\cancel{i_2 = -1.136A}$

$\cancel{V_s = 2(i_1 - i_2)}$

$\cancel{V_s = 2(3.98 + 1.136)}$

$\boxed{V_s = 10.23V}$

Super mesh:

$-50 + 10i_1 + 5i_2 + 4i_1 = 0$

$-50 + 14i_1 + 5i_2 = 0$ — ①

Constraint equation

$3 + V_s/4 = i_2 - i_1$

$3 + 2(i_1 - i_2)/4 = -(i_1 - i_2)$

$3 = -(i_1 - i_2) - (i_1 - i_2)/2$

$\Rightarrow 2 = i_2 - i_1$ — ②

Solving ① and ②

$i_1 = 2.1A$ $i_2 = 4.1A$ $\boxed{i_s = 2.1A}$

$V_s = 2(i_1 - i_2)$

$\boxed{V_s = -4V}$

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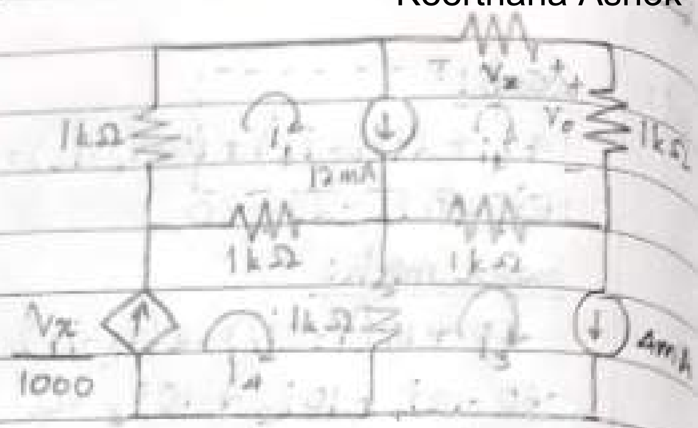
Q6: Determine the voltage V_o..



sol: $i_3 = 4mA$

Super loop:

$1k(i_1) + 1k(i_2) + 1k(i_2) + 1k(i_2 - i_3) + 1k(i_1 - i_4) = 0$

$2ki_1 + 3ki_2 - 1ki_4 - 4 = 0$ — (1)

Loop 3:

$1k(i_3 - i_2) + 1k(i_3 - i_4) = 0$

$8 - 1ki_2 - 1ki_4 = 0$ — (2)

Loop 4:

$1k(i_4 - i_1) + 1k(i_4 - i_3) = 0$

$-1ki_1 + 2ki_4 - 4 = 0$ — (3)

constraint equation : $12m = i_1 - i_2$

~~solving eq (1), (2) and (3)~~

wkt $i_4 = \frac{V_x}{1000}$

Substituting in eq (2)

$8 + V_x + \frac{V_x}{1000} \cdot 1k = 0$

$2V_x = -8$

$V_x = -4V$

$V_x = 1ki_2$

$i_2 = -4mA$ $i_4 = 4mA$ $i_1 = 4mA$ $i_3 = 4mA$

$V_o = 1k\,i_2$

$V_o = 1k(-4m)$

$\boxed{V_o = -4V}$

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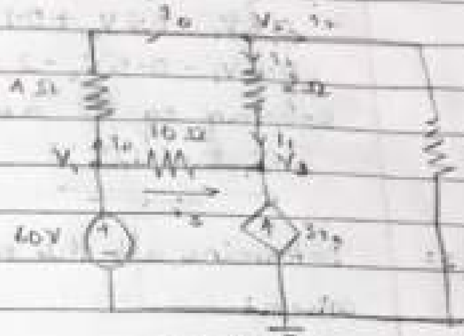
Q1. Use nodal analysis to determine i_0.



Sol: At node V_2:

$i_0 = i_1 + i_2$

$$\frac{V_1 - V_2}{4} = \frac{V_2 - V_3}{2} + \frac{V_2 - 0}{8}$$

$2V_1 - 2V_2 = 4V_2 - 4V_3 + V_2$

$2V_1 - 7V_2 + 4V_3 = 0$

~~At node V_3~~ $V_1 = 60V$

$\therefore 2(60) - 7V_2 + 4V_3 = 0$

$7V_2 - 4V_3 - 120 = 0$ —— (1)

At node V_3:

$3i_0 + i_1 + i_3 = 0$

$$3\left(\frac{60 - V_2}{4}\right) + \frac{V_2 - V_3}{2} + \frac{60 - V_3}{10} = 0$$

$900 - 15V_2 + 10V_2 - 10V_3 + 120 - 2V_3 = 0$

$5V_2 + 12V_3 - 1020 = 0$ —— (2)

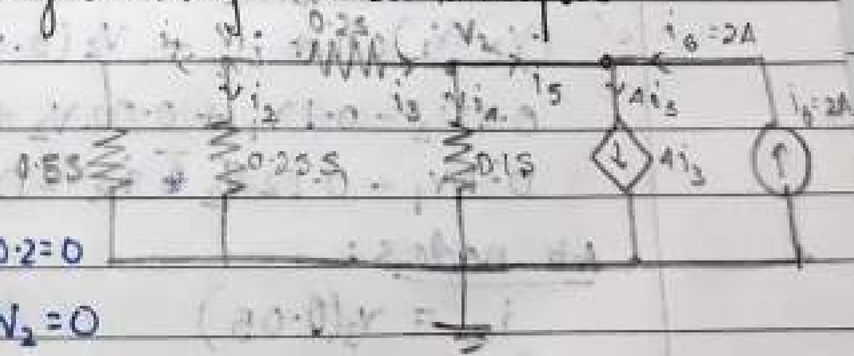
Solving eq (1) and (2)

$V_2 = 53.07V$ $\quad$ $V_3 = 62.88V$

$$i_0 = \frac{V_1 - V_2}{4} = \frac{60 - 53.07}{4}$$

$i_0 = 1.73A$

Q2: Find all the nodal voltages using nodal analysis



Sol: At node V_1:

$i_1 + i_2 + i_3 = 0$

$(V_1 - 0)0.5 + (V_1 - 0)0.25 + (V_1 - V_2)0.2 = 0$

$0.5V_1 + 0.25V_1 + 0.2V_1 - 0.2V_2 = 0$

$0.95V_1 - 0.2V_2 = 0$

At node V_2:

$i_3 = i_4 + i_5 \Rightarrow i_3 = i_4 + (4i_3 - 2)$

$3i_3 + i_4 - 2 = 0$

$3(V_1 - V_2)0.2 + (V_2 - 0)0.1 - 2 = 0$

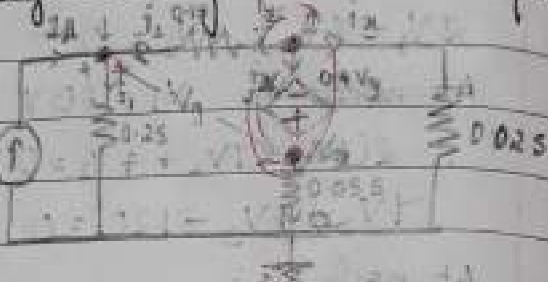
$0.6V_1 - 0.6V_2 + 0.1V_2 - 2 = 0$

$0.6V_1 - 0.5V_2 - 2 = 0$ —— (2)

solving eq (1) and (2)

$V_1 = -1.127\,V$ $\quad V_2 = -5.352\,V$

Q3: Determine all the node voltages using nodal analysis.



Given:

$V_1 - V_3 = V_g$

Sol: Constraint equation:

$V_3 - V_2 = 0.4\,V_g$

wkt $V_g = V_1 - V_3$

$V_3 - V_2 = 0.4V_1 - 0.4V_3$

$0.4V_1 + V_2 - 1.4V_3 = 0$ —— (1)

$V_1 - V_3 = V_g$

$V_3 - V_2 = 0.4\,V_g$

At node 1:

$2 + i_2 = i_1$

$2 + (V_2 - V_1)0.1 = (V_1 - 0)0.25$

$2 + 0.1V_2 - 0.1V_1 = 0.25V_1$

$0.35V_1 - 0.1V_2 - 2 = 0$ —— (2)

At node 2:

$i_2 + i_3 + i_x = 0$

$(V_2 - V_1)0.1 + i_x + V_2(0.02) = 0$

$0.1V_2 - 0.1V_1 + 0.02V_2 + i_x = 0$

$0.1V_1 - 0.12V_2 - i_x = 0$ —— (3)

At node 3:

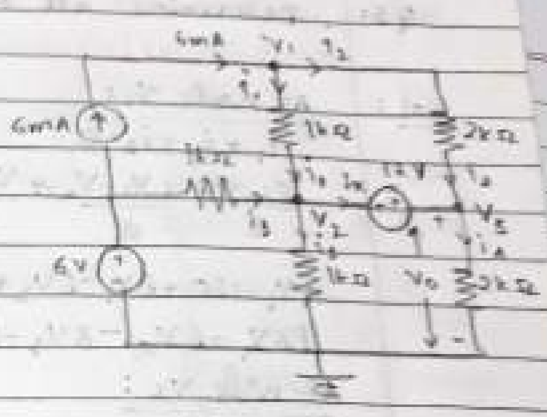
$i_x = V_3(0.05)$

eq (3) $\Rightarrow$ $0.1V_1 - 0.12V_2 - 0.05V_3 = 0$ —— (4)

solving (1), (2) and (4)

$V_1 = 8.16\,V$ $\quad V_2 = 4.49\,V$ $\quad V_3 = 5.54\,V$

Q4: Determine all the node voltage:



sol: At node V_1:

$6m = i_1 + i_2$

$6m = \dfrac{V_1 - V_2}{1k} + \dfrac{V_1 - V_3}{2k}$

$12 = 2V_1 - 2V_2 + V_1 - V_3$

$3V_1 - 2V_2 - V_3 - 12 = 0$ —— (1)

At node V_2:

$i_1 + i_3 = I_x + i_5$

$\dfrac{V_1 - V_2}{1k} + \dfrac{6 - V_2}{1k} = I_x + \dfrac{V_2 - 0}{1k}$

$V_1 - V_2 + 6 - V_2 = 1k(I_x) + V_2$

$V_1 - 3V_2 + 6 = 1k(I_x)$

At node V_3:

$i_2 + I_x = i_4$

$\dfrac{V_1 - V_3}{2k} + I_x = \dfrac{V_3 - 0}{2k}$

$V_1 - V_3 + 2k(I_x) = V_3$

$-V_1 + 2V_3 = 2k(I_x)$

$-V_1 + 2V_3 = 2k\left(\dfrac{V_1 - 3V_2 + 6}{1k}\right)$

$-V_1 + 2V_3 = 2V_1 - 6V_2 + 12$

$3V_1 - 6V_2 - 2V_3 + 12 = 0$ —— (2)

Constraint Equation

$V_3 - V_2 = 12$ —— (3)

Solving eq (1), (2) and (3)

$\therefore V_1 = 10.4\,V$ $\quad V_2 = 2.4\,V$ $\quad V_3 = 14.4\,V$

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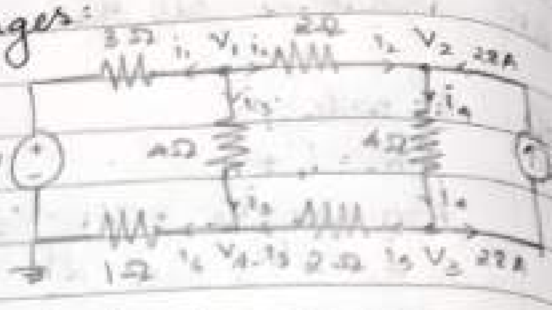
Q5: Determine all the node voltages:



Sol: At node V_1:

$i_1+i_2+i_3=0$

$$\frac{V_1-40}{3}+\frac{V_1-V_2}{2}+\frac{V_1-V_4}{4}=0$$

$4V_1-160+6V_1-6V_2+3V_1-3V_4=0$

$13V_1-6V_2-3V_4-160=0 \Rightarrow 91V_4-26V_3-6V_2-3V_4-160=0$

$-6V_2-26V_3+88V_4-160=0$ —— (1)

At node V_2:

$i_2+28=i_4$

$$\frac{V_1-V_2}{2}+28=\frac{V_2-V_3}{4}$$

$2V_1-2V_2+112=V_2-V_3$

$2V_1-3V_2+V_3+112=0 \Rightarrow 14V_4-4V_3-3V_2+V_3+112=0$

$-3V_2-3V_3+14V_4+112=0$ —— (2)

At node V_3:

$i_4=i_5+28$

$$\frac{V_2-V_3}{4}=\frac{V_3-V_4}{2}+28$$

$V_2-V_3=2V_3-2V_4+28$

$V_2-3V_3+2V_4-112=0$ —— (3)

At node V_4:

$i_3+i_5=i_6$

$$\frac{V_1-V_4}{4}+\frac{V_3-V_4}{2}=\frac{V_4-0}{1}$$

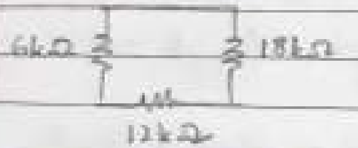
$V_1-V_4+2V_3-2V_4-4V_4=0$

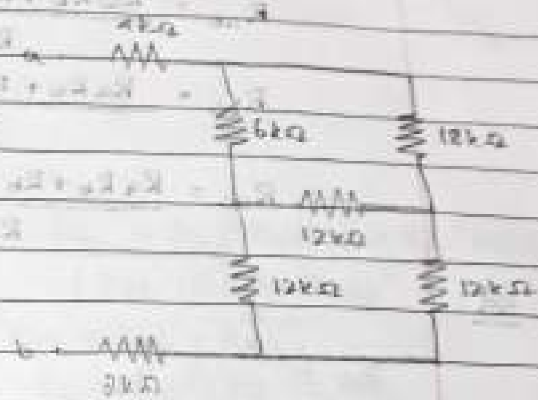
$V_1+2V_3-7V_4=0 \Rightarrow V_1=7V_4-2V_3$

$V_1=38.8V$ $V_2=54.2V$ $V_3=-18V$ $V_4=04V$

1. Find the value of resistance between the terminals a-b of the network shown using delta (Δ) to star (Y) connection.

Sol: Considering



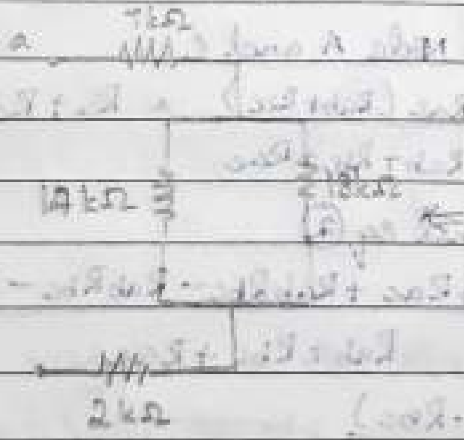


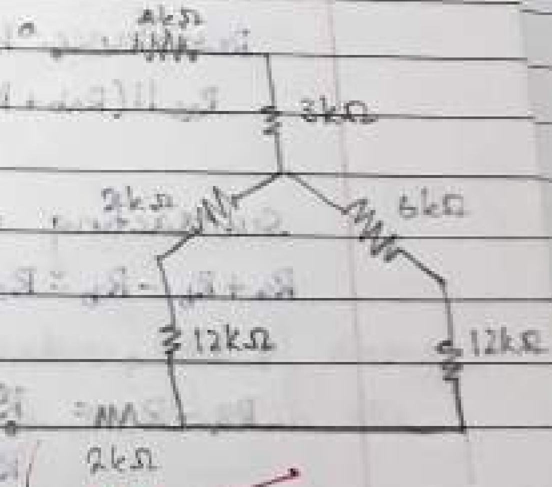
$$R_a = \frac{6k \times 18k}{6k + 18k + 12k} = \frac{108M}{36k}$$

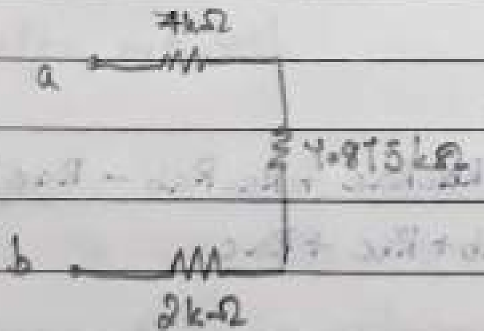
$\underline{R_a = 3k\Omega}$

$$R_b = \frac{6k \times 12k}{6k + 18k + 12k} = \frac{72M}{36k} \qquad \underline{R_b = 2k\Omega}$$

$$R_c = \frac{12k \times 18k}{6k + 18k + 12k} = \frac{216M}{36k} \qquad \underline{R_c = 6k\Omega}$$







$\underline{R_{eq} = 16.875\Omega}$

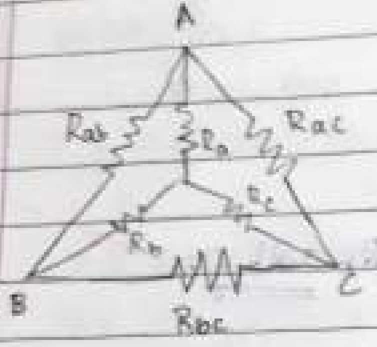
Q2: Derive the expression for R_{ab}, R_{bc} and R_{ac} using star to Delta transformation. Prove that (using Δ to Y equation)

$$R_{ab} = \frac{R_aR_b + R_bR_c + R_cR_a}{R_c}$$

$$R_{bc} = \frac{R_aR_b + R_bR_c + R_cR_a}{R_a}$$

$$R_{ac} = \frac{R_aR_b + R_bR_c + R_cR_a}{R_b}$$

sol:



Resistance between node A and B

$R_{ab} \parallel (R_{ab} + R_{bc})$

$$\Rightarrow \frac{R_{ab}(R_{ac}+R_{bc})}{R_{ab}+R_{bc}+R_{ac}} = R_a + R_b \quad \text{---} (1)$$

Resistance between node B and C

$R_{bc} \parallel (R_{ab} + R_{ac})$

$$\Rightarrow \frac{R_{bc}(R_{ab}+R_{ac})}{R_{ab}+R_{bc}+R_{ac}} = R_b + R_c \quad \text{---} (2)$$

Resistance between Node A and C

$$R_{ac} \parallel (R_{ab}+R_{bc}) \Rightarrow \frac{R_{ac}(R_{ab}+R_{bc})}{R_{ab}+R_{bc}+R_{ac}} = R_a + R_c \quad \text{---} (3)$$

Subtracting eq (1) and eq (2)

$$R_a + \cancel{R_b} - \cancel{R_b} - R_c = \frac{R_{ab}R_{ac} + R_{ab}R_{bc} - R_{ab}R_{bc} - R_{bc}R_{ac}}{R_{ab}+R_{bc}+R_{ac}}$$

$$R_a - R_c = \frac{R_{ac}(R_{ab}-R_{bc})}{R_{ab}+R_{bc}+R_{ac}} \quad \text{---} (4)$$

Adding eq (3) and eq (4)

$$R_a + \cancel{R_c} + R_a - \cancel{R_c} = \frac{R_{ac}R_{ab} + \cancel{R_{ac}R_{bc}} + R_{ac}R_{ab} - \cancel{R_{ac}R_{bc}}}{R_{ab}+R_{bc}+R_{ac}}$$

$$\cancel{2}R_a = \frac{\cancel{2}R_{ab}R_{ac}}{R_{ab}+R_{bc}+R_{ac}}$$

$$R_a = \frac{R_{ab}R_{ac}}{R_{ab}+R_{bc}+R_{ac}} \quad \text{---} (5)$$

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similarly

$$R_b = \frac{R_{bc} R_{ab}}{R_{ab} + R_{bc} + R_{ac}} \quad \text{——— (6)}$$

$$R_c = \frac{R_{bc} R_{ac}}{R_{ab} + R_{bc} + R_{ac}} \quad \text{——— (7)}$$

From eq (5), eq (6) and eq (7)

$$R_a R_b + R_b R_c + R_a R_c = \frac{R_{ab}^2 R_{bc} R_{ac} + R_{bc}^2 R_{ab} R_{ac} + R_{ac}^2 R_{bc} R_{ab}}{(R_{ab} + R_{bc} + R_{ac})^2}$$

$$R_a R_b + R_b R_c + R_a R_c = \frac{R_{ab} R_{bc} R_{ac} (R_{ab} + R_{bc} + R_{ca})}{(R_{ab} + R_{bc} + R_{ca})^2} \quad \text{——— (8)}$$

Dividing eq (8) by eq (7)

$$\boxed{R_{ab} = \frac{R_a R_b + R_b R_c + R_a R_c}{R_c}}$$

similarly dividing eq (8) by eq (5)

$$\boxed{R_{bc} = \frac{R_a R_b + R_b R_c + R_a R_c}{R_a}}$$

similarly dividing eq (8) by eq (6)

$$\boxed{R_{ac} = \frac{R_a R_b + R_b R_c + R_a R_c}{R_b}}$$

Q3: If all the resistors in the star connection are of same value, what will be the value of each resistor in the delta network.

sol: Same value of resistances in star network

i.e., $R_a = R_b = R_c = R$

star to delta network

$$R_{ab} = \frac{R_a R_b + R_b R_c + R_a R_c}{R_c} = \frac{R^2 + R^2 + R^2}{R} = \frac{3R^2}{R}$$

$R_{ab} = 3R$ similarly $R_{bc} = 3R$ $R_{ac} = 3R$

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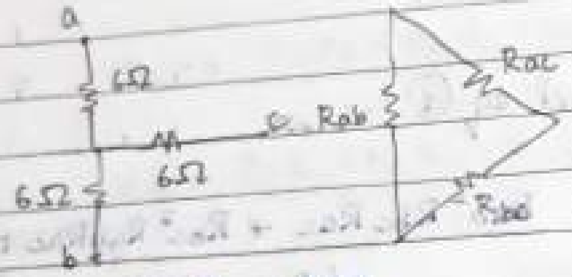
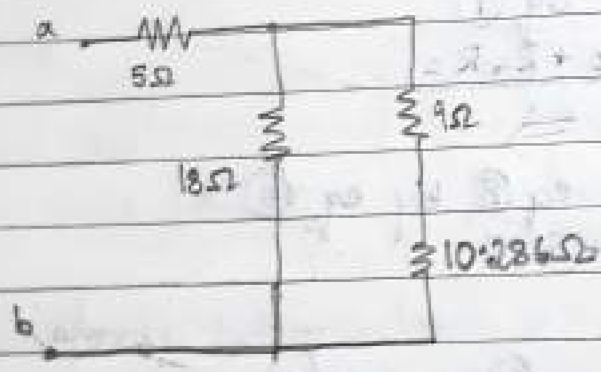
Q4: Find the resistance R_{ab} using Y to Δ transformation:

Sol: considering



$R_a = R_b = R_c = 6\Omega$

$\therefore \; R_{ab} = R_{bc} = R_{ca} = 3R = 3(6) = 18\Omega$



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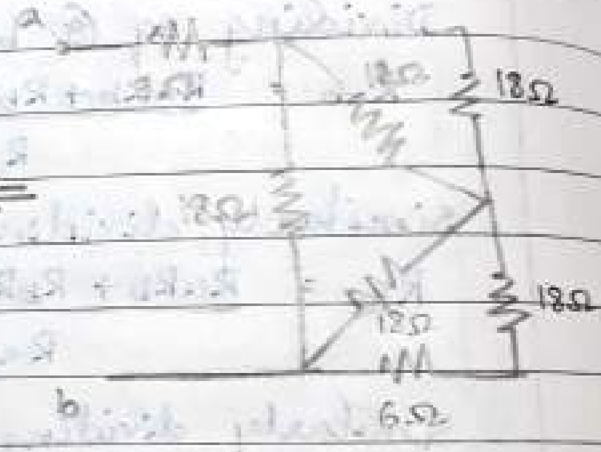
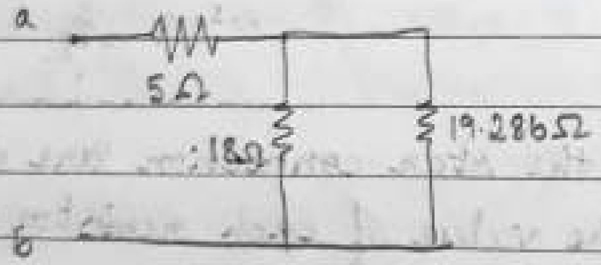
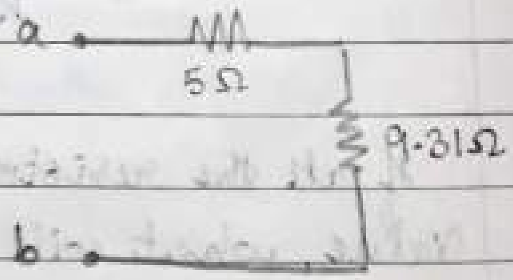


⇓

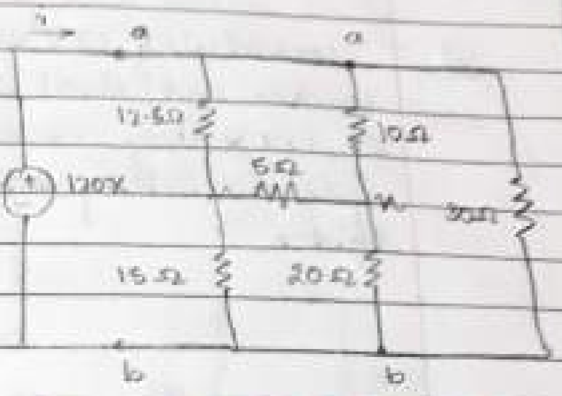


⇒

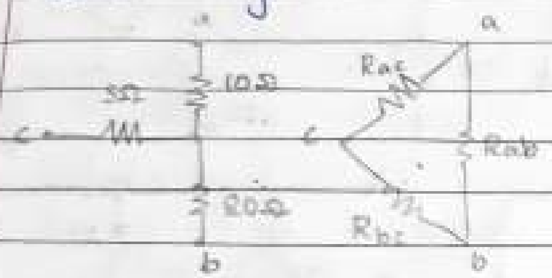


$\therefore \; R_{ab} = 14.31\Omega$

Q5: Obtain the equivalent resistance R_{ab} for the circuit and hence find i.



Sol: considering:



$$R_{ab} = \frac{R_aR_b + R_bR_c + R_aR_c}{R_c}$$

$$R_{ab} = \frac{10(20) + (20)5 + 10(5)}{5}$$

$$\underline{R_{ab} = 70\Omega}$$

$$R_{bc} = \frac{R_aR_b + R_bR_c + R_aR_c}{R_a}$$

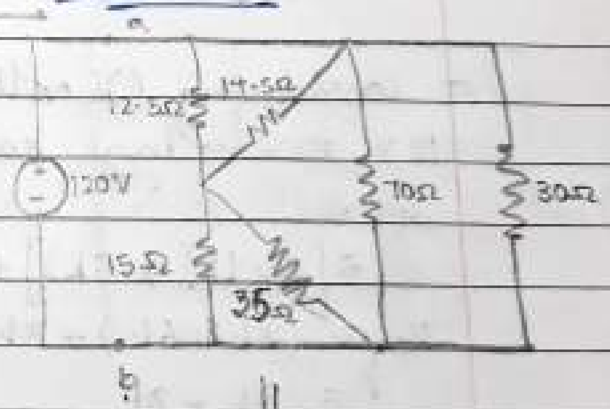
$$R_{bc} = \frac{10(20) + 20(5) + 10(5)}{10}$$

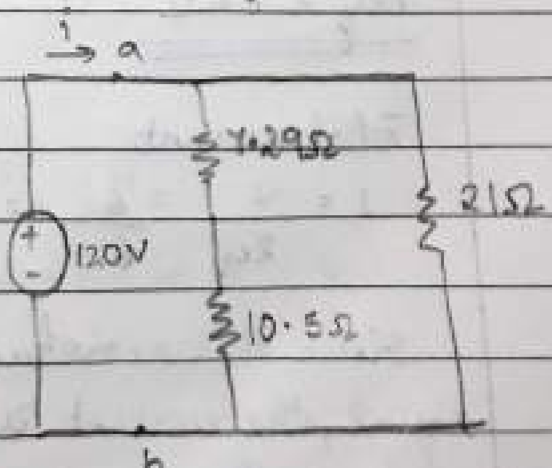
$$\underline{R_{bc} = 35\Omega}$$

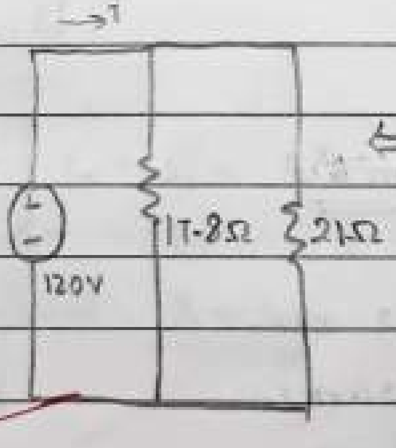
$$R_{ac} = \frac{R_aR_b + R_bR_c + R_aR_c}{R_b}$$

$$R_{ac} = \frac{10(20) + 20(5) + 10(5)}{20}$$

$$\underline{R_{ac} = 17.5\Omega}$$







Resistance

$$\underline{R = 9.634\Omega}$$

$$i = \frac{V}{R}$$

$$i = \frac{120}{9.634}$$

$$\underline{i = 12.45A}$$

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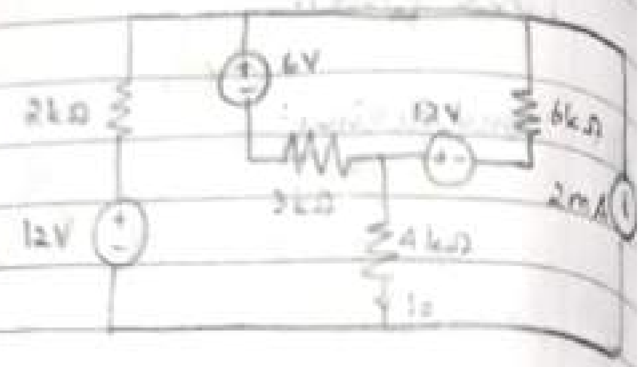
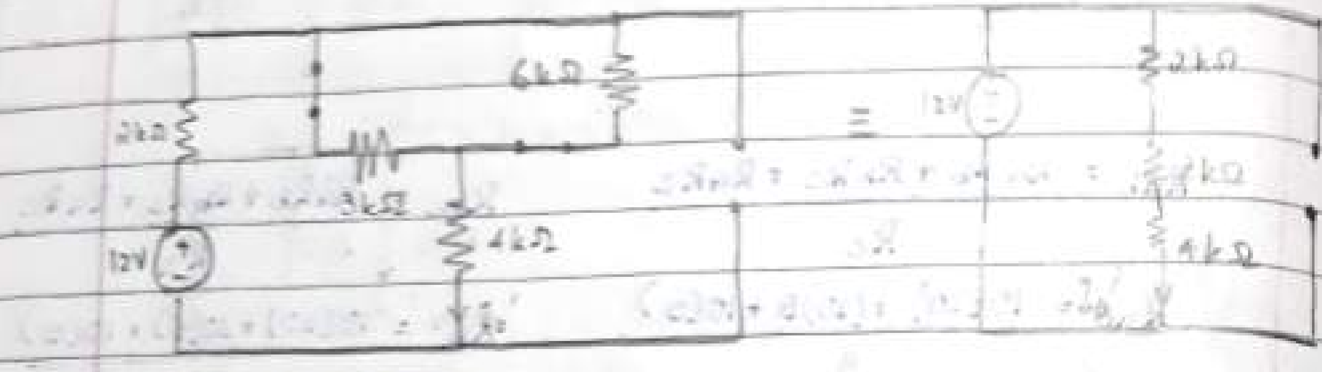
Q1: Find I_o, using Superposition



Sol: – considering 12 V voltage source and short circuiting 6V, 12V voltage source and open circuiting 2mA current source.

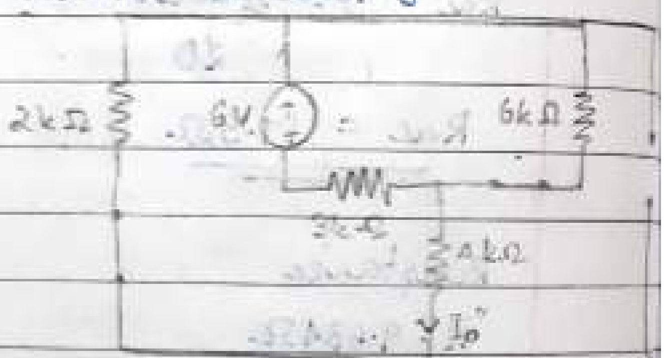


$Req = 2 + 2 + 4$

$\underline{Req = 8k\Omega}$

$I_o' = \frac{V}{Req} = \frac{12}{8k} = \underline{1.5mA}$

– considering 6V voltage source and short circuiting 12V, 12V voltage source and open circuiting 2mA current source.

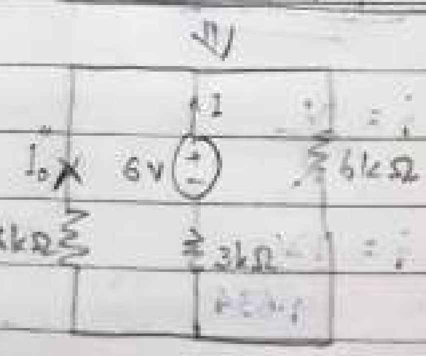


$\Rightarrow [6k \| (2k + 4k)] + 3k$

$Req = (6k \| 6k) + 3k$

$= 3k + 3k$

$\underline{Req = 6k\Omega}$



Total current

$I = \frac{V}{Req} = \frac{6}{6k} = \underline{1mA}$

Since the resistances are equal, the current is same across both $6k\Omega$ resistors

$I_o'' = \frac{1mA}{2} = \underline{-0.5mA}$

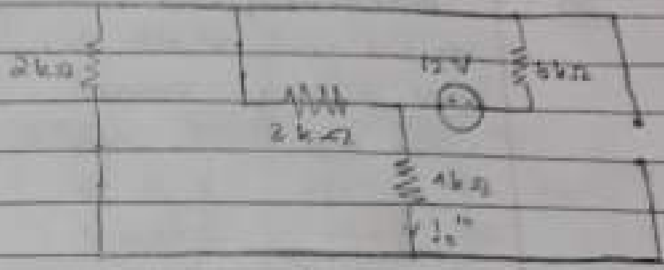
- considering 12V voltage source and short circuiting 6V, 12V voltage source and open circuiting 2mA current source

$\Rightarrow [3k \parallel (2k+4k)] + 6k$

$R_{eq} = (3k \parallel 6k) + 6k$

$= 2k + 6k$

$\underline{R_{eq} = 8k\Omega}$



Total current

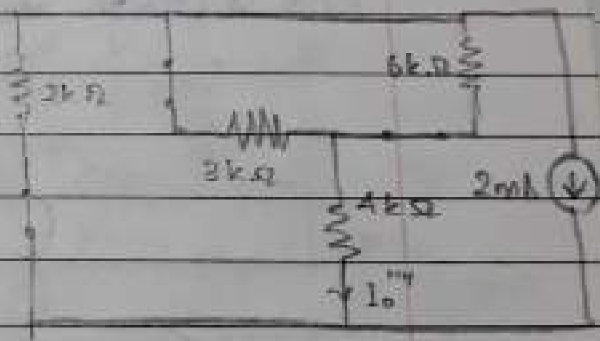
$I = \frac{V}{R_{eq}} = \frac{12}{8k} = \underline{1.5mA}$

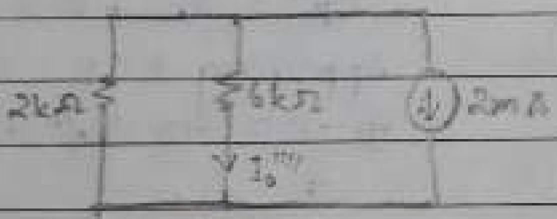
$I_0'' = \frac{I_0 \times 3k}{9k} = \frac{1.5}{3} = \underline{0.5mA}$

- considering 2mA current source and short circuiting 6V, 12V and 12V voltage source.

$\Rightarrow 2k\Omega \parallel [(6k \parallel 3k) + 4k]$

$\Rightarrow 2k\Omega \parallel 6k\Omega$





$I_0''' = \frac{1 \times 2k}{8k} = \frac{2m}{4}$

$\underline{I_0''' = -0.5mA}$

Total current $I_0 = I_0' + I_0'' + I_0''' + I_0''''$

$= 1.5m - 0.5m + 0.5m - 0.5m$

$= \underline{1mA}$

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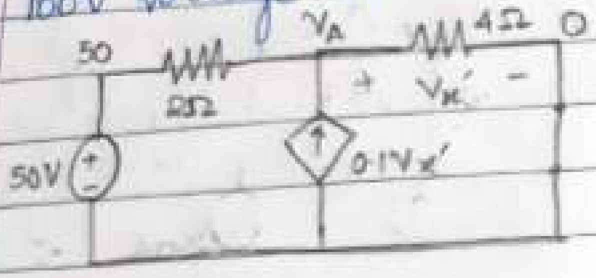
Q2: Find V_x using superposition.

Sol :- considering 50V voltage source and short circuiting 100V voltage source.



$V_x' = V_A - 0 \Rightarrow V_x' = V_A$

Applying KCL

$$\frac{50 - V_A}{2} + 0.1V_x' - \frac{V_A}{4} = 0$$

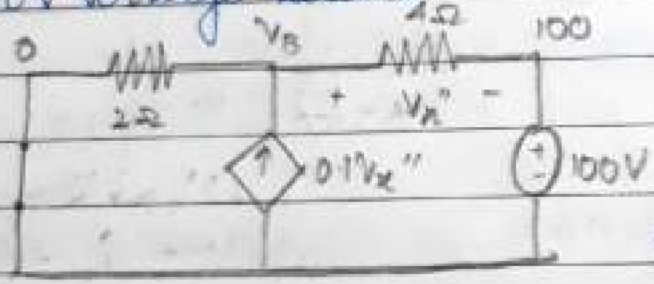
$$100 - 2V_A + 0.4V_x' - V_A = 0$$

$$100 - 3V_A + 0.4x' = 0$$

$$100 - 2.6V_x' = 0$$

$$V_x' = \frac{100}{2.6} = \underline{\underline{38.48\,V}}$$

- considering 100V voltage source and short circuiting 50V voltage source



$V_B - 100 = V_x''$

$V_B = V_x'' + 100$

Applying KCL

$$-\frac{V_B - 100}{4} + 0.1V_x'' - \frac{V_B}{2} = 0$$

$$-V_B + 100 + 0.4V_x'' - 2V_B = 0$$

$$100 - 3V_B + 0.4V_x'' = 0$$

$$100 - 3(V_x'' + 100) + 0.4V_x'' = 0$$

$$-200 - 3V_x'' + 0.4V_x'' = 0$$

$$V_x'' = \frac{-200}{2.6} = \underline{\underline{-76.92V}}$$

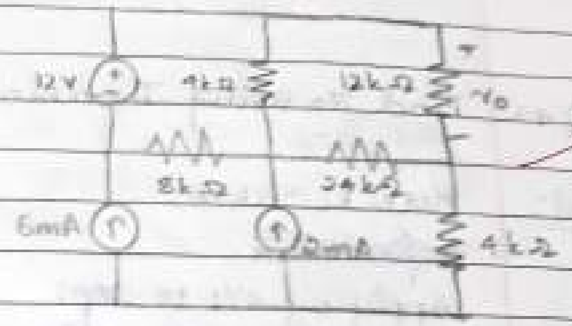
$V_x = -V_x'' + V_x' = -76.92 + 38.48$

$\underline{\underline{V_x = -38.44V}}$

Seen
3/9/18

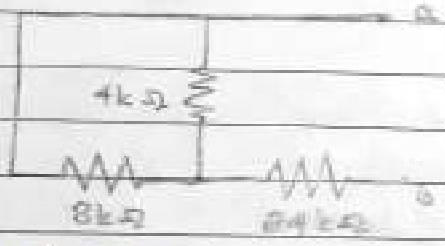
Superposition and Thevenin's Theorem

Q3: Find V_o using Thevenin's Theorem.



sol: Here 12kΩ is the load

To find R_{Th}: 12V voltage source is replaced by short and 6mA and 2mA current source is replaced by open circuit



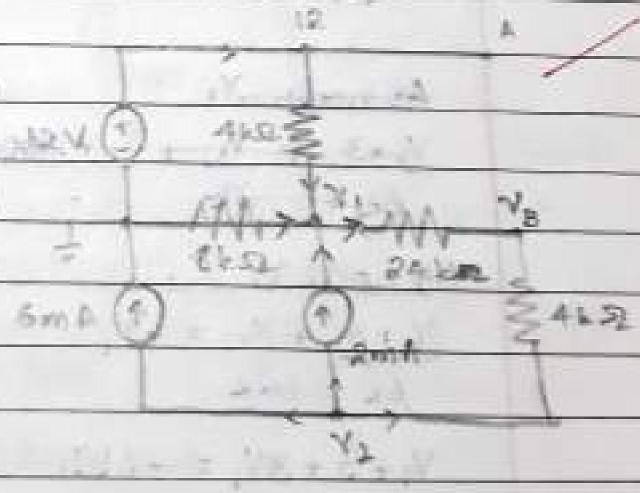
$R_{Th} = (8 \| 4) + 24$

$= 2.64 + 24$

$R_{Th} = 26.64 k\Omega$

At node V_1

$$\frac{-V_1}{8k} + \frac{12 - V_1}{4k} - \frac{V_1 - V_B}{24k} + 2m = 0$$



$$-3V_1 + 72 - 6V_1 - V_1 + V_B + 48 = 0$$

$$10V_1 - V_B - 120 = 0 \quad \text{—— (1)}$$

At node V_2

$$6m + 2m + \frac{V_2 - V_B}{4k} = 0$$

$$32 + V_2 - V_B = 0 \quad \text{—— (2)}$$

At node V_B

$$\frac{V_1 - V_B}{24k} + \frac{V_2 - V_B}{4k} = 0$$

$$V_1 - V_B + 6V_2 - 6V_B = 0$$

$$V_1 + 6V_2 - 7V_B = 0 \quad \text{—— (3)}$$

$V_1 = -8V$

$V_2 = -232V$

$V_B = -200V$

$V_{Th} = V_{AB} = 12 - (-200)$

$V_{Th} = 212V$

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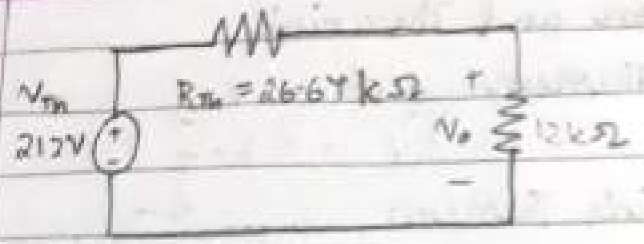
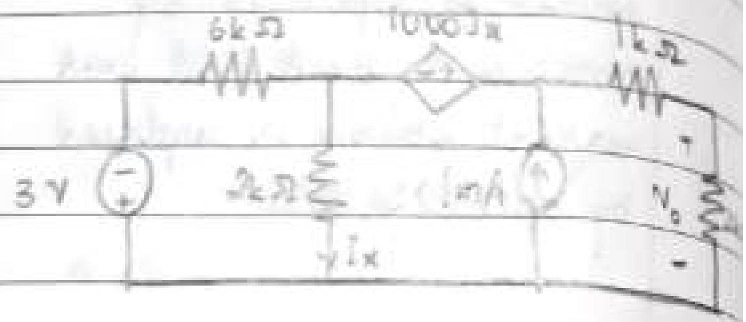


$I = \frac{V_{Th}}{R_{Th} + R_L} \quad \frac{}{(26.67+12)k}$

$I = 5.48\,mA$

$V_o = I(12k) = 65.48V$

Q4: Find v_o using Thevenin's theorem



<u>sol:</u> Here $R_L = 2k\Omega$

~~To find R_{Th}~~

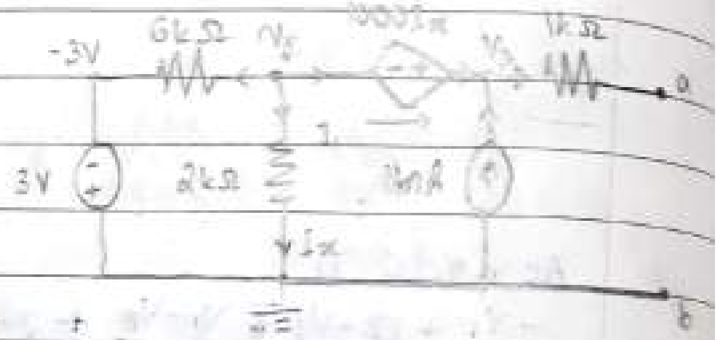
~~Applying KVL to loop 1~~

~~$3 + 6k(I_1) + 2k(I_1 - I_x) = 0$~~

~~$8kI_1 - 2kI_x + 3 = 0$~~ —— (1)

~~Applying KVL to loop 2~~

~~$2k(I_2 - I_x) - 1000I_x$~~



~~At node V_1~~

~~$\frac{V_1+3}{6k} + \frac{V_1}{2k} + \frac{V_1 + 1000I_x}{}$~~

~~constraint equation~~

~~$I_1 - I_2 = I_x$~~

At node V_1

$\frac{V_1+3}{6k} + \frac{V_1}{2k} = -I_1$

$I_1 + 1m = 0$

$V_1 + 3 + 3V_1 = -6kI_1 \quad I_1 = -1m$

$4V_1 + 3 = 6$

$V_1 = \frac{3}{4} V$

$V_2 - V_1 = 1000 I_x$

$V_2 - V_1 = 1000\left(\frac{V_1}{2k}\right)$

$2V_2 - 2V_1 = V_1$

$3V_1 = 2V_2$

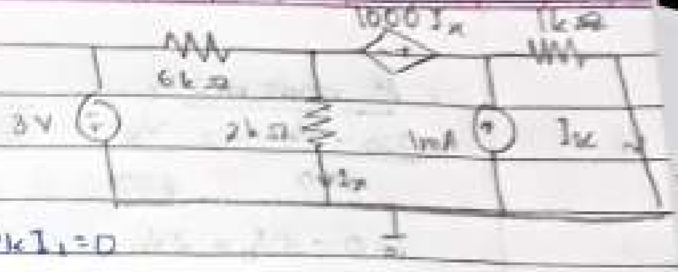
$3\left(\frac{3}{4}\right) = 2V_2$

$V_{Th} = V_2 = \frac{9}{8} V$

Loop 1:

$3 + (6k)I_1 + (2k)(I_1 - I_2) = 0$

$3 + 8kI_1 - 2kI_2 = 0$



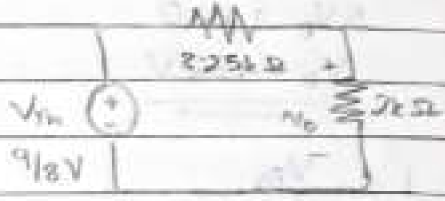
Super mesh:

$-1000(I_1-I_2)-1kI_2$
$+2k(I_2-I_1)=0$

$-kI_1+1kI_2+1kI_2+2kI_2-2kI_1=0$

$-3kI_1+4kI_2=0$

$I_{sc}=I_2=0.346mA$



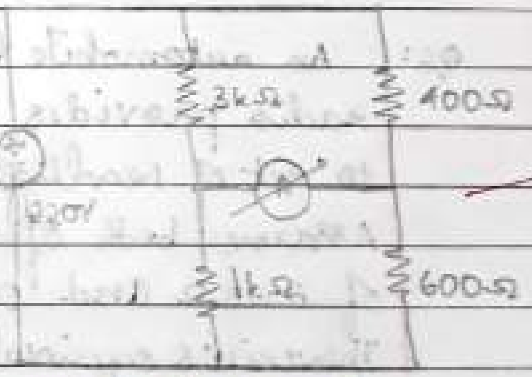
$R_{Th}=\frac{V_{Th}}{I_{sc}}=\frac{9/8}{0.346m}$

$R_{Th}=3.25k\Omega$

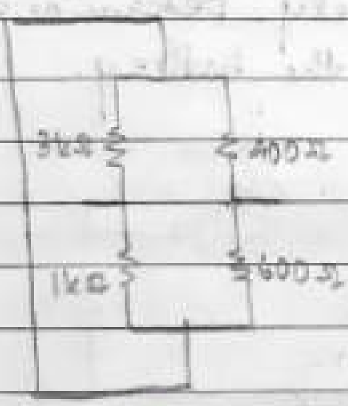
$V_o=\frac{9/8(2k)}{5.25k}$

$V_o=0.428V$

Q5: Find the current indicated by the ammeter if its resistance is 40Ω.



Sol: For R_{Th}



$R_{Th}=(400||600)+(3k||1k)$

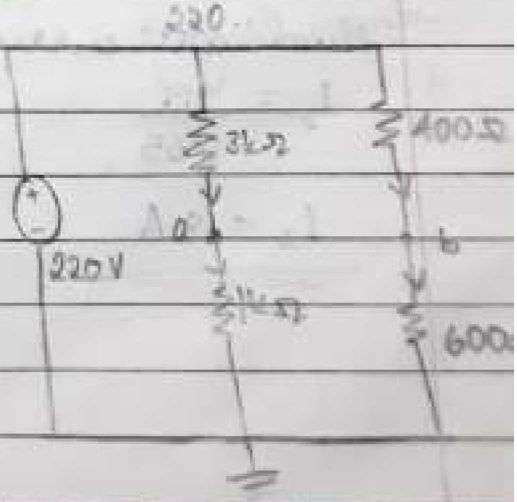
$=\frac{400\times600}{1000}+\frac{3k\times1k}{4k}$

$=240+750$

$R_{Th}=990\Omega$

At node a

$\frac{220-V_a}{3k}=\frac{V_a}{1k}$



$220-V_a=3V_a$

$4V_a=220$

$V_a=55V$

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At node b:

$$\frac{220 - V_b}{400} = \frac{V_b}{600}$$

$660 - 3V_b = 2V_b$

$5V_b = 660$

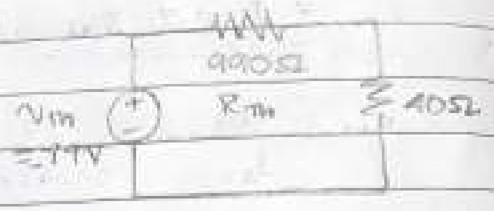
$V_b = 132V$

$V_{Th} = V_{ab}$

$V_{Th} = V_a - V_b$

$V_{Th} = 55 - 132$

$V_{Th} = -77V$



$$I = \frac{V_{Th}}{R_{Th} + R_L}$$

$$I = \frac{-77}{990 + 40}$$

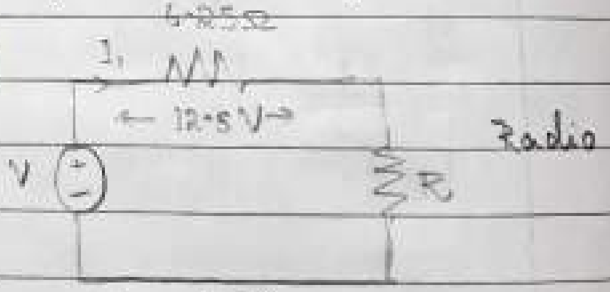
$I = 74.75mA$

Q6: An automobile battery, when connected to a car radio provides 12.5V to the radio. When connected to set of headlights, it provides 11.7V to the headlights. Assume both of them can be modelled as resistors of 6.25Ω and 0.65Ω respectively. Determine the Thevenin's equivalent circuit of the battery.

Sol: ~~Headlight light current~~

$I_1 = \frac{12.5}{6.25}$ (Radio current)

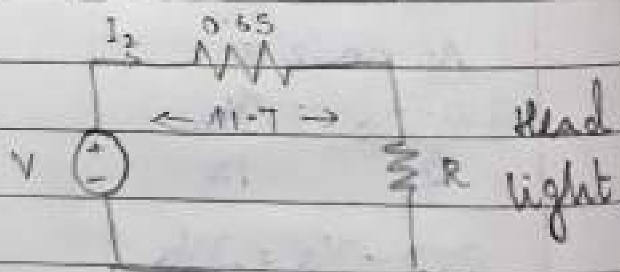
$I_1 = 2A$



Headlight current

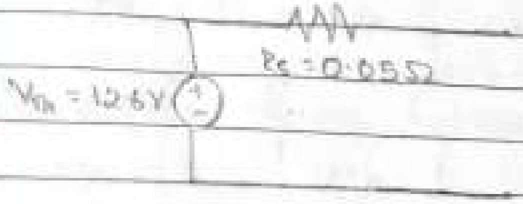
$I_2 = \frac{11.7}{0.65}$

$I_2 = 18A$



$$V - 12.5 - 2R = 0 \quad \text{--- (1)}$$
$$V - 11.7 - 18R = 0 \quad \text{--- (2)}$$

$$\underline{V = 12.6V} \qquad \underline{R = 0.05\,\Omega}$$



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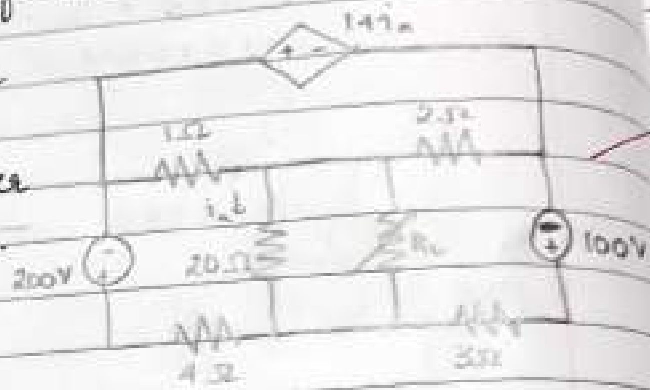
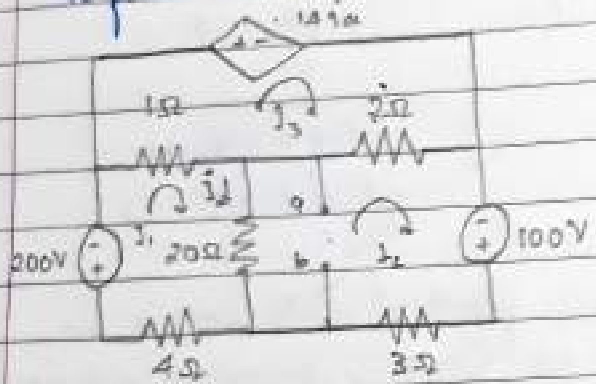
Maximum Power Transfer, Millman's Theorem:

Q1: a. Find the value of R_L for maximum power transfer.
b. Find the maximum power that can be delivered to R_L.



Sol: To find V_{Th}



Mesh 1

$200 + 1(I_1 - I_3) + 20(I_1 - I_2) + 4(I_1) = 0$

$200 + 25I_1 - 20I_2 - I_3 = 0$ — (1)

Mesh 2

$-100 + 3I_2 + 20(I_2 - I_1) + 2(I_2 - I_3) = 0$

$-100 - 20I_1 + 25I_2 - 2I_3 = 0$ — (2)

Mesh 3

$14i_a + 2(I_3 - I_2) + 1(I_3 - I_1) = 0$

$14(I_1 - I_2) + 2I_3 - 2I_2 + I_3 - I_1 = 0$

$13I_1 - 16I_2 + 3I_3 = 0$ — (3)

Constraint equation

$i_a = I_1 - I_2$

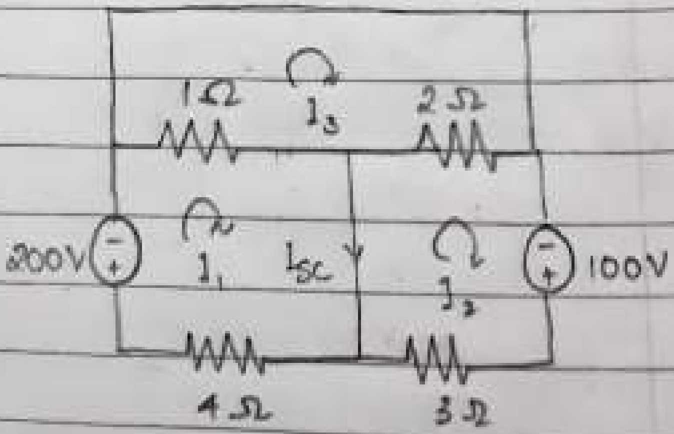
$I_1 = -2.5A \quad I_2 = 5A \quad I_3 = 37.5A$

$i_a = I_1 - I_2 = -2.5 - 5 = -7.5A$

$V_{Th} = i_a(20) \quad V_{Th} = -150V$

To find R_{Th}

Load is shorted hence no current across 20Ω resistor i.e, $i_a = 0$



Mesh 1

$200 + 1(I_1 - I_3) + 4I_1 = 0$

$200 + 5I_1 - I_3 = 0$ ——— (1)

Mesh 2

$-100 + 3I_2 + 2(I_2 - I_3) = 0$

$-100 + 5I_2 - 2I_3 = 0$ ——— (2)

Mesh 3

$2(I_3 - I_2) + 1(I_3 - I_1) = 0$

$-I_1 - 2I_2 + 3I_3 = 0$ ——— (3)

$\underline{\underline{I_1 = -40A}}$

$\underline{\underline{I_2 = 20A}}$

$\underline{\underline{I_3 = 0A}}$

$I_{SC} = I_1 - I_2$

$I_{SC} = -40 - 20$

$\underline{\underline{I_{SC} = -60A}}$

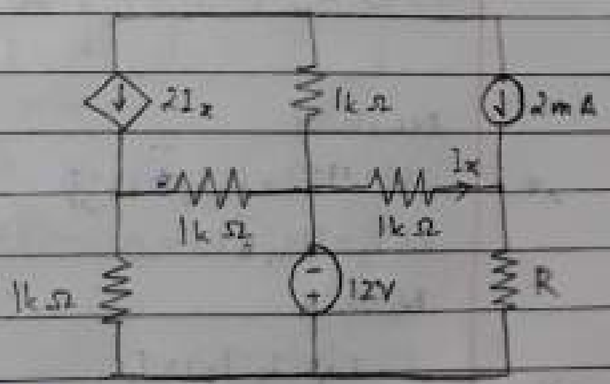
$$R_{Th} = \frac{V_{Th}}{I_{SC}} = \frac{-150}{-60} = \underline{\underline{2.5\Omega}}$$

The delivered to the load is maximum when $R_{Th} = R_L$ $\therefore$ $\underline{\underline{R_L = 2.5\Omega}}$

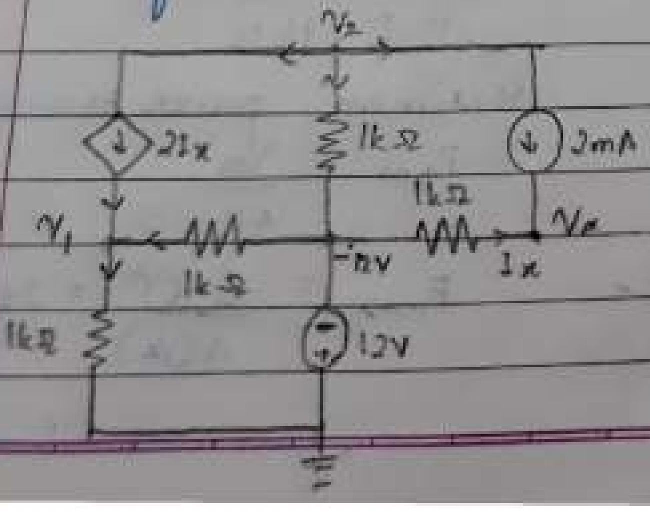
$$P_{max} = \frac{V_{Th}^2}{4R_L}$$

$$P_{max} = \frac{(-150)^2}{4(2.5)} = \underline{\underline{2.25kW}}$$

Q2: Determine the value of R to transfer maximum power. Find the maximum power delivered.



Sol: To find V_{Th}



At V_1

$$2I_x + \frac{-12 - V_1}{1k} = \frac{V_1}{1k}$$

$$\frac{2(-12 - V_a)}{\cancel{1k}} + \frac{-12 - V_1}{\cancel{1k}} = \frac{V_1}{\cancel{1k}}$$

$-36 - 2V_a - 2V_1 = 0$ ——— (1)

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At V_2

$$2I_x + \frac{V_2+12}{1k} + 2m = 0$$

$$\frac{2(-12-V_a)}{1k} + \frac{V_2+12}{1k} + 2m = 0$$

$$-10 - 2V_a + V_2 = 0 \quad \text{--- (2)}$$

At V_a

$$2m + I_x = 0$$

$$2m + \frac{-12 - V_a}{1k} = 0$$

$$2 + -12 - V_a = 0$$

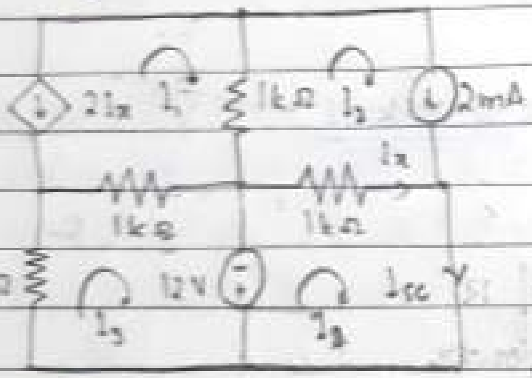
$$\underline{V_a = -10V}$$

$$V_{Th} = V_{ab} = V_a - V_b$$

$$V_{Th} = -10 - 0$$

$$\underline{V_{Th} = -10V}$$

To find R_{Th}



Mesh 1

$$1k\,I_1 + 1k(I_1 - I_3) = 0$$

$$2k\,I_1 - 1k\,I_3 = 0 \quad \text{--- (1)}$$

Mesh 2

$$1k(I_2 - I_4) + 1k(I_2 - I_1) = 0$$

$$1k(2m - I_4) + 1k(2m - I_1) = 0$$

$$4 - 1k\,I_1 - 1k\,I_4 = 0 \quad \text{--- (2)}$$

Mesh 3

$$1k(I_3 - I_1) + 1k\,I_3 - 12 = 0$$

$$2k\,I_3 - 1k\,I_1 - 12 = 0 \quad \text{--- (3)}$$

Mesh 4

$$12 + 1k(I_4 - I_2) = 0$$

$$12 + 1k\,I_4 - 2 = 0$$

$$10 + 1k\,I_4 = 0$$

$$\underline{I_{sc} = I_4 = -10m}$$

$$R_{Th} = \frac{V_{Th}}{I_{sc}} = \frac{-10}{-10m} = \underline{1k\Omega}$$

For maximum power to be transfered $R_{Th} = R_L$

$\therefore R = 1k\Omega$

Maximum power transfered

$$P_{max} = \frac{V_{Th}^2}{4R}$$

$$P_{max} = \frac{(-10)^2}{4(1k)} = \underline{25mW}$$

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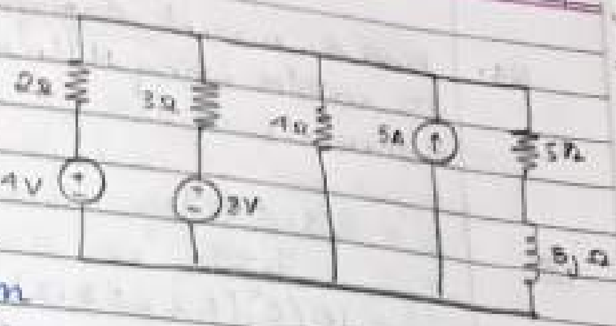
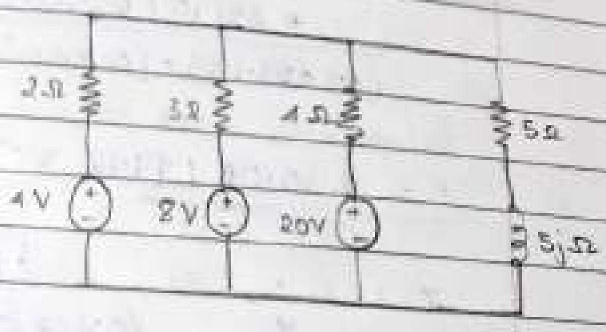
Q3: Use Millman's theorem to find the current through $(5+5j)\Omega$ impedence (Hint - Source transformation)



Sol: 4Ω parallel to $5A$ using source transformation

$V = IR_{sh}$

$V = 5(4) = 20V$

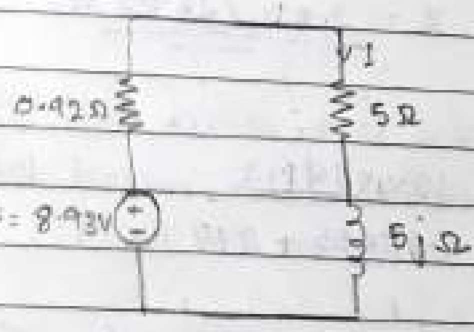


Using Millman's theorem

$$E = \frac{E_1Y_1 + E_2Y_2 + E_3Y_3}{Y_1+Y_2+Y_3}$$

$$E = \frac{4/2 + 8/3 + 20/4}{1/2 + 1/3 + 1/4}$$

$E = 8.93V$



$$Z = \frac{1}{1/2 + 1/3 + 1/4}$$

$Z = 0.92\Omega$

$$I = \frac{E}{Z+(5+5j)} = \frac{8.93}{0.92+5+5j} = \frac{8.93}{5.92+5j}$$

$I = 1.15\angle -40.18^\circ$

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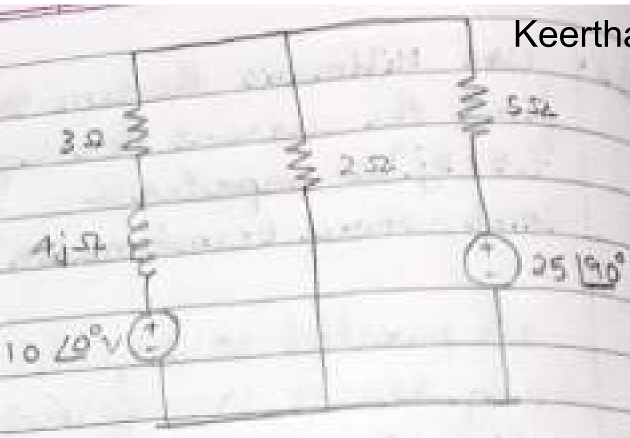
Q4: Find the current through 2Ω resistor using Millman's theorem



Sol:

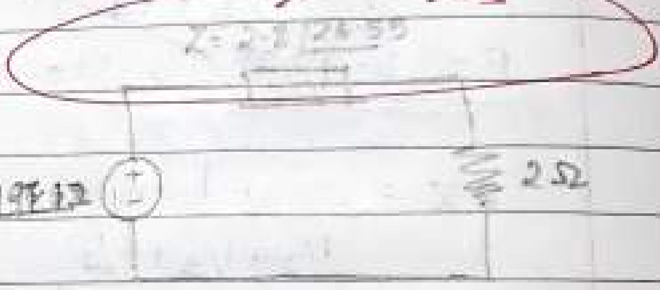
$$E = \frac{E_1 Y_1 + E_2 Y_2}{Y_1 + Y_2}$$

$$= \frac{10\angle 0°(0.2\angle 53.13) + 25\angle 90°(0.2\angle 0°)}{(0.2\angle 53.15) + (0.2\angle 0°)}$$

$$E = 10.08\angle 97.12 \text{ V}$$

$$Z = \frac{1}{Y} = \frac{1}{(0.2\angle 53.15) + (0.2\angle 0°)}$$

$$Z = 2.80\angle 26.55° \ \Omega$$



$$I = \frac{10.08\angle 97.12}{2.8\angle 26.55 + 2\angle 0}$$

$$I = 2.157\angle 81.58 \text{ A}$$

$Z = R + jX_C$

$Z = R + jX_L$

Keerthana A

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Series Resonance

Q1: A series circuit comprises a 10Ω resistance, a $5\mu F$ capacitor and a variable inductance L. The supply voltage is 20 volts at frequency of 318.3Hz. The inductance is adjusted untill the potential difference across the 10Ω resistance is maximum. Determine for the condition

a. the value of inductance L

b. the potential difference across each component.

c. Quality factor.

Sol: Given: $R = 10\Omega$ $\quad f = 318.3Hz$

$C = 5\mu F$ $\quad$ To find: L, Q

$V = 20V$

a. Inductance

If the potential difference across 10Ω resistance is maximum, then there is resonance condition.

$\therefore X_L = X_C$

$$\omega L = \frac{1}{\omega C}$$

$$L = \frac{1}{\omega^2 C} = \frac{1}{(2\pi f)^2 C}$$

$$L = \frac{1}{(2\pi(318.3))^2(5\times10^{-6})} = \underline{50mH}$$

b. potential difference across each component.

$I_{max} = V/R = 20/10 = 2A$

$V_R = I_{max} R = \underline{20V}$

$V_C = I_{max} X_C = 2(1/\omega C) = 2(1/2\pi(318.3)5\times10^{-6}) = \underline{200V}$

$V_L = I_{max} X_L = 2(\omega L) = 2(2\times\pi\times 318.3\times 50\times10^{-3}) = \underline{199.99V}$

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c. Quality factor

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{10}\sqrt{\frac{50\times10^{-3}}{5\times10^{-6}}} = \underline{10}$$

Q2: A coil of 10H and resistance of 50Ω in series with a capacitor is supplied at a constant voltage from a variable frequency source. If the maximum current of 3A occurs at 75Hz. Find the frequencies at which the current is 1A.

sol: given: $L = 10H$; $R = 50\Omega$

$I_{max} = 3A$; $f_r = 75Hz$

$$V = I_{max} R$$

$$V = 3\times50 = 150V$$

$$Z = \frac{V}{I} = \frac{150}{1} = 150\Omega$$

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

$$75 = \frac{1}{2\pi\sqrt{10\times C}}$$

$$\underline{C = 0.45\mu F}$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$Z^2 = R^2 + (X_L - X_C)^2$$

$$150^2 = 50^2 + (\omega(10) - 1/\omega(0.45\mu))^2$$

$$20\times10^3 = \left(10\omega - \frac{1}{0.45\mu\omega}\right)^2$$

$$141.42 = \frac{4.5\times10^{-6}\omega^2 - 1}{0.45\times10^{-6}\omega}$$

$$63.639\times10^{-6}\omega = 4.5\times10^{-6}\omega^2 - 1$$

$$4.5\times10^{-6}\omega^2 - 63.639\times10^{-6}\omega - 1 = 0$$

$$\omega_2 = 478.5 \text{ rad/sec}$$

$$\omega_1 = 464.3 \text{ rad/sec}$$

$$f_2 = \frac{\omega_1}{2\pi} = \frac{478.5}{2\times3.14} = \underline{76.15Hz}$$

$$f_1 = \frac{\omega_2}{2\pi} = \frac{464.3}{2\pi} = \underline{73.89Hz}$$

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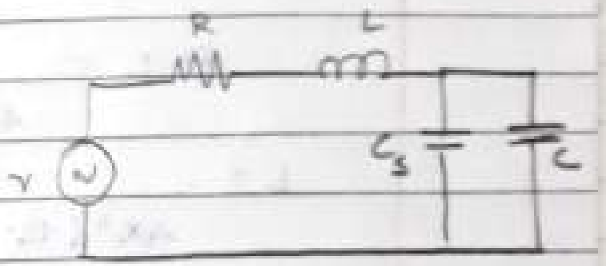
Q3: A coil having inductance L is connected in series with a variable capacitor C. The ~~current~~ circuit possesses stray capacitance C_s which is assumed to be constant and effectively in parallel with the variable capacitor C. When the capacitor is set to 1000pF the resonant frequency of the circuit is 92.5kHz, and when the capacitor is set to 500pF the resonant frequency is 127.8kHz. Determine the values of

a. the stray capacitance C_s

b. the coil inductance L.



$C_{eff} = C_s + C$

Sol: case 1:

$C = 1000pF$

$f_r = 92.5kHz$

wkt

$$f_{r_1} = \frac{1}{2\pi\sqrt{LC_{eff_1}}}$$

$C_{eff_1} = C_s + 1000pF$

$$\Rightarrow LC_{eff_1} = \frac{1}{4\pi^2 f_{r_1}^2} = \frac{1}{4\pi^2(92.5\times10^3)^2}$$

$$L(C_s + 1000\times10^{-12}) = 2.96\times10^{-12} \quad \text{--- (1)}$$

Case 2:

$C = 500pF$

$f_{r_2} = 127.8kHz$

wkt

$$f_{r_2} = \frac{1}{2\pi\sqrt{LC_{eff_2}}}$$

$C_{eff_2} = C_s + 500pF$

$$\Rightarrow LC_{eff_2} = \frac{1}{4\pi^2 f_{r_2}^2} = \frac{1}{4\pi^2(127.8\times10^3)^2}$$

$$L(C_s + 500\times10^{-12}) = 1.55\times10^{-12} \quad \text{--- (2)}$$

Dividing eq ① and eq ⑤

$$\frac{L(C_s + 1000\times10^{-12})}{L(C_s + 500\times10^{-12})} = \frac{2.96\times10^{-12}}{1.55\times10^{-12}}$$

$$1.55\,C_s + 1550\times10^{-12} = 2.96\,C_s + 1480\times10^{-12}$$

$$1.41\,C_s = 70\times10^{-12}$$

$$C_s = 49.64\,pF$$

$$f_{r_1} = \frac{1}{2\pi\sqrt{L C_{eff_1}}}$$

$$\Rightarrow L C_{eff_1} = \frac{1}{4\pi^2 f_{r_1}^2}$$

$$\Rightarrow L = \frac{1}{4\pi^2 f_{r_1}^2 C_{eff_1}}$$

$$L = \frac{1}{4\pi^2 (92.5k)^2 (49.64p + 1000p)} = 2.82\,mH$$

Q4: For a supply voltage of 3V is applied to a series RLC circuit whose resistance is 12Ω, inductance is 7.5mH and capacitance is 0.5μF. Determine:

a. the current flowing at ~~resistance~~ resonance.

b. the current flowing at a frequency 2.5% below the resonant frequency

c. the impedence of the circuit when the frequency is 1% lower than the resonant frequency.

Sol: Given: $V = 3V$ $R = 12\Omega$

$L = 7.5mH$ $C = 0.5\mu F$

a. current flowing at resonance

$$I_{max} = \frac{V}{R} = \frac{3}{12} = 0.25A$$

b. $\delta = 2.5\%$ below $= -0.025$

$Z = R[1 + jQ\delta(2-\delta)]$

$Z = 12[1 + j(10.2)(-0.025)(2+0.025)]$

$Z = 12[1 - 0.516j]$

$\therefore \underline{Z = 12 - 6.19j}$

$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$

$Q = \frac{1}{12}\sqrt{\frac{7.5\times10^{-3}}{0.5\times10^{-6}}}$

$\underline{Q = 10.2}$

$Z = 13.5\angle{-27.28^\circ}$

current $I = \frac{V}{Z} = \frac{3\angle0^\circ}{13.5\angle{-27.28^\circ}} = \underline{0.22\angle27.28^\circ}$

c. $\delta = 1\%$ below $= -0.01$

$Z = R[1 + jQ\delta(2-\delta)]$

$Z = 12[1 + j(-0.01)(10.2)(2.01)]$

$Z = 12[1 - 0.205j]$

$\therefore \underline{Z = 12 - 2.46j}$

$\underline{Z = 12.25\angle{-11.58^\circ}}$

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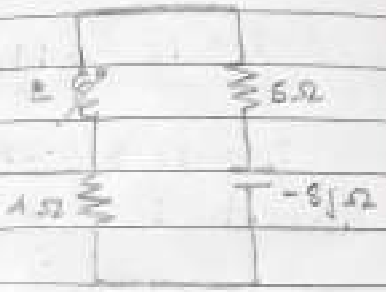
Assignment

Q5: Show that in a series RLC circuit if the frequency is varied untill the voltage across the capacitor reaches its maximum value at $\omega_c = \sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}}$

Q6: A series RLC circuit consists of $R = 100\Omega$, $L = 0.02H$ and $C = 0.02\mu F$. calculate frequency of resonance. A variable frequency sinusoidal voltage of value 50V is applied to the circuit. Find the frequency at which voltage across L and C is maximum. Also calculate voltages a/c L and C at frequency of resonance. Find I_{max}

Parallel Resonance

Q 1: Determine for the parallel network shown in figure, the values of inductance L for which the circuit is resonant at a frequency of 600Hz.



Sol: $Y_T = \frac{R_1}{R_1^2+X_L^2} + \frac{R_2}{R_2^2+X_C^2} + j\left[\frac{X_C}{R_2^2+X_C^2} - \frac{X_L}{R_1^2+X_L^2}\right]$

At resonance $\frac{X_C}{R_2^2+X_C^2} - \frac{X_L}{R_1^2+X_L^2} = 0$

$$\frac{X_C}{R_2^2+X_C^2} = \frac{X_L}{R_1^2+X_L^2}$$

$X_C = 8 \quad R_2 = 5$

$R_1 = 4$

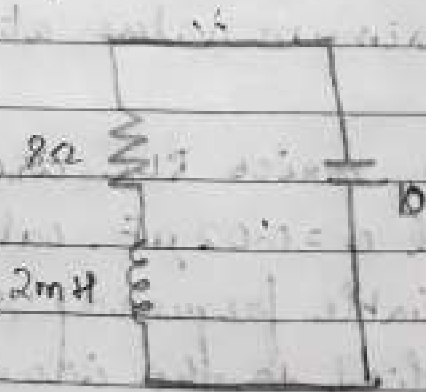
$$\frac{8}{25+64} = \frac{X_L}{16+X_L^2}$$

$$8X_L^2 - 89X_L + 128 = 0$$

$X_L = 9.42\Omega$

$$L = X_L \Rightarrow L = \frac{9.42}{2\pi F} = \frac{9.42}{(2\pi(600))} = \underline{249mH}$$

Q 2: For the parallel network shown in figure determine the resonant frequency. Also find the value of resistance to be connected in series with the 10μF capacitor to change the resonant frequency to 1kHz.



Sol: Resonant frequency

$$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{1}{(2m)(10\mu)} - \frac{8^2}{(2m)^2}}$$

$f_r = 928.02 Hz$

$X_L = \omega L = 2\pi(1k)2m$

$X_L = 12.56\Omega$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi(1k)10\mu} = 15.91\Omega$$

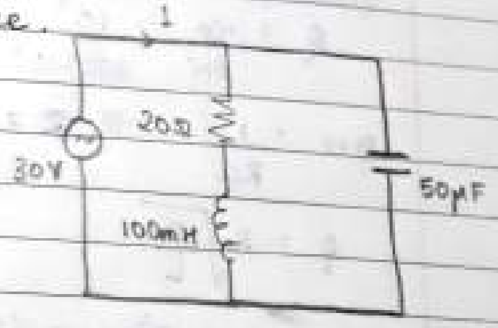
$Y_1 = \frac{R_1}{R_1^2+X_C^2} - j\frac{X_C}{R_1^2+X_C^2} \quad Y_2 = \frac{R_2}{R_2^2+X_L^2} + \frac{jX_L}{R_2^2+X_L^2}$

$Y_1 = \frac{R_1}{R_1^2+jX_C^2} \quad Y_2 = \frac{R_2}{R_2^2+jX_L^2} \quad Y_T = \frac{R_1}{R_1^2+X_C^2} + \frac{R_2}{R_2^2+X_L^2} + j\left[\frac{X_L}{R_2^2+X_L^2} - \frac{X_C}{R_1^2+X_C^2}\right]$

At resonance: $\frac{X_C}{R_2^2+X_C^2} = \frac{X_L}{R_1^2+X_C^2}$

$\frac{1979}{R_2^2+(1979)^2} = \frac{1256}{8^2+(1256)^2} \Rightarrow \underline{R_2 = 5497\Omega}$

Q3: A coil of resistance 20Ω and inductance $100mH$ is connected in parallel with a $50\mu F$ capacitor across a 30V variable frequency supply. Determine:
a. the resonant frequency of the circuit.
b. the dynamic resistance
c. the current at resonance.



Sol: a. Resonant frequency

$$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{1}{(100m)(50\mu)} - \frac{20^2}{(100m)^2}}$$

$$\underline{f_r = 63.66Hz}$$

b. Dynamic resistance

$$R_D = \frac{R^2 + X_L^2}{R} \qquad X_L = \omega L$$

$$X_L = 2\pi f L$$

$$R_D = \frac{(20)^2 + (40)^2}{20} \qquad X_L = 2\pi(63.66)(100m)$$

$$X_L = 40\Omega$$

$$\underline{R_D = 100\Omega}$$

c. current $I = \frac{V}{Z}$ where $Z = \frac{L}{CR}$

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$$Z = \frac{100m}{50\mu \times 20} = \underline{\underline{100\,\Omega}}$$

$$I = \frac{V}{Z} = \frac{30}{100} = \underline{\underline{0.3A}}$$

Q4: A high frequency parallel circuit is required to operate at $\omega_0 = 10^7$ rad/sec with a bandwidth of 200×10^3 rad/sec. Determine the required Q and L when $C = 10pF$.

Sol: Given: $\omega_0 = 10^7$ rad/sec

$BW = 200\times10^3$ rad/sec

$C = 10\times10^{-12}$ F

Quality Factor

$$Q = \frac{\omega_0}{BW} = \frac{10^7}{200\times10^3} = \underline{\underline{50}}$$

$$BW = \frac{1}{RC} \Rightarrow R = \frac{1}{C(BW)} = \frac{1}{10\times10^{-12}\times200\times10^3} = \underline{\underline{500k\,\Omega}}$$

$$Q = R\sqrt{\frac{C}{L}}$$

$$\Rightarrow L = \frac{R^2C}{Q^2} = \frac{(500\times10^3)^2(10\times10^{-12})}{(50)^2} = \underline{\underline{1mH}}$$

Q5: A parallel RLC network, which is driven by a variable frequency of 4A current source has the following values: $R = 1k\Omega$, $L = 100mH$ and $C = 100\mu F$. Find the bandwidth of the network, the half power frequencies and the voltage across the network at half power frequencies.

Sol: Given: $R = 1k\Omega$ $L = 100mH$ $C = 100\mu F$

$I = 4A$

Bandwidth

$$BW = \frac{1}{RC} = \frac{1}{10^3\times100\times10^{-6}} = \underline{10 \text{ rad/sec}}$$

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$$\omega_1 = \frac{-1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

$$= \frac{-1}{2(10^3)(100\mu)} + \sqrt{\left(\frac{1}{2(10^3)(100\mu)}\right)^2 + \frac{1}{10m(100\mu)}}$$

$\omega_1 = 995.01$ rad/sec

$$f_1 = \frac{\omega_1}{2\pi} = \frac{995.01}{2\pi} = \underline{158.36\,Hz}$$

$$\omega_2 = \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

$$= \frac{+1}{2(10^3)(100\mu)} + \sqrt{\left(\frac{1}{2(10^3)(100\mu)}\right)^2 + \frac{1}{10m(100\mu)}}$$

$\omega_2 = 1005.01$ rad/sec

$$f_2 = \frac{\omega_2}{2\pi} = \frac{1005.01}{2\pi} = \underline{159.95\,Hz}$$

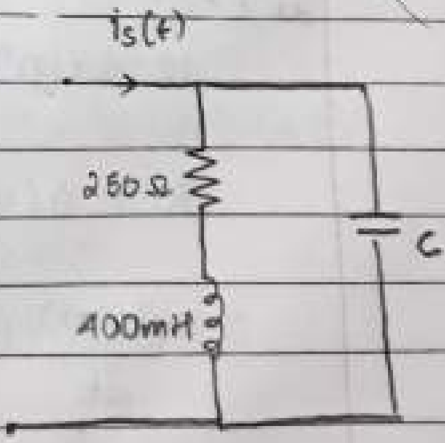
$V = I Z_{max}$

$V = IR$

$V = 4(10^3) = \underline{4000\,V}$

Assignment

Q6. In the circuit given if $i_s(t) = 0.225 + 0.409\cos(2\pi 10^3 t) + 0.36\cos(4\pi 10^3 t) + 0.119\cos(6\pi 10^3 t)$. Determine the value of C so that circuit resonates at 1kHz. Obtain an expression for the output voltage and compare the amplitude at 2kHz and 3kHz with respect to 1kHz component



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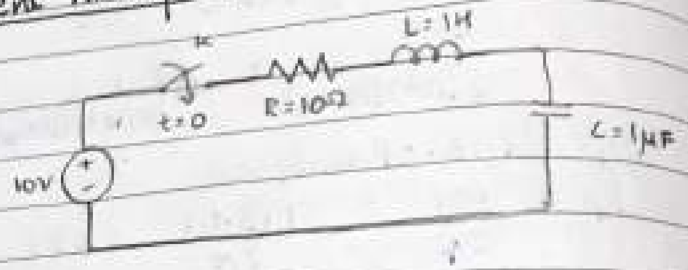
Transient Analysis

Q1: Find:
a. i b. $\frac{di}{dt}$ c. $\frac{d^2i}{dt^2}$

at $t=0^+$



Sol: at $t=0^-$ switch is open
at $t=0$ switch is closed
at $t=0^+$ switch is closed

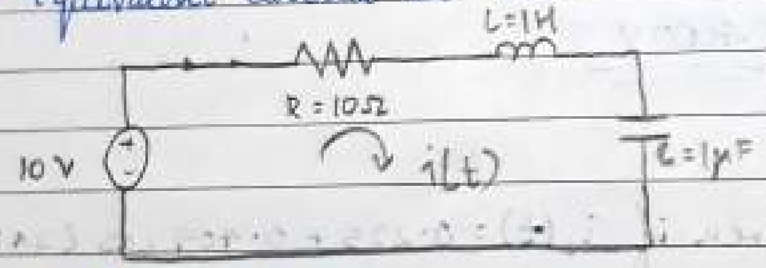
To find: $i(0^+)$; $\frac{di(0^+)}{dt}$; $\frac{d^2i(0^+)}{dt^2}$

Equi: At $t=0^-$ switch is open

$i(0^-)=0$ $V_c(0^-)=0$

$\therefore i(0^-)=i(0^+)=0$ $\therefore V_c(0^-)=V_c(0^+)=0$

Equivalent circuit at $t=0^+$



Applying KVL

$$10 = i(t)R + L\frac{di(t)}{dt} + \int_{0^+}^{t}\frac{1}{C}\, i(\tau)\,d\tau$$

$$10 = i(t)R + L\frac{di(t)}{dt} + V_c(t) \quad \text{--- (1)}$$

at $t=0^+$

$$10 = i(0^+)R + L\frac{di(0^+)}{dt} + V_c(0^+)$$

$$10 = 0(10) + 1\,\frac{di(0^+)}{dt} + 0$$

$$\frac{di(0^+)}{dt} = 10 \text{ A/sec}$$

diff eq ① wrt t

$$0 = \frac{di(t)}{dt}R + L\frac{d^2i(t)}{dt^2} + \frac{i(t)}{C}$$

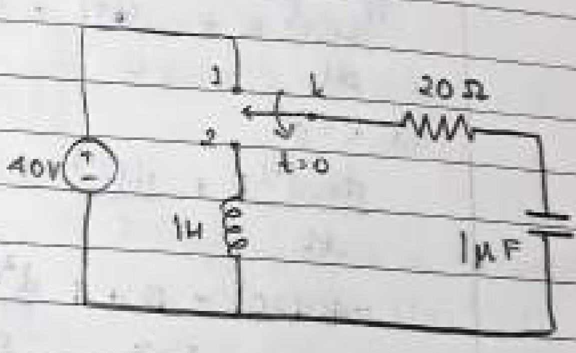
at $t = 0^+$

$$0 = \frac{di(0^+)}{dt}R + L\frac{d^2i(0^+)}{dt^2} + \frac{i(0^+)}{C}$$

$$0 = 10(10) + 1\frac{d^2i(0^+)}{dt^2} + 0$$

$$\frac{d^2i(0^+)}{dt^2} = -100\ A/sec^2$$

Q2: In the network shown, the switch is moved from position 1 to position 2 at $t=0$. The steady state having reached before switching. calculate i, $\frac{di}{dt}$, $\frac{d^2i}{dt^2}$ at $t = 0^+$



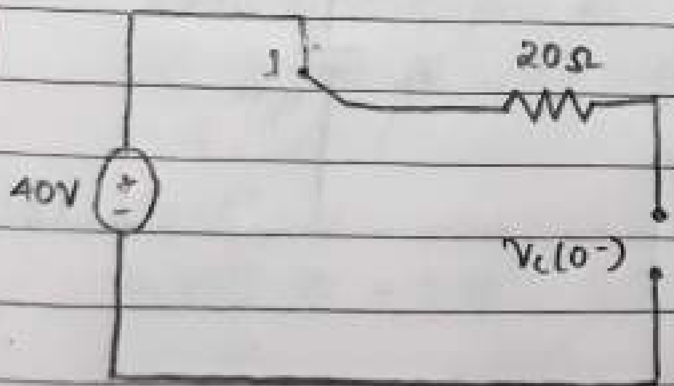
Sol: at $t = 0^-$ position 1 (steady state)

at $t = 0$ position 2

at $t = 0^+$ position 2

To find: $i(0^+)$; $\frac{di(0^+)}{dt}$; $\frac{d^2i(0^+)}{dt^2}$

Equivalent circuit at $t = 0^-$



$V_c(0^-) = 40V$

$\therefore V_c(0^-) = V_c(0^+) = 40V$

$i(0^-) = 0$

$\therefore i(0^-) = i(0^+) = 0$

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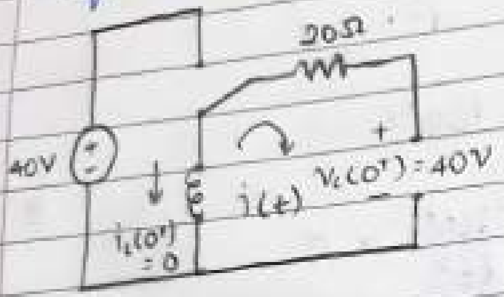
Sol:

Equivalent circuit at $t=0^+$



Applying KVL

$$i(t)R + V_c(t) + L\frac{di(t)}{dt} = 0$$

at $t=0^+$

$$i(0^+)R + V_c(0^+) + L\frac{di(0^+)}{dt} = 0$$

$$0(20) + 40 + 1\frac{di(0^+)}{dt} = 0$$

$$\therefore \frac{di(0^+)}{dt} = -40\ A/sec$$

diff eq ① wrt t.

$$\frac{di(t)}{dt}R + \frac{i(t)}{C} + L\frac{d^2i(t)}{dt^2} = 0$$

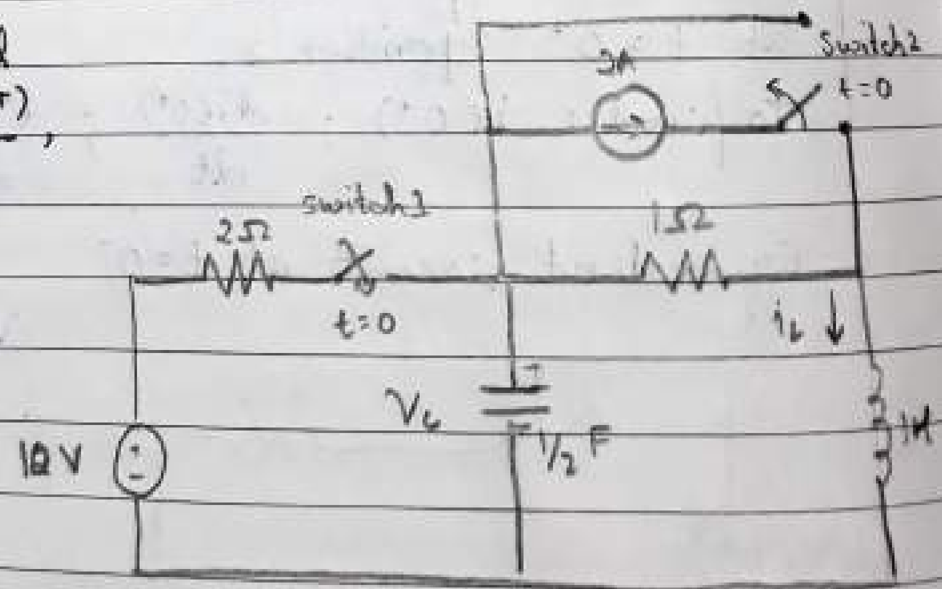
at $t=0^+$

$$\frac{di(0^+)}{dt}R + \frac{i(0^+)}{C} + L\frac{d^2i(0^+)}{dt^2} = 0$$

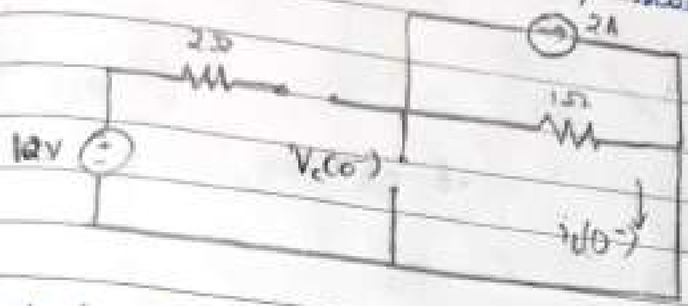
$$-40(20) + 0 + 1\frac{d^2i(0^+)}{dt^2} = 0$$

$$\frac{d^2i(0^+)}{dt^2} = 800\ A/sec^2$$

Q3: For the circuit find $i_L(0^+)$, $V_c(0^+)$, $\frac{di(0^+)}{dt}$, $\frac{dV_c(0^+)}{dt}$.



sol: at $t=0^-$ switch 1 open ; switch 2 closed



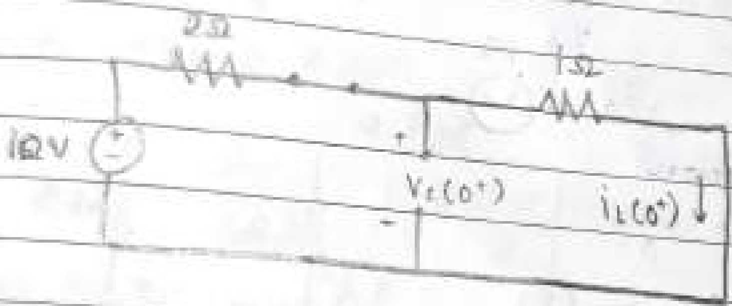
$\because i_L(0^-) = 0$

$\therefore i_L(0^-) = i_L(0^+) = 0$

$V_c(0^-) = -2(1) = -2V$

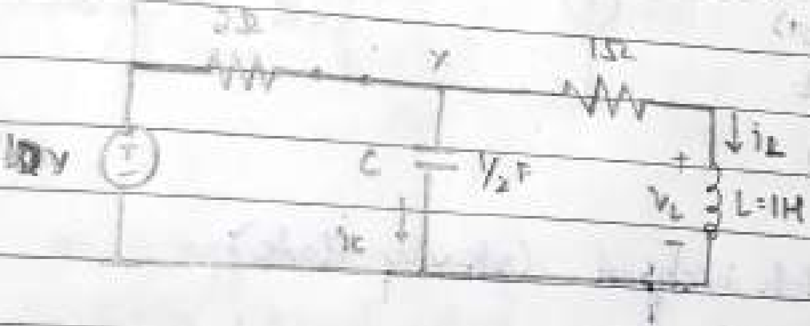
$\therefore V_c(0^-) = V_c(0^+) = -2V$

at $t=0^+$ switch 1 closed ; switch 2 open



$i_L(0^+) = 0$

$V_c(0^+) = -2V$



Applying KVL to loop 2

$V_c(t) - V_L(t) + 1(i_L) = 0$

at $t=0^+$

$V_c(0^+) - V_L(0^+) + i_L(0^+) = 0$

$-2 - V_L(0^+) + 0 = 0$

$\underline{V_L(0^+) = -2V}$

$V_L(0^+) = L\dfrac{di(0^+)}{dt}$

$\therefore \dfrac{di(0^+)}{dt} = \dfrac{V_L(0^+)}{L} = \dfrac{-2}{1} = -2\ A/sec$

At node x applying KCL

$\dfrac{V_c(t) - 10}{2} + i_c(t) + i_L(t) = 0$

at $t = 0^+$

$\dfrac{V_c(0^+) - 10}{2} + i_c(0^+) + i_L(0^+) = 0$

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$$\frac{-2-10}{2} + i_c(0^+) + 0 = 0$$

$$i_c(0^+) = 6A$$

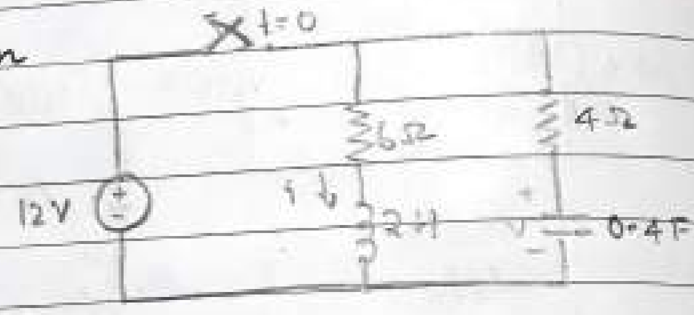
$$i_c(t) = C\,\frac{dv(t)}{dt}$$

at $t = 0^+$

$$i_c(0^+) = C\,\frac{dv(0^+)}{dt}$$

$$\frac{dv(0^+)}{dt} = \frac{i_c(0^+)}{C} = \frac{6}{1/2} = 12\,V/sec$$

Q4: For the circuit shown find



a. $i(0^+)$, $v(0^+)$

b. $\frac{di(0^+)}{dt}$, $\frac{dv(0^+)}{dt}$

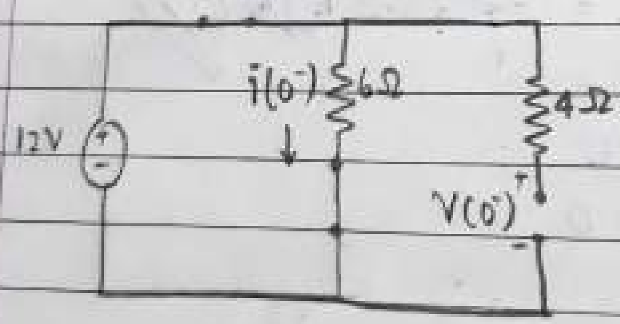
c. $i(\infty)$, $v(\infty)$

Sol: at $t = 0^-$ switch is closed (steady state)

at $t = 0$ switch is open

at $t = 0^+$ switch is open

a. Equivalent circuit at $t = 0^-$



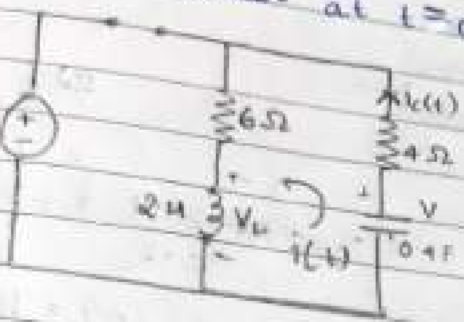
$$i(0^-) = \frac{12}{6} = 2A$$

$$\therefore i(0^-) = i(0^+) = 2A$$

$$V(0^-) = 12V$$

$$\therefore V(0^-) = V(0^+) = 12V$$

b. Equivalent circuit at $t=0^+$



Applying KVL :

$V_L(t) - V(t) + 4i_c(t) + 6i_c(t) = 0$

$V_L(t) - V(t) + 10i_c(t) = 0$

at $t = 0^+$

$V_L(0^+) - V(0^+) + 10i_c(0^+) = 0$

$V_L(0^+) - 12 + 10(2) = 0$

$V_L(0^+) = -8V$

$V_L(t) = L\dfrac{di(t)}{dt}$

at $t = 0^+$

$V_L(0^+) = 2\dfrac{di(0^+)}{dt}$ $\quad\therefore \dfrac{di(0^+)}{dt} = \dfrac{V_L(0^+)}{2} = \dfrac{-8}{2} = -4\,A/sec$

$i_c(t) = C\dfrac{dV(t)}{dt}$ ~~$i_c\ dV(0^+)\ dt$~~

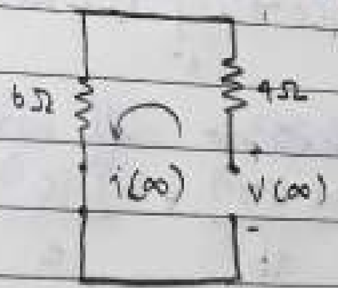
at $t = 0^+$

$i_c(0^+) = 0.4\dfrac{dV(0^+)}{dt}$ $\quad\therefore \dfrac{dV(0^+)}{dt} = \dfrac{i_c(0^+)}{0.4} = \dfrac{2}{0.4} = 5V/sec$

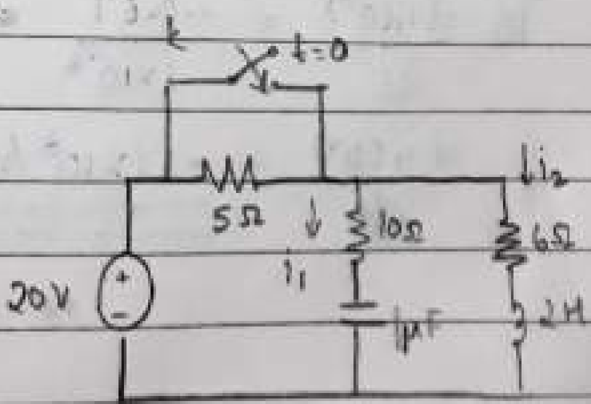
c. as t approaches infinity, the switch is open and the circuit attains steady state.

$i(\infty) = 0$

$V(\infty) = 0$

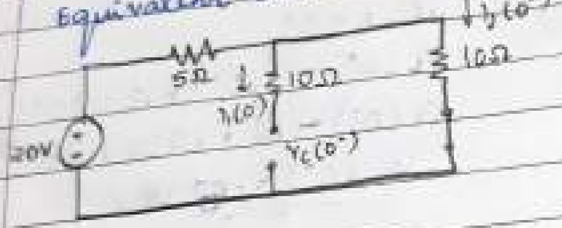


Q5: For the circuit shown, determine, $i_1(0^+)$, $i_2(0^+)$, $\dfrac{di_1}{dt}(0^+)$, $\dfrac{di_2}{dt}(0^+)$



sol: at $t = 0^-$ switch is open
at $t = 0$ switch is closed
at $t = 0^+$ switch is closed

Equivalent circuit at $t = 0^-$



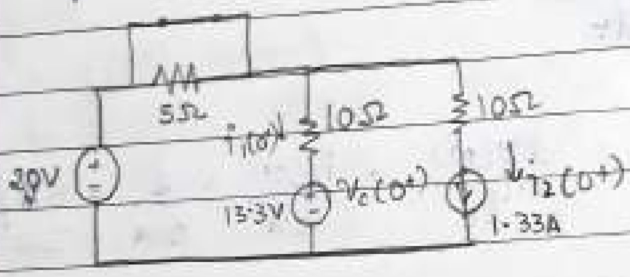
$$i_2(0^-) = \frac{20}{10+5} = 1.33A$$

$$v_c(0^-) = 10 i_2(0^-) = 13.3V$$

$\therefore i_2(0^+) = i_2(0^-) = 1.33A$

$v_c(0^-) = v_c(0^+) = 13.3V$

Equivalent circuit at $t = 0^+$



$$i_1(0^+) = \frac{20 - 13.3}{10} = 0.67A$$

Applying KVL

$$10 i_1(0^+) + \frac{1}{C}\int_{0^+}^{t} i_1(\tau)\,d\tau = 20$$

~~$10 i_1(0^+)$~~ diff wrt t

$$10\frac{di_1(0^+)}{dt} + \frac{i_1(0^+)}{C} = 0$$

$$10\frac{di_1(0^+)}{dt} = \frac{-0.67}{1\times10^{-6}}$$

$$\frac{di_1(0^+)}{dt} = -67\times10^{3}\ A/sec$$

Applying KVL

$$10 i_2(0^+) + 2 \frac{di_2(0^+)}{dt} = 20$$

$$10(1.33) + 2 \frac{di_2(0^+)}{dt} = 20$$

$$\frac{di_2(0^+)}{dt} = 3.35 \, A/sec$$

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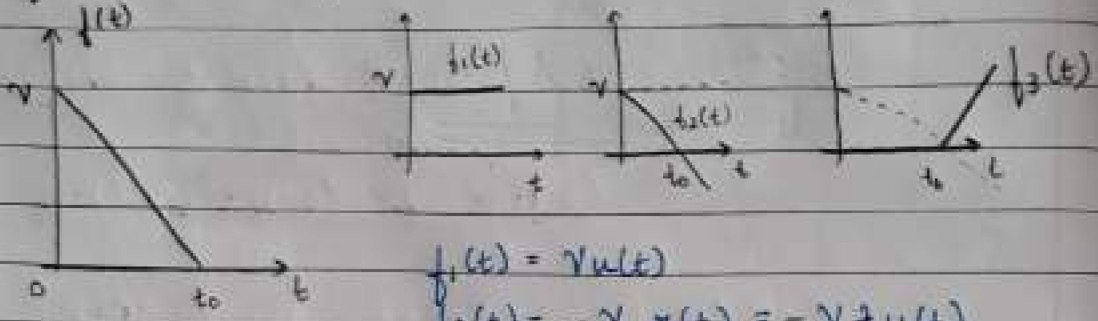
Laplace Transformation and Applications:

1. Synthesize the waveforms shown using standard signals and compute their laplace transforms.

a.



$f_1(t) = Vu(t)$

$f_2(t) = -\frac{V}{t_0} r(t) = -\frac{V}{t_0} t\,u(t)$

$f_3(t) = \frac{V}{t_0} r(t-t_0)$

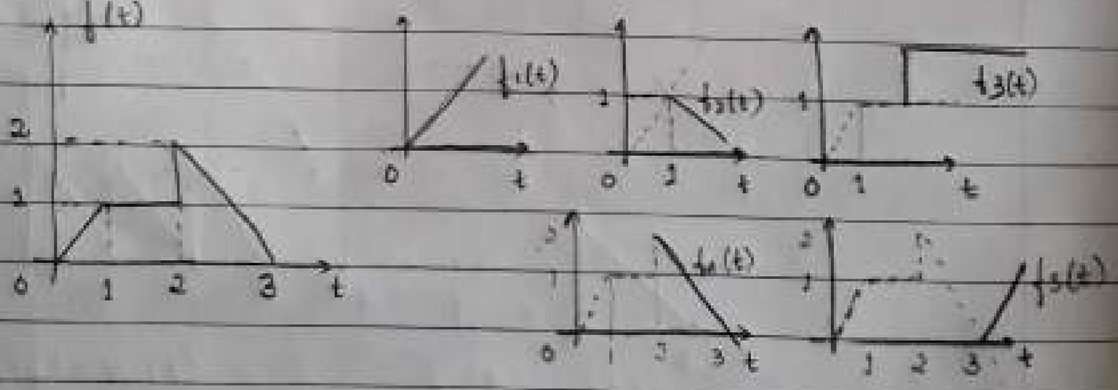
$\therefore f(t) = f_1(t) + f_2(t) + f_3(t)$

$f(t) = Vu(t) - \frac{V}{t_0} r(t) + \frac{V}{t_0} r(t-t_0)$

$L[f(t)] = V L[u(t)] - \frac{V}{t_0} L[r(t)] + \frac{V}{t_0} L[r(t-t_0)]$

$L[f(t)] = \frac{V}{S} - \frac{V}{t_0 s^2} + \frac{Ve^{-t_0 s}}{t_0 s^2}$ //

b.



$f_1(t) = r(t) = t\,u(t)$

$f_2(t) = -r(t-1) = -(t-1)u(t-1)$

$f_3(t) = u(t-2)$

$f_4(t) = -2r(t-2) = -2(t-2)u(t-2)$

$f_5(t) = 2r(t-3) = 2(t-3)u(t-3)$

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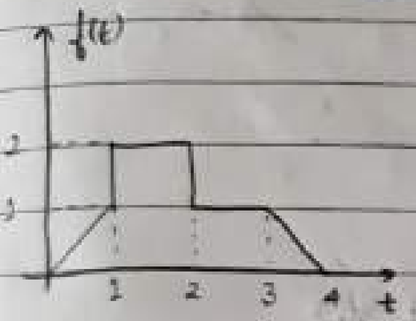
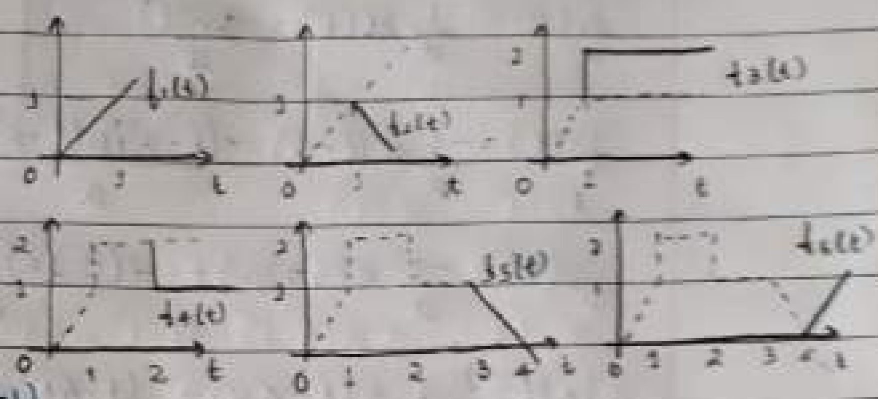
$$f(t) = f_1(t) + f_2(t) + f_3(t) + f_4(t) + f_5(t)$$

$$f(t) = tu(t) - (t-1)u(t-1) + u(t-2) - 2(t-2)u(t-2) + 2(t-3)u(t-3)$$

$$L[f(t)] = \frac{1}{s^2} - \frac{e^{-s}}{s^2} + \frac{e^{-2s}}{s} - \frac{2e^{-2s}}{s^2} + \frac{2e^{-3s}}{s^2}$$

c.





$f_1(t) = r(t) = tu(t)$

$f_2(t) = -r(t-1) = -(t-1)u(t-1)$

$f_3(t) = u(t-1)$

$f_4(t) = -u(t-2)$

$f_5(t) = -r(t-3) = -(t-3)u(t-3)$

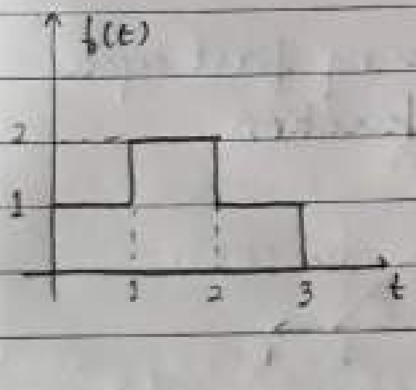
$f_6(t) = r(t-4) = (t-4)u(t-4)$

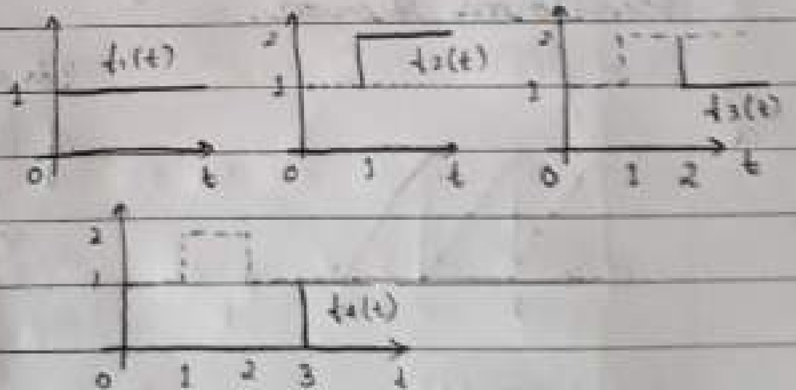
$$\therefore f(t) = f_1(t) + f_2(t) + f_3(t) + f_4(t) + f_5(t) + f_6(t)$$

$$f(t) = tu(t) - (t-1)u(t-1) + u(t-1) - u(t-2) - (t-3)u(t-3) + (t-4)u(t-4)$$

$$L[f(t)] = \frac{1}{s^2} - \frac{e^{-s}}{s^2} + \frac{e^{-s}}{s} - \frac{e^{-2s}}{s} - \frac{e^{-3s}}{s^2} + \frac{e^{-4s}}{s^2}$$

d.





$f_1(t) = u(t)$

$f_2(t) = u(t-1)$

$f_3(t) = -u(t-2)$

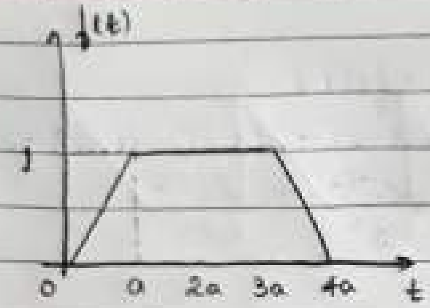
$f_4(t) = -u(t-3)$

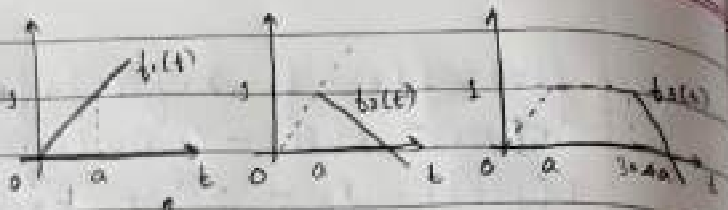
$$f(t) = f_1(t) + f_2(t) + f_3(t) + f_4(t)$$

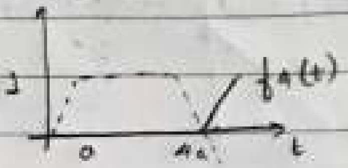
$$f(t) = u(t) + u(t-1) - u(t-2) - u(t-3)$$

$$L[f(t)] = \frac{1}{s} + \frac{e^{-s}}{s} - \frac{e^{-2s}}{s} - \frac{e^{-3s}}{s}$$

e.







$$f_1(t) = \frac{1}{a} r(t) = \frac{t u(t)}{a}$$

$$f_2(t) = -\frac{1}{a} r(t-a) = -\frac{(t-a)u(t-a)}{a}$$

$$f_3(t) = \frac{-1}{a} r(t-3a) = -\frac{(t-3a)u(t-3a)}{a}$$

$$f_4(t) = \frac{1}{a} r(t-4a) = \frac{(t-4a)u(t-4a)}{a}$$

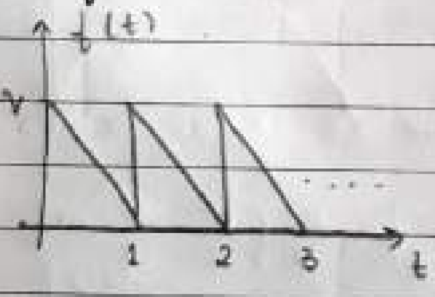
$$f(t) = f_1(t) + f_2(t) + f_3(t) + f_4(t)$$

$$f(t) = \frac{t u(t)}{a} - \frac{(t-a)u(t-a)}{a} - \frac{(t-3a)u(t-3a)}{a} + \frac{(t-4a)u(t-4a)}{a}$$

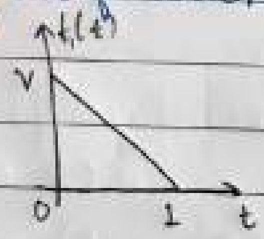
$$L[f(t)] = \frac{1}{as^2} - \frac{e^{-as}}{as^2} - \frac{e^{-3as}}{as^2} + \frac{e^{-4as}}{as^2}$$

2. Find the laplace transform of the periodic function waveforms shown.

a.



considering first cycle of the function.

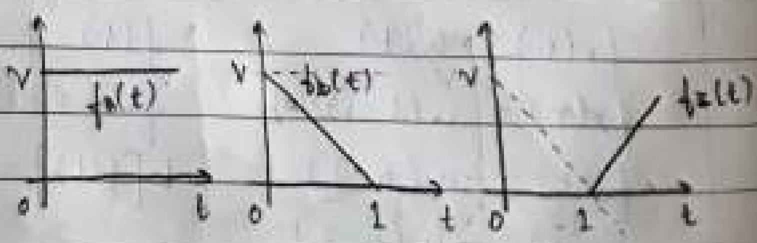


$$f_a(t) = V u(t)$$

$$f_b(t) = -V r(t) = -V t u(t)$$

$$f_c(t) = V r(t-1)$$

$$= V(t-1)u(t-1)$$



1

$$f_1(t) = f_a(t) + f_b(t) + f_c(t)$$

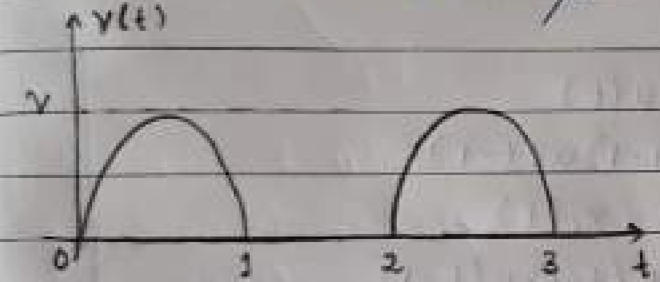
$$f_1(t) = Vu(t) - Vtu(t) + V(t-1)u(t-1)$$

$$F_1(s) = \frac{V}{s} - \frac{V}{s^2} + \frac{Ve^{-s}}{s^2}$$

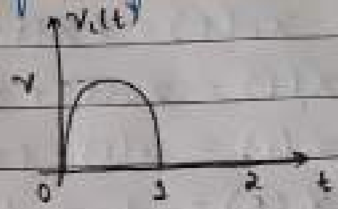
$$F(s) = \frac{F_1(s)}{1-e^{-Ts}} \quad \text{Here } T=1$$

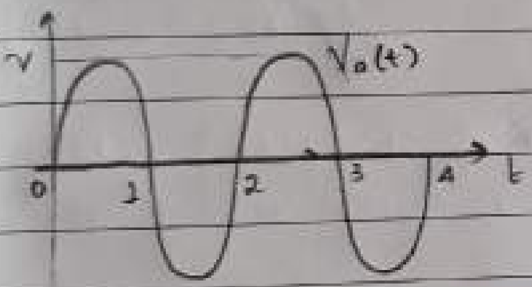
$$\therefore F(s) = \frac{Vs - V + Ve^{-s}}{s^2(1-e^{-s})}$$

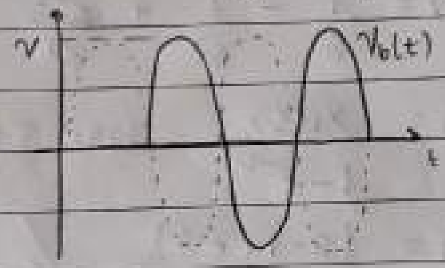
b.



Considering first cycle of the function







$$V_a(t) = V\sin\omega t = V\sin\frac{2\pi}{2}t = V\sin\pi t\, u(t)$$

$$V_b(t) = V\sin\omega(t-\pi) = V\sin\frac{2\pi}{2}(t-\pi) = V\sin\pi(t-\pi)u(t-\pi)$$

$$V_1(t) = V_a(t) + V_b(t)$$

$$V_1(t) = V\sin\pi t\, u(t) + V\sin\pi(t-\pi)u(t-\pi)$$

$$V_1(s) = \frac{V\pi}{s^2+\pi^2} + \frac{V\pi e^{-\pi s}}{s^2+\pi^2}$$

$$\therefore V(s) = \frac{V_1(s)}{1-e^{-Ts}}$$

$$V_s = \frac{V\pi(1+e^{-\pi s})}{(s^2+\pi^2)(1-e^{-2s})}$$

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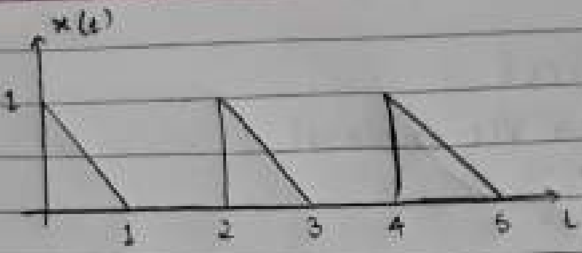
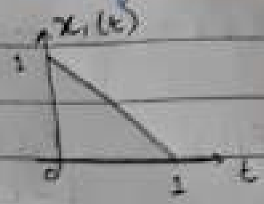
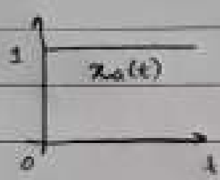
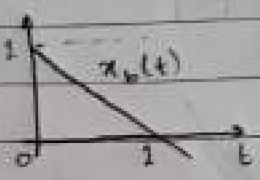
c.

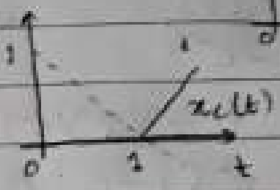


considering first cycle of the function









$x_a(t) = u(t)$

$x_b(t) = -r(t) = -t\,u(t)$

$x_c(t) = +r(t-1) = (t-1)\,u(t-1)$

$x_1(t) = x_a(t) + x_b(t) + x_c(t)$

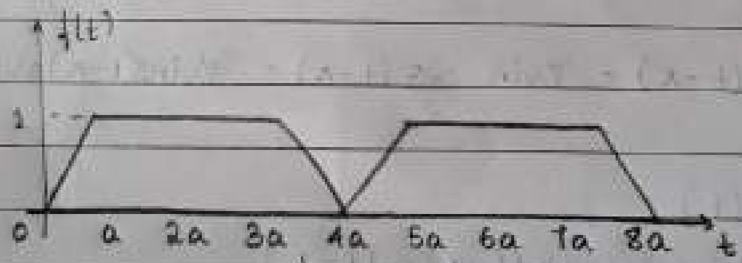
$x_1(t) = u(t) - t\,u(t) + (t-1)\,u(t-1)$

$$X_1(s) = \frac{1}{s} - \frac{1}{s^2} + \frac{e^{-s}}{s^2}$$

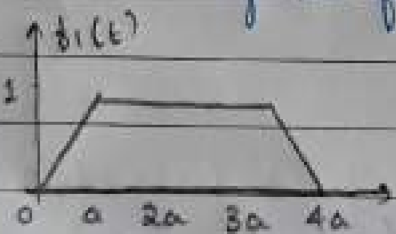
$$X(s) = \frac{X_1(s)}{1-e^{-Ts}} \quad \text{here } T = 2$$

$$X(s) = \frac{(s-1+e^{-s})}{s^2(1-e^{-2s})}$$

d.



Considering the first cycle of the function.



From Q1.e

$$F_1(s) = \frac{(1-e^{-as}-e^{-3as}+e^{-4as})}{as^2}$$

$$F(s) = \frac{F_1(s)}{1-e^{-Ts}} \quad \text{Here } T = 4a$$

$$\therefore F(s) = \frac{1}{as^2}\,\frac{(1-e^{-as}-e^{-3as}+e^{-4as})}{(1-e^{-4as})}$$

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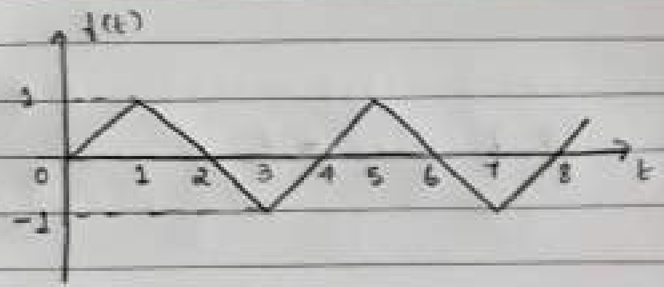
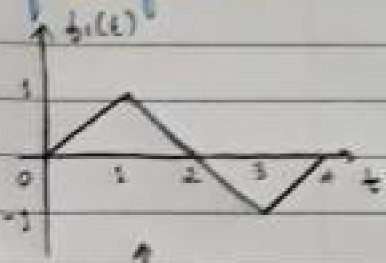
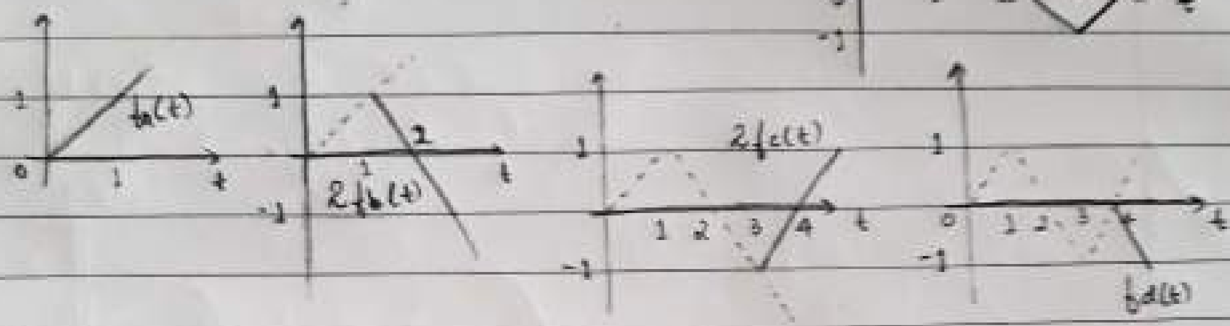
e.



considering first cycle of the function





$f_a(t) = r(t) = t\,u(t)$

$f_b(t) = -r(t-1) = -(t-1)u(t-1)$

$f_c(t) = r(t-3) = (t-3)u(t-3)$

$f_d(t) = -r(t-4) = -(t-4)u(t-4)$

$\therefore f_1(t) = f_a(t) + 2f_b(t) + 2f_c(t) + f_d(t)$

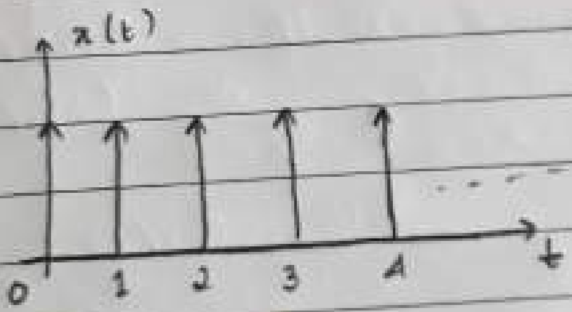
$f_1(t) = t\,u(t) - 2(t-1)u(t-1) + 2(t-3)u(t-3) - (t-4)u(t-4)$

$$F_1(s) = \frac{1}{s^2} - \frac{2e^{-s}}{s^2} + \frac{2e^{-3s}}{s^2} - \frac{e^{-4s}}{s^2}$$

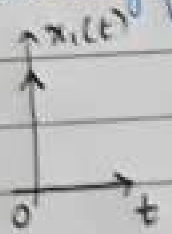
$$F(s) = \frac{F_1(s)}{1-e^{-Ts}} \quad \text{here } T = 4$$

$$\therefore F(s) = \frac{(1-2e^{-s}+2e^{-3s}-e^{-4s})}{s^2(1-e^{-4s})}$$

f.



considering first cycle



$x_1(t) = \delta(t)$

$L[\delta(t)] = 1$

$$\therefore X_1(s) = 1 \Rightarrow X(s) = \frac{X_1(s)}{1-e^{-Ts}} = \frac{1}{1-e^{-s}}$$