UNIT - 1

Measurement Errors

★ **Performance characteristics of instrument:**

1. Accuracy - closeness of measured value to standard value.
2. Resolution - smallest quantity that can be measured by the instrument.
3. Sensitivity - smallest change that can be detected.
4. Precision - repeatitive measurement to get accurate measurement.

★ **ERRORS:**

Deviation from the expected value is called an error.

$$\text{Measured Value} - \text{Actual Value} = \text{Error}$$

Tolerance or marginal error is the small amount that is allowed for in case of miscalculation

- 1. **Absolute error:**

$$E_{absolute} = |\text{Measured Value} - \text{True value}|$$

- 2. **Relative error:**

$$\text{Relative error} = \frac{E_{absolute}}{\text{Actual value}} = \frac{\text{Measured} - \text{Actual}}{\text{Actual value}}$$

$$\text{Relative accuracy} = 1 - \text{Relative Error}.$$

NOTE: Accuracy can be equal to precision but precision is independent of accuracy.

★ Static Errors:

1. Gross Error – Caused due to experiment carelessness or due to the instrumental error.
2. Systematic Error: Caused due to instrument ~~which is~~ which is incorrectly caliberated or used incorrectly.
   - → Instrumental Error
   - → Observed Error
   - → Environmental Error

Q1: The expected value of the voltage across a resistor is 80V, However the measurement gives a value of 79V, calculate: i) absolute error

ii) Percentage error

iii) Relative accuracy

iv) % of accuracy

Sol: Given: actual value = 80 V

measured value = 79 V

i) absolute error

e = actual value − measured value

$e = 80 - 79$

$e = 1\,V$

ii) Percentage error

$$\%\ \text{error} = \frac{\text{actual value} - \text{measured value}}{\text{actual value}} \times 100$$

$$= \frac{80-79}{80} \times 100$$

$$\%\ \text{error} = 1.25\ \%$$

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iii) Relative accuracy

$$= 1 - \left| \frac{\text{Actual value} - \text{Measured value}}{\text{Actual value}} \right|$$

$$= 1 - \left| \frac{80 - 79}{80} \right|$$

$$= 0.9875$$

iv) % of accuracy = 98.75 %

Q2: The table below gives the set of 10 measurements that were recorded in the laboratory. calculate the presision of 6^{th} measurement.

Measurement Number	Measurement Value 'x_n'
1	98
2	95
3	100
4	99
5	96
6	100
7	97
8	96
9	95
10	94

Sol:

$$\overline{x_n} = \frac{98+95+100+99+96+100+97+96+95+94}{10}$$

$$\overline{x_n} = 97$$

$$\text{Precision} = 1 - \left| \frac{x_n - \overline{x_n}}{x_n} \right| = 1 - \left| \frac{100-97}{100} \right|$$

$$P_6 = 1 - 0.03 = 0.97$$

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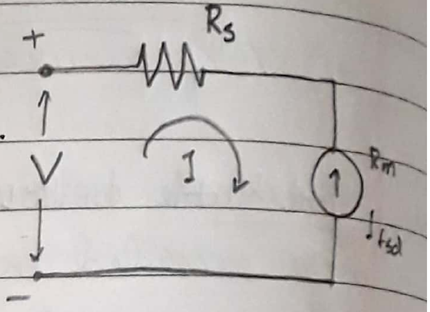
* calibration :
  - To achieve accuracy the instrument must be caliber

* VOLTMETER AND AMMETERS :

⇒ DC voltmeters

$$V = I(R_s + R_m)$$



V - voltage(input) to the maximum fsd.

$R_t = R_s + R_m$

where, R_t = total circuit resistance
R_s - Multiplier resistance
R_m - Meter resistance

fsd (full scale deflec

- sensitivity depends on the range of the instrument that i the full scale deflection.
- DC voltmeter requires a multiplier resistance in series to avoid the excessive current being drawn by the instrument which can damage the instrument.
- Range of the instrument depends on the input voltage.
- To measure multi range of voltages the circuit should be designed with various values of the multiplier resistance
- Sensitivity is given by reciprocal of full scale deflection

$$\text{sensitivity} = \frac{1}{I_{fsd}} \quad (\Omega/V)$$

- Multiplier resistance is given by

$$R_s = \frac{V}{I} - R_m \quad \text{(For fixed voltage range)}$$

Q1: A basic D-Arsonval movement with a full scale deflection of 50μA and internal resistance 500Ω is used as voltmeter. Determine the value of multiplier resistance needed to measure a voltage range of 0-10V.

Sol: Given: $R_m = 500\Omega$ $I = 50\mu A$ $V = 10V$

$$R_S = \frac{V}{I} - R_m$$

$$R_S = \frac{10}{50\times10^{-6}} - 500$$

$$R_S = 200000 - 500$$

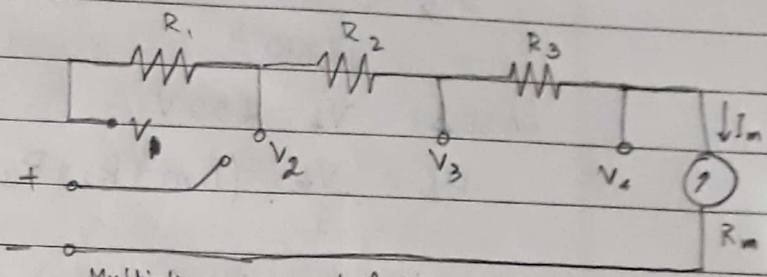
$$\underline{R_S = 199.5k\Omega}$$

Q2 ⇒ **Multirange Voltmeter:**

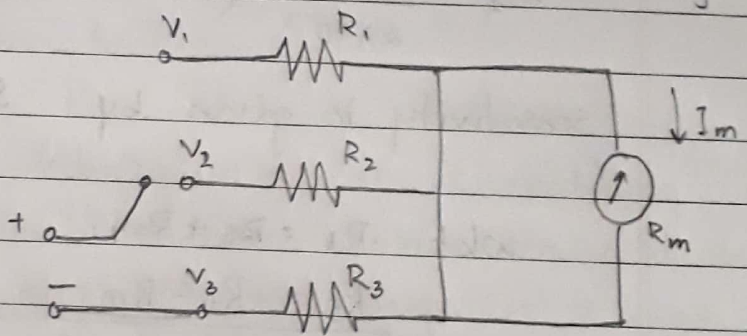
To design a multirange voltmeter ~~we~~ will require various multiplier resistances.

Multiplier resistances connected in series string are usually used.

It is used to measure different set of voltage range.



Multipliers connected in series string



Q1: Convert a basic D-Arsonval movement with an internal resistance of 50Ω and a full scale defection current of 2mA into a multirange DC voltmeter with voltage ranges of 0-10V, 0-50V, ~~and~~ 0-100V, 0-250V.

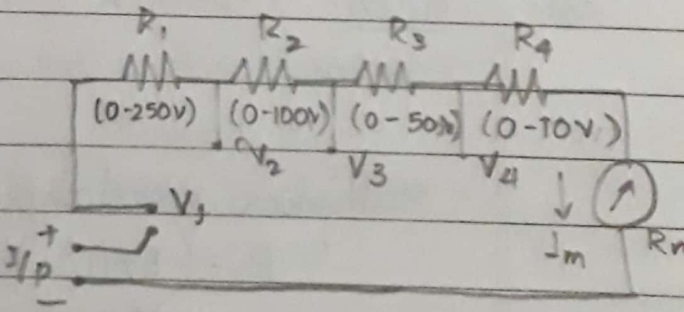
Sol: Given: $R_m = 50\Omega$ $I = 2mA$

CASE 4: $V_4 = 10V$

$$R_4 = \frac{V_4}{I} - R_m$$

$$R_4 = \frac{10}{2\times10^{-3}} - 50$$

$$\underline{R_1 = 4.95k\Omega} \approx \underline{5k\Omega}$$



Multirange DC voltmeter

CASE 3: $V_3 = 50\,V$

$$R_3 = \frac{V_3}{I} - (R_m + R_4)$$

$$R_3 = \frac{50}{2\times10^{-3}} - (50+4950) \Rightarrow R_3 = \cancel{24.95k\Omega}\ 20\,k\Omega$$

CASE 2: $V_2 = 100\,V$

$$R_2 = \frac{V_2}{I} - (R_m + R_3 + R_4)$$

$$R_2 = \frac{100}{2\times10^{-3}} - (25000) \Rightarrow R_2 = 25k\Omega$$

CASE 1: $V_1 = 250\,V$

$$R_1 = \frac{V_1}{I} - (R_m + R_2 + R_3 + R_4)$$

$$R_1 = \frac{250}{2\times10^{-3}} - (50000) \Rightarrow R_1 = 75k\Omega$$

sensitivity is given by $S = \frac{1}{I_{FSD}}$ (Ω/V)

wkt $R_t = R_s + R_m$ — S - sensitivity

$R_s = R_t - R_m$ — V- Maximum voltage range

$$\boxed{R_s = (S\times V) - R_m}$$

Q1: Calculate the value of the multiplier resistance for the 50V range of DC voltmeter that uses 200 μA Ammeter with an interal resistance of 100Ω.

Sol: Given: $I = 200\mu A$ $V = 50V$ $R_m = 100\Omega$

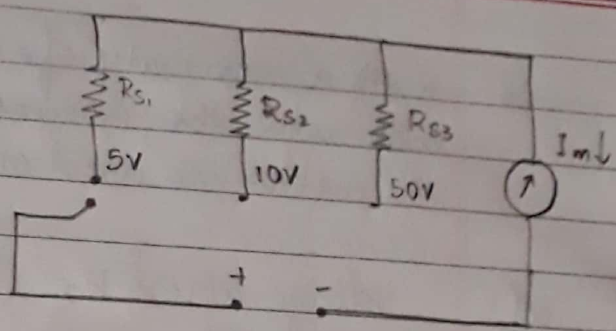
sensitivity $S = \frac{1}{I_{FSD}} = \frac{1}{200\times10^{-6}} = 5\times10^{3}\,\Omega/m$

Multiplier resistance $R_s = (S\times V) - R_m$

$R_s = (5\times10^{3}\times50) - 100$

$R_s = 249.9\,k\Omega \simeq 250k\Omega$

Q2: Calculate the value of multiplier resistance for the multirange DC voltmeter circuit shown given that
$I_{FSD} = 50\mu A$ and $R_m = 1k\Omega$



Sol: Sensitivity: $S = \frac{1}{I_{FSD}} = \frac{1}{50\mu} = \underline{20k\Omega/V}$

$R_{S_1} = SV_1 - R_m$

$R_{S_1} = 20k \times 5 - 1k = \underline{99k\Omega} \approx 100k\Omega$

$R_{S_2} = SV_2 - R_m$

$R_{S_2} = 20k \times 10 - 1k = \underline{199k\Omega} \approx 200k\Omega$

$R_{S_3} = SV_3 - R_m$

$R_{S_3} = 20k \times 50 - 1k = \underline{999k\Omega} \approx 1M\Omega$

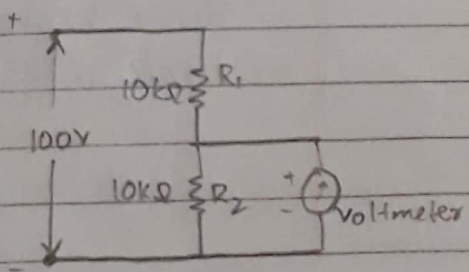
- Loading Effect:

A low sensitivity meter may give a correct reading when measuring voltages in a low resistance circuits, but it is certain to produce unreliable readings in a high resistance circuit.

A voltmeter when connected across two points in a highly resistive circuits acts as a shunt for that portion of the circuit, reducing the total equivalent resistance. This leads to lower reading that what existed.

Mainly occured in low sensitivity instruments.

Q1: A series circuit of R_1 and R_2 is connected to a 100V source. If the voltage across R_2 is to be measured by voltmeters having



a) a sensitivity of 1000 Ω/V

b) a sensitivity of 20000 Ω/V, for which voltmeter will read the accurate value of voltage across R_2. Both the meters are used on the 50V range.

sol: voltage across R_2

$$\frac{10k}{10k+10k} \times 100 = \underline{50V} \quad \text{(actual Voltage)}$$

CASE 1: $S = 1000\Omega/V$

Resistance $= 1000 \times 50 = \underline{50k\Omega}$ for 50V range (meter resist

connecting meter across R_2

$$R_{eq} = \frac{10k \times 50k}{10k + 50k} = \frac{500M}{60k} = 8.33k\Omega$$

$$\text{Voltage} = \frac{R_{eq}}{R_1 + R_{eq}} \times V$$

$$= \frac{8.33k}{10k + 8.33k} \times 100 = \underline{45.43V}$$

CASE 2: $S = 20000\Omega/V$

Resistance $= 20000 \times 50 = \underline{1M\Omega}$ for 50V range (meter resistan

connecting meter across R_2

$$R_{eq} = \frac{1M \times 10k}{1M + 10k} = \frac{10k}{1.01} = 9.9k\Omega$$

$$\text{Voltage} = \frac{R_{eq}}{R_1 + R_{eq}} \times V$$

$$= \frac{9.9k}{10k + 9.9k} \times 100 = \underline{49.74V}$$

Hence higher the sensitivity of the voltmeter more accurate is the readings.

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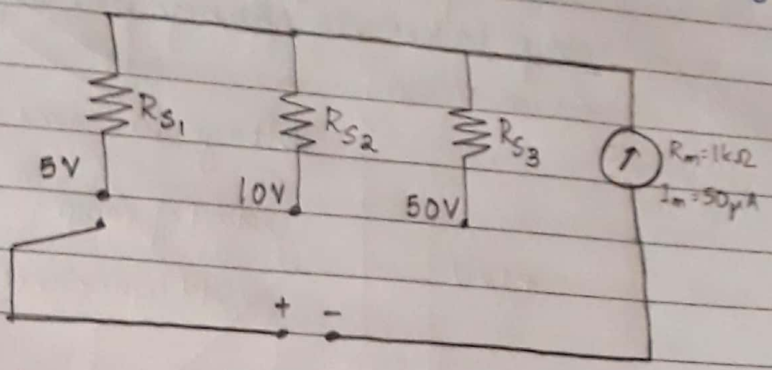
repeated

Q2: Calculate the value of multiplier resistance for the multirange DC voltmeter shown below.

Sol: Given: $R_m = 1k\Omega$

$I_m = 50\mu A$



sensitivity

$$S = \frac{1}{I_m} = \frac{1}{50\mu}$$

$$\underline{S = 20k\Omega/V}$$

$R_{S_1} = SV_1 - R_m = 20k \times 5 - 1k = 99k\Omega$ //

$R_{S_2} = SV_2 - R_m = 20k \times 10 - 1k = 199k\Omega$ //

$R_{S_3} = SV_3 - R_m = 20k \times 50 - 1k = 999k\Omega$ //

⇒ **Digital Voltmeters:**

Merits: Accuracy, flexibility, larger range, more speed, resolution and sensitivity is good.

- Digital ramp voltmeter is counting pulse / time, this is then converted into voltage.

So the time is converted into voltage to get to know the unknown input voltage.

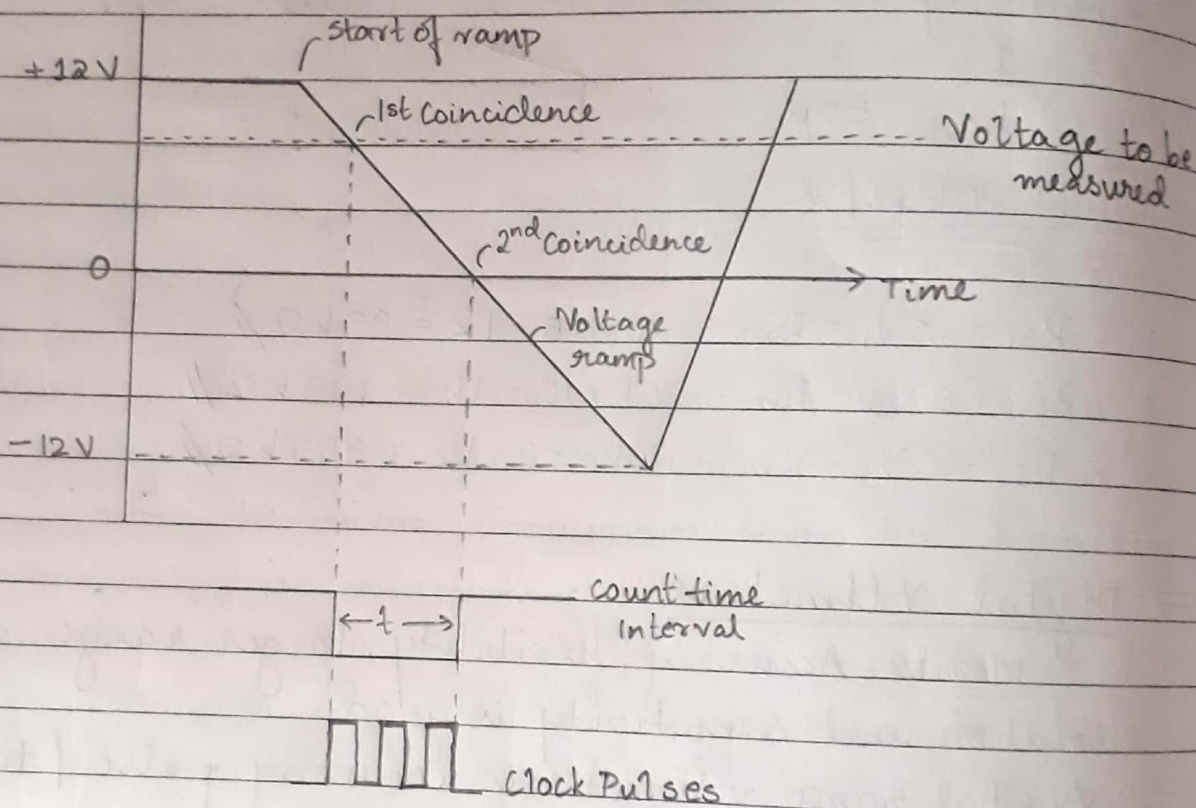
- It reduces human error, eliminates parallax error and increases the reading speed.

- Characteristics:
  - Input range from 1.000V to 1000V
  - Absolute accuracy (±0.005%)
  - Resolution (1μV measured in 1V range)
  - Input resistance typically 10MΩ.
  - calibration internally done, independent of measuring circuit
  - Output in BCD form, for print output further digital processing

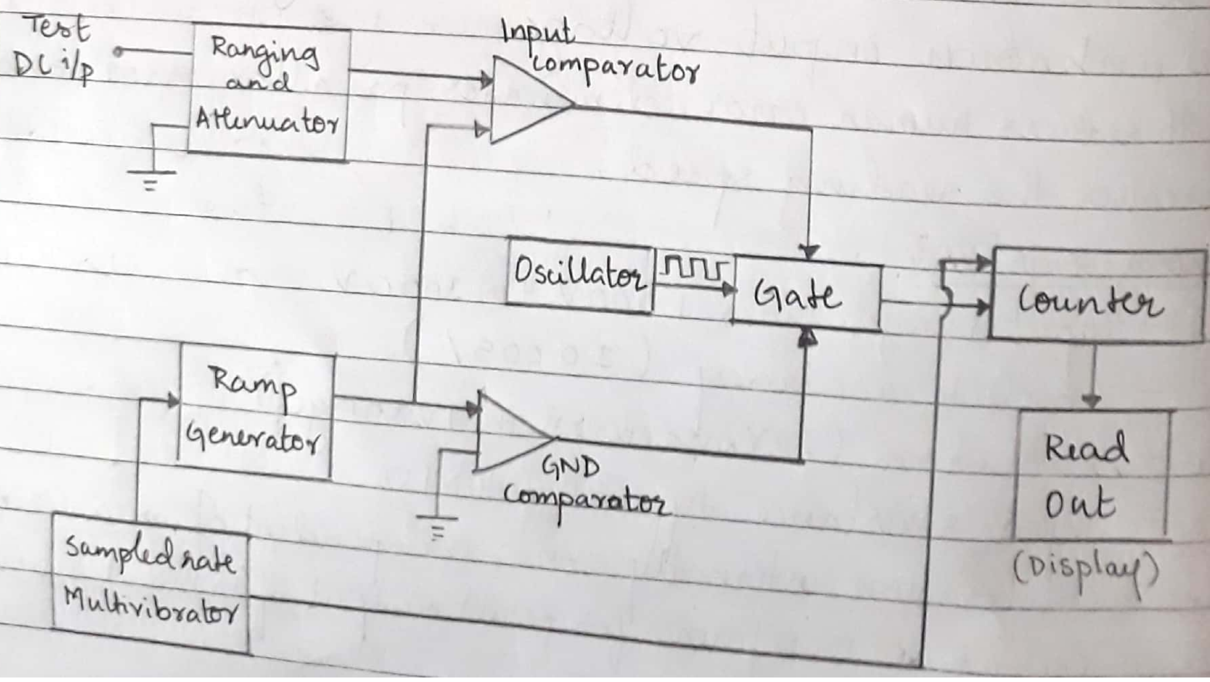
1. **Ramp Technique:**

The operating principle is to measure the time that a ramp takes to change the input level to the ground level or vice versa.

Voltage to Time Conversion



Block Diagram of ramp type DVM



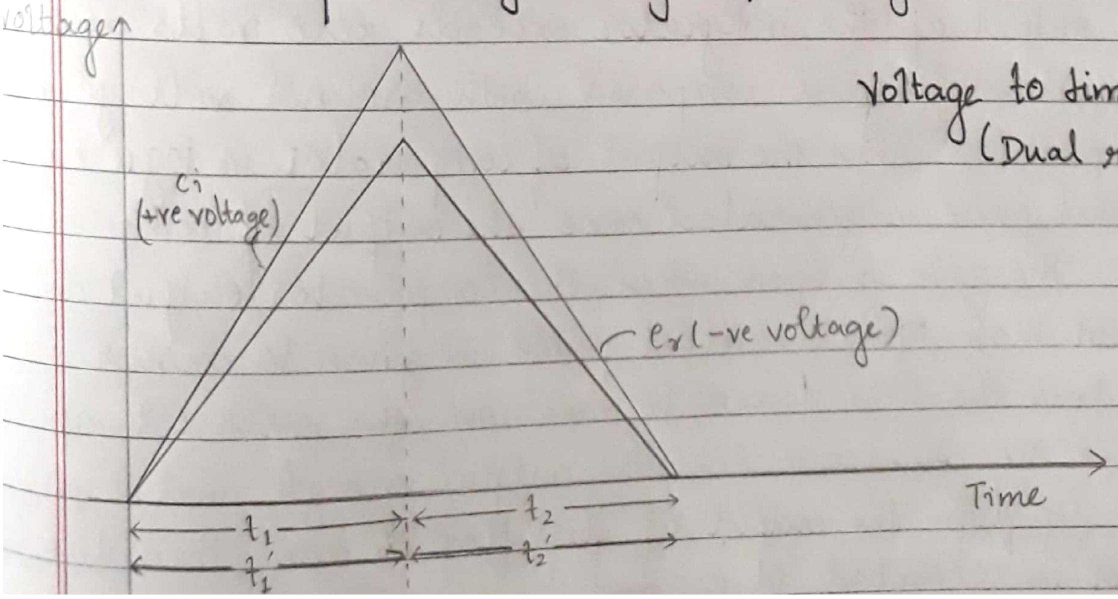
Here a negative ramp is selected.
First the counter is set to 0 and sampled rate multivibrator gives a pulse which initiates the ramp generator generating a ramp voltage. The ramp voltage is continously compared with the voltage that is being measured. Once the two voltages becomes equal the input comparator generates a start pulse which opens the gate. Now it is compared with ground. Once the ramp voltage reaches to zero the ground comparator generates stop pulse closing the gate. The time duration of the gate opening is proportional to the input voltage value.

In the time interval between the start and stop pulses the gate opens and the oscillator drives the counter. The magnitude of the count indicates the magnitude of input voltage, which is displayed by the read out.

Therefore the voltage is converted into time and the time count represents the magnitude of the voltage.

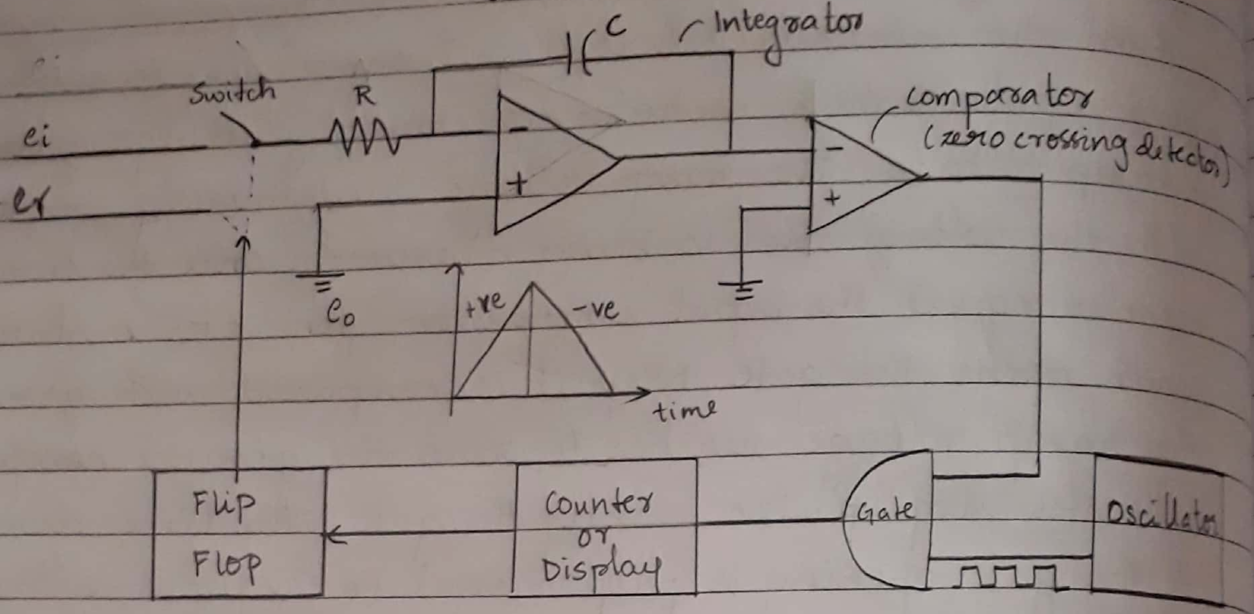
The sample rate circuit provides an initiating pulse for the ramp generator to start its next ramp voltage and a reset pulse is generated which resets to counter to zero.

2. <u>Dual Slope Integrating Type Digital Voltmeter:</u>



Voltage to time conversion.
(Dual ramp DVM)

Block Diagram of Dual Slope DVM



Integrator is a circuit which integrates to get the average of the input which is positive and negative ramp. ~~Here t_1 is the time period of positive ramp and t_2 is the time period of negative ramp~~. It minimises the error.

Positive Ramp → Initially the counter is reset, then the output is given to flip flop. Now the output of flip flop is zero, then the flip flop is connected to e_i and capacitor starts charging.

Now e_i is the input to the integrator, then

$$e_0 = \frac{-1}{RC}\int_0^{t_1} e_i\,dt \quad \text{during charging}$$

The output of the integrator exceeds zero volts and increases, this voltage is compared with ground voltage by the comparator. Then the output of comparator is logic high. Now start pulse is generated once the output of comparator is high. The gate is open when the comparator output and oscillator high. Now the start pulse is given to counter.

Here t_1 is the time taken to measure the positive ramp.

Negative Ramp → Now as the counter is set, the output is high and is given to the flip flop. The output of flip flop is one, then the flip flop is connected to e_r.

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Page

Now e_r is the input to the integrator, then

$$e_o = \frac{1}{RC}\int_0^{t_2} -e_r\,dt$$

The output of the integrator decreases due to discharging of the capacitor. The output is compared with ground voltage by the comparator, since the voltage is less than zero, the output of comparator is logic low (0).

Now the gate is closed as the input is logic low. Now no pulse is given to the counter.

Here t_2 is the time taken to measure negative ramp.

when e_i is the input of the integrator

$$e_o = \frac{-1}{RC}\int_0^{t_1} e_i\,dt \quad \ldots\ldots (1) \quad +\text{ve ramp voltage } (e_i)$$

when e_r is the input of the integrator

$$e_o = \frac{1}{RC}\int_0^{t_2} -e_r\,dt \quad \ldots\ldots (2) \quad -\text{ve ramp voltage } (-e_r)$$

Subtracting eq (2) from eq (1)

$$e_o - e_o = \frac{-1}{RC}\int_0^{t_1} e_i\,dt + \frac{1}{RC}\int_0^{t_2} e_r\,dt$$

$$0 = -e_i t_1 + e_r t_2$$

$$e_i t_1 = e_r t_2$$

$$\therefore \boxed{e_i = e_r \frac{t_2}{t_1}}$$

Oscillator period T and n_1 and n_2 represent number of pulse counted

$$e_i = \frac{n_2 T}{n_1 T} e_r \Rightarrow \boxed{e_i = \frac{n_2}{n_1} e_r} \Rightarrow e_i = K_1 n_2 \quad \left(k_1 = \frac{e_r}{n_1}\right)$$

Q1: An integrator contains a 100kΩ and 1μF capacitor. If the voltage applied to the integrator is 1V, what voltage will be present at the output of integrator after one second. Calculate the Time constant of integrator.

Sol: Given: $R = 100\,k\Omega$ $\quad C = 1\,\mu F$

$t_1 = 1\,sec$ $\quad e_i = 1\,V$

$$e_o = \frac{+1}{RC}\int_0^{t_1} e_i\,dt \quad \text{or} \quad e_o = \frac{e_i t}{RC}$$

$$e_o = \frac{+1}{100\times10^3\times10^{-6}}\int_0^1 dt \qquad e_o = \frac{1\times1}{100\times10^3\times10^{-6}}$$

$$e_o = \frac{+1}{10^5\times10^{-6}} \qquad e_o = 10\,V$$

$$e_o = 10\,V$$

Now if a reference voltage is applied to integrator of the above example at t_1 is 5V. What is the time interval t_2?

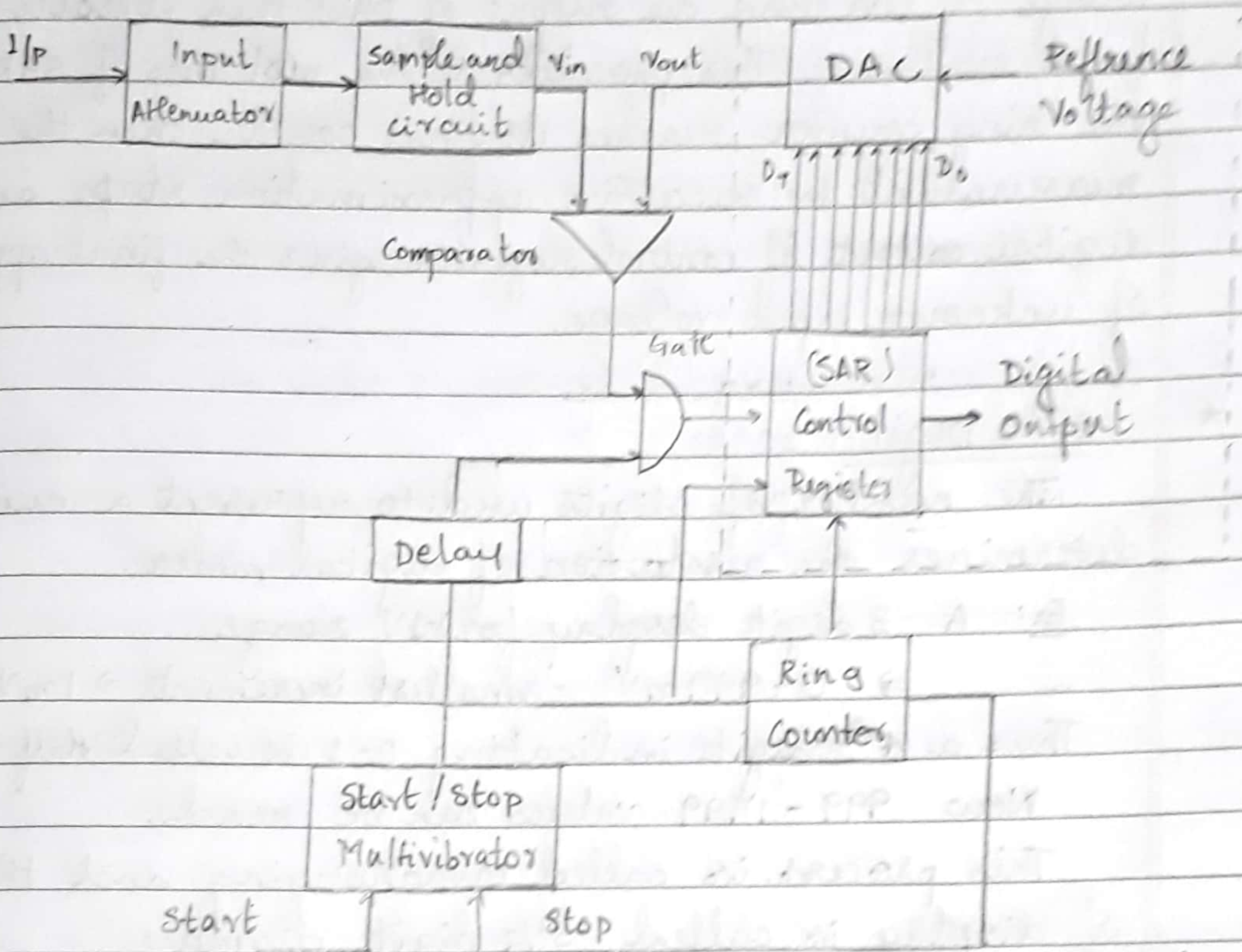
$$\frac{e_i t_1}{RC} = \frac{e_r t_2}{RC}$$

$$1(1) = 5(t_2)$$

$$t_2 = 0.2\,sec$$

* ANALOG TO DIGITAL CONVERSION

Successive Approximation



The input is given to the attenuator followed by sample and hold circuit whose output is given as one of the input to the comparator. First a start pulse is produced by the multivibrator this sets the MSB of control register to 1 (i.e. 10000000), this causes the output of DAC to be 1/2 reference voltage (1/2 V) which is another input to the comparator.

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If the input is greater than the 1/2 V (reference voltage) the MSB is retained to 1, if not then reset to 0. The ring counter increments one count, then the second MSB is set to 1 (i.e, 11000000). Then the output of DAC will be 1/2V + 1/4 V and is compared with the input voltage, if it is less than the output of DAC then the second MSB is reset to 0. (i.e., 10000000). Then the ring counter increases by then the third MSB is set to 1 (i.e., 10100000), now the output of DAC will be 1/2V + 1/8 V and is compared with input voltage, if less than the output of DAC then it results to 0, if not retains 1. This proceeds to the next bits of SAR untill the ring counter reaches its final count, then the measurement by successive approximation stops and the digital output of control register gives the final approximation of unknown input voltage.

★ <u>3 1/2 Digit:</u>

The number of digits used to represent a number determines the resolution of digital meter.

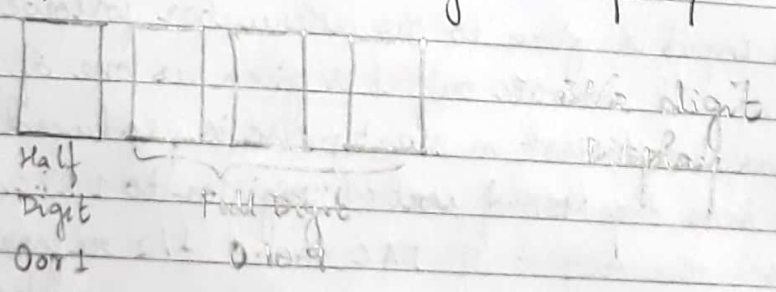
<u>Ex:</u> A 3 digit display for 0-1V range

⇒ 0 - 999mV - smallest increment ⇒ 1mV.

Thus a 4^{th} digit indicating 0-1 is called Half Digit.

Now 999 - 1999 values can be read.

This process is called over-ranging and this type of display is called 3 1/2 digit display



★ **Resolution:**

n - number of full digits

then resolution : $R = \frac{1}{10^n}$

★ **Sensitivity:** smallest change in input a digit meter is able to detect.

Sensitivity : $S = (f_s)_{min} \times R$

$f_{s_{min}}$ - lowest full scale of the meter

R - resolution (in decimal)

Q1: What is the resolution of 3½ digit display on 1V and 10V ranges?

Sol: $n = 3$ $\therefore$ Resolution : $R = \frac{1}{10^n} = \frac{1}{10^3} = 0.001$

Hence the meter cannot distinguish values that differ from each other by less that 0.001 of full scale.

For full scale reading of 1V range.

Resolution = $1 \times 0.001 = 0.001V$

For full scale reading of 10V range.

Resolution = $10 \times 0.001 = 0.01V$

Hence on 10V scale values that differ each other by less than 0.01V cannot be distinguished.

Q2: A 4½ digit voltmeter is used for voltage measurements

i. Find its resolution.

ii. How would 12.98V be displayed on a 10V range?

iii. How would 0.6973 be displayed on 1V and 10V ranges?

Sol: i. $n = 4$ $\therefore$ Resolution : $R = \frac{1}{10^n} = \frac{1}{10^4} = 0.0001$.

ii. There is 5 digit places in 4½ digits

∴ 12.98 would be displayed as 12.980

as resolution for a full scale reading for 10 V range is

$10 \times 0.0001 = 0.001$

iii. Resolution for a full scale reading for 1V range is

$1 \times 0.0001 = 0.0001$

∴ 0.6973 will be displayed as 0.6973

Resolution for a full scale reading for 10V range is

$10 \times 0.0001 = 0.001$

∴ 0.6973 will be displayed as 0.697 instead of 0.69

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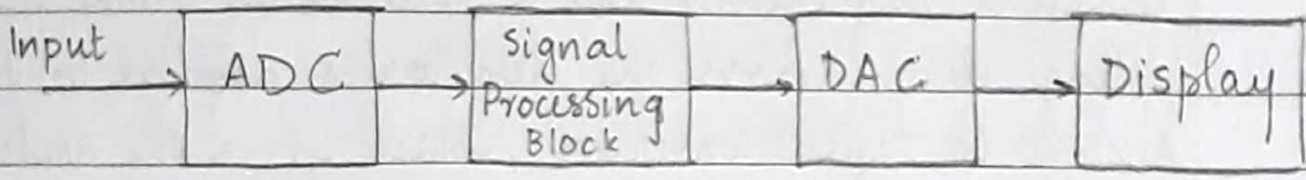
★ Digital Instrument:



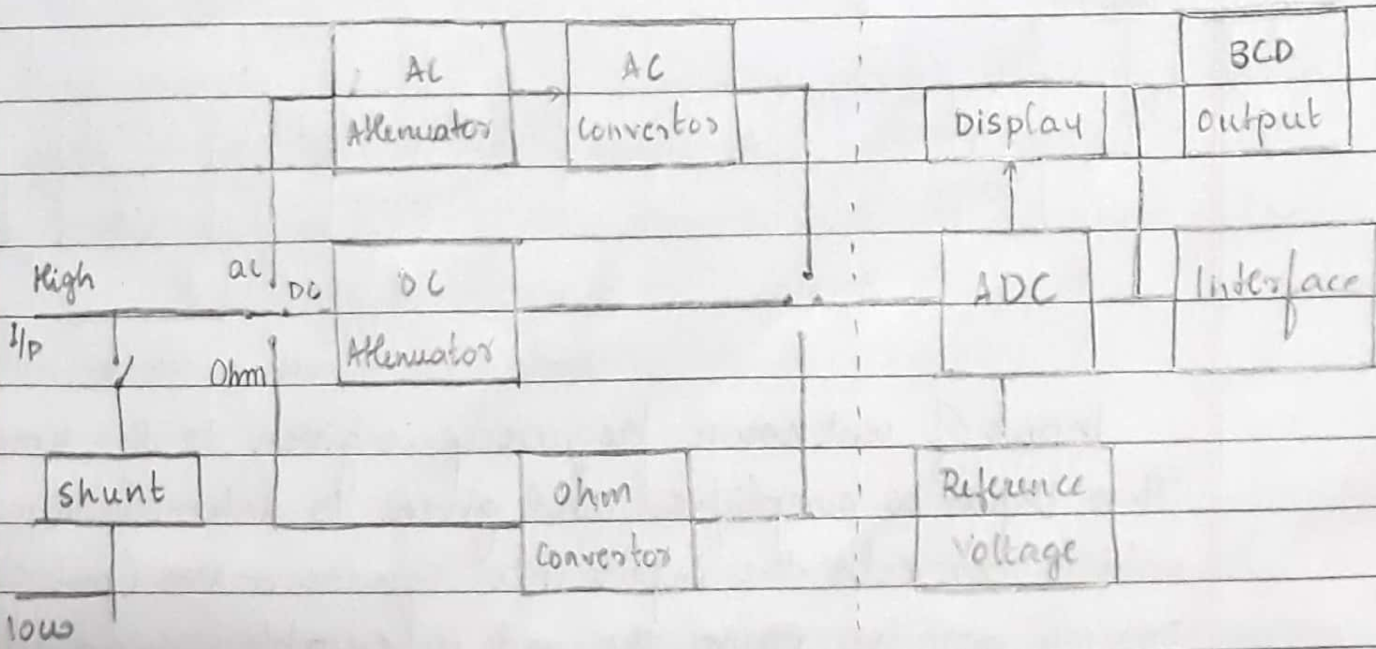
Block Diagram of Digital Instruments

Raw input is analog that is given to the Analog to Digital converter, hence converted into digital.

Signal Processing Block performs some certain operations such as addition, multiplication etc., based on the requirement.

This processed digital output from the signal processing block is given as input to the digital to analog converter which gives analog output which is further given to display as output.

★ Digital Multimeter:



Block Diagram of Digital Multimeter

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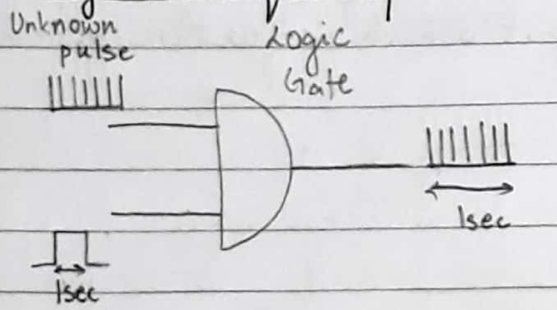
current is converted into voltage by passing it through the low shunt resistance, and the alternating current is converted into dc current by rectifiers and filters.

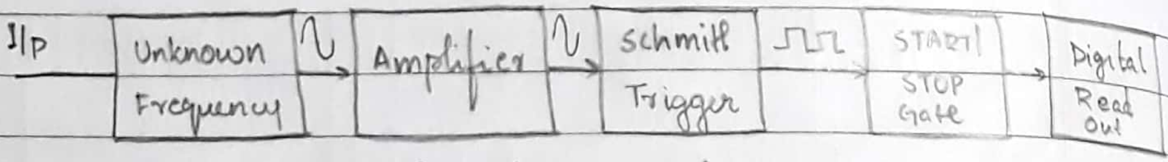
To measure resistance, a low current source is applied across the unknown resistance this gives a dc voltage which is digitalised and is readout as ohms. i.e., the voltage drop across the resistor is applied to the Analog to Digital converter, which gives the value of the unknown resistance.

Digital meters have high accuracy, small in size, and the output can be read out easily when compared to analog meters.

★ <u>Digital Frequency Meter:</u>

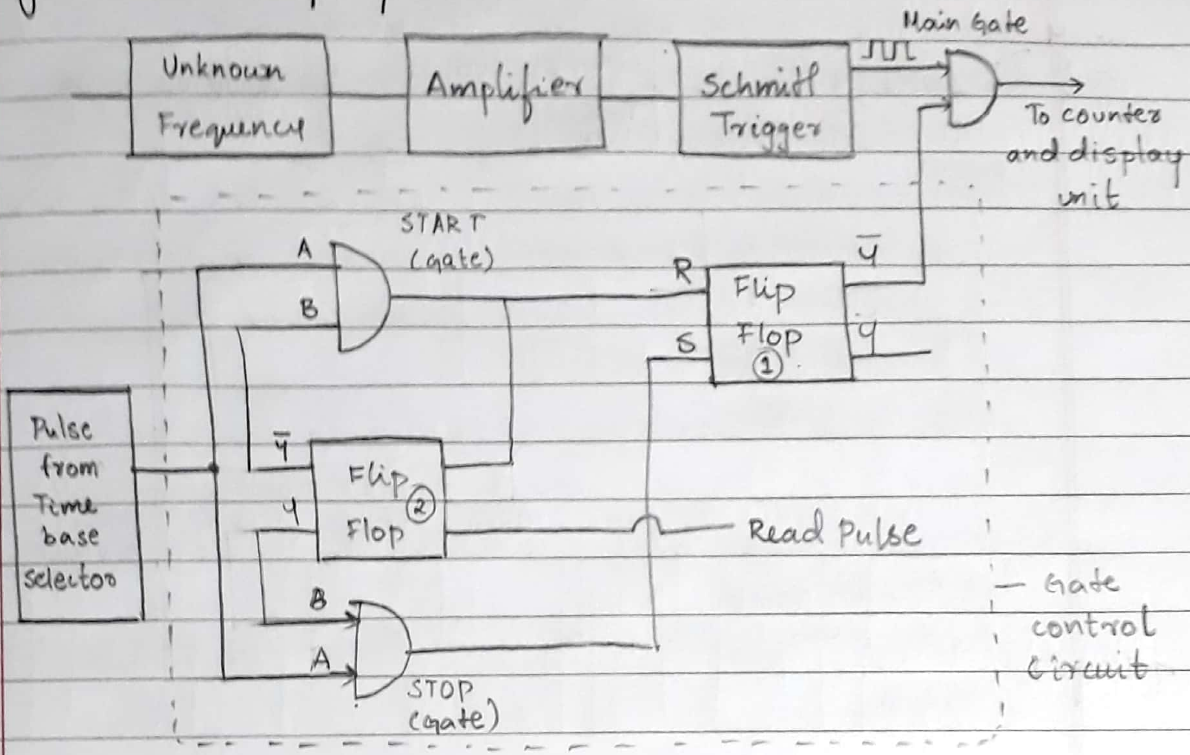


Basic Principle Operation of DFM



Basic block diagram of DFM

Input of unknown frequency is given to the amplifier. This input is amplified and given to schmitt trigger which converts the input into square wave (pulses). During positive pulse the gate is enabled and during negative pulse the gate is disabled. This output of gate is given to the Display.



Pulse from time based selector is 1.

- Assuming output of Flip Flop 2 is set by read pulse then $Y = 1$ $\bar{Y} = 0$

  Stop gate is enabled as $A = B = 1$

  Start gate is disabled as $A = 1$ but $B = 0$

  Since output of stop gate is 1 then $S = 1$ for flip flop 1 then output of flip flop 1 is $Y = 1$, $\bar{Y} = 0$ thus the main gate is off. Hence no output.

- Assuming output of Flip Flop 2 is reset by read pulse then $Y = 0$ $\bar{Y} = 1$

  Stop gate is disabled as $B = 0$, but $A = 1$.

  Start gate is enabled as $A = B = 1$.

  Since output of start gate is 1 then $R = 1$ for flip flop 1 then output of flip flop 1 is $Y = 0$ and $\bar{Y} = 1$ thus the main gate is on. Hence output is obtained at counter.

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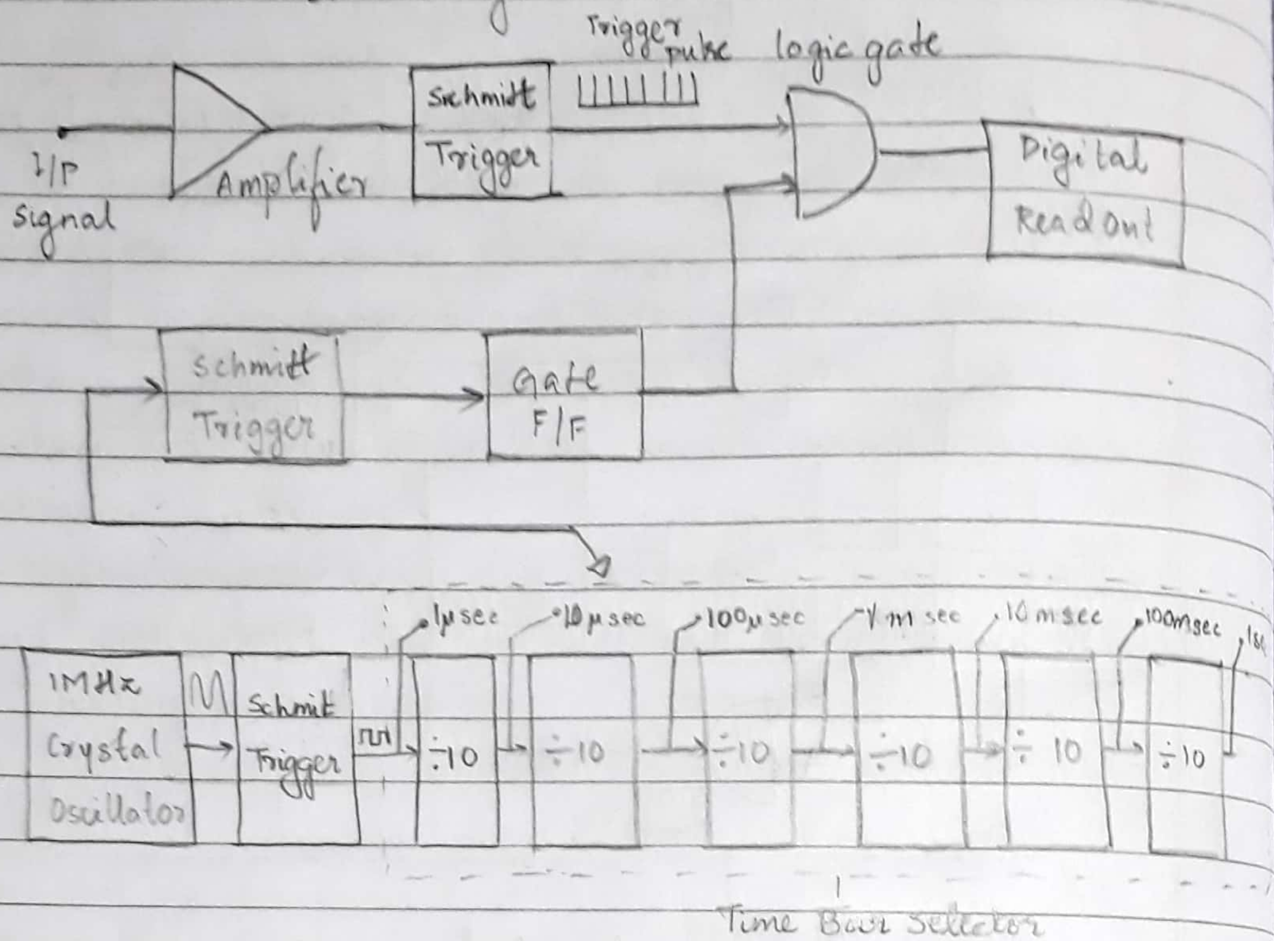
Block Diagram of DFM.



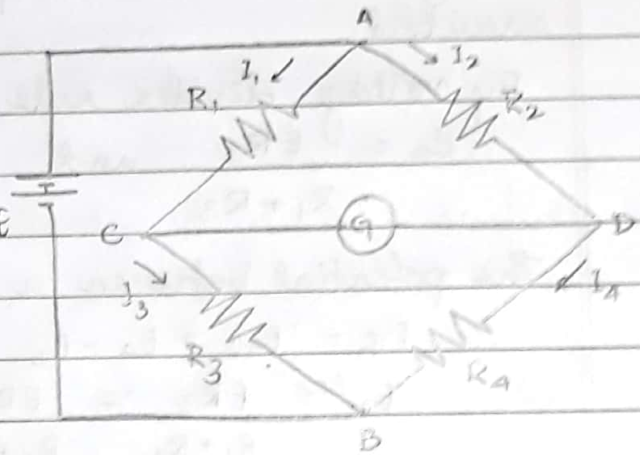
The input signal is amplified and converted to square wave by Schmitt Trigger. The output of time based selector is from the oscillator. The first pulse activates the gate flip flop. This opens the AND gate and the trigger pulses of the input signal for that selected time period is counted.
Similarly for the second pulse from the decade frequency divider changes the state of gate controlled flip flop hence closing the gate, then no output is obtained.
The output corresponds to the number of input pulses received during that particular time interval hence corresponding to the frequency.

UNIT - 3

Measurement of Resistance, Inductance and Capacitance.

★ **WHEATSTONE'S BRIDGE**: (Measurement of Resistance)

– **Balanced Wheatstone's Bridge**



The bridge is said to be balanced when $I_g = 0$, i.e., C and D have equal potential

$$I_1 R_1 = I_2 R_2 \quad \text{—①}$$

(potential between AC = Potential between AD)

For $I_g = 0$ $\quad I_1 = I_3$ and $I_2 = I_4$

$$\therefore \; I_1 = I_3 = \frac{E}{R_1 + R_3} \quad ; \quad I_2 = I_4 = \frac{E}{R_2 + R_4}$$

Substituting I_1 and I_2 in eq ①

$$\frac{\cancel{E} R_1}{R_1 + R_3} = \frac{\cancel{E} R_2}{R_2 + R_4}$$

$$R_1 (R_2 + R_4) = R_2 (R_1 + R_3)$$

$$\cancel{R_1 R_2} + R_1 R_4 = \cancel{R_1 R_2} + R_2 R_3$$

$$R_4 = \frac{R_2 R_3}{R_1} \Rightarrow \boxed{R_x = \frac{R_2 R_3}{R_1}}$$

Sensitivity of a Wheatstone's bridge

$D = S \times I$

D - deflection
S - sensitivity
I - current.

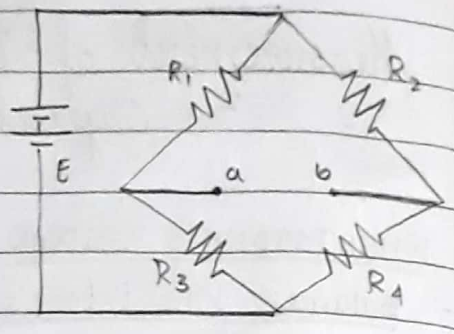
current through the galvanometer causes deflection. More the sensitivity greater is the deflection for the same current.

Unbalanced Wheatstone's bridge:

For unbalanced Wheatstone's bridge $I_g \neq 0$.

Using Thevenin's theorem.

To find thevenin's equivalent voltage Galvanometer is open circuited.



By voltage divider rule

$$E_a = \frac{E R_3}{R_1 + R_3} \quad \text{and} \quad E_b = \frac{E R_4}{R_2 + R_4}$$

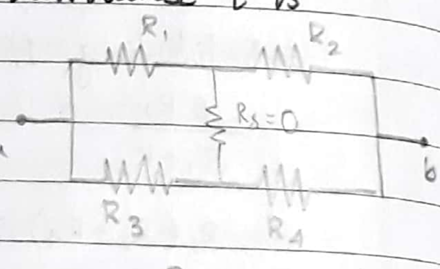
The potential between a and b is the Thevenin's voltage.

$$\therefore \quad E_{Th} = E_{ab} = E_a - E_b$$

$$E_{Th} = \frac{E R_3}{R_1 + R_3} - \frac{E R_4}{R_2 + R_4}$$

$$\boxed{E_{Th} = E\left[\frac{R_3}{R_1 + R_3} - \frac{R_4}{R_2 + R_4}\right]}$$

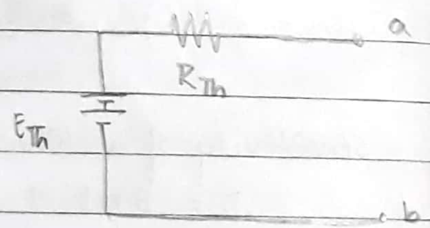
To determine thevenin's equivalent resistance E is short circuited.



$$\therefore \quad R_{Th} = (R_1 \| R_3) + (R_2 \| R_4)$$

$$\boxed{R_{Th} = \frac{R_1 R_3}{R_1 + R_3} + \frac{R_2 R_4}{R_1 + R_4}}$$

Rs - source resistance



Thevenin's equivalent circuit

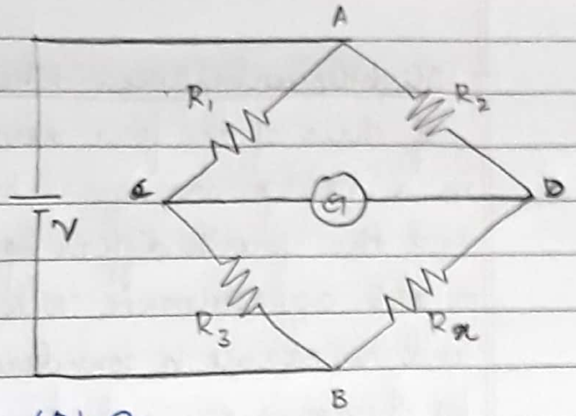
Applying KVL

$$E_{Th} - I_g R_{Th} - I_g R_g = 0$$

$$E_{Th} = I_g (R_{Th} + R_g)$$

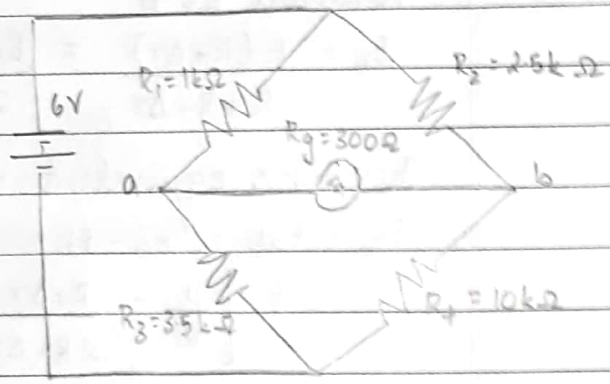
$$\boxed{I_g = \frac{E_{Th}}{R_{Th} + R_g}}$$

Q1: For the given circuit, the following parameters are given: $R_1 = 10k\Omega$, $R_2 = 15k\Omega$ and $R_3 = 40k\Omega$. Find the unknown resistance.



sol: $R_x = \frac{R_2 R_3}{R_1} = \frac{15k \times 40k}{10k} = \underline{60k\Omega}$

Q2: An unbalanced Wheatstone bridge is given. Calculate the current through galvanometer.



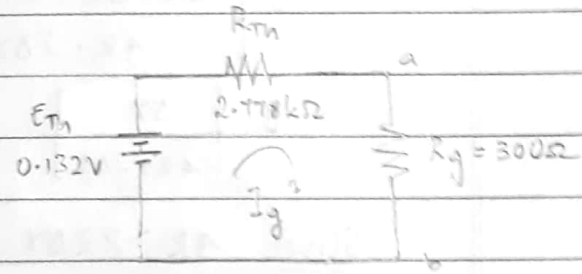
sol: $E_{Th} = E_{ab}$

$$E_{Th} = E\left[\frac{R_3}{R_1+R_3} - \frac{R_4}{R_2+R_4}\right]$$

$$= 6\left[\frac{3.5k}{1k+3.5k} - \frac{10k}{2.5k+10k}\right]$$

$$= 6\left[0.778 - 0.8\right]$$

$$\underline{E_{Th} = 0.132V}$$



$$R_{Th} = \left[\frac{R_1 R_3}{R_1+R_3} + \frac{R_2 R_4}{R_2+R_4}\right]$$

$$R_{Th} = \left[\frac{1k \times 3.5k}{1k+3.5k} + \frac{2.5k \times 10k}{2.5k+10k}\right]$$

$$= \left[0.778 + 2\right]k$$

$$\underline{R_{Th} = 2.778k\Omega}$$

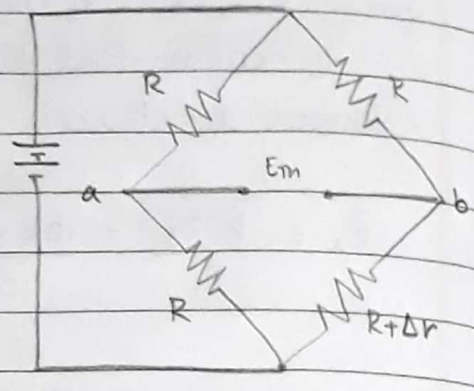
$$I_g = \frac{E_{Th}}{R_{Th} + R_g}$$

$$I_g = \frac{0.132V}{(2.778k\Omega + 300\Omega)}$$

$$\underline{I_g = 42.88\mu A}$$

Slightly unbalanced Wheatstone's Bridge:

If three of the four resistor in a bridge are equal to R and the fourth differs by 5% or less, approximate voltage and resistance is considered for thevenin's equivalent.



Potential at a

$$E_a = \frac{ER}{R+R} = \frac{ER}{2R} = \frac{E}{2}$$

Potential at b

$$E_b = \frac{E(R+\Delta r)}{R+R+\Delta r} = \frac{E(R+\Delta r)}{2R+\Delta r}$$

Thevenin's equivalent voltage

$$E_{Th} = E_{ab} = E_a - E_b$$

$$= E\left[\frac{1}{2} - \frac{R+\Delta r}{2R+\Delta r}\right]$$

$$= E\left[\frac{\cancel{2R}+\Delta r-\cancel{2R}-2\Delta r}{4R+2\Delta r}\right]$$

$$= E\left[\frac{\Delta r}{4R+2\Delta r}\right]$$

Since $4R >> 2\Delta r$

$$\boxed{E_{Th} = \frac{E\Delta r}{4R}}$$

Thevenin's equivalent resistance

$$R_{Th} = \frac{R\times R}{R+R} + \frac{R(R+\Delta r)}{R+R+\Delta r}$$

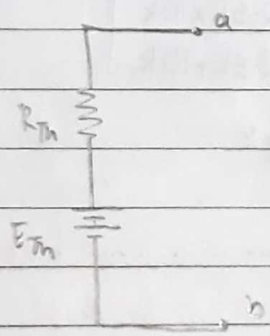
$$= \frac{R^{\cancel{2}}}{2\cancel{R}} + \frac{R(R+\Delta r)}{2R+\Delta r}$$

$$\boxed{R_{Th} = \frac{R}{2} + \frac{R}{2} = R}$$

since $R >> \Delta r$.

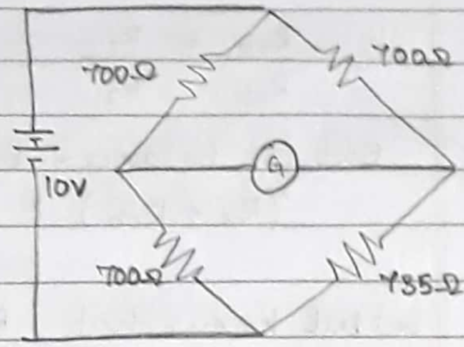
Galvanometer current

$$\boxed{I_g = \frac{E_{Th}}{R_{Th}+R_g}}$$



Keerthana Ashok

Q1: given a center zero 200-0-200μA movement having an internal resistance of 125Ω. Calculate the current through the galvanometer by the approximation method.



Sol: Thevenin's equivalent voltage

$E_{Th} = \frac{E \Delta r}{4R} = \frac{10(35)}{4(700)}$

$E_{Th} = 0.125V$

Given: $R_g = 125\Omega$

$\Delta R = 35\Omega$

$E = 10V$

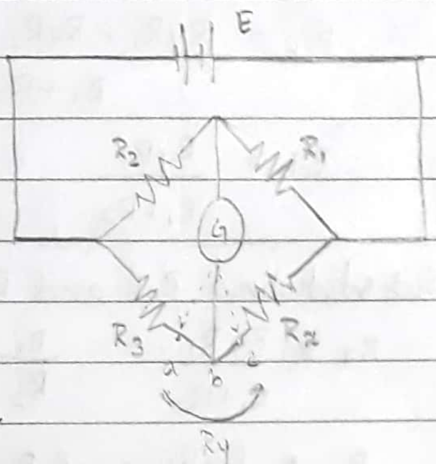
$R_{Th} = 700\Omega = R$

$I_g = \frac{E_{Th}}{R_{Th}+R_g} = \frac{0.125}{700+125} = 0.15mA$

Note: Wheatstone's bridge is not suitable for measuring low and high resistances.

* KELVIN'S BRIDGE:

Used to measure values of resistances below 1Ω.



In the circuit if the galvanometer is connected to point a, then R_y is added to R_x indicating a high value of R_x. When the galvanometer is connected to point c, the R_y is added to R_3, now R_3 is greater than actual. ~~to R_3~~ this leads to ~~higher value~~ lower value of R_x than actual. Hence galvanometer is connected to point b such that the ratio of resistances from c to b and from a to b equals to the ratio of resistances R_1 and R_2

Keerthana Ashok

i.e., $\dfrac{R_{cb}}{R_{ab}} = \dfrac{R_1}{R_2}$

But for balanced bridge the equation is given by

$$(R_x + R_{cb}) = \frac{R_1}{R_2}(R_3 + R_{ab}) \quad \text{—— } (1)$$

but ~~but~~ know that $R_{cb} + R_{ab} = R_y$ and $\dfrac{R_1}{R_2} = \dfrac{R_{cb}}{R_{ab}}$

$$\frac{R_{cb}}{R_{ab}} + 1 = \frac{R_1}{R_2} + 1$$

$$\frac{R_{cb} + R_{ab}}{R_{ab}} = \frac{R_1 + R_2}{R_2}$$

$$\frac{R_y}{R_{ab}} = \frac{R_1 + R_2}{R_2}$$

$$\therefore R_{ab} = \frac{R_y R_2}{R_1 + R_2}$$

Since $R_{ab} + R_{cb} = R_y$

$$R_{cb} = R_y - R_{ab} = R_y - \frac{R_y R_2}{R_1 + R_2}$$

$$R_{cb} = \frac{R_y R_1 + \cancel{R_y R_2} - \cancel{R_y R_2}}{R_1 + R_2}$$

$$R_{cb} = \frac{R_y R_1}{R_1 + R_2}$$

Substituting R_{ab} and R_{cb} in eq (1)

$$R_x + \frac{R_y R_1}{R_1 + R_2} = \frac{R_1}{R_2}\left(R_3 + \frac{R_y R_2}{R_1 + R_2}\right)$$

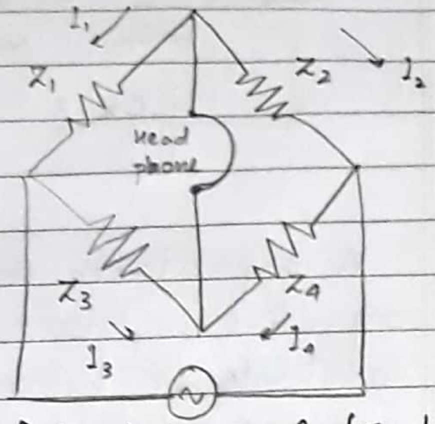
$$R_x + \frac{R_y R_1}{R_1 + R_2} = \frac{R_1 R_3}{R_2} + \frac{R_1 \cancel{R_2} R_y}{\cancel{R_2}(R_1 + R_2)}$$

$$R_x = \frac{R_1 R_3}{R_2} + \cancel{\frac{R_1 R_y}{(R_1 + R_2)}} - \cancel{\frac{R_1 R_y}{(R_1 + R_2)}}$$

$$\boxed{R_x = \frac{R_1 R_3}{R_2}}$$

* AC bridges:

The bridge is similar to a dc bridge except the arms are impedances. The bridge is excited by ac source and the galvanometer is replaced by a detector such as headphones to detect ac.



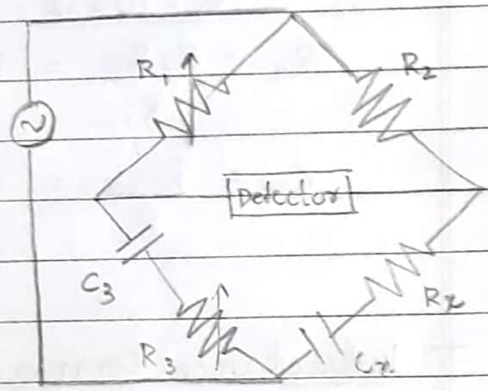
When the bridge is balanced

$$\frac{Z_1}{Z_3} = \frac{Z_2}{Z_4}$$

The bridge must be balanced for both reactance and the resistive component.

- Capacitance comparison bridge:

C_3 is a known standard capacitor in series with R_3.
C_x is unknown capacitor with a small leakage resistance of the capacitor i.e, R_x.



$$Z_1 = R_1$$
$$Z_2 = R_2$$
$$Z_3 = R_3 + j/\omega C_3$$
$$Z_x = R_x + j/\omega C_x$$

The condition for balanced bridge is

$$\frac{Z_1}{Z_3} = \frac{Z_2}{Z_4}$$

$$\therefore \quad Z_1 Z_x = Z_2 Z_3$$

$$R_1\left(R_x - \frac{j}{\omega C_x}\right) = R_2\left(R_3 - \frac{j}{\omega C_3}\right)$$

$$R_1 R_x - \frac{jR_1}{\omega C_x} = R_2 R_3 - \frac{jR_2}{\omega C_3}$$

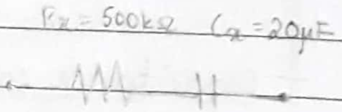
$$\Rightarrow R_1 R_x = R_2 R_3$$

$$\therefore \boxed{R_x = \frac{R_2 R_3}{R_1}}$$

and $\dfrac{R_1}{\omega C_x} = \dfrac{R_2}{\omega C_3}$

$$\boxed{C_x = \frac{R_1 C_3}{R_2}}$$

Q1: A capacitance comparison bridge used to measure a capacitive impedance at a frequency of 2 kHz. The bridge constants at balance are $C_3 = 100\,\mu F$, $R_1 = 10\,k\Omega$, $R_2 = 50\,k\Omega$, $R_3 = 100\,k\Omega$. Find the equivalent series circuit of the unknown impedance.



Sol: Given: $C_3 = 100\,\mu F$ $\quad R_2 = 50\,k\Omega$

$R_1 = 10\,k\Omega$ $\quad R_3 = 100\,k\Omega$

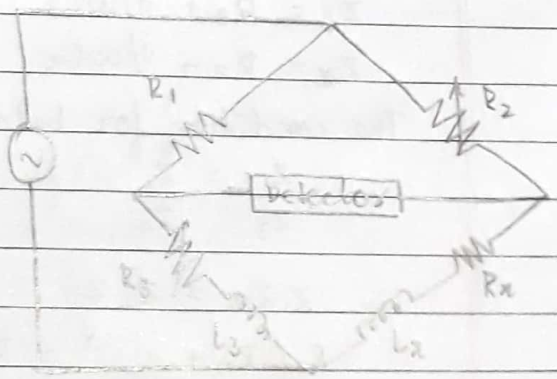
$$R_x = \frac{R_2 R_3}{R_1} = \frac{50k \times 100k}{10k} = \underline{\underline{500\,k\Omega}}$$

$$C_x = \frac{R_1 C_3}{R_2} = \frac{10k \times 100\mu}{50k} = \underline{\underline{20\,\mu F}}$$

- **Inductance comparison Bridge:**

L_x is unknown inductance with internal resistance R_x.

L_3 and R_3 are known standard inductance and resistance.



For balanced bridge

$$\frac{Z_1}{Z_3} = \frac{Z_2}{Z_4}$$

$$\therefore Z_1 Z_x = Z_2 Z_3$$

Here $Z_1 = R_1$, $Z_2 = R_2$, $Z_3 = R_3 + j\omega L_3$, $Z_x = R_x + j\omega L_x$

$$\therefore R_1(R_x + j\omega L_x) = R_2(R_3 + j\omega L_3)$$

$$R_1 R_x + jR_1\omega L_x = R_2 R_3 + jR_2\omega L_3$$

$\Rightarrow R_1 R_x = R_2 R_3$

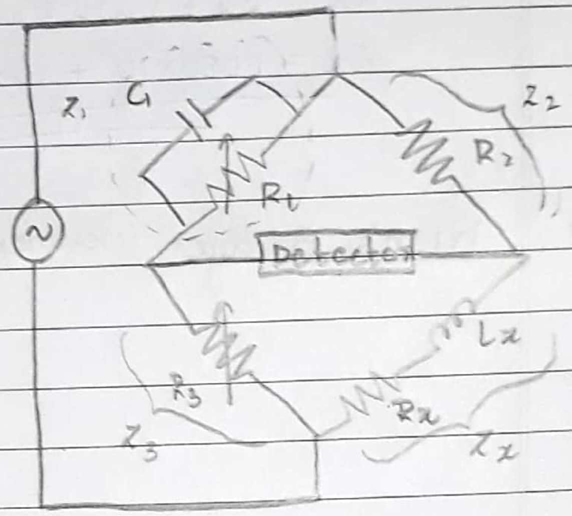
$$\boxed{R_x = \frac{R_2 R_3}{R_1}}$$

$\Rightarrow R_1 j\omega L_x = R_2 j\omega L_3$

$$\boxed{L_x = \frac{R_2 L_3}{R_1}}$$

* MAXWELL'S BRIDGE:

Maxwell's bridge measures an unknown inductance in terms of a capacitor.



For balanced bridge

$$\frac{Z_1}{Z_3} = \frac{Z_2}{Z_x}$$

$$Z_1 Z_x = Z_2 Z_3$$

$$Z_x = \frac{Z_2 Z_3}{Z_1} = Z_2 Z_3 Y_1$$

where $Z_1 = \frac{1}{Y_1}$ $\therefore Y_1 = \frac{1}{R} + j\omega C_1$

$$Z_2 = R_2, \; Z_3 = R_3, \; Z_x = R_x + j\omega L_x$$

$$\therefore R_x + j\omega L_x = R_2 R_3 \left(\frac{1}{R_1} + j\omega C_1\right)$$

$$R_x + j\omega L_x = \frac{R_2 R_3}{R_1} + j\omega C_1 R_2 R_3$$

Equating real and imaginary terms

$$\boxed{R_x = \frac{R_2 R_3}{R_1}} \quad \text{and} \quad \boxed{L_x = R_2 R_3 C_1}$$

Q1: A Maxwell bridge is used to measure an inductive impedance. The bridge constants at balance are

$C_1 = 0.01\mu F$, $R_1 = 470 k\Omega$, $R_2 = 5.1 k\Omega$, $R_3 = 100 k\Omega$