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UNIT- 1

Amplitude Modulation

* **Amplitude Modulation:**

In amplitude modulation the amplitude of the carrier signal (high freqency) changes in accordance with the instantenous value of the modulating or message signal (low frequency).

$c(t)$: high frequency carrier signal

$m(t)$: low frequency modulating/message signal.

* **Time Domain Representation of AM wave:**

Let $m(t)$ be the band limited modulating or message signal

note: band limited signal is a signal present in a particular band of frequency and else where it is zero

Let $c(t)$ be the sinusoidal carrier signal of high frequency is indicated by

$$c(t) = A_c \cos 2\pi f_c t \quad \text{—— (1)}$$

where A_c : Amplitude of carrier signal

f_c : Frequency of carrier signal

In time domain, the amplitude modulated signal $s(t)$ is represented as

$$s(t) = A_c[1 + k_a m(t)] \cos 2\pi f_c t \quad \text{—— (2)}$$

where k_a : Amplitude sensitivity of the modulator (constant)

The envelope of the AM signal $a(t)$ is given by

$$a(t) = A_c[1 + k_a m(t)] \quad \text{—— (3)}$$

The maximum value of $|k_a m(t)|$ is known as the modution index or depth of modulation and is abrevated as μ or m_a.

If the modulation index is multiplied by 100 it is known as percentage modulation.

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Case 1:

Suppose $|k_a m(t)| \leq 1$, for all t —— (4)

Under this condition $1 + k_a m(t)$ is always non negative

i.e, $[1 + k_a m(t)] \geq 0$.

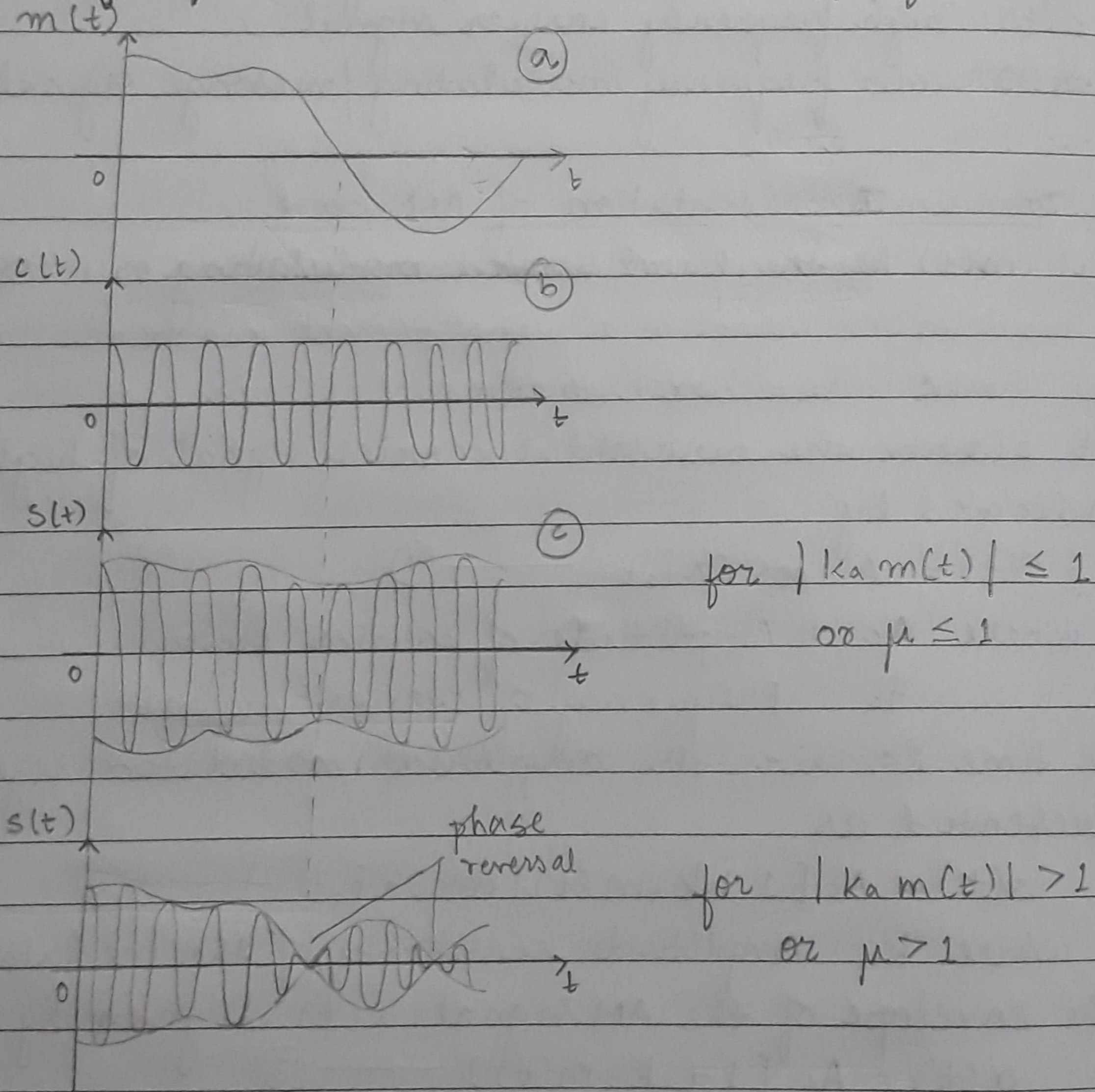
Therefore

$$a(t) = A_c[1 + k_a m(t)], \text{ for all } t \quad \text{—— (5)}$$

Case 2:

Suppose $|k_a m(t)| > 1$, for some t

Using eq (3) to evaluate the envelop of the AM wave



comparing the waveform for $|k_a m(t)| \leq 1$ i.e, fig (c) and $|k_a m(t)| > 1$ i.e, fig (d), the AM was has one to one correspondence with that of modulating or message signal if the percentage modulation is less than 100%.

Here the envelope of modulated signal s(t) follows

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message signal m (t). The one to one correspondence is destroyed if the percentage modulation exceeds 100%.

The distortion in the envelope that is caused due to over modulation, this is called as Envelope Distortion and the waveform is said to be over modulated.

The complexity of the demodulator / detector that can be used to recover the modulating or message signal is greatly simplified if the modulator at the transmitter is designed to produce an AM wave with an envelope that has the same shape as the modulating or message signal. For this requirement we must satisfy the following two conditions:

a. keep percentage modulation less than 100% to avoid envelope distortion.

b. the message bandwidth w is very small when compared to carrier frequency i.e., $f_c >> w$

* **Frequency Domain Representation of AM wave:**

In time domain AM wave can be represented by

$$s(t) = A_c [1 + k_a m(t)] \cos 2\pi f_c t$$

$$s(t) = A_c \cos 2\pi f_c t + A_c k_a m(t) \cos 2\pi f_c t \quad \text{--- } (1)$$

Applying Fourier Transform and Frequency shift property

$$S(f) = \frac{A_c}{2}\left[\delta(f - f_c) + \delta(f + f_c)\right]$$

$$+ \frac{A_c k_a}{2}\left[M(f - f_c) + M(f + f_c)\right]$$

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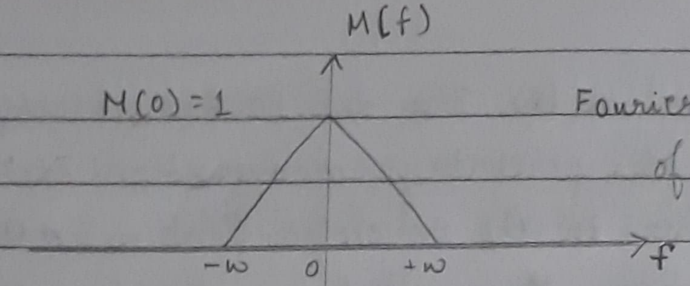
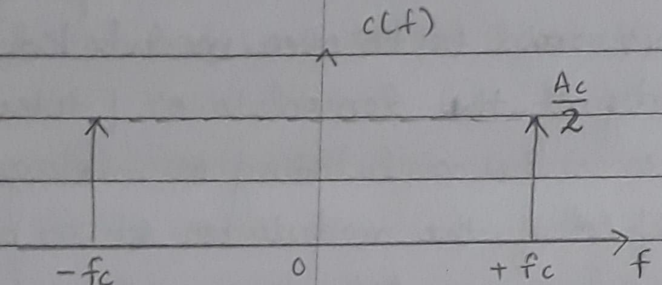
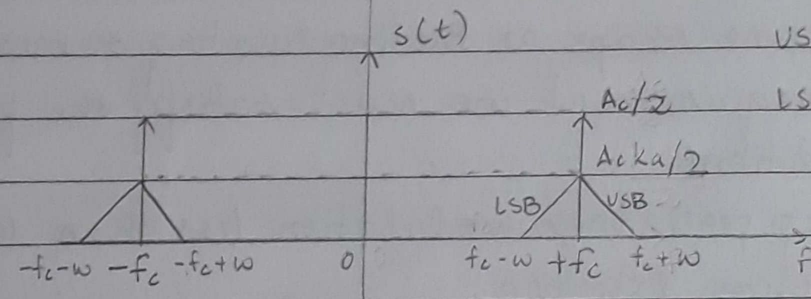


Fourier Transform of m(t)



Fourier Transform of c(t)



USB : $f_c < f < f_c + w$

LSB : $f_c - w < f < f_c$

The side bands are symmetrically distributed with respect to carrier frequency f_c.
For positive frequencies the transmission band width of AM wave is

$$BW = [f_c + w] - [f_c - w]$$

$$BW = 2w \quad Hz$$

where w is the message band width.

- Single Tone AM

Let $m(t)$ be the single tone message or modulating signal given by

$$m(t) = A_m \cos 2\pi f_m t \quad \text{--- (1)}$$

where A_m : peak amplitude

f_m : frequency of message signal

Let $c(t)$ be the high frequency sinusoidal carrier signal given by

$$c(t) = A_c \cos 2\pi f_c t \quad \text{--- (2)}$$

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AM wave in time domain is given by

$$s(t) = A_c [1 + k_a m(t)] \cos 2\pi f_c t \quad \text{--- (3)}$$

Substituting eq (1) in eq (3)

$$s(t) = A_c [1 + k_a A_m \cos 2\pi f_m t] \cos 2\pi f_c t \quad \text{--- (4)}$$

By defination, modulation index $\mu = k_a A_m$ --- (5)

Therefore from eq (4) and eq (5)

$$s(t) = A_c [1 + \mu \cos 2\pi f_m t] \cos 2\pi f_c t \quad \text{--- (6)}$$

$$= A_c \cos 2\pi f_c t + \mu A_c \cos 2\pi f_m t \cos 2\pi f_c t$$

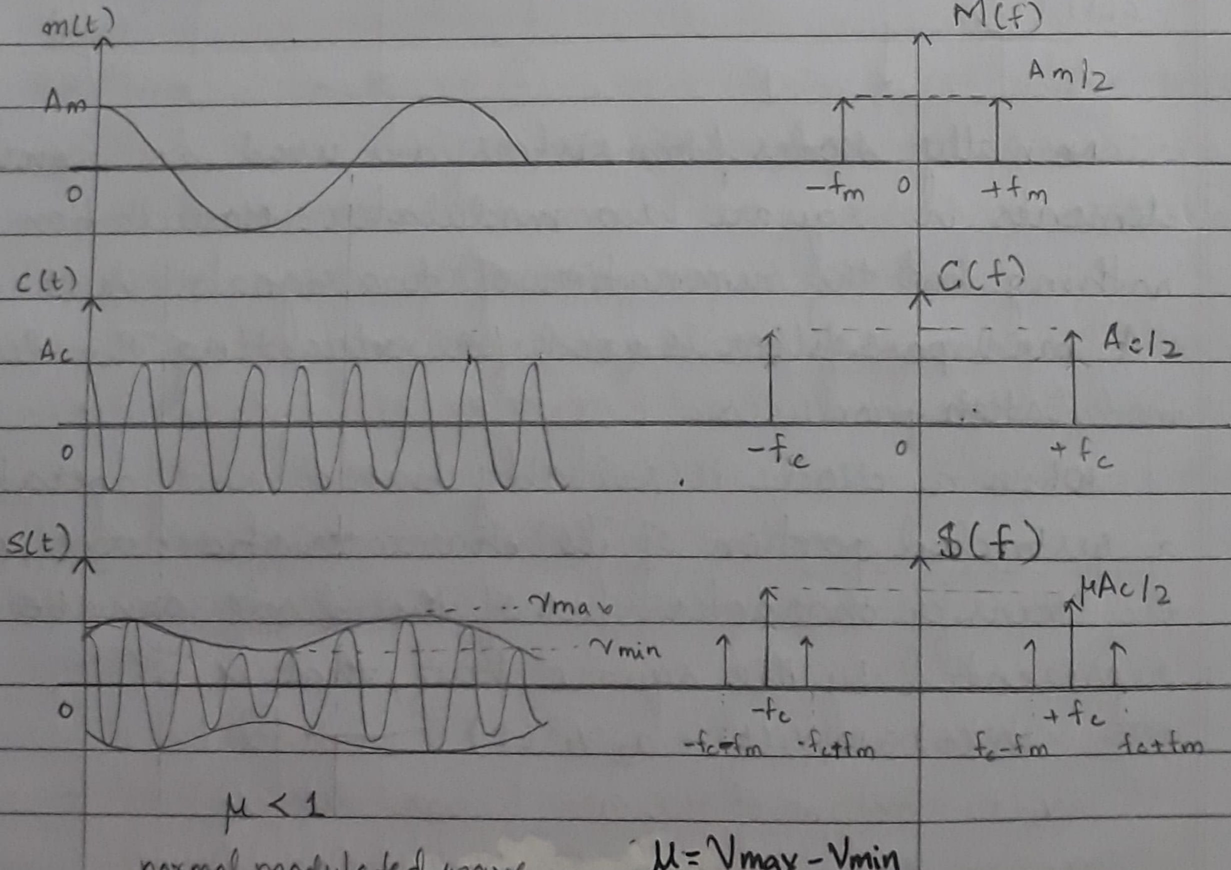
$$s(t) = A_c \cos 2\pi f_c t + \frac{\mu A_c}{2} [\cos 2\pi (f_c + f_m)t + \cos 2\pi (f_c - f_m)t] \quad \text{--- (7)}$$

Applying Fourier Transform for eq (7)

$$S(f) = \frac{A_c}{2} [\delta(f - f_c) + \delta(f + f_c)]$$

$$+ \frac{\mu A_c}{4} [\delta(f - (f_c + f_m)) + \delta(f + (f_c + f_m))]$$

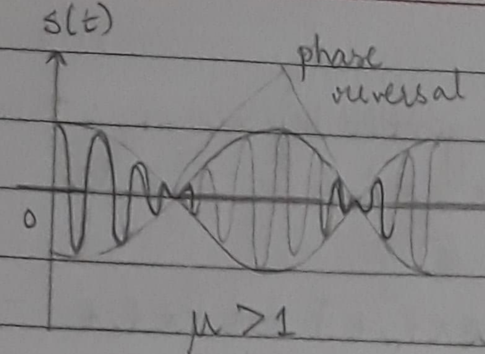
$$+ \frac{\mu A_c}{4} [\delta(f - (f_c - f_m)) + \delta(f + (f_c - f_m))] \quad \text{--- (8)}$$

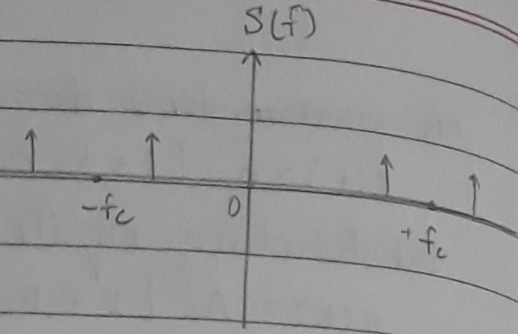


$\mu < 1$

normal modulated wave

$$\mu = \frac{V_{max} - V_{min}}{V_{max} + V_{min}}$$





over modulated wave

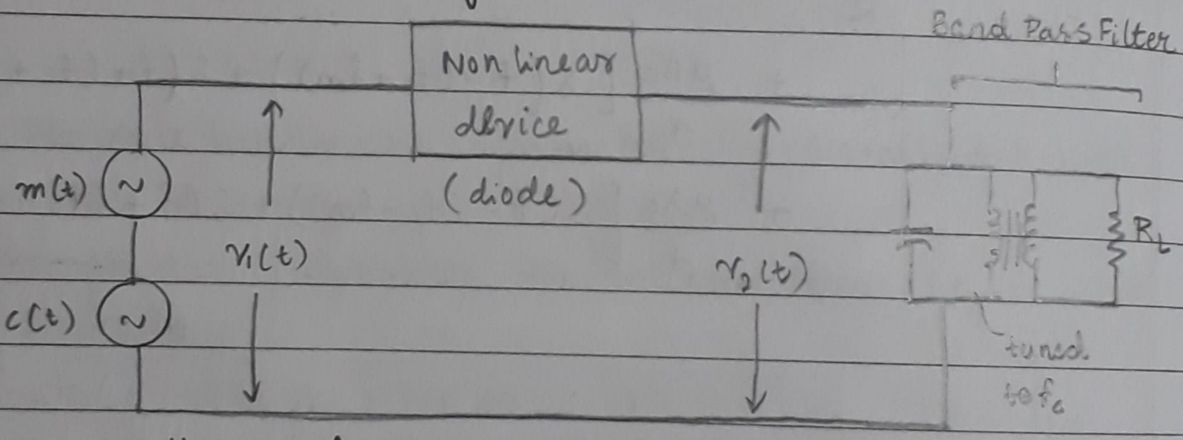
carrier is absent

Side bands are distributed wrt f_c

Drawbacks:

- 66% of the total power is consumed by carrier signal
- Transmission bandwidth $2w$.
- AM envelope are broadly affected by external interference

* AM Generation using square law Modulator:



Normally diodes / transistors are used as nonlinear elements in square law modulator. Modulation is nothing but the summation of two signals: $m(t)$ and $c(t)$

A band pass filter is used for extracting the desired modulation products.

When a diode is suitably biased and operated in a restricted portion of its characteristic curve, then the transfer characteristics of the diode can be represented by the square law, that is:

$$v_2(t) = a_1 v_1(t) + a_2 v_1^2(t) \quad \text{--- (1)}$$

($v_2(t)$: output of the non linear device)

Transfer characteristic of a non linear device

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where a_1 and a_2 are constants, the input voltage $v_1(t)$ consists of the carrier and modulating signals

$$v_1(t) = m(t) + c(t)$$

$$v_1(t) = m(t) + A_c \cos 2\pi f_c t \quad \text{——} \ (2)$$

Substituting eq (2) in eq (1)

$$v_2(t) = a_1[m(t) + A_c \cos 2\pi f_c t] + a_2[m(t) + A_c \cos 2\pi f_c t]^2$$

$$v_2(t) = a_1[m(t) + A_c \cos 2\pi f_c t] + a_2[m^2(t) + A_c^2 \cos^2 2\pi f_c t + 2m(t) A_c \cos 2\pi f_c t] \quad \text{——} \ (3)$$

$$v_2(t) = a_1 m(t) + a_1 A_c \cos 2\pi f_c t + a_2 m^2(t) + a_2 A_c^2\left[\frac{1 + \cos 2(2\pi f_c t)}{2}\right] + 2a_2 A_c m(t) \cos 2\pi f_c t$$

$$v_2(t) = a_1 m(t) + \cos 2\pi f_c t\left[a_1 A_c + 2a_2 A_c m(t)\right] + a_2 m^2(t) + a_2 \frac{A_c^2}{2} + a_2 \frac{A_c^2}{2} \cos 4\pi f_c t$$

$$v_2(t) = \underbrace{\left[a_1 A_c\left[1 + \frac{2a_2}{a_1} m(t)\right] \cos 2\pi f_c t\right]}_{\text{desired}} + a_1 m(t) + a_2 m^2(t) + \frac{a_2 A_c^2}{2}\left[1 + \cos 2\pi (2f_c) t\right] \quad \text{——} \ (4)$$

The first term of eq (4) is the desired AM wave with amplitude sensitivity $k_a = \frac{2a_2}{a_1}$ and the remaining unwanted terms are removed by using tuned band pass filter with following specification:

- center frequency equal to frequency of carrier.
- Band width $BW = 2w$, w is message band width
- frequency of carrier greater than three times the message band width : $f_c > 3w$.

Hence the output of the band pass filter is :

$$s(t) = a_1 A_c\left[1 + k_a m(t)\right] \cos 2\pi f_c t$$

where $k_a = \frac{2a_2}{a_1}$

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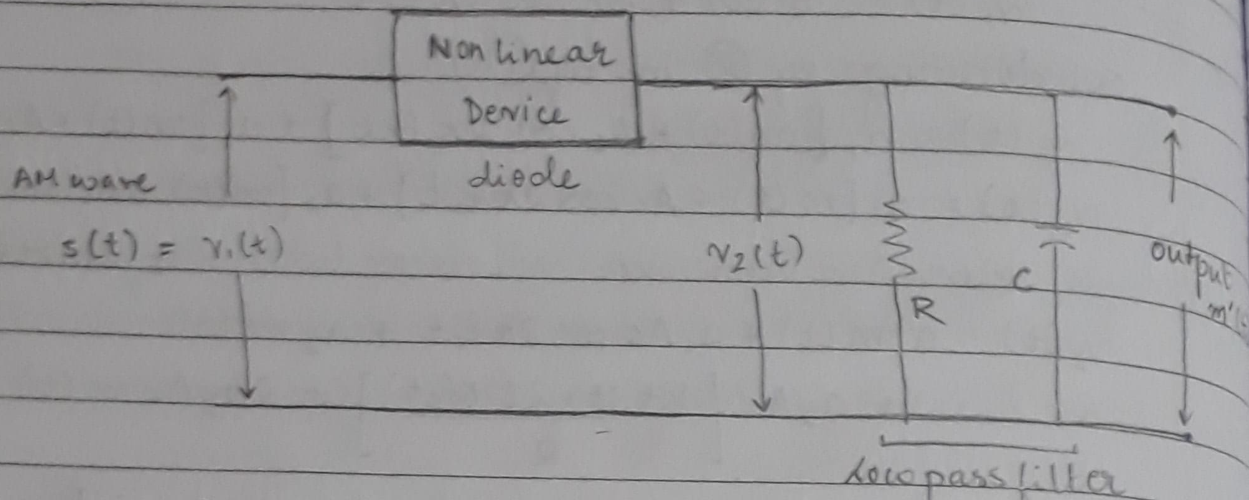
★ **AM detection using Square Law Detector:**

The recovery of m(t) from the modulated signal is called detection or demodulation



Operate the non linear device (diode) in its restricted portion of its characteristic curve.

Then transfer characteristics of a non linear device.

$$v_2(t) = a_1 v_1(t) + a_2 v_1^2(t) \quad \text{—— (1)} \quad \text{square law}$$

The input that is the AM wave is

$$v_1(t) = s(t) = A_c[1 + k_a m(t)] \cos 2\pi f_c t \quad \text{—— (2)}$$

Substituting eq (2) in eq (1)

$$v_2(t) = a_1 A_c[1 + k_a m(t)] \cos 2\pi f_c t + a_2 A_c^2 [1 + k_a m(t)]^2 \cos^2 2\pi f_c t \quad \text{—— (3)}$$

$$v_2(t) = a_1 A_c[1 + k_a m(t)] \cos 2\pi f_c t + a_2 A_c^2 \cos^2 2\pi f_c t\, [1 + k_a^2 m^2(t) + 2k_a m(t)]$$

$$v_2(t) = a_1 A_c[1 + k_a m(t)] \cos 2\pi f_c t + a_2 A_c^2 \left[\frac{1 + \cos 4\pi f_c t}{2}\right] [1 + k_a^2 m^2(t) + 2k_a m(t)]$$

$$v_2(t) = a_1 A_c[1 + k_a m(t)] \cos 2\pi f_c t + \left[\frac{a_2 A_c^2}{2} + \frac{a_2 A_c^2 k_a^2 m^2(t)}{2} + \frac{a_2 A_c^2 2 k_a m(t)}{2}\right] + \frac{a_2 A_c^2}{2} \cos 4\pi f_c t\, [1 + k_a^2 m^2(t) + 2k_a m(t)] \quad \text{—— (4)}$$

desired

High frequency terms (terms related to f_c and $2f_c$) are eliminated by passing the signal $v_2(t)$ through a low pass filter with cut off frequency equal to the message band width w.

The output of the low pass filter is:

$$m'(t) = \frac{a_2 A_c^2}{2} + \frac{a_2 A_c^2}{2} k_a^2 m^2(t) + a_2 A_c^2 k_a m(t) \quad \text{--- (5)}$$

The dc term $a_2 A_c^2/2$ is eliminated by passing it through a dc blocking capacitor.

The output of the dc blocking capacitor is

$$= a_2 A_c^2 k_a m(t) + \frac{a_2 A_c^2}{2} k_a^2 m^2(t) \quad \text{--- (6)}$$

The desired term is: $a_2 A_c^2 k_a m(t)$.

The term $\frac{a_2 A_c^2}{2} k_a^2 m^2(t)$ will cause some distortion in the output.

To reduce this distortion, make the ratio (wanted signal : ~~wanted~~ signal due to distortion) as large as possible.

$$\text{ratio} = \frac{\text{wanted signal}}{\text{signal due to distortion}}$$

$$\text{ratio} = \frac{a_2 A_c^2 k_a m(t)}{a_2 A_c^2 k_a^2 m^2(t)/2}$$

$$\text{ratio} = \frac{2}{k_a m(t)}$$

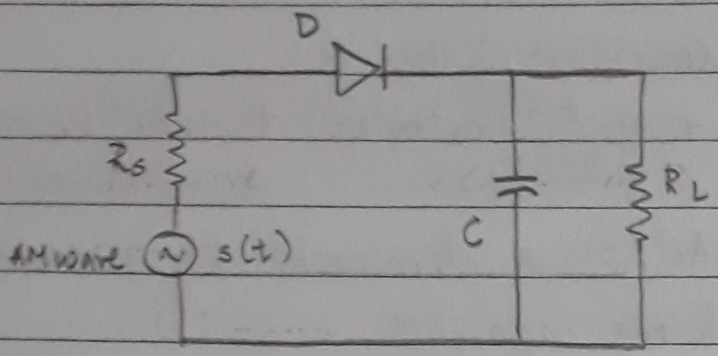
wkt $|k_a m(t)|$ is the modulation index

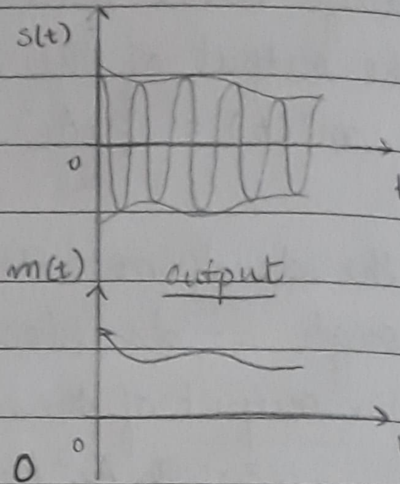
Therefore by limiting the percentage modulation, the amount of distortion can be controlled.

this is possible only ~~when~~ the percentage modulation is very small.

Therefore the distortionless recovery of the message or modulating signal from the incoming AM wave using square law detector is possible only if the applied AM wave is very weak (i.e., percentage modulation is very small)

★ **AM Detection using Envelope detector:**





Assume ideal diode

$R_f = 0$; $R_r = \infty$; $I_r = 0$; $V_f = 0$

Ideally an envelope detector produces an output signal that follows the envelope of the input signal and hence the name.

When the diode is biased in forward direction, the capacitor is charged to its peak value rapidly. For this keep $R_s C$ (time constant) less than $1/f_c$.

i.e., $R_s C << \frac{1}{f_c}$

Similarly when the diode is biased in the reverse direction, the capacitor discharges slowly through R_L. To ensure long discharging period keep:

$$\frac{1}{f_c} << R_L C << \frac{1}{w}$$

where w: message band width.

Therefore, the output voltage of the detector is nothing but, the voltage across the capacitor is very nearly same as the envelope of the AM signal.

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- Analysis:

Input to the envelope detector is

AM wave: $s(t) = A_c\,[1 + k_a m(t)]\cos 2\pi f_c t$ —— (1)

output of the envelope detector is equal to the envelope of the AM wave

output: $a(t) = \sqrt{(\text{Inphase component})^2 + (\text{quadrature component})^2}$

cosine function   sine function

$a(t) = \sqrt{A_c^2\,[1 + k_a m(t)]^2 + (0)^2}$ —— (2)

$a(t) = A_c\,[1 + k_a m(t)]$ —— (3)

$a(t) = A_c + A_c k_a m(t)$

The first term is a dc term, hence it is eliminated by using a dc blocking capacitor.

The output of the dc blocking capacitor is

$A_c k_a m(t)$

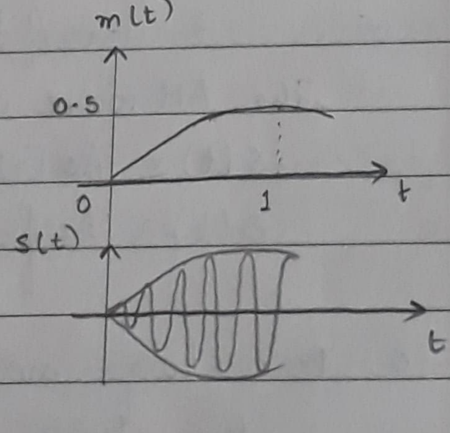
which is nothing but the scaled version of the modulating or message signal $m(t)$.

Q: For the message signal $m(t) = \dfrac{t}{1+t^2}$, find the expression for AM wave when the percentage modulation is:

a. 50%

b. 100%

c. 125%



- Given: $m(t) = \dfrac{t}{1+t^2}$

To plot $m(t)$

Type 1:

t	0	0.2	0.4	0.6	0.8	1.0
$m(t)$	0					0.5

Type 2: to find $m(t)_{max}$

i.e., $\dfrac{dm(t)}{dt} = 0 \Rightarrow \dfrac{1+t^2 - 2t^2}{(1+t^2)^2} = 0 \Rightarrow 1 - t^2 = 0$ $m(t)_{max} = 0.5$

$\Rightarrow t = \pm 1$

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a. Percentage modulation is 50%.

$\mu = 0.5$

wkt $\mu = |k_a m(t)|_{max}$

$\mu = k_a m(t)_{max}$

$k_a = \frac{\mu}{m(t)_{max}}$

$k_a = \frac{0.5}{0.5} = 1$ //

The AM wave is given by

$s(t) = A_c [1 + k_a m(t)] \cos 2\pi f_c t$

$s(t) = A_c \left[1 + \frac{t}{1+t^2}\right] \cos 2\pi f_c t$ //

b. Percentage modulation is 100%.

$\mu = 1$

wkt $\mu = |k_a m(t)|_{max}$

$\mu = k_a m(t)_{max}$

$k_a = \frac{\mu}{m(t)_{max}}$

$k_a = \frac{1}{0.5} = 2$ //

The AM wave is given by

$s(t) = A_c [1 + k_a m(t)] \cos 2\pi f_c t$

$s(t) = A_c \left[1 + \frac{2t}{1+t^2}\right] \cos 2\pi f_c t$ //

c. Percentage modulation is 125%.

$\mu = 1.25$

wkt $\mu = |k_a m(t)|_{max}$

$\mu = k_a m(t)_{max}$

$k_a = \frac{\mu}{m(t)_{max}}$

$k_a = \frac{1.25}{0.5} = 2.5$ //

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The AM wave is given by

$s(t) = A_c[1 + k_a m(t)] \cos 2\pi f_c t$

$s(t) = A_c\left[1 + \frac{2.5t}{1+t^2}\right] \cos 2\pi f_c t$ //

Q: A multitone message signal $m(t) = E_1 \cos 2\pi f_1 t + E_2 \cos 2\pi f_2 t + E_3 \cos 2\pi f_3 t$ modulates the carrier of the form $c(t) = A_c \cos 2\pi f_c t$. Find:

a. time domain expression for the conventional AM wave

b. frequency domain expression and its spectrum.

c. minimum transmission band width.

Given $E_1 > E_2 > E_3$ and $f_3 > f_2 > f_1$.

→ given expressions

$m(t) = E_1 \cos 2\pi f_1 t + E_2 \cos 2\pi f_2 t + E_3 \cos 2\pi f_3 t$.

$c(t) = A_c \cos 2\pi f_c t$

a. time domain expression for the conventional AM wave.

$s(t) = A_c[1 + k_a m(t)] \cos 2\pi f_c t$

$s(t) = A_c\left[1 + k_a (E_1 \cos 2\pi f_1 t + E_2 \cos 2\pi f_2 t + E_3 \cos 2\pi f_3 t)\right] \cos 2\pi f_c t$

$s(t) = A_c \cos 2\pi f_c t + A_c[\mu_1 \cos 2\pi f_1 t + \mu_2 \cos 2\pi f_2 t + \mu_3 \cos 2\pi f_3 t] \cos 2\pi f_c t$

$s(t) = A_c \cos 2\pi f_c t + A_c[\mu_1 \cos 2\pi f_1 t \cos 2\pi f_c t + \mu_2 \cos 2\pi f_2 t \cos 2\pi f_c t + \mu_3 \cos 2\pi f_3 t \cos 2\pi f_c t]$

$$s(t) = A_c \cos 2\pi f_c t + \left[\frac{\mu_1 A_c}{2} \cos 2\pi (f_c - f_1)t + \frac{\mu_1 A_c}{2} \cos 2\pi (f_c + f_1)t\right] + \left[\frac{\mu_2 A_c}{2} \cos 2\pi (f_c - f_2)t + \frac{\mu_2 A_c}{2} \cos 2\pi (f_c + f_2)t\right] + \left[\frac{\mu_3 A_c}{2} \cos 2\pi (f_c - f_3)t + \frac{\mu_3 A_c}{2} \cos 2\pi (f_c + f_3)t\right]$$

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b. Frequency domain expression and its spectrum.

$$s(t) = A_c[1 + k_a m(t)]\cos 2\pi f_c t$$

$$s(t) = A_c\left[1 + k_a(E_1\cos 2\pi f_1 t + E_2\cos 2\pi f_2 t + E_3\cos 2\pi f_3 t)\right]\cos 2\pi f_c t$$

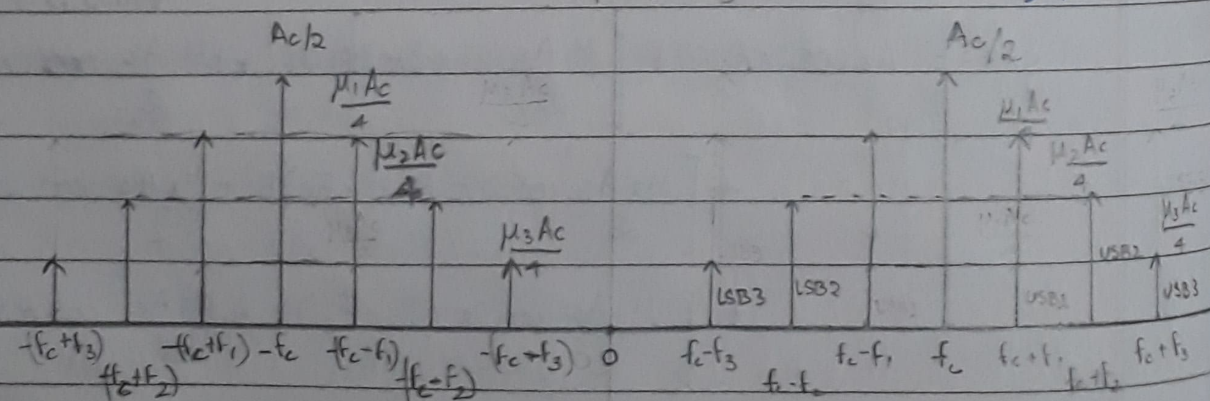
$$s(t) = A_c\cos 2\pi f_c t + A_c[\mu_1\cos 2\pi f_1 t\cos 2\pi f_c t + \mu_2\cos 2\pi f_2 t\cos 2\pi f_c t + \mu_3\cos 2\pi f_3 t\cos 2\pi f_c t]$$

$$s(t) = A_c\cos 2\pi f_c t + \left[\frac{\mu_1 A_c}{2}\cos 2\pi(f_c - f_1)t + \frac{\mu_1 A_c}{2}\cos 2\pi(f_c + f_1)t\right]$$
$$+ \left[\frac{\mu_2 A_c}{2}\cos 2\pi(f_c - f_2)t + \frac{\mu_2 A_c}{2}\cos 2\pi(f_c + f_2)t\right]$$
$$+ \left[\frac{\mu_3 A_c}{2}\cos 2\pi(f_c - f_3)t + \frac{\mu_3 A_c}{2}\cos 2\pi(f_c + f_3)t\right]$$

By applying Fourier transform and time shifting property

$$S(f) = \frac{A_c}{2}[\delta(f - f_c) + \delta(f + f_c)]$$
$$+ \frac{\mu_1 A_c}{4}\left[\delta(f - (f_c - f_1)) + \delta(f + (f_c - f_1)) + \delta(f - (f_c + f_1)) + \delta(f + (f + f_1))\right]$$
$$+ \frac{\mu_2 A_c}{4}\left[\delta(f - (f_c - f_2)) + \delta(f + (f_c - f_2)) + \delta(f - (f_c + f_2)) + \delta(f + (f_c + f_2))\right]$$
$$+ \frac{\mu_3 A_c}{4}\left[\delta(f - (f_c - f_3)) + \delta(f + (f_c - f_3)) + \delta(f - (f_c + f_3)) + \delta(f + (f_c + f_3))\right]$$

$E_1 > E_2 > E_3$



$f_3 > f_2 > f_1$

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c. Minimum transmission band width.

★ **Double Side Band Suppressed Carrier Modulation:**

To conserve the carrier power (66% of the total power) we adopt the Double Side Band Suppressed carrier Modulation.

DSB-SC wave is the product of two signals: modulating or message signal $m(t)$ and carrier signal $c(t)$.

Hence the modulation is by means of multiplication.

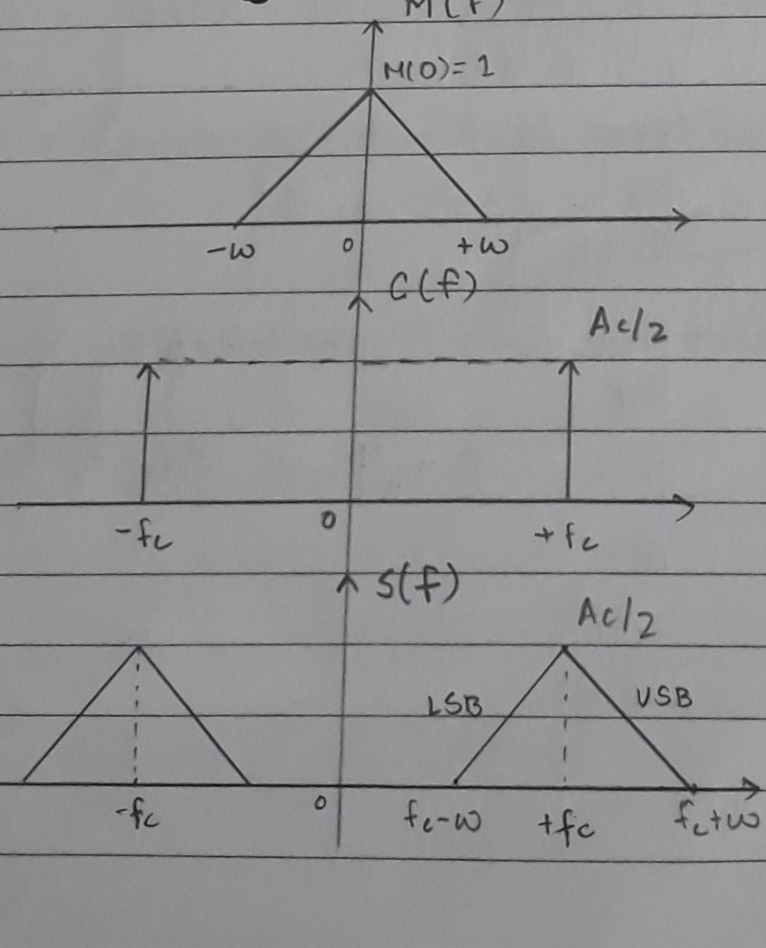
Therefore the DSB-SC modulated wave is given by

$$s(t) = m(t)\,c(t)$$

$$\underset{\text{DSB-SC}}{s(t)} = \underset{\text{band limited}}{\underbrace{m(t)}} A_c \cos 2\pi f_c t$$

Applying Fourier Transform and Frequency Shifting Property

$$S(f) = \frac{A_c}{2}\left[M(f-f_c) + M(f+f_c)\right]$$



In this modulation the carrier is just used for modulation and thus it is not transmitted. Hence it conserves the carrier power (66% of the total power)

BW = 2W

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- Single Tone DSB-SC:

let $m(t) = A_m \cos 2\pi f_m t$

wkt $s_{DSB-SC}(t) = m(t)\, c(t)$

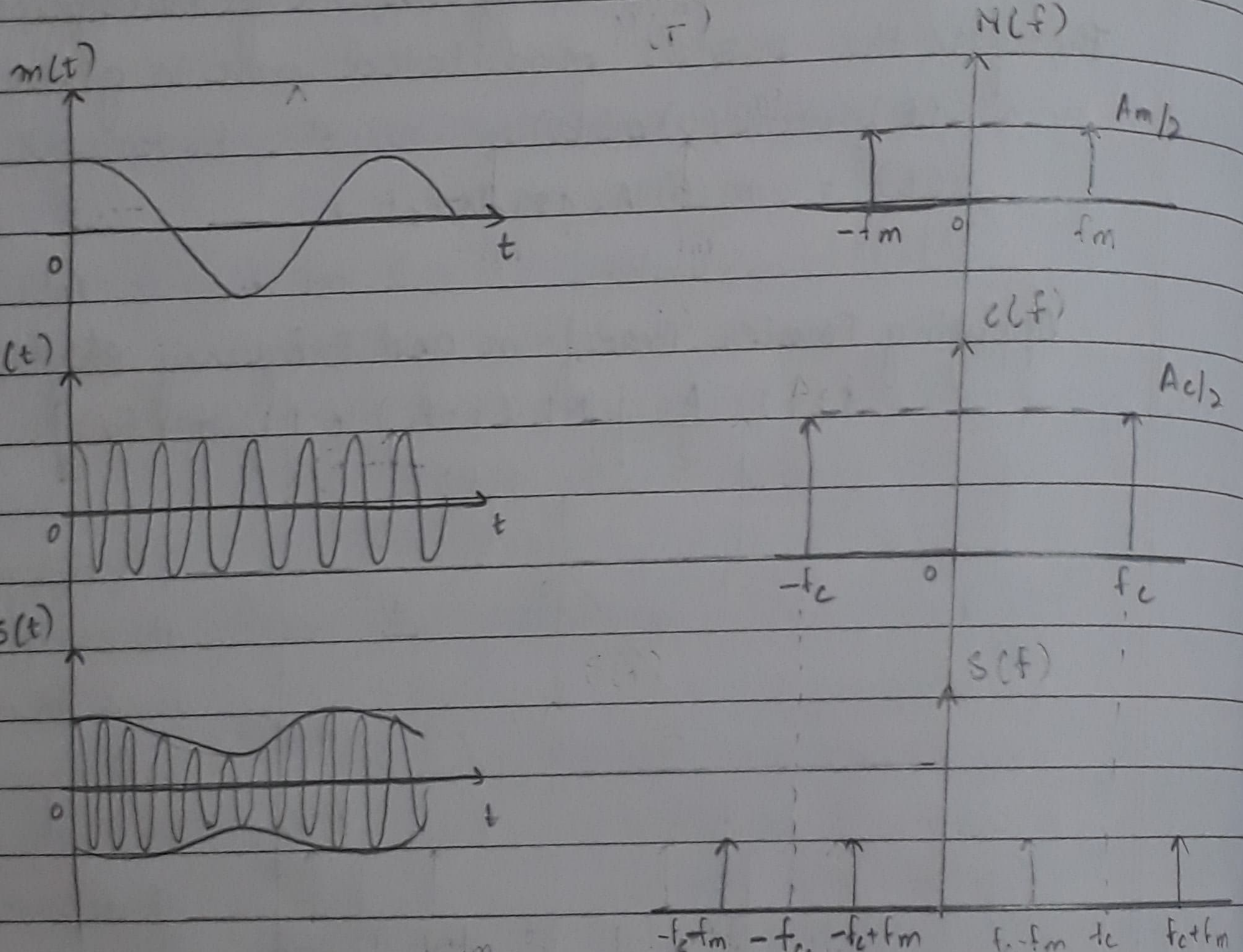
$$s(t) = A_m \cos 2\pi f_m t \; A_c \cos 2\pi f_c t$$

$$= \frac{A_c A_m}{2}\left[\cos 2\pi(f_c+f_m)t + \cos 2\pi(f_c-f_m)t\right]$$

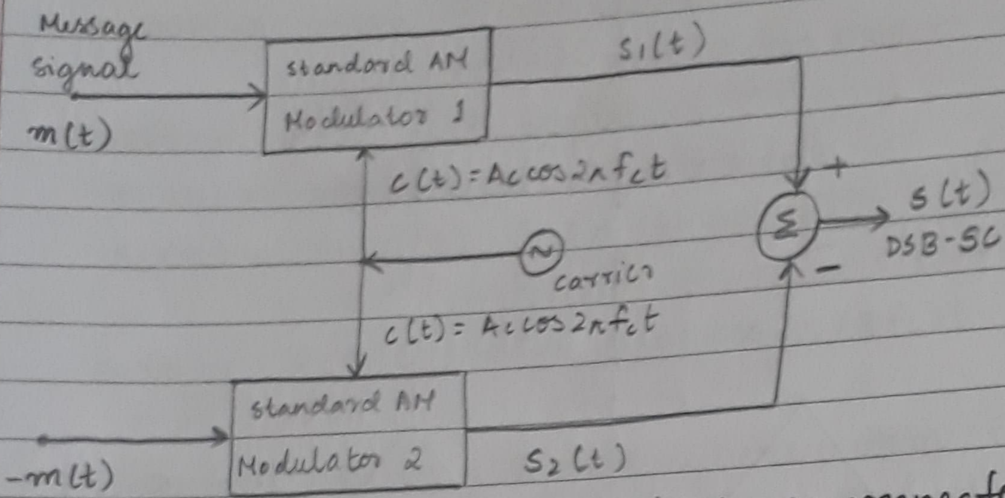
Applying fourier transform

$$S_{DSB-SC}(f) = \frac{A_c A_m}{4}\left[\delta(f-(f_c+f_m)) + \delta(f+(f_c+f_m))\right]$$

$$+ \frac{A_c A_m}{4}\left[\delta(f-(f_c-f_m)) + \delta(f+(f_c-f_m))\right]$$



★ Generation of DSB-SC using Balanced Modulator:



Two standard AM Modulators are connected in balanced configurature and hence the name.

From the figure, output of the std AM Modulator 1 is

$$s_1(t) = A_c[1 + k_a m(t)] \cos 2\pi f_c t \quad \text{---} (1)$$

Similarly the output of the std AM Modulator 2 is

$$s_2(t) = A_c[1 - k_a m(t)] \cos 2\pi f_c t \quad \text{---} (2)$$

Output of the balanced modulator is

$$\underset{DSB\text{-}SC}{s(t)} = s_1(t) - s_2(t) \quad \text{---} (3)$$

$$\underset{DSB\text{-}SC}{s(t)} = [\cancel{A_c \cos 2\pi f_c t} + A_c k_a m(t) \cos 2\pi f_c t] - [\cancel{A_c \cos 2\pi f_c t} - A_c k_a m(t) \cos 2\pi f_c t]$$

$$\underset{DSB\text{-}SC}{s(t)} = 2 A_c k_a m(t) \cos 2\pi f_c t$$

$$\underset{DSB\text{-}SC}{s(t)} = 2 k_a m(t)\, c(t)$$

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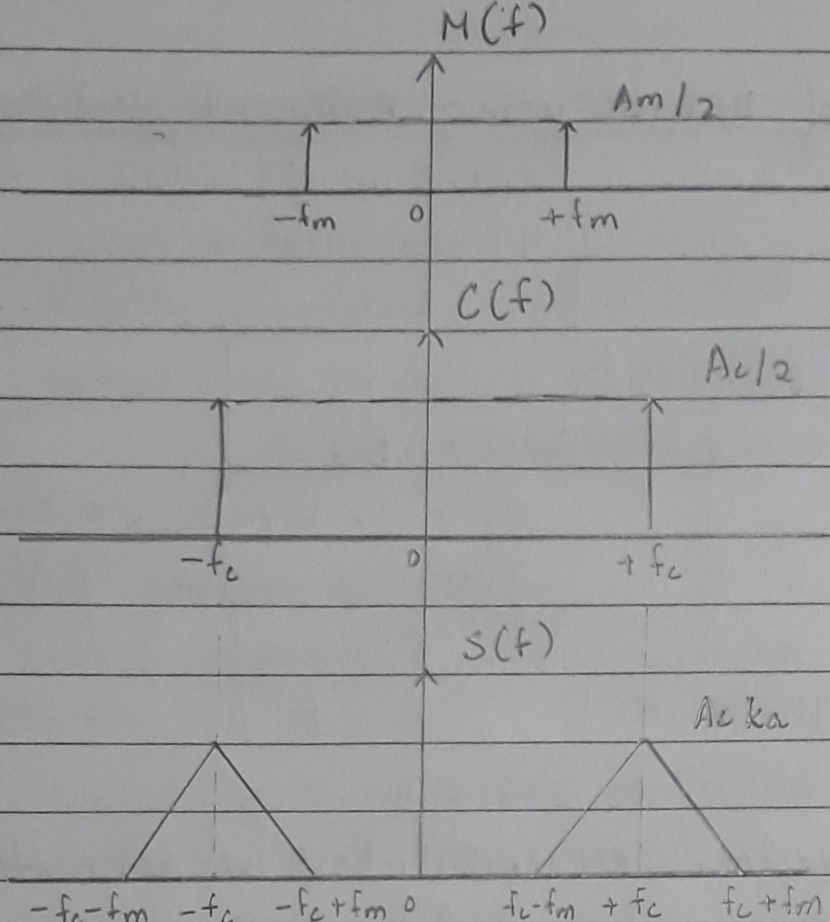
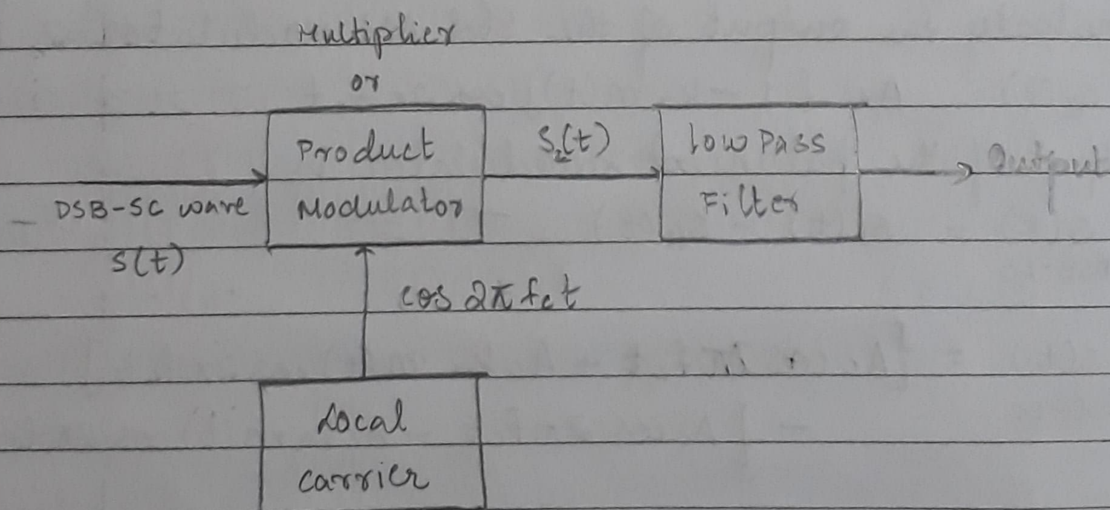


★ <u>Detection of DSB-SC using coherent or Synchronous Detector:</u>



In coherent or synchronous detection the phase and frequency of the local carrier is synchronised or coherent with respect to phase and frequency of the carrier at the modulator.

Coherent detector is a product modulator followed by a low pass filter.

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Output of the product modulator is

$$S_c(t) = s(t) \cdot \cos 2\pi f_c t \quad \text{—— (1)}$$

but DSB-SC wave: $s(t) = c(t)\, m(t)$

$$s(t) = m(t)\, A_c \cos 2\pi f_c t \quad \text{—— (2)}$$

Substituting eq (2) in eq (1)

$$S_c(t) = \cancel{m(t)}\, A_c\, m(t) \cos^2 2\pi f_c t$$

$$S_c(t) = A_c\, m(t) \left[\frac{1 + \cos 2\pi (2f_c)t}{2} \right] \quad \text{—— (3)}$$

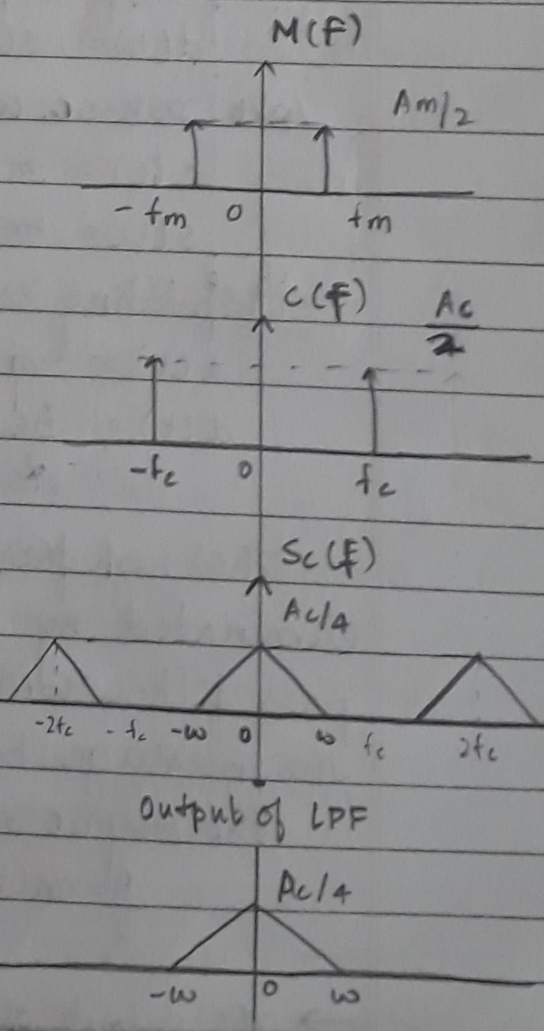
$$S_c(t) = \frac{A_c\, m(t)}{2} + \frac{A_c\, m(t)}{2} \cos 2\pi (2f_c)t$$

The second term is a high frequency term, it also indicates a DSB-SC wave which is distributed with $f = 2f_c$. It is eliminated by passing the signal $s_c(t)$ through a low pass filter.

Output of the low pass filter is

$$\frac{A_c\, m(t)}{2}$$

This is the scaled version of message or modulating signal



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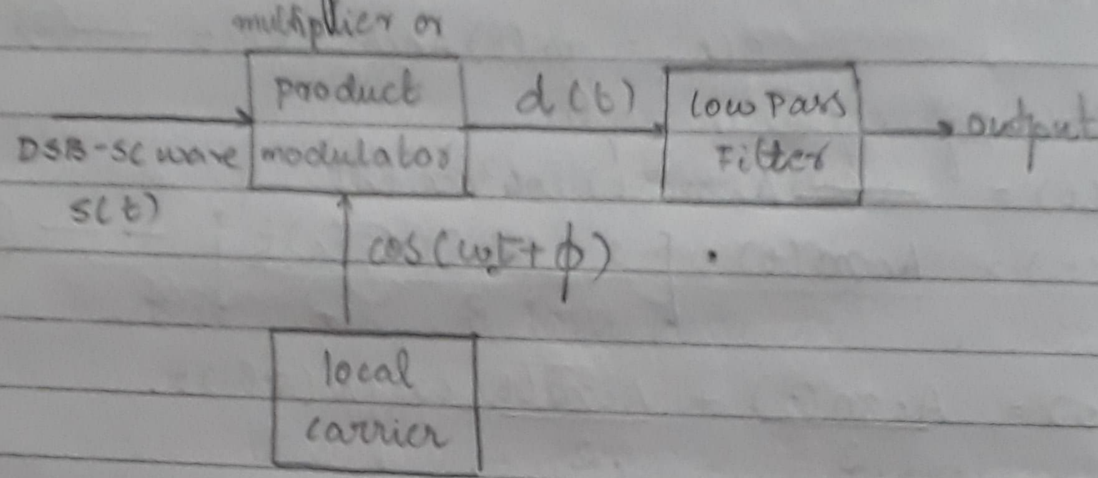
Phase Error

Evaluate the effect of phase error in the local oscillator of synchronous DSB-SC demodulation as shown



The frequency of the local carrier is synchronised with the frequency of the carrier at the modulator but not the phase.

The output of the product modulator / multiplier is

$$d(t) = s(t) \cdot \cos(\omega_c t + \phi) \quad \text{——} \ (1)$$

wkt DSB-SC wave

$$s(t) = m(t)\, c(t)$$

$$s(t) = m(t)\, A_c \cos 2\pi f_c t \quad \text{——} \ (2)$$

Substituting eq (2) in eq (1)

$$d(t) = m(t)\, A_c \cos 2\pi f_c t \cdot \cos(\omega_c t + \phi) \quad \text{——} \ (3)$$

$$d(t) = \frac{A_c}{2} m(t) \left[\cos\phi + \cos(2\omega_c t + \phi)\right] \quad \text{——} \ (4)$$

The high frequency component term (2nd term) is eliminated by passing signal d(t) through a low pass filter with cut off frequency ω, where ω is the message band width.

The output of the low pass filter is

$$\frac{A_c}{2} m(t) \cos\phi \quad \text{——} \ (5)$$

→ The output of the low pass filter is proportional to m(t) when φ is constant

→ The output of the low pass filter reaches it maximum value when $\phi = 0$ (i.e., phase of the local carrier is synchronised wrt the phase of the carrier at the modulator)

i.e., output of LPF $= \frac{A_c}{2} m(t)$

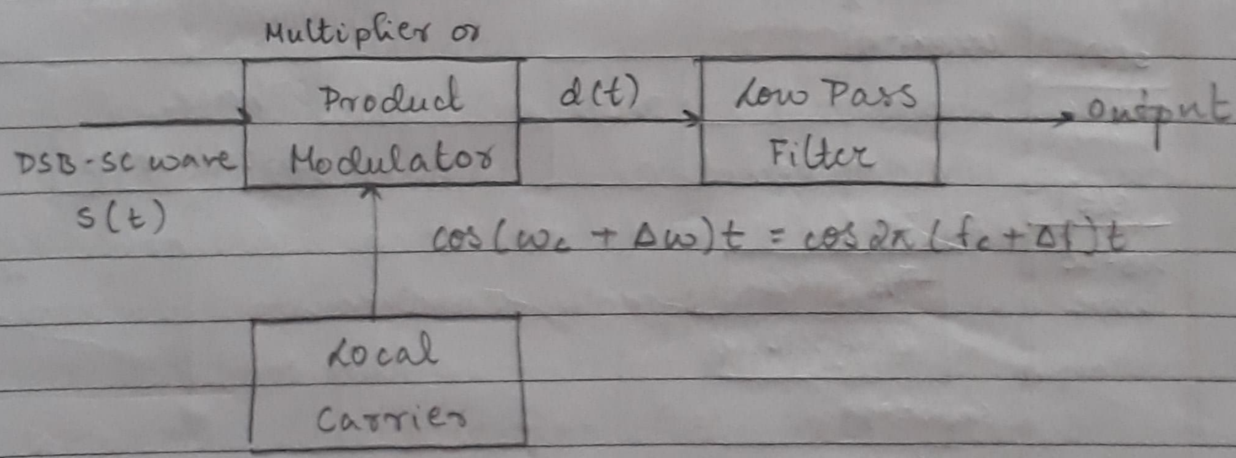
→ The output is completely lost when $\phi = \pm \pi/2$.

Thus the phase error in the local oscillator causes attenuation in the output without any distortion as long as ϕ is a constant but not equal to $\pm \pi/2$.

· If the phase error varies randomly with time then the output will also vary randomly and is undesirable.

- Frequency Error

Evaluate the effect of a small frequency error in the local oscillator on DSB-SC demodulation as shown



The output of the product modulator / multiplier is

$d(t) = s(t) . \cos(\omega_c + \Delta\omega)t$ —— ①

wkt DSB-SC wave

$s(t) = m(t)\, c(t)$

$s(t) = m(t)\, A_c \cos 2\pi f_c t$ —— ②

Substituting eq ② in eq ①

$d(t) = m(t)\, A_c \cos 2\pi f_c t \cos(\omega_c + \Delta\omega)t$ —— ③

$d(t) = \frac{A_c m(t)}{2} \cos \Delta\omega t + \frac{A_c m(t)}{2} \cos(2\omega_c + \Delta\omega)t$ —— ④

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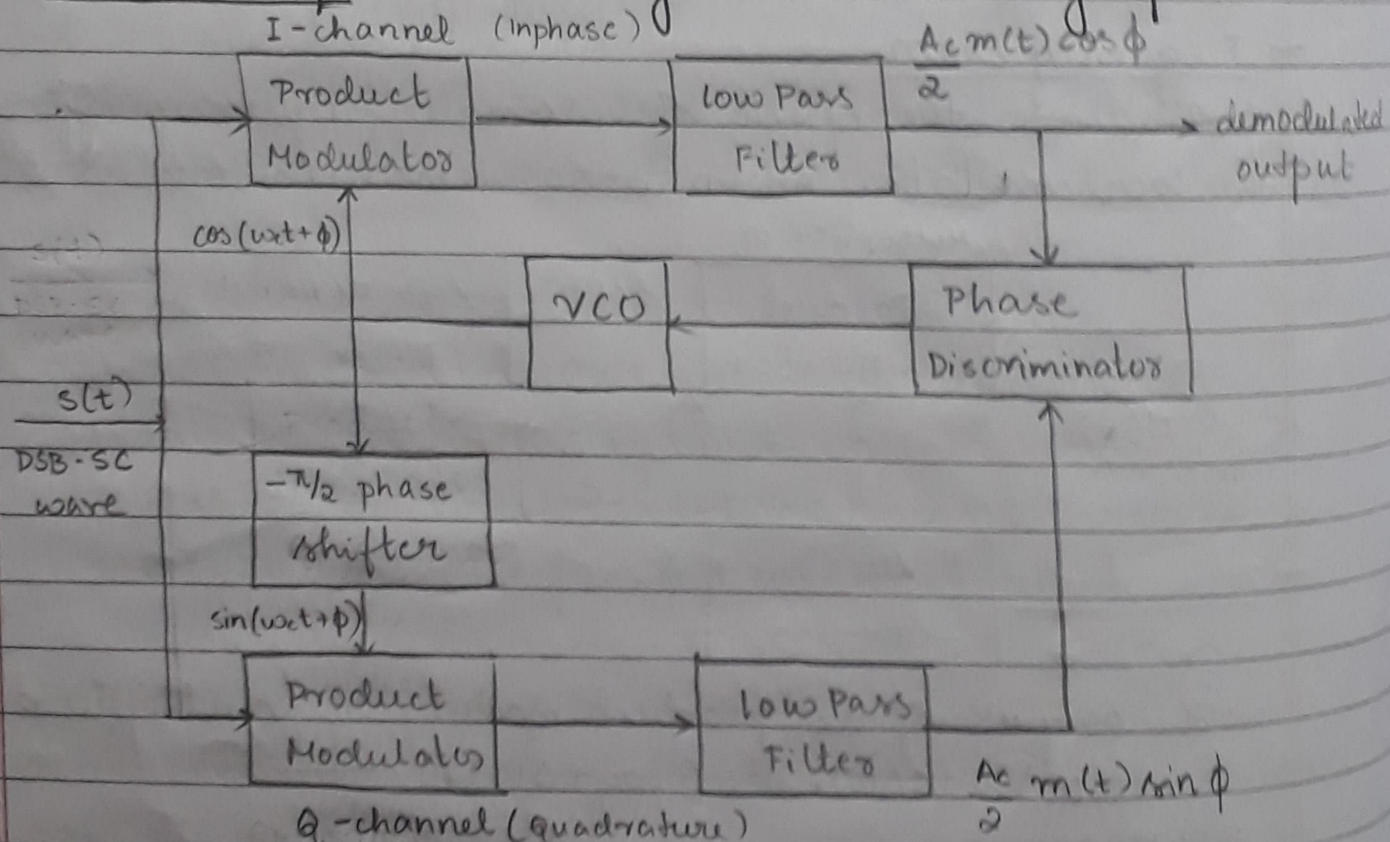
The high frequency component (2nd term) is eliminated by passing signal $d(t)$ through a low pass filter with cut off frequency ω, where ω is the message signal band width.

The output of the low pass filter is

$$\frac{A_c}{2} m(t) \cos \Delta\omega t \quad \text{—— (5)}$$

The presence of a low frequency sinosoid causes distortion at the output which is undesirable. This distortion is due to Beat Effect.

* **Costas Loop**: (Practical Synchronous Receiving System)



Costas Loop is used to make both the carrier signal and the locally generated signal synchronised with frequency and as well as phase.

It consists of two product modulators with a common input $s(t)$ which is DSB-SC wave.

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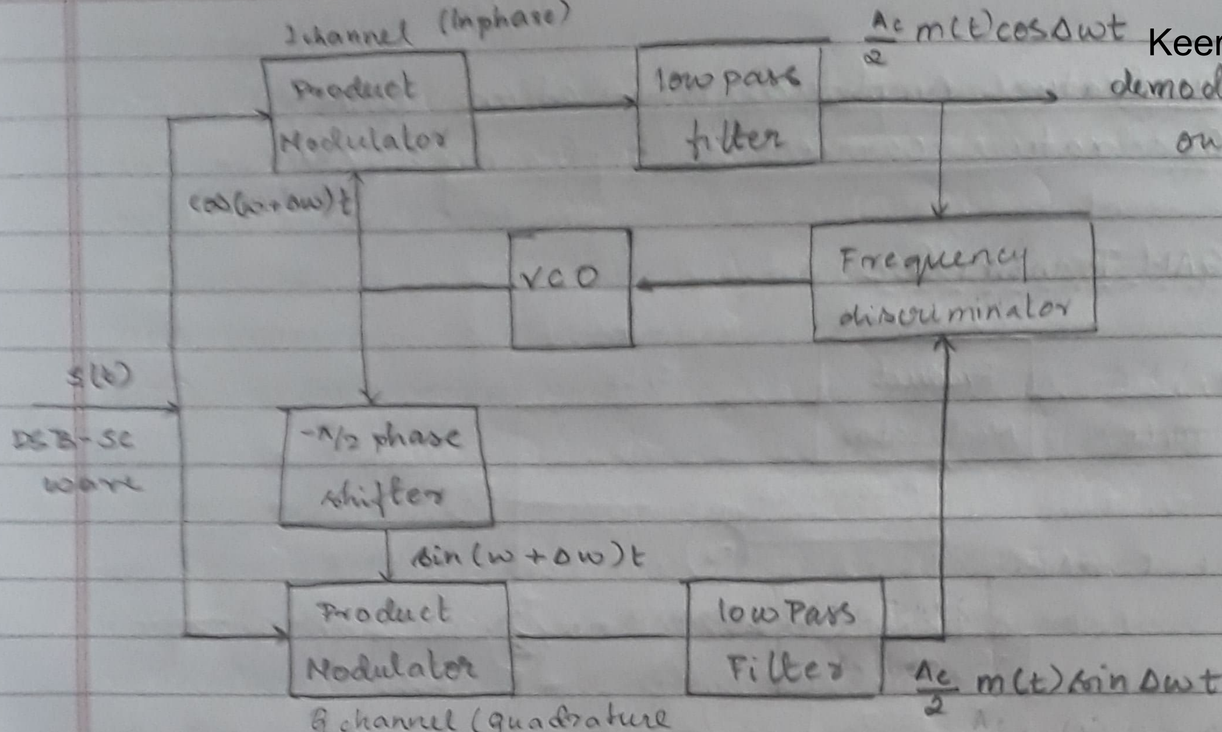

The other input for both the product modulators is taken from the voltage controlled oscillator (VCO) with a -90° phase shift to one of the product modulator to produce the quadrature component from the inphase component just by adding a 90° phase shift.

The output of the VCO is applied as the carrier input of the upper product modulator. Hence the output of the upper product modulator is applied as an input of the upper low pass filter. Therefore the output of this lowpass filter is the scaled version of the modulating signal.

The output of -90° phase shifter is applied as the carrier input of the lower product modulator. Hence its output is applied as an input of the lower low pass filter. This output of the lowpass filter has a 90° phase difference with the output of the upper low pass filter.

The outputs of these two lowpass filters are applied as inputs of the phase/frequency discriminator which produces a DC control signal based on the difference between the signals. This is given as input to VCO to correct the phase/frequency in VCO output

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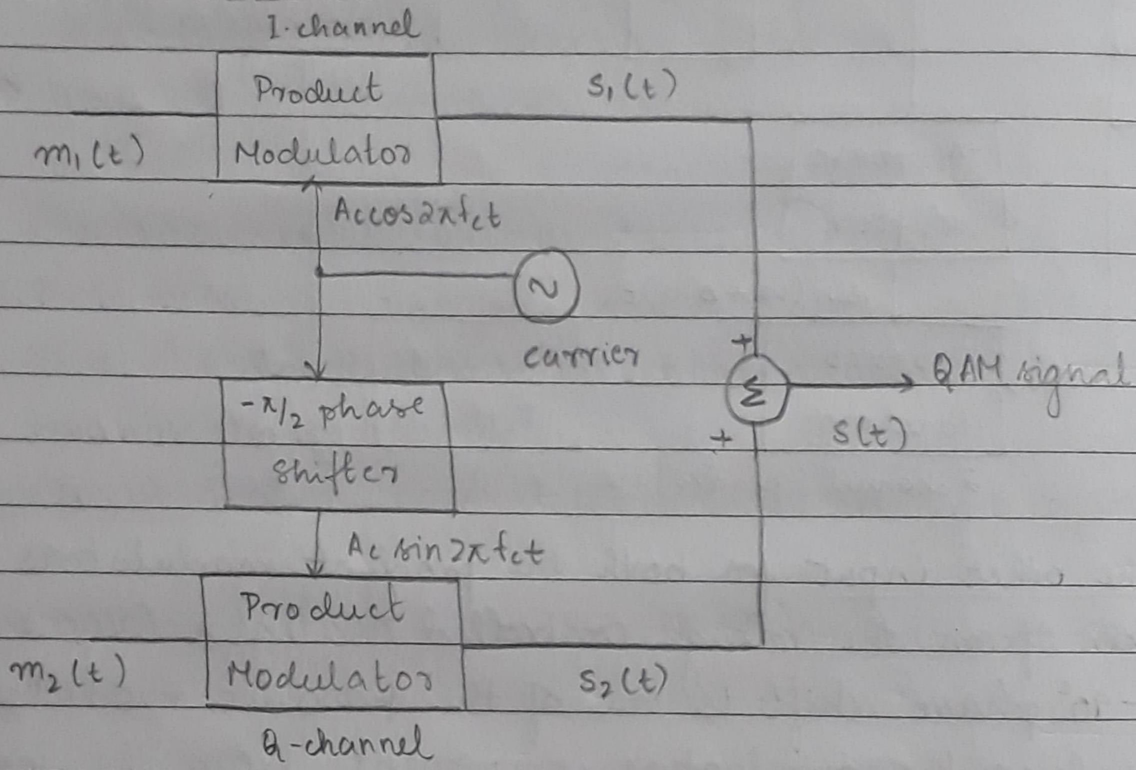
* Quadrature Amplitude Modulation : QAM
Quadrature carrier Multiplexing : QCM

- QAM Transmitter:



The I-channel product modulator receives inphase carrier : $A_c \cos 2\pi f_c t$ and the Q-channel product modulator receives quadrature carrier : $A_c \sin 2\pi f_c t$ which is derived from inphase carrier using $\pi/2$ phase shifter.

Output of I channel product modulator is a DSB-SC wave which is given by.

$$s_1(t) = m_1(t) A_c \cos 2\pi f_c t \quad \text{--- (1)}$$

Similarly the output of Q-channel product modulator is a DSB-SC wave which is given by

$$s_2(t) = m_2(t) A_c \sin 2\pi f_c t \quad \text{--- (2)}$$

Output of the QAM transmitter is

$$s(t) = s_1(t) + s_2(t) \quad \text{--- (3)}$$

Substituting eq (1) and eq (2) in eq (3)

QAM signal

$$s(t) = A_c [m_1(t) \cos 2\pi f_c t + m_2(t) \sin 2\pi f_c t] \quad (4)$$

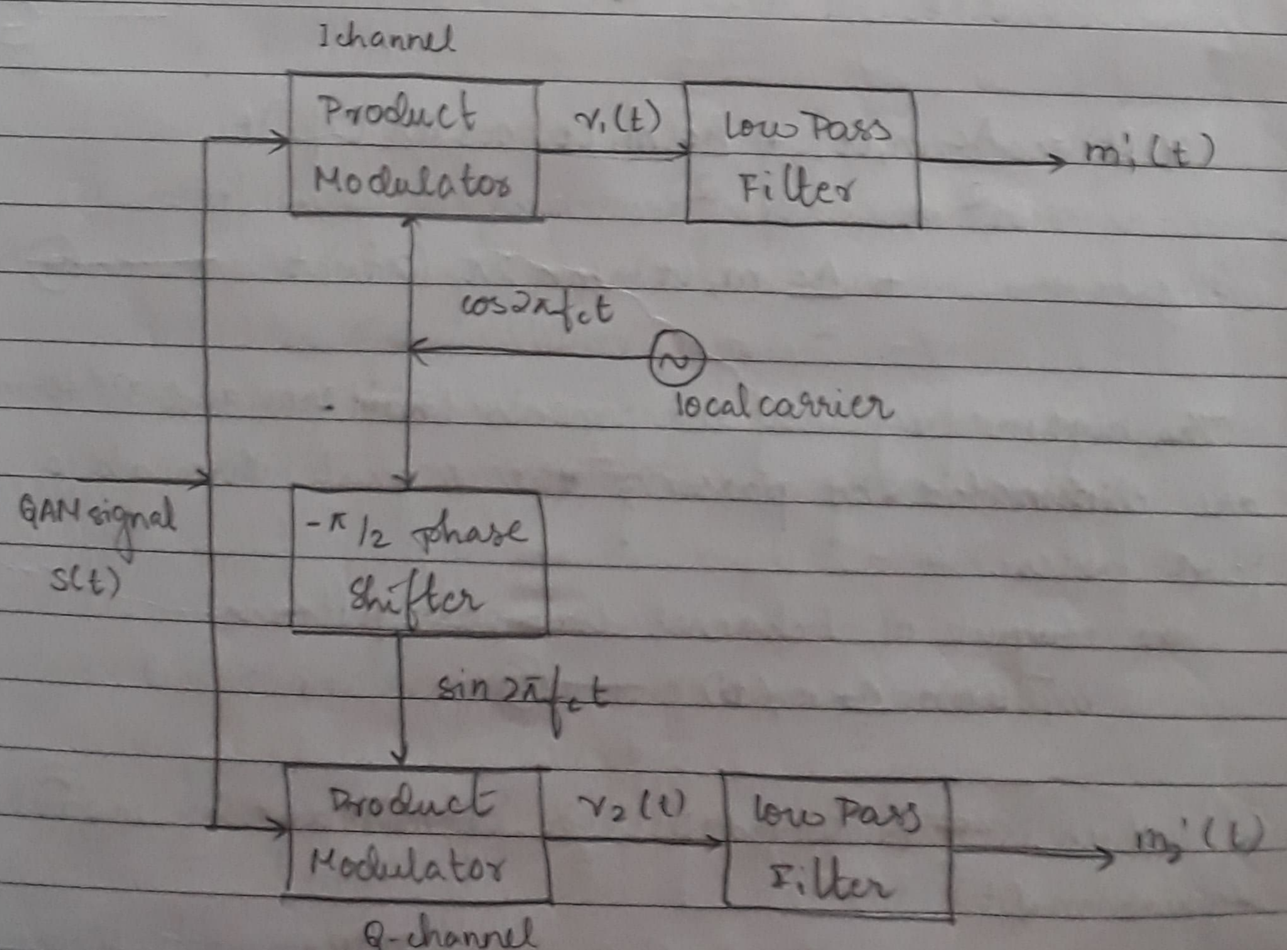
The output of the QAM transmitter is the sum of two product modulator output.

Here, the transmission band width of the QAM signal is 2w where w is the message band width of $m_1(t)$ OR $m_2(t)$ which ever is largest.

- QAM Receiver :

For QAM system to operate satisfactorily it is important to maintain the correct phase and frequency relationships between the carrier and local oscillator used in the transmitter and receiver respectively.

This requirement may be satisfied by using a costas loop.



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The QAM signals are multiplexed signals. s(t) is simultaneously applied to I channel and Q channel coherent detector that are supplied with two local carrier signals of same frequency but inphase quadrature.

→ Analysis

The output of the I channel product modulator is

$v_1(t) = s(t) \cos 2\pi f_c t$ —— ①

but QAM signal is

$s(t) = A_c [m_1(t) \cos 2\pi f_c t + m_2(t) \sin 2\pi f_c t]$ —— ②

Substituting eq ② in eq ①

$v_1(t) = A_c [m_1(t) \cos^2 2\pi f_c t + m_2(t) \sin 2\pi f_c t \cos 2\pi f_c t]$ —— ③

$$v_1(t) = A_c \left[m_1(t) \left(\frac{\cos 2\pi (2f_c) t + 1}{2} \right) + \frac{m_2(t)}{2} \sin 2\pi (2f_c) t \right]$$

$$v_1(t) = \frac{A_c}{2} m_1(t) + \frac{A_c}{2} m_1(t) \cos 2\pi (2f_c) t + \frac{A_c}{2} m_2(t) \sin 2\pi (2f_c) t \quad \text{—— ④}$$

The higher frequency components (2^{nd} term and 3^{rd} term) are eliminated by passing the signal through a low pass filter.

The output of I channel low pass filter is

$m_1''(t) = \frac{A_c}{2} m_1(t)$ —— ⑤

which is the scaled version of message or modulating signal $m_1(t)$

Similarly the output of Q channel low pass filter is

$m_2'(t) = \frac{A_c}{2} m_2(t)$ —— ⑥

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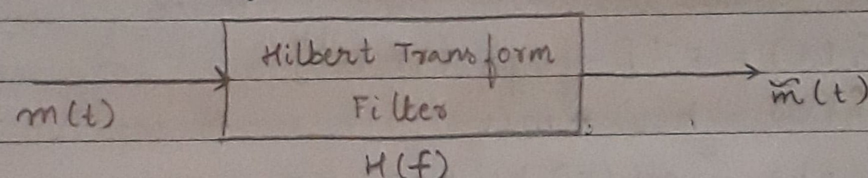
which is the scaled version of message or modulating signal $m_2(t)$.

* **Single Side Band suppressed - carrier Modulation :**

Hilbert Transform

Hilbert transform filter is a filter that phase shifts all frequency component of the input by $\pm\pi/2$ radians without changing the domain.

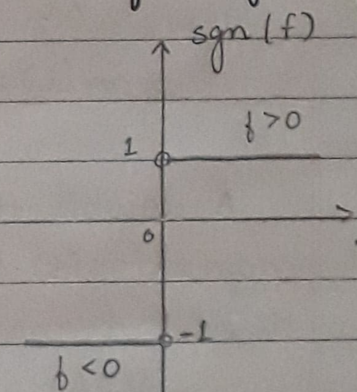


The transformer function of Hilbert transform is given by

$$H(f) = -j\ \mathrm{sgn}(f)$$

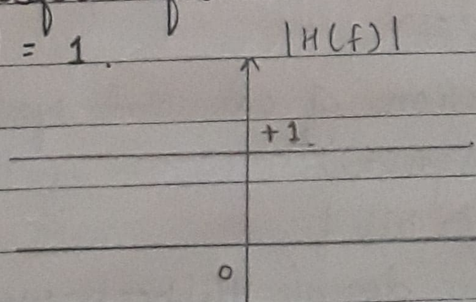
sgn : signum function

$$\mathrm{sgn}(f) = \begin{cases} +1 & ; f > 0 \\ 0 & ; f = 0 \\ -1 & ; f < 0 \end{cases}$$



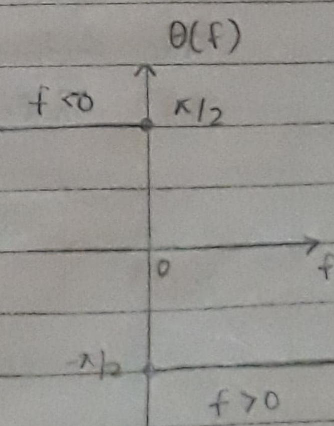
The amplitude response of the Hilbert transform filter is

$$|H(f)| = 1.$$



The angular argument of $H(f)$ is

$$\angle H(f) = \theta(f) = \begin{cases} -\pi/2 & ; f > 0 \\ 0 & ; f = 0 \\ \pi/2 & ; f < 0 \end{cases}$$



If $M(f)$ is the Fourier Transform of $m(t)$ then the output of the Hilbert Transform filter in frequency domain is

$$\hat{M}(f) = M(f)\,H(f)$$

$$\tilde{M}(f) = M(f)\,[-j\,\mathrm{sgn}(f)]$$

$$\hat{M}(f) = -j\,\mathrm{sgn}(f)\,M(f)$$

In time domain the output of the Hilbert Fourier Transform filter is

$$\hat{m}(t) = F^{-1}[-j\,\mathrm{sgn}(f)\,M(f)]$$

$$\hat{m}(t) = h(t) * m(t)$$

$$\hat{m}(t) = \int_{-\infty}^{\infty} m(\tau)\,h(t-\tau)\,d\tau$$

wkt $\dfrac{j}{\pi t} \longleftrightarrow \mathrm{sgn}(f)$

$$\therefore \hat{m}(t) = \int_{-\infty}^{\infty} \frac{m(t-\tau)}{\pi\tau}\,d\tau$$

A double Hilbert system is equivalent to passing the signal $m(t)$ through a cascade system comprising two Hilbert transform filter each having a transfer function as

$$H(f) = -j\,\mathrm{sgn}(f)$$

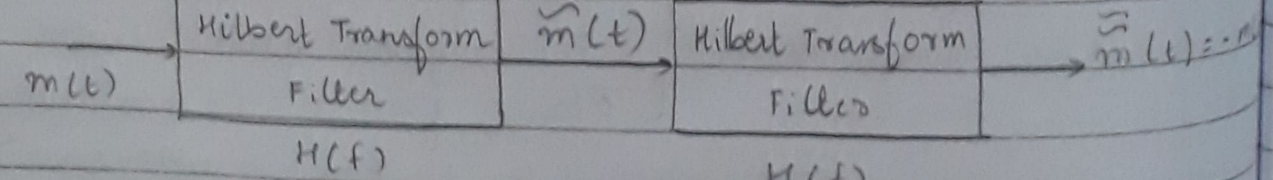
The transfer function of cascaded system is

$$H_{eq}(f) = (-j\,\mathrm{sgn}(f))^2$$

$$H_{eq}(f) = -1 \text{ for all } f$$

The output of the double Hilbert system is

$$\hat{\hat{m}}(t) = -m(t)$$



- Properties

1. Energy or power in a signal $m(t)$ and its Hilbert transform are equal.
2. The signal $m(t)$ and its Hilbert transform are orthogonal to each other that means

$$\int_{-\infty}^{\infty} m(t)\,\hat{m}(t) = 0$$

3. The magnitude spectra of $m(t)$ and its Hilbert transform are identical.
4. The Hilbert transform of an even function is odd and vice versa.
5. The Hilbert transform of $m(\alpha t) = \mathrm{sgn}(\alpha)\,\hat{m}(\alpha t)$
6. Hilbert Transform of convolution of two signals

$$\text{HT of } x(t) * y(t) = x(t) * \hat{y}(t) \text{ or } \hat{x}(t) * y(t)$$

7. Hilbert Transform of a real signal is also real.

Q: Find the Hilbert transform of $m(t) = \cos 2\pi f_c t$

- Given:

$$m(t) = \cos 2\pi f_c t$$

$$\text{HT of } m(t) = \cos(2\pi f_c t - \pi/2)$$

If f_c is positive then add $-\pi/2$ or vice versa

$$\hat{m}(t) = \sin 2\pi f_c t$$

$$\text{FT of } \hat{m}(t) = \hat{M}(f) = H(f)\,M(f)$$

$$\hat{M}(f) = -j\,\mathrm{sgn}(f)\,\underbrace{\frac{1}{2}\left[\delta(f-f_c) + \delta(f+f_c)\right]}_{\text{FT of } \cos 2\pi f_c t}$$

$$\hat{M}(f) = \frac{-j}{2}\left[\mathrm{sgn}(f)\,\delta(f-f_c) + \mathrm{sgn}(f)\,\delta(f+f_c)\right]$$

$$\hat{M}(f) = \frac{1}{2j}\left[1\,\delta(f-f_c) + (-1)\,\delta(f+f_c)\right]$$

Taking inverse Fourier Transform

$$\hat{m}(t) = \sin 2\pi f_c t$$

★ **Time domain representation of SSB-SC wave or SSB-SC wave representation using Hilbert Transform:**

The SSB-SC wave is a band pass signal and can be represented in time domain in cannonical form as

$$s(t)_{SSB-SC} = s_c(t)\cos 2\pi f_c t - s_s(t)\sin 2\pi f_c t) \quad \text{—— (1)}$$

Representation of band pass signal using its lowpass equivalent

$$s(t) = s_c(t)\cos 2\pi f_c t - s_s(t)\sin 2\pi f_c t$$

where $s_c t)$ is the inphase component of SSB-SC wave and $s_s(t)$ is the quadrature component of SSB-SC wave

The inphase component $s_c(t)$ is derived from $s(t)$ by multiplying $s(t)$ with $\cos 2\pi f_c t$ and passing the product through a low pass filter.

Similarly the quadrature component $s_s(t)$ is derived from $s(t)$ by multiplying with $\sin 2\pi f_c t$ and passing the product through LPF.

The Fourier Transform of $s_c(t)$ and $s_s(t)$ that are related to SSB-SC wave are as follows.

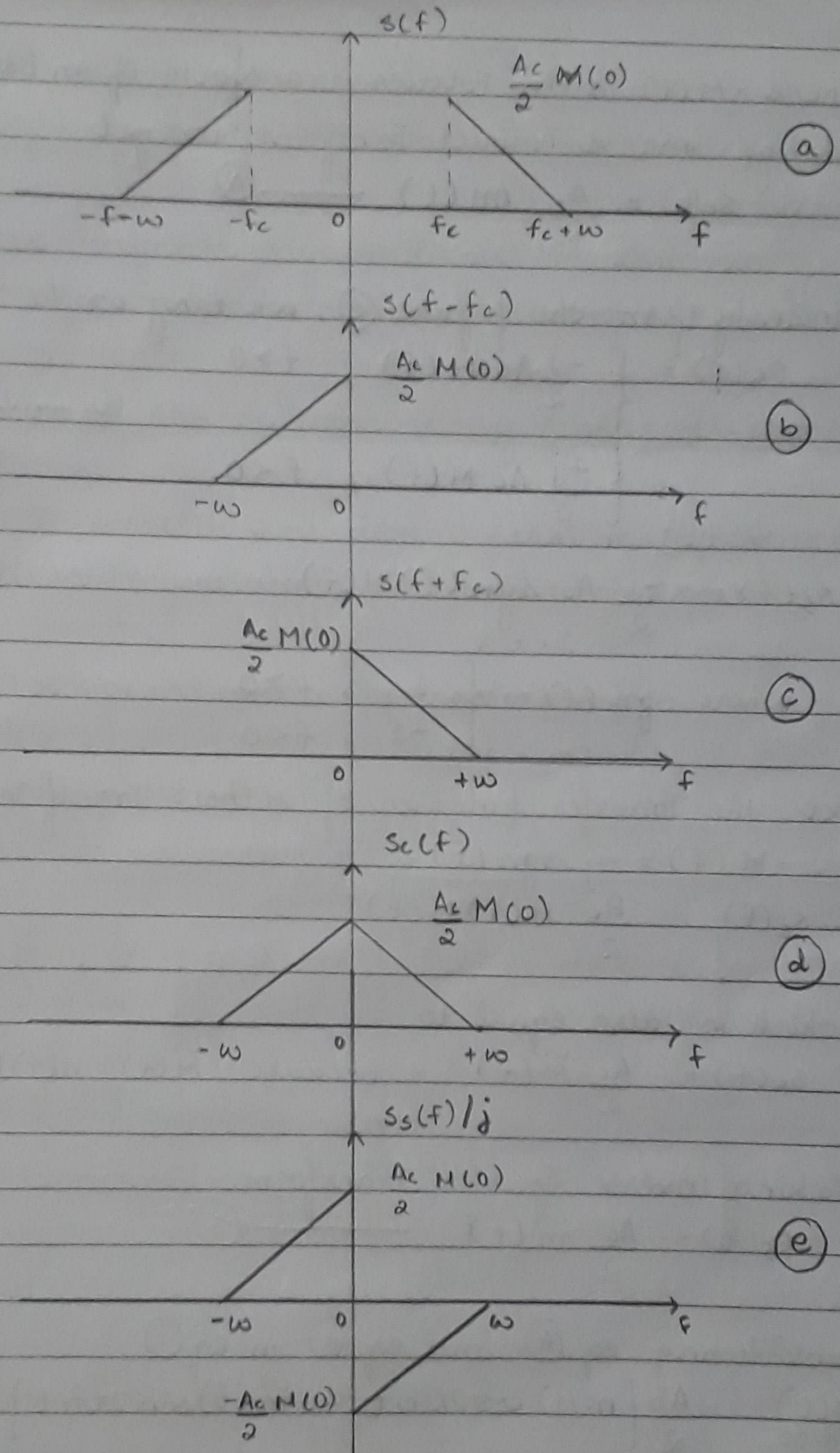
$$S_c(f) = \begin{cases} S(f-f_c) + S(f+f_c) & ; -w \le f \le w \\ 0 & ; \text{elsewhere} \end{cases}$$

Similarly

$$S_s(f) = \begin{cases} -j[S(f-f_c) - S(f+f_c)] & ; -w \le f \le w \\ 0 & ; \text{elsewhere} \end{cases}$$

where $-w \le f \le w$ defines the frequency band occupied by the message or modulating signal.

By considering SSB-SC wave that is obtained by transmitting upper side band (USB)



From the figure (d) we see that the spectrum of inphase component of SSB-SC wave is distributed between $-w$ and w and it looks like a message spectrum with scaling factor $\frac{A_c}{2}$ therefore from figure (d) we may write $s_c(f)$ as

$$S_c(f) = \frac{A_c}{2} M(f)$$

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where $M(f)$ is the Fourier transform of $m(t)$

Taking inverse Fourier Transform, we get

$$s_c(t) = \frac{A_c}{2} m(t) \quad \text{—— (A)}$$

Similarly from the figure (e), we may write

$$s_s(f) = \begin{cases} \frac{-j}{2} A_c M(f) \; ; & f > 0 \\ \frac{+j}{2} A_c M(f) \; ; & f < 0 \end{cases}$$

$$s_s(f) = \frac{-j}{2} A_c \, sgn(f) M(f)$$

where $sgn(f) = \begin{cases} 1 & , f > 0 \\ -1 & , f < 0 \end{cases}$

wkt the Transfer function of Hilbert Transform filter is

$$H(f) = -j \, sgn(f)$$

$$\therefore s_s(f) = \frac{A_c}{2} H(f) M(f)$$

which is also equal to

$$s_s(f) = \frac{A_c}{2} \hat{M}(f) \qquad \text{because } \hat{M}(f) = H(f) M(f)$$

Taking Inverse Fourier Transform

$$s_s(t) = \frac{A_c}{2} \hat{m}(t) \quad \text{—— (B)}$$

Substituting eq (A) and eq (B) in eq (1)

$$s(t)_{SSB-SC} = \frac{A_c}{2} \left[m(t) \cos 2\pi f_c t - \hat{m}(t) \sin 2\pi f_c t \right]$$

[This is valid when USB is considered]

Similarly

$$s(t)_{SSB-SC} = \frac{A_c}{2} \left[m(t) \cos 2\pi f_c t + \hat{m}(t) \sin 2\pi f_c t \right]$$

[This is valid when LSB is considered]

In general

$$s(t) = \frac{A_c}{2}\left[m(t) \cos 2\pi f_c t \mp \hat{m}(t) \sin 2\pi f_c t \right]$$

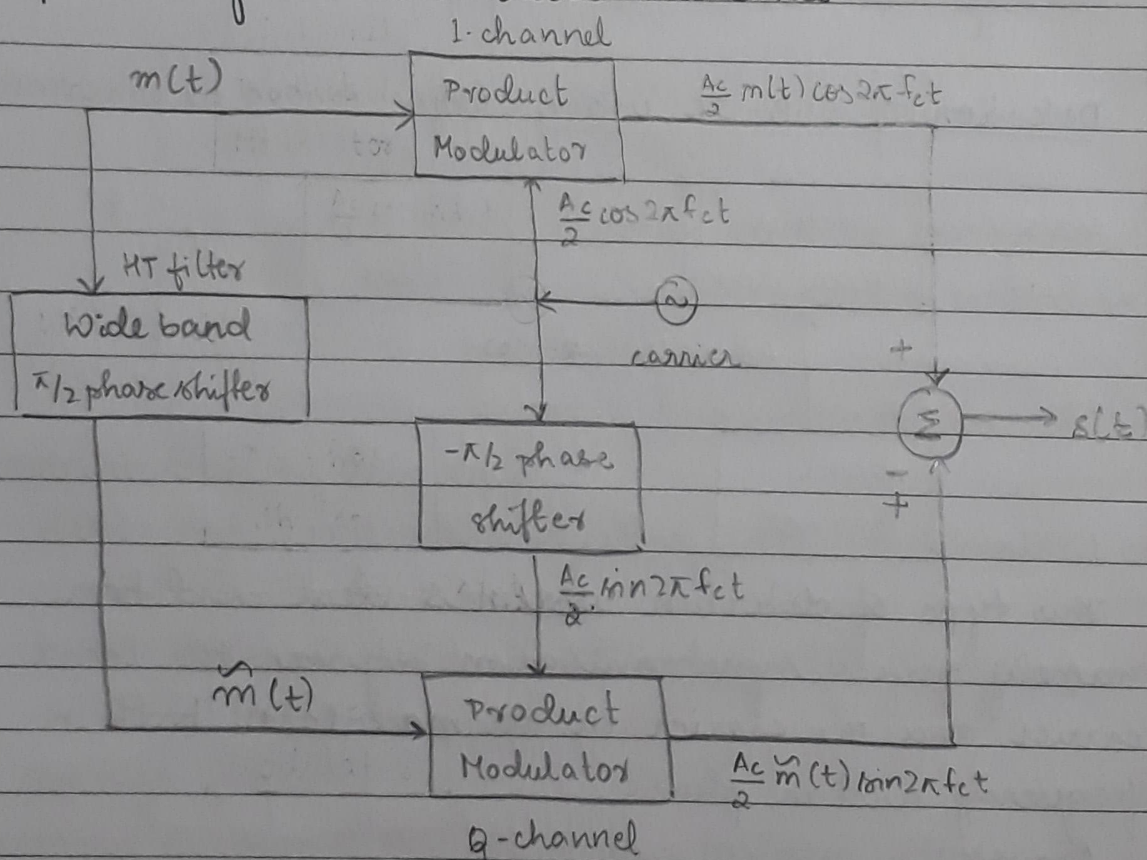
where negative sign indicates transmission of USB and positive sign indicates transmission of LSB.

* <u>Generation of SSB-SC wave using phase discrimination method:</u>

SSB-SC wave in time domain can be represented as

$$s(t) = \frac{A_c}{2}\left[m(t) \cos 2\pi f_c t \mp \hat{m}(t) \sin 2\pi f_c t \right]$$

negative sign indicates : Upper side band
positive sign indicates: lower side band



The quadrature carrier is derived from the inphase carrier using $-\pi/2$ phase shifter.

The Hilbert Transform of $m(t)$ $[\hat{m}(t)]$ is derived from $m(t)$ by using wide band $\pi/2$ phase shifter or Hilbert transform

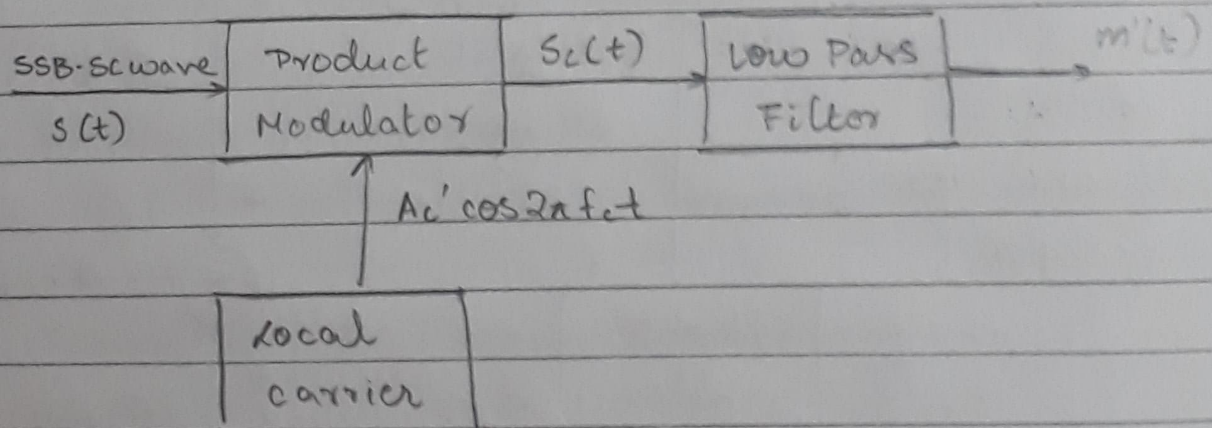
filter that shifts all frequency components by $\pm\pi/2$.
The input to the I channel product modulator is ~~m(t) and~~ the inphase component of the carrier.
i.e., $\frac{A_c}{2} \cdot \cos 2\pi f_c t$.
The input to the Q channel product modulator is HT of $m(t)$ i.e, $\hat{m}(t)$ and the quadrature componet of the carrier by passing the ~~local~~ inphase component through a $-\pi/2$ phase shifter, i.e.
$\frac{A_c}{2} \cdot \sin 2\pi f_c t$.
Hence output of I channel product modulator is

$$\frac{A_c}{2} m(t) \cos 2\pi f_c t$$

output of the Q channel product modulator is

$$\frac{A_c}{2} \hat{m}(t) \sin 2\pi f_c t$$

* Detection of SSB-SC wave: (synchronous or coherrent)



This type of detection assumes ideal condition namely perfect synchronisation between the local carrier and the carrier in the modulator both in frequency and in phase.

Anaysis:
From the figure the output of the product modulator

$$s_c(t) = s(t)\, A_c' \cos 2\pi f_c t \quad \text{--- (1)}$$

wkt $s(t) = \frac{A_c}{2}\left[m(t)\cos 2\pi f_c t - \hat{m}(t) \sin 2\pi f_c t \right]$ --- (2)

Here SSB-SC wave with USB is considered

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Substituting eq ② in eq ①

$$s_o(t) = \frac{A_cA_c'}{2}\left[m(t)\cos^2 2\pi f_ct - \hat{m}(t)\sin 2\pi f_ct \cos 2\pi f_ct\right] \quad — ③$$

$$s(t) = \frac{A_cA_c'}{4} m(t)\left[1+\cos 2\pi(2f_c)t\right] - \frac{A_cA_c'}{4}\hat{m}(t)\left[\sin 2\pi(2f_c)t\right] \quad — ④$$

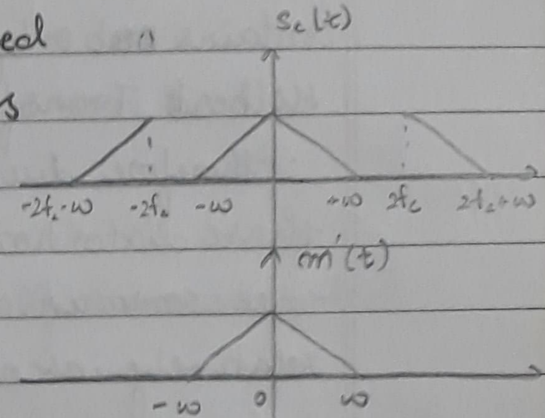
2nd and 3rd Term indicate the SSB-SC wave is distributed at $f = 2f_c$

High frequency components are eliminated by passing the signal through a low pass filter with ω as the cut off frequency.

The output of the low pass filter is

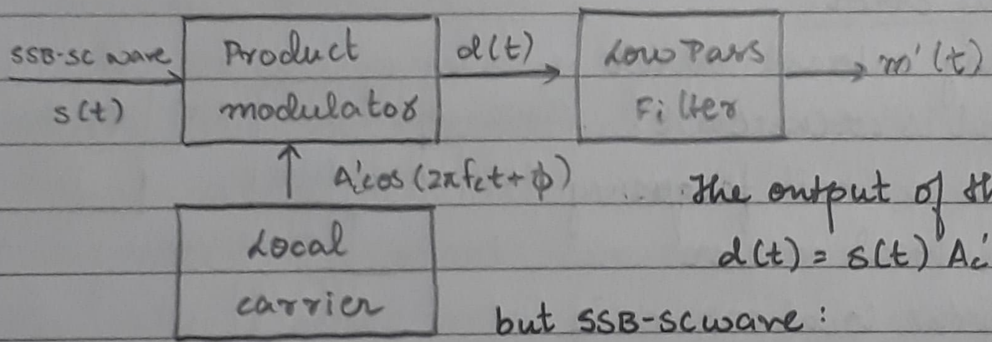
$$m'(t) = \frac{A_cA_c'}{4} m(t) \quad — ④$$

which is the scaled version $m(t)$.



Phase Error:

considering the local carrier whose output is $A_c'\cos(2\pi f_ct+\phi)$ where ϕ is the small phase error in the local carrier.



The output of the product modulator

$$d(t) = s(t)A_c'\cos(2\pi f_ct+\phi) \quad — ①$$

but SSB-SC wave:

$$s(t) = A_c/2\left[m(t)\cos 2\pi f_ct - \hat{m}(t)\sin 2\pi f_ct\right] \quad — ②$$

substituting eq ② in eq ①

$$d(t) = \frac{A_cA_c'}{2}\left[m(t)\cos 2\pi f_ct\cos(2\pi f_ct+\phi) - \hat{m}(t)\sin 2\pi f_ct\cos(2\pi f_ct+\phi)\right] \quad — ③$$

$$d(t) = \frac{A_cA_c'}{4} m(t)\left[\cos\phi + \cos(2\pi(2f_c)t)\right] - \frac{A_cA_c'}{4}\hat{m}(t)\left[\sin\phi + \sin(2\pi(2f_c)t)\right] \quad — ④$$

The high frequency component (2nd and 4th term) is eliminated by passing the signal $d(t)$ through a low pass filter with a cut off frequency ω, where ω is the message signal band width.

The output of the low pass filter is

$$m'(t) = \frac{A_cA_c'}{4} m(t)\cos\phi - \frac{A_cA_c'}{4}\hat{m}(t)\sin\phi$$

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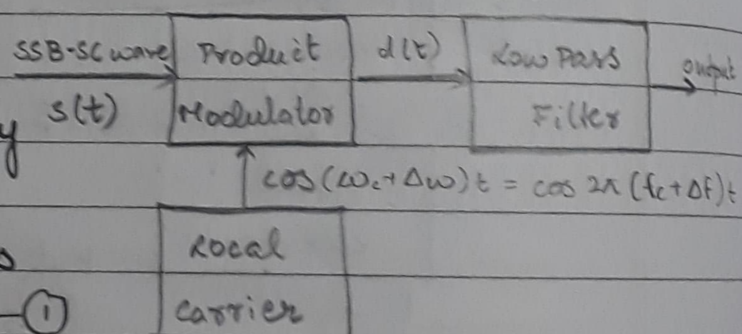
→ when $\phi = 0$, the local carrier phase is perfectly synchronised with that of the carrier at the modulator. Therefore the output of the lowpass filter is: $m'(t) = \frac{A_c A_c'}{4} m(t)$ (scaled version)

→ when $\phi \neq 0$, due to phase error ϕ, the detector output contains ~~not~~ only the message signal $m(t)$ but also its Hilbert Transform $\hat{m}(t)$.

Therefore due to phase error the detector output suffers from phase distortion. This distortion is not severe as far as voice communication is considered because the human ear is relatively insensitive to phase distortion. Whereas in the transmission of music and video signal the phase distortion can be intolerable. Thus the presence of phase distortion give rise to the Donald Duck Voice Effect.

- Frequency Error:

considering a local carrier whose output is $\cos(\omega_c + \Delta\omega)t$ where $\Delta\omega$ is a small frequency error in the local carrier



Output of the local carrier is

$d(t) = s(t)\cos(\omega_c + \Delta\omega)t$ —— ①

For SSB-SC wave: $s(t) = \frac{A_c}{2}\left[m(t)\cos\omega_c t - \hat{m}(t)\sin\omega_c t\right]$ —— ②

Substituting eq ② in eq ①

$$d(t) = \frac{A_c}{2}\left[m(t)\cos\omega_c t\cos(\omega_c + \Delta\omega)t - \hat{m}(t)\sin\omega_c t\cos(\omega_c + \Delta\omega)t\right] \quad ③$$

$$= \frac{A_c m(t)}{4}\left[\cos\Delta\omega t + \cos(2\omega_c + \Delta\omega)t\right] - \frac{A_c\hat{m}(t)}{4}\left[\sin\Delta\omega t + \sin(2\omega_c + \Delta\omega)t\right] \quad ④$$

The high frequency component (2^{nd} and 4^{th} term) are eliminated by passing the signal $d(t)$ through a lowpass filter with cut off frequency ω, where ω is the message signal band width. Hence the output of the lowpass filter is

$$\frac{A_c m(t)}{4}\cos\Delta\omega t - \frac{A_c}{4}\hat{m}(t)\sin\Delta\omega t \quad ⑤$$

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UNIT – 02

Angle Modulation

* **Angle Modulation:**

In time domain angle modulated wave can be represented as

$$s(t) = A_c \cos \theta_i(t) \quad \text{——— (1)}$$

where $\cos \theta_i(t)$ denotes the angle of the modulated carrier and

A_c is the amplitude of the unmodulated carrier and is constant.

– **Frequency Modulation:**

In FM the frequency of the carrier changes in accordance with the instantaneous value of the message signal m(t). (amplitude)

If m(t) is constant, carrier frequency is constant similarly if m(t) varies continuously then the carrier frequency also varies continuously.

Therefore the frequency at any instant known as instanteneous frequency $f_i(t)$ and is defined as:

$$f_i(t) = \frac{1}{2\pi} \frac{d\theta_i(t)}{dt} \quad \text{——— (2)}$$

wkt $\theta_i(t) = 2\pi f_i(t) t$

differentiating wrt time

$$d\theta_i(t) = 2\pi f_i(t)\, dt$$

$$\therefore f_i(t) = \frac{1}{2\pi} \frac{d\theta_i(t)}{dt}$$

The instanteneous frequency $f_i(t)$ changes linearly with the message signal and is represented by.

$$f_i(t) = f_c + k_f\, m(t) \quad \text{--- (3)}$$

where f_c represents the frequency of the unmodulated carrier and the constant k_f represents the frequency sensitivity of modulator and has the unit Hz/Volt.

From eq (2) and eq (3) we have

$$f_i(t) = \frac{1}{2\pi}\frac{d\theta_i(t)}{dt} = f_c + k_f\, m(t) \quad \text{--- (4)}$$

$$d\theta_i(t) = [2\pi f_c + 2\pi k_f\, m(t)]\, dt \quad \text{--- (5)}$$

Hence

$$\theta_i(t) = 2\pi f_c t + 2\pi k_f \int_0^t m(t)\, dt \quad \text{--- (6)}$$

Substituting eq (6) in eq (1)

The frequency modulated wave in time domain can be represented as:

$$\left[s_{FM}(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(t)\, dt\right]\right] \quad \text{--- (7)}$$

Eq (7) reveals that the envelope of FM wave is a constant and is equal to the amplitude of the unmodulated carrier.

- Phase Modulation:

The phase of the carrier changes in accordance with the instantaneous value of the message signal.

In phase modulation, the phase of the unmodulated carrier $\theta_i(t)$ changes linearly with the message signal $m(t)$ that means

$$\theta_i(t) = 2\pi f_c t + k_p\, m(t) \quad \text{--- (8)}$$

where the term $2\pi f_c t$ represents the phase of the unmodulated sinusoidal carrier having the frequency f_c, the constant k_p represents the phase sensitivity of the modulator in rad/volt.

Substituting eq ⑧ in eq ①

The phase modulated wave in time domain can be represented as:

$$\boxed{s_{PM}(t) = A_c \cos[2\pi f_c t + k_p m(t)]} \quad \text{—— (9)}$$

★ **Relation between FM and PM:**

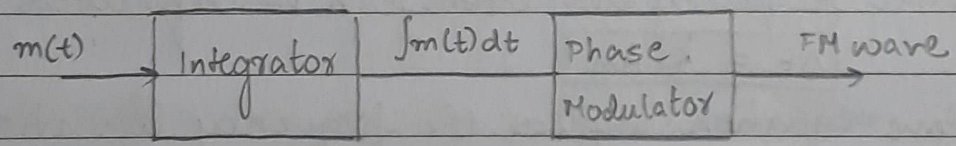
Expression for FM in time domain

$$s_{FM}(t) = A_c \cos[2\pi f_c t + 2\pi k_f \int_0^t m(t)\,dt]$$

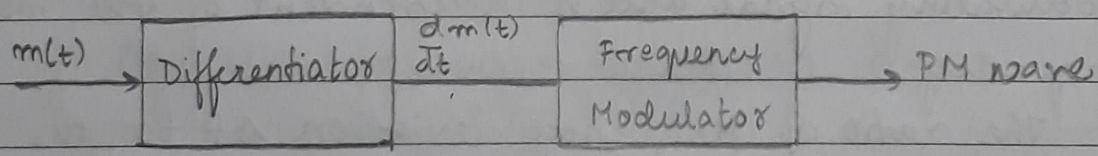
Expression for PM in time domain

$$s_{PM}(t) = A_c \cos[2\pi f_c t + k_p m(t)]$$

– Generation of FM using PM



– Generation of PM using FM



★ **Single tone FM:**

Expression for FM wave in time domain is

$$s_{FM}(t) = A_c \cos[2\pi f_c t + 2\pi k_f \int_0^t m(t)\,dt] \quad \text{—— (1)}$$

Let m(t) be the single tone message or modulating signal and is given by

$$m(t) = A_m \cos 2\pi f_m t \quad \text{—— (2)}$$

where A_m: amplitude of message signal

f_m: frequency of message signal

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Substituting eq ② in eq ①

$$s(t)_{FM} = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t A_m \cos 2\pi f_m t \, dt\right] \quad \text{—— ③}$$

$$s(t)_{FM} = A_c \cos\left[2\pi f_c t + 2\pi k_f A_m \frac{\sin 2\pi f_m t}{2\pi f_m}\right] \quad \text{—— ④}$$

we know that the instantaneous frequency of the resulting FM signal is

$$f_i(t) = f_c + k_f m(t)$$

$$f_i(t) = f_c + k_f A_m \cos 2\pi f_m t \quad \text{—— ⑤}$$

The maximum departure of the instantaneous frequency of the resulting FM signal from the carrier frequency f_c is known as Frequency Deviation Δf and is given by:

$$\Delta f = |f_i(t) - f_c|_{max} = k_f A_m \quad \text{—— ⑥}$$

Therefore the frequency deviation can be defined as:

$$\boxed{\Delta f = k_f A_m}$$

The above equation implies the frequency deviation Δf is proportional to the amplitude of the message or modulating signal and is independent of the modulating frequency.

The ratio of frequency deviation Δf to the modulating frequency f_m is called as Modulation Index and is denoted by m_f or β.

Therefore.

$$m_f \text{ or } \beta = \frac{\Delta f}{f_m} = \frac{k_f A_m}{f_m} \quad \text{—— ⑦}$$

Substituting eq ⑥ and eq ⑦ in eq ④

$$s(t)_{FM} = A_c \cos\left[2\pi f_c t + \frac{\Delta f}{f_m} \sin 2\pi f_m t\right]$$

$$\boxed{s(t)_{FM\,(\text{single tone})} = A_c \cos\left[2\pi f_c t + \beta \sin 2\pi f_m t\right]}$$

If the message signal is band limited then the ratio of frequency deviation Δf to the message bandwidth w is called as deviation ratio and is given by:

$$D = \frac{\Delta f}{w}$$

Depending on the value of β, FM wave is classified as:

a. Narrow Band FM wave (NBFM), where β is small when compared to 1 radian.

b. Wide Band FM wave (WBFM), where β is large when compared to 1 radian.

★ Narrow Band Frequency Modulation:

For single tone message signal, expression for FM wave in time domain is:

$$s_{FM}(t) = A_c \cos\left[2\pi f_c t + \beta \sin 2\pi f_m t\right] \quad \text{(derive)} \quad \text{—— (1)}$$

$$s_{FM}(t) = A_c\left[\cos 2\pi f_c t \cos(\beta \sin 2\pi f_m t) - \sin 2\pi f_c t \sin(\beta \sin 2\pi f_m t)\right] \quad \text{—— (2)}$$

By defination for narrow band FM $\beta << 1$

therefore $\cos(\beta \sin 2\pi f_m t) \approx 1$

and $\sin(\beta \sin 2\pi f_m t) \approx \beta \sin 2\pi f_m t$ —— (3)

substituting eq (3) in eq (1)

$$s_{FM}(t) = A_c\left[\cos 2\pi f_c t - \sin 2\pi f_c t \, \beta \sin 2\pi f_m t\right]$$

$$s_{FM}(t) = A_c \cos 2\pi f_c t - \beta A_c \sin 2\pi f_c t \sin 2\pi f_m t \quad \text{—— (4)}$$

$$s_{FM}(t) = A_c \cos 2\pi f_c t + \frac{\beta A_c}{2} \cos 2\pi (f_c + f_m)t - \frac{\beta A_c}{2} \cos 2\pi (f_c - f_m)t \quad \text{—— (5)}$$

The above expression indicates that the signal has a carrier component and two side bands which is similar to single tone AM signal

$$s_{AM}(t) = A_c \cos 2\pi f_c t + \frac{\mu A_c}{2} \cos 2\pi (f_c + f_m) t + \frac{\mu A_c}{2} \cos 2\pi (f_c -$$

comparing eq ⑤ and eq ⑥

- The bandwidth of the Narrow Band FM wave is equal to the bandwidth of the conventional wave

i.e, $(f_c + f_m) - (f_c - f_m) = 2 f_m$

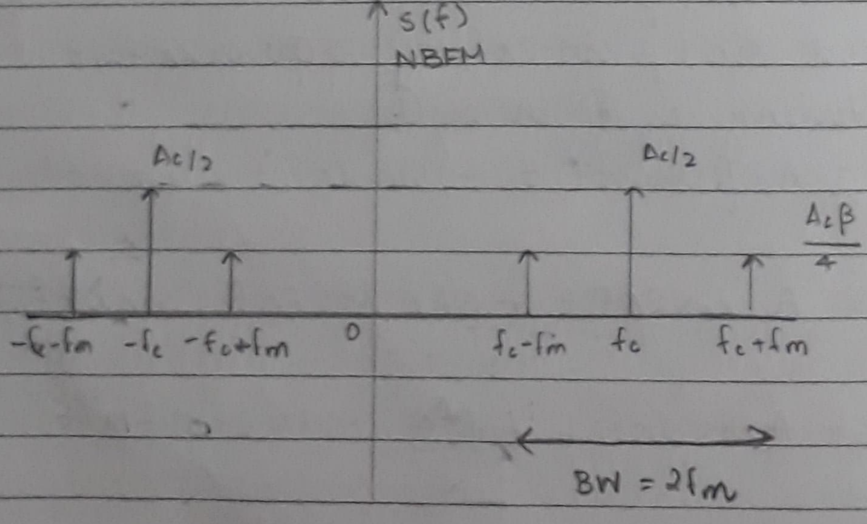
- Both the NBFM and conventional AM the side bands are symmetrically distributed with respect to the carrier frequency f_c
- Both the NBFM and conventional AM wave have the same spectral content.

Applying Fourier Transform to eq ⑤

$$S_{FM}(f) = \frac{A_c}{2} \left[\delta(f - f_c) + \delta(f + f_c) \right]$$

$$+ \frac{\beta A_c}{4} \left[\delta(f - (f_c + f_m)) + \delta(f + (f_c + f_m)) \right]$$

$$- \frac{\beta A_c}{4} \left[\delta(f - (f_c - f_m)) + \delta(f + (f_c - f_m)) \right]$$



Generation of Narrow Band FM wave:

In time domain the expression for FM wave is

$$s_{FM}(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(t)\,dt\right] \quad \text{—— (1)}$$

where $m(t)$ is band limited message signal.

Let $\phi_1(t) = 2\pi k_f \int_0^t m(t)\,dt$ —— (2)

Substituting eq (2) in eq (1)

$$s_{FM}(t) = A_c \cos\left[2\pi f_c t + \phi_1(t)\right] \quad \text{—— (3)}$$

$$s_{FM}(t) = A_c\left[\cos 2\pi f_c t \cos\phi_1(t) - \sin 2\pi f_c t \sin\phi_1(t)\right] \quad \text{—— (4)}$$

For narrow band FM $\phi_1(t) < 1$ radian.

Hence $\cos\phi_1(t) = 1$ and $\sin\phi_1(t) = \phi_1(t)$ —— (5)

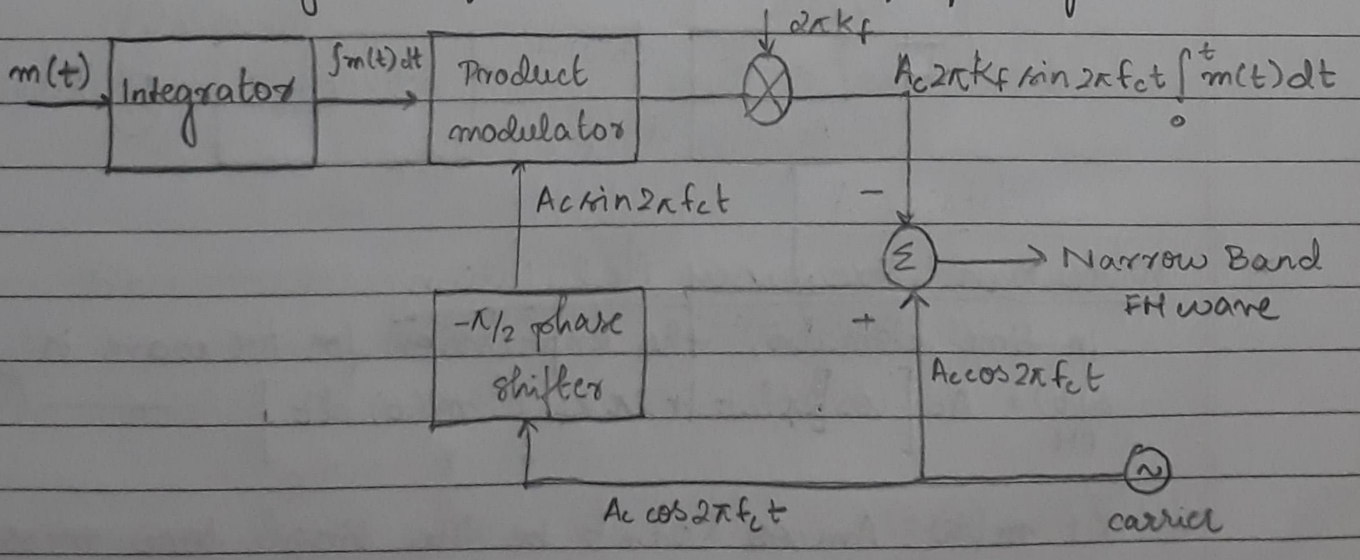
Substituting eq (5) in eq (4).

$$s_{NBFM}(t) = A_c \cos 2\pi f_c t - A_c \sin 2\pi f_c t \,.\, \phi_1(t) \quad \text{—— (6)}$$

From eq (2)

$$s_{NBFM}(t) = A_c \cos 2\pi f_c t - A_c \sin 2\pi f_c t \,.\, 2\pi k_f \int_0^t m(t)\,dt \quad \text{—— (7)}$$

The above equation form the basis for developing the block diagram of a narrow band frequency modulator.



Generation of Narrow Band Phase Modulated Wave:

In time domain the expression PM wave is

$$s_{PM}(t) = A_c \cos[2\pi f_c t + k_p m(t)] \quad \text{--- (1)}$$

$$s_{PM}(t) = A_c \cos 2\pi f_c t \cos k_p m(t) - A_c \sin 2\pi f_c t \sin k_p m(t) \quad \text{--- (2)}$$

For narrow band PM wave

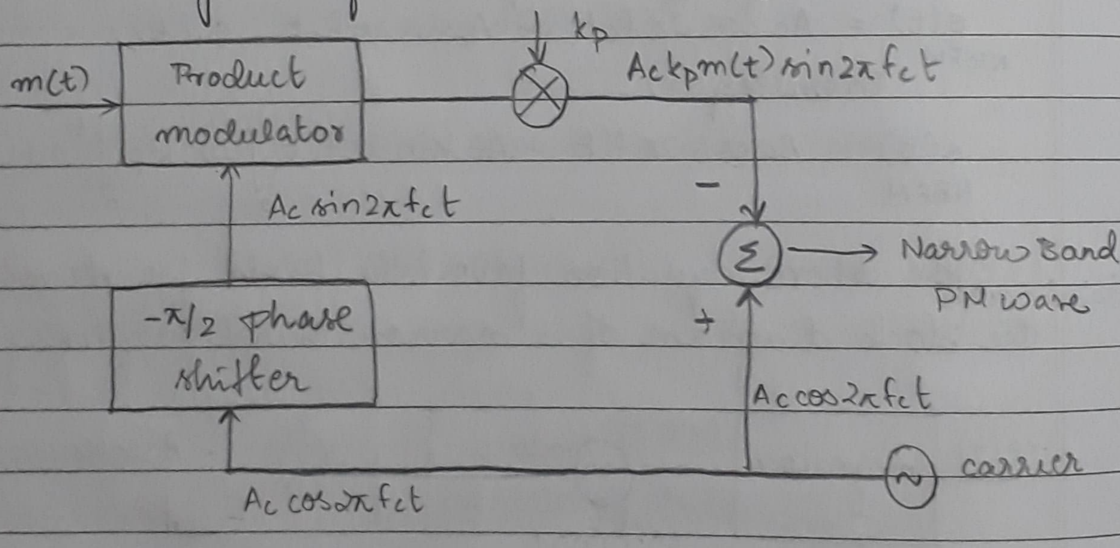
$$|k_p m(t)| << 1$$

$$\left.\begin{aligned} &\text{Therefore } \cos k_p m(t) \approx 1 \\ &\text{and } \sin k_p m(t) \approx k_p m(t) \end{aligned}\right\} \quad \text{--- (3)}$$

Substituting eq (3) in eq (2)

$$s_{PM}(t) = A_c \cos 2\pi f_c t - A_c k_p m(t) \sin 2\pi f_c t \quad \text{--- (4)}$$

The above equation forms the basis for developing the block diagram of a narrow band phase modulator.



★ Wide Band Frequency Modulation:

In time domain, the expression for FM wave is

$$s_{FM}(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(t)\, dt\right] \quad \text{--- (1)}$$

Let $m(t) = A_m \cos 2\pi f_m t$ be the single tone message or modulating signal

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Therefore

$$s(t)_{FM} = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t A_m \cos 2\pi f_m t \, dt\right] \quad \text{--- (2)}$$

$$s(t)_{FM} = A_c \cos\left[2\pi f_c t + 2\pi k_f A_m \frac{\sin 2\pi f_m t}{2\pi f_m}\right]$$

wkt $k_f A_m = \Delta f$: frequency deviation

$$s(t)_{FM} = A_c \cos\left[2\pi f_c t + \frac{\Delta f}{f_m} \sin 2\pi f_m t\right]$$

wkt m_f or $\beta = \frac{\Delta f}{f_m}$: modulation index

Therefore

$$s(t)_{FM} = A_c \cos\left[2\pi f_c t + \beta \sin 2\pi f_m t\right] \quad \text{--- (3)}$$

$$\therefore s(t)_{FM} = A_c \, \text{Real}\left[e^{j2\pi f_c t} \cdot e^{j\beta \sin 2\pi f_m t}\right] \quad \text{--- (4)}$$

The 2nd term is periodic with a period equal to $1/f_m$. Hence it can be represented by a ~~complete~~ complex Fourier series.

A periodic signal $g(t)$ has a complex fourier series given by :

$$g(t) = \sum_{n=-\infty}^{\infty} F_n e^{jn\omega_0 t} \quad \text{--- (5)}$$

where F_n is the complex fourier coefficient given by

$$F_n = \frac{1}{T} \int_{-T/2}^{T/2} g(t) e^{-jn\omega_0 t} dt \quad \text{--- (6)}$$

Since the term $e^{j\beta \sin 2\pi f_m t}$ is a periodic function

$$g(t) = e^{j\beta \sin 2\pi f_m t} \quad \text{--- (7)}$$

where $\omega_0 = 2\pi f_m$

$$T = \frac{1}{f_m}$$

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Substituting eq (7) in eq (5)

$$g(t) = e^{j\beta\sin 2\pi f_m t} = \sum_{n=-\infty}^{\infty} F_n e^{j2\pi f_m n t} \quad (\omega_0 = 2\pi f_m) \qquad (8)$$

$$\text{where } F_n = \frac{1}{1/f_m}\int_{-1/2f_m}^{1/2f_m} e^{j\beta\sin 2\pi f_m t}\, e^{-j2\pi f_m n t}\, dt \qquad (9)$$

putting $2\pi f_m t = x$

$\therefore 2\pi f_m dt = dx$

$$\therefore F_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j(\beta\sin x - nx)}\, dx \qquad (10)$$

$$F_n = J_n(\beta) \qquad (11)$$

where $J_n(\beta)$ is the Bessel Function of First kind and n^{th} order with argument β.

Therefore

$$e^{j\beta\sin 2\pi f_m t} = \sum_{n=-\infty}^{\infty} J_n(\beta)\, e^{j2\pi f_m n t} \qquad (12)$$

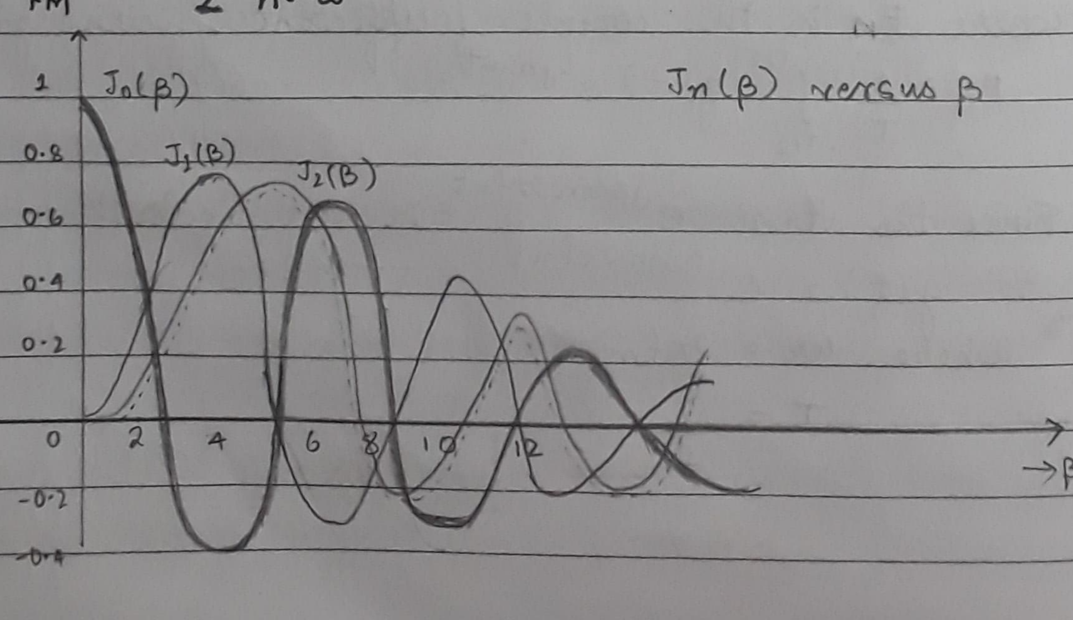
Substituting eq (12) in eq (4)

$$s_{FM}(t) = A_c \,\text{Real}\left[e^{j2\pi f_c t} \sum_{n=-\infty}^{\infty} J_n(\beta)\, e^{j2\pi f_m n t} \right] \qquad (13)$$

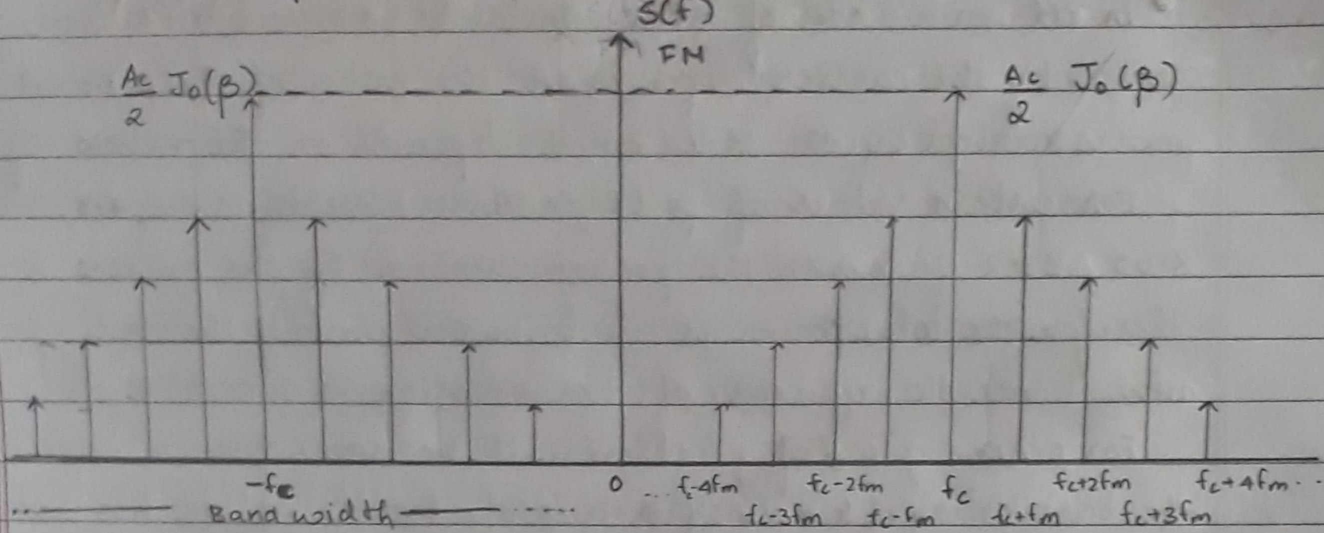
$$s_{FM}(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos 2\pi [f_c + n f_m] t \qquad (14)$$

Applying Fourier Transform

$$s_{FM}(f) = \frac{A_c}{2} \sum_{n=-\infty}^{\infty} J_n(\beta) \left[\delta(f - (f_c + n f_m)) + \delta(f + (f_c + n f_m)) \right] \qquad (15)$$



Amplitude spectrum of FM wave



- The spectrum of FM signal contains the carrier component at the center ($A_c/2 \cdot J_0(\beta)$) and an infinite set of sidebands located symmetrically on either side of the carrier frequency at a frequency separation of $f_m, 2f_m, 3f_m \dots$
  Thus theoretically the FM wave has infinite band width.
- When β is small when compared to 1 radian then only the Bessel coefficients $J_0(\beta), J_1(\beta)$ are significant values. Consequently FM signal effectively consists of carrier components and two side bands $f_c - f_m$ and $f_c + f_m$ (conditions for narrow band FM)
- For $\beta >> 1$, $J_0(\beta), J_1(\beta), J_2(\beta) \dots J_n(\beta)$ where $n \simeq \beta$ have significant values.
  For $J_n(\beta)$ for $n > \beta$ have negligible values.
  Therefore the number of sidebands are decided by the modulation index, β. and hence the band width.
- Since the amplitude of FM wave remains same as that of the unmodulated carrier. Therefore the average power of FM signal is equal to unmodulated carrier.

$$\text{Average power of FM signal} = \text{Average power of unmodulated carrier} = \frac{A_c^2}{2}$$

- For n is odd, odd numbered lower side band is reversed in phase. ($J_{-n}(\beta) = -J_n(\beta)$).

- In FM wave, out of total available power ($A_c^2/2$), the power carried by the carrier depends on the value of $J_0(\beta)$ and the power carried by the side bands depends on the values of $J_n(\beta)$.

For certain values of β (Eigen Values) such as 2.44, 5.552, 8.65, 11.2 for which the power carried by the carrier is zero. Therefore the total power is distributed among the side bands and hence the efficiency of transmission is 100%.

For other values of β there will be some power carried by the carrier and hence efficiency of transmission lies between 33.3% to 100%.

- Transmission Bandwidth:

In FM the number of side bands in a spectrum is decided by the modulation index β. As β increases, number of side bands increases and hence the bandwidth.

For smaller values of β (compared to 1 radian), only $J_0(\beta)$, $J_1(\beta)$ are significant hence the significant side bands are $f_c - f_m$ and $f_c + f_m$. Therefore the transmission bandwidth $B_T = (f_c + f_m) - (f_c - f_m) = 2f_m$

For larger values of β (compared to 1 radian), $J_n(\beta) \simeq 0$, for $n > \beta$, that is $n \simeq \beta$ frequency components are significant. Therefore the side bands are starting from $f_c - nf_m$ to $f_c + nf_m$ or $f_c - \beta f_m$ to $f_c + \beta f_m$

Therefore the transmission bandwidth

$B_T = 2n f_m = 2\beta f_m$.

- Carson's Rule

Carson generalised the bandwidth formula for an FM wave. According to carson, the approximate formula for the bandwidth of an FM signal generated by single tone message signal of frequency f_m is given by

$B_T \simeq 2(\beta + 1) f_m$ ——— (1)

wkt $\beta = \frac{\Delta f}{f_m}$ $\Rightarrow$ $\Delta f = \beta f_m$

Therefore

$$B_T \simeq 2\Delta f + 2f_m \quad \text{--- (2)}$$

$$B_T \simeq 2\Delta f \left[1 + \frac{1}{\beta}\right] \quad \text{--- (3)}$$

For band limited m(t), the deviation ratio is

$$D = \frac{\Delta f}{W} \quad \text{--- (4)}$$

Therefore bandwidth

$$B_T = 2W(1 + D) \quad \text{--- (5)}$$

where W is the message bandwidth.

Q: An angle modulated signal is defined by $s(t) = 10 \cos(2\pi \times 10^6 t + 0.2 \sin 2000\pi t)$ volts is given.

Find i. The power in the modulated signal

ii. Frequency deviation. (Δf)

iii. Phase deviation ($\Delta\theta$)

iv. Approximate transmission bandwidth.

→ For angle modulation

$$s(t) = A_c \cos \theta_i(t)$$

Comparing with the given expression

$$A_c = 10\,V.$$

i. Power in the modulated signal.

$$P = \frac{A_c^2}{2} = \frac{(10)^2}{2} = \underline{50 \text{ watts}}$$

ii. The instantaneous phase of the angle modulated wave is

$$\theta_i(t) = 2\pi \times 10^6 t + 0.2 \sin 2000\pi t.$$

$$\omega_i(t) = \frac{d}{dt}\theta_i(t) = 2\pi \times 10^6 + 0.2(2000\pi)\cos 2000\pi t$$

$$\omega_i(t) = 2\pi f_i(t)$$

Frequency deviation

$$\Delta f = \frac{|\omega_i(t) - \omega_c|_{max}}{2\pi}$$

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$$\Delta f = \frac{|2\pi \times 10^6 + 0.2(2000\pi)\cos 2000\pi t - 2\pi \times 10^6|_{max}}{2\pi}$$

$$\Delta f = |200\cos 2000\pi t|_{max}$$

$$\underline{\Delta f = 200\ Hz}$$

iii. Phase deviation

$$\Delta\theta = |\theta_i(t) - \theta_c|_{max}$$

where $\theta_c = 2\pi \times 10^6 t$

$$\Delta\theta = |2\pi \times 10^6 t + 0.2\sin 2000\pi t - 2\pi \times 10^6 t|_{max}$$

$$\Delta\theta = |0.2\sin 2000\pi t|_{max}$$

$$\underline{\Delta\theta = 0.2 \text{ radians}}$$

iv. Approximate transmission bandwidth.

$$B_T = 2(\Delta f + f_m)$$

wkt $f_m = \frac{2000\pi}{2\pi} = 1000\ Hz$

$$B_T = 2(200 + 1000)$$

$$\underline{B_T = 2400\ Hz}$$

Modulation index

$$\beta = \frac{\Delta f}{f_m} = \frac{200}{1000} = 0.2$$

The bandwidth is small as β is small.

Q: An unmodulated carrier has amplitude of 10V and frequency 100MHz. A sinusoidal waveform of frequency 1kHz, frequency modulates this carrier such that frequency deviation is 75kHz. The modulated waveform passes through zero and is increasing at time t=0. Write the time domain expression for the modulated wave

Given: $A_c = 10V$ $\Delta f = 75kHz$

$f_c = 100MHz$ $f_m = 1kHz$

Since the modulated waveform passes through 0 and is increasing at $t=0$, the FM signal is sine wave and is of the form:

$$s(t) = A_c \sin\left[2\pi f_c t + 2\pi k_f \int_0^t m(t)\,dt\right]$$

If $m(t) = A_m \cos 2\pi f_m t$

$$\therefore s(t) = A_c \sin\left[2\pi f_c t + 2\pi k_f \int_0^t A_m \cos 2\pi f_m t\,dt\right]$$

$$s(t) = A_c \sin\left[2\pi f_c t + 2\pi k_f A_m \frac{\sin 2\pi f_m t}{2\pi f_m}\right]$$

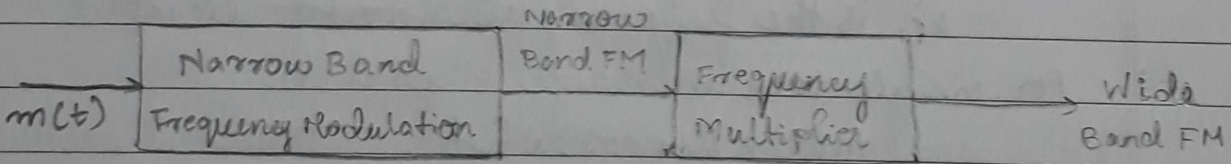
$$\therefore s(t) = A_c \sin\left[2\pi f_c t + \beta \sin 2\pi f_m t\right]$$

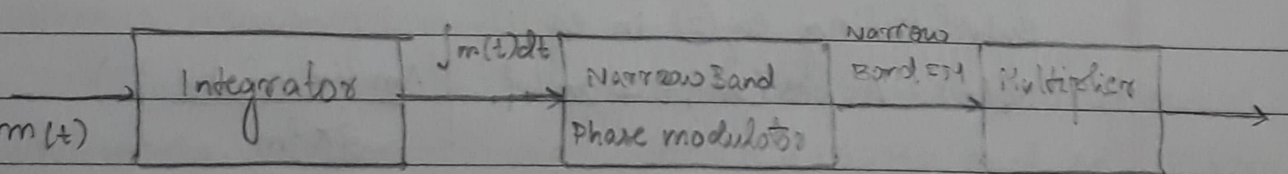
by given information.

$$\beta = \frac{\Delta f}{f_m} = \frac{75k}{1k} = 75$$

$$\therefore s(t) = 10 \sin\left[2\pi(100\times10^6)t + 75 \sin 2\pi(10^3)t\right]$$

* Generation of wide Band Frequency Modulation:
Indirect method (Armstrong method)

a.


b.


A narrow band FM is generated by using an integrator followed by a narrow band Phase modulator. In narrow band PM, the carrier is generated by using a crystal oscillator. (For better stability)

Care must be taken to ensure that $|k_p m(t)| \ll 1$ in order to minimise the distortion in the PM wave.

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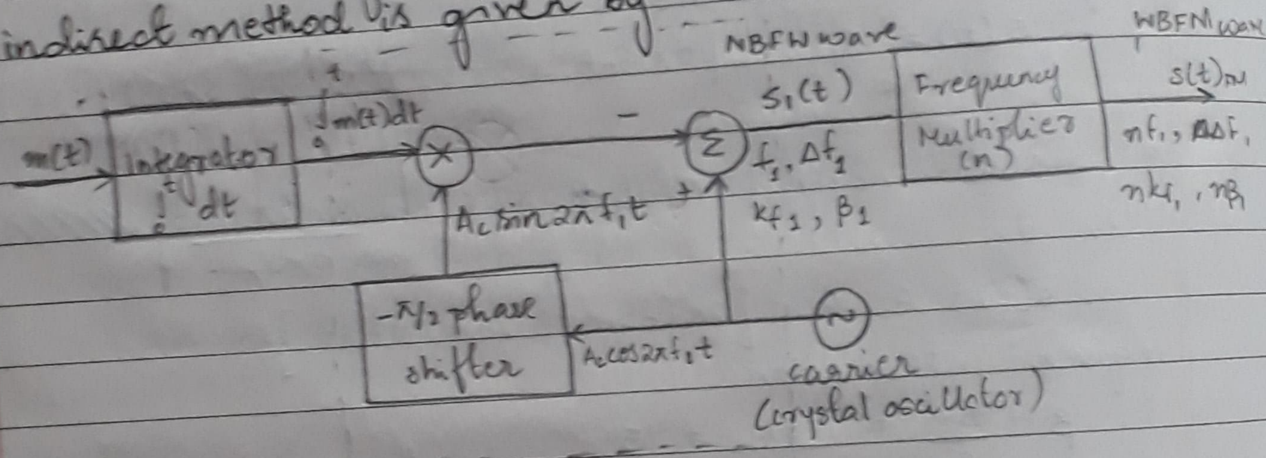
The narrow band FM signal thus generated is applied to the frequency multiplier to increase the frequency deviation to a desired level. The frequency multiplier provides the desired frequency deviation and carrier frequency simultaneously.

The block diagram for generating FM wave using indirect method is given by:



NBPM

The output of the narrow band PM is a narrow band FM wave and is given by:

$$s_1(t) = A_1 \cos\left(2\pi f_1 t + 2\pi k_{f_1} \int^t m(t)\,dt\right) \quad \text{——— (1)}$$

where f_1 is the frequency of the crystal oscillator

k_{f_1} is the frequency sensitivity (constant)

For single tone message signal:

$$m(t) = A_m \cos 2\pi f_m t \quad \text{——— (2)}$$

Substituting eq (2) in eq (1)

$$s_1(t) = A_1 \cos\left(2\pi f_1 t + \cancel{2\pi} k_{f_1} \left[\frac{A_m \sin 2\pi f_m t}{\cancel{2\pi} f_m}\right]\right)$$

$$s_1(t) = A_1 \cos\left(2\pi f_1 t + \beta_1 \sin 2\pi f_m t\right) \quad \text{——— (3)}$$

where $\beta = \dfrac{k_{f_1} A_m}{f_m}$

In the above equation the modulation index β_1 is kept below 0.3 radians to minimise the distortion. (by keeping k_{f_1} small).

The output of the narrow band phase modulator is then multiplied by a frequency multiplier with a multiplication factor n. The output of the multiplier is the wide band FM wave in time domain. It is given by:

$$s_{FM}(t) = A_c \cos\left(2\pi n f_1 t + 2\pi n k_{f_1} \int_0^t m(t)\,dt\right) \quad \text{---} (4)$$

Let $f_c = nf_1$, $k_f = nk_{f_1}$ then

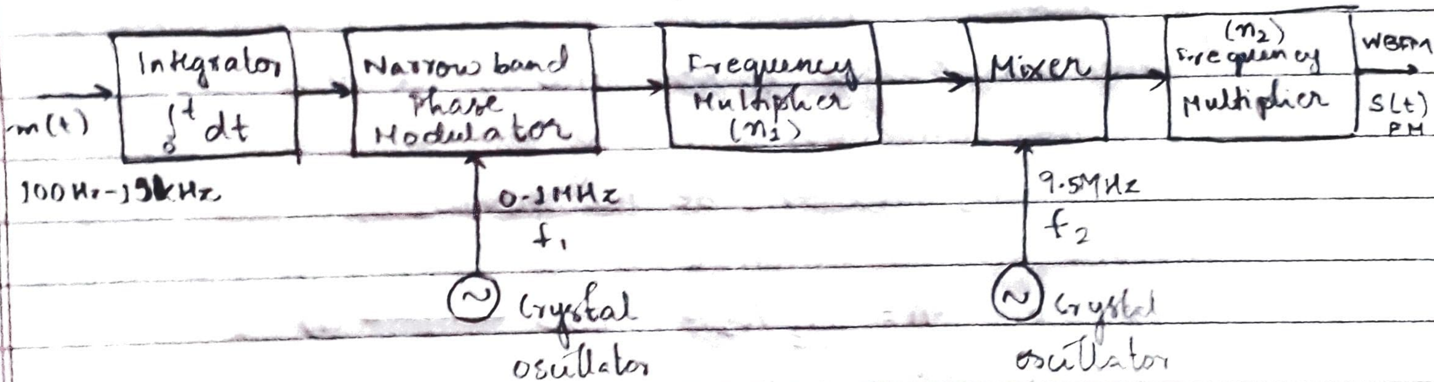
$$s_{FM}(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(t)\,dt\right); \quad \text{---} (5)$$

For single tone message signal

$$s_{FM}(t) = A_c \cos(2\pi f_c t + \beta \sin 2\pi f_m t) \quad \text{---} (6)$$

where $\beta = \frac{\Delta f}{f_m} = \frac{A_m k_f}{f_m} = n\beta_1$

★ **Commercial wide band FM system:**



Mixer = Product modulator followed by band pass filter
Mixer = Frequency translator

For commercial use it is required to transmit audio signal in the frequency range 100 Hz – 15 kHz and $\Delta f = 75$ kHz. Let us assume the final carrier frequency of FM wave be of 100 MHz.

The crystal oscillator connected to narrow band phase modulator produces a sinusoidal carrier of frequency $f_1 = 0.1$ MHz. The output of the Narrow band PM is a narrow band FM wave with modulation index β_1, such that β_1 is taken as 0.2 radian

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(less than 0.3 radians) to minimise harmonic distortion produced by the phase modulator.

The lowest modulation frequency of 100 Hz ~~15kHz~~ produces a frequency deviation of $\Delta f_1 = \beta_1 f_m = 0.2(100) = 20\,Hz$

Similarly the highest modulation frequency of 15kHz produces a frequency deviation of $\Delta f_2 = \beta_1 f_m = 0.2(15k) = 3\,kHz$.

The desired frequency of 75kHz from 20Hz corresponding to $f_m = 100\,Hz$ is obtained by using a total frequency multiplication by a factor $n = \frac{75\,kHz}{20\,Hz} = \frac{\text{desired deviation}}{\text{deviation due to } f_m = 100\,Hz} = 3750$.

The frequency multiplication in a single step will result in a large carrier frequency than the desired carrier frequency of 100 MHz.

To achieve the desired deviation and carrier frequency use multiplier in stages along with mixer.

The heterodyne mixer translates the carrier frequency without changing the frequency deviation Δf.

Let n_1 and n_2 be the frequency multiplication factors for the multipliers as shown in the figure.

Therefore $n = n_1 n_2 = 3750$ —— ①

Let f_2 be the frequency of the local oscillator connected to the mixer. The frequency at the output of the mixture is: $f_2 - n_1 f_1$

output of the first multiplier { The output of the (first) frequency multiplier is $n_1 f_1$

Frequency deviation $= 20\,Hz\,(n_1)$ for $f_m = 100\,Hz$

The output of the second multiplier is $f_c = 100\,MHz$

Input to the second multiplier is f_c/n_2 is equal to the output of the mixer $f_2 - n_1 f_1$

i.e. $\frac{f_c}{n_2} = f_2 - n_1 f_1$ —— ②

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using eq ① and eq ② to solve for n_1 and n_2

given $f_c = 100\,MHz$, $f_1 = 0.1\,MHz$ $f_2 = 9.5\,MHz$

$$n_1\left[\frac{f_c}{f_2 - n_1 f_1}\right] = 3750$$

$$n_1\left[\frac{100\times10^6}{9.5\times10^6 - n_1(0.1\times10^6)}\right] = 3750$$

$$n_1(100) = 3750\,[9.5 - n_1(0.1)]$$

$$n_1(100 + 375) = 35625$$

$$n_1 = \frac{35625}{475} = \underline{\underline{75}}$$

$$n_2 = \underline{\underline{50}}$$

stage	carrier Frequency	Frequency deviation.
output of narrow band PM	$0.1\,MHz$	$20\,Hz$ for $f_m = 100\,Hz$
Output of first multiplier	$0.1\times75 = 7.5\,MHz$ $f_1 n_1$	$20\times75 = 1.5\,kHz$ $\Delta f_1 n_1$
output of first mixer	$f_2 - n_1 f_1$ $= 9.5 - 7.5 = 2\,MHz$	$1.5\,kHz$
Output of second ~~mixer~~ multiplier	$2\times50 = 100\,MHz$ $(f_2 - n_1 f_1)n_2$	$1.5\times50 = 75\,kHz$

$f_c = 100\,MHz$ $f_m = 100\,Hz$

$\Delta f = 75\,kHz$

$$\beta = \frac{75k}{100} = 750$$

$f_m = 15\,kHz$

$$\beta = \frac{75k}{15k} = 5$$

★ **Wide Band FM generation using Direct method:**

In the direct method, the instantaneous carrier frequency is varied directly in accordance with the message signal. A VCO (voltage controlled oscillator) can be used to generate a FM signal.

A VCO has a sinusoidal oscillator having a tank or resonant circuit in which the capacitance will vary in accordance with message or modulating signal.

A fixed capacitor in parallel with a voltage variable capacitor called varactor or varactor diode, can be used in the frequency selective network

The capacitance of a reverse biased varactor diode depends on the voltage applied across its p-n junction. Larger the reverse voltage, smaller the transition capacitance.

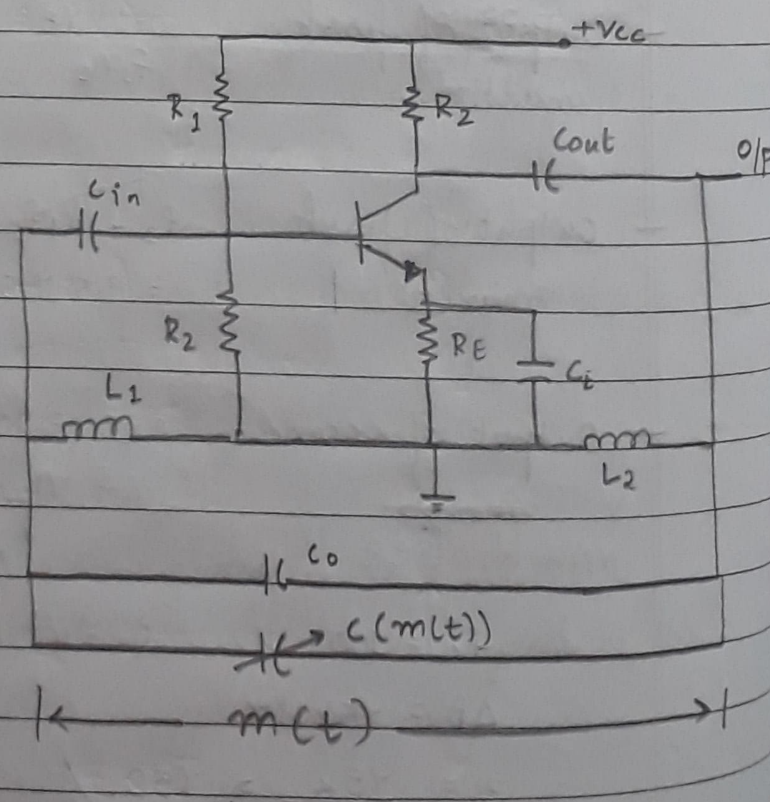
Consider VCO in the form of Hartley oscillator

From the figure the frequency of oscillation is given by

$$f_i(t) = \frac{1}{2\pi\sqrt{(L_1+L_2)C(t)}} \quad \text{—①}$$

where L_1 and L_2 are inductors and the total capacitance, $c(t)$

$$c(t) = C_0 + C(m(t)) \quad \text{—②}$$



HARTLEY OSCILLATOR

Suppose a message or modulating signal $m(t)$ (sinusoidal) with frequency f_m, the total capacitance $c(t)$ is given by

$$c(t) = C_0 + C_m \cos 2\pi f_m t \quad \text{—③}$$

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where C_0 is the total capacitance in the absence of modulation and C_m is the maximum change in the total capacitance.

Substituting eq ③ in eq ①

$$f_i(t) = f_0 \left[1 + \frac{C_m}{C_0} \cos 2\pi f_m t \right]^{-1/2} \quad \text{--- ④}$$

where f_0 is the frequency of the unmodulated carrier.

$$f_0 = \frac{1}{2\pi \sqrt{(L_1 + L_2) C_0}} \quad \text{--- ⑤}$$

Expanding eq ④ using binomial theorem

$$\left[(1+x)^{-1/2} \right] = 1 - \frac{x}{2} \quad ; \quad |x| << 1$$

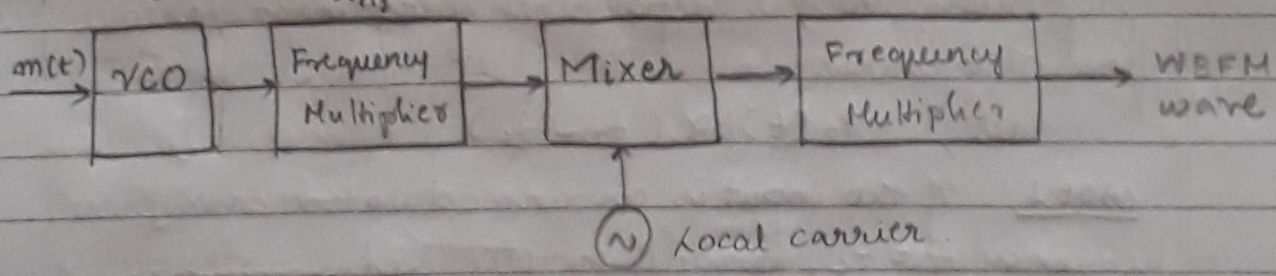
therefore

$$f_i(t) = f_0 \left[1 - \frac{C_m}{2C_0} \cos 2\pi f_m t \right] \quad ; \quad \left| \frac{C_m}{C_0} \right| << 1 \quad \text{--- ⑥}$$

Let $\frac{-C_m}{2C_0} = \frac{\Delta f}{f_0}$

then $f_i(t) = f_0 + \Delta f \cos 2\pi f_m t$

Whenever the frequency deviation generated by the direct method is not sufficient, then use multipliers in stages along with a mixer.



★ FM detection using balanced Frequency Discriminator

In balanced frequency discriminator, two slope detectors are connected in balanced configuration.

The slope detector consists of two parts

a. A tuned or resonant circuit which converts frequency variation into amplitude variation.

b. An envelope detector which detects the amplitude variation and gives the original message signal.

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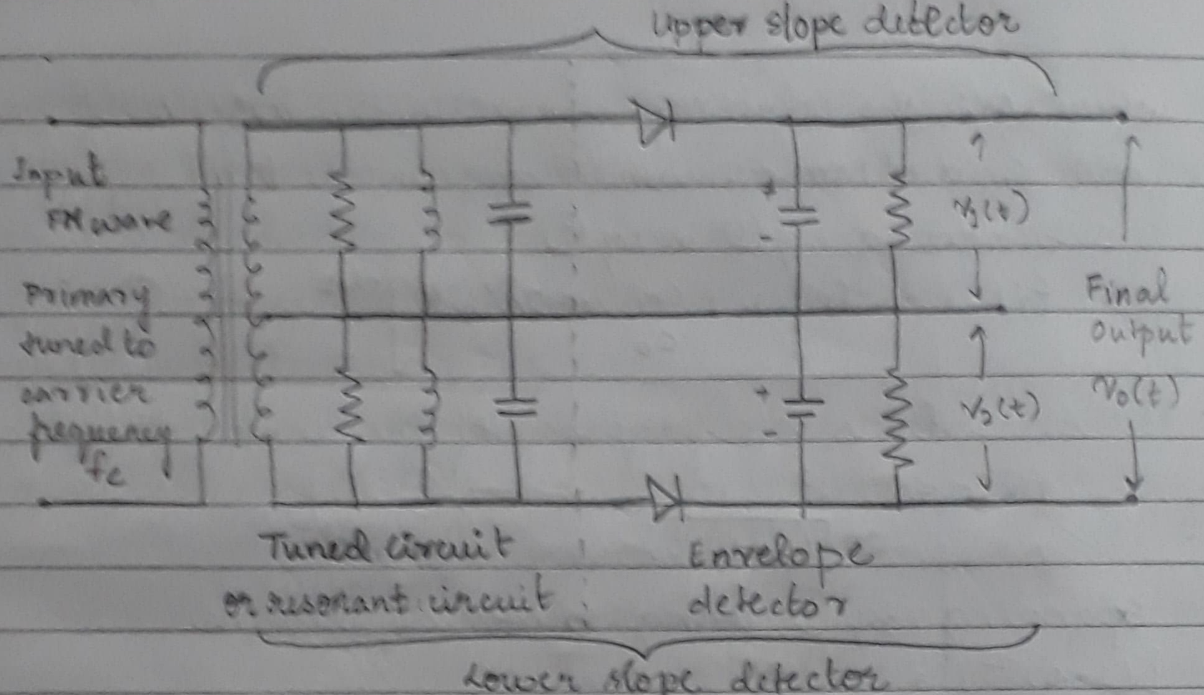
Date
Page



The upper tuned circuit is tuned to a frequency which is greater than f_c.

The ~~lower~~ tuned circuit is tuned to a frequency which is lower than f_c.

The output voltages $v_1(t)$ and $v_2(t)$ are function of frequency. The resultant output voltage $v_o(t)$ is the difference of $v_1(t)$ and $v_2(t)$

$$v_o(t) = v_1(t) \sim v_2(t)$$

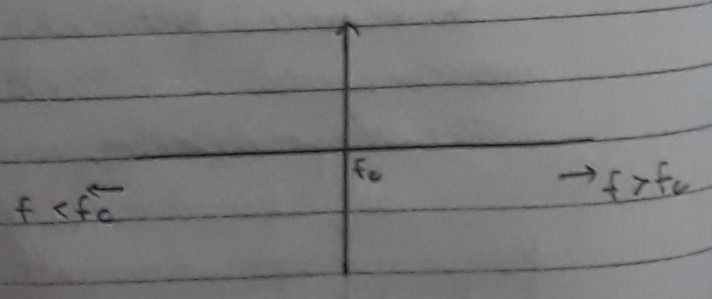
Therefore the output of ideal balanced discriminator is proportional to the departure of the instantaneous frequency from the carrier.

i.e., $v_o(t) \propto (f_i - f_c)$

CASE 1 : If the input is an unmodulated carrier of frequency $f = f_c$, it will supply equal voltages to upper tuned circuit as well as lower tuned circuit

Therefore : $v_1(t) = v_2(t)$

Hence $v_o(t) = 0$.



CASE 1

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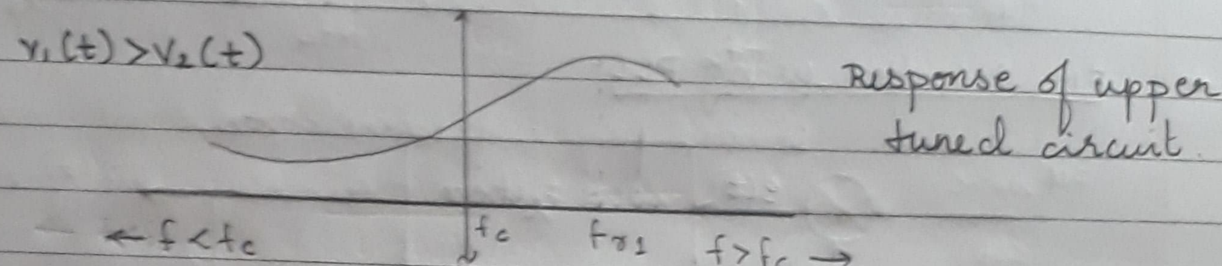
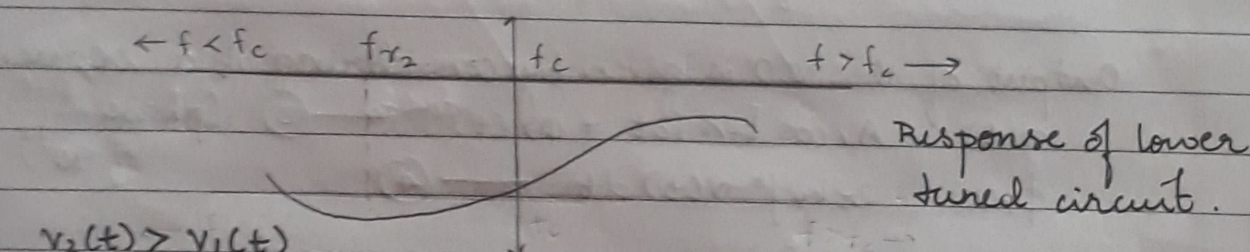
CASE 2: If instantaneous frequency is greater than f_c, since the upper tuned circuit is tuned to a frequency which is greater than f_c, the voltage $v_1(t)$ is greater than $v_2(t)$

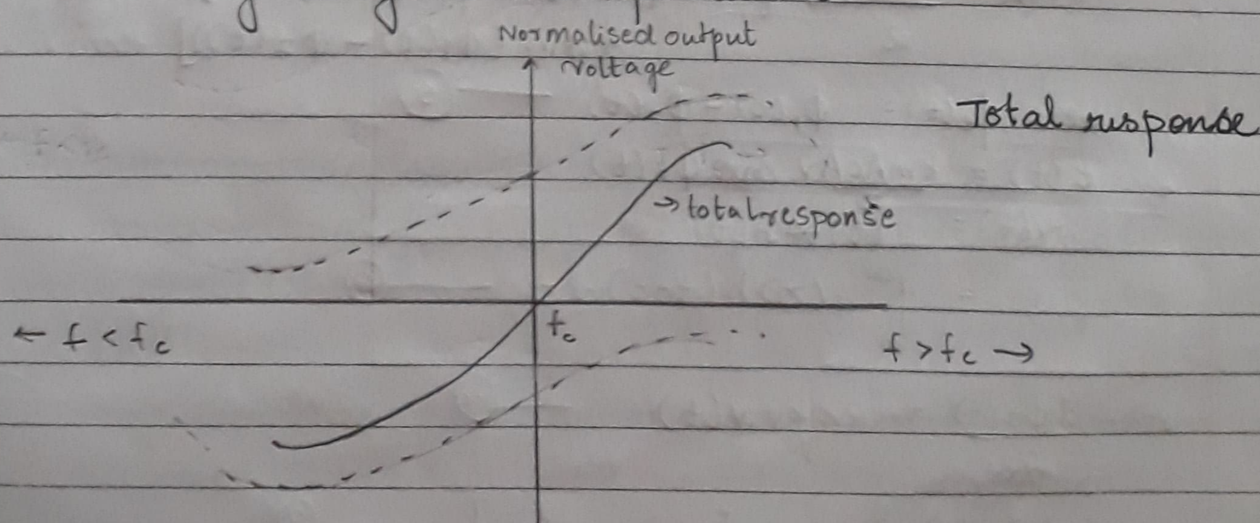


Here, the frequency variation (greater than f_c) is converted into its equivalent amplitude variations and it can be detected by using an envelope detector.

CASE 3: If instantaneous frequency is less than f_c
Since the lower tuned circuit is tuned to a frequency which is less than f_c, the voltage $v_1(t)$ is less than $v_2(t)$



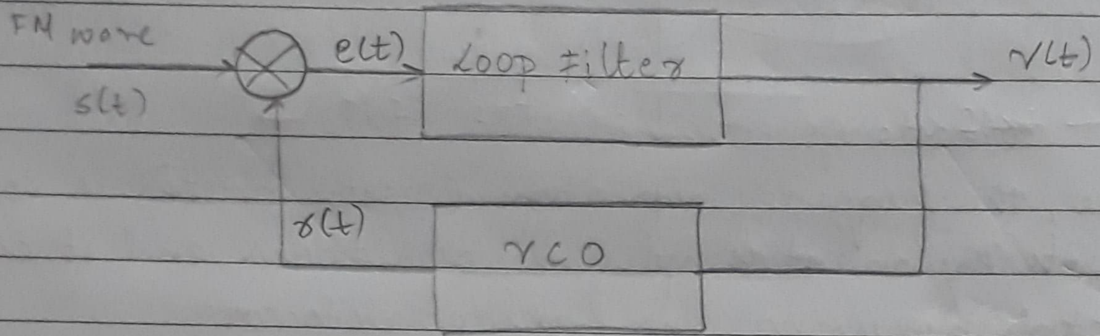
Here the frequency variation (less than f_c) is converted into its equivalent amplitude variations and it can be detected by using an envelope detector



Advantages

A balanced frequency discriminator has sensitivity and a better sensitivity.

★ **FM detection using PLL:**



The FM wave s(t) is given by

$s(t) = A_c \sin[2\pi f_c t + \phi_1(t)]$ —— (1)

$\phi_1(t) = 2\pi k_f \int_0^t m(t)\,dt$ —— (2)

Output of the VCO is given by considering it a frequency modulator

$r(t) = A_v \cos[2\pi f_c t + \phi_2(t)]$ —— (3) (phase difference of 90°)

$\phi_2(t) = 2\pi k_v \int_0^t v(t)\,dt$ —— (4)

Error signal

$e(t) = k_m\, s(t)\, r(t)$

$e(t) = \frac{k_m A_c A_v}{2}\left[\sin(4\pi f_c t + \phi_1(t) + \phi_2(t)) + \sin(\phi_1(t) - \phi_2(t))\right]$ High frequency term is neglected

here $\phi_e(t) = \phi_1(t) - \phi_2(t)$ —— (5)

$e(t) = \frac{k_m A_c A_v}{2} \sin \phi_e(t)$

$v(t) = e(t) * h(t)$

$= \int_{-\infty}^{\infty} e(\tau)\, h(t-\tau)\, d\tau$ —— (6)

$\frac{d\phi_2(t)}{dt} = 2\pi k_v\, v(t)$ —— (7)

$\frac{d\phi_2(t)}{dt} = 2\pi k_v [e(t) * h(t)]$ —— (8)

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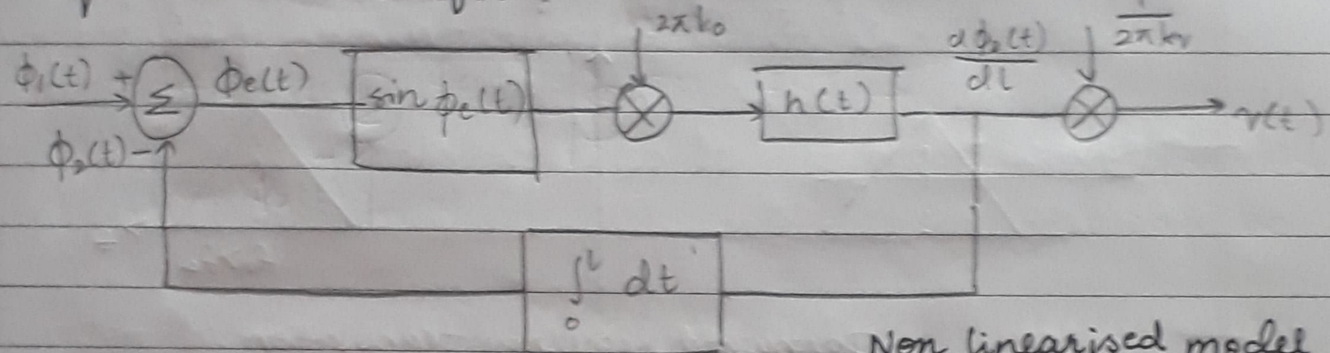
$$\frac{d\phi_2(t)}{dt} = 2\pi k_v \left[\frac{k_m A_c A_v}{2} \sin\phi_e(t) * h(t) \right] \quad \text{——} (9)$$

$$\frac{d\phi_2(t)}{dt} = 2\pi k_0 \left[\sin\phi_e(t) * h(t) \right] \quad \text{——} (10)$$

where $k_0 = \frac{1}{2} k_v k_m A_c A_v$

Equivalent model of PLL



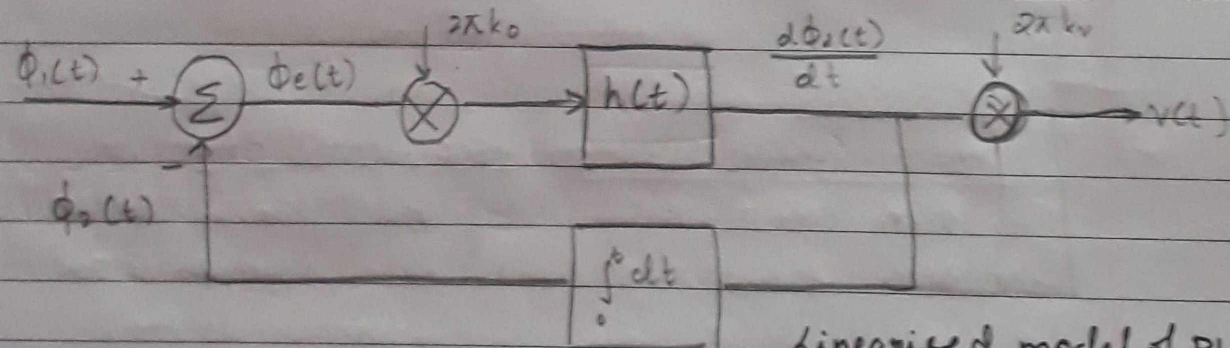
Non linearised model of PLL.

because from eq (7)

$$v(t) = \frac{1}{2\pi k_v} \frac{d\phi_2(t)}{dt}$$

For linearised model of PLL

$\sin\phi_e(t) \simeq \phi_e(t)$



Linearised model of PLL

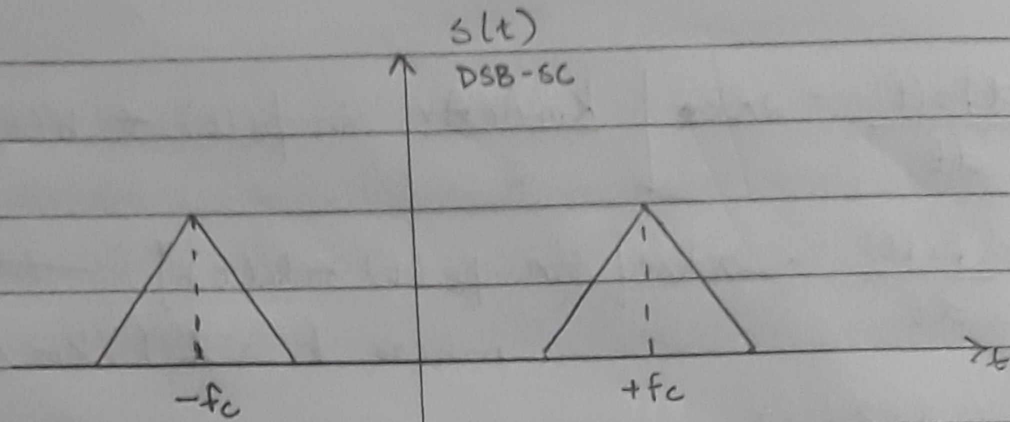
NOTE:

★ Frequency translation :

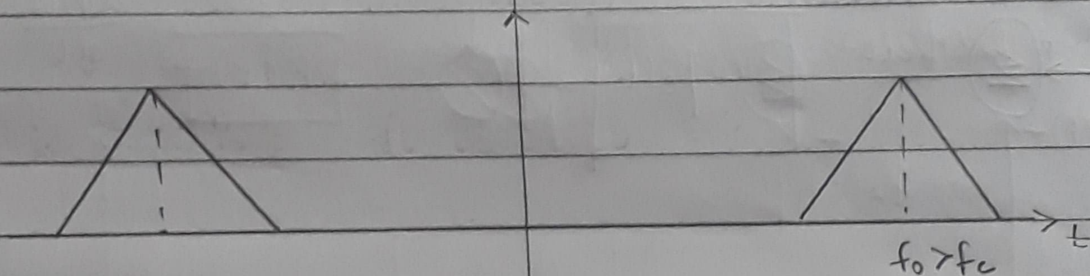
It is the shifting of the spectrum of the modulated wave from f_c (carrier frequency) to some other value greater than f_c or lower than f_c.

greater than f_c : upward frequency translation.

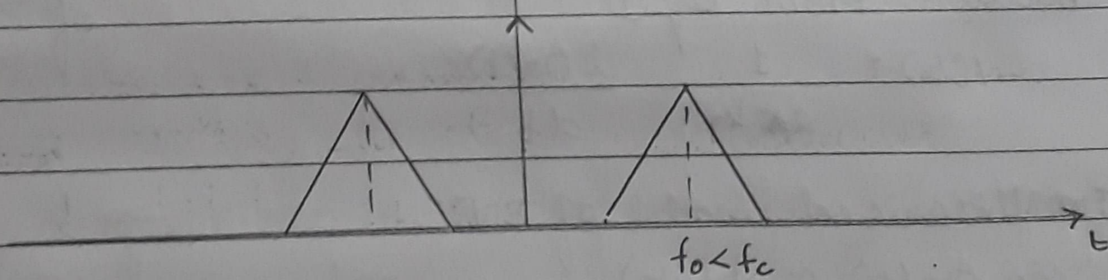
lower than f_c : downward frequency translation.



Upward frequency translation.



Downward frequency translation.



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UNIT - 03

Noise and Noise in Continuous wave Modulation Systems

★ **Thermal Noise:**

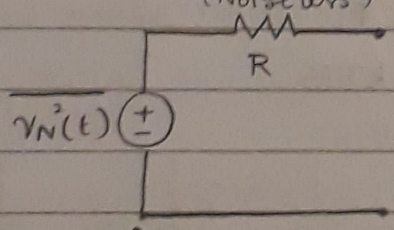
The mean square value of the thermal voltage $V_N(t)$ appearing across the terminals of a resistor of $R\,\Omega$ at temperature $T°K$ in a bandwidth of B Hz is denoted by $E[V_N^2(t)]$ or $\overline{V_N^2(t)}$. It is given by:

$$E[V_N^2(t)] \text{ or } \overline{V_N^2(t)} = 4kTRB \text{ volts}^2$$

where $\overline{V_N^2(t)}$: Ensemble average

k : Boltzmann constant $(1.38 \times 10^{-23}\ J/°K)$

Thus a noisy resistor can be represented by a noiseless resistor in series with a thermal voltage source as shown in the figure



Noise voltage generator (Thevenin's equivalent)

Similarly the mean square value of thermal noise current $I_N(t)$ through a resistor of $R\,\Omega$ at temperature $T°K$ in a bandwidth of B Hz is denoted as $E[I_N^2(t)]$ or $\overline{I_N^2(t)}$. It is given by

$$E[I_N^2(t)] = \overline{I_N^2(t)} = \frac{E[V_N^2(t)]}{R^2}$$

$$E[I_N^2(t)] = \overline{I_N^2(t)} = \frac{4kTRB}{R^2}$$

$$E[I_N^2(t)] = \overline{I_N^2(t)} = 4kTGB \quad \text{where } G = \frac{1}{R}$$

Thus a noisy resistor can be replaced by a conductor $G = 1/R$ in parallel with a thermal current source which is as shown in the figure

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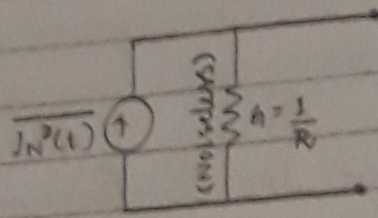
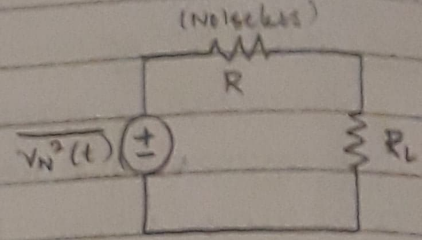


Noise Current Generator
(Norton's Equivalent)

★ <u>Power Calculations:</u>



RMS value of the voltage across the matched load R_L (i.e., $R_L = R$) is given by:

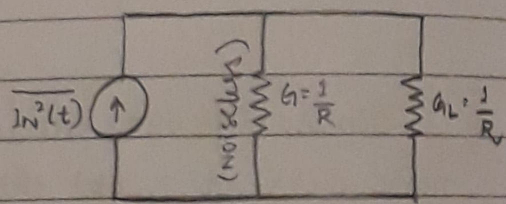
$$V_{rms} = \sqrt{\frac{\overline{V_N^2(t)}}{2}}$$

The maximum power delivered to the load is equal to the available power across R_L is given by:

$$\frac{V_{rms}^2}{R} = \frac{\overline{V_N^2(t)}}{4} \cdot \frac{1}{R}$$

Available power across R_L

$$P_a = \frac{\cancel{4}kTBR}{\cancel{4}R} = kTB$$



RMS value of the current through the matched load G_L (i.e., $G_L = G$) is given by:

$$I_{rms} = \sqrt{\frac{\overline{I_N^2(t)}}{2}}$$

The maximum power delivered to the load is equal to the available power across R_L

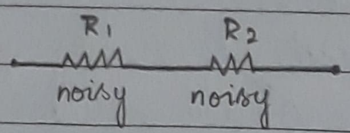
$$I_{rms}^2 R = \frac{\overline{I_N^2(t)}}{4} R = \frac{\overline{I_N^2(t)}}{4G}$$

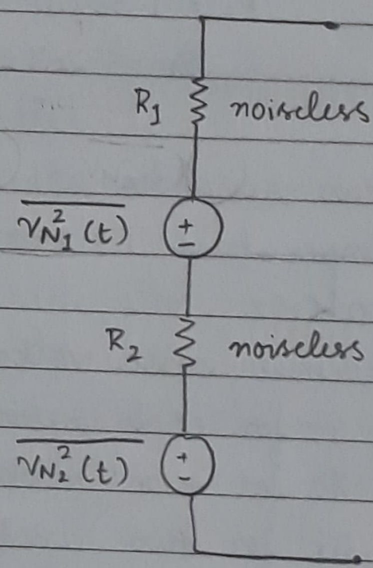
Available power across R_L

$$P_a = \frac{\cancel{4}kTB\cancel{G}}{\cancel{4G}} = kTB$$

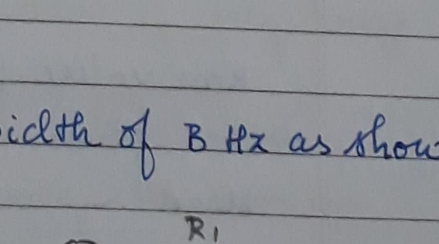
Q: Find the mean square value of the thermal noise voltage when two resistors are connected in
a. series b. parallel

– considering two resistors R_1 and R_2 at the same temperature which are connected in
a. series in a given bandwidth of B Hz as shown





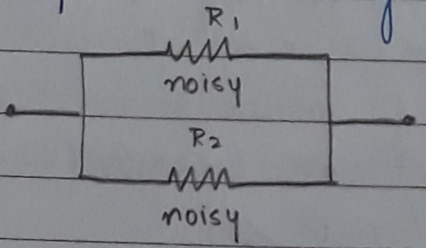
Since the two noise sources are statically independent, the mean square value of the series combination is the sum of individual mean square values of the noise voltages.

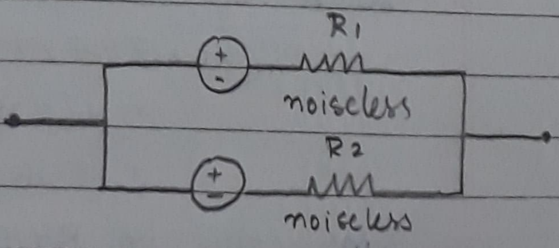


$$\overline{V_N^2(t)} = \overline{V_{N_1}^2(t)} + \overline{V_{N_2}^2(t)}$$
$$= 4kTR_1B + 4kTR_2B$$
$$= 4kTB(R_1 + R_2)$$
$$= 4kTBR_S$$

where $R_S = R_1 + R_2$

b. parallel in a given bandwidth of B Hz as shown





$$\overline{V_N^2(t)} = 4kTB\,[R_1 \| R_2]$$
$$= 4kTB\left[\frac{R_1R_2}{R_1+R_2}\right]$$
$$= 4kTBR_p \quad \text{where } R_p = \frac{R_1R_2}{R_1+R_2}$$

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Q: A receiver is connected to an antenna system that has a noise temperature of T = 100K. Find the noise power that is available from the source over a bandwidth of 20 MHz.

– The noise power available

$$P_a = kTB \qquad \text{where } k = 1.38 \times 10^{-23}$$

$$P_a = 1.38 \times 10^{-23} (100)(20 \times 10^6)$$

$$P_a = 2.76 \times 10^{-14} \text{ watt}$$

Q: Two resistors 28 kΩ and 51 kΩ are at a temperature T = 290 K. Calculate for a bandwidth of 100 kHz, the mean square and rms value of thermal noise voltage:

i. for each resistor
ii. for two resistors in series
iii. for two resistors in parallel

– i. For each resistor

Mean square value for R_1 = 28 kΩ

$$\overline{V_{N_1}^2(t)} = 4kTBR_1$$

$$= 4(1.38 \times 10^{-23})(290)(100 \times 10^3)(28 \times 10^3)$$

$$= 4.48 \times 10^{-11} \text{ volt}^2$$

Rms value for R_1 = 28 kΩ

$$V_{rms} = \sqrt{\overline{V_{N_1}^2(t)}} = 6.69 \times 10^{-6} \text{ volt}$$

Mean square value for R_2 = 51 kΩ

$$\overline{V_{N_2}^2(t)} = 4kTBR_2$$

$$= 4(1.38 \times 10^{-23})(290)(100 \times 10^3)(51 \times 10^3)$$

$$= 8.16 \times 10^{-11} \text{ volt}^2$$

Rms value for R_2 = 28 kΩ

$$V_{rms} = \sqrt{\overline{V_{N_2}^2(t)}} = 9.035 \times 10^{-6} \text{ volt}$$

ii. For two resistors in series

Since the two resistors are statistically independent the mean square value of noise voltage for the two resistors in series is equal to the sum of individual mean square values.

$$\overline{V_{NS}^2(t)} = \overline{V_{N_1}^2(t)} + \overline{V_{N_2}^2(t)}$$

$$= 4kTBR_1 + 4kTBR_2$$

$$= 4kTB(R_1 + R_2)$$

$$= 4(1.38\times10^{-23})(290)(100\times10^3)(28\times10^3 + 51\times10^3)$$

$$= 1.26\times10^{-10}\ \text{volt}^2$$

Rms value is given by

$$V_{rms} = \sqrt{\overline{V_{NS}^2(t)}} = 1.12\times10^{-5}\ \text{volt}$$

ii. For two resistors in parallel

Similarly the mean square value is

$$\overline{V_{NP}^2(t)} = 4kTB[R_1 \| R_2]$$

$$= 4kTB\left[\frac{R_1R_2}{R_1+R_2}\right]$$

$$= 4(1.38\times10^{-23})(290)(100\times10^3)\left[\frac{28(10^3)51(10^3)}{28\times10^3+51\times10^3}\right]$$

$$= 2.89\times10^{-11}\ \text{volt}^2$$

Rms value is given by

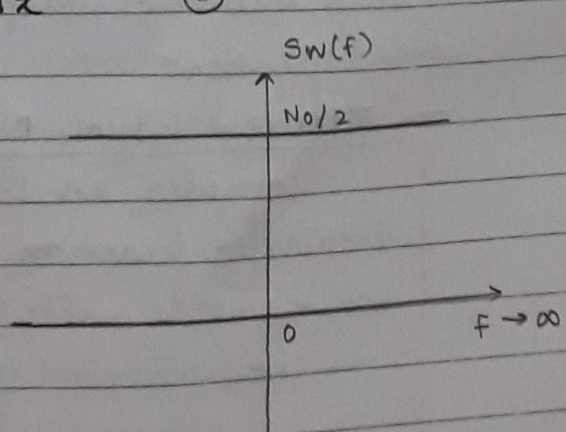
$$V_{rms} = \sqrt{\overline{V_{NP}^2(t)}} = 5.379\times10^{-6}\ \text{volt}$$

* **White Noise**

A random process whose power spectral density is constant and hence independent of frequency is called as white noise.

All the frequency components in white noise contribute equally to noise power. The power spectral density of white noise is given by:

$$S_W(f) = \frac{N_0}{2}\ \text{Watt/Hz} \quad \text{——} (1)$$



In a bandwidth of B Hz the available noise power due to white noise is:

$$P = N_0 B\ \text{Watt} \quad \text{——} (2)$$

Similarly the available noise power of a thermal noise source is

$$P_a = kTB \text{ watt} \quad \text{——— ③}$$

From eq ② and eq ③

$$N_0 \cancel{B} = kT\cancel{B}$$

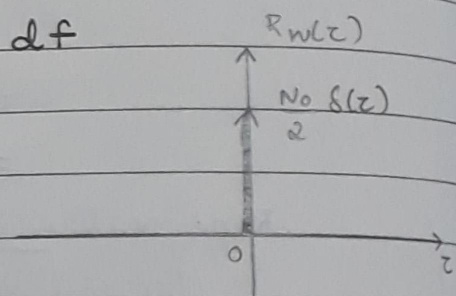
$$N_0 = kT \quad \text{or} \quad \frac{N_0}{2} = \frac{kT}{2} \quad \text{——— ④}$$

We know from Fourier Transform theory that power spectral density (PSD) and Auto correlation function are Fourier Transform pairs.

$$R_W(\tau) \xleftrightarrow{FT} S_W(f)$$

Auto correlation function for white noise is:

$$R_W(\tau) = \int_{-\infty}^{\infty} S_W(f)\, e^{j2\pi f\tau}\, df$$



From eq ①

$$R_W(\tau) = \frac{N_0}{2} \cdot \delta(\tau)$$

The auto correlation function of white noise is a delta function occuring at $\tau = 0$ and of amplitude $N_0/2$.

Since the auto correlation function $R_W(\tau)$ is zero for $\tau \neq 0$. This indicates that any two samples of white noise process separated by τ $(\tau \neq 0)$ are uncorrelated.

If the white noise process is also ?? then the two samples that are uncorrelated are also independent.

Filtered White Noise

Consider an ideal low pass filter with a transfer function $H(f)$. Input to the filter is a white noise with PSD $= \frac{N_0}{2}$.

i.e., $S_W(f) = \frac{N_0}{2}$

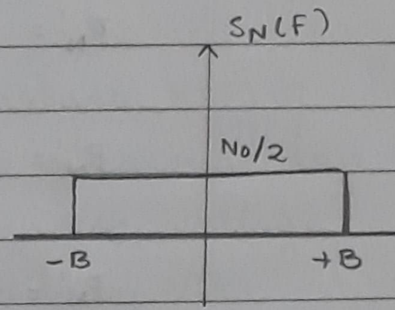
For ideal low pass filter

$$H(f) = \begin{cases} 1 & ; -B < f < B \\ 0 & ; |f| > B \end{cases}$$

The output of the filter

$$S_N(f) = |H(f)|^2 S_W(f)$$

$$S_N(f) = \begin{cases} \frac{N_0}{2} & ; -B < f < B \\ 0 & ; |f| > B \end{cases}$$



- **Noise equivalent bandwidth:**

consider a low pass filter having a response $H(f)$. The input to the filter is a white noise source having a zero mean value and PSD = $N_0/2$.

The PSD of the noise of the output of the filter is

$$S_N(f) = |H(f)|^2 S_W(f) \quad \text{——} (1)$$

$$S_N(f) = |H(f)|^2 \frac{N_0}{2} \quad \text{——} (2)$$

The average noise power P_N is given by

$$P_N = \frac{N_0}{2} \int_{-\infty}^{\infty} |H(f)|^2 df \quad \text{——} (3)$$

since $|H(f)|^2$ is an even function, the above equation can be modified as:

$$P_N = \frac{N_0}{\cancel{2}} \cancel{2} \int_0^{\infty} |H(f)|^2 df \quad \text{——} (4)$$

The practical or general low pass filter is replaced by an ideal low pass filter.

The average output noise power is given by.

$$P_N' = \int_{-\infty}^{\infty} S_N(f)\, df = \int_{-\infty}^{\infty} S_W(f)\, |H(f)|^2 df \quad \text{——} (5)$$

we know that

$$|H(f)|^2 = \begin{cases} |H(0)|^2 & ; |f| \le B_N \\ 0 & ; |f| > B_N \end{cases}$$

From eq (5)

$$P_N' = \int_{-B_N}^{B_N} \frac{N_o}{2} |H(0)|^2 \, df$$

$$P_N' = \frac{N_o}{2} |H(0)|^2 \int_{-B_N}^{B_N} df$$

$$P_N' = \frac{N_o}{2} |H(0)|^2 \cdot 2B_N$$

$$P_N' = N_o |H(0)|^2 B_N \quad \text{———} \ (6)$$

In the above equation, B_N is defined as noise equivalent bandwidth for a low pass filter.

The expression for noise equivalent bandwidth is obtained by equating the eq (4) and eq (6)

$$N_o \int_0^\infty |H(f)|^2 df = N_o |H(0)|^2 B_N$$

Noise equivalent bandwidth is given as

$$B_N = \frac{\int_0^\infty |H(f)|^2 df}{|H(0)|^2}$$

Similarly the noise equivalent bandwidth for a band pass filter is given by

$$B_N = \frac{\int_0^\infty |H(f)|^2 df}{|H(f_c)|^2}$$

where $|H(f_c)|$ is the amplitude response of a filter at the centre frequency, f_c.

* **Noise figure:**

Noise figure is used to measure the noise performance of a linear two port device.

Noise figure of any two port device is defined as the ratio of total available output noise power (due to the device and the source per unit bandwidth) to the available noise power due to source alone per unit band width.

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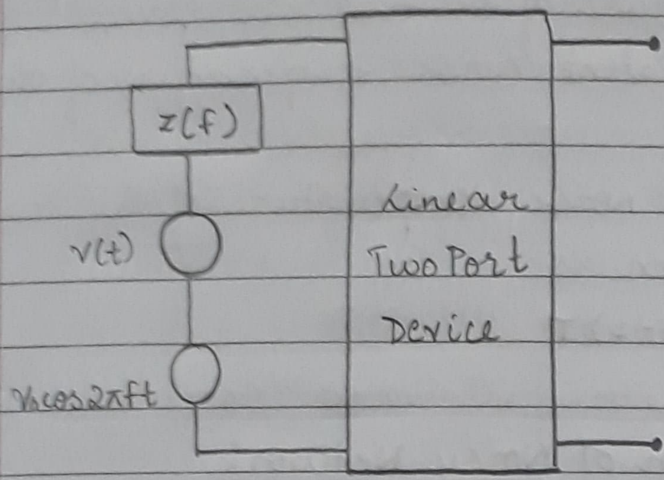
Consider a linear two port device connected to a signal source of impedance $z(f) = R(f) + jX(f)$ at the input as shown in figure:



The resistance $R(f)$ of the source gives rise to a thermal noise voltage represented by $V(t)$.

The output noise of the device has two components; one due to the source and other due to the device itself. The available output noise power in a bandwidth of B_N Hz centred at f is defined as the maximum average noise power at the output of the device in a bandwidth of B_N Hz.

Let $S_{NO}(f)$ be the spectral density of the total noise power at the output of the device. Similarly the spectral density of noise power due to the source at the input of the device is denoted by $S_{NS}(f)$. If $G(f)$ is the available power gain of the device, then the noise figure of the device is given by:

$$F(f) = \frac{S_{NO}(f)}{G(f) S_{NS}(f)}$$

If the two port device is noise free, $S_{NO}(f)$ is given as:

$$S_{NO}(f) = G(f) S_{NS}(f) \quad \text{and} \quad F(f) = 1.$$

In a physical device, noise appearing at the output of the device is the amplified source noise plus the noise introduced by the device itself.

$$\therefore \quad S_{NO}(f) > G(f) S_{NS}(f)$$

$$\Rightarrow \quad F(f) > 1$$

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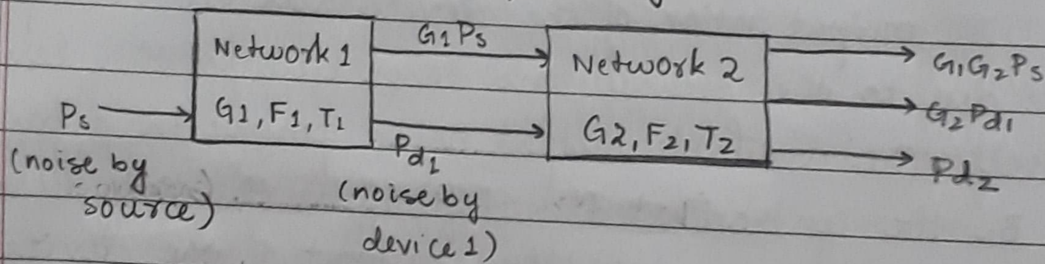
* Equivalent Noise Temperature

Let T be the temperature of the source connected to a two port device.
F be the noise figure of the two port device
T_e be the equivalent noise temperature of the two port device.

The equivalent noise temperature of the two port device is given as:

$$T_e = (F-1)T$$

* Cascade Connection of Noisy Network:



Consider a cascade connection of two noisy two port networks as shown in the figure.
Let G_1 be the available power gain of network 1
G_2 be the available power gain of network 2
T be the equivalent noise temperature of source at the input of network 1
T_1 be the equivalent noise temperature of network 1
T_2 be the equivalent noise temperature of network 2.
F_1 be the noise figure of network 1
F_2 be the noise figure of network 2.

- Available source noise power in a bandwidth of B_N Hz at the input of network 1 is

$$P_s = kTB_N \text{ Watts}$$

- The available noise power available at output of network 1 is amplified by both the networks and given by

$$G_1 G_2 P_s = G_1 G_2 kTB_N$$

- The noise power P_{d_1} introduced by network 1 and amplified by network 2 is given by

$$G_2 P_{d_1} = (k T_1 B_N G_1) G_2$$

- The noise power P_{d_2} introduced by network 2 is given by

$$P_{d_2} = k T_2 B_N G_2$$

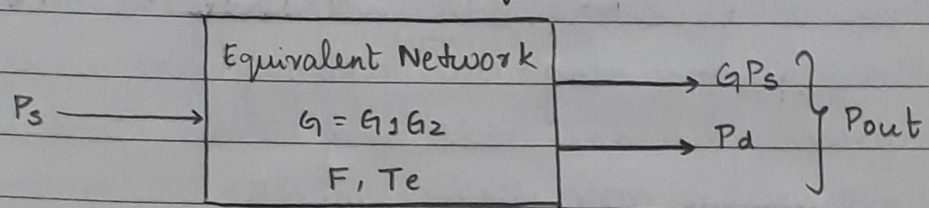
- The output noise power i.e. at the output of network 2 is given by:

$$P_{out} = G_1 G_2 P_s + G_2 P_{d_1} + P_{d_2}$$

$$P_{out} = G_1 G_2 k T B_N + G_1 G_2 k T_1 B_N + G_2 k T_2 B_N$$

$$P_{out} = G_1 G_2 k B_N \left[T + T_1 + \frac{T_2}{G_1} \right] \quad \text{— (A)}$$

consider the equivalent of cascaded pair of networks as shown in the figure.



From the figure, output noise power Pout is given by

$$P_{out} = G P_s + P_d$$

$$P_{out} = G k T B_N + G k T_e B_N$$

$$P_{out} = G k B_N [T + T_e]$$

$$P_{out} = G_1 G_2 k B_N [T + T_e] \quad \text{— (B)}$$

comparing eq (A) and eq (B)

$$T + T_e = T + T_1 + \frac{T_2}{G_1} \quad \text{— (C)}$$

$$T_e = T_1 + \frac{T_2}{G_1} \quad \text{— (D)}$$

wkt $\quad T_e = (F - 1) T$

$$\Rightarrow \frac{T_e}{T} = F - 1$$

$$(F-1) T = T_1 + \frac{T_2}{G_1}$$

$$F - 1 = \frac{T_1}{T} + \frac{T_2}{T G_1}$$

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$$F - \cancel{1} = F_1 - \cancel{1} + \frac{F_2 - 1}{G_1}$$

$$F = F_1 + \frac{F_2 - 1}{G_1}$$

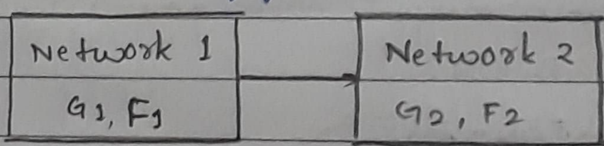
For any number of noisy networks, we can write

$$F = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \frac{F_4 - 1}{G_1 G_2 G_3} + \cdots$$

$$T_e = T_1 + \frac{T_2}{G_1} + \frac{T_3}{G_1 G_2} + \frac{T_4}{G_1 G_2 G_3} + \cdots$$

The above two equations are called Frii's Equation

Q: Two two-port devices are connected in cascade. For the first stage the noise figure and available power gain are 5dB and 12dB respectively. For the second stage the noise figure and available power gain are 15dB and 10 dB. Determine the overall all noise figure in dB.



connected in cascade

For network 1

$F_1 = 5dB$ $\qquad$ $G_1 = 12 dB$

$10\log_{10} F_1 = 5dB$ $\qquad$ $10\log_{10} G_1 = 12 dB$

$F_1 = 3.2$ $\qquad$ $G_1 = 15.85$

For network 2

$F_2 = 15dB$ $\qquad$ $G_2 = 10 dB$

$10\log_{10} F_2 = 15 dB$ $\qquad$ $10\log_{10} G_2 = 10 dB$

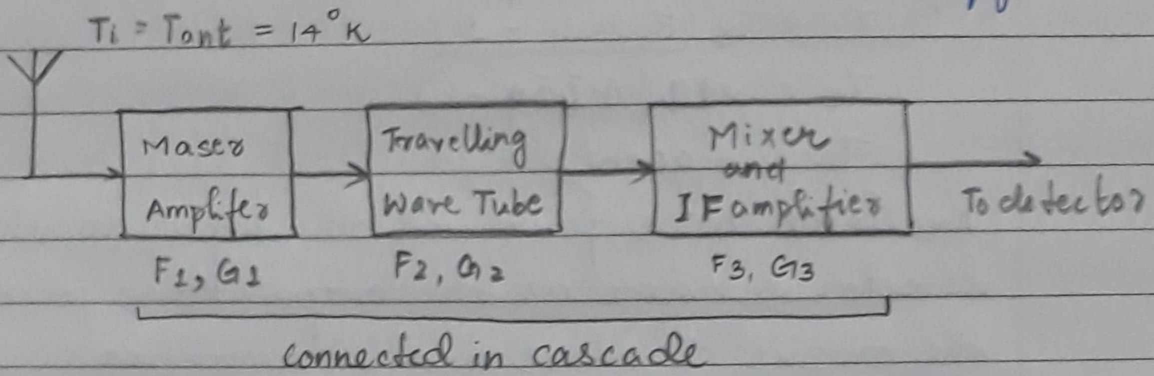
$F_2 = 31.6$ $\qquad$ $G_2 = 10$

Overall Noise figure

$$F = F_1 + \frac{F_2 - 1}{G_1} = 3.2 + \frac{31.6 - 1}{15.85} = 5.13$$

$$10\log_{10} F = 7.10 dB$$

Q: A microwave receiver is as shown in the figure



Equivalent noise temperature Te and available power gain of the Maser amplifier are 4 K and 30dB. Noise figure and available power gain of travelling wave tube are 6dB and 20dB respectively. Noise figure and available power gain of mixer and IF amplifier are 12dB and 40dB respectively. Determine the overall noise figure and equivalent noise temperature of the receiver system. Also find the available noise power at input to the detector in a bandwidth of 1MHz.

— Maser Amplifier

Equivalent noise temperature

$$T_e = (F_1 - 1) T_i$$

To is not given then To is assumed as room temperature $T_0 = 290k$

$$4 = (F_1 - 1)\, 14$$

$$F_1 - 1 = 0.286$$

$$F_1 = 1.286 \qquad G_1 = 30dB = 1000$$

Travelling wave tube

$$F_2 = 6dB = 4 \qquad G_2 = 20dB = 100$$

Mixer and IF amplifier

$$F_3 = 12dB = 15.8 \qquad G_3 = 40dB = 10000$$

Overall noise figure

$$F = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2}$$

$$F = 1.286 + \frac{3}{1000} + \frac{14.8}{100000} = 1.289 = 1.1025\,dB$$

Equivalent Noise temperature

$T_e = T_i(F_{overall} - 1)$

$T_e = 14(1.289 - 1)$

$T_e = 4.04K$

Available noise power at the input to the detector is same as the output noise power at the output of mixer and IF amplifier.

$P_o = Gk(T_i + T_e)B$

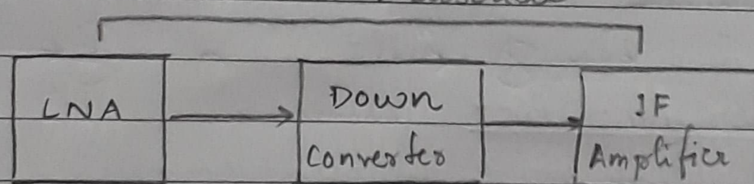
$P_o = G_1 G_2 G_3 k(T_i + T_e)B$

$P_o = 10^3 10^2 10^4 \times (14 + 4.64) 10^6 (1.38 \times 10^{-23})$

$P_o = 25.72 \times 10^{-8}$ Watt

Q: A satellite station consists of a low noise amplifier (LNA), a down converter and an IF amplifier connected in cascade as shown in the figures.

connected in cascade



A low noise amplifier has a gain of 40 dB and equivalent noise temperature of 100 k. The down converter has noise figure of 8dB and a gain of 6 dB. The IF amplifier has a gain of 60dB and noise figure of 14 dB. Find the overall noise figure and overall effective noise temperature of the system.

— Low noise amplifier

$T_e = (F_1 - 1)T_o$

$100 = (F_1 - 1)290$

$F_1 = 1.34$ $G_1 = 40dB = 10^4$

Down converter

$F_2 = 8dB = 6.31$ $G_2 = 6dB = 3.98$

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IF amplifier

$F_3 = 14\,dB = 25.12$ $\quad G_3 = 60\,dB = 10^6$

overall noise figure

$$F = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2}$$

$$F = 1.34 + \frac{6.31 - 1}{10^4} + \frac{25.12 - 1}{10^4 (3.98)}$$

$$F = 1.341$$

$$F_{dB} = 1.274$$

Overall effective noise temperature of the system

$$T_e = T_0 (F_{overall} - 1)$$

$$T_e = 290 (1.341 - 1)$$

$$T_e = 98.89\,k$$

★ Shot Noise

Shot noise arises because current consists of a vast number of discrete charges. The continuous flow of these discrete pulses gives rise to almost white noise.

Unlike thermal noise, this noise is dependent upon the current flowing and has no relationship to the temperature at which the system is operating.

If electrons flow across a barrier, they have discrete arrival times. Those discrete arrivals exhibit shot noise.

It is particularly obvious when current levels are low.

★ Noise in continuous wave modulation systems:

(Noise in communication Receivers):

For the purpose of comparing different continuous wave modulation schemes a figure of merit γ is defined as follows.

Figure of merit

$$\gamma = \frac{(SNR)_{output}}{(SNR)_{channel}}$$

It is desirable to have higher values of figure of merit. Its value depends on modulation and the type of detection used.

Note that:

1. $\gamma > 1$ implies that $(SNR)_o > (SNR)_c$

This range of γ is desirable.

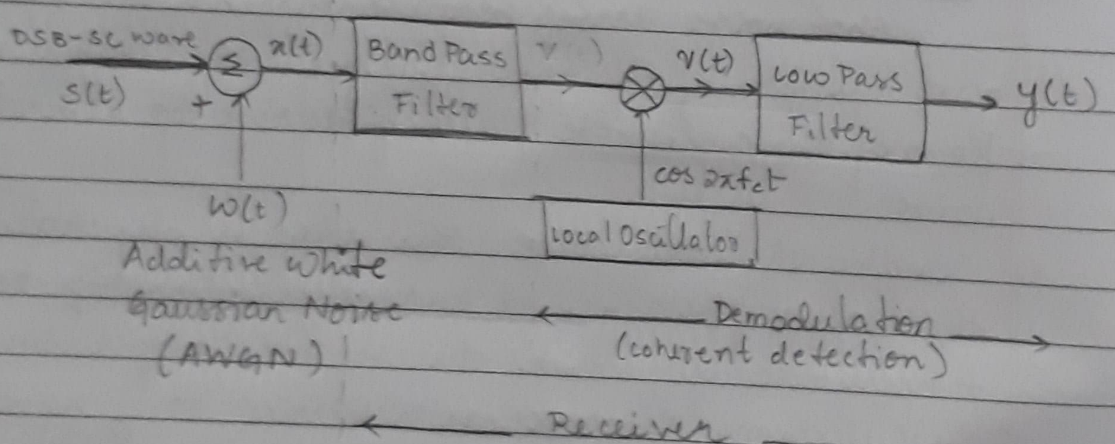
2. $\gamma = 1$ implies that $(SNR)_o = (SNR)_c$

This value of γ is permissible.

3. $\gamma < 1$ implies that $(SNR)_o < (SNR)_c$

This range of γ is not permissible.

- Noise in DSB-SC receivers:



Assume that DSB-SC receiver employs coherent or synchronous detection scheme. The output of the local oscillator is synchronised in frequency and phase of carrier used at the modulator.

The incoming DSB-SC wave is given by:

$$s(t) = m(t)\, c(t)$$

$$s(t) = m(t)\, A_c \cos 2\pi f_c t$$

The message signal $m(t)$ is band limited to w Hz and A_c is the amplitude of carrier.

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The noise at the input of the band pass filter is additive white gaussian noise in nature. After passing through the band pass filter the wide band noise $w(t)$ is converted into a narrow band noise $n(t)$.

(SNR)c :

SNR channel is defined as:

$$(SNR)_c = \frac{\text{Average power of } s(t)}{\text{Average power of } n(t) \text{ in the information Band width}}$$

$$\text{Average power of } s(t) = \overline{s^2(t)}$$
$$= \overline{(m(t)\, A_c \cos 2\pi f_c t)^2}$$
$$= P_m A_c^2 / 2$$

where P_m is the average power of $m(t)$ and $\frac{A_c^2}{2}$ is the average power of carrier

The average power of $n(t)$ in the information band width is $= N_0 W$ $\left[\left(N_0/2\right)(2W)\right]$

where N_0 is the average noise power per unit bandwidth in watts / Hz.

Therefore

$$(SNR)_c = \frac{P_m A_c^2}{2 N_0 W}$$

(SNR)o :

The narrow band noise $n(t)$ can be expressed in terms of inphase and quadrature components with reference to carrier $A_c \cos 2\pi f_c t$.

Narrow band noise $n(t)$ is given by:

$$n(t) = n_c(t) \cos 2\pi f_c t - n_s(t) \sin 2\pi f_c t$$

(cannonical representation of narrow band noise)

where $n_s(t)$: quadrature component

$n_c(t)$: inphase component

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From the figure, the output of the product modulator is given by

$$v(t) = x(t)\cos 2\pi f_c t$$
$$= [n(t) + s(t)] \cos 2\pi f_c t$$
$$= s(t)\cos 2\pi f_c t + n(t)\cos 2\pi f_c t$$
$$= A_c m(t)\cos^2 2\pi f_c t + n_c(t)\cos^2 2\pi f_c t - n_s(t)\sin 2\pi f_c t \cos 2\pi f_c t$$
$$= \frac{A_c m(t)}{2}[1+\cos 4\pi f_c t] + \frac{n_c(t)}{2}[1+\cos 4\pi f_c t] - \frac{n_s(t)}{2}\sin 4\pi f_c t$$

High frequency components are eliminated by passing through the low pass filter.

Output of the low pass filter is:

$$y(t) = \frac{A_c m(t)}{2} + \frac{n_c(t)}{2}$$

$$y(t) = m_d(t) + n_d(t)$$

Average power of $m_d(t) = \overline{m_d^2(t)}$

$$= \overline{\left[\frac{A_c m(t)}{2}\right]^2}$$

$$= \frac{A_c^2 P_m}{4}$$

Similarly the average power of $n_d(t)$

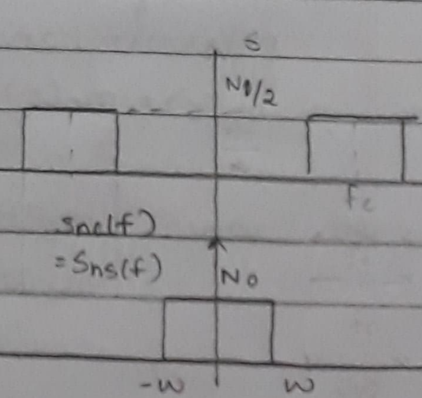
$$= \overline{n_d^2(t)}$$

$$= \overline{\left[\frac{n_c(t)}{2}\right]^2}$$

$$= \frac{1}{4}\text{ Area under PSD curve}$$

$$= \frac{1}{4} N_0 (2W)$$

$$= \frac{N_0 W}{2}$$



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Therefore

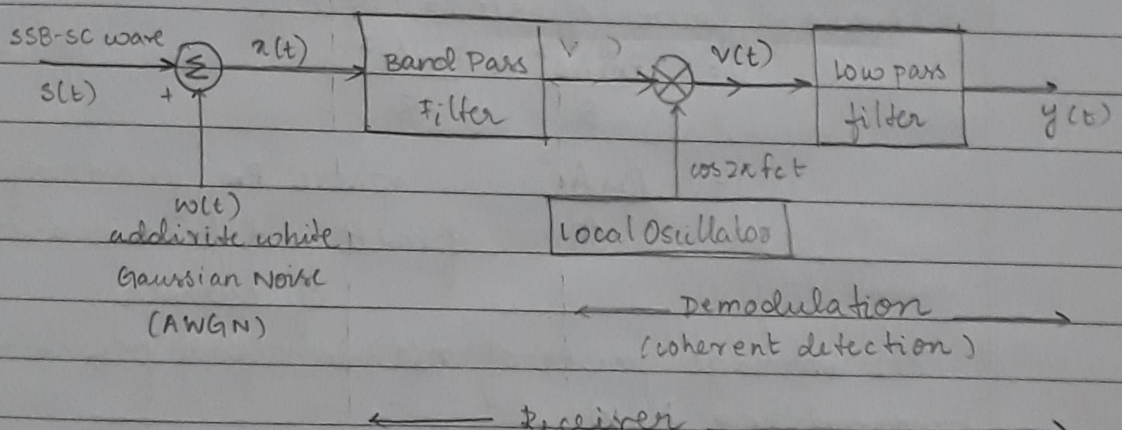
$$(SNR)_0 = \frac{A_c^2}{4}\frac{P_m}{WN_0}\cdot 2 = \frac{A_c^2 P_m}{2N_0W}$$

Figure of merit

$$FOM = \frac{(SNR)_0}{(SNR)_c} = \frac{A_c^2 P_M}{2N_0W} \Big/ \frac{A_c^2 P_M}{2N_0W}$$

$$\boxed{FOM = \gamma = 1}$$

- Noise in SSB-SC receivers:



Assume that SSB-SC receiver employs coherent or synchronous detection scheme. The output of the local oscillator is synchronized in frequency and phase of carrier used at the modulator.

The incoming SSB-SC wave is given by

$$s(t) = \frac{A_c}{2}\left[m(t)\cos 2\pi f_c t + \hat{m}(t)\sin 2\pi f_c t\right]$$

Considering lower side band

The message signal $m(t)$ is band limited to W Hz, A_c is the amplitude of carrier and $\hat{m}(t)$ is the Hilbert transform of the message signal.

The noise at the input of the band pass filter is additive white gaussian noise in nature. After passing through the band pass filter the wide band noise $w(t)$ is converted into a narrow band noise $n(t)$.

(SNR)c:

SNR channel is defined as:

$$(SNR)_c = \frac{\text{Average power of } s(t)}{\text{Average power of } n(t) \text{ in the information bandwidth}}$$

The two components $m(t)$ and $\hat{m}(t)$ are uncorrelated with each other. Therefore their power spectral densities are additive. The $m(t)$ and $\hat{m}(t)$ have the same average power.

They contribute an average power of $P_m A_c^2/8$ each.

Average power of $s(t) = \overline{s^2(t)}$

$$= \frac{P_m A_c^2}{8} + \frac{P_m A_c^2}{8}$$

$$= \frac{P_m A_c^2}{4}$$ where P_m is average power of $m(t)$ and $A_c^2/4$ is average power of carrier.

The average power of $n(t)$ in the information bandwidth $= N_0 W$

where N_0 is the average noise power per unit bandwidth in watt/Hz.

Therefore

$$(SNR)_c = \frac{P_m A_c^2}{4 N_0 W}$$

(SNR)o:

The narrow band $n(t)$ can be expressed in terms of inphase and quadrature components with reference to carrier.

$$n(t) = n_c(t)\cos\left(2\pi\left(f_c - \frac{W}{2}\right)t\right) - n_s(t)\sin\left(2\pi\left(f_c - \frac{W}{2}\right)t\right)$$

where

$n_c(t)$: inphase component

$n_s(t)$: carrier component.

The narrow band noise $n(t)$ differs from carrier frequency f_c by $W/2$ at the midband frequency.

From the figure, the output of the product modulator is

$$r(t) = x(t)\cos 2\pi f_c t$$
$$= [s(t) + n(t)]\cos 2\pi f_c t$$
$$= s(t)\cos 2\pi f_c t + n(t)\cos 2\pi f_c t$$
$$= \frac{A_c}{2} m(t)\cos^2 2\pi f_c t + \frac{A_c}{2}\hat{m}(t)\sin 2\pi f_c t\cos 2\pi f_c t$$
$$+ n_c(t)\cos\left(2\pi\left(f_c - \frac{\omega}{2}\right)t\right)\cos 2\pi f_c t$$
$$+ n_s(t)\sin\left(2\pi\left(f_c - \frac{\omega}{2}\right)t\right)\cos 2\pi f_c t$$
$$= \frac{A_c m(t)}{4}\left[1 + \cos 4\pi f_c t\right] + \frac{A_c\hat{m}(t)}{4}\sin 4\pi f_c t$$
$$+ \frac{n_c(t)}{2}\left[\cos \pi\omega t + \cos 4\pi f_c t\right]$$
$$+ \frac{n_s(t)}{2}\left[\sin \pi\omega t - \sin 4\pi f_c t\right]$$

High frequency components are eliminated by passing through the low pass filter.

Output of the lowpass filter is:

$$y(t) = \frac{A_c m(t)}{4} + \frac{n_c(t)}{2}\cos \pi\omega t + \frac{n_s(t)}{2}\sin \pi\omega t$$

$$y(t) = m_d(t) + n_d(t)$$

Average power of $m_d(t) = \overline{m_d^2(t)}$

$$= \left[\frac{\overline{A_c m(t)}}{4}\right]^2$$
$$= \frac{A_c^2 P_m}{16}$$

Average power of $n_d(t) = \overline{n_d^2(t)}$

$$= \frac{1}{4}\text{ Area under PSD curve}$$
$$= \frac{1}{4} N_0 W$$
$$= \frac{N_0 W}{4}$$

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Therefore

$$(SNR)_o = \frac{A_c^2 P_m}{16} \cdot \frac{4}{N_o W} = \frac{A_c^2 P_m}{4 N_o W}$$

Figure of merit

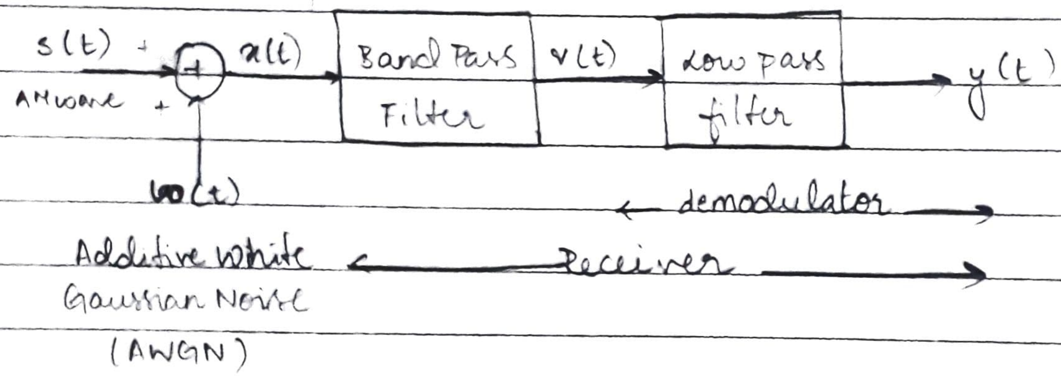
$$FOM = \frac{(SNR)_o}{(SNR)_c} = \frac{A_c^2 P_m}{4 N_o W} \Big/ \frac{A_c^2 P_m}{4 N_o W}$$

$$\boxed{FOM = \gamma = 1}$$

- **Noise in AM receivers:**

In conventional AM, both the side bands and the carrier are transmitted. The incoming wave is given by:

$$s(t) = A_c[1 + k_a m(t)] \cos 2\pi f_c t.$$



The message signal is band limited to w Hz and A_c is the amplitude of carrier.

The noise at the input of the band pass filter is additive white gaussian noise in nature. After passing through the band pass filter the wide band noise $w(t)$ is converted into narrow band noise $n(t)$.

$(SNR)_c$

SNR channel is defined as:

$$(SNR)_c = \frac{\text{Average power of } s(t)}{\text{Average power of } n(t) \text{ in the information bandwidth.}}$$

Average power of $s(t) = \overline{s^2(t)}$

$$= \frac{Ac^2}{2} + \frac{Ac^2 ka^2 Pm}{2}$$

$$= \frac{Ac^2(1 + ka^2 Pm)}{2}$$ where Pm is the average power of $m(t)$

The average power of $n(t)$ in the information bandwidth $= NoW$

where No is the average noise power per unit bandwidth in watt / Hz.

Therefore.

$$(SNR)_c = \frac{Ac^2(1+ka^2 Pm)}{2N_0W}$$

$(SNR)_o$:

The narrow band noise $n(t)$ can be expressed in terms of inphase and quadrature components with reference to carrier.

$$n(t) = n_c(t)\cos 2\pi f_c t - n_s(t)\sin 2\pi f_c t$$

where $n_c(t)$: inphase component

$n_s(t)$: quadrature component

The output of the band pass filter is

$$r(t) = x(t)$$

$$r(t) = s(t) + n(t)$$

$$r(t) = Ac[1 + ka\, m(t)]\cos 2\pi f_c t + n_c(t)\cos 2\pi f_c t - n_s \sin 2\pi f_c t$$

$$r(t) = [Ac + Ac ka\, m(t) + n_c(t)]\cos 2\pi f_c t - n_s \sin 2\pi f_c t$$

Output of the low pass filter (demodulator) is the envelope of the above signal.

$$y(t) = \sqrt{[Ac + Ac ka\, m(t) + n_c(t)]^2 + (n_s(t))^2}$$

$$y(t) \approx Ac + Ac ka\, m(t) + n_c(t)$$

Average power of the demodulated signal is

$$\frac{Ac^2 ka^2 Pm}{2}$$

Average power of noise at the output is WNo.

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Therefore

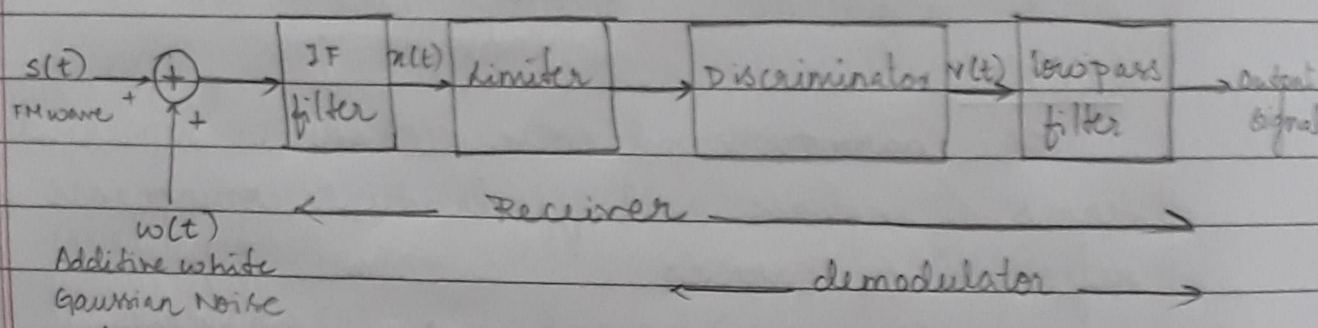
$$(SNR)_o = \frac{A_c^2 k_a P_m}{2} \frac{1}{N_o W} = \frac{A_c^2 k_a P_m}{2 N_o W}$$

Figure of merit

$$FOM = \frac{(SNR)_o}{(SNR)_c} = \frac{A_c^2 k_a^2 P_m}{2 N_o W} \Big/ \frac{A_c^2 (1 + k_a^2 P_m)}{2 N_o W}$$

$$\boxed{FOM = \gamma = \frac{k_a^2 P_m}{1 + k_a^2 P_m}}$$

- Noise in FM receivers:



The incoming FM wave is given by:

$$s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(t)\, dt\right]$$

where A_c : carrier amplitude

f_c : carrier frequency

k_f : frequency sensitivity

$m(t)$: message or modulating signal.

Let $\phi(t) = 2\pi k_f \int_0^t m(t)\, dt$

$$\therefore \quad s(t) = A_c \cos[2\pi f_c t + \phi(t)]$$

The noise at the input of the IF filter is additive white gaussian noise in nature. After passing through the IF filter the wide band noise $w(t)$ is converted into the narrowband noise $n(t)$.

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(SNR)c:

SNR channel is defined as:

$$(SNR)_c = \frac{\text{Average power of } s(t)}{\text{Average power of } n(t) \text{ in the information band width}}$$

Average power of $s(t) = \overline{s^2(t)} = A_c^2 P_m / 2$

where P_m is the average power of $m(t)$ and $A_c^2/2$ is the average power of carrier

The average power of $n(t)$ in the information bandwidth $= N_0 W$.

where N_0 is the average noise power per unit band width in watts | Hz.

Therefore

$$(SNR)_c = \frac{A_c^2 P_m}{2 N_0 W}$$

(SNR)o:

The narrow band $n(t)$ can be expressed as:

$$n(t) = r(t) \cos[2\pi f_c t + \psi(t)]$$

where envelop $r(t)$ and phase $\psi(t)$ are themselves defined in terms of the inphase component $n_c(t)$ and quadrature component $n_s(t)$ as:

$$r(t) = \sqrt{n_c^2(t) + n_s^2(t)}$$

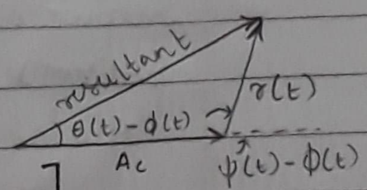
and $$\psi(t) = \tan^{-1}\left(\frac{n_s(t)}{n_c(t)}\right)$$

The total signal at the IF filter output is

$$x(t) = s(t) + n(t)$$

$$= A_c \cos[2\pi f_c t + \phi(t)] + r(t) \cos[2\pi f_c t + \psi(t)]$$

The relative phase $\theta(t)$ of the resultant phasor representing $x(t)$ is obtained as.



$$\theta(t) = \phi(t) + \tan^{-1}\left[\frac{r(t)\sin(\psi(t) - \phi(t))}{A_c + r(t)\cos(\psi(t) - \phi(t))}\right]$$

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The envelope of $x(t)$ is of no interest to us because any envelope variations at the IF filter output is removed by the limiter.

We assume that the carrier to noise ratio measured at the discriminator input is large when compared with unity.

Therefore

$$\theta(t) \simeq \underbrace{\phi(t)}_{\text{signal term}} + \underbrace{\frac{r(t)}{A_c} \sin[\psi(t) - \phi(t)]}_{\text{Noise term}}$$

The signal term $\phi(t)$ is proportional to the integral of the message signal $m(t)$.

i.e., $\phi(t) = 2\pi k_f \int^t m(t)$

Hence the discriminator output is

$$v(t) = \frac{1}{2\pi} \frac{d\theta(t)}{dt}$$

$$v(t) = k_f m(t) + n_d(t)$$

where the noise term $n_d(t)$ is

$$n_d(t) = \frac{1}{2\pi A_c} \frac{d}{dt} \left[r(t) \sin(\psi(t) - \phi(t)) \right]$$

Thus the discriminator output $v(t)$ consists of a scaled version of message signal $m(t)$ along with an additive noise component $n_d(t)$.

The presence of $\phi(t)$ produces components in the power spectrum of the noise $n_d(t)$ at frequencies that lie outside the message band. However such frequency components do not appear at the receiver output as they are rejected by the post detection filter. Hence the message dependent phase $\phi(t)$ is equated to zero.

$$n_d(t) = \frac{1}{2\pi A_c} \frac{d}{dt} \left[r(t) \sin[\psi(t)] \right]$$

The quadrature component of the narrow band noise $n(t)$ is

$$n_s(t) = r(t) \sin[\psi(t)]$$

Therefore

$$n_d(t) = \frac{1}{2\pi A_c} \frac{d n_s(t)}{dt}$$

Hence the average output noise power in an FM receiver depends only on the carrier amplitude A_c and the quadrature noise component $n_s(t)$.

The differentiation of a function with respect to time corresponds to multiplication of its Fourier transform by $j2\pi f$.

The noise process $n_d(t)$ can be obtained by passing $n_s(t)$ through a linear filter with a transfer function of

$$\frac{j2\pi f}{2\pi A_c} = \frac{jf}{A_c}$$

Therefore the power spectral density $S_{N_d}(f)$ of the noise $n_d(t)$ is related to the power spectral density $S_{N_Q}(f)$ of the quadrature noise component $n_s(t)$ as:

$$S_{N_d}(f) = \frac{f^2}{A_c^2} S_{N_Q}(f)$$

With IF filter we assume to have an ideal bandpass characteristics of bandwidth B and midband frequency f_c.

Thus the quadrature component $n_s(t)$ of the narrow band noise $n(t)$ will have the ideal lowpass characteristics.

$$\therefore \quad S_{N_d}(f) = \begin{cases} \dfrac{N_0 f^2}{A_c^2}, & |f| \leq B/2 \\ 0, & \text{otherwise.} \end{cases}$$

W is much smaller than B/2 where B is the transmission bandwidth of the FM signal. Thus the out of band components of noise $n_d(t)$ will be rejected.

Therefore the power spectral density $S_{No}(f)$ of the noise $n_o(t)$ appearing at the receiver output is

$$S_{No}(f) = \begin{cases} \dfrac{N_o f^2}{A_c^2}, & |f| \leq W \\ 0, & \text{otherwise.} \end{cases}$$

Therefore

average power of output noise

$$= \frac{N_o}{A_c^2} \int_{-W}^{W} f^2 \, df$$

$$= \frac{N_o}{A_c^2} \left. \frac{f^3}{3} \right|_{-W}^{W}$$

$$= \frac{2 N_o W^3}{3 A_c^2}$$

average output signal power $= k_f^2 P_m$

Therefore

$$(SNR)_o = k_f^2 P_m \frac{3 A_c^2}{2 N_o W^3} = \frac{3 k_f^2 P_m A_c^2}{2 N_o W^3}$$

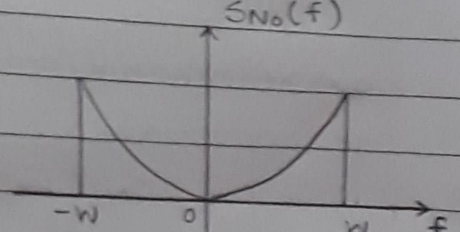
Figure of merit

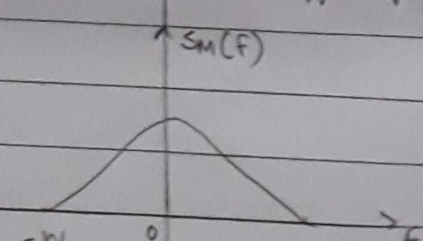
$$FOM = \frac{(SNR)_o}{(SNR)_c} = \frac{3 k_f^2 P_m \cancel{A_c^2}}{\cancel{2} N_o W^3} \Big/ \frac{\cancel{A_c^2} \cancel{P_m}}{\cancel{2} \cancel{N_o} \cancel{W}}$$

$$\boxed{FOM = \gamma = \frac{3 k_f^2}{W^2}}$$

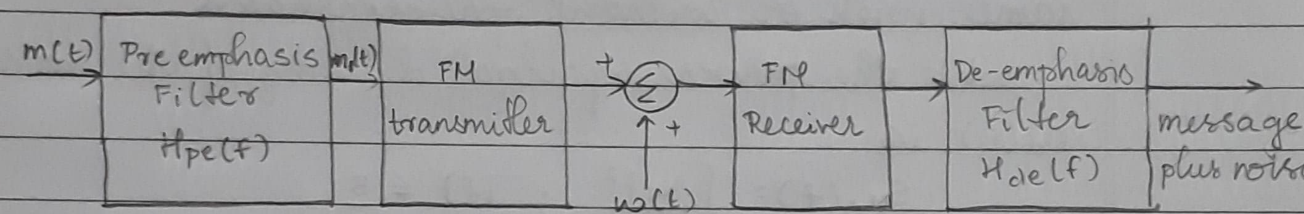
* **Pre-emphasis and De-emphasis:**

The power spectral density of the message usually falls off appreciably at higher frequencies. On the other hand the spectral density of the output noise increases rapidly with frequency. Thus at $f = \pm W$ the relative spectral density of the message is quite low whereas





the output noise is high in comparision. Thus the message is not using the frequency band effectively. One way of improving noise performance is to slightly reduce the bandwidth of the post detection low pass filter so as to reject a large amount of noise power but also a small amount of message power. Hence distortion of the message is caused. Therefore for the efficient use of the allowed frequency band by the usage of the pre-emphasis in the transmitter and de-emphasis in the receiver.



In preemphasis, we artificially emphasize the high frequency components of the message signal prior to modulation in the transmitter and hence the noise is introduced in the receiver. The low frequency and high frequency portions of the power spectral density of the message are equalized such that the message fully occupies the frequency band allotted to it.

At the receiver, inverse operation is performed by de-emphasizing the high frequency components to restore the original signal power distribution. By this the high frequency components of the noise in the output is also reduced.

To produce an undistorted version of the original message at the receiver, the pre-emphasis filter in the transmitter and the de-emphasis filter in the receiver would ideally have transfer functions that are the inverse of each other.

$$H_{de}(f) = \frac{1}{H_{pe}(f)} \qquad -W < f < W$$

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By this choice of transfer functions the average message power at the receiver output is independent of the preemphasis and de emphasis procedure.

The pre-emphasis filter is selected such that the average power of emphasized message signal $m_1(t)$ has the same average power as the original message $m(t)$.

$$\int_{-\infty}^{\infty} |H_{pe}(f)|^2 S_M(f)\, df = \int_{-\infty}^{\infty} S_M(f)\, df$$

$H_{pe}(f)$ ensures that the bandwidth remains same with or without pre-emphasis.

wkt the power spectral density of the noise $n_d(t)$ is:

$$S_{N_d}(f) = \begin{cases} \dfrac{N_0 f^2}{A_c^2} & ; \ |f| \le \dfrac{B}{2} \\ 0 & ; \ \text{else} \end{cases}$$

The power spectral density of the noise at the deemphasis filter is

$$|H_{de}(f)|^2 S_{N_d}(f)$$

Average output noise power with deemphasis

$$= \frac{N_0}{A_c^2} \int_{-\infty}^{\infty} f^2 |H_{de}(f)|^2\, df$$

Therefore improvement in output signal to noise ratio is given by:

$$I = \frac{\text{average output noise power without preemphasis and deemphasis}}{\text{average output noise power with preemphasis and deemphasis}}$$

wkt without pre-emphasis and deemphasis the average output noise power is $2N_0W^3/3A_c^2$

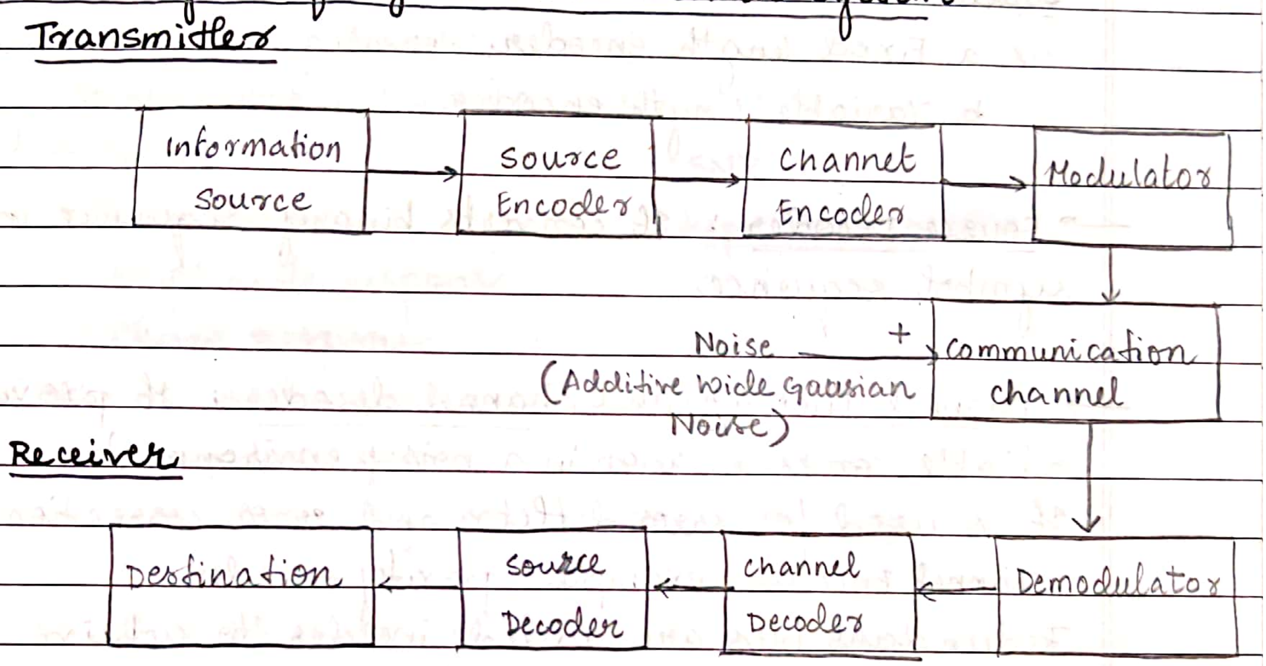
$$\therefore \ I = \frac{2N_0W^3}{3A_c^2} \Big/ \frac{N_0}{A_c^2} \int_{-\infty}^{\infty} f^2 |H_{de}(f)|^2\, df$$

$$I = \frac{2W^3}{3\int_{-\infty}^{\infty} f^2 |H_{de}(f)|^2\, df}$$

UNIT - 04

Sampling Process

★ **Block Diagram of Digital communication system:**



→ **Information source can be:**

a. discrete source : Discrete symbol (group of bits)

b. Binary source : Discrete bits (0's and 1's)

Parameters to define information source are:

- source alphabet : 26 alphabets + 6 special characters = 32 (32 only in case of English alphabets)
- symbol rate : Number of symbols generated per second.
- probability of occurance of a symbol. Equal probability (1/32)
- source entropy : Information content in a long message.
- Information rate.

→ **Source encoder** : It converts symbol sequence into a binary sequence.

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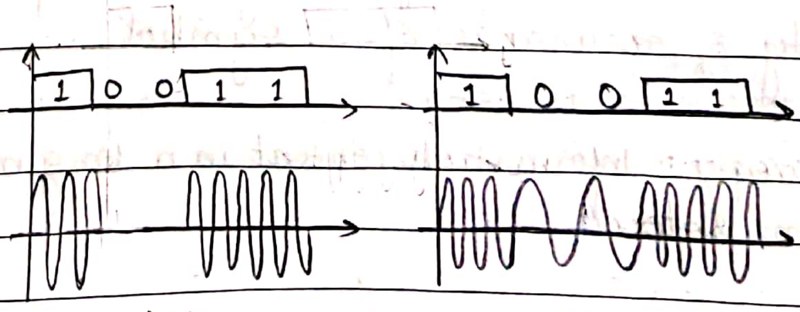
Parameters
- Block size
- code word length
- Encoder efficiency

Source encoder can be:
a. Fixed length encoder: Number of bits per symbol is fixed.
b. Variable length encoder: Number of bits per symbol will vary

→ Source Decoder: It converts binary sequence into symbol sequence

→ Channel Encoder: and channel decoder: It provide reliable communication in a noisy environment. It is used for error detector and error correction.
Channel Encoder example: parity encoder. Redundant bits are added inorder to achive reliable communication.

→ Modulator:
The techniques used for modulation are:
a. ASK: amplitude shift key. (similar to AM)
b. FSK: frequency shift key (similar to FM)
c. PSK: phase shift key (similar to PM)



ASK FSK

Scanned by CamScanner

→ Modem (Modulator - Demodulator) : Parameters of demodulator :
- Probability of error : (Pe) : Recieving zero as one or one as zero
- Power :
- Bandwidth :
- Type of detection : coherrent or non coherrent

→ Communication channel

Parameters to model a channel :

- Power
- Bandwidth
- Amplitude response
- Phase response
- SNR
- type of noise

Types of communication channel

a. Telephone channel : voice communication

a pair of wires : 300 Hz to 3.4 kHz

SNR = 30 dB

It is strongly affected by phase delay hence music and data communication is not possible this can be overcome by equilizers.

coaxial cable : immune to external interference bandwidth is more hence increases the quality but the cost is more.

b. Fiber Optic channels : light carries the information which is transmitted by the optic cable. It offers a very huge bandwidth. As the size is small more optic cables can be occupied. It is cost effective.

c. Micro wave link : It is based on LOS propagation (line of sight propagation). The receiver receives a direct ~~antenna~~ ray from the transmitter. Hence they are situated in higher altitudes. There is no physical link between the receiver and transmitter. The reciever receives a direct ray and ground reflected wave.

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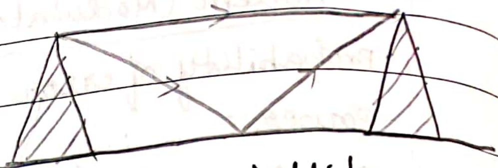

The operating frequency is in the order of GHz.

d. Satellite link: There are various satellites such as GEO, MEO, LEO and HEO satellites. based on the distance from the earth. As the distance increases the area of coverage increases. Due to the altitude it causes round trip delay (RTD). Hence GEO satellites cannot be used for real time applications. Thus GEO satellites are used for entertainment applications. It is at a distance of 27000km.

MEO satellites are at a distance 5000km.

LEO satellites are used for real time applicates. Lesser the distance lesser is the round trip delay (RTD) and launching cost is lesser.

Uplink is at the satellite whereas the downlink is situated on the surface of the earth.

Uplink frequency is higher than the down link frequency.

★ Sampling Process:

Let $x(t)$ be the input signal and f_x be the input frequency, $s(t)$ is the sampling signal and f_s is the sampling frequency.

According to sampling theorem, for the proper recovery of the signal from its samples, the sampling frequency must satisfy the following condition:

$$f_s \geq f_x$$

$$\text{or } T_s \leq \frac{1}{2f_x}$$

The minimum sampling frequency $f_s = 2f_x$ is called the Nyquist Frequency and $T_s = \frac{1}{2f_x}$ is called

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the Nyquist interval.

If $f_s \le 2f_x$, due to overlapping of the side bands $x(t)$ cannot be recovered from its samples without distortion.

★ **Sampling Theorem for lowpass signals:**

" A band limited signal of finite energy which has no frequency components higher than w Hz, is completely described by specifying the values of the signal at instants of time separated by $1/2w$ sec."

" A band limited signal of finite energy which has no frequency components higher than w Hz, may be completely recovered from its samples at the rate of $2w$ samples per second."

The statement 1 gives information about the sampling where as statement 2 gives information about the reconstruction of signal.

Hence: A continuous time signal can be completely represented in its samples and is recovered back if the sampling frequency is greater than or equal twice the highest frequency content of the signal.

★ **Ideal Sampling:** (Verification of sampling theorem)
(or delta sampling or impulse sampling
or instantaneous sampling)

The delta/impulse sampling is accomplished by using a train of impulses as sampling signal.

Let $x(t)$ be the band limited signal, $s(t)$ be the sampling signal (delta signal). A sampled signal is obtained by multiplying $x(t)$ with the sampling function $s(t)$.

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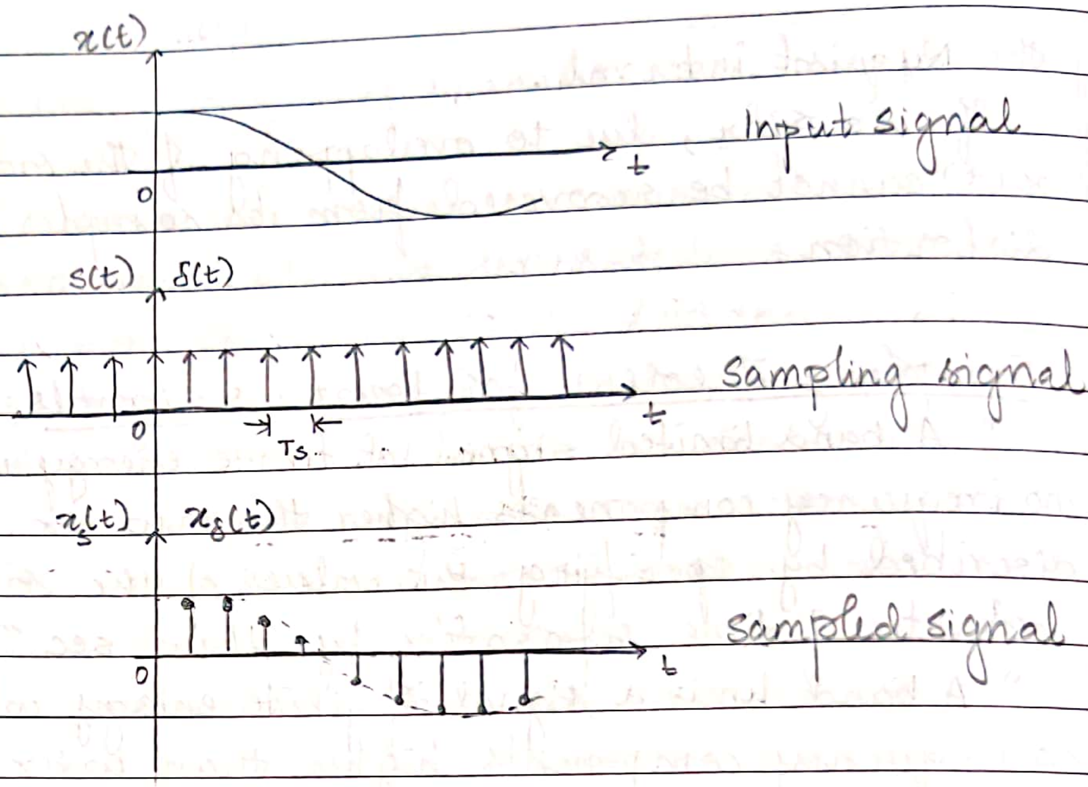

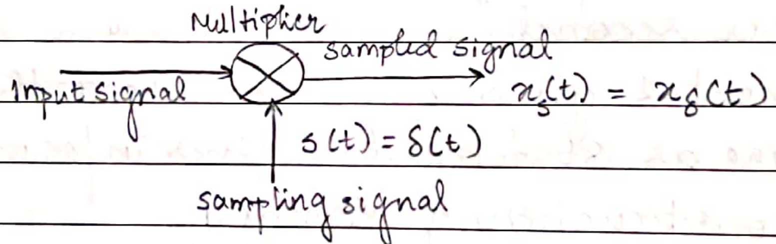

The sampled signal can be written as:

$$x_s(t) = x_\delta(t) = x(t)\cdot s(t) = x(t)\cdot \delta(t) \quad \text{——— (1)}$$

The delta / impulse function can be represented as

$$\delta(t) = s(t) = \sum_{k=-\infty}^{\infty} \delta(t-kT_s) \quad \text{——— (2)}$$

where T_s is the gap between the adjacent samples and is known as sampling interval $(1/f_s)$.

Substituting eq (2) in eq (1)

$$x_s(t) = x_\delta(t) = x(t)\cdot \sum_{k=-\infty}^{\infty} \delta(t-kT_s) \quad \text{——— (3)}$$

Taking Fourier Transform.

$$X_s(f) = X_\delta(f) = X(f) * S(f) = X(f) * \delta(f) \quad \text{——— (4)}$$

wkt fourier Transform of delta function is

$$S(f) = \delta(f) = f_s \sum_{n=-\infty}^{\infty} \delta(f - nf_s) \quad \text{——— (5)}$$

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Substituting eq ⑤ in eq ④

$$X_s(f) = X_\delta(f) = X(f) * f_s \sum_{n=-\infty}^{\infty} \delta(f - nf_s) \quad \text{—— } ⑥$$

Interchange the order of summation and convolution.

$$X_s(f) = X_\delta(f) = f_s \sum_{n=-\infty}^{\infty} X(f) * \delta(f - nf_s) \quad \text{—— } ⑦$$

From the property of delta function as well as the shifting property of Fourier Transform, we have

$$\boxed{X_s(f) = X_\delta(f) = f_s \sum_{n=-\infty}^{\infty} X(f - nf_s)} \quad \text{—— } ⑧$$

$$X_s(f) = X_\delta(f) = f_s X(f) + f_s \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} X(f - nf_s) \quad \text{—— } ⑨$$

The spectrum of the sampled signal is periodic in the frequency with a period equal to sampling frequency but not necessarily continuous.

The process of sampling a signal in a time domain results in a periodic spectrum in frequency domain with a period equal to sampling rate f_s.

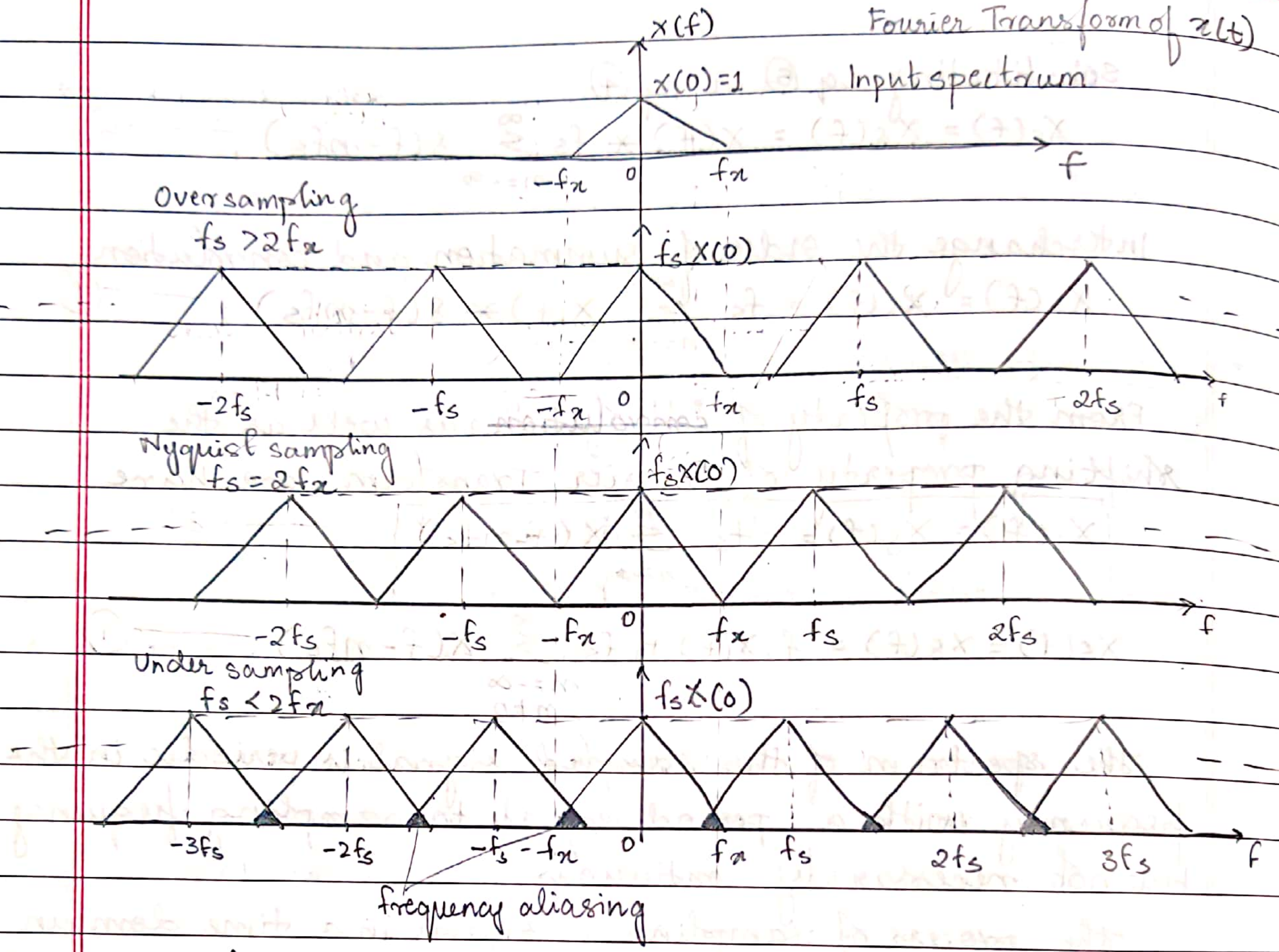
Therefore the sampled signal spectrum is a periodic extension of the input signal spectrum $X(f)$.

Therefore

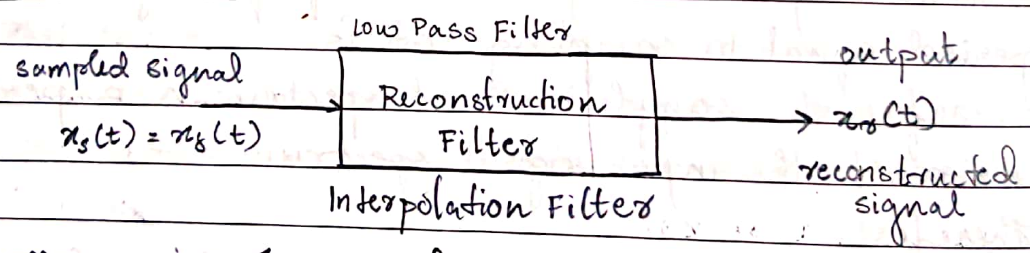
$$X(f - nf_s) = X(f) \text{ at } f = 0, \pm f_s, \pm 2f_s \ldots$$

Assume that the input signal $x(t)$ is band limited

Distortion caused due to overlapping of the sidebands when sampling frequency $f_s < 2f_x$ (under-sampling) is called as frequency aliasing.



- **Reconstruction:**



The process of reconstructing a continuous time signal $x(t)$ from the signal is called reconstruction or interpolation.

The low pass filter is used to recover the original signal from its samples, hence the low pass filter is also known as interpolation filter or reconstruction filter.

The low pass filter passes only low frequencies upto a specified cut-off frequency and rejects all other frequencies above the cut-off frequencies as shown in the figure.

The output of the reconstruction filter is

$x_r(t) = x(t)$ when $f_s \geq 2f_x$ $x(t)$ is band limited.

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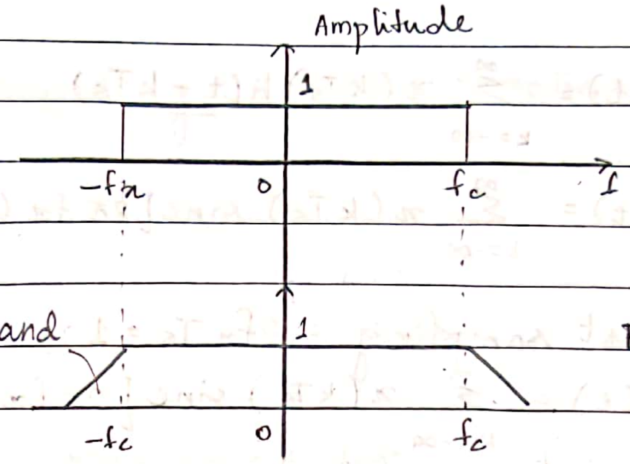


Ideal response of the Low pass filter

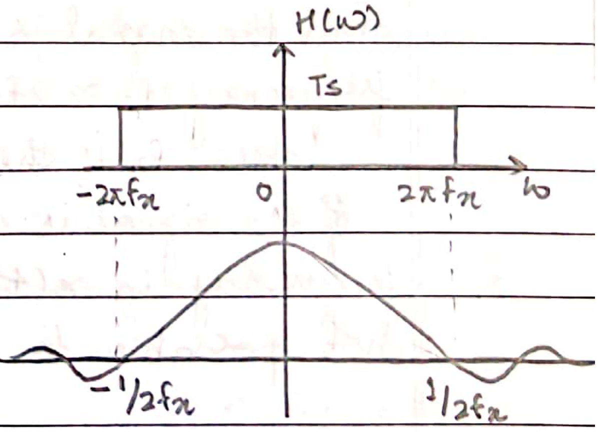
Practical response of the low pass filter

we know than an ideal low pass filter is not physically realisable (the signal abruptly reaches zero)

In practical low pass filter the amplitude response decreases slowly to become zero, i.e., a transition band is present inbetween pass band and the stop band.

To recover $x(t)$ from its samples, pass the sampled signal through an ideal low pass filter of bandwidth f_x Hz and gain of T_s. Therefore the transfer function of reconstruction or interpolation filter is: which is given by

$$H(\omega) = T_s \cdot \text{rect}\left[\frac{\omega}{4\pi f_x}\right]$$



The impulse response of the interpolation filter is

$$h(t) = F^{-1}[H(\omega)]$$

$$h(t) = 2f_x T_s \, \text{sinc}(2\pi f_x t)$$

Assume that the sampling is at nyquist rate

i.e., $f_s = 2f_x$ or $T_s = 1/2f_x$

$$\therefore 2f_x T_s = 1$$

When the sampled signal $x_s(t)$ is applied at the input of the reconstruction filter, then the output of the reconstruction filter is:

$$x_r(t) = x(t) = x_s(t) * h(t)$$

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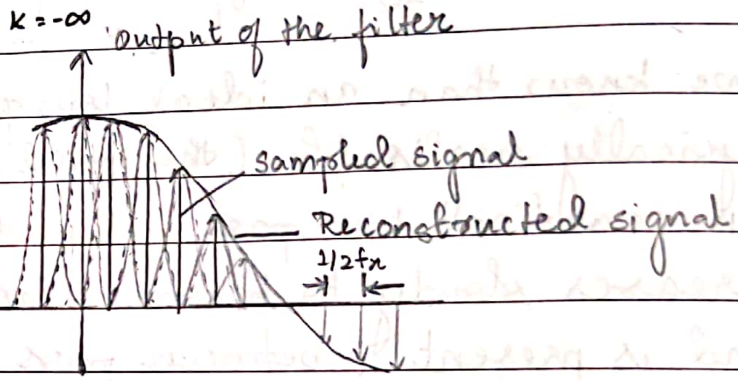
$$x_r(t) = x(t) = \sum_{k=-\infty}^{\infty} x(kT_s)\, h(t-kT_s)$$

$$x_r(t) = x(t) = \sum_{k=-\infty}^{\infty} x(kT_s)\, \mathrm{sinc}[2\pi f_x (t-kT_s)]$$

Under nyquist sampling : $2f_x T_s = 1$

$$x_r(t) = x(t) = \sum_{k=-\infty}^{\infty} x(kT_s)\, \mathrm{sinc}[2\pi f_x t - k\pi] \quad \text{— (A)}$$



Eq (A) is known as interpolation formula which provides values of $x(t)$ between samples as a weighted sum of all the samples used.

To recover the original signal from its samples, assume that the signal is strictly band limited and sampling frequency $f_s \geq 2f_x$.

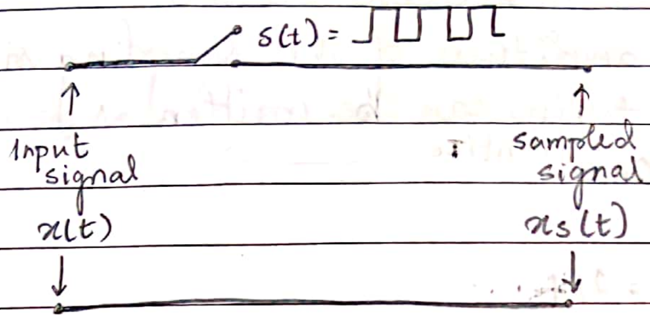
where f_x is the maximum frequency content in $x(t)$.

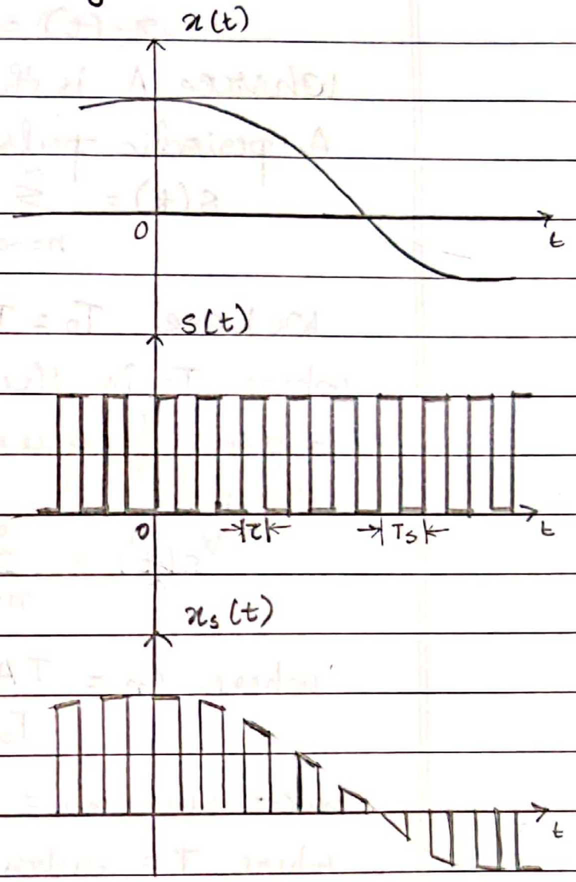
If the signal is not band limited, the maximum frequency in $x(t)$ cannot be predicted. Therefore it is not possible to select a sampling frequency f_s.

★ **Note:**

Ideal sampling is not possible, because a pulse of zero width is not possible to realise physically. Hence we adopt natural sampling.

★ Natural Sampling : (Chopper Sampling)





In natural sampling the pulse has finite width (τ). Natural sampling is also called as chopper sampling because the sampled signal appears to be chopped off from the original signal.

Let $x(t)$ be the analog continuous time signal, sampled at the rate of f_s Hz and f_s is higher than the Nyquist rate so that the sampling theorem is satisfied.

A sampled signal $x_s(t)$ is obtained by the multiplication of the sampling signal and the input signal.

The sampling signal $s(t)$ is a train of periodic pulse of width τ and frequency equal to f_s Hz.

From the fig, we see that when $s(t)$ goes high, switch s is closed. Therefore the output of the sampler is :

$$x_s(t) = x(t)s(t) \quad \text{———} \; (1)$$

$$x_s(t) = x(t)A \qquad \text{when } s \text{ is closed.}$$

If $A = 1$; $x_s(t) = x(t)$

when switch s is opened, output of the sampler
$x_s(t) = 0$. because $s(t) = 0$.

where A is the amplitude of the sampling signal $s(t)$

A periodic pulse train can be written as

$$s(t) = \sum_{n=-\infty}^{\infty} C_n e^{j2\pi nt/T_0} \quad \text{——} ②$$

We have $T_0 = T_s = 1/f_s$

where T_s is the period of $s(t)$ and f_s is the sampling frequency.

Therefore

$$s(t) = \sum_{n=-\infty}^{\infty} C_n e^{j2\pi ntf_s} \quad \text{——} ③$$

$$\text{where } C_n = \frac{TA}{T_0} \text{sinc}(f_n T) \quad \text{——} ④$$

wkt $\text{sinc}(x) = \sin x / x$

where T = width of the pulse $= \tau$

f_n: harmonic frequency $= nf_s$

Therefore

$$C_n = \frac{\tau A}{T_s} \text{sinc}(nf_s\tau) \quad \text{——} ⑤$$

The output of the sampler is given by

$$x_s(t) = x(t) \cdot s(t)$$

$$x_s(t) = \sum_{n=-\infty}^{\infty} \frac{\tau A}{T_s} \text{sinc}(nf_s\tau) e^{j2\pi ntf_s} \cdot x(t) \quad \text{——} ⑥$$

Taking Fourier Transform.

$$X_s(f) = \frac{\tau A}{T_s} \sum_{n=-\infty}^{\infty} \text{sinc}(nf_s\tau) F\left[e^{j2\pi ntf_s} \cdot x(t)\right] \quad \text{——} ⑦$$

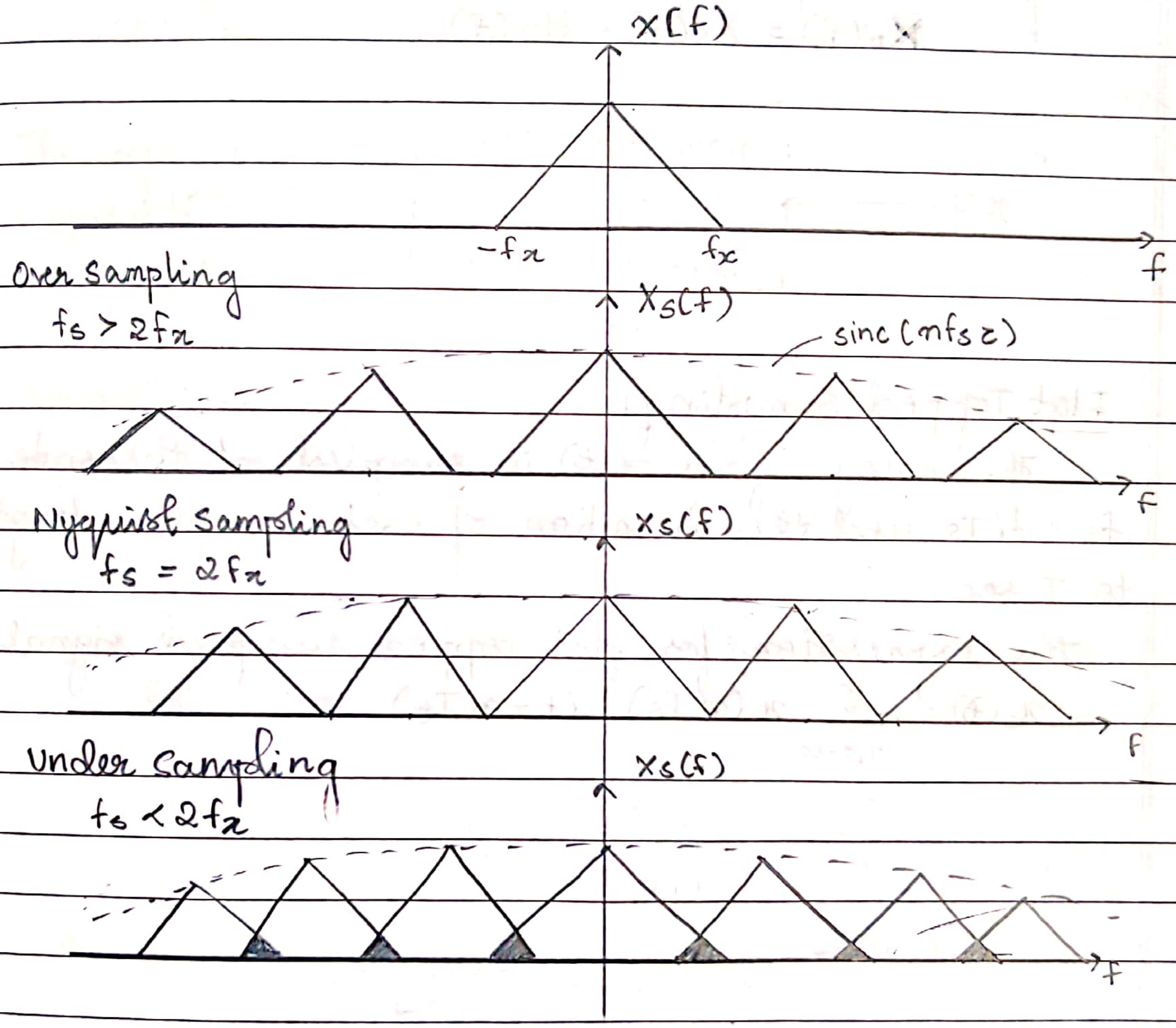
wkt Fourier Transform of $e^{j2\pi ntf_s} x(t)$ is:

$$F\left[e^{j2\pi ntf_s} x(t)\right] \longrightarrow X(f - nf_s) \quad \text{——} ⑧$$

substituting eq ⑧ in eq ⑦

$$x_s(f) = \frac{\tau A}{T_s} \sum_{n=-\infty}^{\infty} \mathrm{sinc}(nf_s\tau)\, X(f - nf_s) \quad \text{--- (9)}$$

when $x(t)$ is band limited.



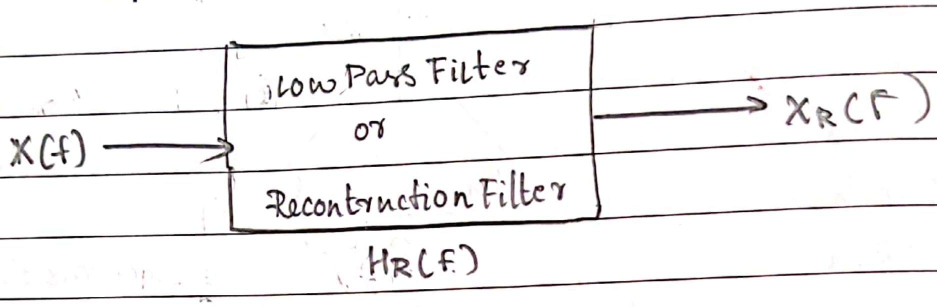
The amplitude of the spectrum decreases due to the presence of sinc function. Thus unlike the spectrum of ideally sampled signal, the spectrum of naturally sampled signal weighted by sinc function.

But the spectrum of ideally sampled signal remains constant throughout the frequency range.

The spectrum of the naturally sampled signal is periodic in f_s and are weighted by sinc function.

- **Reconstruction:**
Pass the naturally sampled signal through an ideal low pass filter with transfer function $H_R(f)$. Therefore the output of LPF is

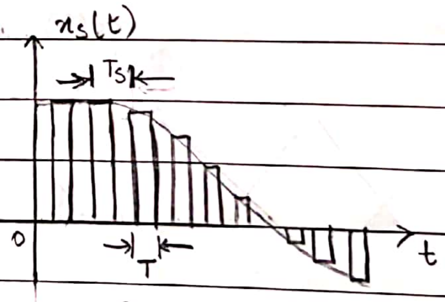
$$X_R(f) = X_S(f) \cdot H_R(f)$$



★ **Flat Topped Sampling:**
The analog signal $x(t)$ is sampled at the rate of $f_s = 1/T_s$ and the duration of each sample is lengthened to T sec.

The expression for flat topped sampled signal is.

$$x_s(t) = \sum_{n=-\infty}^{\infty} x(nT_s)\, h(t - nT_s) \quad \text{——— (A)}$$



In flat top sampling, the amplitude of the sample within the sample duration remains the same. The starting edge of the sample represents the instantaneous value of the input signal $x(t)$.

- "The flat top sampled signal is mathematically equivalent to the convolution of ideally sampled signal and the rectangular pulse signal."

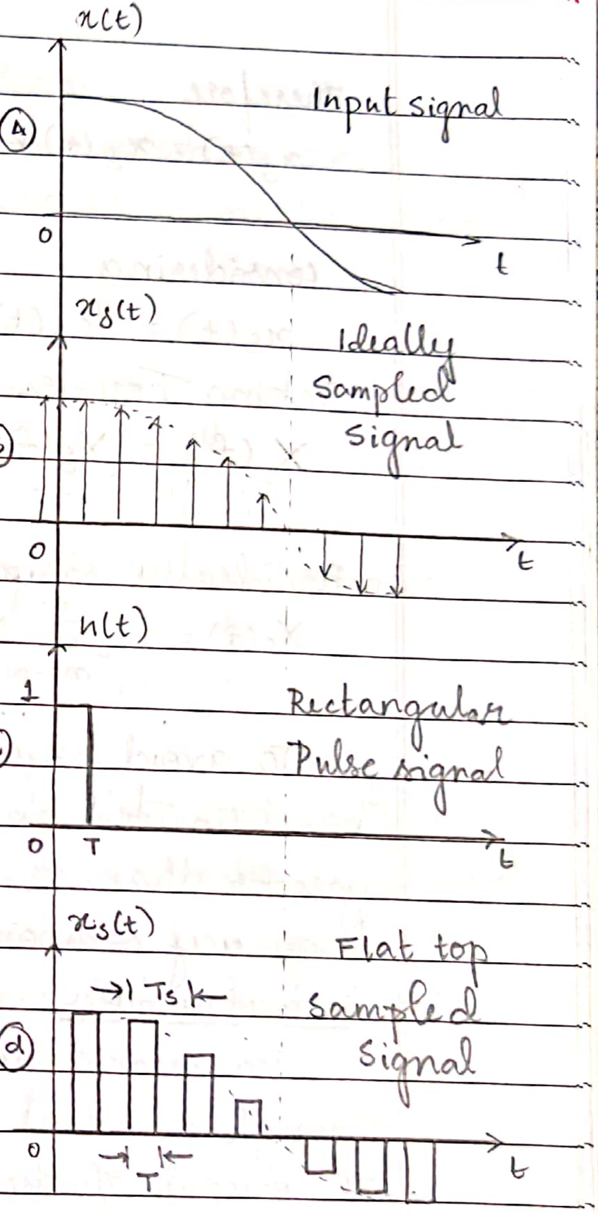
Proof: consider a signal $x(t)$ and its ideally sampled signal $x_\delta(t)$.

Let $h(t)$ be the rectangular pulse function of amplitude unity and duration T seconds and is defined as:

$$h(t) = \begin{cases} 1 & ; 0 \le t \le T \\ 0 & ; \text{else} \end{cases} \quad \text{——} (1)$$

The ideally sampled signal is given by

$$x_\delta(t) = \sum_{n=-\infty}^{\infty} x(nT_s)\,\delta(t-nT_s) \quad \text{——} (2)$$



convolving ideally sampled signal $x_\delta(t)$ with the rectangular pulse signal $h(t)$, we get:

$$x_\delta(t) * h(t) = \int_{-\infty}^{\infty} x_\delta(\tau)\,h(t-\tau)\,d\tau \quad \text{——} (3)$$

Substituting eq (1) and eq (2) in eq (3), we get:

$$x_\delta(t) * h(t) = \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} x(nT_s)\,\delta(\tau-nT_s)\,h(t-\tau)\,d\tau \quad \text{——} (4)$$

Interchanging the order of integration and summation.

$$x_\delta(t) * h(t) = \sum_{n=-\infty}^{\infty} x(nT_s) \int_{-\infty}^{\infty} \delta(\tau-nT_s)\,h(t-\tau)\,d\tau \quad \text{——} (5)$$

Applying the shifting property of delta function.

$$x_\delta(t) * h(t) = \sum_{n=-\infty}^{\infty} x(nT_s)\,h(t-nT_s) \quad \text{——} (B)$$

comparing expression (A) and expression (B), we see that the flat top sampled signal is the convolution of ideally sampled signal and the rectangular pulse signal.

Therefore

$$x_S(t) = x_\delta(t) * h(t) = \sum_{n=-\infty}^{\infty} x(nT_s)\, h(t-nT_s) \quad (6)$$

considering

$$x_S(t) = x_\delta(t) * h(t)$$

Taking Fourier transform.

$X_S(f) = X_\delta(f) \cdot H(f)$ where $H(f)$ is the transfer function of rectangular pulse signal.

For ideally sampled signal

$$X_S(f) = f_s \sum_{n=-\infty}^{\infty} X(f-nf_s)\, H(f)$$

To avoid frequency aliasing assume that $x(t)$ is band limited and the sampling frequency f_s is greater than or equal to twice the maximum frequency component present in the input signal ($f_s \geq 2f_x$)

- Reconstruction:

For reconstruction, pass flat top sampled signal through an ideal low pass or reconstruction filter. The output of the reconstruction filter is:

$$X_R(f) = X_S(f) H_R(f)$$

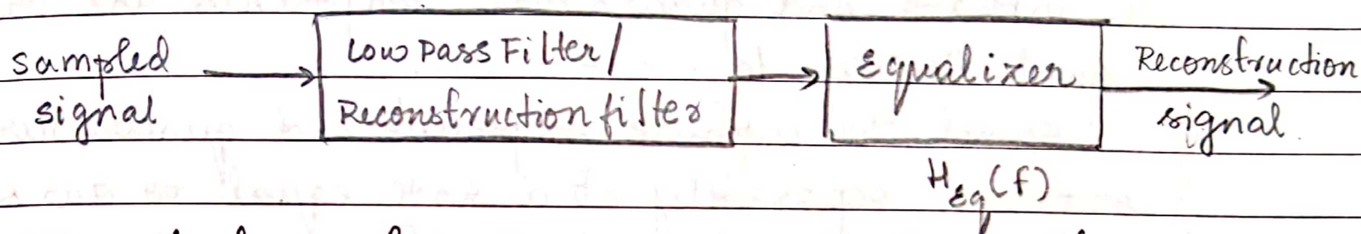
where $H_R(f)$ is the transfer function of the reconstruction filter.

★ Aperture Effect:

By using samples of finite duration, the sampling process introduces amplitude distortion as well as a delay of $T/2$ sec, where T is the duration of the sample. This effect is similar to the variation in transmission with frequency that is caused by the finite size of the scanning aperture in television.

Therefore the distortion caused by lengthening the

samples is called as aperture effect.
This distortion is corrected by connecting an equaliser in cascade with low pass or reconstruction filter.



The ideal amplitude response of the equalizer is

$$\frac{1}{|H_{eq}(f)|} = \frac{1}{T\,\text{sinc}(fT)} = \frac{1}{T}\,\frac{\pi f T}{\sin(\pi f T)}$$

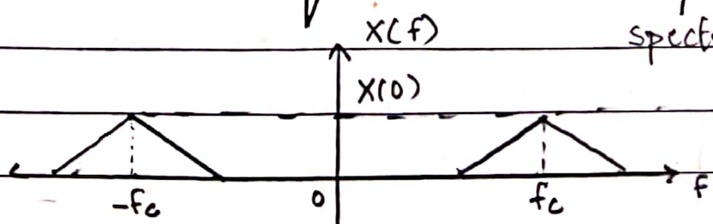
★ <u>Sampling of Band Pass signals</u> (Quadrature sampling):

Let $x(t)$ be the band pass signal (bandwidth of $2W$ and center at f_c). Representing bandpass signal in terms of its inphase or quadrature component or representing bandpass signal using its low pass equivalent.

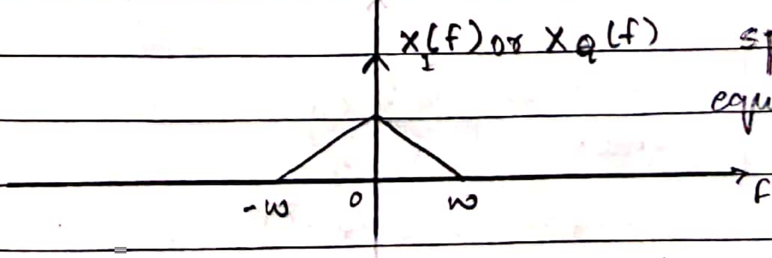
Band pass signal

$$x(t) = x_I(t)\cos 2\pi f_c t - x_Q(t)\sin 2\pi f_c t$$

where $x_I(t)$ is the inphase component
and $x_Q(t)$ is the quadrature component



spectrum of band pass signal



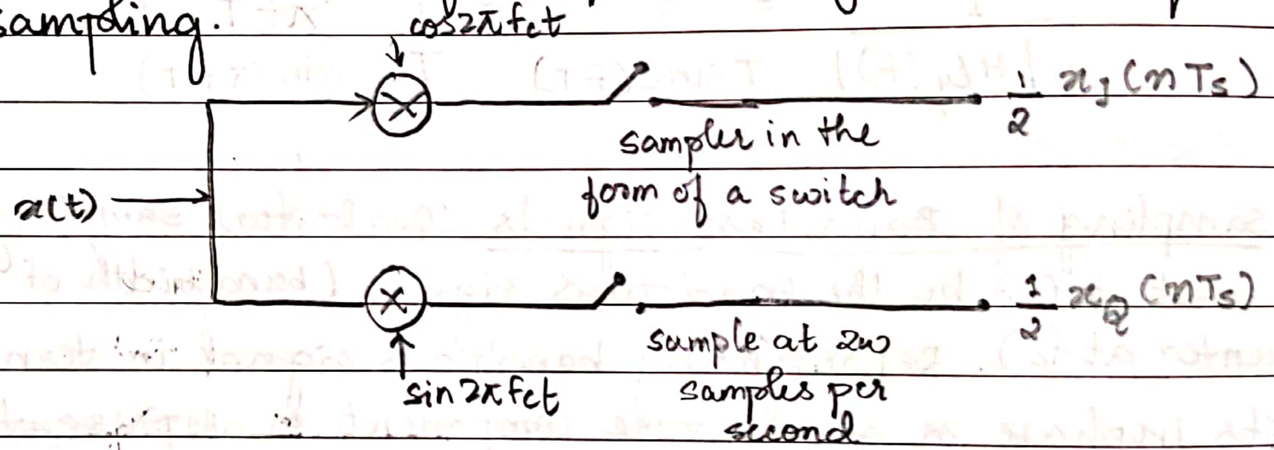
spectrum of low pass equivalents of band pass signal

Inphase or quadrature component are obtained by multiplying the band pass signal $x(t)$ with $\cos 2\pi f_c t$

and $\sin 2\pi f_c t$ respectively ~~by me~~ and suppressing the frequency components by means of appropriate low pass filter.

Under the assumption that $f_c > w$, we see that the inphase and quadrature components are low pass signals limited to $-w \le f \le w$.

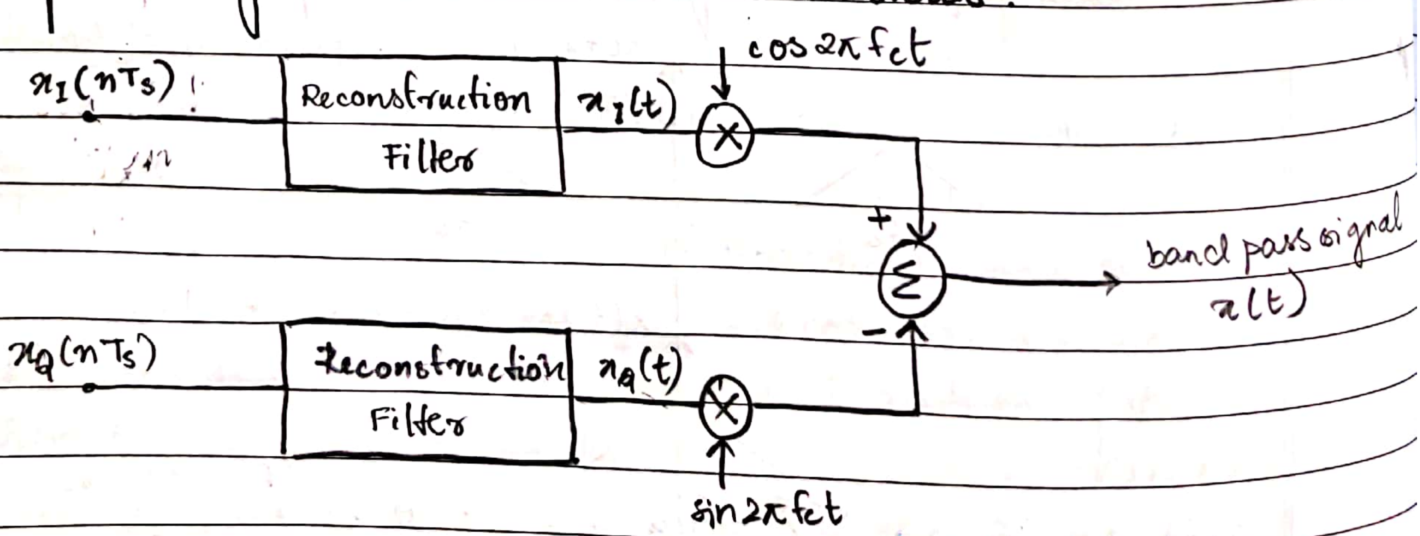
Sample the inphase component and quadrature component separately at a rate equal to $2w$ samples per second. This form of sampling is called quadrature sampling.



- **Reconstruction:**

• Separately pass sampled inphase and quadrature component through the reconstruction filter.

• Multiply them by $\cos 2\pi f_c t$ and $\sin 2\pi f_c t$ respectively and then add the results.



- Sampling theorem of Band Pass Signals:

The bandpass signal with bandwidth $2W$ can be completely represented into and recovered from its samples if it is sampled at the minimum rate i.e., twice the bandwidth of the input signal.

Q: The given message spectrum is sampled at the rate of $f_s = 1.5 f_{max}$ where $f_{max} = 1$ Hz. If the sampled signal is passed through an ideal low pass signal of bandwidth f_{max}, then

i. Find and sketch the spectrum of sampled signal.

ii. Find the spectrum of the reconstructed signal.

x(t) is band limited

- Assume ideal sampling

given $f_s = 1.5 f_{max}$

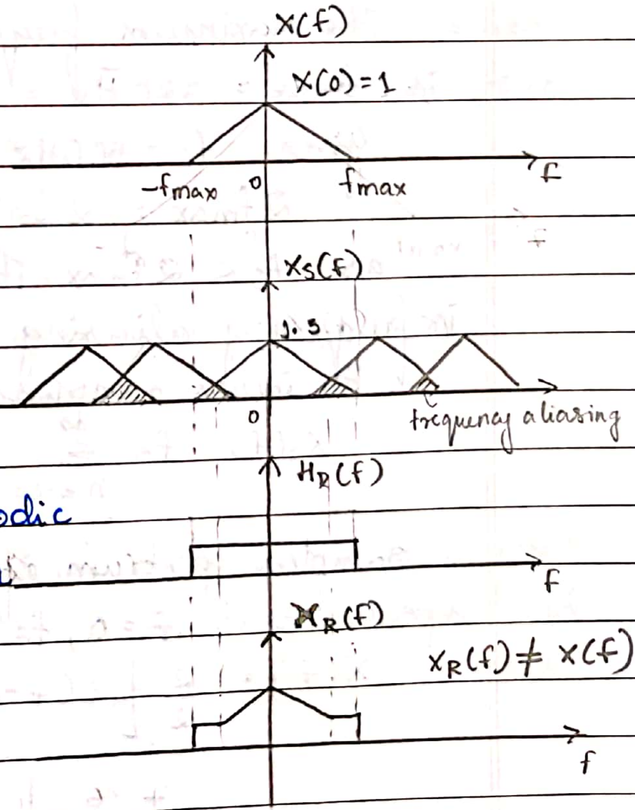
where $f_{max} = 1$ Hz

But here $f_s < 2 f_{max}$

(under sampling)

Hence there is frequency aliasing.

$$X_s(f) = f_s \sum_{n=-\infty}^{\infty} X(f - nf_s)$$



Sampled spectrum is the periodic extension of the input spectrum and input spectrum at

$f = 0,\ f = \pm f_s,\ f = \pm 2f_s \cdots$

$$X_s(f) = 1.5 \sum_{n=-\infty}^{\infty} X(f - 1.5n)$$

Because of frequency aliasing (under sampling) there is distortion in the reconstructed signal.

Therefore $X_R(f) \neq X(f)$

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Q: A signal $x(t) = 2\cos 400\pi t + 6\cos 640\pi t$ is ideally sampled at $f_s = 500Hz$. The sampled signal is passed through an ideal low pass filter with cut off frequency 400 Hz.

i. Find and sketch the spectrum of the sampled signal.

ii. Find the frequency components that will appear across the output of the low pass filter.

— Given

$$x(t) = 2\cos 400\pi t + 6\cos 640\pi t$$

Here $f_1 = \frac{400\pi}{2\pi} = 200 Hz$

$$f_2 = \frac{640\pi}{2\pi} = 320 Hz$$

The maximum frequency component of the signal is $f_{max} = 320 Hz = f_2$

given $f_s = 500Hz$

$$2f_{max} = 2(320) = 640 Hz$$

as $f_s < 2f_{max}$ it is under sampling hence there is frequency aliasing.

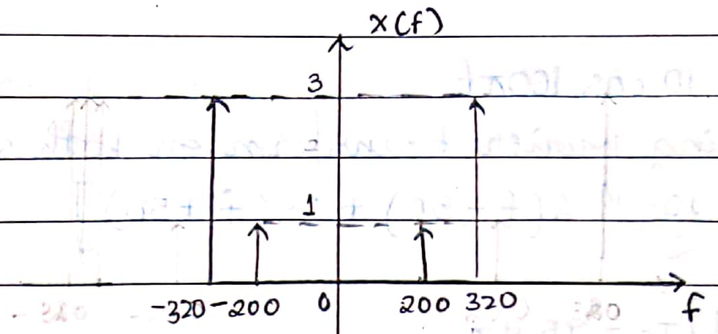
For ideal sampling.

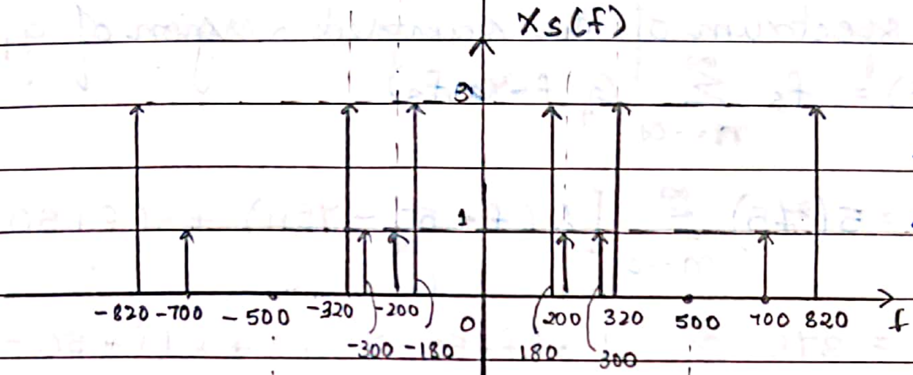
$$X_S(f) = f_s \sum_{n=-\infty}^{\infty} X(f - nf_s)$$

sampled spectrum is the periodic extension of the input spectrum at $f=0$, $f = \pm f_s$; $f = \pm 2f_s$ ....

$$X(f) = \frac{2}{2}\left[\delta(f-200) + \delta(f+200)\right] + \frac{6}{2}\left[\delta(f-320) + \delta(f+320)\right]$$

$$X(f) = [\delta(f-200) + \delta(f+200)] + 3[\delta(f-320) + \delta(f+320)]$$



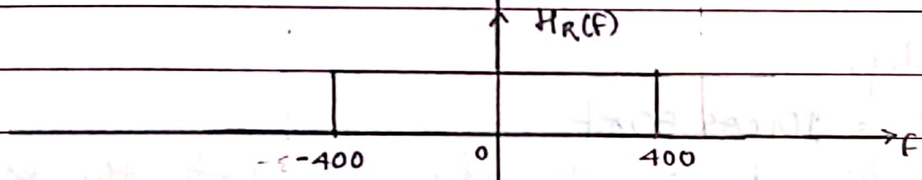


$200 \to 200 + f_s = 700$

$-200 \to -200 + f_s = 300$

$320 \to 320 + f_s = 820$

$-320 \to -320 + f_s = 180$

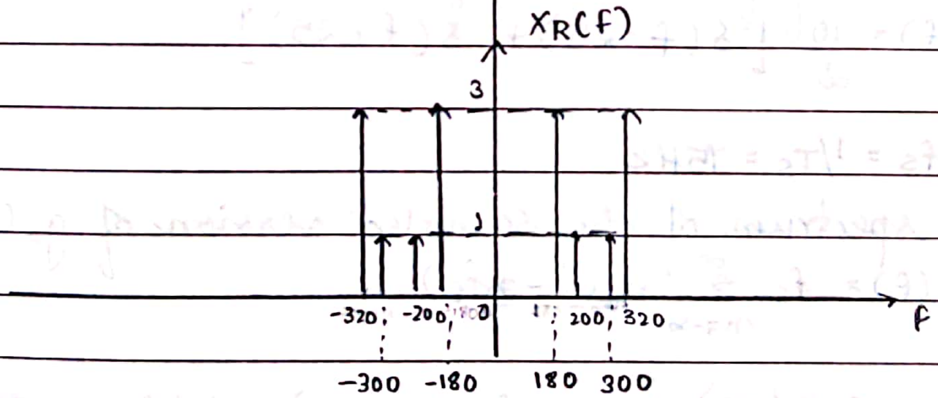


$200 - f_s = -300$

$-200 - f_s = -700$

$320 - f_s = -180$

$-320 - f_s = -820$



Q: The signal $g_1(t) = 10 \cos(100\pi t)$ and signal $g_2(t) = 10\cos 50\pi t$ are both sampled at $t_n = nT_s$ where $n = 0, \pm 1, \pm 2, \pm 3 \dots$ and $f_s = 75$ samples/sec. Show that the two sequences of samples thus obtained are identical.

given:

$g_1(t) = 10\cos(100\pi t)$

$g_2(t) = 10\cos(50\pi t)$

$f_s = 75 Hz$

$g_1(t) = 10\cos 100\pi t$

taking fourier transform on both sides ~~of the~~

$$G_1(f) = \frac{10}{2}\left[\delta(f-50) + \delta(f+50)\right]$$

$f_s = 1/T_s = 75\,Hz$

The spectrum of the sampled version of $g_1(t)$ is

$$G_{1\delta}(f) = f_s \sum_{m=-\infty}^{\infty} G_1(f - mf_s)$$

$$= 5(75) \sum_{n=-\infty}^{\infty} \left[\delta(f-50-75n) + \delta(f+50-75n)\right]$$

$$= 375 \sum_{n=-\infty}^{\infty} \left[\delta(f-50-75n) + \delta(f+50-75n)\right]$$

Similarly.

$g_2(t) = 10\cos 50\pi t$

taking fourier transform on both the sides.

$$G_2(f) = \frac{10}{2}\left[\delta(f-25) + \delta(f+25)\right]$$

$f_s = 1/T_s = 75\,Hz$

The spectrum of the sampled version of $g_2(t)$ is

$$G_{2\delta}(f) = f_s \sum_{n=-\infty}^{\infty} G_2(f - nf_s)$$

$$= 5(75) \sum_{n=-\infty}^{\infty} \left[\delta(f-25-75n) + \delta(f+25-75n)\right]$$

Let $n = l-1$ in first term and $n = k+1$ in second term

$$G_{2\delta}(f) = 375 \sum_{l=-\infty}^{\infty} \delta(f-25-75(l-1)) + 375 \sum_{k=-\infty}^{\infty} \delta(f+25-75(k+1))$$

$$= 375 \sum_{l=-\infty}^{\infty} \delta(f+50-75l) + 375 \sum_{k=-\infty}^{\infty} (f-50-75l)$$

since l and k are any variable it can be replaced by n

$$\therefore G_{2\delta}(f) = 375 \sum_{n=-\infty}^{\infty} \left[\delta(f-50-75n) + \delta(f+50-75n)\right]$$

Therefore $g_1(t)$ and $g_2(t)$ are identical.

Page
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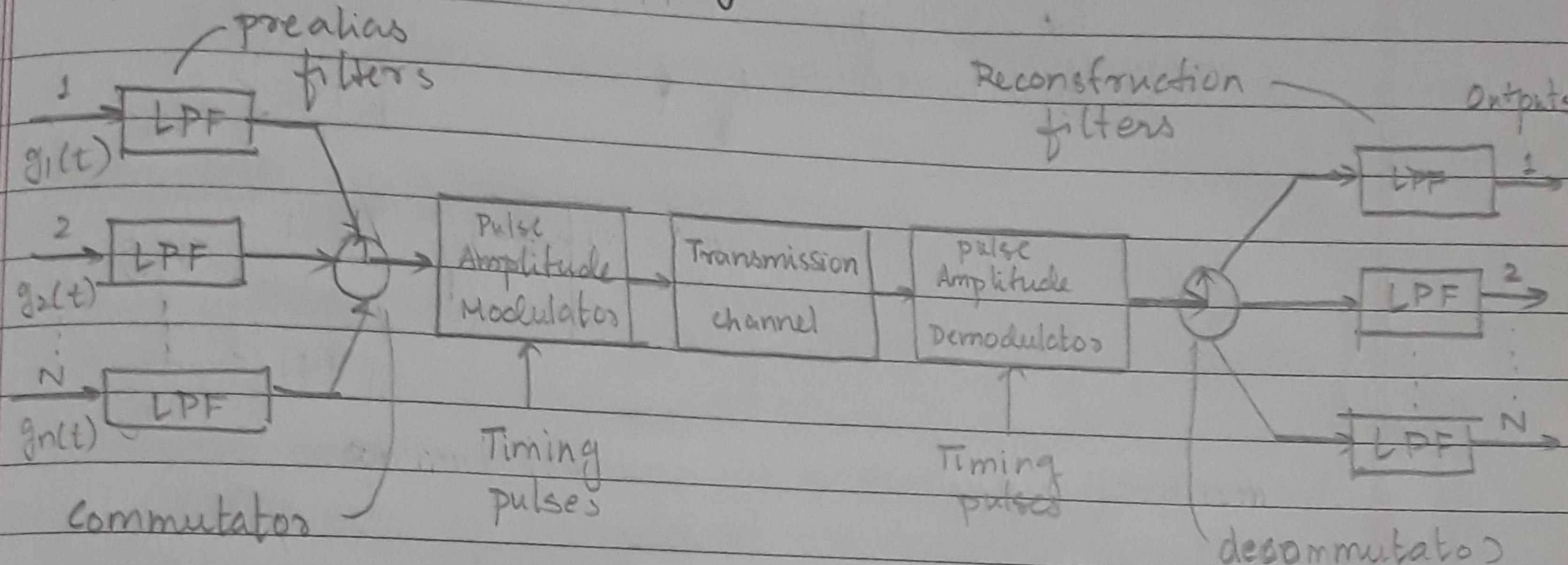
* ## Time division Multiplexing : (TDM)



It is a method of putting multiple data streams in a single signal by ~~separating~~ the signal into multiple segments each having a very small duration.

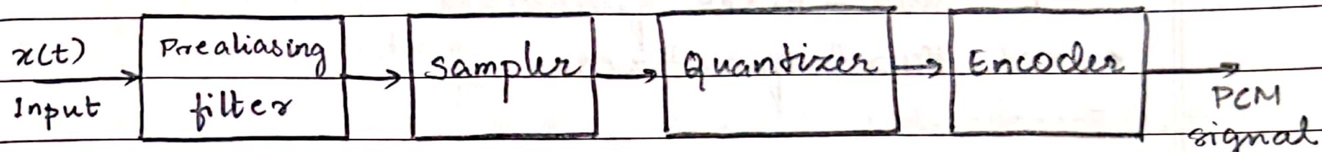
The circuit that combines signals at the source is known as a multiplexer.

Then at the recieving end, the individual signals are separated out by means of a circuit called a demultiplexer.

UNIT - 5

Waveform Coding Techniques

* **Pulse code Modulation: (PCM)**



Pulse code modulation is a technique for rounding off the amplitudes of samples of an analog input waveform.
The three essential operations are sampling, quantizing and encoding.

Sampling: The incoming message signal is passed through a lowpass filter to remove unwanted high frequency components present in it. Thus making the signal band limited to ω Hz. Then this signal is then sampled using a train of rectangular pulses having a width tends to zero.

Quantizing: The analog input g(t) is filtered and sampled to obtain g(nTs). A quantizer rounds off the sample values to a discrete value of quantization levels.
The resulting quantized sample $g_q(nT_s)$ are discrete in time (sampling) and discrete in amplitude (quantizing)

Encoding: An encoder translates quantized samples into digital codewords. In binary encoding each digital word contains n bits. Each bit could be a binary 1 or 0.

Thus Pulse code modulation is analog to digital conversion where the analog samples are represented by digital words in a serial bit stream.

* Quantization:

→ **Uniform Quantization**

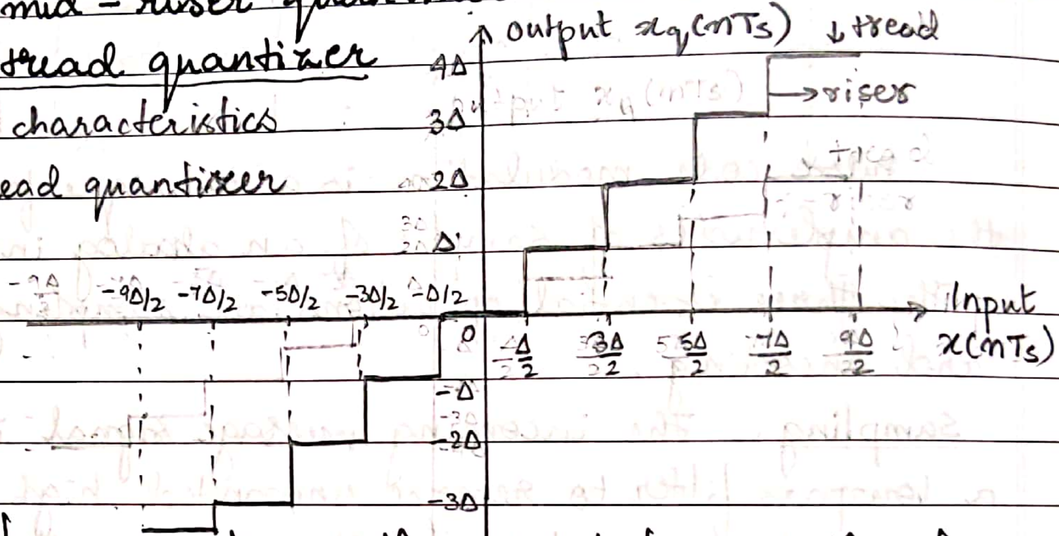
The step size is uniform through out the quantization. Types of uniform quantization are:-

a. mid-tread quantizer

b. mid-riser quantizer.

- **Mid-tread quantizer**

Transfer characteristics of mid tread quantizer



In uniform quantizer the gap between the two quantization levels or step size Δ remains the same throughout the quantization process.

The center of the tread of staircase characteristics coincides with the origin where the quantizer output is zero and hence the name.

– **Analysis:**

Assume that the input to the quantizer varies from $-9\Delta/2$ to $+9\Delta/2$, i.e., the peak to peak value of $x(nT_s)$ is between $-9\Delta/2$ to $+9\Delta/2$.

From the figure we see that when the input is between $-\Delta/2$ and $\Delta/2$, the quantizer output is zero. That means that, if $-\Delta/2 < x(nT_s) < +\Delta/2$, then $x_q(nT_s) = 0$ where Δ is the step size of the quantizer.

Similarly when the input to the quantizer is between $\Delta/2$ and $3\Delta/2$, the quantizer output is $\frac{\Delta/2 + 3\Delta/2}{2} = \Delta$

ie if $\frac{\Delta}{2} < x(nT_s) < \frac{3\Delta}{2}$, then $x_q(nT_s) = \Delta$.

- Quantization error

During the quantization process, the samples are selected on minimum error basis. The error is introduced during the quantization process and this error is referred to as quantization error and is given by:

$$\text{quantization error} : \varepsilon = x_q(nT_s) - x(nT_s)$$

We see that when $x(nT_s)$ is zero, quantizer output is zero i.e., $x_q(nT_s) = 0$ (at the origin)

Therefore the quantization error is zero.

When the input $x(nT_s) = \pm \Delta/2$, the quantizer output $x_q(nT_s) = 0$.

Therefore the quantization error is

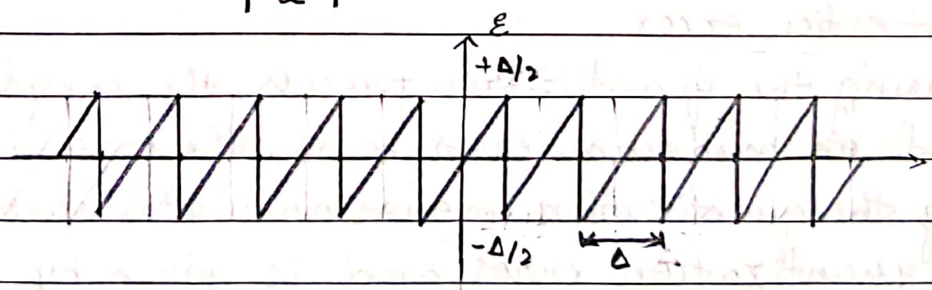
$$\varepsilon = 0 - (\pm \Delta/2)$$

$$\varepsilon = \pm \Delta/2$$

In general for a uniform quantizer, the quantization error is inbetween $\pm \Delta/2$, i.e., $-\Delta/2 \le \varepsilon \le \Delta/2$

In general the maximum quantization error is

$$\varepsilon_{max} = \left|\frac{\Delta}{2}\right|$$



- **Midriser quantizer**

In uniform quantizer the gap between the two quantization levels or step size Δ remains the same throughout the quantization process.

The center of the risers of staircase characteristics coincides with the origin where the quantizer output is zero and hence the name.

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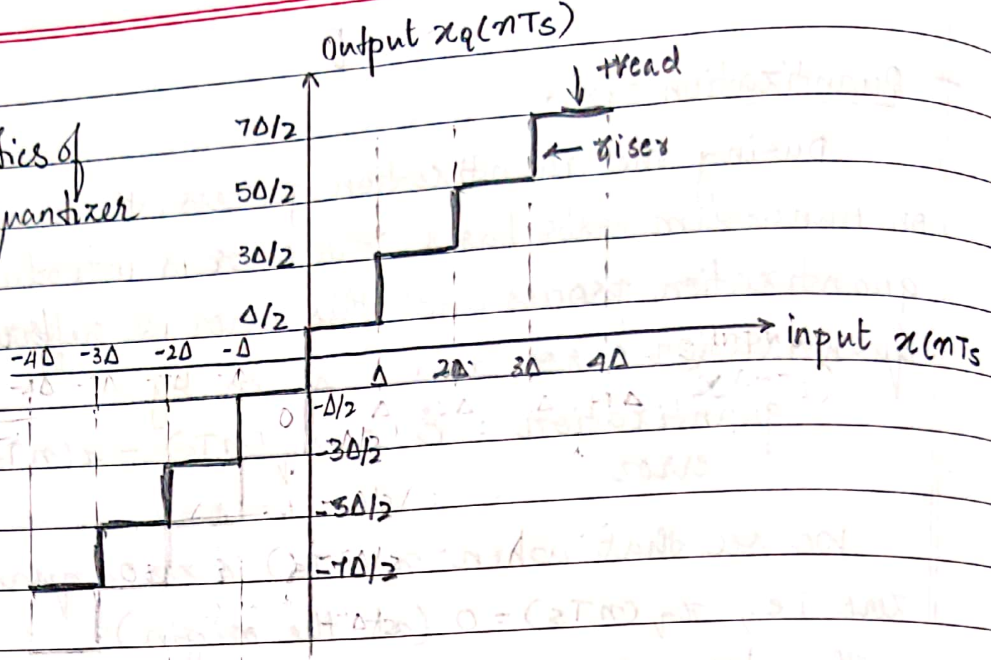


Transfer characteristics of midriser quantizer

- Analysis

Assume that the input to the quantizer varies from -4Δ to $+4\Delta$, i.e, peak to peak value of $x(nT_s)$ is between -4Δ to 4Δ.

From the figure we see that when the input is zero, the quantizer output is zero. This means that if $x(nT_s) = 0$ then $x_q(nT_s) = 0$.

When the input to the quantizer is between 0 and Δ, the quantizer output is $\frac{0+\Delta}{2} = \frac{\Delta}{2}$. i.e,

if $0 < x(nT_s) < \Delta$ then $x_q(nT_s) = \Delta/2$ and so on.

- Quantization error

During the quantization process, the samples are selected on minimum error basis. The error is introduced during the quantization process and this error is reffered to as quantization error and is given by

$$\varepsilon = x_q(nT_s) - x(nT_s)$$

At the origin when $x(nT_s) = 0$, the quantizer output $x_q(nT_s) = 0$. Therefore the quantization error $\varepsilon = 0$.

When the input $x(nT_s) = \pm\Delta$, the quantizer output $x_q(nT_s) = \pm\Delta/2$. Therefore the quantization error is

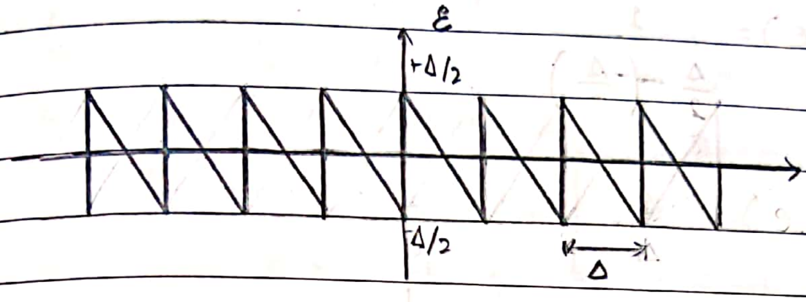
$$\varepsilon = \pm\Delta/2 - \pm\Delta$$

$$\varepsilon = \pm\Delta/2$$

In general for a uniform quatizer, the quantization error is inbetween $\pm \Delta/2$, i.e. $-\Delta/2 \leq \varepsilon \leq \Delta/2$.

In general the maximum quantization error is

$$\varepsilon_{max} = \left|\frac{\Delta}{2}\right|$$



* **Signal to quantization Noise power ratio in PCM system:**

Let $x(nT_s)$ be the inptut to the quantizer and be of continuous amplitude in the range but discrete in time.

If the input varies from $-x_{max}$ to $+x_{max}$, then the dynamic range of the input or the total amplitude range of input is

$$+x_{max} - (-x_{max}) = 2x_{max}$$

If this amplitude range is divided into q levels where q is the quantization level of the quantizer, the step size of the quantizer is given by

$$\Delta = \frac{\text{input range}}{\text{Number of quantization levels}} = \frac{2x_{max}}{q}$$

If the input signal $x(t)$ is normalized then

$+x_{max} = 1$ and $-x_{max} = -1$

Therefore $\boxed{\Delta = \frac{2}{q}}$

If the step size is sufficiently small then it is reasonable to assume that the quantization error will be uniformly distributed random variable, i.e.,

$$-\Delta/2 \leq \varepsilon_{max} \leq \Delta/2$$

where $\varepsilon_{max} = \pm\frac{\Delta}{2} = \left|\frac{\Delta}{2}\right|$

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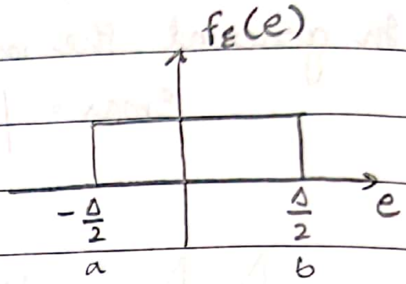
Consider probability density function of uniformly distributed random variable. 'ε'.

wkt $f_\varepsilon(e) = \frac{1}{b-a}$



$$f_\varepsilon(e) = \frac{1}{\frac{\Delta}{2} - \left(-\frac{\Delta}{2}\right)}$$

$$\boxed{f_\varepsilon(e) = \frac{1}{\Delta}}$$

From the figure we see that the quantization error is uniformly distributed over an interval $-\Delta/2$ to $+\Delta/2$.

The probability density function of a uniformly distributed random variable is defined as

$$f_\varepsilon(e) = \begin{cases} 0; & e < -\Delta/2 \\ 1/\Delta; & -\Delta/2 \le e \le +\Delta/2 \\ 0; & e > \Delta/2 \end{cases}$$

- Quantization noise power:

The quantization noise power is the mean square value of uniformly distributed random variable 'ε' where ε is the quantization error.

Quantization Noise Power

$$N_q = \text{mean square value of } e$$
$$= E(e^2)$$
$$= \int_{-\Delta/2}^{\Delta/2} e^2 f_\varepsilon(e)\, de$$
$$= \frac{1}{\Delta}\int_{-\Delta/2}^{\Delta/2} e^2\, de$$
$$= \frac{1}{\Delta} \left.\frac{e^3}{3}\right|_{-\Delta/2}^{\Delta/2} = \frac{1}{\Delta}\left[\frac{\Delta^3}{24} + \frac{\Delta^3}{24}\right]$$

$$\boxed{N_q = \frac{\Delta^2}{12}}$$

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For any uniform quantizer where the step size is uniform through out the quantization process, the quantization noise power is given by $N_q = \Delta^2/12$.

Therefore the signal to quantization noise power ratio is given by

$$\frac{S}{N_q} = \frac{\text{signal Power}}{\text{quantization noise power}} = \frac{P}{\Delta^2/12}$$

where P is the normalised signal power.

Let the quantizer use q quantization levels and let the number of bits n required to code q quantization levels is given by $q = 2^n$, where the system uses binary encoder.

Also, wkt

$$\Delta = \frac{2x_{max}}{q} = \frac{2x_{max}}{2^n}$$

signal to noise power ratio

$$\frac{S}{N_q} = \frac{P}{\Delta^2/12} = \frac{12P\,2^{2n}}{(2x_{max})^2} = \frac{\overset{3}{\cancel{12}}P2^{2n}}{\cancel{4}(x_{max})^2}$$

$$\boxed{\frac{S}{N_q} = \frac{3P\,2^{2n}}{(x_{max})^2}}$$

since the signal power is normalized, therefore the normalized signal power, $P \leq 1$.

Also for normalized input, $x_{max} = 1$

Therefore

$$\frac{S}{N_q} \leq 3(2)^{2n} \qquad \left(\frac{S}{N_q}\right)_{dB} \leq 10\log_{10} 3(2)^{2n}$$

$$\boxed{\left(\frac{S}{N_q}\right)_{dB} \leq (4.8 + 6n)\,dB}$$

Special Case

If the input to the PCM system is sinusoidal in nature, then

$$\frac{S}{N_q} = \frac{3P2^{2n}}{(x_{max})^2}$$

For sinusoidal input $x(t) = Am \sin \omega_m t$

$$P = \frac{V^2}{R} = \frac{(Am/\sqrt{2})^2}{R} = \frac{Am^2}{2R}$$

For $R = 1\Omega$; $P = \frac{Am^2}{2}$

also $x_{max} = Am$

signal to noise power ratio

$$\frac{S}{N_q} = \frac{3Am^2(2)^{2n}}{2Am^2} = 1.5(2)^{2n}$$

$$\left(\frac{S}{N_q}\right)_{dB} = 10 \log\left(1.5(2)^{2n}\right)$$

$$\boxed{\left(\frac{S}{N_q}\right)_{dB} = (1.8 + 6n)\, dB}$$

Q: A television signal with a bandwidth of 4.2MHz is transmitted using binary PCM. The number of quantization levels is 512. Find : i. code word length (number of bits)

ii. transmission band width

iii. final bit rate

iv. output signal to quantization noise power ratio.

given: $f = 4.2MHz$

$q = 512$

i. code word length

wkt $q = 2^n$

$512 = 2^n$

$n = 9$ bit ~~bits~~

ii. transmission bandwidth

Transmission band width and signaling rate (signaling rate is the bit rate, i.e, bits per second)

Suppose the PCM system uses q level quantizer and n bit binary encoder then the relation between them is given by $q = 2^n$.

The signaling rate of the PCM system is the number of bits per second and is given by:

$$\text{signaling rate} = r = \text{number of bits / sec.}$$

$$r = \frac{\text{Number of bits}}{\text{Number of samples}} \quad \frac{\text{Number of samples}}{\text{sec}}$$

$$r = nf_s.$$

The transmission band width of the PCM system is given. half of the signaling rate.

$$\text{Transmission bandwidth} = BW_T = \frac{r}{2} = \frac{nf_s}{2}$$

wkt $f_s \geq 2w$ (to avoid aliasing effect)

where w is the message band width.

$$\boxed{BW \geq nw}$$

Transmission bandwidth

$$BW \geq nw$$

$$BW \geq 9(2\pi(4.2M))$$

$$BW \geq 237.5MHz$$

iii. Final bit rate

$$r = nf_s$$

$$r = 9(4.2M)$$

$$r =$$

Q: Consider an audio signal $x(t) = 3\cos 500\pi t$ as the input to the PCM system.
Find i. S/N_q in dB when the PCM system uses a 10 bit encoder.
ii. the number of bits required to achieve signal to quantization noise power ratio of atleast 40dB.

— i. For sinusoidal input

$$\left(\frac{S}{N_q}\right)_{dB} = (1.8 + 6n)\,dB$$

$$= (1.8 + 60)$$

$$\left(\frac{S}{N_q}\right)_{dB} = 61.8\,dB$$

ii. $$\left(\frac{S}{N_q}\right)_{dB} = 40\,dB = (1.8 + 6n)\,dB$$

$$\therefore 6n = 40 - 1.8$$

$$n = \frac{38.2}{6} = 6.36$$

To achieve $(S/N_q)_{dB}$ of atleast 40dB take $n \simeq 7$

If $n = 6$, gain less than 40dB.

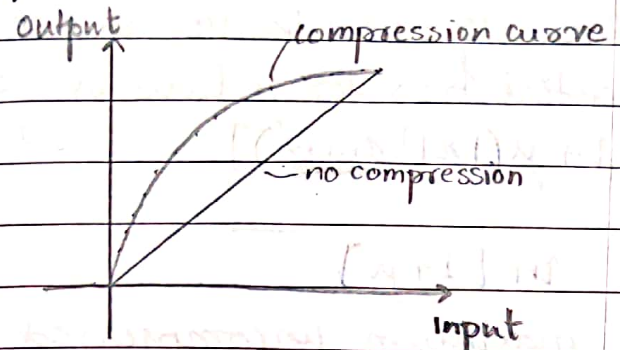
→ **Non Uniform quantization**

For non uniform quantization use smaller step size where the signal is concentrated (near origin) and larger step size where the signal is less concentrated (away from origin) This process is called 'companding'.

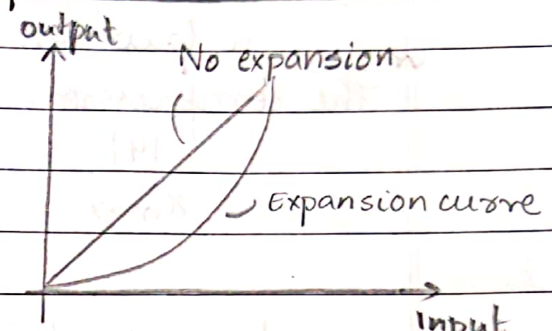
Companding is ~~to~~ compression followed by expansion i.e., compression at the transmitter and expansion at the reciever.

Companding is used to save low frequency components and at the same time high frequency components are sacrificed.

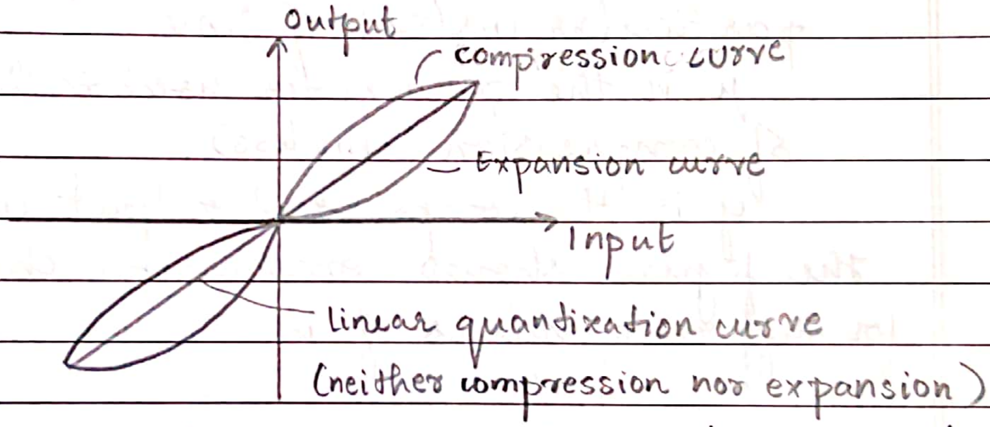
- compressor characteristics:



- Expander characteristics



- compander characteristics



compressor provides higher gain to weak signals and lower gain to strong input signals. Thus weak signals are artificially boosted to improve signal to noise power ratio (S/N_q).

All the artificially boosted by weak signals by compression are brought back to their original position by using an expander at the reciever. The compander characteristics is as shown in the figure, where the straight line indicates neither compression nor expansion and represents the linear quantization curve.

The process of compression is controlled by two laws :
i. μ law companding
ii. A law companding.

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– μ law companding:

μ law companding is used in US and Japan. The compression characteristics for μ law is given by:

$$\frac{|y|}{x_{max}} = \frac{\ln\left[1+\mu\left(|x|/x_{max}\right)\right]}{\ln\left[1+\mu\right]}$$

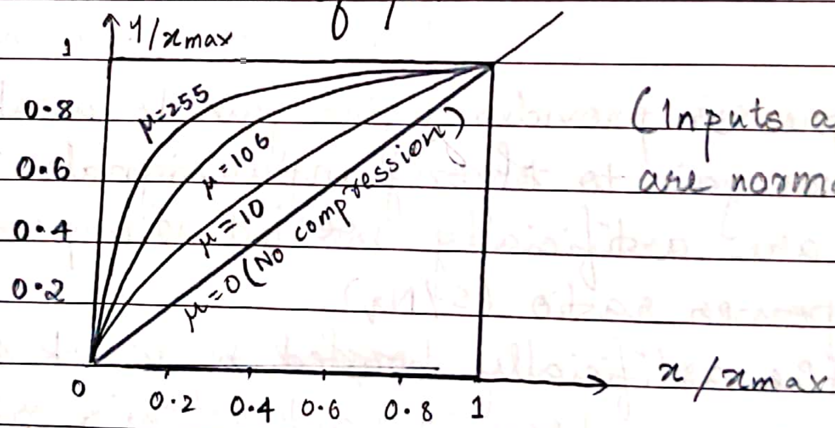
where x_{max} is the maximum uncompressed input

x is the amplitude of the input signal at a particular instant of time.

μ is the parameter used to define the amount of compression (unit less).

y is the compressed output amplitude.

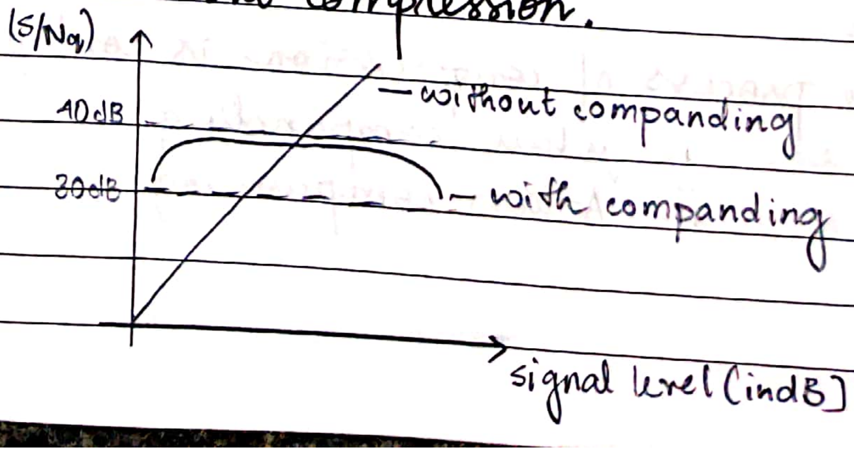
The figure shows compression characteristics for different values of μ.



(Inputs and outputs are normalized)

In μ law companding, compression characteristics is continuous. It is approximately linear for smaller values of the input and logarthmic for higher values of the input.

The curve corresponding to $\mu=0$ indicates uniform quantization without compression.



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The parameter μ determines the range of signal power in which the output signal to quantization noise power ratio of the uniform quantizer with compression is relatively constant.

μ law companding is used for speech and music signals, also in telephone signals in the US, Japan and Canada.

- A law companding:

European countries and India prefer A-law companding to approximate true logarithmic companding.

The compression characteristics for A-law is given by:

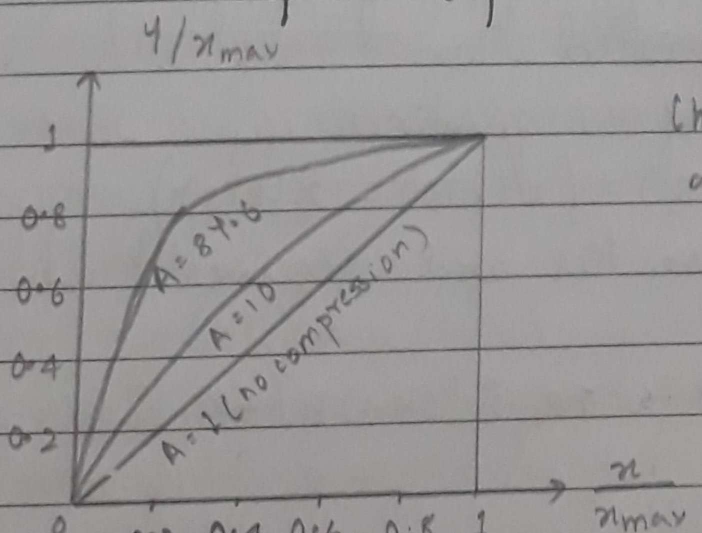
$$\frac{|y|}{x_{max}} = \begin{cases} \dfrac{A|x|/x_{max}}{1+\ln A} & 0 \le \dfrac{|x|}{x_{max}} \le \dfrac{1}{A} \\[2ex] \dfrac{1+\ln[A(|x|/x_{max})]}{1+\ln A} & \dfrac{1}{A} \le \dfrac{|x|}{x_{max}} \le 1 \end{cases}$$

where x_{max} is the maximum uncompressed input

x is the input signal amplitude at a particular instant of time.

y is the compressed output amplitude.

A is the compression parameter.



(Inputs and outputs are normalised.)

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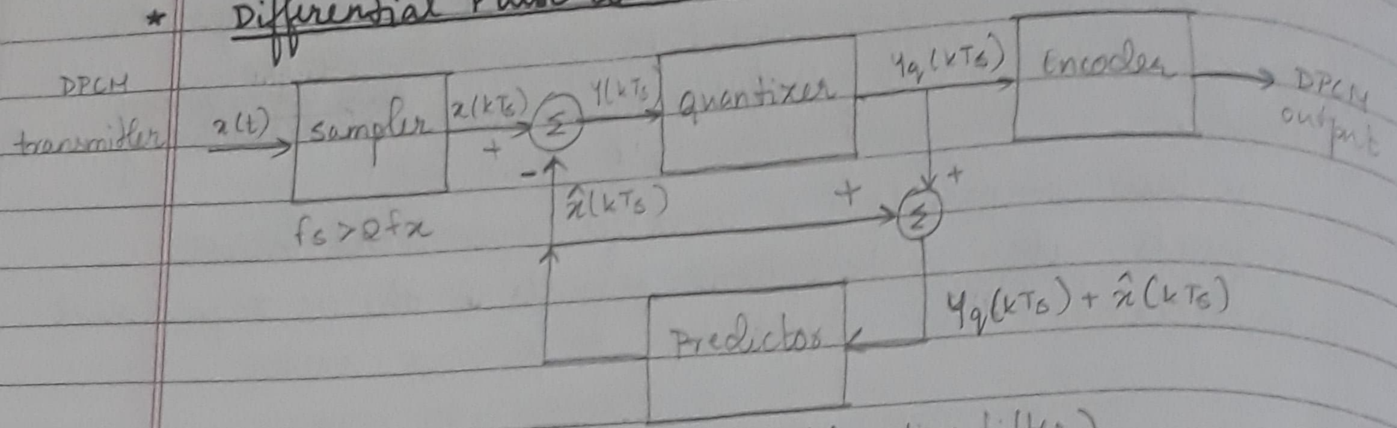
* **Differential Pulse code Modulation: (DPCM)** (theory - textbook)



(a simple n tapped delay line filter)

Analysis:

Let the input signal $x(t)$ is sampled at the rate of f_s which is slightly higher than the Nyquist rate to produce a sequence of correlated samples $x(kT_s)$ where T_s is the sampling interval and k is an integer.

The input to the quantizer is:

$$y(kT_s) = x(kT_s) - \hat{x}(kT_s)$$

where $\hat{x}(kT_s)$ is the predicted value of $x(kT_s)$ and is predicted by the predictor.

The prediction is based on the output $y_q(kT_s)$ and its previous prediction $\hat{x}(kT_s)$.

The difference signal $y(kT_s)$ is quantized, coded and it is transmitted.

Output of the quantizer is:

$$y_q(kT_s) = \left(x(kT_s) - \hat{x}(kT_s)\right)_q$$

Both transmitter and receiver uses a predictor of the form:

$$\hat{x}(kT_s) = a_1\,\hat{x}[(k-1)T_s] + a_2\,\hat{x}[(k-2)T_s] + \cdots \cdots + a_n\,\hat{x}[(k-n)T_s]$$

where $\hat{x}(kT_s)$ is output of predictor

$a_1, a_2, \ldots a_n$ is predictor coefficients or filter coefficients where n is the order of the predictor

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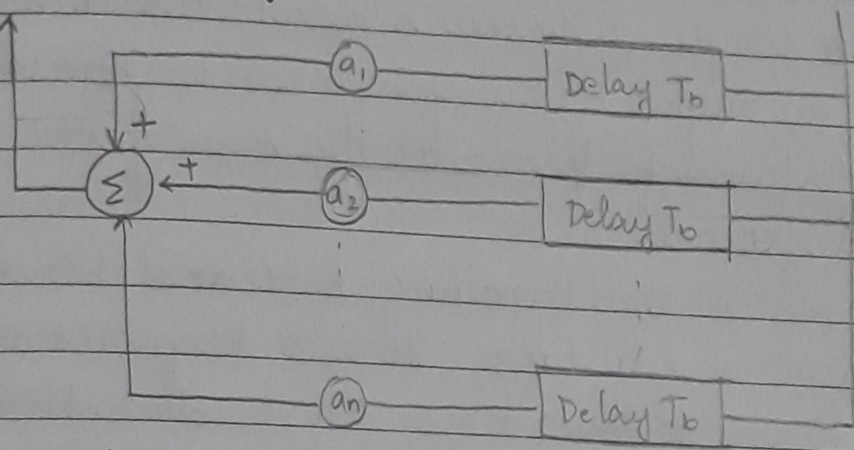
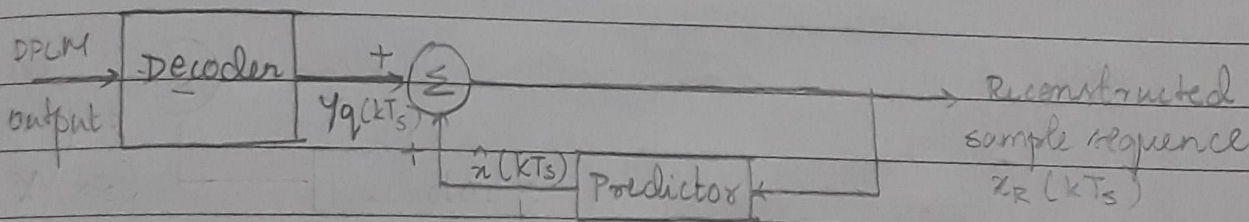
some times a predictor is refered to as a predictor filter or n tapped delay line filter



n tapped delay line filter or predictor



At the reciever the reconstructed signal is

$$x_R(kT_s) = y_q(kT_s) + \hat{x}(kT_s)$$
$$= [x(kT_s) - \hat{x}(kT_s)]_q + \hat{x}(kT_s)$$

- Prediction error

Prediction error is the amount by which the prediction filter fails to predict the input exactly. If the predictor is good then the average power of the prediction error sequence will be smaller than that of the message sequence.

Prediction error is given by:

$$y(kT_s) = e(kT_s) = x(kT_s) - \hat{x}(kT_s)$$

DPCM will yield low mean square error than direct quantizing PCM if the samples are highly correlated. Prediction error reduction is possible as long as the sample to sample correlation is non-zero.

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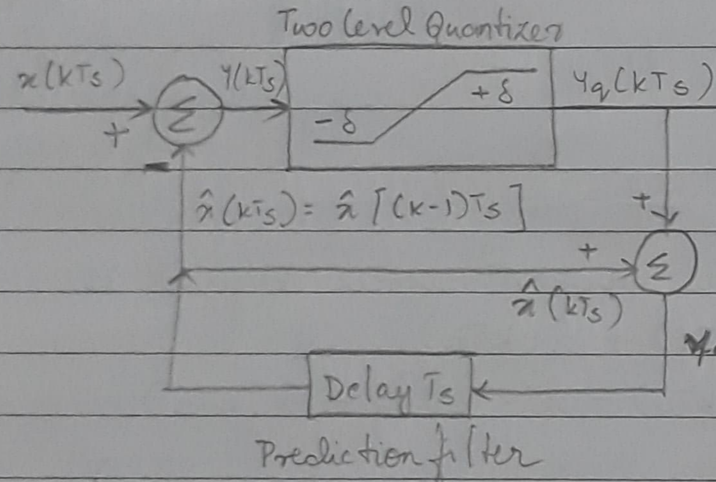
Date

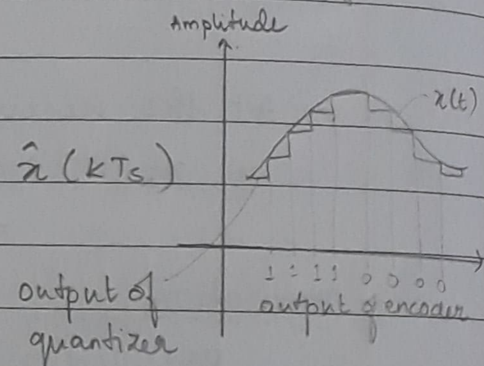
* **Delta Modulation: (DM)**

Delta modulation is the extension of DPCM. Here the sample to sample difference is quantized into two levels. Therefore the output of the quantizer is represented by a single bit which indicates the sign of the sample to sample difference.

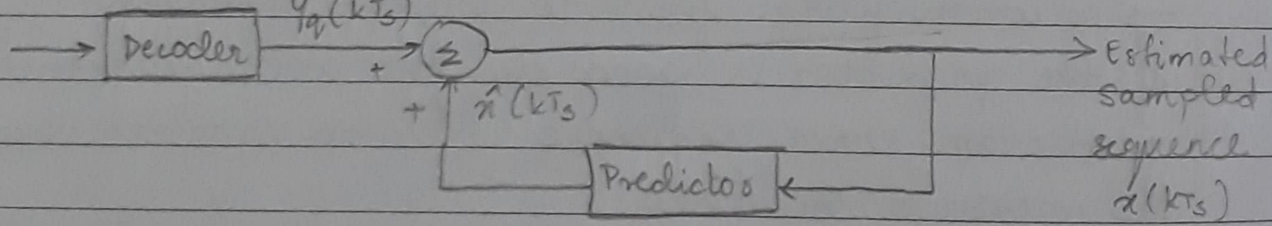
In PCM system the signalling rate and bandwidth requirement is very large. It also requires complicated circuit. Hence to overcome these drawbacks delta modulation is used

– DM transmitter





– DM receiver



The hardware required for modulation at the transmitter and demodulation at the receiver are much simpler in comparision with other PCM system.

At the transmitter the sampled value of $x(kT_s)$ is compared with its predicted value $\hat{x}(kT_s)$ is $\hat{x}((k-1)T_s)$. The difference signal is given by $y(kT_s) = x(kT_s) - \hat{x}(kT_s)$. This difference signal is quantized into one of the two

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values $+\delta$ or $-\delta$. That means, the quantizer produces an output step $+\delta$ if $x(kT_s) \geq \hat{x}(kT_s)$. Similarly $-\delta$ if $x(kT_s) < \hat{x}(kT_s)$.

code $+\delta$ as binary 1 and $-\delta$ as binary zero therefore if it is $+\delta$ binary 1 is transmitted and if it is $-\delta$ binary 0 is transmitted. Hence DM transmitter uses single bit encoder.

At the receiver, the decoded value of the quantized difference signal $y_q(kT_s)$ is added to immediately preceding value of the receiver output. The receiver output at $t=(k-1)T_s$ is $\tilde{x}(k-1)T_s = \hat{x}(kT_s)$

The receiver output is also equal to estimated output

$$\tilde{x}(kT_s) = y_q(kT_s) + \hat{x}(kT_s)$$
$$= [x(kT_s) \sim \hat{x}(kT_s)]_q + \hat{x}(kT_s)$$
$$= \pm\delta + \hat{x}(kT_s)$$

where when difference signal positive : $+\delta$
when difference signal negative : $-\delta$

$$\tilde{x}(kT_s) = \pm\delta + \tilde{x}[(k-1)T_s]$$

From the expression we see that, the present estimated output depends on its previous estimated output

Drawbacks

1. Slope overload distortion

When the step size is too small to accomodate the steep segment of the input signal $x(t)$, there is a large error between the staircase approximated signal and original signal $x(t)$. This error is called slope overload distortion.

Conditions for slope overload distortion:

consider a sinusoidal input signal $x(t)$ which is applied to the δ modulation as input.

$$x(t) = A_m \sin 2\pi f_m t$$

Let δ be the stepsize of the quantizer in δ modulator and T_s be the sampling interval.

The slope of $x(t)$ is maximum when the derivative of $x(t)$ wrt t is maximum. This means

$$\text{max slope of } x(t) = \frac{\text{step size}}{\text{sampling interval}} = \frac{\delta}{T_s}$$

Slope overload distortion will take place if

$$\left|\frac{dx(t)}{dt}\right|_{max} > \frac{\delta}{T_s}$$

i.e., $$\left| A_m 2\pi f_m \cos 2\pi f_m t \right|_{max} > \frac{\delta}{T_s}$$

$$A_m 2\pi f_m > \frac{\delta}{T_s}$$

or $$A_m > \frac{\delta}{2\pi f_m T_s}$$

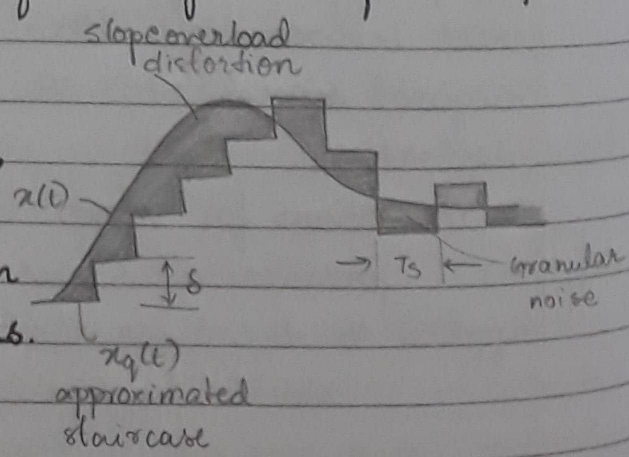
Thus slope overload distortion will not occur if

$$A_m < \frac{\delta}{2\pi f_m T_s}$$

2. Granular noise

Granular noise occurs when δ is too large relative to the local slope characteristics of the input waveform $x(t)$, thereby causing the staircase approximation $x_q(t)$ to hunt around a relatively flat segment of the input waveform.

There is need for large δ to accomodate a wide range, and a small δ is required for the accurate representation of relatively low-level signals. Therefore the optimum δ minimizes the mean-square



value of the quantizing error in linear delta modulation and will be the result of a compromise between slope overload distortion and granular noise.

* T1 carrier system:

The T1 carrier system is designed to accomodate 24 voice channels, primarily for short-distance heavy usage in metropolitan areas. It was pioneered by the Bell system in the US and its introduction lead to the shift to digital communication facilities. It forms the basis for higher order multiplexed systems that are used for either long distance transmission or transmission in heavily populated urban centers.

For companding the T1 system uses a piece-wise linear characteristic to approximate the logarithmic μ-law with the constant $\mu = 255$.

To accomodate the number of output levels each of the 24 voice channels uses a binary code with an 8-bit word. The first bit indicates whether the input voice sample is positive or negative. The next three bits of the code word identify the particular segment inside which the amplitude of the input voice sample lies and the last four bits identify the actual quantizing step inside that segment.

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UNIT - 06

Base Band Shaping for Data Transmisson

* **Representation of Binary data using various formats / Line coding / signaling formats:**

Let us consider a binary sequence

$b_K = 10110100$

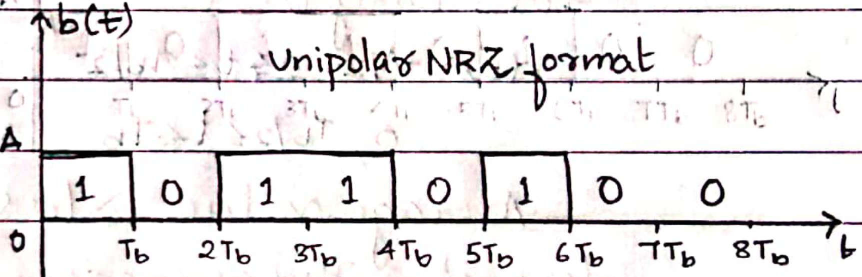
1. **Unipolar NRZ format:**

For binary 1

$b(t) = A \; ; 0 \le t \le T_b$

For binary 0

$b(t) = 0 \; ; 0 \le t \le T_b$



2. **Unipolar RZ format:**

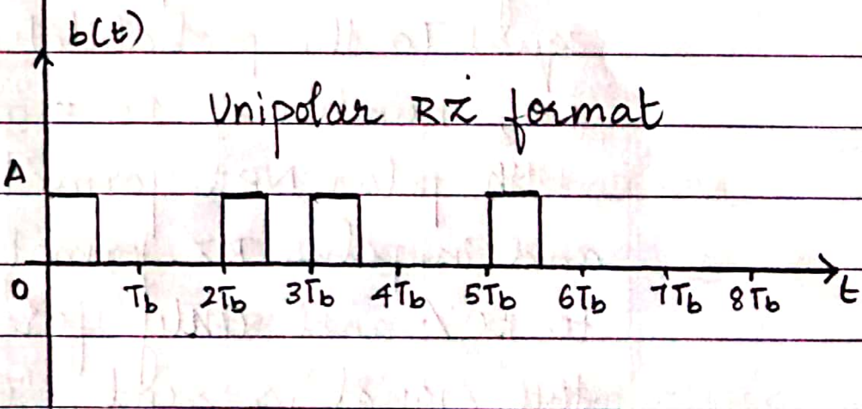
For binary 1

$b(t) = A \; , 0 \le t \le T_b/2$

$\quad\;\; 0 \; ; T_b/2 \le t \le T_b$

For binary 0

$b(t) = 0 \; ; 0 \le t \le T_b$



Both NRZ and RZ format are the average value. compared to NRZ format, in RZ format for binary 1, for 50% of the pulse duration the signal is sent.

Therefore the power consumption is less.

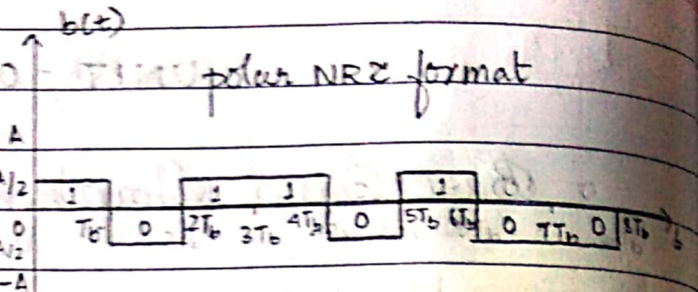
3. <u>Polar NRZ format:</u>

For binary 1

$b(t) = A/2 \;;\; 0 \le t \le T_b$

For binary 0

$b(t) = -A/2 \;;\; 0 \le t \le T_b$



4. <u>Polar RZ format:</u>

For binary 1

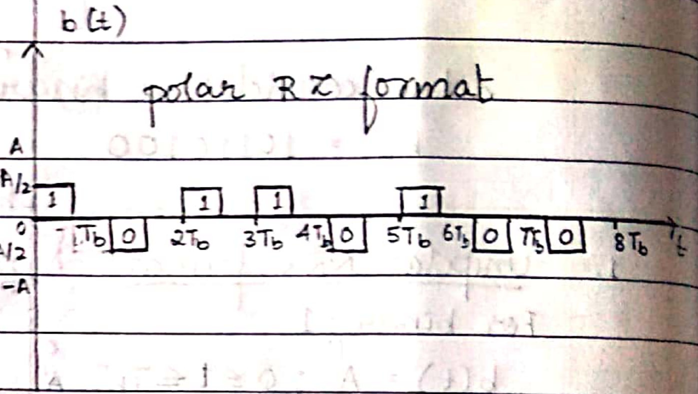
$b(t) = A/2 \;;\; 0 \le t \le T_b/2$

$0 \;;\; T_b/2 \le t \le T_b$

For binary 0

$b(t) = -A/2 \;;\; 0 \le t \le T_b/2$

$0 \;;\; T_b/2 \le t \le T_b$

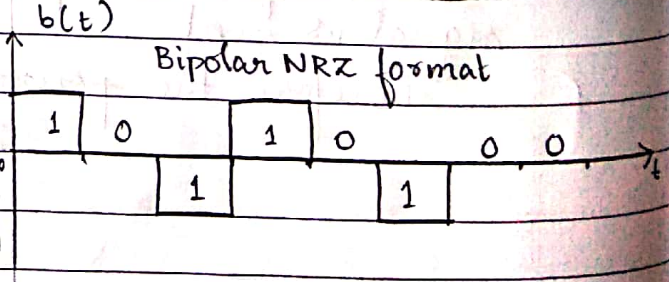


Average value is less when compared to unipolar format. The average value of the format is zero only when the probability of occurance of 1 is equal to the probability of occurance of 0 i.e., number of 1s = number of 0s.

In polar NRZ format the amplitude is reduced (50%) and in polar RZ format the amplitude ~~is~~ reduced to 50% and only for 50% of the pulse duration the signal is sent. Therefore the power consumption is less.

5. <u>Bipolar NRZ format / Alternate Mark Inversion:</u>

In this format the successive ones are represented by pulses with alternate polarity and the zeros are represented by no pulses



The average value of the signal is zero, when the signal has even number of 1s.

6. <u>Split Phase Manchester Coding:</u>

Here binary 1 is transmitted by passing a positive half interval pulse and is followed by a negative half interval pulse.

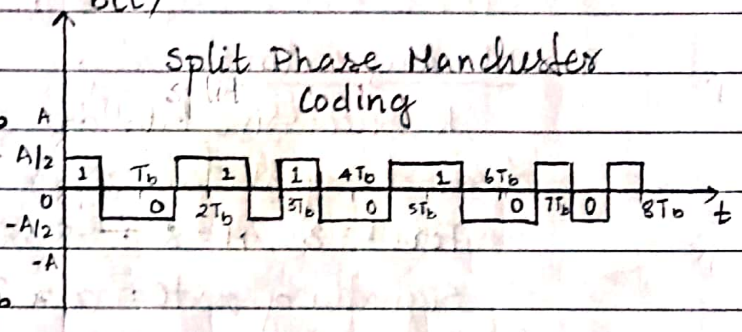
Similarly for binary 0, it is transmitted by passing a negative half pulse followed by a positive half interval pulse.

For binary 1

$$b(t) = \begin{cases} A/2 \,; & 0 \le t \le T_b/2 \\ -A/2 \,; & T_b/2 < t \le T_b \end{cases}$$

For binary 0

$$b(t) = \begin{cases} -A/2 \,; & 0 \le t \le T_b/2 \\ A/2 \,; & T_b/2 \le t \le T_b \end{cases}$$



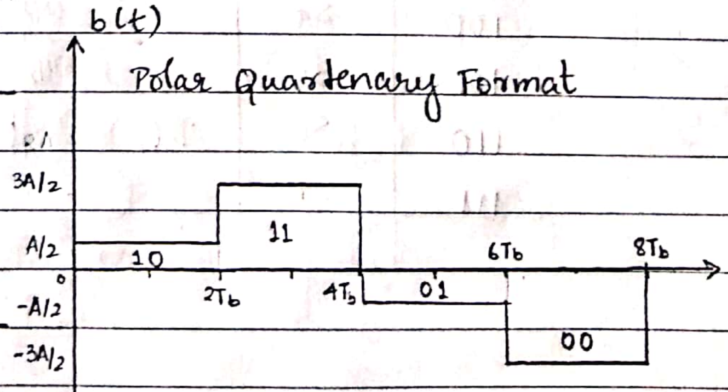
The main advantage of this format is that irrespective of probability of occurance of 1s and 0s, the average value of the signal is zero.

7. <u>Polar Quaternary Format:</u>

This format is used to reduce the signaling rate and hence the bandwidth. The message bits are grouped into blocks of two.

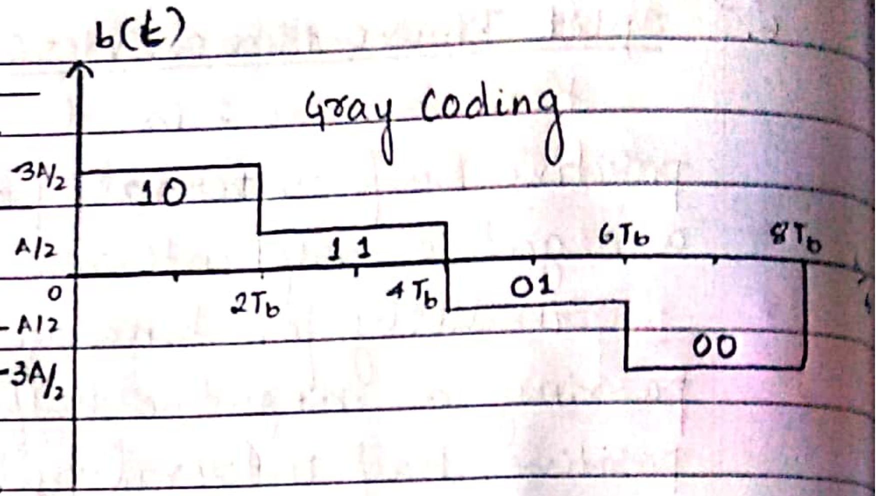
The possible combinations and their amplitude levels are as follows.

Symbols	Message bit combination	Amplitude levels
S_0	00	$-3A/2$
S_1	01	$-A/2$
S_2	10	$A/2$
S_3	11	$3A/2$



8. Grey coding (4 level)

Message bits	Gray code	Amplitude levels
00	00	$-3A/2$
01	01	$-A/2$
10	11	$3A/2$
11	10	$A/2$



9. M - ary coding:

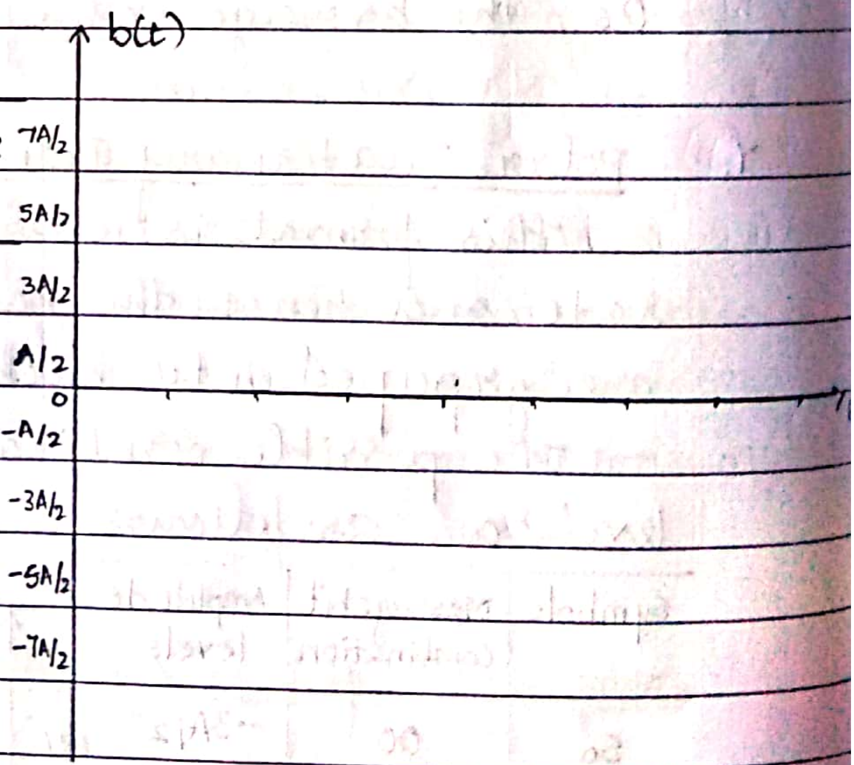
k successive bits are combined to get $M = 2^k$ distinct symbols.

For $k = 3$, $M = 8$: 8 ary coding.

Signalling rate : $r = \frac{r_b}{k} = \frac{r_b}{\log_2 M}$

where $r_b = \frac{1}{T_b}$, where T_b is the bit duration

Group of Message bits	Symbols	Amplitude levels
000	S_0	$-7A/2$
001	S_1	$-5A/2$
010	S_2	$-3A/2$
011	S_3	$-A/2$
100	S_4	
101	S_5	
110	S_6	..
111	S_7	



★ Base band transmission of binary data :

The output of the discrete PAM or data encoder is given by:

$$x(t) = \sum_{k=-\infty}^{\infty} A_k \, g(t - kT_b) \quad \text{--- (1)}$$

where T_b is the duration of input bit and $g(t)$ is the pulse shaping function.

A_k is given by

$$A_k = \begin{cases} a & \text{if } b_k = 1 \\ -a & \text{if } b_k = 0 \end{cases} \quad \text{(2)}$$

This signal $x(t)$ is then passed to transmitting filter ($H_T(f)$) then channel ($H_c(f)$) and receiving filter ($H_R(f)$). (These three are connected in cascade).

Therefore the output of the receiving filter is $y(t)$ and is a noisy replica of the transmitter signal $x(t)$.

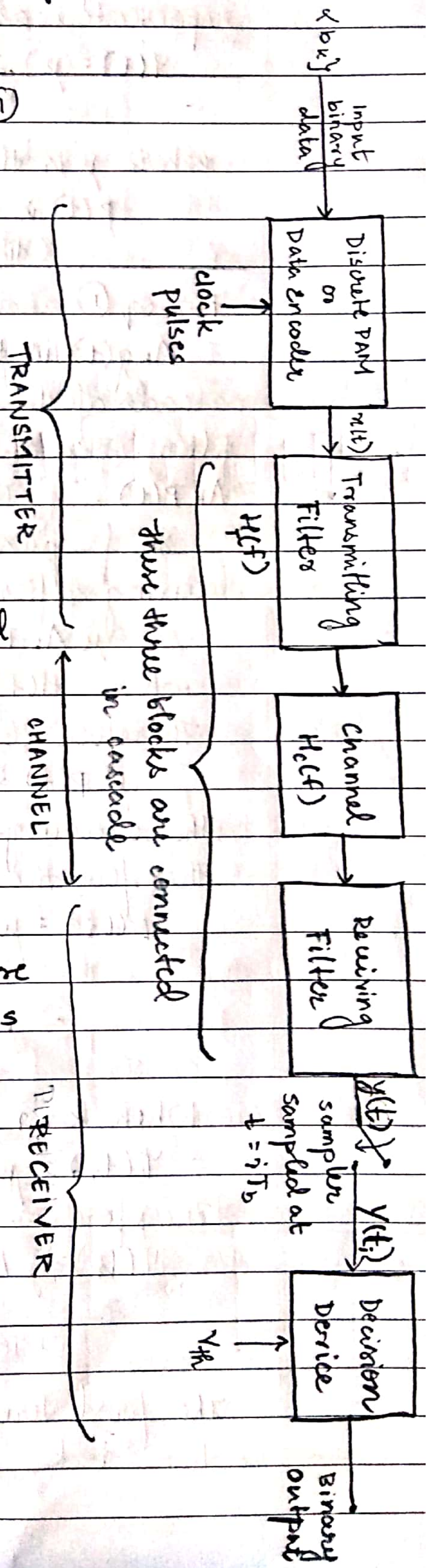
This signal $y(t)$ is sampled at $t = iT_b$. (Sampling instants are synchronised with the clock pulses at the transmitter).

The sampled signal is then given to the decision device as the input.

The decision logic is as follows :

if $y(t_i) > V_{Th}$
  Select symbol or bit 1

if $y(t_i) \leq V_{Th}$
  Select symbol or bit 0



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The output of the receiving filter can be represented as.

$$Y(t) = \mu \sum_{k=-\infty}^{\infty} A_k\, p(t-kT_b) \quad \text{—— ③}$$

where μ is the scaling factor,
$p(t)$ is the pulse shaping function (different from $g(t)$)

For eq ① and eq ③

$A_k g(t)$ is the signal applied to the input of cascade of transmitting, channel and receiving filter. The output of this cascade connection is $A_k P(t)$.

In frequency domain the output of the receiving filter is.

$$\mu A_k P(f) = H(f) \cdot A_k G(f) \quad \text{—— ④}$$

where $H(f) = H_T(f) \cdot H_C(f) \cdot H_R(f)$ (cascade connection) ⑤

Therefore the output of the receiving filter is

$$\mu P(f) = H(f)\, G(f)$$

The receiving filter output is sampled at $t = iT_b$

Therefore the output of the sampler is

$$y(t_i) = \mu \sum_{k=-\infty}^{\infty} A_k P(iT_b - kT_b)$$

$$= \mu \sum_{k=-\infty}^{\infty} A_k P((i-k)T_b) \quad \text{—— ⑥}$$

When $k = i$

$$Y(t_i) = \mu A_i P(0) \quad \text{—— ⑦}$$

Therefore

$$Y(t_i) = \underbrace{\mu A_i P(0)}_{\text{I Term}} + \underbrace{\mu \sum_{\substack{k=-\infty \\ k \neq i}}^{\infty} A_k P(i-k)T_b}_{\text{II Term}} \quad \text{—— ⑧}$$

The first term represents the value of $Y(t_i)$ when $i = k$.

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Normalising p(t) such that P(0) = 1

$$Y(t_i) = \mu A_i + \mu \sum_{\substack{k=-\infty \\ k\neq i}}^{\infty} A_k P(i-k) T_b \quad \text{(9)}$$

I term II term

The first term, μA_i is the contribution of i^{th} transmitted bit.

The second term represents the residual effect of all other bits transmitted before and after the sampling instant.

Thus the presence of output due to other bits interfere with the output of the required bit is called as Inter symbol Interference (ISI). [ISI is the dispersion in the pulse shape due to improper channel response]

If ISI is absent, then the second term will not be present in the above equation.

$$\text{i.e., } y(t_i) = \mu A_i \quad \text{(10)}$$

At $t = iT_b$, the correct bit is A_i and is decoded correctly in the absence of ISI.

It is not possible to eliminate ISI, but ISI can be reduced by:

a. proper design of pulse spectrum
b. proper design of transmitting filter
c. proper design of channel
d. proper design of receiving filter

★ Nyquist Critera for Distortionless Transmission

The pulse shaping function p(t) with Fourier transform given by P(f) that satisfies

$$\sum_{n=-\infty}^{\infty} P(f - nf_b) = T_b \qquad \text{where } T_b = \frac{1}{f_b}$$

has

$$p(iT_b - kT_b) = \begin{cases} 1 & i = k \\ 0 & i \neq k \end{cases}$$

Nyquist criterion for zero ISI.

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Proof: Let us sample $p(t)$ with a period T_b ideally.

$$p_\delta(t) = p(t)\,\delta_\delta(t) \quad \text{—— (1)}$$

Applying Fourier Transform

$$P_\delta(f) = P(f) * \delta_\delta(f)$$

$$P_\delta(f) = P(f) * f_s \sum_{n=-\infty}^{\infty} \delta(f - nf_s) \quad \text{—— (2)}$$

Let $f_s = f_b = 1/T_b$

$$P_\delta(f) = P(f) * f_b \sum_{n=-\infty}^{\infty} \delta(f - nf_b) \quad \text{—— (3)}$$

By convolution property of an impulse function

$$P_\delta(f) = f_b \sum_{n=-\infty}^{\infty} P(f - nf_b) \quad \text{—— (4)}$$

By defination of Fourier Transform wkt

$$P_\delta(f) = \int_{-\infty}^{\infty} p_\delta(t)\, e^{-j2\pi ft}\, dt \quad \text{—— (5)}$$

using eq (1)

$$p_\delta(t) = p(t)\,\delta_\delta(t)$$

$$p_\delta(t) = p(t) \sum_{m=-\infty}^{\infty} \delta(t - mT_b)$$

$$p_\delta(t) = \sum_{m=-\infty}^{\infty} p(mT_b)\, \delta(t - mT_b) \quad \text{—— (6)}$$

Therefore

$$P_\delta(f) = \int_{-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \left[p(mT_b)\,\delta(t - mT_b) \right] e^{-j2\pi ft}\, dt \quad \text{—— (7)}$$

Let $m = i - k$ then $m = 0$ when $i = k$

$m \neq 0$ when $i \neq k$

$$p[(i-k)T_b] = p(mT_b) = \begin{cases} 1 & ; m = 0 \\ 0 & ; m \neq 0 \end{cases} \quad \text{—— (8)}$$

Therefore $p(0) = 1$

From eq (5)

$$P_\delta(f) = \int_{-\infty}^{\infty} p(0)\,\delta(t)\, e^{-j2\pi ft}\, dt$$

By shifting property

$$P_\delta(f) = p(0)\, e^{-j2\pi ft}\Big|_{t=0} = p(0) = 1$$

Therefore from eq (4)

$$1 = f_b \sum_{n=-\infty}^{\infty} P(f - nf_b)$$

$$\sum_{n=-\infty}^{\infty} P(f - nf_b) = T_b \quad \text{where } T_b = \frac{1}{f_b}$$

★ **Correlative coding:**

Adding ISI in a controlled manner or adding redundancy in a controlled manner. It allows the signalling rate of $2B_0$ in the channel of bandwidth B_0. (Nyquist bandwidth).

This is made physically possible by adding ISI in the transmitted signal in a controlled manner and this ISI is known to the receiver and hence the effects of ISI are easily eliminated at the receiver.

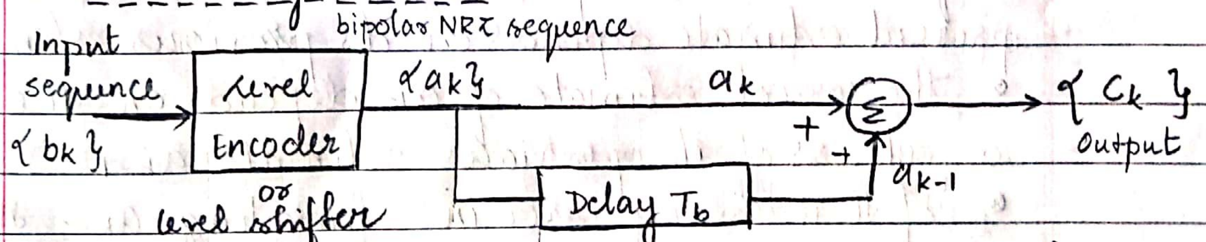
The correlative coding is implemented by:

a. Duo binary signalling
b. Modified duo binary signalling.

— **Duo Binary Signalling:**

Duo binary encoding reduces the maximum frequency of the base band signal. The word duo means to double the transmission capacity of the binary system.

Duo binary encoder



Consider an input sequence $\{b_k\}$ which contains binary symbols 1 and 0. By using a level shifter the sequence is converted into a bipolar NRZ sequence $\{a_k\}$ such that

$$a_k = \begin{cases} 1 & ; \text{ if } b_k = 1 \\ -1 & ; \text{ if } b_k = 0 \end{cases}$$

From the diagram we see that the input to the adder bipolar NRZ sequence a_k and its delayed version (delay of T_b sec) a_{k-1}.

The output of the encoder is a three level signal $(-2, 0, +2)$.

The output of the duo binary encoder is

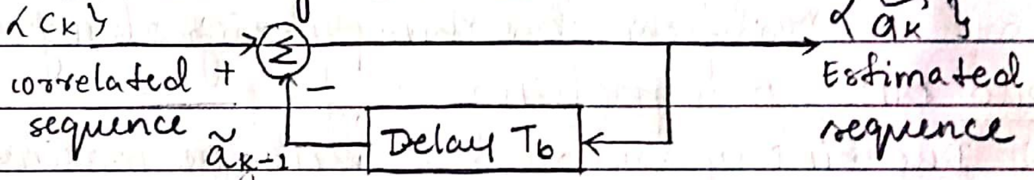
$$c_k = a_k + a_{k-1}$$

a_k	a_{k-1}	c_k
0(-1)	0(-1)	-2
0(-1)	1(1)	0
1(1)	0(-1)	0
1(1)	1(1)	+2

The duo binary encoder converts uncorrelated two level sequence into a correlated three level sequence.

This introduces inter symbol interference into the signal in an artificial manner to reduce the bandwidth

Duo Binary Decoder



Let $\tilde{a}_k$ represent the estimated output of a_k and is given by

$$\tilde{a}_k = c_k - \tilde{a}_{k-1} = \text{output of the decoder}$$

- From the above expression we see that the present estimate depends on its previous estimate.
- The correct estimate of a_k depends on the correctness of its previous estimate a_{k-1}
- If there is any error in estimating a_{k-1} then the error propagates during the estimation process of a_k i.e., the error will propagate in the output sequence and this is called error propagation.

To avoid error propagation precoder is used.

Q: For the input binary data 001101001 construct a duo binary system without precoder.

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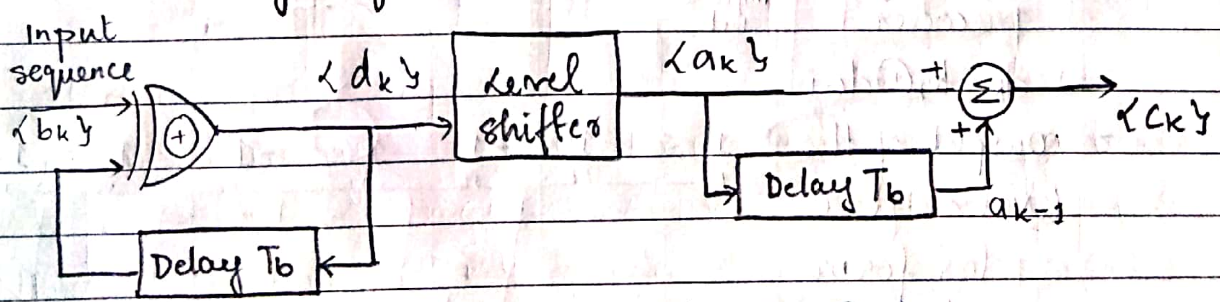
Sequence	k=0	k=1	k=2	k=3	k=4	k=5	k=6	k=7	k=8
– Input Sequence $\{b_k\}$	0	0	1	1	0	1	0	0	1
– Polar form of b_k is a_k (output of the level shifter)	−1	−1	1	1	−1	1	−1	−1	1
– Output of the duo binary encoder $c_k = a_k + a_{k-1}$		−2	0	2	0	0	0	−2	0
		−2−(−1)		2−1				−2−(−1)	
– Estimate of a_k is $\tilde{a}_k = c_k - \tilde{a}_{k-1}$	−1	−1	1	1	−1	1	−1	−1	1
– Estimated output sequence $\{\tilde{b}_k\}$ (decode −1 as 0 and +1 as 1)	0	0	1	1	0	1	0	0	1

Duo binary signal with precoder



Output of the precoder is $d_k = b_k \oplus d_{k-1}$

b_k	d_{k-1}	d_k
0	0	0
0	1	1
1	0	1
1	1	0

$$a_k = \begin{cases} +1 & \text{if } d_k = 1 \\ -1 & \text{if } d_k = 0 \end{cases}$$

The output of the encoder: $c_k = a_k + a_{k-1}$

a_k	a_{k-1}	c_k
0 (-1)	0 (-1)	-2
0 (-1)	1 (1)	0
1 (0)	0 (-1)	0
1 (0)	1 (1)	+2

Decoder

$\{c_k\} \rightarrow$ Rectifier $\xrightarrow{\{|c_k|\}}$ Decision device $\rightarrow$ $b_k = 1$ if $|c_k| < 1$; $b_k = 0$ if $|c_k| > 1$

$\uparrow V_{th} = 1$

From the figure we see that the output of the decision device depends on c_k only. The previous value of the output is not taken into consideration.

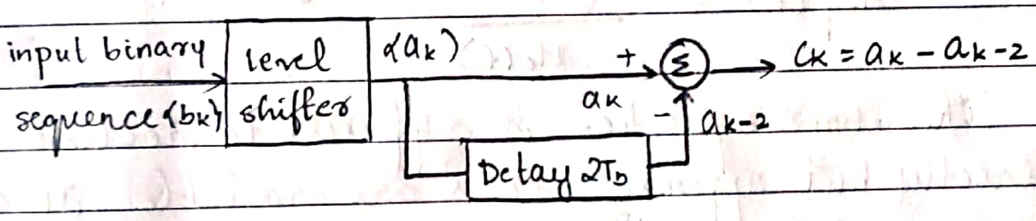
Q: For the input binary data 1011101 obtain the output of the precoder, output of the duo binary encoder and decoder.

sequence	k=-1	k=0	k=1	k=2	k=3	k=4	k=5	k=6
Input binary sequence $\{b_k\}$		1	0	1	1	1	0	1
output of the precoder $d_k = b_k \oplus d_{k-1}$	1 implied bit	0	0	1	0	1	1	0
Output of the level shifter or polar form of d_k [$a_k = +1$ if $d_k = 1$, $a_k = -1$ if $d_k = 0$]	1	-1	-1	1	-1	1	1	-1
output of the encoder $c_k = a_k + a_{k-1}$		0	-2	0	0	0	2	0

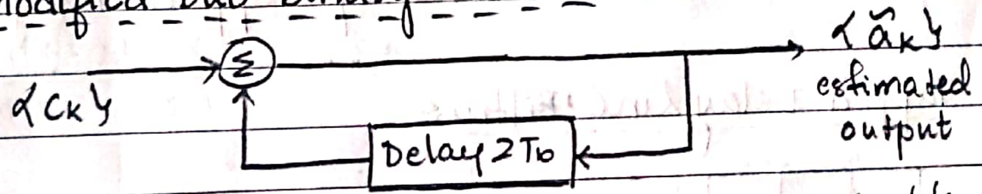
Sequence	k = -1	k = 0	k = 1	k = 2	k = 3	k = 4	k = 5	k = 6
- output of the rectifier is $\lvert c_k \rvert$		0	2	0	0	0	2	0
- output of the decision device $\{b_k\}$		1	0	1	1	1	0	1

- **Modified Duo Binary Signalling:**
  The correlation span is $2T_b$ seconds.

Modified Duo binary encoder



Modified Duo binary decoder



(Problems textbook

Estimation of $a_k = \hat{a}_k = c_k + \hat{a}_{k-2}$

★ **Equalization:**
When the signal is passed through the channel, distortion is introduced in terms of amplitude and delay. This distortion creates the problems of ISI and the detection of the signal also becomes difficult. This distortion can be compensated by using an equalizer.
An equalizer is a filter which corrects the channel distortion. Consider the cascade connection of channel and an equalizer as shown in the fig.

They are connected in cascade

The transfer function of the distortionless system is given as

$$H(f) = k\, e^{-j2\pi f t_0} \quad (1)$$

The transfer function of the cascade connection of the channel and equalizer for distortionless system is

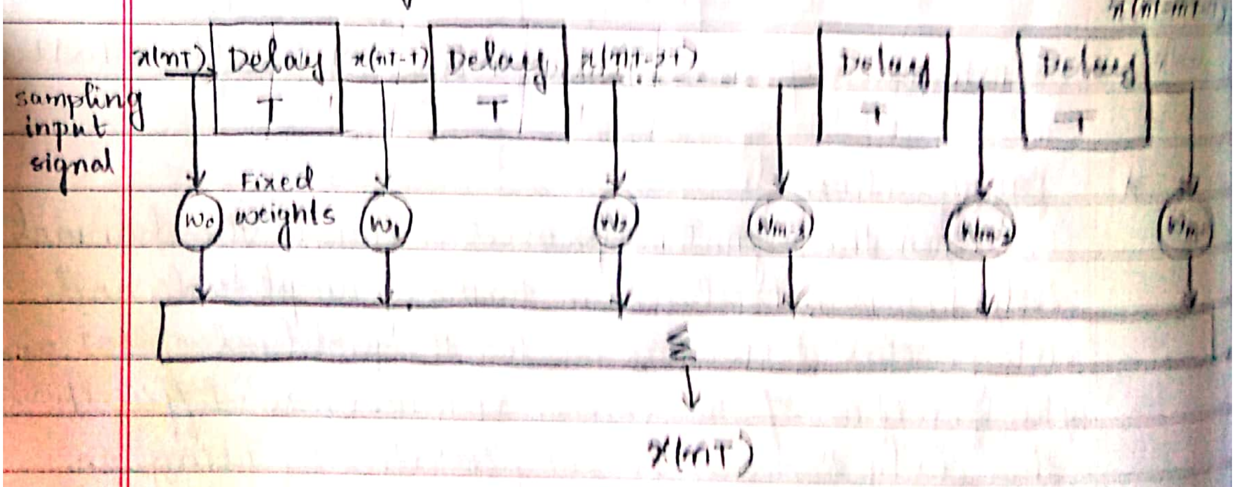
$$H_c(f)\, H_{eq}(f) = H(f) = e^{-j2\pi f t_0} \quad (2)$$

The transfer function of the equalizer for distortionless system is

$$H_{eq}(f) = \frac{e^{-j2\pi f t_0}}{H_c(f)} \quad (3)$$

The above equation is difficult to realize directly but approximations are available. It can be implemented with the help of tapped delay line filters.

- Tapped Delay Line



The output of the delay line filter is

$$y(nT) = \sum_{i=0}^{M-1} W_i\, x(nT - iT)$$

where W_i is the weight of the ith tap

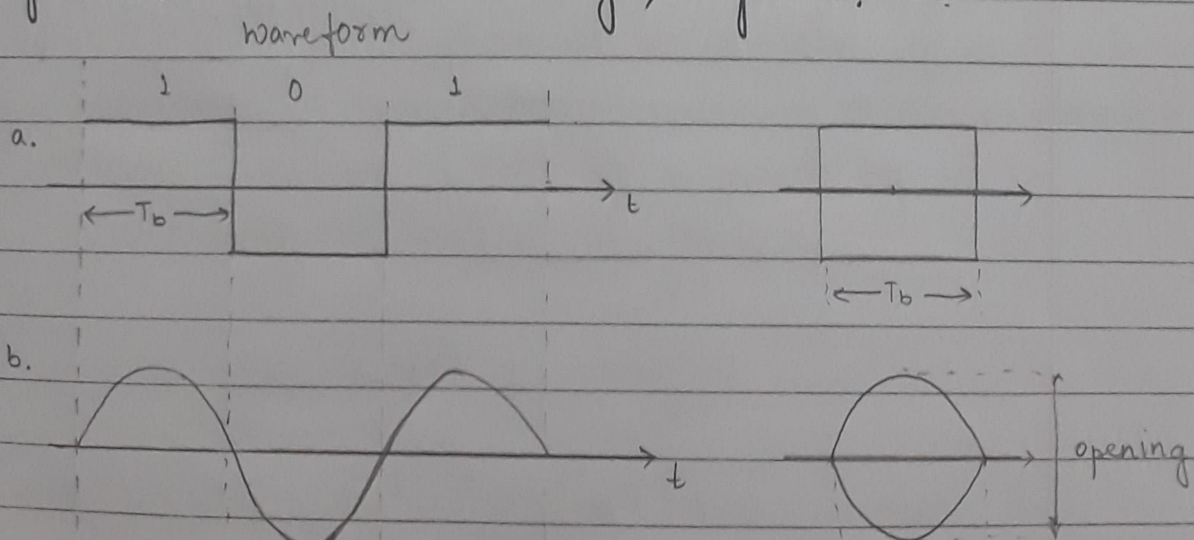
M is the total number of taps.

T is the simple duration of the signal.

The weights are basically symbol coefficients. This filter approximates the equilizer transfer function (eq 3). The approximation will be more accurate if we use more taps in the filter. The weights are calculated as per the characteristics of the channel.

★ **Eye Diagram:**

The ISI and other signal degradations can be studied conveniently on an oscilloscope through what is known as eye diagram. A random binary pulse sequence is sent over the channel. The channel output corresponding to the binary sequence sent is then applied to the vertical deflecting plates of an oscilloscope. A saw tooth waveform at the transmitted bit rate $1/T_b$ is applied to the horizontal deflecting plates of the CRO. The oscilloscope shows the superposition of several traces, which is the input signal (vertical input) cut up every T_b seconds and then superimposed. The oscilloscope pattern thus formed looks like a human eye and hence the name eye diagram.



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Interpretation of eye diagram