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Introduction

* <u>Scalar</u>: A quantity having only magnitude
* <u>Vector</u>: A quantity having both magnitude and direction.

 It is represented as $\vec{R} = |\vec{R}|\hat{a}_R$

 where $\vec{R}$ - is the vector

 $|\vec{R}|$ - magnitude of vector

 $\hat{a}_R$ - unit vector representing the direction.

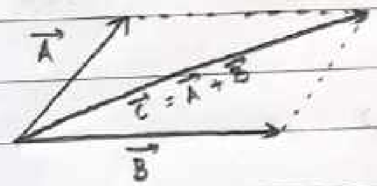
* <u>Vector Algebra</u>:

 Result of two or more vectors can be found by

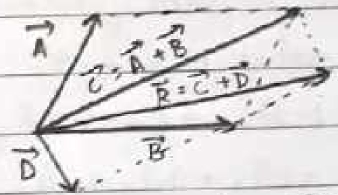
a. <u>Parallelogram law</u>

With two vectors as adjacent sides of the parallelogram drawn from a point, the diagonal of the parallelogram gives the resultant vector.

Only vectors in same plane can be added.



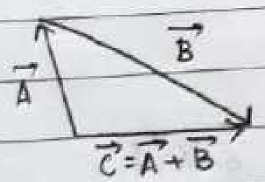
Addition of two vectors $\vec{A}$ and $\vec{B}$



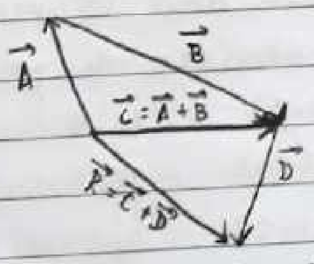
Addition of vectors $\vec{A}, \vec{B}$ and $\vec{D}$

b. <u>Triangle law</u>

With two vectors as two sides of the triangle in same order, the third side in the other order gives the resultant vector.



Addition of vectors $\vec{A}$ and $\vec{B}$



Addition of vectors $\vec{A}, \vec{B}$ and $\vec{D}$

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* Coordinate Systems

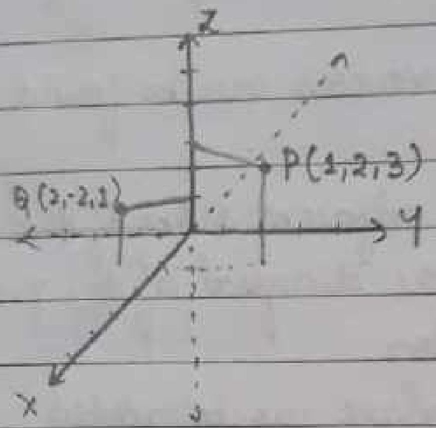
There are three coordinate systems

- a. Rectangular coordinate system / cartesian coordinate system

Three coordinates x, y and z are mutually perpendicular to each other. It is also called as right hand rectangular coordinate system.

Any point P is represented as P(x, y, z).

Ex: P(1, 2, 3) and Q(2, -2, 1)



- Vector from origin to point P

$\vec{P} = (1-0)\hat{a}_x + (2-0)\hat{a}_y + (3-0)\hat{a}_z$

$\vec{P} = 1\hat{a}_x + 2\hat{a}_y + 3\hat{a}_z$

magnitude

$|\vec{P}| = \sqrt{1^2+2^2+3^2} = \sqrt{14}$

Direction

$\hat{a}_P = \frac{\vec{P}}{|\vec{P}|} = \frac{1\hat{a}_x + 2\hat{a}_y + 3\hat{a}_z}{\sqrt{14}}$

- Vector for point Q(2, -2, 1) to point P(1, 2, 3)

$\vec{R}_{QP} = (1-2)\hat{a}_x + (2-(-2))\hat{a}_y + (3-1)\hat{a}_z$

$\vec{R}_{QP} = -1\hat{a}_x + 4\hat{a}_y + 2\hat{a}_z$

magnitude

$|\vec{R}_{QP}| = \sqrt{-1^2+4^2+2^2} = \sqrt{21}$

Direction

$\hat{a}_{QP} = \frac{\vec{R}_{QP}}{|\vec{R}_{QP}|} = \frac{-1\hat{a}_x + 4\hat{a}_y + 2\hat{a}_z}{\sqrt{21}}$

- Surface area and Volume

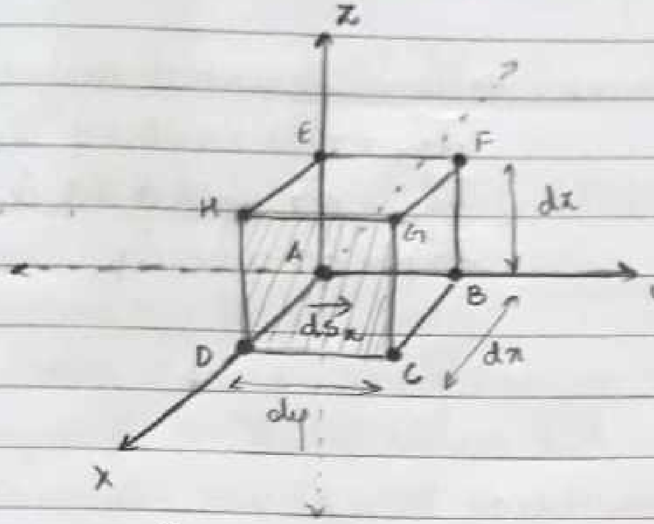
Let us consider the points

A (0,0,0) E (0,0,1)

B (0,1,0) F (0,1,1)

C (1,1,0) G (1,1,1)

D (1,0,0) H (1,0,1)



- differential length along x coordinate : dx
- differential length along y coordinate : dy
- differential length along z coordinate : dz

→ ds - differential surface

$\vec{ds_x} = dy\, dz\, \hat{a}_x$

$\vec{ds_y} = dx\, dz\, \hat{a}_y$

$\vec{ds_z} = dx\, dy\, \hat{a}_z$

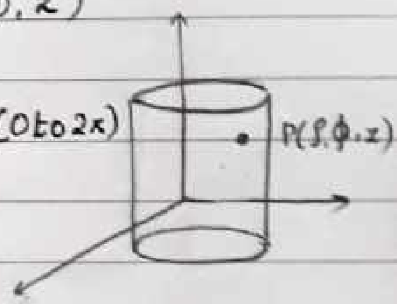
→ dv - differential volume

$dv = dx\, dy\, dz$

→ dL - differential length

$\vec{dL} = dx\,\hat{a}_x + dy\,\hat{a}_y + dz\,\hat{a}_z$

- b. Cylindrical Coordinate system : (ρ, ϕ, z)



ρ - radius of cylinder $(0 \text{ to } \infty)$ (can also be represented as r)

ϕ - determines the arc length (angle) $(0 \text{ to } 2\pi)$

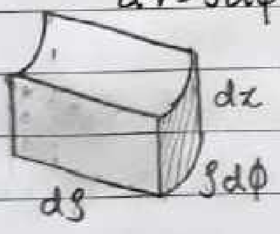
z - height of cylinder $(-\infty, \infty)$

→ dL - differential length

$\vec{dL} = d\rho\,\hat{a}_\rho + \rho\, d\phi\,\hat{a}_\phi + dz\,\hat{a}_z$

→ dv - differential volume

$dv = \rho\, d\phi\, d\rho\, dz$



→ ds - differential surface area

$\vec{ds_\rho} = \rho\, d\phi\, dz\,\hat{a}_\rho$

$\vec{ds_\phi} = d\rho\, dz\,\hat{a}_\phi$

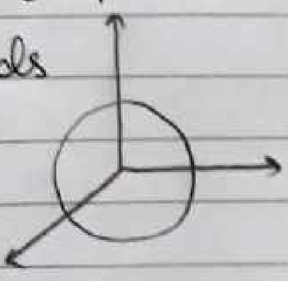
$\vec{ds_z} = d\rho\, \rho\, d\phi\,\hat{a}_z$

- c. Spherical Coordinate System : (r, θ, ϕ) $\quad 0 < \theta < \pi/2$, $0 < \phi < \pi/2$

r - radius of the sphere

θ - angle to move from north to downwards

ϕ - angle to move horizontally.



→ dL - differential length

$\vec{dL} = dr\,\hat{a}_r + r\sin\theta\, d\phi\,\hat{a}_\phi + r\, d\theta\,\hat{a}_\theta$

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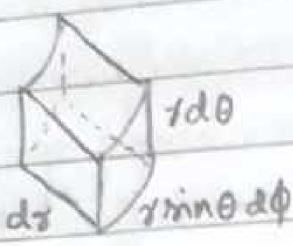
→ dv - differential volume



→ ds - differential surface are[a]

$\overrightarrow{ds_r} = r d\theta\, r\sin\theta\, d\phi\ \hat{a}_r$

$\overrightarrow{ds_\theta} = dr\, r\sin\theta\, d\phi\ \hat{a}_\theta$

$\overrightarrow{ds_\phi} = dr\, r d\theta\ \hat{a}_\phi$

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Unit - 1

STATIC ELECTRIC FIELDS

* **Coulomb's law:**

point charges: charges whose size is very small when compared to the distance between them.

Coulomb's law is used to determine the force acting between any two point charges.

" The magnitude of the electrostatic force of attraction or repulsion between two point charges is directly proportional to the product of magnitudes of charges and inversely proportional to square of distance."

$$F \propto \frac{Q_1 Q_2}{R^2}$$

$$F = k \frac{Q_1 Q_2}{R^2}$$

In free space

$$F = \frac{1}{4\pi\varepsilon_0} \frac{Q_1 Q_2}{R^2} \text{ in Newton.}$$

In any dielectric medium

$$F = \frac{1}{4\pi\varepsilon} \frac{Q_1 Q_2}{R^2}$$

$\varepsilon = \varepsilon_0 \varepsilon_r$

$\varepsilon_0 = 8.854 \times 10^{-12}\ F/m$

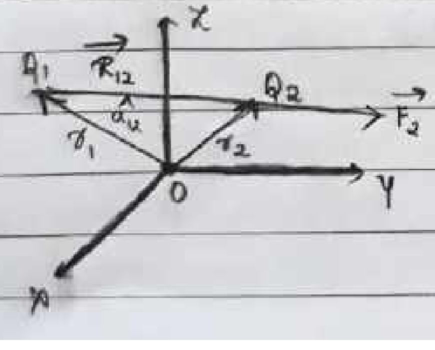
ε_0: absolute permitivity

ε_r: relative permitivity

$$\varepsilon_r = \frac{F_{medium}}{F_{vaccum}}$$

In free space $\varepsilon_r = 1$.

Coulomb's law in vector form



Force exerted by Q_1 on Q_2 : $\vec{F_2}$

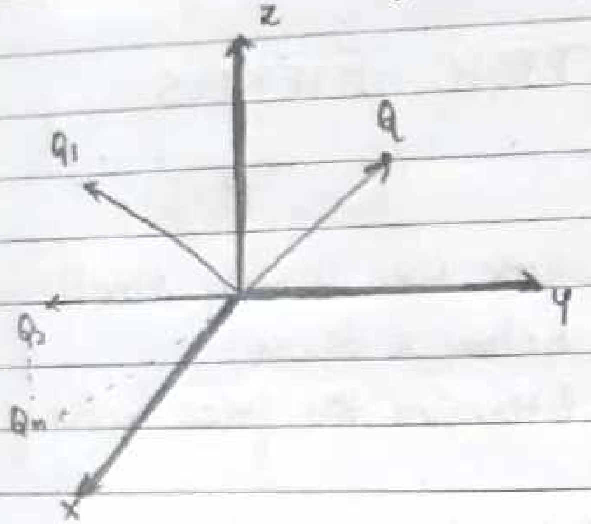
$$\vec{F_2} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R_{12}^2} \hat{a}_{12}$$

$$\hat{a}_{12} = \frac{\vec{r_2} - \vec{r_1}}{|\vec{r_2} - \vec{r_1}|}$$

$$\vec{R}_{12} = \vec{r_2} - \vec{r_1}$$

$$|\vec{R}_{12}| = |\vec{r_2} - \vec{r_1}|$$

Two or more charges acting on a point charge. (Q)



$$\vec{F} = \vec{F_1} + \vec{F_2} + \cdots + \vec{F_n}$$

$$\vec{F} = \frac{Q_1 Q}{4\pi\varepsilon_0 R_1^2}\hat{a}_1 + \frac{Q_2 Q}{4\pi\varepsilon_0 R_2^2}\hat{a}_2 + \cdots + \frac{Q_n Q}{4\pi\varepsilon_0 R_n^2}\hat{a}_n$$

$\vec{R_1} = \vec{r} - \vec{r_1}$ $\quad |\vec{R_1}| = |\vec{r} - \vec{r_1}|$

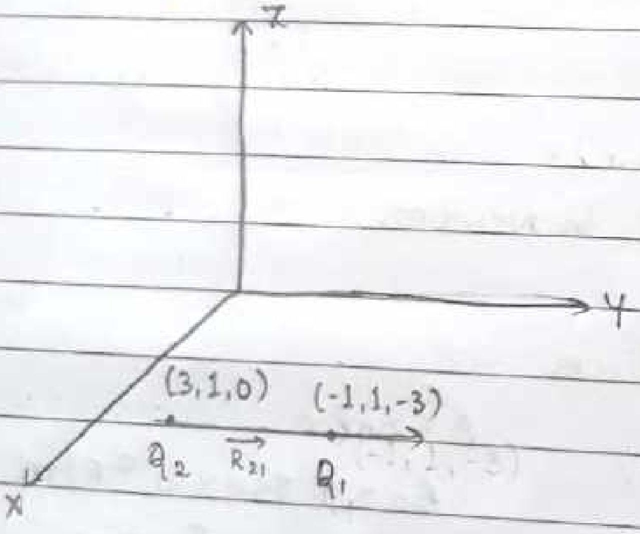
$\vec{R_2} = \vec{r} - \vec{r_2}$ $\quad |\vec{R_2}| = |\vec{r} - \vec{r_2}|$

$\vec{R_n} = \vec{r} - \vec{r_n}$ $\quad |\vec{R_n}| = |\vec{r} - \vec{r_n}|$

$$\hat{a}_1 = \frac{\vec{r} - \vec{r_1}}{|\vec{r} - \vec{r_1}|}$$

Q: Two point charges $Q_1 = 50\mu C$ and $Q_2 = 10\mu C$ are located at $(-1,1,-3)$m and $(3,1,0)$m respectively. Find the force on Q_1.

Sol:



$$\vec{F_2} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R_{21}^2}\hat{a}_{21}$$

$\vec{R_{21}} = (-1-3)\hat{a}_x + (1-1)\hat{a}_y + (-3-0)\hat{a}_z$

$\vec{R_{21}} = -4\hat{a}_x - 3\hat{a}_z$

$|\vec{R_{21}}| = \sqrt{16+9} = \sqrt{25} = 5$

$$\hat{a}_{21} = \frac{\vec{R_{21}}}{|\vec{R_{21}}|} = \frac{-4\hat{a}_x - 3\hat{a}_z}{5}$$

$$\therefore \vec{F_2} = \frac{(50\times10^{-6})(10\times10^{-6})}{4\pi(8.854\times10^{-12})(5)^2}\left(\frac{-4\hat{a}_x - 3\hat{a}_z}{5}\right) \text{ Force on } Q_1.$$

$$\vec{F_2} = -0.144\hat{a}_x - 0.108\hat{a}_z$$

Force on Q_2

$$\vec{F_1} = \frac{Q_1 Q_2}{4\pi\epsilon_0 R_{12}^2}\hat{a}_{12}$$

$$\vec{R_{12}} = (3+1)\hat{a}_x + (1-1)\hat{a}_y + (0+3)\hat{a}_z$$

$$\vec{R_{12}} = 4\hat{a}_x + 3\hat{a}_z$$

$$|\vec{R_{12}}| = \sqrt{16+9} = \sqrt{25} = 5$$

$$\hat{a}_{12} = \frac{\vec{R_{12}}}{|\vec{R_{12}}|} = \frac{4\hat{a}_x + 3\hat{a}_z}{5}$$

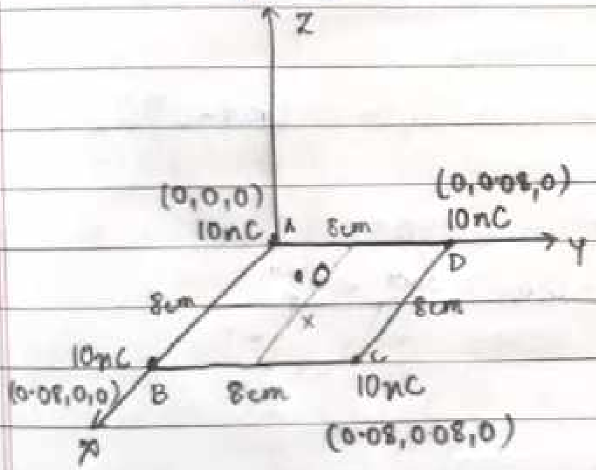
$$\vec{F_1} = \frac{(50\times10^{-6})(10\times10^{-6})}{4\pi(8.854\times10^{-12})(5^2)}\frac{(4\hat{a}_x + 3\hat{a}_z)}{5}$$

$$\vec{F_1} = 0.144\hat{a}_x + 0.108\hat{a}_z$$

Hence the magnitude is same but acts in opposite direction.

Q: Four 10nC positive charges are located in the $x=0$ plane at a corners of a square of side 8cm. A fifth 10nC positive charge is located at a point 8cm distance from the other charges. Calculate the magnitude of the total force on the fifth charge for $\epsilon = \epsilon_0$.

sol:



$$\vec{F} = \vec{F_1} + \vec{F_2} + \vec{F_3} + \vec{F_4}$$

$$\vec{F} = \frac{Q_1 Q}{4\pi\epsilon_0 R_1^2}\hat{a}_1 + \frac{Q_2 Q}{4\pi\epsilon_0 R_2^2}\hat{a}_2 + \frac{Q_3 Q}{4\pi\epsilon_0 R_3^2}\hat{a}_3 + \frac{Q_4 Q}{4\pi\epsilon_0 R_4^2}\hat{a}_4$$

Here $Q_1 = Q_2 = Q_3 = Q_4 = Q = 10nC$

$\cancel{R_1 = R_2 = R_3 = R_4 = 8cm}$

$\cancel{\vec{F} = \frac{4Q^2}{4\pi\epsilon_0 R^2} = \frac{(10\times10^{-9})^2}{\pi(8.854\times10^{-12})(8\times10^{-2})^2}}$

$$\vec{F} = \frac{Q^2}{4\pi\epsilon_0 R_1^2}\hat{a}_1 + \frac{Q^2}{4\pi\epsilon_0 R_2^2}\hat{a}_2 + \frac{Q^2\hat{a}_3}{4\pi\epsilon_0 R_3^2} + \frac{Q^2\hat{a}_4}{4\pi\epsilon_0 R_4^2}$$

$$\vec{F} = \frac{Q^2}{4\pi\epsilon_0}\left[\frac{\hat{a}_1}{R_1^2} + \frac{\hat{a}_2}{R_2^2} + \frac{\hat{a}_3}{R_3^2} + \frac{\hat{a}_4}{R_4^2}\right] \quad \text{—— (1)}$$

X is the mid point.

$AX^2 = 0.04^2 + 0.04^2$

$AX = \sqrt{2(0.04)^2}$

$AX = 0.056$

here coordinates of X is (0.04, 0.04, 0)

$OX^2 = AO^2 - AX^2$

$OX^2 = (0.08)^2 - (0.056)^2$

$OX = 0.057$

coordinates of O is (0.04, 0.04, 0.057)

$\vec{R_1} = (0.04-0)\hat{a}_x + (0.04-0)\hat{a}_y + (0.057 - \cancel{0})\hat{a}_z$

$= 0.04\hat{a}_x + 0.04\hat{a}_y + 0.057\hat{a}_z$

$|\vec{R_1}| = \sqrt{0.04^2 + 0.04^2 + 0.057^2} = 0.0803$

$\hat{a}_1 = \dfrac{\vec{R_1}}{|\vec{R_1}|} = \dfrac{0.04\hat{a}_x + 0.04\hat{a}_y + 0.057\hat{a}_z}{0.0803}$

$\vec{R_2} = (0.04 - 0.08)\hat{a}_x + (0.04 - 0)\hat{a}_y + (0.057 - 0)\hat{a}_z$

$= -0.04\hat{a}_x + 0.04\hat{a}_y + 0.057\hat{a}_z$

$|\vec{R_2}| = \sqrt{-0.04^2 + 0.04^2 + 0.057^2} = 0.0803$

$\hat{a}_2 = \dfrac{\vec{R_2}}{|\vec{R_2}|} = \dfrac{-0.04\hat{a}_x + 0.04\hat{a}_y + 0.057\hat{a}_z}{0.0803}$

$\vec{R_3} = (0.04 - 0.08)\hat{a}_x + (0.04 - 0.08)\hat{a}_y + (0.057 - 0)\hat{a}_z$

$= -0.04\hat{a}_x - 0.04\hat{a}_y + 0.057\hat{a}_z$

$|\vec{R_3}| = \sqrt{-0.04^2 + -0.04^2 + 0.057^2} = 0.0803$

$\hat{a}_3 = \dfrac{\vec{R_3}}{|\vec{R_3}|} = \dfrac{-0.04\hat{a}_x - 0.04\hat{a}_y + 0.057\hat{a}_z}{0.0803}$

$\vec{R_4} = (0.04 - 0)\hat{a}_x + (0.04 - 0.08)\hat{a}_y + (0.057 - 0)\hat{a}_z$

$= 0.04\hat{a}_x - 0.04\hat{a}_y + 0.057\hat{a}_z$

$|\vec{R_4}| = \sqrt{0.04^2 + -0.04^2 + 0.057^2} = 0.0803$

$\hat{a}_4 = \dfrac{\vec{R_4}}{|\vec{R_4}|} = \dfrac{0.04\hat{a}_x - 0.04\hat{a}_y + 0.057\hat{a}_z}{0.0803}$

Substituting in eq ①

here $R_1 = R_2 = R_3 = R_4 = 0.0803$

$$\therefore \vec{F} = \frac{Q^2}{4\pi\varepsilon_0 (R)^2}\left[\frac{\cancel{0.04\hat{a}_x} + \cancel{0.04\hat{a}_y} + 0.057\hat{a}_z - \cancel{0.04\hat{a}_x} + \cancel{0.04\hat{a}_y} + 0.057\hat{a}_z - \cancel{0.04\hat{a}_x} - \cancel{0.04\hat{a}_y} + 0.057\hat{a}_z + \cancel{0.04\hat{a}_x} - \cancel{0.04\hat{a}_y} + 0.057\hat{a}_z}{0.0803}\right]$$

$$\vec{F} = \frac{(10\times10^{-9})^2}{4\pi(8.854\times10^{-12})(0.0803)^3}\left[4(0.057)\hat{a}_z\right]$$

$$\vec{F} = \underline{2.959\times10^{-4}\ \hat{a}_z\ N}$$

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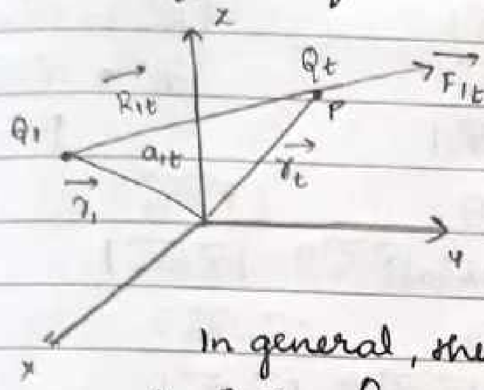
* ## Electric Field Intensity

Let Q_t, test charge be a unit positive charge
The force experienced by a unit positive test charge in a field is called electric field intensity.



$$\vec{F_{1t}} = \frac{Q_1 Q_t}{4\pi\varepsilon_0 R_{1t}^2} \hat{a}_{1t}$$ is the force experienced by Q_t due to Q_1.

$$\frac{\vec{F_{1t}}}{Q_t} = \frac{Q_1}{4\pi\varepsilon_0 R_{1t}^2} \hat{a}_{1t} \quad \text{(Force at point P)}$$

In general, the electric field intensity.

$$\therefore E = \frac{Q}{4\pi\varepsilon_0 R} \hat{a}_R \text{ in } V/m \text{ or } N/C.$$

CASE 1: Electric Field Intensity at Point $P(x, y, z)$ – in cartesian system.

$$E = \frac{Q}{4\pi\varepsilon_0 R^2} \hat{a}_R$$

$$\vec{R} = x\hat{a}_x + y\hat{a}_y + z\hat{a}_z$$

$$|\vec{R}| = \sqrt{x^2+y^2+z^2}$$

$$\hat{a}_R = \frac{x\hat{a}_x + y\hat{a}_y + z\hat{a}_z}{\sqrt{x^2+y^2+z^2}}$$

$$E = \frac{Q}{4\pi\varepsilon_0 (x^2+y^2+z^2)} \left[\frac{x\hat{a}_x + y\hat{a}_y + z\hat{a}_z}{(x^2+y^2+z^2)^{1/2}}\right]$$

$$E = \frac{Q}{4\pi\varepsilon_0} \left[\frac{x}{(x^2+y^2+z^2)^{3/2}} \hat{a}_x + \frac{y}{(x^2+y^2+z^2)^{3/2}} \hat{a}_y + \frac{z}{(x^2+y^2+z^2)^{3/2}} \hat{a}_z\right]$$

CASE 2: Electric Field Intensity at $P(x, y, z)$ when Q is not in the origin i.e, $Q(x_1, y_1, z_1)$.

$$E = \frac{Q}{4\pi\varepsilon_0 R^2} \hat{a}_R$$

$$\vec{R} = (x-x_1)\hat{a}_x + (y-y_1)\hat{a}_y + (z-z_1)\hat{a}_z$$

$$|\vec{R}| = \sqrt{(x-x_1)^2+(y-y_1)^2+(z-z_1)^2}$$

$$\hat{a}_R = \frac{(x-x_1)\hat{a}_x + (y-y_1)\hat{a}_y + (z-z_1)\hat{a}_z}{[(x-x_1)^2+(y-y_1)^2+(z-z_1)^2]^{1/2}}$$

$$\therefore E = \frac{Q}{4\pi\varepsilon_0 [(x-x_1)^2+(y-y_1)^2+(z-z_1)^2]} \left[\frac{(x-x_1)\hat{a}_x + (y-y_1)\hat{a}_y + (z-z_1)\hat{a}_z}{[(x-x_1)^2+(y-y_1)^2+(z-z)^2]^{1/2}}\right]$$

$$E = \frac{Q}{4\pi\varepsilon_0 [(x-x_1)^2+(y-y_1)^2+(z-z_1)^2]^{3/2}} \left[(x-x_1)\hat{a}_x + (y-y_1)\hat{a}_y + (z-z_1)\hat{a}_z\right]$$

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CASE 3: Electric field intensity by n point charges at point

$$E = \frac{Q_1}{4\pi\varepsilon_0 R_1^2}\hat{a}_{R_1} + \frac{Q_2}{4\pi\varepsilon_0 R_2}\hat{a}_{R_2} + \cdots\cdots + \frac{Q_n}{4\pi\varepsilon_0 R_n}\hat{a}_{R_n}$$

here $\vec{R_1} = \vec{r} - \vec{r_1}$ $\quad \vec{R_2} = \vec{r} - \vec{r_2}$ $\quad \cdots\cdots \quad \vec{R_n} = \vec{r} - \vec{r_n}$

$|\vec{R_1}| = |\vec{r} - \vec{r_1}|$ $\quad |\vec{R_2}| = |\vec{r} - \vec{r_2}|$ $\quad |\vec{R_n}| = |\vec{r} - \vec{r_n}|$

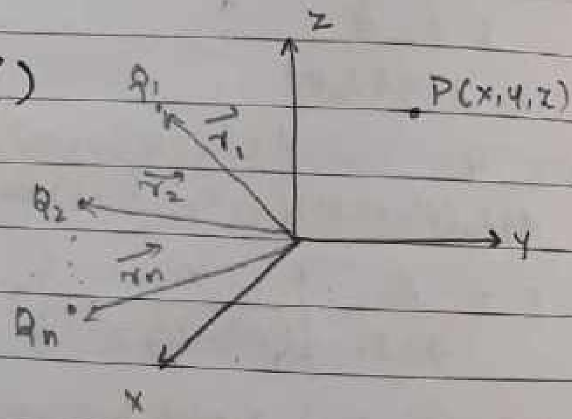
$\hat{a}_{R_1} = \frac{\vec{r} - \vec{r_1}}{|\vec{r} - \vec{r_1}|}$ $\quad \hat{a}_{R_2} = \frac{\vec{r} - \vec{r_2}}{|\vec{r} - \vec{r_2}|}$ $\quad \hat{a}_{R_n} = \frac{\vec{r} - \vec{r_n}}{|\vec{r} - \vec{r_n}|}$

$$\therefore E = \frac{Q_1}{4\pi\varepsilon_0 |\vec{r} - \vec{r_1}|^2}\frac{(\vec{r} - \vec{r_1})}{|\vec{r} - \vec{r_1}|} + \frac{Q_2}{4\pi\varepsilon_0 |\vec{r} - \vec{r_2}|^2}\frac{(\vec{r} - \vec{r_2})}{|\vec{r} - \vec{r_2}|}$$

$$+ \cdots + \frac{Q_n}{4\pi\varepsilon_0 |\vec{r} - \vec{r_n}|^2}\frac{(\vec{r} - \vec{r_n})}{|\vec{r} - \vec{r_2}|}$$

$$E = \frac{Q_1}{4\pi\varepsilon_0 |\vec{r} - \vec{r_1}|^3}(\vec{r} - \vec{r_1}) + \frac{Q_2}{4\pi\varepsilon_0 |\vec{r} - \vec{r_2}|^3}(\vec{r} - \vec{r_2}) + \cdots + \frac{Q_n}{4\pi\varepsilon_0 |\vec{r} - \vec{r_n}|^3}(\vec{r} - \vec{r_n})$$

$$E = \sum_{i=1}^{n} \frac{Q_i}{4\pi\varepsilon_0 |\vec{r} - \vec{r_i}|^3}(\vec{r} - \vec{r_i})$$

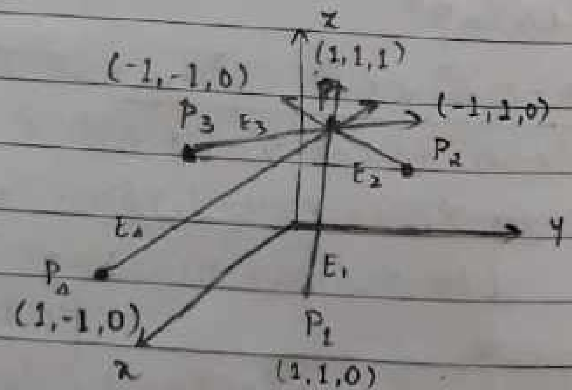


Q: Find E at $P(1,1,1)$ caused by four identical $3\mu C$ charges located at $P_1(1,1,0)$, $P_2(-1,1,0)$, $P_3(-1,-1,0)$, $P_4(1,-1,0)$.

Sol:

$E = E_1 + E_2 + E_3 + E_4$

$$E = \frac{Q_1}{4\pi\varepsilon_0 R_1^2}\hat{a}_{R_1} + \frac{Q_2}{4\pi\varepsilon_0 R_2^2}\hat{a}_{R_2} + \frac{Q_3}{4\pi\varepsilon_0 R_3^2}\hat{a}_{R_3} + \frac{Q_4}{4\pi\varepsilon_0 R_4^2}\hat{a}_{R_4}$$



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Here $Q_1 = Q_2 = Q_3 = Q_4 = 3\mu C$

$$E = \frac{Q}{4\pi\varepsilon_0}\left[\frac{\hat{a}_{R_1}}{R_1^2} + \frac{\hat{a}_{R_2}}{R_2^2} + \frac{\hat{a}_{R_3}}{R_3^2} + \frac{\hat{a}_{R_4}}{R_4^2}\right] \quad \text{—(1)}$$

$\vec{R_1} = (1-1)\hat{a}_x + (1-1)\hat{a}_y + (1-0)\hat{a}_z$

$\vec{R_1} = \hat{a}_z$

$|\vec{R_1}| = \sqrt{1^2} = 1$

$\hat{a}_1 = \frac{\vec{R_1}}{|\vec{R_1}|} = \frac{\hat{a}_z}{1} = \hat{a}_z$

$\vec{R_2} = (1+1)\hat{a}_x + (1-1)\hat{a}_y + (1-0)\hat{a}_z$

$= 2\hat{a}_x + \hat{a}_z$

$|\vec{R_2}| = \sqrt{2^2+1^2} = \sqrt{5}$

$\hat{a}_2 = \frac{\vec{R_2}}{|\vec{R_2}|} = \frac{2\hat{a}_x + \hat{a}_z}{\sqrt{5}}$

$\vec{R_3} = (1+1)\hat{a}_x + (1+1)\hat{a}_y + (1-0)\hat{a}_z$

$= 2\hat{a}_x + 2\hat{a}_y + \hat{a}_z$

$|\vec{R_3}| = \sqrt{2^2+2^2+1} = \sqrt{9} = 3$

$\hat{a}_3 = \frac{\vec{R_3}}{|\vec{R_3}|} = \frac{2\hat{a}_x + 2\hat{a}_y + \hat{a}_z}{3}$

$\vec{R_4} = (1-1)\hat{a}_x + (1+1)\hat{a}_y + (1-0)\hat{a}_z$

$= 2\hat{a}_y + \hat{a}_z$

$|\vec{R_4}| = \sqrt{2^2 + 1^2} = \sqrt{5}$

$\hat{a}_4 = \frac{\vec{R_4}}{|\vec{R_4}|} = \frac{2\hat{a}_y + \hat{a}_z}{\sqrt{5}}$

Substituting in eq (1)

$$E = \frac{3\times10^{-9}}{4\pi(8.854\times10^{-12})}\left[\frac{\hat{a}_z}{1^2} + \frac{2\hat{a}_x+\hat{a}_z}{\sqrt{5}(\sqrt{5})^2} + \frac{2\hat{a}_x+2\hat{a}_y+\hat{a}_z}{3(3)^2} + \frac{2\hat{a}_y+\hat{a}_z}{\sqrt{5}(\sqrt{5})^2}\right]$$

$$E = 26.963 \left[\hat{a}_z + \frac{2\hat{a}_x+\hat{a}_z}{5\sqrt{5}} + \frac{2\hat{a}_x+2\hat{a}_y+\hat{a}_z}{27} + \frac{2\hat{a}_y+\hat{a}_z}{5\sqrt{5}}\right]$$

$$E = \frac{26.963}{5\sqrt{5}\times27}\left[301.87\hat{a}_z + 54\hat{a}_x + 27\hat{a}_z + 22.36\hat{a}_x + 22.36\hat{a}_y + 11.18\hat{a}_z + 54\hat{a}_y + 27\hat{a}_z\right]$$

$$E = 89.32 \times 10^{-3} \left[\hat{a}_x (54 + 22.36) + \hat{a}_y (22.36 + 54) + \hat{a}_z (301.8 + 27 + 2 + 11.18) \right.$$

$$E = 6.82 \hat{a}_x + 6.820 \hat{a}_y + \underline{32.834 \hat{a}_z}$$

* <u>Different types of charge distribution:</u>

a. <u>Point charge:</u>

If the size of the charges is very small when compared to the distance between them are known as point charges.

b. <u>Line charge:</u>

In this distribution the charge is distributed uniformly along a straight line or circumference of a circle.

c. <u>Surface charge:</u>

In this distribution the charge is distributed continuously over some area.

d. <u>Volume charge:</u>

In this distribution the charge is distributed continuously ~~[illegible]~~ over a volume.

For line, surface and volume charges ~~[illegible]~~ it is assumed that the charge is distributed uniformly.

Total charge can be calculated with the help of charge density.

<u>Line charge density</u>

$$\rho_L = \frac{\text{Total charge in C}}{\text{Total length in m}} \quad \text{in C/m.}$$

charge present in a differential length dL

$$dQ = \rho_L \, dL$$

Total charge

$$Q = \int_L \rho_L \, dL$$

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Surface charge | sheet charge

Surface charge density

$$\rho_s = \frac{\text{Total charge in C}}{\text{Total area in } m^2} \quad \text{in } C/m^2$$

charge present in a differential area dS

$$dQ = \rho_s \, dS$$

Total charge

$$Q = \int_S \rho_s \, dS.$$

Volume charge

Volume charge density

$$\rho_v = \frac{\text{Total charge in C}}{\text{Total volume in } m^3} \quad \text{in } C/m^3$$

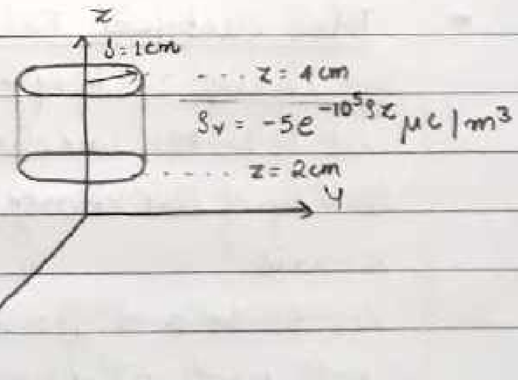
charge present in a differential volume dV

$$dQ = \rho_v \, dV$$

Total charge

$$Q = \int_V \rho_v \, dV.$$

Q: Find the total charge contained in a 2cm length of the electron beam shown in figure.



Sol: Cylindrical coordinate system

$\cancel{d\rho = 1cm}$

$\cancel{dz = 4 - 2 = 2cm}$

differential volume

$$dV = \rho \, d\phi \, d\rho \, dz$$

Total charge

$$Q = \int_V \rho_v \, dV = \int_V -5e^{-10^5 \rho z} \rho \, d\phi \, d\rho \, dz$$

$$Q = \int_{\rho=0}^{0.01} \int_{\phi=0}^{2\pi} \int_{z=0.02}^{0.04} -5e^{-10^5 \rho z} \rho \, d\phi \, d\rho \, dz$$

$$Q = -5\times10^{-6}\int_{\rho=0}^{0.01}\int_{z=0.02}^{0.04} e^{-10^5 z\rho}\,\rho\,\phi\, d\rho\, dz\Big|_0^{2\pi}$$

$$= -5\times10^{-6}\times 2\pi \int_{\rho=0}^{0.01}\int_{z=0.02}^{0.04} e^{-10^5\rho z}\rho\, d\rho\, dz$$

$$= -31.416\times10^{-6}\int_{\rho=0}^{0.01} e^{-10^5\rho z}\cancel{\rho}/(-10^5\cancel{\rho})\,d\rho\Big|_{0.02}^{0.04}$$

$$= -31.416\times10^{-6}\int_{\rho=0}^{0.01}\left[\frac{e^{-10^5\rho(0.04)}}{-10^5} + \frac{e^{-10^5\rho(0.02)}}{10^5}\right]d\rho$$

$$= 31.416\times10^{-11}\int_{\rho=0}^{0.01}\left(e^{-4\times10^3\rho} - e^{-2\times10^3\rho}\right)d\rho$$

$$= 31.416\times10^{-11}\left[\frac{e^{-4\times10^3\rho}}{-4\times10^3} - \frac{e^{-2\times10^3\rho}}{-2\times10^3}\right]_0^{0.01}$$

$$= 31.416\times10^{-11}\left[\frac{e^{-40}}{-4\times10^3} - \frac{e^{-20}}{-2\times10^3} - \frac{1}{-4\times10^3} + \frac{1}{-2\times10^3}\right]$$

$$= 31.416\times10^{-11}\left[-1.062\times10^{-21} + 1.0305\times10^{-12} + 0.25\times10^{-3} - 0.5\right.$$

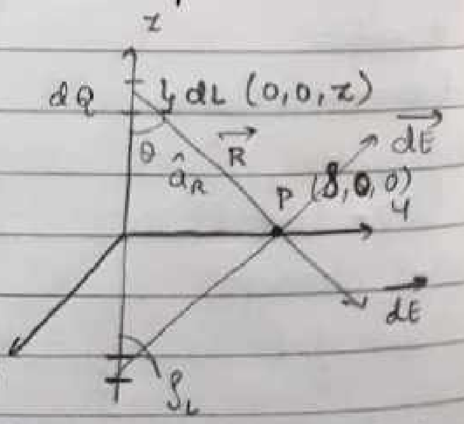
$$= 31.416\times10^{-11}\left[-0.25\times10^{-3}\right]$$

$$\underline{Q = -7.854\times10^{-14}\ C}$$

* <u>Total Electrical Field Intensity due to Infinite length line charge</u> : (in cylindrical coordinate system)

Let ρ_L be the line charge density of line charge along z-axis.

Let dL be a differential length with differential charge dQ.



Let $\vec{dE}$ be the differential electric field intensity due to the differential length dL with differential charge dQ at point P.

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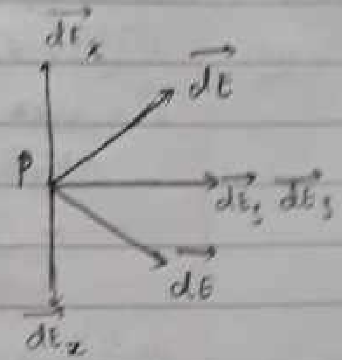
$\vec{R} = \rho\hat{a}_\rho - z\hat{a}_x$

$|\vec{R}| = \sqrt{\rho^2 + z^2}$

$$\hat{a}_R = \frac{\vec{R}}{|\vec{R}|} = \frac{\rho\hat{a}_\rho - z\hat{a}_x}{\sqrt{\rho^2+z^2}}$$

$$E = \frac{Q}{4\pi\varepsilon_0 R^2}\hat{a}_R$$



$$\vec{dE} = \frac{dQ}{4\pi\varepsilon_0(\rho^2+z^2)}\left[\frac{\rho\hat{a}_\rho - z\hat{a}_z}{\sqrt{\rho^2+z^2}}\right]$$

$$E = \int_{z=-\infty}^{\infty} \frac{\rho_L dz(\rho\hat{a}_\rho - z\hat{a}_z)}{4\pi\varepsilon_0(\rho^2+z^2)^{3/2}}$$

$$E = \int_{z=-\infty}^{\infty} \frac{\rho_L dz(\rho\hat{a}_\rho)}{4\pi\varepsilon_0(\rho^2+z^2)^{3/2}}$$

since

$\frac{z}{\rho} = \cot\theta$

$\therefore z = \rho\cot\theta$

$dz = -\rho\,\mathrm{cosec}^2\theta\, d\theta$

Substituting we get

$$E = -\int_{\pi/2}^{-\pi/2} \frac{\rho_L \rho\, \mathrm{cosec}^2\theta(\rho\hat{a}_\rho)d\theta}{4\pi\varepsilon_0(\rho^2+\rho^2\cot^2\theta)^{3/2}}$$

$$E = \frac{-\rho_L\rho^2}{4\pi\varepsilon_0\rho^3}\int_{\pi/2}^{-\pi/2}\frac{\mathrm{cosec}^2\theta\, d\theta\, \hat{a}_\rho}{\mathrm{cosec}^3\theta}$$

$$E = \frac{-\rho_L}{4\pi\varepsilon_0\rho}\int_{\pi/2}^{-\pi/2}\sin\theta\, d\theta\, \hat{a}_\rho$$

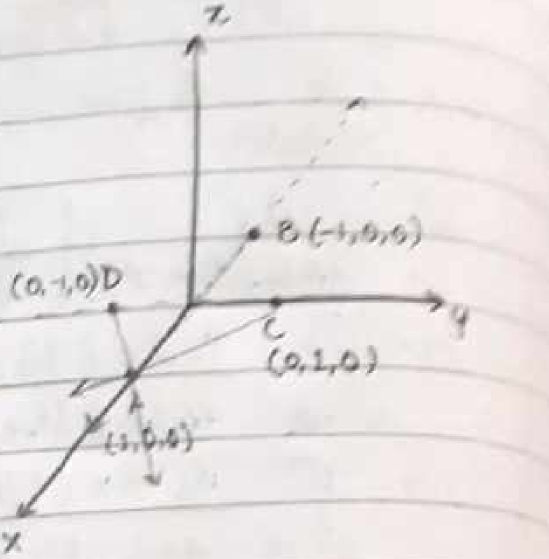
$$E = \frac{-\rho_L}{4\pi\varepsilon_0\rho}\left[-1-(1)\right]\hat{a}_\rho$$

$$E = \frac{\rho_L}{2\pi\varepsilon_0\rho}\hat{a}_\rho$$

$$\boxed{E = \frac{\rho_L}{2\pi\varepsilon_0 R}\hat{a}_\rho}$$

The direction in which the line charge is present in that direction no flux lines exist.

Q: 4 point charges of 50nC each are located at A(1,0,0), B(-1,0,0), C(0,1,0) and D(0,-1,0) in free space find the total force on the charge at A.



<u>Sol:</u> $\vec{F} = \vec{F_1} + \vec{F_2} + \vec{F_3}$

$$\vec{F} = \frac{Q_A Q_1}{4\pi\varepsilon_0 R_1^2}\hat{a}_1 + \frac{Q_A Q_2}{4\pi\varepsilon_0 R_2^2}\hat{a}_2 + \frac{Q_A Q_3}{4\pi\varepsilon_0 R_3^2}\hat{a}_3 \quad \text{— (1)}$$

Here $Q_A = Q_1 = Q_2 = Q_3 = 50nC$

$\vec{R_1} = (1+1)\hat{a}_x + (0-0)\hat{a}_y + (0-0)\hat{a}_z$

$= 2\hat{a}_x$

$|\vec{R_1}| = \sqrt{2^2} = 2$

$\hat{a}_1 = \frac{\vec{R_1}}{|\vec{R_1}|} = \frac{2\hat{a}_x}{2} = \hat{a}_x$

$\vec{R_2} = (1-0)\hat{a}_x + (0-1)\hat{a}_y + (0-0)\hat{a}_z$

$= 1\hat{a}_x - 1\hat{a}_y$

$|\vec{R_2}| = \sqrt{1^2+1^2} = \sqrt{2}$

$\hat{a}_2 = \frac{\vec{R_2}}{|\vec{R_2}|} = \frac{1\hat{a}_x - 1\hat{a}_y}{\sqrt{2}}$

$\vec{R_3} = (1-0)\hat{a}_x + (0+1)\hat{a}_y + ($

$= 1\hat{a}_x + 1\hat{a}_y$

$|\vec{R_3}| = \sqrt{1^2+1^2} = \sqrt{2}$

$\hat{a}_3 = \frac{\vec{R_3}}{|\vec{R_3}|} = \frac{1\hat{a}_x + 1\hat{a}_y}{\sqrt{2}}$

Substituting in eq (1)

$$\vec{F} = \frac{Q^2}{4\pi\varepsilon_0}\left[\frac{\hat{a}_1}{R_1^2} + \frac{\hat{a}_2}{R_2^2} + \frac{\hat{a}_3}{R_3^2}\right]$$

$$= \frac{(50\times10^{-9})^2}{4\pi(8.854\times10^{-12})}\left[\frac{\hat{a}_x}{4} + \frac{\hat{a}_x - \hat{a}_y}{\sqrt{2}(\sqrt{2})^2} + \frac{\hat{a}_x + \hat{a}_y}{\sqrt{2}(\sqrt{2})^2}\right]$$

$$= 0.449\times10^{-6}\left[\frac{\hat{a}_x + \sqrt{2}\hat{a}_x - \sqrt{2}\hat{a}_y + \sqrt{2}\hat{a}_x + \sqrt{2}\hat{a}_y}{4}\right]$$

$$= 0.112\times10^{-6}\left[(1+2\sqrt{2})\hat{a}_x\right]$$

$$\underline{\vec{F} = 428.78\times10^{-3}\,\hat{a}_x\,N}$$

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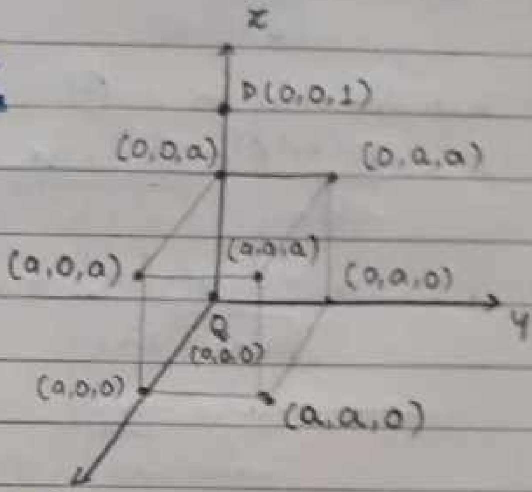
Q: Eight identical point charges of Q C each are located at the corners of a cube of side a, with one charge at origin and with the three nearest charges at (a,0,0), (0,a,0) and (0,0,a). Find an expression for the total vector force on the charge at P (0,0,1) assuming free space.



Sol: $\vec{F} = \vec{F_1} + \vec{F_2} + \vec{F_3} + \vec{F_4} + \vec{F_5} + \vec{F_6} + \vec{F_7} + \vec{F_8}$

$$\vec{F} = \frac{Q^2}{4\pi\epsilon_0}\left[\frac{\hat{a}_1}{R_1^2} + \frac{\hat{a}_2}{R_2^2} + \frac{\hat{a}_3}{R_3^2} + \frac{\hat{a}_4}{R_4^2} + \frac{\hat{a}_5}{R_5^2} + \frac{\hat{a}_6}{R_6^2} + \frac{\hat{a}_7}{R_7^2} + \frac{\hat{a}_8}{R_8^2}\right] \quad \text{——— (1)}$$

→ Due to charge at (a,0,0)

$\vec{R_1} = (0-a)\hat{a}_x + (0-0)\hat{a}_y + (1-0)\hat{a}_z$

$= -a\hat{a}_x + \hat{a}_x$

$|\vec{R_1}| = \sqrt{a^2+1^2} = \sqrt{a^2+1}$

$\hat{a}_1 = \frac{\vec{R_1}}{|\vec{R_1}|} = \frac{-a\hat{a}_x + \hat{a}_z}{\sqrt{a^2+1}}$

→ Due to charge at (a,a,0)

$\vec{R_2} = (0-a)\hat{a}_x + (0-a)\hat{a}_y + (1-0)\hat{a}_z$

$= -a\hat{a}_x - a\hat{a}_y + \hat{a}_z$

$|\vec{R_2}| = \sqrt{a^2+a^2+1^2} = \sqrt{2a^2+1}$

$\hat{a}_2 = \frac{\vec{R_2}}{|\vec{R_2}|} = \frac{-a\hat{a}_x - a\hat{a}_y + \hat{a}_z}{\sqrt{2a^2+1}}$

→ Due to charge at (0,a,0)

$\vec{R_3} = (0-0)\hat{a}_x + (0-a)\hat{a}_y + (1-0)\hat{a}_z$

$= -a\hat{a}_y + \hat{a}_z$

$|\vec{R_3}| = \sqrt{a^2+1}$

$\hat{a}_3 = \frac{\vec{R_3}}{|\vec{R_3}|} = \frac{-a\hat{a}_y + \hat{a}_z}{\sqrt{a^2+1}}$

→ Due to charge at (0,0,0)

$\vec{R_4} = (0-0)\hat{a}_x + (0-0)\hat{a}_y + (1-0)\hat{a}_z$

$= \hat{a}_z$

$|\vec{R_4}| = \sqrt{1^2} = 1$

$\hat{a}_4 = \frac{\vec{R_4}}{|\vec{R_4}|} = \hat{a}_z$

→ Due to charge at (a,0,a)

$\vec{R_5} = (0-a)\hat{a}_x + (0-0)\hat{a}_y + (1-a)\hat{a}_z$

$= -a\hat{a}_x + (1-a)\hat{a}_z$

$|\vec{R_5}| = \sqrt{a^2+(1-a)^2}$

$\hat{a}_5 = \frac{\vec{R_5}}{|\vec{R_5}|} = \frac{-a\hat{a}_x + (1-a)\hat{a}_z}{\sqrt{a^2+(1-a)^2}}$

→ Due to charge at (a,a,a)

$\vec{R_6} = (0-a)\hat{a}_x + (0-a)\hat{a}_y + (1-a)\hat{a}_z$

$= -a\hat{a}_x - a\hat{a}_y + (1-a)\hat{a}_z$

$|\vec{R_6}| = \sqrt{a^2+a^2+(1-a)^2} = \sqrt{2a^2+(1-a)^2}$

$\hat{a}_6 = \frac{\vec{R_6}}{|\vec{R_6}|} = \frac{-a\hat{a}_x - a\hat{a}_y + (1-a)\hat{a}_z}{\sqrt{2a^2+(1-a)^2}}$

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→ Due to charge at (0, a, a)

$\vec{R_7} = (0-0)\hat{a}_x + (0-a)\hat{a}_y + (1-a)\hat{a}_z$

$= -a\hat{a}_y + (1-a)\hat{a}_z$

$|\vec{R_7}| = \sqrt{a^2+(1-a)^2}$

$\hat{a}_7 = \dfrac{-a\hat{a}_y + (1-a)\hat{a}_z}{\sqrt{a^2+(1-a)^2}}$

→ Due to charge at (0, 0, a)

$\vec{R_8} = (0-0)\hat{a}_x + (0-0)\hat{a}_y + (1-a)\hat{a}_z$

$= (1-a)\hat{a}_z$

$|\vec{R_8}| = \sqrt{(1-a)^2} = 1-a$

$\hat{a}_8 = \dfrac{\vec{R_8}}{|\vec{R_8}|} = \dfrac{(1-a)\hat{a}_z}{1-a} = \hat{a}_z$

Substituting in eq ①

$$F = \frac{Q^2}{4\pi\varepsilon_0}\left[\frac{-a\hat{a}_x+\hat{a}_z}{\sqrt{a^2+1}}\left(\frac{1}{a^2+1}\right) + \frac{-a\hat{a}_x - a\hat{a}_y + \hat{a}_z}{\sqrt{2a^2+1}\,(2a^2+1)} + \frac{-a\hat{a}_y+\hat{a}_z}{\sqrt{a^2+1}\,(a^2+1)} + \hat{a}_z + \frac{-a\hat{a}_x+(1-a)\hat{a}_z}{\sqrt{a^2+(1-a)^2}\,(a^2+(1-a)^2)} + \frac{-a\hat{a}_x - a\hat{a}_y + (1-a)\hat{a}_z}{\sqrt{2a^2+(1-a)^2}\,(2a^2+(1-a)^2)} + \frac{-a\hat{a}_y+(1-a)\hat{a}_z}{\sqrt{a^2+(1-a)^2}\,(a^2+(1-a)^2)} + \frac{\hat{a}_z}{(1-a)^2}\right]$$

$$F = \frac{Q^2}{4\pi\varepsilon_0}\left[\frac{-a\hat{a}_x+\hat{a}_z-a\hat{a}_y+\hat{a}_z}{(a^2+1)^{3/2}} + \frac{-a\hat{a}_x+\hat{a}_z-a\hat{a}_x-a\hat{a}_y+\hat{a}_z}{(a^2+(1-a)^2)^{3/2}} + \frac{-a\hat{a}_x-a\hat{a}_y+\hat{a}_z}{(2a^2+1)^{3/2}} + \frac{-a\hat{a}_x-a\hat{a}_y+(1-a)\hat{a}_z}{(2a^2+(1-a)^2)^{3/2}} + \frac{\hat{a}_z}{(1-a)^2} + \hat{a}_z\right]$$

$$F = \frac{Q^2}{4\pi\varepsilon_0}\left[\frac{-a(\hat{a}_x+\hat{a}_y)+2\hat{a}_z}{(a^2+1)^{3/2}} + \frac{-a(\hat{a}_x+\hat{a}_y)+2\hat{a}_z(1-a)}{(a^2+(1-a)^2)^{3/2}} + \frac{-a(\hat{a}_x+\hat{a}_y)+\hat{a}_z}{(2a^2+1)^{3/2}} + \frac{-a(\hat{a}_x+\hat{a}_y)+(1-a)\hat{a}_z}{(2a^2+(1-a)^2)^{3/2}} + \left[\frac{(1-a)^2+}{(1-a)}\right.\right.$$

Q: Let a point charge $Q_1 = 25nC$ be located at $P_1(4,-2,7)$ and a charge $Q_2 = 60nC$ be at $P_2(-3,4,-8)$.

a. If $\varepsilon = \varepsilon_0$ find E at $P_3(1,2,3)$

b. At what point on the y-axis is $E_x = 0$?

Sol: a) $E = E_1 + E_2$

$$E = \frac{Q_1}{4\pi\varepsilon_0 R_1^2}\hat{a}_{R_1} + \frac{Q_2}{4\pi\varepsilon_0 R_2^2}\hat{a}_{R_2} \quad \text{—— (1)}$$

$\vec{R_1} = (1-4)\hat{a}_x + (2+8)\hat{a}_y + (3-7)\hat{a}_z$

$= -3\hat{a}_x + 10\hat{a}_y - 4\hat{a}_z$

$|\vec{R_1}| = \sqrt{9+100+16} = \sqrt{125} = 11.18$

$\hat{a}_1 = \frac{\vec{R_1}}{|\vec{R_1}|} = \frac{-3\hat{a}_x + 10\hat{a}_y - 4\hat{a}_z}{11.18}$

$\vec{R_2} = (1+3)\hat{a}_x + (2-4)\hat{a}_y + (3+8)\hat{a}_z$

$= 4\hat{a}_x - 2\hat{a}_y + 11\hat{a}_z$

$|\vec{R_2}| = \sqrt{16+4+121} = \sqrt{141} = 11.87$

$\hat{a}_2 = \frac{\vec{R_2}}{|\vec{R_2}|} = \frac{4\hat{a}_x - 2\hat{a}_y + 11\hat{a}_z}{11.87}$

Substituting in eq (1)

$$E = \frac{25\times10^{-9}}{4\pi(8.854\times10^{-12})}\left[\frac{-3\hat{a}_x + 10\hat{a}_y - 4\hat{a}_z}{(11.18)^3}\right] + \frac{60\times10^{-9}}{4\pi(8.854\times10^{-12})}\left[\frac{4\hat{a}_x - 2\hat{a}_y + 11\hat{a}_z}{(11.87)^3}\right]$$

$= 0.1604[-3\hat{a}_x + 10\hat{a}_y - 4\hat{a}_z] + 0.3224[4\hat{a}_x - 2\hat{a}_y + 11\hat{a}_z]$

$= -0.48\hat{a}_x + 1.6\hat{a}_y - 0.64\hat{a}_z + 1.29\hat{a}_x - 0.64\hat{a}_y + 3.54\hat{a}_z$

$= 0.81\hat{a}_x + 0.96\hat{a}_y + 2.9\hat{a}_z$

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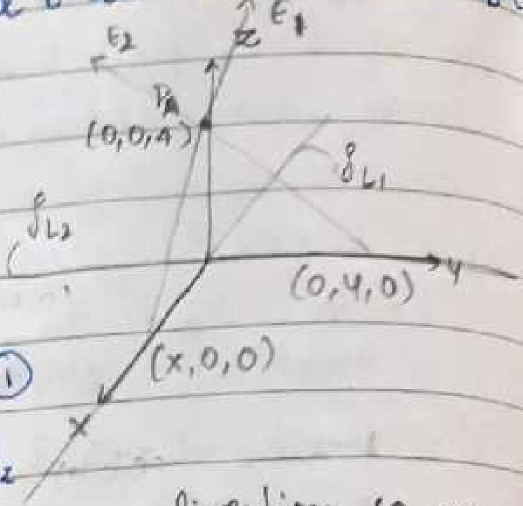
Q: Infinite uniform line charges of 5nC/m lie along the x and y axis in free space. Find E at $P_A(0,0,4)$ and $P_B(0,3,4)$



Sol: $E = \dfrac{\rho_L}{2\pi\varepsilon_0 R}\,\hat{a}_R$

at P_A: $E = E_1 + E_2$

$$E = \frac{\rho_{L1}}{2\pi\varepsilon_0 R_1}\hat{a}_1 + \frac{\rho_{L2}}{2\pi\varepsilon_0 R_2}\hat{a}_2 \quad \text{---} (1)$$

$\vec{R_1} = (0-x)\hat{a}_x + (0-0)\hat{a}_y + (4-0)\hat{a}_z$

$= \cancel{-x\hat{a}_x}^{\,0} + 4\hat{a}_z$ It lies along x direction so no field components will be present in x direction

$|\vec{R_1}| = \sqrt{\cancel{x^2+}16} = 4$

$\cancel{\hat{a}_1 = \dfrac{-x\hat{a}_x + 4\hat{a}_z}{\sqrt{x^2+16}}}$

$\hat{a}_1 = \dfrac{4\hat{a}_z}{4} = \hat{a}_z$

$\vec{R_2} = (0-0)\hat{a}_x + (0-y)\hat{a}_y + (4-0)\hat{a}_z$

$= \cancel{-y\hat{a}_y}^{\,0} + 4\hat{a}_z$ It lies along y direction so no field components will be present in y direction

$|\vec{R_2}| = 4$

$\hat{a}_2 = \dfrac{4\hat{a}_z}{4} = \hat{a}_z$

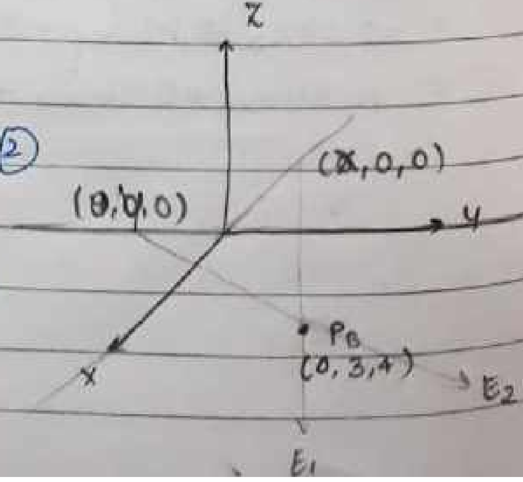
Substituting in eq (1)

$$E = \frac{5\times10^{-9}}{2\pi(8.854\times10^{-12})}\frac{\hat{a}_z}{4} + \frac{5\times10^{-9}}{2\pi(8.854\times10^{-12})}\frac{\hat{a}_z}{4}$$

$\underline{E = 44.94\,\hat{a}_z \text{ V/m}}$

at P_B: $E = E_1 + E_2$

$$E = \frac{\rho_{L1}}{2\pi\varepsilon_0 R_1}\hat{a}_1 + \frac{\rho_{L2}}{2\pi\varepsilon_0 R_2}\hat{a}_2 \quad \text{---} (2)$$



$\vec{R_1} = (0-x)^{\,0}\hat{a}_x + (3-0)\hat{a}_y + (4-0)\hat{a}_z$

$= 3\hat{a}_y + 4\hat{a}_z$

$|\vec{R_1}| = \sqrt{9+16} = \sqrt{25} = 5$

$\hat{a}_1 = \dfrac{\vec{R_1}}{|\vec{R_1}|} = \dfrac{3\hat{a}_y + 4\hat{a}_z}{5}$

$\vec{R_2} = (0-0)\hat{a}_x + (3-3)\hat{a}_y + (4-0)\hat{a}_z$

$= 4\hat{a}_z$

$|\vec{R_2}| = 4$

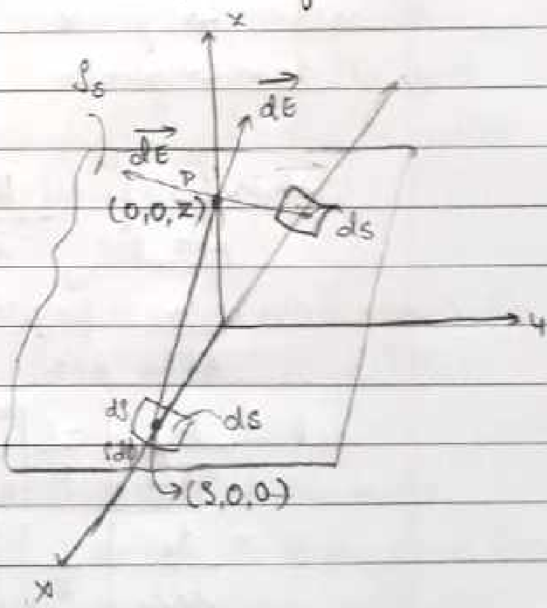
$\hat{a}_2 = \dfrac{\vec{R_2}}{|\vec{R_2}|} = \dfrac{4\hat{a}_z}{4} = \hat{a}_z$

Substituting in eq (2)

$$E = \frac{5\times10^{-9}}{4\pi(8.854\times10^{-12})}\left[\frac{3\hat{a}_y + 4\hat{a}_z}{5}\right] + \frac{5\times10^{-9}}{4\pi(8.854\times10^{-12})}\hat{a}_z$$

$$E = 26.96\,\hat{a}_y + 80.88\,\hat{a}_z$$

* <u>Total Electric field due to infinite sheet of charge:</u>
(in cylindrical coordinate system):



Let ρ_s be the surface charge density of sheet charge.
Let ds be the differential surface with differential charge dQ.
Let $\vec{dE}$ be the differential electric field intensity due to the differential surface ds with differential charge dQ at point P.

$dQ = \rho_s\, ds$

$dQ = \rho_s\, \rho\, d\rho\, d\phi$

$\vec{R} = (0-\rho)\hat{a}_\rho + (0-0)\hat{a}_\phi + (z-0)\hat{a}_z$

$= -\rho\hat{a}_\rho + z\hat{a}_z$

$|\vec{R}| = \sqrt{\rho^2 + z^2}$

$\hat{a}_R = \dfrac{\vec{R}}{|\vec{R}|} = \dfrac{-\rho\hat{a}_\rho + z\hat{a}_z}{\sqrt{\rho^2+z^2}}$

$E = \dfrac{Q}{4\pi\epsilon_0 R^2}\hat{a}_R$

$\vec{dE} = \dfrac{dQ}{4\pi\epsilon_0 R^2}\hat{a}_R$

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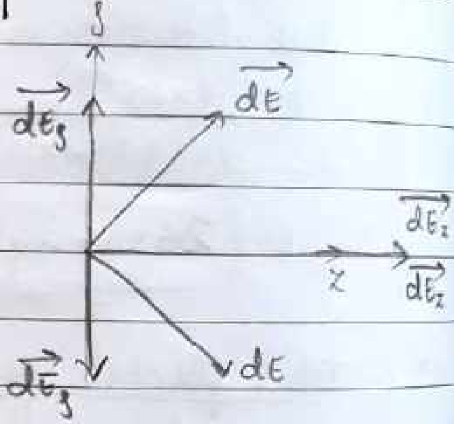
$$\vec{dE} = \frac{\rho_s \rho \, d\rho \, d\phi}{4\pi\varepsilon_0 (\sqrt{\rho^2+z^2})^2} \left(\frac{-\rho \hat{a}_\rho + z\hat{a}_z}{\sqrt{\rho^2+z^2}} \right)$$

$$\vec{dE} = \frac{\rho_s \rho \, d\rho \, d\phi \, (-\rho\hat{a}_\rho + z\hat{a}_z)}{4\pi\varepsilon_0 (\rho^2+z^2)^{3/2}}$$

$$E = \int_S \frac{\rho_s \rho \, d\rho \, d\phi \, (-\rho\hat{a}_\rho^{\to 0} + z\hat{a}_z)}{4\pi\varepsilon_0 (\rho^2+z^2)^{3/2}}$$

Idealy it is uniform charge density ⇒ z is constant. as ρ_s doesnot vary in z direction.

$$E = \int_{\rho=0}^{\infty} \int_{\phi=0}^{2\pi} \frac{\rho_s \rho \, d\rho \, d\phi \, z\hat{a}_z}{4\pi\varepsilon_0 (\rho^2+z^2)^{3/2}}$$



put $\rho^2 + z^2 = \mu^2$

$2\rho \, d\rho = 2\mu \, d\mu$

$\rho \, d\rho = \mu \, d\mu$

as $\rho = 0 \to \mu = z$

$\rho = \infty \to \mu = \infty$

$$\therefore E = \int_{\mu=z}^{\infty} \int_{\phi=0}^{2\pi} \frac{\rho_s \cancel{\mu} \, d\mu \, d\phi \, z\hat{a}_z}{4\pi\varepsilon_0 (\mu^2)^{3/2}}$$

$$E = \frac{\rho_s}{4\pi\varepsilon_0} \int_{\mu=z}^{\infty} \mu^{-2} \, d\mu \int_{\phi=0}^{2\pi} d\phi \, (z\hat{a}_z)$$

$$E = \frac{\rho_s}{4\pi\varepsilon_0} \left[\frac{-1}{\mu} \Big|_z^{\infty} \right] \left[\phi z \hat{a}_z \Big|_0^{2\pi} \right]$$

$$E = \frac{\rho_s}{{}_2\cancel{4\pi}\varepsilon_0} \left[\frac{1}{\cancel{z}} \right] \left[\cancel{2\pi} \cancel{z} \hat{a}_z \right]$$

$$\boxed{E = \frac{\rho_s}{2\varepsilon_0} \hat{a}_z}$$ in V/m

The field components are present only in the perpendicular direction to which the sheet of charge is present. The field components in the direction in which the sheet of charge is zero as they get cancelled mutually

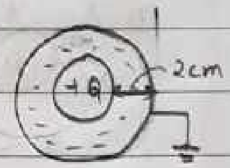
* Electric Flux Density:

Lines of force from a charge is called flux lines.
For positive charge the flux lines are radiated out and for a negative charge the flux lines converge.

Electric flux lines is denoted by ψ and is in C units.

Faraday's Law states that " A positive charge on the inner sphere induces a corresponding negative charge on the outer sphere leading to direct proportionality between the electric flux and the charge on the inner sphere".

Experiment:



Inner sphere is given a known positive charge. The hemispheres are held in such a way that they have about 2cm of dielectric medium between them. The outer sphere is discharged by connecting it to ground momentarily. The outer sphere is seperated carefully such that the induced charge on it is not disturbed. Then negative charge induced on each hemisphere was measured.

Conclusions

- The total charge on the outer sphere is equal in magnitude to the original charge placed in the inner sphere regardless of the dielectric medium.
- The displacement from the inner sphere to the outer sphere was independent of the medium known as displacement flux.

Electric flux density (D) : Flux lines per unit area.

The direction of D at a point is the direction of the flux lines at that point and the magnitude is given by the number of flux lines crossing a surface normal to the lines divided by the surface area.

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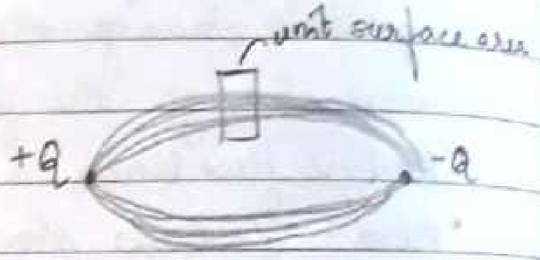
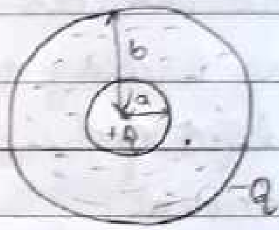
Lines of force per unit surface area is known as ~~unit~~ electric flux density. Lines are normal to the surface.



Electric flux density on the inner sphere.

$$\vec{D} = \frac{\psi}{4\pi a^2}\hat{a}_r = \frac{Q}{4\pi a^2}\hat{a}_r$$



Electric flux density on the outer sphere

$$\vec{D} = \frac{\psi}{4\pi b^2}\hat{a}_r = \frac{Q}{4\pi b^2}\hat{a}_r$$

$$a < r < b$$

$$\vec{D} = \frac{Q}{4\pi r^2}\hat{a}_r \quad \text{——— ①}$$

wkt $\vec{E} = \dfrac{Q}{4\pi \varepsilon_0 R^2}\hat{a}_r$ ——— ②

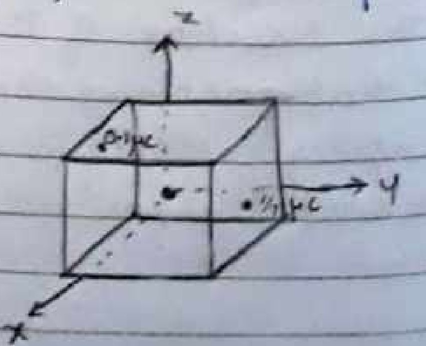
Equating eq ① and eq ②

$$\frac{Q}{4\pi r^2}\hat{a}_r = \frac{Q}{4\pi\varepsilon_0 R^2}\hat{a}_r$$

$$\boxed{\vec{D} = \varepsilon_0 \vec{E}}$$

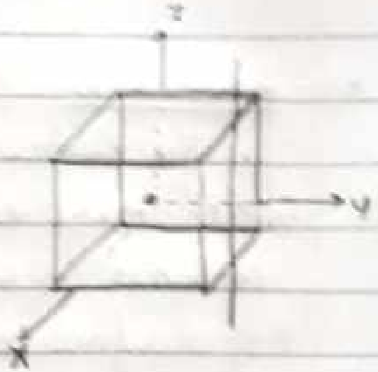
Q: Calculate the total electric flux leaving the cubical surface formed by the six planes $x, y, z = \pm 5$. If the charge distribution is: a. two point charges $0.1\,\mu C$ at $(1, -2, 3)$ and $1/7\,\mu C$ at $(-1, 3, -1)$
b. uniform line charge of $\pi\,\mu C/m$ at $x = 2$, $y = 3$.

Sol: a. Total electric flux leaving the cubical surface is equal to the total charge enclosed by it



$$\psi = 0.1\,\mu C + \frac{1}{7}\,\mu C$$

$$\psi = Q = \underline{0.243\,\mu C}$$



b. The length of the line charge enclosed by the surface is whose charge is responsible for the flux leaving the cubical surface. Length enclosed by the cubical surface is 10 m (side of the cube) of the line charge.

Charge of the 10m line charge is given by

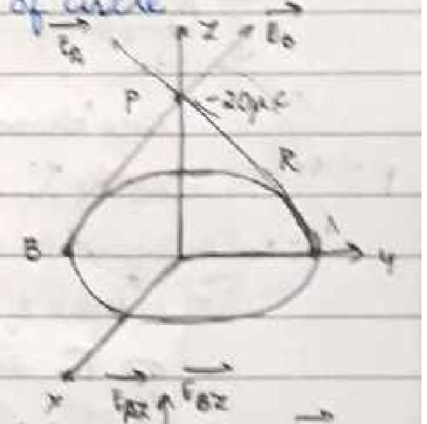
$$Q = \int \rho_L \, dL$$

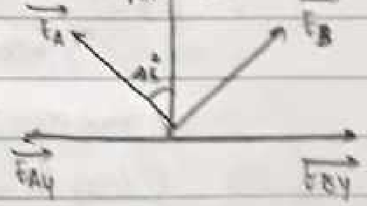
$$Q = \rho_L \int dL = \rho_L L$$

$$Q = (\pi\mu)(10)$$

$$\underline{Q = 31.4\,\mu C}$$

Q: Ten identical charges of 500μc each are separated equally around a circle of radius 2m. Find the force on the charge of −20μc located on the axis 2m from the plane of circle.





sol:

$$\cos 45^\circ = \frac{E_{Ax}}{E_A}$$

$$E_{Ax} = E_A \cos 45^\circ \, \hat{a}_x$$

$$E_t = 10\, E_A \cos 45^\circ \, \hat{a}_x \quad \text{—— (1)}$$

$$E_A = \frac{Q}{4\pi\varepsilon_0 R^2}\, \hat{a}_R$$

$$E_A = \frac{500\times10^{-6}}{4\pi(8.854\times10^{-12})} \frac{1}{(\sqrt{8})^2}$$

$$E_A = 561.7\,kV/m$$

~~$\cos 45^\circ = \frac{E_{Ax}}{E_A}$~~

~~$E_{Ax} = E_A \cos 45^\circ \hat{a}_x$~~

~~$E_t = 10 E_A \cos 45^\circ \hat{a}_x$~~ —— (1)

From eq (1)

$$E_t = 10(561.7\times10^3)(1/\sqrt{2})\,\hat{a}_x$$

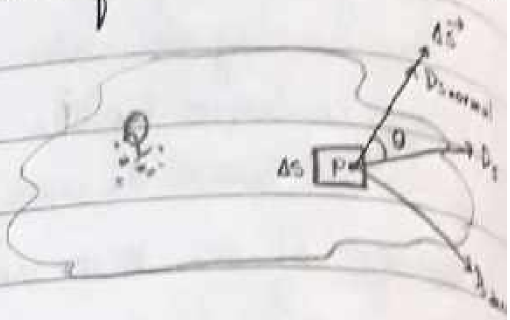
$$\underline{E_t = 3.972\times10^6\ V/m}$$

Force

$$F = E_t Q = 3.972\times10^6(-20\times10^{-6}) = \underline{-79.44\ N}$$

* **Gauss law:** (applicable only for symmetrical distribution of charge)

"The electric flux passing through any surface is equal to the charge enclosed by that surface".



consider a small area at P of surface area ΔS.
No flux passes through the surface in tangential direction, passes on in the normal direction.

Electric flux in ΔS

$$\Delta \psi = \overrightarrow{D}_{s\,normal} \times \Delta S \qquad \cos\theta = \frac{\overrightarrow{D}_{s\,normal}}{D_s}$$

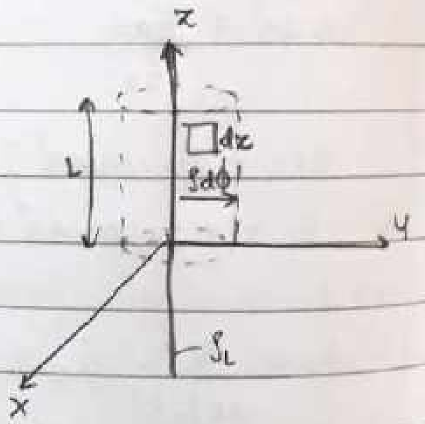
$$\Rightarrow \overrightarrow{D}_{s\,normal} = D_s \cos\theta$$

$$\Delta\psi = D_s \cos\theta \, \Delta S$$

wkt, for dot product of $\overrightarrow{D_s} \cdot \overrightarrow{\Delta S} = D_s \, \Delta S \cos\theta$

$$\therefore \Delta\psi = \overrightarrow{D_s} \cdot \overrightarrow{\Delta S}$$

— **Applying Gauss law to the line charge of uniform charge distribution**



Line charge having charge density ρ_L. consider a Gaussian surface in the form of a cylinder of height L.

$$\oint \overrightarrow{D} \cdot \overrightarrow{ds} = Q$$

$$\int_{top} \overrightarrow{D}\,\overrightarrow{ds}^{\,\nearrow 0} + \int_{bottom} \overrightarrow{D}\,\overrightarrow{ds}^{\,\nearrow 0} + \int_{side} \overrightarrow{D}\,\overrightarrow{ds} = Q$$

($\because$ they are tangential to the line of charge)

$$\int_{side} (D_\rho \hat{a}_\rho + D_\phi \hat{a}_\phi + D_z \hat{a}_z) \cdot (\rho \, d\phi \, dz \, \hat{a}_\rho)$$

$$= \int_{\phi=0}^{2\pi} \int_{z=0}^{L} D_\rho \, \rho \, d\phi \, dz = \int_L \rho_L \, dL$$

$$= D_\rho \, \rho \, 2\pi \, L = \rho_L \int_L dL = \rho_L L$$

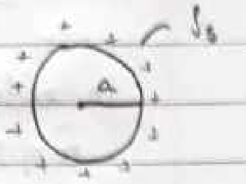
$$\Rightarrow D_\rho \, \rho \, 2\pi = \rho_L$$

$$D_\rho = \frac{\rho_L}{2\pi\rho} \Rightarrow \vec{D_\rho} = \frac{\rho_L}{2\pi\rho}\hat{a}_\rho$$

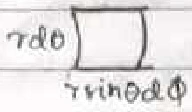
wkt $D = \varepsilon_0 E \Rightarrow \varepsilon_0 E = \frac{\rho_L}{2\pi\rho}\hat{a}_\rho$

$$\therefore \boxed{E = \frac{\rho_L}{2\pi\rho}\hat{a}_\rho}$$

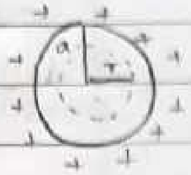
- Applying Gauss law to a spherical shell considering charge present on the surface of a spherical shell.



$$\oiint \vec{D}\cdot\vec{ds} = Q$$



CASE 1 : $r < a$

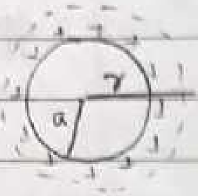


$$\iint (D_r\hat{a}_r + D_\theta\hat{a}_\theta + D_\phi\hat{a}_\phi)\cdot(r^2 d\theta\,\hat{a}_r \sin\theta d\phi) = Q$$

$$D_r(4\pi r^2) = 0$$

$$\therefore D_r = 0 \Rightarrow E = 0$$

CASE 2 : $r > a$



$$\iint (D_r\hat{a}_r + D_\theta\hat{a}_\theta + D_\phi\hat{a}_\phi)\cdot(r^2 d\theta \sin\theta\, d\phi\,\hat{a}_r) = Q$$

$$\iint D_r r^2 \sin\theta\, d\theta\, d\phi = Q$$

$$D_r r^2 \int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} \sin\theta\, d\theta\, d\phi = \int_s \rho_s\, ds$$

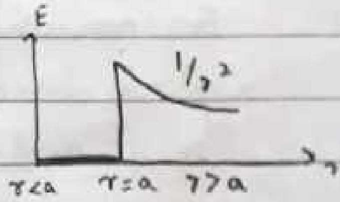
$$\Rightarrow D_r r^2 [2][2\pi] = \rho_s \int_s ds$$

$$D_r r^2 [\cancel{2}][2\pi] = \rho_s\, 4\pi a^2$$

$$D_r = \frac{\rho_s a^2}{r^2}\hat{a}_r \Rightarrow \boxed{E = \frac{\rho_s a^2}{\varepsilon_0 r^2}\hat{a}_r}$$

CASE 3: $r = a$

$$D_r = \frac{\rho_s \cancel{a^2}}{\cancel{r^2}}\hat{a}_r = \rho_s\hat{a}_r \quad \therefore \boxed{E = \frac{\rho_s\hat{a}_r}{\varepsilon_0}}$$



As we move a unit positive charge from infinity towards the shell it will gain some potential energy. This potential energy is maximum at the surface of the shell. Inside the shell the energy is zero, so we can move from one point to another inside the shell without spending energy.

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Application of Gauss law: Differential volume element:

In spherical coordinate system

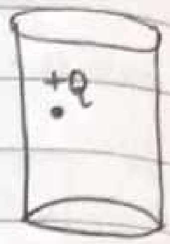
Point charge

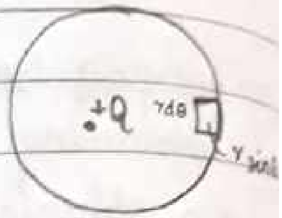
$\vec{D} = D_r \hat{a}_r + D_\theta \hat{a}_\theta + D_\phi \hat{a}_\phi$

$\vec{ds} = r^2 \sin\theta \, d\theta \, d\phi \, \hat{a}_r$

$Q = \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} D_r \, r^2 \sin d\theta \, d\phi \quad \left(\because \oiint \vec{D} \cdot \vec{ds} = Q \right)$

$\therefore \underline{Q = Q}$





In cylindrical coordinate system

$\vec{D} = D_\rho \hat{a}_\rho + D_\phi \hat{a}_\phi + D_z \hat{a}_z$

$\vec{ds} = \rho \, d\phi \, dz \, \hat{a}_\rho$

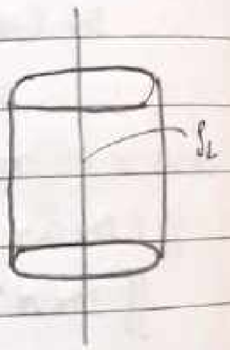
$Q = \oiint D_\rho \, \rho \, d\phi \, dz$

Line charge

Line charge of charge density ρ_L

$\oiint \vec{D} \cdot \vec{ds} = Q$

$\oiint D_\rho \, \rho \, d\phi \, dz = \int_L \rho_L \, dL = Q$



$Q = \int_L \rho_L \, dl$ line charge.

$Q = \int_S \rho_s \, dS$ surface charge

$Q = \int_V \rho_v \, dv$ volume charge

For nonuniform charge distribution gauss law cannot be applied directly. Hence we consider a very small differential element and then we calulate for the entire volume.

Electric flux passing through a Gaussian surface enclosing no charge is zero.

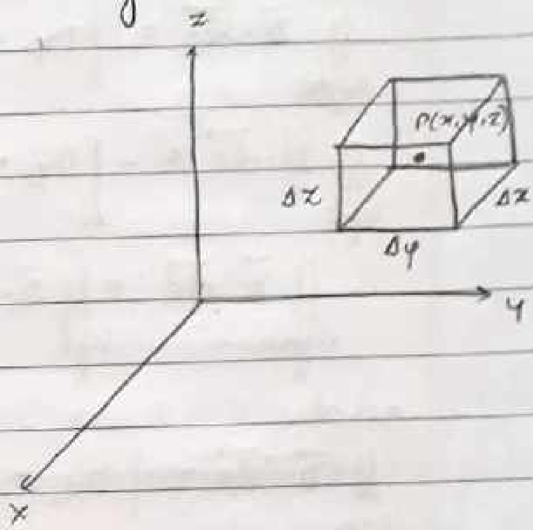
considering a cubical surface enclosing charge Q

$\Delta v = \Delta x \, \Delta y \, \Delta z$

electric flux flowing ~~through~~ from the cube

$\vec{D} = D_x \hat{a}_x + D_y \hat{a}_y + D_z \hat{a}_z$

$\oint\!\!\oint \vec{D} \cdot \vec{ds} = Q_{enclosed}$



$$\iint_{front} \vec{D} \cdot \vec{ds} + \iint_{back} \vec{D} \cdot \vec{ds} + \iint_{right} \vec{D} \cdot \vec{ds}$$

$$+ \iint_{left} \vec{D} \cdot \vec{ds} + \iint_{top} \vec{D} \cdot \vec{ds} + \iint_{bottom} \vec{D} \cdot \vec{ds} = Q$$

— $\iint_{front} \vec{D} \cdot \vec{ds} = D_x \, \Delta y \, \Delta z$

$\vec{D_x} = D_{x_0} + \frac{\partial D_x}{\partial x} \frac{\Delta x}{2}$

$\therefore \iint_{front} \vec{D} \cdot \vec{ds} = \left[D_{x_0} + \frac{\partial D_x}{\partial x} \frac{\Delta x}{2} \right] \Delta y \, \Delta z$

— $\iint_{back} \vec{D} \cdot \vec{ds} = D_x \left(-\Delta y \, \Delta z \right)$

$\vec{D}_x = D_{x_0} - \frac{\partial D_x}{\partial x} \frac{\Delta x}{2}$

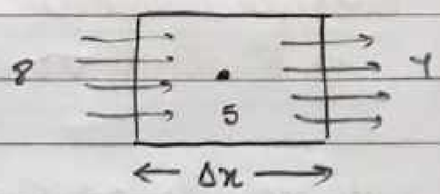
$\therefore \iint_{back} \vec{D} \cdot \vec{ds} = -\left[D_{x_0} - \frac{\partial D_x}{\partial x} \frac{\Delta x}{2} \right] \Delta y \, \Delta z$

At the centroid

$\vec{D_0} = D_{x_0} \hat{a}_x + D_{y_0} \hat{a}_y + D_{z_0} \hat{a}_z$

Ex:

8 lines entering, 5 lines at centroid and 7 lines leaving. i.e, 8 lines enter and 3 lines are cancelled by other components then 2 more lines are added with the entered lines are emerged out.



$\Rightarrow \iint_{front} \vec{D} \cdot \vec{ds} + \iint_{back} \vec{D} \cdot \vec{ds}$

$$= \left[\cancel{D_{x_0}} + \frac{\partial D_x}{\partial x} \frac{\Delta x}{2} \right] \Delta y \, \Delta z - \left[\cancel{D_{x_0}} - \frac{\partial D_x}{\partial x} \frac{\Delta x}{2} \right] \Delta y \, \Delta z$$

$$\therefore \iint_{front} \vec{D} \cdot \vec{ds} + \iint_{back} \vec{D} \cdot \vec{ds} = \frac{\partial D_x}{\partial x} \Delta x \, \Delta y \, \Delta z = \frac{\partial D_x}{\partial x} \Delta v$$

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Similarly

$$\iint_{right} \vec{D}\cdot\vec{ds} = \left[D_{y_0} + \frac{\partial D_y}{\partial y}\frac{\Delta y}{2}\right]\Delta x\,\Delta z$$

$$\iint_{left} \vec{D}\cdot\vec{ds} = -\left[D_{y_0} - \frac{\partial D_y}{\partial y}\frac{\Delta y}{2}\right]\Delta x\,\Delta z$$

$$\therefore \iint_{right} \vec{D}\cdot\vec{ds} + \iint_{left} \vec{D}\cdot\vec{ds} = \frac{\partial D_y}{\partial y}\Delta y\,\Delta x\,\Delta z = \frac{\partial D_y}{\partial y}\Delta v$$

and

$$\iint_{top} \vec{D}\cdot\vec{ds} = \left[D_{z_0} + \frac{\partial D_z}{\partial z}\frac{\Delta z}{2}\right]\Delta x\,\Delta y$$

$$\iint_{bottom} \vec{D}\cdot\vec{ds} = -\left[D_{z_0} - \frac{\partial D_z}{\partial z}\frac{\Delta z}{2}\right]\Delta x\,\Delta y$$

$$\therefore \iint_{top} \vec{D}\cdot\vec{ds} + \iint_{bottom} \vec{D}\cdot\vec{ds} = \frac{\partial D_z}{\partial z}\Delta z\,\Delta x\,\Delta y = \frac{\partial D_z}{\partial z}\Delta v$$

$$\boxed{\therefore \oiint \vec{D}\cdot\vec{ds} = \left(\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}\right)\Delta v = Q_{en}}$$

div $\vec{D}$ $$\lim_{\Delta v\to 0} \frac{\oiint \vec{D}\cdot\vec{ds}}{\Delta v} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} = \lim_{\Delta v\to 0}\frac{Q}{\Delta v}$$

Maxwell's first equation

$$\boxed{\nabla\cdot D = \rho_v}$$

Divergence theorem

$$\boxed{\oiint \vec{D}\cdot\vec{ds} = \iiint_v \nabla D\, dv}$$

If no charge is enclosed by a volume

$\rho_v = 0$ but $\rho_v = \nabla\cdot D \Rightarrow \vec{D} = 0$

If +ve charge is enclosed by a volume

$\rho_v > 0$ then $\vec{D}$ is positive (diverging lines)

If -ve charge is enclosed by a volume

$\rho_v < 0$ then $\vec{D}$ is negative (converging lines)

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$$\nabla = \frac{\partial}{\partial x}\hat{a}_x + \frac{\partial}{\partial y}\hat{a}_y + \frac{\partial}{\partial z}\hat{a}_z$$

$$\vec{D} = D_x\hat{a}_x + D_y\hat{a}_y + D_z\hat{a}_z$$

$$\boxed{\nabla\cdot\vec{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_x}{\partial z}}$$

Q1: In free space let $D = 8xyz^4\hat{a}_x + 4x^2z^4\hat{a}_y + 16x^2yz^3\hat{a}_z$ pC/m^2

a. find the total electric flux passing through the rectangular surface $z=2$ and x varies from 0 to 2 and y varies from 1 to 3 in the $\hat{a}_z$ direction.

b. find E at P(2,-1,3)

c. find an approximate value for the total charge contained in an incremental sphere located at P(2,-1,3) and having a volume of 10^{-12} m^3.

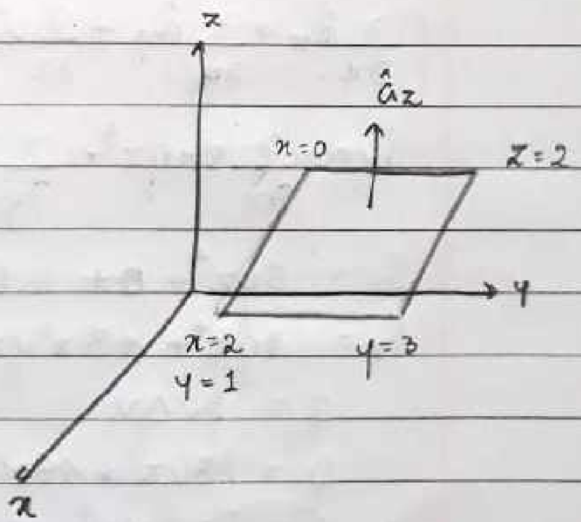
Sol: Given: $D = \underbrace{8xyz^4}_{D_x}\hat{a}_x + \underbrace{4x^2z^4}_{D_y}\hat{a}_y + \underbrace{16x^2yz^3}_{D_z}\hat{a}_z$ pC/m^2

a. Electric flux

$\psi = Q$

$\oint_S \vec{D}\cdot\vec{ds} = Q_{en}$

$\vec{ds} = dx\,dy\,\hat{a}_z$



$$\int_{x=0}^{2}\int_{y=1}^{3}\vec{D}(dx\,dy)\hat{a}_z$$

$$= \int_{x=0}^{2}\int_{y=1}^{3} 16x^2yz^3\,dx\,dy$$

$$= \int_{x=0}^{2} 8x^2y^2z^3\,dx\Big|_1^3$$

$$= \int_{x=0}^{2} 8x^2z^3\,dx\,[3^2-1^2]$$

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$$= \int_{x=0}^{2} 64 x^2 z^3 dx$$

$$= 64 \frac{x^3}{3} z^3 \Big|_0^2$$

$$= 21.33 z^3 (2^3 - 0)$$

$$Q_{en} = 170.67 z^3 = 170.67 (2)^3$$

$$\underline{Q_{en} = 1.365 nC}$$

b. E at P(2, -1, 3)

$$E = \frac{D}{\varepsilon_0} = \frac{8xyz^4 \hat{a}_x + 4x^2z^4 \hat{a}_y + 16x^2yz^3 \hat{a}_z}{\varepsilon_0}$$

$$E = \frac{8(2)(-1)(3)^4}{8.854 \times 10^{-12}} \hat{a}_x + \frac{4(2)^2(3)^4}{8.854 \times 10^{-12}} \hat{a}_y + \frac{16(2)^2(-1)(3)^3}{8.854 \times 10^{-12}} \hat{a}_z$$

$$E = -146.37 \times 10^{12} \hat{a}_x + 146.37 \times 10^{12} \hat{a}_y - 195.16 \times 10^{12} \hat{a}_z$$

c. $\nabla \cdot D = \rho_v$

$$\left(\frac{\partial}{\partial x} \hat{a}_x + \frac{\partial}{\partial y} \hat{a}_y + \frac{\partial}{\partial z} \hat{a}_z\right) \cdot \left(8xyz^4 \hat{a}_x + 4x^2z^4 \hat{a}_y + 16x^2yz^3 \hat{a}_z\right) = \rho_v$$

$$\rho_v = \frac{\partial}{\partial x} 8xyz^4 + \frac{\partial}{\partial y} 4x^2z^4 + \frac{\partial}{\partial z} 16x^2yz^3$$

$$\rho_v = 8yz^4 + 0 + 16(3)x^2yz^2$$

$$\rho_v = 8yz^4 + 48x^2yz^2 = 8(-1)3 + 48(2)^2(-1)(3)^2$$

$$Q = \rho_v \Delta v$$

$$Q = (8yz^4 + 48x^2yz^2) 10^{-12}$$

$$Q = [8(-1)(3)^4 + 48(2)^2(-1)(3)^2] 10^{-12} \quad \text{at } P(2,-1,3)$$

$$Q = -2376 \times 10^{-12}$$

$$\underline{Q = -2.376 nC}$$

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Q2: Given a 60 μC point charge located at the origin. Find the total electric flux passing through

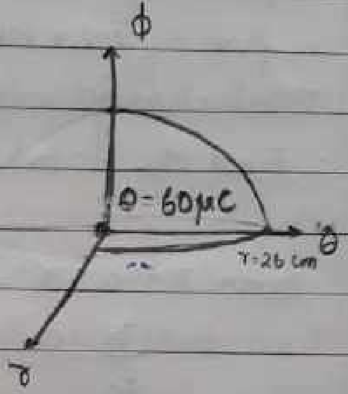
a. that portion of the sphere $r = 26\,cm$ bounded by θ varies from 0 to $\pi/2$, ϕ varies from 0 to $\pi/2$.

b. the closed surface defined by $\rho = 26\,cm$ and $z = \pm 26\,cm$

c. the plane $z = 26\,cm$.

sol: a. $\theta = 0$ to $\pi/2$ and $\phi = 0$ to $\pi/2$

So 1/8 th of the sphere



Flux

$$\psi = \frac{Q}{\frac{4\pi r^2}{A}}$$

$$A = \int_{\theta=0}^{\pi/2} \int_{\phi=0}^{\pi/2} r^2 \sin\theta \, d\theta \, d\phi$$

$$A = \int_{\theta=0}^{\pi/2} r^2 \sin\theta \, d\theta \, [\pi/2 - 0]$$

$$A = r^2 (-\cos\theta |_0^{\pi/2} (\pi/2)$$

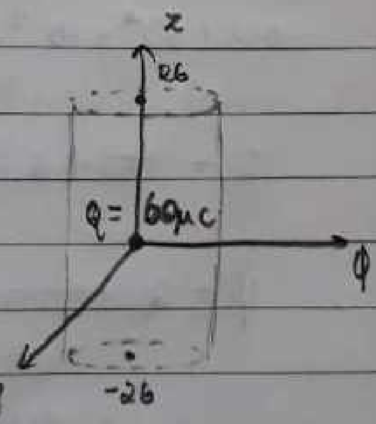
$$A = r^2 [-0+1] (\pi/2] = \frac{\pi r^2}{2}$$

$$\therefore \psi = \frac{Q}{\frac{4\pi r^2}{\frac{\pi r^2}{2}}} = \frac{Q}{8} = \frac{60\mu C}{8} = \underline{\underline{\cancel{7}\cdot 5\mu C}}$$

b. $\rho = 26\,cm$ $\quad z = \pm 26\,cm$

Total flux is equal to total charge enclosed by the surface

$$\underline{\underline{\psi = Q}} = 60\mu C$$



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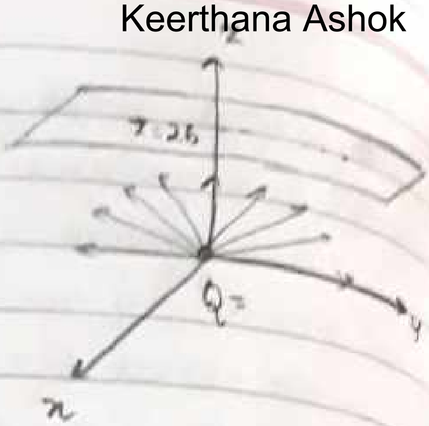
c. Due to infinite length of the sheet hence $\psi = \frac{Q}{2}$.



$$\psi = \frac{60\mu C}{2} = \underline{30\mu C}$$

Q3: Given the electric flux density $D = 0.3r^2 \hat{a}_r \, nC/m^2$ in free space. a. Find E at point $P(r=2, \theta=25°, \phi=90°)$.
b. Total charge within the sphere $r=3$
c. Find the total electric flux leaving the sphere $r=4$

sol: Given $D = 0.3r^2 \hat{a}_r \, nC/m^2$

a. $E = \frac{D}{\varepsilon_0} = \frac{0.3r^2 \hat{a}_r \times 10^{-9}}{8.854 \times 10^{-12}} = \frac{0.3(2)^2 \times 10^{-9}}{8.854 \times 10^{-12}} \hat{a}_r$

$$E = \underline{135.5 \, \hat{a}_r \, V/m}$$

b. $\oiint \vec{D} \cdot \vec{ds} = Q$

$\vec{ds} = r^2 \sin\theta \, d\theta \, d\phi$

$\therefore \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} 0.3r^2 \hat{a}_r \times 10^{-9} . \, r^2 \sin\theta \, d\theta \, d\phi \, \hat{a}_r = Q$

$\therefore \int_{\theta=0}^{\pi} 0.3r^4 \sin\theta \, d\theta \times 10^{-9} \cancel{\hat{a}_r} [2\pi - 0] = Q$

$0.3(3)^4 \times 10^{-9} \, 2\pi \, [-\cos\theta|_0^{\pi} \, \cancel{\hat{a}_r} = Q$

$Q = 152.68 [1+1] \cancel{10^{-9}} = \underline{305.36 \, nC}$

c. $\psi = Q = 0.3r^4 \, 2\pi \int_{\theta=0}^{\pi} \sin\theta \, d\theta \times 10^{-9}$

$= 0.3(4)^4 \times 10^{-9} \times 2\pi \, [-\cos\theta|_0^{\pi}$

$\psi = 965.097 \, nC$

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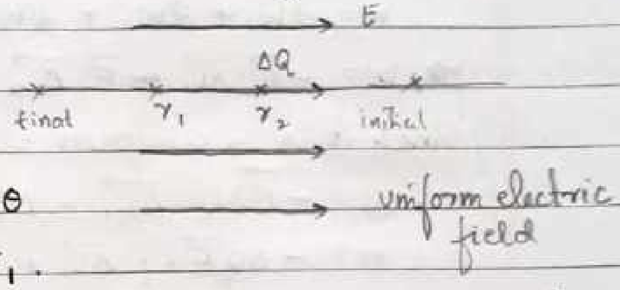
Unit - 2

ENERGY AND POTENTIAL

* **Workdone:**

An charge in an uniform electric field experiences certain force. Hence the charge is displaced due this force in the same direction.



$$E = F/Q$$

$$\therefore F = \Delta Q\, E$$

An external force is applied to move the charge from r_2 to r_1.

Therefore work done is given by

Work done = − Force × distance

$$= -E\,\Delta Q\,(r_2 - r_1)$$

$$\therefore \boxed{dW = -E\,\Delta Q\,dr}$$

(negative because the force is applied in the opposite direction of the electricfield).

Work done = − Force × displacement

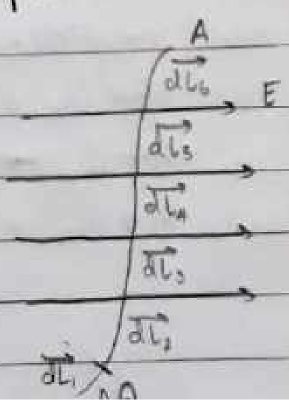
$$\boxed{\vec{dW} = -\vec{F}\cdot\vec{dL}}$$

$\therefore$ Total work done

$$W = \int_{\text{initial}}^{\text{final}} dW = \int_{\text{initial}}^{\text{final}} \vec{F}\cdot\vec{dL}$$

The work done does not depend on the path taken. It depends only on the initial and final point.

Let us consider a test charge ΔQ. to move the test charge from B to A across a uniform electric field E.



wkt $dW = -\vec{F}\cdot\vec{dL}$

$\therefore\ dW_1 = -\vec{F}\cdot\overrightarrow{\Delta L_1}$

$dW_2 = -\vec{F}\cdot\overrightarrow{\Delta L_2}$

$dW_3 = -\vec{F}\cdot\overrightarrow{\Delta L_3}$

$dW_4 = -\vec{F}\cdot\overrightarrow{\Delta L_4}$

$dW_5 = -\vec{F}\cdot\overrightarrow{\Delta L_5}$

$dW_6 = -\vec{F}\cdot\overrightarrow{\Delta L_6}$

Hence the total work done to move ΔQ from B to A.

$\therefore\ W = dW_1 + dW_2 + dW_3 + \cdots + dW_6$

$\Rightarrow W = -\vec{F}\cdot\overrightarrow{\Delta L_1} - \vec{F}\cdot\overrightarrow{\Delta L_2} - \vec{F}\cdot\overrightarrow{\Delta L_3} - \cdots - \vec{F}\cdot\overrightarrow{\Delta L_6}$

Since it is a uniform electric field

$W = -\vec{E}\Delta Q\cdot\overrightarrow{\Delta L_1} - \Delta Q\vec{E}\cdot\overrightarrow{\Delta L_2} - \cdots - \Delta Q\vec{E}\cdot\overrightarrow{\Delta L_6}$

$W = -\Delta Q\vec{E}\cdot\left[\overrightarrow{\Delta L_1} + \overrightarrow{\Delta L_2} + \cdots + \overrightarrow{\Delta L_6}\right]$

$W = -\Delta Q\vec{E}\cdot\overrightarrow{L_{BA}}$

similarly wkt

$W = -\int_{initial}^{final}\vec{F}\,\overrightarrow{dL}$

$W = -\int_B^A \Delta Q\vec{E}\cdot\overrightarrow{dL}$

$W = -\Delta Q\vec{E}\int_B^A\overrightarrow{dL}$

$W = -\Delta Q\vec{E}\cdot\overrightarrow{L_{BA}}$

Hence the workdone depends only on the initial and final points.

Q1: Given the electric field $\vec{E} = \frac{1}{z^2}\left[8xyz\,\hat{a}_x + 4x^2z\,\hat{a}_y - 4x^2y\,\hat{a}_z\right]$ V/m. Find the differential amount of work done in moving a 6nC charge by a distance of $2\mu m$ starting at $P(2,-2,3)$ and proceeding in the direction $\hat{a}_L$ equal to

a. $6/7\,\hat{a}_x + 3/7\,\hat{a}_y + 2/7\,\hat{a}_z$

b. $6/7\,\hat{a}_x - 3/7\,\hat{a}_y - 2/7\,\hat{a}_z$

c. $3/7\,\hat{a}_x + 6/7\,\hat{a}_y$

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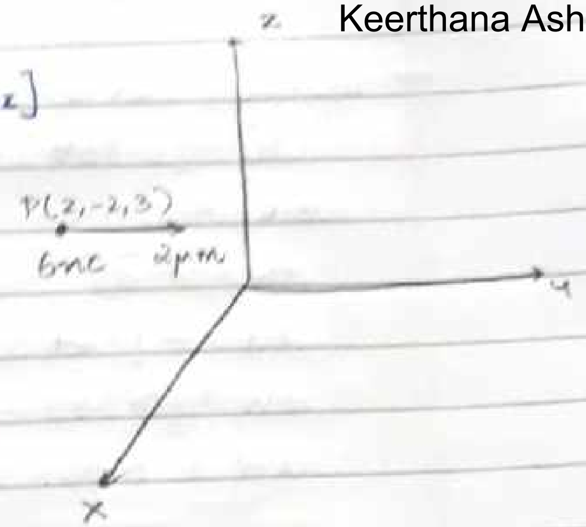


sol: given: $\vec{E} = \frac{1}{z^2}[8xyz\,\hat{a}_x + 4x^2z\,\hat{a}_y - 4x^2y\,\hat{a}_z]$

$Q = 2nC$ at $P(2,-2,3)$

Electric field at $P(2,-2,3)$

$$\vec{E} = \frac{1}{3^2}\left[8(2)(-2)(3)\hat{a}_x + 4(2)^2(3)\hat{a}_y - 4(2)^2(-2)\hat{a}_z\right]$$

$$\vec{E} = -10.67\hat{a}_x + 5.33\hat{a}_y + 3.56\hat{a}_z$$

$$\vec{F} = Q\vec{E}$$

$$\vec{F} = 6\times10^{-9}[-10.67\hat{a}_x + 5.33\hat{a}_y + 3.56\hat{a}_z]$$

$$\vec{F} = [-64.02\hat{a}_x + 31.98\hat{a}_y + 21.36\hat{a}_z]\,nN$$

$$\vec{dL} = dL\,\hat{a}_L$$

i. $\vec{dL} = (2\times10^{-6})(6/7\hat{a}_x + 3/7\hat{a}_y + 2/7\hat{a}_z)$

$\vec{dL} = [1.71\hat{a}_x + 0.86\hat{a}_y + 0.57\hat{a}_z]10^{-6}$

work done

$dW = -\vec{F}.\vec{dL}$

$dW = -[-64.02\hat{a}_x + 31.98\hat{a}_y + 21.36\hat{a}_z]10^{-9}[1.71\hat{a}_x + 0.86\hat{a}_y + 0.57\hat{a}_z]$

$dW = [0.109 - 0.027 - 0.012]\times10^{-12}\,Nm$

$dW = 0.07\times10^{-12}\,Nm$

ii. $\vec{dL} = (2\times10^{-6})(6/7\hat{a}_x - 3/7\hat{a}_y - 2/7\hat{a}_z)$

$\vec{dL} = [1.71\hat{a}_x - 0.86\hat{a}_y - 0.57\hat{a}_z]10^{-6}$

work done

$dW = -\vec{F}.\vec{dL}$

$dW = -[-64.02\hat{a}_x + 31.98\hat{a}_y + 21.36\hat{a}_z]10^{-9}[1.71\hat{a}_x - 0.86\hat{a}_y - 0.57\hat{a}_z]$

$dW = [0.109 + 0.027 + 0.012]\times10^{-12}$

$dW = 0.148\times10^{-12}\,Nm$

iii. $\vec{dL} = (2\times10^{-6})[\,3/7\hat{a}_x + 6/7\hat{a}_y]$

$\vec{dL} = [0.86\,\hat{a}_x + 1.714\,\hat{a}_y]\,10^{-6}$

Work done

$dW = -\vec{F}.\vec{dL}$

$dW = -[-64.02\hat{a}_x + 31.98\hat{a}_y + 21.36\hat{a}_z]\,10^{-9}\,[0.86\,\hat{a}_x + 1.71\hat{a}_y]$

$dW = [0.055 - 0.054]\,10^{-12}$

$dW = 0.001\times10^{-12}\ J$

* **Potential:**

Potential is the workdone to move unit charge ~~to move~~ from one point to another.

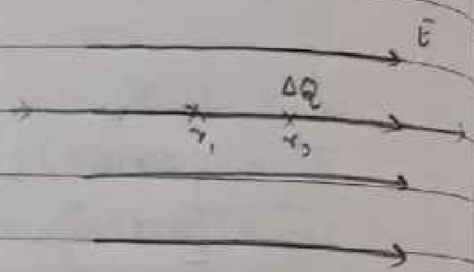
Workdone = $-$ Force $\times$ distance

$dW = -F\,dr$

$dW = -\vec{F}.\vec{dL}$

$dW = -\Delta Q\vec{E}.\vec{dL}$

$\frac{dW}{\Delta Q} = -\vec{E}\cdot\vec{dL}$



$\frac{\text{workdone}}{\text{charge}}$ = Potential = $-\vec{E}.\vec{dL}$

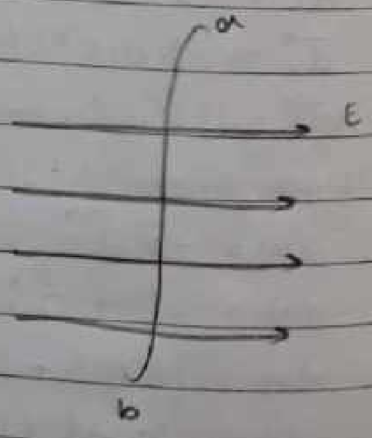
For differential potential

$\Delta V = V(r_1) - V(r_2)$

$\Delta V = -\int_{r_2}^{r_1}\vec{E}.\vec{dL}$

$\Delta V = V_{ab} = V(a) - V(b)$

$\Delta V = -\int_b^a \vec{E}.\vec{dL}$



Potential increases on moving against the electric field.

Potential is independent of the path taken it depends only on the final and initial points

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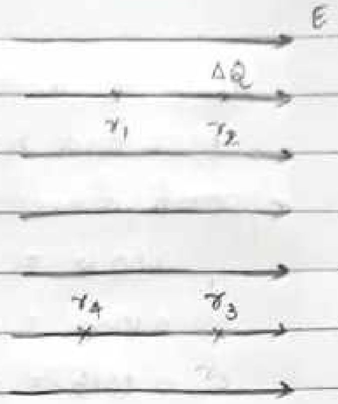
Let us consider a uniform electric field E and unit test positive charge ΔQ.

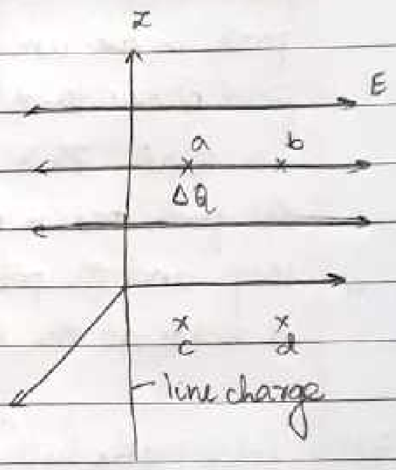


Movement of a test charge in a closed path, then there is no workdone hence workdone is zero.

Moving charge from r_2 to r_1 and moving charge from r_3 to r_4 same amount of work is done.

On moving charge from r_2 to r_3 or r_1 to r_4 no work is done as the points are at same potential.

Work done in moving a unit test positive charge from a to b in a uniform electric field E.



$$W = -\int_{initial}^{final} \vec{F}\, \vec{dS}$$

$$\vec{F} = \vec{E}\Delta Q$$

For a line charge

$$\vec{E} = \frac{S_L}{2\pi\varepsilon_0 S}\hat{a}_S$$

In cylindrical system

$$\therefore \vec{dL} = dS\,\hat{a}_S$$

Therefore workdone to move from a to b

$$W = -\int_a^b \frac{S_L}{2\pi\varepsilon_0 S}\Delta Q\, dS$$

$$W = -\Delta Q \frac{S_L}{2\pi\varepsilon_0}\ln\left(\frac{b}{a}\right)$$

Similarly workdone to move from b to a

$$W = -\int_b^a \frac{S_L}{2\pi\varepsilon_0 S}\Delta Q\, dS$$

$$W = -\Delta Q\frac{S_L}{2\pi\varepsilon_0}\ln\left(\frac{a}{b}\right) = \Delta Q\frac{S_L}{2\pi\varepsilon_0}\ln\left(\frac{b}{a}\right)$$

Hence the amount of work done in moving the charge from a to b and b to a is same but the workdone is in the opposite direction to each other.

a to b = c to d   b to d = c to a = 0

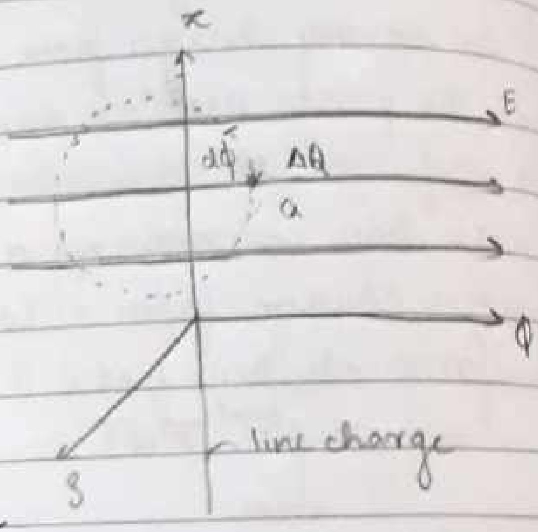
a to b = −(b to a)

a to b = −(d to c)

b to a = d to c

<u>Conservative Field:</u>

work done in moving a unit test positive charge ΔQ from a to a in a circular path in an uniform electric field E due to a line charge is zero. This is because the path taken is closed loop and the potential difference is zero at the same point.

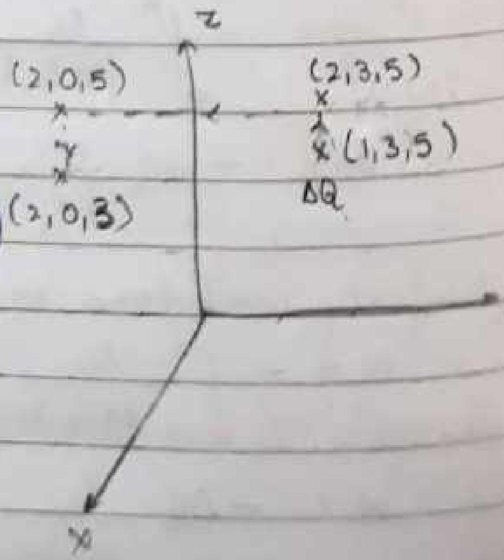


Q1: Let $E = y\hat{a}_x$ at certain instant of time and calculate the work required to move a 3C charge from (1,3,5) to (2,0,3) along the straight line segments joining
  a. (1,3,5) to (2,3,5) to (2,0,5) to (2,0,3)
  b. (1,3,5) to (1,3,3) to (1,0,3) to (2,0,3)

Sol: $W = -\int_{initial}^{final} \vec{F}\, d\vec{L}$

$$W = -\int_{initial}^{final} 3(y\hat{a}_x)(dx\hat{a}_x + dy\hat{a}_y + dz\hat{a}_z)$$

a. $W = -\int_{initial}^{final} 3y\, dx$



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From (1,3,5) to (2,3,5)

$$W_1 = -\int_{(1,3,5)}^{(2,3,5)} 3y\,dx$$

$$W_1 = -3yx \Big|_{(1,3,5)}^{(2,3,5)}$$

$$W_1 = -3(3)[(2)-(1)]$$

$$\underline{W_1 = -9\,J}$$

From (2,3,5) to (2,0,5)

$$W_2 = -\int_{(2,3,5)}^{(2,0,5)} 3y\,dx$$

$$W_2 = -3yx \Big|_{(2,3,5)}^{(2,0,5)}$$

$$W_2 = -3(0)[(2)-(2)]$$

$$\underline{W_2 = 0\,J}$$

From (2,0,5) to (2,0,3)

$$W_3 = -\int_{(2,0,3)}^{(2,0,5)} 3y\,dx$$

$$W_3 = -3yx \Big|_{(2,0,3)}^{(2,0,5)}$$

$$W_3 = -3(0)[2-2]$$

$$\underline{W_3 = 0\,J}$$

$\therefore$ Total work done is equal to

$$W = W_1 + W_2 + W_3$$

$$\underline{W = -9\,J}$$

b. $$W = -\int_{\text{initial}}^{\text{Final}} 3y\,dx$$

From (1,3,5) to (1,3,3)

$$W_1 = -\int_{(1,3,5)}^{(1,3,3)} 3y\,dx$$

$$W_1 = -3yx \Big|_{(1,3,5)}^{(1,3,3)}$$

$$W_1 = -3(3)[1-1]$$

$$\underline{W_1 = 0}$$

From (1,3,3) to (1,0,3)

$$W_2 = -\int_{(1,3,3)}^{(1,0,3)} 3y\,dx$$

$$W_2 = -3yx\Big]_{(1,3,3)}^{(1,0,3)}$$

$$W_2 = -3(3)[1-1] = \underline{0\,J}$$

From (1,0,3) to (2,0,3)

$$W_3 = -\int_{(1,0,3)}^{(2,0,3)} 3y\,dx$$

$$W_3 = -3yx \Big|_{(1,0,3)}^{(2,0,3)}$$

$$W_3 = -3(0)[2-1]$$

$$\underline{W_3 = 0}$$

$\therefore$ Total work done is equal to

$$W = W_1 + W_2 + W_3$$

$$\underline{W = 0\,J}$$

Hence $E = y\hat{a}_x$ is not a conservative ~~plane~~ field.

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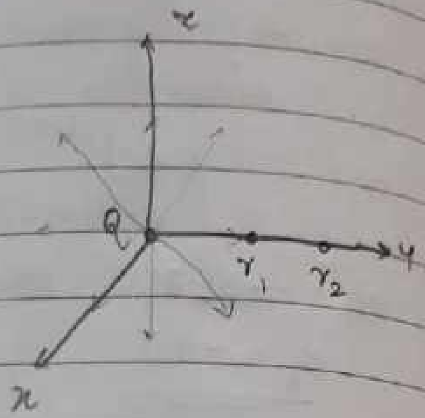
* <u>Potential difference</u>

Potential difference between the points r_1 and r_2 with an unit test charge at the origin.



$$\hat{V}_{12} = -\int_{r_2}^{r_1} \vec{E} \cdot \vec{dl}$$

Here $\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2} \hat{a}_r$

and $\vec{dl} = dr\hat{a}_r$

$$\therefore V_{12} = -\int_{r_2}^{r_1} \frac{Q}{4\pi\varepsilon_0 r^2} dr$$

$$V_{12} = \frac{-Q}{4\pi\varepsilon_0} \int_{r_2}^{r_1} \frac{1}{r^2} dr$$

$$V_{12} = \frac{-Q}{4\pi\varepsilon_0} \left. \frac{-1}{r} \right|_{r_2}^{r_1}$$

$$\boxed{V_{12} = \frac{Q}{4\pi\varepsilon_0} \left[\frac{1}{r_1} - \frac{1}{r_2} \right]}$$

i.e., $V_{12} = V_1 - V_2$

Absolute potential is the potential at any point in the space of point charge.

<u>CASE 1:</u>

When $r_2 = \infty$ i.e, a unit test positive charge is from

$$V_{12} = \frac{Q}{4\pi\varepsilon_0 r_1}$$ from infinity to point r_1.

$\therefore$ Potential at a point

$$\boxed{V_r = \frac{Q}{4\pi\varepsilon_0 r}}$$ Absolute potential.

At infinity: $V_r = \frac{Q}{4\pi\varepsilon_0(\infty)} = 0$ // ~~Absolute potential~~

If in case there is some potential at infinity (i.e., no zero reference at infinity) then it is added with the potential at that point.

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$$\boxed{V = \frac{Q}{4\pi\varepsilon_0 r} + C_1}$$ where C_1 is the potential at infinity.

For a line charge we consider some potential at infinity whereas for point charge we consider zero potential at infinity.

Q1: An electric field is expressed in rectangular coordinates by $E = 6x^2\hat{a}_x + 6y\hat{a}_y + 4\hat{a}_z$ V/m. Find:

a. V_{MN} if points M and N are specified by $M(2,6,-1)$ and $N(-3,-3,2)$.

b. V_M if $V=0$ at $Q(4,-2,-35)$

c. V_N if $V=2$ at $P(1,2,-4)$

This method is used when type of charge distribution is not given.

Sol: Given: $E = 6x^2\hat{a}_x + 6y\hat{a}_y + 4\hat{a}_z$

a. $V_{MN} = V_M - V_N$

$\vec{dl} = dx\,\hat{a}_x + dy\,\hat{a}_y + dz\,\hat{a}_z$

$V_{MN} = -\int_N^M \vec{E}\cdot\vec{dl}$

~~$\vec{dl} = (2+3)\hat{a}_x + (6+3)\hat{a}_y + (-1-2)\hat{a}_z$~~

~~$\vec{dl} = 5\hat{a}_x + 9\hat{a}_y - 3\hat{a}_z$~~

$$V_{MN} = -\left[\int_N^M 6x^2\,dx + \int_N^M 6y\,dy + \int_N^M 4\,dz\right]$$

$$= -\left[\int_{-3}^{2} 6x^2\,dx + \int_{-3}^{6} 6y\,dy + \int_{2}^{-1} 4\,dz\right]$$

$$= -\left[2x^3\Big|_{-3}^{2} + 3y^2\Big|_{-3}^{6} + 4z\Big|_{2}^{-1}\right]$$

$$= -\left[2[8+27] + 3[36-9] + 4[-1-2]\right]$$

$$= -[70 + 81 - 12]$$

$$V_{MN} = -139\ V$$

b. $V_{MQ} = V_M - V_Q$

given $V_Q = 0$

$\therefore V_{MQ} = V_M = -\int_Q^M \vec{E} \cdot \vec{dl}$

$$V_M = -\left[\int_Q^M 6x^2 dx + \int_Q^M 6y\,dy + \int_Q^M 4\,dz \right]$$

$$V_M = -\left[2x^3 \Big|_4^2 + 3y^2 \Big|_{-2}^{6} + 4z \Big|_{-35}^{-1} \right]$$

$$V_M = -\left[2(8-64) + 3(36-4) + 4(-1+35) \right]$$

$$V_M = -\left[-112 + 96 + 136 \right]$$

$$\underline{\underline{V_M = -120\,V}}$$

c. $V_{NP} = V_N - V_P$

given $V_P = 2$

$\therefore V_N = V_{NP} + V_P = -\int_P^N \vec{E} \cdot \vec{dl} + V_P$

$$V_N = -\left[\int_P^N 6x^2 dx + \int_P^N 6y\,dy + \int_P^N 4\,dz \right] + 2$$

$$V_N = -\left[2x^3 \Big|_1^{-3} + 3y^2 \Big|_2^{-3} + 4z \Big|_{-4}^{2} \right] + 2$$

$$V_N = -\left[2(-27-1) + 3(9-4) + 4(2+4) \right] + 2$$

$$V_N = -\left[-56 + 15 + 24 \right] + 2$$

$$\underline{\underline{V_N = 19\,V}}$$

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Q2: A 15nC point charge at the origin in freespace. calculate V_1 if point P_1 is located at $P_1(-2,3,-1)$ and

a. $V=0$ at $(6,5,4)$

b. $V=0$ at ∞

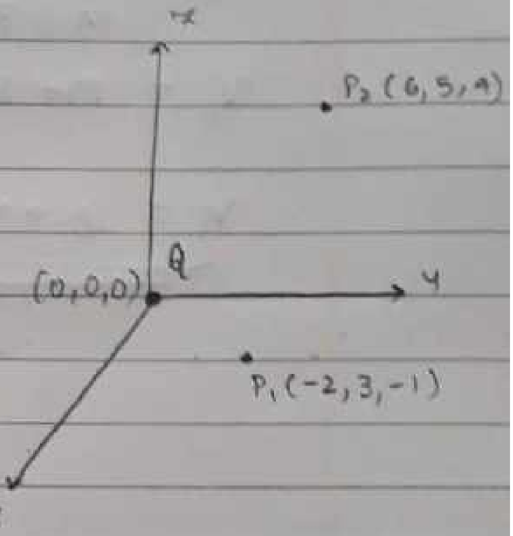
c. $V=5V$ at $(2,0,4)$

Sol: a. $P_1(-2,3,-1)$ $P_2=(6,5,4)$ and $V_2=0$

$$r_1 = \sqrt{(-2-0)^2+(3-0)^2+(-1-0)^2}$$

$$r_1 = \sqrt{4+9+1} = \sqrt{14}$$

$$\underline{r_1 = 3.742}$$



$$r_2 = \sqrt{(6-0)^2+(5-0)^2+(4-0)^2}$$

$$r_2 = \sqrt{36+25+16} = \sqrt{77}$$

$$\underline{r_2 = 8.775}$$

$$V_{12} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

$$V_{12} = \frac{15\times10^{-9}}{4\pi(8.854\times10^{-12})}\left[\frac{1}{3.742} - \frac{1}{8.775}\right]$$

$$V_{12} = 134.8\,[0.267 - 0.114]$$

$$\underline{V_{12} = 20.624\,V}$$

b. $V=0$ at ∞

$$V_1 = \frac{Q}{4\pi\varepsilon_0 r_1} = \frac{15\times10^{-9}}{4\pi(8.854\times10^{-12})3.742}$$

$$\underline{V_1 = 36.03\,V}$$

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c. $V = 5V$ at $(2, 0, 4)$

$r_2 = \sqrt{(2-0)^2 + (0-0)^2 + (4-0)^2}$

$r_2 = \sqrt{4+16} = \sqrt{20}$

$r_2 = 4.472$

$V_{12} = V_1 - V_2$

$V_{12} = V_1 - 5$

$V_1 = V_{12} + 5$

$$V_1 = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r_1} - \frac{1}{r_2}\right] + 5$$

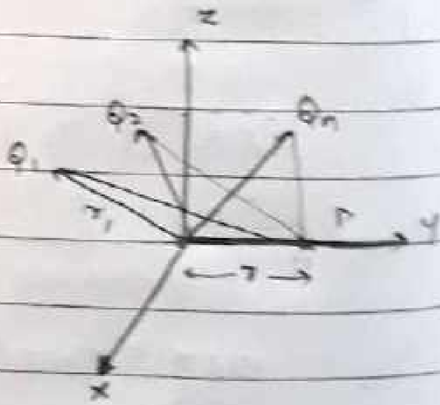
$$V_1 = \frac{15\times10^{-9}}{4\pi(8.854\times10^{-12})}\left[\frac{1}{3.742} - \frac{1}{4.472}\right] + 5$$

$$V_1 = 134.8\left[0.292 - 0.224\right] + 5$$

$$\underline{\underline{V_1 = 10.9\,V}}$$

* **Potential due to 'n' point charges:**

Potential at point P due to the point charges $Q_1, Q_2, \ldots, Q_n$.



$$V_1 = \frac{Q_1}{4\pi\varepsilon_0|r - r_1|}$$

$$V_2 = \frac{Q_2}{4\pi\varepsilon_0|r - r_2|}$$

$\vdots$

$$V_n = \frac{Q_n}{4\pi\varepsilon_0|r - r_n|}$$

Therefore the total potential at point P is

$$V = \frac{Q_1}{4\pi\varepsilon_0|r - r_1|} + \frac{Q_2}{4\pi\varepsilon_0|r - r_2|} + \cdots + \frac{Q_n}{4\pi\varepsilon_0|r - r_n|}$$

$$\boxed{V = \sum_{m=1}^{n} \frac{Q_m}{4\pi\varepsilon_0|r - r_m|}}$$

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* Potential due to line charge

$$V = \int_L \frac{\rho_L \, dL}{4\pi\varepsilon_0 |r - r_1|}$$

* Potential due to sheet of charge

$$V = \iint_S \frac{\rho_S \, dS}{4\pi\varepsilon_0 |r - r_1|}$$

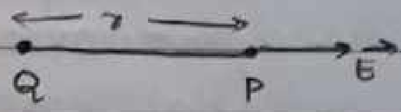
* Potential due to volume charge

$$V = \iiint_v \frac{\rho_v \, dv}{4\pi\varepsilon_0 |r - r_1|}$$

* Potential gradient:

potential at point P

$$V = \frac{Q}{4\pi\varepsilon_0 r}$$



differentiating wrt r

$$\frac{dV}{dr} = \frac{-Q}{4\pi\varepsilon_0 r^2}$$

$$\frac{dV}{dr} \hat{a}_r = \frac{-Q}{4\pi\varepsilon_0 r^2} \hat{a}_r$$

$$\therefore \boxed{\vec{E} = -\frac{dV}{dr} \hat{a}_N}$$

On moving a charge in perpendicular direction to the electric field no work is done. $\Rightarrow V = 0$. If the force acts in normal direction then there will be maximum displacement. hence maximum potential rise.

Equipotential surfaces

Point charges

- spherical coordinate system: concentric spheres with the point charge as center.

On moving in any direction if there is a change in the potential then there exists an electric field components. whereas if there is not change in the potential then the electric field components in that direction is zero.

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$$|E_x| = -\frac{\partial V}{\partial x}\bigg|_{y,z} \qquad |E_y| = \frac{-\partial V}{\partial y}\bigg|_{x,z} \qquad |E_z| = \frac{-\partial V}{\partial z}\bigg|_{y,x}$$

$$E = \frac{-\partial V}{\partial x}\hat{a}_x - \frac{\partial V}{\partial y}\hat{a}_y - \frac{\partial V}{\partial z}\hat{a}_z$$

$$E = -\left[\frac{\partial}{\partial x}\hat{a}_x + \frac{\partial}{\partial y}\hat{a}_y + \frac{\partial}{\partial z}\hat{a}_z\right] V$$

$$E = -\nabla \cdot V$$

$$\boxed{E = \text{grad } V}$$

On moving the direction of electric field potential decreases and on moving in the direction opposite to electric field potential increases.

Q1: The potential field $V = 2x^2y - 5z$ and a point $P(-4,3,6)$ find the several values of point P: the potential V, the electric field intensity E, the electric flux density D and the volume charge density δv.

Sol: a. the potential V

$$V = 2x^2y - 5z$$

$$V = 2(-4)^2 3 - 5(6)$$

$$V = 96 - 30$$

$$\underline{V = 66V}$$

b. the electric field intensity E

$$E = -\nabla V$$

$$E = -\left[\frac{\partial}{\partial x}\hat{a}_x + \frac{\partial}{\partial y}\hat{a}_y + \frac{\partial}{\partial z}\hat{a}_z\right](2x^2y - 5z)$$

$$E = -\left[4xy\,\hat{a}_x + 2x^2\,\hat{a}_y - 5\hat{a}_z\right]$$

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$$E = -\left[4(-4)3\hat{a}_x + 2(-4)^2\hat{a}_y \mp 5\hat{a}_z \right]$$

$$E = 48\hat{a}_x \mp 32\hat{a}_y + 5\hat{a}_z$$

c. the electric flux density D

$$D = E\varepsilon_0$$

$$D = 8.854\times10^{-12}\left[48\hat{a}_x - 32\hat{a}_y + 5\hat{a}_x\right]$$

$$D = \left[0.425\,\hat{a}_x - 0.283\,\hat{a}_y + 0.044\,\hat{a}_z\right]\times10^{-9}$$

d. the volume charge density

$$\rho_v = \nabla \cdot D$$

$$\rho_v = \left(\frac{\partial}{\partial x}\hat{a}_x + \frac{\partial}{\partial y}\hat{a}_y + \frac{\partial}{\partial z}\hat{a}_z\right)\left(-4xy\hat{a}_x - 2x^2\hat{a}_y + 5\hat{a}_z\right)\frac{10^{-9}\varepsilon_0}{\varepsilon_0}$$

$$\rho_v = \left[-4y - 0 + 0\right]10^{-9} \quad (8.854\times10^{-12})$$

$$\rho_v = +106\times10^{-21}\,C/m^3$$

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Unit - 03

Current and Conductors

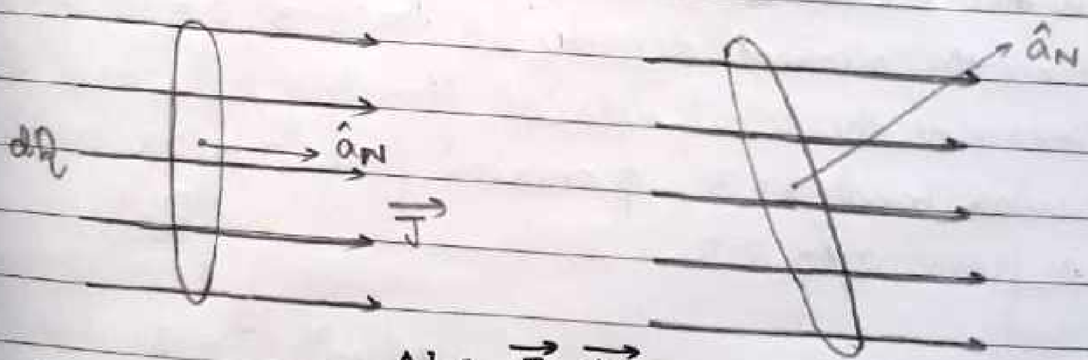
* **Current:**

current is the rate of flow of charges per unit time

$$I = \frac{dQ}{dt} \quad \text{in A}$$

* **Current Density:**

current density is defined as the current flowing per unit area. $\vec{J} = \frac{I}{\vec{ds}}$ in A/m^2



$$\Delta I = \vec{J} \cdot \vec{ds}$$

∴ Total current

$$I = \int_S \vec{J} \cdot \vec{ds}$$

* **Relation between current I and velocity $\vec{v}$.**

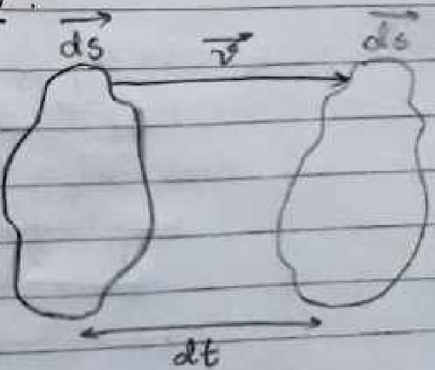
n - charges per unit volume

$\vec{v}$ - velocity

$\vec{ds}$ - differential surface area

dt - time

$q = 1.6 \times 10^{-19}$ charge of an electron



$$dQ = nq\vec{v}\,dt\,\vec{ds}$$

$$\therefore \boxed{I = \frac{dQ}{dt} = nq\vec{v}\,\vec{ds}} \Rightarrow \boxed{\vec{J} = \frac{I}{\vec{ds}} = nq\vec{v} = \rho\vec{v}}$$

because $nq = \rho_v$

* **conductors:**

- Within a conductor there is zero charge density and surface charge density resides on the exterior surface as the charges reside on the surface of the conductor.
- No charge and no electric field may exist at any point within a conducting material.
- External electric field intensity has two components. Since static conditions are assumed, the tangential electric field intensity and electric flux density are zero. Since $E=0$ inside, the flux leave surface normally $\Rightarrow D_N = \rho_s$

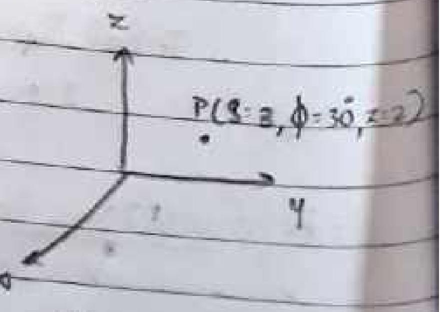
Q1: Given the vector current density $\vec{J} = 10\rho^2 z\,\hat{a}_\rho - 4\rho\cos^2\phi\,\hat{a}_\phi$ mA/m²
a. Find the current density at $P(\rho=3, \phi=30^\circ, z=2)$.
b. Determine the total current flowing outward through the circular band $\rho=3$, ϕ varies from 0 to 2π and z varies from 2 to 2.8.

Sol: $\vec{J} = 10\rho^2 z\,\hat{a}_\rho - 4\rho\cos^2\phi\,\hat{a}_\phi$ mA/m²

a. $P(\rho=3, \phi=30^\circ, z=2)$

$$\vec{J} = 10(3^2)(2)\,\hat{a}_\rho - 4(3)\cos^2(30^\circ)\,\hat{a}_\phi \text{ mA/m}^2$$

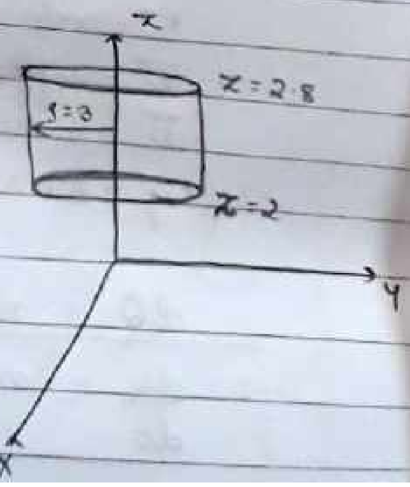
$$\vec{J} = \underline{180\,\hat{a}_\rho - 9\,\hat{a}_\phi \text{ mA/m}^2}$$



b. $\rho=3$; $\phi = 0$ to 2π; $z = 2$ to 2.8

$$I = \int_s \vec{J}\cdot\vec{ds}$$

Here $\vec{ds}_\rho = \rho\, d\phi\, dz\, \hat{a}_\rho$



$$\cancel{I = \int_{\phi=0}^{2\pi}\int_{z=2}^{2.8} (180\,\hat{a}_\rho - 9\,\hat{a}_\phi)\cdot(\rho\, d\phi\, dz\, \hat{a}_\rho)}$$

$$\cancel{I = \int_{\phi=0}^{2\pi}\int_{z=2}^{2.8} 180\rho\, d\phi\, dz}$$

$$\cancel{I = 180(3)\int_{z=2}^{2.8} \phi\Big|_0^{2\pi}\, dz}$$

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$\cancel{I = 540 [2\pi] \; z \Big|_{2}^{2.8}}$

$\cancel{I = 3.39 \times 10^3 [2.8 - 2]}$

$\cancel{I =}$

$$I = \int_s (10 \rho^2 z \,\hat{a}_\rho - 4 \rho \cos^2\phi \,\hat{a}_\phi)(\rho\, d\phi\, dz\, \hat{a}_\rho)$$

$$I = \int_{\phi=0}^{2\pi} \int_{z=2}^{2.8} 10 \rho^3 z \, d\phi \, dz$$

$$I = 10(3)^3 \int_{\phi=0}^{2\pi} \frac{z^2}{2}\Big|_{2}^{2.8} d\phi$$

$$I = 270 \left[\frac{2.8^2}{2} - \frac{2^2}{2} \right] \left[\phi \Big|_0^{2\pi} \right]$$

$$I = 270(1.92)[2\pi]$$

$$\underline{I = 3.257\,A}$$

* <u>continuity Equation</u>:

When some charge or current flows out of a surface of volume same amount of charge reduces in that volume.

When the current density lines is present then there is some flow of charges.

$$\therefore \nabla \cdot \vec{J} \neq 0 \Rightarrow \frac{-dQ}{dt}$$

$$\oint_s \vec{J} \cdot \vec{ds} = I = \frac{dQ_{out}}{dt} = -\frac{dQ_{in}}{dt}$$

$$\therefore \oint_s \vec{J} \cdot \vec{ds} = \frac{-dQ}{dt}$$

wkt $Q = \int_v \rho_v \, dv$

$$\therefore \oint_s \vec{J} \cdot \vec{ds} = -\frac{d}{dt} \int_v \rho_v \, dv$$

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$$\oiint_S \vec{J}\cdot\vec{ds} = -\int_V \frac{\partial \rho_v}{\partial t}\, dv$$

but $\oiint \vec{J}\cdot\vec{ds} = I$

$$\oiint \vec{J}\cdot\vec{ds} = \iiint_V \rho_v \, dv$$

$$\oiint \vec{J}\cdot\vec{ds} = \iiint_V \nabla\cdot\vec{J}\, dv$$

$$\int_V \nabla \vec{J}\, dv + \int_V \frac{\partial \rho_v}{\partial t}\, dv = 0$$

$$\int_V \left[\nabla \vec{J} + \frac{\partial \rho_v}{\partial t}\right] dv = 0$$

$$\therefore \boxed{\nabla\cdot\vec{J} = -\frac{\partial \rho_v}{\partial t}}$$

NOTE: 1. On moving in the radial direction current increases.

2. Spherical system $\nabla = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 J_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta J_\theta) + \frac{1}{r\sin\theta}\frac{\partial J_\phi}{\partial\phi}$

Ex: $\vec{J} = \frac{1}{r} e^{-t}\hat{a}_r$ $t = 1\text{sec}$ $r = 5m$, $r = 6m$

$$I = \oiint_S \vec{J}\cdot\vec{ds}$$

$$I = \oiint \left(\frac{1}{r} e^{-t}\hat{a}_r\right)\left(r^2 \sin\theta\, d\theta\, d\phi\, \hat{a}_r\right)$$

$$I = \frac{1}{r} e^{-t}(4\pi r^2)$$

at $r = 5$ m : $I = 23.1$ A

at $r = 6$ m : $I = 27.7$ A

$$\nabla\cdot\vec{J} = -\frac{\partial \rho_v}{\partial t} = \nabla\cdot\left(\frac{1}{r^2} e^{-t}\hat{a}_r\right) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{1}{r} e^{-t}\hat{a}_r\right)$$

Page

$$\therefore -\frac{\partial \rho_v}{\partial t} = \frac{e^{-t}}{r^2}$$

therefor the volume charge density is

$$\rho_v = -\int_v \frac{1}{r^2} e^{-t}\, dt = -\frac{(-e^{-t})}{r^2}$$

$$\rho_v = \frac{1}{r^2} e^{-t} \; C/m^3$$

wkt $J = \rho_v v$

$$\therefore \text{velocity } v_r = \frac{J_r}{\rho_v} \quad \frac{1/r\, e^{-t}}{1/r^2\, e^{-t}} = r \; m/s$$

Hence the velocity is greater at $r = 6$ than it is at $r = 5$. Therefore some force is accellrating the charge density in an outward direction.

Therefore here we can conclude that

current density : $J \propto \frac{1}{r}$

charge desity : $\rho_v \propto \frac{1}{r^2}$

velocity : $v \propto r$

total current : $I \propto r$

and all the quantities vary as e^{-t}.

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Q1: Current density is given in cylindrical coordinates as $\vec{J} = -10^6 z^{1.5}\,\hat{a}_z\ A/m^2$ in the region $0 \le \rho \le 20\mu m$; for $\rho \ge 20\mu m$, $\vec{J} = 0$.

a. Find the total current passing the surface $z = 0.1m$ in the $\hat{a}_z$ direction.

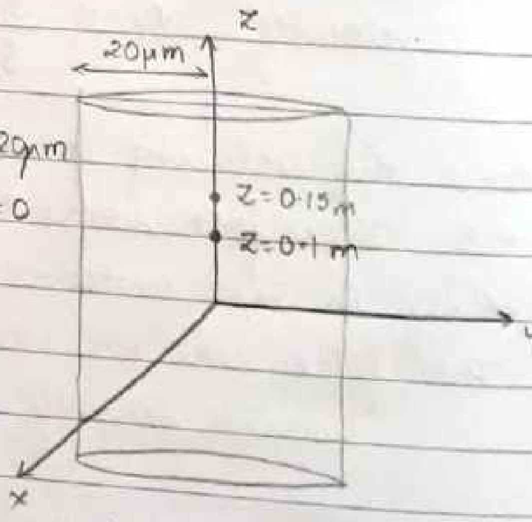
b. If the charge velocity is $2\times10^6 m/s$ at $z = 0.1m$ find ρ_v.

c. If the volume charge density at $z = 0.15m$ is $-2000\,C/m^3$ find the charge velocity.

Sol: cylindrical system

$$\vec{ds} = \rho\, d\rho\, d\phi\, \hat{a}_z$$

Given: $\vec{J} = -10^6 z^{1.5}\,\hat{a}_z\ A/m^2$



a. $$I = \int_S \vec{J}\cdot\vec{ds}$$

$$I = \int_{\rho=0}^{20\mu}\int_{\phi=0}^{2\pi} -10^6 z^{1.5}\,\hat{a}_z\ \rho\, d\rho\, d\phi\, \hat{a}_z$$

$$I = -10^6 z^{1.5}\int_{\rho=0}^{20\mu}\int_{\phi=0}^{2\pi} \rho\, d\rho\, d\phi$$

$$I = -10^6 (0.1)^{1.5}\,[2\pi]\int_{\rho=0}^{20\mu} \rho\, d\rho$$

$$I = 198.69\times10^3 \left[\frac{\rho^2}{2}\right]_0^{20\mu}$$

$$I = 198.69\times10^3 \left[\frac{(20\mu)^2}{2}\right]$$

$$\underline{I = 36.73\,\mu A}$$

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* Metallic Conductors:

- On applying an electric field the electron present gets excited and is accelerated with **drift** velocity due to the force experienced. This electron then excites another electron and it slows down. Later this accelerated electron excites another electron and so on.

Force experienced by the electron is given by

$$\vec{F_e} = Q\vec{E}$$

$$\vec{F_e} = -q\vec{E}$$

but $\vec{v_d} \propto \vec{E}$ $\quad v_d$ - drift velocity

$$\vec{v_d} = -\mu_e \vec{E} \qquad \mu_e \text{ - mobility}$$

but wkt

$$\vec{J} = \rho_v \vec{v}$$

$$\therefore \vec{J} = \rho_v(-\mu_e \vec{E})$$

$$\vec{J} = -nq\mu_e \vec{E}$$

$$\boxed{\vec{J} = \sigma \vec{E}} \qquad \text{where } \sigma = -nq\mu_e$$

conductivity

$$\sigma_{cu} = 5.8 \times 10^7 \text{ S/m}$$

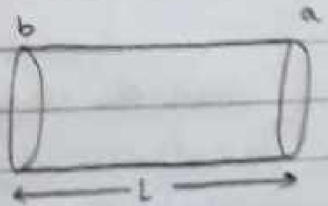
$$\sigma_{Ag} = 6.17 \times 10^7 \text{ S/m}$$

When conductivity of the conductor is maximum the resistance offered by that conductor is minimum.

i.e., $\sigma = \infty \Rightarrow R = 0$.

Ohm's law:

Let us apply a uniform electric field E to a conductor of length L.



By Ohm's law we know that

$$V = IR \quad \text{—— (1)}$$

$$V_{ab} = -\int_a^b \vec{E} \cdot \vec{dl}$$

$$V_{ab} = -\vec{E} \cdot \int_a^b \vec{dl} \qquad \text{(because } \vec{E} \text{ is uniform)}$$

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$\therefore V_{ab} = -\vec{E}\cdot\vec{L}_{ba} = \vec{E}\cdot\vec{L}_{ab}$

$V_{ab} = EL\cos\theta$

but since the angle θ between $\vec{E}$ and $\vec{L}$ is zero

hence $\cos\theta = 1$.

$\therefore V_{ab} = EL$

As electric field is uniform, the current density lines is also uniform.

Current is given by

$$I = \int_s \vec{J}\cdot\vec{ds}$$

$$I = \vec{J}\cdot\int_s \vec{ds}$$

$$I = \vec{J}\cdot\vec{S}$$

Since the angle between the normal of the surface and the current density lines is zero. Hence $\cos\phi = 1$.

$\therefore I = JS$

substituting in eq 1

$$EL = R(JS)$$

$$\therefore R = \frac{EL}{JS}$$

$$R = \frac{\cancel{E}L}{\sigma \cancel{E} S} \quad (\because J = \sigma E)$$

$$\boxed{R = \frac{L}{\sigma S}} \Rightarrow \boxed{R = \frac{\rho L}{S}}$$

When the electric field applied is not uniform, then resistance is

$$R = \frac{V}{I} = \frac{-\int \vec{E}\cdot\vec{dL}}{\int_s \vec{J}\cdot\vec{dS}}$$

Q1: Find the resistance of a 1 mile length of copper wire which has a diameter of 0.0508 inch.

Sol: Given: $L = 1 \text{ mile} = 1609.3\, m$

$d = 0.0508 \text{ inch} = 1.29 \times 10^{-3}\, m$

$\therefore r = 0.645 \times 10^{-3}\, m$

$S = \pi r^2 = 1.3076 \times 10^{-6}\, m$

$$R = \frac{L}{\sigma S} = \frac{1609.3}{5.8 \times 10^{7} \times 1.3076 \times 10^{-6}}$$

$$R = 21.2198\, \Omega$$

Q2: Find the magnitude of the current density in a sample of silver for which $\sigma = 6.17 \times 10^{7}\, S/m$, $\mu = 0.0056\, m^2/Vs$, if

a. $V_d = 1.5\, \mu m/s$

b. the electric field intensity is $1\, mV/m$.

c. the sample is a cube $2.5\, mm$ on a side having a voltage of $0.4\, mV$ between opposite faces.

d. the sample is a cube $2.5\, mm$ on a side carrying a total current of $0.5\, A$.

Sol: Given: $\sigma = 6.17 \times 10^{7}\, S/m$, $\mu = 0.0056\, m^2/V sec$.

a. $V_d = 1.5\, \mu m/s$

$V_d = \mu_e E$

$1.5 \times 10^{-6} = 0.0056 (E)$

$E = 0.267 \times 10^{-3}\, V/m$

$J = \sigma E$

$J = 6.17 \times 10^{7} (0.267 \times 10^{-3})$

$$J = 16.474 \times 10^{3}\, A/m^2$$

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b. $E = 1\times10^{-3}\, V/m$

$J = \sigma E$

$J = (6.17\times10^{7})(10^{-3})$

$J = \underline{61.7\, kA/m^2}$

c. side $= 2.5\,mm$ $\quad V = 0.4\,mV$

$V = \int \vec{E}\cdot\vec{dL} = EL$

$0.4m = E(2.5m)$

$\therefore E = \underline{0.16\, V/m}$

$J = \sigma E = (6.17\times10^{7})(0.16)$

$J = \underline{9.87\, MA/m^2}$

d. side $= 2.5\,mm$; $I = 0.5$

$A = (2.5\times10^{-3})^2$

$A = \underline{6.25\times10^{-6}}\, A$

$J = \frac{I}{A} = \frac{0.5}{6.25\times10^{-6}}$

$J = \underline{80\, kA/m^2}$

* **Boundary conditions at conductor - free space**

— considering a Gaussian surface S_1

$\oint \vec{D}\cdot\vec{ds} = Q$

but since no charge is enclosed.

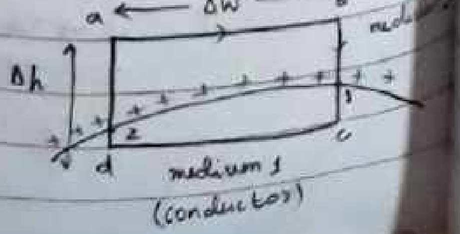
$\oint \vec{D}\cdot\vec{ds} = 0$

$\therefore \vec{D} = 0$

$\Rightarrow \boxed{\vec{E} = 0}$

Hence electric field intensity inside the conductor is zero.

— Workdone in a closed path in a conservative field is zero.



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$$\therefore \oint \vec{E}\cdot\vec{dl} = 0$$

$$\Rightarrow \int_a^b \vec{E}\cdot\vec{dl} + \int_b^1 \vec{E}\cdot\vec{dl} + \cancel{\int_1^c \vec{E}\cdot\vec{dl}} + \cancel{\int_c^d \vec{E}\cdot\vec{dl}} + \cancel{\int_d^2 \vec{E}\cdot\vec{dl}} + \int_2^a \vec{E}\cdot\vec{dl} = 0$$

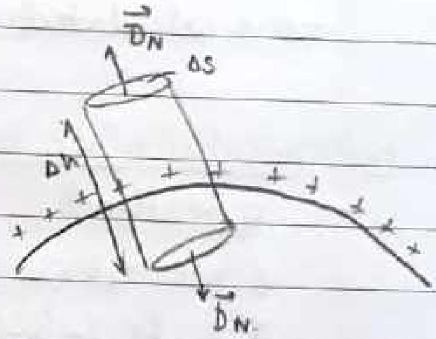
inside the conductor

$$\therefore \int_a^b \vec{E}\cdot\vec{dl} + \int_b^1 \vec{E}\cdot\vec{dl} + \int_2^a \vec{E}\cdot\vec{dl} = 0$$

$$E_{t(freespace)}\cdot\Delta w - E_N \cancel{\frac{\Delta h}{2}} + E_N \cancel{\frac{\Delta h}{2}} = 0 \qquad \Delta h \to 0$$

$$\boxed{E_{t(freespace)} = 0} \Rightarrow \boxed{D_{t(freespace)} = 0}$$

- considering a gaussian volume.



$$\oint \vec{D}\cdot\vec{ds} = Q$$

$$\int_{top} \vec{D}\cdot\vec{ds} + \underbrace{\cancel{\int_{botton} \vec{D}\cdot\vec{ds}}}_{0\text{ inside the conductor}} + \underbrace{\cancel{\int_{side} \vec{D}\cdot\vec{ds}}}_{0\text{ as }\Delta h\to 0} = \rho_v \Delta v$$

$$\int \vec{D}\cdot\vec{ds} = \rho_v\,\Delta s\,\Delta h$$

as Δh tends to zero, it is like a sheet of charge

$$\therefore \int \vec{D}\cdot\vec{ds} = \rho_s \Delta s$$

$$\therefore D_{N(freespace)}\cdot\cancel{\Delta s} = \rho_s \cancel{\Delta s}$$

$$\Rightarrow \boxed{D_{N(freespace)} = \rho_s}$$

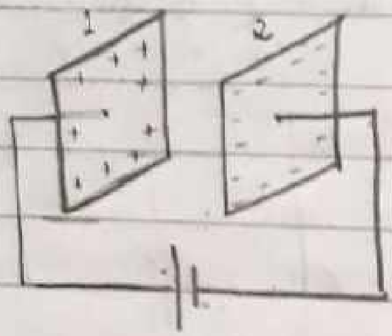
$$\therefore \boxed{E_{N(freespace)} = \frac{\rho_s}{\varepsilon}}$$

Therefore the electric field from the surface are always normal to the surface of the conductor.

NOTE:

The potential difference between any points on the surface of the conductor is zero. Hence it is equipotential surface.

★ **Capacitor:**



Let us connect two conductors to a battery. It induces charges on the conductors.

$$C = \frac{|Q|}{|V|} = \frac{\iiint_v \rho_v \, dv}{-\int_2 \vec{E} \cdot \vec{dL}}$$

Q method
- ρ_v is known
- E is calculated

V method
- V is known
- $E = -\nabla V$

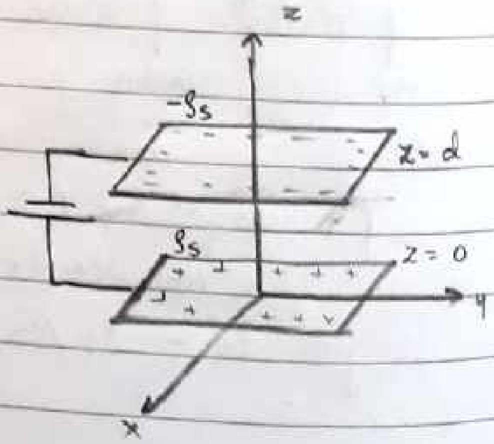
Ex: Parallel plate capacitor

— Q method

$E = E_1 + E_2$

$$E = \frac{\rho_s}{2\varepsilon}(+\hat{a}_z) + \frac{-\rho_s}{2\varepsilon}(-\hat{a}_z)$$

$$E = \frac{\rho_s}{\varepsilon}\hat{a}_z$$



$$V_0 = -\int_{\text{upper}}^{\text{lower}} \vec{E} \cdot \vec{dl}$$

$$V_0 = -\int_{z=d}^{z=0} \frac{\rho_s}{\varepsilon}\hat{a}_z \cdot dz\, \hat{a}_z$$

$$V_0 = \frac{-\rho_s}{\varepsilon} z \Big|_d^0$$

$$\boxed{V_0 = \frac{\rho_s d}{\varepsilon}}$$

$\therefore$ $Q = \int_s \rho_s \, dS = \rho_s \int_s dS = \rho_s S$

$$C = \frac{Q}{V_0} = \frac{\rho_s S}{\rho_s d/\varepsilon} = \frac{\varepsilon S}{d}$$

$$\therefore \boxed{C = \frac{\varepsilon S}{d}}$$

$\varepsilon = \varepsilon_0 \varepsilon_r$ F/m

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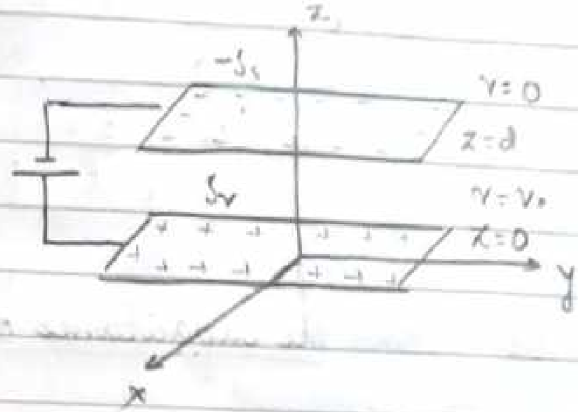
- V method:

Laplace equation

$\nabla^2 V = 0$

$$\nabla^2 V = \cancel{\frac{\partial^2 V}{\partial x^2}} + \cancel{\frac{\partial^2 V}{\partial y^2}} + \frac{\partial^2 V}{\partial z^2} = 0$$

no change in potential in this direction



$$\therefore \frac{\partial^2 V}{\partial z^2} = 0$$

Integrating wrt z

$$\frac{\partial V}{\partial z} = A$$

where A and B are constants.

Integrating wrt z

$$V = Az + B \quad \text{—— (1)}$$

Substituting the boundary condition

at V = 0 : $0 = Ad + B$

at V = V_0 : $V_0 = B$ $\quad\therefore A = -V_0/d$

Substituting in eq (1)

$$V = -\frac{V_0}{d} z + V_0$$

wkt $E = -\nabla V$

$$\therefore E = -\left[\cancel{\frac{\partial V}{\partial x}}\hat{a}_x + \cancel{\frac{\partial V}{\partial y}}\hat{a}_y + \frac{\partial V}{\partial z}\hat{a}_z\right]$$

$\therefore E = -A$

$$E = \frac{V_0}{d}\hat{a}_z \qquad D = \epsilon E = \frac{\epsilon V_0}{d}\hat{a}_z$$

$D = D_N \hat{a}_N$

$$\Rightarrow D_N = \frac{\epsilon V_0}{d} = \rho_s$$

$$Q = \int_S \rho_s \, ds = \rho_s S \quad \therefore \boxed{Q = \frac{\epsilon V_0 S}{d}}$$

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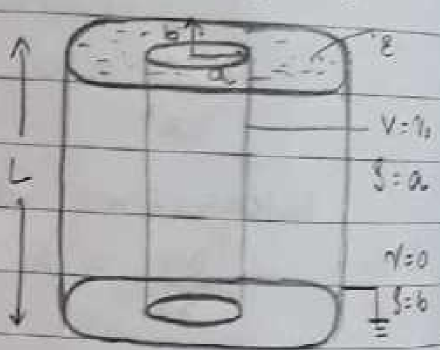
$$c = \frac{Q}{V} = \frac{\varepsilon V_0 S}{V_0 d}$$

$$\therefore \boxed{c = \frac{\varepsilon S}{d}}$$

Ex: the capacitance of coaxial transmission line of length L.

- V method



Laplace equation

$$\nabla^2 V = 0$$

$$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial V}{\partial \rho}\right) + \cancel{\frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2}} + \cancel{\frac{\partial^2 V}{\partial z^2}} = 0$$

No change in potential in ϕ and z direction

$$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial V}{\partial \rho}\right) = 0$$

$$\frac{\partial}{\partial \rho}\left(\rho \frac{\partial V}{\partial \rho}\right) = 0$$

Integrating wrt ρ

$$\rho \frac{\partial V}{\partial \rho} = A$$

$$\frac{\partial V}{\partial \rho} = \frac{A}{\rho}$$

Integrating wrt ρ

$$V = A \log \rho + B \quad \text{——— (1)}$$

Substituting boundary condition

at $V=0$ and $\rho = b$: $0 = A \log b + B \Rightarrow B = -A \log b$

at $V = V_0$ and $\rho = a$: $V_0 = A \log a + B$

$$V_0 = A \log a - A \log b$$

$$V_0 = A \log\left(\frac{a}{b}\right)$$

$$\therefore A = \frac{V_0}{\log(a/b)}$$

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$$B = -\frac{V_0}{\log(a/b)} \log b$$

substituting in eq ①

$$V = \frac{V_0}{\log(a/b)} \log \rho - \frac{V_0}{\log(a/b)} \log b$$

wkt $E = -\nabla V$

$$E = -\left(\frac{\partial V}{\partial \rho} \hat{a}_\rho + \frac{1}{\rho} \frac{\partial V}{\partial \phi} \hat{a}_\phi + \frac{\partial V}{\partial z} \hat{a}_z \right)$$

$$E = -\left[\frac{V_0}{\log(a/b)} \frac{1}{\rho} \right] \hat{a}_\rho$$

$$E = \frac{V_0}{\rho \log(b/a)} \hat{a}_\rho$$

$$D = \varepsilon E = \frac{\varepsilon V_0}{\rho \log(b/a)} \hat{a}_\rho$$

$$D = D_N \hat{a}_N$$

$$\Rightarrow D_N = \frac{\varepsilon V_0}{\rho \log(b/a)} = \rho_s$$

$$Q = \int_S \rho_s ds = \rho_s \int_S ds$$

$$Q = \rho_s (2\pi a L)$$

$$C = \frac{Q}{V} = \frac{\rho_s 2\pi a L}{V_0}$$

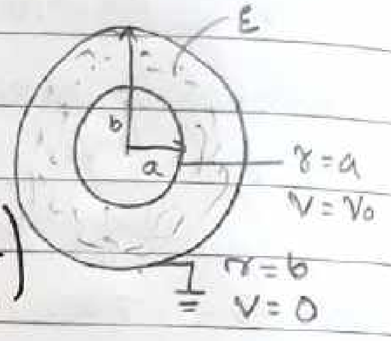
$$C = \frac{\varepsilon \cancel{V_0}}{\rho \log(b/a)} \frac{2\pi a L}{\cancel{V_0}} \quad (\because D_N = \rho_s)$$

For inner conductor $\rho = a$

$$\therefore \boxed{C = \frac{2\pi \varepsilon L}{\log(b/a)}}$$

Ex: The capacitance of concentric spheres.

– V method:



Laplace equation:

$$\nabla^2 V = 0$$

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) + \cancel{\frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial V}{\partial \theta}\right)}^{\,0}$$

$$+ \cancel{\frac{1}{r^2 \sin^2\theta}\frac{\partial^2 V}{\partial \phi^2}}^{\,0} = 0$$

Potential doesnot vary in θ and ϕ direction.

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) = 0$$

$$\therefore \frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) = 0$$

Integrating wrt r

$$r^2 \frac{\partial V}{\partial r} = A$$

$$\frac{\partial V}{\partial r} = \frac{A}{r^2}$$

Integrating wrt r

$$V = \frac{-A}{r} + B \quad \text{——} \ (1)$$

substituting the boundary conditions.

At $V = 0$ and $r = b$: $\quad 0 = -\frac{A}{b} + B \Rightarrow B = A/b$

At $V = V_0$ and $r = a$: $\quad V_0 = -\frac{A}{a} + B$

$$V_0 = -\frac{A}{a} + \frac{A}{b}$$

$$V_0 = A\left[\frac{1}{b} - \frac{1}{a}\right]$$

$$\therefore A = \frac{V_0}{\left[\frac{1}{b} - \frac{1}{a}\right]} \quad \Rightarrow B = \frac{V_0}{b\left[\frac{1}{b} - \frac{1}{a}\right]}$$

substituting in eq ①

$$V = \frac{-V_0}{r\left[\frac{1}{b}-\frac{1}{a}\right]} + \frac{V_0}{b\left[\frac{1}{b}-\frac{1}{a}\right]}$$

wkt $E = -\nabla V$

$$E = -\left(\frac{\partial V}{\partial r}\hat{a}_r + \frac{1}{r}\cancel{\frac{\partial V}{\partial \theta}}\hat{a}_\theta + \cancel{\frac{\partial V}{\partial \phi}}\hat{a}_\phi\right)$$

$$E = -\left[\frac{V_0}{r^2\left[\frac{1}{b}-\frac{1}{a}\right]}\right]\hat{a}_r$$

$$E = \frac{V_0}{r^2\left[\frac{1}{a}-\frac{1}{b}\right]}\hat{a}_r$$

$$D = \varepsilon E = \frac{V_0 \varepsilon}{r^2\left[\frac{1}{a}-\frac{1}{b}\right]}\hat{a}_r$$

$$D = D_N \hat{a}_N$$

$$\Rightarrow D_N = \frac{\varepsilon V_0}{r^2\left[\frac{1}{a}-\frac{1}{b}\right]} = \rho_s$$

$$Q = \int_s \rho_s ds = \rho_s \int_s ds$$

$$Q = \rho_s (4\pi a^2)$$

$$C = \frac{Q}{V} = \frac{\rho_s 4\pi a^2}{V_0}$$

$$C = \frac{\varepsilon \cancel{V_0}}{r^2\left[\frac{1}{a}-\frac{1}{b}\right]} \frac{4\pi a^2}{\cancel{V_0}}$$

For inner sphere $r = a$

$$\boxed{C = \frac{4\pi\varepsilon}{\left[\frac{1}{a}-\frac{1}{b}\right]}}$$

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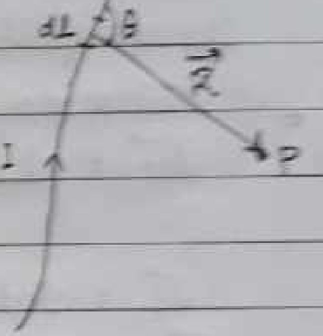
UNIT - 4

Magnetic Fields

charges in rest produce electric field and charges in motion produce magneticfield around them.

* **Biot's Savart's Law:**



It states that, "the magnitude of magnetic field intensity produced by the differential element at any point P is directly proportional to the product of the current, the magnitude of differential length and the sine of the angle between the conductor and the line jointing to the point P from the differential length and is inversely proportional to the square of the distance between them."

The direction of the magnetic field intensity is normal to the plane containing the differential element and the line joining the differential element and the point P. Right hand screw rule can be used to find the direction.

$$dH \propto \frac{I\, dL \sin\theta}{R^2}$$

$$dH = \frac{k\, I\, dL \sin\theta}{R^2}$$

where $k = \frac{1}{4\pi}$

$$\therefore \boxed{\vec{dH} = \frac{I\, dL \sin\theta}{4\pi R^2}\, \hat{a}_R}$$

$\hat{a}_R$ - resultant

$$\vec{dL} \times \hat{a}_R = dL\, 1 \sin\theta\, \hat{a}_R$$

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$$\vec{dH} = \frac{I\vec{dL} \times \hat{a}_R}{4\pi R^2}$$

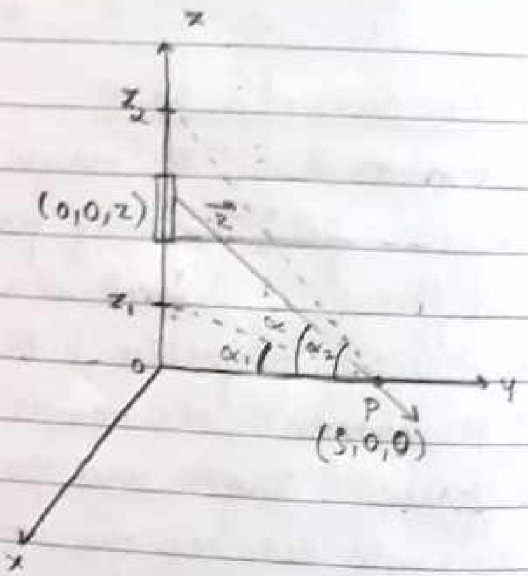
$$\therefore \boxed{H = \oint \vec{dH} = \oint \frac{I\vec{dL} \times \hat{a}_R}{4\pi R^2} \ A/m}$$

Finite length filament*

Application of Biot Savart's law:

considering a differential current element $I\vec{dL}$.

The magnetic field intensity at P due to $z_1 z_2$ is



$$\vec{dH} = \frac{I\vec{dL} \times \hat{a}_R}{4\pi R^2} \quad \text{—— (1)}$$

$$I\vec{dL} = I\,dz\,\hat{a}_z$$

$$\vec{R} = \rho\hat{a}_\rho - z\hat{a}_z$$

$$\hat{a}_R = \frac{\rho\hat{a}_R - z\hat{a}_z}{\sqrt{\rho^2+z^2}}$$

$$\therefore R = \sqrt{\rho^2+z^2}$$

Substituting in eq (1)

$$\vec{dH} = \frac{I\vec{dL} \times \hat{a}_R}{4\pi(\rho^2+z^2)}$$

$$I\vec{dL} \times \hat{a}_R = \begin{vmatrix} \hat{a}_\rho & \hat{a}_\phi & \hat{a}_z \\ 0 & 0 & I\,dz \\ \frac{\rho}{\sqrt{\rho^2+z^2}} & 0 & \frac{-z}{\sqrt{\rho^2+z^2}} \end{vmatrix}$$

$$I\vec{dL} \times \hat{a}_R = -\hat{a}_\phi\left[\frac{-I\,dz\,\rho}{\sqrt{\rho^2+z^2}}\right]$$

$$\vec{dH} = \frac{\rho I\,dz\,\hat{a}_\phi}{4\pi(\rho^2+z^2)\sqrt{\rho^2+z^2}}$$

$$\vec{dH} = \frac{\rho I\,dz\,\hat{a}_\phi}{4\pi(\rho^2+z^2)^{3/2}}$$

$$\therefore \vec{H} = \int_{z_1}^{z_2} \frac{\rho I\,dz\,\hat{a}_\phi}{4\pi(\rho^2+z^2)^{3/2}}$$

Put $z = \rho\tan\theta$

then $dz = \rho\sec^2\theta\,d\theta$

$\therefore$ as $z \to z_1$, $\theta \to \alpha_1$

as $z \to z_2$, $\theta \to \alpha_2$

$$\therefore \vec{H} = \int_{\alpha_1}^{\alpha_2} \frac{\rho I \, \hat{a}_\phi \, \rho \sec^2\theta}{4\pi(\rho^2 + \rho^2\tan^2\theta)^{3/2}} \, d\theta$$

$$\vec{H} = \frac{1}{4\pi} \int_{\alpha_1}^{\alpha_2} \frac{\rho^2 \sec^2\theta \, d\theta}{\rho^3(1+\tan^2\theta)^{3/2}} \, \hat{a}_\phi$$

$$\vec{H} = \frac{1}{4\pi\rho} \int_{\alpha_1}^{\alpha_2} \frac{\sec^2\theta \, d\theta}{\sec^3\theta} \, \hat{a}_\phi$$

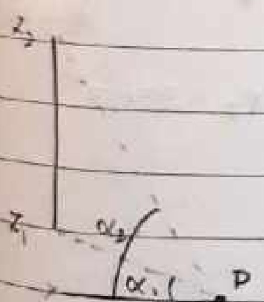
$$\vec{H} = \frac{1}{4\pi\rho} \int_{\alpha_1}^{\alpha_2} \cos\theta \, d\theta \, \hat{a}_\phi$$

$$\vec{H} = \frac{1}{4\pi\rho} \left[\sin\theta \right]_{\alpha_1}^{\alpha_2}$$

$$\boxed{\vec{H} = \frac{1}{4\pi\rho} [\sin\alpha_2 - \sin\alpha_1] \, \hat{a}_\phi}$$

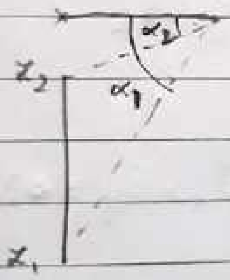
ρ = normal distance from the point P to the conductor.

case 1:



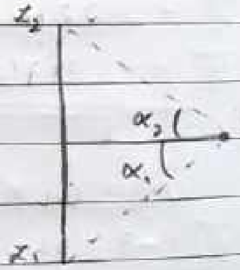
α_1 and α_2 are positive

case 2:



α_1 and α_2 are negative.

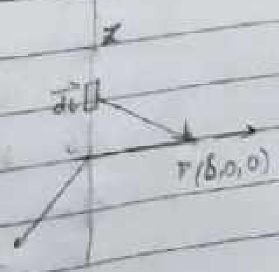
case 3:



α_2 is positive and α_1 is negative

$\hat{a}_\phi = \hat{a}_L \times \hat{a}_\rho$ $\quad$ $\hat{a}_L$ - direction in which the conductor is aligned

$\hat{a}_\phi = \hat{a}_z \times \hat{a}_\rho$



For an infinite length conductor

$$\vec{H} = \frac{1}{4\pi\rho} [\sin\alpha_2 - \sin\alpha_1] \hat{a}_\phi$$

where $\alpha_1 = -\pi/2$ and $\alpha_2 = \pi/2$

$$\vec{H} = \frac{1}{4\pi\rho} [\sin\pi/2 - \sin(-\pi/2)] \hat{a}_\phi$$

$$\vec{H} = \frac{1}{4\pi\rho} [2] \hat{a}_\phi \quad \therefore \boxed{\vec{H} = \frac{1}{2\pi\rho} \hat{a}_\phi \ A/m}$$

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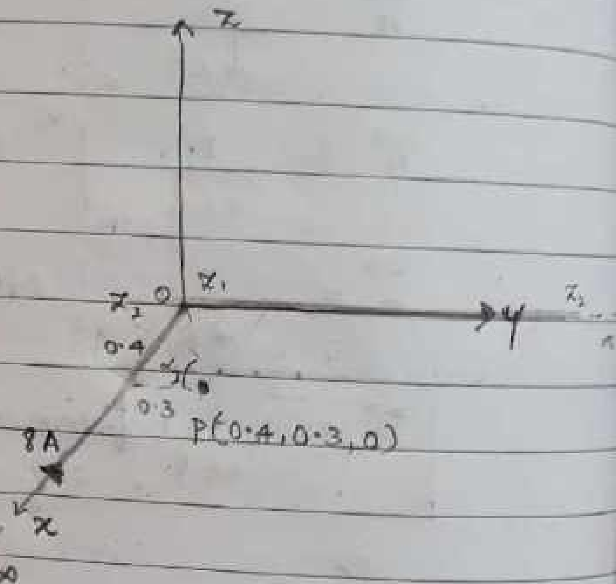
Q1: Determine $\vec{H}$ at $P_2(0.4, 0.3, 0)$ in the field of an 8A filamentary current directed inward from infinity to the origin on the positive x axis and then outward to infinity along the y axis

Sol: Total magnetic field is



$\vec{H} = \vec{H_1} + \vec{H_2}$

$\vec{H_1} = \dfrac{I}{4\pi R_1}(\sin\alpha_2 - \sin\alpha_1)\,\hat{a}_\phi$

$\vec{R_1} = 0.3\,\hat{a}_y$

$|\vec{R_1}| = 0.3$

$\vec{H_1} = \dfrac{8}{4\pi(0.3)}[-\sin\alpha_1 + \sin(-\pi/2)]\,\hat{a}_\phi$

$\vec{H_1} = 2.122\,[-\sin\alpha_1 - 1]\,\hat{a}_\phi$

$\vec{H_1} = 2.122\,[-\sin 53.13^\circ - 1]\,\hat{a}_\phi$

$\vec{H_1} = -3.8196\,\hat{a}_\phi$ A/m

$\alpha_1 = \tan^{-1}\left(\dfrac{0.4}{0.3}\right)$

$\alpha_1 = 53.13^\circ$

$\hat{a}_\phi = \hat{a}_L \times \hat{a}_s = \hat{a}_x \times \hat{a}_y = \hat{a}_z$

$\vec{H_1} = -3.8196\,\hat{a}_z$ A/m

$\vec{H_2} = \dfrac{I}{4\pi R_2}(\sin\alpha_2 - \sin\alpha_1)\,\hat{a}_\phi$

$\vec{R_2} = 0.4\,\hat{a}_x$

$|\vec{R_2}| = 0.4$

$\vec{H_2} = \dfrac{8}{4\pi(0.4)}[\sin(-\pi/2) - \sin\alpha_1]\,\hat{a}_\phi$

$\alpha_1 = \tan^{-1}\left(\dfrac{0.3}{0.4}\right)$

$\alpha_1 = 36.87^\circ$

$\vec{H_2} = 1.592\,[-1 - \sin 36.87^\circ]\,\hat{a}_\phi$

$\vec{H_2} = -2.544\,\hat{a}_\phi$

$\vec{H_2} = -2.544\,\hat{a}_z$ A/m

$\hat{a}_\phi = \hat{a}_L \times \hat{a}_s$

$= \hat{a}_y \times \hat{a}_x$

$= -\hat{a}_z$

$\vec{H} = \vec{H_1} + \vec{H_2}$

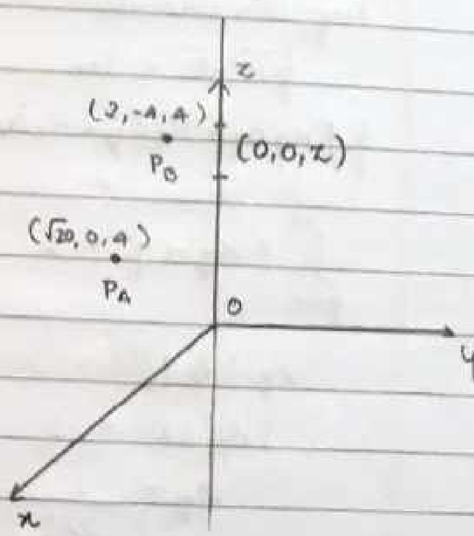
$\vec{H} = -3.8196\,\hat{a}_x - 2.544\,\hat{a}_x$

$\vec{H} = -6.3636\,\hat{a}_x$ A/m

Q: A current filament carrying 15A in the $\hat{a}_x$ direction lies along the entire x-axis. Find $\vec{H}$ in rectangular coordinates

a. $P_A(\sqrt{20}, 0, 4)$

b. $P_B(2, -4, 4)$



Sol: a. $P_A(\sqrt{20}, 0, 4)$

$\vec{R} = (\sqrt{20}\hat{a}_x - (4-z)\hat{a}_x)$

In the direction of conductor the field components is zero

$\vec{R} = \sqrt{20}\,\hat{a}_x$

$\Rightarrow |\vec{R}| = \sqrt{20}$

For an infinite conductor

$\vec{H} = \dfrac{I}{2\pi R}\hat{a}_\phi$

$\hat{a}_\phi = \hat{a}_L \times \hat{a}_R = \hat{a}_x \times \hat{a}_x = \hat{a}_y$

xy = z

yz = x

zx = y

$\vec{H} = \dfrac{15}{2\pi(\sqrt{20})}\hat{a}_\phi = 0.534\,\hat{a}_y$ //

b. $P_B(2, -4, 4)$

$\vec{R} = (2\hat{a}_x - 4\hat{a}_y - (4-z)\hat{a}_x)$

$\vec{R} = 2\hat{a}_x - 4\hat{a}_y$

$|\vec{R}| = \sqrt{2^2+4^2} = \sqrt{20}$ $\therefore \hat{a}_R = \dfrac{2\hat{a}_x - 4\hat{a}_y}{\sqrt{20}}$

For an infinite conductor

$\vec{H} = \dfrac{I}{2\pi R}\hat{a}_\phi$

$\hat{a}_\phi = \hat{a}_L \times \hat{a}_R = \hat{a}_x \times \left[\dfrac{2\hat{a}_x - 4\hat{a}_y}{\sqrt{20}}\right]$

$\therefore \hat{a}_\phi = \dfrac{2\hat{a}_y + 4\hat{a}_x}{\sqrt{20}}$

$\vec{H} = \dfrac{15}{2\pi(\sqrt{20})}\left[\dfrac{2\hat{a}_y + 4\hat{a}_x}{\sqrt{20}}\right]$

$\therefore \vec{H} = 0.238\,\hat{a}_y + 0.477\,\hat{a}_x$

* Ampere's law :

Ampere's circuital law states that the line integral of magnetic field intensity H about any closed path is exactly equal to the direct current enclosed by that path.

$$\oint \vec{H}\cdot\vec{dl} = I_{en}.$$

a. **For infinite length filament :**

Ampere's circuital law is given by

$$\oint \vec{H}\cdot\vec{dl} = I_{en}$$

$$\oint (H_\rho \hat{a}_\rho + H_\phi \hat{a}_\phi + H_z \hat{a}_z)\cdot(\rho\, d\phi\, \hat{a}_\phi) = I$$

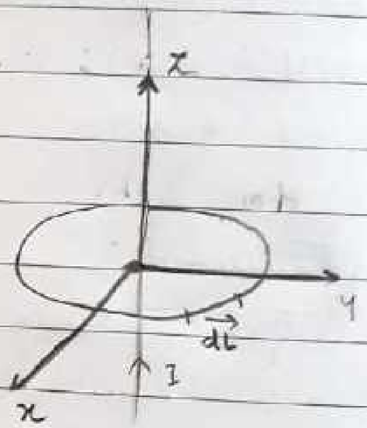
$$\int_{\phi=0}^{2\pi} \rho H_\phi\, d\phi = I$$

$$\rho H_\phi [\phi]_0^{2\pi} = I$$

$$\rho H_\phi\, 2\pi = I$$

$$H_\phi = \frac{I}{2\pi\rho}$$

$$\therefore \boxed{\vec{H} = \frac{I}{2\pi\rho}\hat{a}_\phi}$$

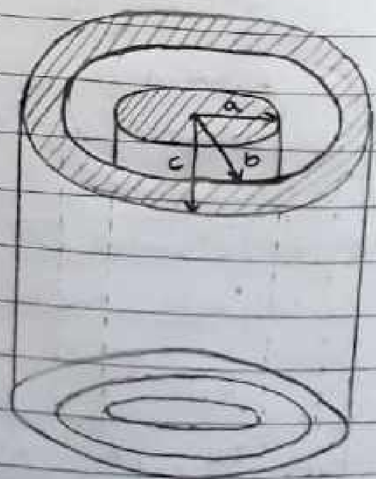


b. **For co axial cable:**

Let the radius of inner conductor be 'a', the radius of inner radius of outer conductor be 'b' and the outer radius of the outer conductor be 'c'.

Let the radius of the ampere's closed path be 'ρ'.

Let the current in inner conductor be I and the current in outer conductor be −I.



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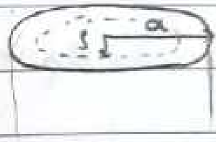
CASE 1 : Inside the inner conductor : $\rho < a$

By ampere's circuital law

$$\oint \vec{H} \cdot \vec{dl} = I_{en}$$



I_{en} by the closed path $\rho < a$ is

$$I_{en} = I\left(\frac{\pi \rho^2}{\pi a^2}\right)$$

$$\therefore H_\phi \rho 2\pi = I\left(\frac{\cancel{\pi} \rho^2}{\cancel{\pi} a^2}\right)$$

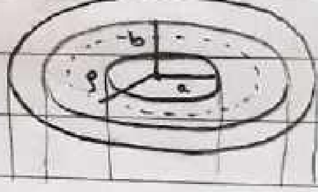
$$H_\phi = \frac{I\rho}{2\pi a^2}$$

$$\therefore \boxed{\vec{H} = \frac{I\rho}{2\pi a^2}\, \hat{a}_\phi}$$

CASE 2 : Inbetween the inner and outer conductor : $a < \rho < b$

By ampere's circuital law

$$\oint \vec{H} \cdot \vec{dl} = I_{en}$$



Here current enclosed by the closed path of radius $a < \rho < b$ is : $I_{en} = I$

$$\therefore H_\phi \rho 2\pi = I$$

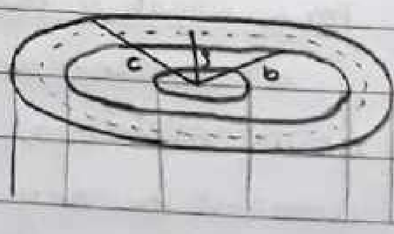
$$H_\phi = \frac{I}{2\pi \rho}$$

$$\therefore \boxed{\vec{H} = \frac{I}{2\pi \rho}\, \hat{a}_\phi}$$

CASE 3 : Inside the outer conductor : $b < \rho < c$

By ampere's circuital law

$$\oint \vec{H} \cdot \vec{dl} = I_{en}$$



Here current enclosed by the closed path of radius $b < \rho < c$ is

$$I_{en} = I + (-I)\left(\frac{\pi \rho^2 - \pi b^2}{\pi c^2 - \pi b^2}\right)$$

$$\therefore I_{en} = I\left[1 - \frac{(\rho^2 - b^2)}{c^2 - b^2}\right] = I\left[\frac{c^2 - \cancel{b^2} - \rho^2 + \cancel{b^2}}{c^2 - b^2}\right] = I\left[\frac{c^2 - \rho^2}{c^2 - b^2}\right]$$

$$\therefore H_\phi \, \rho \, 2\pi = I\left[\frac{c^2-\rho^2}{c^2-b^2}\right]$$

$$H_\phi = \frac{I}{2\pi\rho}\left[\frac{c^2-\rho^2}{c^2-b^2}\right]$$

$$\therefore \boxed{\vec{H} = \frac{I}{2\pi\rho}\left[\frac{c^2-\rho^2}{c^2-b^2}\right]\hat{a}_\phi}$$

CASE 4: Outside the outer conductor: $\rho > c$

By ampere's circuital law

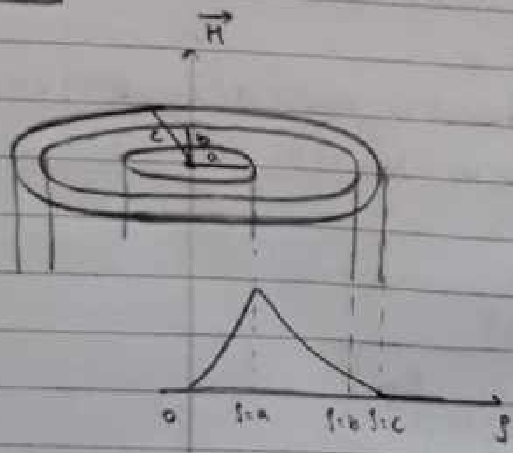
$$\oint \vec{H}\cdot\vec{dl} = I_{en}$$

Here current enclosed by the closed path of radius $\rho > c$ is

$$I_{en} = I + (-I) = 0$$

$$\therefore H_\phi \, \rho \, 2\pi = 0$$

$$\therefore \boxed{\vec{H} = 0}$$



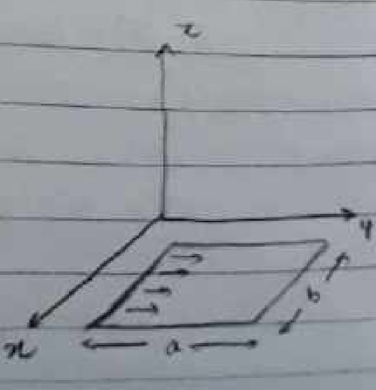
c. For a sheet of current:

k : current density (2 dimension)

Let us consider a sheet of current in x-y plane.

Let the length and breadth of the conductor be 'a' and 'b' respectively.

Let the current flow in y direction.



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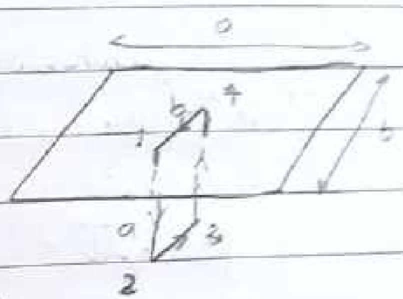
$\vec{K} = K_y \hat{a}_y$

Along the breadth (along y direction).

$K_y = \frac{I}{b}$ (current density = $\frac{\text{current}}{\text{length}}$: line conductor)

$\therefore I = K_y b$



Let us consider a closed ampere's path (1234).

By Ampere's circuital law

$\oint \vec{H} \cdot \vec{dl} = I_{en}$

$\int_1^2 \vec{H} \cdot \vec{dl} + \int_2^3 \vec{H} \cdot \vec{dl} + \int_3^4 \vec{H} \cdot \vec{dl} + \int_4^1 \vec{H} \cdot \vec{dl} = I_{en}$

$(-H_z)(-a) + (-H_x)(-b) + (H_z)(a) + (H_x)(b) = k_y b$

$H_z a + H_x b + H_z a + H_x b = k_y b$

~~Above the sheet~~ due to multiple filaments ~~together~~ a sheet is formed. Hence all the H_x components get added up (as in the same direction) (above and below the sheet) and the H_z components cancel each other mutually.

$$\therefore H = \begin{cases} H_x & z > 0 \\ -H_x & z < 0 \end{cases}$$

$\Rightarrow 2 H_x b = k_y b$

$\therefore H = \frac{k_y}{2} \hat{a}_x$

For general formula

here: $\hat{a}_x = \hat{a}_y \times \hat{a}_z$

$\therefore \hat{a}_x = \vec{k} \times \hat{a}_N$

$$\therefore \boxed{\vec{H} = \frac{1}{2} \vec{k} \times \hat{a}_N}$$

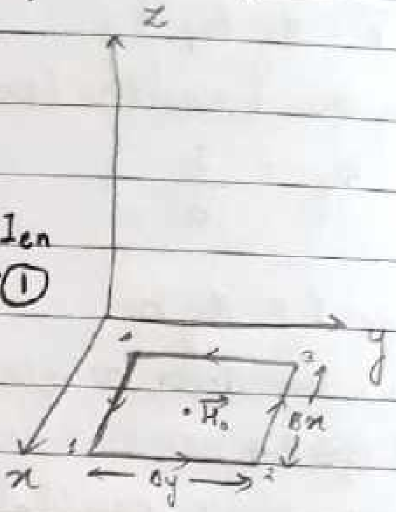
Application of Ampere's law or point form of Ampere's law.

★ Curl:

By ampere's circuital law

$$\oint \vec{H}\cdot\vec{dl} = I_{en}$$

$$\int_1^2 \vec{H}\cdot\vec{dl} + \int_2^3 \vec{H}\cdot\vec{dl} + \int_3^4 \vec{H}\cdot\vec{dl} + \int_4^1 \vec{H}\cdot\vec{dl} = I_{en} \quad \text{--- (1)}$$



at the center of the path

$$\vec{H_0} = H_{x_0}\hat{a}_x + H_{y_0}\hat{a}_y + H_{z_0}\hat{a}_x$$

$$\therefore \int_1^2 \vec{H}\cdot\vec{dl} = H_y \Delta y = \left[H_{y_0} + \frac{\Delta x}{2}\frac{\partial H_y}{\partial x}\right]\Delta y \qquad H_y = H_{y_0} + \frac{\Delta x}{2}\frac{\partial H_y}{\partial x}$$

$$\int_3^4 \vec{H}\cdot\vec{dl} = H_y(-\Delta y) = -\left[H_{y_0} - \frac{\Delta x}{2}\frac{\partial H_y}{\partial x}\right]\Delta y \qquad H_y = H_{y_0} - \frac{\Delta x}{2}\frac{\partial H_y}{\partial x}$$

$$\int_2^3 \vec{H}\cdot\vec{dl} = H_x(-\Delta x) = -\left[H_{x_0} + \frac{\Delta y}{2}\frac{\partial H_x}{\partial y}\right]\Delta x \qquad H_x = H_{x_0} + \frac{\Delta y}{2}\frac{\partial H_x}{\partial y}$$

$$\int_4^1 \vec{H}\cdot\vec{dl} = H_x \Delta x = \left[H_{x_0} - \frac{\Delta y}{2}\frac{\partial H_x}{\partial y}\right]\Delta x \qquad H_x = H_{x_0} - \frac{\Delta y}{2}\frac{\partial H_x}{\partial y}$$

substituting in eq (1), we get

$$\left[\cancel{H_{y_0}} + \frac{\Delta x}{2}\frac{\partial H_y}{\partial x}\right]\Delta y + \left[\cancel{-H_{y_0}} + \frac{\Delta x}{2}\frac{\partial H_y}{\partial x}\right]\Delta y$$

$$+\left[\cancel{-H_{x_0}} - \frac{\Delta y}{2}\frac{\partial H_x}{\partial y}\right]\Delta x + \left[\cancel{H_{x_0}} - \frac{\Delta y}{2}\frac{\partial H_x}{\partial y}\right]\Delta x = I_{en}$$

$$\frac{\partial H_y}{\partial x}\Delta x\Delta y - \frac{\partial H_x}{\partial y}\Delta x\Delta y = I_{en}$$

wkt $I = \int \vec{J}\cdot\vec{ds}$

$$I = (J_x\hat{a}_x + J_y\hat{a}_y + J_z\hat{a}_z)\cdot(\Delta x\Delta y\ \hat{a}_x)$$

$$I = J_z \Delta x \Delta y$$

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$$\therefore \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right) \Delta x \Delta y = J_z \Delta x \Delta y$$

$$\therefore \boxed{\lim_{\Delta x \Delta y \to 0} \frac{\oint \vec{H} \cdot \vec{dl}}{\Delta x \Delta y} = \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = J_z}$$

as $\Delta x \Delta y \to 0$ the path reduces and is hence a point.

Similarly when the path is in y-z plane.

$$\boxed{J_x = \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = \lim_{\Delta y \Delta z \to 0} \frac{\oint \vec{H} \cdot \vec{dl}}{\Delta y \Delta z}}$$

$y \to x \to z$

when the path is in x-z plane

$$\boxed{J_y = \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = \lim_{\Delta z \Delta x \to 0} \frac{\oint \vec{H} \cdot \vec{dl}}{\Delta z \Delta x}}$$

Hence

$$\boxed{\vec{J} = \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right)\hat{a}_x + \left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right)\hat{a}_y + \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right)\hat{a}_z}$$

that is.

$$\vec{J} = \vec{\nabla} \times \vec{H} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix}$$ cartesian form.

general formula:

$$\vec{\nabla} \times \vec{A} = \begin{vmatrix} a_u & a_v & a_w \\ \frac{1}{h_u}\frac{\partial}{\partial u} & \frac{1}{h_v}\frac{\partial}{\partial v} & \frac{1}{h_w}\frac{\partial}{\partial w} \\ H_u & H_v & H_w \end{vmatrix} \frac{1}{h_u h_v h_w}$$

For	u	v	w	h_u	h_v	h_w
i. cartesian system	x	y	z	1	1	1
ii. Cylindrical system	ρ	ϕ	z	1	ρ	1
iii. spherical system	r	θ	ϕ	1	r	$r\sin\theta$

By ampere's circuital law

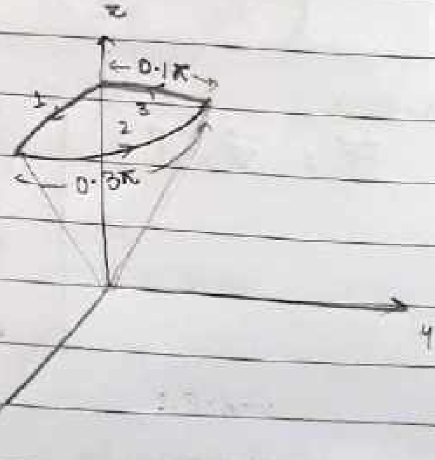
$$\oint \vec{H} \cdot \vec{dl} = I$$

but wkt $I = \int \vec{J} \cdot \vec{ds}$

$$\therefore \oint \vec{H} \cdot \vec{dl} = \int_s \vec{J} \cdot \vec{ds}$$

$$\Rightarrow \boxed{\oint \vec{H} \cdot \vec{dl} = \int (\vec{\nabla} \times \vec{H}) \cdot \vec{ds}}$$ stroke's law

Q1: A portion of a sphere shown, the surface is specified by $r = 4$, θ varies from 0 to 0.1π, ϕ varies from 0 to 0.3π. closed path forming its perimeter is composed of three circular arcs. Given the field $H = 6r\sin\phi\,\hat{a}_r + 18r\sin\theta\cos\phi\,\hat{a}_\phi$. Evaluate each side by stroke's theorem.



sol: By stroke's law

$$\oint \vec{H} \cdot \vec{dl} = \int_s (\nabla \times H) \cdot ds$$

$$\underline{LHS} = \int_1 \vec{H} \cdot \vec{dl} + \int_2 \vec{H} \cdot \vec{dl} + \int_3 \vec{H} \cdot \vec{dl}$$

$$= \int_1 (6r\sin\phi\,\hat{a}_r + 18r\sin\theta\cos\phi)\hat{a}_\phi \cdot (r\,d\theta\,\hat{a}_\theta)$$

$$+ \int_2 (6r\sin\phi\,\hat{a}_r + 18r\sin\theta\cos\phi)\hat{a}_\phi \cdot (r\sin\theta\,d\phi\,\hat{a}_\phi)$$

$$+ \int_3 (6r\sin\phi\,\hat{a}_r + 18r\sin\theta\cos\phi)\hat{a}_\phi \cdot (r\,d\theta\,\hat{a}_\theta)$$

$r = 4$

$0 \le \theta \le 0.1\pi$

$0 \le \phi \le 0.3\pi$

$$= \int_2 18r^2\sin^2\theta\cos\phi\,d\phi$$

$$= 18r^2\sin^2\theta \int_{\phi=0}^{0.3\pi} \cos\phi\,d\phi$$

$$= 18(4)^2\sin^2(18^\circ)\left[\sin\phi\right]_0^{0.3\pi} = \underline{22.2\,A}$$

RHS : $\nabla \times \vec{H} = \begin{vmatrix} \hat{a}_u & \hat{a}_v & \hat{a}_w \\ \frac{1}{h_u}\frac{\partial}{\partial u} & \frac{1}{h_v}\frac{\partial}{\partial v} & \frac{1}{h_w}\frac{\partial}{\partial w} \\ H_u & H_v & H_w \end{vmatrix} \frac{1}{h_u h_v h_w}$

Here spherical coordinate system

$\therefore u\ v\ w = r\ \theta\ \phi$

and $h_u\ h_v\ h_w = 1\ r\ r\sin\theta$

$$\therefore \nabla \times \vec{H} = \begin{vmatrix} \hat{a}_r & \hat{a}_\theta & \hat{a}_\phi \\ \frac{\partial}{\partial r} & \frac{1}{r}\frac{\partial}{\partial \theta} & \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi} \\ H_r & H_\theta & H_\phi \end{vmatrix} \frac{1}{r^2\sin\theta}$$

$$\nabla \times \vec{H} = \frac{1}{r^2\sin\theta} \begin{vmatrix} \hat{a}_r & \hat{a}_\theta & \hat{a}_\phi \\ \frac{\partial}{\partial r} & \frac{1}{r}\frac{\partial}{\partial \theta} & \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi} \\ 6r\sin\phi & 0 & 18r\sin\theta\cos\phi \end{vmatrix}$$

$$\nabla \times \vec{H} = \frac{1}{r^2\sin\theta}\Big[\hat{a}_r\Big(\frac{1}{\cancel{r}}\frac{\partial}{\partial\theta}(18\cancel{r}\sin\theta\cos\phi) - 0\Big) - \hat{a}_\theta\Big(\frac{\partial}{\partial r}(18r\sin\theta\cos\phi) - \frac{1}{\cancel{r}\sin\theta}\frac{\partial}{\partial\phi}(6\cancel{r}\sin\phi)\Big) + \hat{a}_\phi\Big(0 - \frac{1}{r}\frac{\partial}{\partial\theta}(6r\sin\phi)\Big)\Big]$$

$$\nabla \times \vec{H} = \frac{1}{r^2\sin\theta}\Big[18\cos\theta\cos\phi\,\hat{a}_r - 18\sin\theta\cos\phi\,\hat{a}_\theta + \frac{6\cos\phi}{\sin\theta}\hat{a}_\theta - 0\,\hat{a}_\phi\Big]$$

$\vec{ds} = r\,d\theta\ r\sin\theta\,d\phi\ \hat{a}_r$

$\therefore \vec{ds} = r^2\sin\theta\,d\theta\,d\phi\,\hat{a}_r$

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$$\therefore \int_S (\nabla \times \vec{H}) \cdot \vec{ds} = \int_S \left[\frac{1}{r^2 \sin\theta} \left(18 \cos\theta \cos\phi \, \hat{a}_r - 18 \sin\theta \cos\phi \, \hat{a}_\theta + \frac{6\cos\theta}{\sin\theta} \hat{a}_\phi \right) \cdot \left(r^2 \sin\theta \, d\theta \, d\phi \, \hat{a}_r \right) \right]$$

$$\therefore \int_S (\nabla \times \vec{H}) \cdot \vec{ds} = \int_S \frac{1}{r^2 \sin\theta} \left(18 \cos\theta \cos\phi \, r^2 \sin\theta \, d\theta \, d\phi \right)$$

$$= 18 \int_{\theta=0}^{0.1\pi} \int_{\phi=0}^{0.3\pi} \cos\theta \cos\phi \, d\theta \, d\phi$$

$$= 18(4) \int_{\theta=0}^{0.1\pi} \cos\theta \left[\sin\phi \right]_0^{0.3\pi} d\theta$$

$$= 72 \int_{\theta=0}^{0.1\pi} \cos\theta \left[\sin(0.3\pi) - 0 \right] d\theta$$

$$= 58.25 \int_{\theta=0}^{0.1\pi} \cos\theta \, d\theta$$

$$= 58.25 \left[\sin\theta \right]_0^{0.1\pi}$$

$$= \underline{\underline{18 \text{ A}}}$$

doubt

UNIT - 5

Magnetic Forces and Time Varying Fields

Magnetic force experienced by a point charge in a magnetic field when stationary is zero. (because $v=0$).

CASE 1: Point charge

When a point charge moves with a velocity v in a magnetic field it experiences a magnetic force given by

$$\vec{F}_m = Q\, v\, B \sin\theta\, \hat{a}_N$$

$$\boxed{\vec{F}_m = Q\, \vec{v} \times \vec{B}}$$

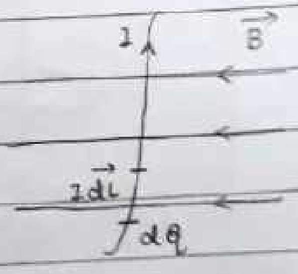
The force experienced is perpendicular to the direction in which the point charge is moving with velocity v and also perpendicular to the magnetic field.

When both electric field and magnetic field is present then the force experience is

$$F = F_e + F_m \qquad \text{(Lorentz Force equation)}$$

$$\boxed{\vec{F} = Q(\vec{E} + \vec{v} \times \vec{B}) \text{ in N.}}$$

CASE 2: A conductor carrying current I is placed in a magnetic field B.



Let us consider a current element $I\vec{dl}$ with a differential charge dQ

wkt $\vec{J} = \rho_v \vec{v}$

$$I\vec{dl} = \vec{J}\, dV$$

$$\therefore I\vec{dl} = \rho_v \vec{v}\, dV \qquad v\text{ - drift velocity}$$

$$I\vec{dl} = dQ.\vec{v}$$

The force experienced by the differential current element is

$$\therefore \vec{dF} = dQ.\vec{v} \times \vec{B}$$

$$\therefore \vec{dF} = I\vec{dl} \times \vec{B}$$

Hence the total force experienced is

$$\boxed{\vec{F} = \oint I\vec{dl} \times \vec{B}}$$

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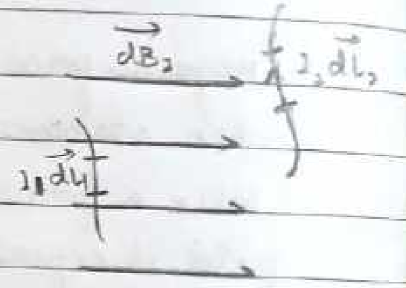
CASE 3: Let $\overrightarrow{dB}$ be the magnetic field produced by a current element $I_2\overrightarrow{dL_2}$. Then the force experienced by another differential current element $(I_1\overrightarrow{dL_1})$ due to this differential magnetic field is

$$d(d\vec{F_1}) = I_1\overrightarrow{dL_1} \times \overrightarrow{dB_2}$$

wkt $\vec{B} = \mu\vec{H}$

$\therefore \overrightarrow{dB} = \mu\,\overrightarrow{dH}$



By Biot Savart's law

$$\overrightarrow{dB} = \frac{\mu I\,\overrightarrow{dL} \times \hat{a}_R}{4\pi R^2}$$

Force experienced by differential current element $I_2\overrightarrow{dL_2}$

$$d\vec{F} = I_2\overrightarrow{dL_2} \times \vec{B}$$

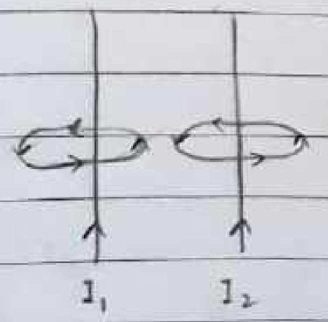
$$\therefore d(d\vec{F_1}) = \frac{I_1\overrightarrow{dL_1} \times \mu I_2\overrightarrow{dL_2} \times \hat{a}_{R21}}{4\pi R_{21}^2}$$

$$\vec{F_1} = \frac{\mu I_1 I_2}{4\pi}\oint_{L_1}\overrightarrow{dL_1} \times \oint_{L_2}\frac{\overrightarrow{dL_2} \times \hat{a}_{21}}{R_{21}^2}$$

Similarly

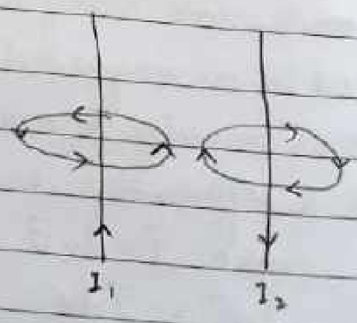
$$\vec{F_2} = \frac{\mu I_1 I_2}{4\pi}\oint_{L_2}\overrightarrow{dL_2} \times \oint_{L_1}\frac{\overrightarrow{dL_1} \times \hat{a}_{12}}{R_{12}^2}$$

a.



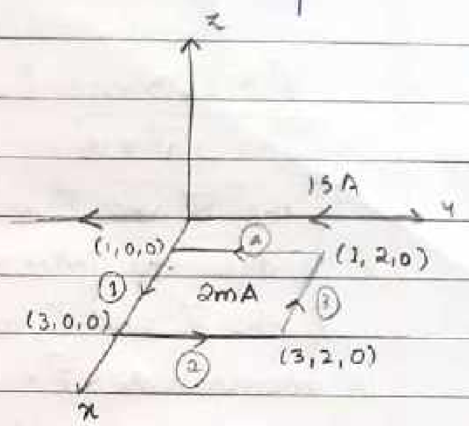
The conductors attract each other if the current in both the conductors is in same direction.

b.



The conductors repel each other if the current in both the conductors are in opposite direction.

Q1: We have a square loop of wire on a $z=0$ plane carrying 2mA with a field of an infinite filament on the y axis, with current 15 A. Find total force on the loop.



Sol: For infinite filament

$$\vec{H} = \frac{1}{2\pi\rho}\hat{a}_\phi$$

$$\hat{a}_\phi = \hat{a}_L \times \hat{a}_\rho$$

$$\hat{a}_\phi = -\hat{a}_y \times \hat{a}_x = \hat{a}_z$$

$$\therefore \vec{H} = \frac{I}{2\pi x}\hat{a}_z$$

$$\Rightarrow \vec{B} = \frac{\mu I}{2\pi x}\hat{a}_z$$

Force

$$\vec{F} = \oint_L I\,\vec{dL} \times \vec{B}$$

$$\vec{F} = \oint_L I\,\vec{dL} \times \frac{15\mu}{2\pi x}\hat{a}_z$$

$$\vec{F} = -\oint_L \frac{15\mu\,\hat{a}_z}{2\pi x} \times I\,\vec{dL}$$

$$\vec{F} = \frac{-15\mu(2m)}{2\pi}\oint_L \frac{\hat{a}_z \times \vec{dL}}{x}$$

$$\vec{F} = -6\times10^{-9}\left[\int_{x=1}^{3}\frac{\hat{a}_z}{x}\times dx\,\hat{a}_x + \int_{y=0}^{2}\frac{\hat{a}_z}{3}\times dy\,\hat{a}_y + \int_{x=3}^{1}\frac{\hat{a}_z}{x}\times dx\,\hat{a}_x + \int_{y=2}^{0}\frac{\hat{a}_z}{1}\times dy\,\hat{a}_y\right]$$

$$\vec{F} = -6\times10^{-9}\left[\int_{x=1}^{3}\frac{dx}{x}\hat{a}_y + \int_{y=0}^{2}\frac{-dy}{3}\hat{a}_x + \int_{x=3}^{1}\frac{dx}{x}\hat{a}_y + \int_{y=2}^{0}\frac{-dy}{1}\hat{a}_x\right]$$

$$\vec{F} = -6\times10^{-9}\left[\ln(3/1)\,\hat{a}_y - \frac{1}{3}(2-0)\hat{a}_x + \ln(1/3)\,\hat{a}_y - (0-2)\hat{a}_x\right]$$

$$\vec{F} = -6\times10^{-9}\left[\frac{4}{3}\hat{a}_x\right] = -8\,\hat{a}_x \text{ nN}$$

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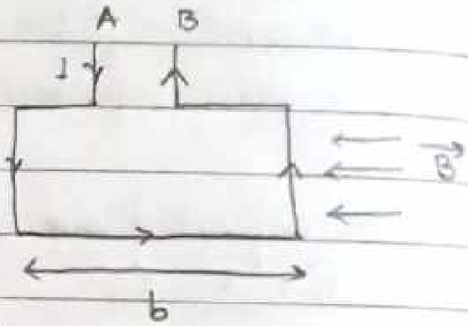
★ **Force and torque on a closed loop:**



wkt

$$\vec{F} = \oint_L I\, \vec{dl} \times \vec{B}$$

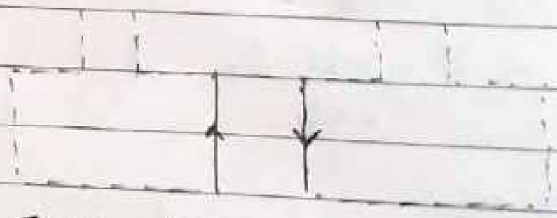
$$\vec{F} = ILB\sin\theta\, \hat{a}_N$$

$$\boxed{\vec{F} = Q\vec{v} \times B}$$

If the $\vec{B}$ and $\vec{I}$ are in same direction then the force experienced is zero i.e., $\vec{F} = 0$. $(\because \theta = 0)$

If the $\vec{B}$ and $\vec{I}$ are perpendicular then the force experienced is maximum $\vec{F} = BIL\,\hat{a}_N$ $(\because \theta = 90°)$

These forces lead to torque.



Force acts upwards   Force acts downwards

$$\tau = \vec{r} \times \vec{F}$$

$$\tau = \tau_1 + \tau_2$$

$$\tau = \frac{b}{2} \times \vec{F} + \frac{b}{2} \times \vec{F}$$

$$\tau = \frac{b}{2}F + \frac{b}{2}F$$

$$\tau = Fb = BILb = BIab$$

$$\tau = BIS$$

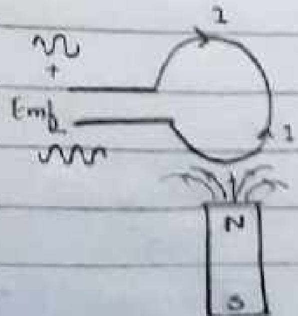
$$\Rightarrow \tau = \underbrace{mB \text{ or } \mu B}_{\text{dipole moment}}$$

$$\boxed{\tau = \vec{m} \times \vec{B}}$$

Application: current meter.

★ **Time varying Fields:** (using Faraday's laws)

CASE 1: Change in Magnetic Flux in a space the emf is induced and current is induced in the opposite direction to the current produced.



$$emf = \frac{-d\phi}{dt} \Rightarrow V_{emf} = \oint_L \vec{E} \cdot \vec{dL}$$

$$\phi = \int_S \vec{B} \cdot \vec{ds}$$

$$\therefore V_{emf} = \oint_L \vec{E} \cdot \vec{dL} = \frac{-d\phi}{dt}$$

$$V_{emf} = \oint_L \vec{E} \cdot \vec{dL} = -\frac{d}{dt}\left(\int_S \vec{B} \cdot \vec{ds}\right)$$

$$\oint_L \vec{E} \cdot \vec{dL} = -\int_S \frac{\partial}{\partial t}(\vec{B} \cdot \vec{ds}) \quad \text{—— (1)}$$

$$\oint_L \vec{H} \cdot \vec{dL} = \int_S (\nabla \times H) ds \quad : \text{point form}$$

Also $$\oint_L \vec{E} \cdot \vec{dL} = \int_S (\nabla \times E) ds \quad \text{—— (2)}$$

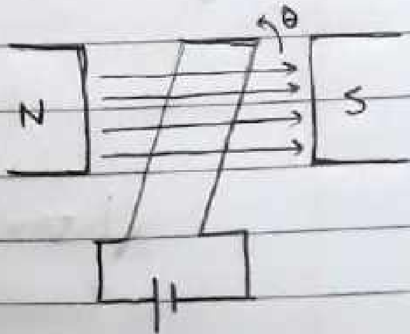
Hence from eq (1) and eq (2)

$$\int_S (\nabla \times E) ds = -\int_S \frac{\partial}{\partial t}(\vec{B} \cdot \vec{ds})$$

$$\boxed{\nabla \times E = -\frac{\partial B}{dt}}$$

The decreasing magnetic field produces the change in the electric field.

CASE 2: θ_1 changes to θ_2 as the plane rotates some distance dL.



wkt $\vec{F} = Q(\vec{V} \times \vec{B})$

$$dW = \vec{F} \cdot \vec{dL}$$
$$= Q(\vec{V} \times \vec{B}) \cdot \vec{dL}$$

$$W = \oint_L \vec{F} \cdot \vec{dL} = \oint_L (Q\vec{V} \times \vec{B}) \cdot \vec{dL}$$

$$\frac{W}{Q} = V_{emf} = \oint_L (\vec{V} \times \vec{B}) \vec{dL}$$

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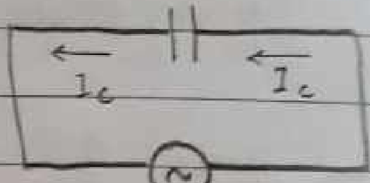
* **Displaced current or Modified Ampere's law:**

$$\oint \vec{H}\cdot\vec{dl} = I_{en} + I_d$$

$$I_{en} = \begin{cases} I_c \ ; & S_1 \\ 0 \ ; & S_2 \end{cases}$$

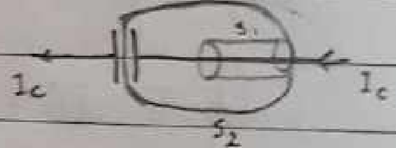


$V_0 = V_m \sin \omega t$

Point form of Ampere's law

$$\nabla \times H = J$$

$$\nabla \cdot \nabla \times H = \nabla \cdot J = -\frac{\partial \rho_v}{\partial t}$$



$$0 = \nabla \cdot J = -\frac{\partial \rho_v}{\partial t}$$

$$\nabla \cdot J \neq 0$$

$$\nabla \times H = J + G$$

$$\nabla \cdot (\nabla \times H) = \nabla \cdot J + \nabla \cdot G$$

$$0 = \nabla \cdot J + \nabla G$$

$$\frac{\partial \rho_v}{\partial t} = \nabla \cdot G$$

$$\nabla \cdot G = \frac{\partial}{\partial t}(\nabla \cdot D) \qquad (\because \nabla \cdot D = \rho_v)$$

$$\nabla \cdot G = \nabla \cdot \frac{\partial D}{\partial t}$$

$$\therefore G = \frac{\partial D}{\partial t}$$

$$\boxed{\nabla \times H = J + \frac{\partial D}{\partial t}}$$

$$\oint \vec{H}\cdot\vec{dl} = \int_s (\nabla \times H)\, ds$$

$$\int_s (\nabla \times H)\, ds = \int_s J \cdot ds + \int_s \frac{\partial D}{\partial t}\, ds$$

$$\boxed{\oint_s \vec{H}\cdot\vec{dl} = I_c + I_D}$$

* Maxwell's Equation :

1. Static Field:

Point form or Differential Form	Integral Form
→ $\nabla \cdot D = \rho_v$	$\oint \vec{D} \cdot \vec{ds} = Q$
→ $\nabla \cdot B = 0$	$\oint \vec{B} \cdot \vec{ds} = 0$
→ $\nabla \times H = J$	$\oint \vec{H} \cdot \vec{dl} = I_{en}$
→ $\nabla \times E = 0$	$\oint \vec{E} \cdot \vec{dl} = 0$

2. Time varying fields:

a. $$\oint_L \vec{E} \cdot \vec{dl} = -\int_s \frac{\partial B}{\partial t} ds$$

$$\oint \vec{H} \cdot \vec{dl} = \int_s (\nabla \times H) ds$$

or $$\int \vec{E} \cdot \vec{dl} = \int_s (\nabla \times E) ds$$

b. $$\int (\nabla \times E) ds = \int -\frac{\partial B}{\partial t} ds$$

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

c. $$\oint \vec{H} \cdot \vec{dl} = I_c + I_D$$

$$\oint \vec{H} \cdot \vec{dl} = I_c + \int_s \frac{\partial D}{\partial t} ds$$

$$\int (\nabla \times H) ds = \int_s J ds + \int_s \frac{\partial D}{\partial t} ds$$

$$\nabla \times H = J + \frac{\partial D}{\partial t}$$

$\frac{\partial D}{\partial t}$ → Displacement current density.

d. $\oint_s \vec{D} \cdot \vec{ds} = Q$

$$\oint_s \vec{D} \cdot \vec{ds} = \int_v \rho_v \, dv$$

$$\oint_s \vec{D} \cdot \vec{ds} = \int_v (\nabla \cdot D \, dV)$$

$$\nabla . D = \rho_v$$

e. $\oint_s \vec{B} \cdot \vec{ds} = \int_s (\nabla \cdot B) \, ds$

$$\nabla . B = 0$$

UNIT - 6

Electromagnetics Waves

★ **WAVE PROPAGATION IN FREE SPACE:**

A time varying current produces a time varying magnetic field which generates electricfield in the direction normal to its orientation. Hence producing a uniform plane wave.

From Maxwell's Equation

$$\nabla \times E = \frac{-\partial B}{\partial t}$$

$$\nabla \times E = -\mu_0 \frac{\partial H}{\partial t} \quad \text{——— (1)} \quad \text{because } B = \mu_0 H$$

$$\nabla \times H = J + \frac{\partial D}{\partial t}$$

In free space $J = 0$

$$\therefore \nabla \times H = \varepsilon_0 \frac{\partial E}{\partial t} \quad \text{——— (2)} \quad \text{because } D = \varepsilon_0 E$$

From eq (1)

$$\nabla \times \nabla \times E = \nabla \times \left(-\mu_0 \frac{\partial H}{\partial t} \right)$$

$$\nabla \cdot (\cancel{\nabla \cdot E}) - \nabla^2 E = -\mu_0 \frac{\partial}{\partial t} (\nabla \times H) \quad \left[\text{wkt } \nabla \cdot E = \rho_v \text{ but } \rho_v = 0 \right]$$

(free space)

Substituting eq (2)

$$-\nabla^2 E = -\mu_0 \frac{\partial}{\partial t} \left(\varepsilon_0 \frac{\partial E}{\partial t} \right)$$

$$\nabla^2 E = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2} \qquad c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

$$\therefore \nabla^2 E = \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2}$$

$$\nabla^2 E = \frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + \frac{\partial^2 E}{\partial z^2} = \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2} \quad \text{——— (3)}$$

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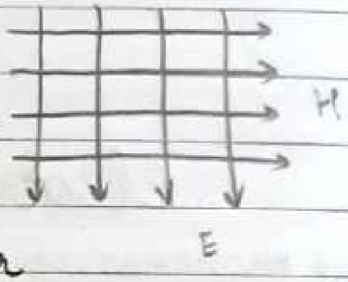
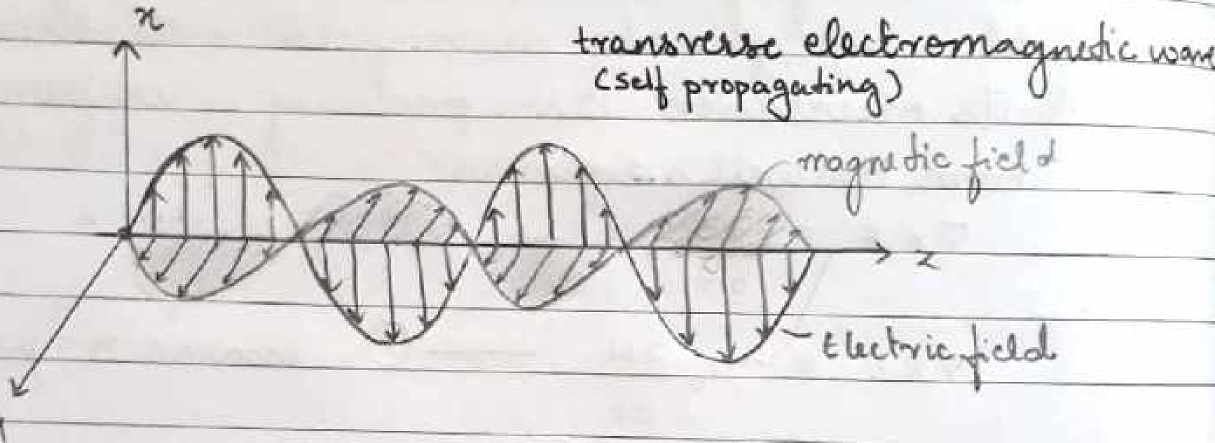
But magnetic and electric fields are in same plane but are perpendicular to each other. and the uniform plane propagates in the perpendicular direction to them.





Here let us assume an electric field is varying in x direction, hence thee y and z components is zero.

Therefore from eq (3) $\qquad E = E_x \hat{a}_x \quad E_y = 0 \quad E_z = 0$

$$\nabla^2 E = \frac{\partial^2 E}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2}$$

$$\text{i.e.,} \quad \frac{\partial^2 E_x}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2 E_x}{\partial t^2} \quad \text{—— (4)}$$

similarly $\nabla^2 H = \frac{\partial^2 H_y}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2 H_y}{\partial t^2}$

because magnetic field varies in y direction, hence the x and z components are zero i.e., $H = H_y \hat{a}_y \quad H_x = 0 \quad H_z = 0$

$E = E(r)e^{j\omega t}$ where $E(r) = E_{x_0} e^{-jkz}$

diff partially wrt t

$$\frac{\partial E}{\partial t} = E(r)\, j\omega\, e^{j\omega t}$$

$$\frac{\partial^2 E}{\partial t^2} = E(r)\, j\omega\, j\omega\, e^{j\omega t} = -E(r)\,\omega^2 e^{j\omega t} = -\omega^2 E$$

substituting in eq (4)

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$$\frac{\partial^2 E_x}{\partial z^2} = \frac{1}{c^2}(-\omega^2 E)$$

Here $\frac{\omega}{c} = k$; in freespace $k = k_0$

$$\therefore \frac{\partial^2 E_x}{\partial z^2} = -k^2 E$$

$$\frac{\partial^2 E_x}{\partial z^2} + k^2 E = 0$$

$$\boxed{\nabla^2 E + k^2 E = 0}$$ Helmholtz Equation

$$(\nabla^2 + k^2) E = 0$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) E + k^2 E = 0$$

$$\frac{\partial^2 E_x}{\partial x^2} + k^2 E_x = 0$$

$$\left[\frac{\partial^2 E_x}{\partial x^2} = -k^2 E_x\right]$$ is the representation of the assumed electromagnetic wave.

the solution for the above equation is given by

$$E_x(z) = \underbrace{E_{x_0} e^{-jk_0 z}}_{\text{Forward propagation}} + \underbrace{E'_{x_0} e^{jk_0 z}}_{\text{Backward Propagation}}$$: phasor form equation of a wave

$$E_x(z,t) = E_{x_0} e^{-jk_0 z} e^{j\omega t} + E'_{x_0} e^{jk_0 z} e^{j\omega t}$$

$$= E_{x_0} e^{j(\omega t - k_0 z)} + E'_{x_0} e^{j(\omega t + k_0 z)}$$

$$\boxed{E_x(z,t) = E_{x_0} \cos(\omega t - k_0 z + \phi_1) + E'_{x_0} \cos(\omega t + k_0 z + \phi_2)}$$

Similarly

$$\boxed{H_y(z,t) = H_{y_0} \cos(\omega t - k_0 z + \phi_1) + H'_{y_0} \cos(\omega t + k_0 z + \phi_2)}$$

wave number in free space : $k_0 = \omega/c$ rad/m

wave length in free space : $\lambda = \frac{2\pi}{k_0}$

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★ Relation between E and H:

wkt in free space

$E_x(z,t) = E_{x0} \cos(\omega t - k_0 z) + E_{x0}' \cos(\omega t + k_0 z)$

considering

$E_x(z,t) = E_{x_0} \cos(\omega t - k_0 z)$: forward propagation.

$\tilde{E}_x = E_x(z,t) = E_{x_0} e^{-jk_0 z} e^{j\omega t}$ — (1)

$\tilde{H}_y = H_y(z,t) = H_{y_0} e^{-jk_0 z} e^{j\omega t}$ — (2)

By maxwell's equation.

$$\nabla \times \tilde{E} = -\frac{\partial B}{\partial t}$$

$$\nabla \times \tilde{E} = -\mu_0 \frac{\partial \tilde{H}}{\partial t} \quad \text{— (3)}$$

From eq (1)

diff partially wrt z

$$\frac{\partial E_x}{\partial z} = E_{x_0} (-jk_0) e^{-jk_0 z} e^{j\omega t}$$

$$\therefore \frac{\partial \tilde{E}_x}{\partial z} = -jk_0 \tilde{E}_x$$

From eq (2)

diff partially wrt t

$$\frac{\partial H_y}{\partial t} = H_{y_0} (j\omega) e^{-jk_0 z} e^{j\omega t}$$

$$\therefore \frac{\partial \tilde{H}_y}{\partial t} = j\omega \tilde{H}_y$$

Substituting in eq (3), we get

$$\nabla \times \tilde{E} = -\mu_0 j\omega \tilde{H}_y$$

$$\tilde{H}_y = \frac{-1}{j\omega\mu_0} (\nabla \times \tilde{E})$$

Here $\nabla \times \tilde{E}_x = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ 0 & 0 & \partial/\partial z \\ E_x & 0 & 0 \end{vmatrix} = \frac{\partial E_x}{\partial z} \hat{a}_y$

$$\therefore \tilde{H}_y = \frac{-1}{j\omega\mu_0}\left(\frac{\partial E_x}{\partial x}\hat{a}_y\right)$$

$$\tilde{H}_y = \frac{-1}{\cancel{j}\omega\mu_0}\left(-\cancel{j}k_0\tilde{E}_x\right)$$

$$\tilde{H}_y = \frac{k_0\tilde{E}_x}{\omega\mu_0}$$

$$\frac{\tilde{E}_x}{\tilde{H}_y} = \frac{\omega\mu_0}{k_0}$$

but $k_0 = \frac{\omega}{c} = \frac{\omega}{1/\sqrt{\mu_0\varepsilon_0}} = \omega\sqrt{\mu_0\varepsilon_0}$

$$\frac{\tilde{E}_x}{\tilde{H}_y} = \frac{\cancel{\omega}\mu_0}{\cancel{\omega}\sqrt{\mu_0\varepsilon_0}}$$

$$\therefore \boxed{\eta_0 = \frac{\tilde{E}_x}{\tilde{H}_y} = \sqrt{\frac{\mu_0}{\varepsilon_0}}}$$

where η_0 is intrinsic impedance.

$$\eta_0 = \frac{\tilde{E}_x}{\tilde{H}_y} = 377\,\Omega$$

* **WAVE PROPAGATION IN DIELECTRIC MEDIUM:**

In metalic conductors : $\frac{\sigma}{\omega\varepsilon} >> 1 \Rightarrow \sigma >> 1$

In dielectric medium : $\frac{\sigma}{\omega\varepsilon} << 1 \Rightarrow \sigma << 1$

There are no free charge carriers, hence

$\rho_v = 0 \Rightarrow J = 0 \Rightarrow \sigma = 0$

From Faraday's law, we have

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\nabla \times E = -\mu\frac{\partial H}{\partial t} \quad \text{——— (1)}$$

because $B = \mu H$

$$\nabla \times H = J + \frac{\partial D}{\partial t}$$

$$\nabla \times H = \sigma E + \varepsilon\frac{\partial E}{\partial t} \quad \text{——— (2)}$$

$E = E_{x_0} e^{-jkz} e^{j\omega t} = E_{x_0} e^{j\omega t}$

diff partially wrt t

$\frac{\partial E}{\partial t} = E_{x_0} j\omega e^{j\omega t} = j\omega E$

$H = H_{y_0} e^{-jkz} e^{j\omega t} = H_{y_0} e^{j\omega t}$

diff partially wrt t

$\frac{\partial H}{\partial t} = H_{y_0} j\omega e^{j\omega t} = j\omega H$

From eq (1)

$\nabla \times E = -\mu \frac{\partial H}{\partial t}$

but $\frac{\partial H}{\partial t} = j\omega H$

$\therefore \nabla \times E = -\mu j\omega H$

From eq (2)

$\nabla \times H = \sigma E + \varepsilon \frac{\partial E}{\partial t}$

but $\frac{\partial E}{\partial t} = j\omega E$

$\therefore \nabla \times H = \sigma E + \varepsilon j\omega E$

Taking curl on both sides

$\nabla \times \nabla \times E = -\mu j\omega (\nabla \times H)$

$\nabla(\cancel{\nabla \cdot E}^{\,0}) - \nabla^2 E = -\mu j\omega (\sigma + \varepsilon j\omega) E$

because $\rho_v = 0 \Rightarrow J = 0$ as $\nabla \cdot D = \rho_v$ hence $\nabla \cdot E = 0$

$\therefore \nabla^2 E = \mu j\omega (\sigma + \varepsilon j\omega) E$

$\nabla^2 E = \frac{\partial^2 E}{\partial z^2} = \underbrace{\mu j\omega (\sigma + \varepsilon j\omega)}_{(jk)^2} E$: wave equation

$\therefore \nabla^2 E = \frac{\partial^2 E}{\partial z^2} = (jk)^2 E$

$E_x(z) = E_{x_0} e^{-jkz} + E_{x_0}' e^{jkz}$

$$\boxed{jk = \alpha + j\beta}$$

where k : propagation constant (complex value)

α : attenuation constant

β : phase constant.

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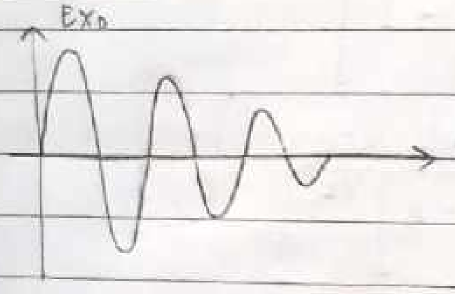
$$E_x(z) = E_{x_0} e^{-(\alpha+j\beta)z} = E_{x_0} e^{-\alpha z} e^{-j\beta z}$$

$$\boxed{E_x(z,t) = E_{x_0} e^{-\alpha z} \cos(\omega t - \beta z)}$$: real part



Amplitude reduces exponentially.

wkt $jk = \alpha + j\beta$

and $jk = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$

complex permittivity

$$\varepsilon = \varepsilon' - j\varepsilon'' = \varepsilon_0\varepsilon_r' - j\varepsilon_0\varepsilon_r''$$

As there are no charge carriers in dielectric medium $\sigma = 0$.

$$jk = \sqrt{j\omega\mu[\sigma + j\omega(\varepsilon' - j\varepsilon'')]} = \alpha + j\beta$$

squaring on both sides

$$j\omega\mu[\sigma + j\omega(\varepsilon' - j\varepsilon'')] = \alpha^2 - \beta^2 + 2j\alpha\beta$$

$$\alpha^2 - \beta^2 + 2j\alpha\beta = -\omega^2\mu\varepsilon' + j\omega^2\mu\varepsilon''$$

Equating the real and imaginary parts.

$$\alpha^2 - \beta^2 = -\omega^2\mu\varepsilon' \quad \text{—— (1)} \qquad 2\alpha\beta = \omega^2\mu\varepsilon'' \quad \text{—— (2)}$$

considering

$$(\alpha^2+\beta^2)^2 = \alpha^4 + \beta^4 + 2\alpha^2\beta^2$$

$$(\alpha^2-\beta^2)^2 = \alpha^4 + \beta^4 - 2\alpha^2\beta^2$$

$$\Rightarrow (\alpha^2+\beta^2)^2 - (\alpha^2-\beta^2)^2 = 4\alpha^2\beta^2$$

$$(\alpha^2+\beta^2)^2 = (\alpha^2-\beta^2)^2 + (2\alpha\beta)^2$$

substituting eq (1) and eq (2)

$$(\alpha^2+\beta^2)^2 = (-\omega^2\mu\varepsilon')^2 + (\omega^2\mu\varepsilon'')^2$$

taking square root on both sides.

$$\alpha^2+\beta^2 = \sqrt{(-\omega^2\mu\varepsilon')^2 + (\omega^2\mu\varepsilon'')^2} \quad \text{—— (3)}$$

Adding eq (1) and eq (3)

$$2\alpha^2 = \sqrt{(-\omega^2\mu\varepsilon')^2 + (\omega^2\mu\varepsilon'')^2} - \omega^2\mu\varepsilon'$$

$$2\alpha^2 = \sqrt{(\omega^2\mu\varepsilon')^2\left[1 + \left(\frac{\varepsilon''}{\varepsilon'}\right)^2\right]} - \omega^2\mu\varepsilon'$$

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$$2\alpha^2 = \omega^2\mu\varepsilon'\sqrt{\left[1+\left(\frac{\varepsilon''}{\varepsilon'}\right)^2\right]} - \omega^2\mu\varepsilon'$$

$$\alpha^2 = \frac{\omega^2\mu\varepsilon'}{2}\left[\sqrt{\left[1+\left(\frac{\varepsilon''}{\varepsilon'}\right)^2\right]} - 1\right]$$

Taking squareroot on both sides

$$\boxed{\alpha = \omega\sqrt{\frac{\mu\varepsilon'}{2}}\left[\sqrt{1+\left(\frac{\varepsilon''}{\varepsilon'}\right)^2} - 1\right]^{1/2}}$$

Similarly subtracting eq ① from eq ③

$$2\beta^2 = \sqrt{(-\omega^2\mu\varepsilon')^2 + (\omega^2\mu\varepsilon'')^2} + \omega^2\mu\varepsilon'$$

$$2\beta^2 = \sqrt{(\omega^2\mu\varepsilon')^2\left[1+\left(\frac{\varepsilon''}{\varepsilon'}\right)^2\right]} + \omega^2\mu\varepsilon'$$

$$2\beta^2 = \omega^2\mu\varepsilon'\sqrt{\left[1+\left(\frac{\varepsilon''}{\varepsilon'}\right)^2\right]} + \omega^2\mu\varepsilon'$$

$$\beta^2 = \frac{\omega^2\mu\varepsilon'}{2}\left[\sqrt{1+\left[\frac{\varepsilon''}{\varepsilon'}\right)} + 1\right]$$

Taking squareroot on both sides

$$\boxed{\beta = \omega\sqrt{\frac{\mu\varepsilon'}{2}}\left[\sqrt{1+\left(\frac{\varepsilon''}{\varepsilon'}\right)^2} + 1\right]^{1/2}}$$

Propagation velocity

$$v_p = \frac{\omega}{\beta}$$

Wavelength : $\lambda = \dfrac{2\pi}{\beta}$

Intrinsic impedance : $\eta = \sqrt{\dfrac{\mu}{\varepsilon}} = \sqrt{\dfrac{\mu}{\varepsilon' - j\varepsilon''}}$

but $\varepsilon'' = \sigma/\omega$

$$\therefore \eta = \sqrt{\frac{\mu}{\varepsilon' - j\sigma/\omega}} = \sqrt{\frac{\omega\mu}{\omega\varepsilon' - j\sigma}} = \sqrt{\frac{j\omega\mu}{j\omega\varepsilon' + \sigma}}$$

Dielectric Medium

- Good dielectric medium

$$\frac{\sigma}{\omega\varepsilon'} << 1.$$

- Perfect dielectric medium

$$\varepsilon'' = 0$$

$$\Rightarrow \alpha = 0 \text{ and } \beta = \omega\sqrt{\mu\varepsilon'}$$

* **WAVE PROPAGATION IN GOOD CONDUCTORS:**

$$\frac{\sigma}{\omega\varepsilon'} \text{ or } \frac{\varepsilon''}{\varepsilon'} >> 1 \Rightarrow \sigma >> \omega\varepsilon$$

Propagation constant

$$jk = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$$

as $\sigma >> \omega\varepsilon$ hence $j\omega\varepsilon$ can be neglected.

$$jk = \sqrt{j\omega\mu\sigma}$$

$$jk = \sqrt{\omega\mu\sigma}\cdot\sqrt{j}$$

$$jk = \sqrt{2\pi f\mu\sigma}\cdot\frac{1}{\sqrt{2}}[1+j]$$

$$0+j = 1\angle 90^\circ$$

$$\sqrt{j} = 1\angle 45^\circ$$

$$\sqrt{j} = \cos 45^\circ + j\sin 45^\circ$$

$$\sqrt{j} = \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}$$

$$\sqrt{j} = \frac{1}{\sqrt{2}}[1+j]$$

$$\Rightarrow \alpha + j\beta = \sqrt{\pi f\mu\sigma}\,(1+j)$$

because $jk = \alpha + j\beta$

Equating real and imaginary parts.

$$\alpha = \sqrt{\pi f\mu\sigma} \text{ and } \beta = \sqrt{\pi f\mu\sigma}$$

Wave equation is given by

$$E_x(z,t) = E_{x_0} e^{-\alpha z}\cos(\omega t - \beta z)$$

Substituting α and β, we get

$$E_x(z,t) = E_{x_0} e^{-\sqrt{\pi f\mu\sigma}\, z}\cos(\omega t - \sqrt{\pi f\mu\sigma}\, z)$$

at $z = 0$

$$E_x(z,t) = E_{x_0}\cos\omega t$$

Current density is given by

$$J_x = \sigma E_x$$

$$J_x = \sigma E_0 e^{-z\sqrt{\pi f\mu\sigma}}\cos(\omega t - z\sqrt{\pi f\mu\sigma})$$

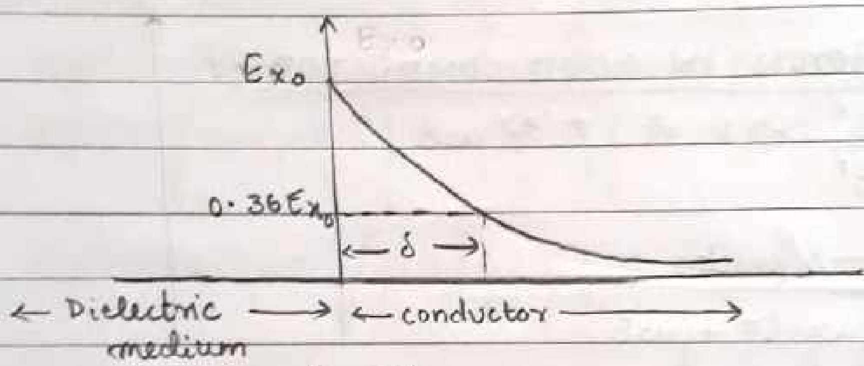
When $z = 1/\sqrt{\mu f \pi \sigma}$

then $e^{-z\sqrt{\pi f \mu \sigma}} = e^{-1} = 0.3678$

$$z = \frac{1}{\sqrt{\mu f \pi \sigma}} = \delta = \frac{1}{\alpha} = \frac{1}{\beta}$$

where δ is the depth of penetration / skin depth.

$$\therefore \boxed{E_x(z,t) = E_{x_0} e^{-z/\delta} \cos(\omega t - z/\delta)} \ \text{V/m}$$



propagation velocity

$$v_p = \frac{\omega}{\beta} = \omega\delta$$

Wavelength : $\lambda = \dfrac{2\pi}{\beta} = 2\pi\delta$

Intrinsic impedance

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon'}}$$