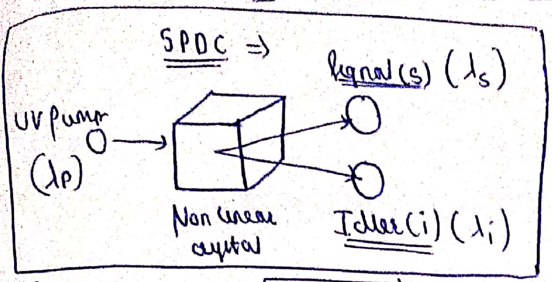
QT209 Assignment 6 ⇒

③ ⓐ (i) Non-degenerate SPDC process ⇒ ① Signal + idler photons are of different wavelengths
Generally frequency of signal photon > Frequency of idler photon

$\Rightarrow \boxed{\lambda_{signal} < \lambda_{idler}}$ — ①



② But, Energy conservatⁿ should be satisfied ⇒

$E_{pump} = E_{signal} + E_{idler}$

$h\nu_{pump} = h\nu_{signal} + h\nu_{idler}$

$$\boxed{\frac{1}{\lambda_{pump}} = \frac{1}{\lambda_{signal}} + \frac{1}{\lambda_{idler}}} \quad - ②$$

(i) Non degenerate SPDC ⇒ $\boxed{\lambda_s \neq \lambda_i}$

(ii) Degenerate SPDC ⇒ $\boxed{\lambda_s = \lambda_i}$

③ $\lambda_{pump} \Rightarrow 400\,nm$, $\boxed{\lambda_{signal} \; \& \; \lambda_{idler}}$ can take any values satisfying ①, ② + ③ $\boxed{k_{pump} = k_s + k_i}$ ⇒ Critical phase matching

$\boxed{n \Rightarrow \text{refractive index of non linear crystal}}$

$$\boxed{\frac{n(\lambda_p, \theta)}{\lambda_p} = \frac{n(\lambda_s)}{\lambda_s} + \frac{n(\lambda_i)}{\lambda_i}} - ③$$

(ii) Degenerate SPDC process ⇒

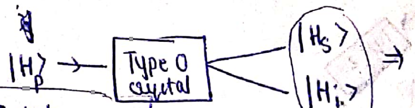
① Signal + idler photons are of same wavelengths.

$\Rightarrow \boxed{\lambda_s = \lambda_i}$

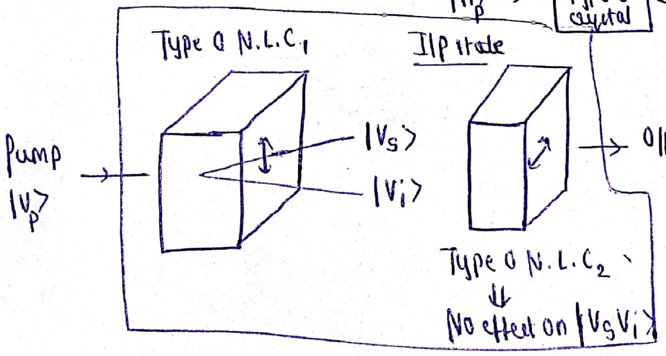
② $\boxed{E_{pump} = E_{signal} + E_{idler}} \Rightarrow \frac{1}{\lambda_{pump}} = \frac{1}{\lambda_s} + \frac{1}{\lambda_i} \Rightarrow \frac{1}{400} = \frac{2}{\lambda_s} \Rightarrow \boxed{\lambda_s = \lambda_i = 800\,nm}$

ⓑ Type 0 non linear optical crystal ⇒ Input (pump) states + O/P states (idler + signal) which we get when we pass i/p photon through non linear crystal as same i.e $\boxed{\text{Pump state} = \text{Signal state} = \text{Idler state}}$

ex ⇒

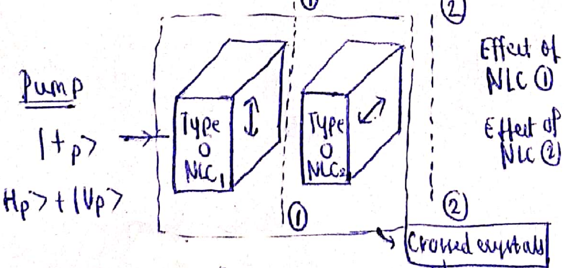


⇒ O/P state ⇒ $|H_s H_i\rangle$



O/P ⇒ $\boxed{|V_s V_i\rangle \Rightarrow \text{Not an entangled state}}$

To create an entangled state ⇒ We can use pump state ⇒ $\frac{|H_p\rangle + |V_p\rangle}{\sqrt{2}} \Rightarrow |+_p\rangle$



Effect of NLC ① ⇒ @ ①-① ⇒ $|V_p\rangle \longrightarrow |V_s V_i\rangle$

Effect of NLC ② ⇒ @ ②-② ⇒ $|H_p\rangle \longrightarrow |H_s H_i\rangle$

∴ $\boxed{\text{final O/P state} \Rightarrow \frac{|V_s V_i\rangle + |H_s H_i\rangle}{\sqrt{2}}}$ ⇒ entangled state

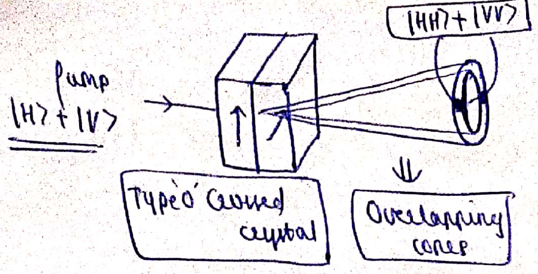
∴ for two Type 0 NLC (Non linear crystal)

we can't generate an entangled state @ O/P if I/P state is $|V\rangle$ or $|H\rangle$

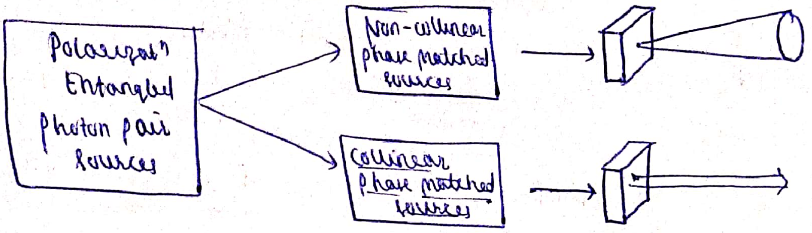
we can generate entangled state @ O/P only when I/P state is $\boxed{|+\rangle_p \text{ or } |-\rangle_p}$ or $\boxed{\alpha|H_p\rangle + \beta|V_p\rangle}$

$\boxed{\text{Note}}$ ⇒ ① $\boxed{\text{We can generate an entangled state for i/p } |V\rangle \text{ or } |H\rangle, \text{ provided the two crystals used are Type 2}}$

ex ⇒ I/P state ⇒ $|V_p\rangle$, ⇒ $\boxed{\text{O/P state} \Rightarrow \frac{|V_s H_i\rangle + |H_s V_i\rangle}{\sqrt{2}}}$



(c)

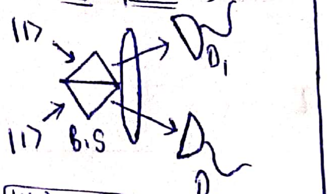


⇒ Photons pairs emitted in cone

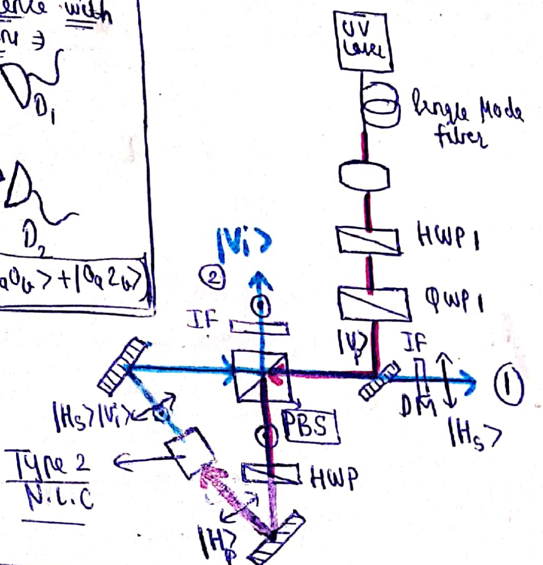
⇒ Signal & idler photons co-propagate

(1) Schematic Using Non collinear-sources ⇒ Polarization Sagnac Interferometer ⇒

HOM Interference with single photons ⇒



$$|\psi\rangle_{o/p} = \frac{1}{\sqrt{2}}\left(|2_a 0_b\rangle + |0_a 2_b\rangle\right)$$



HWP ⇒ Half wave plate

DM ⇒ Dichroic Mirror

PBS ⇒ Polarizing Beam Splitter

(1) & (2) ⇒ Coincidence detector

IF ⇒ Interference filter

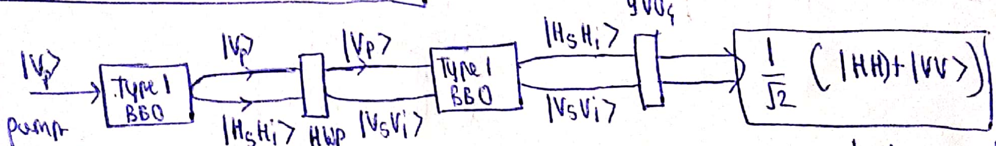
$$|\psi\rangle_{o/p} = \frac{1}{\sqrt{2}}\left(|H_s\rangle_1 |V_i\rangle_2 + e^{i\phi}|V_s\rangle_1 |H_i\rangle_2\right)$$

In photon number state $|\psi\rangle = \frac{1}{\sqrt{2}}\left(|2,0\rangle + |0,2\rangle\right)$

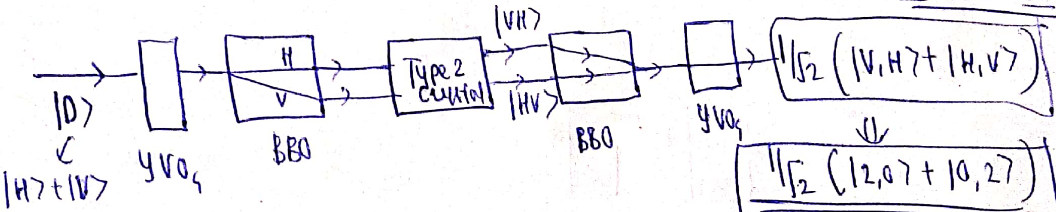
↳ Required State

(*) Expt. Setup for polarization Sagnac interferometer type II down conversion

(2) Schematic Using Collinear Sources ⇒



↳ Not asked state



↳ Required State

② ⓐ Signal to Noise Ratio $\Rightarrow$ $\boxed{SNR = \frac{S}{N}}$ $S \Rightarrow$ signal

$N \Rightarrow$ Noise

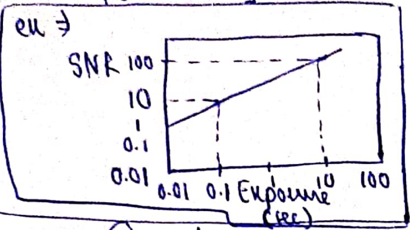
Input - SNR $\Rightarrow$ $\boxed{SNR_{in} = \frac{S_{in}}{N_{in}}}$ $\Rightarrow$ $S_{in} \Rightarrow$ Mean No. of photons seen by detector $= \underline{10} = \mu$

$N_{in} \Rightarrow$ Expected Noise from signal $\Rightarrow \sqrt{S_{in}} = \sqrt{\mu}$

$\therefore SNR_{in} = \frac{\mu}{\sqrt{\mu}} = \sqrt{\mu} = \sqrt{10} = \underline{3.16}$

ⓑ Exposure $\Rightarrow$ Amount of light (i.e no. of photons) detected by detector (sensor) per sec

By increasing the Exposure time in seconds the SNR value can be increased. $\Rightarrow$ $\boxed{SNR \propto \text{Exposure in seconds}}$



for $\uparrow$ SNR to 31.6 $\Rightarrow \mu_{new} \approx 1000$,

we know for given setup $\underline{\mu = 10}$

$\therefore$ Exposure time should be increased by $\boxed{100 \text{ sec}}$ in sec $\rightarrow$ $\mu_{new} = 10 \times 100 = \underline{1000}$

ⓒ Assuming Poissonian distribution for arrival of photons emitted by weak coherent light source,

Probability to emit n photons $\Rightarrow$ $\boxed{P_n = \frac{e^{-\mu}\mu^n}{n!}}$

$\mu \Rightarrow$ Mean photon number of each emitted pulse

$P_n \Rightarrow$ Probability to emit n photons

$P_n = \frac{e^{-10} \times 10^n}{n!}$, $P_1 = \frac{e^{-10} \times 10^1}{1!} = \boxed{4.5399 \times 10^{-5}}$

ⓓ In general, attenuated level is set so that intensity of emitted pulse approaches single photon regime $\Rightarrow$ Criteria $\Rightarrow$ $\boxed{\mu = \sum_{n=0}^{\infty} P_n n < 1}$

Attenuation $\Rightarrow$ Reduces intensity of light $\rightarrow$ Reduces no. of photons per sec

Attenuation Factor $= 10 = \frac{\text{I/P intensity}}{\text{O/P intensity}} \approx \frac{\mu_{i/p}}{\mu_{o/p}} \Rightarrow \boxed{\mu_{o/p} = \frac{\mu_{i/p}}{10} = \frac{10}{10} = \underline{1}}$

Ideal case of true single photon source $\Rightarrow$ $\boxed{P_1 = 1, \ P_{n \neq 1} = 0}$

$\because P_1$ for $\mu = 1 \Rightarrow P_1 = \frac{e^{-1} \times 1}{1!} = \boxed{0.3678}$

$\because P_1 + P_{n \neq 1} = 1 \Rightarrow \boxed{P_{n \neq 1} = 0.6322}$

$\therefore$ No, even if laser source is attenuated by factor of 10, it would not become ideal single photon source

Ideally, for $P_1 = 1 \Rightarrow \frac{e^{-\mu}\mu^1}{1!} \Rightarrow 1 = \frac{\mu}{e^{\mu}} \Rightarrow \boxed{\mu = e^{\mu}} \rightarrow$ $\boxed{\text{No real sol}^n \text{ of } \mu \text{ for this equat}^n}$

Practically, $\Rightarrow$ $\boxed{\mu < 1}$ $\Rightarrow$ intensity of emitted pulse approaches single photon regime

$\therefore$ For $\boxed{\text{Attenuation factor} > 10}$ $\Rightarrow$ $\boxed{\text{intensity of emitted pulse approaches single photon regime}}$

ⓔ Detection efficiency $\Rightarrow 50\% \Rightarrow \eta_D$

$\therefore$ Mean no. of photons detected by detector $\Rightarrow \mu \times 0.5 = 10 \times 0.5 = \underline{5}$

$\therefore$ Mean no. of photons = Mean no. electrons $= 0.5 \times 10 = \underline{5}$

Again Assuming Poisson distribution $\Rightarrow$ Probability distribution funct^n for electrons $= \boxed{P_{en} = \frac{e^{-\mu}\mu^n}{n!} = \frac{e^{-5}5^n}{n!}}$

① Raw key rate depends on ① Mean photon no. (μ)
② Channel transmission efficiency $\eta_{ch} = \exp(-\alpha . L)$
③ Detector efficiency ($\eta_{detector}$)

$\therefore \boxed{R_{SK} \sim \mu\, \eta_{ch}\, \eta_{det}} \Rightarrow R_{raw}$, & $\boxed{\text{Sifted key rate} \Rightarrow 1/2\, R_{raw}}$

ⓐ Given parameters $\Rightarrow$ $\alpha = 0.2\, dB/km$, $P_{optical} = 1 \times 10^{-15}\, W$, $\lambda = 1550 \times 10^{-9}\, m$

$\eta_D = 0.25$, $\Delta t = 1 \times 10^{-6}\, s$, $P_{error}^{optical} = 0.1$

$\boxed{\text{Dark count rate} = 10^{+3}\ \text{counts}/s} = 10^{-3}$ counts / μsec

$P_{optical} = \frac{N_{Sent}\, hc}{\lambda} \Rightarrow N_S = \frac{\lambda P_o}{hc} = \frac{1550 \times 10^{-9} \times 1 \times 10^{-15}}{6.63 \times 10^{-34} \times 3 \times 10^{8}} = 7.79 \times 10^{-3}$ photons / μsec

$P_{received} = \frac{N_r\, hc}{\lambda}$

Propagation loss / km (α) $\times$ Length of optical fibre (L) $= 10 \log_{10} \left(\frac{P_{optical}}{P_{received}} \right)$

$$\alpha \times L = 10 \log_{10} \left(\frac{N_S}{N_r} \right)$$

$$0.2 L = 10 \log_{10} \left(\frac{N_S}{N_r} \right) \Rightarrow \frac{N_S}{N_r} = 10^{0.02L}$$

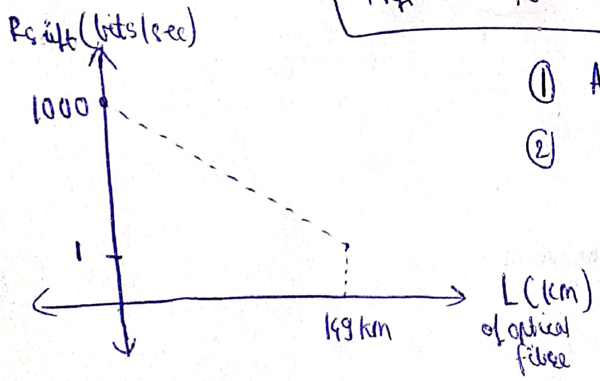
$$\therefore \boxed{N_r = N_S \times 10^{-0.02L}\ \text{photons} / \mu sec}$$

$$\therefore R_{sift} = 1/2\, \eta_D \times N_r$$

$$= 1/2 \times 0.25 \times N_S \times 10^{-0.02L} = \underline{0.97375} \times 10^{-3} \times 10^{-0.02L}\ \text{per}\ \mu sec$$

$$\therefore R_{sift} = 10^{-(0.02L + 3.02)}\ \text{per}\ \mu sec$$

$$\boxed{R_{sift} = 10^{(2.98 - 0.02L)}\ \text{bits per sec}}$$



① At $L = 0 \Rightarrow R_{sift} = 10^{2.98} \simeq \underline{1000}$ bits per s

② At $R_{sift} = 1 = 10^{(2.98 - 0.02L)} \Rightarrow 2.98 = 0.02L$

$\boxed{L = 149\, km}$

$\boxed{\text{As } L \uparrow \Rightarrow R_{sift} \downarrow}$

Hence, depending on the distance of communication no. of sifted key bits per sec $\downarrow$ as the dist. $\uparrow$

Hence there is a length of communication limit if ~~no.~~ R_{sift} value is fixed for given above parameters

(b) $QBER = \dfrac{\text{No. of wrong bits (after sifting)}}{\text{No. of total bits (after sifting)}}$

for single detector $(n=1)$ ⇒

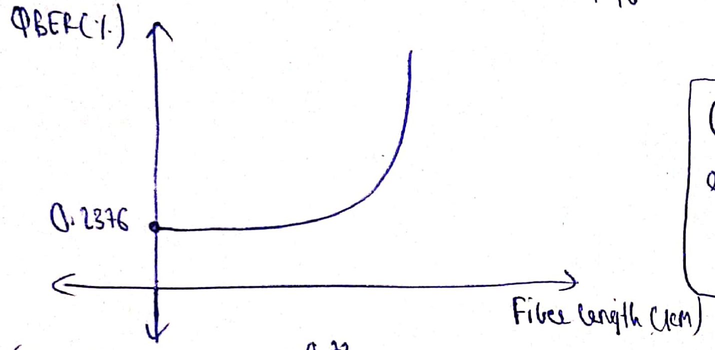
$$= \frac{P_{detection} \cdot P_{optical\,error} + n \times 0.5 \times P_{dark}}{P_{detect^n} + n \cdot P_{dark}}$$

$$\therefore \quad P_{detect^n} = N_r \eta_0 \Delta t = 7.79 \times 10^{-3} \times 10^{-0.02L} \times 0.25 \times 1 \times 10^{-6}$$

$$= 1.9475 \times 10^{-(0.02L+3)} = 10^{-(0.02L+2.72)}$$

$$QBER = \frac{10^{-(0.02L+2.72)} \times 0.1 + 0.5 \times 10^{-3}}{10^{-(0.02L+2.72)} + 10^{-3}}$$

$$\therefore \quad QBER = \frac{10^{-(0.02L+3.72)} + 0.5 \times 10^{-3}}{10^{-(0.02L+2.72)} + 10^{-3}} = \boxed{\frac{10^{-(0.02L+0.72)} + 0.5}{10^{-(0.02L-0.28)} + 1}} \Rightarrow \frac{N}{D}$$



① As $L \uparrow$ both N & $D \downarrow$ but $D \downarrow$ at faster rate than N, & hence QBER slowly $\uparrow$ with length initially

② At $L \to \infty \Rightarrow QBER \approx 1$

QBER in % ⇒ 100%

$$(QBER)@L=0 = \frac{10^{-0.72} + 0.5}{10^{+0.28} + 1} = \frac{0.6905}{2.905} = \underline{0.2376}$$