

Question 1:

The intervals between the rewards hold significance. **The sign of the rewards ensure the presence of adequate intervals while making the reward structure more intuitive.** The discounted return for a state-action pair is given as:

$$R_t = \sum_{k=0}^{\infty} \gamma^k r_{t+k+1}$$

If we add a constant C to all the rewards in the above equation we get

$$\begin{aligned} R_t' &= \sum_{k=0}^{\infty} \gamma^k (r_{t+k+1} + C) \\ R_t' &= \sum_{k=0}^{\infty} [\gamma^k r_{t+k+1} + \gamma^k C] \\ R_t' &= \sum_{k=0}^{\infty} \gamma^k r_{t+k+1} + \sum_{k=0}^{\infty} \gamma^k C \\ R_t' &= R_t + \sum_{k=0}^{\infty} \gamma^k C \end{aligned}$$

We observe that adding a constant C to each step reward causes the discounted return to increase by a constant K, where

$$K = \sum_{k=0}^{\infty} \gamma^k C$$

This is evidence that adding a constant value to the rewards does not affect the relative values of the states. This supports the earlier claim that intervals are more significant in the reward structure as opposed to the signs of individual rewards, which can be done away with by adding a constant value to all the rewards.