Homework 4:

Tuesday, April 9, 2024 18:58

1) $\overline{(A \uparrow A) \downarrow (B \uparrow B)} \equiv (\bar{A} \downarrow \bar{A}) \uparrow (\bar{B} \downarrow \bar{B})$

$\Rightarrow \overline{(\bar{A} \downarrow \bar{B})}$

$\Rightarrow \bar{\bar{A}} \uparrow \bar{\bar{B}}$

$\Rightarrow A \uparrow B$

$\Rightarrow \overline{(\bar{A} \downarrow \bar{A})} \uparrow \overline{(\bar{B} \downarrow \bar{B})}$

$\Rightarrow \bar{\bar{A}} \uparrow \bar{\bar{B}}$

$\Rightarrow A \uparrow B$

$\therefore$, Proved.

2) $(A \uparrow A) \uparrow (((A \uparrow B) \uparrow (A \uparrow B)) \uparrow ((A \uparrow B) \uparrow (A \uparrow B))) = A$

$\Rightarrow \bar{A} \uparrow (AB \uparrow AB)$

$\Rightarrow \overline{\bar{A} \uparrow (AB)}$

$\Rightarrow \bar{\bar{A}} \downarrow \overline{AB}$

$\Rightarrow A \downarrow (\bar{A} \downarrow \bar{B})$

$\Rightarrow A \downarrow \bar{A}$ is true

$\Rightarrow$ True $\downarrow \bar{B}$

$\Rightarrow$ True

3) Given:- $B\bar{A} \Rightarrow$ False

$\bar{B} + A \Rightarrow$ True

$B = BA$

$(A+\bar{B})(\bar{A}+A\bar{B}) + \bar{A}B(A+B)$

$A\bar{A} + AA\bar{B} + \bar{A}\bar{B} + A\bar{B}\bar{B} + A\bar{A}B + \bar{A}BB$

$A\bar{A} + A\bar{B} + \bar{A}\bar{B} + A\bar{B} + | A\bar{A}B + \bar{A}B$

$A\bar{B} + \overline{AB} + \overline{ABB}$

$A\bar{B} + \bar{B}$

$\bar{B}$

$B \Rightarrow A$

4&5) Distributivity:

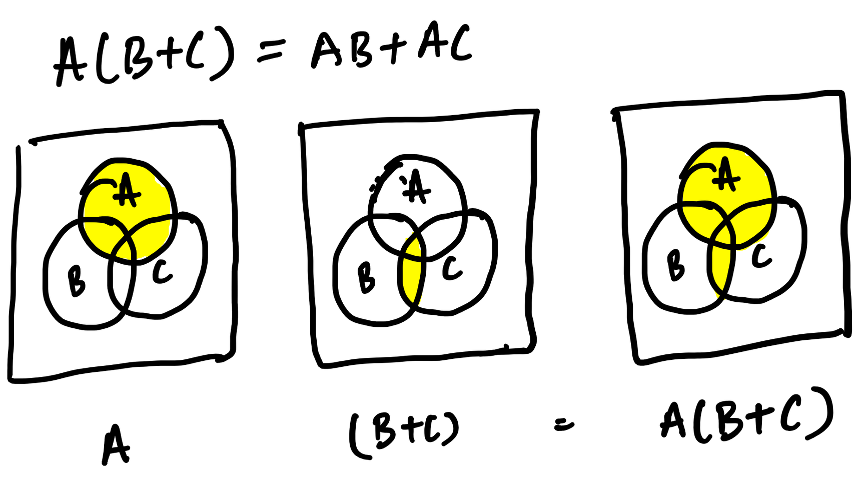
$A(B+C) = AB+AC$, $A+BC = (A+B)(B+C)$

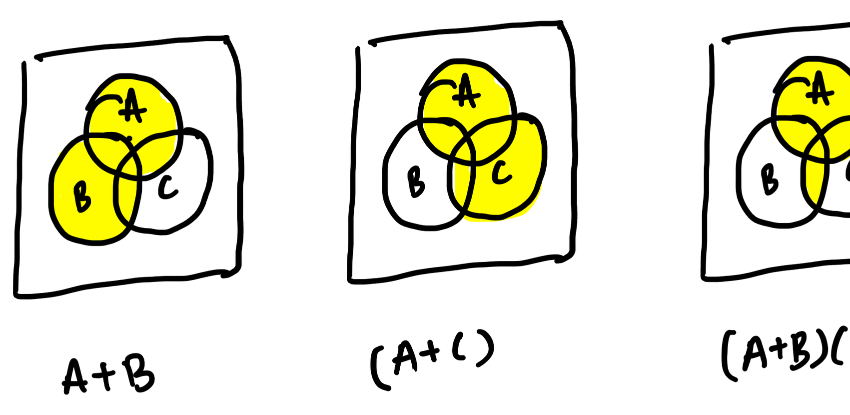
A	B	C	B+C	A(B+C)	AB	AC	AB+AC	A+B	A+C	(A+B)(A+C)	BC	A+BC
T	T	T	T	T	T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	T	T	T	T	F	T
T	F	T	T	T	F	T	T	T	T	T	F	T
T	F	F	F	F	F	F	F	T	T	T	F	T
F	T	T	T	F	F	F	F	T	T	T	T	T
F	T	F	T	F	F	F	F	T	F	F	F	F
F	F	T	T	F	F	F	F	F	T	F	F	F
F	F	F	F	F	F	F	F	F	F	T	F	T

$A(B+C) \equiv AB+AC$ and $A+BC = (A+B)(B+C)$

$\therefore$, The identity holds

$A(B+C) = AB+AC$





Duality:

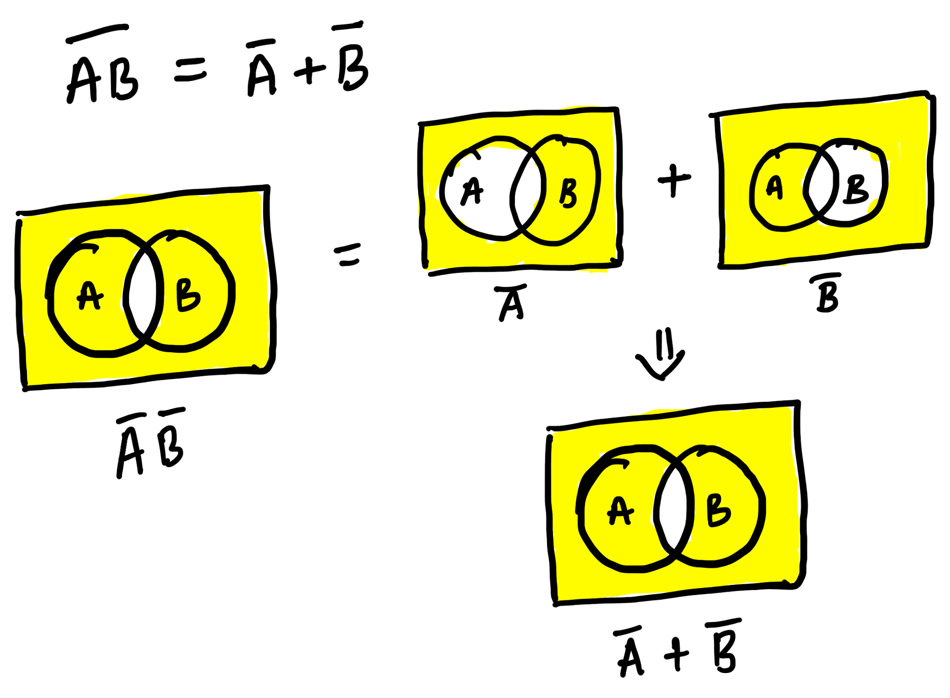
If $C = AB$ then $\bar{C} = \bar{A} + \bar{B}$

If $D = A+B$ then $\bar{D} = \bar{A}\bar{B}$

A	B	AB=C	$\bar{C}$	$\bar{A}$	$\bar{B}$	$\bar{A}+\bar{B}$	D=A+B	$\bar{D}$	$\bar{A}\bar{B}$
T	T	T	F	F	F	F	T	F	F
T	F	F	T	F	T	T	T	F	F
F	T	F	T	T	F	T	T	F	F
F	F	F	T	T	T	T	F	T	T

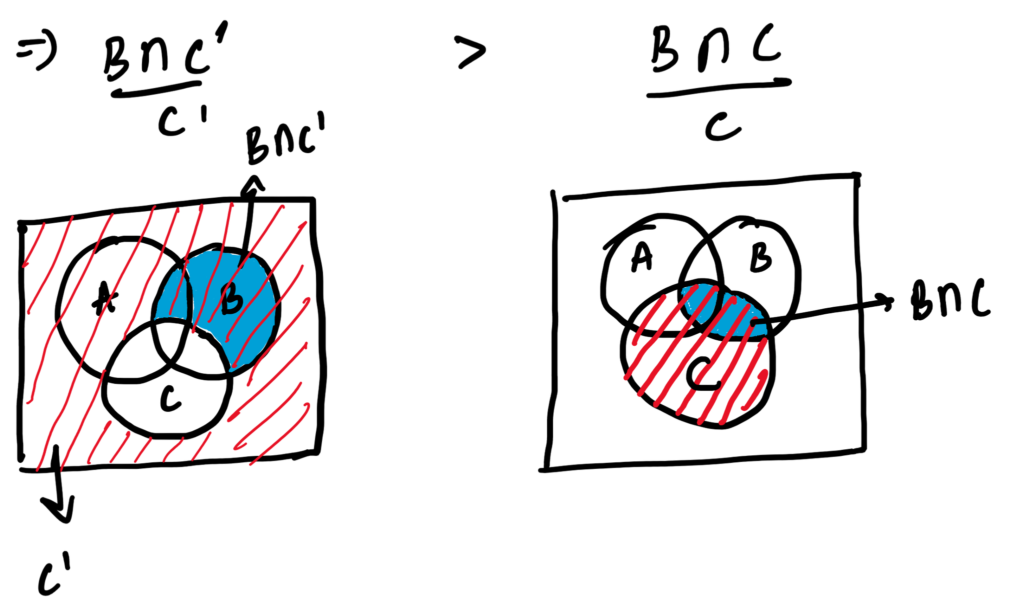
$\therefore$, From the table above the identity holds.

$\overline{AB} = \bar{A} + \bar{B}$



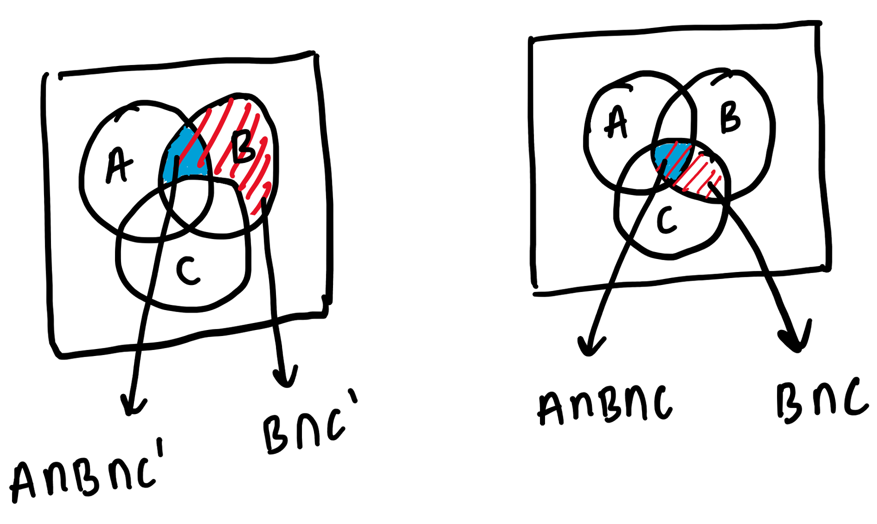
6) (i) $B|C' > B|C$

$\Rightarrow \frac{B \cap C'}{C'} > \frac{B \cap C}{C}$

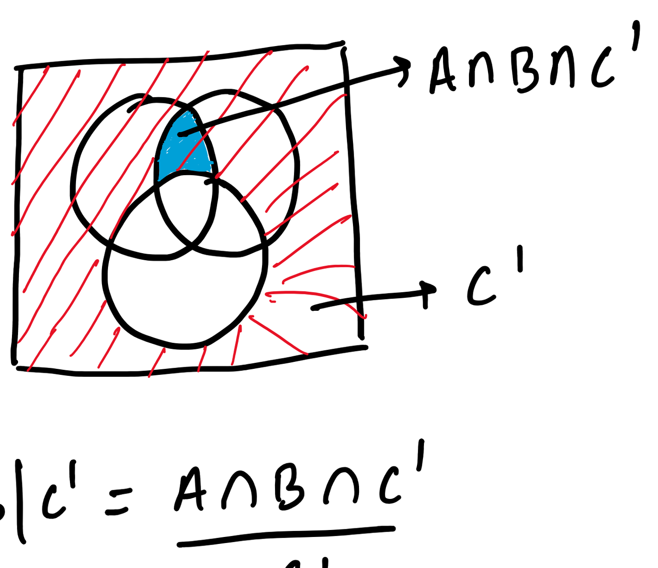


(ii) $A|BC' = A|BC$

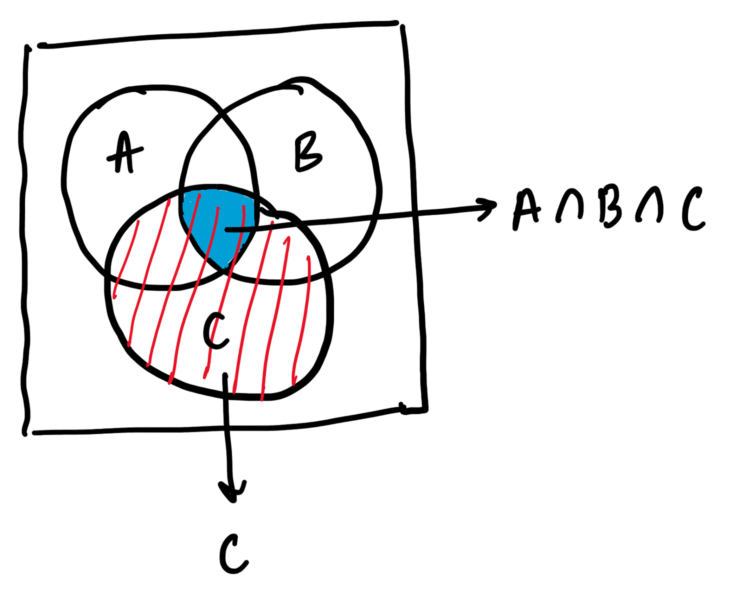
$\frac{A}{B \cap C'} = \frac{A \cap B \cap C}{B \cap C}$



To prove:



$AB|C' = \frac{A \cap B \cap C'}{C'}$



$AB|C$

7) Three fundamental desiderata of Probability Theory are:

↳ Additivity $[P(A+B) = P(A) + P(B)]$

↳ Non-Negativity $[P(A) \geq 0]$

↳ Normalization $[\sum P(A) = 1]$