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A Comparative Study on Thresholding Methods in Wavelet-based Image Denoising

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Abstract

Wavelet-based image denoising is an important technique in the area of image noise reduction. Wavelets have their natural ability to represent images in a very sparse form which is the foundation of wavelet-based denoising through thresholding. This paper explores properties of several representative thresholding techniques in wavelets denoising, such as VisuShrink, SureShrink, BayesShrink and Feature Adaptive Shrinkage. A quantitative comparison between these techniques through PSNR (Peak Signal-to-Noise Ratio) is also given.

Keywords: Discrete wavelet transform; image denoising; thresholding

1. Introduction

In the diverse fields, scientists are faced with the problem of recovering original images from incomplete, indirect and noisy images. Traditionally, we can find many noise reduction methods, many of them are in spatial or frequency domain by filtering [1-7]. Spatial Low-pass filters will not only smooth away noise but also blur edges in signals and images while the high-pass filters can make edges even sharper and improve the spatial resolution but will also amplify the noisy background. The conventional Fast Fourier Transform (FFT) based image denoising method is essentially a low pass filtering technique in which edge is not as sharp in the reconstruction as it was in the original image--the edge information is

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spread across frequencies because of the FFT basis functions, which are not being localized in time or space [8].

The localized nature of the wavelet transforms both in time and space results in denoising with edge preservation. The discrete wavelet transform (DWT) can decompose an image into a form with a series of coefficients. Small coefficients are dominated by noise, while coefficients have a large absolute value carry more signal information than noise. Replacing noisy coefficients below a certain threshold value by zero may lead to a noise removed result. Early work on thresholding the DWT transform coefficients can be found in [9].

In this paper, properties of several representative thresholding techniques are explored, the thresholding techniques we investigated are VisuShrink, SureShrink, BayesShrink and Feature Adaptive Shrinkage. The performance of these thresholding methods in wavelet denoising is also evaluated. The paper is organized as follows. Section 2 describes fundamental concepts of wavelet transform and wavelet based denoising. Section 3 explains several thresholding techniques in wavelet denoising. Experiments and results are given in Section 4. Finally a conclusion is given in Section 5.

2. Discrete Wavelet Transform and Image Denoising

The discrete wavelet transform (DWT) is identical to a hierarchical sub-band system where the sub-bands are logarithmically spaced in frequency and represent octave-band decomposition. By applying a two dimensional DWT to an image, the image *s* is divided into four sub-bands, these four sub bands arise from convolving rows and columns with low-pass filter L and high-pass filter H and down sampling by two. This kind of two-dimensional DWT leads to a decomposition of approximation coefficients CA_j at level j in four components: the approximation CA_{j+1} at level $j + 1$ and the details in three orientations: horizontal, vertical, and diagonal, if we denote A_j as the approximation coefficients in level j, and D_i as all the detail coefficients in three directions, we can obtain the following:

$$S = A_j + \sum_{i}^{j} D_i$$

(1)

As we have mentioned in before, noise are mainly dominates the detail coefficients in DWT, that is D_i in (1), if we can set a reasonable threshold λ, we can drop all the noise contaminated detail coefficients to zero to remove noise from our images. The following question is how we use the threshold to reduce noise in an image and the crucial ingredient in this procedure is: how to decide the value of the threshold λ? There are many threshold selection methods exist, we will explain and compare some typical methods of them in detail.

3. Threshold selection for Wavelet Coefficients

The threshold selection is the core of the whole wavelet shrinkage. There are many threshold selection methods exist, but in this paper, we mainly focus on some representative ones. They are VisuShrink, SURE, BayesShrink and Feature Adaptive Shrinkage.

- VisuShrink

VisuShrink is thresholding by applying the Universal threshold proposed by Donoho and Johnstone [9]. This threshold is given by

$$\lambda_{UT} = \sigma\sqrt{2logM} \qquad (2)$$

where σ is the noise variance and M is the number of pixels in the image. It is proved that the maximum of any M values i.i.d. as $N(0, \sigma^2)$ will be smaller than the universal threshold with high probability, with the probability approaching 1 as the value of M increases. Thus, with high probability, a pure noise signal is estimated as being identically 0.

- SURE Shrink

The SURE (Stein's Unbiased Risk Estimate) method can generate thresholds λ_n under a risk rule which minimizes Stein's Unbiased Risk Estimate with the shrinkage function and with the level of multiresolution [10]. For example, the threshold on level j with soft thresholding can be written as:

$$\lambda_j^S = \arg\min_{\lambda \geq 0} SURE^S(\lambda, W_{j,k}) \tag{3}$$

The SureShrink is an automatic procedure in which λ_j^S is estimated from decomposition coefficients $W_{j,k}$ at level j to minimize the unbiased estimate of MSE:

$$SURE_j(\lambda, W_{j,k}) = k\sigma^2 + \sum_{k=1}^{k} \min(W_{j,k}^2, \lambda^2) - 2\sigma^2 \sum_{k=1}^{k} \{|W_{j,k}| \leq \lambda\} \tag{4}$$

For the range $0 \leq \lambda \leq \sqrt{2\log k}$, where k=$2^j$. However if most of the wavelet coefficients are zeros, the universal threshold is to be preferred.

- BayesShrink

The BayesShrink rule uses a Bayesian mathematical framework for images to produce subband-dependent thresholds which are nearly optimal for soft thresholding [11]. The threshold for every subband j is:

$$\lambda_j = \frac{\hat{\sigma}^2}{\hat{\sigma}_x} \tag{5}$$

where $\hat{\sigma}^2$ is the estimate noise variance, and $\hat{\sigma}_x{}^2$ is the estimated signal variance on the subband j. The noise variance is estimated as the median absolute deviation of the diagonal detail coefficients on level 1, the HH_1 subband by:

$$\hat{\sigma}^2 = \left[\frac{\mathrm{median}(|W_{j,k}|)}{0.6745}\right]^2 \tag{6}$$

The estimate of the signal standard deviation is

$$\hat{\sigma}_x = \sqrt{\max(\hat{\sigma}_y{}^2 - \hat{\sigma}^2, 0)} \tag{7}$$

where

$$\hat{\sigma}_y{}^2 = \frac{1}{N_j} \sum_{j=1}^{N_j} w_j^2 \tag{8}$$

is an estimate of the variance of the observations, with N_j being the number of the wavelet coefficients w_j on the subband j. This method has been proposed for use with soft thresholding.

- Feature Adaptive Shrinkage (FAS)

By finding the energy in localize area in the wavelet domain, we can get the information of edge or structural information of signal. The smoother the image, the lower is the energy. The discontinuity detection is based on finding the energy in a window (R x R). The Feature Adaptive shrinkage rule and shrinkage function are (9) and (10) [12]:

$$\lambda_{j,k}^2 = \frac{1}{R^2}\sum_{m=-R}^{R}\sum_{n=-R}^{R} d_{m,n}^2 \tag{9}$$

$$\hat{d}_{j,k} = \begin{cases} d_{j,k}\left(1-\alpha\frac{\lambda^2}{s_{j,k}^2}\right), & \text{if } s_{j,k}^2 \geq \beta\lambda^2 \\ 0, & \text{else} \end{cases} \tag{10}$$

where $\lambda^2 = 4\sigma^2 \log R$ and $d_{j,k}$ is the central pixel of window. It should be mention that if $d_{j,k}$ is at the left or right boundary of level j wavelet coefficients, the boundary conditioning is required. Here [18] uses a window (R x R) for finding $s_{j,k}^2$. The α and β are found using a set of test images which serve as training samples, α =0.1 and β is found as 0.3.

4. Experiments and Results

The wavelet domain denoising thresholding algorithms compared are VisuShrink, SUREShrink, BayesShrink and Feature Adaptive Shrink. In our implementation, hard thresholding, soft thresholding and semi-soft thresholding are used to evaluate these threshold selection algorithms. The wavelet used in our experiment is discrete Meyer filter four levels, since this filter result in better denoising performance [13], we test the differences between different decomposition levels. The experiments are conducted on several natural gray scale test images like Lena, Barbara of size 512 × 512 at different noise levels σ = 10,20,30,35. We also compare the wavelet-based denoising methods with two spatial denoising methods which are mean filtering and median filtering. The objective quality of the reconstructed image is measured by:

$$PSNR = 10\log\frac{255^2}{MSE}\,dB \tag{11}$$

where MSE is the mean square error between the original and the denoised image with size m×n:

$$MSE = \frac{1}{m\times n}\sum_{i=1}^{m}\sum_{j=1}^{n}[x(i,j)-\hat{x}(i,j)]^2 \tag{12}$$

The corresponding PSNR of each method is in Table 1.

Table 1. PSNR comparative results of different denoising algorithms

	Visu Hard	Visu Soft	B-M hard	B-M soft	Semi soft	SURE	Bayes	FAS	Mean Filtering	Median Filtering
	Lena									
$\sigma = 10$	35.35	34.45	35.52	34.64	36.29	33.46	**37.29**	**36.78**	36.07	34.79
$\sigma = 20$	34.19	33.71	34.70	34.24	34.31	33.22	**35.85**	**34.73**	33.63	32.14
$\sigma = 30$	33.64	33.27	33.98	33.85	33.10	33.00	**34.97**	**33.45**	32.13	30.62
$\sigma = 35$	33.38	33.11	33.64	33.68	32.67	32.91	**34.61**	**33.00**	31.59	30.03
	Barbara									
$\sigma = 10$	31.37	30.82	31.33	30.84	32.97	30.32	**34.12**	**34.29**	31.99	31.28
$\sigma = 20$	30.72	30.44	31.08	30.70	31.45	30.21	**32.48**	**32.25**	30.90	29.93

$\sigma = 30$	30.41	30.23	30.75	30.53	30.60	30.11	**31.61**	**31.24**	30.04	28.93
$\sigma = 35$	30.31	30.16	30.59	30.47	30.26	30.06	**31.27**	**30.75**	29.63	28.50
	Boat									
$\sigma = 10$	33.21	33.25	32.98	32.10	34.70	31.10	**35.50**	**35.08**	34.59	33.76
$\sigma = 20$	32.21	31.57	32.54	31.87	33.02	30.98	**34.03**	**33.29**	32.77	31.58
$\sigma = 30$	31.60	31.17	32.08	31.62	31.95	30.83	**33.20**	**32.21**	31.49	30.15
$\sigma = 35$	31.40	31.04	31.87	31.52	31.53	30.78	**32.88**	**31.81**	30.94	29.60

5. Conclusion

In this paper, we evaluated several thresholding methods in image denoising. The test images corrupted with white Gaussian noise. We considered a set of thresholding methods of wavelet coefficients as well as the more traditional approaches using spatial filters.

Based on our experiments, we conclude that BayesShrink and Feature-Adaptive Shrink are the best wavelet-based denoising methods in methods we considered. None of other methods in our experiments produced lesser MSE or higher PSNR than them.

We also compared the wavelet-based methods with two spatial filter-based methods. Our results indicate that wavelet-based methods can give better results than the spatial ones. But actually we must say, to combine the wavelet-based methods and spatial filter-based methods can generate a good result, the feature adaptive shrink is such a method.

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