Assignment 2: Heuristic, Tabu Search and Simulated Annealing
(Due Date : June 30,2016- midnight)

This assignment can be approached in groups. Each group consists of two students.

Question 1:
Assume a 25 × 25 two-dimensional maze. Each square in the maze has an (x,y) coordinate, with the bottom-left corner being (0,0) and the top-right corner being (24,24). Each position in the maze can be either empty or blocked. In addition, there are two "special" positions, the starting position and the exit position, both of which are selected randomly.
The agent can only move up, down, left or right, but never diagonally. It also cannot enter blocked positions or move outside the maze. Its objective is to find a path from its starting position to the exit position, preferably the cheapest one. The cost of a path is the number of positions the agent has to move through, including the starting and exit position.

An example of a maze is given in Figure 1.
Requirements

- Implement code to find a path from the starting position to the exit position using (1) Breath-First Search, (2) Depth-First Search, and (3) A* Search. For the A* Search, you must define an appropriate heuristic function, and justify your choice. Your implementation should output information about the search, including the complete path, its cost, and the number of nodes explored (or squares checked) before finding it.

The above three search heuristics represent the minimum requirements for this question.
Deliverables
You are expected to turn in a short paper (the shorter the better) which should include the following:

1 A short description of your implementations of the search methods. In particular, for the A* Search, explain and justify your chosen heuristic function.
2 Sample output of each search method you implemented on the maze of Figure 1.
3 You will have to test each search technique three times:
3.1 With the agent starting at S and ending at E1
3.2 With the agent starting at S and ending at E2
3.3 With the agent starting at (0,0) and ending (24,24)

Question 2:
The game of Conga was developed by Matin Franke, and published by "Das Spiel Hamburg" in 1998. It is a two-player game played on a square 4X4 board, as shown in Figure1.

Player 1

(1,4)	(2,4)	(3,4)	(4,4)
(1,3)	(2,3)	(3,3)	(4,3)
(1,2)	(2,2)	(3,2)	(4,2)
(1,1)	(2,1)	(3,1)	(4,1)

Player 2

Figure 1: A Conga board and two players.

Initially, Player 1 has ten black stones in (1,4) and Player 2 has ten white stones in (4,1). The players alternate turns. On each turn, a player chooses a square with some of his stones in it, and picks a direction to move them, either horizontally, vertically or diagonally. The move is done by removing the stones from the square and placing one stone in the following square, two in the next one, and the others into the last one. The stones can only be moved in consecutive squares that are not occupied by the opponent; if a direction has less than three squares not occupied by the opponent in a row, then all remaining stones are placed in the last empty square. If a square has no neighbouring squares that are not occupied by the opponent, then the stones in that square cannot be moved.

You can move stones from a square to a (series of) neighbouring square(s), provided the squares you are moving to are not occupied by the opponent. It makes no difference if the squares are free or occupied by yourself. You do not need any of the squares you are moving to be free; they could all already be occupied by your other stones. The only distinction is between squares that are occupied by the opponent (which block you) and squares that are not occupied by the opponent (which you can move to). For example, let's say you have a number of stones in square (1,4) and you want to move them right.

There can be four possible cases:
1) The opponent occupies square (2,4). You cannot move right. That is the example in Figure 4.
2) The opponent occupies square (3,4), but square (2,4) is either free or occupied by yourself. You move all the stones from (1,4) to (2,4).
3) The opponent occupies square (4,4), but squares (2,4) and (3,4) are either free or occupied by yourself. You move one stone from (1,4) to (2,4), and all other stones in

(3,4). That is the example in Figure 3.
4) The opponent doesn't occupy any squares in that row, and all squares are either free or occupied by yourself. You move one stone from (1,4) to (2,4), two stones to (3,4), and all other stones in (4,4). That is the example in Figure 2.
The goal of the game is to block the opponent, so that he has no legal moves. In other words, all of the opponents' stones must be trapped in squares that are all surrounded by the player's stones.

Requirements
For this question, you must design and implement a computer program to play the Conga game. The agents implemented should use the Minimax search algorithm and Alpha-Beta pruning, as well as some evaluation function to limit the search. They should also have a reasonable response time. Since it is unlikely that you will discover an optimal evaluation function on the first try, you will have to consider several evaluation functions and present them in your report.
If you wish, you may implement improvements of the basic Minimax/Apha-Beta agent, such as an iterative deepening Minimax agent or the Null-Window Alpha-Beta pruning, for bonus points.
Your implementation should output information about the search, including the depth and the number of nodes explored.
In order to write the report, you will also need to implement a Random Agent, or an agent that always plays a random legal move.

Deliverables
You are expected to turn in a short paper (the shorter the better) which should include the following:
1) A short description of your program.
2) A description of your implementation of the search agent.
3) An analysis and comparison of the different evaluation functions you implemented, based on relevant criteria, along with appropriate conclusions.
4) Sample output of your agent using the "best" (according to your previous analysis) evaluation function you implemented, playing against the Random
Agent. Explain and/or justify key moves your agent makes.
You must also send by email the code you implemented. All this material must be handed over to the TA in charge of this project, prior to the due date.

Your grade in this question will be function of the quality and completeness of the report you hand in. Handing in the project after the deadline gives you automatically 0.
Notes on the game:
1. While researching the game of Conga, you may realize that there is a second victory condition. A player can win by making a line four squares long containing the same number of stones. This victory condition has been removed from the game for this project, in order to simplify the evaluation function.
2. A consequence of this simplification is that the game is now much harder to win.
In fact, two players of equal strength can play for hours without being able to end the game. However, it is still possible for a player to defeat a player of lesser skill, or for an

agent to defeat another agent with a shallower search and a less accurate evaluation function. And it is definitely possible for a rational agent to defeat the Random Agent, in as little as 30 moves.

3. Given the requirement of making your agent play against the Random Agent, you may come up with the idea of somehow optimizing their agent to play better against that opponent specifically, by exploiting its random, non-optimal play. That is not acceptable for this project. The goal of the project is to design an agent that can play rationally against any opponent, not one that is optimized to play only against the Random Agent.

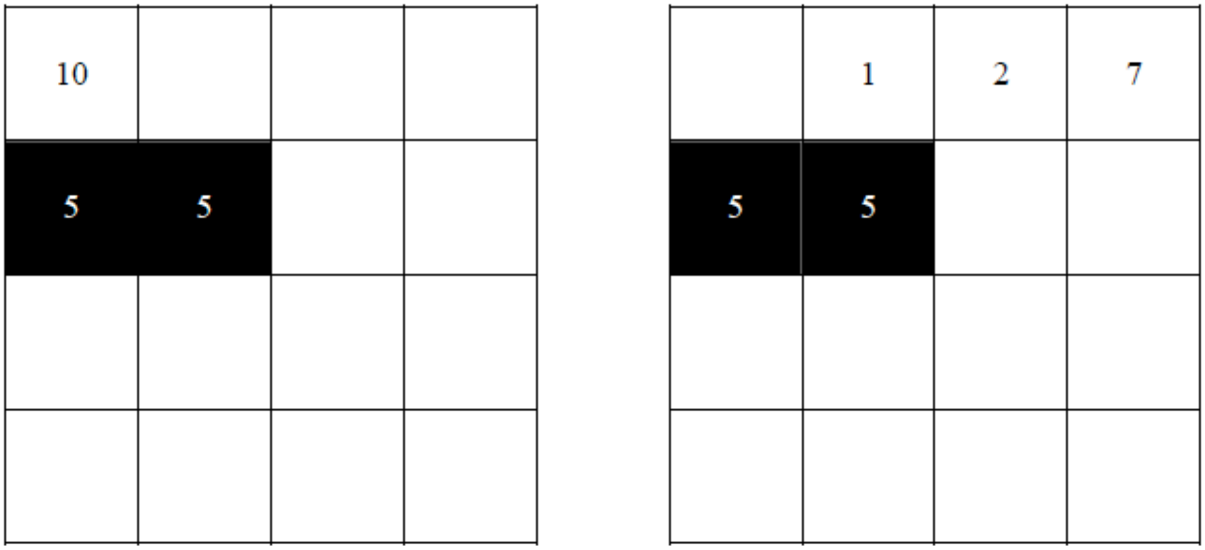


Figure 2: From the setup of the left board, white has only one legal move. The result of that move is shown in the right board.

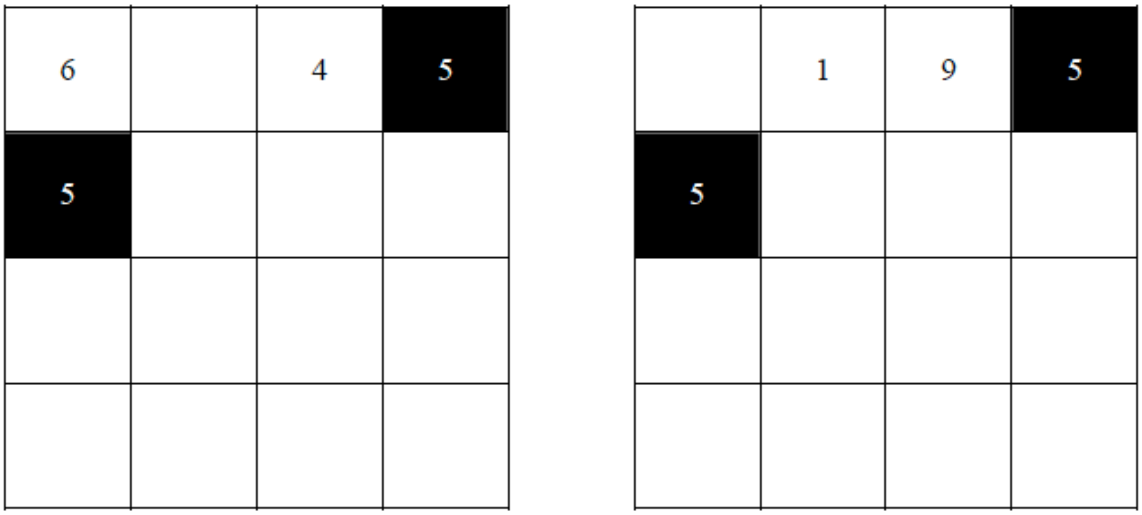


Figure 3: From the setup of the left board, white has six legal moves, and decides to move (1,4) to the right. The result of that move is shown in the right board.

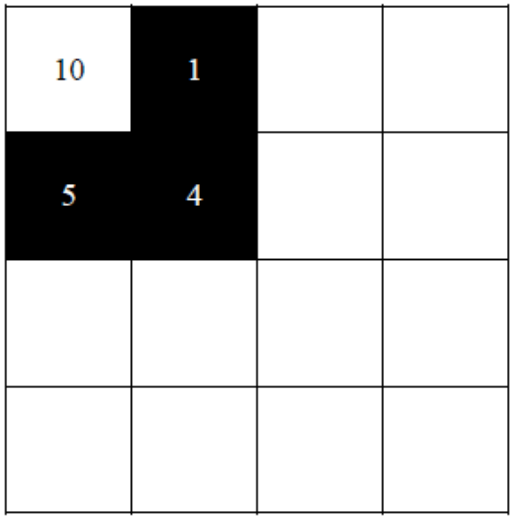


Figure 4: In the setup of this board, white has no legal moves. Black has won.

Question 3:

This is one of the QAP (quadratic assignment problem) test problems of Nugent et al. 20 departments are to be placed in 20 locations with five in each row (see below). The objective is to minimize costs between the placed departments. The cost is (flow * rectilinear distance), where both flow and distance are symmetric between any given pair of departments. Below are the flow and distance matrices, respectively. The optimal solution is 1285 (or 2570 if you double the flows).

3.3.1 Layout of department locations

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20

1. Code a simple TS to solve the problem. To do this you need to encode the problem as a permutation, define a neighborhood and a move operator, set a tabu list size and select a stopping criterion. Use only a recency based tabu list and no aspiration criteria at this point.
2. Run your TS.
3. Perform the following changes on your TS code (one by one) and compare the results.
   - Change the initial starting point (initial solution) 10 times
   - Change the tabu list size – smaller and larger than your original choice
   - Change the tabu list size to a dynamic one – an easy way to do this is to choose a range and generate a random uniform integer between this range every so often

(i.e., only change the tabu list size infrequently)
- Add one or more aspiration criteria such as best solution so far, or best solution in the neighborhood, or in a number of iterations
- Use less than the whole neighborhood to select the next solution
- Add a frequency based tabu list and/or aspiration criteria (designed to encourage the search to diversify)

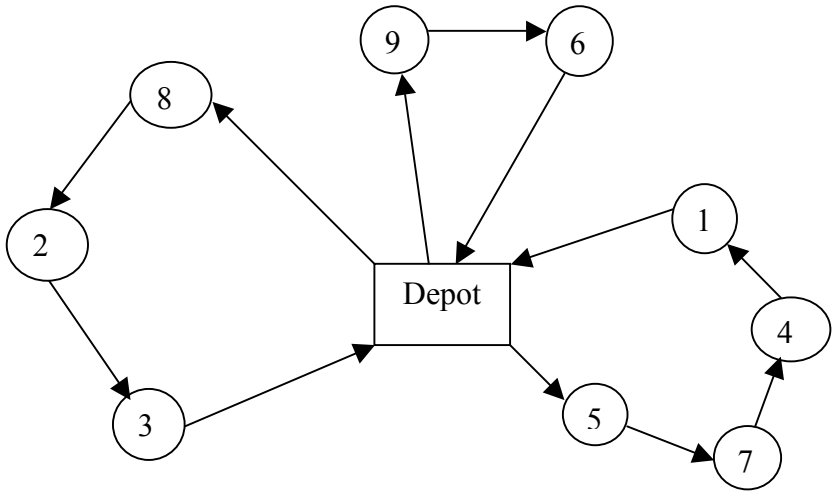
4. Requirements and deliverables (as above)

Question 4:

Simulated Annealing can be used to solve The Vehicle Routing Problem (VPR) defined by having *m* vehicles at a depot that need to service customers in ***c*** cities. The travel distances between every two cities are defined by a matrix ***D*** with element d_{ij} denoting distance between cities *i* and *j*. The travel distances from the depot to each of the cities are given by a vector ***Depot***. Each customer *j* has a service time s_j The VPR consists of determining the routes to be taken by a set of ***m*** vehicles satisfying the following conditions:

- Starting and ending at the depot,
- Having minimum cost (travel+service),
- Each customer is visited exactly once by exactly one vehicle.

The figure below illustrate possible routes for a 9 customers and 3 vehicles problem



a) Assuming that the number of required routes is fixed and that it's equal to the number of vehicles. Simulated Annealing could be applied in order to solve this problem, it's required to:
   i. Find a suitable solution representation ,
   ii. Define a suitable neighborhood operator,
   iii. Define the objective function used to calculate the cost of a solution .

b) How would the problem formulation change if we add the constraint that the total duration of any route shouldn't exceed a preset bound T?

Question 5:
The capacitated vehicle routing problem (CVRP) is one of the elemental problems in supply chain management. The objective of CVRP is to deliver a set of customers with known demands on minimum-cost vehicle routes originating and terminating at a delivery depot. CVRP is a difficult combinatorial problem, since it contains both the bin packing problem and the traveling salesperson problem as special cases. Simulated Annealing presents a good stochastic approach to this problem.

A. Develop a Simulated Annealing program using a language of your choice to solve this problem. You must use set A of the Augerat et al. problem instances to test and demonstrate your solution:

http://neo.lcc.uma.es/vrp/vrp-instances/capacitated-vrp-instances/

B. Compare your results with the optimal solutions reported for these instances .

C. Run your code under the following control scenarios:
- **i. initial temperature of 1000, and final temperature of 1, and alpha factor of 0.85, with geometric temperature updating, and 100 iterations per temperature**
- **ii. initial temperature of 600, and final temperature of 100, and alpha factor 0f 0.96, with geometric temerature updating, and 100 iterations per temperature.**

Please refer to the attached article for more insight and solution ideas on this problem.

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You will work in gropus of 2 to solve the problems in this assignment.

Marking Scheme:

i. The five questions are equally weighted
ii. Solution description (40%): strategy (15%), algorithm/pseudo code (15), and hand worked sample 10%
iii. Code and output sample 30%
iv. Memory and time complexity analysis 20%
v. Working code (demo) 10%

>>>>>>>>>>>>>>>>>> Include the names of your group <<<<<<<<<<<<<<<<<<

A Simulated Annealing Algorithm for the Capacitated Vehicle Routing Problem.

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A Simulated Annealing Algorithm for The Capacitated Vehicle Routing Problem

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Abstract

The Capacitated Vehicle Routing Problem (CVRP) is a combinatorial optimization problem where a fleet of delivery vehicles must service known customer demands from a common depot at a minimum transit cost without exceeding the capacity constraint of each vehicle. In this paper, we present a meta-heuristic approach for solving the CVRP based on simulated annealing. The algorithm uses a combination of *random* and *deterministic operators* that are based on problem knowledge information. Experimental results are presented and favorable comparisons are reported.

1 Introduction

The vehicle routing problem (VRP) is a generalization of the traveling salesman problem, and was initially proposed by Dantzig et al. [13] in order to find the optimum routing of a fleet of gasoline delivery trucks between a bulk terminal and a large number of service stations supplied by the terminal. The *Capacitated VRP* (CVRP) is a variation of the problem where all the customers correspond to deliveries and the demands are deterministic, known in advance, and may not be split. The vehicles are identical and based at a single central depot, and only the capacity restrictions for the vehicles are imposed. The objective is to minimize a weighted function of the number of routes and their travel time while serving all customers [30]. Other variations of the problem add pickup and delivery constraints (VRPPD) and time windows constraints (VRPTW).

The vehicle routing problem is an important combinatorial optimization problem that has various real-life applications. The problem has been shown to be related to practical problems such as solid waste collection, street cleaning, bus routing, dial-a-ride systems, routing of maintenance units, transports for handicapped, and modern telecommunication networks.

The CVRP is known to be NP-hard (in the strong sense), and generalizes the well known *Traveling Salesman Problem (TSP)*, calling for the determination of a minimum-cost simple circuit visiting all the vertices of a graph, and arising when there is one vehicle only with the capacity C not smaller than the demand at each vertex [30]. Therefore no exact algorithm has been shown to consistently solve CVRP instances with more than 25 customers [11].

A variety of optimization approaches have been applied to the CVRP [6, 18, 19, 30]. Clarke et al. [12] proposed a heuristic for the VRP based on the notion of savings where a distance saving is generated if two routes can be feasibly merged into a single route. Baldacci et al. [5] proposed a two-commodity network flow formulation by deriving a new lower bound from linear programming LP relaxation. The algorithm is based on the extension to CVRP of the two-commodity flow formulation for the TSP. Other researchers have proposed optimization approaches that are based on the use of graph theoretical relaxations such as b-matchings [22], K-trees [14], dynamic programming [16], and set partitioning and column generation [1]. Branch-and-cut algorithms based on the "two-index formulation of the CVRP" have been proposed as well [2, 3, 8, 30, 24, 26]. Valid linear inequalities are used as cutting planes to strengthen a linear programming relaxation at each node of a branch-and-bound tree. Other solutions that are based on meta-heuristics have been proposed as well such as simulated annealing algorithms [23], tabu search [9, 15, 29], genetic algorithms [4, 7, 25], honey bees mating optimization [20], and ant colony algorithm [21, 27, 28]. A comprehensive survey of the problem is provided in [30].

In this paper, we present an effective simulated annealing algorithm for solving the capacitated vehicle routing problem (CVRP). The proposed algorithm combines the strong global search ability of simulated annealing with problem knowledge information in order to provide effective and competitive solutions. The remainder of the paper is organized as follows. Section 2 describes the CVRP problem while section 3 introduces simulated annealing. Section 4 formulates the *annealing CVRP* and describes the *formulation, cool-*

ing schedule, the neighborhood function, and the cost function. The *CVRP annealing algorithm* is described in section 5 while *experimental results* are presented in section 6. We *conclude* with remarks in section 7.

2 Problem description

The CVRP involves determining a set of routes, starting and ending at the depot v_0, that cover a set of customers. Each customer has a given demand and is visited exactly once by exactly one vehicle. All vehicles have the same capacity and carry a single kind of commodity. No vehicle can service more customers than its capacity C permits. The objective is to minimize the total distance traveled or the number of vehicles used, or a combination of both. Thus, the CVRP is reduced to partitioning the graph into m simple circuits where each circuit corresponds to a vehicle route with a minimum cost such that: (1) each circuit includes the depot vertex; (2) each vertex is visited by exactly one circuit; (3) the sum of the demands of the vertices by a circuit does not exceed the vehicle capacity C.

Formally, the CVRP can be defined as follows: Given a complete undirected graph $G = (V, E)$ where $V = v_0, v_1, ..., v_n$ is a vertex set and $E = (v_i, v_j)/v_i, v_j \in V, i < j$ is an edge set. Vertex v_0 denotes the depot, and it is from where m identical vehicles of capacity C must serve all the cities or customers, represented by the set of n vertices $v_1, ..., v_n$. Each edge is associated with a non-negative cost, distance or travel time c_{ij}, between customers v_i and v_j. Each customer v_i has non-negative demand of goods q_i and drop time δ_i (time needed to unload all goods). Let $V_1, ..., V_m$ be a partition of V, a route R_i is a permutation of the customers in V_i specifying the order of visiting them, starting and finishing at the depot v_0. The cost of a given route $R_i = v_{i0}, v_{i1}, ..., v_{ik+1}$, where $v_{ij} \in V$ and $v_{i0} = v_{ik+1} = 0$ (0 denotes the depot), is given by:

$$Cost(R_i) = \sum_{j=0}^{k} c_{j,j+1} + \sum_{j=0}^{k} \delta_j \qquad (1)$$

and the cost of the problem solution (S) is:

$$F(S)_{CVRP} = \sum_{i=1}^{m} Cost(R_i). \qquad (2)$$

3 Simulated Annealing

Simulated annealing is a global stochastic method that is used to generate approximate solutions to very large combinatorial problems and was first introduced by Kirkpatrick et al. [17]. The annealing algorithm begins with an initial feasible configuration and proceeds to generate a neighboring solution by perturbing the current solution. If the cost of the neighboring solution is less than that of the current solution, the neighboring solution is accepted; otherwise, it is accepted or rejected with probability $p = e^{-\frac{\Delta_C}{T}}$. The probability of accepting inferior solutions is a function of the *temperature*, T, and the change in cost between the neighboring solution and the current solution, Δ_C. The temperature is decreased during the optimization process and thus the probability of accepting a worse solution decreases as well. The set of parameters controlling the initial temperature, stopping criterion, temperature decrement between successive stages, and the number of iterations for each temperature is called the *cooling schedule*. Typically, at the beginning of the algorithm, the temperature T is large and an inferior solution has a high probability of being accepted. During this period, the algorithm acts as a random search to find a promising region in the solution space. As the optimization process progresses, the temperature decreases and there is a lower probability of accepting an inferior solution. The algorithm behaves like a down hill algorithm for finding the local optimum of the current region.

4 Problem Formulation

The proposed method starts with a graph of demands and generates, through a sequence of transformations, a set of sub-optimal routes. The key elements in implementing the annealing CVRP algorithm are: 1) the definition of the initial configuration, 2) the definition of a neighborhood on the configuration space and perturbation operators exploring it; 3) the choice of the cost function; and 4) a cooling schedule. In what follows, we describe our annealing algorithm with reference to Figure 1.

4.1 Configuration Representation

In order to solve the CVRP problem, we propose the configuration shown in Figure 1(a). The representation is based on a vector where each cell $m \in 1, 2, 3, ..., n$ corresponds to a customer and cell 0 denotes the depot. Each route starts and ends at the depot. For example, Figure 1(b) encodes the following three routes: $<v_0, v_1, v_3, v_7, v_0>$, $<v_0, v_2, v_4, v_0>$ and $<v_0, v_6, v_5, v_8, v_0>$.

In order to speed-up the operation, we maintain hash tables that represent the routes gener-

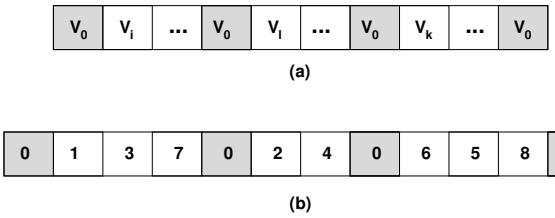

Figure 1: (a) Configuration Representation, (b) Sample Configuration

ated as well as the corresponding capacity. Thus, in Figure 1, the following hash indexes are maintained for the routes: $(r_0, 0, 1, 3, 7, 0)$, $(r_1, 0, 2, 4, 0)$, and $(r_2, 0, 6, 5, 8, 0)$. Similarly, the following hash indexes are maintained for the capacity of each route: (r_0, total demand in r_0), (r_1, total demand in r_1), and (r_2, total demand in r_2).

4.2 Initial Configuration

The initial solution is generated deterministically using a greedy algorithm that iterates over the list of cities and constructs initial non-optimal feasible routes based on a first-fit approach. The algorithm, shown in Figure 2, proceeds as follows. If there are n customers $1, 2, 3, ..., n$, the algorithm assigns the first customer in the list to a new route as long as its demand doesn't violate the capacity constraint of a vehicle and sets the status of the node to be visited. The algorithm proceeds to the next customer in the list. If it encounters a customer demand that violates the capacity constraint, the customer is skipped and the following customer in the list is processed. If the route capacity is exceeded, then a new route is allocated and the algorithm repeats until all customers have been assigned to routes.

```
InitialSolution()
{
    i ← 0
    do {
        Allocate a new route r_i.
        j ← 0
        ∀ v_j {
            if (v_j.visited == false && C(r_j) < Q) then
                r_i ← r_i ∪ v_i
                r_i.visited ← true
            }
        j++
}
```

Figure 2: Initial Solution

4.3 Neighborhood Transformation

The annealing algorithm uses two transformations in order to explore the design space. In what follows, we present both transformations.

4.3.1 Move

The *move* transformation finds *five* pairs of customers $< v_i, v_{i+1} >$ that have the shortest distances closest to each other including the depot. This is done by computing all distances between each pair of customers on all generated routes, including the distances to the depot. The transformation next selects five random customers that *exclude* the depot and the customers at positions v_{i+1}. The random customers are removed from their routes, and deterministically inserted into random routes. The transformation selects a random route and inserts the random customers in the routes based on the capacity constraint. Thus, for every random customer, a random route is selected and if the customer demand does not violate the route capacity it will be inserted in the new route.

4.3.2 Replace Highest Average

The *replace highest average* transformation calculates the average distance of every pair of customers in the graph. Thus, for each vertex v_i in a route, the method computes $d_i = \frac{d_{i-1,i}+d_{i,i+1}}{2}$ and selects the five vertices with the largest average distances and removes them from their routes. The transformation picks next five random routes and inserts the five selected customers in the route with the resulting minimum cost.

4.4 Cost Function

The objective of the algorithm is to minimize the cost of all routes. Thus, the objective is to minimize the following problem cost function:

$$f_S = \sum_{i=1}^{m} Cost(R_i). \quad (3)$$

4.5 Cooling Schedule

The cooling schedule is the set of parameters controlling *the initial temperature*, *the stopping criterion*, *the temperature decrement* between successive stages, and *the number of iterations* for each temperature. The cooling schedule was empirically determined. Thus, the initial temperature, T_{init}, was set to 5000 while the temperature reduction multiplier, α, was set to 0.99. The number of iterations, M, was determined to be 5 while the iteration multiplier, β was

set to 1.05. The algorithm stops when the temperature, T_f, is below 0.001.

5 CVRP Annealing Algorithm

Each configuration represents an intermediate route that has a different cost. During every annealing iteration, the neighborhood of the configuration is explored. The algorithm must ensure the following:

1. Each route originates and terminates at the depot;
2. The total demand for each route is within the capacity limit;
3. The maximum traveling distance is not surpassed;
4. The number of routes generated does not exceed the number of vehicles.

The algorithm, shown in Figure 3, starts by selecting an initial configuration and then a sequence of iterations is performed. In each iteration a new configuration is generated in the neighborhood of the original configuration by replacing vertices that have the largest distances to other vertices as well as by moving the vertices with the highest average distance to their neighbors. The *move* transformation is applied 80% of the time while the *replace highest transformation* is applied in every iteration. The solution is always feasible as the neighborhood transformations use a constructive approach that generates feasible routes. The variation in the cost functions, Δ_C, is computed and if negative then the transition from C_i to C_{i+1} is accepted. If the cost function increases, the transition is accepted with a probability based on the Boltzmann distribution. The temperature is gradually decreased from a sufficiently high starting value, $T_{init} = 5000$, where almost every proposed transition, positive or negative, is accepted to a freezing temperature, $T_f = 0.001$, where no further changes occur.

```
Annealing_CVRP()
{
   S_0 = Initial solution
   α = 0.99            //Temperature reduction multiplier
   β = 1.05            // Iteration multiplier
   M_0 = 5             //Time until next parameter update
   BestS = Best solution
   T = 5000
   CurrentS = S_0
   CurrentCost = Cost(CurrentS)
   BestCost = Cost(BestS)
   Time = 0
   do {
      M = M_0
      do {
         NewS = Neighbor(CurrentS);
         NewCost=Cost(NewS)
         Δ_Cost = NewCost − CurrentCost
         If (Δ_Cost < 0)
            CurrentS=NewS
            CurrentCost=Cost(CurrentS);
            If (NewCost < BestCost) then
               BestS=NewS
               BestCost = Cost(BestS)
         else if (Random < e^(−Δ_Cost/T) then
            CurrentS=NewS
            CurrentCost = Cost(CurrentS);
         M = M − 1
      } while (M ≥ 0)
      Time = Time + M_0;
      T = α ∗T;
      M_0 = β ∗ M_0;
   } while (Time > MaxTime and T > 0.001);
   Return(BestS);
}
```

Figure 3: Annealing CVRP Algorithm

6 Experimental Results

The proposed algorithm was implemented using Java on a Pentium Core 2 duo 1.80 GHZ with 1 GB of RAM, and tested on various instances from series A and E that are available at `www.branchandcut.org`.

6.1 A Benchmarks

The first set of benchmarks that we have attempted are the A instances from Augerat [3]. The first field in the name refers to the problem type where A refers to the *asymmetric capacitated vehicle routing problem* (ACVRP) instances. The second field is a three-digit integer that denotes the number of vertices of the problem graph, including the depot vertex. The third field is a one-digit integer that denotes the number of available vehicles. All examples were proposed by Augerat [3]. For example, `A-n32-k5` identifies the classical 32-customers Euclidean instance with 5 available vehicles. All examples are Euclidean, with integer edge costs following the TSPLIB standard. For the instances in the class A, both customer locations and demands are random. The algorithm solved all problems in under 15 minutes. Each benchmark was solved for 10 times and the best, worst and average results are reported in Table 2. All results were within 5% of the reported optimum or best answer with one case where our algorithm found a new optimum, `E-n23-k3`.

6.2 E Benchmarks

We have attempted three large benchmarks that are part of the E benchmark series. The first field in the name is the problem type where E refers to Euclidean single-depot complex vehicle routing problem instances (SCVRP). The second field is a three-digit integer that denotes the number of vertices of the prob-

Benchmark	Best Known Optimal	Simulated Annealing			Std Deviation σ	Difference from Optimal
		Best	Worst	Average		
E051-05e	524.61	555.14	573.09	562.72	6.73	5.82%
E076-10e	835.26	910.32	974.55	940.06	19.89	7.92%
E101-08e	826.14	1051.34	1091.17	1073.94	11.9	26.72%

Table 1: Results for the E instances

lem graph. The third field is a two-digit integer that denotes the number of available vehicles. All examples were proposed by Christofides and Eilon [10]. For example, `E051-05e` identifies the classical 50-customers Euclidean instance with 5 available vehicles. The examples are relatively large and include 50, 75, and 100 cities. Each benchmark was attempted for 10 times. The running time for the algorithm was 13 minutes in all cases. We compare our results in Table 1 to the reported optimal routes. It can be shown that the results of the first benchmark are very close to the reported optimal and differs by about 5% for the 50 cities and 8% for the 75 cities. The results were 26.72% from the optimal solution in the case of the 100 cities.

7 Conclusion

We have presented a meta-heuristic solution for the *Capacitated Vehicle Routing Problem*. The problem is solved based on simulated annealing using a combination of *random* and *deterministic operators* that are based on problem knowledge information. The algorithm was implemented and various benchmarks were attempted. The reported results are promising and future directions will focus on parallelization in order to further improve the results.

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Benchmark	Best Known Optimal	Simulated Annealing			Std Deviation σ	Difference from Optimal
		Best	Worst	Average		
E-n23-k3	569	568.5625	571.2862	569.3499	0.93	-0.08%
A-n32-k5	784	791.0789	845.704	818.0189	19.11	0.9%
A-n33-k5	661	685.577	704.602	696.9174	6.945	3.72%
A-n33-k6	742	748.08	767.53	756.3908	7.23	0.82%
A-n34-k5	778	802.614	819.388	812.5855	5.166	3.16%
A-n36-k5	799	837.477	883.809	862.2379	15.65	4.82%
A-n37-k5	669	697.943	745.564	725.473	15.58	4.33%
A-n37-k6	949	963.995	1000.84	981.1535	12.97	1.58%
A-n38-k5	730	733.945	773.824	763.19	11.76	0.54%
A-n39-k5	822	848.322	877.3145	865.427	9.75	3.2%
A-n39-k6	831	859.147	924.254	893.18	20.48	3.39%
A-n44-k6	937	951.802	1020.155	989.044	24.68	1.58%
A-n45-k6	944	982.829	1035.32	1010.898	14.55	4.11%
A-n45-k7	1146	1199.50	1313.74	1269.13	34.79	4.67%
A-n53-k7	1010	1044.91	1118.59	1087.58	22.51	3.46%
A-n54-k7	1167	1192.69	1275.43	1250.78	26.07	2.2%
A-n55-k7	1073	1144.85	1203.59	1175.15	21	6.7%
A-n60-k9	1408	1455.63	1567.34	1511.54	41	3.38%
A-n61-k9	1035	1064.76	1145.74	1101.70	23.63	2.88%
A-n62-k8	1290	1484.81	1633.13	1547.42	61.35	15.1%
A-n63-k9	1634	1698.63	1733.40	1715.08	12.23	3.96%

Table 2: Results for the A instances

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