(3V) Source

Gate

(5V)

Doain

Drain

(2) Doain

Gate —|E|

(5) Sauoce

Gain expression:

$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left(V_{GS} - V_{TH} \right)^2$$

$$\boxed{I_D + i_d = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left(V_{GS} + v_{gs} - V_{TH} \right)^2}$$

$\boxed{v_{gs}}$ = small incement

$$\Rightarrow \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left((V_{GS} - V_{TH})^2 + v_{gs}^2 + 2 v_{gs} (V_{GS} - V_{TH}) \right)$$

$\underbrace{}_{K}$

$$\boxed{I_D + i_d = \frac{1}{2} K \left[V_{GS} - V_{TH} \right]^2 + \frac{1}{2} K v_{gs}^2 + K v_{gs} (V_{GS} - V_{TH})}$$

$$\Rightarrow K v_{gs} \left(\frac{v_{gs}}{2} + (V_{GS} - V_{TH}) \right)$$

$K v_{gs} (V_{GS} - V_{TH})$

overdrive voltage

$V_{GS} - V_{TH} \gg \frac{v_{gs}}{2}$

$v_{gs} \ll 2(V_{OV})$
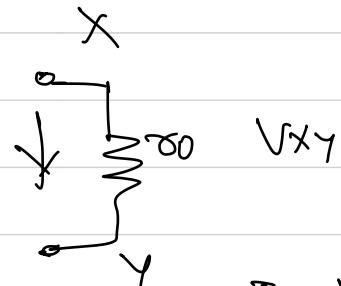
$$\boxed{\mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH}) v_{gs} = i_d}$$

Small signal analysis:

$$id = \underbrace{\mu_n C_{ox} \frac{W}{L}(V_{GS}-V_{TH})}_{\text{DC parameter}} v_{gs}$$

↓

o Drain

o Source

V_{XY}

$$I = \frac{V_{XY}}{r_0}$$
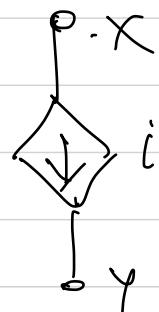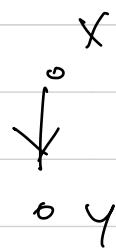
Gate (v_g)

Drain (vd)

Source (v_s)

v_g vd

v_s

$$\boxed{\mu_n C_{ox} \frac{W}{L}\underbrace{(V_{GS}-V_{TH})}_{g_m} v_{gs}}$$

$g_m v_{gs}$

$$ID = \frac{1}{2} K (V_{GS}-V_{TH})^2$$

$$\frac{dI}{dV_{GS}} = K(V_{GS}-V_{TH})$$

$$dI = K(V_{GS}-V_{TH}) \, dV_{GS}$$

$\underbrace{(id)}$ ⟵ $\underbrace{(vgs)}$

channel length modulation:

$$I_D = \frac{1}{2} k \left(V_{GS} - V_{TH}\right)^2 \left[1 + \lambda V_{DS}\right]$$

$$\frac{dI_D}{\downarrow}$$

small change $\underline{id}$ = $\underbrace{(1 + \lambda V_{DS}) \; k \; (V_{GS} - V_{TH}) \; \underline{dV_{GS}}}_{}$ ① v_{gs}

$$+ \underbrace{\frac{1}{2} k \left(V_{GS} - V_{TH}\right)^2 \lambda \; \underline{dV_{DS}}}_{} \quad v_{ds}$$

②

v_g v_d

v_s

MOSFET in triode:

$$I_D = \mu_n C_{ox} \frac{W}{L} \left(V_{GS} - V_{TH}\right) V_{DS}$$

② v_{ds} ① v_{gs}

$$id = dI_D = \underbrace{k\left(V_{GS} - V_{TH}\right) \underline{dV_{DS}}}_{} + \underbrace{(k V_{DS}) \underline{dV_{GS}}}_{}$$

v_g drain (v_d)

$g_m v_{gs}$ $\frac{v_{ds}}{r_o}$

v_s

$$\boxed{\begin{array}{l} g_m = k V_{DS} \\[2mm] r_o = \dfrac{1}{k(V_{GS} - V_{TH})} \end{array}}$$

transconductance (g_m)
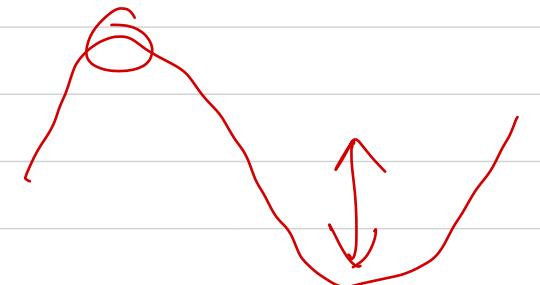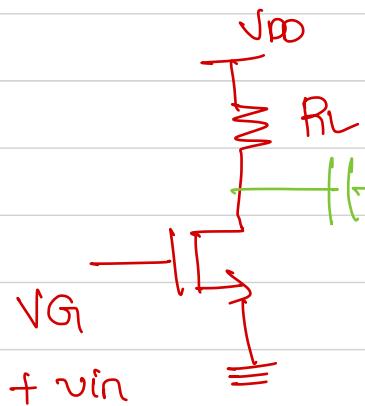
$$= \frac{dI_D}{dV_{GS}}$$
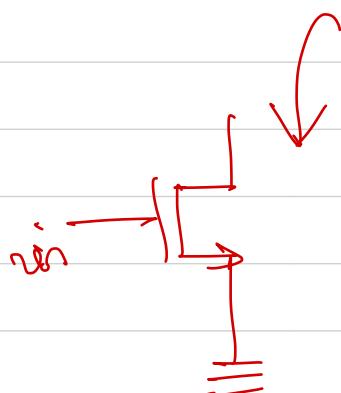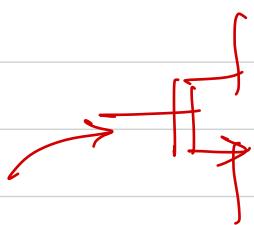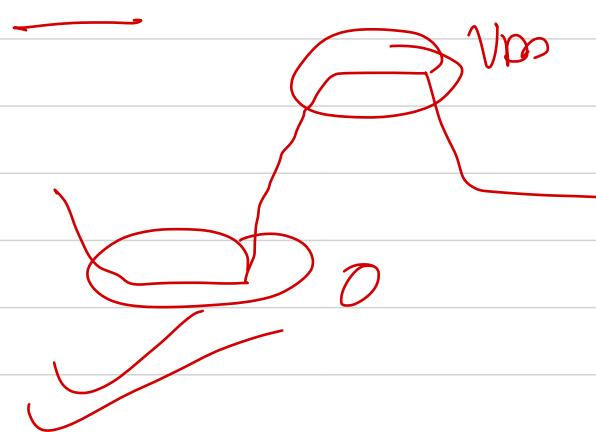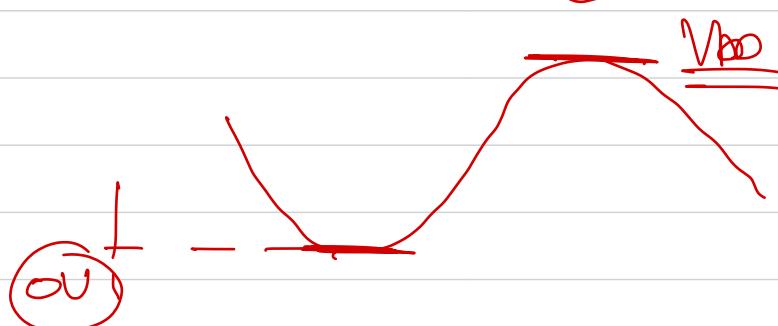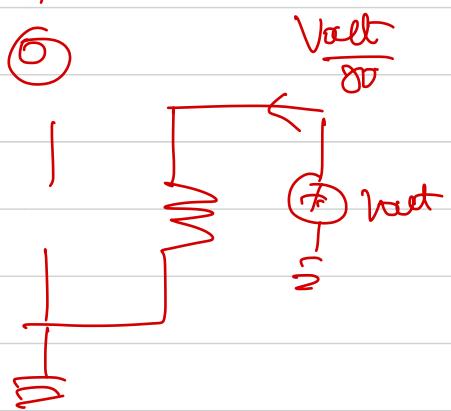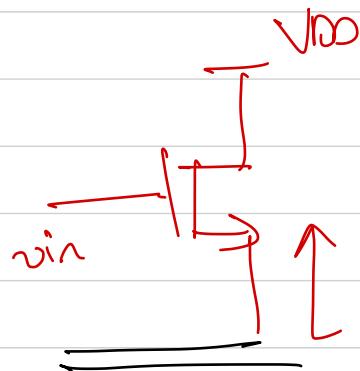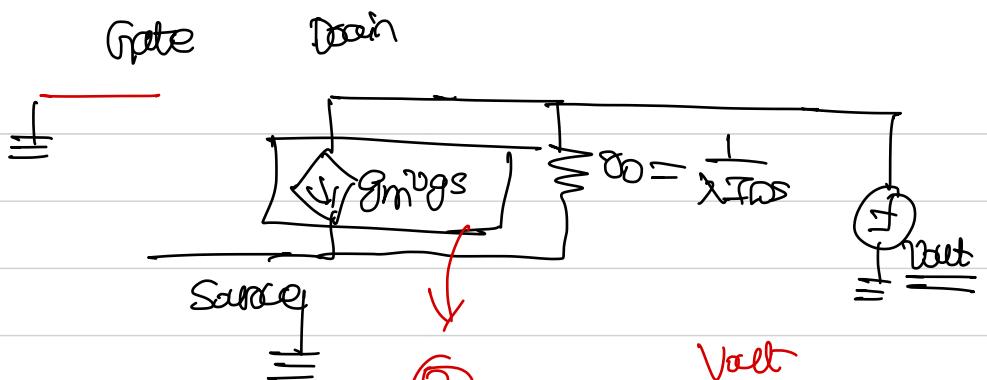
Saturation: (No CLM)

$$g_m = \frac{2 I_D}{(V_{GS} - V_{TH})} \Bigg| \sqrt{2 I_D \mu_n C_{ox} \frac{W}{L}} \Bigg| \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})$$

V_{DD}

R_L

V_G

$+ v_{in}$

$v_{gs} = v_{in}$

$\overline{OUT}$

$\frac{V_{DD}}{}$

V_{DD}

O

v_{in}

Gate   Drain

⏚ ___

$v_i \, g_m v_{gs}$ $r_0 = \dfrac{1}{\lambda I_{DS}}$ $\downarrow i$ v_{out}

Source ⏚

○

$\dfrac{v_{out}}{r_0}$

V_{DD}

v_{in} ↑

$\dfrac{v_{out}}{r_0}$ $\bigoplus i$ v_{out}

$\dfrac{1}{g_m} \| r_0$

$= \dfrac{r_0}{1 + g_m r_0}$

output imp $= r_0$

$= \dfrac{1}{\lambda I_{DS}}$

⏚ ___

$\downarrow i$ r_0 ⏚

i → $-g_m v_c$

$\bigoplus v_c$

$g_m v_c + \dfrac{v_c}{r_0} = i$

$\dfrac{v_c}{i} = \dfrac{r_0}{1 + g_m r_0}$
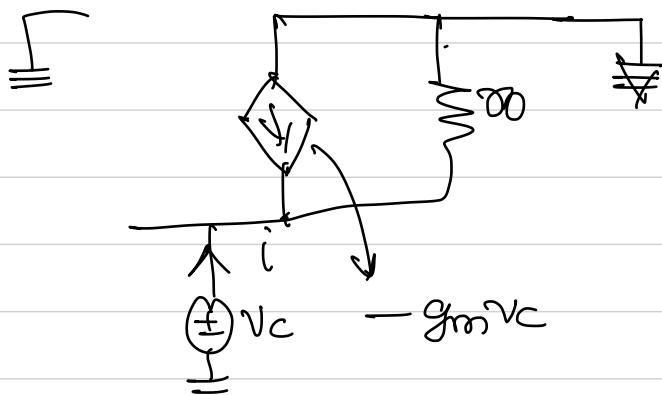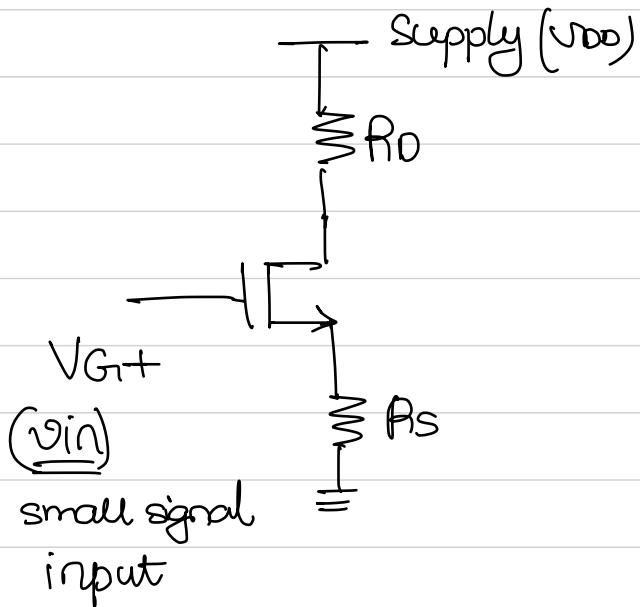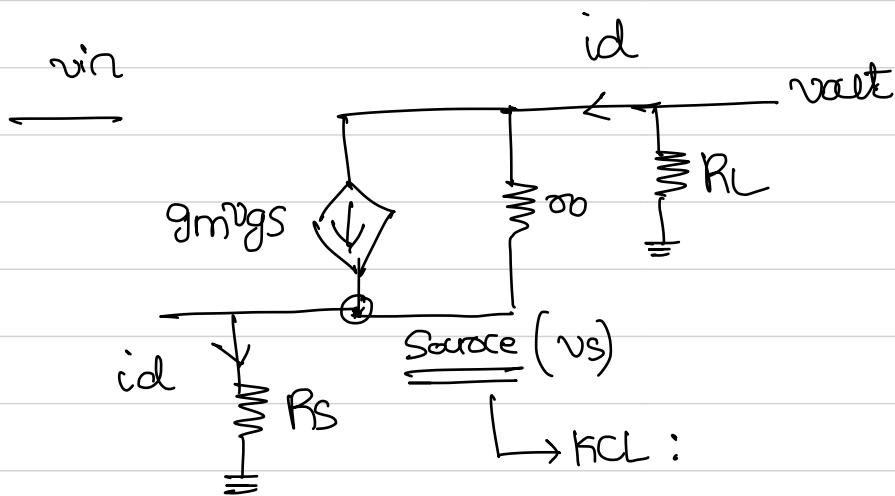
Task:

Find gain, impedance from source and drain terminal
(include channel length modulation)

Supply (V_{DD})

R_D

$V_G +$
(vin)
small signal
input

R_S

(8a) Gain:

vin

id

vout

$g_m v_{gs}$

r_o

R_L

Source (v_s)

id

R_S

→ KCL:

$$\boxed{\frac{-v_{out}}{R_L} = g_m(v_{in} - v_s) + \frac{v_{out} - v_s}{r_o} = \frac{v_s}{R_s}}$$

$$g_m(v_{in} - v_s) + \frac{v_{out} - v_s}{r_o}$$

$$= \frac{v_s}{R_s}$$

$$\Rightarrow \quad g_m v_{in} + \frac{v_{out}}{r_o} = v_s\left[\frac{1}{r_o} + \frac{1}{R_s} + g_m\right]$$

$$\Rightarrow \quad g_m r_o v_{in} + v_{out} = \frac{v_s(R_s + r_o + g_m r_o R_s)}{R_s}$$

$$\Rightarrow \quad v_S = \frac{(g_m r_o v_{in} + v_{out})R_S}{R_S + r_o + g_m R_S r_o}$$

$$-\frac{v_{out}}{R_L} = \frac{v_S}{R_S} = \frac{g_m r_o v_{in} + v_{out}}{R_S + r_o + g_m R_S r_o}$$

$$\Rightarrow \quad -v_{out}(R_S + r_o + g_m R_S r_o) = g_m r_o R_L v_{in} + v_{out} R_L$$

$$\Rightarrow \quad -v_{out}(R_S + r_o + g_m R_S r_o + R_L) = g_m R_L r_o v_{in}$$

$$\frac{v_{out}}{v_{in}} = gain = \frac{-g_m R_L r_o}{R_S + r_o + R_L + g_m R_S r_o}$$

if no CLM:

$$\frac{-g_m R_L}{1 + g_m R_S}$$

impedance from drain:

$$i_x = \frac{v_x}{R_L} + i_d$$

$$i_d = \frac{v_S}{R_S} = g_m(-v_S) + \frac{v_x - v_S}{r_o} \qquad \underline{\qquad} \text{ KCL at source node}$$

$$\Rightarrow \quad v_S\left[\frac{r_o + R_S + g_m R_S r_o}{R_S r_o}\right] = \frac{v_x}{r_o}$$

$$\Rightarrow \quad \frac{v_s}{R_s} = \frac{v_x}{\infty + R_s + g_m R_s \infty}$$

$$i_x = \frac{v_x}{R_L} + \frac{v_x}{\infty + R_s + g_m R_s \infty}$$

$$\Rightarrow \quad \boxed{\frac{R_L(\infty + R_s + g_m R_s \infty)}{R_L + R_s + \infty + g_m R_s \infty}}$$

impedance from source:

$$i_x = i_s + g_m(v_x) + \frac{v_x - v_{out}}{\infty}$$

① (KCL at source node)

KCL at drain node:

$$② \quad -g_m v_x + \frac{v_x - v_{out}}{\infty} = \frac{v_{out}}{R_L}$$

$$\Rightarrow \left(\frac{g_m \infty + 1}{\infty}\right) v_x = \left(\frac{R_L + \infty}{R_L \infty}\right) v_{out}$$

$$\Rightarrow \frac{v_{out}}{R_L} = \frac{(g_m \infty + 1) v_x}{R_L + \infty}$$

$$i_x = i_s + g_m v_x + \frac{v_x - v_{out}}{\infty} \overbrace{}^{} \underbrace{}_{} \frac{v_{out}}{R_L}$$

$$\Rightarrow \quad \frac{v_x}{R_s} + \frac{(g_m \infty + 1) v_x}{R_L + \infty}$$

$$\boxed{\frac{v_x}{i_x} = \frac{R_s(R_L + \infty)}{R_L + \infty + (g_m \infty + 1) R_s}}$$