1. given $ab=bc$ $a,b,c \in$ field

To Prove $a=c$

$a=a\cdot 1$ By multiplicative Identity (Rule)

$a=a\cdot(b\cdot b^{-1})$ By rule $x\cdot x^{-1}=1$

$a=(a\cdot b)\cdot b^{-1}$ By associativity of multiplaction

$a=(b\cdot c)\cdot b^{-1}$ from Question (given)

$a=c\cdot(b\cdot b^{-1})$ By associativity of multiplication

$a=c\cdot 1$ By rule $(x\cdot x^{-1})=1$

$a=c$ By multiplicative Identity (Rule)

Hence proved

2. given $a+b=b+c$ $(a,b,c)\in$ field

To prove $a=c$

$a=a+0$ By additive Identity (Rule)

$a=a+(b+(-b))$ By Inverse Rule of addition

$a=(a+b)+(-b)$ By Associative law of Addition

$a=(b+c)+(-b)$ from question (given)

$a=((-b)+b)+c$ By Associative law of Addition

$a=0+c$ By Inverse Rule $(x+(-x)=0)$

$a=c$ By Additive Identity.

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3) To prove: Any subfield of $(\mathbb{C},+,\cdot)$ must contain every rational number

given $\mathbb{C} \longrightarrow$ (complex number) ~~sub~~field $(\mathbb{C},+,\cdot)$ that follow all 9 axioms of field.

and let, Number set's be

$\mathbb{N}$ — Natural numbers

$\mathbb{Z}$ — Integers

$\mathbb{Q}$ — Rational numbers

$\mathbb{R}$ — Real number

Let F, be a field acc. (3),(7) axioms of field 0,1 must belong to Field F $\cancel{0,1 \in F}$ $0 \& 1 \in F$

and from given F also subfield $(\mathbb{C},+,\cdot)$ and F is under usual operations of '+' and '·' of complex number

Now, by closure property

$$+ : F \times F \rightarrow F \qquad \text{and } 0,1 \in F$$

then,

$$0+1=1 \in F$$

$$1+1=2 \in F$$

$$1+2=3 \in F$$

$$\vdots$$

$$1+(k-2)=k-1 \in F$$

$$1+(k-1)=k \in F$$

Since $k \in F$,

$$1+k \in F$$

(or by mathematical Induction)

Therefore the above result is true for $\forall\, k \in \mathbb{N}$

$\Longleftrightarrow \mathbb{N} \subseteq F$.

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Now According to Axiom 4, $\forall x \in F$,
$\forall x \in \mathbb{C}$

$$\exists\ x' = -x \in F,$$

either $x \in F$ (all elements are +ve) or $x \in$ elements common from both ~~[illegible]~~ $(\mathbb{C},+,\cdot)$ and $(F,+,\cdot)$ must belongs to $F, \mathbb{C}$ then,

$$-x \in F$$

~~[illegible]~~ such that,

$$x+(-x)=0=(-x)+x$$

$$\therefore \quad \mathbb{Z} \subseteq F$$

Acc. to axiom 8, $\forall x \in F^*$, $x^{-1} \in \mathbb{C}$

$$\exists \quad x^{-1} = \frac{1}{x}$$

$$x \cdot \frac{1}{x} = 1 = \frac{1}{x} \cdot x$$

$$\therefore \quad \mathbb{Z} \cup \left\{ \pm\frac{1}{1}, \pm\frac{1}{2}, \ldots \right\} \subseteq F$$

Here Acc. to field axiom every element $\forall x \in \mathbb{C}$ $\exists\ x'$ and that should $\in F, \mathbb{C}$

By field def:

$$\because: F \times F \longrightarrow F \qquad [m,n \in F]$$

$$m \cdot \frac{1}{n} = \frac{m}{n} \in F$$

where $m \in \mathbb{Z}$ and $\frac{1}{n} \in \left\{ \pm\frac{1}{1}, \pm\frac{1}{2}, \ldots \right\}$

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Thus, ~~Every~~ ~~when~~ Any Field 'F' is a subfield of $(\mathbb{C},+,\cdot)$ such that Every rational number is an element in F.

<u>used axioms :</u>

3. $\exists\, 0, \in F : \forall x \in F : x+0=x$

4. $\forall x \in F : \exists\, -x \in F : x+(-x)=0$

7. $\exists\, 1 \in F : \forall x \in F : x\cdot 1 = x$

8. $\forall x \in F \,\&\, (x \neq 0) : \exists\, x^{-1} \in F$ s.t $x \cdot x^{-1} = 1$

Such that $\longrightarrow$ (s.t) $\longrightarrow$:

$\exists \longrightarrow$ there exist

$\forall \longrightarrow$ for all/each

$\in \longrightarrow$ belongs to

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