Even and odd numbers are an interesting concept. In decimal and binary, it is very easy to recognize even and odd numbers just by observing the last digit – if that is even/odd, the entire number is even/odd. What is the mathematical basis for this? We are not so lucky in the ternary number system (radix = 3). Design a simple mathematical method to determine whether a given n-bit ternary number is even or odd $(A_{n-1}A_{n-2}A_{n-3}...A_0)_3$, $(A_i \leq 2)$. Explain the derivation of this method. Now, design a digital circuit that accepts a 4-digit ternary number as input (total 8 binary lines because each digit will need two lines as 00, 01, 10 for 0, 1 and 2), and outputs 1 if the number is odd and 0 if the number is even. Explain the design procedure and choices you make.

There are four independent logic variables A, B, C and D. We are tasked with making a Boolean function and a circuit that outputs logic-1 when *either* one or more of the following conditions is/are *explicitly* met (in cases not explicitly described by the statements, it will be considered as an unmet condition):

1. When A is true, B is true
2. When B is true, either A is true, or C and D are both false
3. When either C is true or A and B are both false, D is true
4. If B is false when either one of C or D is true, then A is true, and either C is false, or D is true

In case *all the conditions* are unmet, the function outputs a zero. Make a truth-table, k-map, simplified logical expression and logic circuit (sum of products) for this function.

Let's say we are asked to design a circuit for a vending machine that dispenses candy for Rs. 2. The input consists of a coin slot that can accept Rs. 1 and Rs. 2 coins. The deposit of these coins by the user is detected by a circuit that gives out two outputs x and y – when Rs. 1 is inserted, y goes to one, and when Rs. 2 is inserted, x goes to one, for one clock cycle. x and y stay at zero by default. Only one coin can be entered at once. Once the user has finished inserting the coins, she can press a button to obtain the candies and change (if any). This button also generates a pulse of 1 when pressed and remains zero otherwise. Design a circuit such that it takes x, y and button press as inputs and outputs the count of candies to be provided, and if Rs. 1 change is to be dispensed or not. The circuit should be able to hold information of up to Rs. 30, i.e., at any time after inserting between 0 and 30, if the button is pressed, we should know how many candies to dispense, and whether to dispense change.