A – Information and Communication – 3

Q1)

$$x(t) = e^{-t/2} \cdot u(t)$$

considering $u(t)$ as

$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

(a) $|X(\omega)|$

WKT, $x(t) = e^{-t/2} \cdot u(t)$

and $X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$.

$$\Rightarrow X(\omega) = \int_{0}^{\infty} e^{-t/2} \cdot e^{-j\omega t} \cdot u(t) dt$$

$$X(\omega) = \int_{-\infty}^{0} e^{-t/2} \cdot e^{-j\omega t} \cdot (0) + \int_{0}^{\infty} e^{-t/2} \cdot e^{-j\omega t} dt$$

$$X(\omega) = \left[\frac{e^{-(1/2 + j\omega)t}}{-(1/2 + j\omega)} \right]_{0}^{\infty}$$

$$X(\omega) = \left[\frac{1}{-(1/2 + j\omega) \times \infty} \cancel{(e^{-j\omega t})} + \frac{1}{1/2 + j\omega} \right]$$

bounded

(0 × bounded = 0)

$$X(\omega) = \frac{2}{1 + 2j\omega}$$

$$X(\omega) = \frac{2}{1 + 2j\omega} \times \frac{(1 - 2j\omega)}{1 - 2j\omega}$$

$$X(\omega) = \frac{2 - (2j\omega)2}{1 + 4\omega^2} = \frac{2 - 4j\omega}{1 + 4\omega^2}$$

$$X(\omega) = \frac{2}{1 + 4\omega^2} - j\frac{4\omega}{1 + 4\omega^2}$$

$$X(\omega) = Re\{X(\omega)\} + Im\{X(\omega)\}$$

$$Re\{X(\omega)\} = \frac{2}{1 + 4\omega^2}, \quad Im\{X(\omega)\} = \frac{-4\omega}{1 + 4\omega^2}$$

$$|X(\omega)| = \sqrt{(Re\{X(\omega)\})^2 + (Im\{X(\omega)\})^2}$$

$$|X(\omega)| = \sqrt{\left(\frac{2}{1+4\omega^2}\right)^2 + \left(\frac{-4\omega}{1+4\omega^2}\right)^2}$$

$$|X(\omega)| = \sqrt{\frac{4+16\omega^2}{1+16\omega^4+8}} = \sqrt{\frac{4\cancel{(1+4\omega^2)}}{(1+4\omega^2)^{\cancel{2}}}}$$

$$|X(\omega)| = \frac{2}{\sqrt{1+4\omega^2}}$$

(b) $\angle X(\omega)$

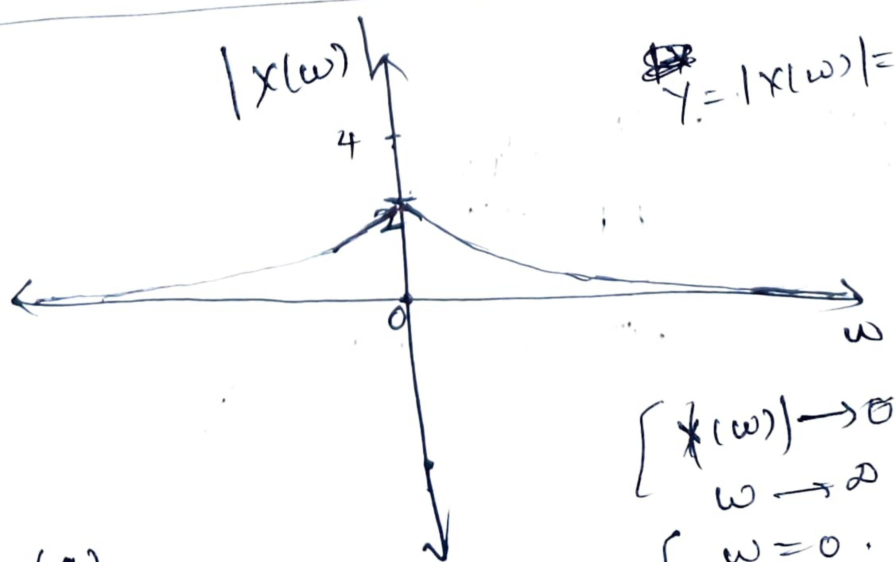
WKT,

$$\angle X(\omega) = \tan^{-1}\left(\frac{Im\{X(\omega)\}}{Re\{X(\omega)\}}\right) = \tan^{-1}\left(\frac{\frac{-4\omega}{1+4\omega^2}}{\frac{2}{1+4\omega^2}}\right)$$

$$\angle X(\omega) = \tan^{-1}\left(\frac{-4\omega}{2}\right) = \tan^{-1}(-2\omega)$$

(c) $= \dfrac{2}{1+4\omega^2}$

(d) $= \dfrac{-4\omega}{1+4\omega^2}$



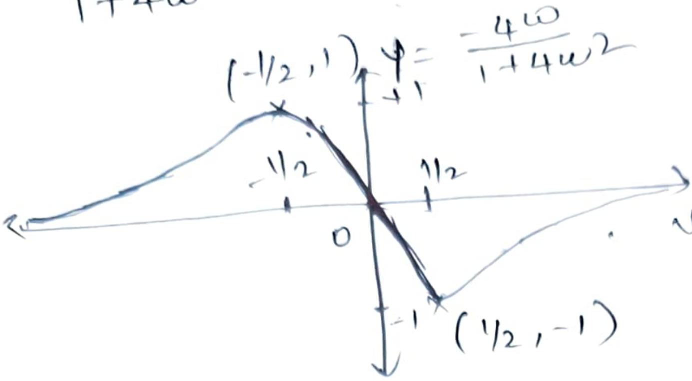
$Y = |X(\omega)| = \dfrac{2}{\sqrt{1+4\omega^2}}$

$\left[\begin{array}{l} |X(\omega)| \to 0 \\ \omega \to \infty \end{array}\right]$

$\left[\begin{array}{l} \omega = 0 \\ X(\omega) = 2 \end{array}\right]$

(a)

(d) $\frac{-4\omega}{1+4\omega^2} = y = I_m\{X(\omega)\}$



$$\frac{dy}{d\omega} = \frac{(1+4\omega^2)(-4) - (4\omega)(8\omega)}{(1+4\omega^2)^2} = 0$$

$8\omega^2 = 1+4\omega^2$

$4\omega^2 = 1$

$\omega^2 = \pm\frac{1}{2}$

$$y = \frac{-4(1/2)}{1+\cancel{4}\times\frac{1}{\cancel{4}}}$$

$= \frac{-\cancel{2}}{\cancel{2}} = -1$

$y = (-1)$ minimum

after this

Then ~~decreases~~

Increases

$$\frac{\cancel{4}^{2}\times\frac{1}{\cancel{2}}}{1+\cancel{4}\times\frac{1}{\cancel{4}}} = \frac{2}{2}$$

$y = 1$

$y = +1$ (maximum)

after this

Then decreases.

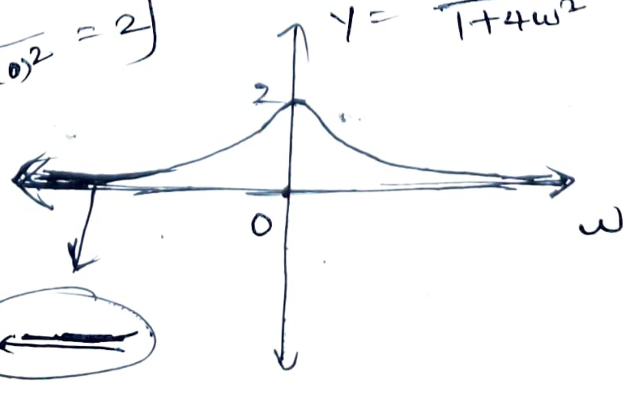
(c) $Re\{X(\omega)\} = y = \frac{2}{1+4\omega^2}$

$\omega = 0 \Rightarrow y = \frac{2}{1+4(0)^2} = 2$

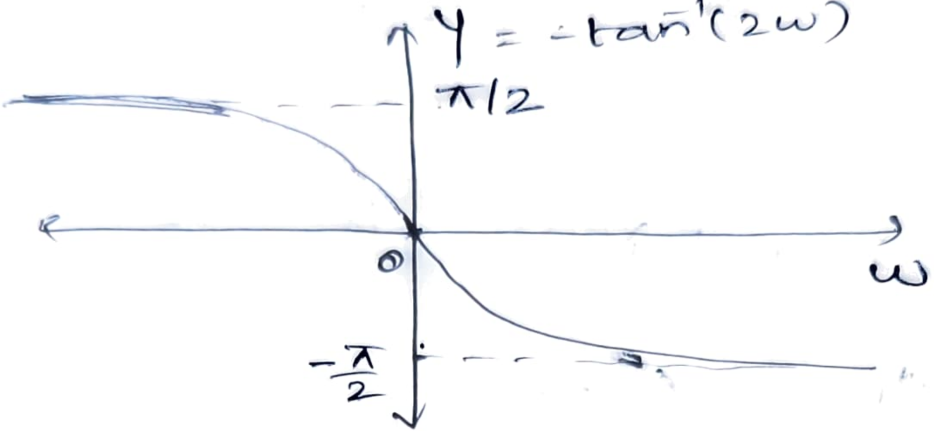
$y = 2$

$\omega \to \infty$

$y \to 0$



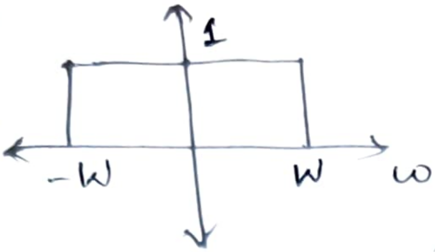
(b) $\angle X(\omega) = y = \tan^{-1}(-2\omega) = -\tan^{-1}(2\omega)$

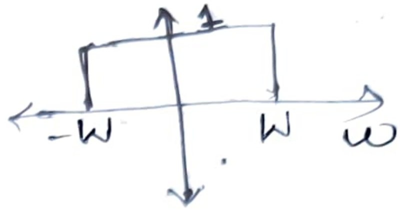


given, $Re\{X(\omega)\}$ $\qquad\qquad$ $Im\{X(\omega)\}$

2)

(a)





$$Re\{X(\omega)\} = \begin{cases} 1 & -W < \omega < W \\ 0 & \text{other} \end{cases}$$

$$Im\{X(\omega)\} = \begin{cases} 1 & -W < \omega < W \\ 0 & \text{other} \end{cases}$$

$$X(\omega) = \begin{cases} 1+j(1) & -W < \omega < W \\ 0 & \text{other} \end{cases}$$

$$X(\omega) = Re\{X(\omega)\} + j\,Im\{X(\omega)\} \Rightarrow |X(\omega)| = \sqrt{(1)^2+(1)^2}$$

$$|X(\omega)| = \sqrt{(1)^2+(1)^2} = \sqrt{Re\{X(\omega)\} + Im\{X(\omega)\}}$$

$$|X(\omega)| = \begin{cases} \sqrt{2} & -W < \omega < W \\ 0 & \text{other} \end{cases}$$

$$\angle X(\omega) = \tan^{-1}\left(\frac{Im\{X(\omega)\}}{Re\{X(\omega)\}}\right)$$

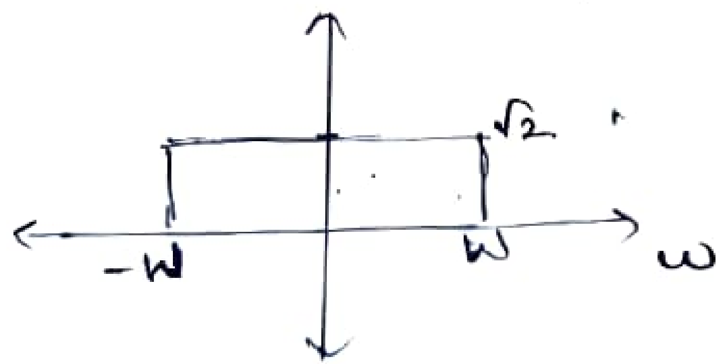
$$\angle X(\omega) = \tan^{-1}\left(\frac{1}{1}\right) = 45^\circ = \frac{\pi}{4}$$

$$\angle X(\omega) = \frac{\pi}{4} \quad (-W \leq \omega < W)$$

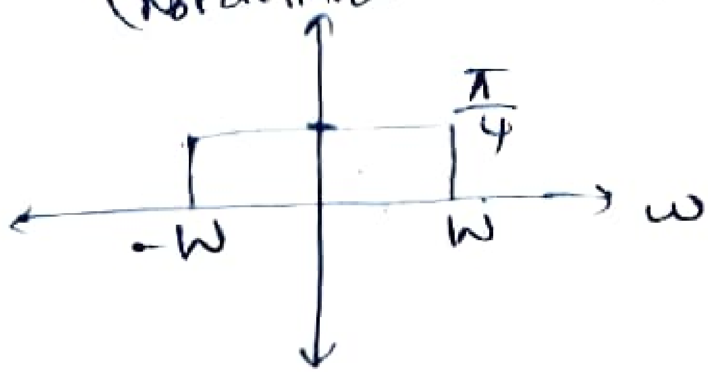
$\angle X(\omega) =$ undefined. (other ω)

Plots

$$y = |X(\omega)| = \begin{cases} \sqrt{2} & -W < \omega < W \\ 0 & \text{otherwise} \end{cases}$$



$$y = \angle X(\omega) = \begin{cases} \frac{\pi}{4} & -W < \omega < W \\ \text{Not defined} & \text{otherwise} \end{cases}$$



(b) By property of Fourier transform

For a real signal $x(t)$

It Fourier Tranform must satisfy

$$X(-\omega) = X^*(\omega)$$

$$X(-\omega) = \begin{cases} 1+j & -W < \omega < W \\ 0 & \text{otherwise} \end{cases}$$

$$x^*(\omega) = \begin{cases} 1-j & -W < \omega < W \\ 0 & \text{otherwise} \end{cases}$$

So,

No, $x(t)$ is not real, because

$$x^*(\omega) \neq X^*(-\omega)$$

(4) $C(n,k)$

(a)

Let $c \in C(n,k)$

In general if $c \in C(n,k)$

$$cH^T = 0.$$

but The received vector v may contain (include) error so,

received vector 'r' can be written as

$$r = c + e$$

where c is code word and e is the error pattern.

So, we might get some other (output) value instead of 0 for rH^T

let,

$$s = rH^T = (c+e)H^T = eH^T + 0$$

So, Instead of all 2^n codewords an $(n-k) \times n$ matrix is enough called H

The estimated code word for r is

$r = c + e$

$r + e = c + e + e$

$\hat{c} = r + e = c$ (or) $\hat{c} = r - e = c$

$r = c + e$

$$\left.\begin{aligned} rH^T &= cH^T + eH^T \\ s = rH^T &= eH^T + 0 \end{aligned}\right\} \text{as writtern in before page}$$

So, we can get e by multiplying r with H^T. because we have equivalent table for e, eH^T by this from the given table There's a corresponding error for error occurred so, The corresponding e for eH^T is our equired error.

Here we can saving space because in standard array we need to store $(2^k).2^{n-k} = 2^n$ elements but here 2^k is enough. here $k < n$ (k is no.of message bits and n is No.of message bits + parity)

(b) given $r = [0,0,1,1;0,1,1]$

$r.H^T = (c+e)H^T = eH^T$

$$\Rightarrow [0,0,1,1,0,1,1] \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

$r.H^T = [1+1, 1, 1+1, 1]$

$rH^T = eH^T = [0101]$ given [from the table]

The corresponding e will be $e = [0101000]$

So, The actual code word is

$$r = c + e$$

$$r + e = c + e + e \quad \text{(modulo 2 addition)}$$

$$r + e = c$$

$$c = [0011011] + [0101000]$$

$$c = [0110011].$$

5(a) B-psk Binary phase shift keying.

Q-psk Quadrature phase shift keying

(i) Bpsk

~~No. of Symbols~~

No of bits per symbol

1 bit per symbol

$\Rightarrow 1 \Rightarrow \log_2 2$

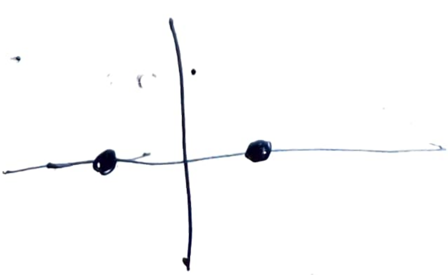
Qpsk.

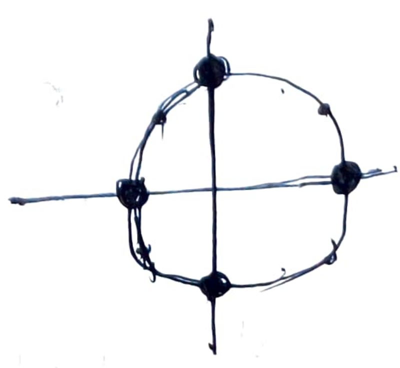
2 bits per symbol

No of bits per symbol

$\Rightarrow 2 \Rightarrow \log_2 4$

graph/more info





$\frac{360}{2} = 180°, (\cancel{180}+180)$ 370°

$\frac{360}{4} = 90°, 180°, 270, \cancel{360}$ 360°

(ii)

0° and 180°

(0) (1)

0°, 90°, 180°, 270°

(00) (01) (11) (10)

$\Delta\phi = \frac{2\pi}{2} = 180°$

$\Delta\phi = \frac{2\pi}{n}$

$\Delta\phi = \frac{2\pi}{4} = 90°$

→ $\Delta\phi$ is phase shift between neighbouring points. (in constellation)

(iii) Bandwidth efficiencey.

as we know

BPSK transmits 1 bit per symbol

QPSK transmits 2 bits per symbol

For n-bit message transmission QPSK need only $\frac{n}{2}$ symbols whereas BPSK needs n symbols.

So, In QPSK more information is packed into the same time interval compared to BPSK.

→ higher symbol rate → more rapid changes → wider bandwidth

each symbol change in time creates a change in the waveform → these changes determine frequencey content of the signal.

here QPSK has [more bit rate / Less symbol rate] compared to BPSK

→ Lower symbol rate narrower band width.

and also

Bandwidth efficiencey $\eta = \frac{\text{bits per symbol}}{\text{symbol deration}}$

$$\boxed{\eta_{BPSK} = \frac{1}{2} \cdot \eta_{QPSK}}$$

because, QPSK is more bandwidth efficient than BPSK

QPSK uses half the Bandwidth of BPSK

5(a)
(iv)

Susceptibility to noise and 5(b)

generally as we know
Bpsk uses only 2 points on constellation

(0° and 180°)

Theseare Far apart in phase

→ So even if noise disturbs a single bit it's unlikely* to be confused for the wrong bit

Bpsk is like a yes/No signal with a large gap - very easy to distinguish

and worstcase corrupted (by high noise) only 1 bit is corrupted

→ but in Qpsk 2 bits could be corrupted instead of 1 and also more likely to corrupted and unlikely to distinguish where the (noise) error is when compared to Bpsk.

Let's see how

Qpsk uses 4 points each representing 2 bits, also placed very closer (90° apart)

since these points are closer in phase a small amount of noise might push the signal to the wrong symbol.

The symbol Error rate for QPSK is

slightly more ~~and~~.

∴ QPSK is slightly more vulnerable (susceptible)

to noise compare to BPSK.

The larger distance b/w the points, the less susceptible the system is to noise

In QPSK the small amount of noise could cause the received symbol to shift into the wrong quadrant, leading to higher probability error (closer spacing of constellation points)

5

(b) If both modulation schemes are operating under the same bandwidth and power conditions.

~~some theory already written in~~

~~Susceptibility to noise that also follow this.~~

• Similarly BPSK uses only 2 phases
minimum distance is 2A

for QPSK $d_{min} = A\sqrt{2}$

QPSK is more vulnerable to noise, hence more (BER) Bit error rate compare to BPSK at same power and band width the QPSK may have higher efficiency of bandwidth because it can pack more (2) bits per symbol and BPSK is more noise resistant due to large symbol spacing

So, under same power and Bandwidth BPSK has a lower BER because it's effective signal to noise ratio (SNR) per bit is higher

6) $b(t) \in \{0,1\}$

Carrier wave $= \cos(2\pi f_c t)$

f_c is the carrier frequency

For bpsk let π phase diff should be there

$b(t)=0$ Phase shift $\phi(b(t))=0$

$b(t)=1$ Phase shift $\phi(b(t))=\pi$

Let $x(t) = A.\cos(2\pi f_c t + \phi(b(t)))$

Let $A=1$, since carier wave has

$A=1$, $x(t) = A.\cos(2\pi f_c t + \pi(b(t)))$

So, when $b(t)=0$ $x(t) = A\cdot\cos(2\pi f_c t \cdot \pi(0))$

$x(t) = A\cdot\cos(2\pi f_c t)$

$x(t) = \cos(2\pi f_c t)$

$b(t)=1$

$x(t) = A\cos(2\pi f_c t + \pi)$

$= -A\cos(2\pi f_c t)$

$x(t) = -\cos(2\pi f_c t)$

This is how BPSK modulates.

given binary sequence $[1,0,1,1,0]$

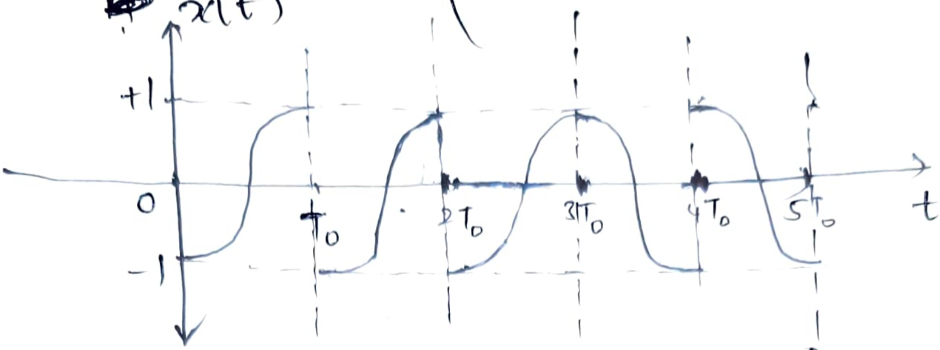
x-axis divided into 5-equal intervals $T_0 = \Delta t$

y-axis ranging $(-1, 0, +1)$

at $t=0$ to T_b (taking complete cos wave)

$x(t)$ considering 1st bit

taking $x(t) = \{ \cos(2\pi f_c t + \pi(b(t))) \}$



Phase change $180° \to 0° \to 180° \to 180° \to 0°$

$b(t) = (1), (0), (1), (1), (0)$

7.
(a) given $m(t) = 5\cos(2\pi \cdot 1000t)$

$c(t) = 10\cos(2\pi \cdot 100000t)$

$\boxed{f_m = 1000\text{Hz}}$

The message modulates in I-component of Signal

modulated signal $= x(t) = m(t) \cdot c(t)$ $[2\cos A\cos B = \cos(A+B) + \cos(A-B)]$

$V_{DSB}(t) = x(t) = 5\cos(2\pi 1000t) \cdot 10\cos(2\pi 100000t)$

$x(t) = \frac{50}{2} \times 2\cos(2\pi 1000t)\cos(2\pi 100000t)$

$x(t) = 25 \times (\cos(2\pi t(101000)) + \cos(2\pi t(99000)))$

$x(t) = 25[\cos(2\pi \cdot 101000t) + \cos(2\pi \cdot 99000t)]$

In DSBSC modulation the modulated signal has 2 Side bands.

upper sideband (USB): centered at $f_c + f_m$

lower side band (LSB): centered at $f_c - f_m$

(b)

(B) Bandwidth $= 2 f_m = 2 B_{message}$

$BW = 2000\text{Hz}.$

f_m is the highest frequency component msg signal.

(c) The frequency spectrum of modulated signal is

→ These are 2 cos signals in $v_{DSB}(t)$
each cos signal gives 2 frequency (+ve & -ve) in Fourier transform δ (terms)
The fourier transform of them
→ each term (cosine) represents single frequency in spectrum
is

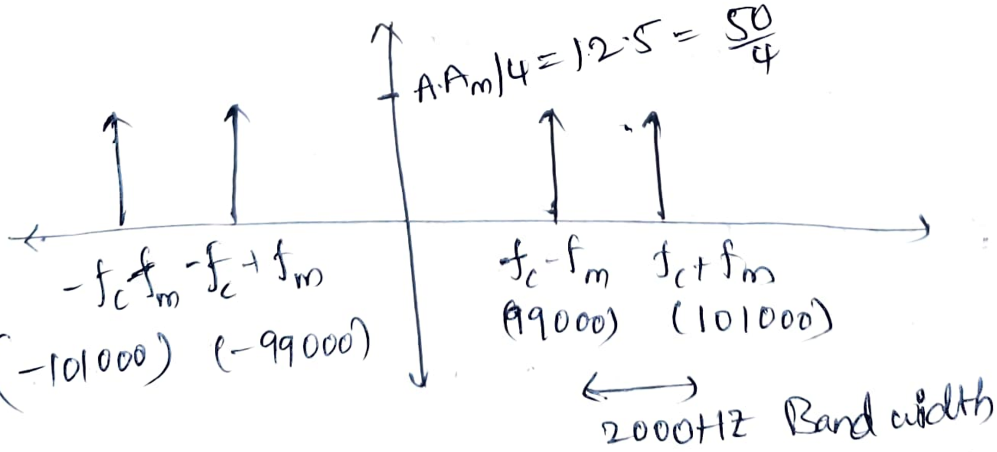
$$v_{SB}(f) = \int_{-\infty}^{\infty} v_{DSB}(t)\,dt$$

$$v_{DSB}(f) = \int_{-\infty}^{\infty} 25\frac{\left(e^{+2\pi 101000t - st} + e^{-2\pi 101000t + st}\right)}{2}\,dt$$

$$+ \int_{-\infty}^{\infty} \frac{25}{2}\left[e^{2\pi 99000t - st} + e^{-2\pi 99000t + st}\right]$$

$$v_{DSB}(f) = 12.5\left[\delta(f-101{,}000) + \delta(f+101000)\right]$$

$$+ 12.5\left[\delta(f-99000) + \delta(f+99{,}000)\right]$$



→ The frequency spectrum of $v_{DSB}(t)$ is a plot of amplitude vs frequency,
→ real signal spectrum is symmetric about $f=0$
→ The carrier frequency at $f_c = 100\,kHz$ is suppressed

→ no component at f_c, which is characteristic of DSB-SC

8)

$$S(t) = 4\cos(2\pi \cdot 10^6 t) \cdot \cos(2\pi\ 10^3 t)$$

(a)

WKT, $2\cos A \cos B = \cos(A+B) + \cos(A-B)$

$$s(t) = 2\left[\cos 2\pi(10^6 + 10^3)t + \cos 2\pi(10^6 - 10^3)t\right]$$

$$s(t) = 2\left[\cos 2\pi(1001000)t + \cos 2\pi\ (999000)t\right]$$

$f_c + f_m = 1001000\ Hz$

$f_c - f_m = 999000\ Hz$

by this $\quad f_c = 10^6\ Hz = 1MHz$

$\quad\quad\quad f_m = 10^3 = 1KHz$

(b)

(BW) Bandwidth $= 2 \cdot f_m = 2 \cdot B_{message}$

$= 2 * 10^3$ $\quad\quad$ f_m is highest freq component in msg signal.

$BW = 2KHZ$

(3) WKT, In modulation

(a)

The modulation property

$$x(t) \cdot \cos(\omega_0 t) \xleftrightarrow{F\{\}} \frac{1}{2}\left[X(\omega+\omega_0) + X(\omega-\omega_0)\right]$$

$X(f)$ is Fourier transform of $x(t)$

$$F\{x(t) \cdot \cos(2\pi f_0 t)\}$$

$$x(t)\left(\frac{e^{2\pi j f_0 t} + e^{-2\pi j f_0 t}}{2}\right)$$

$$\frac{x(t) \cdot e^{2\pi j f_0 t}}{2} + \frac{x(t) e^{-2\pi j f_0 t}}{2}$$

$$x(t) \xrightarrow{F} X(f)$$

$\therefore$ Similarly, $x(t) \cdot e^{j2\pi f_0 t} \xrightarrow{F} X(f-f_0)$

$$x(t) \cdot e^{-j2\pi f_0 t} \xrightarrow{F} X(f+f_0)$$

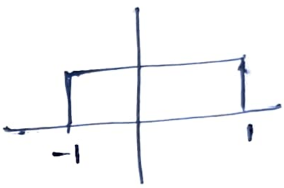
$$\Rightarrow \frac{X(f-f_0)}{2} + \frac{X(f+f_0)}{2}$$

Fourier transform of the given signal

→ The modulation has made a baseband to passband signal.
without changing the bandwidth

→ made copies of the original signal
FT at both $+f_0$ and $-f_0$ frequencies

(b) $x(t) = rect(t/2)$

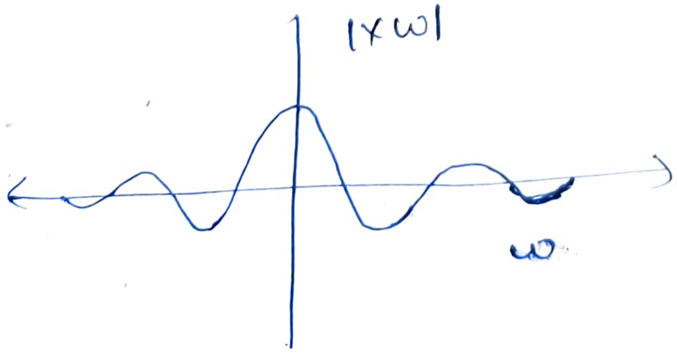


$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega_0 t} dt$$

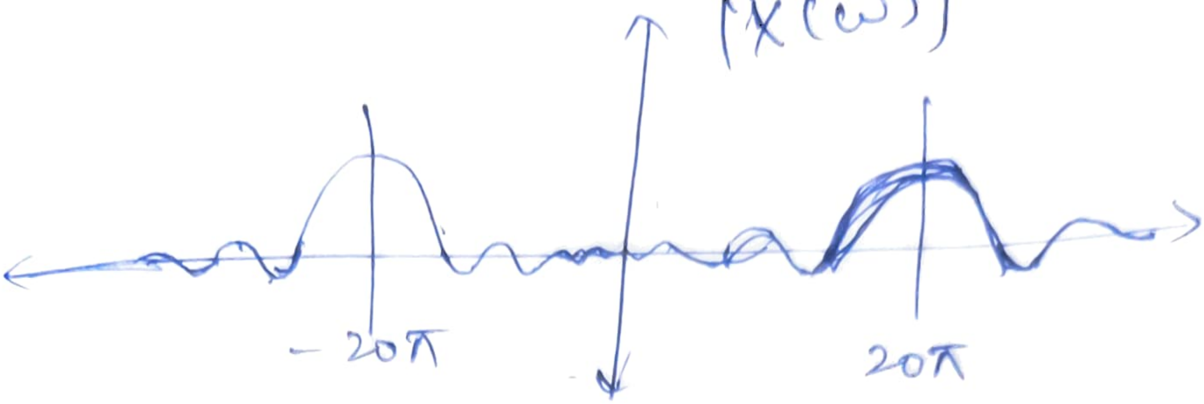
$$= \int_{-1}^{1} e^{-j\omega_0 t} dt$$

$$X(\omega) = \left[\frac{e^{-j\omega_0 t}}{-j\omega_0} \right]_{-1}^{1} = \frac{e^{-j\omega_0} - e^{j\omega_0}}{-j\omega_0}$$

$$X(\omega) = \frac{2j\sin\omega}{j\omega} = \frac{2\sin\omega}{\omega}$$



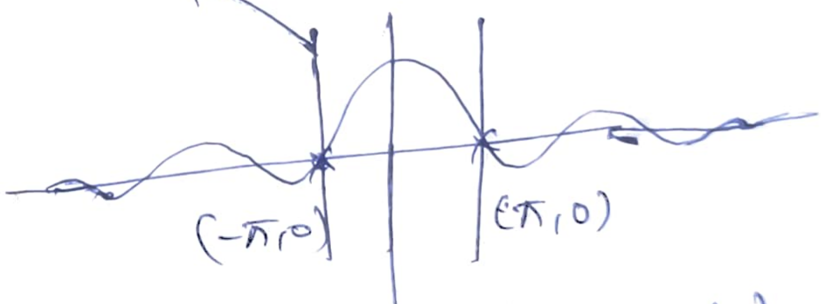
by (a) part fft of $x(t)\cos(20\pi t)$ is



$$\frac{\sin(20\pi-\omega)}{20\pi-\omega}+\frac{-\sin(20\pi+\omega)}{20\pi+\omega}$$

The signal is never 0 it only decays its fft bandwidth is infinite

The maximum energy concenrtation is in $(-\pi,\pi)$ region taking π as band with



Taking this as bandwith π (input)

Now, band width of signal ($x(t)\cos(20\pi t)$) becomes twice hence 2π.