

dc model parameters

$$\text{Linear : } I_{DS} = \beta_N \left\{ (V_{GS} - V_{THN}) V_{DS} - \frac{V_{DS}^2}{2} \right\} \quad \beta_N = kP_N \cdot \frac{W}{L}$$

$$\text{Saturation : } I_{DS} = \frac{\beta_N}{2} (V_{GS} - V_{THN})^2 [1 + \lambda_n V_{DS}] \quad \lambda_N$$

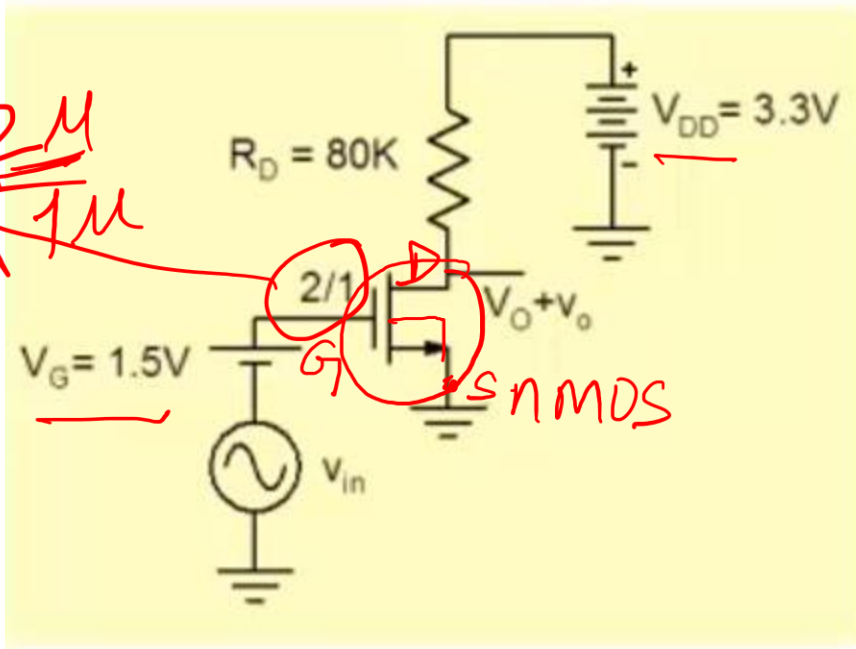
$$V_{THN} = V_{THN0} + \gamma (\sqrt{2\phi_F + V_{SB}} - \sqrt{2\phi_F})$$

$$\left[\begin{aligned} V_{THN0} &= 1V; \gamma = 0.7 V^{1/2}; 2\phi_F = 0.7V; \\ KP_N &= 100 \mu A / V^2; L = 1 \mu m; \lambda = 0.01 V^{-1} \end{aligned} \right]$$

Common Source Amplifier with Resistive load

Calculate Bias point \rightarrow ac sources are shorted

$$\frac{W}{L} = \frac{2\mu}{1\mu}$$



✓ dc analysis

Assume : MOS is in saturation

$$I_{DSQ} = \frac{\beta_n}{2} (V_{GS} - V_{TN})^2 = 25\mu A$$

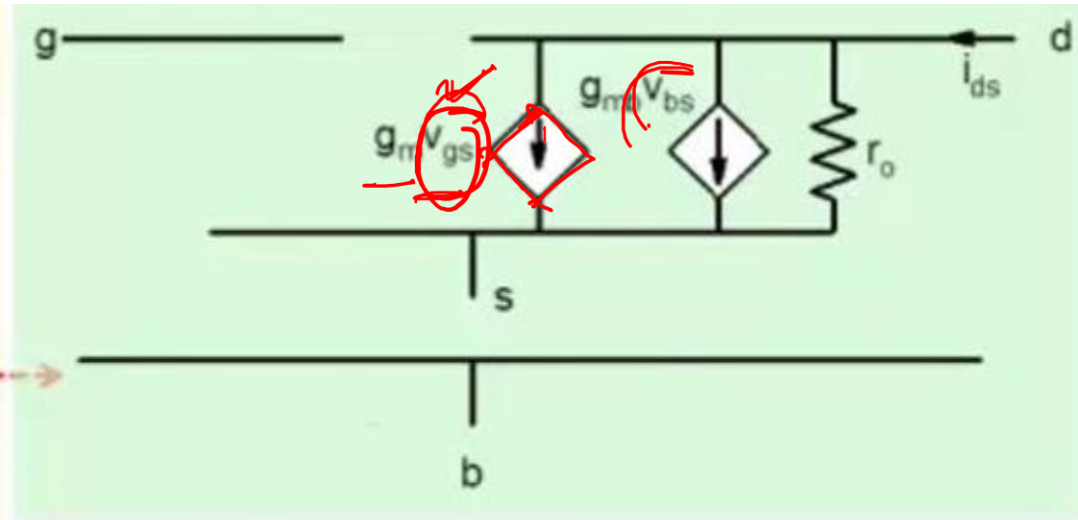
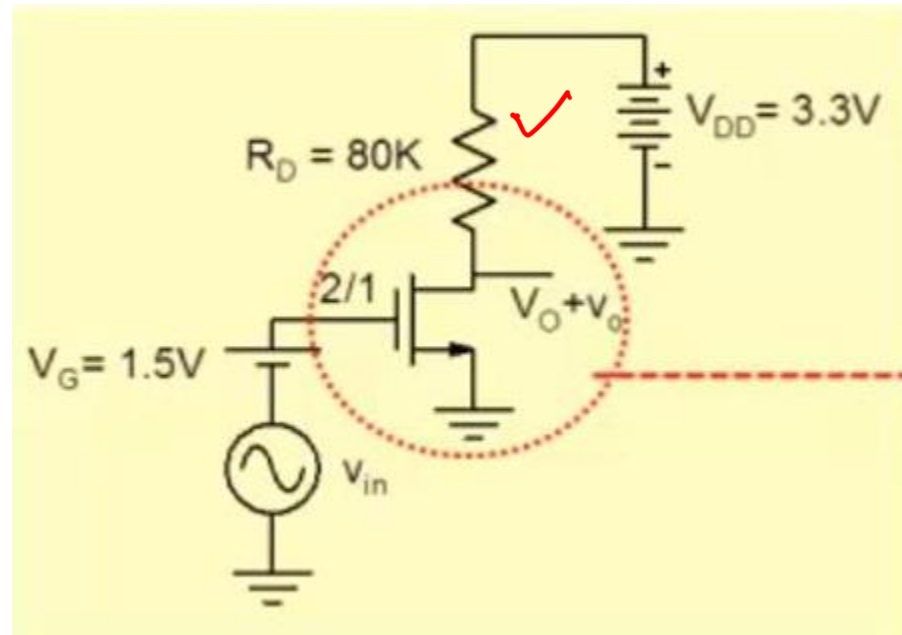
$$V_{DSQ} = V_{DD} - I_{DSQ} \times R_D = 1.3V$$

$$3.3V - 25\mu A \times 80K = 1.3V$$

Bias point $\Rightarrow \{I_{DSQ}, V_{DSQ}\}$ ✓

$$V_{sat} = V_{GSQ} - V_{TN} = 0.5V \text{ so Tr. is in Saturation}$$

Small Signal Model → dc sources are shorted



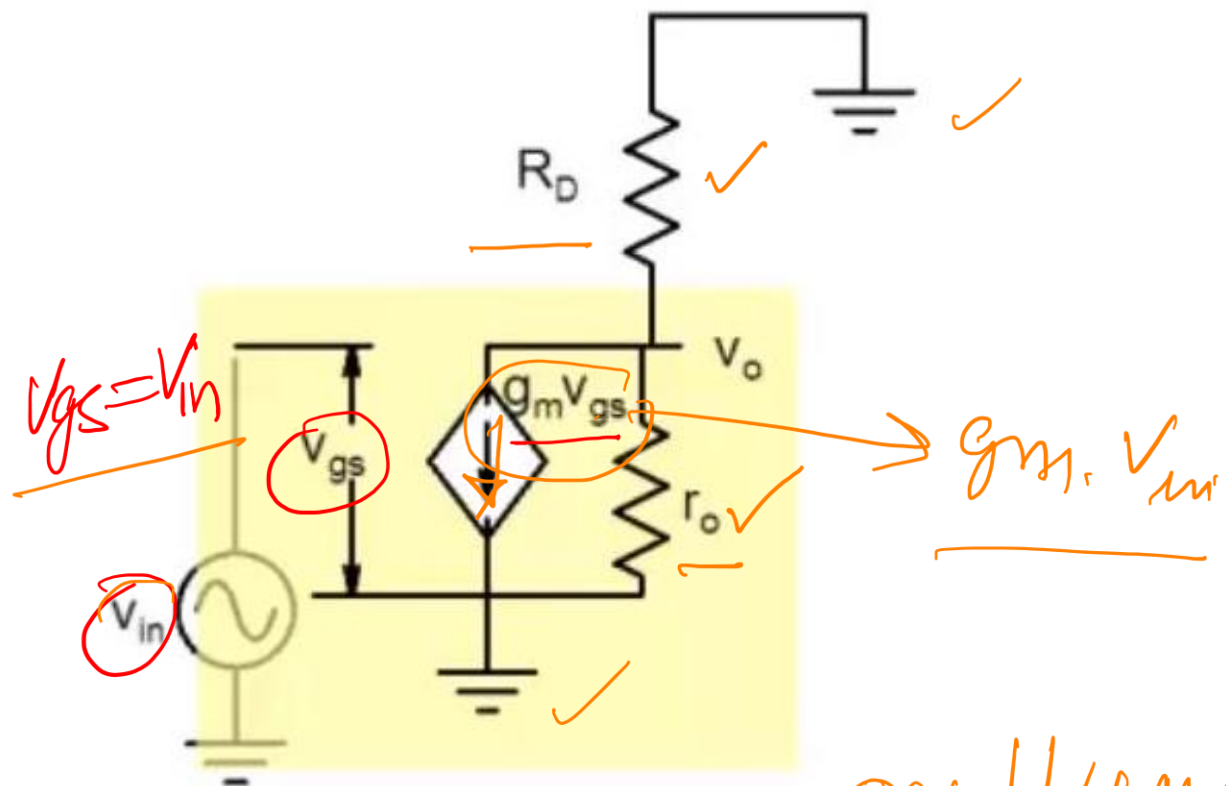
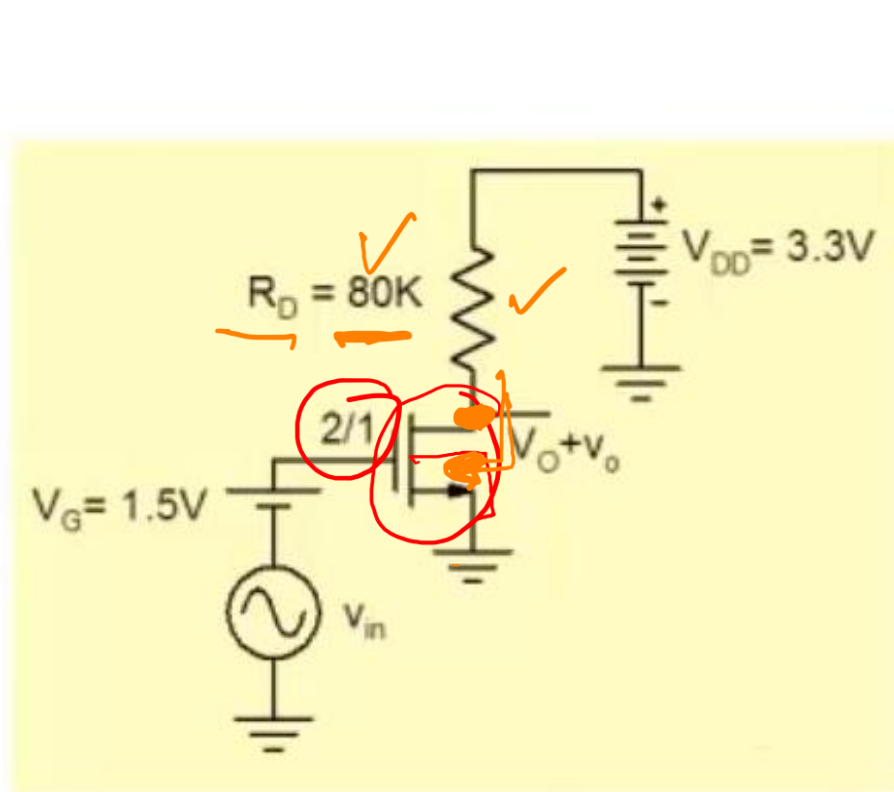
$$g_m = \frac{2I_{DSQ}}{V_{GSQ} - V_{THN}} = \sqrt{2I_{DSQ}\beta} = 100 \mu A/V$$

$$r_o = \frac{1}{\lambda_n I_{DSQ}} = 4 M\Omega$$

$$g_{mb} = g_m \cdot \eta = 41.83 \mu A/V$$

→ Now, we have small-signal terms / values

1. Voltage Gain



Body is connected to most negative potential, $V_{bs} = 0$.

$$v_o = -g_m v_{in} \times R_D \parallel r_o$$

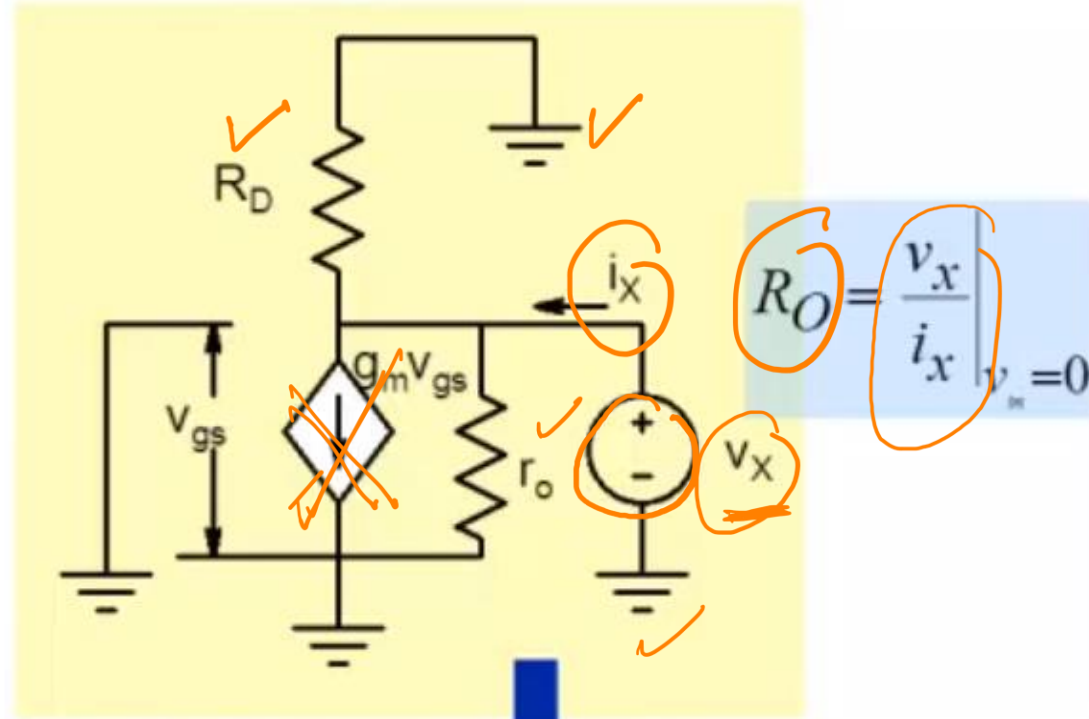
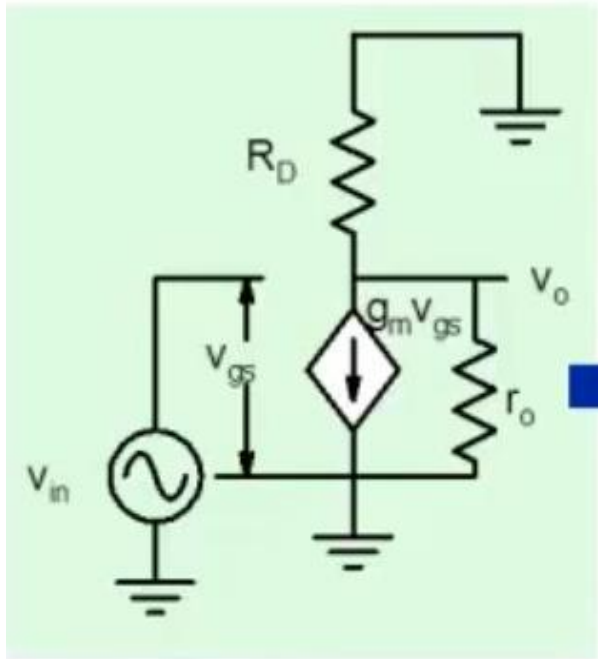
$$A_V = \frac{v_o}{v_{in}} = -g_m \times R_D \parallel r_o \approx -g_m R_D = -8$$

$$80K \parallel 4M\Omega \approx 80K$$

$$R_D \parallel r_o \approx R_D$$

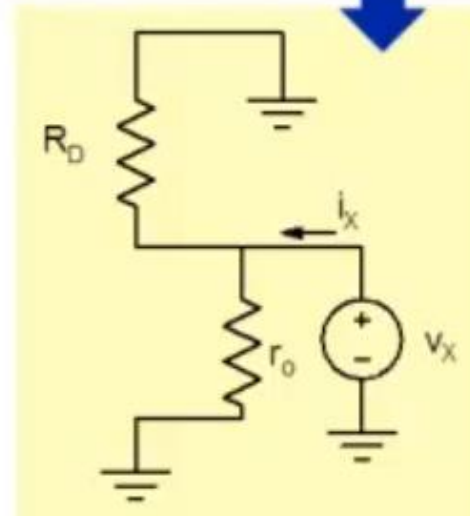
2. Output Resistance

To calculate o/p resistance \rightarrow Remove i/p source
 $V_{gs}=0$
 $I=0$

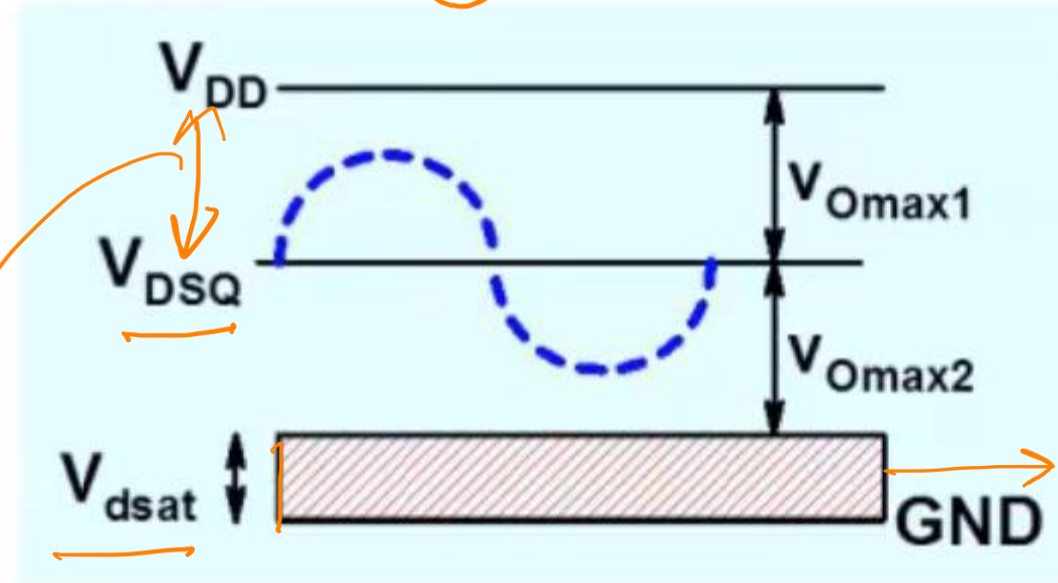
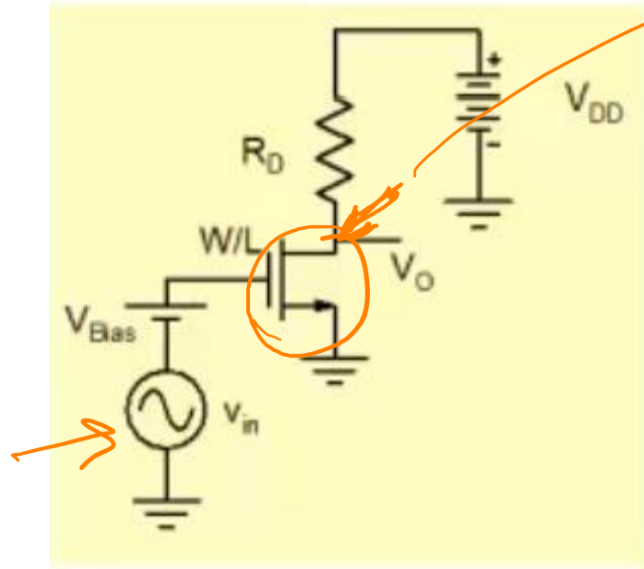


$$R_O = \left. \frac{v_x}{i_x} \right|_{V_{in}=0}$$

$$R_O = R_D \parallel r_o \approx R_D = 80K$$



3. Output Voltage Swing



Ensure MOS remain in saturation

$$v_{Omax1} = V_{DD} - V_{DSQ}$$

$$v_{Omax2} = V_{DSQ} - V_{dsat}$$

$$v_{Omax} = \text{Min}\{v_{Omax1}; v_{Omax2}\}$$

[However, this will not match with actual results/values]

Voltage swing limited by harmonic distortion

- Harmonic distortion in CS amplifier occurs because the relationship between drain current and gate voltage is nonlinear.

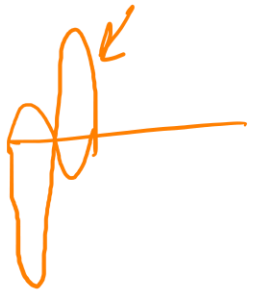
Bias

$$I_{DSQ} + \underset{\text{signal}}{i_{ds}} = \frac{\beta}{2} \times (V_{GSQ} + v_{gs} - V_T)^2$$

$$i_{ds} = g_m v_{gs} + \left(\frac{0.5}{V_{GSQ} - V_T} \right) \times g_m v_{gs}^2$$

Re-arrangement

amplitude



Let's take

$$v_{gs} = a_o \sin(2\pi f_o t)$$

$$i_{ds} = g_m \underbrace{a_o}_{\text{wanted}} \sin(2\pi f_o t) + \left(\frac{a_o^2 g_m}{4V_{GSQ} - V_T} \right) \cos(2\pi 2f_o t)$$

wanted

$$\left(\frac{a_o^2 g_m}{4V_{GSQ} - V_T} \right)$$

$$\left(\frac{a_o^2 g_m}{4(V_{GSQ} - V_T)} \right) \cos(2\pi 2f_o t)$$

x

$$HD_2(\%) =$$

$$\frac{\frac{a_o^2}{4(V_{GSQ} - V_T)}}{a_o} \times 100$$

$$HD_2(\%) = \frac{a_o / 4}{V_{GSQ} - V_T} \times 100$$

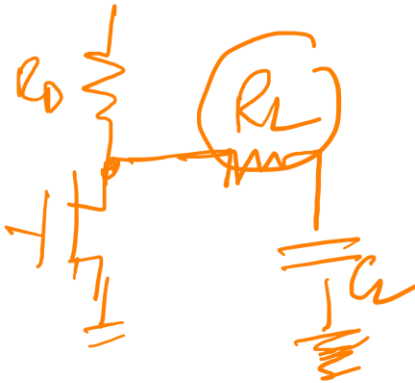
Amplitude distortion

Cont'd (dc) & eliminated by C_{eq}

Output Voltage Swing

$$v_{in} = a_o \sin(2\pi f_o t)$$

$$HD_2 (\%) = \frac{a_o / 4}{V_{GSQ} - V_T} \times 100 = \frac{v_{in}}{V_{dsat}} \times 25$$



$$v_{in} = \frac{HD_2}{25} \times V_{dsat}$$

put for the sake of completeness

Now, $A_v = \frac{v_o}{v_{in}}$

$$v_o = A_v \times v_{in} \cong g_m R_D \parallel R_L \times v_{in}$$

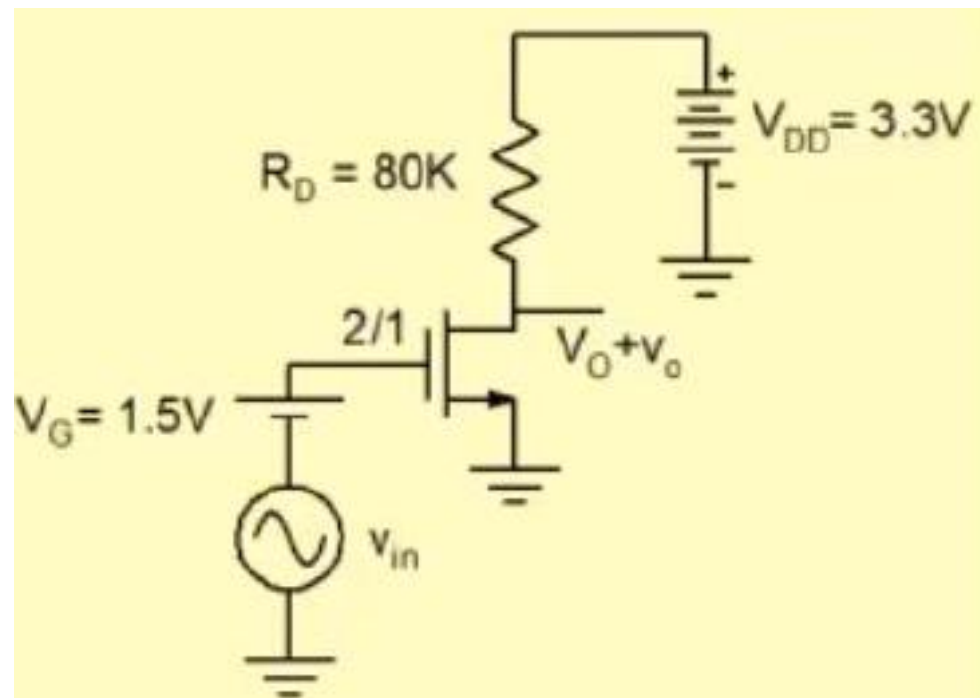
$$g_m = \frac{2I_{DSQ}}{V_{GSQ} - V_T}$$

$$v_{o\max 3} \cong I_{DSQ} R_D \parallel R_L \times \frac{HD_2}{12.5}$$

(This is the swing determined by the distortion)

$$v_{o\max 1} = V_{DD} - V_{DSQ} = I_{DSQ} R_D$$

$$v_{o\max 3} < v_{o\max 1}$$



$$V_{DSQ} = \underline{1.3V}; \underline{V_{sat} = 0.5V}$$

$$\underline{v_{o\max}} \leq V_{DSQ} - V_{sat} = \underline{0.8V} \checkmark$$

$$\underline{v_{o\max 3}} \cong I_{DSQ} R_D \parallel \cancel{R_L} \times \frac{HD_2}{12.5}$$

$$= 0.8V \text{ for } HD_2 = 5\%$$

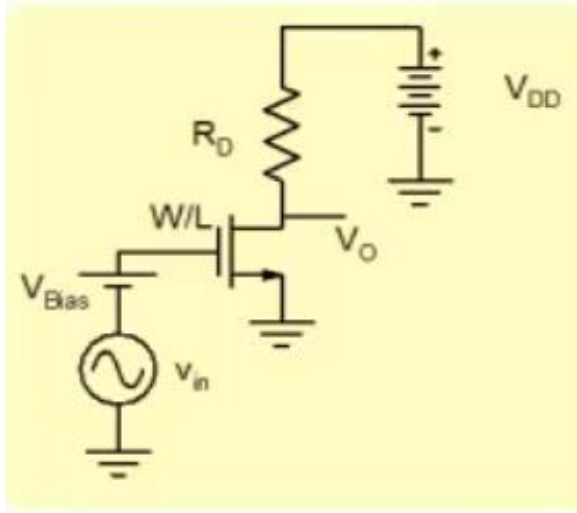
$$25 \mu A \cdot 80K \cdot \frac{5}{12.5}$$

$$\underline{25 \times 10^{-6}} \cdot \underline{80 \times 10^3} \cdot \frac{5}{12.5}$$

$$= 0.8V$$

How to get the maximum swing (at given V_{DD})

Design Problem



$$v_o \cong I_{DSQ} R_D \parallel R_L \times \frac{HD_2}{12.5} \quad \checkmark$$

($v_{o\max 1}$)

$$v_o \leq V_{DSQ} - V_{sat}$$

($v_{o\max 2}$)

Optimum drain-source bias

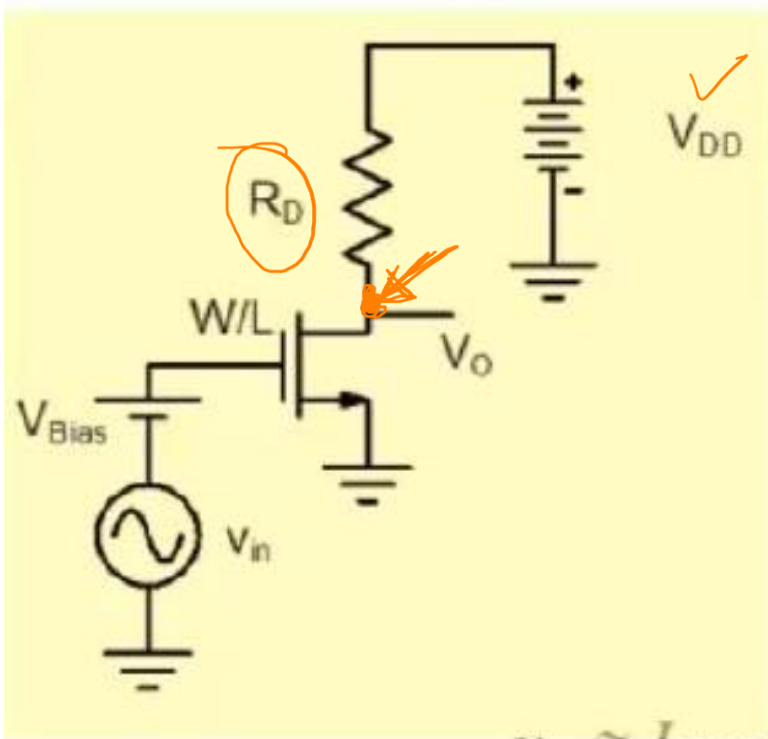
$$I_{DSQ} R_D \times \frac{HD_2}{12.5} \times \frac{1}{1 + R_D/R_L} = V_{DSQ} - V_{sat}$$

$$V_{DSQ} = \frac{V_{DD} \times \frac{HD_2}{12.5} + V_{sat}}{1 + \frac{HD_2}{12.5}}$$

$$v_{o\max} = \frac{V_{DD} - V_{sat}}{1 + \frac{12.5}{HD_2}}$$

$$\chi = 12.5 \times (1 + R_D/R_L)$$

Voltage Gain (including voltage swing)



$$A_v = -g_m R_D$$

$$g_m = \sqrt{2I_{DSQ}\beta}$$

$$R_D = \frac{V_{DD} - V_{DSQ}}{I_{DSQ}}$$

$$A_v = \sqrt{\frac{2\beta}{I_{DSQ}}} \times (V_{DD} - V_{DSQ})$$

$$Max\ C_{ov} \times \frac{W}{L}$$

$$v_o \cong I_{DSQ} R_D \parallel R_L \times \frac{HD_2}{12.5}$$

$$v_o \leq V_{DSQ} - V_{sat}$$

$$V_{DSQ} \geq V_{gs} + V_{sat}$$

$$I_{DSQ} = \frac{\beta}{2} (V_{gs} - V_T)^2$$

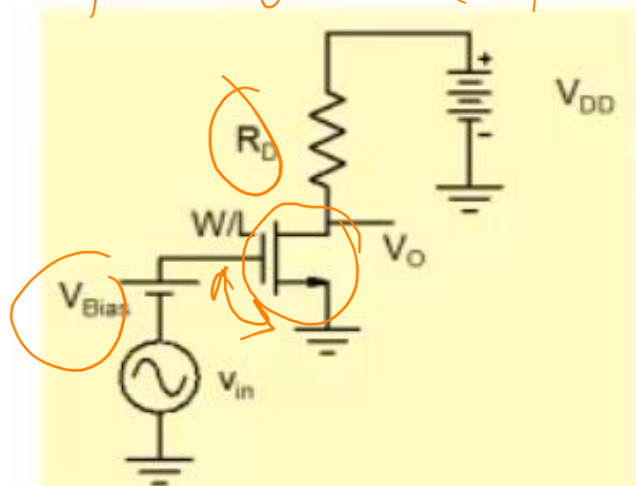
$$\sqrt{\frac{2I_{DSQ}}{\beta}} = (V_{gs} - V_T)$$

Trade off Gain - voltage swing

$$A_v \leq \sqrt{\frac{2\beta}{I_{DSQ}}} \times (V_{DD} - v_{om} - \sqrt{\frac{2I_{DSQ}}{\beta}})$$

max voltage swing

How to improve gain (for example)



$I_{DS} = 25\mu A$ and $v_{om} = 0.1V$,
 $R_D = 108k$

W (μm)	A_v	V_{GS} (V)
2	10.8	1.5
5	18.24	1.316
10	26.6	1.22
20	38.47	1.158
50	62	1.1
100	88.5	1.07

High Width
Cap
B W

$$A_v \leq \sqrt{\frac{2\beta}{I_{DSQ}}} \times (V_{DD} - v_{om} - \sqrt{\frac{2I_{DSQ}}{\beta}})$$

① fix $W = 2\mu m$ and $v_{om} = 0.1V$

I_{DS} (μA)	A_v	R_D (M Ω)	V_{GS} (V)
100	4.4	0.022	2
50	7.05	0.05	1.707
25	10.8	0.108	1.5
10	18.2	0.29	1.316
5	26.6	0.59	1.224
2	43.2	1.53	1.14
1	62	3.1	1.1

Low current or large size is needed to obtain large gain.

Gain saturates as transistor gets closer to threshold voltage

for example

Low swing
May be we want to use this Amp in first stage

$V_{TH} \approx 1V$

Further decrease

Sub- V_T

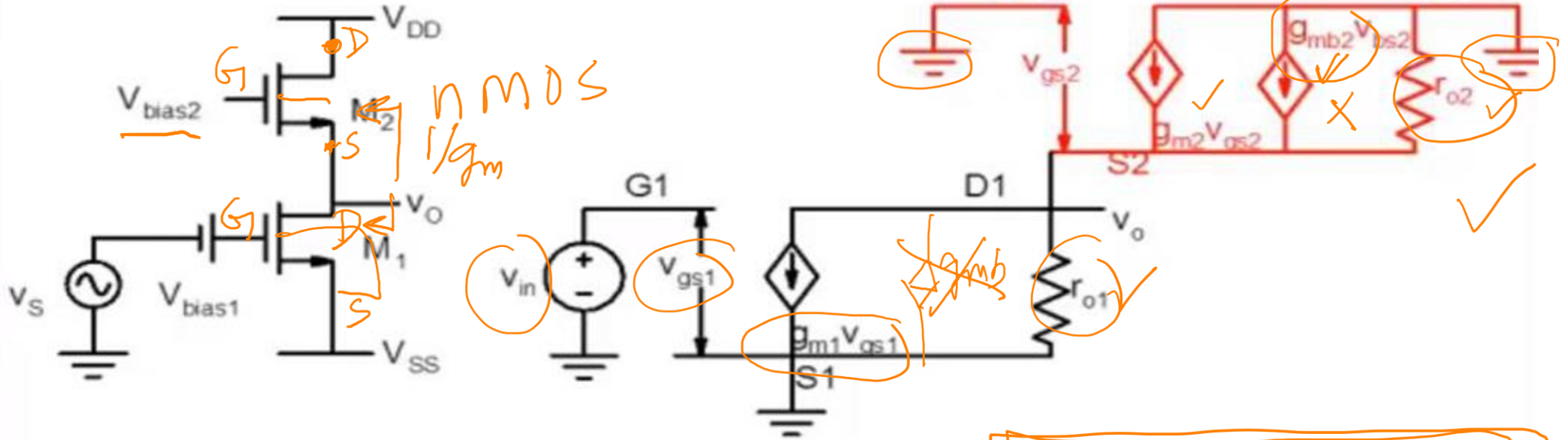
High R

Not good for fabrication

CS amplifier with Active Load

CS Amplifier with Active Load (NMOS)

Small-Signal Model



$$v_o = -g_{m1}v_{in} \times \left\{ r_{o1} \parallel r_{o2} \parallel \frac{1}{g_{m2}(1+\eta)} \right\}$$

$$A_v \cong -\frac{g_{m1}}{g_{m2}(1+\eta)}$$

$$= \frac{\sqrt{2KP \times (W/L)_1 \times I_{DSQ1}}}{\sqrt{2KP \times (W/L)_2 \times I_{DSQ1}}} \times \frac{1}{1+\eta} = \sqrt{\frac{(W/L)_1}{(W/L)_2}} \times \frac{1}{1+\eta}$$

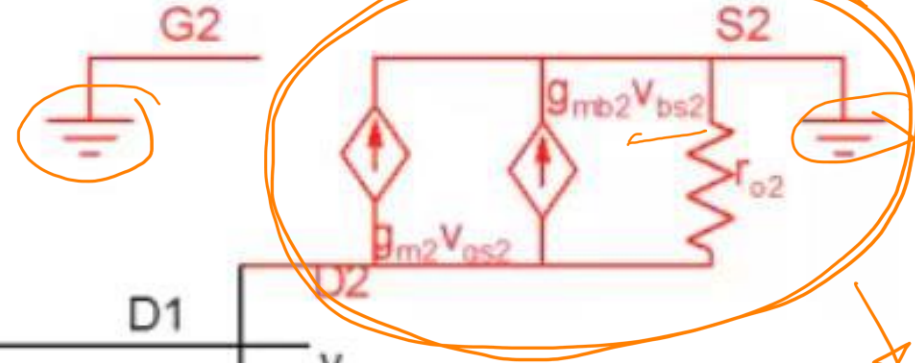
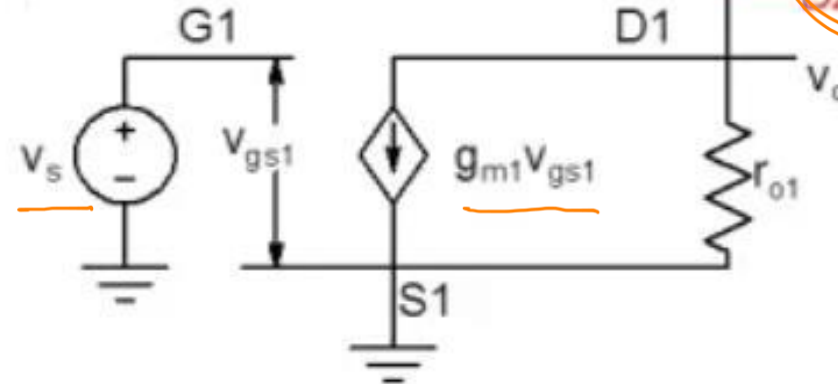
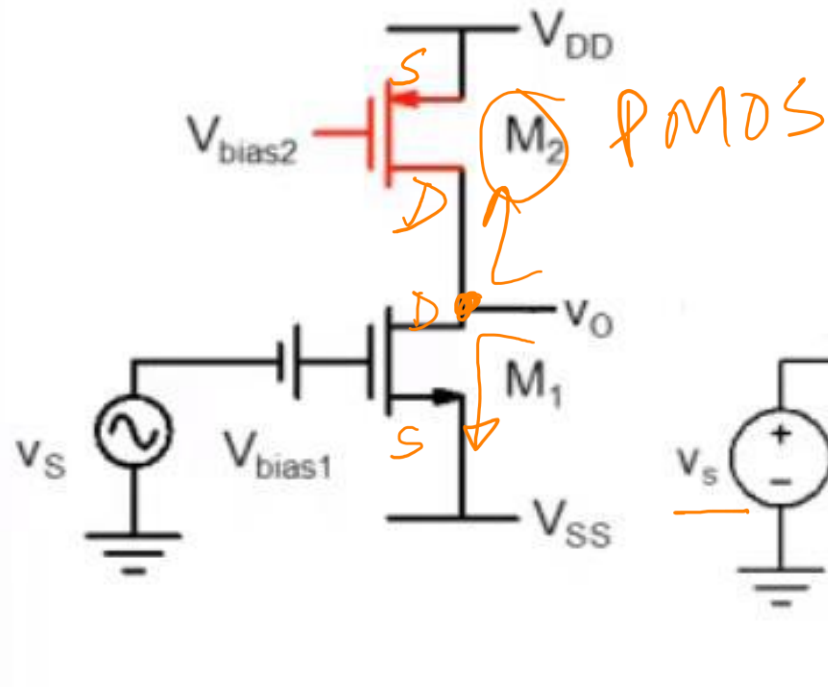
Handwritten calculation:

$$\frac{100}{1} \times \frac{1}{1+1.5} = 10/2.5 = 4$$

Annotation: $g_{m2}(1+\eta)$

CS Amplifier with PMOS active load

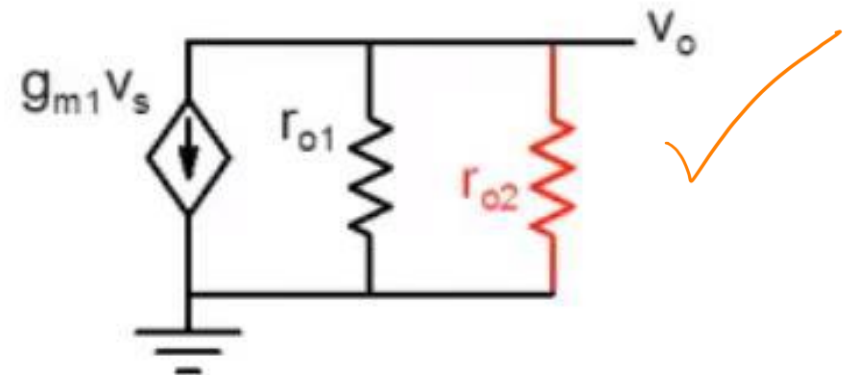
Small-signal Model



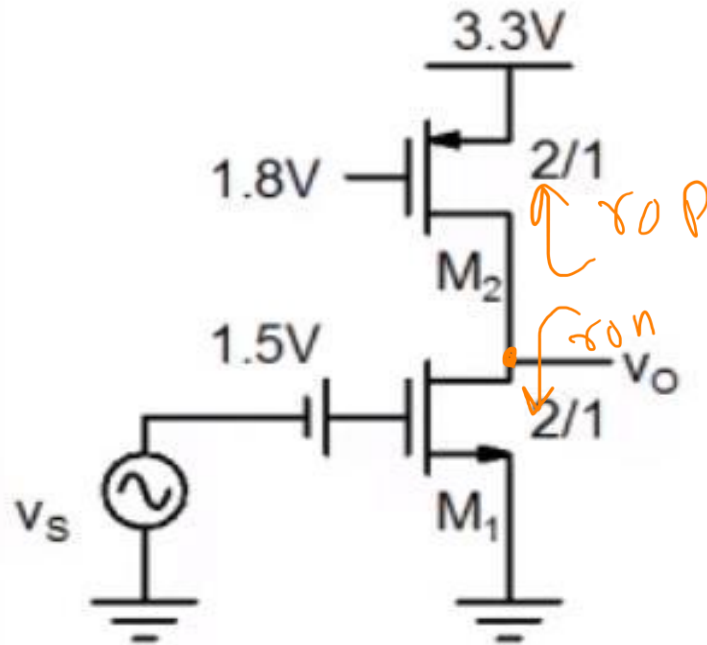
r_{o2}
~~ro2~~



$$A_V = -g_{m1} \times \underline{r_{o1}} \parallel \underline{r_{o2}}$$



Example



$$\underline{I_{DSQ} = 25\mu A; V_{DSQ} = 1.65(?)}$$

$$g_{mn} = g_{mp} = 100\mu A/V ; \underline{r_{on} = r_{op} = 4M\Omega}$$

$$|A_v| = g_{mn} \times r_{on} \parallel r_{op}$$

$$\boxed{= -200} \checkmark$$

$$\boxed{R_o = r_{on} \parallel r_{op} = 2M\Omega}$$

Large gain is easily obtained without requiring large resistors which are difficult to fabricate