SIGNALS & SYSTEMS

Assignment-1

2024102014

1)
(a) $|x^2(t)|$

$f(t) = |x^2(t)|$

$f(-t) = |x^2(-t)|$

$f(t) \neq f(-t) \neq -f(t)$

$\therefore$ The given Signal is neither an even nor odd function. because

The above signal may or may not considered as same without knowing $f(t)$ or $x(t)$.

* for this extent cannot say.

(b) $x(t^2)$

$f(t) = x(t^2)$

$f(-t) = x((-t)^2) = x(t^2)$

$\therefore$ The given Signal is even signal

(c) $x(t) + x(-t)$

$f(t) = x(t) + x(-t)$

$f(-t) = x(-t) + x(-(-t)) = x(t) + x(-t)$

$\therefore$ The given Signal is even Signal

(d) $e^{x(t)}$

$y(t) = e^{x(t)}$

$y(-t) = e^{x(-t)}$

$\therefore$ The above signal, cannot state even or odd.

If the function $x(t)$ Is even then, The whole Signal is even.

If the function $x(t)$ is odd then, the whole signal is neighter even nor odd.

* cannot be labeled as an even or odd without knowing $x(t)$.

e) $x(t) - x(-t)$

$$f(t) = x(t) - x(-t)$$

$$f(-t) = x(-t) - x(t)$$

$$f(-t) = -(x(t) - x(-t))$$

$$f(-t) = -f(t)$$

$\therefore$ The above signal is odd since $f(-t) = -f(t)$.

To proove that for any $x(t)$, there exist two signals, one is even and other is odd. Such that

$$x(t) = x_e(t) + x_o(t).$$

Last Question.

WKT, $x(t)+x(-t)=x_e(t)$

then,

Let $x_e(t)=\frac{1}{2}(x(t)+x(-t))$ also even f^n

WKT, $x(t)-x(-t)=x_o(t)$

then,

Let $x_o(t)=\frac{1}{2}(x(t)-x(-t))$ also odd f^n

$x_e(t)+x_o(t)=\frac{x(t)}{2}+\frac{x(t)}{2}+\cancel{\frac{x(-t)}{2}}-\cancel{\frac{x(-t)}{2}}$

$x_e(t)+x_o(t)=2\times\frac{1}{2}x(t)=x(t)$

$\therefore$ for every signal $x(t)$ $\exists$ an even and an odd functions $x_e(t)$ and $x_o(t)$ respectively.

Such that $x(t)=x_e(t)+x_o(t)$

Verification :-

$x(t)=x_e(t)+x_o(t)$ — ①

$x(-t)=x_e(-t)+x_o(-t)$

$x(-t)=x_e(t)-x_o(t)$ — ②

from ① & ②.

①+② $2x_e(t)=x(t)+x(-t)$

$x_e(t)=\frac{x(t)+x(-t)}{2}$ — ③

①−② $2x_o(t)=x(t)-x(-t)$

$x_o(t)=\frac{x(t)-x(-t)}{2}$ — ④

③+④ $\Rightarrow x_e(t)+x_o(t)=\frac{x(t)}{2}+\frac{x(t)}{2}-\cancel{\frac{x(-t)}{2}}+\cancel{\frac{x(-t)}{2}}$

$x_e(t)+x_o(t)=x(t)$

$\therefore$ Hence showed.

2)
$$x(t) = \frac{1}{2}\cos(2\pi f_1 t + \phi_1) + e^{j(2\omega_2 t + \phi_2)}$$

$$x(t) = \frac{1}{2}\cos(2\pi f_1 t + \phi_1) + e^{j(4\pi f_2 t + \phi_2)}$$

Let ~~us~~ seperate $x(t)$ into two parts

$$\underbrace{\frac{1}{2}\cos(2\pi f_1 t + \phi_1)}_{\text{First term}} + \underbrace{e^{j(4\pi f_2 t + \phi_2)}}_{\text{Second term}}$$

$$T_1 = \frac{1}{f_1} \qquad T_2 = \frac{1}{2f_2}$$

1st term:- euler form

$$\frac{1}{2}\cos(2\pi f_1 t + \phi_1) = \frac{1}{4}\left(e^{j(2\pi f_1 t + \phi_1)} + e^{-j(2\pi f_1 t + \phi_1)}\right)$$

$$x(t) = \frac{1}{4}e^{j(2\pi f_1 t + \phi_1)} + \frac{1}{4}e^{-j(2\pi f_1 t + \phi_1)} + e^{j(4\pi f_2 t + \phi_2)}$$

For the Signal to be periodic ratio of Time period of 1st and 2nd term to be a rational number.

$$\frac{f_1}{2f_2} = \frac{p}{q}$$

if $\frac{p}{q}$ is rational $x(t)$ is periodic

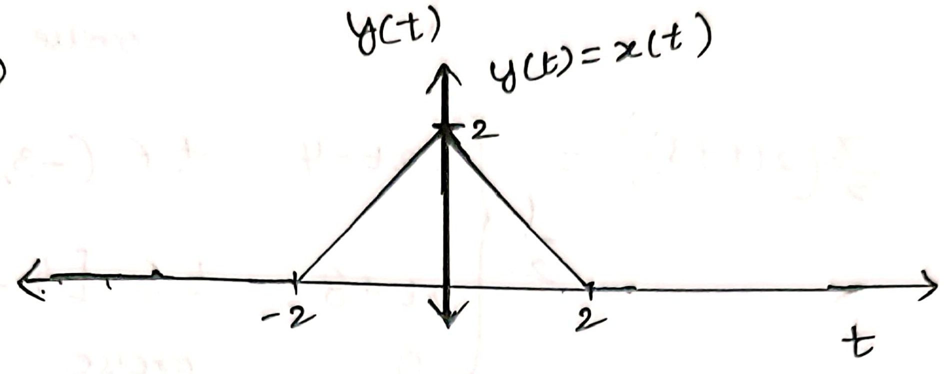
if $\frac{p}{q}$ is irrational $x(t)$ is non-periodic

* ϕ_1 and ϕ_2 do not affect the periodicity of the signal. ϕ_1 and ϕ_2 only shifts the Signal.

* If we add more terms of sinusoidal's will also require the ratios of frequency or time periods to be rational for the signal to remain periodic

3) $x(t) = \begin{cases} 2-t & \text{if } t \in [0,2], \\ 2+t & \text{if } t \in [-2,0], \\ 0 & \text{otherwise} \end{cases}$

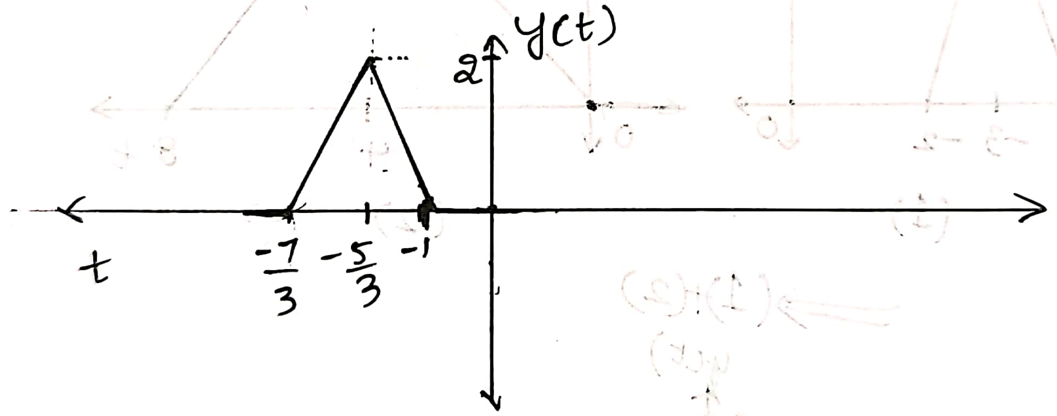
(a)



(b) $y(t) = x(3t+5)$

$x(3t+5) = \begin{cases} -3t-3 & 3t+5 \in (0,2] \\ 7+3t & 3t+5 \in [-2,0] \\ 0 & \text{elsewhere} \end{cases}$

$x(3t+5) = \begin{cases} -3t-3 & t \in (\frac{-5}{3}, -1] \\ 7+3t & t \in [\frac{-7}{3}, \frac{-5}{3}] \\ 0 & \text{else where} \end{cases}$



(c) $y(t) = x(2-0.5t) + 0.5x(2(t+3))$

* The y(t) signals into two parts by addition.
* Signal 1 contains x(t) scaling by 2 and reversing time axis and adding 2.
* Signal 2 contains x(t) time scaling by 0.5 and amplified by $\frac{1}{2}$ and shifted by some constant

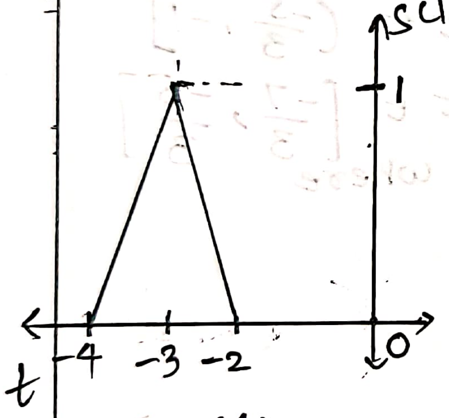
1. $x(2-\frac{t}{2}) = \begin{cases} \frac{t}{2} & t \in [0,4) \\ 4-\frac{t}{2} & t \in [4,8] \\ 0 & \text{elsewhere} \end{cases}$

2. $\frac{x}{2}\left(2(t+3)\right) = \frac{1}{2}\begin{cases} 2-(2t+6) & 2t+6 \in (0,2] \\ 2+(2t+6) & 2t+6 \in [-2,0] \\ 0 & \text{orelse} \end{cases}$

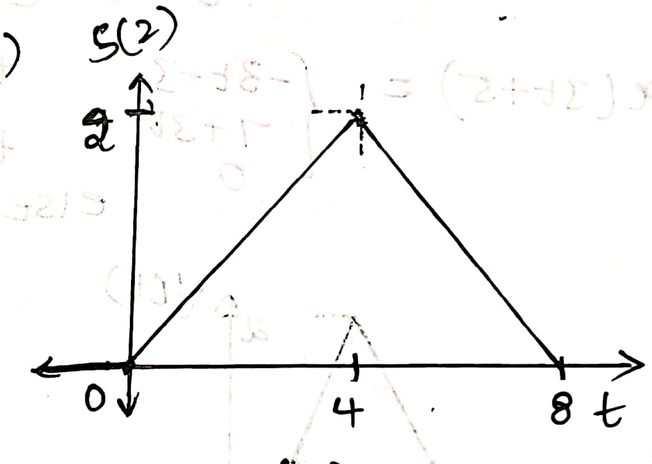
$$\frac{x}{2}\left(2(t+3)\right) = \frac{1}{2}\begin{cases} -2t-4 & t \in (-3,-2] \\ 2t+8 & t \in [-4,-3] \\ 0 & \text{orelse} \end{cases}$$

$$x\left(2(t+3)\right) = \begin{cases} -t-2 & t \in (-3,-2] \\ t+4 & t \in [-4,-3] \\ 0 & \text{orelse} \end{cases}$$

<u>adding</u> <u>1 & 2.</u>

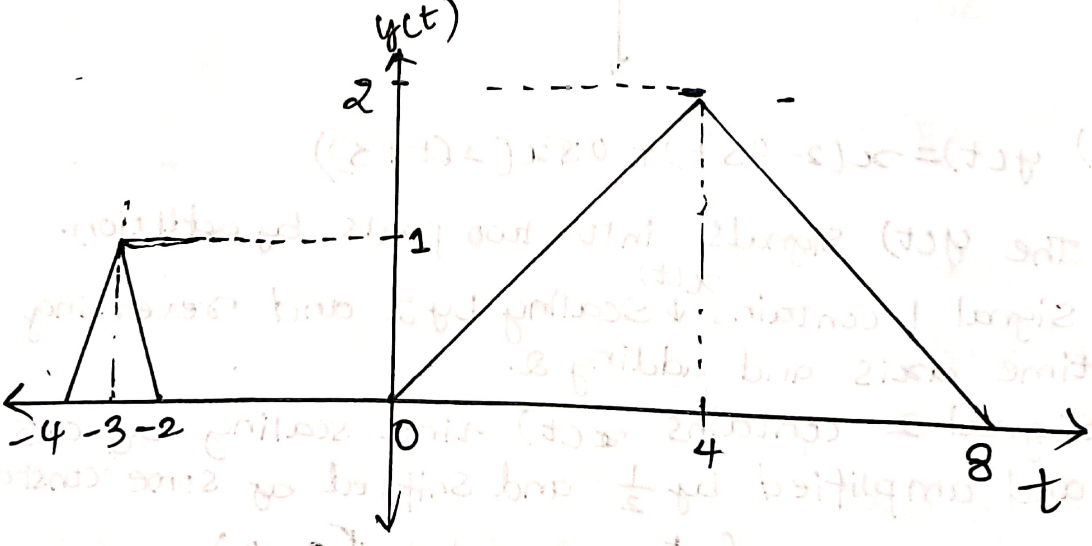


(1)



(2)

$\Longrightarrow (1)+(2)$



(d) $y(t) = x(t)x(t+1)$

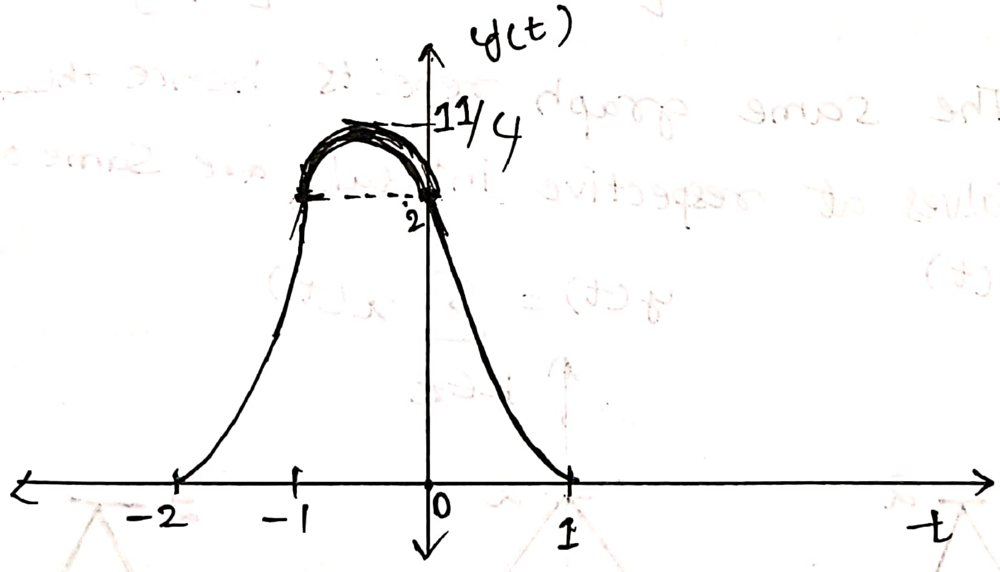
Product of the $x(t)$ and the same $x(t)$ shifted by -1.

$$x(t) \begin{cases} 2-t & t \in (0,2] \\ 2+t & t \in [-2,0] \\ 0 & \text{elsewhere} \end{cases}$$

$$x(t+1) \begin{cases} -t+1 & t \in (-1,1] \\ 3+t & t \in [-3,-1] \\ 0 & \text{elsewhere} \end{cases}$$

$$x(t) \cdot x(t+1) \begin{cases} (-t+1)(2-t) & (0,1] \ni t \\ (2+t)(-t+1) & [-1,0] \ni t \\ (2+t)(3+t) & [-2,-1] \ni t \\ 0 & \text{elsewhere} \end{cases}$$

$$x(t) \cdot x(t+1) \begin{cases} t^2-3t+2 & t \in (0,1] \\ 2-t^2-t & t \in (-1,0] \\ 6+t^2+5t & t \in [-2,-1] \end{cases}$$



(e) $y(t) = \sum_{k \in \mathbb{Z}} x(t-6k)$:

* endless summation of the Signal $x(t)$ shifted by multiple of 6 frontend and Backend because "$k \in \mathbb{Z}$"

* the graph reapeats like a sinad for every such intsant. (Sinusoid)

* So, The inverval Never be common because of shifting by multiple of "6".

So, we can add all the summation of grap in a single graph.

$$y(t) = \sum_{k \in \mathbb{Z}} x(t - 6k)$$

k=1

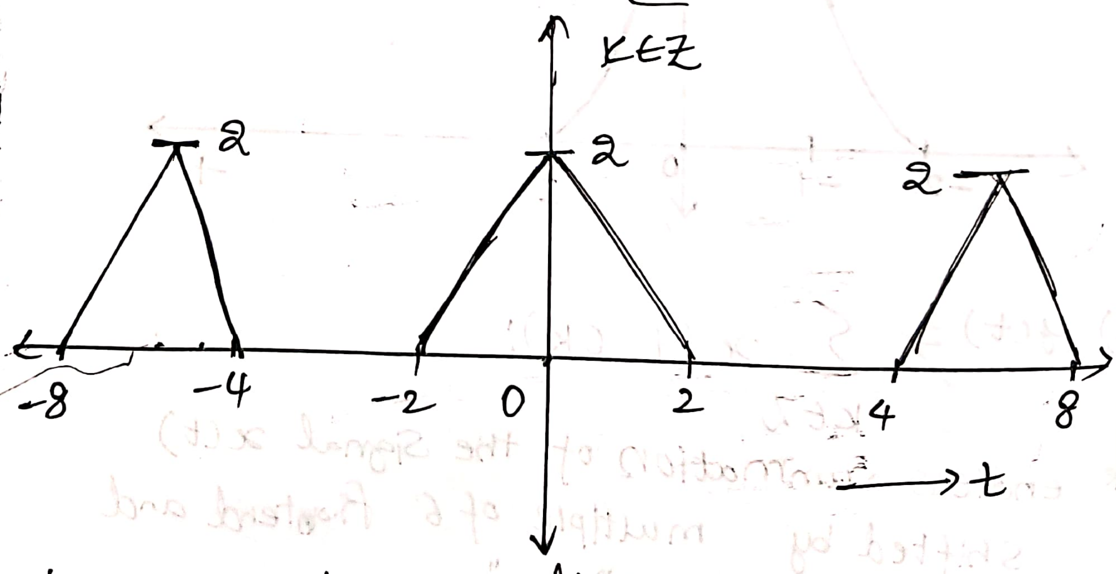
$$x(t-6) = \begin{cases} 8-t & t \in (6,8] \\ -4+t & t \in [4,6] \end{cases}$$

k=0

$$x(t) = \begin{cases} 2-t & t \in (0,2] \\ t+2 & t \in [-2,0] \end{cases}$$

∴ The same graph repeats hence the Values at respective intesals are Same of y(t)

$$y(t) = \sum_{k \in \mathbb{Z}} x(t)$$



∴ It continues endless. as same

(f) $y(t) = \lim_{t \to 0} x(t)$

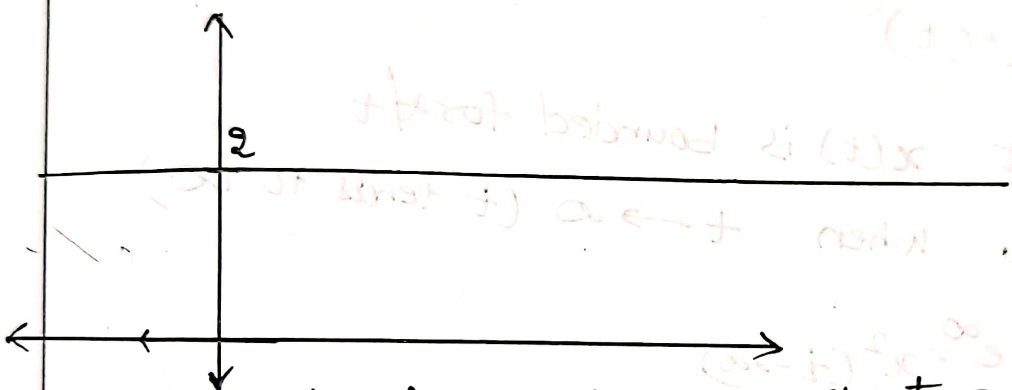
$$x(t) \begin{cases} 2-t & t \in (0,2] \\ 0 & \text{elsewhere} \\ 2+t & t \in [-2,0] \end{cases}$$

when $t \to 0$ $x(t) \to$ constant (in all cases)

$\cancel{x(t)}$

$$\lim_{x \to 0} x(t) \begin{cases} 2-0 \\ 0 \\ 2+0 \end{cases}$$

(dosen't depend on Interval)



graph of y(t) is a constant graph with Slope "1".

4) Question

(a) $y(t) = e^t x^2(t)$

LTI :-

Scaling:-

$y(t) = e^t x^2(t)$

$a \cdot y(t) = a \cdot e^t x^2(t)$

i/p $a \cdot x(t)$ $=$ o/p $e^t \cdot a^2 \cdot x(t)$

o/p $= a \cdot y(t) \neq e^t \cdot a^2 \cdot x(t)$ (when i/p $= a \cdot x(t)$)

y(t) donot show Scaling property so, it does not follow Linear system.

Hence, ∴ y(t) not a Time Invariant System.

Causal: i/p $x(t) \Rightarrow x(\tau)$

o/p $e^t \cdot x^2(t) = y(t) \Rightarrow e^{\tau} \cdot x^2(\tau) = y(\tau)$

o/p $y(t)$

In $x(\tau)$ $\nexists$ $\tau > t$ so, the signal is causal.

$\therefore$ The value of $y(t)$ depends on $x(\tau) \forall \tau \in t$ and not on $x(\tau)$ such that $x(\tau) \leq t$ $\tau > t$

hence it is a causal system.

bounded:-

$y(t) = e^t x^2(t)$

If let $x(t)$ is bounded for $\forall t$.

then, when $t \to \infty$ (t tends to ∞)

$y(t) = e^{\infty} \cdot x^2(t \to \infty)$

$y(t) = \infty$

Not a bounded Signal(system)

(b) **LTI** $y(t) = t - x(t)$

Linear:-

$y_1(t) = t - x_1(t)$

$y_2(t) = t - x_2(t)$

$y_1(t) + y_2(t) = t - (x_1(t) + x_2(t))$

$2t - x_1(t) - x_2(t) \neq t - x_1(t) - x_2(t)$

$\therefore$ It is not an Linear (Not following addition)

$\therefore$ the given system is not a LTI and does not follow scaling property.

so [Not (linear time invariant)] (LTI)

Causal

$y(t) = t - x(t) \Rightarrow y(\tau) = \tau - x(\tau)$

$\therefore$ The value of $y(t)$ depends on $x(\tau)\ \forall\ \underbrace{\tau \in t,\ \tau \le t}$ and not on $x(\tau)$

Suchthat $\tau > t$

$\therefore$ it is a causal system.

bounded:-

$y(t) = t - x(t)$

Let $x(t)$ is bounded $\forall t$

then, when $t \to \infty$

$y(t) \longrightarrow \infty \quad [\infty - x(t \to \infty)]$

$y(t) = \infty$

Not a bounded Signal (system).

(c) $y(t) = \dfrac{e^{2^{x(t)}}}{t}$

LTI ; Scaling:-

$a y(t) = \dfrac{e^{2^{a \cdot x(t)}}}{t} \quad (i/p = a \cdot x(t))$

$a \cdot \dfrac{e^{2^{x(t)}}}{t} \neq \dfrac{e^{2^{a \cdot x(t)}}}{t}$

Not following scaling condition.

So, Not a Linear System

$\therefore$ Not a Linear Time invariant System

Causal

$y(t) = \dfrac{e^{2^{x(t)}}}{t} \Rightarrow y(\tau) = \dfrac{e^{2^{x(\tau)}}}{\tau}$

$\therefore$ The value of $y(t)$ depends on $x(\tau)\ \forall$ $\tau \le t, \tau \in t$ and not depends on $x(\tau)$ Suchthat $\tau > t$

$\therefore$ it is a causal System.

bounded:- $y(t) = \dfrac{e^{2^{x(t)}}}{t}$

let $x(t)$ is bounded for $\forall t$

but $y(t) \longrightarrow \infty$ when $t=0$.

So, $y(t)$ is not a bounded system

(d) $y(t) = \displaystyle\int_{-\infty}^{t/2} h(t-\tau)x(\tau)d\tau$

LTI scaling

$a \cdot y(t) = \displaystyle\int_{-\infty}^{t/2} a \cdot h(t-\tau)x(\tau)d\tau$ [i/0 = $a \cdot x(t)$]

$\displaystyle\int_{-\infty}^{t/2} h(t-\tau) \cdot a \cdot x(\tau)d\tau = \int_{-\infty}^{t/2} a \cdot h(t-\tau) \cdot x(\tau)d\tau$

it shows scaling property.

adding:-

$y_1(t) = \displaystyle\int_{-\infty}^{t/2} h(t-\tau) \cdot x_1(\tau)d\tau$

$y_2(t) = \displaystyle\int_{-\infty}^{t/2} h(t-\tau) \cdot x_2(\tau)d\tau$

$y_1(t) + y_2(t) = \displaystyle\int_{-\infty}^{t/2} h(t-\tau) \cdot (x_1(\tau) + x_2(\tau))d\tau$

$\displaystyle\int_{-\infty}^{t/2} h(t-\tau) \cdot x_1(\tau)d\tau + h(t-\tau) \cdot x_2(\tau)d\tau$

$= \displaystyle\int_{-\infty}^{t/2} h(t-\tau) \cdot (x_1(\tau) + x_2(\tau))d\tau$

It shows additivity also

$y(t)$ is a linear system.

time Variant :-

For $x(t-t_0)$

$$y(t-t_0) = \int_{-\infty}^{(t-t_0)} h(t-t_0-\tau)\cdot x(\tau)\,d\tau$$

$[\text{i/p } x(\tau-t_0)]$

$$y_{\text{shifted}}(t) = \int_{-\infty}^{t} h(t-t_0-\tau)\cdot x(\tau-t_0)\,d\tau$$

$$y(t-t_0) = \int_{\tau=-\infty}^{\tau=(t-t_0)/2} h(t-t_0-\tau)\cdot x(\tau)\, d\tau$$

$$y_{shifted}(t) = \int_{\tau=-\infty}^{\tau=t/2} h(t-\tau)\, x(\tau-t_0)\, d\tau$$

$a=\tau-t_0$

$$y_s(t) = \int_{-\infty}^{\frac{t}{2}-t_0} x(a)\cdot h(t-t_0-a)\, da.$$

$$y_s(t) = \int_{-\infty}^{t/2-t_0} x(\tau)\cdot h(t-t_0-\tau)\, d\tau$$

$$y_s(t) \neq y(t-t_0)$$

So, $y(t)$ is Not an Linear time invariant.

$y_{shifted}(t) \neq y(t-t_0)$ Then $y(t)$ is time variant
So, $y(t)$ is not "LTI"

Causal:-

$$y(t)=\int_{-\infty}^{t/2} h(t-\tau)x(\tau)\,d\tau$$

at $t<0$ then $y(t)$ depends on $x(\tau)$, for some $\tau>t$, Hence Not causal

The value of $x(t)$ is depending on $\tau >$ "t" (at integral $\int_{-\infty}^{t/2}$) But Acc. to definition $x(t)$ should not depend on $\tau >$ "t"

So, system $y(t)$ is $\boxed{\text{not causal}}$ (If current time is $(-t)$ the $x(t)$ depends on limit $(-t/2)$ (which is greater than $-t$ so (depends on future value))

Bounded:-

$$y(t)=\int_{-\infty}^{t/2} h(t-\tau)x(\tau)\,d\tau$$

$\int_{-\infty}^{\infty} h^2(t)\,dt < \infty$ because

The system $y(t)$ is not bounded

The area under the given functional graph may or may or be finite. because.

(For Example when $h(t-\tau)*x(\tau)$ the is a constant then area under $y(t) \longrightarrow \infty = \infty$

So, the $y(t)$ System is not a bounded system.

(e) $y(t)=\frac{d}{dt}x(t+t_0)$, $t_0 \in$ constant

(e) **LTI:-**

Scaling

$$a.y(t) = a.\frac{d}{dt}x(t+t_0)$$

$$\frac{d}{dt}.a.x(t+t_0) = a.\frac{d}{dt}x(t+t_0) \quad [i/P\ a.x(t)]$$

y(t) follows scaling property.

adding-

$$y_1(t) = \frac{d}{dt}x_1(t+t_0)$$

$$y_2(t) = \frac{d}{dt}x_2(t+t_0)$$

$$y_1(t) + y_2(t) = \frac{d}{dt}(x_1(t+t_0) + x_2(t+t_0))$$

$$\frac{d}{dt}x_1(t+t_0) + \frac{d}{dt}x_2(t+t_0) = \frac{d}{dt}(x_1(t+t_0) + x_2(t+t_0))$$

y(t) follows additivity.

time invariant:-

$$y(t) = \frac{d}{dt}x(t+t_0)$$

$$y(t-t_0) = \frac{d}{dt}x(t-t_0) \quad (o/P)$$

$$x(t-t_0) \Rightarrow (i/P) \Rightarrow \frac{d}{dt}x(t-t_0) = y.$$

y(t) is time invarriant

∴ This concludes y(t) is linear time invariant system.

Causal

The system output depends on the value of $x(t+t_0)$ (future value)

Hence, the system is non-causal

at $t>0$ $y(t)$ depends on $x(\tau)$ where $\boxed{\tau > t}$ then, (non-causal)

Boundedness

Let $x(t)$ bounded but some cases when graph of $x(t)$ has sharp edges where differentiation is not defined so and also for square wave function if we differentiate we get impulse (unbounded)

* $y(t)$ is unbounded.

(a) $y(t) = x(t) = \begin{cases} 2-t & [0\ 2] \\ 2+t & [-2, 0] \\ 0 & \end{cases}$

~~by the abovegraph~~

$$\text{Energy}_\infty = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$\text{Energy}_\infty = \int_{-\infty}^{-2+\delta} |x(t)|^2 dt + \int_{-2}^{0} |x(t)|^2 + \int_{0-\delta}^{2} |x(t)| + \int_{2+\delta}^{\infty} |x(t)|^2 dt$$

$$\text{Energy}_\infty = 0 + \cancel{\quad} +$$

$$E_\infty = \left[\frac{(2+t)^3}{3}\right]_{-2}^{0} + \left[\frac{(2-t)^3}{3}\right]_{0}^{2}$$

$$E_\infty = \frac{8}{3} + \frac{8}{3} = \frac{16}{3}$$

E_∞ = finite so, $P_\infty = 0$ (avg)

(b) $y(t) = x(3t+5) = \begin{cases} -3t-3 & (-\frac{5}{3}, -1 \\ 7+3t & [-\frac{7}{3}, -\frac{5}{3} \\ 0 & \end{cases}$

$$E_\infty = \int_{-\infty}^{\infty} |x(3t+5)|^2 dt$$

$$E_\infty = \int_{-\infty}^{-\frac{7}{3}} \cancel{|x(3t+5)|^2 dt} \to 0 + \int_{-\frac{7}{3}}^{-5/3} (7+3t)^2 dt$$

$$+ \int_{-\frac{5}{3}}^{-1} (-3t-3)^2 dt + \int_{-1}^{\infty} \cancel{|x(3t+5)|^2 dt} \to 0$$

$$E_\infty = \left[\frac{(7+3t)^3}{3\times 3}\right]_{-7/3}^{-5/3} + \left[\frac{-(-3t-3)^3}{3\times 3}\right]_{-5/3}^{-1}$$

$$= \frac{8}{9} - 0 + 0 + \frac{8}{9}$$

$$E_\infty = \frac{16}{9} \qquad \text{so,}$$

E_∞ = finite $\quad P_{avg} = 0$

(e) $y(t) = \sum_{k \in Z} x(t-6k)$

* $y(t)$ graph is continous and positive (periodic) and endless

so $E_\infty = \infty$ $\quad E_\infty = \int_{-\infty}^{x} y^2(t) + \int_{-2}^{0} y^2(t) + \int_{0}^{2} y^2(t) + \int_{4}^{6} y^2(t)$

$\int_{6}^{8} y^2(t) \cdots\cdots$

$P_{avg} = \lim_{t \to \infty} \frac{1}{t} \int E_\infty$

* Hence $y(t)$ is perfect periodic signal with period of time "6 units".

So, the $P_{avg} = P_{\text{Timeperiod}}$ (P at Total)

$$P_{avg} = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} (x(t))^2 \, dt$$

$$P_{avg} = \frac{1}{T_0} \int_{-3}^{3} (x(t))^2 \, dt$$

$$P_{avg} = \frac{1}{6} \cdot \left[0 + \frac{16}{3} + 0 \right]$$

$$P_{avg} = \frac{8}{9}$$