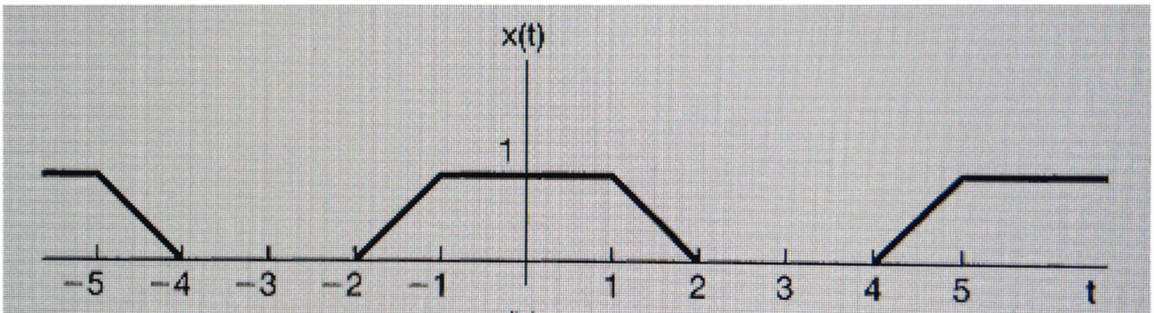
3.22 (a)

(b)



(b)

−5 marks

x(t) is periodic with T = 6

Defining from [−3, 3]

$$x(t) = \begin{cases} 0, & -3 \le t < -2 \\ t+2, & -2 \le t < -1 \\ 1, & -1 \le t < 1 \\ -t+2, & 1 \le t < 2 \\ 0, & 2 \le t < 3 \end{cases}$$

$$\boxed{a_k = \frac{1}{T} \int_{\langle T \rangle} x(t) e^{-jk\omega_0 t} \, dt}$$

$, \omega_0 = \frac{2\pi}{T}$

$$a_0 = \frac{1}{T}\underbrace{\int_{\langle T\rangle} x(t)\,dt}$$

$\hookrightarrow$ Area of Trapezium

$\frac{1}{2}((1+5)(1))$

$= 3$

$$a_0 = \frac{1}{6} \times 3$$

$$\boxed{a_0 = \frac{1}{2}}$$

$$a_k = \frac{1}{T}\int x(t)e^{-jk\omega_0 t}\,dt$$

$$= \frac{1}{6}\int_{-2}^{2} x(t)e^{-jk\omega_0 t}\,dt$$

$$= \frac{1}{6}\left[\int_{-2}^{-1}(t+2)e^{-jk\omega_0 t}\,dt + \int_{-1}^{1} e^{-jk\omega_0 t}\,dt + \int_{1}^{2}(-t+2)e^{-jk\omega_0 t}\,dt\right]$$

$$= \frac{1}{6}\left[\frac{(t+2)e^{-jk\omega_0 t}}{-jk\omega_0} + \frac{1.e^{-jk\omega_0 t}}{(-jk\omega_0)^2}\right]_{-2}^{-1}$$

$$+\left[\frac{e^{-jk\omega_0 t}}{-jk\omega_0}\right]_{-1}^{1}+\left[\frac{(-t+2)e^{-jk\omega_0 t}}{-jk\omega_0}+\frac{-1e^{jk\omega_0 t}}{(-jk\omega_0)^2}\right]_{1}^{2}$$

After applying limits

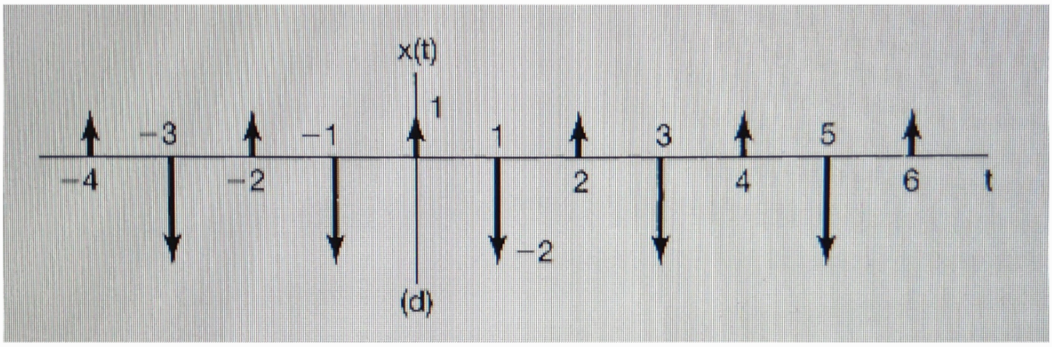
$$=\frac{\cos(k\omega_0)-\cos(2k\omega_0)}{3k^2\omega_0^2}$$

$$=\frac{6}{\pi^2k^2}\sin\left(\frac{\pi k}{2}\right)\sin\left(\frac{\pi k}{6}\right),\quad k\neq 0$$

$$\boxed{a_k=\begin{cases}0, & k\in \text{even}\\[2ex] \dfrac{6}{\pi^2k^2}\sin\left(\dfrac{\pi k}{2}\right)\sin\left(\dfrac{\pi k}{6}\right), & k\in\text{odd}\end{cases}}$$

↳ since fx^n is odd, even harmonics will be zero which we can validate by expression as well

(d)



– 5 maks

This part is repeating

$\boxed{T = 2}$

$$x(t) = \delta(t) - 2\delta(t-1)$$

$$a_k = \frac{1}{T}\int x(t)e^{-jk\omega_0 t}\,dt$$

$$a_k = \frac{1}{2}\int_0^2 \left(\delta(t) - 2\delta(t-1)\right)e^{-jk\omega_0 t}\,dt$$

$$= \frac{1}{2}\left(1 - 2e^{-jk\omega_0}\right)$$

$$\omega_0 = \frac{2\pi}{2}$$

$$\boxed{\omega_0 = \pi}$$

$$= \frac{1}{2}\left[1 - 2\left(\cos(k\pi) - \underbrace{\sin(k\pi)}_{0}\right)\right]$$

$$\boxed{a_k = \frac{1}{2}\left(1 - 2(-1)^k\right)} \quad , \; k \neq 0$$

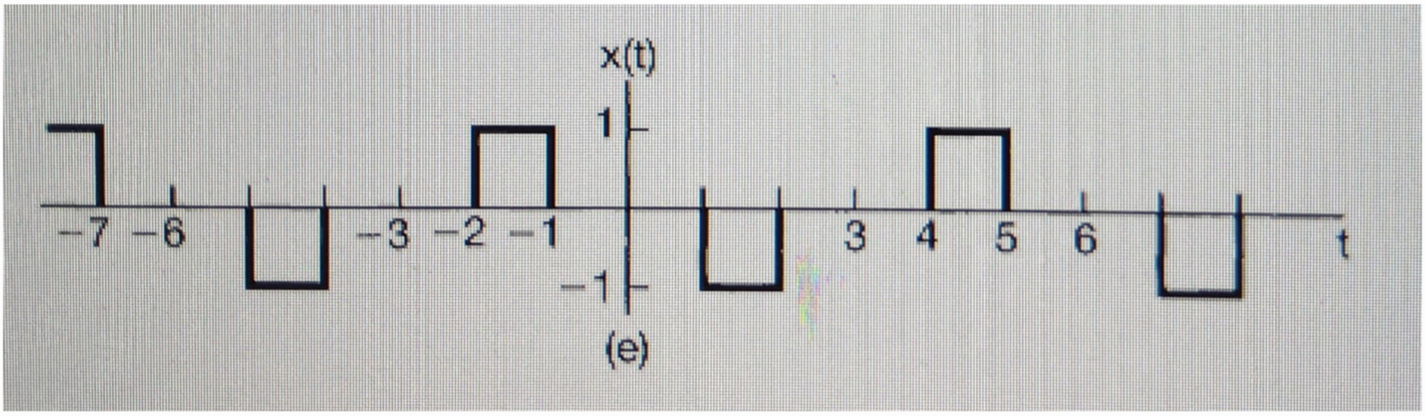
$$a_0 = \frac{1}{T}\int x(t)\,dt$$

$$= \frac{1}{T}\int_0^2 [\delta(t) - 2\delta(t-1)]\,dt$$

$$= \frac{1}{2}(1-2)$$

$$\boxed{a_0 = -\frac{1}{2}}$$

(e) ← 5 marks



↳ This part is repeating

$$\boxed{T = 6}$$

$$\omega_0 = \frac{2\pi}{6}$$

$$\omega_0 = \frac{\pi}{3}$$

$$x(t) = \begin{cases} 0, & -3 < t < -2 \\ 1, & -2 \le t < -1 \\ 0, & -1 \le t < 1 \\ -1, & 1 \le t < 2 \\ 0, & 2 \le t < 3 \end{cases}$$

$$a_0 = \frac{1}{T}\int_{\langle T\rangle} \underbrace{x(t)\,dt}$$

$\Rightarrow$ +ve & -ve square cancels each other so its 0

$$\boxed{a_0 = 0}$$

$$a_k = \frac{1}{6}\left[\int_{-2}^{-1} e^{-jk\omega_0 t}\,dt - \int_{1}^{2} e^{-jk\omega_0 t}\,dt\right]$$

$$a_k = \frac{1}{6}\left[\left(\frac{e^{-jk\omega_0 t}}{-jk\omega_0}\right)_{-2}^{-1} - \left(\frac{e^{-jk\omega_0 t}}{-jk\omega_0}\right)_{1}^{2}\right]$$

$$= \frac{-1}{6jk\omega_0}\left[\left(e^{jk\omega_0} - e^{2jk\omega_0}\right) - \left(e^{-2jk\omega_0} - e^{-jk\omega_0}\right)\right]$$

$$= \frac{-1}{6jk\omega_0}\left[\left(e^{jk\omega_0} + e^{-jk\omega_0}\right) - \left(e^{2jk\omega_0} + e^{-2jk\omega_0}\right)\right]$$

$$= -\frac{1}{6jk\omega_0}\left[2\cos(k\omega_0) - 2\cos(2k\omega_0)\right]$$

$$= \frac{1}{3jk\omega_0}\left[\cos(2k\omega_0) - \cos(k\omega_0)\right]$$

$$\boxed{a_k = \frac{1}{jk\pi}\left[\cos(2k\pi/3) - \cos\left(k\frac{\pi}{3}\right)\right]}$$

3.22) — 5 marks

b $x(t)$ is periodic with period 2

$$x(t) = e^{-t} \quad -1 < t < 1$$

$$a_k = \frac{1}{2}\int_{-1}^{1} e^{-t} e^{-jk\omega_0 t}\, dt$$

$$= \frac{1}{2}\int_{-1}^{1} e^{-(1+jk\omega_0)t}\, dt$$

$$= \frac{-1}{2(1+jk\omega_0)}\left[e^{-(1+jk\omega_0)t}\right]_{-1}^{1}$$

$$= \frac{-1}{2(1+jk\omega_0)}\left[e^{-(1+jk\omega_0)} - e^{(1+jk\omega_0)}\right]$$

$$= \frac{-1}{2(1+jk\omega_0)}\left[e^{-1}\cdot e^{-jk\omega_0} - e^{1}\cdot e^{jk\omega_0}\right]$$

$$= \frac{-1}{2(1+jk\pi)}\left[e^{-1}\cdot e^{-jk\pi} - e\cdot e^{jk\pi}\right]$$

$$= \frac{-1\times(-1)^k}{2(1+jk\pi)}\left(e^{-1} - e\right)$$

$$\boxed{a_k = \frac{(-1)^k}{2(1+jk\pi)}\left(e - e^{-1}\right) \quad \forall k}$$

3.22 — 10 marks

c)

$$x(t) = \begin{cases} \sin(\pi t), & 0 \le t \le 2 \\ 0, & 2 < t \le 4 \end{cases}$$

$$\boxed{T = 4}$$

$$\omega_0 = \frac{2\pi}{4}$$

$$\omega_0 = \pi/2$$

$$a_k = \frac{1}{4}\int_0^2 \sin(\pi t)\, e^{-jk\omega_0 t}\, dt$$

$$= \frac{1}{4}\int_0^2 \left[\frac{e^{j\pi t} - e^{-j\pi t}}{2j}\right] e^{-jk\omega t}\, dt$$

$$= \frac{1}{8j}\int_0^2 \left[e^{j(\pi - k\omega)t} - e^{-j(\pi + k\omega)}\right] dt$$

$$= \frac{1}{8j}\left[\frac{e^{j(\pi - k\omega)t}}{j(\pi - k\omega)} + \frac{e^{-j(\pi + k\omega)t}}{j(\pi + k\omega)}\right]_0^2$$

$$= \frac{1}{8j}\left[\frac{e^{j(\pi - k\omega)2} - 1}{j(\pi - k\omega)}\right] + \frac{1}{8j}\left[\frac{e^{-j(\pi + k\omega)2} - 1}{j(\pi + k\omega)}\right]$$

$$= \frac{1}{8j}\left[\frac{e^{2j\pi} \cdot e^{-2jk\omega} - 1}{j(\pi - k\omega)} + \frac{e^{-2j\pi} \cdot e^{-2jk\omega} - 1}{j(\pi + k\omega)}\right]$$

$$= \frac{1}{8j}\left[\frac{e^{-jk\pi} - 1}{j(\pi - k\pi/2)} + \frac{e^{-jk\pi} - 1}{j(\pi + k\pi/2}\right]$$

$$= \frac{-1}{8}\left[\frac{(-1)^k - 1}{\pi - k\pi/2} + \frac{(-1)^k - 1}{\pi + k\pi/2}\right]$$

$$= \frac{1}{4\pi}\left[\frac{1-(-1)^k}{2-k} + \frac{1-(-1)^k}{2+k}\right]$$

$$= \frac{1}{4\pi}\left[\frac{2+k-(-1)^k(2+k) + 2-k-(-1)^k(2-k)}{4-k^2}\right]$$

$$= \frac{1}{4\pi}\left[\frac{4-4(-1)^k}{4-k^2}\right]$$

$$= \frac{1}{\pi}\left[\frac{1-(-1)^k}{4-k^2}\right] \quad , \quad k \neq \pm 2$$

3.25) a) $\cos(4\pi t) \rightarrow \dfrac{e^{j4\pi t} + e^{-j4\pi t}}{2}$ since $\omega = 4\pi$, $k = 1$

$a_1 = a_{-1} = \dfrac{1}{2}$

b) $\sin(4\pi t) \rightarrow \dfrac{e^{j4\pi t} - e^{-j4\pi t}}{2j}$

$b_1 = \dfrac{1}{2j}$ or $\dfrac{-j}{2}$

$b_{-1} = \dfrac{-1}{2j}$ or $\dfrac{j}{2}$

c) $z(t) = x(t)\,y(t)$

Then FS coeffs are:

$C_2 = \sum a_k b_{2-k}$

$= a_1 b_1 = \dfrac{-j}{4}$

$C_0 = \sum a_k b_{-k}$

$= a_1 b_{-1} + a_{-1} b_1$

$= 0$

$C_{-2} = \sum a_k b_{-2-k}$

$= a_{-1} b_{-1} = \dfrac{j}{4}$

d)

$z(t) = x(t)\,y(t) = (\cos 4\pi t)(\sin 4\pi t)$

$= \dfrac{1}{2}\sin(8\pi t)$

$= a_0 + 2$
(0)

We got the same thing as in part (c).

3.26) a) If $a_k = a^*_{-k}$, then $x(t)$ is real.

so $a^*_{-k} = -j\left(\frac{1}{2}\right)^{|-k|} = -j\left(\frac{1}{2}\right)^{|k|} \neq a_k$

so $x(t)$ is NOT real.

b) if $x(t)$ is even, $x(t) = x(-t)$

$\Rightarrow a_k = a_{-k}$.

$a_{-k} = j\left(\frac{1}{2}\right)^{|-k|} = j\left(\frac{1}{2}\right)^{|k|} = a_k$

$\therefore$ $x(t)$ is even.

c) if $x(t)$ has FS coeffs a_k,

$\frac{dx(t)}{dt}$ has FS coeffs $a'_k = jk\omega_0 a_k$

so $a'_{-k} = j(-k)\omega_0 a_{-k} = -jk\omega_0 a_k = -a'_k$

since $a'_{-k} = -a'_k$, it is ODD.

so $\frac{d}{dt} x(t)$ is not EVEN.

3.34) $$H(s) = \int_{-\infty}^{\infty} h(t) e^{-st} dt$$

$$= \int_{-\infty}^{\infty} e^{-(4|t| + st)} dt$$

$$= \int_{-\infty}^{0} e^{(4-s)t} dt + \int_{0}^{\infty} e^{-(4+s)t} dt$$

$$= \left[\frac{e^{(4-s)t}}{4-s} \right]_{-\infty}^{0} + \left[\frac{-e^{-(4+s)t}}{4+s} \right]_{0}^{\infty}$$

Note that only for $Re(s) \in (-4, 4)$ does this fn. actually converge.

$$= \left[\frac{1}{4-s} \right] + \left[\frac{1}{4+s} \right]$$

Technically we need $s = j\omega$, so $H(j\omega) = \frac{1}{4-j\omega} + \frac{1}{4+j\omega}$

a) Given $T = 1 \Rightarrow \omega = 2\pi$, and since $b_k = a_k H(jk\omega)$ [And $a_k = 1$ here]

$$b_k = 1 \cdot \left(\frac{1}{4 - jk\omega} + \frac{1}{4 + jk\omega} \right)$$

b) Since $a_{2k} = 0$, $a_{2k+1} = 1$

$$b_{2k} = 0, \quad b_{2k+1} = -\left(\frac{1}{4 - jk\omega} + \frac{1}{4 + jk\omega} \right)$$

c) $$a_k = \frac{\sin(k\pi/2)}{k\pi}$$

so $$b_k = \frac{\sin(k\pi/2)}{k\pi} \cdot \left(\frac{1}{4 - jk\omega} + \frac{1}{4 + jk\omega} \right)$$

3.41)

1. $a_k = a_{-k} \Rightarrow \boxed{x(t) = x(-t)}$

Period, $T=3$

$\Rightarrow \omega = \frac{2\pi}{T} = \frac{2\pi}{3}$

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j\omega t}$$

$$= 2\sum_{k=0}^{\infty} a_k e^{\frac{2\pi j t}{3}}$$

2. $a_k = a_{k+2} \Rightarrow$ Summing over all k's and multiplying with $e^{j\omega t}$ on both sides,

$$\sum_{k=-\infty}^{\infty} a_k e^{j\omega t} = \sum_{k=-\infty}^{\infty} a_{k+2} e^{j\omega t}$$

$$= e^{-2j\omega t}\sum_{k=-\infty}^{\infty} a_{k+2} e^{j(\omega+2)t}$$

$$\Rightarrow \boxed{x(t) = x(t)e^{-2j\omega t} = x(t)e^{-j\frac{4\pi}{3}t}}$$

This means that $x(t)$ is non-zero only when $e^{-j\frac{4\pi}{3}t} = 1 \Rightarrow t = 0, \pm\frac{3}{2}, \pm 3, \ldots$

3. $\int_{-0.5}^{0.5} x(t)\,dt = 1 \Rightarrow$ Since in this range $x(t)$ is non-zero only at $t=0 \Rightarrow x(t) = \delta(t)$ in this range

So, $x(t) = \delta(t)$ in $-0.5 < t < 0.5$

4. $\int_{1}^{2} x(t)\,dt = 2 \Rightarrow$ Similarly $x(t) = 2\delta\left(t - \frac{3}{2}\right)$ in $1 < t < 2$

Since the period is 3, $\int_{3n-0.5}^{3n+0.5} x(t)\,dt = 1 \;\forall\; n \in (-\infty, \infty)$

which means $x(t) = \delta(t-3n)$

& $\int_{3n+1}^{3n+2} x(t)\,dt = 2 \;\forall\; n \in (-\infty, \infty)$

$x(t) = 2\delta(t - 3n - 3/2)$

$$\therefore\; x(t) = \sum_{n=-\infty}^{\infty} \delta(t-3n) + 2\sum_{n=-\infty}^{\infty} \delta\left(t - 3n - \frac{3}{2}\right)$$

3.40 SOLUTION & RUBRIC

Rubric → Pencil

3.40) a)

$$x(t) = a_0 + 2\sum_{k=1}^{\infty} (B_k \cos(k\omega_0 t) - C_k \sin(k\omega_0 t))$$

$$= a_0 + 2\sum B_k \left[\frac{e^{jk\omega_0 t} + e^{-jk\omega_0 t}}{2}\right]$$

(2)

$$+ C_k \left[\frac{e^{jk\omega_0 t} - e^{-jk\omega_0 t}}{2j}\right]$$

$$x(t) = a_0 + \sum e^{jk\omega_0 t}[B_k + jC_k] + \sum e^{-jk\omega_0 t}[B_k - jC_k]$$

$$Ev(x(t)) = \frac{x(t) + x(-t)}{2} = a_0 + \sum e^{jk} \ldots$$

$$= a_0 + \sum_{k=1}^{\infty} B_k e^{jk\omega_0 t} + \sum_{k=1}^{\infty} B_k e^{-jk\omega_0 t}$$

$$= \underbrace{a_0 + \sum_{k=1}^{\infty} B_k e^{jk\omega_0 t} + \sum_{k=-\infty}^{-1} B_{-k} e^{jk\omega_0 t}}$$

(2)

↓ Compare with

$$Ev(x(t)) = \alpha_0 + \sum_{k=1}^{\infty} \alpha_k e^{jk\omega_0 t} + \sum_{k=-\infty}^{-1} \alpha_k e^{jk\omega_0 t}$$

⇓

$$\boxed{\alpha_k = \begin{cases} a_0, & k = 0 \\ B_k, & k > 0 \\ B_{-k}, & k < 0. \end{cases}}$$

(1)

Similarly for odd,

$$\text{Odd}(x(t)) = \frac{x(t) - x(-t)}{2} = \sum_{k=1}^{\infty} i e^{jk\omega_0 t}(c_k) + \underbrace{\sum_{k=1}^{\infty} i e^{-jk\omega_0 t}(-c_k)}$$

(a)

$$= \underbrace{\sum_{k=1}^{\infty} c_k(i) e^{jk\omega_0 t} + \sum_{k=-\infty}^{-1}(-c_{-k})(i) e^{jk\omega_0 t}}.$$

(2) ⇓ Comparing this with

$$\text{Odd}(x(t)) = \sum_{k=1}^{\infty} \beta_k e^{jk\omega_0 t} + \sum_{k=-\infty}^{-1} \beta_k e^{+jk\omega_0 t}.$$

Obviously:

(1)

$$\boxed{\beta_k = \begin{cases} 0, & k = 0. \\ i c_k, & k > 0. \\ -i c_{-k}, & k < 0. \end{cases}}$$

(#).

Tot = 10.

3.45 (b) From the definition of α_k in part (a), it's clear that $\boxed{\alpha_k = \alpha_{-k}}$ ✓ (2.5)

$$\beta_k = \begin{cases} 0, & k = 0 \\ i c_k, & k > 0 \\ -i c_{-k}, & k < 0 \end{cases} ; \quad \beta_{-k} = \begin{cases} 0, & k = 0 \\ i c_{-k}, & k < 0 \\ -i c_k, & k > 0 \end{cases} \quad (-i c_k)$$

Compare

So, $\boxed{\beta_k = -\beta_{-k}.}$ ✓ (2.5)

Tot = 5

7(c)

$(\omega_0 = \frac{2\pi}{3})$

$$x(t) = a_0 + 2\sum_{k=1}^{\infty}\left[B_k \cos\left(\frac{2\pi kt}{3}\right) - C_k \sin\left(\frac{2\pi kt}{3}\right)\right]$$

$$z(t) = d_0 + 2\sum\left[E_k\left(\cos\left(\frac{2\pi kt}{3}\right)\right) - F_k \sin\left(\frac{2\pi kt}{3}\right)\right]$$

Reqd : $y(t) = A(a_0 + d_0) + 2\sum_{k=1}^{\infty}\left[\left(B_k + \frac{E_k}{2}\right)\cos\left(\frac{2\pi kt}{3}\right) + F_k \sin\left(\frac{2\pi kt}{3}\right)\right]$

$\boxed{\text{Logic : Try \& interpolate } y(t) \text{ as combinations of even \& odd parts of the given signal.}}$

Even parts :

$Ev(x(t)) = a_0 + 2\sum B_k \cos(k\omega_0 t)$. Here, $\omega_0 = 2\pi/3$.

$Ev(z(t)) = d_0 + 2\sum E_k \cos(k\omega_0 t)$

Odd parts :

$Odd(x(t)) = -2\sum C_k \sin(2\pi kt/3)$ (2)

$Odd(z(t)) = -2\sum F_k \sin(\omega_0 kt)$.

In the reqd $y(t)$, Consider the second term :

$$2\sum\left[\left(B_k + \frac{E_k}{2}\right)\cos(\omega_0 kt) + F_k \sin\left(\frac{2\pi kt}{3}\right)\right]$$
$$= (Ev(x(t)) - a_0) + \left(\frac{Ev(z(t)) - d_0}{2}\right) - Odd(z(t))$$
$$+ (A(a_0 + d_0)) \quad \text{first term.}$$

(2)

we get $\Rightarrow$ $\boxed{\left(\frac{7}{2}d_0 + 3a_0\right) + Ev(x(t)) + \frac{Ev(z(t))}{2} - Odd(z(t))}$ (*)

Need to be simplified further !!

$a_0 = \frac{\int x(t)dt}{T} = \frac{\frac{1}{2}+2+1}{3} = 4/3$, (1)

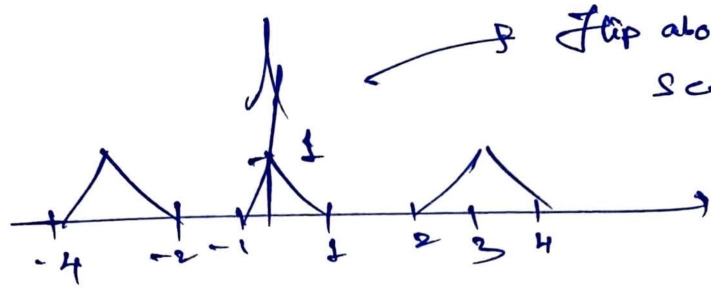
& $d_0 = \frac{0}{T} = 0$. (1)

(1)

$\therefore \boxed{y(t) = 1 + Ev(x(t)) + \frac{1}{2} Ev(2x(t)) - Odd(2x(t))}$ (#)

Plotting :

Even(x(t)) =



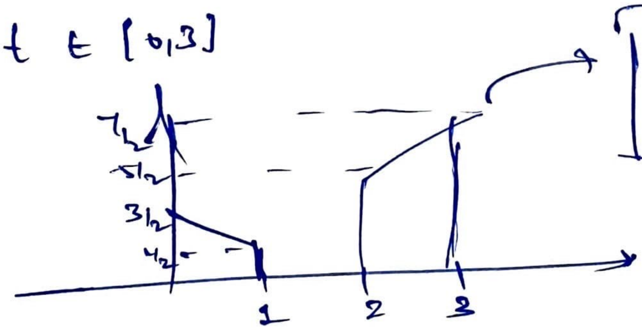
→ Flip about y axis & scale amplitude by 2.

$\frac{Ev(2x(t))}{2} = \begin{cases} 1/2, & t \in [0,1] \\ -1, & t \in [1,2] \\ 1/2, & t \in [2,3] \end{cases}$

& $Odd(x(t)) = \begin{cases} -1, & t \in [0,1] \\ 0, & t \in (1,2) \\ +1, & t \in (2,3). \end{cases}$

Total plotting = (3).

So, for $t \in [0,3]$



$\boxed{\text{Limits are very imp (\#)}}$

$\boxed{\text{Tot} = 10}$

& Combining all, we get :

(#) Final plot.

NeSS – A2

Problem – 2

- 9.21

(a) $x(t) = e^{-2t}u(t) + e^{-3t}u(t)$

$$X(s) = \int_{-\infty}^{\infty} e^{-2t}u(t)e^{-st}dt + \int_{-\infty}^{\infty} e^{-3t}u(t)e^{-st}dt$$

$$= \int_{0}^{\infty} e^{-(s+2)t}dt + \int_{0}^{\infty} e^{-(s+3)t}dt$$

$$= \frac{1}{s+2}(1-0) + \frac{1}{s+3}(1-0)$$

$$= \frac{1}{s+2} + \frac{1}{s+3} = \boxed{\frac{2s+5}{(s+2)(s+3)} \quad . \quad Re\{s\} > -2}$$

(b) $x(t) = e^{-4t}u(t) + e^{-5t}(\sin 5t)u(t)$

$$X(s) = \int_{-\infty}^{\infty} e^{-4t}u(t)e^{-st}dt + \int_{-\infty}^{\infty} e^{-5t}\sin(5t)u(t)e^{-st}dt$$

$$= \int_{0}^{\infty} e^{-(4+s)t}dt + \int_{0}^{\infty} e^{-(5+s)t}\sin(5t)dt$$

$\downarrow$ $\frac{1}{s+4}$, $Re\{s\} > -4$

$\downarrow$ $\frac{1}{s+5-j5}$, $Re\{s\} > -5$

$$X(s) = \frac{s^2 + 15s + 70}{s^3 + 14s^2 + 90s + 100} \quad , \quad Re\{s\} > -4$$

(c) $x(t) = e^{2t}u(t) + e^{3t}u(-t)$

$$X(s) = \int_{-\infty}^{\infty} e^{2t}u(\bar{t})e^{-st}dt + \int_{-\infty}^{\infty} e^{3t}u(t)e^{-st}dt$$

$$= \int_{-\infty}^{0} e^{(2-s)t}dt + \int_{-\infty}^{0} e^{(3-s)t}dt$$

$$= \frac{1}{s-2} + \frac{1}{s-3} = \frac{2s-5}{(s+2)(s-3)} \quad , \quad Re\{s\} < 2 .$$

(d) $x(t) = te^{-2|t|}$

$$X(s) = \int_{-\infty}^{\infty} te^{-2|t|} \cancel{dt}\, e^{-st} dt$$

$$= \int_{-\infty}^{0} te^{(2-s)t} dt + \int_{0}^{\infty} te^{-(2+s)t} dt$$

We have $e^{(2-s)t} u(t) \xrightarrow{\mathcal{L}} \frac{1}{s-2}$, $Re\{s\} < 2$

$e^{(2+s)t} u(t) \xrightarrow{\mathcal{L}} \frac{1}{s+2}$, $Re\{s\} > -2$

$$te^{-2|t|} \xleftrightarrow{\mathcal{L}} -\frac{d}{ds}\left[\frac{2s}{s^2-4}\right] = \frac{2s^2+8}{(s^2-4)^2}, \quad -2 < Re\{s\} < 2$$

(e) $x(t) = |t| e^{-2|t|}$

$te^{-2t} u(t) \xrightarrow{\mathcal{L}} \frac{1}{(s+2)^2}$, $Re\{s\} > -2$

$-te^{2t} u(-t) \xrightarrow{\mathcal{L}} \frac{-1}{(s-2)^2}$, $Re\{s\} < 2$

$\therefore$ $|t| e^{-2|t|} \xrightarrow{\mathcal{L}} \frac{-4s}{(s+2)^2(s-2)^2}$, $Re\{s\} \in (-2, 2)$

(f) $x(t) = |t| e^{2t} u(-t)$

$x(t) = -te^{2t}$ for $t \in [-\infty, 0]$.

$-te^{2t} u(-t) \xrightarrow{\mathcal{L}} \frac{-1}{(s-2)^2}$, $Re\{s\} < 2$

(g) $x(t) = u(t) - u(t-1) \Rightarrow X(s) = \frac{1-e^{-s}}{s}$, All s.

(h) $$X(s) = \frac{se^{-s} - 1 + e^{-s}}{s^2} + \frac{e^{-2s} - se^{s} - 1 + e^{s}}{s^2}, \text{ All } s.$$

(i) $X(s) = 1 + \frac{1}{s}$, $Re\{s\} > 0$

$$\int_{-\infty}^{\infty} \delta(3t) e^{-st} dt$$

(j) $X(s) = 1 + \frac{1}{s}$, $Re\{s\} > 0$

• 9.26

$X_1(s) = \frac{1}{s+2}$, $X_2(s) = \frac{1}{s+3}$ when $x_1(t) = e^{-2t}u(t)$

$Re\{s\} > -2$, $Re\{s\} > -3$. ~~tot~~ $x_2(t) = e^{-3t}u(t)$

$$x_1(t-2) \xleftrightarrow{\mathcal{L}} e^{-2s} X_1(s) = \frac{e^{-2s}}{s+2}, \quad Re\{s\} > -2 \quad 1$$

$$x_2(-t+3) \xleftrightarrow{\mathcal{L}} e^{-3s} X_2(-s) = \frac{e^{-3s}}{3-s}, \quad Re\{s\} > -3. \quad 1$$

Mult 1

• 9.29 ROC 1

(a) $x(t) = e^{-t}u(t) \xleftrightarrow{\mathcal{L}} X(s) = \frac{1}{s+1}$, $Re\{s\} > -1$

$h(t) = e^{-2t}u(t) \xleftrightarrow{\mathcal{L}} H(s) = \frac{1}{s+2}$, $Re\{s\} > -2$. 2

(b) $Y(s) = X(s) \times H(s) = \frac{1}{s+1} \times \frac{1}{s+2}$, $Re\{s\} > -1$ 1

(c) $Y(s) = \frac{1}{(s+1)(s+2)} = \frac{1}{s+1} - \frac{1}{s+2}$

$\Rightarrow y(t) = e^{-t}u(t) - e^{-2t}u(t)$ 2

(d) $y(t) = e^{-t}u(t) * e^{-2t}u(t)$

$$y(t) = \int_{-\infty}^{\infty} e^{-\tau}u(\tau)\, e^{-2(t-\tau)}u(t-\tau)\, d\tau \quad 2$$

$$= \int_{0}^{t} e^{-\tau} \times e^{-2(t-\tau)}\, d\tau$$

$$= e^{-2t} \int_0^t e^{-\tau} e^{2\tau}\, d\tau$$

$$= e^{-2t}(e^{t} - 0)$$

$$= (e^{-t} - e^{-2t})\, u(t).$$