Assignment-1 (AEC)

Solutions.

Q1: Transcient and Ac analysis of RC (Theory part):

$$V(t) = V(t=\infty) + \left[V(t=0) - V(t=\infty)\right] e^{-t/\tau}$$

(For any 1st order / single pole systems)

$V(t=\infty)$: Voltage at $t=\infty$

$V(t=0)$: Voltage at $t=0^+$

(a)

$V(t=0^+)$: Capacitor acts as short circuit

$\hookrightarrow = 0$

$V(t=\infty)$: Capacitor steady state acheived

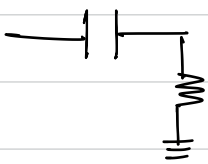
$i = C\frac{dV_{out}}{dt}$

$i=0$ means $V_{out} = V_p$

$\tau = RC$

pole $= -\frac{1}{RC}$ (make net impedance ∞)

(b)



$V_{out}(t=0^+)$ — V_p

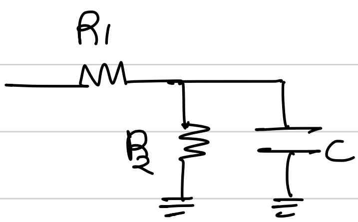
$V_{out}(t=\infty)$ — 0

$V_{out} = V_p e^{-t/RC}$

zero $= 0$ (i must be 0, means $\frac{1}{sC} = \infty$ happens if $s=0$)

pole $= -\frac{1}{RC}$

(c)



$t=0^+$, $R_2 \| \frac{1}{sC} \sim \frac{1}{sC}$ ($s \sim \infty$ means $\frac{1}{sC} \sim 0$)

$V_{out}(t=0^+) \Rightarrow 0$

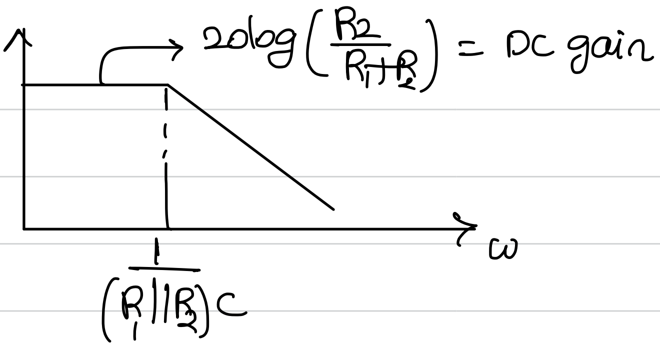
$t \to \infty$, $R_2 \| \frac{1}{sC} \sim R_2$ ($s \sim 0$ means $\frac{1}{sC} \sim \infty$)

$V_{out}(t=\infty) = \frac{R_2}{R_1+R_2} V_p$

$\tau = (R_1 \| R_2)C$

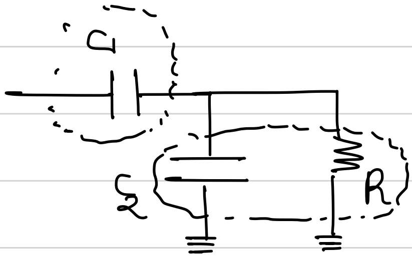
pole $= \frac{-1}{(R_1 \| R_2)C}$

Bodeplot :



Impedance 1 (I_1)

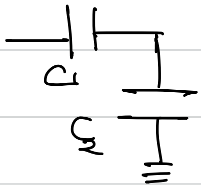
(d)



Impedance 2 (I_2)

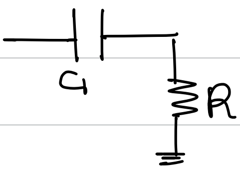
at $t = 0^+$,

$\frac{1}{sC_2} \| R \sim \frac{1}{sC_2}$ (equivalent to capacitance at $s \to \infty$ means $\frac{1}{sC_2} \to 0$)



$V_{out} = \frac{C_1 V_p}{C_1 + C_2}$

at $t \to \infty$, $\frac{1}{sC_2} \| R \sim R$ (equivalent to Resistance at $s \to 0$ means $\frac{1}{sC_2} \to \infty$)



$V_{out}(t = \infty) = 0$ (steady state means no current should flow through C_1. $I(C_1) = 0 = I(R)$)

$\tau = R(C_1 + C_2)$

For calculating zeros : $I_1 = 1/sC_1$

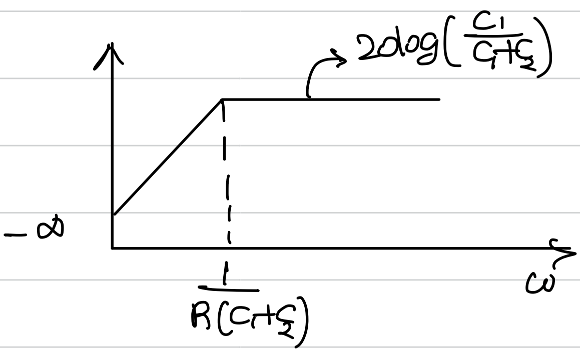
$I_2 = \frac{1}{sC_2} \| R$

make $z_1 = \infty$ without making z_2 to ∞

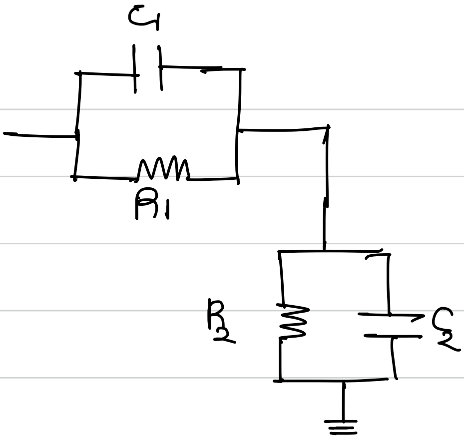
(or) make $z_2 = 0$ without making z_1 to 0

zero, at $s = 0$

pole at $s = \frac{-1}{R(C_1 + C_2)}$



(e)

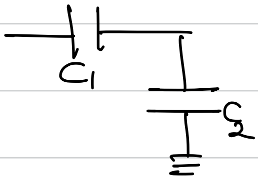


at $t = 0^+$, $s \sim \infty$ means

Resistance || capacitance $(1/sC)$

$\sim$ capacitance $(1/sC)$

Circuit will look like:

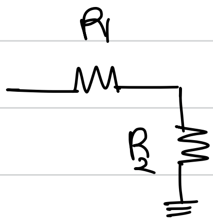


$$V_{out} = \frac{C_1}{C_1 + C_2} V_P$$

at $t = \infty$ $(s = 0)$ means

Resistance || capacitance $(1/sC)$

$\sim$ Resistance



$$V_{out} = \frac{R_2}{R_1 + R_2} V_P$$

zeros: make $Z_1 = \infty$ means at $s = \frac{-1}{R_1 C_1}$

make $Z_2 = 0$ means at $s = \infty$

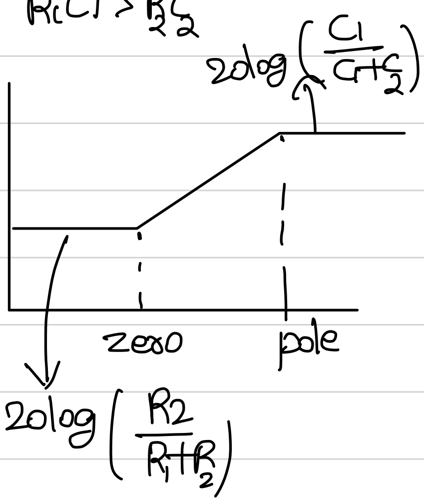
pole: (make input, ground)

find output impedance

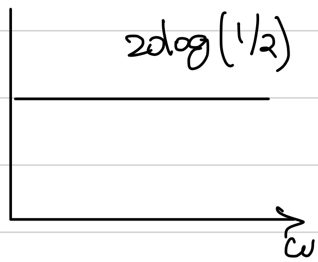
pole at $s = \frac{-1}{[R_1 || R_2](C_1 + C_2)}$

$$\Rightarrow \frac{-(R_1 + R_2)}{R_1 R_2 (C_1 + C_2)}$$

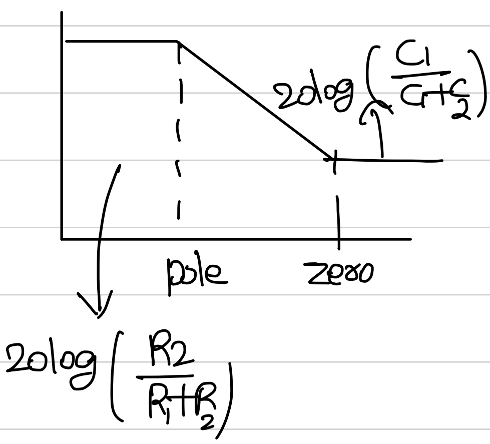
$R_1 C_1 > R_2 C_2$



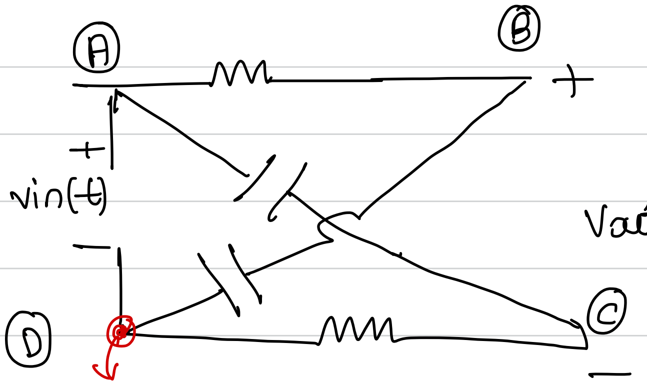
$R_1 C_1 = R_2 C_2$



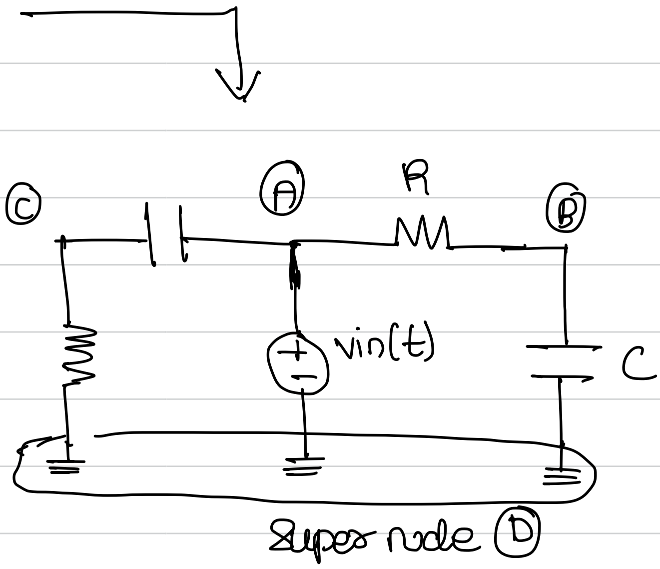
$R_1 C_1 < R_2 C_2$



(f)



assume this node to be grounded



$$V_{out}(t) = V(B) - V(C)$$

$$V(B) = v_{in}(1 - e^{-t/RC}) \quad \text{(LPF)}$$

$$V(C) = v_{in}e^{-t/RC} \quad \text{(HPF)}$$

$$V_{out}(t) = v_{in} - 2v_{in}e^{-t/RC}$$

AC analysis:

$$V_B(s) = \frac{1}{1+j\omega RC} v_{in}(s)$$

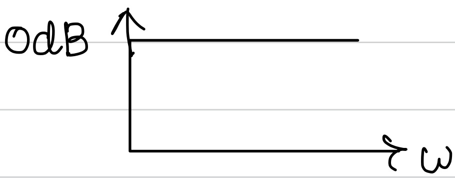
$$V_C(s) = \frac{j\omega RC}{1+j\omega RC} v_{in}(s)$$

$$V_{out}(s) = \frac{1-j\omega RC}{1+j\omega RC} v_{in}(s)$$

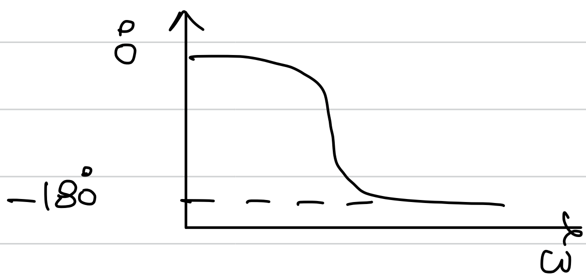
$$\text{pole} = \frac{-1}{RC}$$

$$\text{zero} = \frac{1}{RC}$$

Bode magnitude plot vs ω:



phase plot vs ω



$$\text{phase} = -2\tan^{-1}(\omega RC)$$