25/10/2024

A complete metric space is equivalent to every Cauchy sequence is convergent.

A set A, which is a subset of X, is dense if the closure of A equals X.

The closure of A consists of A itself plus all limit points of A.

A set is nowhere dense if the interior of its closure is empty.

Q is Dense in R but N is nowhere dense in R.

Closure of N is N itself and int(N) is empty.

Closure of Q is R. And int(R) is R.

A set A is nowhere dense in X if and only if for every non-empty open set G in X, the intersection of (X \ Ā) and G is not empty. If this intersection were empty, G would be a subset of Ā. Since G is open, int(G) is not empty. However, A being nowhere dense means int(Ā) is empty, leading to a contradiction. Therefore, the intersection cannot be empty.

This also implies that Every Non-Empty set contains an open ball disjoint from A.

A set is of the First Category if it is a countable union of nowhere dense else it is of Second Category.

Baire Category

A complete metric space is always Second Category

Proof

Let X is complete Metric Space.

let {An} be a family of nowhere dense sets.

To show : There exists a x belongs X s.t x doesn’t belongs to An for all N.

A1 is nowhere dense Let y is an open set in X there exists a ball $U(a1,r1)$ such that $U(a1,r1)$ intersection $A1$ = empty.

Let a set closed $F1 = B(q1,r1/2)$

$int(F1) \neq$ empty.

as $A2$ is nowhere dense

let a ball $U2(a2,r2)$ be a ball in $int(F1)$

so $U2(a2,r2)$ intersection $A2$ is empty

$F2 = B(a2,r2/2)$ and so on ....