3.22

(a)

$$x(t) = \sum_{k \in \mathbb{Z}} a_k \cdot e^{jk\omega t}$$

(b)

the given graph has period of 6

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{6}$$

$$x(t) = \begin{cases} 1 & t \in [6n-1, 6n+1] \\ 2-t & t \in [6n+1, 6n+2] \\ t+2 & t \in [6n-2, 6n-1] \\ 0 & \text{otherwise} \end{cases}$$

$$a_k = \frac{1}{T} \int_{\langle T \rangle} e^{-jk\omega t} \cdot x(t)$$

$$a_k = \frac{1}{6} \left[\int_{-2}^{-1} (t+2) e^{-jk\omega_0 t} + \int_{-1}^{1} e^{-jk\omega_0 t} \, dt + \int_{1}^{2} (2-t) e^{-jk\omega t} \, dt \right]$$

1 + 2

$$\left[\int_1^2 (2-t)\left(e^{jk\omega_0 t}\right) + \int_1^2 (2-t)\, e^{-jk\omega_0 t}\, dt\right] - ③$$

$$① + ② = ③$$

$$a_k = \left[\frac{e^{-jk\omega t}}{-jk\omega t}\right]_{-1}^{1} + 2\left(\frac{\cos k\omega_0}{(k\omega_0)^2} - \frac{\sin k\omega_0}{k\omega_0} - \frac{\cos 2k\omega_0}{(k\omega_0)^2}\right)$$

$$\frac{2\sin(k\omega)}{k\omega} + \frac{4\sin(2k\omega)}{k\omega} - \frac{4\sin(k\omega)}{k\omega} - 2\int_1^2 t\cos(k\omega t)\,dt$$

$$\frac{-2\sin(k\omega)}{k\omega} + \frac{4\sin(2k\omega)}{k\omega} - 2\left[\frac{t\sin(k\omega t)}{k\omega} - \int\frac{\sin(k\omega t)}{k\omega}\right]_1^2$$

$$-\frac{2\sin(k\omega)}{k\omega} + \frac{4\sin(2k\omega)}{k\omega} - 2\left[\frac{t\sin(k\omega t)}{k\omega} + \frac{\cos(k\omega t)}{k^2\omega^2}\right]_1^2$$

$$\rightarrow \cancel{\frac{-2\sin(k\omega)}{k\omega}} + \cancel{\frac{4\sin(2k\omega)}{k\omega}} - \cancel{\frac{4\sin(2k\omega)}{k\omega}} - \frac{2\cos(2k\omega t)}{k^2\omega^2}$$

$$+\frac{2\sin(k\omega)}{k\omega} + \frac{2\cos(k\omega)}{k^2\omega^2}$$

$$\longrightarrow \cancel{\frac{2\sin(2k\omega)}{k\omega}} - \frac{2\cos(2k\omega t)}{k^2\omega^2} + \frac{2\cos(k\omega)}{k^2\omega^2}$$

$$x(t) = \sum_{k\in\mathbb{Z}} a_k e^{jk\omega t}$$

$$\longrightarrow a_k = \frac{1}{6}\left(\frac{2\cos(k\omega)}{k^2\omega^2} - \frac{2\cos(2k\omega t)}{k^2\omega^2}\right) \quad \forall k\in\mathbb{Z}$$

(d)

$$x(t) = \begin{cases} \delta(t-2n) & t = 2n \text{ (even)} \\ -2\cdot\delta(t-2n-1) & t = 2n+1 \text{ (odd)} \\ 0 & \text{otherwise} \end{cases}$$

$$a_k = \frac{1}{2}\left(\int_0^1 \delta(t-2n)e^{-jk\omega t}\,dt + \int_1^2 \delta(t-2n-1)(-2)\cdot e^{-jk\omega t}\,dt\right.$$

let $n=0$

$$a_k = \frac{1}{2}\int_0^1 \delta(0)\cdot e^{-jk\omega(0)}\,dt + \int_1^2 \delta(0)(-2)\cdot e^{-jk\omega(1)}\,dt$$

$$a_k = \frac{1}{2}\left[1\right] + \frac{-\cancel{2}}{\cancel{2}}\left[e^{-jk\omega}\right]$$

$$a_k = \frac{1}{2} - \left[e^{-jk\omega}\right] \quad \forall k \in Z$$

$$x(t) = \sum_{k \in Z} a_k \cdot e^{jk\omega t}$$

(e)

$$x(t) = \begin{cases} 1 & [6n+4, 6n+5] \\ -1 & [6n+1, 6n+2] \\ 0 & \text{otherwise} \end{cases}$$

$$a_k = \frac{1}{3} \cdot \frac{1}{2}\left(\int_1^2 -e^{-jk\omega t}\,dt\right) + \frac{1}{2}\int_4^5 e^{-jk\omega t}\,dt$$

$$a_k = \frac{1}{3} \cdot \frac{1}{2} \cdot \left[\frac{e^{-jk\omega t}}{+k\omega j}\right]_1^2$$

$$a_k = \frac{1}{3} \cdot \frac{1}{2}\left[\frac{+e^{-2jk\omega}}{k\omega} - \frac{e^{+jk\omega}}{k\omega}\right] + \frac{1}{3} \cdot \frac{1}{2}\left[\frac{-e^{-5jk\omega t}}{k\omega} + \frac{e^{+4jk\omega t}}{k\omega}\right]$$

$$a_k = \frac{1 \cdot 1}{3 \cdot 2 k\omega j}\left[e^{jk\omega} - e^{-5jk\omega} - e^{-2jk\omega} + e^{4jk\omega t}\right] \quad \forall k \in Z$$

$$a_k = \frac{1}{6k\omega j}\left[-e^{-6jk\omega} \cdot e^{+jk\omega} + e^{jk\omega} + e^{6jk\omega} \cdot \left[e^{2jk\omega t} - e^{-2jk\omega t}\right]\right]$$

$$a_k = \frac{1}{6}\left(\frac{e^{jk\omega_0}}{-jk\omega_0} - \frac{e^{2jk\omega_0}}{-jk\omega_0}\right) - \frac{1}{6}\left(\frac{e^{-2jk\omega_0}}{-jk\omega_0} - \frac{e^{-jk\omega_0}}{-jk\omega_0}\right)$$

$$a_k = \frac{\sin(2k\omega_0)}{3jk\omega_0} - \frac{\cos k\omega_0}{3jk\omega_0}$$

$$x(t) = \sum_{k \in \mathbb{Z}} a_k \cdot e^{jk\omega t}.$$

3.22

(b) $x(t) = e^{-t}$, $-1 < t < 1$

$$a_k = \frac{1}{2}\int_{-1}^{1} e^{-t} \cdot e^{-jk\omega t}\, dt$$

$$a_k = \frac{1}{2}\int_{-1}^{1} e^{(-jk\omega-1)t}\, dt$$

$$a_k = \frac{1}{2(-jk\omega-1)} \cdot \left[e^{(-jk\omega-1)t} \right]_{-1}^{1}$$

$$a_k = \frac{1}{2(-jk\omega-1)} \cdot \left[e^{-jk\omega-1} - e^{+jk\omega+1} \right] \quad \forall k \in Z$$

$[\text{as } \omega_0 = \pi]$

k even one case

k odd other case

$$a_k = \frac{\frac{1}{2}\left((-1)^k\left(e^{1} - e^{-1}\right)\right)}{(j\pi k + 1)} \Rightarrow x(t) = \sum_{k \in \mathbb{Z}} a_k \cdot e^{jk\omega t}$$

3.22
(c)

$$x(t) = \begin{cases} \sin \pi t & 0 \le t \le 2 \\ 0 & 02 \le t \le 4 \end{cases}$$

$$a_k = \frac{1}{4}\left[\int_0^2 \sin\pi t \cdot e^{-j\omega t k}\, dt\right]$$

$$a_k = \frac{1}{4}\left(\int_0^2 \left(\frac{e^{jt\pi} - e^{-jt\pi}}{2j}\right) e^{-jk\omega t}\, dt\right]$$

$$a_k = \frac{1}{2 \cdot 4j}\left[\frac{e^{jt(\pi - k\omega)}}{j(\pi - k\omega)} - \frac{e^{-jt(\pi + k\omega t)}}{-j(\pi + k\omega t)}\right]_0^2$$

$$a_k = \frac{1}{8j}\left[\frac{e^{2j(\pi - k\omega)}}{j(\pi - k\omega)} - \frac{e^{-2j(\pi + k\omega t)}}{-j(\pi + k\omega t)} + \frac{1}{j(\pi - k\omega)} + \frac{-1}{j(\pi + k\omega)}\right]$$

$$a_k = \frac{1}{8j}\left[\frac{e^{2j(\pi - k\omega)}}{j(\pi - k\omega)} - \frac{e^{-2j(\pi + k\omega t)}}{-j(\pi + k\omega t)} + \frac{1}{j(\pi - k\omega)} - \frac{1}{j(\pi + k\omega)}\right]$$

$\forall\, k \in \mathbb{Z}$ $$x(t) = \sum_{k \in \mathbb{Z}} a_k \cdot e^{jk\omega t} \qquad \left[a_k = \frac{\pi}{4j} \cdot \frac{(e^{-2k\omega} - 1)}{\pi^2 - k^2\omega^2}\right]$$

3.25

(a) $x(t) = \cos(4\pi t)$

$\frac{1}{2}[2\cos(4\pi t)]$

$\frac{1}{2}[\cos(4\pi t) + j\sin(4\pi t) + \cos(4\pi t) - j\sin(4\pi t)]$

$\frac{1}{2}\left[e^{j4\pi t} + e^{-j4\pi t}\right]$ $k = 4, -4$

(b) $x(t) = \sin(4\pi t)$

$\frac{1}{2j}[2j\sin(4\pi t)]$

$\frac{1}{2j}\left[e^{4j\pi t} - e^{-4j\pi t}\right]$ $k = 4, -4$

$$z(t) = x(t) \cdot y(t)$$

$$z(t) = \left(\frac{1}{2}, 0, 0, 0, \frac{1}{2}\right) \cdot \left(\frac{1}{2j}, 0, 0, 0, \frac{1}{2j}\right)$$

---

c). $z(t) = x(t) \cdot y(t)$

$$c_k = \sum_{l \in Z} a_l \, b_{k-l}$$

$$c_1 = a_{-1} b_2 \quad a_0 b_{1-0} + a_1 b_0 + a_2 b_{-1}$$

$$c_1 = 0$$

$$c_0 = a_{-4} b_4 \ldots \ldots a_0 b_0 \quad a_1 b_{-1} \quad a_2 b_{-2} \quad a_3 b_{-3} \quad a_4 b_{-4}$$

$$c_0 = \left[\frac{1}{4j} + 0 + 0 + 0 + 0 - \frac{1}{4j}\right]$$

$$c_0 = 0$$

$$c_{-1} = \sum_{l \in Z} a_l \, b_{k-l}$$

$$c_{-1} = a_{-2} b_1 + a_{-1} b_0 + b_0 a_{-1} + a_4 b_{-5}$$

$$a_{-4} \cdot b_3 + b_{-4} a_3 + b_4 a_{-5}$$

$$c_{-1} = 0$$

so.... on

$c_8 = a_0 b_8 + a_4 b_4 + = \left(\frac{1}{2}\right)\left(\frac{1}{2j}\right) = \frac{1}{4j}$

$c_{-8} = a_{-4} b_{-4} = \left(\frac{1}{2}\right)\left(-\frac{1}{2j}\right) = \frac{-1}{4j}$

other than c_8, c_{-8} will be zero.

(d) $z(t) = x(t) \cdot y(t)$

$= \frac{2}{2} \cdot \cos(4\pi t) \sin(4\pi t)$

$z(t) = \frac{1}{2} \cdot \sin(8\pi t)$.

$z(t) = \frac{1}{2}\left[\frac{e^{8\pi j} - e^{-8\pi j}}{2j}\right]$

$z(t) = \frac{1}{4j}\left(e^{8\pi j}\right) - \frac{1}{4j}\left(e^{-8\pi j}\right)$

$c_{-8} = -\frac{1}{4j}, \quad c_8 = \frac{1}{4j}$

3.26

$a_k = \begin{cases} 2 & k = 0 \\ j\left(\frac{1}{2}\right)^{|k|} & \text{otherwise} \end{cases}$

$a_0 = 2$

$a_1 = \frac{j}{2}, \quad a_2 = \frac{j}{4}, \quad a_3 = \frac{j}{8}, \quad a_{-2} = \frac{j}{4}$

(a) when, $x(t)$ real

$a_k = (a_{-k})^*$

But, $a_2 \neq (a_{-2})^* \Rightarrow \frac{j}{4} \neq -\frac{j}{4}$

Hence $x(t)$ is not real.

(b) According to question.

$$x(t) = 2 + \sum_{k \in Z \setminus \{0\}} j\left(\frac{1}{2}\right)^{|k|} \cdot e^{j\omega k t}$$

$$x(t) = \left\{ \cdots\cdots j\left(\frac{1}{2}\right)^{2} \cdot e^{-j\omega k t} + j\left(\frac{1}{2}\right)^{1} \cdot e^{-j\omega k t} + 2(1) + j\left(\frac{1}{2}\right)^{2} e^{j\omega k t} + j\left(\frac{1}{2}\right)^{1} e^{j\omega k t} \cdots\cdots \right\}$$

$$x(-t) = \left\{ \cdots\cdots j\left(\frac{1}{2}\right)^{2} e^{j\omega k t} + \left(\frac{1}{2}\right)^{1} j e^{j\omega k t} + 2 + j\left(\frac{1}{2}\right)^{2} e^{-j\omega k t} + j\left(\frac{1}{2}\right)^{1} e^{-j\omega k t} \right\}$$

$x(t) = \cancel{\&} x(-t)$

$\therefore$ Hence given $x(t)$ is even.

(c)

$$\underset{y(t)}{\frac{dx(t)}{dt}} = \left\{ \cdots\cdots j\left(\frac{1}{2}\right)^{1}(-j\omega k) \cdot e^{-j\omega k t} + 2(1) + j\left(\frac{1}{2}\right)(j\omega k) \cdot e^{j\omega k t} \cdots \right\}$$

$$\underset{y(-t)}{\frac{dx(-t)}{dt}} = \left\{ \cdots\cdots j\left(\frac{1}{2}\right)(-j\omega k) e^{+j\omega k t} + 2(1) + j\left(\frac{1}{2}\right)(j\omega k) e^{-jk\omega t} \cdots\cdots \right\}$$

$y(-t) \neq y(t)$

Hence $\frac{d}{dt} x(t)$ is (not even)

3.34

$$h(t) = e^{-4|t|}$$

Convolution

$\uparrow$

(a) $$y(t) = \sum_{n=-\infty}^{\infty} \delta(t-n) * h(t)$$

(or)

Since $y(t)$ is an output of LTI system with impulse response and $x(t)$ can also convertible to exponential function

We can use.

$$y(t) = b_k \cdot e^{jk\omega t}$$

$$b_k = a_k \cdot H(jk\omega)$$

$x(t)$ can be represented as $\sum_{k \in Z} a_k e^{jk\omega t}$

$$a_k = \frac{1}{T} \int_{\langle T \rangle} \sum_{n=\infty}^{-\infty} \delta(t-n)$$

$$a_k = 1 \cdot \int_{-0.5}^{0.5} \delta(t) e^{-jk\omega t} dt$$

[in sigma n is $\sum_{-\infty}^{\infty}$ but only 1 'n' value is in between the limits of integrals]

$$a_k = e^{-jk\omega(0)} \cancel{\int_{-0.5}^{0.5} \delta(t)}$$

$$a_k = 1.$$

$$H(s) = \int_{-\infty}^{\infty} e^{-4|t|} \cdot e^{-st}$$

$$H(s) = \int_{0}^{\infty} e^{-4t} e^{-st} + \int_{-\infty}^{0} e^{4t} \cdot e^{-st}$$

$$H(s) = \left[\frac{e^{-(s+4)t}}{\;} \right]_0^{\infty} + \left[\frac{e^{-(s+4)t}}{-(s-4)} \right]_{-\infty}^{0}$$

$s+4>0$ $\quad\quad$ $s-4<0$

$\sigma > -4$ $\quad\quad$ $\sigma < 4$

$$H(s) = \frac{1}{s+4} - \frac{1}{s-4}$$

$$H(s) = \frac{-8}{s^2-16}$$

$$y(t) = \sum_{k \in Z} a_k \cdot e^{jk\omega t} \cdot H(s)$$

$[s = jk\omega]$

$$y(t) = \sum_{k \in Z} 1 \cdot e^{jk\omega t} \cdot \left(\frac{-8}{(jk\omega)^2 - 16} \right)$$

(b) $\quad x(t) = \sum_{n=-\infty}^{\infty} (-1)^n \delta(t-n)$

when $(-1)^n$ is added the period becomes 2

and then

$$a_k = \frac{1}{2} \int_0^1 \delta(t) e^{-jk\omega t} dt + \int_1^2 -\delta(t) e^{-jk\omega t}$$

$$a_k = \frac{1}{2} \left((1) \cdot e^{-jk\omega(0)} - e^{-jk\omega(1)} \cdot (1) \right)$$

$$a_k = \frac{1}{2} \left(1 - e^{-jk\omega} \right)$$

from previous $H(s) = \frac{-8}{s^2-16}$

$$b_k = a_k \cdot H(jwk)$$

$$b_k = \left(\frac{-8}{(jwk)^2 - 16} \right) \left(\frac{1 - e^{-jk\omega}}{2} \right)$$

$$y(t) = \sum_{k=-\infty}^{\infty} b_k e^{jk\omega t}$$

(c) $x(t) = \begin{cases} 1 & [n-0.25,\ n+0.25] \\ 0 & \text{otherwise} \end{cases}$

$y(t) = x(t) * h(t)$ (or) $y(t) = b_k \cdot e^{jk\omega t}$

$b_k = a_k \cdot H(jk\omega)$

$x(t) = a_k \cdot e^{jk\omega t}$

$$a_k = \frac{1}{T} \int_{\langle T \rangle} e^{-jk\omega t} \cdot x(t)\, dt$$

$$a_k = \left[\int_{-1/4}^{1/4} e^{-jk\omega t} \cdot (1) + 0 \right] \frac{1}{1}$$

$$a_{k\omega} = \frac{e^{\frac{jk\omega}{4}} - e^{-\frac{jk\omega_0}{4}}}{jk\omega} \quad \left[\omega_0 = \frac{2\pi}{(1)}\right]$$

$$a_k = \frac{\cancel{2j} \sin\left(\frac{k\pi}{2}\right)}{\cancel{2j} k\pi} = \frac{\sin\left(\frac{k\pi}{2}\right)}{k\pi}$$

$$H(s) = \frac{-8}{s^2 - 16}$$

$$H(jk\omega) = \frac{-8}{(jk\omega)^2 - 16}$$

$$b_k = a_k \cdot \frac{-8}{}$$

$$b_k = \left(\frac{\sin\left(\frac{k\pi}{2}\right)}{k\pi}\right)\left(\frac{-8}{(jk\omega)^2 - 16}\right)$$

$$y(t) = \sum_{k \in \mathbb{Z}} b_k \cdot e^{jk\omega t}$$

$$b_k = \left(\frac{\sin\left(\frac{k\pi}{2}\right)}{k\pi}\right)\left(\frac{-8}{(jk\omega)^2 - 16}\right)$$

9.21

(a) $x(t) = e^{-2t}u(t) + e^{-3t}u(t)$

$$X(s) = \int_{-\infty}^{\infty} \left[e^{-2t}u(t) + e^{-3t}u(t)\right] dt \cdot e^{-st} dt.$$

$$X(s) = \int_{-\infty}^{\infty} e^{-2t-st} \cdot u(t) + \int_{-\infty}^{\infty} e^{-3t-st} \cdot u(t)$$

$$X(s) = \int_{[0]}^{\infty} e^{-(2+s)t} \cdot \cancel{u(t)}^{1} + \int_{0}^{\infty} e^{-(3+s)t} \cdot \cancel{u(t)}^{1}$$

$$X(s) = \left[\frac{e^{-(s+2)}}{-(s+2)}\right]_0^{\infty} \left[\frac{e^{-(s+3)}}{-(s+3}\right]_0^{\infty}$$

$\sigma + 2 > 0$

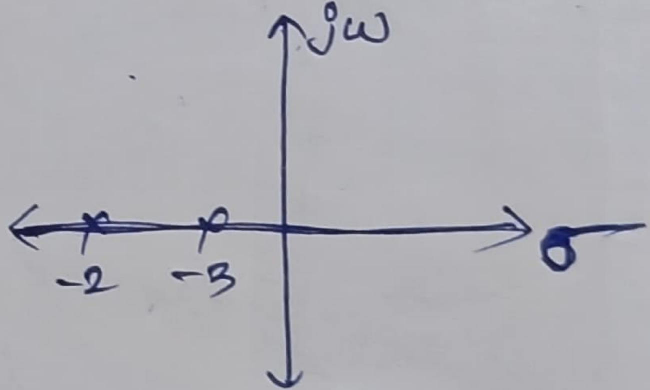
$\sigma > -2$

$\sigma + 3 > 0$

$\sigma > -3$

ROC $\sigma > -2$

$$X(s) = \frac{1}{2+s} + \frac{1}{3+s}$$



(b) $x(t) = e^{-4t}u(t) + e^{-5t}(\sin 5t)u(t)$

$$X(s) = \int_{-\infty}^{\infty} e^{-st}\cdot\left[e^{-4t}u(t)\right] + \int_{-\infty}^{\infty} \frac{e^{-st}}{2j}\cdot\left[e^{j5t} - e^{-j5t}\right]\cdot e^{-5t}u(t)$$

$$X(s) = \int_{0}^{\infty} e^{-st-4t}\,\cancel{u(t)}^{1} + \frac{1}{2j}\left[\int_{0}^{\infty} e^{-st+j5t-5t}\,\cancel{u(t)}^{1} - \int_{0}^{\infty} e^{-st-j5t-5t}\,\cancel{u(t)}^{1}\right]$$

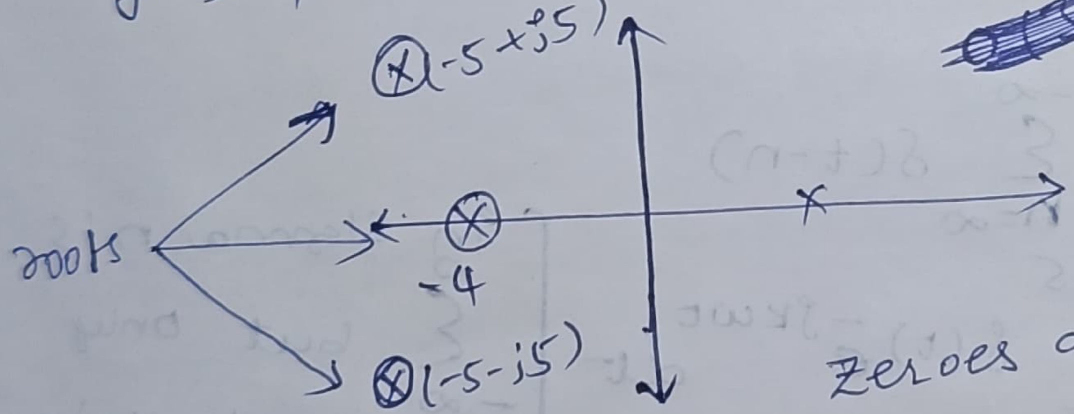
$$X(s) = \left[\frac{e^{-st-4t}}{-(s+4)}\right]_0^{\infty} + \frac{1}{2j}\left[\frac{e^{-t(s-j5+5)}}{[s-j5+5]}\right]_0^{\infty} - \frac{1}{2j}\left[\frac{e^{-t(s+5j+5)}}{(s+5j+5)}\right]_0^{\infty}$$

$$X(s) = \frac{1}{s+4} + \frac{1}{2j}\left[\frac{-1}{s+j5+5} + \frac{1}{s-j5+5}\right] =$$

$s+4>0$ $\qquad$ $\sigma+5>0$ $\qquad$ $\sigma+5>0$

$\sigma+4>0$ $\qquad$ $\sigma>-5$ $\qquad$ $\therefore\ \sigma>-5$

$\sigma>-4$



ROC $\sigma > -4$

Zeroes are $\frac{i\sqrt{55}-15}{2}$, $\frac{-i\sqrt{55}-15}{2}$

$$X(s) = \frac{s^2+70+155}{(s+j5+5)(s-j5+5)(s+4)}$$

(c) $x(t) = e^{2t}u(-t) + e^{3t}u(-t)$

$$X(s) = \int_{-\infty}^{\infty} \left[e^{2t}u(-t) + e^{3t}u(-t)\right]e^{-st}\,dt.$$

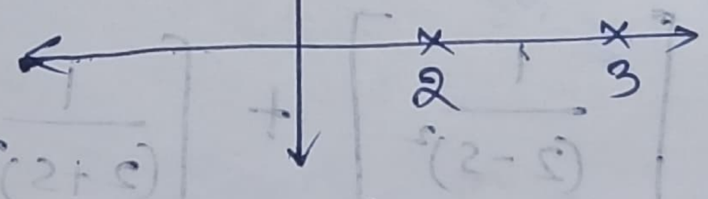
$$X(s) = \int_{-\infty}^{0} e^{2t-st}\,u(-t)\,dt + \int_{-\infty}^{0} e^{3t-st}\,u(-t)\,dt.$$

$$X(s) = \left[\frac{e^{(2-s)t}}{2-s} + \frac{e^{(3-s)t}}{3-s}\right]_{-\infty}^{0}$$

$2-(\sigma) > 0 \ \cap (3-\sigma) > 0$

$$X(s) = \frac{1}{2-s} + \frac{1}{3-s}$$

ROC: $\sigma < 2$



(d) $x(t) = te^{-2|t|}$

$$X(s) = \int_{-\infty}^{\infty} te^{-2|t|}\cdot e^{-st}\,dt$$

$$X(s) = \int_{-\infty}^{0} te^{+2t-st}\cdot dt \ \int_{0}^{\infty} te^{-2t-st}\,dt$$

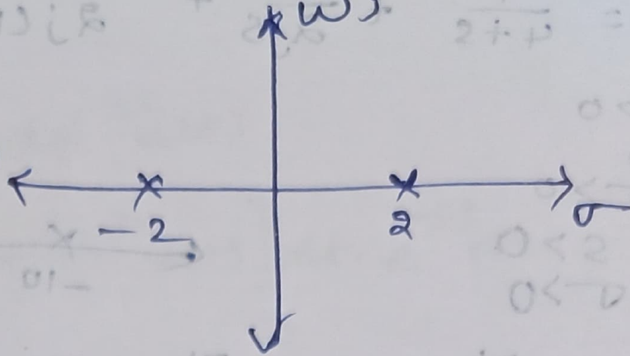
$$X(s) = \left[t\cdot\frac{e^{2t-st}}{(2-s)} - \frac{e^{2t-st}}{(2-s)^2}\right]_{-\infty}^{0} + \left[t\cdot\frac{e^{-2t-st}}{-2-s} - \frac{e^{-2-s}}{(-2-s)^2}\right]_{0}^{\infty}$$

$$X(s) = 0 \ \frac{-1}{(2-s)^2} + 0 + \frac{1}{(-2-s)^2} = \frac{-8s}{(2-s)^2(2+s)^2}$$

$\sigma > -2$

$\sigma < 2,$

ROC: $\sigma \in (-2,2)$



(e) $x(t) = |t| e^{-2|t|}$

$$X(s) = \int_{-\infty}^{0} -t\, e^{2t-st}\, dt + \int_{0}^{\infty} t\, e^{-2t-st}\, dt$$

$$X(s) = \left[-t \cdot \frac{e^{2t-st}}{2-s}\right]_{-\infty}^{0} + \int_{-\infty}^{0} \frac{e^{2t-st}}{2-s} + \int_{0}^{\infty} t\, e^{-2t-st}\, dt$$

$$X(s) = \left[-\frac{t \cdot e^{2t-st}}{2-s} + \frac{e^{2t-st}}{(2-s)^2}\right]_{-\infty}^{0} + \left[\frac{t \cdot e^{-2t-st}}{-2-s} - \frac{e^{-2t-st}}{(-2-s)^2}\right]_{0}^{\infty}$$

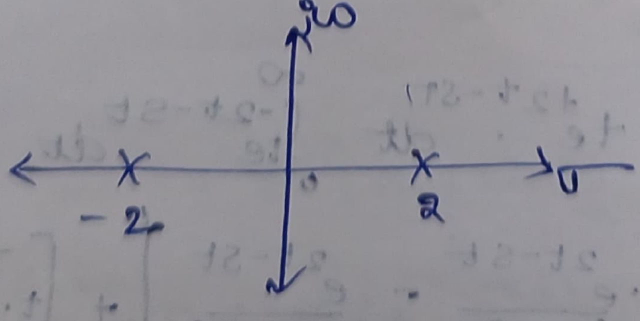
$$X(s) = \left[\frac{1}{(2-s)^2}\right] + \left[\frac{1}{(2+s)^2}\right] = \frac{8+2s^2}{(2-s)^2(2+s)^2}$$

$2-\sigma > 0$

$2+\sigma > 0$

ROC $\sigma \in (-2,2)$

No Zeroes



(f) $x(t) = |t| e^{2t} u(-t)$

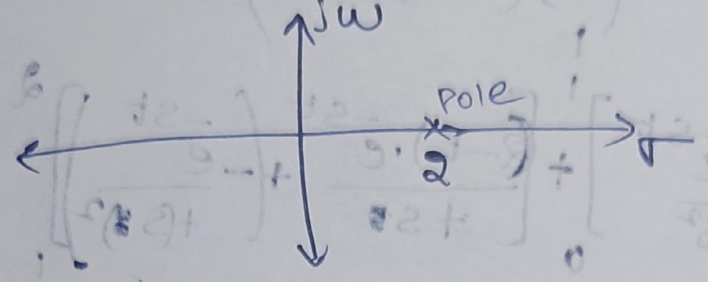
$$X(s) = \int_{-\infty}^{0} -t\, e^{2t}\, u(-t)$$

$$X(s) = \left[-t \cdot \frac{e^{2t-st}}{(2-s)} + \frac{e^{2t-st}}{(2-s)^2} \right]_{-\infty}^{0}$$

$$X(s) = \frac{1}{(2-s)^2}$$

$2 - \sigma > 0$

ROC $\sigma < 2$.



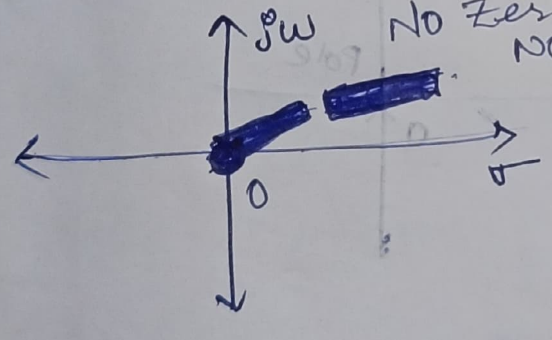
No Zeroes.

(g) $x(t)$

$$X(s) = \int_{-\infty}^{+\infty} x(t) \cdot e^{-st}\, dt$$

$$X(s) = \int_{0}^{1} x(t)\, e^{-st}\, dt + 0$$

$$X(s) = \left[\frac{e^{-st}}{-s} \right]_0^1 = \frac{e^{-s}}{-s} + \frac{1}{s} = \frac{1-e^{-s}}{s}$$



No Zeros
No roots.

$\sigma \in$

(*) ROC $\cdot \{R\}$

entire complex plane.

bcz function is always converging at any value.

$\frac{1-e^{-s}}{s}$ when $s \to 0$

$\frac{0}{0}$ then use L-hospital. (when $\lim_{s \to 0}$

$$\lim_{s \to 0} \frac{\cancel{0} + e^{-s}}{1} = 1 \text{ (constant)}$$

No Zero No root

(b) $x(t)$

$$X(s) = \int_{-\infty}^{\infty} x(t)\cdot e^{-st}\,dt$$

$$X(s) = \int_{0}^{1} t\cdot e^{-st} + \left(+\int_{1}^{2} (2-t)\cdot e^{-st}\right)$$

$$X(s) = \left[\frac{t\cdot e^{-st}}{-s} - \frac{e^{-st}}{s^2}\right]_0^1 + \left[\frac{(2-t)\cdot e^{-st}}{+s} + \left(-\frac{e^{-st}}{+s^2}\right)\right]_1^2$$

$$X(s) = \cancel{\frac{e^{-s}}{-s}} - \cancel{\frac{e^{-s}}{s^2}} + \frac{1}{s^2} + 0 + 0 - \frac{e^{-2s}}{s^2} - \cancel{\frac{e^{-s}}{+s}} + \cancel{\frac{e^{-s}}{s^2}}$$

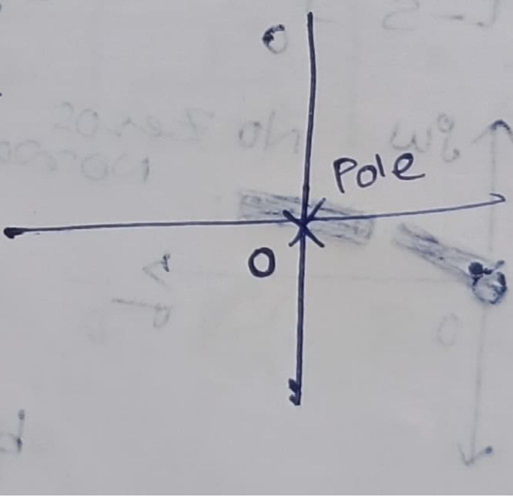
$$X(s) = \frac{-e^{-2s}+1}{s^2} - \frac{2e^{-s}}{s}$$

$$X(s) = \frac{-s2e^{-s} - e^{-2s} + 1}{s^2}$$

$$X(s) = \frac{-2\cdot s\cdot e^{-s} - e^{-2s} + 1}{s^2}$$

For $\forall\ s$

ROC $\sigma \in \{R\}$



(i)

$x(t) = \delta(t) + u(t)$

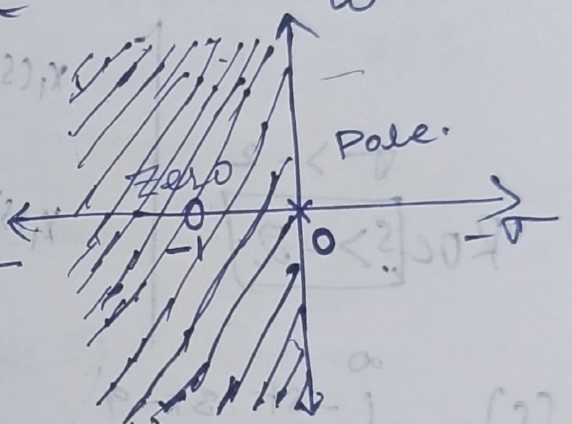
$$X(s) = \int_{-\infty}^{\infty} [\delta(t) + u(t)] e^{-st} dt.$$

$$X(s) = \int_{-\infty}^{\infty} \delta(0) e^{-s(0)} + \int_{0}^{\infty} e^{-st} \cancel{u(t)}'$$

$$X(s) = 1 + \left[\frac{e^{-st}}{-s}\right]_0^{\infty}$$

$$X(s) = 1 + \frac{1}{s} = \frac{s+1}{s} \quad [\text{when, } s<0]$$

$+\ s > 0$



ROC $s > 0$

ROC $\sigma > 0$

(ii) $x(t) = \delta(3t) + u(3t)$

$$X(s) = \int_{-\infty}^{\infty} [\delta(3t) + u(3t)] e^{-st} dt.$$

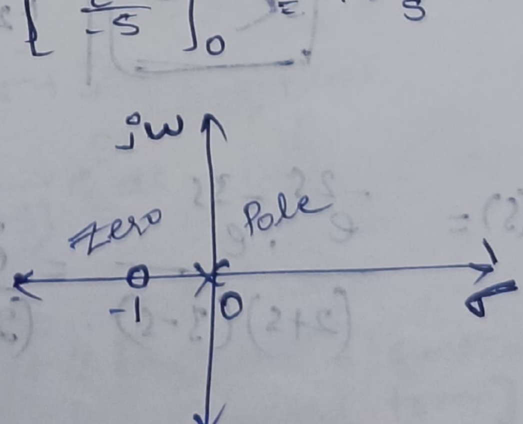
$$X(s) = \int_{-\infty}^{\infty} \delta(3\cancel{t}) \cancel{e^{-st}} dt + \int_{0}^{\infty} u(\cancel{3}t) e^{-st} dt$$

$$X(s) = 1 + \left[\frac{e^{-st}}{-s}\right]_0^{\infty} = 1 + \frac{1}{s} = \frac{s+1}{s}$$

$-s < 0$

$\sigma > 0$

$s > 0$



$$\boxed{\begin{matrix} \text{ROC} & s>0 \\ & \sigma>0 \end{matrix}}$$

9.26

$$y(t) = x_1(t-2) * x_2(-t+3)$$

$$\Big\downarrow L\cdot T \qquad\qquad \Big\downarrow L\,T$$

$$Y(s) = X_1(s)\cdot X_2(s)$$

$$X_1(s) = \int_{-\infty}^{\infty} e^{-st}\cdot e^{-2t+4}\,u(t) = e^{4}\int_{2}^{\infty} e^{-st-2t}\cdot u(t-2)$$

$$X_1(s) = e^{4}\cdot\left[0 + \frac{e^{-2s-4}}{2+s}\right]_{2}^{\infty}$$

$\sigma > -2$

$\text{ROC}\ \boxed{s > -2}$

$$X_1(s) = \frac{e^{-2s}}{2+s}$$

$$X_2(s) = \int_{-\infty}^{\infty} e^{-st}\cdot e^{3t-9}\cdot u(-t+3)\,dt = e^{-9}\int_{-\infty}^{3} e^{-st+3t}\cdot u(-t+3)$$

$$X_2(s) = e^{-9}\cdot\left[0 + \frac{e^{(-s+3)3}}{-s+3}\right]_{-\infty}^{3}$$

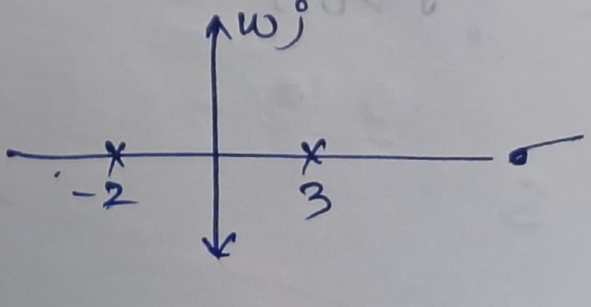
$\sigma < +3$

$\text{ROC}\ \boxed{s < +3}$

$$X_2(s) = \cancel{e^{-9}}\cdot\frac{e^{-3s}\cdot\cancel{e^{9}}}{-s+3} = \frac{e^{-3s}}{-s+3}$$

$$Y(s) = \frac{e^{-2s}\cdot e^{-3s}}{(2+s)(3-s)} = \frac{e^{-5s}}{(2+s)(3-s)}$$

ROC combine $\sigma \in (-2\ \ 3)$



9.29

$$x(t) = e^{-t}u(t) \qquad h(t) = e^{-2t}u(t)$$

(a) $$X(s) = \int_{-\infty}^{\infty} e^{-t-st} u(t) \qquad H(s) = \int_{-\infty}^{\infty} e^{-2t-st} u(t)$$

$$X(s) = \int_{0}^{\infty} e^{-t-st} \cancel{u(t)}\, dt, \qquad H(s) = \int_{0}^{\infty} e^{-2t-st} \cancel{u(t)}\, dt$$

$$X(s) = \left[\frac{e^{-(1+s)t}}{-(1+s)}\right]_0^{\infty} \qquad H(s) = \left[\frac{e^{-(2+s)t}}{-(2+s)}\right]_0^{\infty}$$

$$X(s) = \frac{1}{1+s} \qquad H(s) = \frac{1}{2+s}$$

(b) $$y(t) = x(t) * h(t)$$

convolution

(L-T)

(L-T)

$$y(s) \quad X(s)\cdot H(s)$$

$$Y(s) = \left(\frac{1}{1+s}\right)\left(\frac{1}{2+s}\right)$$

(c) $$Y(s) = \frac{1}{(1+s)(2+s)}$$

$$y(t) = \frac{1}{2\pi j} \int_{\sigma+j(-\infty)}^{\sigma+j(\infty)} \frac{1}{(1+s)(2+s)} \cdot e^{st}\, ds$$

$$y(t) = \frac{1}{2\pi j}\left[\int_{\sigma+j(-\infty)}^{\sigma+j(\infty)} \frac{1}{(1+s)} e^{st}\, ds - \int_{\sigma+j(-\infty)}^{\sigma+j(\infty)} \frac{1}{(2+s)} e^{st}\, ds\right]$$

Hence it is [the above 2 terms are in general form]

~~y(t) =~~

$$y(t) = e^{-t}u(t) + \left(-e^{-2t}u(t)\right) = u(t)\left[e^{-t} - e^{-2t}\right]$$

(d) $x(t) \underset{\text{convolution}}{*} h(t) = y(t) = \int_{-\infty}^{\infty} x(t)\,h(k-t)$

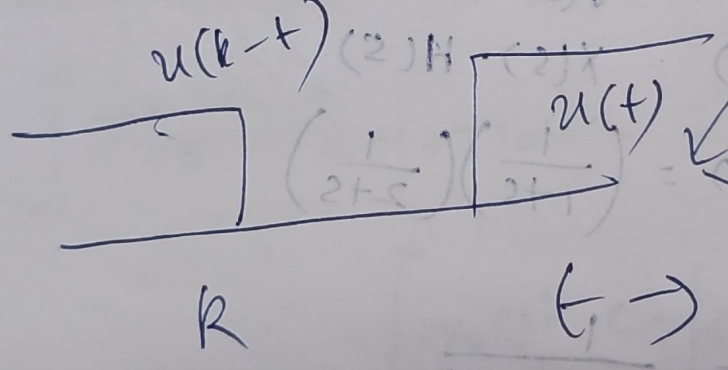
$$y(k) = \int_{-\infty}^{\infty} e^{-t}u(t)\cdot e^{-2(k-t)}\cdot u(k-t)\,dt$$

$$y(k) = \int_{0}^{k} e^{-t}\overset{1}{u(t)}\cdot e^{-2(k-t)}\,\overset{1}{u(k-t)} + \int_{k}^{\infty} e^{-t}(1)\cdot e^{-2(k-t)}\,u(k$$

$$y(k) = \int_0^k e^{-2k+t} = \left[\frac{e^{-2k+t}}{1}\right]_0^k = e^{-k} - e^{-2k}$$

$$y(k) = e^{-k} - e^{-2k}.\quad u(k) \quad \forall k>0$$

$$\Rightarrow \quad y(t) = e^{-t} - e^{-2t} \quad \forall\, t>0$$



$$y(k) = \begin{cases} e^{-k} - e^{-2k}, & \forall k>0 \\ 0, & \forall k<0 \end{cases}$$

$$y(t) = \begin{cases} e^{-t} - e^{-2t}, & \forall\, t \geq 0 \\ 0, & \forall t<0 \end{cases}$$