Bipolar Junction Transistor

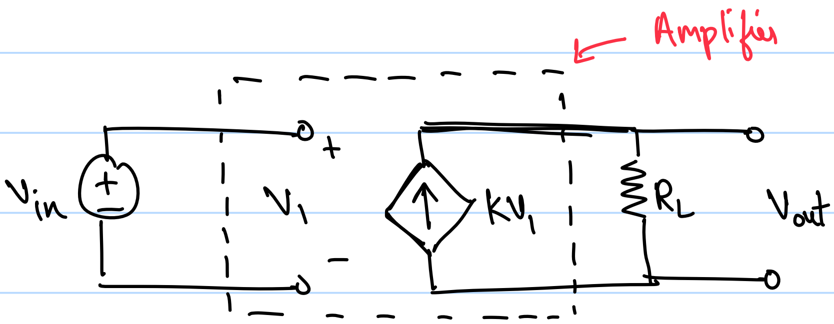
→ Structure of BJT

→ BJT Operations

→ Amplifier :

- A Basic Amplifier :-

We can easily construct a amplifier using dependent sources. ex:



$$V_{out} = (KV_1) R_L$$

$$V_1 = V_{in} \text{ (obv)}$$

$$\Rightarrow V_{out} = K R_L V_{in}$$

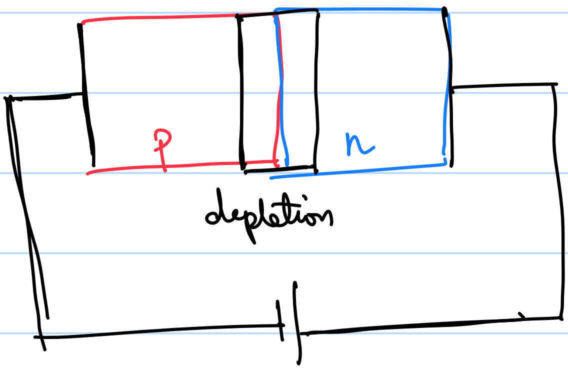
$$\Rightarrow V_{out} / V_{in} = K R_L$$

- If $KR_L > 1$, then this circuit can act like an amplifier
- Problem: We do not have dependent sources easily available.
- Therefore we use transistors to build amplifiers.

→ Few Observations : (From PN)

- Carrier Injection :

In a reverse biased PN junction,

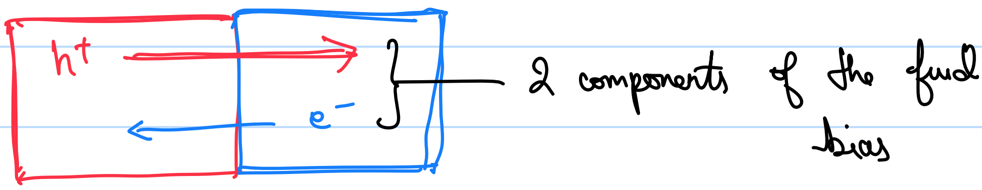


- Suppose some electrons are injected into the depletion region, The electric field in the depletion region is directed from n to p.
Therefore electrons will go to the n side and towards the +ve terminal of the battery

- As such we can maintain a constant current from the depletion to the n section and to the battery

- Effect Of Asymmetric Doping :

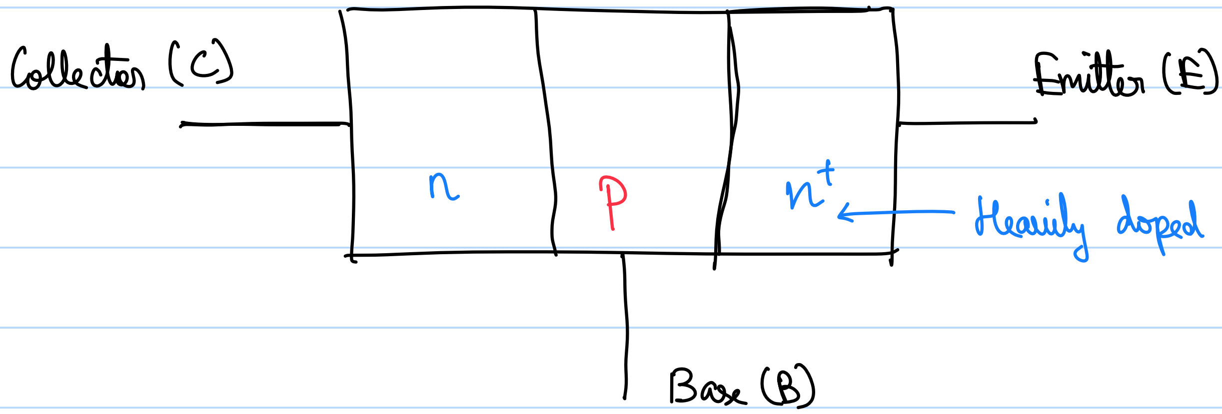
In a forward biased pn junction



- If $N_D >> N_A$, the diffusion current is heavily dominated by electrons.

→ Structure and Symbol of BJT :-

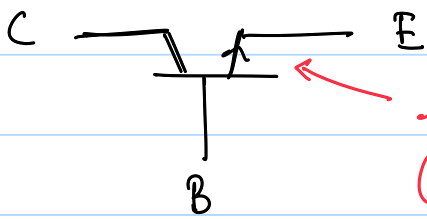
- 3 terminal device w/ 3 semiconductor region.



- Since n^+ is heavily doped, the device is asymmetric.

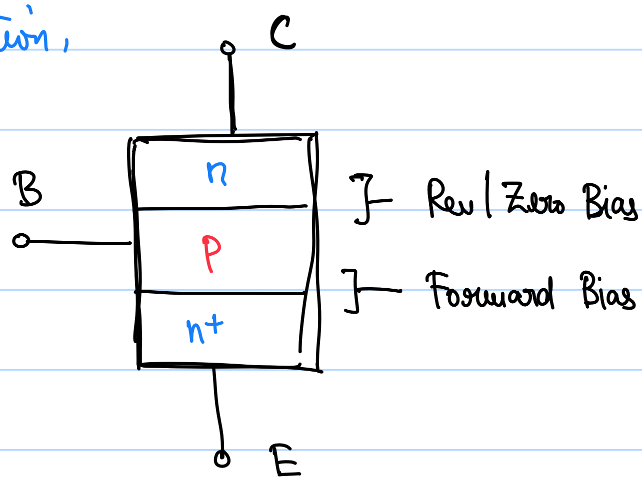
- It can be seen that C is the input terminal and E is the output, from the names given.

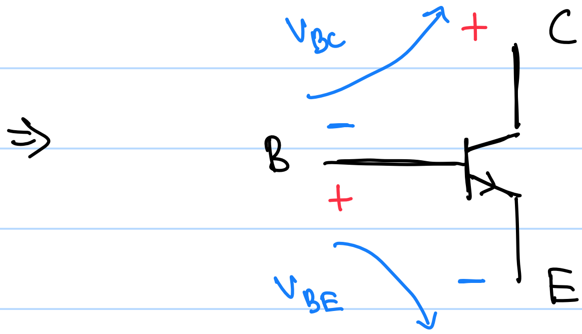
- Symbol



Since the device must be asymmetric (Specific for npn transistor).

• For amplification,





$V_{BE} > 0.7\,V$

$V_{BC} \leq 0$

$\underline{V_C > V_B > V_E}$

This is termed as forward active biasing / region.

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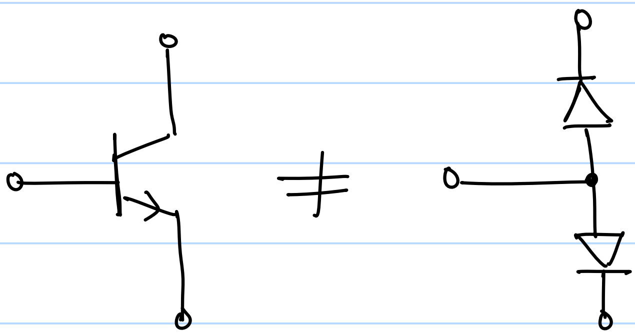
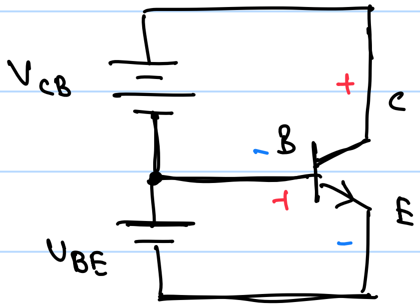
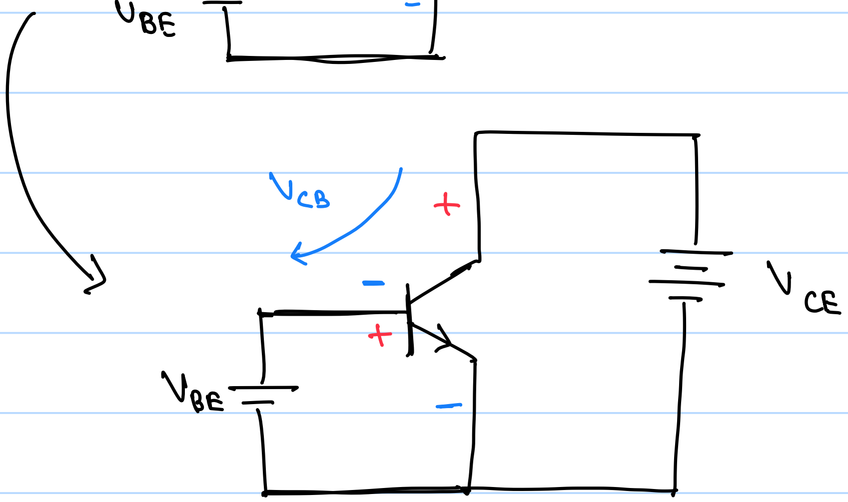


• The base region is very thin in a real BJT.

→ Operation of BJT:-



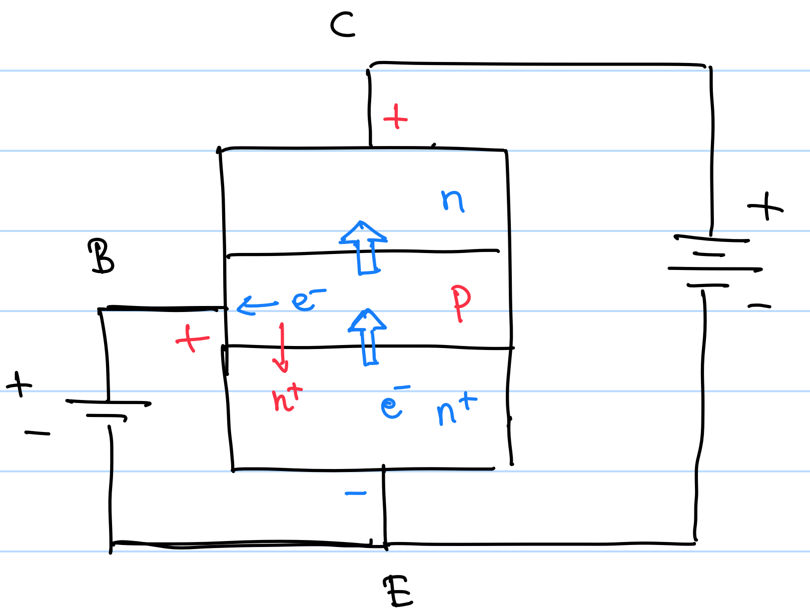
$V_{CB} > V_{BE}$



Better for studying Amplifiers

$$V_{CE} = V_{CB} - V_{BE} \geq 0$$

$$\Rightarrow \quad V_{CB} \geq V_{BE} > 0.7V$$



- Since Base-Emitter Junction (BEJ) is fwd biased, current will flow in the BE loop.

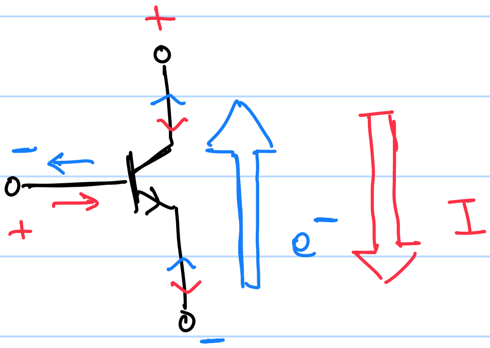
- This BE current will be dominated by electrons (due to highly doped n^+).

- Since CBJ is in reverse bias, it will have a depletion region.

- The electrons coming from the emitter has 2 options:
  1) Exit through the base terminal and get absorbed by the battery.
  2) Go to the CBJ depletion region.

- If the electron enters the CBJ depletion region, it will be swept across into the Collector region (Carrier injection), ie, the Collector region "collects" the electrons, after which they go to the battery.

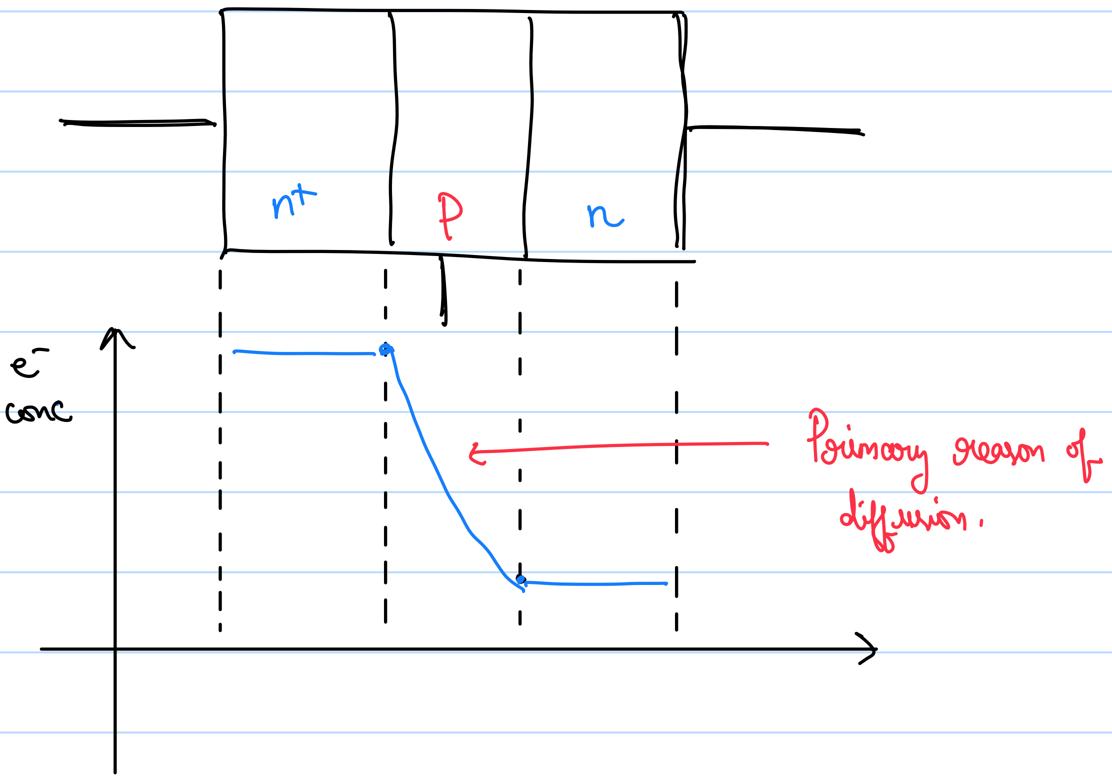
- Most electrons from the EBJ will go to the CBJ. This is why base is very thin.



$\longrightarrow$ Define this current as I_{CE}

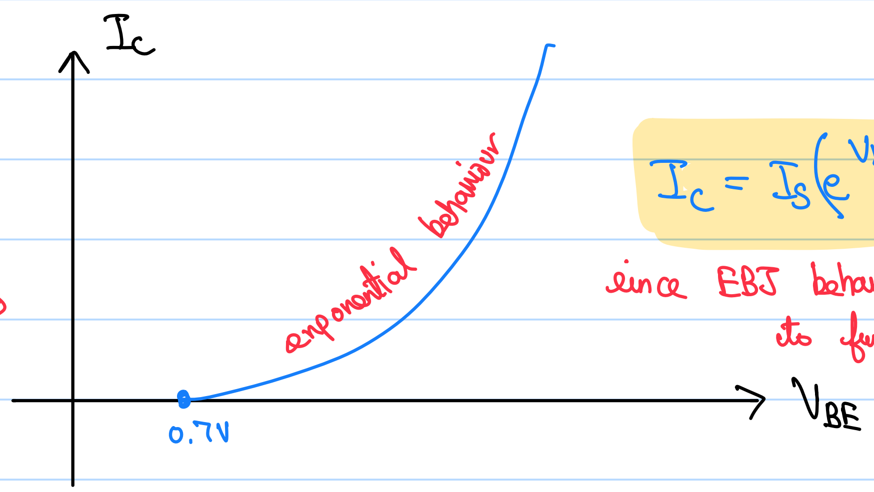
- The base region is thin and sustains little voltage across it
  $\Rightarrow$ Negligible drift current.

- So the current flow I_{CB} is majorly governed by diffusion



- If V_{BE} increases, the current across EBJ increases (fwd bias).
  - If i in EBJ, i in CBJ also increases $\Rightarrow$ Collector current increases.

Keeping V_{CE} constant we get →



$$\boxed{I_c = I_s\left(e^{V_{BE}/V_T} - 1\right)}$$

since EBJ behaviour is ||| to fwd biased diode behaviour

- $$I_S = A_E \frac{q n_i^2 D_n}{W_B N_B}$$ $\Rightarrow I_S \propto T$ (By n_i^2)

D_n ← Diffusivity of e^-

N_B ← Doping of Base

A_E ↑ Emitter Area

W_B ↑ Base Width

- $I_c = I_s\left(e^{\frac{V_{BE}}{V_T}} - 1\right) \Rightarrow$ I_c is dependent of V_{BE}
  ↳ Voltage Dependent Current

- If $\Delta V_{BE} = +60\ mV$, the current increases by a factor of 10. (Same as fwd biased diode)

- $I_c = I_s\left(e^{V_{BE}/V_T} - 1\right) \simeq I_s e^{V_{BE}/V_T}$ (same approximation as diode)

  $\Rightarrow$ $\boxed{V_{BE} = V_T \ln\left(I_c/I_s\right)}$

Example:

The cross sectional area of a transistor is doubled and V_{BE} is decreased by 60mV. What is the change in the collector current?

We know, $I_s \propto A$. $\therefore I_s \longrightarrow 2I_s$

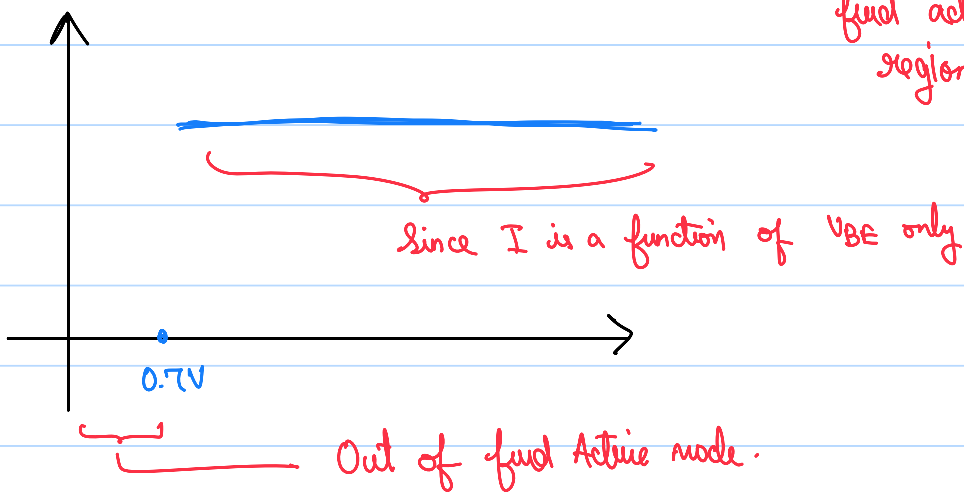
$\Rightarrow I_c = 2I_s\left(e^{V_{BE}/V_T} - 1\right) \Rightarrow I_{C2} = 2I_{C1}$

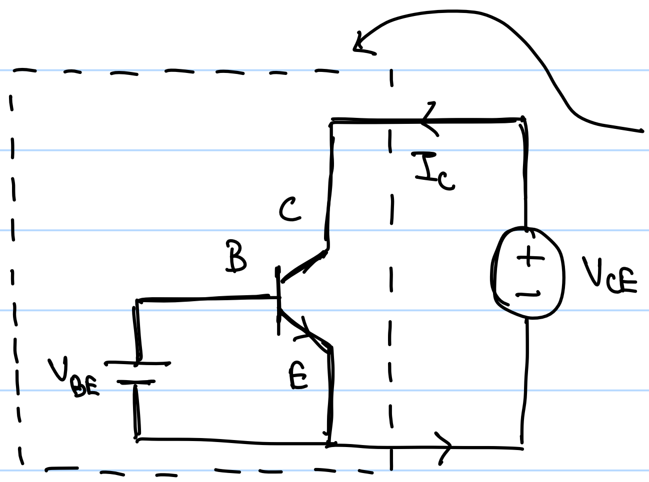
Since 60mV decrease in V_{BE} creates a 10 times decrease in I_c.

$$I_{C3} = \frac{2I_{C1}}{10} \Rightarrow 0.2 I_{C1}$$

$\therefore$ The new collector current is 0.2 times the initial one.

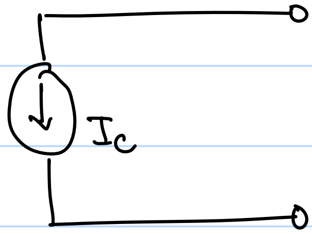
- Now if we kept V_{BE} constant and vary V_{CE} (within the fwd active region)



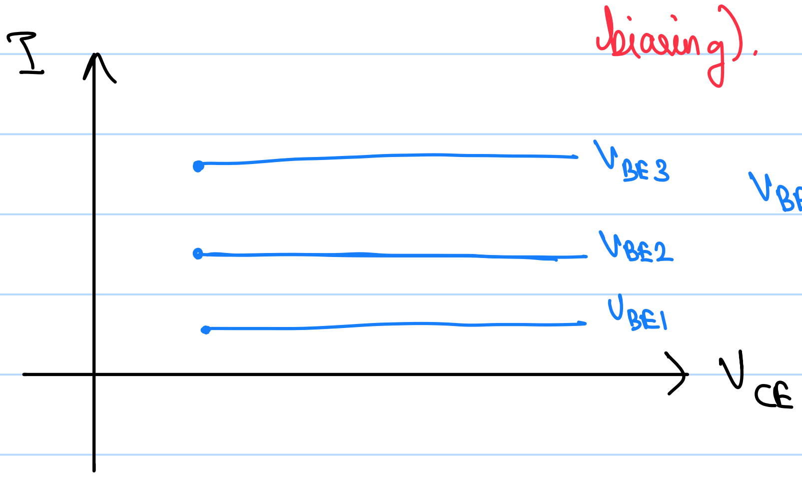


- This is a device such that a variation in V_{CE} will NOT create a change in current, ie, An ideal current source.

=>

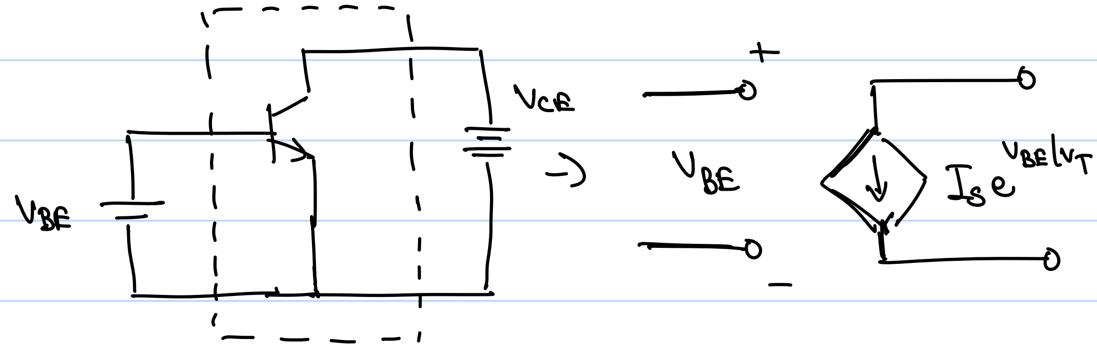


If V_{BE} is kept constant, the transistor can act as a current source (provided fwd active biasing).



$V_{BE3} > V_{BE2} > V_{BE1}$

Therefore, the device is also a Voltage dependent current source.

$\Rightarrow$ 

- **Terminal Currents:**

1) **Collector Current:**

$$I_C = I_S e^{V_{BE}/V_T}$$

2) **Base Current:**

From the electrons that come from the emitter, it was seen that a majority of them will go into the collector region.

- However a small amount of the electrons may exit the device through the base terminal and get absorbed by the battery, or recombine with the holes of the base region

- The above 2 components form the base current.

- Base current $\propto$ Collector Current

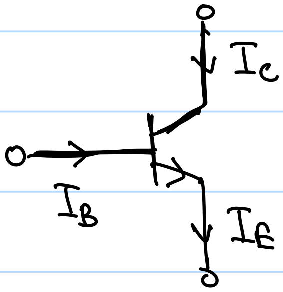
$\Rightarrow I_B \propto I_C$

$\Rightarrow \boxed{I_C = \beta I_B}$ In practice β: 50 - 200.

- β is termed as the current gain of the transistor (common emitter current gain - Sedra Smith).

- $$I_B = \frac{I_S}{\beta}\left(e^{V_{BE}/V_T} - 1\right)$$

- I_B is directed into the base region



By KCL we get,

$$I_E = I_C + I_B$$

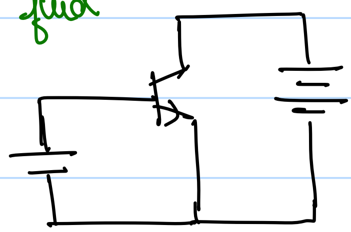
$$= \left(\frac{\beta+1}{\beta}\right) I_S e^{V_{BE}/V_T}$$

Since usually $\beta >> 1$,

$$I_E \simeq I_S e^{V_{BE}/V_T} = I_C$$

Example:

A BJT has a collector current of 1 mA and $I_S = 10^{-16}$ A. How much is V_{BE}, given fwd active region.



ANS:

Given, $I_C = 1\text{mA}$

$$I_C = I_S e^{V_{BE}/V_T} \Rightarrow V_{BE} = V_T \ln \frac{I_C}{I_S}$$

$$V_{BE} = (26) \ln \frac{10^{-3}}{10^{-16}} \text{ mV}$$

$$V_{BE} = 26 \ln 10^{13} \text{ mV}$$

$$= 338 \ln 10 \text{ mV}$$

$$= 338 \times 2.303 \text{ mV}$$

$$= 778.273 \text{ mV}$$

$$\simeq \underline{\underline{0.78 \text{ V}}}$$

<u>Example</u>: In the prev. scenario, if β is known to be 50, find base current.

<u>Ans</u>:

Given $I_C = 1\text{mA}$ & $\beta = 50$.

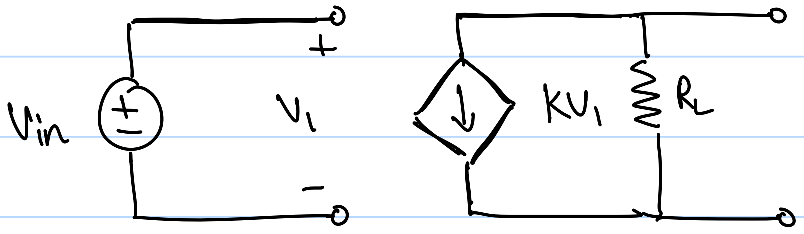
$$\beta = \frac{I_C}{I_B}$$

$$I_B = \frac{I_C}{\beta} = \frac{1}{50} \text{ mA}$$

$$= \underline{\underline{0.02\text{mA}}}$$

→ <u>BJT as an Amplifier</u> :-

- We know that,

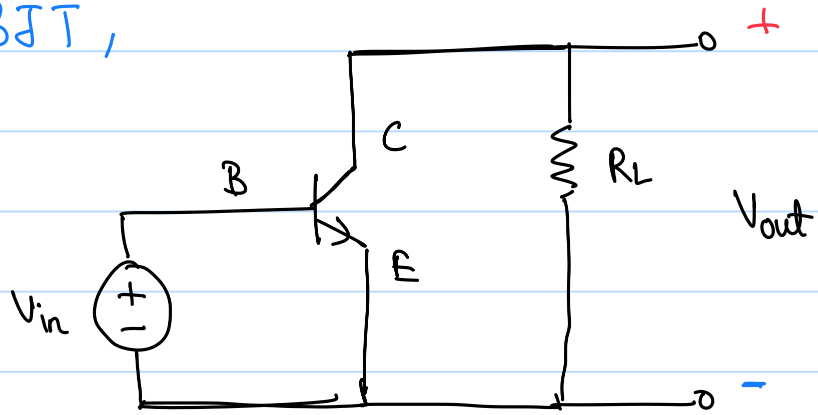


$$V_{out} = \underline{\underline{-KR_L V_{in}}}$$

inverted output due to direction of KV_1.

is a simple amplifier

- Using a BJT,



starting place for making a BJT Amplifier

Clearly, the design is flawed and will not work. Analysing the circuit,

Assume $I_S = 5\times10^{-16}$ A (Standard for BJT)

$$I_C = I_S\left(e^{V_{in}/V_T} - 1\right) = (5\times10^{-16})\left(e^{V_{in}/V_T} - 1\right) \text{ A}$$

$$V_{out} = -R_L(5\times10^{-16})\left(e^{V_{in}/V_T} - 1\right) \text{ V}$$

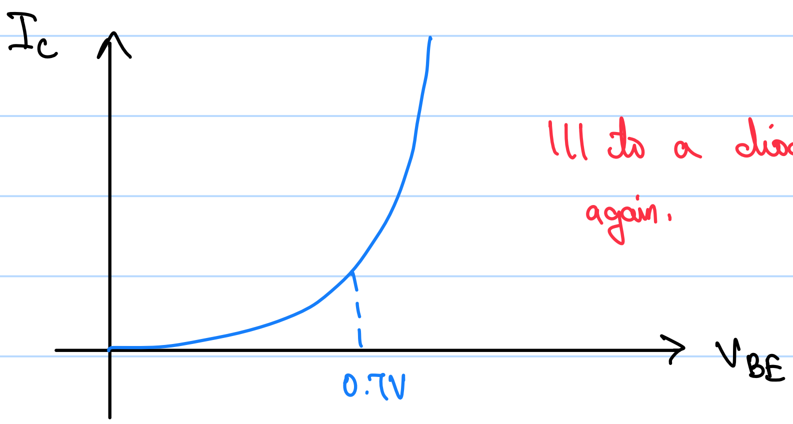
Take an example signal V_{in} et V_{in} starts at zero and goes to 10 mV at time t → Doubtful considering that V_{BE} should be > 0.7V.

After time t in V_{out},

$$\begin{aligned} V_{out} &= -R_L I_S\left(e^{10mV/V_T} - 1\right) \\ &= -R_L(5\times10^{-16})\left(e^{10/26} - 1\right) \\ &\approx -R_L(2.35\times10^{-16}) \text{ V} \end{aligned}$$

Therefore, for any reasonable gain in the output, R_L must be impractically large. → Not a usable amplifier.

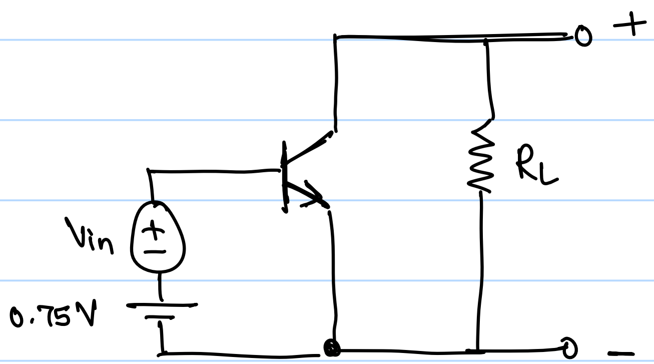
- I-V Characteristics of a BJT :-



||| to a diode fwd bias, again.

- By the above graph we can see why our I_C was very low in our calculations.

- To combat this, maybe we can introduce a battery in series with V_{in}.



- If $V_{in} = 0$, $V_{BE} = 0.75V$, which can produce a significant amount of current,

$$I_C = I_S\left(e^{V_{BE}/V_T} - 1\right)$$

$$= (5\times10^{-5})\left(e^{750/26} - 1\right)$$

$$\simeq \underline{1.7mA}$$

When V_{in} goes to 10 mV,

$$I_C = (5\times10^{-5})(e^{760/23}-1)$$

$$\simeq 2.48 \text{ mA}$$

$$\Delta I_C = 2.48 - 1.7 = 0.78 \text{ mA} \longrightarrow \text{significant}$$

Therefore, we can use a practical value of resistance to create a significant gain in the output.

If gain = 10, $\Delta V_{out} = 10\,\Delta V_{in}$, $\Delta V_{in} = 10\text{ mV}$

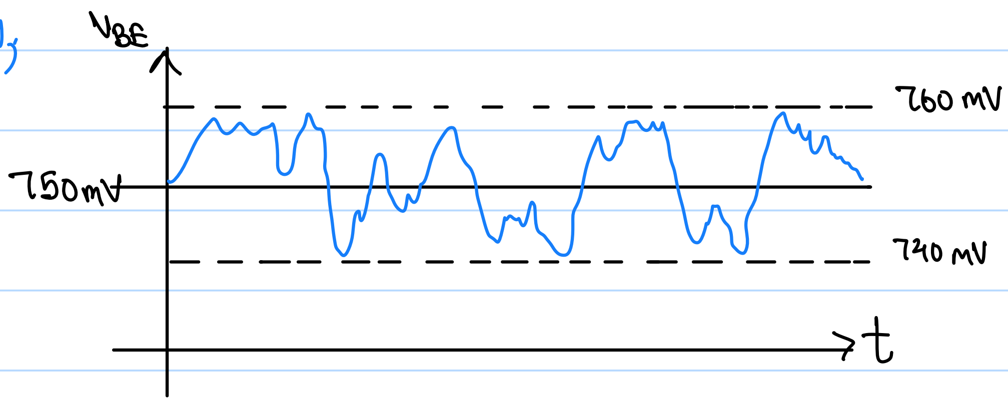
$\Rightarrow \Delta V_{out} = 100\text{ mV}$

$$\Delta V_{out} = R_L\,\Delta I_C = 100\text{mV}$$

$$R_L = \frac{100\text{mV}}{\Delta I_C} = \frac{100}{0.78} \simeq 128\,\Omega$$

An R_L of $\underline{\underline{128\,\Omega}}$ will give us a gain of 10 in this case.

If we use this model with a V_{in} signal that varies by 10 mV,



We will get a gain of 10, ie, an output signal of amplitude 100 mV

- This process of adding a voltage source so that the transistor can amplify, is called biasing the transistor

- Biasing is done to move the transistor into the forward active region before applying the input voltage.

- The base values of V_{BE}, I_C, I_B ... is defined as operating point.

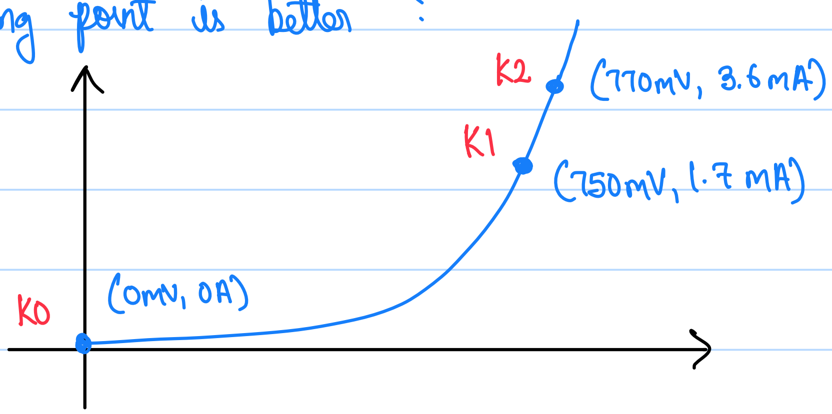
In the prev. situation, $V_{BE} = 0.75V$ and $I_C = 1.7mA$ is our operating point.

→ <u>Observations</u> :-

① A bipolar transistor can act as a voltage dependent current source, since its collector current is exponentially related to the base-emitter voltage.

② The operating point determines how the transistor responds.

③ Which operating point is "better"?



Obviously K0 is a bad OP as seen before, but which is better among K1 and K2 ?

In K2,

At $V_{in} = 0\,mV$, At $V_{in} = 10\,mV$

$$I_c = I_S\left(e^{770/23} - 1\right) \qquad I_c = I_S\left(e^{780/23} - 1\right)$$

$$\simeq 3.6\,mA \qquad \simeq 5.3\,mA$$

$$\therefore \Delta V_{in} = 10\,mV \Rightarrow \Delta I_c = 5.3 - 3.6 = 1.7\,mA$$

If $\Delta V_{out} = 100\,mV$

$$R_L = \frac{\Delta V_{out}}{\Delta I_c} = \frac{100}{1.7} = 58.823$$

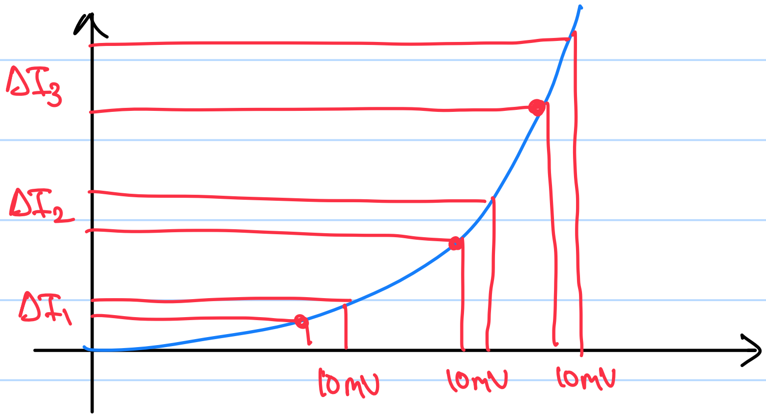
Therefore, the case K2 requires a lesser resistance requirement, ie, has a stronger response to the change

- An increase in the biasing voltage of V_{BE} results in an increase in the sensitivity of the amplifier.

- This increase in sensitivity is denoted by transconductance.

- <u>Transconductance</u>:

- In the forward active region, if we plot I_c vs V_{BE}



We see that $\Delta I_1 < \Delta I_2 < \Delta I_3$

$\Rightarrow$ Slope of the relation is increasing $\left(\frac{dI_C}{dV_{BE}}\right)$

- The slope $\frac{dI_C}{dV_{BE}}$ is termed as transconductance. (g_m)

In the scenario we have $\Delta I_1 = 0.78\,mA$ and $\Delta I_2 = 1.7\,mA$ each for a ΔV_{in} of $10\,mV$. Assume this ΔV_{in} to be small, calculate the transconductance at OP 1 and OP 2.

$$g_m = \frac{dI_C}{dV_{BE}} \simeq \frac{\Delta I_C}{\Delta V_{BE}} \quad (\text{for small } \Delta V_{BE})$$

$$g_{m1} = \frac{0.78\,mA}{10\,mV} = 0.078\,A/V = 0.078\,S \quad (\text{siemens})$$

$$g_{m2} = \frac{1.7\,mA}{10\,mV} = 0.17\,A/V = 0.17\,S$$

- Since ckt $I_C = I_S\left(e^{V_{BE}/V_T} - 1\right)$

$$g_m = \frac{d\,I_S\left(e^{V_{BE}/V_T} - 1\right)}{dV_{BE}} = I_S\,\frac{de^{V_{BE}/V_T}}{dV_{BE}}$$

$$g_m = \frac{I_S}{V_T}\,e^{V_{BE}/V_T}$$ $\longrightarrow$ Transconductance in itself is exponentially dependent on V_{BE}

☆ $\boxed{g_m = \frac{I_C}{V_T}}$

- If $I_C = 1mA$ at room temp, $g_m = 0.04\,S$.

- The very first circuit made for amplification failed because of low transconductance. The biasing step increased the transconductance.

- If $g_m = 0 \longrightarrow$ No Amplification.

- $g_m > 0 \Rightarrow \underbrace{\text{need certain } I_C \Rightarrow \text{need certain } V_{BE}}_{\text{From the desired OP}}$

Using the new formula, Calculate transconductance of OP1 and OP2 in the prev. scenario.

$$g_{m1} = \frac{I_{C1}}{V_T} = \frac{1.7mA}{26mV} = 0.0653\,S \qquad g_{m2} = \frac{I_{C2}}{V_T} = \frac{3.6mA}{26mV} = 0.1384\,S$$

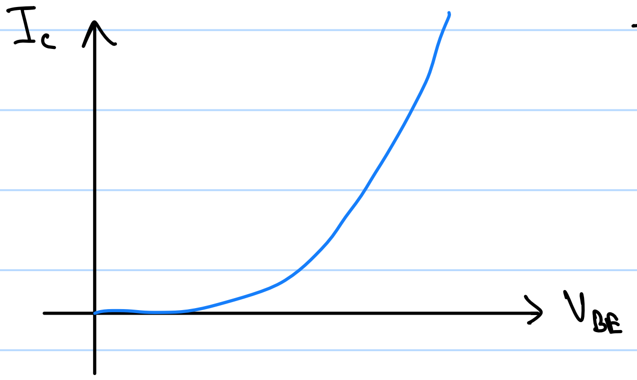
Old $g_{m1} = 0.078$ Old $g_{m2} = 0.17\,S$

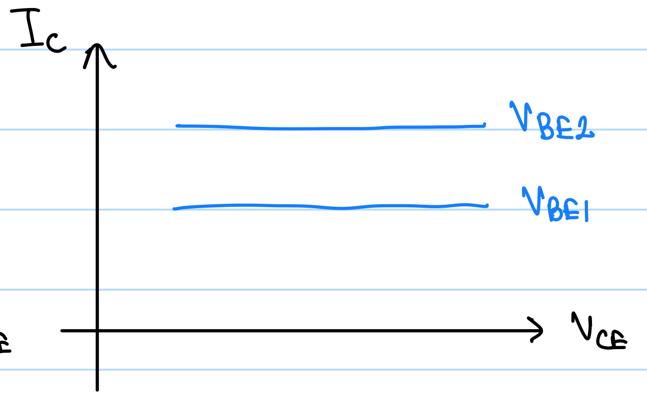
- Therefore, from this discussion, we get the fact that the response of the amplifier is stronger, greater the biasing voltage.

- Tradeoff is power consumption from the biasing voltage source.

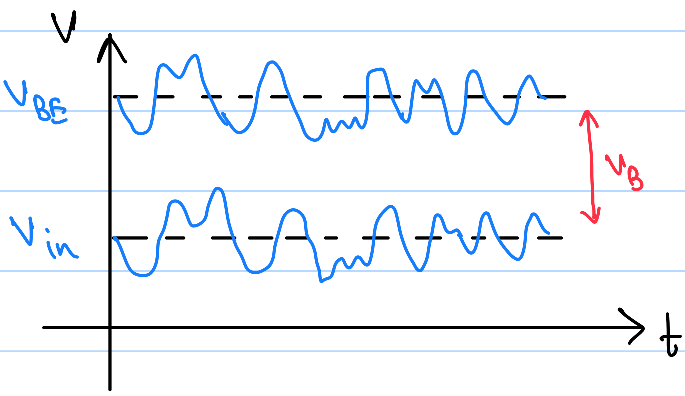
- The approximation in the derivative can be taken if V_{BE} is a small fraction of V_T.

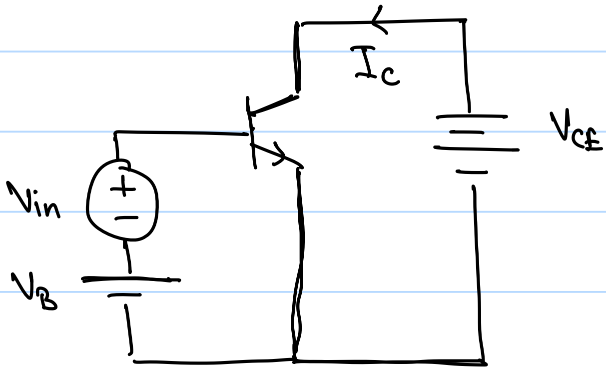
- **IV Characteristics :-**

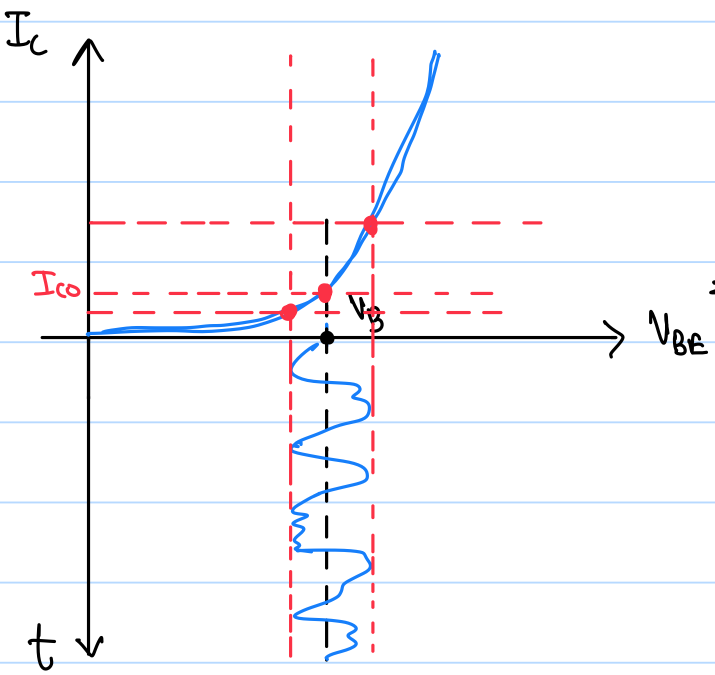




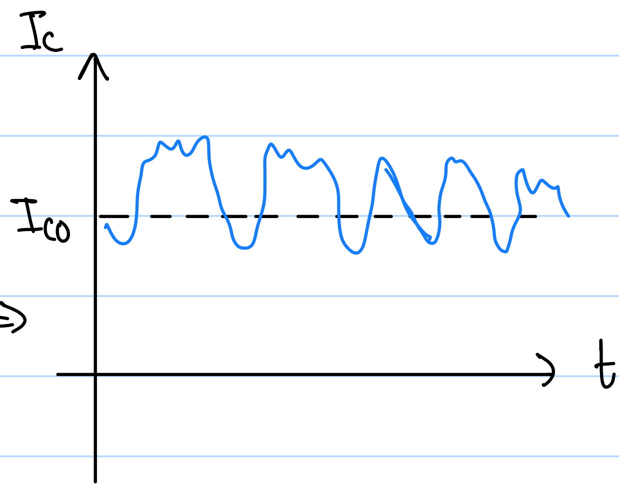
- **Combining Time Response w/ IV Characteristics :-**



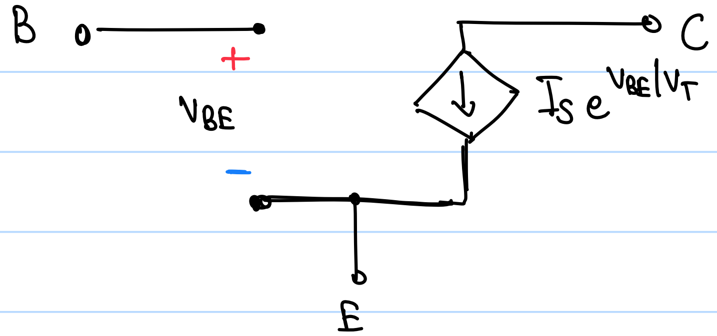




$\Rightarrow$



→ Simple BJT Models :-



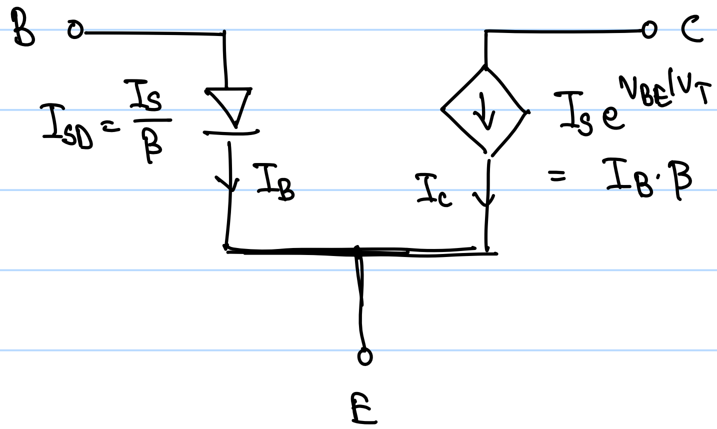
The current source goes directly over the base into the emitter due to the presence of base current

(Not very accurate obv.)

- To model the base current, w.k.t,

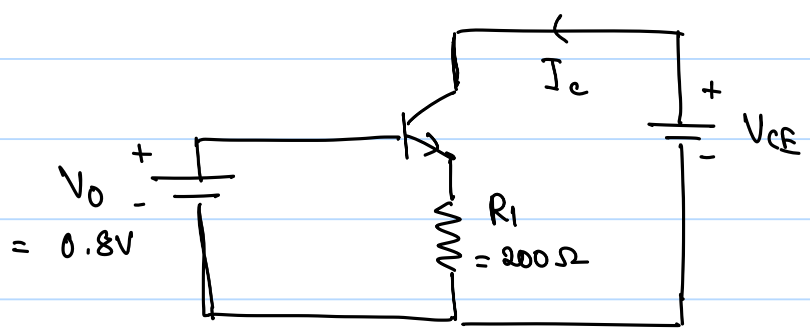
$$I_B = \frac{I_S}{\beta} e^{V_{BE}/V_T}$$

We can model this behaviour with a diode of rev. sat current $\frac{I_S}{\beta}$.



- This model is valid only for forward active biasing.

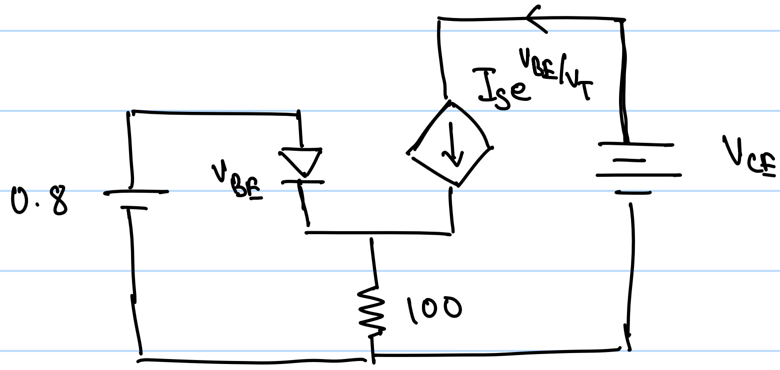
Example:



$I_S = 5 \times 10^{-16}$ A

Find I_C

Applying the model,



Let $I_E \simeq I_C$. By KVL,

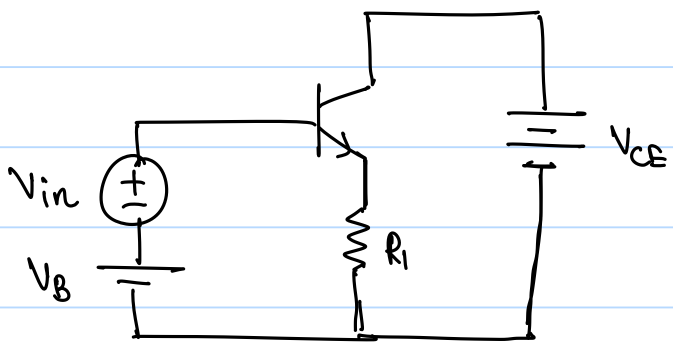
$$V_0 = V_{BE} + R_1 I_E \simeq V_{BE} + R_1 I_C$$

W.k.t $\quad V_{BE} = V_T \ln \frac{I_C}{I_S}$

$$\Rightarrow \quad V_0 = V_T \ln \frac{I_C}{I_S} + R_1 I_C$$

We can solve this by iteration by assuming a value for I_C in $V_T \ln I_C/I_S$ and solving for I_C's new value.

Example:



$V_{in} = V_M \sin \omega t$

Find I_C

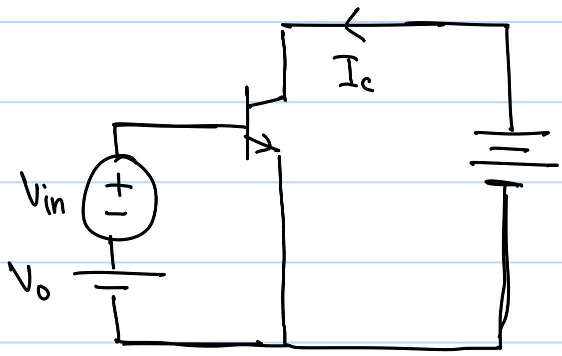
From prev. qn, we can see that

$$V_{in} + V_0 = V_T \ln \frac{I_C}{I_S} + R_1 I_C$$

$$= \quad V_m \sin\omega t + V_0 = V_T \ln \frac{I_C}{I_S} + R_1 I_C$$

Here I_C is a function of time, ie, it is not constant. Hence we cannot use the iterative method.

If we use a simpler situation,



By KVL,

$$V_{in} + V_0 = V_{CE}$$

$$= \quad V_{in} + V_0 = V_T \ln \frac{I_C}{I_S}$$

$$= \quad I_C = I_S \, e^{\frac{V_{in} + V_0}{V_T}} =$$

$$= \quad I_C = I_S \exp\left(V_m \sin\omega t + V_0 / V_T\right)$$

$$= \underbrace{I_S \exp\left(V_0/V_T\right)}_{\text{const}} \exp\left(V_m \sin\omega t / V_T\right)$$

Therefore, the behaviour of I_C is complicated in general case / large signal models. (exponential of a sinusoid)

Small Signal Operation:

The signal perturbes the operating point by only a small amount, ie,

$$v_m << V_{Bias}$$

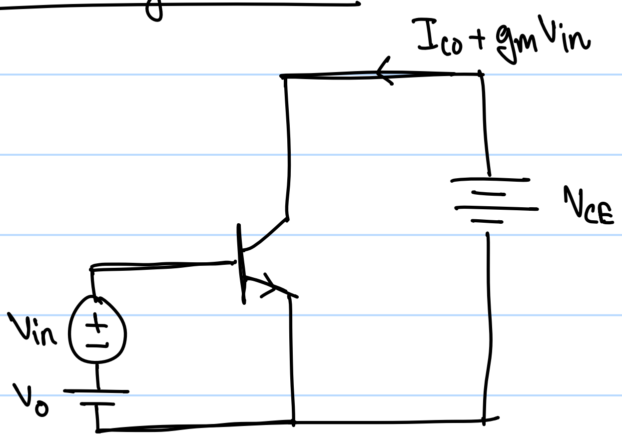
$$I_C = I_S \exp(V_0/V_T)\exp(v_m \sin\omega t/V_T)$$

$$\simeq \underbrace{I_S \exp(V_0/V_T)}_{\text{OP current}}\left(1 + \frac{v_m \sin\omega t}{V_T}\right) \qquad \underline{v_m << V_T}$$

$$= I_C = I_{C0} + \underbrace{\frac{I_{C0}}{V_T}}_{g_m} v_m \sin\omega t$$

- In small signal operations,
  - I_C varies sinusoidally
  - $I_C = I_{C0} + g_m v_m \sin\omega t$ → (Can be observed using Time and IV Analysis.

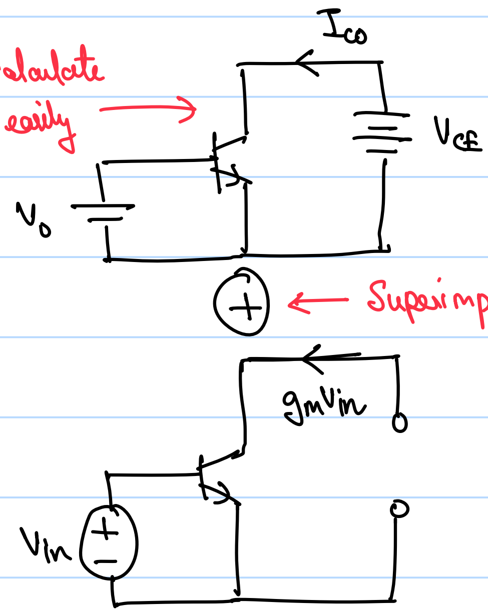
Small Signal Model:



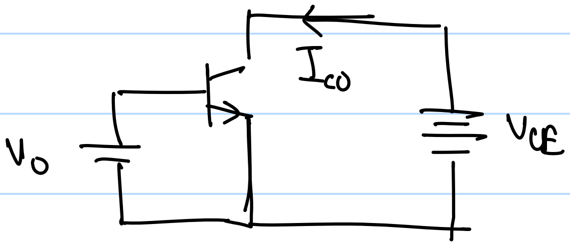
small signal operation

$\equiv$

can calculate I_{C0} easily →



← Superimpose



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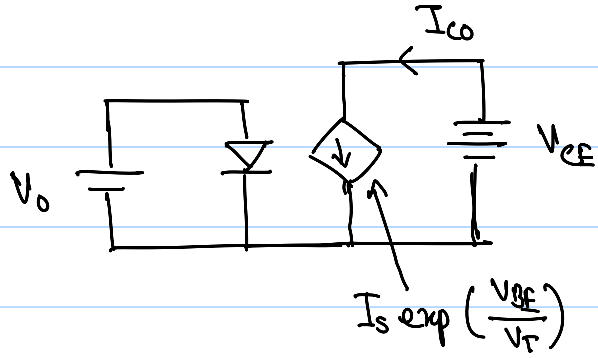

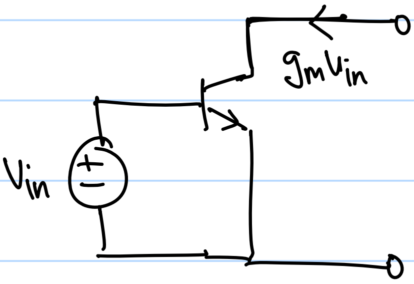

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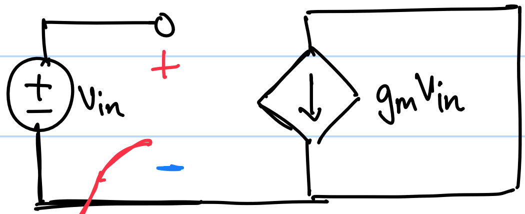


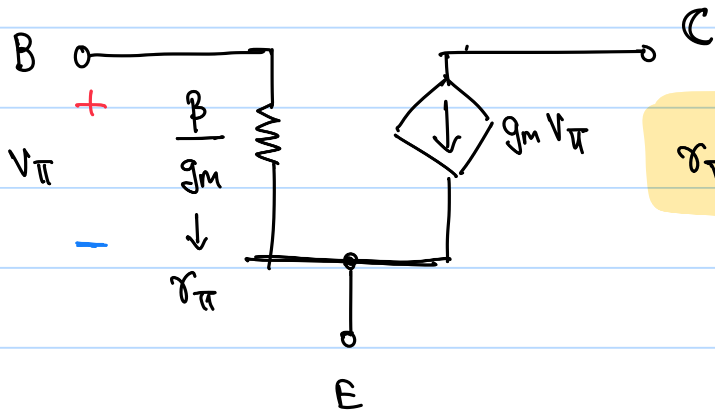
Small Signal Model

The base region is open

To model base current,

$$I_B = I_C/\beta = I_{CO}/\beta + g_m V_{in}/\beta$$

$\Rightarrow$

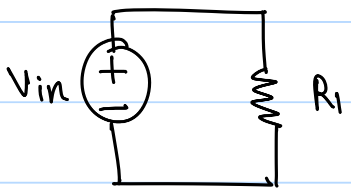


$$\boxed{r_\pi = \frac{\beta}{g_m}}$$

Small Signal Model

Example:

Find the small signal model of a linear resistor



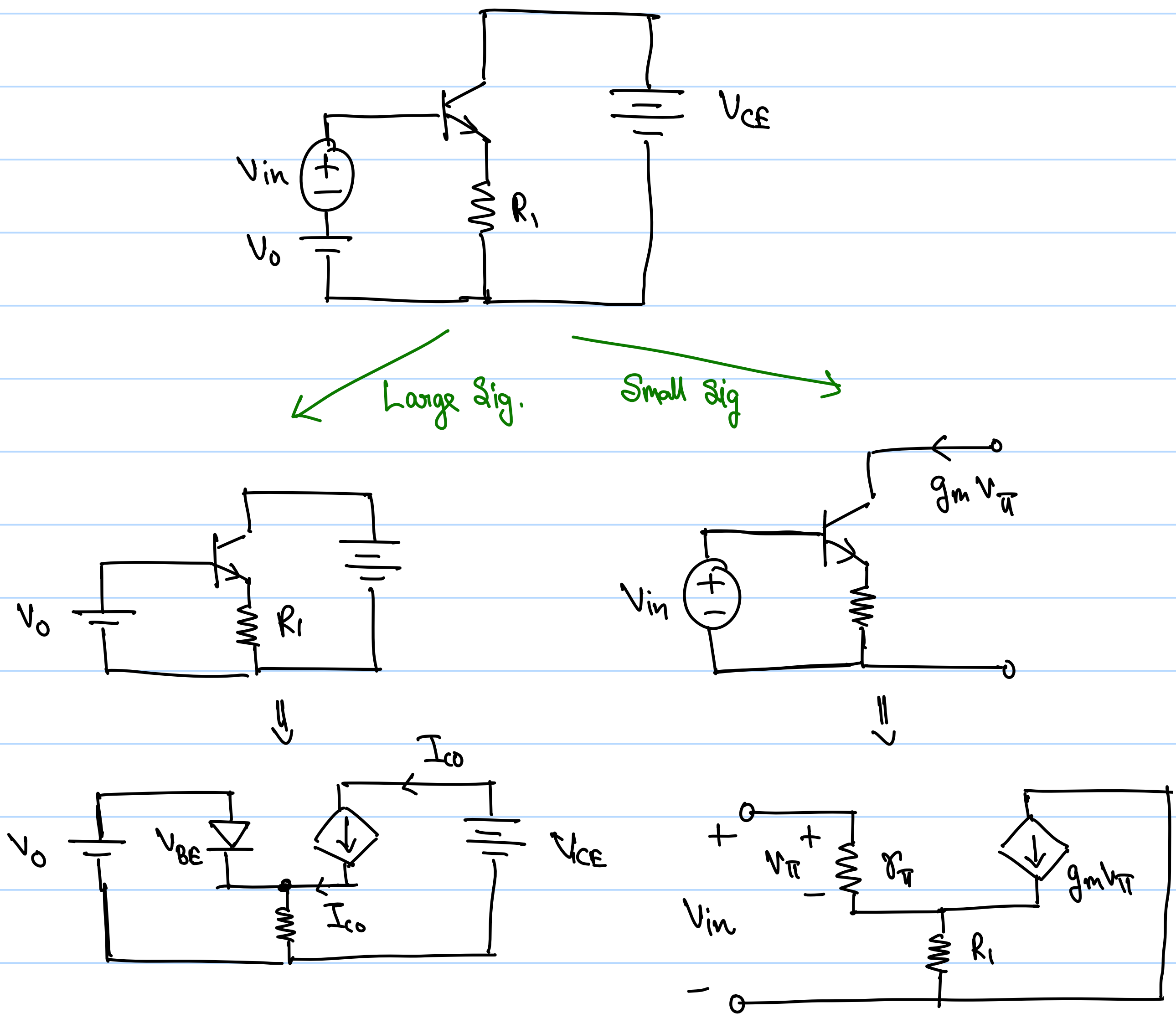
$$I = \frac{V_{in}}{R_1} \Rightarrow \Delta I = \frac{\Delta V_{in}}{R_1} \rightarrow$$

ie, it's the same model

since there is no diff in the relation

- In our small signal model, all the constant voltage sources are set to zero, ie, short circuited.
  |||ly all constant current sources are opened.

Example: Coming back to a prev. scenario,



By KVL,

$$V_0 = V_{BE} + R_1 I_{C0}$$

$$V_0 = V_T \ln \frac{I_{C0}}{I_S} + R_1 I_{C0}$$

By KVL,

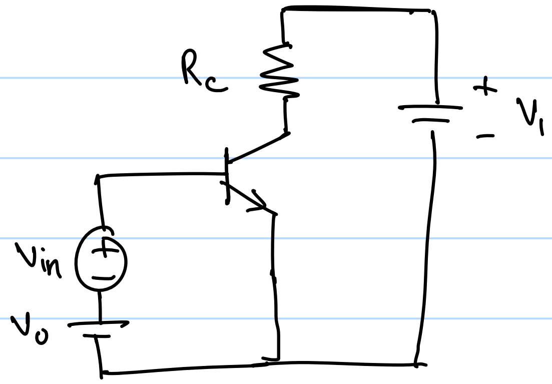
$V_\pi = \Delta V_{BE}$

$g_m V_\pi = \Delta I_C$

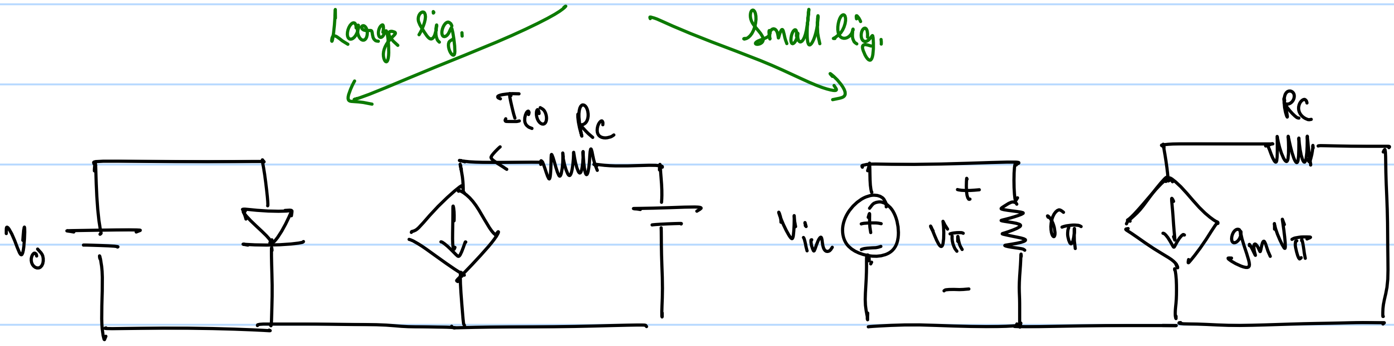
$$V_{in} = V_\pi + R_1 g_m V_\pi$$

$$= V_\pi (1 + g_m R_1)$$

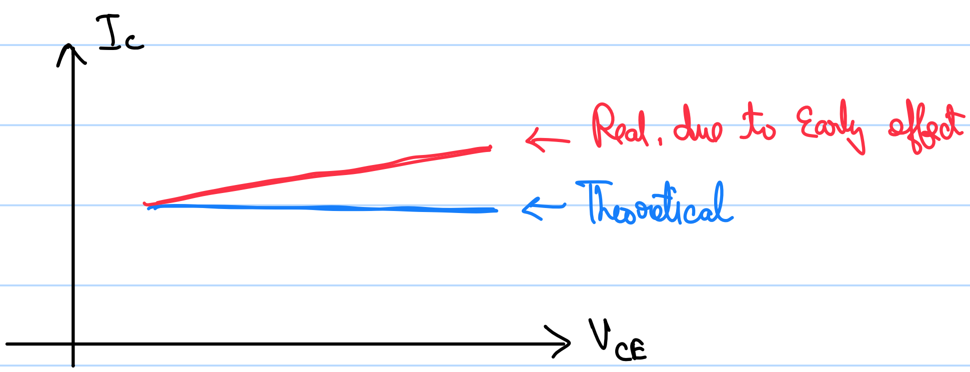
Example 2:



Model this circuit.

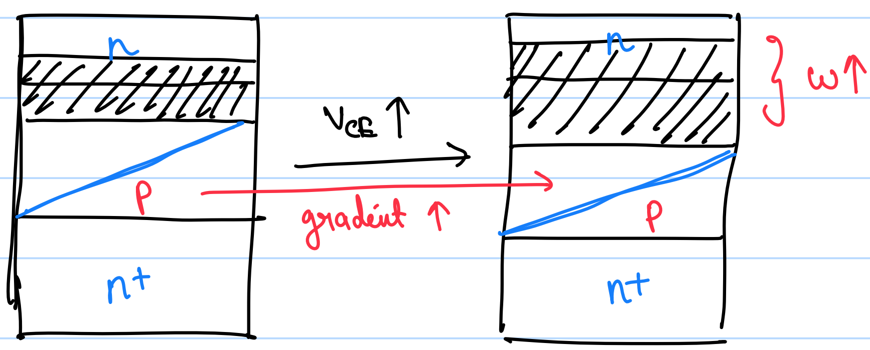


→ Early Effect :-



- We have seen that I_c does not depend on V_{CE}, but in reality there is some linear relation b/w them.

- This is due to the width of the CBJ depletion region in reverse bias.



Also, $I_S = \dfrac{A q n_i^2 D_n}{N_A W_B}$ → Decreases w/ V_{CE} ↑

To take into account that linear relation,

$$\boxed{I_c = I_S \left(e^{V_{BE}/V_T} + 1\right)\left(1 + \frac{V_{CE}}{V_A}\right)}$$

Where V_A is termed as Early Voltage, a transistor dependent quantity.

◦ Early Effect in Small Signal Operation :-

Note: In all models till now, it was assumed that the transistor is always in fwd. active mode.

- To create a small signal model, apply a voltage change and calculate the current changes across the circuit. Then model those changes w/ appropriate devices.

- $$g_m = \frac{dI_c}{dV_{BE}} = \frac{I_S}{V_T} \exp\left(\frac{V_{BE}}{V_T}\right)\left(1 + \frac{V_{CE}}{V_A}\right)$$

$$= \boxed{g_m = \frac{I_c}{V_T}}$$

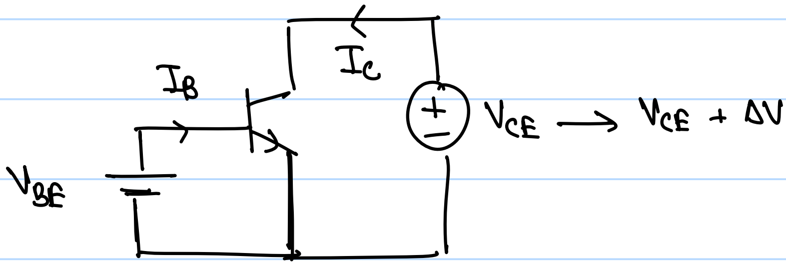
→ g_m is still affected by Early effect. That effect is in I_c.

r_π : $I_B = \dfrac{I_c}{\beta} \Rightarrow \Delta I_B = \dfrac{\Delta I_c}{\beta}$

$$\Rightarrow \boxed{r_\pi = \frac{\beta}{g_m}}$$

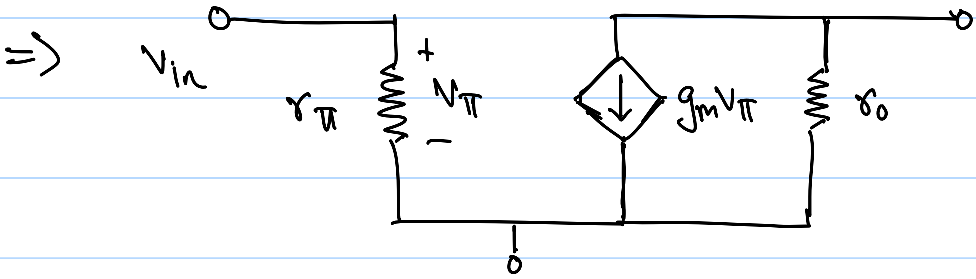
Still our previous small signal model is wrong as V_{CE} is not taken into account anywhere.

- Including Early Effect in the circuit,

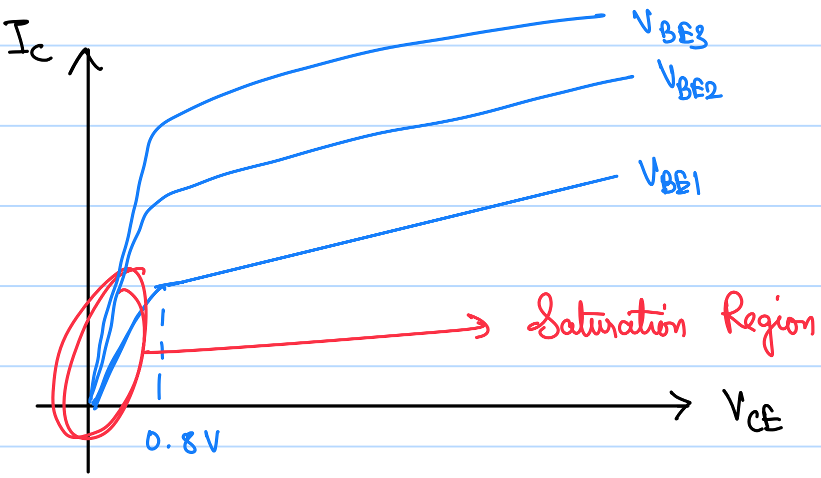


$$\Delta I_C \text{ (due to EE)} = I_S e^{V_{BE}/V_T} \left(\frac{\Delta V}{V_A}\right) \longrightarrow \text{linear dependence on } \Delta V$$

$\Rightarrow$



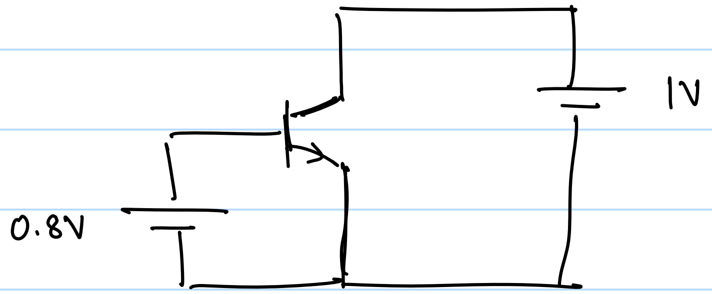
$$\boxed{r_o = \frac{V_A}{I_S e^{V_{BE}/V_T}} \approx \frac{V_A}{I_C}}$$



When V_{CE} drops below 0.7V, the transistor exits fwd active region and falls into Saturation Mode.

- Soft Saturation : $V_{CE} \in (0.5V, 0.8V)$
  Deep Saturation : $V_{CE} < 0.5V$

Example:



$I_S = 5\times10^{-16}A$, $\beta = 100$, $V_A = 5V$, $I_C = ?$, $g_m = ?$, $r_\pi = ?$, $r_o = ?$

$$I_C = I_S\left(\exp(V_{BE}/V_T)\right)\left(1 + \frac{V_{CE}}{V_T}\right) = 13.8\ mA$$

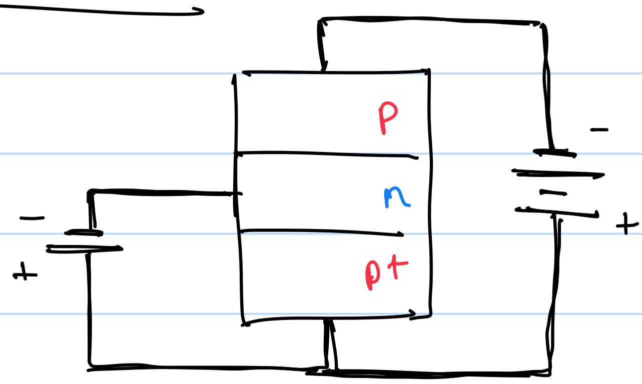
$$g_m = \frac{I_C}{V_T} = \frac{13.8}{26} = \underline{0.53\ S}$$

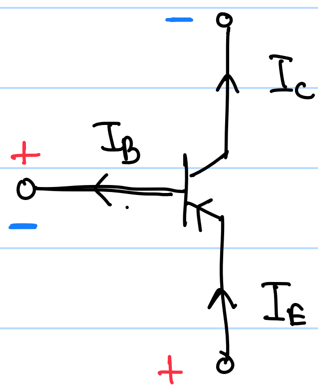
$$r_\pi = \frac{\beta}{g_m} = \frac{100}{0.53} = \underline{1900\ \Omega}$$

$$r_o = \frac{V_A}{I_C} = \frac{5}{13.8\times10^{-3}} = \underline{362\ \Omega}$$

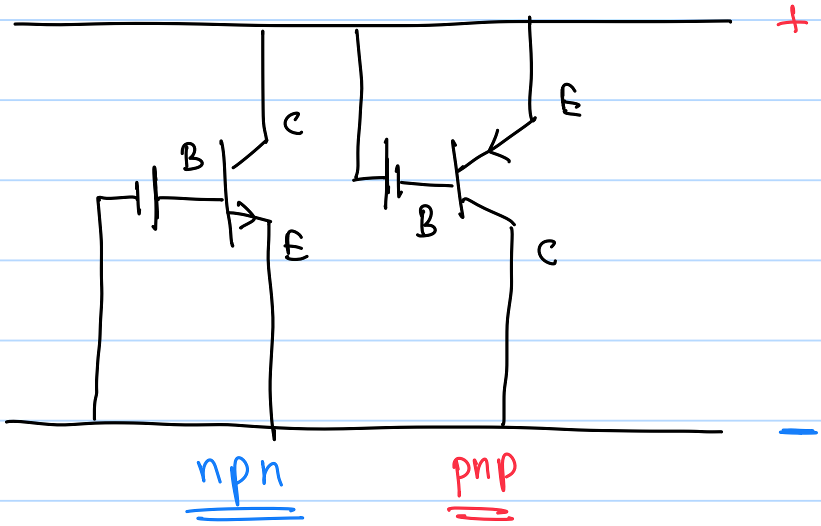
In the above example, note that $\underline{g_m r_o >> 1}$ – Keep in mind for later.

→ PNP Transistor:-

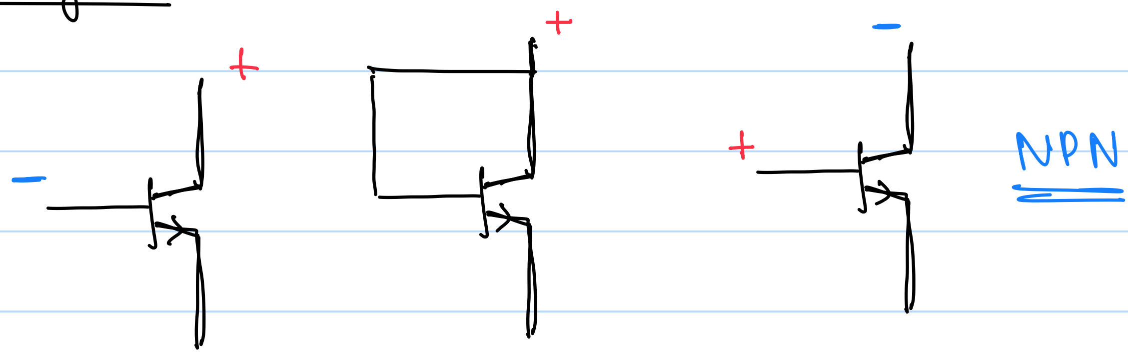




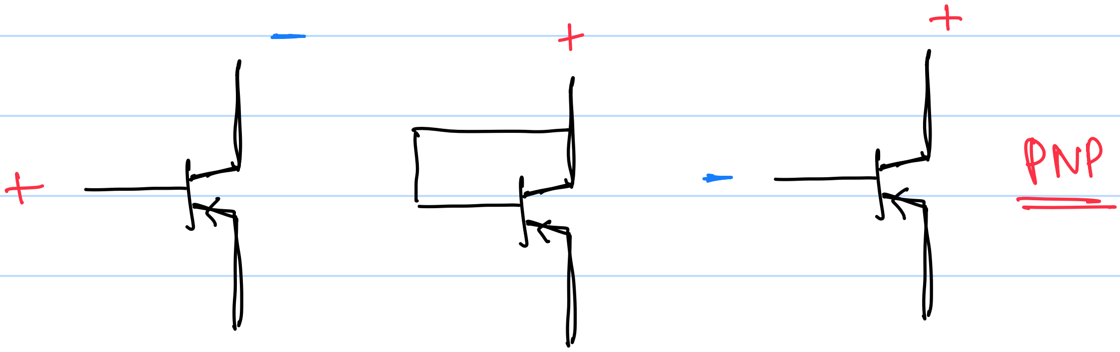
- Fwd Active Region : $V_{EB} > 0$, $V_{EC} < V_{EB}$



- Visualization :-

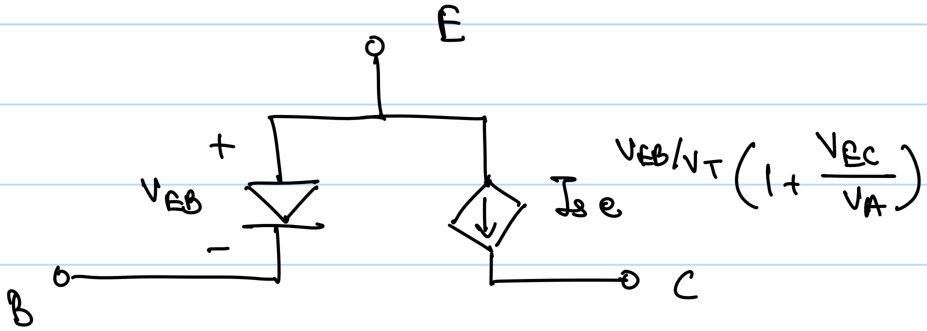


Active Edge of Saturation Saturation



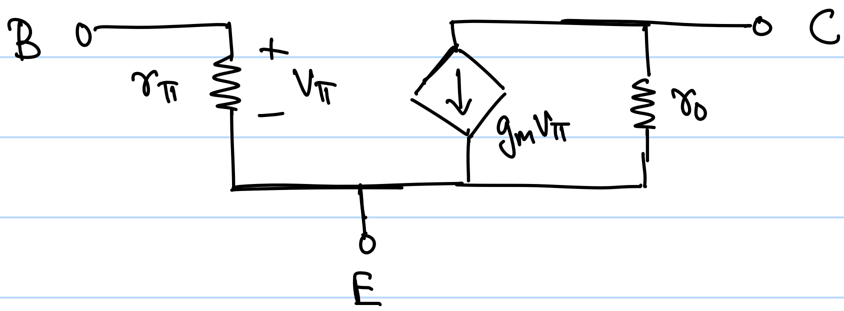
Active Edge of Saturation Saturation

- **Large Signal Model :-**



- **Small Signal Model :-**

- The effects of small signal approximation is identical to npn.

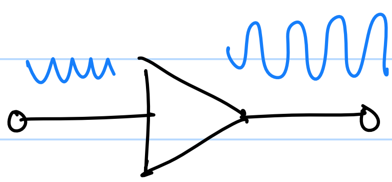


→ **Amplifier Design :**

1) Topology 2) Biasing 3) Small Signal Properties

- **Amplifier Characteristics :-**

- **Voltage Gain :**

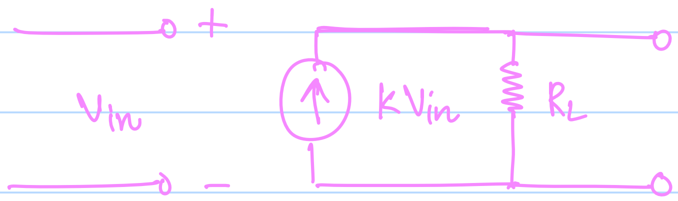


$$A_v = \frac{\text{o/p Amplitude}}{\text{i/p Amplitude}}$$

- Power Consumption :-

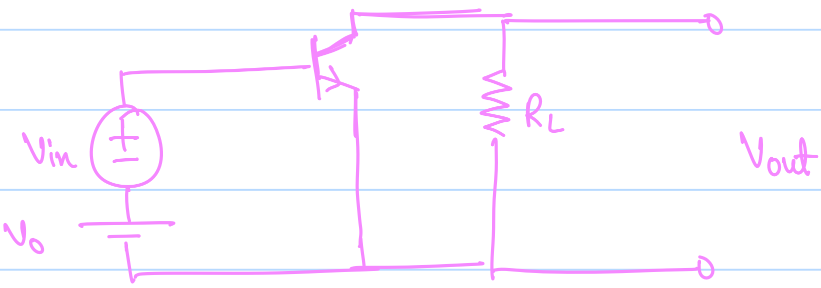
Biasing circuitry need external power.

Note: We designed an amplifier in the beginning,



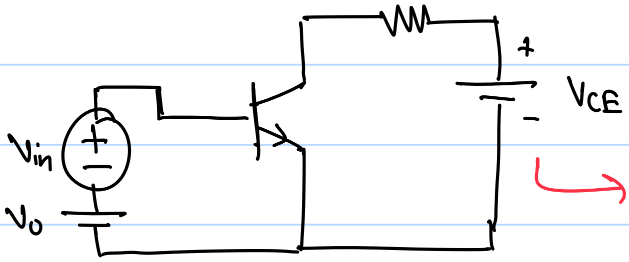
$$A_v = \frac{V_o}{V_i} = \underline{\underline{KR_L}}$$

We also noted that a BJT can be used to create the voltage dependent current source, as below



But, all this time, we assumed that the BJT is in fixed bias mode, which is not guaranteed.

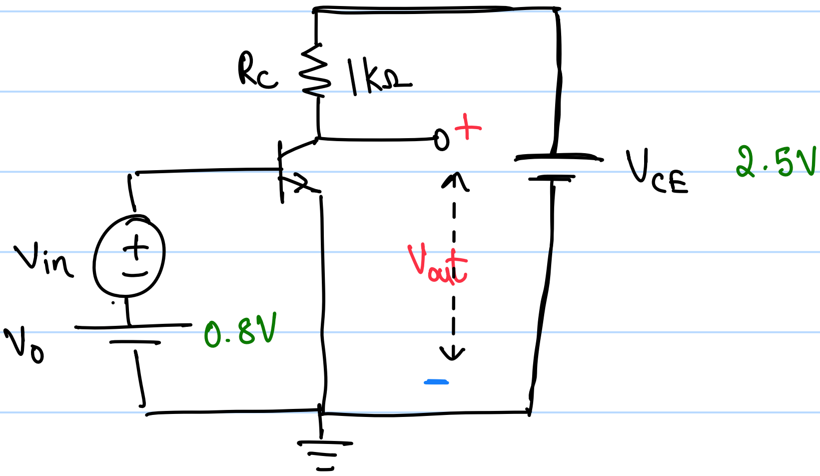
⇒



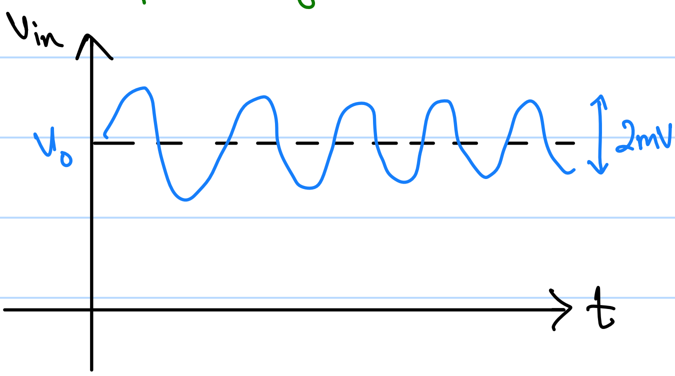
↳ To maintain fixed bias.

This circuit is termed as having a common emitter topology.

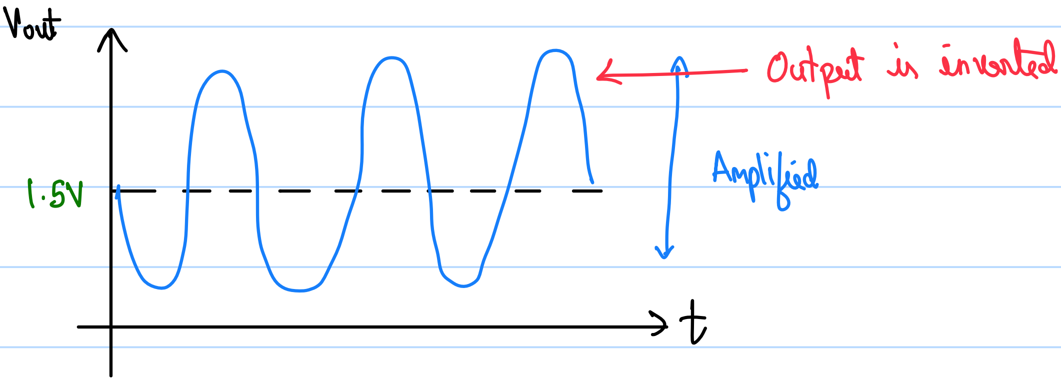
→ Common Emitter Topology :-



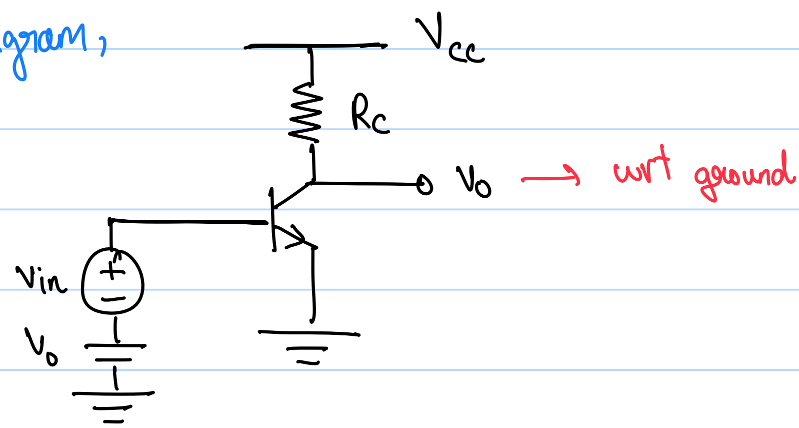
Let V_{in} have an amplitude of 2mV, $I_{CO} = 1mA$



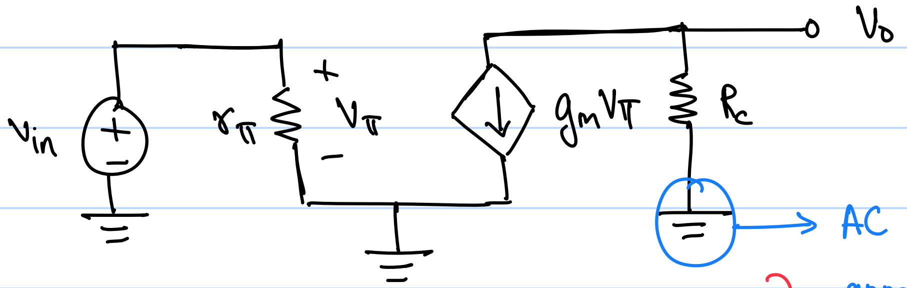
At operating point, $V_0 = 2.5 - (1mA \times 1k\Omega) = 1.5V$



Simplified Diagram,



Small signal Voltage gain calculation,



AC ground, since it appeared due to small signal analysis.

Here $V_{in} = V_\pi$,

$$V_o = -R_C \cdot I_C$$

$$\Rightarrow V_o = -R_C g_m V_{in}$$

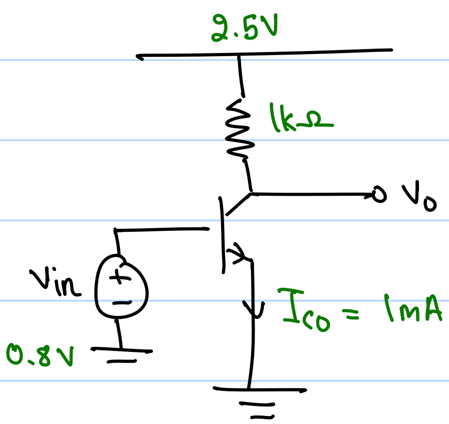
$$\Rightarrow A_V = -\underline{\underline{g_m R_C}}$$

Neglecting Early effect.

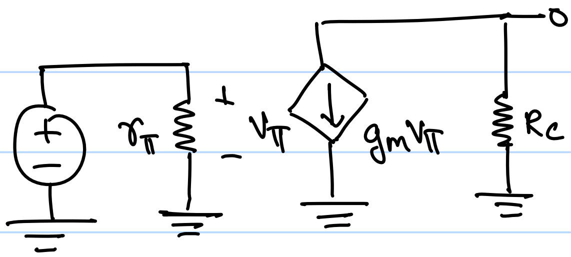
$$\Rightarrow \boxed{A_V = -g_m X}$$

X - Total resistance b/w collector and AC ground

Example: Find gain if $V_{CC} = 2.5V$, $\Delta V_{in} = 2mV$, $I_{CO} = 1mA$, $R_C = 1k\Omega$, $V_o = 0.8V$



$\Rightarrow$



$$g_m = \frac{I_{CO}}{V_T} = \frac{1}{26} = 0.038 \ S$$

$$V_{out} = -R_C g_m V_\pi = -R_C g_m V_{in}$$

$$\Delta V_{out} = -R_C g_m \Delta V_{in}$$

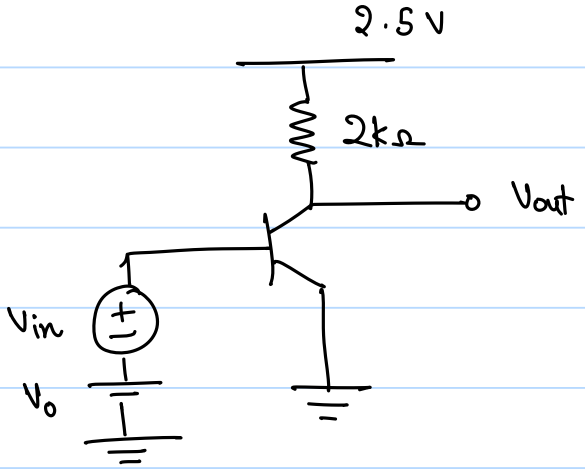
$$\Rightarrow \quad \text{gain} = -R_C g_m$$

$$= -(1 \times 10^3)(0.038)$$

$$= \underline{\underline{-38}}$$

<u>Example 2:</u> Suppose we want to double the gain, does doubling R_C work?

If new $R_C = 2k\Omega$,



At operating point,

$$V_{CE} = 2.5 - R_C I_{C0}$$

$$= 2.5 - (2 \times 10^3)(1 \times 10^{-3})$$

$$= 0.5\text{ V} < 0.8\text{ V}$$

→ V_{CE} is less than V_{BE}

→ Saturation region

- Therefore in this design there is an upper limit on the gain we can get just by increasing R_C.

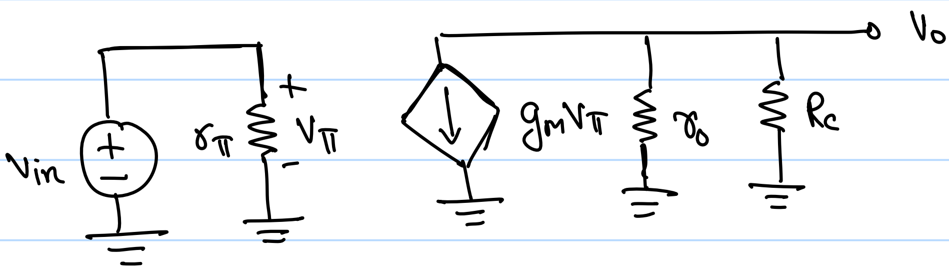
Example: What if we double g_m by doubling I_C?

$I_C R_C$ doubles $\Rightarrow$ Reduces V_{CE} $\rightarrow$ Same reason as before.

- Therefore, there is a strict upper limit on gain.

- Inclusion Of Early Effect :-

- Early Effect changes our small signal model.



The total resistance seen from the output is $r_o \parallel R_C$.

$$\Rightarrow \boxed{\text{gain} = g_m \left(\frac{R_C r_o}{R_C + r_o} \right)} \longrightarrow \text{Reduction in Gain}$$

- We know, $g_m = \dfrac{I_C}{V_T}$, $r_o = \dfrac{V_A}{I_C}$

$$\Rightarrow g_m r_o = \frac{\cancel{I_C}}{V_T} \times \frac{V_A}{\cancel{I_C}} = \frac{V_A}{V_T}$$

$$\Rightarrow \boxed{g_m r_o = \frac{V_A}{V_T} \rightarrow \text{Intrinsic Gain of the transistor}}$$

- Intrinsic gain is the max possible gain of the transistor. ($R_C \rightarrow \infty$).

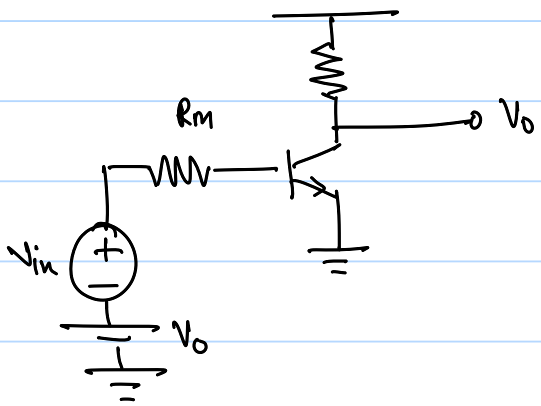
- In a CE topology,

  1) i/p applied to base 2) o/p taken from collector

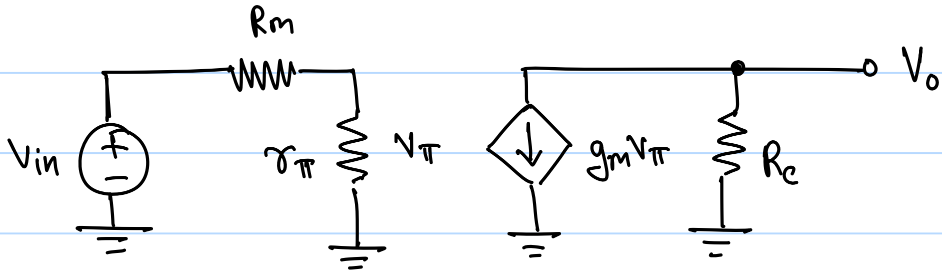
- <u>Impedances</u> :-

  - In the amplifier, there may be some impedances at the input and output.

<u>Example</u>: The input source has a series resistance of R_M.



SSM,



$$V_\pi = \frac{V_{in}\, r_\pi}{R_M + r_\pi}$$ $\left(\frac{r_\pi}{R_M + r_\pi} - \text{Attenuation factor} \right)$

$$V_o = -\, g_m V_\pi R_C$$

$$\Rightarrow \frac{V_o}{V_{in}} = -\frac{g_m R_C r_\pi}{R_M + r_\pi}$$ $\longrightarrow$ Reduced gain

- The resistance R_M causes an impedance at the i/p.

- Input Impedance : To calculate i/p impedance,

1. Set all independent sources to zero

2. Apply a small signal voltage source b/w the input terminals. Calculate the current supplied by the source.

$$R_M = \frac{V_{in}}{I_x}$$

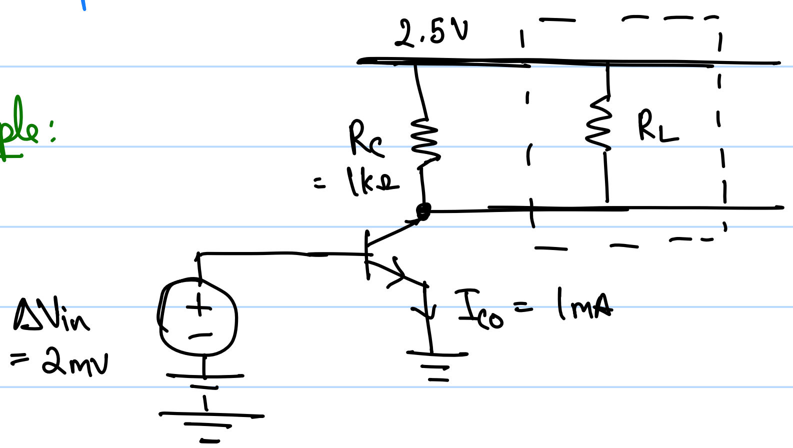
$$I_x r_\pi = V_{in}$$

$$\Rightarrow R_M = r_\pi \longrightarrow \text{Does not depend on } R_C, r_o$$

- Output Impedance :-

Suppose at the output, we have a low impedance device attached.

Example:



$R_L = 8\Omega$

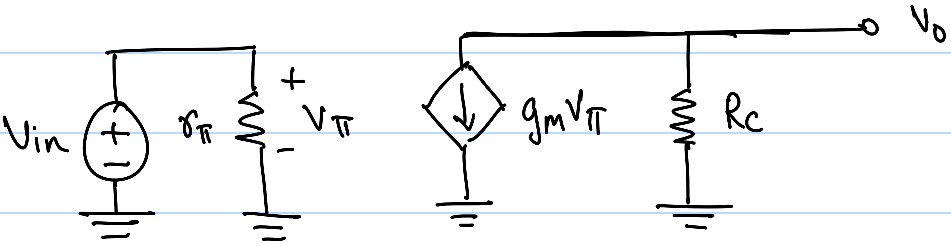
$\beta = 100$

$$A_V = -g_m (R_C \parallel R_L)$$

$$= -(0.038)\left(\frac{1000 \times 8}{1000 + 8}\right) \simeq -0.3 \rightarrow \text{Attenuation}$$

The output impedance is measured by

SSM:

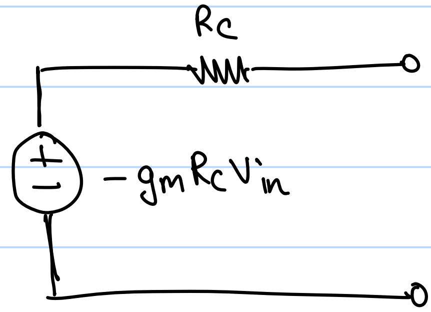


We find the Thevenin Equivalent of this circuit,

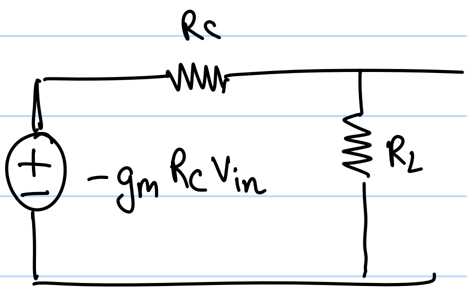
$$V_{out} = -R_C g_m V_\pi$$

$$R_{eq} = R_C$$

$\Rightarrow$



If load resistance is R_L, voltage delivered to the load is given by,



$$V_L = \frac{R_L}{R_L + R_C}(-g_m R_C V_{in})$$

$$\Rightarrow \boxed{\text{gain} = -g_m R_C \underbrace{\frac{R_L}{R_L + R_C}}_{\text{Attenuation factor}}}$$

- Calculation of o/p impedance (Rout) is ||| to i/p impedance, with $V_{in} = 0$.

- $R_{out} = R_C$ (or) $R_C \parallel r_o$ → Early Effect

- i/p and o/p impedances are observed when connecting circuitry to the i/p or o/p of the Amplifier.

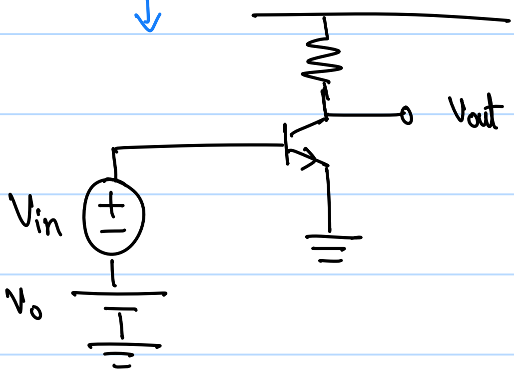
○ **Gain Variation** :-

The gain of an Amplifier is given by

$$A_V = g_m R_C = \frac{-I_C R_C}{V_T}$$

– This gain can vary due to
1) Variation of temperature. (V_T)
2) Variation of process (Manufacturing, R_C)
3) Variation due to signal amplitude

↓



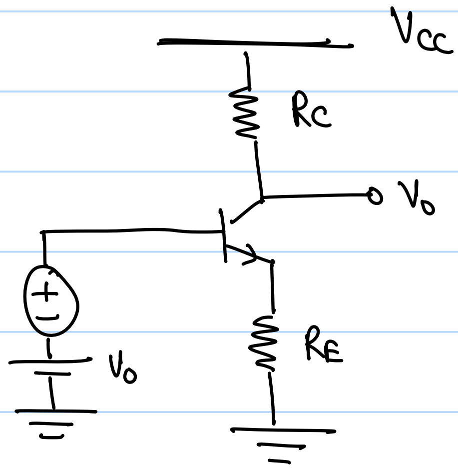
If V_{in} is a large signal, there will be significant variation in I_C, which affects the gain.

⇒ gain ∝ instantaneous V_{in}

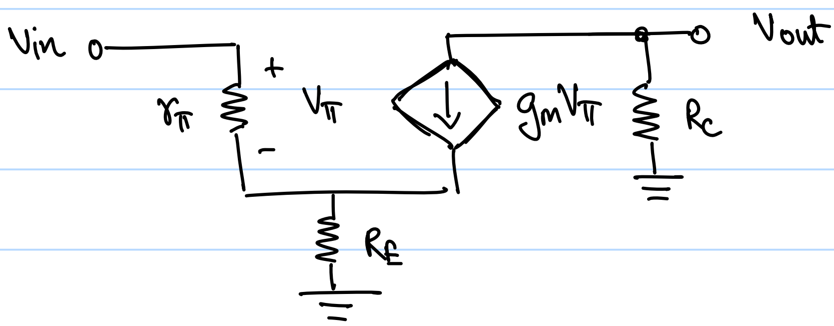
↳ This causes non-linear behaviour/ distortion in o/p.

- To fix this, we have emitter degeneration

→ CE stage w/ Emitter Degeneration :-



- Gain of this design can be calculated as follows,



By KCL,

$$g_m V_\pi + \frac{V_{out}}{R_C} = 0$$

$$V_\pi = -\frac{V_{out}}{g_m R_C}$$

KCL at Emitter node,

$$\frac{V_\pi}{r_\pi} + g_m V_\pi = I_E$$

$\Rightarrow$ Voltage across R_E $= \left(\frac{V_\pi}{r_\pi} + g_m V_\pi\right) R_E$

$$= \left(\frac{-V_{out}}{g_m R_C r_\pi} - \frac{\cancel{g_m} V_{out}}{\cancel{g_m} R_C}\right) R_E$$

By KVL

$$V_{in} = V_\pi + \left(\frac{-V_{out}}{g_m R_C r_\pi} - \frac{V_{out}}{R_C} \right) R_E$$

$$V_{in} = \frac{-V_{out}}{g_m R_C} - \frac{V_{out} R_E}{g_m R_C r_\pi} - \frac{V_{out} R_E}{R_C}$$

$$\Rightarrow \frac{1}{A_v} = -\left(\frac{1}{g_m R_C} + \frac{R_E}{g_m R_C r_\pi} + \frac{R_E}{R_C} \right)$$

$$= -\left(\frac{1}{g_m R_C} + \frac{R_E}{\beta R_C} + \frac{R_E}{R_C} \right)$$

$$\Rightarrow A_v = \frac{\beta g_m R_C}{\beta + (\beta+1) g_m R_E}$$

$$= \frac{R_C}{\frac{1}{g_m} + \left(\frac{\beta+1}{\beta}\right) R_E}$$

$$\Rightarrow \boxed{A_v = \frac{R_C}{\frac{1}{g_m} + R_E}} \quad \beta >> 1$$

- If R_E is large enough, variation in g_m will not affect overall gain. (R_E dominates over g_m).