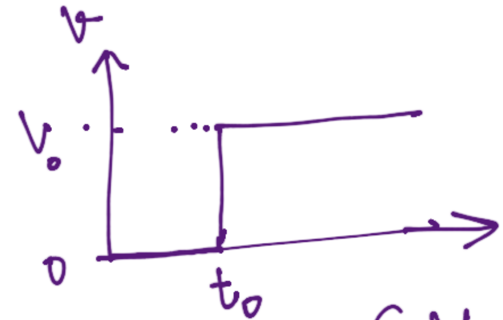
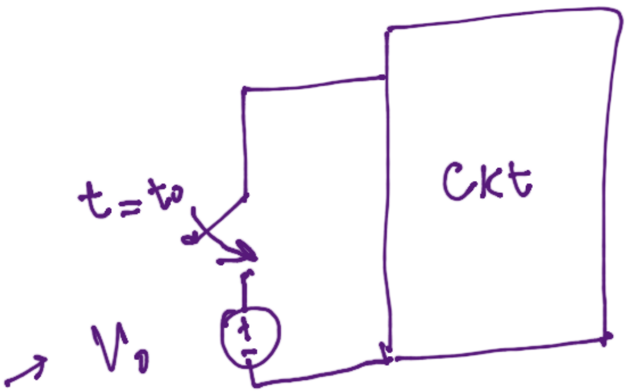
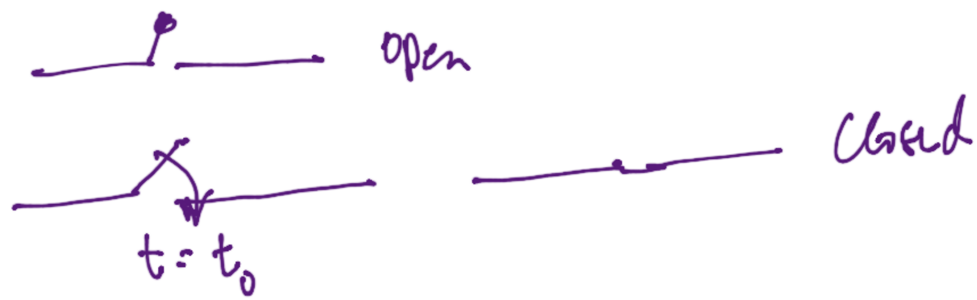
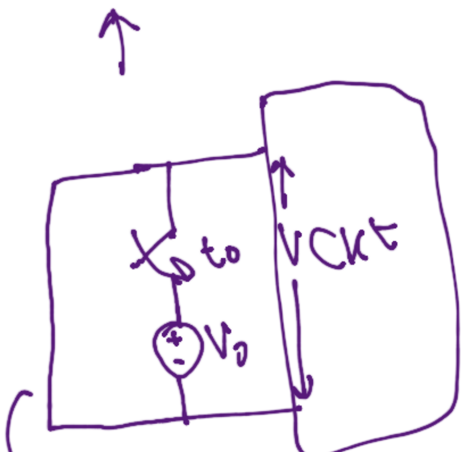
Unit Step Function



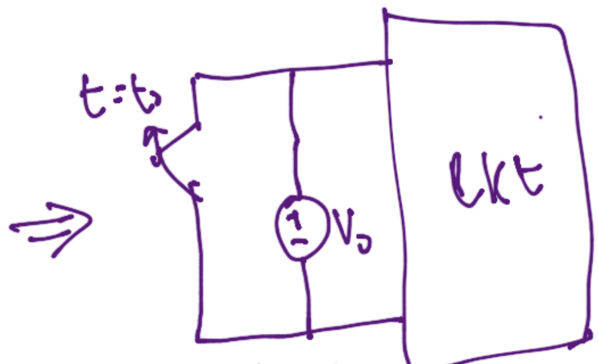
$$u(t-t_0) = \begin{cases} V_0 & t > t_0 \\ 0 & t < t_0 \end{cases}$$







$t < t_0$ $(V=0)$



$t > t_0$

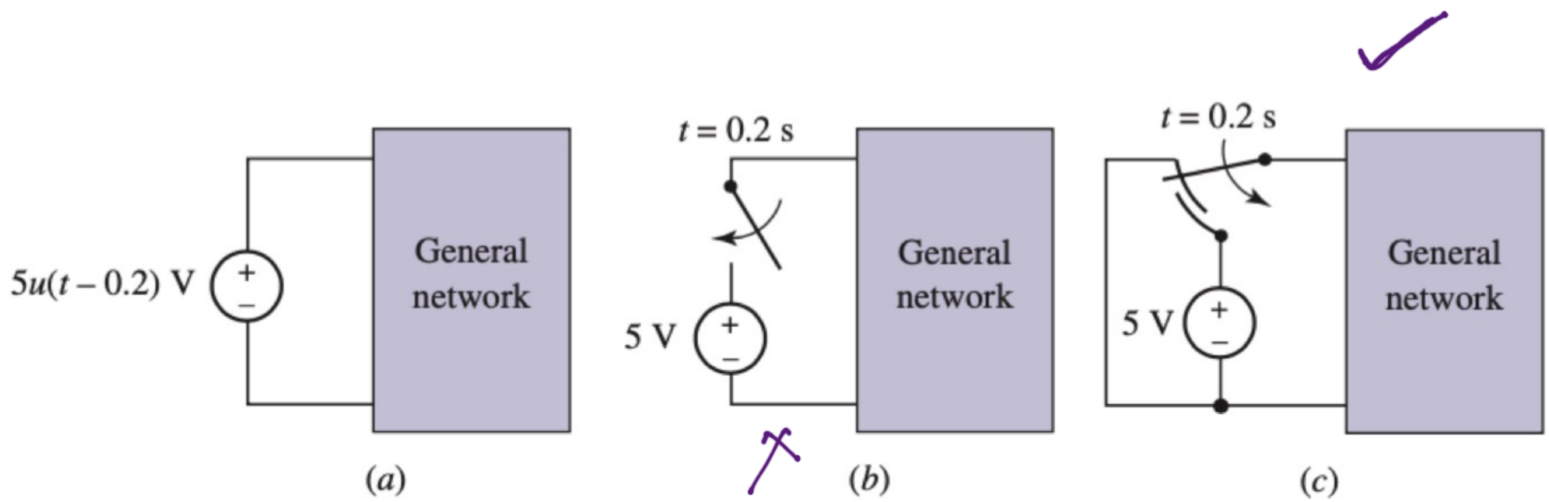
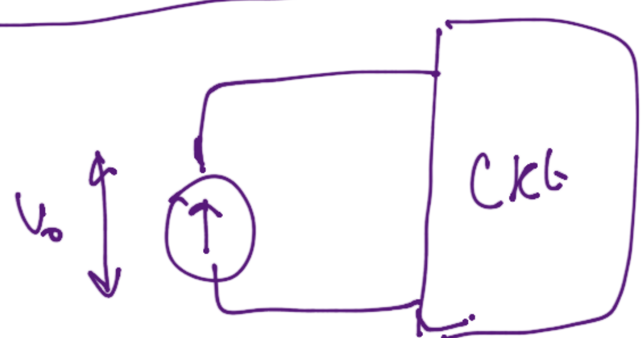


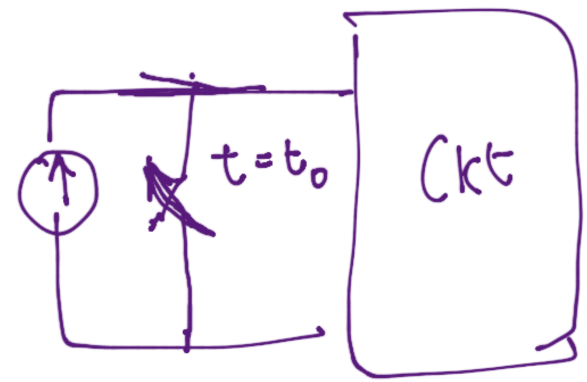
FIGURE 8.28 (*a*) A voltage-step forcing function is shown as the source driving a general network. (*b*) A simple circuit which, although not the exact equivalent of part (*a*), may be used as its equivalent in many cases. (*c*) An exact equivalent of part (*a*).

Current Source

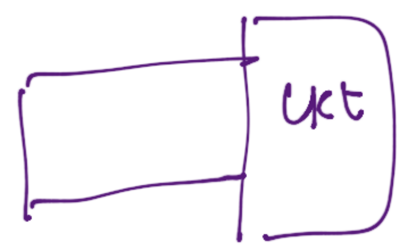


$t > t_0$

Approx.



$t < t_0$

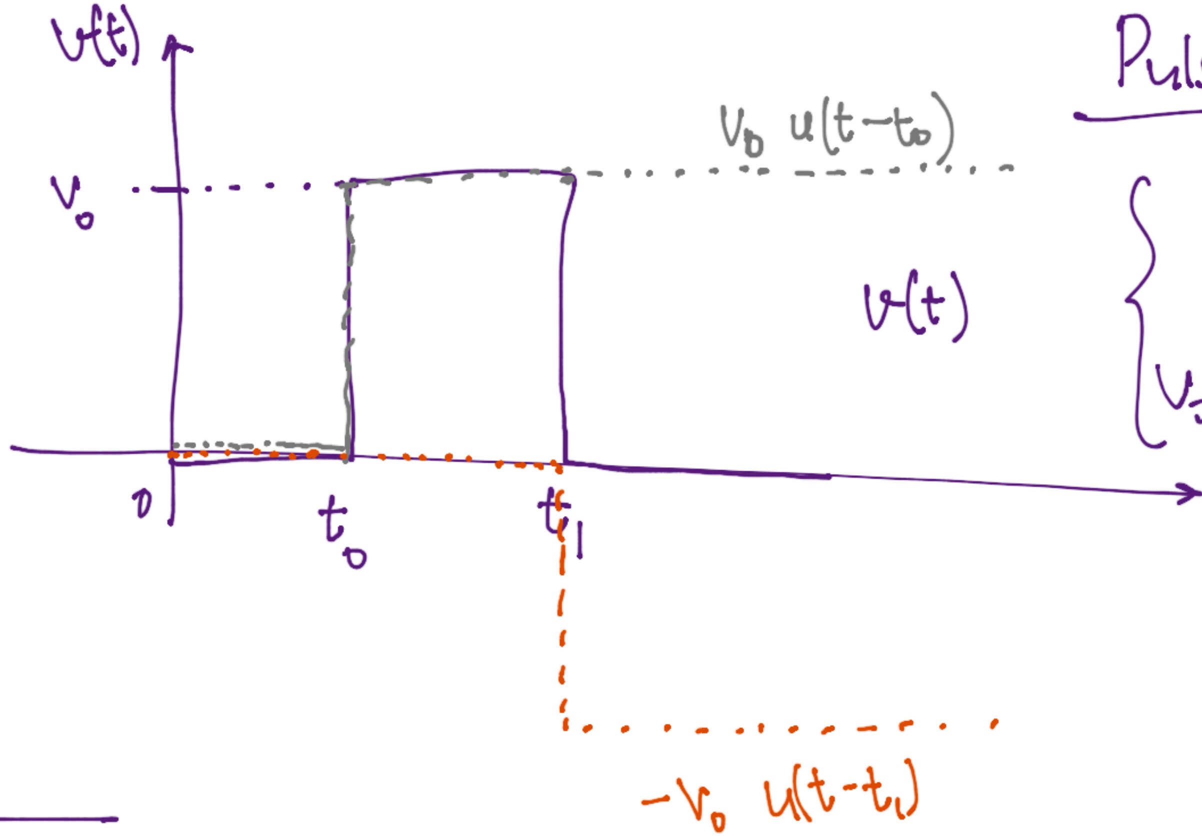


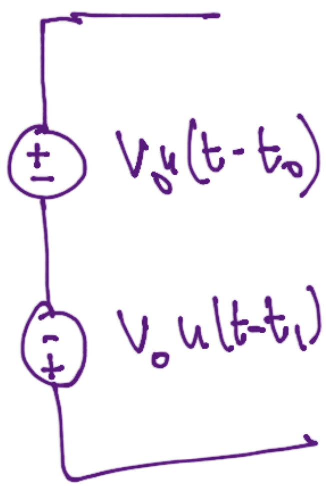
$I = 0$

$V = ?$

Pulse

$$v(t) = \begin{cases} 0 & t < t_0 \\ 0 & t > t_1 \\ V_0 & t_0 < t < t_1 \end{cases}$$





General Solution : Natural Response + Forced Response.

First order

$$\frac{di}{dt} + Pi = Q$$

$\uparrow$ Forcing function.

$$\Rightarrow \quad di + Pi\,dt = Q\,dt \qquad \text{——— } (1)$$

Integrating factor $e^{\int P dt} = e^{P\int dt} = e^{Pt}$ $\quad \left(\begin{array}{l} P = \text{const.} \\ L, C, R \end{array} \right)$

(1) * Int. factor

$$\underbrace{e^{Pt} di + Pi e^{Pt} dt}\ = Q e^{Pt} dt$$

$$\Rightarrow \int d(ie^{Pt}) = \int Q e^{Pt} dt \Rightarrow$$

After integration

$$\Rightarrow \quad i e^{pt} = \int \phi e^{pt} dt + A$$

Solution

$$i = \underbrace{e^{-pt} \int \phi e^{pt} dt}_{\mathrm{I}} + \underbrace{A e^{-pt}}_{\mathrm{II}}$$

II $\quad A e^{-pt} = \text{Natural response} = i_n$

'p' $\quad RC \,,\, \frac{L}{R}$

$i_n \longrightarrow 0$ at $t \longrightarrow \infty$

I $\quad i_f = e^{-pt} \int \phi e^{pt} dt$

Forced response.

Steady state resp.

Particular solution.

If $\phi =$ const. $\quad i_f = \cancel{e^{-pt}} \phi p^{-1} \cancel{e^{pt}} = \frac{\phi}{p}$

So, if $V_s \to$ const (Sources)
(or I_s)

$$\boxed{i = \frac{Q}{P} + Ae^{-Pt} = i_f + i_n}$$

General V_s, $I_s \to$ not const.

$$\widehat{i = i_f + i_n}$$

$$\boxed{i = e^{-Pt}\int Q e^{Pt} dt + Ae^{-Pt}}$$

L-R

Ex 8.8

$$\boxed{i_n = A e^{-t/\tau}}$$

Determine $i(t)$ for all values of time in the circuit of Fig. 8.37.

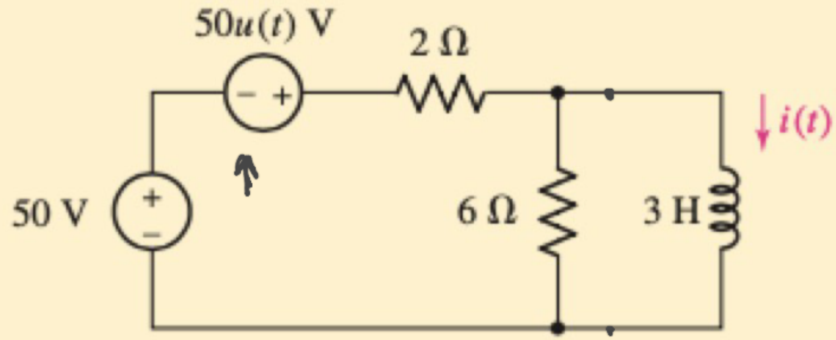
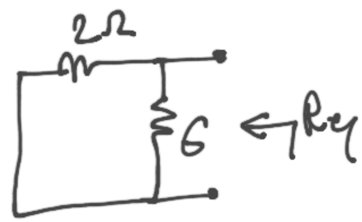


FIGURE 8.37 The circuit of Example 8.8.

R-L type circuit

Step 1   P or τ R_{eq}



$$= \frac{12}{8} = \frac{3}{2} = 1.5$$

$$L = 3H \Rightarrow \tau = \frac{L}{R_{eq}} = \frac{3}{3/2} = \underline{2\ sec}$$

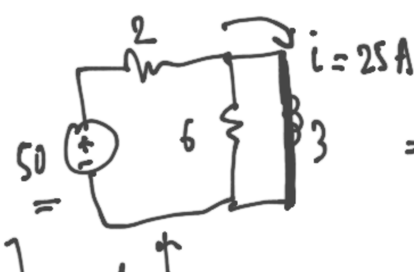
Step 2   Natural respons

$$i_n = A e^{-t/2}$$

$$\cancel{i_n = 25 e^{-t/2}}$$

Apply Initial condition to complete solution Not.

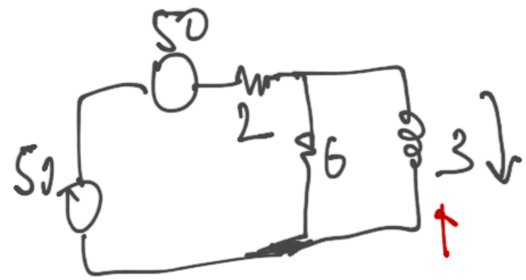
Initial (∞) = [t < 0 sec]



$\Rightarrow i_0$ (initial condition)

Satisfy the $\boxed{\text{total response}}$ not just the natural respon *

Forced response



Forced respon

↓

L → wire.

$V_s = 50+50 = 100$

$R_s = 2\,\Omega$

$\frac{V_s}{R} = \frac{100}{2} = 50\,A$

$i_f = 50\ A$

$i = i_f + i_n$

$i = 50 + A e^{-t/2}$

Check

$t \to \infty \qquad i = 50$ ✓

$t = 0 \qquad 25 = 50 + A e^{0} \Rightarrow A = 25 - 50 = -25$

$$\boxed{i = 50 - 25 e^{-t/2}}$$

Steps for Finding Solution (R-C)

L-R ; R-C

Time const.

$\frac{L}{R_{eq}} = \tau$

1. With all independent sources zeroed out, simplify the circuit to determine R_{eq}, C_{eq}, and the time constant $\tau = R_{eq}C_{eq}$.

$L \to S.C$ $i_L(0)$ -

Initial

2. Viewing C_{eq} as an open circuit, use dc analysis methods to find $v_C(0^-)$, the capacitor voltage just prior to the discontinuity.

Forced response
$C \to O.C$
$L \to S.C$
L, $i_f = V/R$
C $i_f = V/R$

$L \to S.C$
V/R

3. Again viewing C_{eq} as an open circuit, use dc analysis methods to find the forced response. This is the value approached by $f(t)$ as $t \to \infty$; we represent it by $f(\infty)$.
4. Write the total response as the sum of the forced and natural responses: $f(t) = f(\infty) + Ae^{-t/\tau}$. = Total response

$i_L(0^-) = i_L(0^+)$

5. Find $f(0^+)$ by using the condition that $v_C(0^+) = v_C(0^-)$. If desired, C_{eq} may be replaced by a voltage source $v_C(0^+)$ [a short circuit if $v_C(0^+) = 0$] for this calculation. With the exception of capacitor voltages (and inductor currents), other voltages and currents in the circuit may change abruptly.

$A = ?$

Find constants

6. $f(0^+) = f(\infty) + A$ and $f(t) = f(\infty) + [f(0^+) - f(\infty)]e^{-t/\tau}$, or total response = final value + (initial value − final value) $e^{-t/\tau}$.

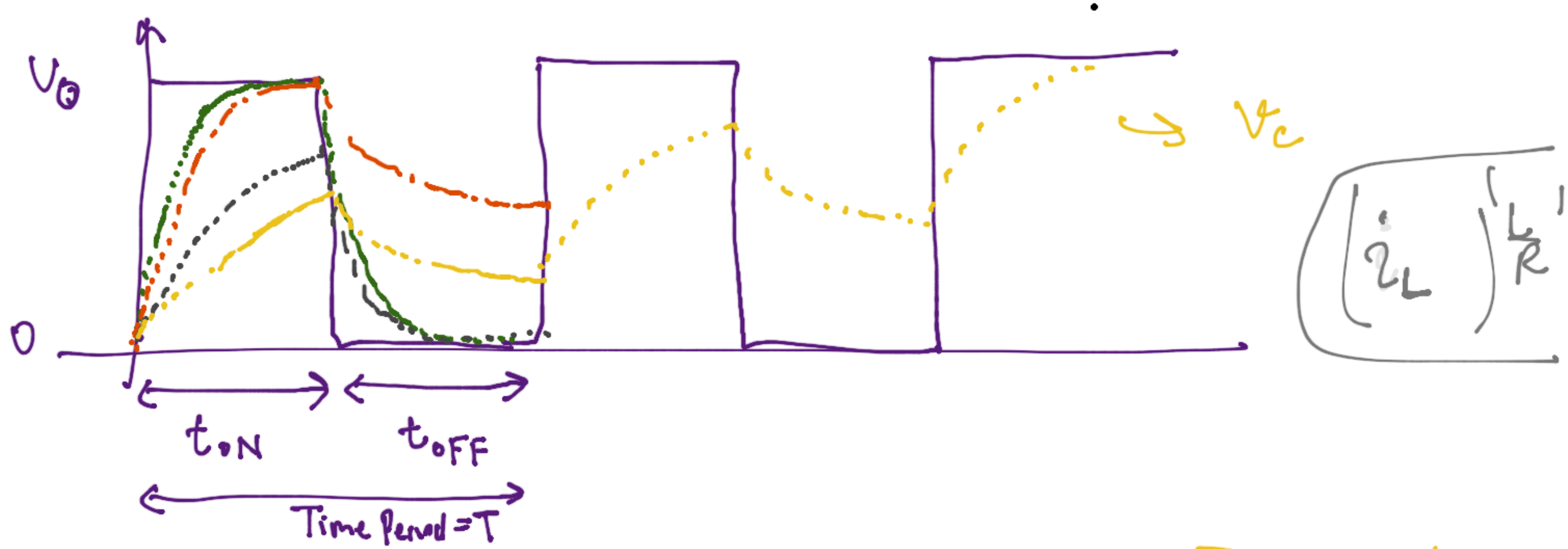
Steps for find Solution (R-L)

1. With all independent sources zeroed out, simplify the circuit to determine R_{eq}, L_{eq}, and the time constant $\tau = L_{eq}/R_{eq}$.
2. Viewing L_{eq} as a short circuit, use dc analysis methods to find $i_L(0^-)$, the inductor current just prior to the discontinuity.
3. Again viewing L_{eq} as a short circuit, use dc analysis methods to find the forced response. This is the value approached by $f(t)$ as $t \to \infty$; we represent it by $f(\infty)$.
4. Write the total response as the sum of the forced and natural responses: $f(t) = f(\infty) + Ae^{-t/\tau}$.
5. Find $f(0^+)$ by using the condition that $i_L(0^+) = i_L(0^-)$. If desired, L_{eq} may be replaced by a current source $i_L(0^+)$ [an open circuit if $i_L(0^+) = 0$] for this calculation. With the exception of inductor currents (and capacitor voltages), other currents and voltages in the circuit may change abruptly.
6. $f(0^+) = f(\infty) + A$ and $f(t) = f(\infty) + [f(0^+) - f(\infty)]\, e^{-t/\tau}$, or total response = final value + (initial value − final value) $e^{-t/\tau}$.

Switching Circuit

HW

R-C



τ = Time const..

t_{OFF} & t_{ON} → varying :

$t_{OFF}, t_{ON} \gg \tau$

$t_{ON} \gg \tau$; $t_{OFF} < \tau$

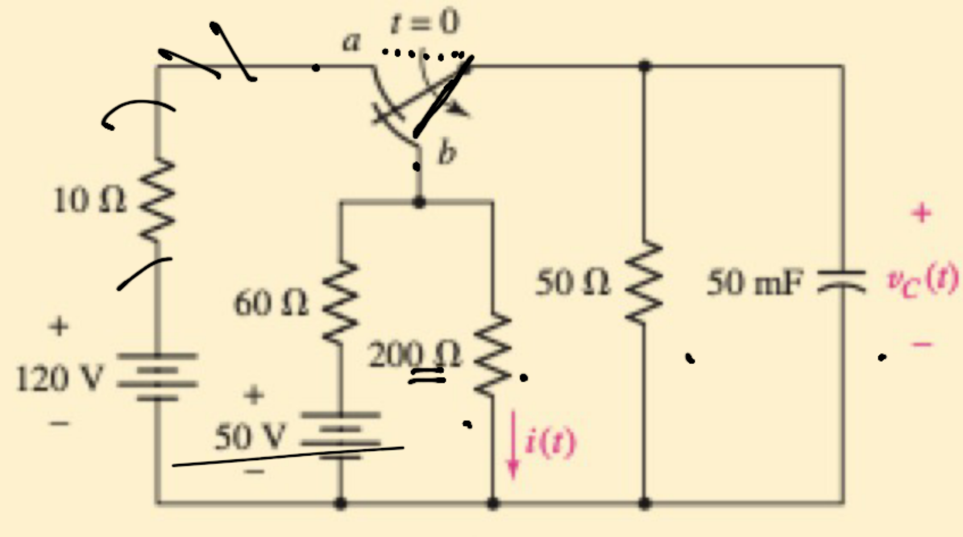
$t_{ON} < \tau$; $t_{OFF} \gg \tau$

$t_{ON} < \tau$, $t_{OFF} < \tau$

Ex 8.10

(R-C type)

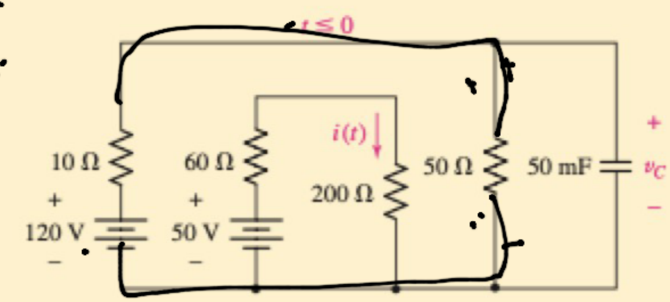
Find the capacitor voltage $v_C(t)$ and the current $i(t)$ in the 200 Ω resistor of Fig. 8.42 for all time.



(a)

t < 0

Initial condition.



(b)

$V_c(0) =$

$t \to \infty$ C: O.C

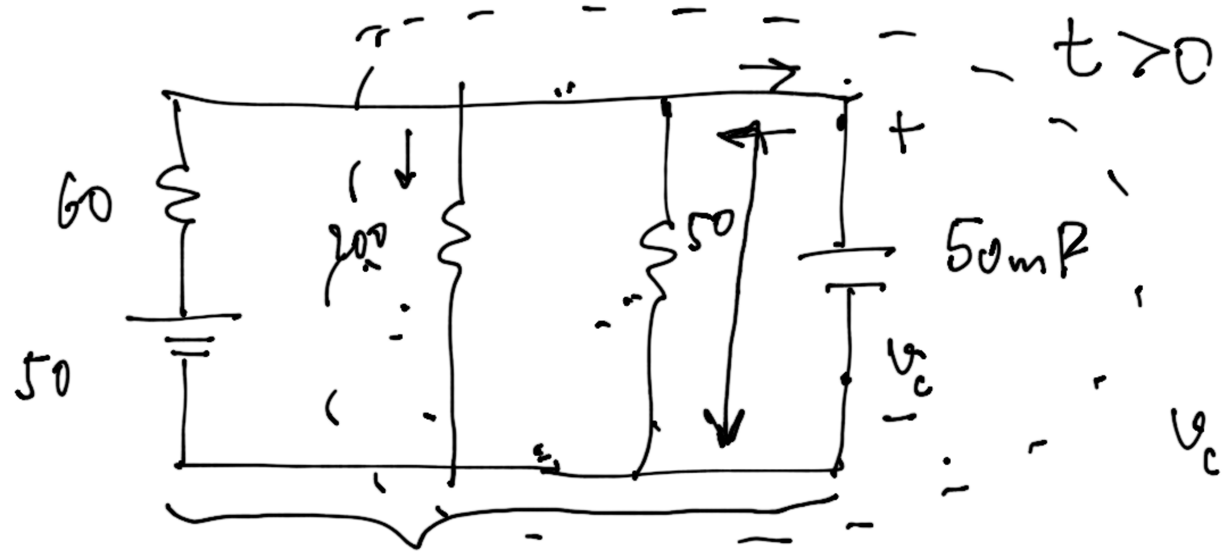
$V_c(0) = \frac{120}{60} * 50 = 100V$

$V_c(0^-) = V_c(0^+)$

$= \underline{100V}$

200Ω : $i_R(0^-) \neq i_R(0^+)$

t > 0 $\quad i(0^+)$



$v_c(t) =$

1. $\tau = (60 \| 200 \| 50) * 50m$

$= 1.2$ sec.

$v_{cn} = A e^{-t/1.2}$

2. Cap $(t \to \infty) \to$ O.C

$V_{cf} = \frac{V*R}{R_{eq}} = \frac{50V * (200 \| 50)}{60 + (200 \| 50)}$

$= 20V$

3. $V_c = V_{cf} + V_{cn} = 20 + A e^{-t/1.2} = V_c$

Find A using initial

$(t=0)$ $v_c = 100 = 20 + Ae^{0}$

$A = 80$

$$\boxed{v_c = 20 + 80e^{-t/1.2}}$$

- - - - - -

i (cal condition) $\quad \underset{\text{(without cap.)}}{\underset{I_{002}}{i}} = \dfrac{50}{60 + 50\|200} * \dfrac{50}{200+50} = 0.1A$

Ckt has Active i & v will have = (forced + natural respon)

$\tau = 1.2$ $\quad i = i_f + i_n = 0.1 + \underline{K}e^{-t/1.2}$

$0.5 = 0.1 + Ke^{-t/1.2}$

$t>0$ $v_c =$

$K = ?.$

Example