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Networks, Signals & Systems, 2024.

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What is a signal? → Something which has utility for some specific purpose.

→ The signal itself may be important totally (like AC/DC voltage signal) or it may carry some information that we seek / can-use. (like telephone signal)

Examples of various signals:

1. EM waves
2. Voltage/current signals
(2a) Sequence of pulses sent to a computer
4. Sound waves
5. Photos, Videos
6. Vibrational signals like seismic signals
7. Chemical signals – smell/taste.
8. Volume of a tank over time
9. Attention level of a student over class-time
10. No. of people who vote in different age groups.
11. Temperature variation in different places in Hyderabad.
12. Many many such examples..

→ These are all classes of signals. Each of these have one or more 'independent' parameter & one or more dependent parameters

→ If we see each signal as a function from a domain to a codomain, the 'independent' parameters are in the 'domain' of the function which represents the signal, the value of the function is the dependent parameter, which is in the co-domain

Eg: 1. Electrical signal (voltage/current) generally represents Amplitude of voltage at any given time.

$\underline{V(t)} \rightarrow$ notation

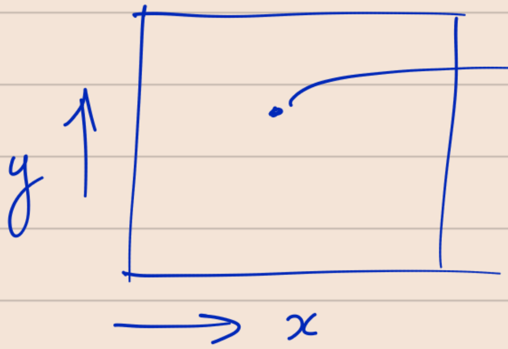
t - denotes time, $V(t) \rightarrow$ value of voltage at time t.

V: Time $\longrightarrow$ Voltage levels
($\in$ Real numbers) ($\in$ Real numbers)

Here, Time t is the independent variable, Voltage is the dependent variable.

E.g 2: Picture / Photo (assume digital photo).

A digital photo on a screen is simply a set of (R, G, B) values for each 'pixel' of the screen



$$\underbrace{f(x,y)}_{} = \underline{(r,g,b)}$$

(x, y) capturing the position of the picture (independent)

3 values capturing the color. (dependent).

f: 2D-space $\longrightarrow$ 3D color space

this function describes the 'photo' signal.

E.g: 3: Video

(A 'moving' sequence of images.)

So we have video signal: $f(x, y, t) \longrightarrow (r, g, b)$

(extra variable. time)

Note: Observe that this style of writing a signal as a function captures not one particular signal (like only one electrical signal, one particular photo, etc.) but all signals in a class of signals (i.e. a category of signals) are write-able in this way.

That is, for every electrical signal,
time is independent parameter, while amplitude is dependent parameter.

H.W: Try writing the other signals presented before as functions of independent variables resulting in the values of dependent variables. (Identify what the dep./indep. variables are, & what is the domain/co-domain).

In this course:

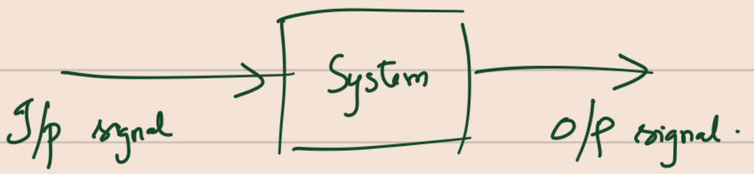
We will deal with signals which have domain is the set of real numbers (denoted by $\mathbb{R}$), representing time. The co-domain can be complex numbers ($\mathbb{C}$) or real numbers ($\mathbb{R}$).

→ Thus, we will generally have that the independent variable is time t.

→ Thus, signals for this course will generally look like $f : \mathbb{R} \to \mathbb{R}$ & written as $f(t)$. Observe that this is also easy for us to plot on 2D plane (like paper, board, etc)

Systems:

→ Converts/modifies signals to obtain other signals



→ The input & output signals may not necessarily be of the same type.

→ We might have more than one input/output signals also.

Examples:

(a) electrical signal $\rightarrow$ [System] $\rightarrow$ Sound signal or EM wave.

Sound signal $\rightarrow$ Loudspeakers

EM wave $\rightarrow$ Radio transmitters

(b) Mechanical signal $\rightarrow$ [System] $\rightarrow$ Electrical signal

(eg:- Dynamo, piezoelectric).

(c) sound, touch, smell, taste, light $\rightarrow$ [human being] $\rightarrow$ action

$\rightarrow$ In this course, we will look at single-input, single-output systems, where the input & output signals are of the same type

Examples:

(1) $x(t) \rightarrow$ [Delay system] $\rightarrow y(t) = \underbrace{x(t-t_0)}$

This means,

" the value of the function y at time t, is the same as the value of function x, at time t. "

(2) $x(t) \rightarrow$ [Amplify] $\rightarrow A\,x(t)$

(3) Series of systems

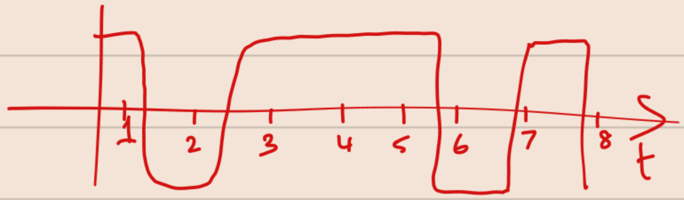
$x(t) \rightarrow$ [delay] $\rightarrow$ [amplify] $\rightarrow A\,x(t-t_0)$

(4) $x(t) \rightarrow$ [time scaling] $\rightarrow y(t)$

Time scaling means?.

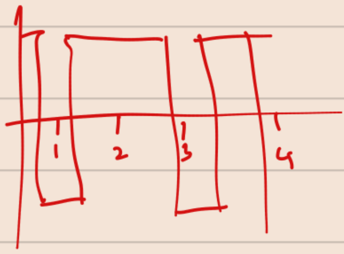
Consider a signal like this

$x(t)$ =

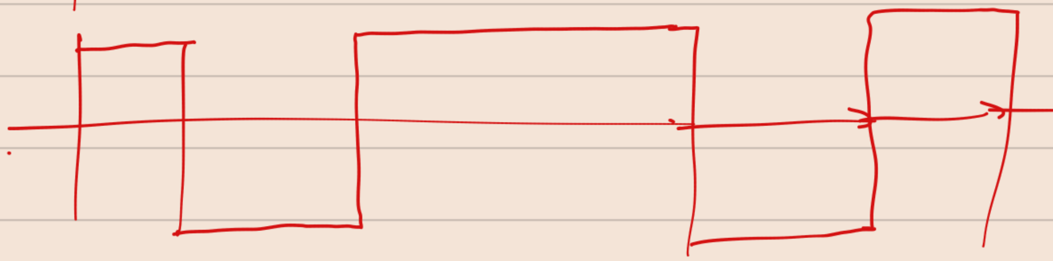


Suppose we want to compress or stretch this signal in time

→ 'Compress'



→ 'expand'



How to write $y(t)$ in these cases?

Compress: $y(t) = x(2t)$ (Value of y at time t is value of x at time $2t$)

Expand: $y(t) = x(t/2)$ (" " " " " t " " " " " $t/2$)

(5) System that tracks changes in input. (rate of change)

Suppose we want to track changes in $x(t)$

→ If signal $x(t)$ is not changing much, we want to keep $|y(t)|$ small ($|y(t)|$ = absolute value of $y(t)$)

→ Otherwise, if $x(t)$ is experiencing lot of changes, $|y(t)|$ should be large.

$$x(t) \longrightarrow \boxed{\text{Differentiator}} \longrightarrow y(t)$$

$$y(t) = \frac{d}{dt} x(t).$$

$x(t)$:  , $y(t)$ = 

(6) **'Moving average' system (or) Smoothing system:**

Suppose we want to 'smooth' out the fast changes in $x(t)$

$$x(t) \longrightarrow \boxed{\text{Moving average/ Smoothing}} \longrightarrow y(t) = \int_{\tau = t-\varepsilon}^{t+\varepsilon} x(\tau)\, d\tau$$

The 'integrator' is another system defined as follows. (It 'accumulates' the effect of $x(t)$ until the specific time instant)

$$x(t) \longrightarrow \boxed{\text{Integrator}} \longrightarrow y(t) = \int_{\tau=-\infty}^{t} x(\tau)\, d\tau$$

Exercise:

Think about the 2D versions of these systems ((ie) if our input was a picture & the output was another picture).

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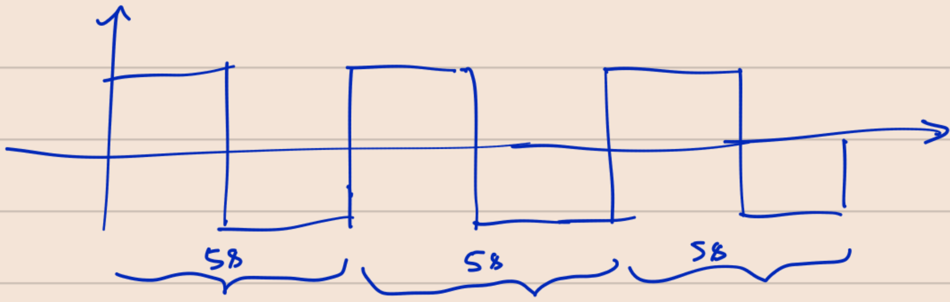
Signals in time variable: Various Classifications:

(1) ↳ Continuous-time (mostly our focus in this course)
↳ Discrete-time (arises from sampling continuous signals as well as naturally)

② Periodic → signal repeats after some finite time

$$x(t+T) = x(t), \ \forall t$$



$x(t+10) = x(t)$
$x(t+5) = x(t)$ → smallest T is 5.

This smallest T s.t $x(t+T) = x(t), \forall t$ is called $T = 5s$.

Non-periodic: (No such T exists)

③ Even & odd signals:

A signal is called 'even' → $x(t) = x(-t), \forall t$

- - - - - - 'odd' → $x(t) = -x(-t), \forall t$

Remark: Any signal can be written as the sum of an even signal & an odd signal

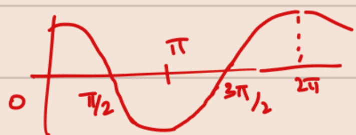
$$x(t) = x_e(t) + x_o(t), \text{ where } x_e(t) \triangleq \frac{x(t) + x(-t)}{2}.$$
$$\& \quad x_o(t) \triangleq \frac{x(t) - x(-t)}{2}.$$

(Note: " $\triangleq$ " is notation for "is defined as")

Some input signal & signal classes:

① (Real-valued) Sinusoidal signals..

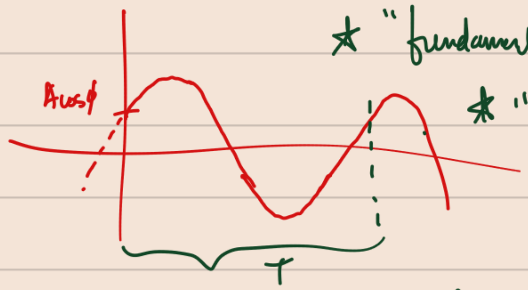
$x(t) = \cos t$



→ (periodic signal with max amplitude = 1, period = 2π (can change) phase shift = 0)

More generally $x(t) = A\cos(\underbrace{\tfrac{2\pi}{T}t + \phi}) \rightarrow$ " instantaneous phase of the sinusoid "

$\phi \triangleq$ phase shift



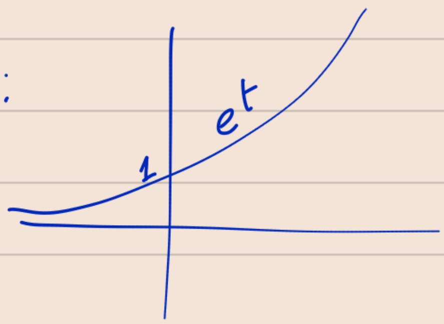
★ " fundamental period " $= T$

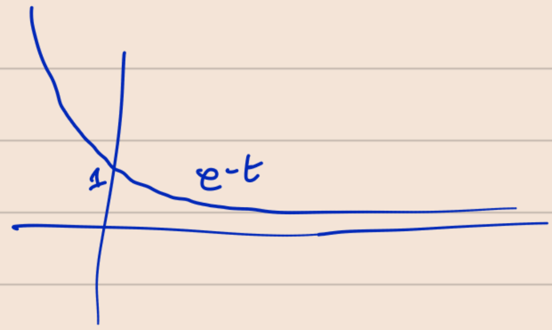
★ " fundamental angular frequency " of the sinusoid $\omega_0 \triangleq 2\pi/T$.

We then write $x(t) = A\cos(\omega_0 t + \phi)$.

$\boxed{\text{Third quantity: " fundamental \underline{ordinary} freq " } \triangleq 1/T}$

Exponential signals:





More generally $x(t) = Ce^{at}$

$\underline{C, a}$ being real numbers.

$(a \neq 0)$

Complex sinusoid signals:

$$x(t) = e^{j\omega_0 t} = \cos\omega_0 t + j\sin\omega_0 t.$$

(both real & imaginary parts of complex sinusoids are sinusoids)

Complex exponential signals

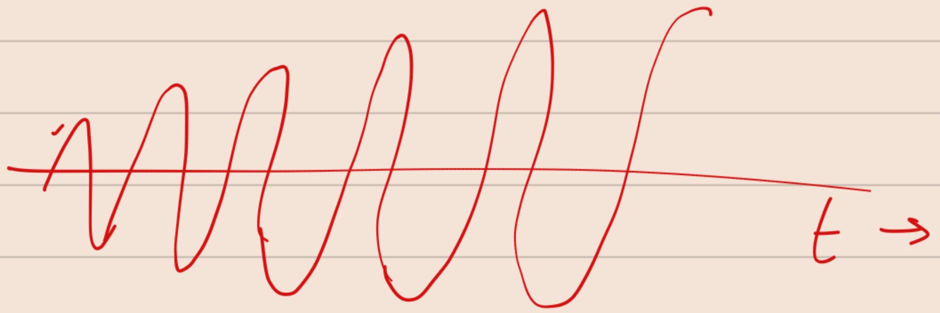
$$x(t) = Ce^{at}, \qquad a, C \in \mathbb{C} \text{ [set of complex numbers]}$$

Let $C = |C|e^{j\theta}$ be the polar representation of C.

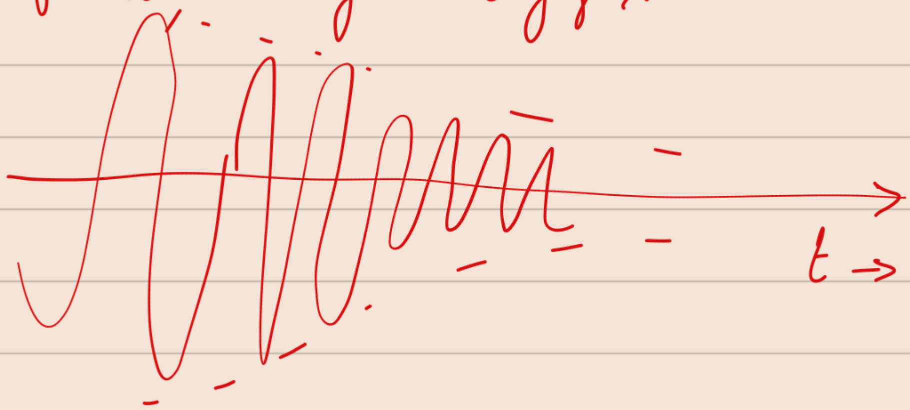
Let $a = r + j\omega_0$ be the rectangular representation of a.

Thus $x(t) = |C|e^{j\theta} \cdot e^{(r+j\omega_0)t} = |C|e^{rt} \cdot e^{j(\omega_0 t + \theta)}$.

Now, if $r > 0$, then the real & imaginary parts of $x(t)$ are growing sinusoids.



If $r < 0$ then they are decaying sinusoids



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Some important quantities associated with a signal.

Energy of a signal : (We are defining a new quantity here).

Energy of signal $x(t)$ in an interval (t_1, t_2)

$$\triangleq \int_{t_1}^{t_2} x^2(t)\,dt$$

(Total) Energy of the signal $\triangleq \int_{-\infty}^{\infty} x^2(t)\,dt$. (denoted by E_∞)

* If $E_\infty < \infty$ (i.e. energy of $x(t)$ is finite), then we call $x(t)$ as a (finite) energy signal.

(Average) Power of the signal $x(t)$

$$P_\infty \triangleq \lim_{T\to\infty} \frac{1}{T} \underbrace{\int_{-T/2}^{T/2} |x(t)|^2 dt}_{\leq E_\infty \text{ for every } T}$$

Thus $P_\infty = 0$, if $E_\infty < \infty$.
$\Rightarrow$ finite energy signal have zero average power

* If $E_\infty = \infty$, but $P_\infty < \infty$ ((i.e) power is finite) then $x(t)$ is called a (finite) power signal
→ Example: Periodic signals.

HW: Show that, for a signal with period T_0, $E_\infty = \infty$ but

$$P_\infty = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x^2(t)\, dt.$$

(i.e for calculating average power it is enough to restrict to one period)

* We could also have signals which have infinite energy & infinite power. Example: Diverging signals.

Special signals of interest:

Impulse (Dirac delta) signal:

Consider a very short 'rectangular' pulse at time $t=0$, of width Δ and area under the signal $= 1$.

$$\text{(i.e)} \quad \delta_\Delta(t) \triangleq \begin{cases} 1/\Delta, & \text{for } t \in [0,\Delta] \\ 0, & \text{otherwise.} \end{cases}$$

Observe that $\int_{t=-\infty}^{\infty} \delta_\Delta(t)\, dt = \int_{t=0}^{\Delta} 1/\Delta \, dt$

$$= \frac{1}{\Delta} [t]_0^\Delta = 1.$$

Now consider the 'limiting' version of the signal $\delta_\Delta(t)$ as $\Delta \to 0$. We get a signal whose area $= 1$, but 'width' $= 0$.

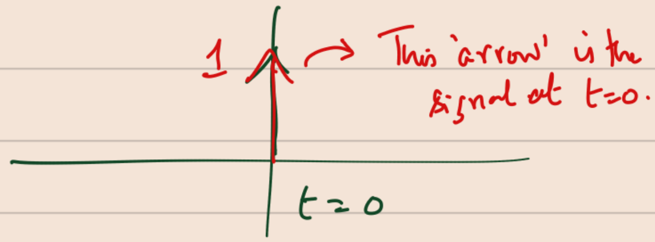
This is our impulse function $\delta(t)$. Formally it is defined as follows.

Defn: The impulse function, $\delta(t)$ is that function such that $\delta(t) = 0, \forall t \neq 0$ and $\int_{-\infty}^{\infty} \delta(t)\, dt = 1$.

(Since the area of the signal is "unity" $(=1)$, this is also called unit impulse).

Pictorially it is represented as

(This is just a representation)



Unit step function:

Now consider a signal $u(t)$, obtained by integrating the impulse from $-\infty$ to t.

$$u(t) \triangleq \int_{-\infty}^{t} \delta(z)\, dz .$$

Observe that for $t < 0$, this signal $u(t) = 0$.

And for $t > 0$, $u(t) = 1$.

At $t = 0$, there is a discontinuity. This signal is called the unit-step function

Formally: $u(t) = \begin{cases} 1, & \forall t > 0 \\ 0, & \forall t < 0 \end{cases}$

Also know: Scaled, shifted impulse function. Shifted unit step → representation & figure (see text/notes from class)

— x —

Some important classes of systems (notes to be added).