

Fundamentals of Electronics

**ECE 101**



# Digital Electronics

# Digital computing

## In digital computing:

- $2^{10}$  is referred to as K (kilo)
- $2^{20}$  as M (mega)
- $2^{30}$  as G (giga)
- $2^{40}$  as T (tera)
- 1 byte = 8 bits (one byte can store one character, e. g. 'A' or 'x' or '\$')
- Thus,  $4K = 2^{12} = 4,096$  and  $16M = 2^{24} = 16,777,216$ .

| <i>n</i> | <i>2<sup>n</sup></i> | <i>n</i> | <i>2<sup>n</sup></i> | <i>n</i> | <i>2<sup>n</sup></i> |
|----------|----------------------|----------|----------------------|----------|----------------------|
| 0        | 1                    | 8        | 256                  | 16       | 65,536               |
| 1        | 2                    | 9        | 512                  | 17       | 131,072              |
| 2        | 4                    | 10       | 1,024 (1K)           | 18       | 262,144              |
| 3        | 8                    | 11       | 2,048                | 19       | 524,288              |
| 4        | 16                   | 12       | 4,096 (4K)           | 20       | 1,048,576 (1M)       |
| 5        | 32                   | 13       | 8,192                | 21       | 2,097,152            |
| 6        | 64                   | 14       | 16,384               | 22       | 4,194,304            |
| 7        | 128                  | 15       | 32,768               | 23       | 8,388,608            |

# Complement of numbers: Radix and Diminished radix

$$r' \text{ s complement} = \{(r^n)_{10}\}_r - N$$

$$(r-1)' \text{ s complement} = \{(r^n)_{10} - 1\}_r - N$$

The n is the number of digits in the number.

The N is the given number.

The r is the radix or base of the number.

**Examples: 1.  $(1011000)_2$**

$$\text{1's complement: } \{(2^7)_{10} - 1\} - (1011000)_2 = \{(128)_{10} - 1\}_2 - (1011000)_2 = 1111111_2 - 1011000_2 = 0100111$$

$$\text{2's complement: } \{(2^7)_{10}\} - (1011000)_2 = \{(128)_{10}\}_2 - (1011000)_2 = 10000000_2 - 1011000_2 = 0101000_2$$

**2.  $(155)_{10}$**

$$\text{9's complement: } \{(10^3)_{10} - 1\} - (155)_{10} = (1000 - 1) - 155 = 999 - 155 = (844)_{10}$$

$$\text{10's complement: } \{(10^3)_{10}\} - (155)_{10} = (1000) - 155 = 1000 - 155 = (845)_{10}$$

**3.  $(174)_8 = ?$**

# Subtraction with complements

1. Add the minuend  $M$  to the  $r$ 's complement of the subtrahend  $N$ . **Mathematically:**  $M + (r^n - N) = M - N + r^n$ .
2. If  $M > N$ , the sum will produce an end carry  $r^n$ , which can be discarded; what is left is the result  $M - N$ .
3. If  $M < N$ , the sum does not produce an end carry and is equal to  $r^n - (N - M)$ , which is the  $r$ 's complement of  $(N - M)$ . To obtain the answer in a familiar form, **take the  $r$ 's complement of the sum and place a negative sign in front.**

Example 1: Using 10's complement, subtract **72532 - 3250**

$$\begin{array}{rcl} M & = & 72532 \\ \text{10's complement of } N & = & + \underline{96750} \\ \text{Sum} & = & 169282 \\ \text{Discard end carry } 10^5 & = & - \underline{100000} \\ \text{Answer} & = & 69282 \end{array}$$

# Subtraction with complements

Example 2: Using 10's complement, subtract **3250 - 72532**

$$\begin{array}{r} M = \quad 03250 \\ 10\text{'s complement of } N = + \underline{27468} \\ \text{Sum} = \quad 30718 \end{array}$$

the answer is  $-(10\text{'s complement of } 30718) = -69282$ .

Example 3: Given the two binary numbers  $X = 1010100$  and  $Y = 1000011$ , perform the subtraction (a)  $X - Y$  and (b)  $Y - X$  by using 2's complements.

# Subtraction with complements

Example 3: Given the two binary numbers  $X = 1010100$  and  $Y = 1000011$ , perform the subtraction (a)  $X - Y$  and (b)  $Y - X$  by using 2's complements.

$$\begin{array}{rcl} \text{(a)} & X = & 1010100 \\ & 2\text{'s complement of } Y = + & \underline{0111101} \\ & \text{Sum} = & 10010001 \\ & \text{Discard end carry } 2^7 = - & 10000000 \\ & \text{Answer: } X - Y = & 0010001 \end{array}$$

$$\begin{array}{rcl} \text{(b)} & Y = & 1000011 \\ & 2\text{'s complement of } X = + & 0101100 \\ & \text{Sum} = & 1101111 \end{array}$$

There is no end carry. Therefore, the answer is  $Y - X = -(2\text{'s complement of } 1101111) = -0010001$ .

# Subtraction with complements

Example 4: Given the two binary numbers  $X = 1010100$  and  $Y = 1000011$ , perform the subtraction (a)  $X - Y$  and (b)  $Y - X$  by using 1's complements.

(a)  $X - Y = 1010100 - 1000011$

$$\begin{array}{r} X = \quad 1010100 \\ 1\text{'s complement of } Y = + \underline{0111100} \\ \text{Sum} = \quad 10010000 \\ \text{End-around carry} = + \quad 1 \\ \text{Answer: } X - Y = \quad 0010001 \end{array}$$

(b)  $Y - X = 1000011 - 1010100$

$$\begin{array}{r} Y = \quad 1000011 \\ 1\text{'s complement of } X = + \underline{0101011} \\ \text{Sum} = \quad 1101110 \end{array}$$

There is no end carry. Therefore, the answer is  $Y - X = -(1\text{'s complement of } 1101110) = -0010001$ .



# Signed binary numbers

Computers must represent everything with binary digits.

It is customary to represent the sign with a bit placed in the leftmost position of the number.

The convention is to make the sign bit 0 for positive and 1 for negative.

$01001 = 9 \text{ or } +9$

$11001 = 25 \text{ or } -9$       **Signed-magnitude convention**

## Signed-complement system

**A negative number is indicated by its complement**

- Either the 1's or the 2's complement, but the 2's complement is the most common

$01001 = 9 \text{ or } +9$

|                                       |          |
|---------------------------------------|----------|
| signed-magnitude representation:      | 10001001 |
| signed-1's-complement representation: | 11110110 |
| signed-2's-complement representation: | 11110111 |

| Decimal | Signed-2's Complement | Signed-1's Complement | Signed Magnitude |
|---------|-----------------------|-----------------------|------------------|
| +7      | 0111                  | 0111                  | 0111             |
| +6      | 0110                  | 0110                  | 0110             |
| +5      | 0101                  | 0101                  | 0101             |
| +4      | 0100                  | 0100                  | 0100             |
| +3      | 0011                  | 0011                  | 0011             |
| +2      | 0010                  | 0010                  | 0010             |
| +1      | 0001                  | 0001                  | 0001             |
| +0      | 0000                  | 0000                  | 0000             |
| -0      | —                     | 1111                  | 1000             |
| -1      | 1111                  | 1110                  | 1001             |
| -2      | 1110                  | 1101                  | 1010             |
| -3      | 1101                  | 1100                  | 1011             |
| -4      | 1100                  | 1011                  | 1100             |
| -5      | 1011                  | 1010                  | 1101             |
| -6      | 1010                  | 1001                  | 1110             |
| -7      | 1001                  | 1000                  | 1111             |
| -8      | 1000                  | —                     | —                |

# Arithmetic Addition/Subtraction

Generally, addition/subtraction involves comparison of signs and magnitudes and performing operations.

- If the signs are the same, we add the two magnitudes and give the sum the common sign.
- If the signs are different, we subtract the smaller magnitude from the larger and give the difference the sign of the larger magnitude.
- The same procedure applies to binary numbers in signed-magnitude representation.
- The rule for adding numbers in the signed-complement system does not require a comparison or subtraction, but only addition.
- The addition of two signed binary numbers (negative numbers represented in signed-2's-complement form) is obtained from the addition of the two numbers, including their sign bits. A carry out of the sign-bit position is discarded.

# Arithmetic Addition/Subtraction

- The addition of two signed binary numbers (negative numbers represented in signed-2's-complement form) is obtained from the addition of the two numbers, including their sign bits. **A carry out of the sign-bit position is discarded.**

$$\begin{array}{r} + 6 \quad 00000110 \\ +13 \quad 00001101 \\ \hline +19 \quad 00010011 \end{array}$$

$$\begin{array}{r} - 6 \quad 11111010 \\ +13 \quad 00001101 \\ \hline + 7 \quad 00000111 \end{array}$$

$$\begin{array}{r} + 6 \quad 00000110 \\ -13 \quad 11110011 \\ \hline - 7 \quad 11111001 \end{array}$$

$$\begin{array}{r} - 6 \quad 11111010 \\ -13 \quad 11110011 \\ \hline -19 \quad 11101101 \end{array}$$

- In order to obtain a correct answer, we must ensure that the result has a sufficient number of bits to accommodate the sum.
- Subtraction:** Take the 2's complement of the subtrahend (including the sign bit) and add it to the minuend (including the sign bit). A carry out of the sign-bit position is discarded.

# Arithmetic Addition/Subtraction

- **Subtraction:** Take the 2's complement of the subtrahend (including the sign bit) and add it to the minuend (including the sign bit). A carry out of the sign-bit position is discarded.

$$(\pm A) - (+B) = (\pm A) + (-B);$$

$$(\pm A) - (-B) = (\pm A) + (+B).$$

- Changing a positive number to a negative number is easily done by taking the 2's complement of the positive number.
- The reverse is also true.
- **Binary numbers in the signed-complement system are added and subtracted by the same basic addition and subtraction rules as unsigned numbers. Therefore, computers need only one common hardware circuit to handle both types of arithmetic.**

# Information: Codes

## Binary Coded Decimal (BCD)

$$(185)_{10} = (0001\ 1000\ 0101)_{\text{BCD}} = (10111001)_2$$

BCD numbers are decimal numbers and not binary numbers, although they use bits in their representation.

| Decimal<br>Symbol | BCD<br>Digit |
|-------------------|--------------|
| 0                 | 0000         |
| 1                 | 0001         |
| 2                 | 0010         |
| 3                 | 0011         |
| 4                 | 0100         |
| 5                 | 0101         |
| 6                 | 0110         |
| 7                 | 0111         |
| 8                 | 1000         |
| 9                 | 1001         |

## BCD Addition

|    |       |       |       |       |       |
|----|-------|-------|-------|-------|-------|
| 4  | 0100  | 4     | 0100  | 8     | 1000  |
| +5 | +0101 | +8    | +1000 | +9    | 1001  |
| 9  | 1001  | 12    | 1100  | 17    | 10001 |
|    |       | +0110 |       | +0110 |       |
|    |       | 10010 |       | 10111 |       |

# ASCII Code

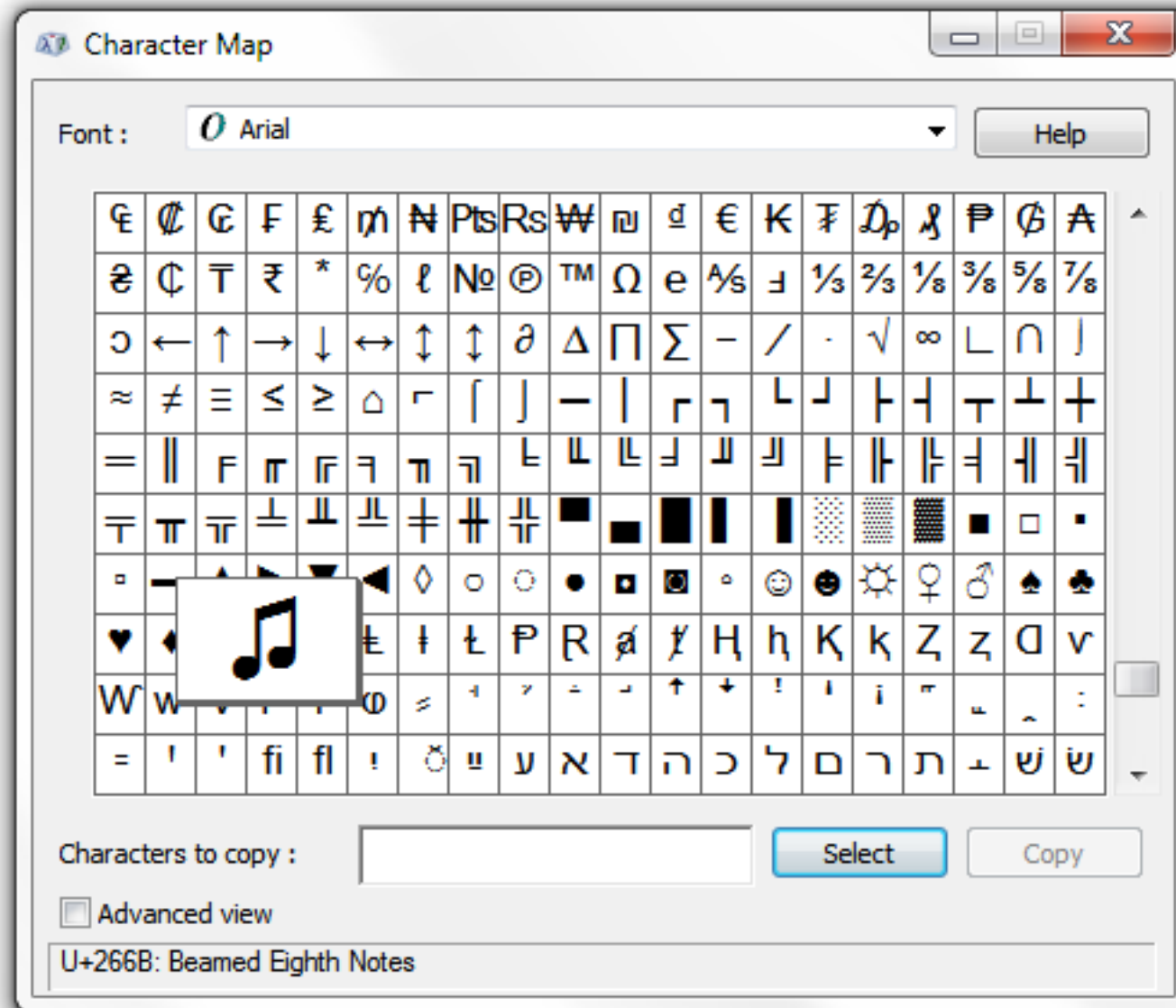
American Standard Code for Information Interchange (ASCII)

| <i>b<sub>4</sub>b<sub>3</sub>b<sub>2</sub>b<sub>1</sub></i> | <i>b<sub>7</sub>b<sub>6</sub>b<sub>5</sub></i> |     |     |     |     |     |     |     |
|---|--|-----|-----|-----|-----|-----|-----|-----|
|   | 000  | 001 | 010 | 011 | 100 | 101 | 110 | 111 |
| 0000  | NUL  | DLE | SP  | 0   | @   | P   | `   | p   |
| 0001  | SOH  | DC1 | !   | 1   | A   | Q   | a   | q   |
| 0010  | STX  | DC2 | “   | 2   | B   | R   | b   | r   |
| 0011  | ETX  | DC3 | #   | 3   | C   | S   | c   | s   |
| 0100  | EOT  | DC4 | \$  | 4   | D   | T   | d   | t   |
| 0101  | ENQ  | NAK | %   | 5   | E   | U   | e   | u   |
| 0110  | ACK  | SYN | &   | 6   | F   | V   | f   | v   |
| 0111  | BEL  | ETB | ‘   | 7   | G   | W   | g   | w   |
| 1000  | BS   | CAN | (   | 8   | H   | X   | h   | x   |
| 1001  | HT   | EM  | )   | 9   | I   | Y   | i   | y   |
| 1010  | LF   | SUB | *   | :   | J   | Z   | j   | z   |
| 1011  | VT   | ESC | +   | ;   | K   | [   | k   | {   |
| 1100  | FF   | FS  | ,   | <   | L   | \   | l   |     |
| 1101  | CR   | GS  | —   | =   | M   | ]   | m   | }   |
| 1110  | SO   | RS  | .   | >   | N   | ^   | n   | ~   |
| 1111  | SI   | US  | /   | ?   | O   | —   | o   | DEL |

| Control Characters |                     |     |                           |
|--------------------|---------------------|-----|---------------------------|
| NUL                | Null                | DLE | Data-link escape          |
| SOH                | Start of heading    | DC1 | Device control 1          |
| STX                | Start of text       | DC2 | Device control 2          |
| ETX                | End of text         | DC3 | Device control 3          |
| EOT                | End of transmission | DC4 | Device control 4          |
| ENQ                | Enquiry             | NAK | Negative acknowledge      |
| ACK                | Acknowledge         | SYN | Synchronous idle          |
| BEL                | Bell                | ETB | End-of-transmission block |
| BS                 | Backspace           | CAN | Cancel                    |
| HT                 | Horizontal tab      | EM  | End of medium             |
| LF                 | Line feed           | SUB | Substitute                |
| VT                 | Vertical tab        | ESC | Escape                    |
| FF                 | Form feed           | FS  | File separator            |
| CR                 | Carriage return     | GS  | Group separator           |
| SO                 | Shift out           | RS  | Record separator          |
| SI                 | Shift in            | US  | Unit separator            |
| SP                 | Space               | DEL | Delete                    |

# Unicode

## Universal Coded Character Set



UTF-8

UTF-16

UTF-32

# Information storage and transfer

## Register

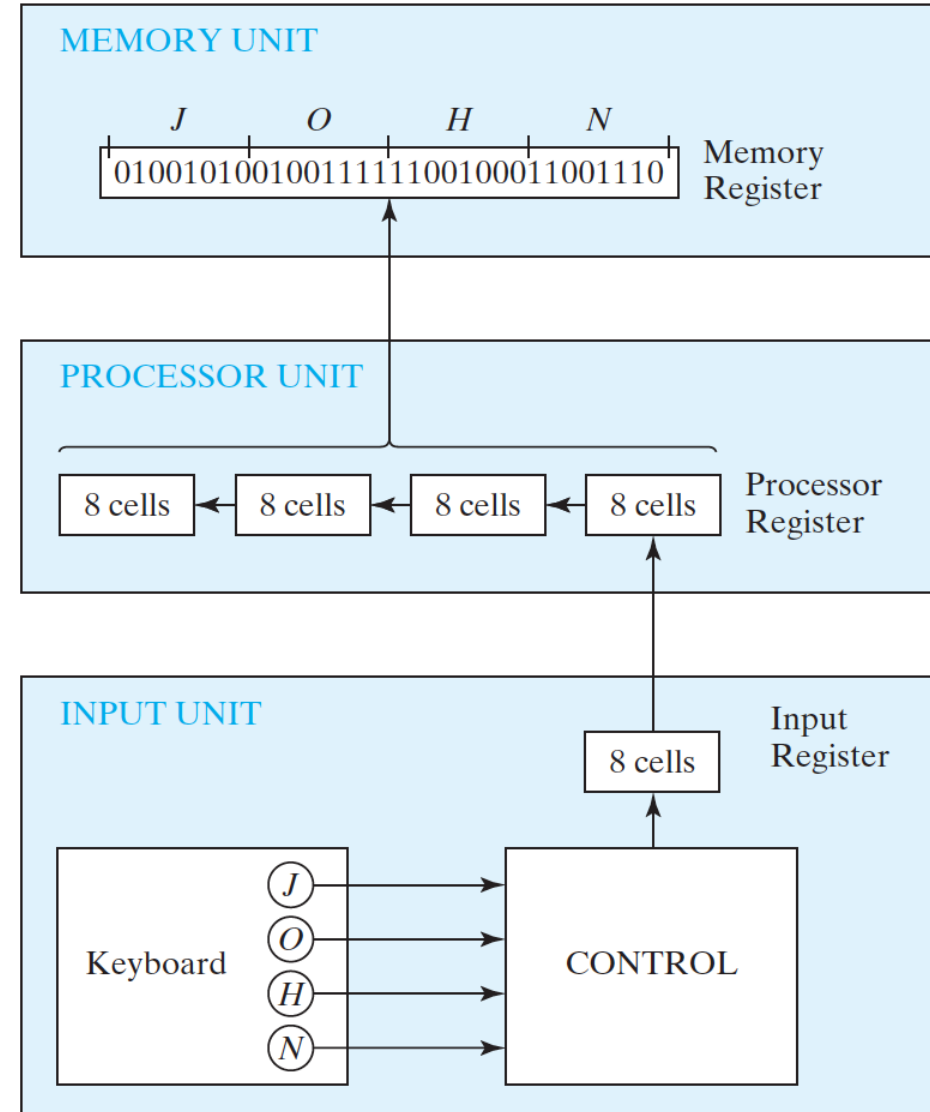
A binary cell is a device that possesses two stable states and is capable of storing one bit (0 or 1) of information.

**A register is a group of binary cells.**

A register with  $n$  cells can store any discrete quantity of information that contains  $n$  bits.

1100001111001001

## Register transfer





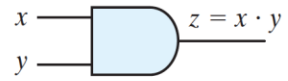
# Binary Logic/Boolean Algebra

- **Field:** binary numbers (0, 1)
- **Operation(s):**  $\wedge$  (AND),  $\vee$  (OR), and  $\neg$  ("complement" or "not")
  - $a \vee (b \vee c) = (a \vee b) \vee c$        $a \wedge (b \wedge c) = (a \wedge b) \wedge c$       **Associativity**
  - $a \vee b = b \vee a$        $a \wedge b = b \wedge a$       **Commutativity**
  - $a \vee (a \wedge b) = a$        $a \wedge (a \vee b) = a$       **Absorption**
  - $a \vee 0 = a$        $a \wedge 1 = a$       **Identity**
  - $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$        $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$       **Distributivity**
  - $a \vee \neg a = 1$        $a \wedge \neg a = 0$       **Complements**

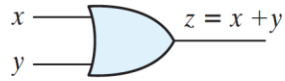
## Truth Tables of Logical Operations

| AND |     |             | OR  |     |         | NOT |      |
|-----|-----|-------------|-----|-----|---------|-----|------|
| $x$ | $y$ | $x \cdot y$ | $x$ | $y$ | $x + y$ | $x$ | $x'$ |
| 0   | 0   | 0           | 0   | 0   | 0       | 0   | 1    |
| 0   | 1   | 0           | 0   | 1   | 1       | 1   | 0    |
| 1   | 0   | 0           | 1   | 0   | 1       |     |      |
| 1   | 1   | 1           | 1   | 1   | 1       |     |      |

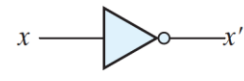
# Digital logic gates



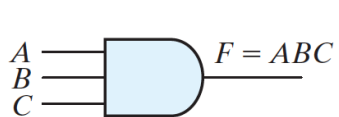
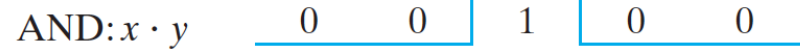
(a) Two-input AND gate



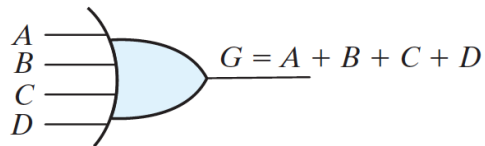
(b) Two-input OR gate



(c) NOT gate or inverter



(a) Three-input AND gate



(b) Four-input OR gate

AND



$$F = x \cdot y$$

| $x$ | $y$ | $F$ |
|-----|-----|-----|
| 0   | 0   | 0   |
| 0   | 1   | 0   |
| 1   | 0   | 0   |
| 1   | 1   | 1   |

OR



$$F = x + y$$

| $x$ | $y$ | $F$ |
|-----|-----|-----|
| 0   | 0   | 0   |
| 0   | 1   | 1   |
| 1   | 0   | 1   |
| 1   | 1   | 1   |

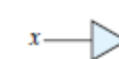
Inverter



$$F = x'$$

| $x$ | $F$ |
|-----|-----|
| 0   | 1   |
| 1   | 0   |

Buffer



$$F = x$$

| $x$ | $F$ |
|-----|-----|
| 0   | 0   |
| 1   | 1   |

NAND



$$F = (xy)'$$

| $x$ | $y$ | $F$ |
|-----|-----|-----|
| 0   | 0   | 1   |
| 0   | 1   | 1   |
| 1   | 0   | 1   |
| 1   | 1   | 0   |

NOR



$$F = (x + y)'$$

| $x$ | $y$ | $F$ |
|-----|-----|-----|
| 0   | 0   | 1   |
| 0   | 1   | 0   |
| 1   | 0   | 0   |
| 1   | 1   | 0   |

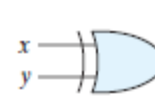
Exclusive-OR  
(XOR)



$$F = xy' + x'y = x \oplus y$$

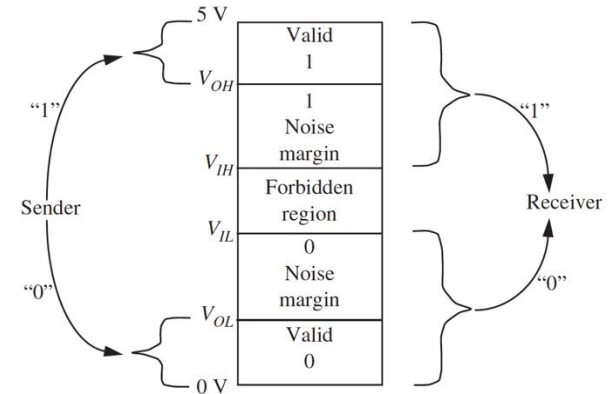
| $x$ | $y$ | $F$ |
|-----|-----|-----|
| 0   | 0   | 0   |
| 0   | 1   | 1   |
| 1   | 0   | 1   |
| 1   | 1   | 0   |

Exclusive-NOR  
or  
equivalence



$$F = xy + x'y' = (x \oplus y)'$$

| $x$ | $y$ | $F$ |
|-----|-----|-----|
| 0   | 0   | 1   |
| 0   | 1   | 0   |
| 1   | 0   | 0   |
| 1   | 1   | 1   |



Apart from gates, we have registers, counters, memory elements.

# Basic Theorems of Boolean Algebra

## *Postulates and Theorems of Boolean Algebra*

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|                           |     |                             |     |                           |
|---------------------------|-----|-----------------------------|-----|---------------------------|
| Postulate 2               | (a) | $x + 0 = x$                 | (b) | $x \cdot 1 = x$           |
| Postulate 5               | (a) | $x + x' = 1$                | (b) | $x \cdot x' = 0$          |
| Theorem 1                 | (a) | $x + x = x$                 | (b) | $x \cdot x = x$           |
| Theorem 2                 | (a) | $x + 1 = 1$                 | (b) | $x \cdot 0 = 0$           |
| Theorem 3, involution     |     | $(x')' = x$                 |     |                           |
| Postulate 3, commutative  | (a) | $x + y = y + x$             | (b) | $xy = yx$                 |
| Theorem 4, associative    | (a) | $x + (y + z) = (x + y) + z$ | (b) | $x(yz) = (xy)z$           |
| Postulate 4, distributive | (a) | $x(y + z) = xy + xz$        | (b) | $x + yz = (x + y)(x + z)$ |
| Theorem 5, DeMorgan       | (a) | $(x + y)' = x'y'$           | (b) | $(xy)' = x' + y'$         |
| Theorem 6, absorption     | (a) | $x + xy = x$                | (b) | $x(x + y) = x$            |

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### **Duality Principle:**

- Every algebraic expression deducible from the postulates of Boolean algebra remains valid if the operators and identity elements are interchanged.
- If the dual of an algebraic expression is desired, we simply interchange OR and AND operators and replace 1's by 0's and 0's by 1's.

# Basic Theorems of Boolean Algebra

**THEOREM 1(a):**  $x + x = x$ .

| Statement                 | Justification  |
|---------------------------|----------------|
| $x + x = (x + x) \cdot 1$ | postulate 2(b) |
| $= (x + x)(x + x')$       | 5(a)           |
| $= x + xx'$               | 4(b)           |
| $= x + 0$                 | 5(b)           |
| $= x$                     | 2(a)           |

**THEOREM 1(b):**  $x \cdot x = x$ .

| Statement            | Justification  |
|----------------------|----------------|
| $x \cdot x = xx + 0$ | postulate 2(a) |
| $= xx + xx'$         | 5(b)           |
| $= x(x + x')$        | 4(a)           |
| $= x \cdot 1$        | 5(a)           |
| $= x$                | 2(b)           |

**THEOREM 2(a):**  $x + 1 = 1$ .

| Statement                 | Justification  |
|---------------------------|----------------|
| $x + 1 = 1 \cdot (x + 1)$ | postulate 2(b) |
| $= (x + x')(x + 1)$       | 5(a)           |
| $= x + x' \cdot 1$        | 4(b)           |
| $= x + x'$                | 2(b)           |
| $= 1$                     | 5(a)           |

**THEOREM 2(b):**  $x \cdot 0 = 0$  by duality.

**THEOREM 3:**  $(x')' = x$ . From postulate 5, we have  $x + x' = 1$  and  $x \cdot x' = 0$ , which together define the complement of  $x$ . The complement of  $x'$  is  $x$  and is also  $(x')'$ .

# Basic Theorems of Boolean Algebra

**THEOREM 6(a):**  $x + xy = x$ .

| Statement                 | Justification  |
|---------------------------|----------------|
| $x + xy = x \cdot 1 + xy$ | postulate 2(b) |
| $= x(1 + y)$              | 4(a)           |
| $= x(y + 1)$              | 3(a)           |
| $= x \cdot 1$             | 2(a)           |
| $= x$                     | 2(b)           |

**THEOREM 6(b):**  $x(x + y) = x$  by duality.

## Operator Precedence

The operator precedence for evaluating Boolean expressions is

- (1) parentheses
- (2) NOT
- (3) AND
- (4) OR.

$$(x + y)' = x'y'$$

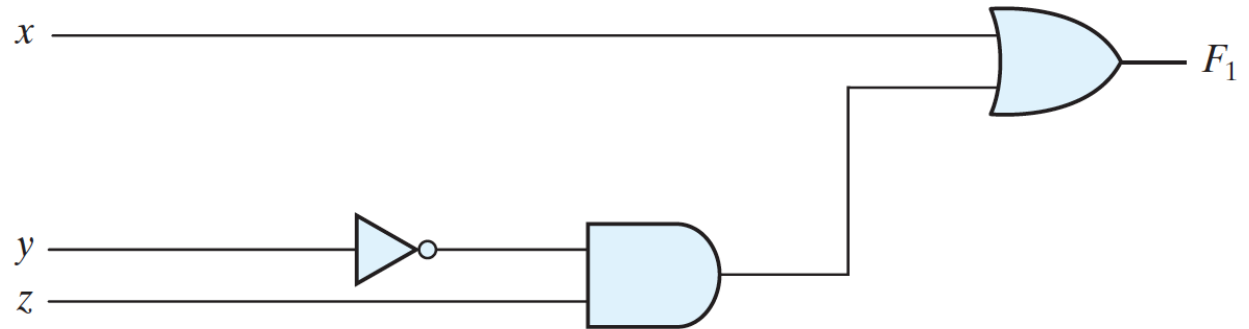
$$(xy)' = x' + y'$$

# Boolean Functions

$$f : \{0, 1\}^k \rightarrow \{0, 1\}$$

$$F_1 = x + y'z$$

| <b><i>x</i></b> | <b><i>y</i></b> | <b><i>z</i></b> | <b><i>F</i><sub>1</sub></b> |
|-----------------|-----------------|-----------------|-----------------------------|
| 0               | 0               | 0               | 0                           |
| 0               | 0               | 1               | 1                           |
| 0               | 1               | 0               | 0                           |
| 0               | 1               | 1               | 0                           |
| 1               | 0               | 0               | 1                           |
| 1               | 0               | 1               | 1                           |
| 1               | 1               | 0               | 1                           |
| 1               | 1               | 1               | 1                           |



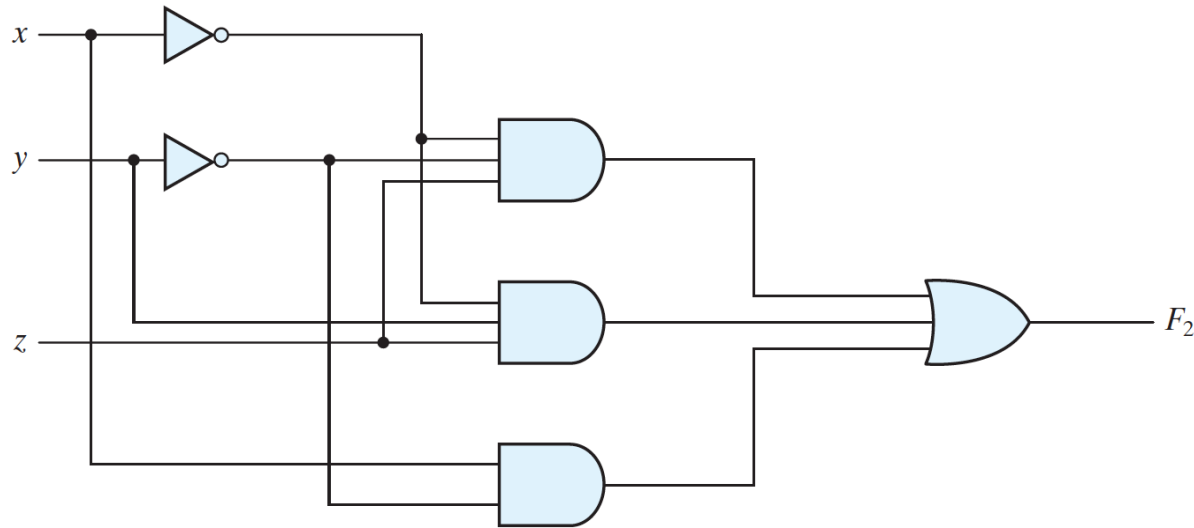
$$F_2 = x'y'z + x'yz + xy'$$

# Boolean Functions

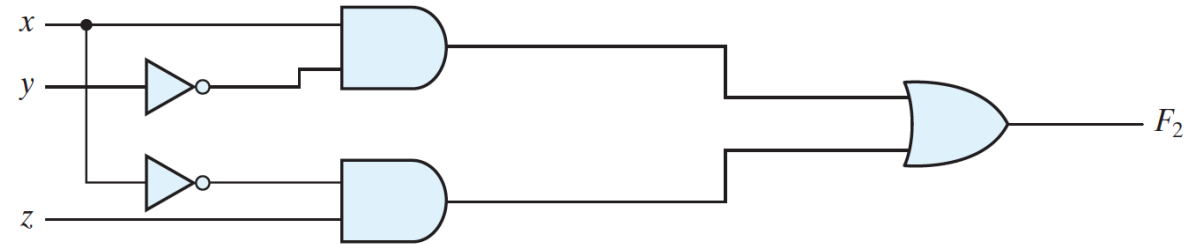
$$f : \{0, 1\}^k \rightarrow \{0, 1\}$$

$$F_2 = x'y'z + x'yz + xy'$$

$$F_2 = x'y'z + x'yz + xy' = x'z(y' + y) + xy' = x'z + xy'$$



(a)  $F_2 = x'y'z + x'yz + xy'$



(b)  $F_2 = xy' + x'z$

# Other Boolean Functions

Truth Tables for the 16 Functions of Two Binary Variables

| <i>x</i> | <i>y</i> | <i>F</i> <sub>0</sub> | <i>F</i> <sub>1</sub> | <i>F</i> <sub>2</sub> | <i>F</i> <sub>3</sub> | <i>F</i> <sub>4</sub> | <i>F</i> <sub>5</sub> | <i>F</i> <sub>6</sub> | <i>F</i> <sub>7</sub> | <i>F</i> <sub>8</sub> | <i>F</i> <sub>9</sub> | <i>F</i> <sub>10</sub> | <i>F</i> <sub>11</sub> | <i>F</i> <sub>12</sub> | <i>F</i> <sub>13</sub> | <i>F</i> <sub>14</sub> | <i>F</i> <sub>15</sub> |
|----------|----------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|------------------------|------------------------|------------------------|------------------------|------------------------|------------------------|
| 0        | 0        | 0                     | 0                     | 0                     | 0                     | 0                     | 0                     | 0                     | 0                     | 1                     | 1                     | 1                      | 1                      | 1                      | 1                      | 1                      | 1                      |
| 0        | 1        | 0                     | 0                     | 0                     | 0                     | 1                     | 1                     | 1                     | 1                     | 0                     | 0                     | 0                      | 0                      | 1                      | 1                      | 1                      | 1                      |
| 1        | 0        | 0                     | 0                     | 1                     | 1                     | 0                     | 0                     | 1                     | 1                     | 0                     | 0                     | 1                      | 1                      | 0                      | 0                      | 1                      | 1                      |
| 1        | 1        | 0                     | 1                     | 0                     | 1                     | 0                     | 1                     | 0                     | 1                     | 0                     | 1                     | 0                      | 1                      | 0                      | 1                      | 0                      | 1                      |

Boolean Expressions for the 16 Functions of Two Variables

| Boolean Functions   | Operator Symbol          | Name         | Comments                            |
|---|--------------------------|--------------|-------------------------------------|
| <i>F</i> <sub>0</sub> = 0                                 |                          | Null         | Binary constant 0                   |
| <i>F</i> <sub>1</sub> = <i>xy</i>                         | <i>x</i> • <i>y</i>      | AND          | <i>x</i> and <i>y</i>               |
| <i>F</i> <sub>2</sub> = <i>xy</i> '                       | <i>x</i> / <i>y</i>      | Inhibition   | <i>x</i> , but not <i>y</i>         |
| <i>F</i> <sub>3</sub> = <i>x</i>                          |                          | Transfer     | <i>x</i>                            |
| <i>F</i> <sub>4</sub> = <i>x</i> ' <i>y</i>               | <i>y</i> / <i>x</i>      | Inhibition   | <i>y</i> , but not <i>x</i>         |
| <i>F</i> <sub>5</sub> = <i>y</i>                          |                          | Transfer     | <i>y</i>                            |
| <i>F</i> <sub>6</sub> = <i>xy</i> ' + <i>x</i> ' <i>y</i> | <i>x</i> ⊕ <i>y</i>      | Exclusive-OR | <i>x</i> or <i>y</i> , but not both |
| <i>F</i> <sub>7</sub> = <i>x</i> + <i>y</i>               | <i>x</i> + <i>y</i>      | OR           | <i>x</i> or <i>y</i>                |
| <i>F</i> <sub>8</sub> = ( <i>x</i> + <i>y</i> )'          | <i>x</i> ↓ <i>y</i>      | NOR          | Not-OR                              |
| <i>F</i> <sub>9</sub> = <i>xy</i> + <i>x</i> ' <i>y</i> ' | ( <i>x</i> ⊕ <i>y</i> )' | Equivalence  | <i>x</i> equals <i>y</i>            |
| <i>F</i> <sub>10</sub> = <i>y</i> '                       | <i>y</i> '               | Complement   | Not <i>y</i>                        |
| <i>F</i> <sub>11</sub> = <i>x</i> + <i>y</i> '            | <i>x</i> ⊃ <i>y</i>      | Implication  | If <i>y</i> , then <i>x</i>         |
| <i>F</i> <sub>12</sub> = <i>x</i> '                       | <i>x</i> '               | Complement   | Not <i>x</i>                        |
| <i>F</i> <sub>13</sub> = <i>x</i> ' + <i>y</i>            | <i>x</i> ⊃ <i>y</i>      | Implication  | If <i>x</i> , then <i>y</i>         |
| <i>F</i> <sub>14</sub> = ( <i>xy</i> )'                   | <i>x</i> ↑ <i>y</i>      | NAND         | Not-AND                             |
| <i>F</i> <sub>15</sub> = 1                                |                          | Identity     | Binary constant 1                   |



# Algebraic manipulation

**Complement of a Function:**

$$\begin{aligned}(A + B + C)' &= (A + x)' \\ &= A'x' \\ &= A'(B + C)' \\ &= A'(B'C') \\ &= A'B'C'\end{aligned}$$

$$(A + B + C + D + \cdots + F)' = A'B'C'D' \dots F'$$

$$(ABCD \dots F)' = A' + B' + C' + D' + \cdots + F'$$

**Examples:**

$$F_1' = (x'yz' + x'y'z)' = (x'yz')'(x'y'z)' = (x + y' + z)(x + y + z')$$

$$\begin{aligned}F_2' &= [x(y'z' + yz)]' = x' + (y'z' + yz)' = x' + (y'z')'(yz)' \\ &= x' + (y + z)(y' + z') \\ &= x' + yz' + y'z\end{aligned}$$

# Algebraic manipulation

## Examples:

$$F_1 = x'yz' + x'y'z.$$

The dual of  $F_1$  is  $(x' + y + z')(x' + y' + z)$ .

Complement each literal:  $(x + y' + z)(x + y + z') = F_1'$ .

$$F_2 = x(y'z' + yz).$$

The dual of  $F_2$  is  $x + (y' + z')(y + z)$ .

Complement each literal:  $x' + (y + z)(y' + z') = F_2'$ .

# Canonical and Standard Forms

## Minterms and Maxterms

### Product (AND), Sum (OR)

- Each maxterm is obtained from an OR term of the  $n$  variables, with each variable being unprimed if the corresponding bit is a 0 and primed if a 1
- Each maxterm is the complement of its corresponding minterm and vice versa.

| $x$ | $y$ | $z$ | Minterms |             | Maxterms       |             |
|-----|-----|-----|----------|-------------|----------------|-------------|
|     |     |     | Term     | Designation | Term           | Designation |
| 0   | 0   | 0   | $x'y'z'$ | $m_0$       | $x + y + z$    | $M_0$       |
| 0   | 0   | 1   | $x'y'z$  | $m_1$       | $x + y + z'$   | $M_1$       |
| 0   | 1   | 0   | $x'yz'$  | $m_2$       | $x + y' + z$   | $M_2$       |
| 0   | 1   | 1   | $x'yz$   | $m_3$       | $x + y' + z'$  | $M_3$       |
| 1   | 0   | 0   | $xy'z'$  | $m_4$       | $x' + y + z$   | $M_4$       |
| 1   | 0   | 1   | $xy'z$   | $m_5$       | $x' + y + z'$  | $M_5$       |
| 1   | 1   | 0   | $xyz'$   | $m_6$       | $x' + y' + z$  | $M_6$       |
| 1   | 1   | 1   | $xyz$    | $m_7$       | $x' + y' + z'$ | $M_7$       |

# Representation of Boolean Functions

**Minterms:** A Boolean function can be expressed algebraically from a given truth table by forming a minterm for each combination of the variables that produces a 1 in the function and then taking the OR of all those terms.

## Example:

| x | y | z | Function $f_1$ | Function $f_2$ |
|---|---|---|----------------|----------------|
| 0 | 0 | 0 | 0              | 0              |
| 0 | 0 | 1 | 1              | 0              |
| 0 | 1 | 0 | 0              | 0              |
| 0 | 1 | 1 | 0              | 1              |
| 1 | 0 | 0 | 1              | 0              |
| 1 | 0 | 1 | 0              | 1              |
| 1 | 1 | 0 | 0              | 1              |
| 1 | 1 | 1 | 1              | 1              |

$$f_1 = x'y'z + xy'z' + xyz = m_1 + m_4 + m_7$$

$$f_2 = x'yz + xy'z + xyz' + xyz = m_3 + m_5 + m_6 + m_7$$

$$f_1' = x'y'z' + x'yz' + x'yz + xy'z + xyz'$$

$$\begin{aligned} f_1 &= (x + y + z)(x + y' + z)(x' + y + z')(x' + y' + z) \\ &= M_0 \cdot M_2 \cdot M_3 \cdot M_5 \cdot M_6 \end{aligned}$$

$$\begin{aligned} f_2 &= (x + y + z)(x + y + z')(x + y' + z)(x' + y + z) \\ &= M_0 M_1 M_2 M_4 \end{aligned}$$

- Form a maxterm for each combination of the variables that produces a 0 in the function,
- Then form the AND of all those maxterms.

Boolean functions expressed as a sum of minterms or product of maxterms are said to be in canonical form

## Sum of minterms

The minterms whose sum defines the Boolean function are those which give the 1's of the function in the truth table.

## Product of maxterms

The maxterms whose product defines the Boolean function are those which give the 0's of the function in the truth table.

## Sum of minterms representation: $F = A + B'C$

$$A = A(B + B') = AB + AB'$$

$$\begin{aligned} A &= AB(C + C') + AB'(C + C') \\ &= ABC + ABC' + AB'C + AB'C' \end{aligned}$$

$$B'C = B'C(A + A') = AB'C + A'B'C$$

$$F = A + B'C$$

$$= ABC + ABC' + AB'C + AB'C' + A'B'C$$

$$F = A + B'C$$

$$\begin{aligned} F &= A'B'C + AB'C + AB'C + ABC' + ABC \\ &= m_1 + m_4 + m_5 + m_6 + m_7 \end{aligned}$$

$$F(A, B, C) = \Sigma(1, 4, 5, 6, 7)$$

## Product of maxterms representation:

$$F = xy + x'z$$

$$\begin{aligned} F &= xy + x'z = (xy + x')(xy + z) \\ &= (x + x')(y + x')(x + z)(y + z) \\ &= (x' + y)(x + z)(y + z) \end{aligned}$$

$$x' + y = x' + y + zz' = (x' + y + z)(x' + y + z')$$

$$x + z = x + z + yy' = (x + y + z)(x + y' + z)$$

$$y + z = y + z + xx' = (x + y + z)(x' + y + z)$$

$$\begin{aligned} F &= (x + y + z)(x + y' + z)(x' + y + z)(x' + y + z') \\ &= M_0 M_2 M_4 M_5 \end{aligned}$$

$$F(x, y, z) = \Pi(0, 2, 4, 5)$$

# Conversion between minterm and maxterm forms

$$F(A, B, C) = \Sigma(1, 4, 5, 6, 7)$$

$$F'(A, B, C) = \Sigma(0, 2, 3) = m_0 + m_2 + m_3$$

$$F = (m_0 + m_2 + m_3)' = m'_0 \cdot m'_2 \cdot m'_3 = M_0 M_2 M_3 = \Pi(0, 2, 3)$$

$$m'_j = M_j$$

Truth Table for  $F = xy + x'z$

$$F = xy + x'z$$
$$F(x, y, z) = \Sigma(1, 3, 6, 7)$$

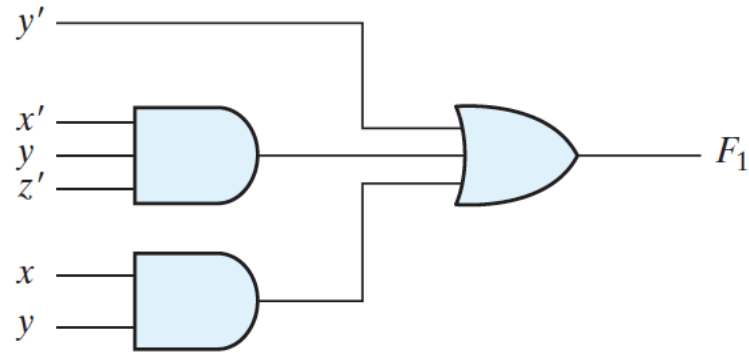
| <b>x</b> | <b>y</b> | <b>z</b> | <b>F</b> |
|----------|----------|----------|----------|
| 0        | 0        | 0        | 0        |
| 0        | 0        | 1        | 1        |
| 0        | 1        | 0        | 0        |
| 0        | 1        | 1        | 1        |
| 1        | 0        | 0        | 0        |
| 1        | 0        | 1        | 0        |
| 1        | 1        | 0        | 1        |
| 1        | 1        | 1        | 1        |

Minterms

Maxterms

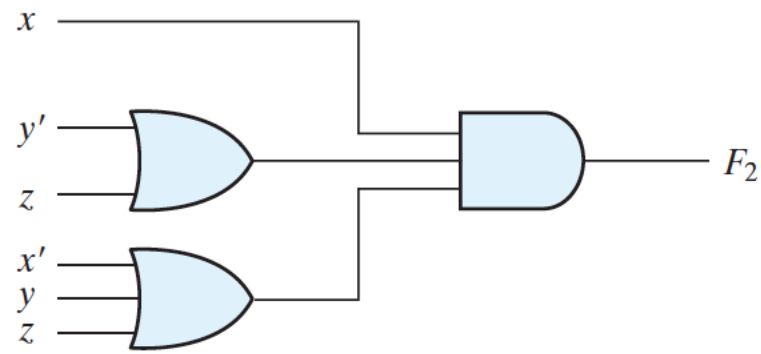
## Standard Forms: SOPs and POSs

$$F_1 = y' + xy + x'yz'$$



(a) Sum of Products






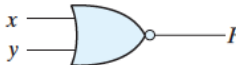


$$F_2 = x(y' + z)(x' + y + z')$$



(b) Product of Sums



# Digital Logic Gates

| Name                         | Graphic symbol  | Algebraic function                   | Truth table  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
|------------------------------|---|--------------------------------------|--|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| AND                          |    | $F = x \cdot y$                      | <table><tr><th>x</th><th>y</th><th>F</th></tr><tr><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>1</td><td>0</td></tr><tr><td>1</td><td>0</td><td>0</td></tr><tr><td>1</td><td>1</td><td>1</td></tr></table> | x | y | F | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 1 |
| x                            | y   | F                                    |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 0                            | 0   | 0                                    |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 0                            | 1   | 0                                    |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 1                            | 0   | 0                                    |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 1                            | 1   | 1                                    |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| OR                           |    | $F = x + y$                          | <table><tr><th>x</th><th>y</th><th>F</th></tr><tr><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>1</td><td>1</td></tr><tr><td>1</td><td>0</td><td>1</td></tr><tr><td>1</td><td>1</td><td>1</td></tr></table> | x | y | F | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 |
| x                            | y   | F                                    |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 0                            | 0   | 0                                    |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 0                            | 1   | 1                                    |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 1                            | 0   | 1                                    |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 1                            | 1   | 1                                    |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| Inverter                     |    | $F = x'$                             | <table><tr><th>x</th><th>F</th></tr><tr><td>0</td><td>1</td></tr><tr><td>1</td><td>0</td></tr></table>   | x | F | 0 | 1 | 1 | 0 |   |   |   |   |   |   |   |   |   |
| x                            | F   |                                      |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 0                            | 1   |                                      |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 1                            | 0   |                                      |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| Buffer                       |    | $F = x$                              | <table><tr><th>x</th><th>F</th></tr><tr><td>0</td><td>0</td></tr><tr><td>1</td><td>1</td></tr></table>   | x | F | 0 | 0 | 1 | 1 |   |   |   |   |   |   |   |   |   |
| x                            | F   |                                      |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 0                            | 0   |                                      |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 1                            | 1   |                                      |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| NAND                         |    | $F = (xy)'$                          | <table><tr><th>x</th><th>y</th><th>F</th></tr><tr><td>0</td><td>0</td><td>1</td></tr><tr><td>0</td><td>1</td><td>1</td></tr><tr><td>1</td><td>0</td><td>1</td></tr><tr><td>1</td><td>1</td><td>0</td></tr></table> | x | y | F | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 0 |
| x                            | y   | F                                    |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 0                            | 0   | 1                                    |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 0                            | 1   | 1                                    |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 1                            | 0   | 1                                    |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 1                            | 1   | 0                                    |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| NOR                          |    | $F = (x + y)'$                       | <table><tr><th>x</th><th>y</th><th>F</th></tr><tr><td>0</td><td>0</td><td>1</td></tr><tr><td>0</td><td>1</td><td>0</td></tr><tr><td>1</td><td>0</td><td>0</td></tr><tr><td>1</td><td>1</td><td>0</td></tr></table> | x | y | F | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 |
| x                            | y   | F                                    |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 0                            | 0   | 1                                    |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 0                            | 1   | 0                                    |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 1                            | 0   | 0                                    |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 1                            | 1   | 0                                    |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| Exclusive-OR (XOR)           |  | $F = xy' + x'y$<br>$= x \oplus y$    | <table><tr><th>x</th><th>y</th><th>F</th></tr><tr><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>1</td><td>1</td></tr><tr><td>1</td><td>0</td><td>1</td></tr><tr><td>1</td><td>1</td><td>0</td></tr></table> | x | y | F | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 0 |
| x                            | y   | F                                    |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 0                            | 0   | 0                                    |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 0                            | 1   | 1                                    |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 1                            | 0   | 1                                    |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 1                            | 1   | 0                                    |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| Exclusive-NOR or equivalence |  | $F = xy + x'y'$<br>$= (x \oplus y)'$ | <table><tr><th>x</th><th>y</th><th>F</th></tr><tr><td>0</td><td>0</td><td>1</td></tr><tr><td>0</td><td>1</td><td>0</td></tr><tr><td>1</td><td>0</td><td>0</td></tr><tr><td>1</td><td>1</td><td>1</td></tr></table> | x | y | F | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 1 |
| x                            | y   | F                                    |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 0                            | 0   | 1                                    |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 0                            | 1   | 0                                    |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 1                            | 0   | 0                                    |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 1                            | 1   | 1                                    |  |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |

**Thank you**