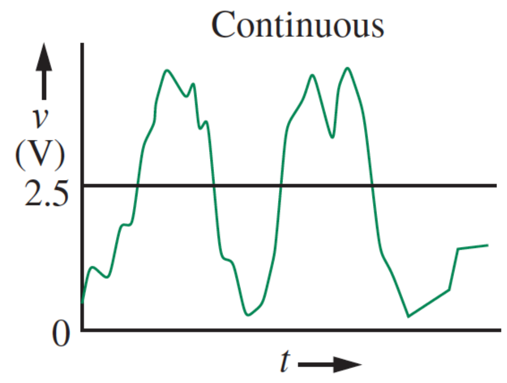
Fundamentals of Electronics

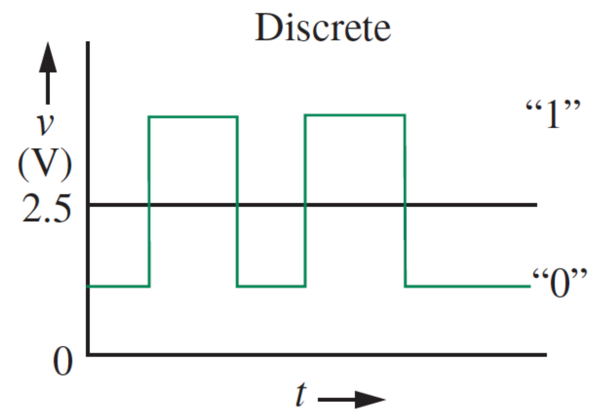
ECE 101

Digital Electronics

Introduction to digital systems

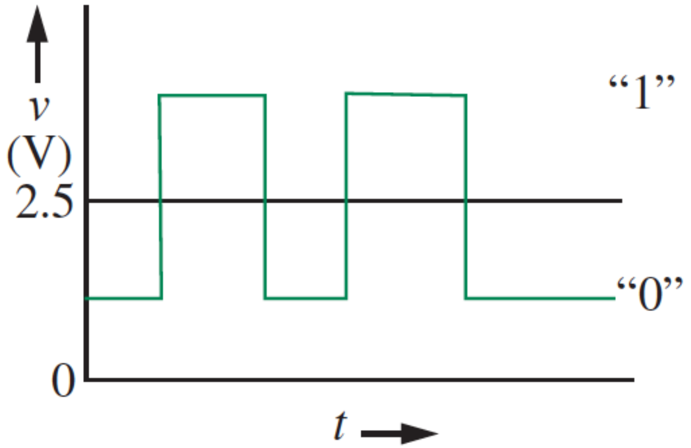


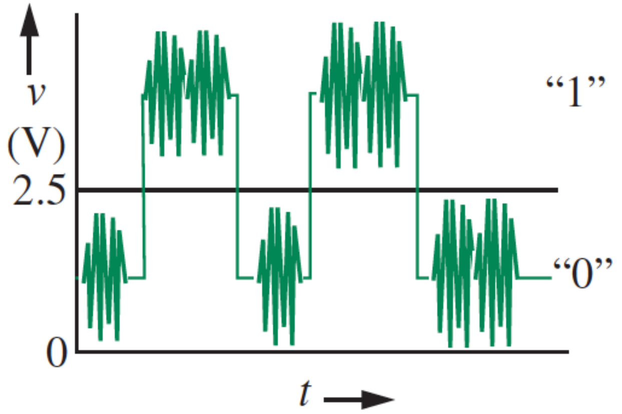
- Analog systems
- Computation: mathematical and logical operations.
- Digital electronics
  - All desired mathematical and logical operations are digitized and carried out using digital circuits.

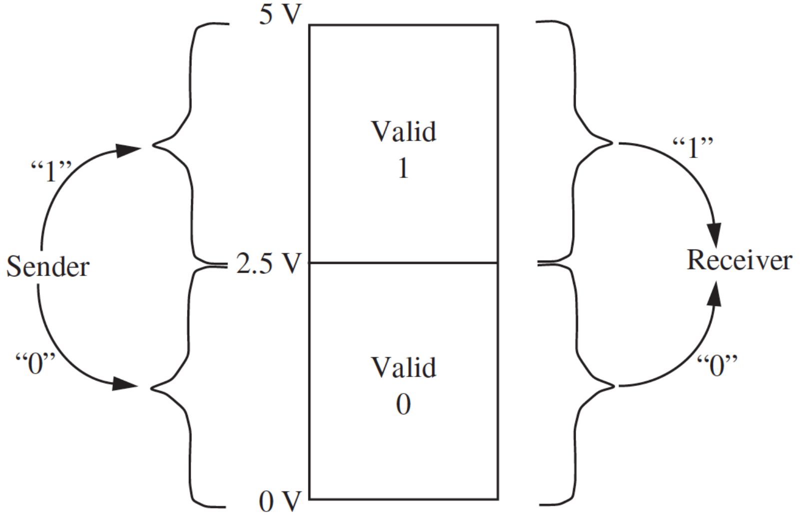


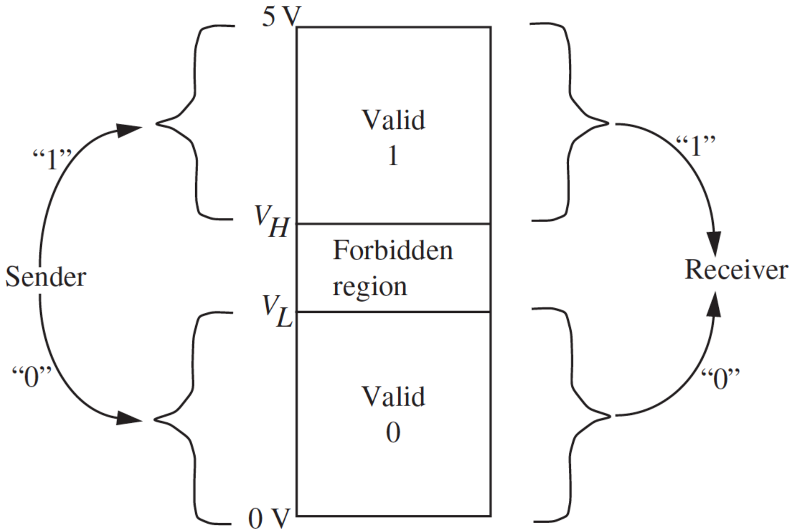
- Number system: binary numbers
- In digital realm, electronic signal is a binary number
- Analysis: Boolean algebra

Digital Signals

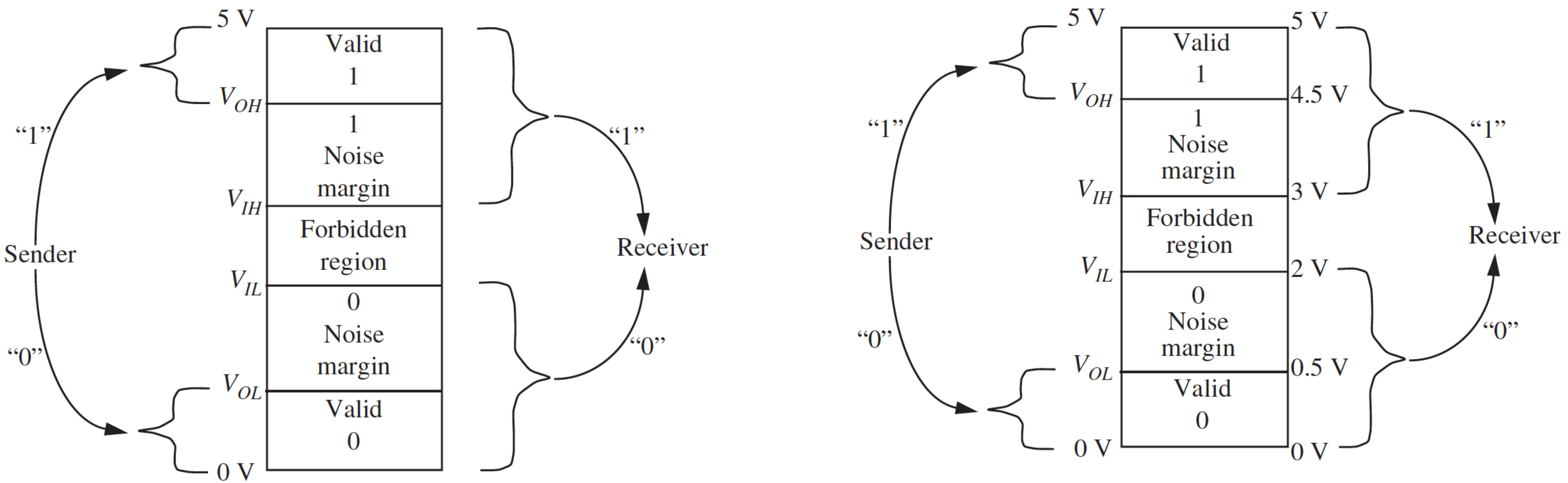








Digital Signals: considering noise margin



Boolean Logic

$$\textit{If} \quad X \textit{ is TRUE} \quad \textit{AND} \quad Y \textit{ is TRUE} \quad \textit{then} \quad Z \textit{ is TRUE} \quad \textit{else} \quad Z \textit{ is FALSE.}$$

$$Z = X \quad AND \quad Y. \qquad\qquad Z = X \cdot Y = XY.$$

X	Y	Z
0	0	0
0	1	0
1	0	0
1	1	1

$$\textit{If} \quad (A \textit{ is TRUE}) \quad OR \quad (B \textit{ is NOT TRUE}) \quad \textit{then} \quad (C \textit{ is TRUE}) \textit{ else } (C \textit{ is FALSE})$$

$$C = A + \overline{B}.$$

Boolean Algebra

Algebra is the abstract study of number systems and operations within them

- **Field**
- **Operation(s)**

Boolean Algebra

- **Field:** binary numbers (0, 1)
- **Operation(s):** $\wedge$ (AND), $\vee$ (OR), and $\neg$ ("complement" or "not")

$a \vee (b \vee c) = (a \vee b) \vee c$	$a \wedge (b \wedge c) = (a \wedge b) \wedge c$	**Associativity**
$a \vee b = b \vee a$	$a \wedge b = b \wedge a$	**Commutativity**
$a \vee (a \wedge b) = a$	$a \wedge (a \vee b) = a$	**Absorption**
$a \vee 0 = a$	$a \wedge 1 = a$	**Identity**
$a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$	$a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$	**Distributivity**
$a \vee \neg a = 1$	$a \wedge \neg a = 0$	**Complements**

Number System and Binary numbers

Discrete elements of information are represented with groups of bits called binary codes/numbers.

Any number is a series of coefficients in a certain system

$$a_5a_4a_3a_2a_1a_0.\, a_{-1}a_{-2}a_{-3}$$

$$10^5a_5 + 10^4a_4 + 10^3a_3 + 10^2a_2 + 10^1a_1 + 10^0a_0 + 10^{-1}a_{-1} + 10^{-2}a_{-2} + 10^{-3}a_{-3}$$

In binary number system, what is: $\mathbf{(11010.11)_2}$

$$1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 + 1 \times 2^{-1} + 1 \times 2^{-2} = 26.75$$

In general (base-r):

$$a_n \cdot r^n + a_{n-1} \cdot r^{n-1} + \cdots + a_2 \cdot r^2 + a_1 \cdot r + a_0 + a_{-1} \cdot r^{-1} + a_{-2} \cdot r^{-2} + \cdots + a_{-m} \cdot r^{-m}$$

Number System: some examples

$$(4021.2)_5 = 4 \times 5^3 + 0 \times 5^2 + 2 \times 5^1 + 1 \times 5^0 + 2 \times 5^{-1} = (511.4)_{10}$$

$$(127.4)_8 = 1 \times 8^2 + 2 \times 8^1 + 7 \times 8^0 + 4 \times 8^{-1} = (87.5)_{10}$$

$$(B65F)_{16} = 11 \times 16^3 + 6 \times 16^2 + 5 \times 16^1 + 15 \times 16^0 = (46{,}687)_{10}$$

$$(110101)_2 = 32 + 16 + 4 + 1 = (53)_{10}$$

Number System: some examples

Summation:

$$
\begin{array}{lr}
 & 101101 \\
 & +100111 \\ \hline
\text{sum:} & 1010100
\end{array}
$$

Subtraction:

$$
\begin{array}{lr}
 & 101101 \\
 & -100111 \\ \hline
\text{difference:} & 000110
\end{array}
$$

Multiplication:

$$
\begin{array}{lr}
 & 1011 \\
 & \times\ 101 \\ \hline
 & 1011 \\
\text{partial product:} & 0000 \\
 & 1011 \\ \hline
\text{product:} & 110111
\end{array}
$$

Number System: Decimal to Binary

Convert $(41)_{10}$ to binary

	Integer Quotient	Coefficient
$41/2 =$	20	$a_0 = 1$
$20/2 =$	10	$a_1 = 0$
$10/2 =$	5	$a_2 = 0$
$5/2 =$	2	$a_3 = 1$
$2/2 =$	1	$a_4 = 0$
$1/2 =$	0	$a_5 = 1$

Integer	Remainder
41	
20	1
10	0
5	0
2	1
1	0
0	1 101001 = answer

$$(41)_{10} = (a_5 a_4 a_3 a_2 a_1 a_0)_2 = (101001)_2.$$

Number System: Decimal to other base

Convert $(153)_{10}$ to octal

$$\begin{array}{r|l} 153 & \\ 19 & 1 \\ 2 & 3 \\ 0 & 2 = (231)_8 \end{array}$$

Convert $(0.6875)_{10}$ to binary

	Integer		Fraction	Coefficient
$0.6875 \times 2 =$	1	+	0.3750	$a_{-1} = 1$
$0.3750 \times 2 =$	0	+	0.7500	$a_{-2} = 0$
$0.7500 \times 2 =$	1	+	0.5000	$a_{-3} = 1$
$0.5000 \times 2 =$	1	+	0.0000	$a_{-4} = 1$

$$(0.6875)_{10} = (0.\, a_{-1}\, a_{-2}\, a_{-3}\, a_{-4})_2 = (0.1011)_2.$$

Convert $(0.513)_{10}$ to octal

$$\begin{aligned} 0.513 \times 8 &= 4.104 \\ 0.104 \times 8 &= 0.832 \\ 0.832 \times 8 &= 6.656 \\ 0.656 \times 8 &= 5.248 \\ 0.248 \times 8 &= 1.984 \\ 0.984 \times 8 &= 7.872 \end{aligned}$$

$$\boxed{(0.513)_{10} = (0.406517\ldots)_8}$$

Octal and Hexadecimal Numbers

$$\begin{array}{ccccccccccl} (10 & 110 & 001 & 101 & 011 & \cdot & 111 & 100 & 000 & 110)_2 & = (26153.7406)_8 \\ 2 & 6 & 1 & 5 & 3 & & 7 & 4 & 0 & 6 & \end{array}$$

Decimal (base 10)	Binary (base 2)	Octal (base 8)	Hexadecimal (base 16)
00	0000	00	0
01	0001	01	1
02	0010	02	2
03	0011	03	3
04	0100	04	4
05	0101	05	5
06	0110	06	6
07	0111	07	7
08	1000	10	8
09	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

$$\begin{array}{ccccccc} (10 & 1100 & 0110 & 1011 & \cdot & 1111 & 0010)_2 \\ 2 & C & 6 & B & & F & 2 \end{array}$$

$$= (2C6B.F2)_{16}$$

$$(673.124)_8 = \begin{array}{ccccccc} (110 & 111 & 011 & \cdot & 001 & 010 & 100)_2 \\ 6 & 7 & 3 & & 1 & 2 & 4 \end{array}$$

$$(306.D)_{16} = \begin{array}{ccccc} (0011 & 0000 & 0110 & \cdot & 1101)_2 \\ 3 & 0 & 6 & & D \end{array}$$

Digital computing

In digital computing:

n	2^n	n	2^n	n	2^n
0	1	8	256	16	65,536
1	2	9	512	17	131,072
2	4	10	1,024 (1K)	18	262,144
3	8	11	2,048	19	524,288
4	16	12	4,096 (4K)	20	1,048,576 (1M)
5	32	13	8,192	21	2,097,152
6	64	14	16,384	22	4,194,304
7	128	15	32,768	23	8,388,608

- 2^{10} is referred to as K (kilo)
- 2^{20} as M (mega)
- 2^{30} as G (giga)
- 2^{40} as T (tera)
- 1 byte = 8 bits (one byte can store one character, e. g. 'A' or 'x' or '$')
- Thus, 4K = 2^{12} = 4,096 and 16M = 2^{24} = 16,777,216.

Complement of numbers: Radix and Diminished radix

r' s= complement= $\{(\mathbf{r^n})_{10}\}_r$ - **N**

(r-1)' s complement= $\{(\mathbf{r^n})_{10} - 1\}_r$ - **N**

The n is the number of digits in the number.
The N is the given number.
The r is the radix or base of the number.

Examples: **1. $(1011000)_2$**

1's complement: $\{(2^7)_{10}-1\} - (1011000)_2 = \{(128)_{10}-1\}_2 - (1011000)_2 = 1111111_2 - 1011000_2 = 0100111$

2's complement: $\{(2^7)_{10}\} - (1011000)_2 = \{(128)_{10}\}_2 - (1011000)_2 = 10000000_2 - 1011000_2 = 0101000_2$

2. $(155)_{10}$

9's complement: $\{(10^3)_{10}-1\} - (155)_{10} = (1000-1) - 155 = 999 - 155 = (844)_{10}$

10's complement: $\{(10^3)_{10}\} - (155)_{10} = (1000) - 155 = 1000 - 155 = (845)_{10}$

3. $(174)_8$ = ?

Thank you