LA Assignment 3

February 7, 2025

1. Show that a subset S of a vector space V defined over a field F is a subspace if and only if $\forall \vec{\alpha}, \vec{\beta} \in S$, $c \in F$, we have that $c\vec{\alpha} + \vec{\beta} \in S$.

2. Show that all 2×2 Hermitian matrices over $\mathbb{C}^{2\times 2}$ are of the following form:
$$M = \begin{bmatrix} a & x + yi \\ x - yi & b \end{bmatrix}$$
where $a, b, x, y \in \mathbb{R}$. Additionally, show that the set of Hermitian matrices $\mathcal{H} \subseteq \mathbb{C}^{n\times n}$ is not a vector subspace of $\mathbb{C}^{n\times n}$. What if $\mathbb{C}$ was replaced by $\mathbb{R}$?

3. Show that the subspace spanned by a non-empty subset S of a vector space V is the set of all linear combinations of vectors in S.

4. Show that if $(W_i)_{i=1}^k$ are subspaces of a vector space V, then $\sum_{i=1}^k W_i$ is a subspace, and is spanned by the vector set formed by $\bigcup_{i=1}^n W_i$.

5. Let $A \in F^{m\times n}$, $S_A = \{X \in F^n \mid AX = \vec{0}\}$, that is, S_A is the solution space of $AX = \vec{0}$. Find the number of linearly independent $X \in S_A$.

6. Let V be a vector space spanned by $(\vec{\beta}_i)_{i=1}^n$. Then prove that any independent set of vectors in V is finite and contains no more than n elements.