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Kinematic Sensitivity Evaluation of Revolute and Prismatic 3-DOF Delta Robots

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***Abstract*—The Delta parallel robot establishes itself as one of the most successful parallel robot designs thanks to its intrinsic properties. Generally, based on the type of actuator, the 3-DOF Delta robot falls into two groups, namely, revolute-input and prismatic-input. The main aim of this paper consists in evaluating and comparing the kinematic performance of the two foregoing types of Delta robots. So far, different kinematic sensitivity indices have been proposed. In this paper, by employing different kinematic sensitivity indices such as manipulability, dimensionally kinematic sensitivity and weighted kinematic sensitivity indices. the kinetostatic performance of the aforementioned robots are compared over a certain workspace. The obtained results showed that the sensitivity of prismatic Delta robot is high in the center of the workspace and the sensitivity increases when the end-effector moves to edges from the center. However sensitivity of the revolute Delta robot is totally different. It is high on the edges and it decreases by moving to center. weighted kinematic sensitivity results show that geometrical features of the robots are very important and it changes the sensitivity values and trend greatly. Moreover, in order to enhance the kinematic performance of the robots, the configuration of the robots are redesigned using optimization of the kinematic sensitivity indices. The results of this paper revealed that kinematic sensitivity indices can be employed in the design step of the robot to reach an optimal kinematic sensitivity in the workspace.**

***Index Terms*—Kinematic sensitivity indices, Delta Robot, Weighted kinematic sensitivity, Sobol's method, Kinematic modeling.**

I. Introduction

Parallel robots are closed-loop kinematic chains connecting a fixed base of the robot to its moving platform. This class of robots has a variety of intrinsict characteristics, such as high velocity capability, high rigidity, agility, and capability of carrying large loads [1]. Since advent of parallel robots, they have been employed in many different applications like, flight simulation [2], 3D printing [3], and pick and place applications [4].

A research group, led by Reymond Clavel, invented the original 3-DOF rotational Delta robot in the early 1980s at the Ecole Polytechnique Federale de Lausanne [5]. The industrial demand at that time was to pick-and-place light and small objects very quickly, so this robot was designed to satisfy industrial requirements [6]. The concept of this 3-DOF Delta robots is to limit the mobility of the end-effector to translational with no rotation by using parallelograms [1]. Many industries use Delta robots because of their superior characteristics, like high velocity and acceleration, high stiffness, and precision. The other types of 3-DOF Delta robot is the prismatic Delta in which active revolute joints are replaced with prismatic joints. In this type of robot, because of less active linkages, rigidity is higher but the total precision depends on active joints uncertainties. In order to make a good choice between these two kinds of Delta robots, an assessment should be taken. These robots are used in many fields, such as 3D printing, pharmaceutical industries, haptic controllers [7] and medical applications [8].

Many researchers have contributed to sensitivity analysis of robots to measure and compare different robots precision and certainty, among those, are manipulability [9] and dexterity [10] that has been used for decades. These indices are suggested to evaluate robot output according to its architectural features, but these indices have some drawbacks, like the impossibility of propounding a solitary invariant metric for the special Euclidean group [11]. Later, in 2010 Cardou et al. [11] proposed kinematic sensitivity index to go over these limitations. However, this index also has some issues. One important issue is that in the determination of the indices, the effect of every joint are taken equal which is ideal and real sensitivity index should entail a different proportion of uncertainty for each different active joint. The kinematic sensitivity indices have been applied on variety of robots such as 3-RPR planar parallel manipulator [12], [13] and 3-UPU parallel mechanism [14].

Lately, an improved method is suggested which employs statistical sensitivity analysis to calculate geometry oriented kinematic sensitivity [15]. Sensitivity analysis methods are statistical methods which calculate the effect of each input uncertainty on the objectives. These methods are used in many different fields, such as economics, applied science, and engineering. In robotics, some sensitivity analysis researches are done, for example, the sensitivity of n-revolute robots to geometric and design parametrs [16], length and angular variations of the manipulator [17] and kinematic parameters [18] are implemented. Weighted kinematic sensitivity uses Sobol's sensitivity analysis method to determine the weight matrix. This method has been used to evaluate parallel robots, such as 4-DOF Delta robot [15]. In the latter study, it has been

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shown that the improved method considers the geometrical features and different joint's uncertainties which suggest an index closer to the reality [15].

The performances of parallel mechanisms are highly affected by their kinematic sensitivity. So far, numerous examples reported that a careful design optimization can lead to significant improvements over the kinematic performances [19]. In the literature, different criteria have been used to optimize the performance of the robots such as kinetostatic performance using evolutionary techniques [20], [21] , greatest continuous circle in the constant-orientation workspace [22] and kinematic sensitivity [23]. However, far too little attention has been paid to using a reliable kinematic sensitivity indices as a criterion for the optimization of the robot structure.

In this paper, precision and certainty, as important factors in industrial and medical applications, are investigated for both prismatic and revolute 3-DOF Delta robots. To this end, kinematic modeling of revolute and prismatic 3-DOF Delta manipulator is implemented. Formulation of manipulability, dexterity, kinematic sensitivity, and weighted kinematic sensitivity is presented. Different sensitivity indices are calculated and Weighted kinematic sensitivity of 3-DOF parallel Delta manipulators are implemented and compared for the first time.

This paper is organized as follows. First, kinematic sensitivity indices are presented in Section II. Then, kinematic modeling of Delta robots is implemented in Section III. Section IV is dedicated to the results and discussion. The Optimization of kinematic sensitivity of the robot to the design parameters of the it is presented in Section V. Finally, the paper concludes by some remark and hints as ongoing works.

II. Kinematic Sensitivity Indices

The kinematics sensitivity indices are mathematical expressions defining the kinematic performances of the manipulator such as velocity and acceleration which are related to the design parameters. Generally, a high or very low kinematic sensitivity of the manipulator is undesirable. More precisely, a high sensitivity would describe that a small changes at the actuator joint space would be amplified into a large orientation change of the end-effector. On the other hand, a very low kinematic sensitivity would indicate that the orientation of the end-effector is not controllable because a change in the joint coordinates would cause no effect on the end-effector orientation [24].

Consider $\dot{\boldsymbol{\theta}}$ and $\dot{\mathbf{x}}$ as small variations in the actuator joint coordinates and end-effector orientation coordinates respectively. In general, differentiating the loop-closure equations of such a parallel manipulator results in its first-order kinematics relationships, which can be formulated as follows:

$$\dot{\mathbf{x}} = \mathbf{J}\dot{\boldsymbol{\theta}} \tag{1}$$

where $\mathbf{J}$ is the Jacobian matrix. Generally, the Jacobian matrix describes a linearized matrix mapping from velocities in joint space to velocities in Cartesian space. Here, for the sake of quick reference, we introduce only an overview of some of the numerous performance indices which are based on the Jacobian matrix and by the means of theses indices, the kinematic performance of two types of 3-DOF Delta robot, namely, revolute-input and prismatic-input are investigated.

A. Manipulability

Generally, the transformation which maps the hypersphere in the actuators errors space into an ellipsoid is called as manipulability. Ellipsoid is in the generalized coordinates' error space. Yoshikawa [9] introduced the manipulability index for a robot as follows:

$$m(\mathbf{J}) = \sqrt{|\mathbf{J}\mathbf{J}^T|} \tag{2}$$

As shown in Eq. (2), the concept of manipulability is defined on the basis of the Jacobian matrix, while the rotational and translational parts of dimensionally non-homogeneous Jacobain matrix are merged together. As a result, manipulability indicates physically inapplicable interpretations about the mechanisms, as it is normalizing unit-inconsistent quantities.

B. Indices for dimensionally nonhomogeneous Jacobian

As aforementioned, generally for a parallel manipulator which performs both position and orientation tasks, the components of the Jacobian matrix are dimensionally nonhomogeneous. In order to overcome the latter problem, using a geometric standpoint, kinematic sensitivity is defined as the maximum displacement/rotation of the end-effector of the mechanism, under a unit norm displacement/rotation in the joint space which have the general form of [11]:

$$\sigma_{r_{c,f}} \equiv \max_{\|\boldsymbol{\theta}\|_c=1} \|\phi\|_f, \text{ and } \sigma_{p_{c,f}} \equiv \max_{\|\boldsymbol{\theta}\|_c=1} \|\mathbf{p}\|_f \tag{3}$$

where $\sigma_{r_{c,f}}$ and $\sigma_{p_{c,f}}$ denote the rotational and point-displacement kinematic sensitivity, respectively. Moreover, vector $\mathbf{p}$ and scalar ϕ are the decomposition of the $\dot{\mathbf{x}}$ into variation of position and orientation of the moving platform, respectively. In the latter equation, notion c and f can be 2 or ∞, which leads to 4 individual indices. It has been proved in [25], [26], using $c = \infty$ and $f = 2$ leads to dependence of frame that indices are calculated in which and consequently infinity constraints norm and second norm objective function bears the most meaningful case. Hence, for the aim of this paper, the aforementioned indices are considered in the case of $\sigma_{\infty,2}$ for both translation and rotation indices.

C. Weighted kinematic sensitivity Indices

As shown in previous Sections, two aforementioned indices have considered equal proportion of errors in the actuated joint coordinates. But in reality, all manipulators have uncertainties which related to small manufacturing errors and clearances [27]. Hence, weighted kinematic sensitivity approach [15] is proposed to take account the difference in sensitivity of manipulator to the active joints uncertainties. According to

Eqs. (1) and (3), the constraint $c = \infty$ represents effect of unit bounds of the small actuator displacement errors on the set of possible end-effector pose error and can be rewritten as follows:

$$\|\mathbf{Kx}\|_\infty \leq \mathbf{1} \tag{4}$$

To consider different uncertainties portion, a sensitivity weighted matrix should be taken into account. To this end, in [15], the following equation is proposed:

$$\|\mathbf{Kx}\|_\infty \leq \mathbf{W1} \tag{5}$$

where $\mathbf{K}$ is $\mathbf{J}^{-1}$ and $\mathbf{W}$ is a weighted matrix which is derived based on statistics approaches [15]. In order to have a better interpretation on the weighted kinematic sensitivity, Fig. 1 gives an example of comparison between different proposed indices. As it can be inferred from the latter figure, manipulability and global kinematic sensitivity consider a sphere in the joint workspace, while weighted approach provides an ellipsoid which represents the effect of different uncertainties in the actuated joints.

Now, in order to apply the aforementioned indices on two Delta robots, first, the inverse kinematics problem is a crucial issue for analyzing the robots kinematic sensitivity. Although the main focus of this paper consists in analyzing the kinematic sensitivity of the aforementioned robots, but as the Jacobian matrix serves as the first step in the kinematic sensitivity analyzing, therefore, in the next Section, kinematic modeling of these two robots is briefly, presented.

III. Kinematic Modeling

As shown in the previous Section, Jacobian matrix plays an vital role in kinematic analysis of the robot and it serves as the first step of analysis. Thus, in this Section, kinematic modeling of each aforementioned Delta robots is broadly presented and then the kinematic sensitivity of these robots is evaluated. In Sec.III-A, kinematic modeling of 3-DOF prismatic Delta robot is presented and Sec.III-B devoted to 3-DOF revolute Delta robot.

A. 3-DOF prismatic Delta

The schematic of the inverse kinematic of the robot is illustrated in Fig. 2. Based on the latter figure, let $\vec{OE} = \mathbf{p}$, $\vec{OA_i} = \mathbf{a}_i$, $\vec{A_iB_i} = \boldsymbol{\rho}_i$, $\vec{B_iC_i} = \mathbf{L}_i$ and $\vec{EC_i} = \mathbf{c}_i$. Considering the closed-loop kinematic chains $O - A_i - B_i - C_i - E$, for $i = 1, 2$ and 3, leads to the following equation:

$$\mathbf{p} = \mathbf{a}_i + \boldsymbol{\rho}_i + \mathbf{L}_i - \mathbf{c}_i \tag{6}$$

Resorting to Eq. (6), one can write $\mathbf{L}_i = \mathbf{p} + \mathbf{c}_i - \mathbf{a}_i - \boldsymbol{\rho}_i$. Assuming that the length of arms, $\mathbf{L}_i$, is constant:

$$l_i^2 = \|\mathbf{L}_i\|^2 = \mathbf{L}_i^T\mathbf{L}_i \tag{7}$$

Now, three constraints equations for the 3-DOF prismatic Delta robot can be derived from Eq. (7). In order to introduce the first time derivative in joint space and in Cartesian space which the Jacobian matrix maps between one can take the time derivative of the constraints equations which gives:

$$\mathbf{J}'\dot{\mathbf{p}} = \mathbf{K}'\dot{\boldsymbol{\rho}} \tag{8}$$

For the sake of simplicity, matrices $\mathbf{J}'$ and $\mathbf{K}'$, finnaly, can be written as:

$$\mathbf{J}' = \begin{bmatrix} x & y+\alpha & z+\rho_1 \\ x+\beta & y+\gamma & z+\rho_2 \\ x-\beta & y+\gamma & z+\rho_3 \end{bmatrix} \tag{9a}$$

$$K'_{ij} = \begin{cases} -(z+\rho_i) & i = j \\ 0 & i \neq j \end{cases} \tag{9b}$$

where,

$$\alpha = \frac{\sqrt{3}}{3}\left(\frac{d_B}{2} - d_E\right) \tag{10a}$$

$$\beta = \frac{1}{2}\left(d_E - d_B\right) \tag{10b}$$

$$\gamma = \frac{\sqrt{3}}{6}(d_E - \frac{d_B}{2}) \tag{10c}$$

$$d_E = \left\|\vec{C_1C_2}\right\| = \left\|\vec{C_2C_3}\right\| = \left\|\vec{C_1C_3}\right\| \tag{10d}$$

$$d_B = \left\|\vec{A_1A_2}\right\| = \left\|\vec{A_2A_3}\right\| = \left\|\vec{A_1A_3}\right\| \tag{10e}$$

Finally, the corresponding Jacobian matrix can be written as $\mathbf{J} = \mathbf{J}'^{-1}\mathbf{K}$.

B. 3-DOF revolute Delta

The schematic representation of a 3-DOF revolute Delta robot is illustrated in Fig. 3. Let $\vec{OE} = \mathbf{p}$, $\vec{OA_i} = \boldsymbol{\rho}_i$, $\vec{A_iB_i} = \mathbf{a}_i$, $\vec{B_iC_i} = \mathbf{b}_i$, $\vec{C_iE} = \mathbf{c}_i$. From the latter figure, one can readily write the following closed-loop kinematic chains $O - A_i - B_i - C_i - E$, for $i = 1, 2, 3$:

$$\mathbf{p} = \boldsymbol{\rho}_i + \mathbf{a}_i + \mathbf{b}_i + \mathbf{c}_i \tag{11}$$

To derive the Jacobian matrix, one can take the time derivative of Eq. (11). Thus, Differentiating the latter equation with respect to time results in:

$$\dot{\mathbf{a}}_i + \dot{\mathbf{b}}_i - \dot{\mathbf{p}} = \mathbf{0} \tag{12}$$

As $\dot{\mathbf{a}}_i = \dot{\theta}_i\left(\mathbf{e}_i \times \mathbf{a}_i\right)$ and $\dot{\mathbf{b}} = \boldsymbol{\omega}' \times \mathbf{b}_i$, where $\boldsymbol{\omega}'$ is the angular velocity of second link, Eq. (12) can be written as:

$$\dot{\theta}_i\left(\mathbf{e}_i \times \mathbf{a}_i\right) - \left(\boldsymbol{\omega}' \times \mathbf{b}_i\right) = \dot{\mathbf{p}} \tag{13}$$

Dot product both sides of Eq. (13) by vector $\mathbf{b}$ leads to:

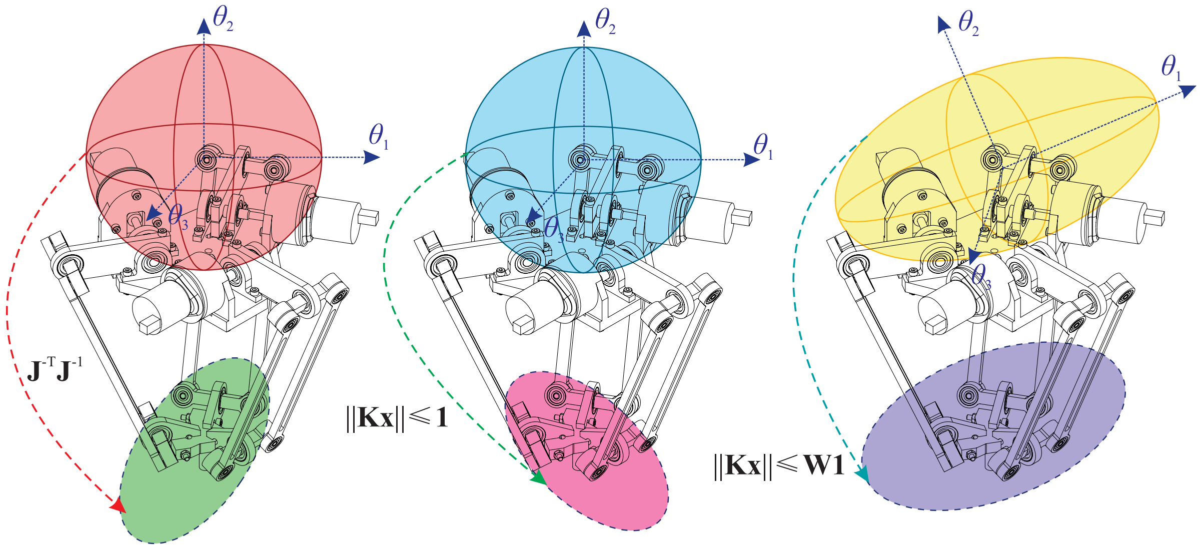
$$\dot{\theta}_i\left(\mathbf{e}_i \times \mathbf{a}_i\right) \cdot \mathbf{b}_i = \dot{\mathbf{p}} \cdot \mathbf{b}_i \tag{14}$$

The matrix form of the kinematic relation can be written as:

$$\mathbf{J}'\dot{\boldsymbol{\theta}} = \mathbf{K}'\dot{\mathbf{p}} \tag{15}$$

where,

$$J'_{ij} = \begin{cases} 0 & i \neq j \\ \left(\mathbf{e}_i \times \mathbf{a}_i\right) \cdot \mathbf{b}_i & i = j \end{cases} \tag{16}$$



(a) Manipubility (b) Standard kinematic sensitivity (c) Weighted kinematic sensitivity

Fig. 1: Graphical comparison between different proposed kinematic sensitivity indices.

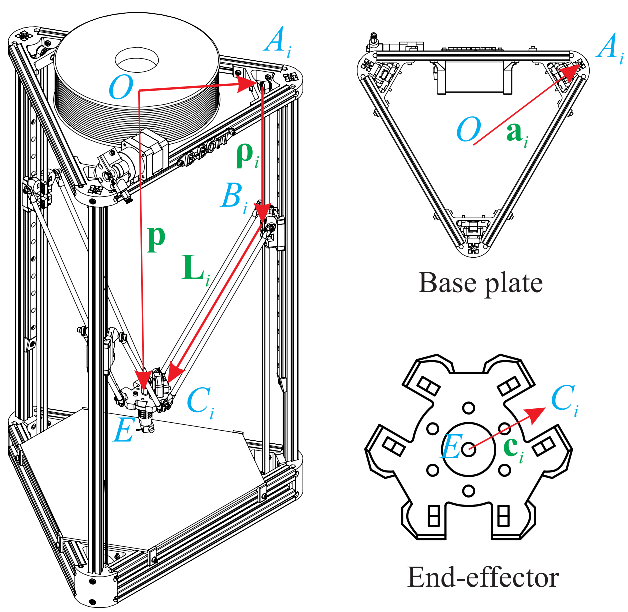


Fig. 2: CAD model of the 3-DOF prismatic Delta parallel manipulator.

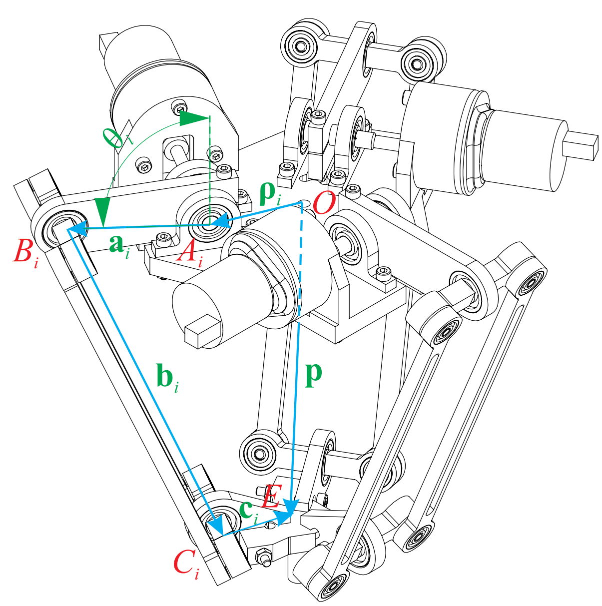


Fig. 3: CAD model of the 3-DOF revolute Delta parallel manipulator.

$$\left[\dot{\boldsymbol{\theta}}\right] = \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix}, \ [\mathbf{K}'] = \begin{bmatrix} \mathbf{b}_1^T \\ \mathbf{b}_2^T \\ \mathbf{b}_3^T \end{bmatrix} \tag{17}$$

Finally, the corresponding Jacobian matrix can be written as $\mathbf{J} = \mathbf{J}'^{-1}\mathbf{K}$.

IV. Result and Discussion

In this section, performance of two different actuated robots are investigated. To this end, a clear and definite workspace for two robots is considered and different sensitivity indices are calculated and compared in the workspace. The corresponding workspace is a flat surface parallel to xy plane. In order to have a smooth graph, sensitivity indices are calculated in 40×40 equally distributed points through the plane. In each point, Jacobian of the prismatic robot is calculated by Eqs. (15),

(16) and (17) and Jacobian of the revolute robot is calculated by Eqs. (8), (9) and (10). When the Jacobian is known, manipulability index can be obtained by Eq. (2). Global kinematic sensitivity, and weighted kinematic sensitivity are calculated by Eq. (4) and Eq. (5), respectively.

As the two robots have different types of actuators therefore, their Jacobian and kinematic sensitivity indices are unit-inconsistent quantities. In order to compare these two, all indices are normalized with respect to their maximum values. For the sake of having a better illustration, results are shown both in contour and surface format.

Manipulability of the prismatic and revolute 3-DOF Delta robot are shown in Figs. 4(a) and 4(b) respectively. These results show that trend of manipulability in these two robots are opposite of eachother. In the center of workspace, manipulability of prismatic Delta is at its maximum value, while the maximum of the manipulability of the revolute Delta is in the edges of the workspace and its minimum value is in the center. Global kinematic sensitivity of two robots are illustrated in Figs. 4(c) and 4(d). From these results it can be concluded that the sensitivity of the prismatic Delta is high at the center and as the distance from the center is increasing the sensitivity is reducing, while the sensitivity of the revolute Delta is low at the center of the corresponding workspace and as the robot moves to the edges, sensitivity gets higher. These results are similar to the results obtained from manipulability, while the results obtained from weighted kinematic sensitivity show a different trend which illustrate the effect of considering geometrical features that has taken into account in the weighted kinematic sensitivity index. The weighted kinematic sensitivity results are shown in Figs. 4(e) and 4(f). The sensitivity of the prismatic Delta shows different values from those two other indices, but total trend is similar. The weighted kinematic sensitivity of revolute Delta is totally different from other sensitivity indices. Weighted kinematic sensitivity considers the geometrical features of the robots and suggests a more realistic sensitivity index.

V. Optimization of Kinematic Sensitivity

In this Section, an application of kinematic sensitivity analysis indices is presented. One of the effective parameters on the kinematic sensitivity of the robot is the geometric design parameters, such as links length. Therefore, here, design parameters of the robots are redesigned based on the kinematic sensitivity. More precisely, design parameters of the robots are analyzed to reach optimized kinematic sensitivity over the workspace. To do so, for the prismatic-input robot, a $\pm 10\%$ tolerance for the length of the two links namely $\vec{BC}$ -or $\|\mathbf{L}\|$- and $\vec{CE}$ -or $\|\mathbf{c}\|$- (see Fig. 2) is considered and consequently, the Standard Deviation (SD) of the kinematic sensitivity indices over the workspace is derived. Fig. 5 shows the relation between the kinematic sensitivity indices and the tolerance of the two aforementioned links. Same procedure is applied for the revolute-input robot with the difference that $\pm 10\%$ tolerance is considered for the $\vec{AB}$ -or $\|\mathbf{a}\|$- and $\vec{BC}$ -or $\|\mathbf{b}\|$- (see Fig. 3) and the obtained results is presented are Fig. 6.

As it can be inferred from the aforementioned figures, for the revolute-input robot, the tolerance of the link's length have a smaller effect on the kinematic sensitivity, while in the prismatic-input, it exhibits a higher impact on the kinematic sensitivity. Another important implication of Fig. 6 is that increasing $\|\mathbf{a}\|$ leads to an increment in kinematic sensitivity, while this trend is vise versa for the $\|\mathbf{b}\|$. The foregoing issue could be attributed this fact that $\|\mathbf{a}\|$ is directly connected to the actuators and the uncertainties in the actuators directly applied to this link, while the second link, $\|\mathbf{a}\|$ is closer the end-effector and therefore, less proportion of the uncertainties is related to this link. Similarly, same conclusion can be drawn from Fig. 5, where increasing the length of the link $\|\mathbf{L}\|$ increases the kinematic sensitivity of the robot, while the link $\|\mathbf{c}\|$ have a contrariwise effect. It can be concluded from this Section that as an application of the kinematic sensitivity indices, in the design step of the robot, it can be used in order to optimize the design parameters of the robot.

VI. Conclusion

The purpose of the current study was to evaluate the kinematic performance of two families of 3-DOF Delta robots, namely, revolute-input and prismatic-input. In order to compare the kinematic sensitivity of these two robots, different criteria such as manipulability, dimensional and weighted kinematic sensitivity indices are used for evaluating the robots. A rectangular meshes of robot workspace was taken into account and the aforementioned indices were derived over the workspace. Based on the kinematic sensitivity indices, the kinematic performance of two robots were compared. Dimensional and weighted kinematic sensitivity indices showed a totally different trend for two different robots. In order to satisfy different sensitivity criteria, it is very important to choose a proper precision to make sensitivity in the critical points negligible. Moreover, as an application of the kinematic sensitivity, by means of optimization of the sensitivity indices, the design parameters of robot, i.e. length of links, were redesigned. The obtained results revealed that kinematic sensitivity indices can be used to optimized the design parameters of the robots and reach minimum of the kinematic sensitivity. In future investigations, it might be possible to use the reported results in this paper in controlling uncertainties of actuated joints of robots. Therefore, a further study with more focus on uncertainties in the passive joints is suggested.

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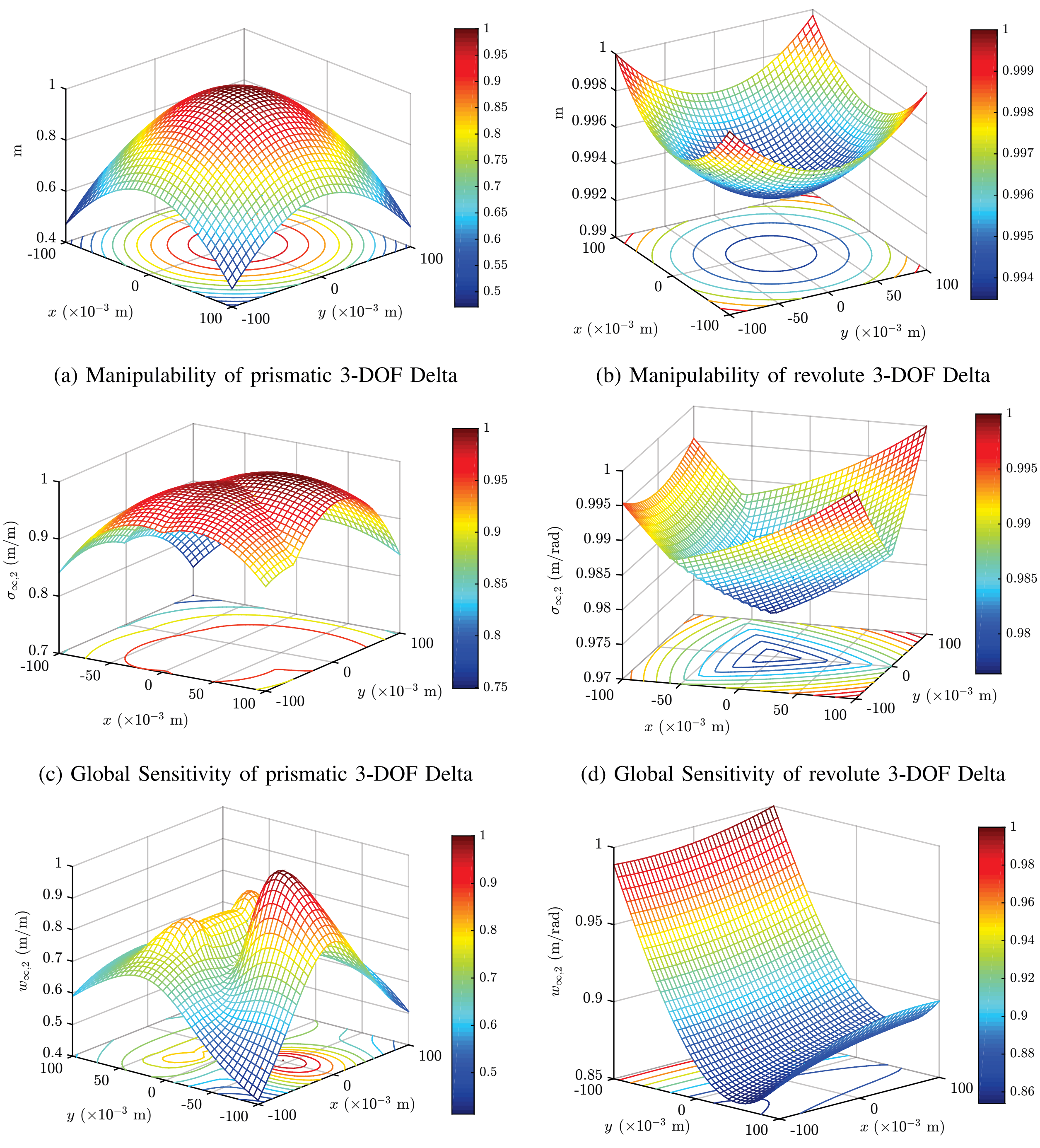

(a) Manipulability of prismatic 3-DOF Delta

(b) Manipulability of revolute 3-DOF Delta

(c) Global Sensitivity of prismatic 3-DOF Delta

(d) Global Sensitivity of revolute 3-DOF Delta

(e) Weighted Sensitivity of prismatic 3-DOF Delta

(f) Weighted Sensitivity of revolute 3-DOF Delta

Fig. 4: Kinematic sensitivity of prismatic and revolute 3-DOF Delta robots

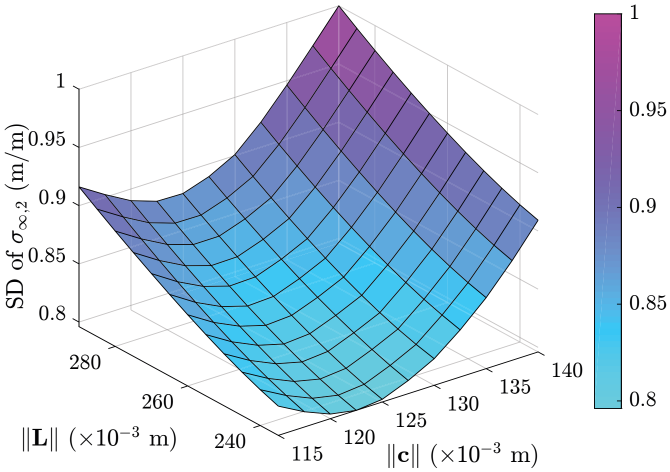

Fig. 5: The effect of design parameters on the kinematic sensitivity indices of the prismatic 3DOF Delta robot.

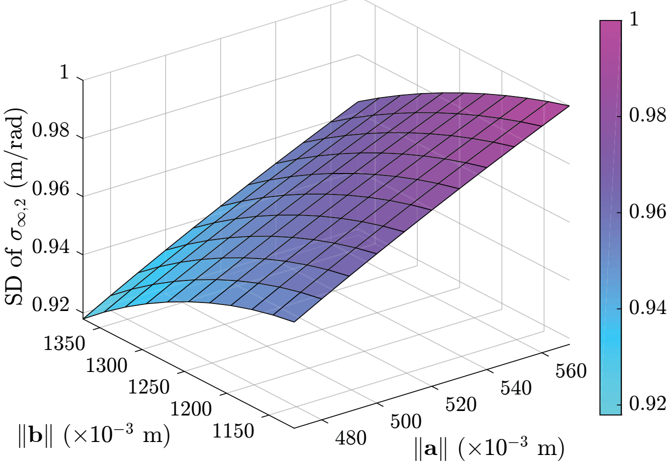

Fig. 6: The effect of design parameters on the kinematic sensitivity indices of the revolute 3DOF Delta robot.

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