A New Approach for Kinematics-based Design of 3-RRR Delta Robots with a Specified Workspace

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***Abstract*—One of the most significant issues in design of the parallel robots is to determine proper kinematic parameters leading to a desired workspace. The aim of this research is to propose a new approach for kinematics-based design of 3-RRR Delta robots considering a specified workspace. To this end, a new concept, called the Maximum Surrounded Workspace (MSW),will be presented, which is the basis of the new design methodology utilized in this paper. Additionally, the kinematic parameters of some of the prominent industrial 3-RRR Delta robots and their relationships have carefully been examined which are then will be employed in the proposed design procedure. The obtained real-world results of using the proposed approach for designing a 3-RRR Delta robot sample, reveal not only the proper performance of the suggested method, but also its simplicity and efficiency.**

Keywords—Kinematics; Workspace; Manipulator Design; Delta robot;

I. INTRODUCTION

In recent years, a considerable amount of research has been focused on the design and development of parallel manipulators (PMs) due to their advantages over serial manipulators in terms of high precision, velocity, stiffness and payload capacity. These advantages encourage their wide range of exploitations in industrial applications, flight simulators and milling machines, etc. However, one of the major drawbacks of PMs is their limited workspace[1].A parallel robot typically is a closed-loop kinematic chain mechanism whose end-effect or is linked to the fixed base by some independent kinematic chains[2].In PMs' family, different types of architectures have been suggested for relevant applications[3].Clavel's3-RRR Delta robot is one of the most famous parallel platforms with high acceleration [4], [5] whose pluses resulted into its vast industrial employment. This robot, see Fig. 1, with three translational DOF has a wide range of applications as diverse as packaging industry, production lines, medical and pharmaceutical industry, etc.

One of the most notable issues in the process of design of parallel robots is their workspace. In the literature, different methods to determine the workspace of a parallel robot have been suggested using geometrical and/or algebraic approaches providing results numerically or analytically [1], [6], [7], [8], [9].The first investigations of robot workspace were reported in [6] and [7]. Most of the numerical techniques to determine workspace of PMs are based on the discretization of the pose parameters in order to determine the workspace boundary [8], [9]. With discretization method, the inverse kinematics are solved for each potential pose of the end-effect or, and the designer verifies that all the constraints are satisfied.Moreover, some studies proposed optimization techniques to determine workspace.

Although deriving the workspace of a parallel robot is a hard problem, the design of a parallel robot for a given workspace is, however, more challenging. Therefore, any algorithmic technique to solve this topic is welcome, see[10], [11], [12] and [13].A GA-based method has been proposed in [14] tooptimal dimensional synthesis of the Delta parallel robots with a prescribed workspace. In [15] an approach to design of Delta robots for a desired workspace is suggested considering swing range of spherical joints. However, the proposed method is provided for 3-PPP Delta robot architecture. Analysis of a 3-RRR Delta robot kinematic response, has been accomplished [16], in an attempt to reach the desired workspace through variations of robot parameters and swing range of ball-joints.

In the most common industrial applications of Delta robot such as pick-and place for assembly and material handling, the desired workspace is a right circular cylinder with a specified height and base radius. In this paper, a novel geometrical

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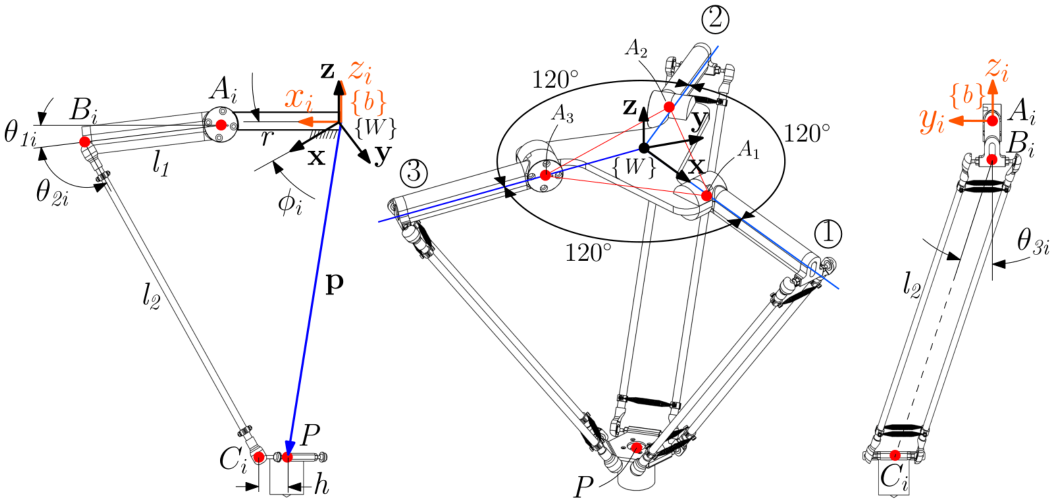


Fig. 1. Geometric parameters of the 3-RRR Delta robot.

approach for kinematics-based design of 3-RRR Delta robot for a given cylindrical workspace is presented. Toward this goal, the geometric representation of the desired workspace for a 3-RRR Delta robot is discussed and a new concept, called the Maximum Surrounded Workspace (MSW) is proposed. The geometric parameters of the MSW is derived and the surrounded radius of the reachable workspace section is determined. Furthermore, a statistical analysis for some well-known industrial 3-RRR Delta robots is provided to explore the relationships exist between their design parameters. The obtained relationships is then exploited in the proposed design methodology.

The remainder of this paper is organized as follows. After the above introduction, kinematic analysis of 3-RRR Delta robots will be discussed in Sec. II. The workspace analysis of the mentioned platform will then be introduced and the concept of the MSW will be proposed in Sec. III. Next, the design procedure of the robot with a desired workspace will be explained. After that, in order to illustrate the performance of the suggested approach, a real case study will be presented in Sec. IV. Finally, the paper ends with some concluding remarks.

II. Kinematic Analysis of 3-RRR Delta robot

A Delta robot is a type of parallel robot which consists of a moving platform connected to a fixed base through three parallel kinematic chains. Each chain contains a revolute joint actuated by a rotational motor which are mounted on the base platform. Movements are transmitted to the moving end-effect or platform through parallelograms formed by bars and spherical joints, as shown in Fig. 1.

Here, for convenience, the forward and inverse kinematics problems are briefly introduced. As it can be showed in Fig. 1, $\{w\}$is the fixed world coordinate system located at the center of the regular triangle $A_1A_2A_3$ and $\{b\}$ is the body coordinate system. Note that, all three chain of the Delta robot are identical in length. The geometric parameters of the robot are l_1, l_2, r, h and $\phi_i(i=1,2,3)$,which are depicted in Fig. 1. In addition,θ_{1i}, θ_{2i}, $\theta_{3i}(i=1,2,3)$ arethe joint angles that define the configuration of each chain.

Suppose that $P$is a center point of the end-effector, then the position vector ${}^b\mathbf{p}$in frame $\{b\}$can be written as follows:

$$ {}^b\mathbf{p} = \begin{Bmatrix} r-h+l_1\cos\theta_{1i}+l_2\cos\theta_{3i}\cos(\theta_{1i}+\theta_{2i}) \\ l_2\sin\theta_{3i} \\ -l_1\sin\theta_{1i}-l_2\cos\theta_{3i}\sin(\theta_{1i}+\theta_{2i}) \end{Bmatrix}, i=1,2,3. \quad (1) $$

Additionally, the position vector of P in the world coordinate system,$\{w\}$, can be represented as follows:

$$ {}^w\mathbf{p} = {}^w_b\mathbf{R}(\phi_i)\,{}^b\mathbf{p} = (X_P \quad Y_P \quad Z_P)^T \quad ,i=1,2,3. \quad (2) $$

where ${}^w_b\mathbf{R}(\phi_i)$is the orthonormal rotation matrix of frame $\{b\}$relative to $\{w\}$. According to Eq. (2), by some mathematical manipulations to eliminate the passive joint variables such as θ_{2i} and θ_{3i}, the constraint equation of each kinematic chain can be written as

$$ ({X_P}^2-{X_i}^2)+({Y_P}^2-{Y_i}^2)+({Z_P}^2-{Z_i}^2)={l_2}^2 \quad ,i=1,2,3. \quad (3) $$

where

$$ \begin{cases} X_i = (r-h+l_1\cos\theta_{1i})\cos\phi_i \\ Y_i = (r-h+l_1\cos\theta_{1i})\sin\phi_i \quad ,i=1,2,3. \\ Z_i = -l_1\sin\theta_{1i} \end{cases} \quad (4) $$

Eq. (3) represents three spheres centered at (X_i, Y_i, Z_i) with same radius, l_2. The intersection of these spheres is a point which is the solution of this system of equations. These three constraint equations yield the kinematics equations for the 3-RRR Delta robot.

A. Forward Kinematics Problem

Forward kinematics (FK)refers to the use of the kinematics equations of a robot to compute the position of the end-effect or from specified values of the joint parameters. The FK solution for parallel robots is generally hard. It requires the solution of multiple coupled nonlinear algebraic equations which has been derived previously (Eq. (3)). In general, there are two valid solutions, which means that, for specified joint angles, the moving platform can have two possible configurations with respect to the fixed base. For more details see [17].

B. Inverse Kinematics Problem

Inverse kinematics (IK) refers to the use of the kinematics equations of a robot to determine the joint parameters that provide a desired position of the end-effect or.

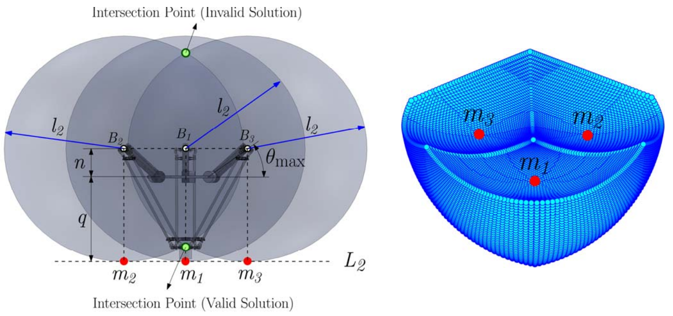


Fig. 2. Scheme of the workspace of the 3-RRR Delta robot.

Eq. (3) can be expressed as a function of joint space variableθ_{1i} as follows:

$$D_i \sin\theta_{1i} + E_i \cos\theta_{1i}\,.+F_i = 0 \tag{5}$$

where

$$\begin{cases} D_i = 2l_1 Z_P \\ E_i = 2(r-h)l_1 - 2l_1 X_P\cos\phi_i - 2l_1 Y_P \sin\phi_i \\ F_i = (r-h)^2 - 2(r-h)X_P\cos\phi_i - 2(r-h)Y_P\sin\phi_i \\ \quad +{l_1}^2 - {l_2}^2 + {X_P}^2 + {Y_P}^2 + {Z_P}^2 \end{cases} \tag{6}$$

By replacing of sine and cosine terms of Eq. (5) with tangent of half angle, the following result will be obtained for θ_{1i}

$$\theta_{1i} = 2\tan^{-1}\left(\frac{-D_i \pm \sqrt{{D_i}^2 + {E_i}^2 - {F_i}^2}}{F_i - E_i}\right) \tag{7}$$

Considering Eq. (7), one can see that there are eight IK solutions for a given position of the moving platform. For our robot the valid configuration occurs when the sign "±" in Eq. (7) is "−", i.e., the configuration as depicted in Fig. 2. For more details on the IK model of the Delta robot see [17] and [18].

III. Workspace Analysis

The workspace of the considered parallel Delta robot can be defined as a reachable region covered by center point of end-effect or. As observed in Fig. 2, the 3-RRR Delta robot's workspace have some irregular protuberances and dentswhich where the robot's configuration is permanently in or near the singular position. To dispel concerns about singularity and also facilitate in kinematics-based design of the Delta robot, the notion of *the Maximum Surrounded Workspace*(MSW) is suggested. The geometric parameters and characteristics of the MSW will be elaborated in next section.

A. The Maximum Surrounded Workspace

As shown in Fig. 3, the MSW is a solid volume obtained by rotating the area$S$ around the $\mathbf{z}$ axis of the robot.As it can be seen in Fig. 3, $S$is the area in $\mathbf{xz}$-plane(frame $\{w\}$) which is enclosed by two lines L_1 and$L_2$and an arc of circle C as follows:

$$\begin{cases} L_1:\ x = 0 \\ L_2:\ z = -q \\ C:\ (x-e)^2 + (z+n)^2 = {l_2}^2 \end{cases} \tag{8}$$

where

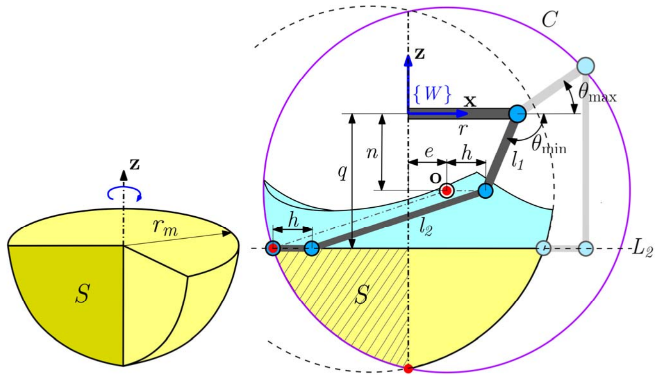


Fig. 3. The MWS of the 3-RRR Delta robot.

$$\begin{cases} q = l_2 - l_1\sin\theta_{max} \geq 0 \\ e = r - h + l_1\cos\theta_{min} \geq 0 \\ n = l_1 \sin\theta_{min} \geq 0 \end{cases} \tag{9}$$

where θ_{max} and θ_{min} are the positive and negative rotational limits of the active arms' joints, respectively. Eqs. (9) are generated due to mechanical interferences constraints. As depicted in Fig. 2, the line $L_2$is the intersection of the $\mathbf{xz}$-plane and the plane passing through points m_1, m_2 and m_3.These points are illustrated in Fig. 2. The surrounded radius of the reachable workspace section can be written as follows (see Fig. 3):

$$r_m = -e + \sqrt{{l_2}^2 - (z+n)^2} \quad , -n - \sqrt{{l_2}^2 - e^2} \leq z \leq -q \tag{10}$$

IV. Design of The Delta Robot with a Specified Workspace

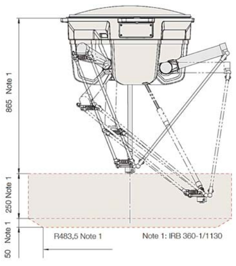
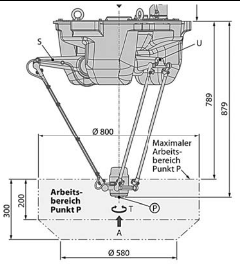
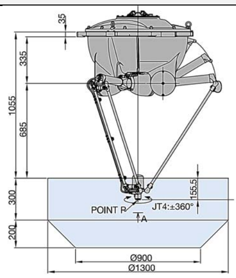
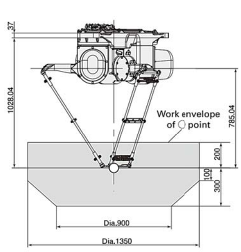
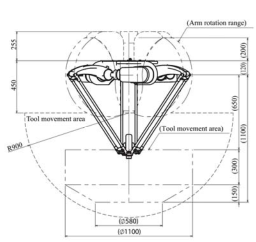
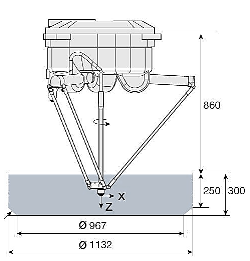
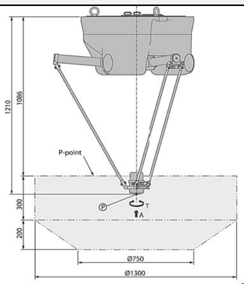
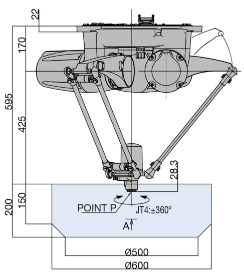
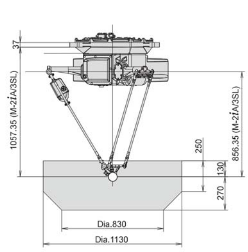
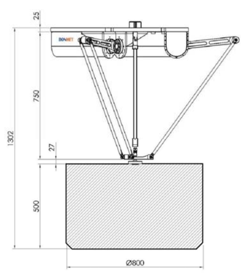
A. Formulation of The Suggested Approach

As discussed earlier, the desired workspace is a right circular cylinder with a height $H$and a base of radius R which is inscribed in the MSW. In the suggested approach, the geometric parameters of the robot are extracted based on the dimensions of this cylinder. As illustrated in Fig. 4, we define angle α to describe relationship between the desired workspace and the MSW as follows:

$$\begin{cases} H = l_2\cos\alpha - q + n \\ R = l_2 \sin\alpha - e \end{cases} \quad ,0 < \alpha < \pi/2 \tag{11}$$

Eq. (11) is a linear system with two equations and four variables l_1, l_2, r and h.To ensure that this system of equations has one solution, another two auxiliary equations have been defined as follows:

TABLE I. STATISTICAL ANALYSIS OF SOME PROMINENT INDUSTRAIL 3-RRR DELTA ROBOTS

	ABB-IRB 360/1	YASKAWA-MPP3S	KAWASAKI-YF03N	FANUC-M3iA	OMRON-R6Y3
					
K_1 [a]	≅2.28	≅2.69	≅2.43	≅2.23	≅2.47
K_2	≅4.41	≅2.14	≅2.73	≅1.81	≅3.6
	ABB-IRB 340 IRC5	**YASKAWA-MPP3**	**KAWASAKI-YS02N**	**FANUC-M2iA/3SL**	**BONMET-BR-2-Delta-R800**
					
K_1	≅2.22	≅2.66	≅2.0	≅2.31	≅2.42
K_2	≅4.33	≅2.28	≅1.98	≅1.91	≅2.6

[a] All values are extracted from datasheets and CAD models of the depicted robots, available in their websites.

$$\begin{cases} K_1 = l_2/l_1 \\ K_2 = r/h \end{cases} \tag{12}$$

K_1 and K_2 are two significant parameters in the kinematic-based design of the 3-RRR Delta robot. To find the design parameters of the robot, one should solve Eq. (11) and (12) simultaneously which leads to:

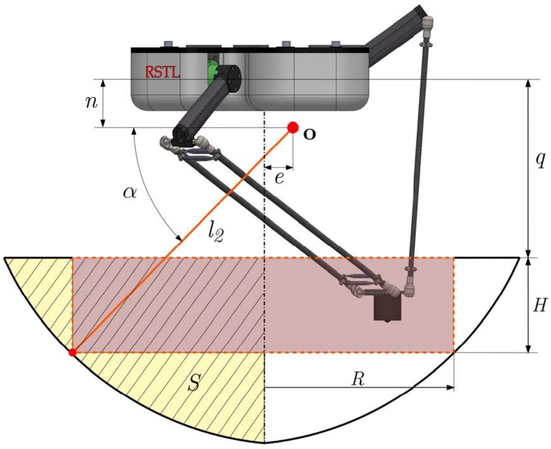


Fig. 4. The Specified Workspace Parameters

$$\begin{cases} l_1 = \dfrac{H}{U + K_1(\cos\alpha - 1)} \\ l_2 = \dfrac{K_1 H}{U + K_1(\cos\alpha - 1)} \\ h = -\dfrac{H\cos\theta_{\min} + RU + K_1[R(\cos\alpha - 1) - H\sin\alpha]}{(K_2 - 1)[U + K_1(\cos\alpha - 1)]} \\ r = -\dfrac{K_2\{H\cos\theta_{\min} + RU + K_1[R(\cos\alpha - 1) - H\sin\alpha]\}}{(K_2 - 1)[U + K_1(\cos\alpha - 1)]} \end{cases} \tag{13}$$

where

$$U = \sin\theta_{\max} + \sin\theta_{\min} \tag{14}$$

According to Eq. (11), in Eq. (13) all α angles lie in the interval $0 < \alpha < \pi/2$ which satisfy $e \geq 0$ are acceptable. As it can be observed in Eq. (13), if the value of α is increased, then the geometric parameters l_1, l_2, r and h are increased as well. Therefore, downsizing of the geometric parameters of the robot is obtained by selecting the smallest value of α.

In Table 1 a statistical analysis for some famous industrial 3-RRR Delta robots has been presented based on K_1 and K_2 parameters. As it can be seen in Table 1, the interval changes for these parameters are $2 < K_1 < 2.7$ and $1.8 < K_2 < 4.5$. These intervals will be used in the suggested design process.

B. Design Procedure by the Suggested Approach

Figure 5 demonstrates the suggested design procedure to determine the geometric parameters of the parallel robot. In order to illustrate the advantages of the suggested method for design of parallel Delta robots, a real design sample problem will be addressed as discussed in the following section.

C. Practical Design Example

As an example to clarify benefits of the suggested method for design of the Delta robot, a cylindrical workspace with radius $R = 550\,mm$ and height $H = 300\,mm$as a desired workspace is taken into account. The objective is to calculate the geometric parameters l_1, l_2, r and h.

Assume that the α angle is45°, i.e., $\alpha = 45°$. Letting $K_1 = 2.47$,K_2 =3.6,$\theta_{\max} = 60°$ and $\theta_{\min} = 90°$, the geometric parameters of the robot will be derived based on Eq. (13) as

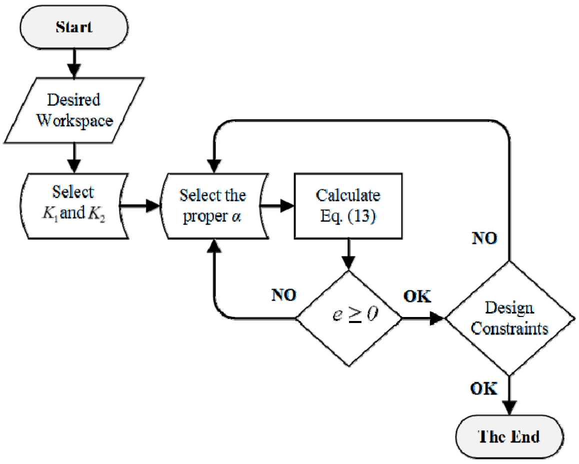


Fig. 5. Design Flowchart by the Suggested Approach

$$\begin{cases} l_1 = 262.56\,mm \\ l_2 = 684.53\,mm \\ h = -35.16\,mm \\ r = -126.58\,mm \end{cases} \quad (15)$$

The obtained results, however, do not satisfy the relation $e \geq 0$. Consequently, we select a new α angleas$\alpha = 49°$ which leads to:

$$\begin{cases} l_1 = 295.13\,mm \\ l_2 = 728.98\,mm \\ h = 0.064\,mm \\ r = 0.231\,mm \end{cases} \quad (16)$$

The above results are the minimum admissible design parameters for the robot satisfying the constraint$e \geq 0$. The obtained results show the values of$h$ and r are too smallwhich are not suitable from the view point of mechanical design due to mechanical interferences and design constraints. For example, the minimum required base radius is$r = 200\,mm$, therefore by increasing α to 53.7°, the following results will be obtained.

$$\begin{cases} l_1 = 349.53\,mm \\ l_2 = 863.34\,mm \\ h = 56.07\,mm \\ r = 201.86\,mm \end{cases} \quad (17)$$

The above resulted values support the fact that the design procedure has been employed successfully. The workspace of the designed 3-RRR Delta robot is depicted in Fig. 6. Moreover, Fig. 7 shows the MSW, in which the desired workspace is embedded completely.

V. Conclusions

In the current study, a novel approach for kinematics-based design of the 3-RRR Delta robots with a given workspace was proposed. The foundation of the suggested design method is based on a new suggested concept called the Maximum Surrounded Workspace. Additionally, a statistical analysis for some prominent industrial 3-RRR Delta robots was presented

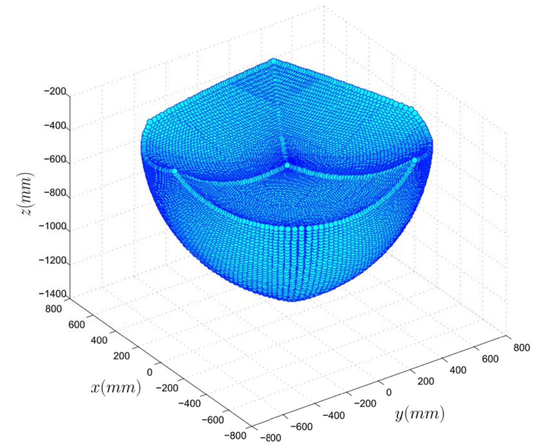


Fig. 6. The workspace of the design example

(1

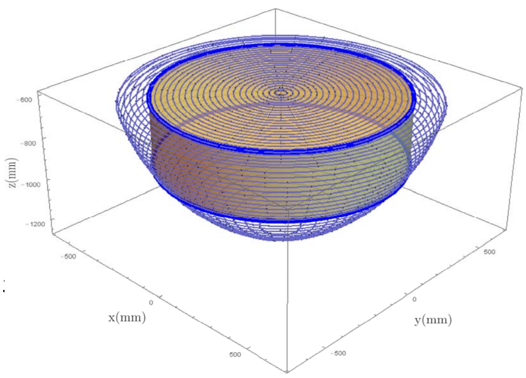


Fig. 7. The Maximum Surrounded Workspace of the design example

to find the relationships exist between their kinematic parameters. The derived relationships were then augmented in the suggested design technique. The obtained results of a practical design example proved the merits of the suggested method in terms of good performance, and facility of the suggested design manner for 3-RRR Delta robot(15)

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