

Algebra

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August 2023

SECTION A

1. If $\sin \theta = 0$ then the value of $\tan^2 \theta + \cot^2 \theta$ is
 - (a) 2
 - (b) 4
 - (c) 1
 - (d) $\frac{10}{9}$
2. The value(s) of k for which the quadratic equation $3x^2 - kx + 3 = 0$ has equal roots, is (are)
 - (a) 6
 - (b) -6
 - (c) +-6
 - (d) 9
3. $5\tan^2\theta - 5\sec^2\theta = \underline{\hspace{2cm}}$
4. If α, β are zeroes of the polynomial $2x^2 - 5x - 4$ then $\frac{1}{\alpha} + \frac{1}{\beta}$
5. In Figure1, a tower stands vertically on the ground. From a point on the ground, which is 80m away from the foot of the tower, the angle of elevation of the tower is found to be 30° . Find the height of the tower.
6. Solve $9x^2 - 6a^2x + a^4 - b^4$ using quadratic formula.
7. Show that $\cos(38^\circ)\cos(52^\circ) - \sin(38^\circ)\sin(52^\circ) = \cos(90^\circ)$.
8. Prove that

$$\frac{\sin\theta}{\cot\theta + \csc\theta} = 2 + \frac{\sin\theta}{\cot\theta - \csc\theta}. \quad (1)$$

9. given $15\cot(A)=8$, find the values of $\sin(A)$ and $\sec(A)$.

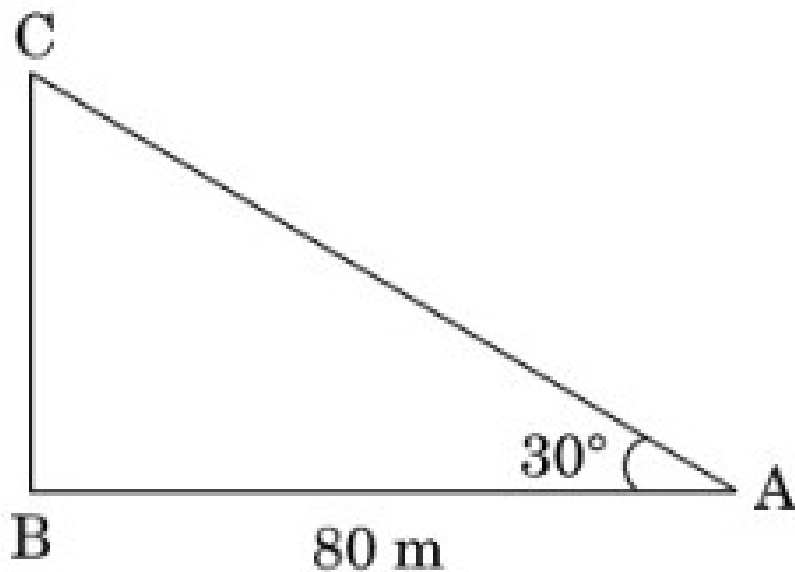


Figure 1: as.jpeg

10. The angles of depression of the top and bottom of a tower as seen from the top of a $60\sqrt{3}m$ high cliff are 45° and 60° respectively. Find the height of the tower.(Use $\sqrt{3} = 1.73$)
11. A and B jointly finish a piece of work in 15 days. When they work separately, A takes 16 days less than the number of days taken by B to finish the same piece of work. Find the number of days taken by B to finish the work.
12. If the polynomial

$$f(x) = 3x^4 - 9x^3 + x^2 + 15x + k \quad (2)$$

is completely divisible by $3x^2 - 5$, then find the value of k . Using the quotient, so obtained, find two zeroes of the

13. Find all the zeroes of the polynomial

$$f(x)x^4 - 8x^3 + 23x^2 - 28x + 12 \quad (3)$$

if two of its zeroes are 2 and 3.

14. Find the value of m for which the quadratic equation

$$(m - 1)x^2 + 2(m - 1)x + 1 = 0 \quad (4)$$

has two real and equal roots.

15. Solve the following quadratic equation for x

$$\sqrt{3}X^2 + 10x + 7\sqrt{3} = 0 \quad (5)$$

16. The product of Rehan's age(in years) 5 years ago and his age 7 years from now, is one more than twice his age. Find his present age.
17. The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60° . If the tower is 50 m high, then find the height of the building.
18. From a point on a bridge across a river, the angles of depression of the banks on opposite sides of the river are 30° and 60° respectively. If the bridge is at a height of 3 m from the banks, then find the width of the river
19. In figure2 Gadisar Lake is located in the Jaisalmer district of Rajasthan. It was built by the King of Jaisalmer and rebuilt by Gadsingh in 14th century. The lake has many Chatris. One of them is shown below .



Figure 2: ak.jpg

Observe the picture. From a point A h m above from water level, the

angle of elevation of top of Chhatri (point B) is 45° and angle of depression of its reflection in water (point C) is 60° . If the height of Chhatri above water level is (approximately) 10m, then

- draw a well-labelled figure based on the above information
- find the height(h) of the point A above water level. (Use $\sqrt{3} = 1.73$)

20. solve the quadratic equation $x^2 + 2\sqrt{2}x - 6 = 0$ for x

21. In figure 3 From a point A on a bridge across a river, the angles of depression of the banks on opposite sides of the river are 30° and 45° . If the bridge is at a height of 8 m from the banks, then find the width of the river.

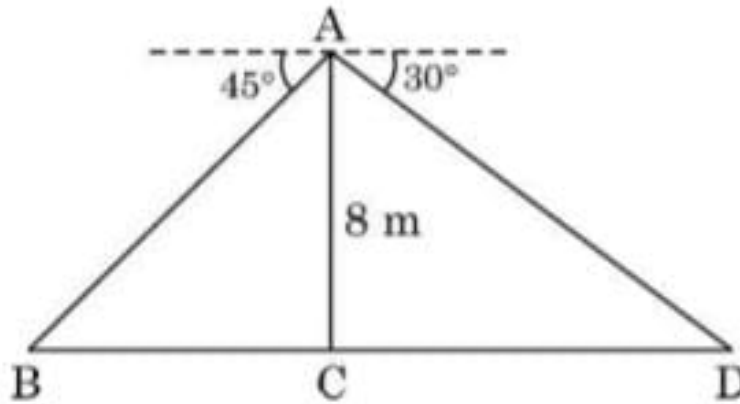


Figure 3: su.jpg

22. A 2-digit number is such that the product of its digits is 24. If 18 is subtracted from the number, the digits interchange their places. Find the number.

23. The difference of the squares of two numbers is 180. The square of the smaller number is 8 times the greater number. Find the two numbers.

24. Case Study-1

Kite Festival Kite festival is celebrated in many countries at different times of the year. In India, every year 14th January is celebrated as International Kite Day. On this day many people visit India and participate in the festival by flying various kinds of kites. The picture given below, the kites flying together.

In Fig. 5, the angles of elevation of two kites (Point A and B) from the hands of a man (Point C) are found to be 30° and 60° respectively. Taking $AD = 50\text{m}$ and $BE = 60\text{m}$, find

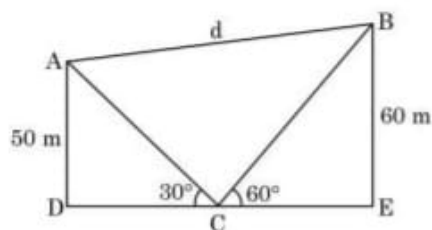
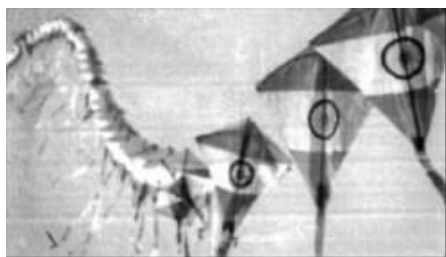


Figure 4: kites

- (a) the lengths of strings used (take them straight) for kites A and B as shown in the figure
- (b) the distance 'd' between these two kites

25. Solve the quadratic equation for x:

$$x^2 - 2ax - (4b^2 - a^2) = 0 \quad (6)$$

$$x^2 - 2ax - (4b^2 - a^2) = 0$$

26. If the quadratic equation

$$(1 + a^2)x^2 + 2abx + (b^2 - c^2) = 0 \quad (7)$$

has equal and real roots, then prove that : $b^2 = c^2(1 + a^2)$

27. Two boats are sailing in the sea 80 m apart from each other towards a cliff AB. The angles of depression of the boats from the top of the cliff are 30° and 30° respectively, as shown in Figure 5. Find the height of the cliff.
28. The angle of elevation of the top Q of a vertical tower PQ from a point X on the ground is 30° . From a point Y, 40 m vertically above X, the angle of elevation of the top Q of tower PQ is 45° . Find the height of the tower PQ and the distance PX. [Use $\sqrt{3} = 1.73$]

29. Find the value of 'k' for which the quadratic equation

$$2kx^2 - 40 + 25 = 0 \quad (8)$$

has real and equal roots.

30. solve for x: $\frac{5}{2}x^2 + \frac{2}{5} = 1 - 2x$

31. An aeroplane at an altitude of 200 metres observes the angles of depression of opposite points on the two banks of a river to be 45° and 60° . Find the width of the river. (Use $\sqrt{3} = 1.732$)
32. find the value of 'p' for which the quadratic equation $(px - 4)(x - 2)$

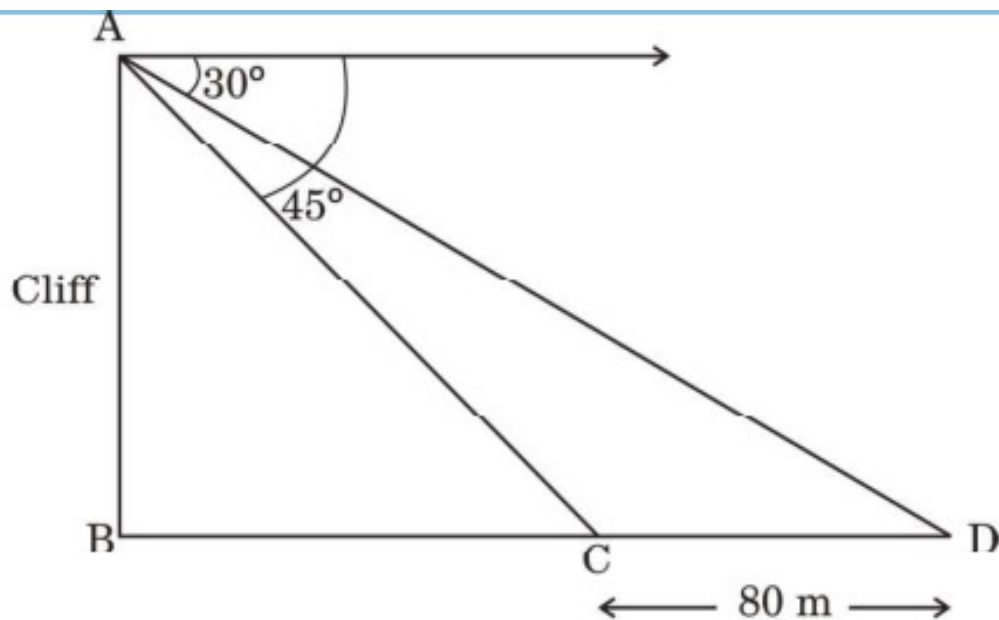


Figure 5: figure5

33. Had Aarush scored 8 more marks in a Mathematics test, out of 35 marks, 7 times these marks would have been 4 less than square of his actual marks, How many marks did he get in the test?
34. From the top of an 8 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45° . Determine the height of the tower. (Take $\sqrt{3} = 1.732$).
35. Find the roots of the quadratic equation

$$9x^2 - 6\sqrt{2}x + 2 = 0 \quad (9)$$

36. The product of two consecutive odd positive integers is 255. Find the integers, by formulating a quadratic equation.
37. Find the value(s) of k for the quadratic equation,

$$(k + 3)x^2 + kx + 1 = 0 \quad (10)$$

, to have two real and equal roots.

38. As observed from the top of a lighthouse 60 m high from the sea level, the angles of depression of two ships are 45° and 60° . If one ship is exactly behind the other on the same side of the lighthouse, then find the distance between the two ships. [Use $\sqrt{3} = 1.732$]

39. At a point on the level ground, the angle of elevation of the top of a vertical tower is found to be α , such that $\tan \alpha = \frac{5}{12}$. On walking 192 m towards the tower, the angle of elevation β is such that $\tan \beta = \frac{3}{4}$. Find the height of the tower.

40. $\tan^{-1} \frac{1}{\sqrt{3}} - \cot^{-1} \frac{-1}{\sqrt{3}}$

41. Show that the relation R in the set R of all real numbers, defined $R = \{(a, b) : a \leq b^2\}$

42. Two angles of a triangle are $\cot^{-1} 2$ and $\cot^{-1} 3$. The third angle of the triangle is ?

43. Solve for x:

$$\sin^{-1}(1 - x) - 2 \sin^{-1} x = \frac{\pi}{2} \quad (11)$$

44. Find the present value of a perpetuity of ₹ 18,000 at the end of 6 months is worth 8% p.a. compounded semi-annually.

45. Find the effective rate which is equivalent to nominal rate of 10% p.a. compounded monthly.

[given that: $1.00833^{12} = 1.1047$]

46. Abhay bought a mobile phone for ₹ 30,000. The mobile phone is estimated to have a scrap value of ₹ 3,000 after a span of 3 years. Using the linear depreciation method, find the book value of the mobile phone at the end of 2 years.

47. Madhu exchanged her old car valued at ₹ 1,50,000 with a new one priced at ₹ 65,000. She paid ₹ x as down payment and the balance in 20 monthly equal instalments of ₹ 21,000. The rate of interest offered to her is 9% p.a. Find the value of x. [Given that: $1.0075^{-20} = 0.86118985$]

48. Calculate the EMI under 'Flat Rate System' for a loan of ₹ 5,00,000 with 10% annual interest rate for 5 years.

49. A machine costing ₹ 2,00,000 has effective life of 7 years and its scrap value is ₹ 30,000. What amount should the company put into a sinking fund earning 5 % p.a., so that it can replace the machine after its usual life ? Assume that a new machine will cost ₹ 3,00,000 after 7 years. [Given that: $(1.05)^7 = 1.407$]

50. A start-up company invested ₹ 3,00,000 in shares for 5 years. The value of this investment was ₹ 3,50,000 at the end of second year, ₹ 3,80,000 at the end of third year and on maturity, the final value stood at ₹ 4,50,000. Calculate the Compound Annual Growth Rate (CAGR) on the investment [Given that: $(1.5)^{\frac{1}{5}} = 1.084$]