DSML Intermediate : Time Complexity and Sorting III

Recap

$f(N) = N^3 + 10N^2 + 8$

$= N^3 + N^2$

$\Rightarrow O(N^3)$

Quizzes

Big-O:

0. Calculate no of iterations / operations in the program
1. Drop off constants
2. Take only the highest power in terms of the input (N)

In [ ]:

In [ ]:

In [ ]:

1. Drop off constants
2. Take only the highest power in terms of the input (N)

Quiz - 2

Pseudo Code

```
for (i = 0; i < N; i++) {
    for (j = 0; j < N; j++) {
        if (numbers[i] < numbers[j]) {
            ++eqPerms
        }
        else {
            ++neqPerms
        }
    }
}
```

Pseudo Code

=> independent of programming lang.

for i in range(N)

start = 0
end = N-1
inc = 1

for (i = 0 ; i < N ; i++)

start cond^n inc

In [ ]:

Quiz-3

$N^2 + N$

$N^2 + N$

N^2

In [ ]:

In [ ]:

In [ ]:

1. Drop off constants
2. Take only the highest power in terms of the input (N)

Quiz - 2

```
for (i = 0; i < N; i++) {
    for (j = 0; j < N; j++) {
        if (numbers[i] < numbers[j]) {
            ++eqPerms
        }
        else {
            ++neqPerms
        }
    }
}
```

i += 1
j += 1
list
eqPerms += 1
neqPerms += 1
} int

```
for i in range(N):
    for j in range(N):
        if numbers[i] < numbers[j]:
            eqPerms += 1
        else:
            neqPerms += 1
```

Python

In [ ]:

1. Drop off constants
2. Take only the highest power in terms of the input (N)

[a,b] => b-a+1

Quiz - 2

```
for (i = 0; i < N; i++)  {
   for (j = 0; j < N; j++) {
      if (numbers[i] < numbers[j]) {
         ++eqPerms
      }
      else {
         ++neqPerms
      }
   }
}
```

[0, N-1]

=> N-1-0+1

=> N

```
for i in range(N):
    for j in range(N):
        if numbers[i] < numbers[j]:
            eqPerms += 1
        else:
            neqPerms += 1
```

i	j	Cnt Iterations
0	[0, N)	N-0 = N
1	[0, N)	N
⋮	⋮	⋮
N-1	[0, N)	N

N(N) = N^2

In [ ]:

1. Drop off constants
2. Take only the highest power in terms of the input (N)

Quiz - 2

```
for (i = 0; i < N; i++)  {
   for (j = 0; j < N; j++) {
      if (numbers[i] < numbers[j]) {
         ++eqPerms
      }
      else {
         ++neqPerms
      }
   }
}
```

0 Ith ✓

1 Constants removed ✓

2

$2N^2$

```
for i in range(N):
    for j in range(N):
        if numbers[i] < numbers[j]:
            eqPerms += 1
        else:
            neqPerms += 1
```

$1(N^2)$

$3N^2$

$\Rightarrow O(N^2)$

In [ ]:

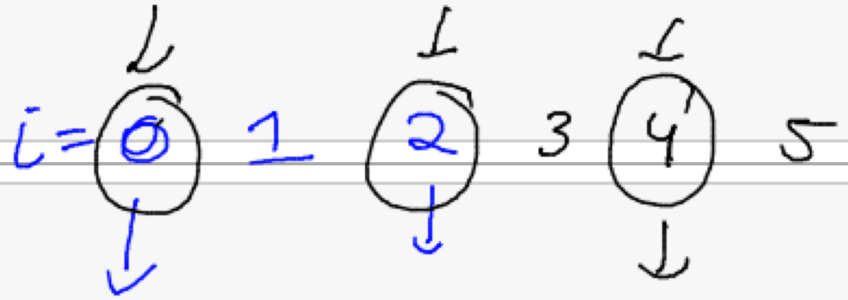
```
            eqPerms += 1
        else:
            neqPerms += 1
```

Quiz - 4

```
for (i = 0; i < N; ++i) {
   if ((i % 2) == 0) {
      outVal[i] = inVals[i] * i
   }
}
```

i = 0 → N

N = 6




```
for i in range(N):
    if i % 2 == 0:
        outVal[i] = inVal[i] * i
```

N

1
2
3

i has to be even.

$f(N) = \frac{N}{2} + c$

In [ ]:

In [ ]:

In [ ]:

In [ ]:


```
for (i = 0; i < N; ++i) {
   if ((i % 2) == 0) {
      outVal[i] = inVals[i] * i
   }
}
```

Iteration N

```
for i in range(N):
    if i % 2 == 0:
        outVal[i] = inVal[i] * i
```

O(N)

In [ ]:

```
i = 0                                   1
while i < N:                            → N+1
    if i % 2 == 0:                      → N
        outVal[i] = inVal[i] * i        → N/2
    i += 1                              → N
```

$1 + N + 1 + N + \frac{N}{2} + N$

$= \frac{3}{2}N + \frac{N}{2} + 2$

In [ ]:

In [ ]:

In [ ]:

In [ ]:

In [ ]:

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In [ ]:

In [ ]:

Sorting

Bubble Sort

In [1]:
```
def bubble_sort(a):
    j = 0
    N = len(a) # get the length
    while j < N - 1:
        if a[j] > a[j + 1]:
            # swap larger with smaller
            a[j], a[j + 1] = a[j + 1], a[j]
        j += 1
    return a
```

In [2]:
```
bubble_sort([10, 6, 14, 20, 2, 19])
```

Out[2]: [6, 10, 14, 2, 19, 20]

Largest element was settled/bubbled.

In [ ]:

In [ ]:

```
        j += 1
    print(a)
    print('-'*30)

    j = 0
    while j < N - 1:
        if a[j] > a[j + 1]:
            # swap larger with smaller
            a[j], a[j + 1] = a[j + 1], a[j]
        j += 1

    print(a)
    print('-'*30)
    return a
```

6, 10, 14, 2, 19, 20

In [6]:
```
bubble_sort([10, 6, 14, 20, 2, 19])
```

```
[6, 10, 14, 2, 19, 20]
------------------------------
[6, 10, 2, 14, 19, 20]
------------------------------
```

6, 10, 2, 14, 19, 20

Out[6]:
```
[6, 10, 2, 14, 19, 20]
```

In [ ]:

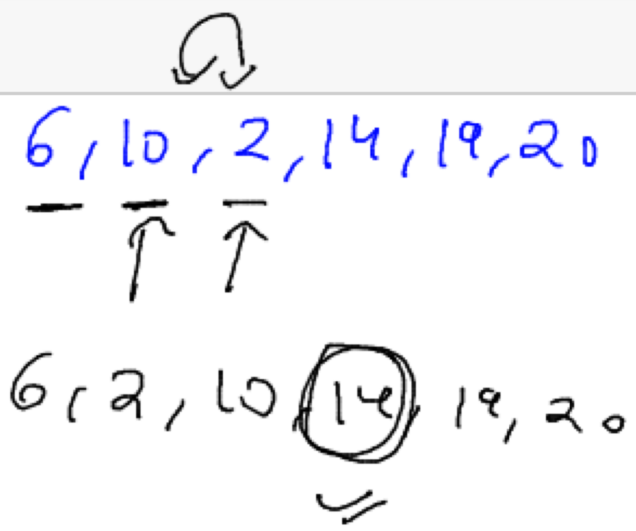
In [ ]:

```
    print(a)
    print('-'*30)

    j = 0
    while j < N - 1:
        if a[j] > a[j + 1]:
            # swap larger with smaller
            a[j], a[j + 1] = a[j + 1], a[j]
        j += 1

    print(a)
    print('-'*30)
    return a
```

In [8]:
```
bubble_sort([10, 6, 14, 20, 2, 19])
```

```
[6, 10, 14, 2, 19, 20]
------------------------------
[6, 10, 2, 14, 19, 20]
------------------------------
[6, 2, 10, 14, 19, 20]
------------------------------
```

Out[8]: [6, 2, 10, 14, 19, 20]

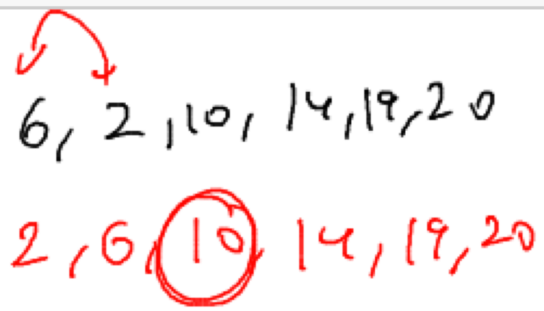


In [ ]:

```
    print(a)
    print('-'*30)

    j = 0
    while j < N - 1:
        if a[j] > a[j + 1]:
            # swap larger with smaller
            a[j], a[j + 1] = a[j + 1], a[j]
        j += 1

    print(a)
    print('-'*30)
    return a
```

In [10]:
```
bubble_sort([10, 6, 14, 20, 2, 19])
```

```
[6, 10, 14, 2, 19, 20]
------------------------------
[6, 10, 2, 14, 19, 20]
------------------------------
[6, 2, 10, 14, 19, 20]
------------------------------
[2, 6, 10, 14, 19, 20]
------------------------------
```



Out[10]:
```
[2, 6, 10, 14, 19, 20]
```

In [ ]:

```
    print(a)
    print('-'*30)

    j = 0
    while j < N - 1:
        if a[j] > a[j + 1]:
            # swap larger with smaller
            a[j], a[j + 1] = a[j + 1], a[j]
        j += 1

    print(a)
    print('-'*30)
    return a
```

n = 5

In [11]:
```
bubble_sort([9, 8, 7, 6, 5])
```

```
1 [8, 7, 6, 5, 9]
  ------------------------------
2 [7, 6, 5, 8, 9]
  ------------------------------
3 [6, 5, 7, 8, 9]
  ------------------------------
4 [5, 6, 7, 8, 9]
  ------------------------------
```

n - 1 iterations

Out[11]:
```
[5, 6, 7, 8, 9]
```

In [ ]:

In [15]:
```
bubble_sort([9, 8, 7, 6, 5])
```

```
[8, 7, 6, 5, 9]
------------------------------
[7, 6, 5, 8, 9]
------------------------------
[6, 5, 7, 8, 9]
------------------------------
[5, 6, 7, 8, 9]
------------------------------
```

Out[15]: [5, 6, 7, 8, 9]

start end inc=

for (j=0; j<N-1; j++)

for j in range(N-1)

In [ ]:
```
def bubble_sort(a):
    N = len(a) # get the length
    for i in range(N - 1):
        j = 0
        while j < N - 1:
            if a[j] > a[j + 1]:
                # swap larger with smaller
                a[j], a[j + 1] = a[j + 1], a[j]
            j += 1
        print(a)
        print('-'*30)
    return a
```

In [ ]:

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14 / 14

In [16]:
```
def bubble_sort(a):
    N = len(a) # get the length
    for i in range(N - 1):
        for j in range(N - 1):
            if a[j] > a[j + 1]:
                # swap larger with smaller
                a[j], a[j + 1] = a[j + 1], a[j]
            j += 1
        print(a)
        print('-'*30)
    return a
```

In [17]:
```
bubble_sort([11, 10, 9, 8, 7, 6, 5])
```

```
[10, 9, 8, 7, 6, 5, 11]
------------------------------
[9, 8, 7, 6, 5, 10, 11]
------------------------------
[8, 7, 6, 5, 9, 10, 11]
------------------------------
[7, 6, 5, 8, 9, 10, 11]
------------------------------
[6, 5, 7, 8, 9, 10, 11]
------------------------------
[5, 6, 7, 8, 9, 10, 11]
------------------------------
```

Out[17]:
```
[5, 6, 7, 8, 9, 10, 11]
```

15 / 15

In [1

```
    N = len(a) # get the length
    for i in range(N - 1):
        for j in range(N - 1):
            if a[j] > a[j + 1]:
                # swap larger with smaller
                a[j], a[j + 1] = a[j + 1], a[j]
            j += 1
        print(a)
        print('-'*30)
    return a
```

$(a-b)^2 = a^2 + b^2 - 2ab$

$(N-1)^2$

$= N^2 + X - 2N$

$= O(N^2)$

Worst Case

N = 7

In [17]:
```
bubble_sort([11, 10, 9, 8, 7, 6, 5])
```

6 it

```
i=0 [10, 9, 8, 7, 6, 5, 11]
----------
i=1 [9, 8, 7, 6, 5, 10, 11]
----------
i=2 [8, 7, 6, 5, 9, 10, 11]
----------
i=3 [7, 6, 5, 8, 9, 10, 11]
----------
i=4 [6, 5, 7, 8, 9, 10, 11]
----------
i=5 [5, 6, 7, 8, 9, 10, 11]
----------
```

Out[17]: [5, 6, 7, 8, 9, 10, 11]

i	j	# iterations
0	[0, N-1)	N-1
1	[0, N-1)	N-1
2	⋮	⋮
⋮	⋮	⋮
N-2	[0, N-1)	N-1
# N-1		(N-1)(N-1)

In [ ]:

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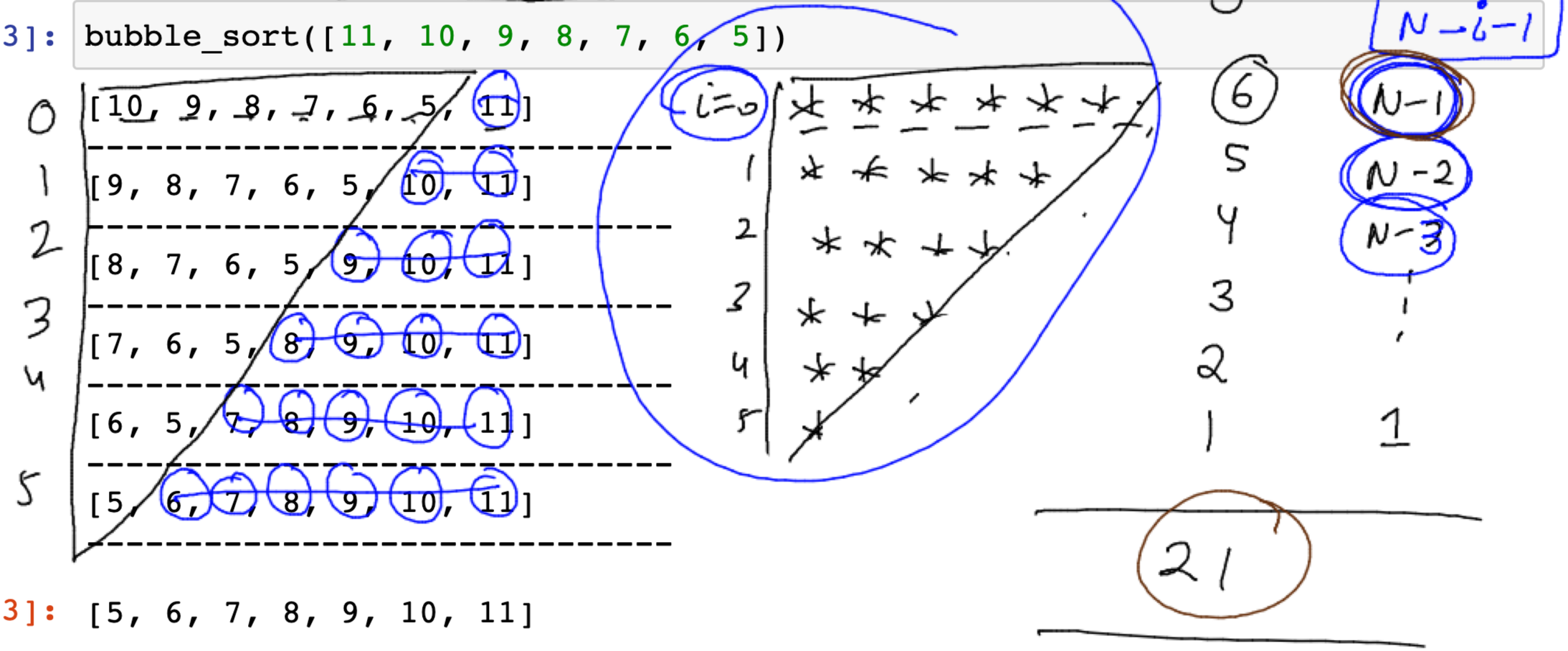
In [42]:
```
def bubble_sort(a):
    N = len(a) # get the length
    for i in range(N - 1):
        for j in range(N - i - 1):
            if a[j] > a[j + 1]:
                # swap larger with smaller
                a[j], a[j + 1] = a[j + 1], a[j]
        print(a)
        print('-'*30)
    return a
```

$1+2+\cdots+N = \frac{N(N+1)}{2}$

$\frac{6*7}{2} = \frac{42}{2} = 21$

$N = 7$

In [43]:
```
bubble_sort([11, 10, 9, 8, 7, 6, 5])
```



Out[43]: [5, 6, 7, 8, 9, 10, 11]

In [44]:
```
bubble sort([5, 4, 1, 2, 7, 8, 0, 10])
```

In [ ]:
```
    N = len(a) # get the length
    for i in range(N - 1):
        for j in range(N - i - 1):
            if a[j] > a[j + 1]:
                # swap larger with smaller
                a[j], a[j + 1] = a[j + 1], a[j]
            j += 1
        print(a)
        print('-'*30)
    return a
```

In [18]:
```
bubble_sort([11, 10, 9, 8, 7, 6, 5])
```

```
[10, 9, 8, 7, 6, 5, 11]
------------------------------
[9, 8, 7, 6, 5, 10, 11]
------------------------------
[8, 7, 6, 5, 9, 10, 11]
------------------------------
[7, 6, 5, 8, 9, 10, 11]
------------------------------
[6, 5, 7, 8, 9, 10, 11]
------------------------------
[5, 6, 7, 8, 9, 10, 11]
------------------------------
```

Out[18]: [5, 6, 7, 8, 9, 10, 11]

In [ ]:

Out[2

In [22]:
```
bubble_sort([5, 4, 1, 2, 7, 8, 0, 10])
```

```
[4, 1, 2, 5, 7, 0, 8, 10]
--------------------------------
[1, 2, 4, 5, 0, 7, 8, 10]
--------------------------------
[1, 2, 4, 0, 5, 7, 8, 10]
--------------------------------
[1, 2, 0, 4, 5, 7, 8, 10]
--------------------------------
[1, 0, 2, 4, 5, 7, 8, 10]
--------------------------------
[0, 1, 2, 4, 5, 7, 8, 10]
--------------------------------
[0, 1, 2, 4, 5, 7, 8, 10]
--------------------------------
```

last ? pass →

Out[22]:
```
[0, 1, 2, 4, 5, 7, 8, 10]
```

In [ ]:

In [ ]:

In [ ]:

```
[1, 2, 0, 4, 5, 7, 8, 10]
-----------------------------
[1, 0, 2, 4, 5, 7, 8, 10]
-----------------------------
[0, 1, 2, 4, 5, 7, 8, 10]
-----------------------------
[0, 1, 2, 4, 5, 7, 8, 10]
-----------------------------
```

Out[22]: `[0, 1, 2, 4, 5, 7, 8, 10]`

In [26]:
```
bubble_sort([1, 2, 3, 5, 4, 7, 6])
```

```
[1, 2, 3, 4, 5, 6, 7]
-----------------------------
[1, 2, 3, 4, 5, 6, 7]
-----------------------------
[1, 2, 3, 4, 5, 6, 7]
-----------------------------
[1, 2, 3, 4, 5, 6, 7]
-----------------------------
[1, 2, 3, 4, 5, 6, 7]
-----------------------------
[1, 2, 3, 4, 5, 6, 7]
-----------------------------
```

Out[26]: `[1, 2, 3, 4, 5, 6, 7]`

In [ ]:

```
[0, 1, 2, 4, 5, 7, 8, 10]
------------------------------
[0, 1, 2, 4, 5, 7, 8, 10]
------------------------------
```

Out[22]:
```
[0, 1, 2, 4, 5, 7, 8, 10]
```

In [27]:
```
bubble_sort([1, 2, 3, 5, 4, 7, 6])
```

```
[1, 2, 3, 4, 5, 6, 7]
------------------------------
[1, 2, 3, 4, 5, 6, 7]
------------------------------
[1, 2, 3, 4, 5, 6, 7]
------------------------------
[1, 2, 3, 4, 5, 6, 7]
------------------------------
[1, 2, 3, 4, 5, 6, 7]
------------------------------
[1, 2, 3, 4, 5, 6, 7]
------------------------------
```

Out[27]:
```
[1, 2, 3, 4, 5, 6, 7]
```

In [ ]:

In [ ]:

In [33]:
```
def optimized_bubble_sort(a):
    N = len(a) # get the length
    for i in range(N - 1):
        for j in range(N - i - 1):
            if a[j] > a[j + 1]:
                # swap larger with smaller
                a[j], a[j + 1] = a[j + 1], a[j]
            j += 1
        print(a)
        print('-'*30)
    return a
```

unhappy student ?

In [34]:
```
optimized_bubble_sort([1, 2, 3, 4, 5, 6])
```

all happy
=> no swaps !!

```
[1, 2, 3, 4, 5, 6]
------------------------------
[1, 2, 3, 4, 5, 6]
------------------------------
[1, 2, 3, 4, 5, 6]
------------------------------
[1, 2, 3, 4, 5, 6]
------------------------------
[1, 2, 3, 4, 5, 6]
------------------------------
[1, 2, 3, 4, 5, 6]
------------------------------
```

Out[34]:
```
[1, 2, 3, 4, 5, 6]
```

Worst case time complexity:

In [3

```
    N = len(a) # get the length
    for i in range(N - 1):
        num_swaps = 0
        for j in range(N - i - 1):
            if a[j] > a[j + 1]:
                # swap larger with smaller
                a[j], a[j + 1] = a[j + 1], a[j]
                num_swaps += 1
            j += 1
        if num_swaps == 0:
            break
        print(a)
        print('-'*30)
    return a
```

HW:
Try making
code simpler

In [39]:
```
optimized_bubble_sort([4, 5, 4, 2, 5, 6])
```

Can you use a boolean variable?

```
[4, 4, 2, 5, 5, 6]
------------------------------
[4, 2, 4, 5, 5, 6]
------------------------------
[2, 4, 4, 5, 5, 6]
------------------------------
```

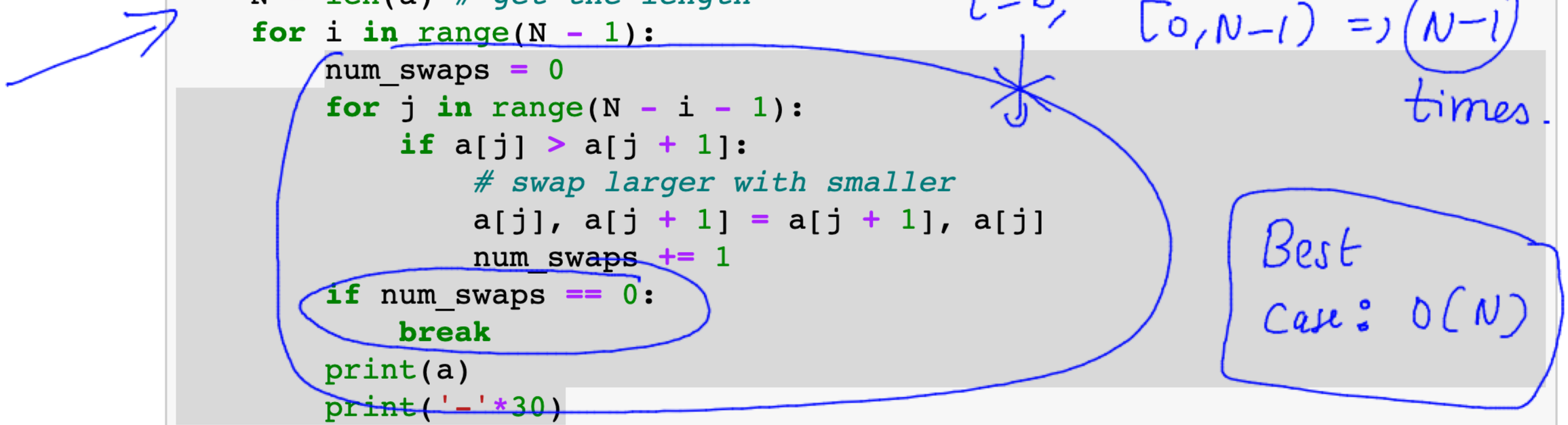
Out[39]:
```
[2, 4, 4, 5, 5, 6]
```

Worst case time complexity:

In [ ]:

Optimized Bubble Sort

In [46]:
```
def optimized_bubble_sort(a):
    N = len(a) # get the length
    for i in range(N - 1):
        num_swaps = 0
        for j in range(N - i - 1):
            if a[j] > a[j + 1]:
                # swap larger with smaller
                a[j], a[j + 1] = a[j + 1], a[j]
                num_swaps += 1
        if num_swaps == 0:
            break
        print(a)
        print('-'*30)
    return a
```



In [47]:
```
optimized_bubble_sort([4, 5, 4, 2, 5, 6])
```

```
[4, 4, 2, 5, 5, 6]
------------------------------
[4, 2, 4, 5, 5, 6]
------------------------------
[2, 4, 4, 5, 5, 6]
------------------------------
```

```
------------------------------
[7, 6, 5, 8, 9, 10, 11]
------------------------------
[6, 5, 7, 8, 9, 10, 11]
------------------------------
[5, 6, 7, 8, 9, 10, 11]
------------------------------
```

Out[41]: `[5, 6, 7, 8, 9, 10, 11]`

Best case for this code?

In [42]:
```
def bubble_sort(a):
    N = len(a) # get the length
    for i in range(N - 1):
        for j in range(N - i - 1):
            if a[j] > a[j + 1]:
                # swap larger with smaller
                a[j], a[j + 1] = a[j + 1], a[j]
        print(a)
        print('-'*30)
    return a
```

=> $O(N^2)$ worst & best case.

In [43]:
```
bubble_sort([11, 10, 9, 8, 7, 6, 5])
```

```
[10, 9, 8, 7, 6, 5, 11]
------------------------------
[9, 8, 7, 6, 5, 10, 11]
------------------------------
[8, 7, 6, 5, 9, 10, 11]
------------------------------
```

```
------------------------------
[7, 6, 5, 8, 9, 10, 11]
------------------------------
[6, 5, 7, 8, 9, 10, 11]
------------------------------
[5, 6, 7, 8, 9, 10, 11]
------------------------------
```

Out[41]:
```
[5, 6, 7, 8, 9, 10, 11]
```

In [42]:
```
def bubble_sort(a):
    N = len(a) # get the length
    for i in range(N - 1):
        for j in range(N - i - 1):
            if a[j] > a[j + 1]:
                # swap larger with smaller
                a[j], a[j + 1] = a[j + 1], a[j]
        print(a)
        print('-'*30)
    return a
```

In [43]:
```
bubble_sort([11, 10, 9, 8, 7, 6, 5])
```

```
[10, 9, 8, 7, 6, 5, 11]
------------------------------
[9, 8, 7, 6, 5, 10, 11]
------------------------------
[8, 7, 6, 5, 9, 10, 11]
------------------------------
```


```
[5, 6, 7, 8, 9, 10, 11]
------------------------------
```

Out[41]: `[5, 6, 7, 8, 9, 10, 11]`

In [42]:
```
def bubble_sort(a):
    N = len(a) # get the length
    for i in range(N - 1):
        for j in range(N - i - 1):
            if a[j] > a[j + 1]:
                # swap larger with smaller
                a[j], a[j + 1] = a[j + 1], a[j]
        print(a)
        print('-'*30)
    return a
```

In [43]:
```
bubble_sort([11, 10, 9, 8, 7, 6, 5])
```

```
[10, 9, 8, 7, 6, 5, 11]
------------------------------
[9, 8, 7, 6, 5, 10, 11]
------------------------------
[8, 7, 6, 5, 9, 10, 11]
------------------------------
[7, 6, 5, 8, 9, 10, 11]
------------------------------
[6, 5, 7, 8, 9, 10, 11]
------------------------------
[5, 6, 7, 8, 9, 10, 11]
```


```
[5, 6, 7, 8, 9, 10, 11]
------------------------------
```

Out[41]: `[5, 6, 7, 8, 9, 10, 11]`

In [42]:
```
def bubble_sort(a):
    N = len(a) # get the length
    for i in range(N - 1):
        for j in range(N - i - 1):
            if a[j] > a[j + 1]:
                # swap larger with smaller
                a[j], a[j + 1] = a[j + 1], a[j]
        print(a)
        print('-'*30)
    return a
```

In [43]:
```
bubble_sort([11, 10, 9, 8, 7, 6, 5])
```

```
[10, 9, 8, 7, 6, 5, 11]
------------------------------
[9, 8, 7, 6, 5, 10, 11]
------------------------------
[8, 7, 6, 5, 9, 10, 11]
------------------------------
[7, 6, 5, 8, 9, 10, 11]
------------------------------
[6, 5, 7, 8, 9, 10, 11]
------------------------------
[5, 6, 7, 8, 9, 10, 11]
```

```
[5, 6, 7, 8, 9, 10, 11]
------------------------------
```

Out[41]: [5, 6, 7, 8, 9, 10, 11]

In [42]:
```
def bubble_sort(a):
    N = len(a) # get the length
    for i in range(N - 1):
        for j in range(N - i - 1):
            if a[j] > a[j + 1]:
                # swap larger with smaller
                a[j], a[j + 1] = a[j + 1], a[j]
        print(a)
        print('-'*30)
    return a
```

After 1 Complete pass

In [43]:
```
bubble_sort([11, 10, 9, 8, 7, 6, 5])
```

```
[10, 9, 8, 7, 6, 5, 11]
------------------------------
[9, 8, 7, 6, 5, 10, 11]
------------------------------
[8, 7, 6, 5, 9, 10, 11]
------------------------------
[7, 6, 5, 8, 9, 10, 11]
------------------------------
[6, 5, 7, 8, 9, 10, 11]
------------------------------
[5, 6, 7, 8, 9, 10, 11]
```

Optimized Bubble Sort

In [46]:
```
def optimized_bubble_sort(a):
    N = len(a) # get the length
    for i in range(N - 1):
        num_swaps = 0
        for j in range(N - i - 1):
            if a[j] > a[j + 1]:
                # swap larger with smaller
                a[j], a[j + 1] = a[j + 1], a[j]
                num_swaps += 1
        if num_swaps == 0:
            break
        print(a)
        print('-'*30)
    return a
```

In [47]:
```
optimized_bubble_sort([4, 5, 4, 2, 5, 6])
```

```
[4, 4, 2, 5, 5, 6]
------------------------------
[4, 2, 4, 5, 5, 6]
------------------------------
[2, 4, 4, 5, 5, 6]
------------------------------
```

Out[47]:
```
[2, 4, 4, 5, 5, 6]
```

You have left the meeting

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