Combinatorics

Combinatorics ⇒ Maths. Why?

1. Current understanding poll.
2. Quizzes improve your level
3. Goal of session ⇒ Good knowledge and intuitive understanding of Combinatorics.

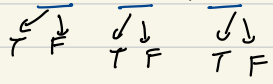
Imp in DS ML Journey.

Q1. Given three T/F questions.
Count total ways to answer questions.

T/F

2 & 2 & 2

$2 * 2 * 2 = 8$



T T T	F T T
T T F	F T F
T F T	F F T
T F F	F F F

Binary → Decimal

$2 * 2 * 2 = 8$.

1/0 1/0 1/0

0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

Q2. 10 Girls and 7 Boys in a hall.
Count no. of ways to pick 1 boy and 1 girl.

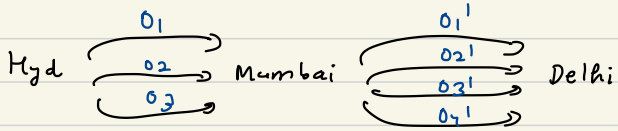
G1	
G2	B1
G3	B2
G4	B3
G5	B4
G6	B5
G7	B6
G8	B7
G9	
G10	

add

(G1, B1) or (G1, B2) or (G1, B3) or --- (G1, B7)
(G2, B1) or (G2, B2) or (G2, B3) or ---- (G2, B7)
10 G. ⋮
(G10, B1) or (G10, B2) or ----- (G10, B7)

$\Rightarrow 10 * 7 = 70$ possibilities.

Q3.

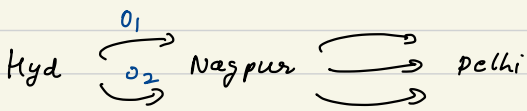


Hyd to Delhi?

O1 or O2 or O3

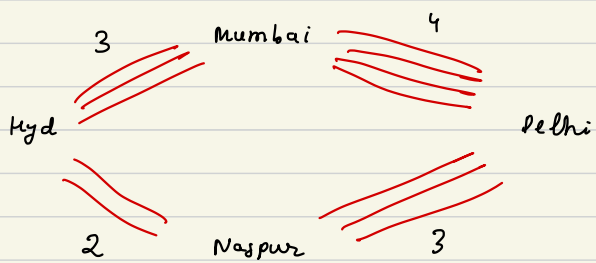
4 4 4 $3 * 4 = 12$

Q4.



$2 * 3 = 6$

Q5.



Hyd → Mumbai → Delhi OR Hyd → Nag → Delhi

$3 * 4 + 2 * 3$

$12 + 6 = 18$

Q6.

Pens : 3 P_1, P_2, P_3

Books : 5 B_1, B_2, B_3, B_4, B_5 You can only gift one of the foll. combos, count total no. of ways to form gifts:

Flowers : 7

Chocolate : 3

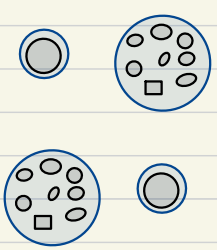
Rings : 3

multiply

- (1 pen and 1 book) OR ⇒ $3 * 5 = 15$
- (1 flower and 1 chocolate) OR ⇒ $7 * 3 = 21$
- (1 ring) ⇒ 3

39

Permutation = **Arrangement** of objects.



$(i,j) \neq (j,i)$

Q.7 Count the no. of ways to arrange 3 characters a, b & c.

$3 \times 2 \times 1 = 6.$

a b c

a	a	a	✗
a	b	c	✓
a	b	a	✗

a	b	c
a	c	b
b	a	c
b	c	a
c	a	b
c	b	a

Q.8. Count no. of ways to arrange 4 char: a, b, c and d.

$4 * 3 * 2 * 1 = 24$

a, b, c, d

a b c d	b ___	c ___	d ___
a b d c	b ___	c ___	d ___
a c b d	b ___	c ___	d ___
a c d b	b ___	c ___	d ___
a d b c	b ___	c ___	d ___
a d c b	b ___	c ___	d ___

☆ No. of ways to arrange N distinct objects in N places

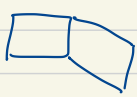
$N * (N-1) * (N-2) \ldots * 1$

$= N!$ $= fact(N)$

Q9. Count the no. of ways to arrange 0 distinct characters.

3 → 100 Rs

$3! = 6$



$0! = 1$

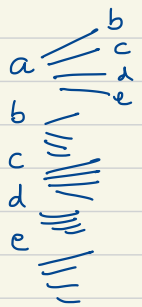
empty state

1 way.

Q16. Given 5 diff char, in how many ways can we arrange them in 2 places?

$5 \times 4 = 20.$

a b c d e



$$\frac{5!}{(5-2)!} = \frac{5!}{3!} = \frac{120}{6} = 20.$$

N distinct obj. & 3 positions $\Rightarrow N*(N-1)*(N-2)$ (3−1)

4 pos $\Rightarrow N*(N-1)\cdots(N-3)$ (4−1)

5 pos $\Rightarrow N(N-1)(N-2)(N-3)(N-4)$ (5−1)

6 pos $\Rightarrow N(N-1)(N-2)(N-3)(N-4)(N-5)$ (6−1)

K pos $\Rightarrow$ $N*(N-1)*\cdots(N-K+1)$

$N-(K-1) \Rightarrow N-K+1$

$n=5$, $k=2$

$$\frac{5*4*\cancel{3}*\cancel{2*1}}{\cancel{3*2*1}} = \frac{5!}{3!}$$

(5−2)

distinct N obj and K pos.

$$\frac{N(N-1)(N-2)\cdots(N-K+1)\,(N-K)(N-K-1)\cdots 3\cdot 2\cdot 1}{(N-K)(N-K-1)\cdots 3\cdot 2\cdot 1}$$

$$= \boxed{\frac{N!}{(N-K)!}} = {}^{n}P_{k}$$

$$\frac{6*5*4*3*2*1}{3*2*1} = \frac{6!}{3!}$$

Q11. Given 5 characters, how many ways to arrange in 3 places.

$$\underline{5} * \underline{4} * \underline{3} = 60$$

a b c d e

$$_{N}P_{K} = \frac{N!}{(N-K)!}$$

$$_{5}P_{3} = \frac{5!}{(5-3)!} = \frac{120}{2} = 60$$

Combinations

Selection of objects

Order does not matter.

D K R B S

N=5 Dhoni, Kohli, Rohit, Bumrah, Shami

$(i,j) = (j,i)$

Dhoni, Kohli, Rohit

K=3

D, K, R

K, D, R

R, D, K

K, R, D

R, K, D

D, R, K

Q12. Select 3 batsmen from a pool of 4 cricketers.

C_1, C_2, C_3, C_4

Selecting ⇓ Combination

C_1, C_2, C_3
C_1, C_3, C_2
C_2, C_1, C_3
C_2, C_3, C_1
C_3, C_1, C_2
C_3, C_2, C_1

C_1, C_3, C_4
C_1, C_4, C_3
C_3, C_1, C_4
C_3, C_4, C_1
C_4, C_1, C_3
C_4, C_3, C_1

C_1, C_2, C_4
C_1, C_4, C_2
C_2, C_1, C_4
C_2, C_4, C_1
C_4, C_1, C_2
C_4, C_2, C_1

C_2, C_3, C_4
C_2, C_4, C_3
C_3, C_2, C_4
C_3, C_4, C_2
C_4, C_2, C_3
C_4, C_3, C_2

n=4
k=3

Total arrangement = $4 * 3 * 2 = 24 = {}^4P_3 = \frac{4!}{(4-3)!} = \frac{24}{1}$

Permutation ↑

Total combinations $= {}^{n}C_{k} = \frac{24}{6} = 4 = \frac{{}^{n}P_{k}}{k!}$

$$ {}^{n}C_{k} = \frac{{}^{n}P_{k}}{k!} $$

$n = 6$ Pool of 6 cricketers

$k = 1$

Select 1 C_1 or C_2 or C_3 or C_4 or C_5 or C_6

$\Rightarrow 6$

$k = 2$ Select 2 Total arrangements.

${}^{6}P_{2} = \frac{6!}{4!}$

$6 * 5 = 30$ $= 30.$

C_1, C_2	C_2, C_1	C_3, C_1	C_4, C_1	C_5, C_1	C_6, C_1
C_1, C_3	C_2, C_3	C_3, C_2	C_4, C_2	C_5, C_2	C_6, C_2
C_1, C_4	C_2, C_4	C_3, C_4	C_4, C_3	C_5, C_3	C_6, C_3
C_1, C_5	C_2, C_5	C_3, C_5	C_4, C_5	C_5, C_4	C_6, C_4
C_1, C_6	C_2, C_6	C_3, C_6	C_4, C_6	C_5, C_6	C_6, C_5

Total combinations $= \frac{30}{2} = 15$

$k = 3$

$$6 * 5 * 4 = 120$$

$n = 6$
$k = 3$

6 distinct obj, 3 places $\Rightarrow$ ${}^{6}P_{3}$ arrangements

C_1, C_2, C_3
C_1, C_3, C_2
C_2, C_1, C_3
C_2, C_3, C_1
C_3, C_1, C_2
C_3, C_2, C_1

$3! = 6$

$$\text{Total selection} = \frac{\text{Total arrangement of n obj, k places}}{\text{Total arrangement of k obj, k places.}} = \frac{120}{6}$$

$$= 20.$$

Q→ Given 5 players, select (order does not matter.) 2 batsmen.

$${}^{5}C_{2} = \frac{{}^{5}P_{2}}{2!} = \frac{5*4}{2} = \frac{20}{2} = 10.$$

${}^{5}C_{2}$ ← Choose / select.

$$\boxed{{}^{n}C_{k} = \frac{{}^{n}P_{k}}{k!}}$$

$P_1 P_2$ $P_1 P_3$ $P_1 P_4$ $P_1 P_5$
$P_2 P_3$ $P_2 P_4$ $P_2 P_5$
$P_3 P_4$ $P_3 P_5$
$P_4 P_5$

If order matters

$P_1 P_2 \neq P_2 P_1$

$n = 5$

$k = 2$

$$^{n}P_{k} = \frac{5!}{(5-2)!} = \frac{5!}{3!}$$

$$= \frac{120}{6} = 20$$

$P_1 P_2 = P_2 P_1$

$$^{n}C_{k} = \frac{^{n}P_{k}}{k!} = \frac{20}{2!} = \frac{20}{2} = 10.$$

$l = [\,]$

```
def fact(n):

    if n==0:
        return 1

    res = n * fact(n-1)
    global l
    l.append(res)

    return res
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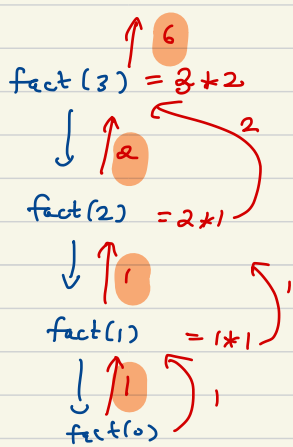


$T(1) = 1$

$$T(N) = T(N/2) + 1$$
$$= T(N/4) + 2$$
$$= T(N/8) + 3$$
$$\vdots$$
$$= T(N/2^K) + K$$

$\log_a a^x = x$

$\log_2 8 = 3$

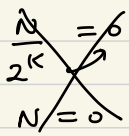
$\log_2 2^3 = 3$

$\frac{N}{2^K} = 1$

$\Rightarrow 2^K = N$

$\Rightarrow \log_2 2^K = \log_2 N$

$\Rightarrow \boxed{K = \log_2 N}$



$\Rightarrow T(N) = T(1) + \log_2 N$

$= 1 + \log_2 N.$

$\Rightarrow \boxed{O(\log_2 N)}$

$T(N/2) = 2T(N/4) + 1$

$T(N) = 2\boxed{T(N/2)} + 1$

$= 4T(N/4) + (2+1)$ 3 $\quad 4-1 \quad 2(2T(N/4)+1)$

$= 8T(N/8) + (4+2+1)$ 7 $\quad 8-1$

$= 2^K T(N/2^K) + (2^K - 1)$

$T(1) = 1$

$\frac{N}{2^K} = 1 \Rightarrow K = \log_2 N$

$\Rightarrow T(N) = N(1) + N - 1$

$= 2N - 1 \Rightarrow O(N).$