Coordinate

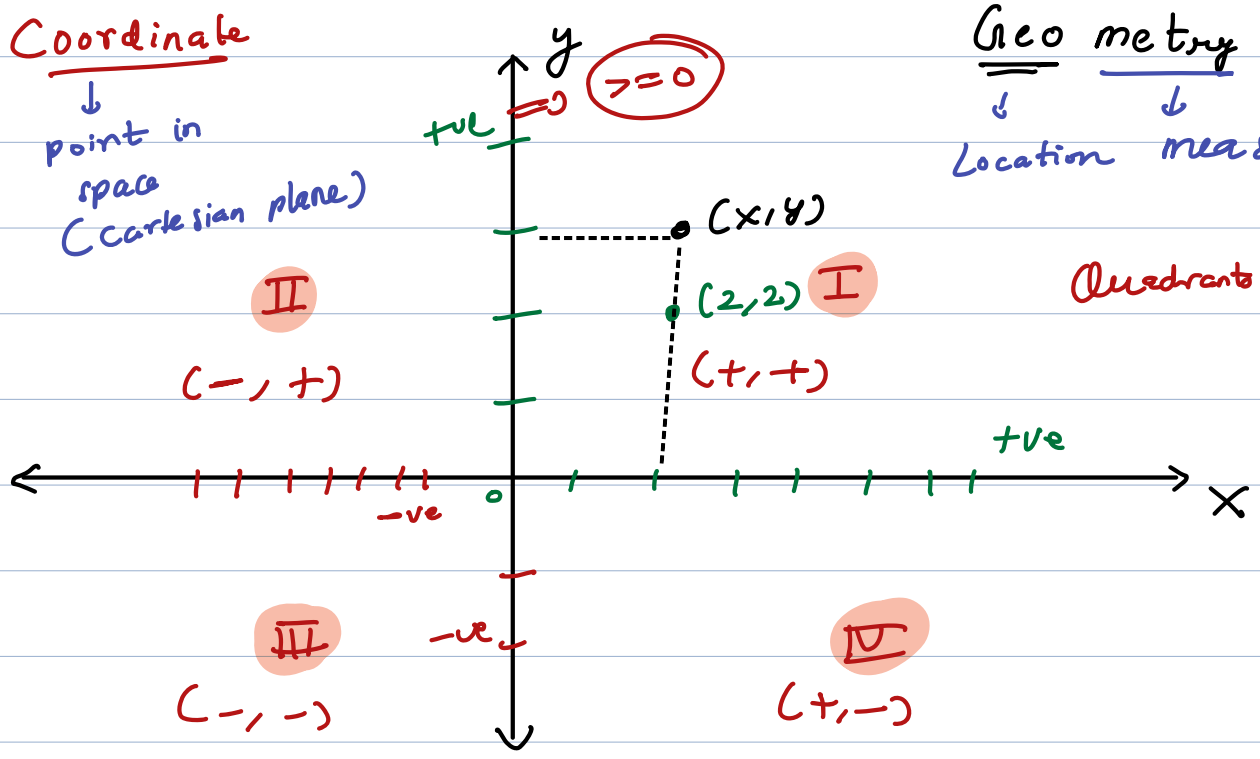
↓

point in space (Cartesian plane)

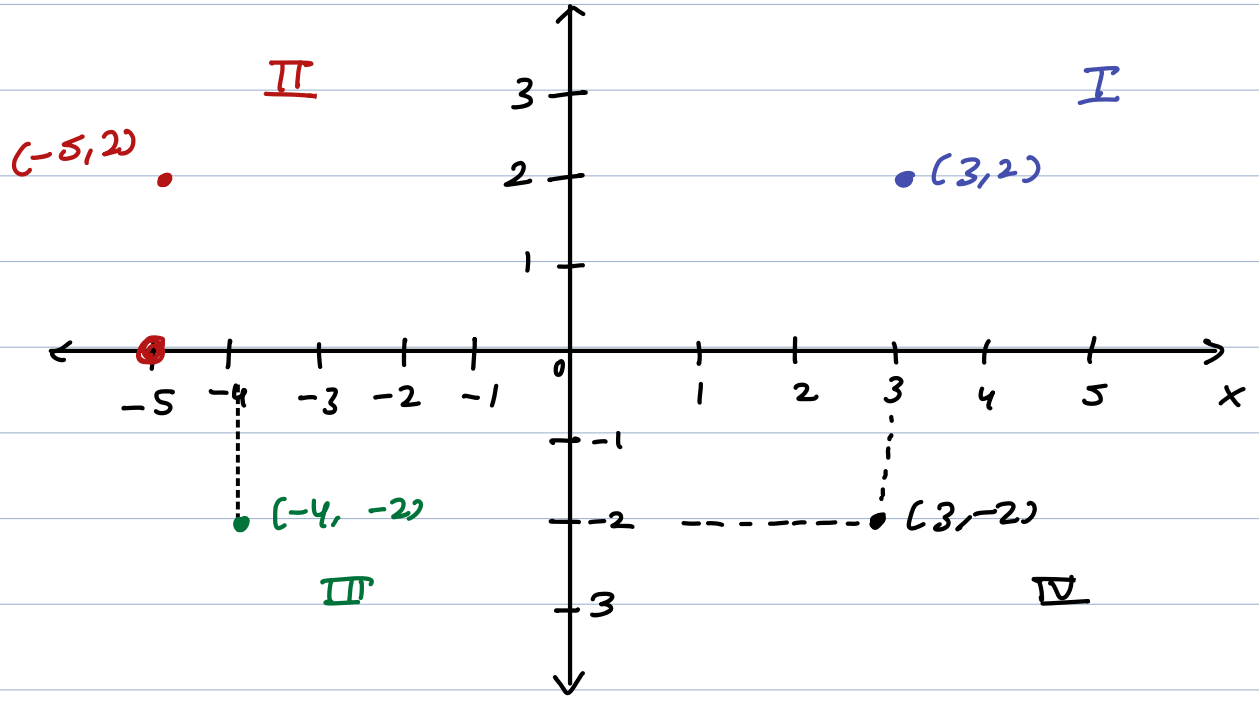
Geometry

↓ Location

↓ measurements.

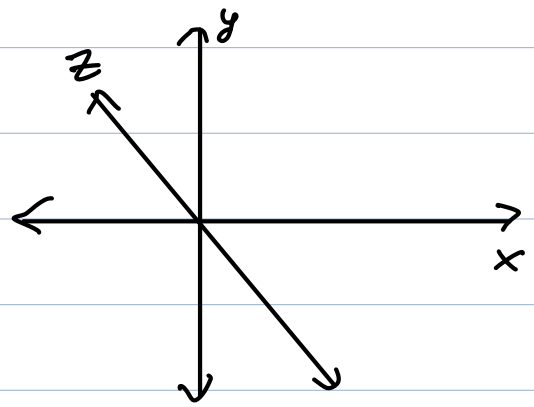


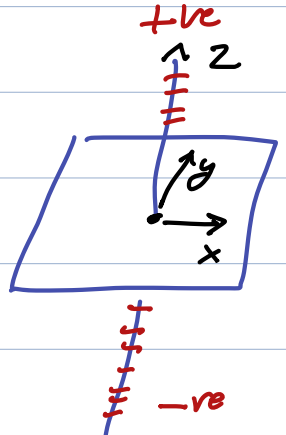
Quadrants => 4.



(-5, 2, -7)

x, y, z

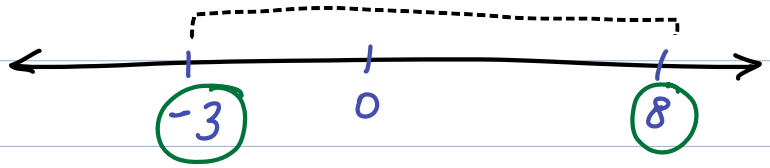




Distance

$8 - (-3) = 8 + 3 = 11$

1D



$x_1 = -3$

$x_2 = 8$

$x_2 - x_1 = 8 - (-3)$

$= 11$

$x_1 = 8$

$x_2 = -3$

$x_2 - x_1 = -3 - 8 = -11$

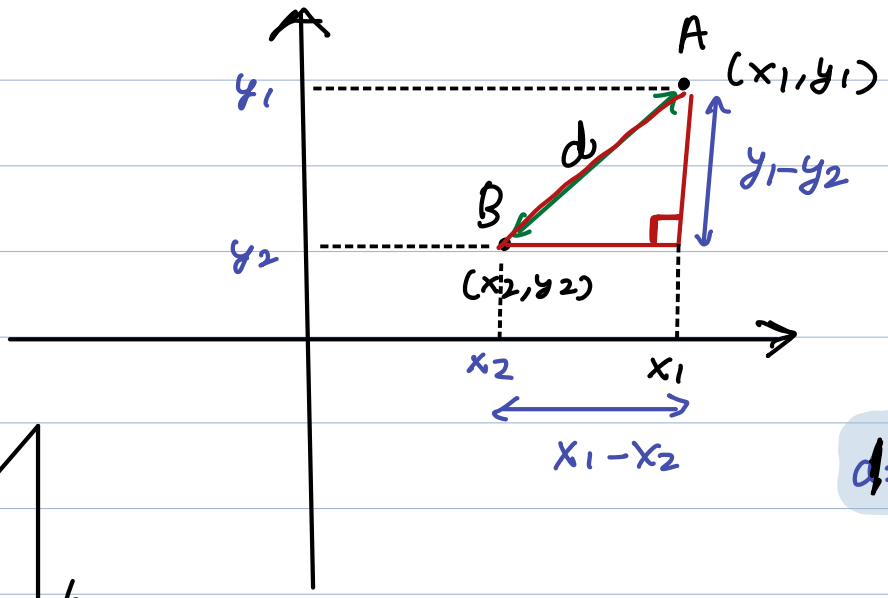
$|-11| = 11$

$d = |x_2 - x_1|$

$3^2 = 9$

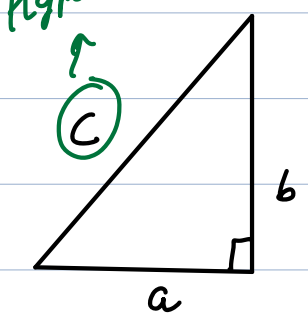
$(-3)^2 = 9$

2D



$d^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2$

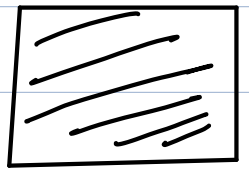
$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$



Pythagoras Theorem: $c^2 = a^2 + b^2$

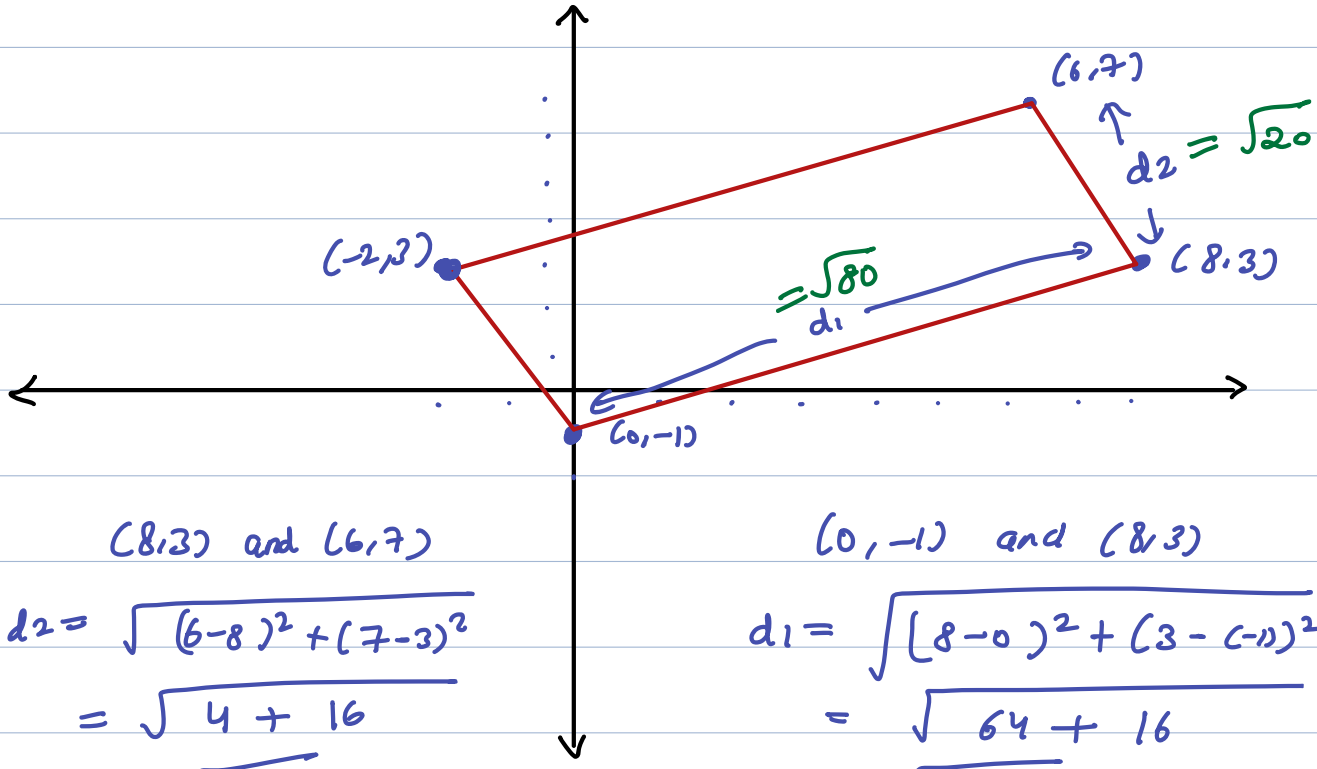
$c = \sqrt{a^2 + b^2}$

Quiz-2 Area of Rectangle



$l \times b$

(0,−1), (6,7), (−2,3), (8,3)



(8,3) and (6,7)

$d_2 = \sqrt{(6-8)^2 + (7-3)^2}$

$= \sqrt{4 + 16}$

$= \sqrt{20}$

(0,−1) and (8,3)

$d_1 = \sqrt{(8-0)^2 + (3-(-1))^2}$

$= \sqrt{64 + 16}$

$= \sqrt{80}$

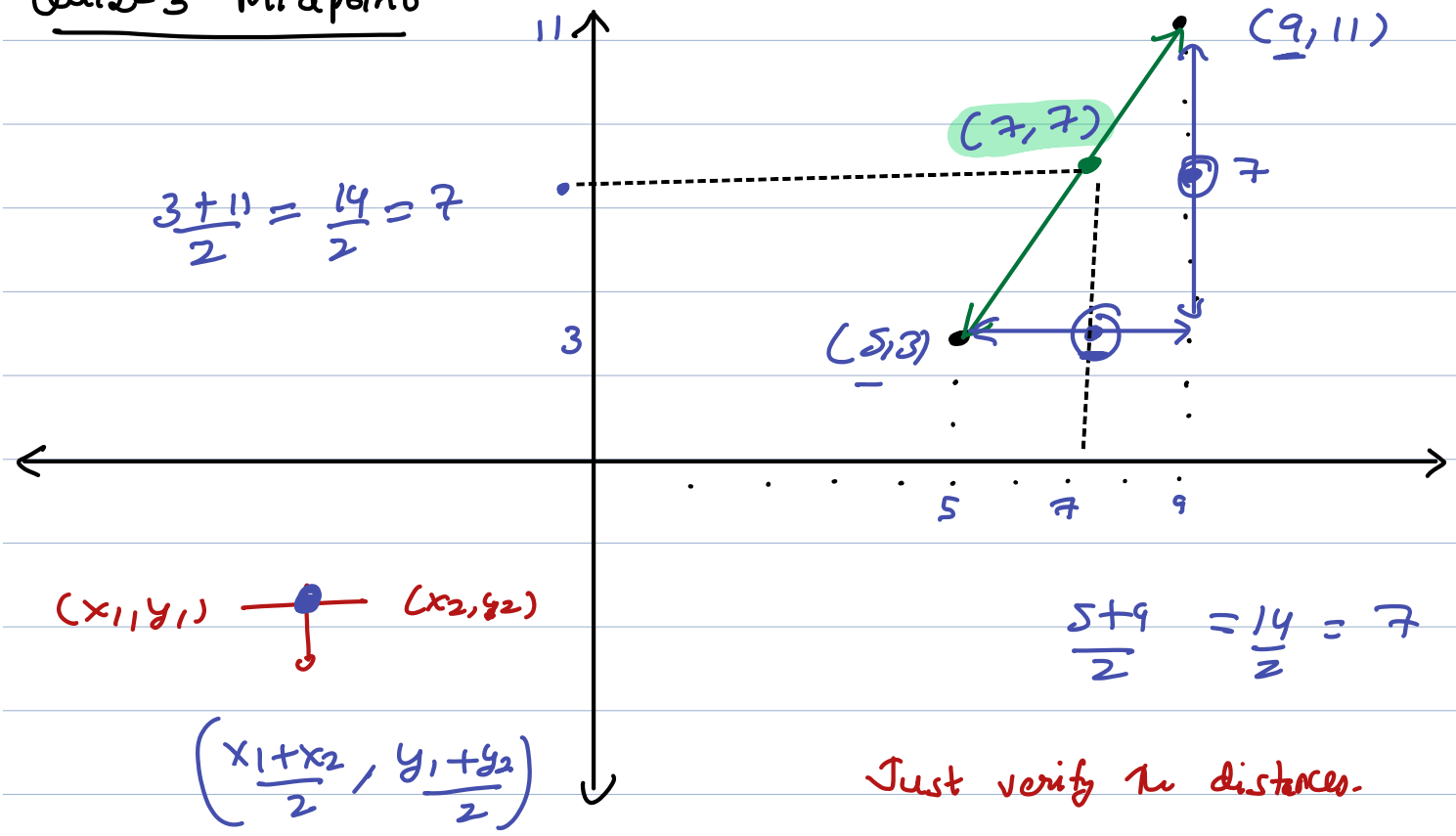
Area $= d_1 \times d_2$

$= \sqrt{80} \times \sqrt{20}$

$= \sqrt{1600} = 40$

(5,3) and (9,11)

Quiz-3 Midpoint



$\frac{3+11}{2} = \frac{14}{2} = 7$

(x_1, y_1) — (x_2, y_2)

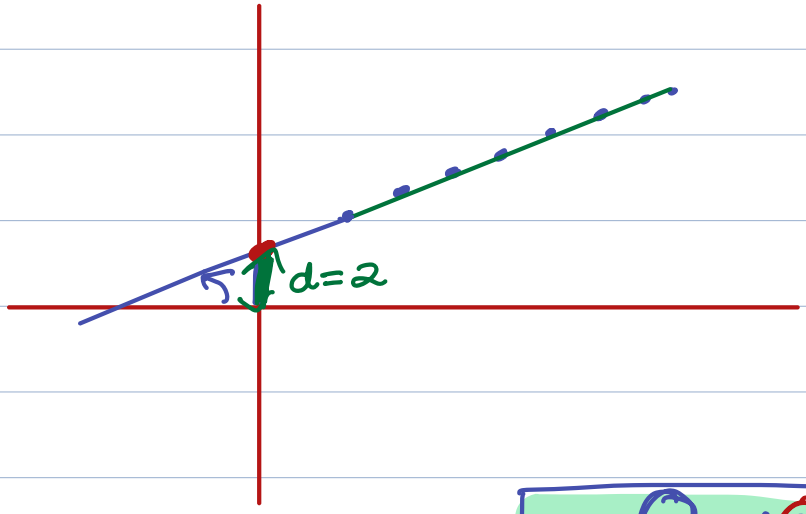
$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$\frac{5+9}{2} = \frac{14}{2} = 7$

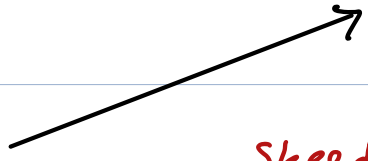
Just verify in distances.

Straight Lines ?

collection of points which follow a simple equation.



Vector



50 km/hr.

Speed

Velocity ⇒ speed + dirn

Slope ?

$y = mx + c$

intercept with y-axis

slope = m ⇒ rate of change of y

wrt. x

$y = x$ $\quad \begin{cases} x = 1 \to 2 \\ y \;\; 1 \to 2 \end{cases}$

$y = 2x$ $\quad \begin{cases} x = 1 \to 2 \\ y = 2 \to 4 \end{cases}$ $\quad \Delta x = 2 - 1 = 1$, $\Delta y = 4 - 2 = 2$

$y = -2x$ $\quad x = 1 \to 2$ $\quad \Delta x = 2 - 1 = 1$

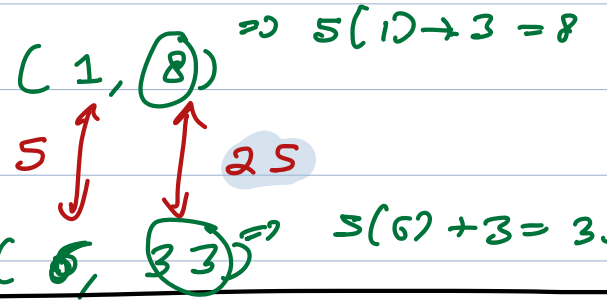
$y = -2 \to -4$ $\quad \Delta y = -4 - (-2) = -4 + 2 = -2$

Quia $\quad y = 5x + 3$

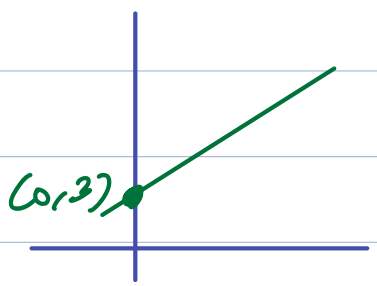
$m = 5$ $\quad \Delta x = 5$

$\Delta y = m \, \Delta x$ $\quad \Rightarrow \quad m = \dfrac{\Delta y}{\Delta x}$

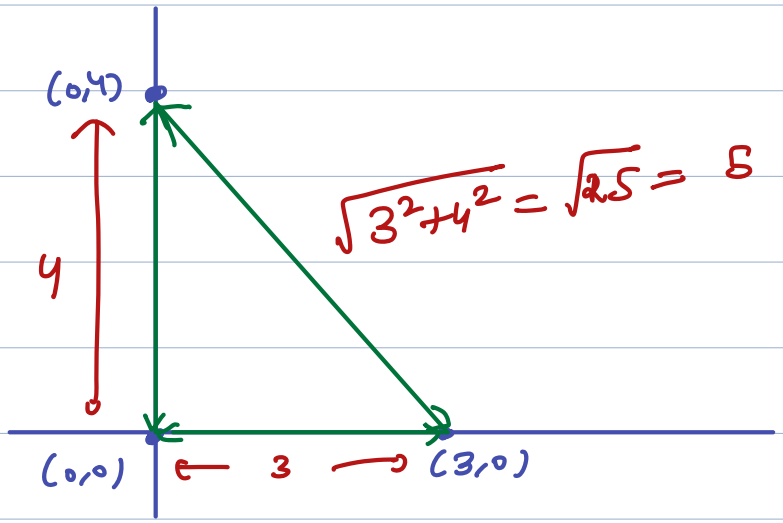
$\Delta y = 5(5) = 25$



$x = 0 \quad \Rightarrow \quad y = 5(0) + 3 = 3$



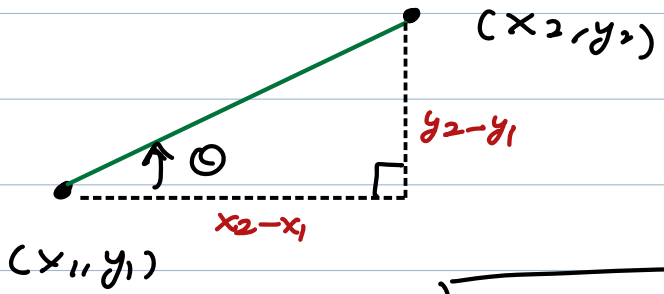
Quiz (0,4), (0,0), (3,0)



$5+4+3 = 12.$

Slope using 2 pts on a line.

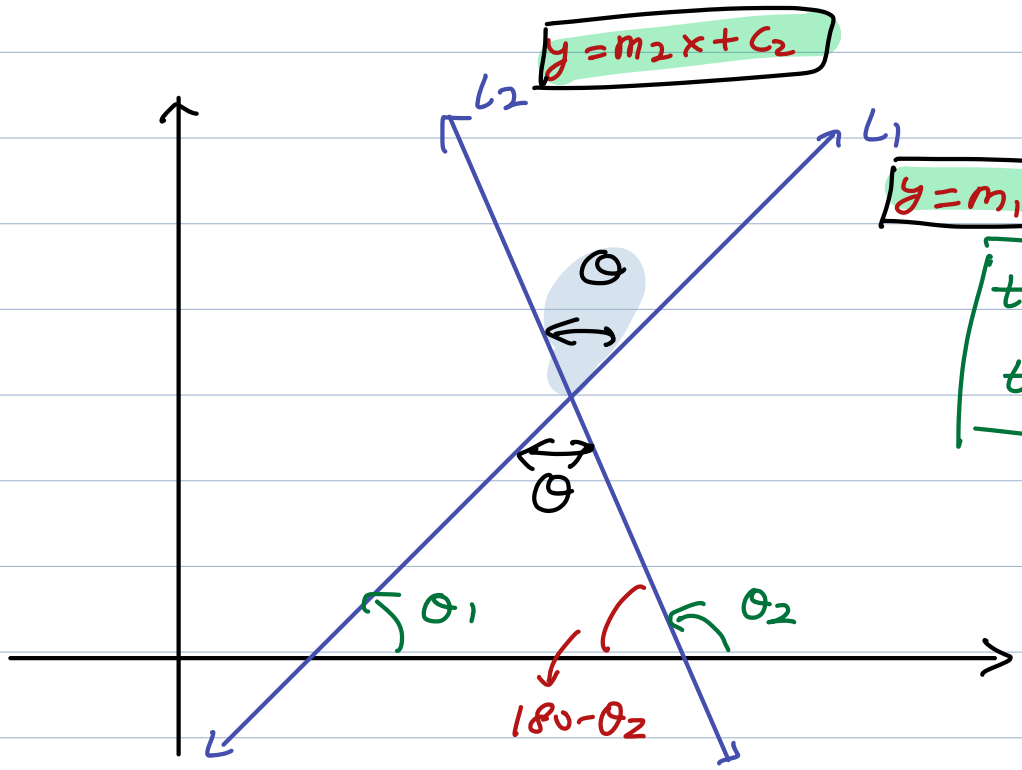
$$m = \frac{\Delta y}{\Delta x}$$



Revise Basic Trigonometry.

$$m = \tan\theta = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{perpendicular}}{\text{base}}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$



$\tan\theta_1 = m_1$

$\tan\theta_2 = m_2$

$\theta_2 = \tan^{-1}(m_2)$

$\theta_1 = \tan^{-1}(m_1)$

$\theta + \theta_1 + (180 - \theta_2) = 180$

$\Rightarrow \theta = \theta_2 - \theta_1$

Take tan both sides

$$\theta = \tan^{-1}(m_2) - \tan^{-1}(m_1)$$

II $\tan\theta = \tan(\theta_2 - \theta_1)$ — Formula

$$\Rightarrow \tan\theta = \frac{\tan\theta_2 - \tan\theta_1}{1 + \tan\theta_1 \tan\theta_2}$$

put $\tan\theta_1 = m_1$, $\tan\theta_2 = m_2$

$$\Rightarrow \tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2}$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{m_2 - m_1}{1 + m_1 m_2}\right)$$

Gives the angle given the value.

$\tan 45^\circ = 1$

$\tan^{-1}(1) = 45^\circ$

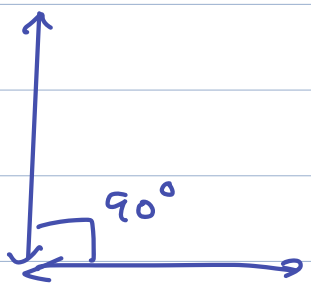
When are 2 lines parallel?

$\tan\theta = 0$

$\Rightarrow \frac{m_2 - m_1}{1 + m_1 m_2} = 0$

$\Rightarrow \boxed{m_2 = m_1}$

When are 2 lines perpendicular?



$\tan 90° = \frac{1}{0}$

$\frac{1}{\tan 90°} = 0$

$\frac{1}{\tan\theta} = \frac{1 + m_2 m_1}{m_2 - m_1} = 0$

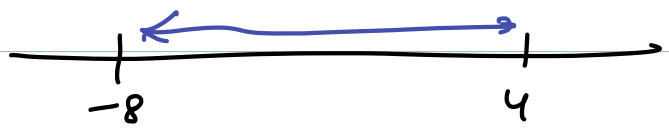
$\Rightarrow 1 + m_1 m_2 = 0$

$\Rightarrow \boxed{m_1 m_2 = -1}$

HW:
Revise Trigonometric Function.

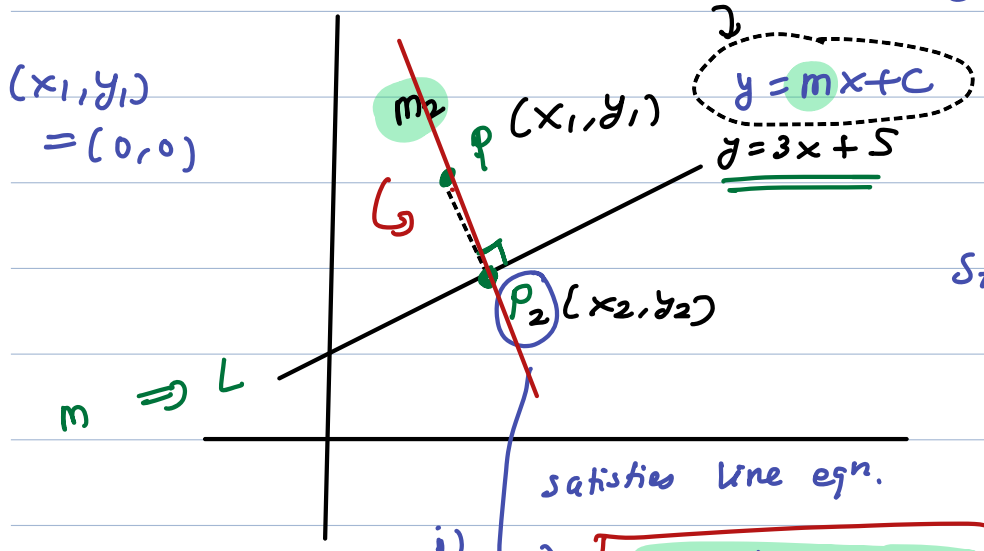
Quiz

$|4 - (-8)| = |12| = 12.$



Distance of a point from a line

$y = 3x + 5$



Step-1 : Find P_2 on line L which is at 90° to line L. (perpendicular)

Step 2 : Get the distance b/w P & P_2.

i) $y_2 = mx_2 + c$ → (1)

$m = 3$
$c = 5$

ii) $m_2 = \dfrac{y_2 - y_1}{x_2 - x_1}$ ⇒ slope of ⊥r line

iii) $m \cdot m_2 = -1 \Rightarrow m\left(\dfrac{y_2 - y_1}{x_2 - x_1}\right) = -1$

① $y_2 = 3x_2 + 5$

② $y_2 = \dfrac{-x_2}{3} + 0$ $(x_1, y_1) = (0,0)$

2 eqn, 2 variables.

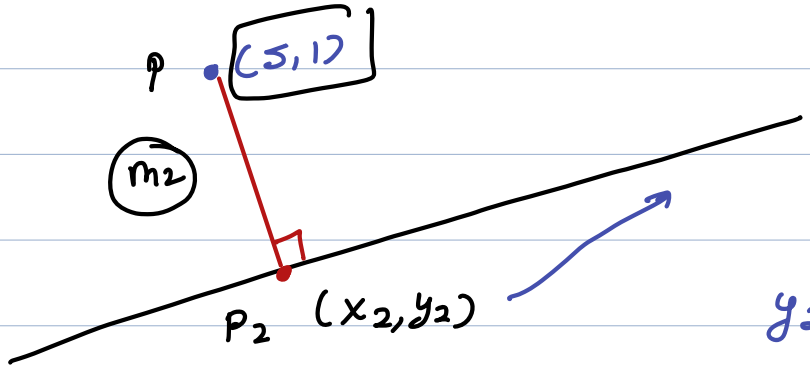
$\Rightarrow m y_2 - m y_1 = x_1 - x_2$

$\Rightarrow m y_2 = x_1 - x_2 + m y_1$

$\Rightarrow y_2 = \dfrac{(x_1 - x_2) + m y_1}{m}$

$\Rightarrow \boxed{y_2 = \dfrac{(x_1 - x_2)}{m} + y_1}$ → (2)

$(5,1)$ from line $y = 3x + 1$.



$3 \cdot m_2 = -1$

$\Rightarrow m_2 = \frac{-1}{3}$

$y_2 = 3x_2 + 1 \longrightarrow (1)$

$$m_2 = \frac{y_2 - 1}{x_2 - 5} = \frac{-1}{3}$$

$\Rightarrow 3(y_2 - 1) = 5 - x_2$

$\Rightarrow \boxed{x_2 = 5 - 3(y_2 - 1)} \longrightarrow (2)$

Put x_2 in (1)

$y_2 = 3(5 - 3y_2 + 3) + 1$

$\Rightarrow y_2 = 15 - 9y_2 + 9 + 1$

$\Rightarrow y_2 = -9y_2 + 25$

$\Rightarrow 10\, y_2 = 25$

$\Rightarrow \boxed{y_2 = \frac{25}{10}} = \frac{5}{2}$

Put y_2 in (2)

$\Rightarrow x_2 = 5 - 3\left(\frac{25}{10} - 1\right)$

$= 5 - 3\left(\frac{15}{10}\right)$

$x_2 = \frac{50 - 45}{10} = \boxed{\frac{5}{10}}$

$(5,1)$ from $\left(\frac{1}{2}, \frac{5}{2}\right)$

$$\sqrt{\left(5-\frac{1}{2}\right)^2+\left(\frac{5}{2}-1\right)^2}$$

$$=\sqrt{\left(\frac{9}{2}\right)^2+\left(\frac{3}{2}\right)^2}$$

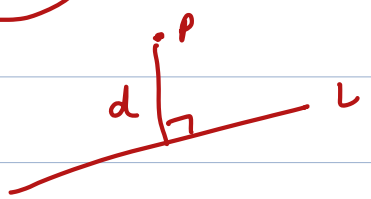
$$=\sqrt{\frac{81}{4}+\frac{9}{4}} \qquad =\sqrt{\frac{90}{4}}=\frac{45}{2}$$

$$=\sqrt{\frac{45}{2}} \quad \checkmark$$

$$\frac{15}{\sqrt{10}}=\sqrt{\frac{225}{10}}=\sqrt{\frac{45}{2}} \quad \checkmark$$

Shortcut $L: y=mx+c \Rightarrow \boxed{y-mx-c=0}$

$P=(x_1,y_1)$



$$\boxed{d=\frac{|y_1-mx_1-c|}{\sqrt{1+m^2}}}$$

$(5,1)$ from line $y=3x+1$.

$y_1=1$, $x_1=5$, $m=3$, $c=1$

$$d=\frac{|1-3(5)-1|}{\sqrt{1+3^2}}=\frac{|1-15-1|}{\sqrt{10}}=\frac{15}{\sqrt{10}}$$

$(0,0)$ from $y = 3x + 5$ $\quad m = 3$

$\Rightarrow y - 3x - 5 = 0$

$$d = \frac{|0 - 3(0) - 5|}{\sqrt{1 + 3^2}} = \frac{|5|}{\sqrt{10}} = \frac{5}{\sqrt{10}}$$

HW: Verify whether it's matching without shortcut.