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HOMEWORK 9

1) Let us consider number of possible password for each case:

- 2 digit and 4 lowercase letters: $10^2 \times 26^4 \times {}^6C_2$

- 3 digits and 3 lowercase letter: $10^3 \times 26^3 \times {}^6C_3$

- 4 digits and 2 lowercase letters: $10^4 \times 26^2 \times {}^6C_4$

Thus the total number of possible passwords:
$(10^2 \times 26^4 \times {}^6C_2) + (10^3 \times 26^3 \times {}^6C_3) + (10^4 \times 26^2 \times {}^6C_4)$

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2) There are 12! ways to order 12 people around the table. If we rearrange 3 people for any given side, the seating still will remain the same. Rearranging for each of the 4 sides of the table, we need to divide the answer by $(3!)^4$. Also, since the positioning of groups relative to each other does not affect seating, and there are 4 sides, we must also divide by 4!.

So final answer is: $\frac{12!}{(3!)^4 \cdot 4!} = \frac{12!}{31104} = 15400$

There are 15400 ways to seat 12 people at the square shaped table which seats 3 people on each side.

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3) a) We are picking 7 backpacks out of the 11 backpacks to put each textbook in $^{11}C_7$ combinations.
And because the textbooks and backpacks are unique, we must account for the arrangements and locations of the textbooks. There are 7! ways to order the textbooks.

- Thus the number of ways is: $^{11}C_7 \times 7! = \underline{^{11}P_7}$

b) Because the backpacks are identical, it does not matter which backpack we put each textbook in, and the order and placement of each textbook does not matter.
So there is 1 way to distribute the 7 textbooks
- Thus the number of ways is: $\underline{1}$

c) Because the textbooks are identical, we don't need to account for which order they are arranged.
But because the backpacks are all unique, we must account for all the possible combinations to pick 7 backpacks out of 11 backpacks to place each textbook in, which is $^{11}C_7$.
- Thus the number of ways is: $\underline{^{11}C_7} =$

d) Since there is no distinction for the textbooks or the backpacks, it does not matter which textbook goes into which backpack, and the arrangement of textbooks do not matter. So there is effectively only 1 way to distribute the identical textbooks into the identical backpacks.
- Thus the number of ways is: $\underline{1}$

4) Since "SEERESS" is a 7-letter string, we need to find the number of strings for length 6 and 7.

For length 7: $\frac{7!}{3!3!1!} = 140$

So there are 140 strings of length 7.

For length 6:

- If we omit an R, we form $\frac{6!}{3!3!} = 20$ strings
- If we omit an E, we form $\frac{6!}{3!2!1!} = 60$ strings
- If we omit an S, we form $\frac{6!}{3!2!1!} = 60$ strings

Based on the 3 cases, we can form a total of 140 strings of length 6.

$140 + 140 = 280$ strings total for length 6 or length 7.

In conclusion, summing up the strings of length 7 and 6, we have a grand total of 280 strings for lengths 6 or greater

5) a) Players are picking 7 distinct numbers from a set of 80 numbers.

So $|S| = \binom{80}{7} = {}^{80}C_7 = \frac{80!}{7!\,73!}$

$\Rightarrow |S| = 3,176,716,400$

b) The event is that the player chooses 7 winning number. Thus, the players are picking 7 numbers from a set of 11 winning numbers, since organiser pick a set of 11 distinct integers.

So $|E| = \binom{11}{7} = {}^{11}C_7 = \frac{11!}{7!\,4!} = 330$

$\Rightarrow |E| = 330$

c) Probability of winning $= P(E) = \frac{|E|}{|S|} = \frac{{}^{11}C_7}{{}^{80}C_7}$

$\Rightarrow$ Thus the probability of winning is : $\frac{{}^{11}C_7}{{}^{80}C_7}$

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5) d) For sample space S' organiser choose 11 distinct numbers from the set $\{1, 2, \dots, 80\}$

Thus $|S'| = {}^{80}C_{11}$

The event E' is that the player has a winning set, which is guaranteed if the organisers draw any set of 11 that includes the player's chosen numbers that are part of a winning set.

In E', we choose 7 numbers from the organizers set that is part of the 7 numbers in the players set, and we choose 4 numbers out of the 73 numbers that are NOT in the player's set.

Thus $|E'| = {}^{7}C_{7} \times {}^{73}C_{4} = 1 \times {}^{73}C_{4}$

So $|E'| = {}^{73}C_{4}$

e) $P(E') = \frac{E'}{S'} = \frac{{}^{73}C_{4}}{{}^{80}C_{11}}$

So the probability of winning for (d) is : $\frac{{}^{73}C_{4}}{{}^{80}C_{11}}$

Based on calculator:

$P(E') = \frac{3}{28879240}$ and $P(E) = \frac{3}{28879240}$

So the two probabilities calculated in (c) and (e) are equal

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6) Sample space = S

$n(S) = 6 \times 6 \times 6 = 216$

A = set of possibilities where sum of dice is less than 17

The maximum sum is 18 from 1 combination which is (6,6,6)

Then, the combination where the sum is 17 is (5,6,6), (6,6,5) and (6,5,6). So there are 3 possibilities of the sum being 17.

So, there are 4 combinations of choices where the sum is greater than or equal to 17.

Thus $n(A) = n(S) - 4$

$\therefore n(A) = 216 - 4$

$\therefore n(A) = 212$

Therefore $P(A) = \frac{n(A)}{n(S)} = \frac{212}{216} = \frac{53}{54}$

Thus the probability that the sum of dice is less than 17 is approximately $\frac{53}{54}$.

7) There are 2 events:

- The first event is that at least one die came up with a 6.

Let O_6 be the set of all combinations of 2 dices where at least one die came up with a 6. So we have:

Thus $O_6 = \{(1,6), (2,6), (3,6), (4,6), (5,6), (6,6),$
$(6,5), (6,4), (6,3), (6,2), (6,1)\}$

So $|O_6| = 11$

- The second event is that the sum of the two dices is 7.

Let S_7 be the set of all combinations of 2 dices where at least one die came up with a 6 AND the sum of the two dices is 7. So we have:

$S_7 = \{(1,6), (6,1)\}$

So $|S_7| = 2$

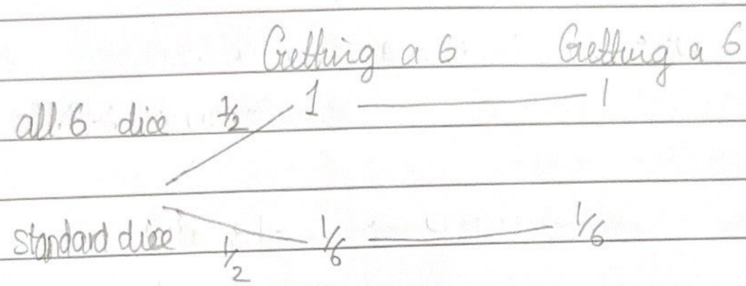
In this case, S_7 is a subset of O_6, because $(1,6) \in O_6$ and $(6,1) \in O_7$

So $|S_7 \cap O_6| = 2$

Therefore, $Pr[S_7 | O_6] = 2/11$.

- In conclusion, with the given information, the probability is $2/11$

8) a) Let $P(F)$ = Probability of rolling a 6 the first time
Let $P(S)$ = Probability of rolling a 6 the second time.
Let's have 2 cases:
Case 1: 50% chance to get all 6-faced die. All 6-dice has a 100% chance to get 6. Thus $1 \cdot \frac{1}{2} = \frac{1}{2}$
Case 2: 50% chance to get standard die. There's a $\frac{1}{6}$ to get a 6. Therefore $\frac{1}{2} \cdot \frac{1}{6} = 12$
Thus $P(F) = \frac{7}{12}$



Branch 1 = $P(F) = \frac{1}{2} \times 1 \times 1 = \frac{1}{2}$
Branch 2 = $P(S) = \frac{1}{2} \times \frac{1}{6} \times \frac{1}{6} = \frac{1}{72}$
Thus $P(F \cap S) = \frac{1}{2} + \frac{1}{72} = \frac{37}{42}$

Since we are looking for $P(S|F)$

$$P(S|F) = \frac{P(F \cap S)}{P(F)} = \frac{\left(\frac{37}{72}\right)}{\left(\frac{7}{12}\right)} = \frac{37}{72} \times \frac{12}{7} = \frac{37}{42}$$

Thus if you roll the same die, the probability that the probability that the next roll is also a 6 is $\frac{37}{42}$

8) b) No, the events are NOT independent.

Two events, let's say A and B, are independent if the occurrence of A does not affect the occurrence of B, which means that $P(A|B)=P(A)$.

However in our case if we have already observed a 6, the probability of rolling another 6 with the same die is $\frac{37}{42}$, not the original $1/6$.

Thus rolling a 6 in turn increases the probability of rolling a 6 again

Given that the first roll's outcome changes the probability of the second roll, We can conclude that the events are NOT independent.