☆ Min. Edit Distance :

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$X = x_1 x_2 \ldots x_i$ $\quad |X| = i$ where $i \geq 1$

$Y = y_1 y_2 \ldots y_j$ $\quad |Y| = j$ where $j \geq 1$

$MED(X, Y)$

$$= \text{Min} \begin{cases} MED(x_1 x_2 \ldots x_{i-1}, y_1 y_2 \ldots y_j) + D(x_i), \\ MED(x_1 x_2 \ldots x_{i-1}, y_1 y_2 \ldots y_{j-1}) + C(x_i, y_j), \\ MED(x_1 x_2 \ldots x_i, y_1 y_2 \ldots y_{j-1}) + I(y_j) \end{cases}$$

☆ 3 possibilities for MED(X, Y)

① Find MED from first $(i-1)$ characters of X and j characters of Y. To do this we also need to Delete i^{th} character from X.

i.e. $\underbrace{MED(x_1 x_2 \ldots x_{i-1}, y_1 y_2 \ldots y_j)}_{\text{Remaining Subproblem}} + D(x_i)$

Remaining Subproblem — Cost to delete x_i from string X

② We can change X_i to Y_j and the remaining subproblem is to find MED from $(i-1)$ characters of X and $(j-1)$ characters of Y.

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i.e. $\underbrace{MED(x_1 x_2 .. x_{i-1}, y_1 y_2 .. y_{j-1})}_{\text{Remaining Subproblem.}} + C(x_i, y_j)$

Cost to change x_i to y_j

③ We can find MED(X, Y) by

→ Inserting y_j to the X and thus remaining subproblem is

$$\underbrace{MED(x_1 x_2 .. x_i, y_1 y_2 .. y_{j-1})}_{\text{Remaining Sub-problem}} + \underbrace{I(y_j)}_{\text{Insertion of } y_j \text{ to the result}}$$

→ Thus, we need to find Minimum out of these 3 options.

$$MED(X, Y) = \text{Min} \begin{cases} MED(x_1 x_2 .. x_{i-1}, y_1 y_2 .. y_j) + D(x_i), \\ MED(x_1 x_2 .. x_{i-1}, y_1 y_2 .. y_{j-1}) + C(x_i, y_j), \\ MED(x_1 x_2 .. x_i, y_1 y_2 .. y_{j-1}) + I(y_j) \end{cases}$$

$X = x_1 x_2 .. x_i$

$Y = y_1 y_2 .. y_j$

where $i \geq 1$

$j \geq 1$

→ Create T [length of X + 1] [length of Y + 1]

Say m = X.length

→ 0^{th} row & column n = Y.length

for i = 0 to m
{
    T[i][0] = i;
}
for j = 1 to n
{
    T[0][j] = j;
}

Note :
→ Cost of delete = 1
→ Cost of Insert = 1
→ Cost of change = 2

↓

Why? Because i = 0 indicates X = Λ , j = 0 indicates Y = Λ

∴ MED(Λ, String) = |string|

MED(String, Λ) = |String|

↓

Why? Because to Convert Λ to String, we need to Insert characters of String to Λ.

E.g. MED(Λ, "RAM") = 3

Λ → Insert 3 characters ⇒ MED = 3

→ Similarly MED("xyz", ∧) = 3
Because to convert xyz to ∧, we need to delete all characters

→ <u>For Remaining Rows and Columns</u> :

for i = 1 to m
{
  for j = 1 to n
  {

$$T[i][j] = Min \begin{Bmatrix} T[i-1][j] + 1, \\ T[i-1][j-1] + R, \\ T[i][j-1] + 1, \end{Bmatrix}$$

  }   Here K = 0 if x[i] = y[j]
}   K = 2 if x[i] ≠ y[j]

<u>E.g.</u> X = RAM   Y = ROM

j →

	T	j=0	R j=1	O j=2	M j=3
	j=0	0	1	2	3
i ↓ R	j=1	1	0	1	2
A	j=2	2	1	2	3
M	i=3	3	2	3	2

0^{th} Row, 0^{th} column

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$$\rightarrow T[1][1] = Min \begin{cases} T[0][1]+1, \\ T[0][0]+k, \\ T[1][0]+1 \end{cases}$$

$$= Min\{1+1, 0+k, 1+1\}$$

$$= Min\{2, 0+0, 2\}$$ Here $k=0$

$\therefore x[1]=y[1]$

$= 'R'$

$$= Min\{2, 0, 2\}$$

$$= 0$$

$$\rightarrow T[1][2] = Min \begin{cases} T[0][2]+1, \\ T[0][1]+k, \\ T[1][1]+1 \end{cases}$$

$$= Min\{2+1, 1+k, 0+1\}$$

$$= Min\{3, 1+2, 0+1\}$$

Here $k=2$

because $x[1] \neq y[2]$

$'y' \neq 'o'$

$$= Min\{3, 3, 1\}$$

$$= 1$$

$i = 1$ $\quad$ $X = $ 'R'

$j = 2$ $\quad$ $Y = $ 'RO'

☆ **Logic**

Choice 1, $T[0][2] + 1$

i.e. delete 1^{st} character from X

i.e. $X = \wedge$ and find

$$\text{MED}(\underset{\substack{\uparrow \\ \text{First } (i-1) \\ \text{chars of } X}}{\wedge}, \underset{\substack{\uparrow \\ \text{First } j \text{ of } Y}}{\text{'RO'}}) + \underset{\substack{\uparrow \\ \text{cost of delete}}}{1}$$

$= 2 + 1$

$= 3$

☆

Choice 2 $i = 1$, $j = 2$ Here $X[1] \neq Y[2] \Rightarrow k = 2$

→ Change i^{th} character of X by j^{th} char. of Y.

i.e. 1^{st} char. of X by 2^{nd} char of Y.

i.e. $X = $ 'O'

∴ Remaining Subproblem

$$\text{MED}(\underset{\substack{\uparrow \\ \text{First}(i-1) \\ \text{chars. of } X}}{\wedge}, \underset{\substack{\uparrow \\ \text{First } (j-1) \\ \text{chars. of } Y}}{\text{'R'}}) = 1$$

i.e. $(i-1)^{th}$ char. of X and $(j-1)^{th}$ char. of Y

→ Thus total cost $= \text{MED}(\wedge, \text{'R'}) + 2$

$= 1 + 2$

$= 3$

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☆ Choice 3 $i=1, j=2$

→ Insert j^{th} char. of Y to the result i.e. 2^{nd} char. of Y i.e. 'O' to the result

∴ Remaining Sub-problem is

MED('R', 'R')

First (i) characters of X

First (j-1) Characters of Y

i.e. $T[i][j-1] + 1$

$= \underbrace{T[1][1]} + 1$

$= 0 + 1$

$= 1$

i.e. We can convert R to RO by cost 1

i.e. Add y_2 i.e. 'O' and the remaining Subproblem is MED('R', 'R') = 0

→ Thus $Min\{3, 3, 1\} = 1$

$\therefore \boxed{T[1][2] = 1}$

→ $T[1][3]$

$x[1]$ $y[3]$

'R' $\neq$ 'M'

$\therefore \boxed{k = 2}$

$$T[1][3] = Min\begin{cases} T[0][3] + 1, \\ T[0][2] + 2, \\ T[1][2] + 1 \end{cases}$$

$$= Min\{3+1, 2+2, 1+1\}$$
$$= Min\{4, 4, 2\}$$

$\therefore \boxed{T[1][3] = 2}$

→ This indicates

$i = 1$ 'R'

$j = 3$ 'ROM'

'R' → 'ROM' can be easily done by inserting 'O', 'M' to 'R'

i.e. Total cost = 2

While other options are costly.

☆ T[2][1] Here X[2] = 'A' y[1] = 'R'

'A' $\neq$ 'R' $\Rightarrow$ k = 2

$$T[2][1] = \text{Min}\begin{Bmatrix} T[1][1] + 1, \\ T[1][0] + 2, \\ T[2][0] + 1 \end{Bmatrix}$$

$$= \text{Min}\{0+1,\ 1+2,\ 2+1\}$$

$$= \text{Min}\{1, 3, 3\}$$

$$= 1$$

☆ T[2][2] Here X[2] = 'A' y[2] = 'O'

'A' $\neq$ 'O' $\Rightarrow$ k = 2

$$\therefore T[2][2] = \text{Min}\begin{Bmatrix} T[1][2] + 1, \\ T[1][1] + 2, \\ T[2][1] + 1 \end{Bmatrix}$$

$$= \text{Min}\{1+1,\ 0+2,\ 1+1\}$$

$$= \text{Min}\{\underbrace{2, 2, 2}\}$$

$$= 2$$

every possibility is equal here

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$\rightarrow$ $T[2][3]$ Here $X[2] = $ 'A' $Y[3] = $ 'M'

$\therefore$ $K = 2$ (As 'A' $\neq$ 'M')

$$T[2][3] = \text{Min}\left\{\begin{array}{l} T[1][3]+1, \\ T[1][2]+2, \\ T[2][2]+1 \end{array}\right\}$$

$$= \text{Min}\{2+1,\ 1+2,\ 2+1\}$$

$$= 3$$

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★ $T[3][1]$ $\quad x[3] = \text{'M'}$

$y[1] = \text{'R'}$

$x[3] \neq y[1] \Rightarrow K = 2$

$$T[3][1] = \text{Min}\begin{Bmatrix} T[2][1] + 1, \\ T[2][0] + 2, \\ T[3][0] + 1 \end{Bmatrix}$$

$$= \text{Min}\{1+1,\ 2+2,\ 3+1\}$$

$$= \text{Min}\{2, 4, 4\}$$

$$= \boxed{2}$$

★ $T[3][2]$ $\quad x[3] \neq y[2]$ i.e. $\text{'M'} \neq \text{'O'}$

$\therefore K = 2$

$$T[3][2] = \text{Min}\begin{Bmatrix} T[2][2] + 1, \\ T[2][1] + 2, \\ T[3][1] + 1 \end{Bmatrix}$$

$$= \text{Min}\{2+1,\ 1+2,\ 2+1\}$$

$$= \boxed{3}$$

★ $T[3][3]$, Here $x[3] = y[3]$ i.e. $\text{'M'} = \text{'M'}$

$\Rightarrow K = 0$

$$T[3][3] = \text{Min}\begin{Bmatrix} T[2][3] + 1, \\ T[2][2] + 0, \\ T[3][2] + 1 \end{Bmatrix} = \text{Min}\{3+1,\ 2+0,\ 3+1\}$$

$$= \text{Min}\{4, 2, 4\}$$

$$= \boxed{2}$$