Total No. of Questions—8] [Total No. of Printed Pages—6

Seat No.	

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S.E. (Mechanical/Auto/S/W) (I Sem.) EXAMINATION, 2018
ENGINEERING MATHEMATICS—III
(2015 PATTERN)

Time : Two Hours **Maximum Marks : 50**

***N.B.* :—** (*i*) Answer Q. No. **1** *or* Q. No. **2**, Q. No. **3** *or* Q. No. **4**, Q. No. **5** *or* Q. No. **6**, Q. No. **7** *or* Q. No. **8**.

(*ii*) Neat diagrams must be drawn wherever necessary.

(*iii*) Figures to the right indicate full marks.

(*iv*) Assume suitable data, if necessary.

(*v*) *All* questions are compulsory.

1. (*a*) Solve any *two* of the following : [8]

(*i*) $\left(D^2 + 2D + 1\right) y = xe^{-x} \cos x$

(*ii*) $\dfrac{d^2 y}{dx^2} + 3\dfrac{dy}{dx} + 2y = e^{e^x}$ (using method of variation of parameter)

(*iii*) $(3x+2)^2 \dfrac{d^2 y}{dx^2} + 3(3x+2)\dfrac{dy}{dx} - 36y = 3x^2 + 4x + 1.$

(*b*) Using suitable Fourier transform, solve the following equation : [4]

$$\int_0^\infty f(x) \sin \lambda x \, dx = \begin{cases} 1 - \lambda & 0 \le \lambda \le 1 \\ 0 & \lambda \ge 1 \end{cases}.$$

Or

2. (*a*) Solve any *one* : [4]

(*i*) Find Laplace transform of :

$$e^{-4t}\int_0^t \frac{\sin 3t}{t}\, dt.$$

(*ii*) Find Inverse Laplace transform of $\dfrac{s}{s^2+6s+25}$.

(*b*) Using Laplace transform solve the D.E. : [4]

$$y'' + 4y' + 13y = \frac{1}{3}e^{-2t}\sin 3t,\ \ y(0) = 1,\ \ y'(0) = -2.$$

(*c*) A body of weight W = 1 N is suspended from a spring streches it 4 cm. If the weight is pulled down 8 cm below the equilibrium position and then released : [4]

(*i*) Set up a differential equation.

(*ii*) Find the position and velocity as function of time.

3. (*a*) The first four moments of a distribution about the value 5 are 2, 20, 40 and 50. From the given information obtain the first four central moments and coefficient of skewness and kurtosis. [4]

(*b*) In a certain factory turning out razor blades, there is a small chance of $\dfrac{1}{500}$ for any blade to be defective. The blades are supplied in a packet of 10. Use Poisson distribution to calculate the approximate number of packets containing : [4]

(*i*) no defective blades in a consignment

(*ii*) two defective blades

in a consignment of 10,000 packets.

(*c*) Find the directional derivative of the function $\phi = e^{2x - y - z}$ at (1, 1, 1) in the direction of tangent to the curve : [4]

$x = e^{-t}, \; y = 2 \sin t + 1, \; z = t - \cos t$ at $t = 0$.

Or

4. (*a*) Find the regression line of y on x for the following data : [4]

x	y
10	18
14	12
18	24
22	6
26	30
30	36

(*b*) Prove the following (any *one*) : [4]

(*i*) $\nabla\left[\bar{b} \cdot \nabla\left(\frac{1}{r}\right)\right] = \frac{3\bar{r}\left(\bar{b} \cdot \bar{r}\right)}{r^5} - \frac{\bar{b}}{r^3}$

(*ii*) $\nabla^2 \left(r^n \log r\right) = \left[n(n+1)\log r + 2n + 1\right] r^{n-2}$.

(*c*) Show that the vector field : [4]

$$\bar{F} = \left(2xz^3 + 6y\right)\bar{i} + (6x - 2yz)\,\bar{j} + \left(3x^2 z^2 - y^2\right)\bar{k}$$

is irrotational and hence find scalar function ϕ such that $\bar{F} = \nabla\phi$.

5. (*a*) Find the work done in moving a particle once round the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$, $z = 0$ under the field of force given by : [5]

$$\bar{F} = (2x - y + z)\,\bar{i} + \left(x + y - z^2\right)\bar{j} + (3x - 2y + 4z)\,\bar{k}.$$

(*b*) Use divergence theorem to evaluate : [4]

$$\iint_S \left(x\,\bar{i} + y\,\bar{j} + z^2\,\bar{k}\right) \cdot d\bar{S}$$

where S is the curved surface of the cylinder $x^2 + y^2 = 4$, bounded by the planes $z = 0$ and $z = 2$.

(*c*) Evaluate : [4]

$$\iint \left(\nabla \times \bar{F}\right) \cdot \hat{n}\, dS$$

where S is the plane surface of a lamina bounded by $x = 0$, $y = 0$, $x = 1$, $y = 1$, $z = 2$ and .

$$\bar{F} = y^2\,\bar{i} + x^2\,\bar{j} + z\,\bar{k}.$$

Or

6. (*a*) Evaluate : [4]

$$\int_C \bar{F} \cdot d\bar{r}$$

where

$$\bar{F} = \sin y\,\bar{i} + x(1 + \cos y)\,\bar{j}$$

and C is the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $z = 0$.

(*b*) Show that : [4]

$$\iiint_V \frac{dv}{r^2} = \iint_S \frac{\bar{r} \cdot \hat{n}}{r^2}\, ds.$$

(*c*) Evaluate : [5]

$$\iint_S \text{curl}\, \bar{F} \cdot \hat{n}\, ds$$

for the surface of the paraboloid

$$z = 9 - \left(x^2 + y^2\right)$$

where

$$\bar{F} = \left(x^2 + y - 4\right) \bar{i} + 3xy\, \bar{j} + \left(2xz + z^2\right) \bar{k}.$$

7. (*a*) If $\dfrac{\partial^2 y}{\partial t^2} = c^2 \dfrac{\partial^2 y}{\partial x^2}$ represents the vibrations of a string of length l fixed at both ends, find the solution with boundary conditions : [7]

(*i*) $y(0, t) = 0$

(*ii*) $y(l, t) = 0$

(*iii*) $\left(\dfrac{\partial y}{\partial t}\right)_{t=0} = 0$

(*iv*) $y(x, 0) = klx^2, \quad 0 \le x \le l.$

(*b*) Solve $\dfrac{\partial u}{\partial t} = k\dfrac{\partial^2 u}{\partial x^2}$ if [6]

(*i*) $u(0, t) = 0$

(*ii*) $u_x(l, t) = 0$

(*iii*) $u(x, t)$ is bounded

(*iv*) $u(x, 0) = \dfrac{2x}{l}, \quad 0 \le x \le l.$

Or

8. (*a*) A rectangular plate with insulated surface is 4 cm wide and so long to its width that it may be consider infinite in length. If the temperature of the short edge $y = 0$ is given by : [6]

$$\begin{aligned} u &= 2x & 0 \le x \le 2 \\ &= 2(4 - x) & 2 \le x \le 4 \end{aligned}$$

two two long edges $x = 0$, $x = 4$ as well as other short edge are kept at 0°C then find $u(x, y)$.

(*b*) Use Fourier transform to solve the equation $\dfrac{\partial u}{\partial t} = \dfrac{\partial^2 u}{\partial x^2}$ $0 < x < \infty$, $t > 0$, subject to conditions : [7]

(*i*) $u(0, t) = 0, \ t > 0$

(*ii*) $u(x, 0) = \begin{cases} 1 & 0 < x < 1 \\ 0 & x > 1 \end{cases}$

(*iii*) $u(x, t)$ is bounded.