Total No. of Questions—**8**] [Total No. of Printed Pages—**6**

Seat No.	

[5559]-160

S.E. (Elect. & Inst.) (First Sem.) EXAMINATION, 2019

ENGINEERING MATHEMATICS

Paper III

(2015 PATTERN)

Time : Three Hours **Maximum Marks : 80**

***N.B.* :—** (*i*) Figures to the right indicate full marks.

(*ii*) Neat diagrams must be drawn wherever necessary.

(*iii*) Use of non-programmable, electronic pocket calculator is allowed.

(*iv*) Assume suitable data, if necessary.

1. (*a*) Solve (any *two*) : [8]

(*i*) $(D^2 + 2D + 1)y = e^{-x} + \cos x$.

(*ii*) $(D^2 - 6D + 9)y = \dfrac{e^{3x}}{x^2}$ by variation of parameters method.

(*iii*) $(2x+1)^2 \dfrac{d^2y}{dx^2} - 6(2x+1)\dfrac{dy}{dx} + 16y = 8(2x+1)^2$.

P.T.O.

(*b*) Solve the following differential equation by Laplace transform method : [4]

$$\frac{d^2y}{dt^2}+2\frac{dy}{dt}+5y=e^{-t}\sin t$$

given that $y(0)=0, \quad y'(0)=1$.

Or

2. (*a*) A resistance of 50 ohms, an inductor of 2 henries and a 0.005 farad capacitor are connected in series with e.m.f. of 40 volts and an open switch. Find the instanteneous charge and current after the switch is closed at $t = 0$, assuming that at that time, charge on capacitor is 4, coulomb. [4]

(*b*) Solve (any *one*) : [4]

(*i*) Find :

$$L\left[t\, e^{-4t}\sin 3t\right].$$

(*ii*) Find :

$$L^{-1}\left[\frac{1}{s^2(s+1)}\right].$$

(*c*) Evaluate : [4]

$$\int_0^\infty t\, e^{-t}\sin t\, dt.$$

3. (*a*) Solve the integral equation : [4]

$$\int_0^{\infty} f(x) \cos \lambda x \, dx = \begin{cases} 1-\lambda, & 0 \le \lambda \le 1 \\ 0, & \lambda \ge 1 \end{cases}.$$

(*b*) Attempt any *one* : [4]

(*i*) Find Z-transform of $f(k) = \dfrac{2^k}{k}, k \ge 1$.

(*ii*) Find inverse Z-transform of $F(z) = \dfrac{z^2}{(z^2+1)}, |z| > 1$.

(*c*) Find directional derivative of $\phi = xy^2 + yz^3$ at (2, –1, 1) along the line $2(x-2) = (y+1) = (z-1)$. [4]

Or

4. (*a*) Prove any *one* : [4]

(*i*) $\nabla^2 \left[\nabla.(r^{-2} \; \overline{r}) \right] = 2r^{-4}$

(*ii*) $\nabla \times \left[\dfrac{1}{r} (r^2 \overline{a} + (\overline{a} \; . \; \overline{r}) \overline{r}) \right] = 0$.

(*b*) For the curve $x = e^t \cos t, y = e^t \sin t, z = e^t$ find the velocity and acceleration of the particle moving on the curve at t = 0. [4]

(*c*) Solve : [4]

$$f(k+2) + 3\, f(k+1) + 2f(k) = 0$$

given $f(0) = 0, f(1) = 1$.

5. Attempt any *two* :

(*a*) Evaluate the line integral of vector point function,

$$\overline{\mathrm{F}} = (x^2 - y^2)\,\overline{i} + 2xy\,\overline{j}$$

along the curve $y^2 = x$ from the point (0, 0) to (1, 1) in X-Y-plane. [6]

(*b*) Using stoke's theorem, evaluate the integral $\iint\limits_{S} \nabla \times \overline{\mathrm{F}}.\overline{ds}$ where :

$$\overline{\mathrm{F}} = y\overline{i} + z\overline{j} + x\overline{k}$$

and 'S' is the surface of paraboloid $z = 1 - x^2 - y^2,\ z \geq 0$ above X-Y-plane. [7]

(*c*) Evaluate the integral :

$$\oiint\limits_{S} (4x\overline{i} - 2y^2\overline{j} + z^2\overline{k}).\,\overline{ds},$$

over the surface of cylinder $x^2 + y^2 = 4$ from $z = 0$ to $z = 3$ closed at both ends. [6]

Or

6. Attempt any *two* :

(*a*) Use Green's Lemma to evaluate the integral $\oint\limits_{C} (xydx + y^2dy)$. Over the area bounded by curves $y = x^2$ and line $y = x$ in Ist quadrant. [6]

(*b*) Evaluate using Stoke's theorem,

$$\oint_C \left[(x^2 + y^2)\bar{i} + (x^2 - y^2)\bar{j}\right].\, d\bar{r},$$

where 'C' is the boundary of region bounded by circles, $x^2 + y^2 = 4$ and $x^2 + y^2 = 16$ in X-Y-plane. [7]

(*c*) Evaluate :

$$\oiint_S (x^3\bar{i} + y^3\bar{j} + z^3\bar{k}).\overline{ds},$$

using divergence theorem, where 'S' the surface of the sphere $x^2 + y^2 + z^2 = 16$. [6]

7. (*a*) If $\phi + i\psi$ is complex potential for an electric field (analytic function) and $\phi = -2xy + \dfrac{y}{x^2 + y^2}$. Find ψ. [4]

(*b*) Evaluate :

$$\int_{2+4i}^{5-5i} (z+1)\, dz,$$

along the st. line joining the points $z = 2 + 4i$ and $z = 5 - 5i$. [5]

(*c*) Find the bilinear transformation which maps points 1, 0, i of *z*-plane onto the points $\infty, -2, -\frac{1}{2}(1 + i)$ of *w*-plane. [4]

Or

8. (*a*) Find the condition on a, b, c and d under which :

$$u = ax^3 + bx^2y + cxy^2 + dy^3$$

is harmonic function. [4]

(*b*) Evaluate :

$$\oint_C \frac{z+4}{z^2+2z+5}\, dz,$$

where 'C' is the circle $|z-2i| = 3/2$. [5]

(*c*) Obtain the image of st. line $y = x$ under the transformation $w = \dfrac{z-1}{z+1}$. [4]