Total No. of Questions—8] [Total No. of Printed Pages—5

Seat No.	

[5252]-541

S.E. (Elec./Inst. & Cont.) (First Semester) EXAMINATION, 2017

ENGINEERING MATHEMATICS-III

(2015 PATTERN)

Time : Two Hours **Maximum Marks : 50**

***N.B.* :—** (*i*) Figures to the right indicate full marks.

(*ii*) Neat diagrams must be drawn wherever necessary.

(*iii*) Use of electronic pocket calculator is allowed.

(*iv*) Assume suitable data, if necessary.

1. (*a*) Solve any *two* : [8]

(1) $\dfrac{d^3y}{dx^3} + 3\dfrac{d^2y}{dx^2} - 4y = 6e^{-2x}$

(2) $(D^2 + 1)y = 2 \cot x$ by variation of parameters method.

(3) $x^2\dfrac{d^2y}{dx^2} + 5x\dfrac{dy}{dx} + 3y = \log x$.

(*b*) Solve by Laplace transform method : [4]

$$\frac{d^2y}{dt^2} + y = t$$

with $y(0) = 1$, $y'(0) = -2$

Or

2. (*a*) A capacitor of 10^{-3} farads is in series with an emf of 20V. and an inductor of 0.4 H. At $t = 0$, the charge Q and current I are zero, find Q at any time t. [4]

P.T.O.

(*b*) Solve any *one* : [4]

(1) $L\left[e^{-4t}\int_0^t t\sin 3t\,dt\right]$.

(2) $L^{-1}\left[\dfrac{1}{s^2(s^2+1)}\right]$ by convolution theorem.

(*c*) Find Laplace transform of : [4]

$$[t^4\, U\,(t-2)]$$

3. (*a*) Show that : [4]

$$\int_0^\infty \frac{\lambda \sin \lambda x}{(4+\lambda^2)(9+\lambda^2)}d\lambda = \frac{\pi}{10}(e^{-2x}-e^{-3x}).$$

(*b*) Attempt any *one* : [4]

(*i*) Find Z-transform of $f(k) = \begin{cases} 2k & k < 0 \\ \left(\dfrac{1}{2}\right)^k & k = 0, 2, 4, 6, ... \\ \left(\dfrac{1}{3}\right)^k & k = 1, 3, 5, ... \end{cases}$

(*ii*) Find $Z^{-1}\left(\dfrac{z(z+1)}{z^2-2z+1}\right), |z|>1\cdot$

(*c*) If directional derivative of $\phi = axy + byz + czx$ at (1,1,1) has maximum magnitude 4 in direction parallel to x-axis, find a, b and c. [4]

Or

4. (*a*) Prove (any *one*) : [4]

(*i*) $\nabla \times \left(\frac{\bar{a} \times \bar{r}}{r^n}\right) = \frac{2-n}{r^n}\bar{a} + \frac{n(\bar{a}\,.\,\bar{r})\bar{r}}{r^{n+2}}$.

(*ii*) $\nabla^2\left(\nabla\,.\left(\frac{\bar{r}}{r^2}\right)\right) = \frac{2}{r^4}$.

(*b*) Find the tangent to the curve : [4]

$x = t^2 + 1,\ y = 4t - 3,\ z = 2t^2 - 6t$ at $t = 1$ and $t = 2$.

(*c*) Solve the difference equation : [4]

$12\ f(k + 2) - 7\ f(k + 1) + f\ (k) = 0\ \ k \geq 0,\ f(0) = 0,\ f(1) = 3$.

5. Attempt any *two* :

(*a*) If $\overline{\mathrm{F}} = (2x + y^2)\,\bar{i} + (3y - 4x)\,\bar{j}$ evaluate $\int_{\mathrm{C}} \overline{\mathrm{F}}.\,d\bar{r}$ along the parabolic curve $y = x^2$ joining (0, 0) and (1, 1). [6]

(*b*) Evaluate $\iint_s (\nabla \times \overline{\mathrm{F}}).\,d\bar{s}$ where $\overline{\mathrm{F}} = (x^3 - y^3)\,\bar{i} - xyz\,\bar{j} + y^3\bar{k}$ and s is the surface $x^2 + 4y^2 + z^2 - 2x = 4$ above $x = 0$. [6]

(*c*) Evaluate :

$$\iint_{\mathrm{S}} xz^2 dydz + (yx^2 - z^2)\,dzdx + (2xy + y^2 z)\,dxdy$$

where S is the surface enclosing a region bounded by hemisphere $x^2 + y^2 + z^2 = 4$ above XoY–plane. [7]

Or

6. Attempt (any *two*) :

(*a*) Using Green's theorem evaluate $\int_c \overline{\mathrm{F}} \cdot d\overline{r}$ where $\overline{\mathrm{F}} = 3y\,\overline{i} + 2x\overline{j}$ and c is the boundary of region bounded by $y = 0$, $y = \sin x$, $x = 0$, $x = \pi$. [6]

(*b*) Evaluate $\iint_{\mathrm{S}} (\nabla \times \overline{\mathrm{F}}) \cdot d\overline{\mathrm{S}}$ where $\overline{\mathrm{F}} = (x^2 + y - 4)\overline{i} + 3xy\overline{j} + (2xz + z^2)\,\overline{k}$ over the surface of hemisphere $x^2 + y^2 + z^2 = 16$ above XoY plane. [6]

(*c*) Using Gauss divergence theorem evaluate the surface integral

$$\iint_{\mathrm{S}} (4xz\ \overline{i} - y^2\overline{j} + yz\,\overline{k}) \cdot d\overline{\mathrm{S}}$$

over the cube bounded by the plane $x = 0$, $x = 2$, $y = 0$, $y = 2$, $z = 0$, $z = 2$. [7]

7. (*a*) If $v = \dfrac{-y}{x^2 + y^2}$ find u such that $f(z) = u + iv$ is analytic. Determine $f(z)$ in terms of z. [4]

(*b*) Evaluate $\int_c \dfrac{4z^2 + z}{(z-1)^2} dz$, where c is the circle $|z - 1| = 2$. [5]

(*c*) Find the bilinear transformation which maps the points 1, i, $2i$ of z-plane onto points $-2i$, 0, 1 of w-plane. [4]

Or

8. (*a*) Show that $u(x, y) = y + e^x \cos y$ is harmonic function. Find its harmonic coujugate. [4]

(*b*) Evaluate $\int_c \frac{2z^3 + z + 5}{(z-5)^3} dz$ where C is $\frac{x^2}{16} + \frac{y^2}{4} = 1$. [5]

(*c*) Find the map of the straight line $2y = x$ under the transformation $w = \frac{2z-1}{2z+1}$. [4]