Total No. of Questions—**8**] [Total No. of Printed Pages—**5**

| **Seat No.** | |
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# [5459]-110

## S.E. (Civil) (First Semester) EXAMINATION, 2018

## ENGINEERING MATHEMATICS—III

## (2015 PATTERN)

**Time : Two Hours** **Maximum Marks : 50**

***N.B.* :—** (*i*) Attempt Q. No. **1** *or* **2**, Q. No. **3** *or* **4**, Q. No. **5** *or* **6**, Q. No. **7** *or* **8**.

(*ii*) Figures to the right indicate full marks.

(*iii*) Neat diagrams must be drawn wherever necessary.

(*iv*) Use of electronic pocket calculator is allowed.

(*v*) Assume suitable data, if necessary.

**1.** (*a*) Solve any *two* of the following : [8]

(*i*) $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^{4x}$

(*ii*) $(D^2 + 4)y = \sec 2x$

(by method of variation of parameters)

(*iii*) $x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 4y = x^4$

(*b*) Solve the following system by Gauss elimination method : [4]

$$2x_1 + 4x_2 - 6x_3 = -4$$
$$x_1 + 5x_2 + 3x_3 = 10$$
$$x_1 + 3x_2 + 2x_3 = 5$$

P.T.O.

*Or*

**2.** (*a*) The differential equation satisfied by a beam, uniformly loaded with one end fixed and second subjected to a tensile force P is given by : [4]

$$EI\frac{d^2y}{dx^2} - Py = -\frac{W}{2}x^2$$

Show that the elastic curve for the beam under conditions $y = 0, \frac{dy}{dx} = 0$ where $x = 0$ is given by :

$$y = \frac{W}{2P}\left[x^2 + \frac{2}{n^2} - \frac{e^{nx}}{n^2} - \frac{e^{-nx}}{n^2}\right]$$

where $EI = \frac{P}{n^2}$.

(*b*) Given $\frac{dy}{dx} + \frac{y}{x} = \frac{1}{x^2}$; $y(1) = 1$, find $y(1.1)$ by Euler's modified method taking $h = 0.1$. [4]

(*c*) Solve the following system by Cholesky's method : [4]

$$9x_1 + 6x_2 + 12x_3 = 17.4$$
$$6x_1 + 13x_2 + 11x_3 = 23.6$$
$$12x_1 + 11x_2 + 26x_3 = 30.8.$$

**3.** (*a*) Calculate first three moments about the mean for the following tabulated data : [4]

| $x$ | $f$ |
|---|---|
| 61 | 5 |
| 64 | 18 |
| 67 | 42 |
| 70 | 27 |
| 73 | 8 |

(*b*) A manufacturer of cotter pins knows that 2% of his product is defective. If he sells cotter pins in boxes of 100 pins, then using Poisson distribution, find the probability that a box contains at least 4 defective pins. [4]

(*c*) Find the directional derivative of $\phi = e^{2x} \cos yz$ at (0, 0, 0) in the direction of tangent to the curve $x = a \sin \theta$, $y = a \cos \theta$, $z = a\theta$ at $\theta = \frac{\pi}{4}$. [4]

*Or*

**4.** (*a*) Prove the following identity (any *one*) : [4]

(*i*) $\nabla . [r \nabla r^{-n}] = n(n-2)r^{-n-1}$

(*ii*) $\nabla^2 (r^5 \log r) = (30 \log r + 11)r^3$

(*b*) If $\bar{F}_1 = (y+z)\bar{i} + (z+x)\bar{j} + (x+y)\bar{k}$ and $\bar{F}_2 = (x^2 - yz)\bar{i} + (y^2 - zx)\bar{j} + (\bar{z}^2 - xy)\bar{k}$, then show that $\bar{F}_1 \times \bar{F}_2$ is solenoidal. [4]

(*c*) Obtain regression lines for the following data : [4]

$n = 5, \Sigma x_i = 30, \Sigma y_i = 40,$

$\Sigma x_i^2 = 220, \Sigma y_i^2 = 340, \Sigma x_i y_i = 214.$

**5.** (*a*) Find the work done by the force $\bar{F} = (2y+3)i + xz\hat{j} + (yz - x)\hat{k}$, when it moves from the point (0, 0, 0) to (2, 1, 1) along the curve $x = 2t^2, y = t, z = t^3$. [5]

(*b*) Evaluate $\iint_S \bar{F} . \hat{n} dS$ where $\bar{F} = 4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}$ and S is the surface bounding the region $x = 0$, $y = 0$, $z = 0$ and $x = a$, $y = a$ and $z = a$. [4]

(c) Evaluate $\iint_S \nabla \times \overline{F} \cdot d\overline{S}$, where $\overline{F} = (x - z)i + (x^3 + yz)\hat{j} - 3xy^2\hat{k}$ and S is the surface of the cone $z = 2 - \sqrt{x^2 + y^2}$ above the $xy$ plane. [4]

*Or*

**6.** (a) A vector field is given by : [5]

$$\overline{F} = (x^3 + e^{2y})i + 2x(e^{2y} + 3)\hat{j},$$

Using Green's Lemma, evaluate :

$$\int_C \overline{F} \cdot d\overline{r}$$

where C is the circle $x^2 + y^2 = a^2$.

(b) Using Stokes' theorem, evaluate : [4]

$\iint_S \nabla \times \overline{F} . dS$, where $\overline{F} = y^2 i + y\hat{j} - xz\hat{k}$

and S is the upper half of the sphere $x^2 + y^2 + z^2 = a^2$ and $z \geq 0$.

(c) Use divergence theorem to evaluate : [4]

$$\iint_S (y^2z^2 i + z^2x^2\hat{j} + x^2y^2\hat{k}) . d\overline{S},$$

where S is the upper part of the sphere $x^2 + y^2 + z^2 = 9$ above $xoy$-plane.

**7.** (a) A homogeneous rod of conducting material of length 200 cms has its ends kept at zero temperature and the temperature initially is : [7]

$$u(x, 0) = x, \quad 0 \leq x \leq 100$$
$$= 200 - x, \quad 100 \leq x \leq 200$$

Find the temperature $u(x, t)$ at any time $t$.

(*b*) A string is stretched and fastened to two points $l$ apart. Motion is started by displaying the string in the form $u = 2a \sin\left(\frac{\pi x}{l}\right)$ from which it is released at time $t = 0$. Find the displacement $u(x, t)$ from one end. $\left(\text{Use wave equation } \frac{\partial^2 u}{\partial t^2} = C^2 \frac{\partial^2 u}{\partial x^2}\right)$ [6]

*Or*

**8.** (*a*) Solve $\frac{\partial u}{dt} = k\frac{\partial^2 u}{\partial x^2}$ if : [7]

(*i*) $u(0, t) = 0$

(*ii*) $u(l, t) = 0$

(*iii*) $u(x, t)$ is bounded and

(*iv*) $u(x, 0) = \frac{u_0 x}{l}, 0 \le x \le l$

(*b*) A rectangular plate with insulated surfaces is 10 cm wide and so long compared to its width that it may be considered infinite in length without introducing an appreciable error. If the temperature along short edge $y = 0$ is given by $u(x, 0) = 50 \sin\left(\frac{\pi x}{10}\right)$, $0 \le x \le 10$, while the two long edges $x = 0$ and $x = 10$ as well as the other short edge kept at 0°C. Find the steady state temperature $u(x, y)$. [6]