Total No. of Questions—**8**] [Total No. of Printed Pages—**5**

Seat No.	

[5559]-119

S.E. (Mech/Auto./S/W) (I Sem.) EXAMINATION, 2019

ENGINEERING MATHEMATICS—III

(2015 PATTERN)

Time : Two Hours **Maximum Marks : 50**

***N.B.* :—** (*i*) Neat diagrams must be drawn wherever necessary.

(*ii*) Figures to the right indicate full marks.

(*iii*) Use of electronic pocket calculator is allowed.

(*iv*) Assume suitable data, if necessary.

1. (*a*) Solve any *two* of the following differential equations : [8]

(*i*) $\dfrac{d^2y}{dx^2} + 6\dfrac{dy}{dx} + 9y = e^{-3x} \cos 4x + 6e^{2x}$

(*ii*) $x^2\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} + 16y = x^2 + 2^{\log x} + 4 \cosh(\log x)$

(*iii*) $\dfrac{d^2y}{dx^2} + y = \operatorname{cosec} x$, (by using method of variation of parameters)

(*b*) Solve the integral equation : [4]

$$\int_0^\infty f(x) \cos \lambda x \, dx = e^{-2\lambda}, \lambda > 0$$

P.T.O.

Or

2. (*a*) A 8 lb weight is placed at one end of a spring suspended from the ceiling. The weight is raised to 5 inches above the equilibrium position and left free. Assuming the spring cosntant 12 lb/ft, find the equation of motion, the displacement function, amplitude and period. [4]

(*b*) Solve any *one* of the following : [4]

(*i*) $L[t\, et^{2t} \cos 3t]$

(*ii*) $L^{-1}\left[\dfrac{2s+7}{s^2+4s+29}\right]$.

(*c*) Solve the differential equation by Laplace transform method : [4]

$$\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + y = te^t$$

where $y(0) = 0, y'(0) = 3$.

3. (*a*) The first four moments of a distribution about the value 2.5 are 1, 10, 20 and 25. Obtain first four central moments. Also calculate coefficient of skewness (β_1) and coefficient of kurotsis (β_2). [4]

(*b*) A dice is thrown five times. If getting an odd number is a success, then what is the probability of getting : [4]

(*i*) four successes

(*ii*) at least four successes.

(*c*) Find the directional derivative of $\phi = xy^2 + yz^2 + zx^2$ at (1, 1, 1) along the vector $\bar{i} + 2\bar{j} + 2\bar{k}$. [4]

Or

4. (*a*) Obtain the regression line of y on x for the following data : [4]

x	y
1	2
2	5
3	3
4	8
5	7

(*b*) Prove the following (any *one*) : [4]

(*i*) $\nabla \cdot \left(\frac{\overline{a} \times \overline{r}}{r}\right) = 0$

(*ii*) $\nabla^2 (r^9 \log r) = (90 \log r + 19) r^7$.

(*c*) Show that the vector field : [4]

$$\overline{F} = (x^2 - yz)\,\overline{i} + (y^2 - zx)\overline{j} + (z^2 - xy)\overline{k}$$

is irrotational. Also find the scalar potential ϕ such that $\overline{F} = \nabla\phi$.

5. (*a*) Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = zi + xj + yk$ and C is the arc of the curve $\vec{r} = \cos ti + \sin tj + tk$ from t = 0 to t = 2π. [5]

(*b*) Using Gauss divergence theorem, evaluate $\iiint_V \nabla \cdot \vec{F}\, dV$ where $\vec{F} = 2x^2yi - y^2j + 4xz^2k$ over the region bounded by the cylinder $y^2 + z^2 = 9$ and the plane $z = 2$ in the first octant. [4]

(*c*) Using Stoke's theorem evaluate $\iint_S \nabla \times \vec{F} \cdot \hat{n}\, dS$ where $\vec{F} = (x + y)\, i + (y + z)\, j - xk$ and S is the surface of the plane $2x + y + z = 2$ which is in the first octant. [4]

Or

6. (*a*) Using Green's theorem, evaluate $\int_C e^{-x}(\sin y\, dx + \cos y\, dy)$ where 'C' is the rectangle with vertices $(0, 0)$ $(\pi, 0)$, $\left(\pi, \frac{\pi}{2}\right)$, $\left(0, \frac{\pi}{2}\right)$. [4]

(*b*) Using Gauss divergene theorem, evaluate

$$\iint_S [(x^2 - yz)\, dydz + (y^2 - xz)\, dx\, dz + (z^2 - xy) dx\, dy]$$

taken over rectangular parallelopiped $0 \le x \le a$, $0 \le y \le b$, $0 \le z \le c$. [4]

(*c*) Using stoke's theorem evaluate $\iint_S \nabla \times \vec{F} \cdot \hat{n}\, dS$. Where $\vec{F} = yi + zj + xk$ over the surface $x^2 + y^2 = 1 - z$, $z > 0$. [5]

7. (*a*) Solve the wave equation $\frac{\partial^2 u}{\partial t^2} = C^2 \frac{\partial^2 u}{\partial x^2}$ under the conditions : [7]

(*i*) $u(0, t) = 0$

(*ii*) $u(4, t) = 0$

(*iii*) $\frac{\partial u}{\partial t} = 0$ when $t = 0$

(*iv*) $u(x, 0) = 25$.

(*b*) Solve $\frac{\partial u}{\partial t} = C^2 \frac{\partial^2 u}{\partial x^2}$ under the conditions : [6]

(*i*) $u(0, t) = 0$

(*ii*) $u(2, t) = 0$

(*iii*) $u(x, 0) = x, \; 0 < x < 2$

Or

8. (*a*) Solve $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$, given that : [6]

(*i*) $V(0, y) = 0$

(*ii*) $V(C, y) = 0$

(*iii*) $V \to 0$ as $y \to \infty$

(*iii*) $V = V_0$ when $y = 0$.

(*b*) Use fourier transform to solve the equation [7]

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial t^2}, 0 < x < \infty, t > 0$$

subejct to conditions :

(*i*) $u(0, t) = 0, \; t > 0$

(*ii*) $u(x, 0) = \begin{cases} 6 & 0 < x < 1 \\ 0 & x > 1 \end{cases}$

(*iii*) $u(x, t)$ is bounded.