Total No. of Questions—**8**] [Total No. of Printed Pages—**5**

Seat No.	

[5459]-119

S.E. (Mech./Prod./Auto) EXAMINATION, 2018

(Common to Mech. & Mech. S/W)

ENGINEERING MATHEMATICS—III

(2015 PATTERN)

Time : Two Hours **Maximum Marks : 50**

***N.B.* :—** (*i*) Neat diagrams must be drawn wherever necessary.

(*ii*) Figures to the right indicate full marks.

(*iii*) Use of electronic pocket calculator is allowed.

(*iv*) Assume suitable data, if necessary.

1. (*a*) Solve any *two* of the following differential equations : [8]

(*i*) $\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} + y = x^2 + x + 1$

(*ii*) $x^2\dfrac{d^2y}{dx^2} - 3x\dfrac{dy}{dx} + 3y = \sin\left(\sqrt{3}\log x\right) + x^3$

(*iii*) $\dfrac{d^2y}{dx^2} + 4y = \tan 2x$, by using the method of variation of parameters.

(*b*) Solve the integral equation : [4]

$$\int_0^\infty f(x)\sin\lambda x\, dx = 4e^{-6\lambda}, \lambda > 0.$$

Or

2. (*a*) A 4 lb weight is suspended at one end of the spring suspended from ceiling. The weight is raised to $\left(\frac{5}{12}\right)$ feet above the equilibrium position and left free. Assuming the spring constant is 8 lb/ft, find the equation of motion, displacement function, amplitude and period. [4]

(*b*) Solve any *one* of the following : [4]

(*i*) Evaluate the integral $\int_0^\infty e^{-4t}\, t \cos t\, dt$, by using concept of Laplace transform.

(*ii*) Obtain $L^{-1}\left[\frac{s+1}{(2s-1)(s+2)}\right]$.

(*c*) Solve the following differential equation by using the Laplace transform method : [4]

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 4y = te^{-2t}$$

where $y(0) = 0$, $y'(0) = 2$.

3. (*a*) Calculate the first four moments about the mean of the following frequency distribution : [4]

x	f
0	1
1	8
2	28
3	56
4	70
5	56
6	28
7	8
8	1

(*b*) 200 students appeared in a certain examination obtained average marks 50% with standard deviation 5%. How many students do you expect to obtain more than 60% of marks, supposing that the marks are distributed normally. [4]

(Given : At $z = 2$, A = 0.4772)

(*c*) Find the directional derivative of $\phi = x^2 - y^2 - 2z^2$ at the point P(2, –1, 3) in the directional PQ, where the point Q is Q(5, 6, 4). [4]

Or

4. (*a*) Obtain the regression line of y on x for the following data : [4]

x	y
5	10
1	11
10	5
3	10
9	6

(*b*) Prove the following (any *one*) : [4]

(*i*) $\nabla \cdot \left(r \nabla \dfrac{1}{r^5}\right) = \dfrac{15}{r^6}$

(*ii*) $\nabla^2 \left[\dfrac{1}{r} \log r\right] = \dfrac{-1}{r^3}$

(*c*) Show that the vector field : [4]

$$\bar{F} = (8xy + z^4)\bar{i} + (4x^2 - z)\bar{j} + (4xz^3 - y)\bar{k}$$

is irrotational. Also find the scalar ϕ such that $\bar{F} = \nabla\phi$.

5. (*a*) Evaluate $\int_C \vec{F} . d\vec{r}$ where $\vec{F} = e^y i + x(1 + e^y) j$ and 'C' is the curve of $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, z = 0$. [5]

(*b*) Evaluate $\iint_S [(z^2 - x)\, dy\, dz - xy\, dx\, dz + 3z\, dx\, dy]$ where S is closed surface of region bounded by $x = 0$, $x = 3$, $z = 0$, $z = 4 - y^2$ by using Gauss divergence theorem. [4]

(*c*) By using Stokes' theorem evaluate $\iint_S \nabla \times \overline{F} . \hat{n}\, dS$ where S is the curved surface of the paraboloid $x^2 + y^2 = 2z$ bounded by the plane $z = 2$ where $\vec{F} = 3(x - y)\, i + 2xzj + xyk$. [4]

Or

6. (*a*) Using Green's theorem evaluate $\int_C [\cos x \sin y - 4y) dx + \sin x \cos y\, dy]$ where C is the circle $x^2 + y^2 = 1$. [5]

(*b*) Using Gauss divergence theorem evaluate $\iint_S (lx + my + nz) dS$ where l, m, n are direction cosines of the outer normal to the surface $x^2 + y^2 + z^2 = 4$. [4]

(*c*) By using Stokes' theorem prove that : [4]

$$\int_C (\vec{a} \times \vec{r}) . d\vec{r} = 2\vec{a} . \iint_S d\vec{S}.$$

7. (*a*) Solve the equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$, under conditions : [7]

(*i*) $u(0, t) = 0$

(*ii*) $u(\pi, t) = 0$

(*iii*) $\frac{\partial u}{\partial t} = 0$ when $t = 0$

(*iv*) $u(x, 0) = 2x, 0 < x < \pi$.

(*b*) Solve $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$, under the condition : [6]

(*i*) $u(0, t) = 0$

(*ii*) $u(l, t) = 0$

(*iii*) $u(x, 0) = 100\frac{x}{l} \quad 0 < x < l$.

Or

8. (*a*) Solve the equation : [6]

$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ which satisfies the conditions :

$u(0, y) = u(\pi, y) = 0$ for all y

$u(x, 0) = k \qquad 0 < x < \pi, \ \lim_{y\to\infty} u(x, y) = 0 \qquad 0 < x < \pi$

(*b*) Use Fourier transform to solve the equation : [7]

$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < \infty, \quad t > 0$ subject to conditions

(*i*) $u(0, t) = 0, \ t > 0$

(*ii*) $u(x, 0) = \begin{cases} 2 & 0 < x < 1 \\ 0 & x > 1 \end{cases}$

(*iii*) $u(x, t)$ is bounded.