Total No. of Questions—8]

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S.E. (E&TC/Electronics Engg.) (II Sem.) EXAMINATION, 2017 ENGINEERING MATHEMATICS-III

(2015 **PATTERN**)

Time: Two Hours

Maximum Marks: 50

- Answer Q. No. 1 or Q. No. 2, Q. No. 3 or *N.B.* :— Q. No. 4, Q. No. 5 or Q. No. 6, Q. No. 7 or Q. No. 8.
 - Neat diagrams must be drawn wherever necessary. (ii)
 - Figures to the right indicate full marks. (iii)
 - Use of logarithmic tables, slide rule, Mollier charts, electronic (iv)pocket calculator and steam tables is allowed.
 - Assume suitable data, if necessary. (v)
- Solve any two: 1. (a)

(i)
$$(D^2 - 7D + 6)y = e^{2x}$$

- $(ii) \quad (D^2 + 4)y = x \sin x$
- (iii) $(x+2)^2 \frac{d^2y}{dx^2} + 3(x+2) \frac{dy}{dx} + y = 4 \sin \log (x+2)$
- (*b*) Find the Fourier sine transform of the function: [4] $f(x) = e^{-x}, \ x > 0.$

P.T.O.

- 2. (a) A circuit consists of an inductance L and condenser of capacity C in series. An alternating e.m.f. $E \sin nt$ is applied to it at time t=0, the initial current and charge on the condenser being zero and $w^2 = \frac{1}{LC}$, find the current flowing in the circuit at any time for $w \neq n$. [4]
 - (b) Find inverse z-transform (any one): [4]

(i)
$$F(z) = \frac{z}{(z-1)(z-3)}, |z| > 3$$

(ii)
$$F(z) = \frac{z}{\left(z - \frac{1}{4}\right)\left(z - \frac{1}{5}\right)}.$$

(by inversion integral method).

(c) Solve the following difference equation: [4]

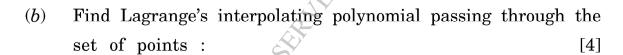
$$f(k+1) + \frac{1}{4}f(k) = \left(\frac{1}{4}\right)^k$$
;

$$f(0) = 0, \quad k \ge 0$$

3. (a) Using fourth order Runge-Kutta method, solve the differential equation: [4]

$$\frac{dy}{dx} = x + y + xy$$

with y(0) = 1 to get y(0.1) taking h = 0.1.





(c) Find the directional derivative of :
$$\phi = x^2 - y^2 + 2z^2$$
 [4]

the point (1, 2, 3) in the direction of $4\overline{i} - 2\overline{j} + \overline{k}$.

(i)
$$\nabla \left(\frac{\overline{a}.\overline{r}}{r^2}\right) = \frac{\overline{a}}{r^2} - \frac{2(\overline{a}.\overline{r})\overline{r}}{r^4}$$

(i)
$$\nabla \left(\frac{\overline{a}.\overline{r}}{r^2}\right) = \frac{\overline{a}}{r^2} - \frac{2(\overline{a}.\overline{r})\overline{r}}{r^4}$$
(ii)
$$\nabla \left(\overline{a}.\nabla\frac{1}{r}\right) = \frac{3(\overline{a}.\overline{r})\overline{r}}{r^5} - \frac{\overline{a}}{r^3}$$

$$\overline{F} = (6xy + z^3)\overline{i} + (3x^2 - z)\overline{j} + (3xz^2 - y)\overline{k}$$

is irrotational. Find scalar potential φ such that $\overline{F}=\nabla \varphi$.

(c) By using Trapezoidal rule, evaluate :
$$\int_{0}^{2} \frac{1}{1+x^{4}} dx$$
 [4]

$$\int_{0}^{2} \frac{1}{1+x^4} dx$$

taking $h = \frac{1}{2}$, correct upto 3-decimal places.

Evaluate $\int \overline{F} \cdot d\overline{r}$ for (a) **5.**

[4]

$$\overline{F} = 3x^2 \overline{i} + (2xz - y)\overline{j} + z\overline{k}$$

along straight line joining (0, 0, 0) and (2, 1, 3).

(*b*) Evaluate:

[4]

$$\iint\limits_{\mathbf{S}} \left[(4x + 3yz^2)\overline{i} - (x^2z^2 + y)\overline{j} + (y^2 + 2z)\overline{k} \right] . d\overline{\mathbf{S}}$$

where S is the surface of the sphere $x^2 + y^2 + z^2 = 9$.

Apply Stokes' theorem to evaluate $\oint \overline{F}.d\overline{r}$ where : [5]

$$\overline{F} = xy^2 \overline{i} + y\overline{j} + xz^2 \overline{k}$$

and C is the boundary of rectangle:

x = 0, y = 0, x = 1, y = 2 in z = 0 plane.

Using Green's lemma evaluate: 6. (a)

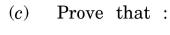
$$\int_{C} (xy - x^2) dx + x^2 y dy$$

(*b*)

$$\iint (\nabla \times \overline{F}) \cdot \hat{n} \ ds,$$

the cone $z = 4 - \sqrt{x^2 + y^2}$, above the xoy plane.

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$$\iint\limits_{S} (\phi \nabla \Psi - \Psi \nabla \phi) . d\overline{S} = \iiint\limits_{V} (\phi \nabla^{2} \Psi - \Psi \nabla^{2} \phi) dV$$

7. Show that the function: (a)

$$f(z) = u + iv$$

with constant modulus and constant amplitude is constant in each case.

(*b*) Evaluate: [4]

$$\oint_C \frac{4z^2 + z}{z^2 - 1} dz$$

where C is the circle $|z-1| = \frac{1}{2}$

bilinear transformation (c) Find which the maps the points:

$$z = 1, i, 2i$$

onto the points w = -2i, 0, 1 respectively. J = u + iv $u - v = x^2 - y^2 - 2xy$ 5

8. (*a*) If [5]

$$f(z) = u + iv$$

is analytic, find f(z), if

$$u-v=x^2-y^2-2xy.$$

Evaluate: (*b*) [4]

$$\oint_{C} \frac{(z^{2} + \cos^{2} z)}{\left(z - \frac{\pi}{4}\right)^{3}} dz,$$
where C is circle $|z| = 1$.

Find the map of the straight line y = x under the transformation

[4]