Seat	
No.	3

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## S.E. (Civil) (First Semester) EXAMINATION, 2018

## ENGINEERING MATHEMATICS—III

## (2015 **PATTERN**)

Time: Two Hours

Maximum Marks: 50

- N.B. :— (i) Attempt Q. No. 1 or 2, Q. No. 3 or 4, Q. No. 5 or 6, Q. No. 7 or 8.
  - (ii) Figures to the right indicate full marks.
  - (iii) Neat diagrams must be drawn wherever necessary.
  - (iv) Use of electronic pocket calculator is allowed.
  - (v) Assume suitable data, if necessary.
- 1. (a) Solve any two of the following:

[8]

$$(i) \qquad \frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^{4x}$$

$$(ii) \quad (D^2 + 4)y = \sec 2x$$

(by method of variation of parameters)

(iii) 
$$x^2 \frac{d^2 y}{dx^2} - 2x \frac{dy}{dx} - 4y = x^4$$

(b) Solve the following system by Gauss elimination method: [4]

$$2x_1 + 4x_2 - 6x_3 = -4$$

$$x_1 + 5x_2 + 3x_3 = 10$$

$$x_1 + 3x_2 + 2x_3 = 5$$

2. (a) The differential equation satisfied by a beam, uniformly loaded with one end fixed and second subjected to a tensile force P is given by:

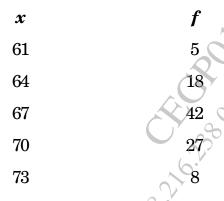
$$EI\frac{d^2y}{dx^2} - Py = -\frac{W}{2}x^2$$

Show that the elastic curve for the beam under conditions y = 0,  $\frac{dy}{dx} = 0$  where x = 0 is given by :

$$y = \frac{W}{2P} \left[ x^2 + \frac{2}{n^2} - \frac{e^{nx}}{n^2} - \frac{e^{-nx}}{n^2} \right]$$

where  $EI = \frac{P}{n^2}$ 

- (b) Given  $\frac{dy}{dx} + \frac{y}{x} = \frac{1}{x^2}$ ; y(1) = 1, find y(1.1) by Euler's modified method taking h = 0.1. [4]
- (c) Solve the following system by Cholesky's method : [4]  $9x_1 + 6x_2 + 12x_3 = 17.4$   $6x_1 + 13x_2 + 11x_3 = 23.6$   $12x_1 + 11x_2 + 26x_3 = 30.8.$
- 3. (a) Calculate first three moments about the mean for the following tabulated data: [4]



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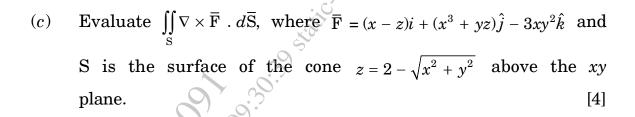
- A manufacturer of cotter pins knows that 2% of his product (*b*) is defective. If he sells cotter pins in boxes of 100 pins, then using Poisson distribution, find the probability that a box contains at least 4 defective pins. [4]
- Find the directional derivative of  $\phi = e^{2x} \cos yz$  at (0, 0, 0) in (c)the direction of tangent to the curve  $x = a \sin \theta$ ,  $= a \cos \theta$ ,  $z = a \theta \text{ at } \theta = \frac{\pi}{4}$ . [4]

- (a) Prove the following identity (any one) 4. [4]

  - $\nabla \cdot [r \ \nabla \ r^{-n}] = n(n-2)r^{-n-1}$   $\nabla^2 \ (r^5 \ \log \ r) = (30 \ \log \ r + 11)r^3$
  - If  $\overline{\mathbf{F}}_1 = (y+z)\overline{i} + (z+x)\overline{j} + (x+y)\overline{k}$  and  $\overline{\mathbf{F}}_2 = (x^2-yz)\overline{i} + (y^2-zx)\overline{j} + (y^2-zx)\overline{j}$ (*b*)  $(\overline{z}^2 - xy)\overline{k}$ , then show that  $\overline{F}_1 \times \overline{F}_2$  is solenoidal. [4]
  - Obtain regression lines for the following data: (c) $n=5, \Sigma x_i=30, \Sigma y_i=40,$  $\Sigma x_i^2 = 220, \ \Sigma y_i^2 = 340, \ \Sigma x_i y_i = 214.$
- Find the work done by the force  $\overline{F} = (2y + 3)i + xz\hat{j} + (yz x)\hat{k}$ , **5.** (a)when it moves from the point (0, 0, 0) to (2, 1, 1) along the curve  $x = 2t^2$ , y = t,  $z = t^3$ . [5]
  - Evaluate  $\iint_{S} \overline{F} \cdot \hat{n} dS$  where  $\overline{F} = 4x\hat{i} 2y^2\hat{j} + z^2\hat{k}$  and S is the surface (*b*) bounding the region x = 0, y = 0, z = 0 and x = a, x = a and z = a.

    [4]

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A vector field is given by: **6.** (a)

$$\overline{\mathbf{F}} = (x^3 + e^{2y})i + 2x(e^{2y} + 3)\hat{j},$$

Using Green's Lemma, evaluate:

$$\int_{\Gamma} \overline{\mathbf{F}} \cdot d\overline{r}$$

where C is the circle  $x^2 + y^2 = a^2$ . (b) Using Stokes' theorem, evaluate : [4] $\iint_{S} \nabla \times \overline{F} \cdot dS, \text{ where } \overline{F} = y^{2}i + y\hat{j} - xz\hat{k}$ 

and S is the upper half of the sphere  $x^2 + y^2 + z^2 = a^2$  $z \geq 0$ .

[5]

Use divergence theorem to evaluate : (c) $\iint_{S} (y^2 z^2 i + z^2 x^2 \hat{j} + x^2 y^2 \hat{k}) . d\overline{S},$ 

where S is the upper part of the sphere  $x^2 + y^2 + z^2 = 9$  above xoy-plane.

**7.** (a)A homogeneous rod of conducting material of length 200 cms has its ends kept at zero temperature and the temperature initially is: [7]

$$u(x, 0) = x,$$
  $0 \le x \le 100$   
= 200 - x,  $100 \le x \le 200$ 

Find the temperature u(x, t) at any time t.

A string is stretched and fastened to two points l apart. Motion (*b*) is started by displaying the string in the form  $u = 2a \sin\left(\frac{\pi x}{I}\right)$ from which it is released at time t = 0. Find the displacement u(x, t) from one end. (Use wave equation  $\frac{\partial^2 u}{\partial t^2} = C^2 \frac{\partial^2 u}{\partial x^2}$ ) [6]

Or

8. (a) Solve 
$$\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}$$
 if :  
(i)  $u(0, t) = 0$   
(ii)  $u(l, t) = 0$   
(iii)  $u(x, t)$  is bounded and

- (iv)  $u(x, 0) = \frac{u_0 x}{l}, 0 \le x \le l$
- A rectangular plate with insulated surfaces is 10 cm wide and (*b*) so long compared to its width that it may be considered infinite in length without introducing an appreciable error. If the temperature along short edge y = 0 is given by u(x, 0) =50  $\sin\left(\frac{\pi x}{10}\right)$ ,  $0 \le x \le 10$ , while the two long edges x = 0eds x, y). and x = 10 as well as the other short edge kept at 0°C. Find the steady state temperature u(x, y). [6]