Total No. of Questions—8] [Total No. of Printed Pages—7

Seat No.	

[5152]-541

S.E. (Electrical & Instru.) (I Sem.) EXAMINATION, 2017

ENGINEERING MATHEMATICS—III

(Common With Instru. & Control)

(2015 PATTERN)

Time : Two Hours **Maximum Marks : 50**

***N.B.* :—** (*i*) Figures to the right indicate full marks.

(*ii*) Use of electronic pocket calculator is allowed.

(*iii*) Assume suitable data, if necessary.

(*iv*) Neat diagrams must be drawn wherever necessary.

1. (*a*) Solve any *two* : [8]

(*i*) $(D^2 + D + 1)y = x \sin x$

(*ii*) $(2x + 3)^2 \dfrac{d^2y}{dx^2} - 2(2x + 3)\dfrac{dy}{dx} - 12y = 6x$

(*iii*) $(D^2 + 3D + 2)y = \sin e^x$

using method of variation of parameters.

(*b*) Solve the following differential equation by using Laplace transform : [4]

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = te^{-t}, \quad y(0) = 1, \quad y'(0) = -2.$$

P.T.O.

Or

2. (*a*) An electric current consists of an inductance 0.1 henry a resistance R of 20 ohms and a condenser of capacitance C of 25×10^{-6} farads. If the differential equation of electric circuit is :

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = 0,$$

then find the charge q and current p at any time t given that at $t = 0$, $q = 0.05$ columbs, $i = 0$. [4]

(*b*) Solve any *one* : [4]

(*i*) Find $L\left[\int_0^t \frac{\sin t}{t}\, dt\right]$

(*ii*) Find $L^{-1}\left[\frac{3s+1}{(s+1)^4}\right]$

(*c*) Evaluate the following integral using Laplace transform : [4]

$$\int_0^\infty t\, e^{-3t} \sin t\, dt.$$

3. (*a*) Find inverse sine transform if : [4]

$$F_s(\lambda) = \frac{1}{\lambda} e^{-a\lambda}.$$

(*b*) Attepmt any *one* : [4]

(*i*) Find z-transform of

$$f(k) = \frac{2^k}{k!}, \quad k \geq 0.$$

(*ii*) Find the inverse z-transform of :

$$\frac{z(z+1)}{z^2 - 2z + 1}, \quad |z| > 1.$$

(*c*) Find directional derivative of [4]

$$\phi = xy^2 + yz^3$$

at (1, –1, 1) along the vector

$$i + 2j + 2k.$$

Or

4. (*a*) Attempt any *one* : [4]

(*i*) Prove that :

$$\bar{b} \times \nabla\left(\bar{a} \cdot \nabla \log r\right) = \frac{\bar{b} \times \bar{a}}{r^2} - \frac{2\left(\bar{a} \cdot \bar{r}\right)\left(\bar{b} \times \bar{r}\right)}{r^4}.$$

(*ii*) $$\nabla \times \left(\frac{\bar{a} \times \bar{r}}{r^3}\right) = \frac{-\bar{a}}{r^3} + \frac{3\left(\bar{a} \cdot \bar{r}\right)\bar{r}}{r^5}.$$

(*b*) Show that : [4]

$$\bar{F} = \left(6xy + z^3\right)\bar{i} + \left(3x^2 - z\right)\bar{j} + \left(3xz^2 - y\right)\bar{k}$$

is irrotational. Find Scalar ϕ such that $\bar{F} = \nabla\phi$.

(*c*) Obtain $f(k)$ given that : [4]

$$f(k + 1) + \frac{1}{2}f(k) = \left(\frac{1}{2}\right)^k, \; k \geq 0, \; f(0) = 0.$$

5. Attempt any *two* :

(*a*) Verify Green's theorem in plane for [6]

$$\int_C \left(xy + y^2\right)dx + x^2 dy$$

where C is the boundary of the closed region bounded by $y = x$ and $y = x^2$.

(*b*) Evaluate : [6]

$$\iint_S \left(xi + yj + z^2k\right). d\bar{S}$$

where S is the curved surface of the cylinder

$$x^2 + y^2 = 4$$

bounded by the planes $z = 0$ and $z = 2$.

(c) Verify Stokes' theorem for

$$\overline{F} = (2x - y)i - yz^2 j - y^2 zk$$

over the surface of hemisphere

$$x^2 + y^2 + z^2 = 1$$

above the *xoy* plane. [7]

Or

6. Attempt any *two* :

(a) Find the work done in moving a particle from (1, –2, 1) to (3, 1, 4) in a force field [6]

$$\overline{f} = (2xy + z^3)i + x^2 j + 3xz^2 k$$

(b) Prove that : [6]

$$\iiint_V \frac{1}{r^2}\, dV = \iint_S \frac{1}{r^2}\overline{r} \cdot d\overline{S}$$

where S is closed surface enclosing the volume V. Hence evaluate :

$$\iint_S \frac{xi + yj + zk}{r^2} \cdot d\overline{S}$$

where S is surface of the sphere

$$x^2 + y^2 + z^2 = a^2.$$

(*c*) Verify Stokes' theorem for [7]

$$\bar{F} = y^2 i + xyj - xzk$$

where S is the hemisphere :

$$x^2 + y^2 + z^2 = a^2, \ z \geq 0.$$

7. (*a*) If $\phi + i\psi$ is complex potential for an electric field (which is analytic) and

$$\phi = -2xy + \frac{y}{x^2 + y^2},$$

find the function ψ. [4]

(*b*) Evaluate : [5]

$$\oint_C \frac{z+4}{(z+1)^2 (z+2)^2} dz,$$

where 'C' is a circle $|z + 1| = \frac{1}{2}$.

(*c*) Find the bilinear transformation, which maps point 1, 0, i of z-plane onto the points ∞, -2, $-\frac{1}{2}(1 + i)$ of w-plane. [4]

Or

8. (*a*) Show that analytic function with constant amplitude is constant. [4]

(b) Evaluate : [5]

$$\int_{2+4i}^{5-5i} (z+1)\,dz,$$

along the line joining points $(2 + 4i)$ and $(5 - 5i)$.

(c) Find the image of Hyperbola [4]

$$x^2 - y^2 = 1,$$

under the transformation $w = \frac{1}{z}$.