Total No. of Questions—8] [Total No. of Printed Pages—4

Seat No.	

[5459]-160

S.E. (Electrical Engineering & Instru.) (I Sem.) EXAMINATION, 2018

ENGINEERING MATHEMATICS—III

(2015 PATTERN)

Time : Two Hours **Maximum Marks : 50**

***N.B.* :—** (*i*) Figures to the right indicate full marks.

(*ii*) Use of electronic pocket calculator is allowed.

(*iii*) Neat diagrams must be drawn wherever necessary.

(*iv*) Assume suitable data, if necessary.

1. (*a*) Solve any *two* : [8]

(*i*) $\dfrac{d^2y}{dx^2} - y = x \sin x$

(*ii*) $(D + 1)^2 y = e^{-x}$ by variation of parameter method.

(*iii*) $x^2 \dfrac{d^2y}{dx^2} - 3x\dfrac{dy}{dx} + 3y = x^2 \sin(\log x)$

(*b*) Solve by Laplace-transform method : [4]

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 5y = e^{-t} \sin t$$

with $y(0) = 0$ and $y'(0) = 1$.

P.T.O.

Or

2. (*a*) An emf E sin pt is applied at $t = 0$ to a circuit containing a capacitance C and inductance L. Current I satisfies the equation

$$L\frac{dI}{dt} + \frac{1}{C}\int I\,dt = E\sin pt$$ if $p^2 = \frac{1}{LC}$ and initially the current I and charge Q are zero then show that the current at time t is $\frac{Et}{2L}\sin pt$ where $I = -\frac{dQ}{dt}$. [4]

(*b*) Solve any *one* : [4]

(*i*) Evaluate :

$$\left[\int_0^{\infty}\frac{\cos 6t - \cos 4t}{t}\,dt\right].$$

(*ii*) $L^{-1}\left[\frac{1}{s^4(s+5)}\right]$ by convolution theorem.

(*c*) Find Laplace transform of $\cosh t\ \delta(t - 4)$. [4]

3. (*a*) Find the Fourier transform of the function : [4]

$$f(x) = \begin{cases} 1 - x^2 & |x| \le 1 \\ 0 & \text{otherwise} \end{cases}.$$

(*b*) Attempt any *one* : [4]

(*i*) Find z-transform of $f(k) = \left(\frac{1}{4}\right)^{|k|}\ \forall\ k$

(*ii*) Find inverse z-transform of $f(z) = \frac{z}{\left(z - \frac{1}{4}\right)\left(z - \frac{1}{5}\right)}$ for $\frac{1}{5} < |z| < \frac{1}{4}$.

(*c*) In what direction, the directional derivative of $\phi = x^2yz^3$ is maximum from the point $(2, 1, -1)$? What is its magnitude ? [4]

Or

4. (*a*) Prove that (any *one*) : [4]

(*i*) $\nabla^4(r^2 \log r) = \dfrac{6}{r^2}$

(*ii*) $\nabla \times \left(\dfrac{\overline{a} \times \overline{r}}{r^3}\right) = \dfrac{-\overline{a}}{r^3} + \dfrac{3(\overline{a}\,.\overline{r})\overline{r}}{r^5}$

(*b*) Find a, b, c, so that $\overline{\mathrm{F}} = (x + 2y + az)\overline{i} + (bx - 3y - z)\overline{j} + (4x + cy + 2z)\overline{k}$ is irrotational. [4]

(*c*) Obtain inverse z-transform of $\mathrm{F}(z) = \dfrac{1}{(z-3)(z-4)}$ $|z| > 3$ by inversion integral method. [4]

5. Attempt any *two* :

(*a*) Evaluate $\int_{\mathrm{C}} \overline{\mathrm{F}}\,.\,d\overline{r}$ for $\overline{\mathrm{F}} = 3x^2\overline{i} + (2xz - y)\overline{j} + z\overline{k}$ along the following curve $x = \alpha t^2, y = t, z = 4t^2 - t$ from $t = 0$, $t = 1$. [6]

(*b*) Using Stokes' theorem evaluate : [7]

$$\int_{\mathrm{C}} (x + y)dx + (2x - z)dy + (y + z)dz$$

where C is the curve of intersection of $x^2 + y^2 + z^2 - 2ax - 2ay = 0$ and $x + y = 2a$.

(*c*) Evaluate $\iint_{\mathrm{S}} (z^2 - x)\, dy\, dz - xy\, dz\, dx + 3z\, dx\, dy$ where S is the closed surface of region bounded by $x = 0$, $x = 3$, $z = 0$, $z = 4 - y^2$. [6]

Or

6. Attempt any *two* :

(*a*) Using Green's theorem evaluate $\int_{\mathrm{C}} \overline{\mathrm{F}}.d\overline{r}$ where $\overline{\mathrm{F}} = x\overline{i} + y\overline{j}$ over the first quadrant of the circle $x^2 + y^2 = a^2$. [6]

(*b*) Evaluate $\iint_S (\nabla \times \bar{F}).d\bar{S}$ where $\bar{F} = 3(x - y)\bar{i} + 2xz\bar{j} + xy\bar{k}$ over the surface of the paraboloid $x^2 + y^2 = 2z$ bounded by the plane $z = 2$. [7]

(*c*) Find $\iint_S \bar{F}.d\bar{S}$ where S is the sphere $x^2 + y^2 + z^2 = 9$ and $\bar{F} = (4x + 3yz^2)\bar{i} - (x^2z^2 + y)\bar{j} + (y^3 + 2z)\bar{k}$. [6]

7. (*a*) If $u - v = x^3 + 3x^2y - 3xy^2 - y^3$, find an analytic function $f(z) = u + iv$. [4]

(*b*) Evaluate $\oint_C \frac{z+2}{z^2+1} dz$ where C is the circle $|z + i| = \frac{1}{2}$. [5]

(*c*) Find the bilinear transformation which maps the points $-i$, 0, $(2 + i)$ of z-plane onto the points 0, $-2i$, 4 of the w-plane. [4]

Or

8. (*a*) Find an analytic function $f(z)$ whose imaginary part is $r^n \sin n\theta$. [4]

(*b*) Evaluate : [5]

$$\oint_C \frac{\sin^2 z}{\left(z - \frac{\pi}{6}\right)^3} dz$$

where C is the circle $|z| = \frac{3}{2}$.

(*c*) Show that the map $w = \frac{2z+3}{z-4}$ transforms the circle $x^2 + y^2 - 4x = 0$ into the straight line $4u + 3 = 0$. [4]