Assignment 3

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Problem Statement - Tune a proportional controller using root locus for a generalized second order system to have a specific settling time and maximum overshoot for step reference input

Let's take our system to be -

$$H(s) = \frac{Y(s)}{U(s)} = \frac{s+7}{s(s+5)(s+15)(s+20)}$$

And design requirements to be,
Peak overshoot < 5%
Rise time < 1s

Now we know for generalized second order system,

$$T.F = \frac{k\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

where

k=dc gain

ζ=damping factor

ω_n=undamped natural frequency

We know from the formula of peak overshoot that to have a overshoot less than 5% we need to have ,

$$e^{-\zeta\pi/\sqrt{1-\zeta^2}} \times 100\%$$

Damping constant > 0.7

And from rise time formula,

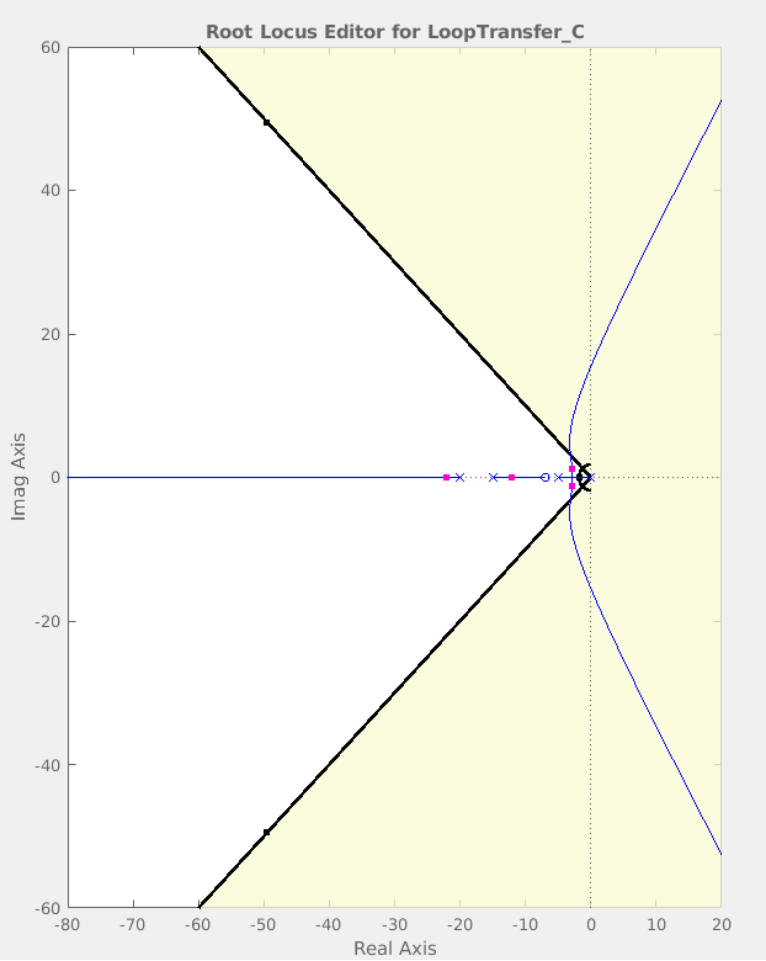
$$\text{So rise time } (t_r) = \frac{\pi - \theta}{\omega_d}$$

We know that wn > 1.8

So now we can use these to control the parameters,

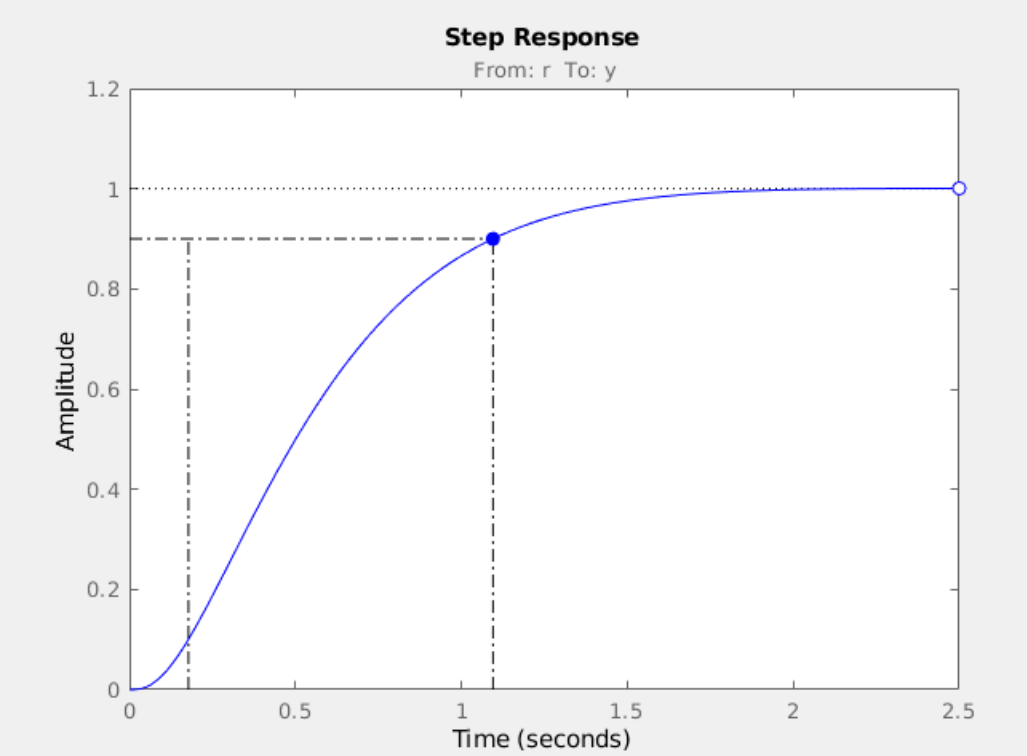
Using Control Designer Tool -

By adding the above design requirement, we get the following root locus graph,

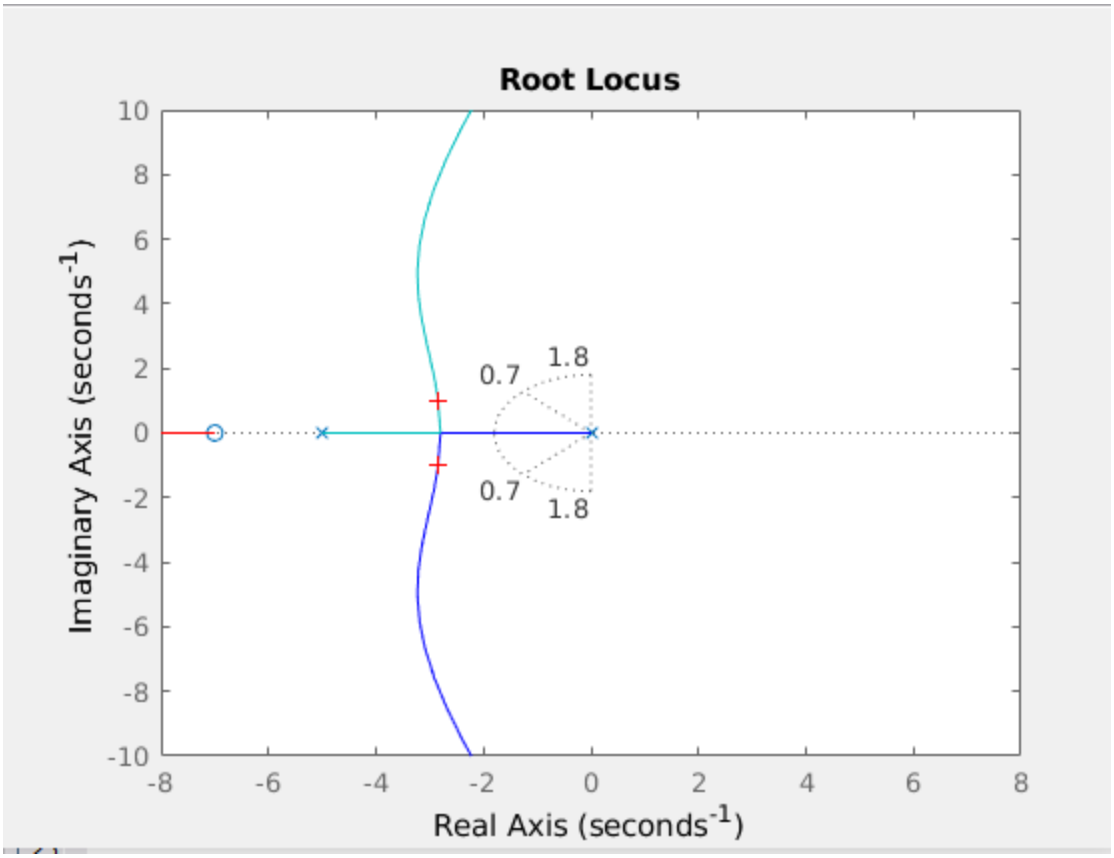


Where white area is acceptable region for second order system constraints. But this will work in our case also as we can design dominant poles of our system to be within range designed for second order and those will determine the characteristic of our system,

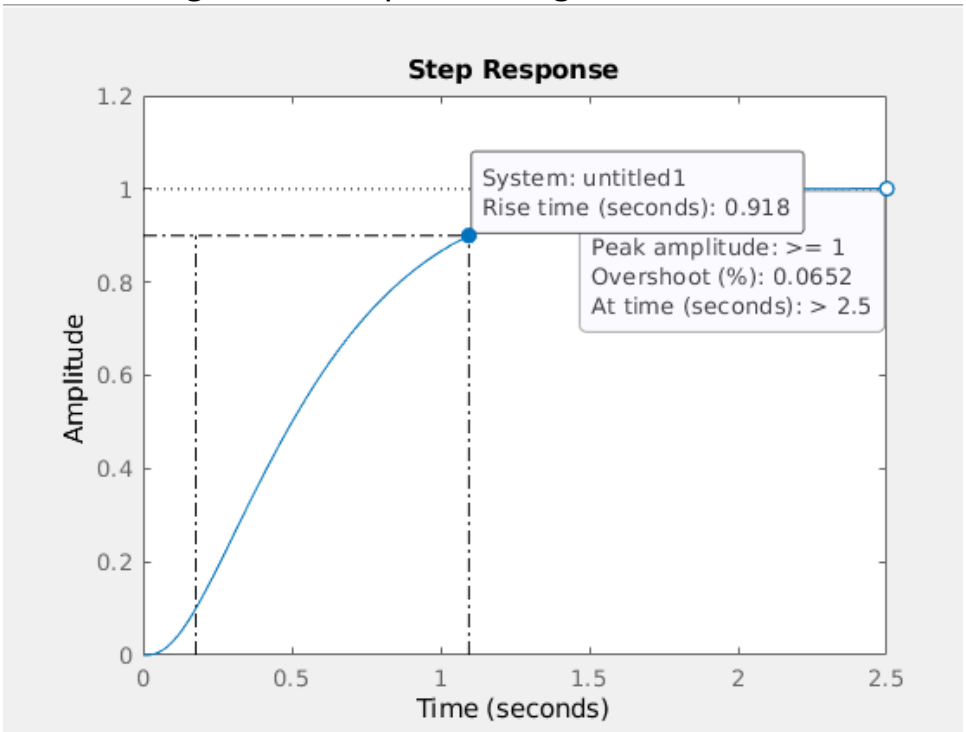
For choosing the poles as shown in above diagram as pink we get,
Gain (k) = 370.4
Rise time = 0.918
Peak Overshoot = 0.0652
Step Response =



Using MATLAB
Using the attached code, we get following root locus and our constraints as dotter lines. We need to select points outside dotted semicircle for wn>1.8 and withing angled line for damping constant > 0.7



On selecting the above points we get similar results as following,



Results -

So, using generalized second order system we tuned our system for specific design requirements of rise time and peak overshoot. Same was verified using MATLAB.