LDPC Codes for Compressed Sensing

Akshit Kumar, *EE14B127* Sachin Agrawal, *EE14B104*

***Abstract*—We present a simplified interpretation of a bridge between compressive sensing and channel coding. We demonstrate a mathematical connection and transfer of performance guarantees from the domain of channel coding to compressive sensing for binary symmetric channels (BSC).We provide a method for deterministic construction of "good" measurement matrices for compressive sensing. We illustrate the obtained results on a small example using the parity check matrix of a (7,4) Hamming Code.**

***Keywords*—Compressive sensing, Channel decoding, Linear programming relaxation, LDPC codes, basis pursuit**

I. Introduction

Recently the research community has been really excited by the theory of sparse signal recovery or compressive sensing, as it is better known. This field was started by two seminal papers in this area by *Candes et al.*[3] and *Donoho*[4]. They presented a setup called "decoding by linear programming", henceforth called compressive sensing linear programming decoding (**CS-LPD**).

During the same time, *Feldman et al.*[2] considered the problem of decoding a binary linear code over a binary-input memory-less channel, a problem which they formulated as a integer linear program along with presenting a linear programming relaxation for it, henceforth called channel coding linear programming decoding (**CC-LPD**)

A connection between **CS-LPD** and **CC-LPD** was presented by *Dimakis et al.*[1]. The general form of the results presented said that if a given parity-check matrix is "good" for **CC-LPD** then the same matrix (considered over the reals) is a "good" measurement matrix for **CS-LPD**.

In this paper, we present an intuitive and simplified understanding of the bridge between **CS-LPD** and **CC-LPD**. We substantiate the theory developed with a small example verifying the guarantee for Binary Symmetric Channel(BSC).

The organization of this paper is as follows: In Section II we give an introduction to the compressive sensing. We give the intuition for using the ℓ_1 minimization and give conditions for equivalence between the NP-hard **CS-OPT** problem and the linear programming relaxation **CS-LPD**. In Section III, we review the channel coding problem along with it's linear programming relaxation.

In Section IV, we present the bridge between **CS-LPD** and **CC-LPD** and provide a more intuitive and simplified understanding of the bridge. In Section V, we illustrate the claim of Section IV, by using the parity check matrix of a (7,4) Hamming Code H as a sensing matrix. Since H can correct upto 1 bit flip errors under **CC-LPD**, we can use it as a sensing matrix to recover all 1-sparse vectors under **CS-LPD**. So we take a random 1-sparse vector(say e), calculate the corresponding y given by $y = He$ and pass that y and H into a solver in MATLAB for ℓ_1-minimization, to get the estimate $\hat{e}$. This results in perfect recovery of all 1-sparse vectors. The case of 2-sparse vectors is also dealt with. We show that for the 2 sparse case, recovery is possible in certain cases only.

II. Compressive Sensing

A. Motivation

Compressive Sensing addresses the following problem : there exists a *signal* represented by an n-dimensional vector x. We observe the image of x under a linear transformation $x \mapsto Ax$, where A is an m x n sensing matrix. The goal is to now recover x from the observed y. One of the motivations behind studying the area of compressive sensing can be addressed by a problem in the area of image signal processing. In general consider the problem of a camera which when capturing images, records a large number of pixels. Then we realize that we cannot store this data, so we decide to compress the data. One intuition behind compressive sensing comes from this redundancy of compressing a large number of measurements. Instead of making such a large number of measurements in the first place is it possible to make smaller number of measurements to record the same data or at least similar data? This is the problem compressive sensing tries to solve. The signal need not be sparse in time domain. Compressive sensing can be used on signals which are sparse in the frequency domain or some other domain as well. In most practical cases, we find that the signal is sparse in some domain.

B. The Setup

Let $\boldsymbol{H_{CS}}$ be a real matrix of dimension $\boldsymbol{m}$ x $\boldsymbol{n}$, called the measurement matrix and the let $\boldsymbol{y}$ denote a real valued vector containing $\boldsymbol{m}$ measurements. The solution to the compressive problem can be posed as an optimization problem of finding the sparsest real vector $\boldsymbol{x}$ with $\boldsymbol{n}$ components which satisfies $\boldsymbol{H_{CS}.x = y}$ and can be written as

$$\begin{aligned} &\min_{\boldsymbol{x}} \|\boldsymbol{x}\|_0 \\ &\text{s.t. } \boldsymbol{H_{CS}.x = y} \end{aligned} \tag{CS-OPT}$$

In Compressive Sensing the additional information we have is that $\boldsymbol{x}$ is $\boldsymbol{s}$-sparse i.e. has at most a given number $\boldsymbol{s}$ of non-zero entries. Initially the sparsity consideration may not seem reasonable but it occurs in many contexts.

If we try to recover x, one natural approach to solving the minimization problem is *brute-force* search : we consider all the subsets I of the index set {1,2,3,...n} - first the empty set, then n singleton sets {1},...,{n}, then $\frac{n(n-1)}{2}$ 2-element subsets and so on and so forth, each time we solve the system of linear equations $H_{CS}.x = y$, $x_j = 0$ when $j \notin I$. We terminate when we find the first solution as it would be the sparset solution to the linear equation given by $H_{CS}.x = y$. As we can see this approach has exponential complexity and the algorithm is NP-hard.

In the context of **CS-OPT**,we relax the optimization problem by substituting the ℓ_0 "norm" by the closest convex norm which is the ℓ_1 norm., which turns into a linear program and can be solved in polynomial time. This relaxation is also known as *basis pursuit*.

$$\begin{aligned} &\min_{x} \|x\|_1 \\ &\text{s.t. } H_{CS}.x = y \end{aligned} \tag{CS-LPD}$$

The intuition behind relaxing **CS-OPT** with an ℓ_1 minimization as opposed to ℓ_2 minimization is that ℓ_1 minimization promotes sparse solutions. The intuitive argument for it can be given with the help of the figure given below. In the case of ℓ_2 minimization, solution is obtained at the point where the ℓ_2 norm ball intersects the hypothesis and the solution has both the x_1 and x_2 components. On the other hand, in the case of ℓ_1 minimization, solution is obtained at the corner points which makes one of the coordinates to go to 0, hence leading to sparsity. On extending this to higher dimensions, we can see that ℓ_1 minimization leads to solutions where many variables have 0 value and hence leads to sparse solutions.

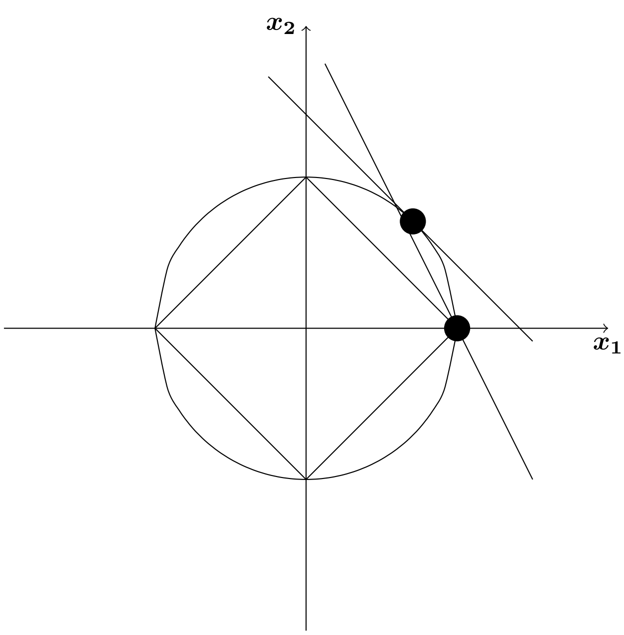

Fig. 1. ℓ_1 minimization vs ℓ_2 minimization

C. *Equivalence between* **CS-LPD** *and* **CS-OPT**

The theory of compressive sensing revolves around one central question : under what conditions would the solution obtained by the original NP-hard problem (**CS-OPT**) and the relaxed linear program (**CS-LPD**) yield the same solution (or close to each other).
Let us say that a sensing measurement matrix H_{CS} is s-good or "good",if in the ℓ_1 minimization case, it recovers correctly all s-sparse signals x. The nullspace characterization can be used to give a necessary and sufficient condition for a matrix to be "good".

Definition II.1. Let H be some matrix. We denote the set of row and column indices of H by $\mathcal{J}(H)$ and $\mathcal{I}(H)$ respectively.

Definition II.2. Let $\mathcal{S} \subseteq \mathcal{I}(H_{CS})$ and let $C \in \mathbb{R}_{\geq 0}$. We say that H_{CS} has the nullspace property $NSP_{\mathbb{R}}^{\leq}(S, C)$ and write $H_{CS} \in NSP_{\mathbb{R}}^{\leq}(S, C)$, if

$$C.\, \|v_S\|_1 < \|v_{\bar{S}}\|_1$$

for all $v \in Ker_{\mathbb{R}}(H_{CS})$

Definition II.3. Let $k \in \mathbb{Z}_{\geq 0}$ and let $C \in \mathbb{R}_{\geq 0}$. We say that H_{CS} has the nullspace property $NSP_{\mathbb{R}}^{\leq}(k, C)$ and write $H_{CS} \in NSP_{\mathbb{R}}^{\leq}(k, C)$, if

$$H_{CS} \in NSP_{\mathbb{R}}^{\leq}(S, C)$$

for all $\mathcal{S} \subseteq \mathcal{I}(H_{CS})$ with $|S| \leq k$

For C = 1, we can simplify the nullspace property for a matrix H_{CS} to be s-good with the following necessary and sufficient condition:
$\|v_I\|_1 < \frac{1}{2}\|v\|_1$ for all $v \in \mathbb{R}^n : Av = 0, v \neq 0, I \subset \{1, ..., n\}, |I| \leq s$
An intuitive way to understand this if $v \in Ker(H_{CS})$ then $\|v\|_{s,1}$, the sum of s-largest magnitudes of entries in v should be strictly less than half of the sum of magnitudes of all entries.

One point that we need to observe is that vectors in nullspace of H_{CS} have more than k coordinates as non-zero entries. For $C \geq 1$, atleast $2k$ coordinates must be non-zero. It can also be shown that Definition II.2 is a necessary and sufficient condition for a measurement matrix to be "good" for k-sparse vectors and the estimate given by **CS-OPT** is the same as the estimate given by **CS-LPD** which is formulated as a theorem as given.

Theorem 1. Let H_{CS} be a measurement matrix. Further, assume that $y = H_{CS}.x$ and that x has at most k nonzero elements, i.e., $\|x\|_0 \leq k$. Then the estimate $\hat{e}$ produced by **CS-LPD** will be equal to the estimate $\hat{e}$ produced by **CS-OPT** if H_{CS} is "good" i.e. H_{CS} has the strict nullspace property for $C = 1$ and can be written in a compact mathematical form as $H_{CS} \in NSP_{\mathbb{R}}^{<}(k, C = 1)$.

D. Applications

Since it's inception in 2006, the theory of compressive sensing has found wide range interest from the research community and found various applications in the field of ranging from astronomy to biology, from communications to medicine, and from image and video signal processing to radar. Specifically to the domain of the communication networks, compressive sensing is being applied to detection and estimation theory, data collection in sensor networks and network monitoring. One example which is closely related to the paper is from the field of linear coding. Compressive sensing provides a way to combine the output of multiple transmitters in an error correcting fashion, so at to make possible the recovery of the original signal even if a significant portion of the output is corrupted. For instance, a 1000 bit information stream can be encoded with the help of a linear code into a 3000 bit stream and then even if a few hundred bits are corrupted (say 300 to 400), the original message can be recovered perfectly. This connection with compressive sensing arises owing to the viewpoint of seeing the corruption as a sparse signal (having 300 non-zero values in a stream of 3000 bits).

III. Channel Code Linear Program Decoder(CC-LPD)

This is the second linear program(LP) we will formulate. Our goal of course is to find a connection between the two linear programs. Here, we describe a LP approach to decoding block codes.

Consider a code with $[n, k, d]$ code with a generator matrix G and a parity check matrix H_{CC}. Here,we are talking only about binary codes. Thus, the code vector is restricted to take the values only 0 and 1.
A vector $c \in \{0, 1\}^n$ lies in the codespace if and only if

$$H_{CC}.c = 0$$

Contrary to general convention, c here is a column vector.

The maximum likelihood decoder can correct upto $\lceil \frac{d}{2} \rceil - 1$ errors. We give an LP formulation which does slightly worse than this but can be solved easily using simplex algorithm. Let r denote the received vector and y denote the transmitted vector. We define the log-likelihood ratio of the i^{th} bit in the received vector as:

$$\gamma_i = ln\left(\frac{Pr[r_i|y_i = 0]}{Pr[r_i|y_i = 1]}\right)$$

The sign of the LLR tells us whether the ith bit is more likely to be a 1 or a 0. If γ_i is positive then y_i is more likely to be a 0. The LLR can be thought of as the cost of setting a bit to be 1. We define n LP variables called f_i. These should take values only $\{0, 1\}$. MLD tries to minimise the overall cost. Hence, the problem can be posed as:

$$\min_f \sum_{i=1}^{n} \gamma_i f_i \quad \text{s.t. } f \in \mathcal{C} \tag{CC-MLD}$$

$\mathcal{C}$ represents the code space. This problem can be relaxed to the convex hull of the code vectors. This problem will also give the same solution as an LP over a convex feasible region has its optima at one of the corner points. However, the description of such a polytope would have exponential complexity. This is because we would need to enumerate all the extreme points of the convex hull to describe it. This basically means listing out 2^k codewords. To relax this further we need the following result:

Theorem 2. $conv(A \cap B) \subseteq conv(A) \cap conv(B)$
Consider the set: $S_j = \{x \mid H_{CC,j}.x = 0(mod\ 2)\}$. There exist m such sets, one for each row of H_{CC}. We can represent the codespace as a intersection of all of these sets.

$$\begin{aligned} conv(C_{CC}) &= conv(S_1 \cap S_2 \cap \dots S_m) \\ conv(C_{CC}) &\subseteq conv(S_1) \cap conv(S_2) \cap \dots conv(S_m) \end{aligned} \tag{1}$$

Let us call the Polytope defined by H_{CC} as $\mathcal{P}(H_{CC})$.

$$\mathcal{P}(H_{CC}) = \cap_{i=1}^{m} conv(S_i)$$

. Thus the following problem is a relaxation of the original problem:

$$\min_f \sum_{i=1}^{n} \gamma_i f_i \quad \text{s.t. } f \in \mathcal{P}(H_{CC}) \tag{CC-LPD}$$

Every codeword in the codespace is a vertex of the polytope($\mathcal{P}$). However, the polytope also contains some additional corner points which maybe fractional and are not valid codewords. Thus, CC-LPD may solve to one of these points instead of to a valid codeword. If CC-LPD solves to a valid codeword, then it is the same solutions as that of the maximum likelihood decoder. This is because CC-LPD is a relaxtion of CC-MLD. The set of vectors which lie in $\mathcal{P}$ are called pseudo vectors. There are also alternative definitions of pseudo-weights based on M-covers of tanner graphs. However, we do not need such definitions presently.
The next question is when does CC-LPD give the same solution as that of CC-MLD. For this we need to introduce the concept of fractional distance. Let d_{frac} denote the fractional distance of a code.

$$d_{frac} = \min \sum_{i=1}^{N} |y_i - f_i| \quad \text{s.t } y \in \mathcal{C}, f \in \mathcal{V}(\mathcal{P}), y \neq f \tag{2}$$

In the above equation $\mathcal{V}(\mathcal{P})$ represents the vertices of $\mathcal{P}$. Note that for a given code, d_{frac} depends on the parity check matrix. For two different parity matrices of the same code, we get a different polytope $\mathcal{P}$ and hence a different fractional distance. It is clear that d_{frac} is an upper bound on the actual distance d(as d_{frac} becomes equal to d if we restrict f to belong to those vertices of $\mathcal{P}$ which are in the codespace. Hence, d_{frac} can be viewed as a relaxation d). It can then

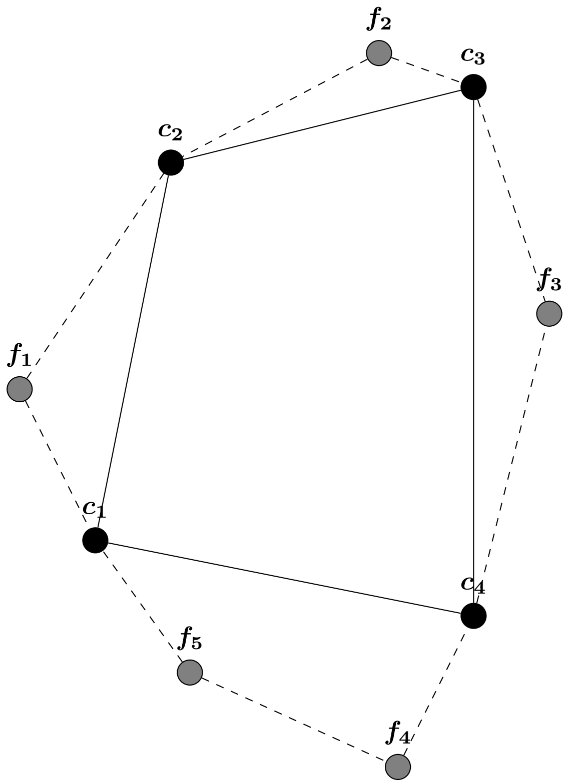

Fig. 2. The darkened lines represent the convex hull of the codespace, and the dotted lines represent the polytope $\mathcal{P}(H_{CC})$. The gray nodes represent fractional vertices and the black ones represent valid codewords

be shown that CC-LPD can correct upto $\left\lceil \frac{d_{frac}}{2} \right\rceil - 1$ errors. Hence, we are able to correct slightly lesser number of errors. As an example RM(1,3) code has $d = 4$, but $d_{frac} = 3$. In this case both CC-LPD1 and CC-LPD2 can correct only one error. However, for larger codes, there is a loss in the number of correctable errors.

The above condition does not help us form the bridge between CC-LPD and CS-LPD. For this we need to represent the set of pseudocodes as a cone.

Definition III.1. The fundamental cone $\mathcal{K}(H_{CC})$ is defined as the set of vectors $\nu = (\nu_1, \nu_2, \ldots, \nu_n) \in \mathbb{R}^n$such that $\forall$ $1 \leq i \leq n$ and $1 \leq j \leq m$, we have the following:

$$\nu_i \geq 0$$

$$\sum_{i' \neq i} h_{ji'} \nu_{i'} \geq h_{ji} \nu_i$$

.

The conic hull of H_{CC} is denoted by $\mathcal{K}$. As an example consider the conic hull of the $[7, 4, 3]$ hamming code. We take the parity check matrix in the following form:

$$H = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

Note that the the parity check matrix has some redundant rows in it. Another point of importance is that the fundamental cone depends on the parity check matrix and its form. The conic hull of the (7,4) code is given by

$$\begin{aligned} &\mathcal{K} = (\nu_1, \ldots, \nu_7) \in \mathbb{R}^n \\ &\text{ s.t } \nu_i \geq 0 \ \forall \ 1 \leq i \leq 7 \\ &\nu_1 = \nu_2 = \nu_3, \ \nu_5 = \nu_6 = \nu_7 \\ &2\nu_1 \geq \nu_4, \ 2\nu_5 \geq \nu_4 \end{aligned} \tag{3}$$

IV. Bridge between CS-LPD and CC-LPD

Now we begin to form the bridge between CC-LPD and CS-LPD. The significance of this paper is that it provides the first deterministic way to form a sensing matrix with some performance guarantees. Past constructions of sensing matrices relied on expander graphs or randomized algorithms. By deterministic way, we mean that the method to form the sensing matrix has polynomial time complexity. The first result is that if a parity check matrix can correct s bit-flipping errors under **CC-LPD**, then the same parity check matrix is s-good. That is, if there exists a s-sparse x which acts as a solution to

$$y = H_{CC} x$$

, then **CS-LPD** will decode to it. Note that the result holds only one way and not the other way round. That is if we have a sensing matrix then we cannot use it as a LDPC parity check matrix. To show this result we need to first prove the following lemma:

Lemma 3. Let $\mathcal{S}$ denote the set of indices flipped over during transmission over a BSC channel. If the parity check matrix H_{CC} is such that for all vectors(ω) in the fundamental cone of the parity matrix, the following condition holds:

$$||\omega_{\mathcal{S}}||_1 < ||\omega_{\bar{\mathcal{S}}}||_1 \tag{4}$$

. Then the **CC-LPD** will decode to the transmitted codeword.

Although the condition in (4) is necessary and sufficient for correct decoding, we only require to show the sufficiency of the above result. To prove the same we assume that the channel performs equally for all codewords. That is the channel is symmetric in terms of the codeword transmitted. Hence we assume that the all zero codeword was transmitted. Let $+L > 0$ denote the log likelihood ratio associated with a received 0 and $-L < 0$ denote the log likelihood associated with a received 1. So for all indices(i) in the set $i \in \mathcal{S}$, $\gamma_i = -L$ and

$\forall\ i \in \bar{\mathcal{S}}, \gamma_i = -L$. Now consider the cost of any arbitrary vector in the cone denoted by ω.

$$\begin{aligned} <\gamma,\omega> &= \sum_{i\in\bar{\mathcal{S}}}(L).\omega_i + \sum_{i\in\mathcal{S}}(-L).\omega_i \\ &= L.||\omega_{\bar{\mathcal{S}}}||_1 - L.||\omega_{\mathcal{S}}||_1 > 0 \end{aligned} \quad (5)$$

In the above proof, we note that the fundamental cone consist of positive coordinates only. Hence, we can say that $\omega_i = ||\omega_i||_1$. The second line of the proof comes from the condition mentioned in (4). Hence, we see that the cost for all vectors in the fundamental cone is higher than for the all zero vector(Note that the cost of the all zero vector is 0). Thus, the CC-LPD will decode to the all zero codeword, provided the condtion in (4) is met. This lemma is very similar to the null space property for compressive sensing (at $C = 1$) given in Definition II.2. This is the first concrete piece of intuition which shows that there might be a connection between **CS-LPD** and**CC-LPD**. Of course we have restricted ourselves to a BSC channel for now. Performance measures for other channel are not as tight as they for the BSC case.

Now we present the main result of this paper. We show that there exists a relation between CC-LPD and CS-LPD. To show this, we prove that corresponding to each point in the nullsace of H_{CS}, there exists a point in the fundamental cone($\mathcal{K}$) of H_{CS}.

Lemma 4. Let H be a zero one measurement matrix. Then

$$\nu \in\ Nullsp(H) \implies |\nu| \in \mathcal{K}(H)$$

Let $\omega = |\nu|$. We show that ω is in the fundamental cone of H. Thus, corresponding to every ν, we have an $\omega = |\nu|$ such that ν lies in the fundamental cone of H. Recall that for ω to lie in the fundamental cone of H_{CS} it must satisfy the following conditions:

$$\omega_i \geq 0 \forall i \in 1, 2, \ldots, n$$

and

$$\sum_{i'\neq i} h_{ji'}\nu_{i'} \geq h_{ji}\nu_i$$

. Let the set of all indices with non-zero mappings in the jth row be given by $\mathcal{I}_j$. Note that since we are talking about zero-one matrices we can reduce the second condition as:

$$\begin{aligned} &\omega_j \leq \sum_{i'\in\mathcal{I}_j, i'\neq i} \omega_{i'} \\ &\forall \text{ rows j, i} \in \mathcal{I}_j \end{aligned} \quad (6)$$

Now a vector ν which belongs to the nullspace of H satisfies the property given by $\sum_i h_{j,i}\nu_i = 0$ for all rows j. This implies that $\sum_{i\in\mathcal{I}_j} \nu_i = 0$. Thus we get the following:

$$\begin{aligned} \omega_i &= |\nu_i| = |-\sum_{i'\in\mathcal{I}_j, i\neq i'} \nu_{i'}| \\ &\leq \sum_{i'\in\mathcal{I}_j, i'\neq i} |\nu_{i'}| \\ &\leq \sum_{i'\in\mathcal{I}_j, i'\neq i} \omega_{i'} \end{aligned} \quad (7)$$

This is nothing but the condition for the vector $|\nu| = \omega$ to lie in the fundamental($\mathcal{K}$) of H_{CS}.

A. Interpretation of the Result

Now we offer some insight as to how to interpret this result. This lemma gives a one way result. That is we can associate a point in the nullspace of H_{CS} with a point in the fundamental cone of H_{CS}. The other way round result need not hold. Recall the condition in (4) and the condition in Definition II.2. Consider a H_{CS}, which we want to use as a sensing matrix. However, we do not know if this matrix is an $s - good$ sensing matrix. Let us say that we know that this matrix is a good LDPC code. Hence, we can say that every point in the fundamental cone of H_{CS} satisfies property (4) upto a particular cardinality of $\mathcal{S}$, where $\mathcal{S}$ is the set of bits which were flipped during transmission. By lemma 4, we know that every point in the nullspace of H_{CS} has a map to a point in the fundamental cone given by the mod of the vector(mod being the positive values of each coordinate). So for a point ν in the nullspace of H, $|\nu| \in \mathcal{K}(H)$. Now we know that $|\nu|$ satisfies property (4), since H is a good LDPC code. Hence, ν will satisfy the nullspace property for $C = 1$. This holds for all such points in the nullspace of H. Hence, by virtue of H being a a good LDPC code, we can say that H is also a good sensing matrix.

This translates to some form of performance guarantees. If a point is problematic for CC-LPD, that is it has a point with a low pseudo weight, then there will be points which will be problematic even for the CS-LPD problem. If CS-LPD can correct s errors over a BSC channel, then this would translate to the corresponding H being an s-good sensing matrix. Performance guarantees are not so direct for other channel models but we can still get some performance guarantees even from other channel models.

V. Demonstration with an Example

The central theme of the paper is the transfer of guarantee from channel coding domain to the compressive sensing domain. Particularly for a Binary Symmetric Channel (BSC), if a parity check matrix can correct any s bit flipping errors under the **CC-LPD** then the same matrix taken as a measurement matrix over the reals can be used to recover all s-sparse error signals under **CS-LPD**.

A. Correct Recovery of $\mathbf{1}$-sparse vector

To illustrate this point with an example, we consider a rather small parity check matrix given by $(7,4)$ Hamming code. Let us denote the $(7,4)$ Hamming code parity check matrix by $\boldsymbol{H}$.

$$\boldsymbol{H} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}$$

The minimum weight of $(7,4)$ Hamming code is $\mathbf{3}$. Since $\boldsymbol{k} = \lceil \frac{d_{min}}{2} \rceil - \mathbf{1}$, we get $\boldsymbol{k} = \mathbf{1}$. This means that this parity check code is a single bit error correcting code. Therefore, we can use this matrix as sensing matrix of dimension $\mathbf{3}$x$\mathbf{7}$ to recover all $\mathbf{1}$-sparse vectors which solve the under-determined system of linear equations given by $\boldsymbol{y} = \boldsymbol{H}.\boldsymbol{x}$.
We take the measurement vector $\boldsymbol{y}$ to be

$$\boldsymbol{y} = \begin{bmatrix} 0 \\ 0.9245 \\ 0.9245 \end{bmatrix}$$

We make use of the l1-magic solver to solve for ℓ_1-minimization problems. On solving the system of equations, we get the following value of $\hat{\boldsymbol{x}}$ as the estimate of $\boldsymbol{x}$, the original signal.

$$\hat{\boldsymbol{x}} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0.9244 \\ 0 \\ 0 \end{bmatrix}$$

We can clearly see from the estimate of $\boldsymbol{x}$, $\hat{\boldsymbol{x}}$ that it is $\mathbf{1}$-sparse and is complaint with the theoretical results that we established in the Section IV.

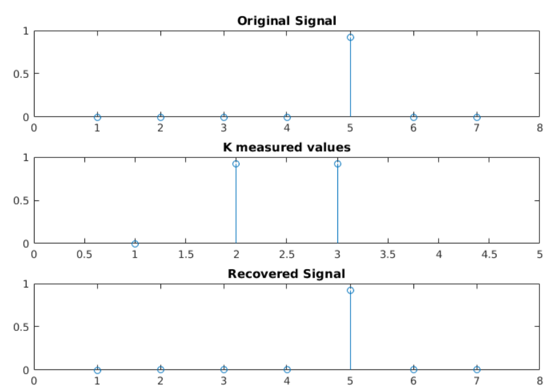

Fig. 3. Plots showing the original signal, the measurement signal and the recovered signal for $\mathbf{1}$-sparse signal and using the $(7,4)$ as the sensing matrix for compressive sensing.

B. Recovery of $\boldsymbol{p}$-sparse matrices where $\boldsymbol{p} > \boldsymbol{s}$

The results guarantee that recovery of $\mathbf{1}$-sparse signals, but the same sensing matrix can also recover $\boldsymbol{m}$-sparse signals, for $\boldsymbol{m} > \mathbf{1}$ for certain cases. In the following two examples we present the cases where the parity check matrix $\boldsymbol{H}$ is able to correctly and incorrectly recover the signal $\boldsymbol{x}$ given the measurement vector $\boldsymbol{y}$ and sensing matrix $\boldsymbol{H}$.

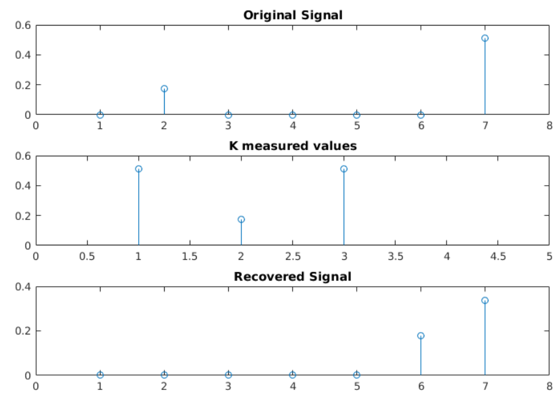

Fig. 4. This illustrates the case of incorrect recovery. We can clearly see that the estimate of the original signal is not the same as the original signal.

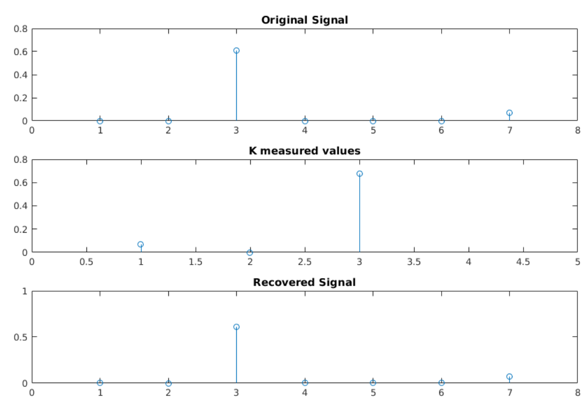

Fig. 5. This illustrates the case of approximately correct recovery of the original signal. We can clearly see that the estimate of the original signal is approximately the same as the original signal.

VI. Conclusion

In summary, we showed that a code which can correct $\boldsymbol{s}$ errors using CC-LPD for transmission over binary symmetric channel acts as an $\boldsymbol{s}$-good matrix for CS-LPD. Hence, we presented a deterministic polynomial time algorithm to form a zero-one measurement matrix. Note that most methods of generating sensing matrices rely on expander graphs or randomised algorithms. If the code can correct a set of errors

at indices denoted by $\mathcal{S}$, then the sensing matrix can recover the sparse solution with non-zero values at $\mathcal{S}$, provided this solution is the sparsest available solution. Note that this result is one way. That is a good LDPC zero one matrix will make a good sensing matrix. This holds for finding a zero one sensing matrix in polynomial time.A word of caution. The paper claims to have found a deterministic method to find a s-good sensing matrix. However, the method to form these LDPC matrices is not deterministic. Since, LDPC codes have been studied in detail, we can leverage our prior knowledge of LDPC codes to form a sensing matrix.
We also showed that performance guarantees for binary symmetric channel are tight between CC-LPD and CS-LPD.We also gave the intuition behind the existence of relation between performance guarantees of CC-LPD and CS-LPD.

We also present a small example which not only verifies the theory presented in the paper, but also helps to explain the results in the paper to some degree. The code used is a $(7,4)$ Hamming code. Hence it can correct only one error and obtain a 1-sparse solution to CS-LPD if such a solution exists.
There are various methods suggested to expand the theory presented in the paper. One thing is to introduce the concept of pseudo weights and find how the performance guarantees translate for various channels(AWGN, BEC). The concept of M-cover of a graph and the equivalent definition of fractional weights using M-cover of the graph. Another way to expand the presented theory is to expand the set of sensing matrices from zero one matrices to matrices over the set of complex numbers which have their modulo to be equal to 1. These things have not been covered in this paper.

VII. Acknowledgment

The authors would like to thank Prof.Pradeep Sarvepalli for giving us the opportunity to work on this project. We would also like to than Prof. Radhakrishna Ganti for giving us a formal introduction to the field of compressive sensing.

References

[1] Dimakis, Roxana, Pascal *LDPC codes for Comressed Sensing*, arXiv: https://arxiv.org/pdf/1012.0602.pdf

[2] J. Feldman, *Decoding error-correcting codes via linear programming* Ph.D. dissertation, Dept. of Electrical Engineering and Computer Science, Massachusetts Institute of Technology, Cambridge, MA, 2003.

[3] E. J. Candes and T. Tao, *Decoding by linear programming* IEEE Trans. Inf. Theory, vol. 51, no. 12, pp. 42034215, Dec. 2005.

[4] D. Donoho, *Compressed sensing* IEEE Trans. Inf. Theory, vol. 52, no. 4, pp. 12891306, Apr. 2006.

[5] Ben-Tal and Nemrovski *Lectures on Modern Convex Optimisation*, Fall semester 2013.

[6] P. O. Vontobel and R. Koetter, *Graph-cover decoding and finite-length analysis of message-passing iterative decoding of LDPC codes* CoRR, http://www.arxiv.org/abs/cs.IT/0512078, Dec. 2005

[7] l1-magic solver *https://statweb.stanford.edu/ candes/l1magic/*, solving compressive sensing pproblems

[8] Christine Kelly, Deepak Sridhara *Pseudocodewords on Tanner Graphs*, IEEE transactions on information theory, 2007

[9] Ralf koetter, Ching, Pascal, et. all *Characterizations of pseudo-codewords of LDPC codes*

[10] Dimakis, Tehrani and Caire, *Optimal Deterministic Compressed Sensing Matrices*

[11] Richard Baraniuk, *Compressive sensing of Videos: Tutorial*, Short course on comressive sensing.

[12] Terrence Tao, *Terrence Tao's blog, 2007 April*, https://terrytao.wordpress.com/2007/04/13/compressed-sensing-and-single-pixel-cameras/

[13] Nana Traore, Tobias, et. all *Message Passing algorithm and Linear program decoding for LDPC and linear block codes*, Aalborg university

[14] Duarte, Davenport, Eldar, Kutyniok *Introduction to Comressive sensing*

[15] R.G.Gallager,*Low-Density Parity-Check Codes.* M.I.T. Press, Cambridge, MA, 1963, available online under http://justice.mit.edu/ people/gallager.html.