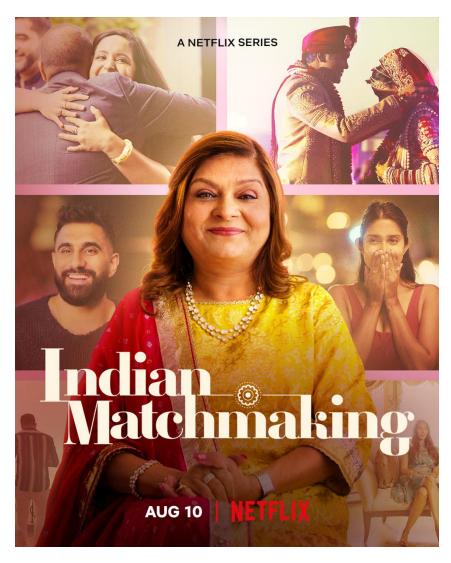
Marketplace Operations: From Resource Allocation to Recommendations

Akshit Kumar

Columbia Business School

Joint work with Omar Besbes and Yash Kanoria





Match demand location with fulfillment center



Match demand location with fulfillment center



Match customers with service providers



Match demand location with fulfillment center



Recommend movies/songs to consumers



Match customers with service providers



Match demand location with fulfillment center



Recommend movies/songs to consumers



Match customers with service providers







Match demand location with fulfillment center



Recommend movies/songs to consumers



Match customers with service providers





Match demand location with fulfillment center



Recommend movies/songs to



Match customers with service providers





Match demand location with fulfillment center



Research Questions

Recommend movies/songs to



Match customers with service providers





Match demand location with fulfillment center



Match customers with service providers



Research Questions

1. How should platforms allocate limited resources dynamically and efficiently?





Match demand location with fulfillment center



Match customers with service providers

Part I

Research Questions

- 1. How should platforms allocate limited resources dynamically and efficiently?
- 2. What are the fundamental driver of algorithmic performances in these marketplaces?



Match demand location with fulfillment center



Recommend movies/songs to consumers



Match customers with service providers







Match demand location with fulfillment center



Recommend movies/songs to consumers



Match customers with service providers





Part II

Research Questions

1. What is the value of personalized recommendations in marketplaces?



Recommend movies/songs to consumers



Match customers with service providers









We study prototypical models of dynamic resource allocation problem (focus on the multisecretary prob)





We study prototypical models of dynamic resource allocation problem (focus on the multisecretary prob)



Shape of the reward distribution is a fundamental driver of algorithmic performance





We study prototypical models of dynamic resource allocation problem (focus on the multisecretary prob)



Shape of the reward distribution is a fundamental driver of algorithmic performance



Workhorse policies can be highly suboptimal and nearoptimal algorithms are overly specified





We study prototypical models of dynamic resource allocation problem (focus on the multisecretary prob)



Shape of the reward distribution is a fundamental driver of algorithmic performance



Workhorse policies can be highly suboptimal and nearoptimal algorithms are overly specified



Design a simple and near-optimal policy called Repeatedly Act using Multiple Simulations (RAMS)







We study a prototypical utility model which comprises of public and private utility components





We study a prototypical utility model which comprises of public and private utility components



Identify subtle interplay of personalized recommendations and capacity constraints on social welfare





We study a prototypical utility model which comprises of public and private utility components



Identify subtle interplay of personalized recommendations and capacity constraints on social welfare



Personalized recos may have *little to no* benefit in markets with uncapacitated supply (think movies, songs)





We study a prototypical utility model which comprises of public and private utility components



Identify subtle interplay of personalized recommendations and capacity constraints on social welfare



Personalized recos may have *little to no* benefit in markets with uncapacitated supply (think movies, songs)



Personalized recos unlock *significant* social welfare in capacitated supply settings (think AirBnB, college admits)



Part I

Dynamic Resource Allocation: Algorithmic Design Principles & Spectrum of Achievable Performances

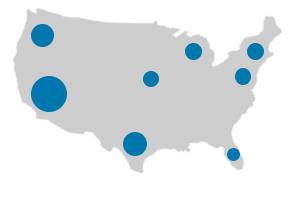
Forthcoming in Operations Research (2024)

Dynamic Resource Allocation

Algorithmic Design Principles & Spectrum of Achievable Performances

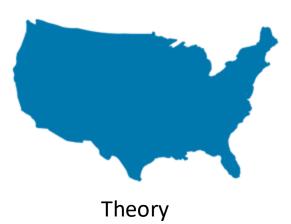


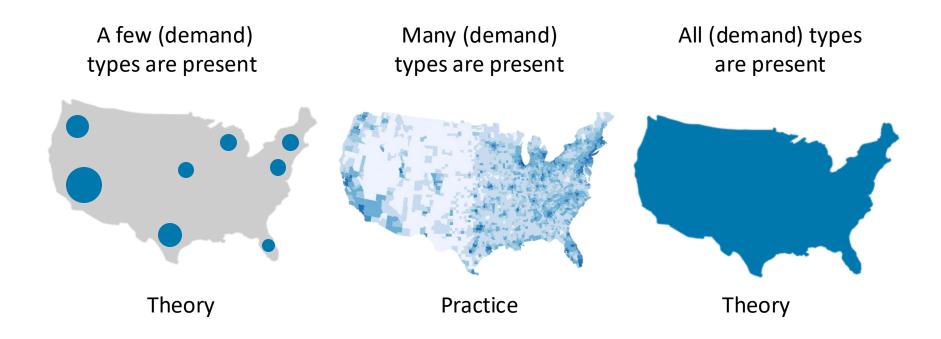
A few (demand) types are present

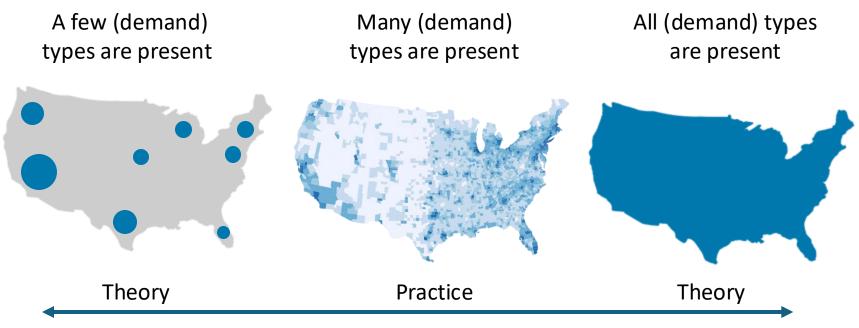


Theory

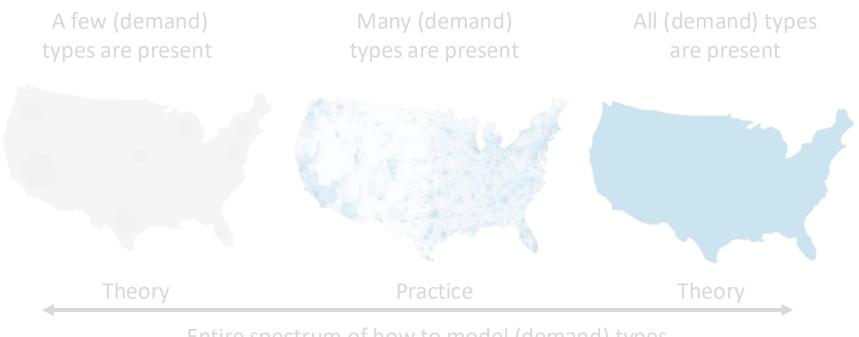
All (demand) types are present





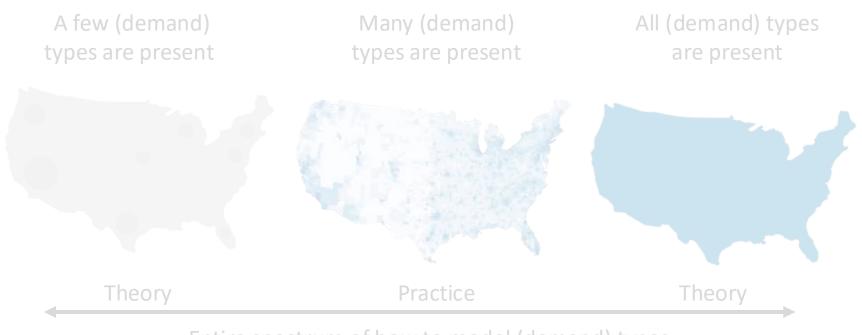


Entire spectrum of how to model (demand) types



Entire spectrum of how to model (demand) types

1. What is the interplay between the distribution of request types and achievable algorithmic performance?

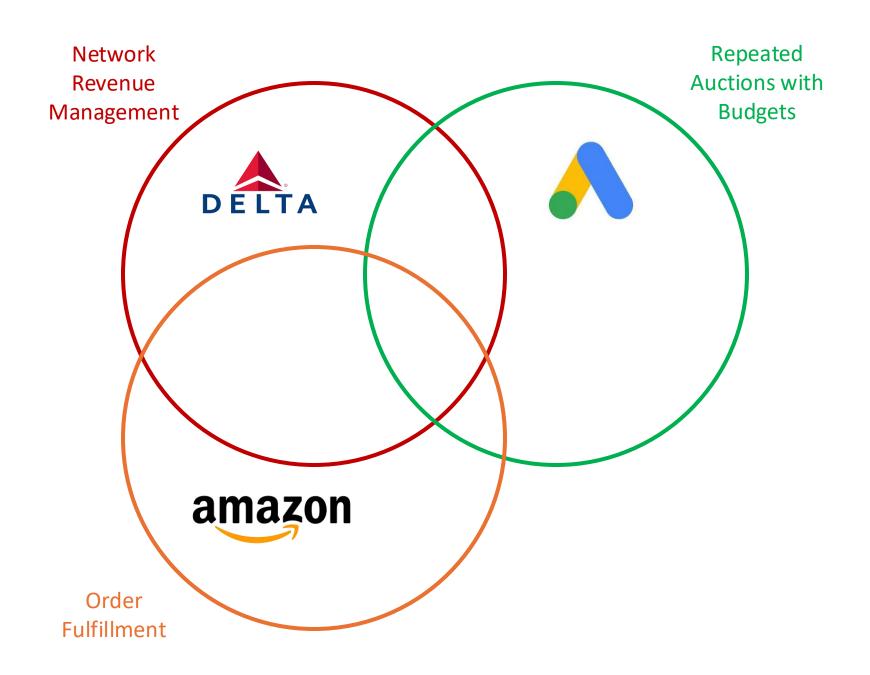


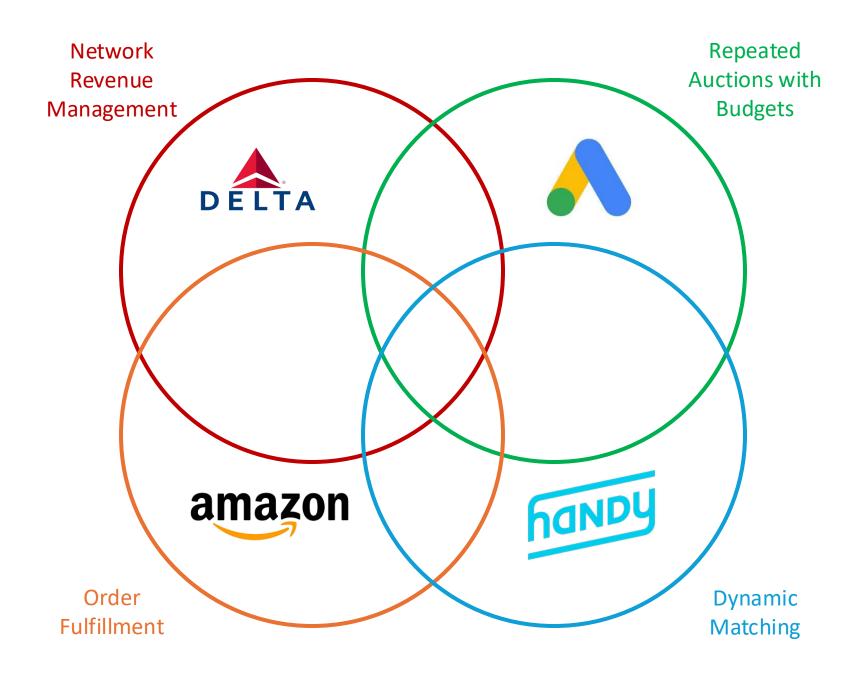
Entire spectrum of how to model (demand) types

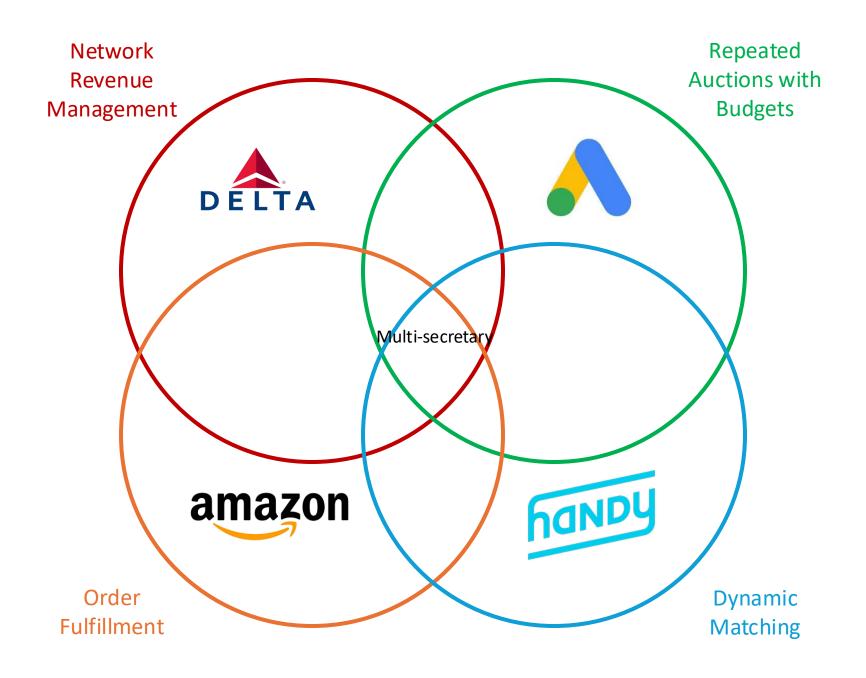
- 1. What is the interplay between the distribution of request types and achievable algorithmic performance?
- 2. Can we design a **unified**, **simple** and **near-optimal** algorithms which works for all type distributions?

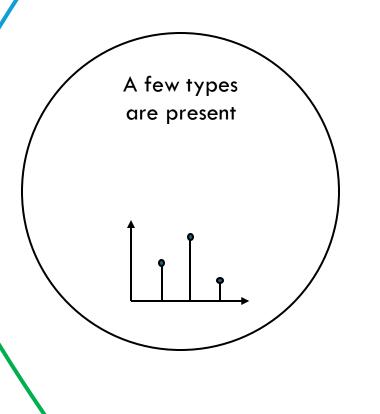


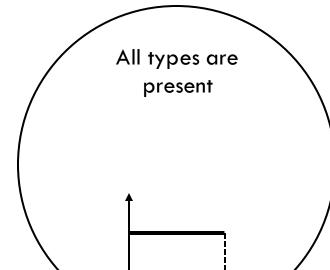








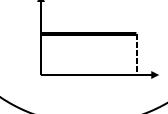




A few types are present Bounded Regret

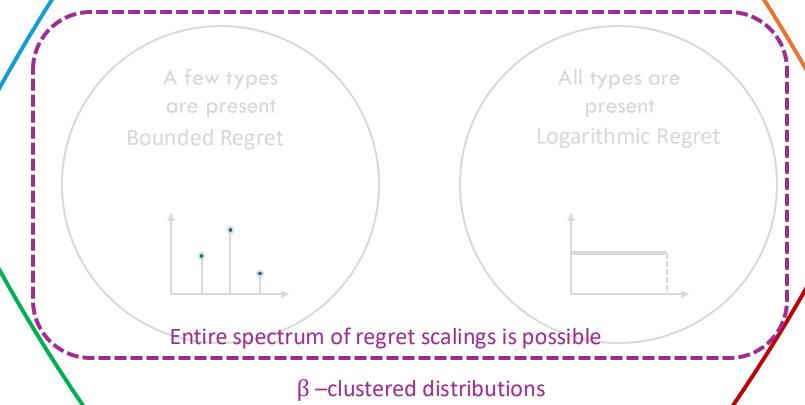
All types are present

Logarithmic Regret



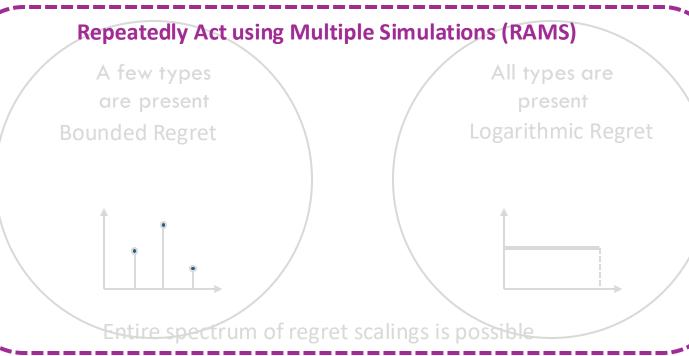


Regret is the additive gap b/w the value of hindsight opt. and value under some algorithm

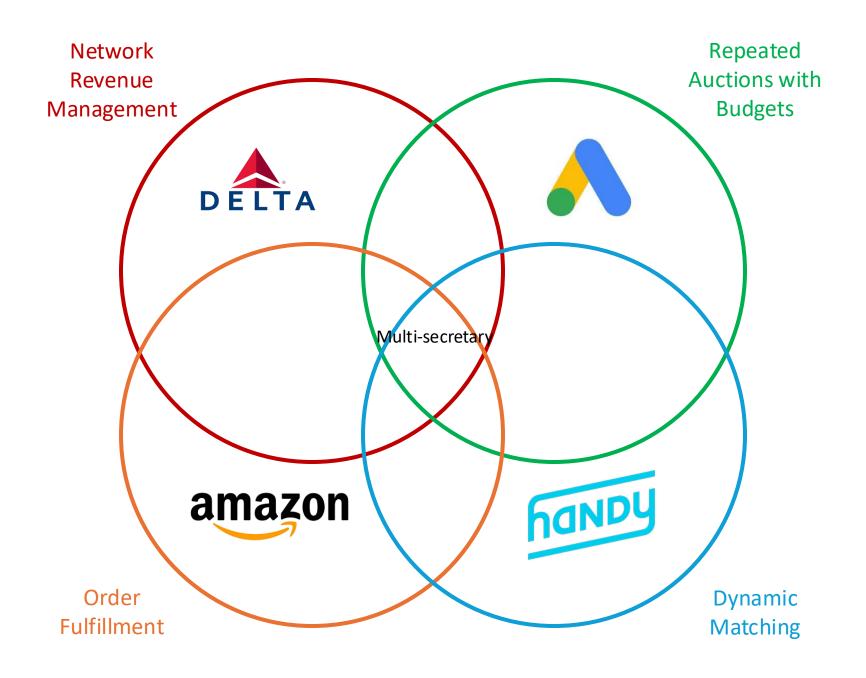


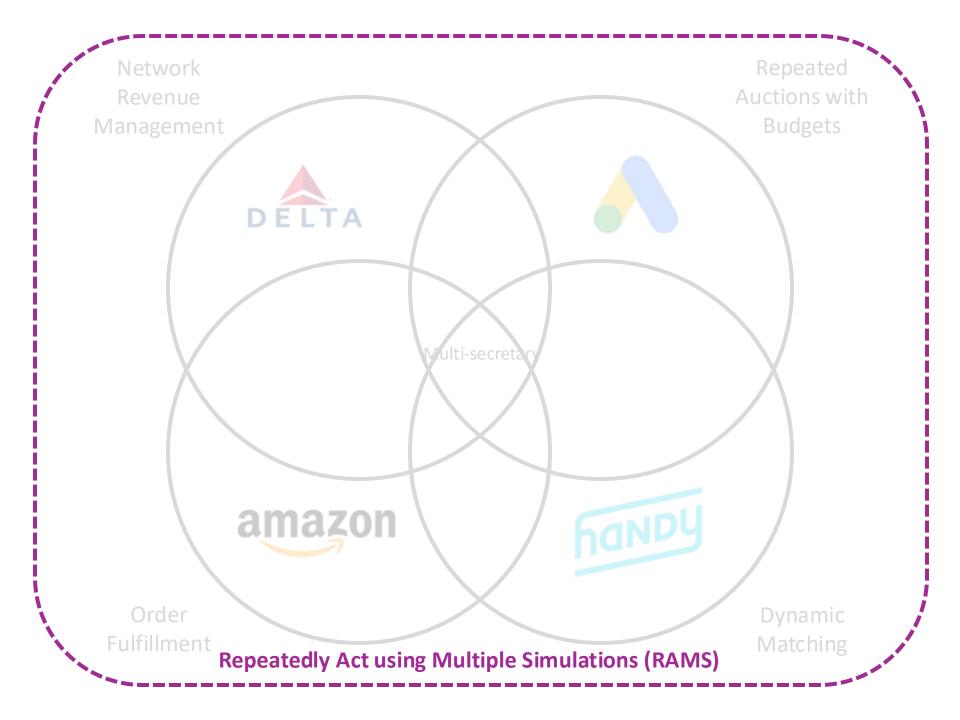
Regret is the additive gap b/w the value of hindsight opt. and value under some algorithm

one algorithm to solve them all



β –clustered distributions

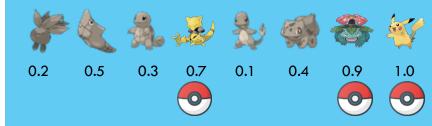




• Given a sequence of T values and budget B



• Given a sequence of T values and budget B, the DM wants to select the top B values



- Given a sequence of T values and budget B, the DM wants to select the top B values
- The values arrive in an online fashion



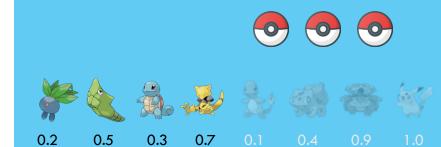
- Given a sequence of T values and budget B, the DM wants to select the top B values
- The values arrive in an online fashion



- Given a sequence of T values and budget B, the DM wants to select the top B values
- The values arrive in an online fashion



- Given a sequence of T values and budget B, the DM wants to select the top B values
- The values arrive in an online fashion



- Given a sequence of T values and budget B, the DM wants to select the top B values
- The values arrive in an online fashion



- Given a sequence of T values and budget B, the DM wants to select the top B values
- The values arrive in an online fashion



- Given a sequence of T values and budget B, the DM wants to select the top B values
- The values arrive in an online fashion



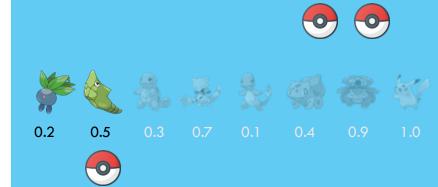
- Given a sequence of T values and budget B, the DM wants to select the top B values
- The values arrive in an online fashion



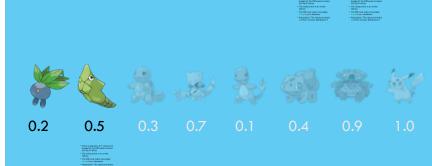
- Given a sequence of T values and budget B, the DM wants to select the top B values
- The values arrive in an online fashion
- The DM must make irrevocable accept/reject decisions



- Given a sequence of T values and budget B, the DM wants to select the top B values
- The values arrive in an online fashion
- The DM must make irrevocable accept/reject decisions



- Given a sequence of T values and budget B, the DM wants to select the top B values
- The values arrive in an online fashion
- The DM must make irrevocable accept/reject decisions
- Assumption: The values are drawn i.i.d from a known distribution F



- Given a sequence of T values and budget B, the DM wants to select the top B values
- The values arrive in an online fashion
- The DM must make irrevocable accept/reject decisions
- Assumption: The values are drawn i.i.d from a known distribution F
- Performance Metric: Minimize Regret

```
Regret(\pi) = (Expected Maximum Value in Hindsight) – (Expected Value under \pi)
```

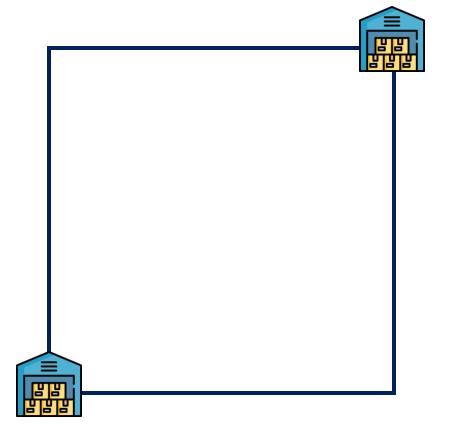
- Given a sequence of T values and budget B, the DM wants to select the top B values
- The values arrive in an online fashion
- The DM must make irrevocable accept/reject decisions
- Assumption: The values are drawn i.i.d from a known distribution F
- Performance Metric: Minimize Regret Regret (π) = (Expected Maximum Value in

Hindsight) – (Expected Value under π)

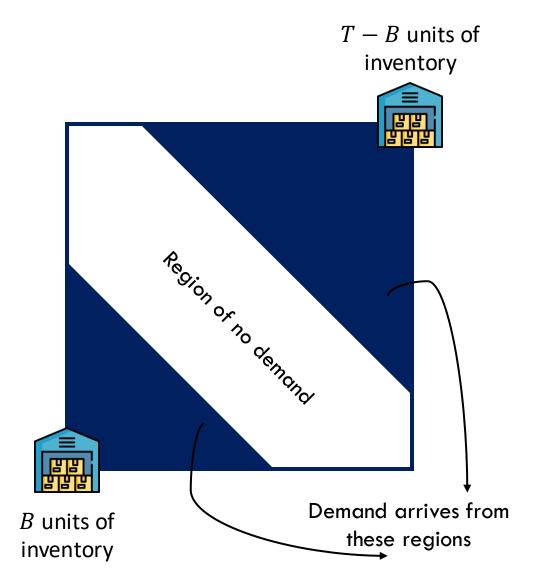
Value in hindsight = 0.7 + 0.9 + 1.0 = 2.60.2 0.5 0.3 0.7 0.1 0.4 0.9 1.0

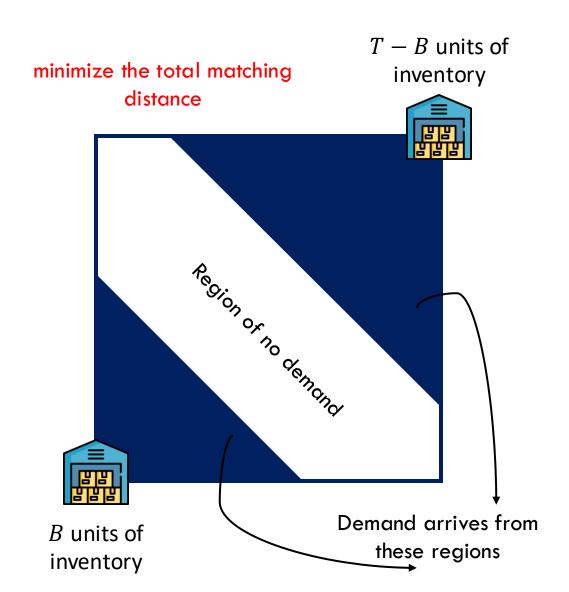
Value for policy $\pi = 0.5 + 0.4 + 0.9 = 1.8$

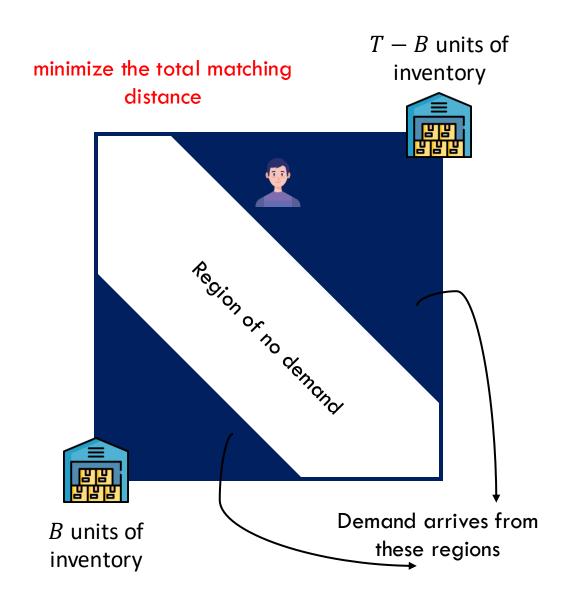
T - B units of inventory

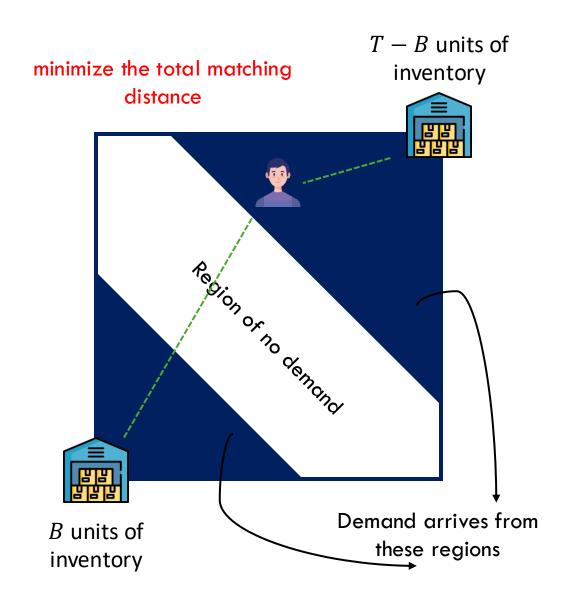


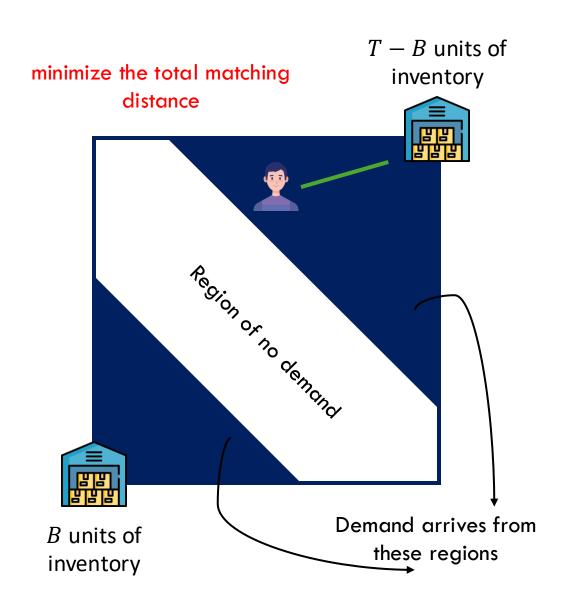
B units of inventory

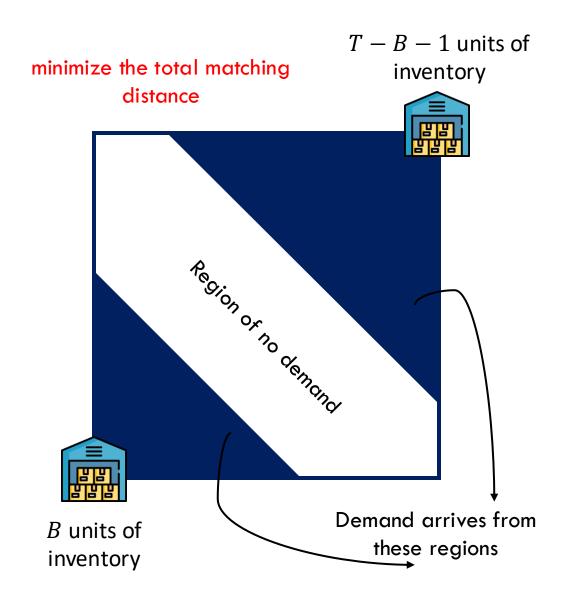


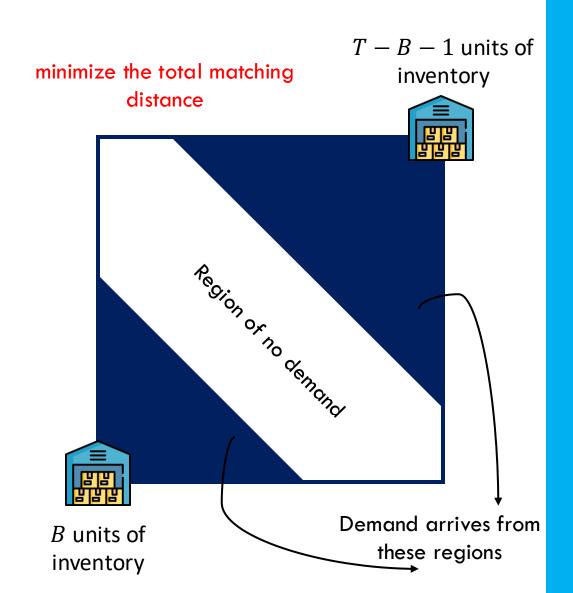


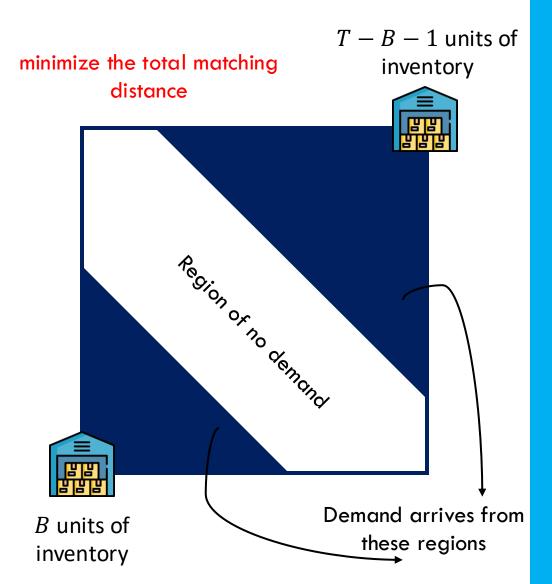


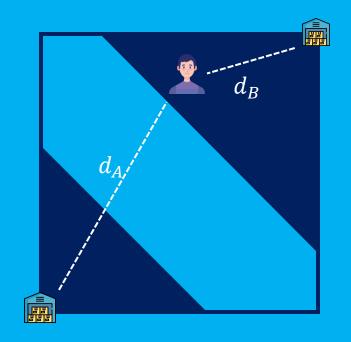


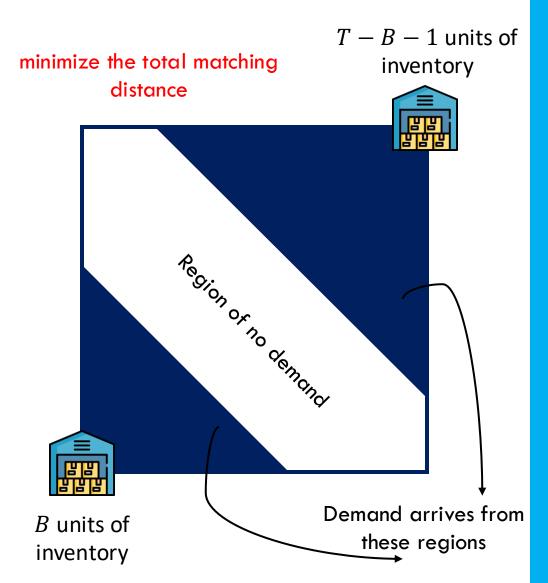


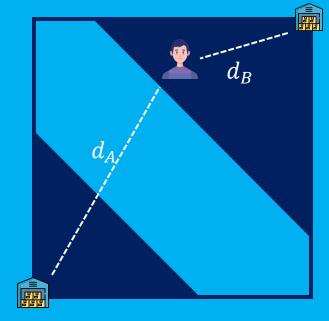






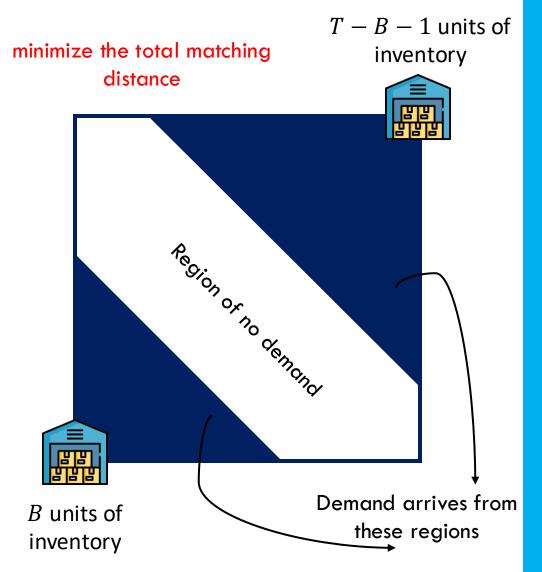


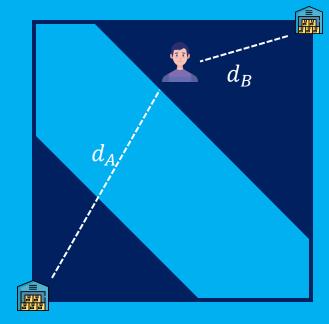




$$r = d_B - d_A + L$$

minimize total <u>maximize total</u> matching distance matching reward



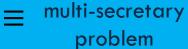


$$r = d_B - d_A + L$$

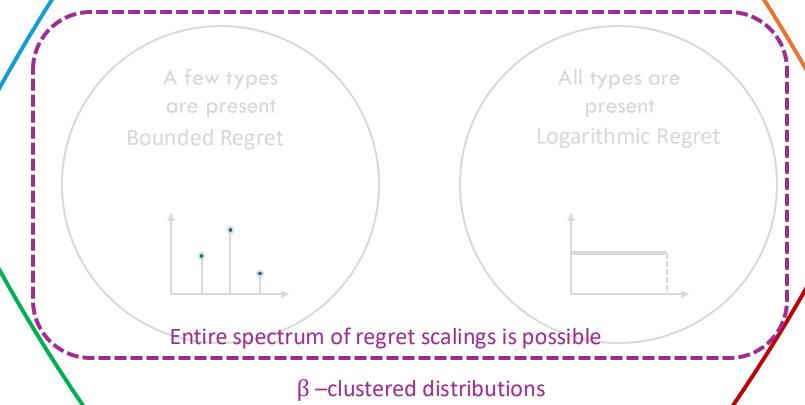
minimize total <u>maximize total</u> matching distance matching reward

Dynamically choose the highest B reward values, given the reward distribution

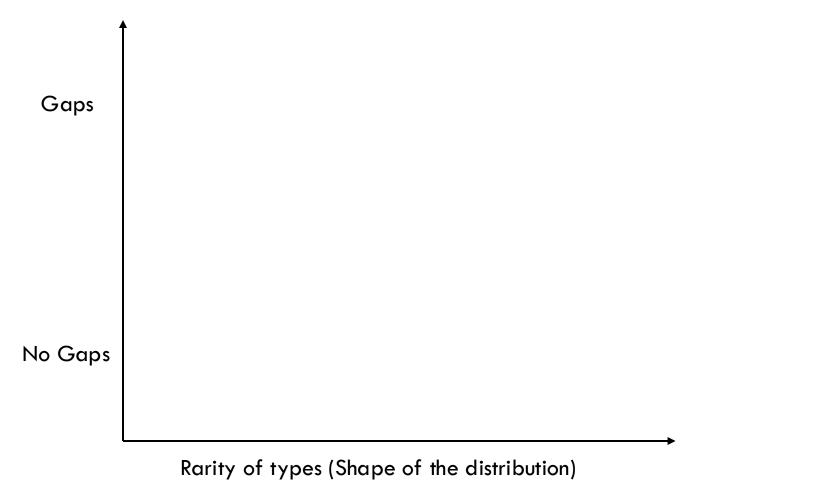
maximize total = r

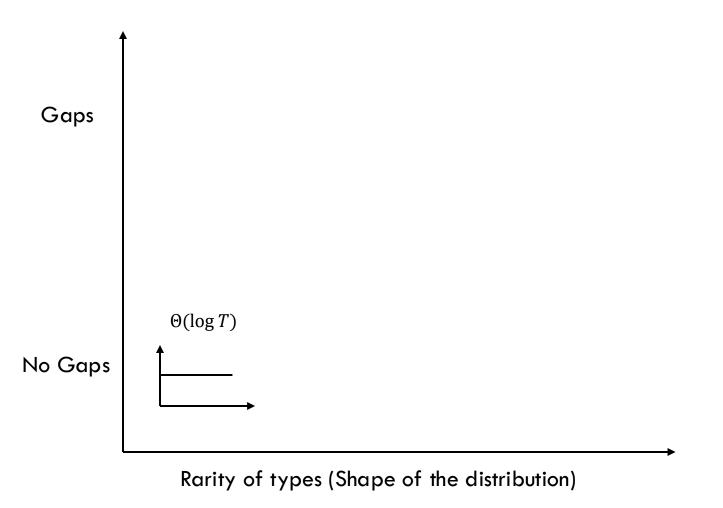


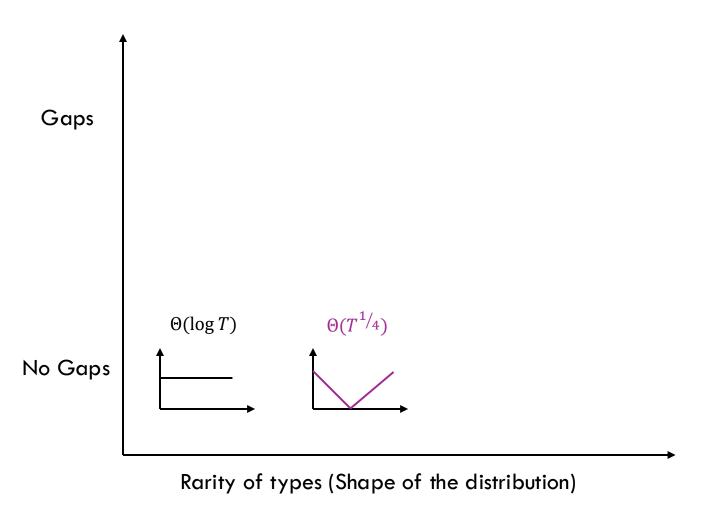
Multi-secretary Problem

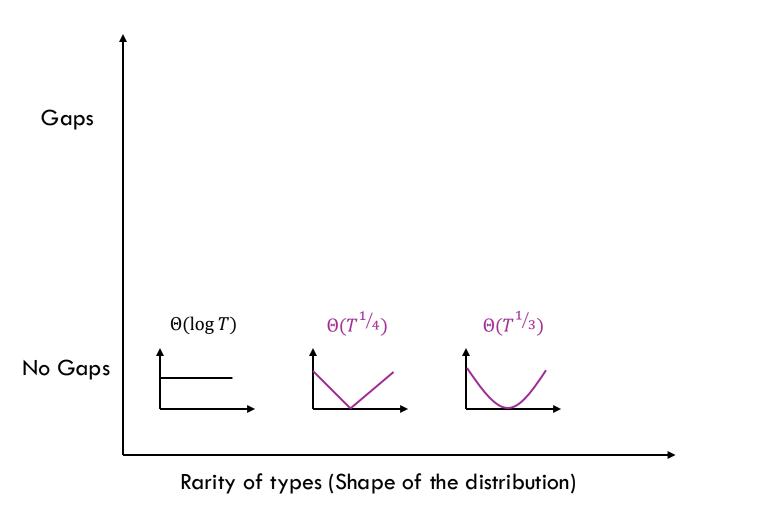


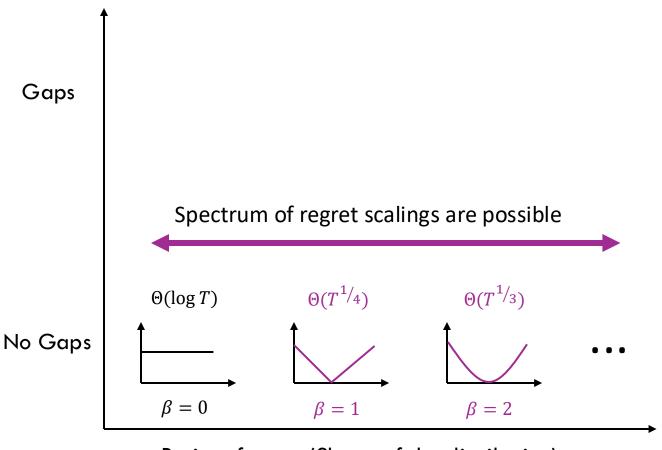
Regret is the additive gap b/w the value of hindsight opt. and value under some algorithm

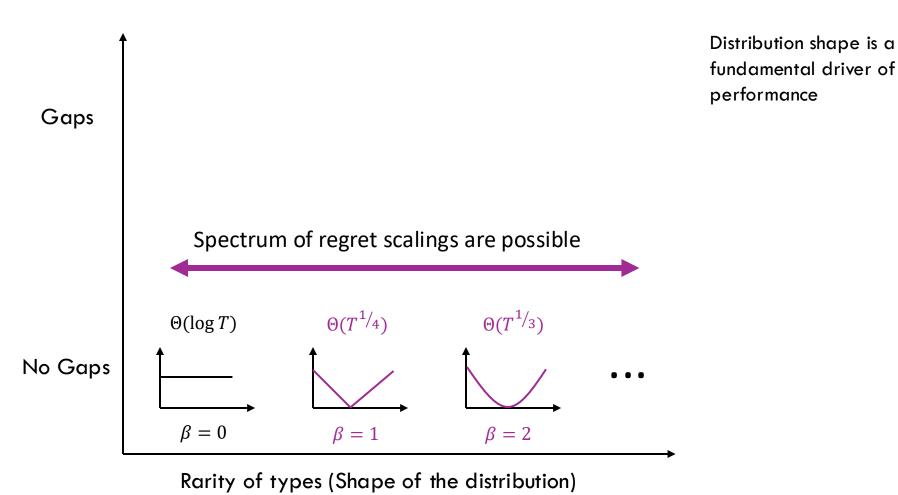


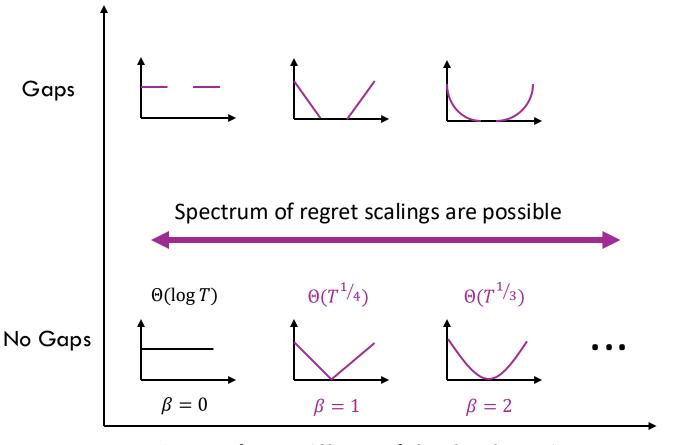




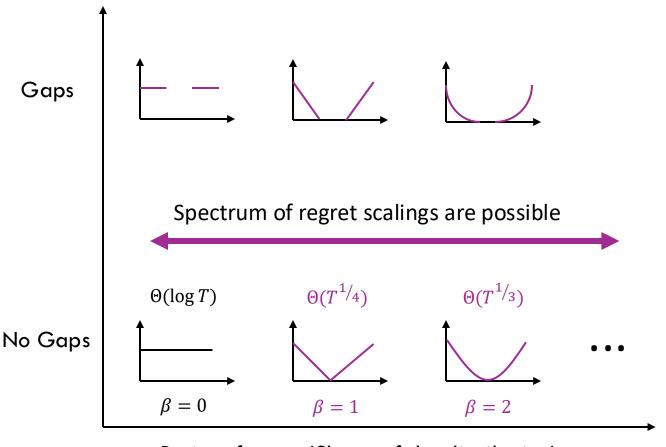






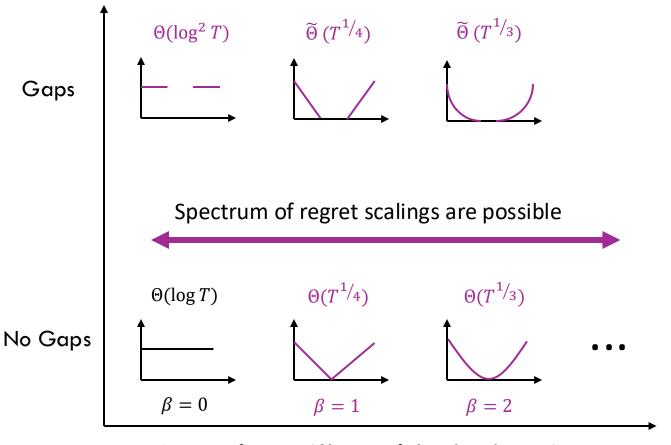


Distribution shape is a fundamental driver of performance



Distribution shape is a fundamental driver of performance

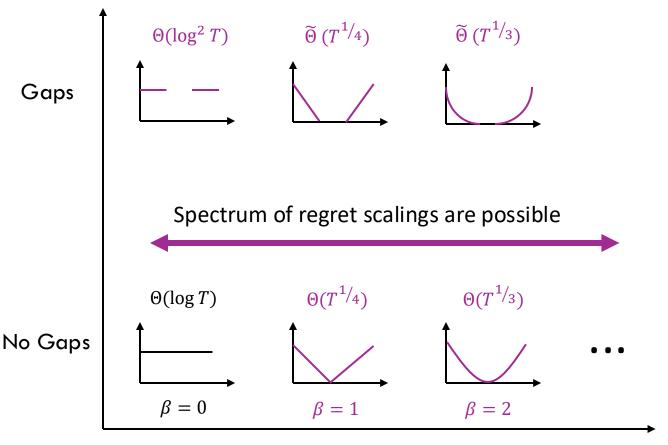
Dealing with gaps in an algorithmic challenge



Distribution shape is a fundamental driver of performance

Dealing with gaps in an algorithmic challenge

Conservativeness with respect to gaps (CwG) principle enables near-optimal performance



Distribution shape is a fundamental driver of performance

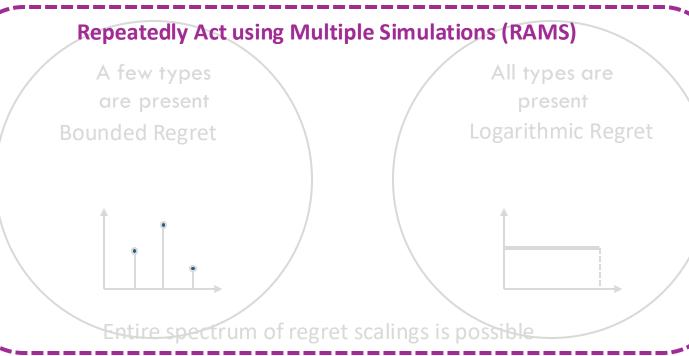
Dealing with gaps in an algorithmic challenge

Conservativeness with respect to gaps (CwG) principle enables near-optimal performance

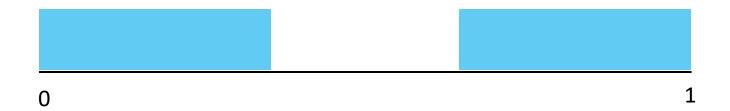
Use RAMS to operationalize CwG

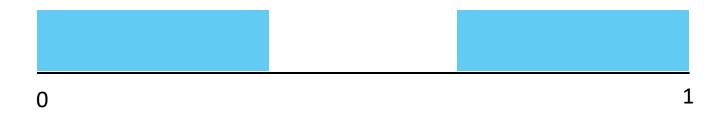
Multi-secretary Problem

one algorithm to solve them all

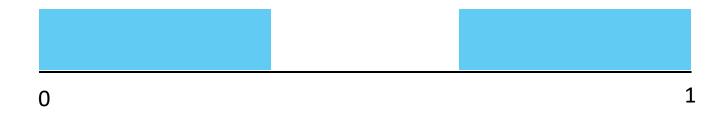


β –clustered distributions



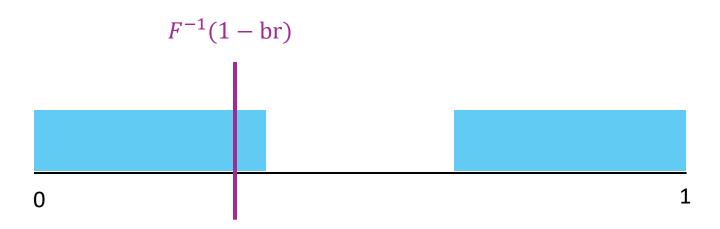


Certainty Equivalent Control computes the budget ratio



Certainty Equivalent Control computes the budget ratio

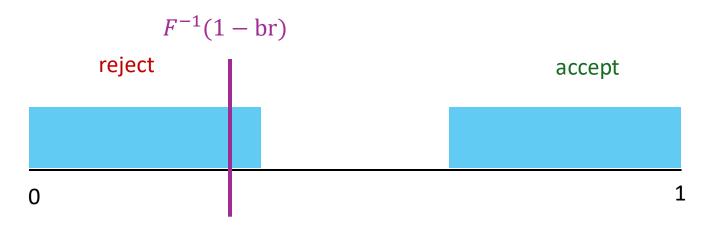
br = Budget Ratio = (Remaining Budget) / (Remaining Time)



Certainty Equivalent Control computes the budget ratio

br = Budget Ratio = (Remaining Budget) / (Remaining Time)

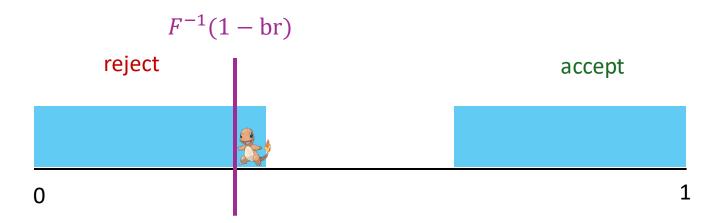
Accept if the request type value is more than $F^{-1}(1-br)$, else reject the request



Certainty Equivalent Control computes the budget ratio

br = Budget Ratio = (Remaining Budget) / (Remaining Time)

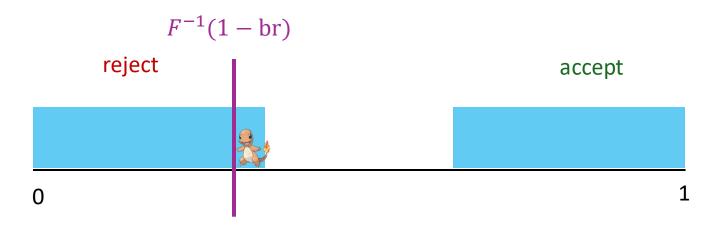
Accept if the request type value is more than $F^{-1}(1-br)$, else reject the request



Certainty Equivalent Control computes the budget ratio

br = Budget Ratio = (Remaining Budget) / (Remaining Time)

Accept if the request type value is more than $F^{-1}(1-br)$, else reject the request

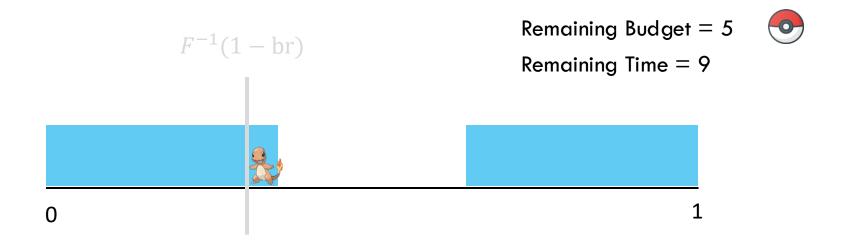


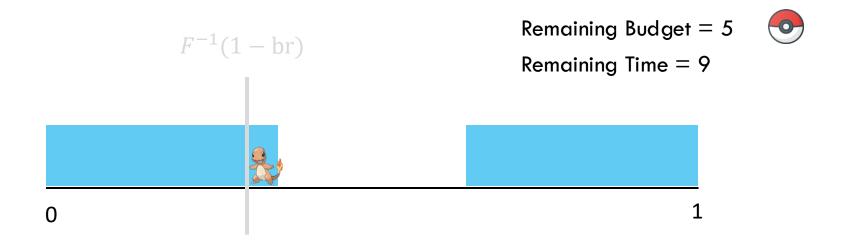
Certainty Equivalent Control computes the budget ratio

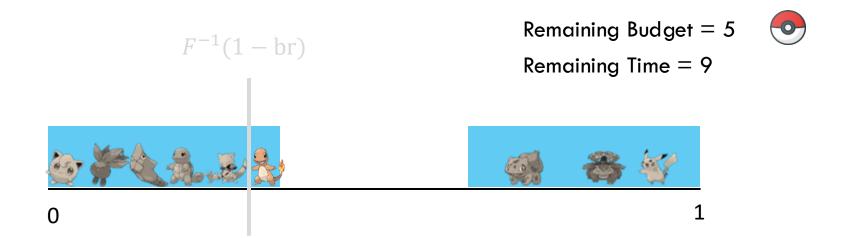
br = Budget Ratio = (Remaining Budget) / (Remaining Time)

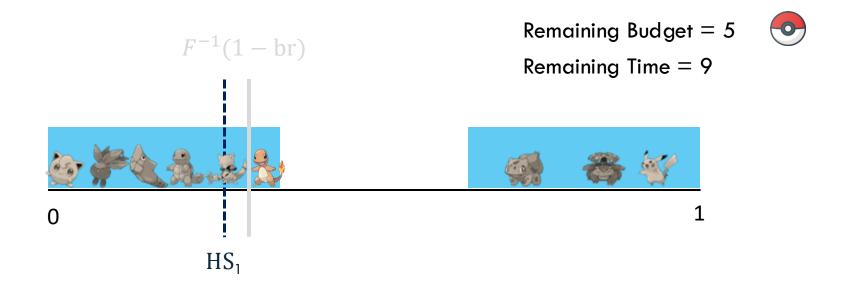
Accept if the request type value is more than $F^{-1}(1-br)$, else reject the request

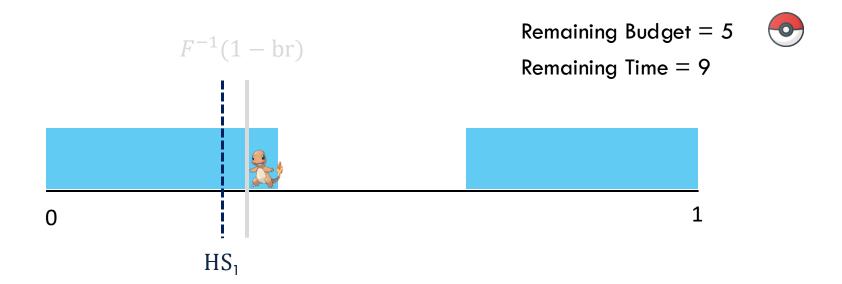
Regret(CE) = $\Omega(\sqrt{T})$ (highly sub-optimal regret scaling)

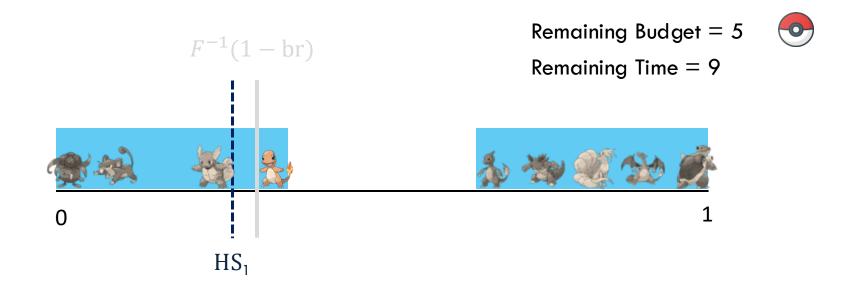


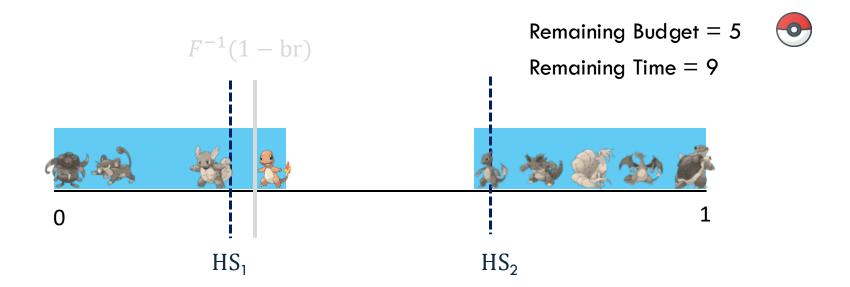


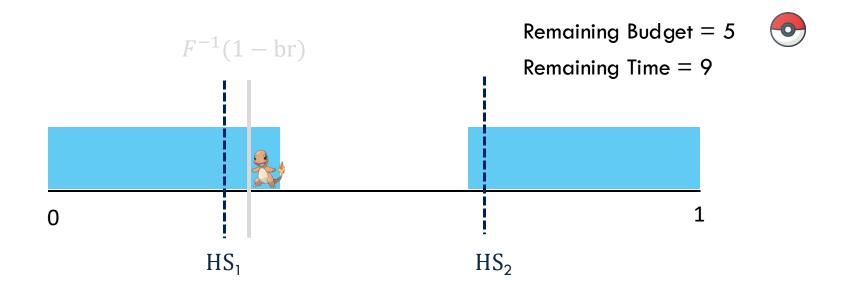


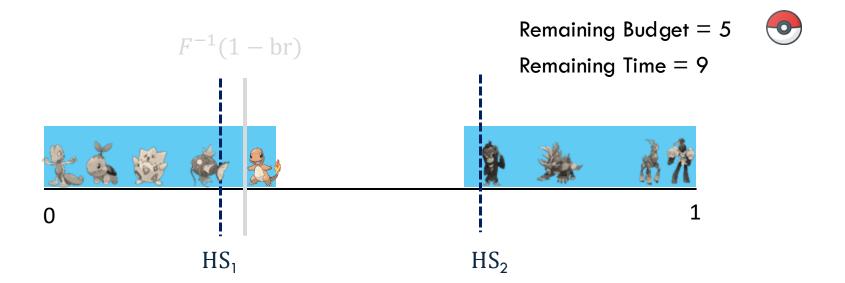


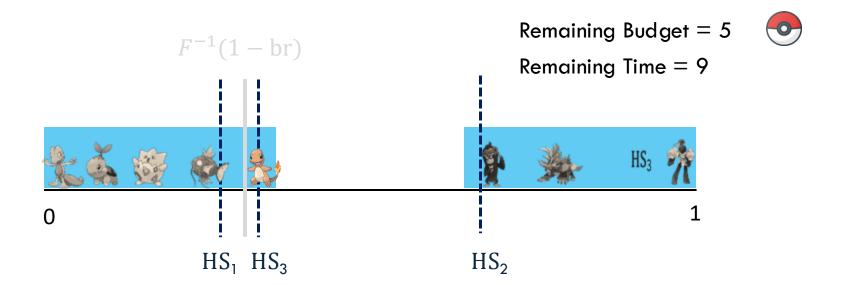


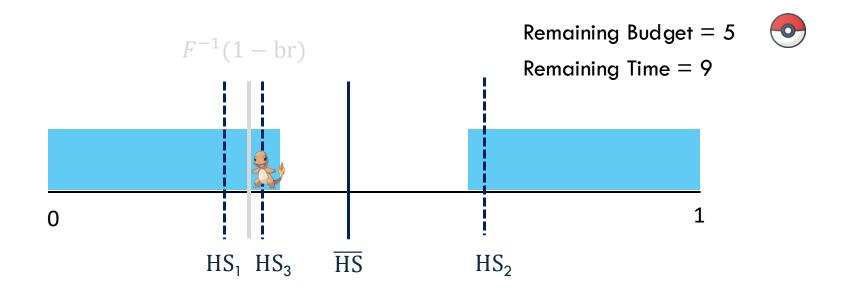


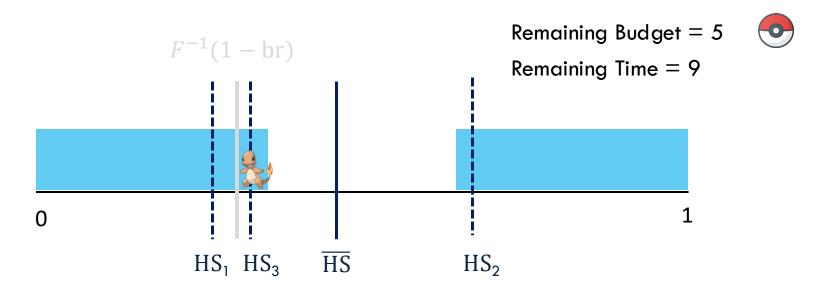




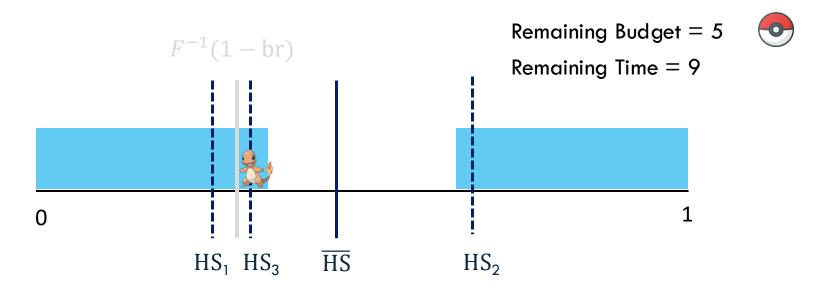






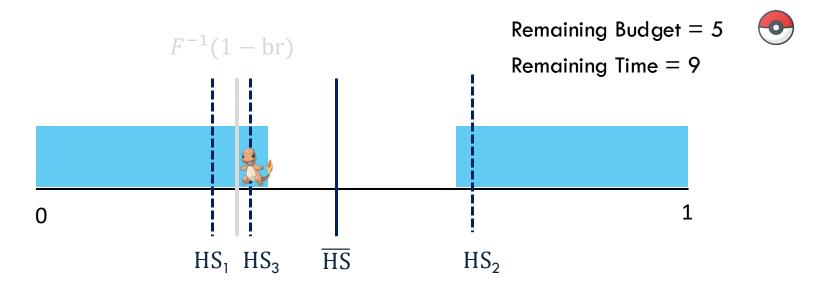


Accept if the request type value is more than \overline{HS} , else reject the request



Accept if the request type value is more than \overline{HS} , else reject the request

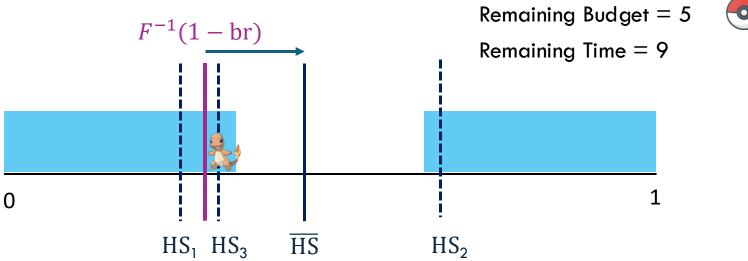
 $Regret(RAMS) = O(\log^2 T)$



Accept if the request type value is more than \overline{HS} , else reject the request

 $Regret(RAMS) = O(\log^2 T)$

Regret(CE) = $\Omega(\sqrt{T})$ (highly sub-optimal regret scaling)

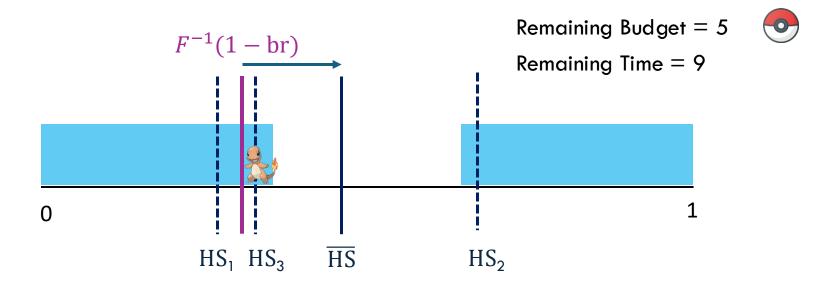


Accept if the request type value is more than HS, else reject the request

Conservativeness with respect to Gaps Principle

If the CE threshold $F^{-1}(1-br)$ is close to a gap, use the gap as the threshold. Otherwise continue using the CE threshold.

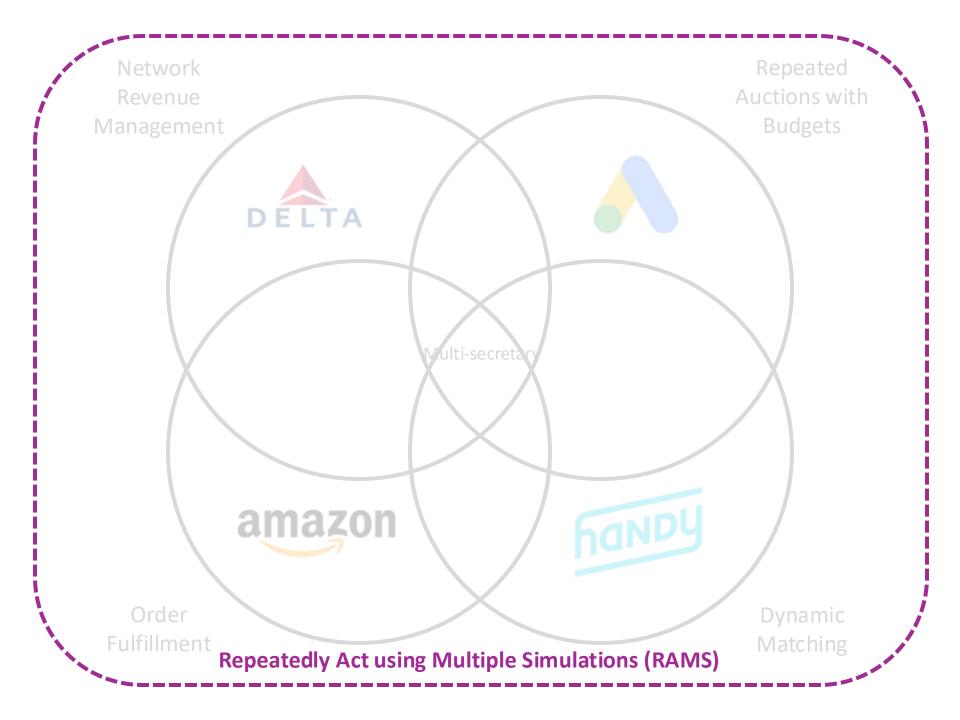




Accept if the request type value is more than \overline{HS} , else reject the request

Connections to "Dual Averaging"

The different HS thresholds are the shadow prices of the budget for different scenarios, the bid price is computed by averaging the HS thresholds



State (Budget) B_t and feasible set of actions A_t

State (Budget) B_t and feasible set of actions A_t



State (Budget) B_t and feasible set of actions A_t

Request $\theta_t = (r_t, c_t)$ arrives at time t



State (Budget) B_t and feasible set of actions A_t

Request $\theta_t = (r_t, c_t)$ arrives at time t

Simulate multiple request scenarios



State (Budget) B_t and feasible set of actions A_t Request $\theta_t = (r_t, c_t)$ arrives at time t

Simulate multiple request scenarios

$$\begin{array}{ll} \theta_t, \theta_{t+1}^{(1)}, \theta_{t+2}^{(1)}, ..., \theta_T^{(1)} & \text{Scenario 1} \\ \theta_t, \theta_{t+1}^{(2)}, \theta_{t+2}^{(2)}, ..., \theta_T^{(2)} & \text{Scenario 2} \\ \vdots & \vdots & \end{array}$$

$$\theta_t, \theta_{t+1}^{(m)}, \theta_{t+2}^{(m)}, \dots, \theta_T^{(m)}$$
 Scenario m



State (Budget) B_t and feasible set of actions A_t

Request $\theta_t = (r_t, c_t)$ arrives at time t

Simulate multiple request scenarios

$$\begin{array}{ll} \theta_t, \theta_{t+1}^{(1)}, \theta_{t+2}^{(1)}, \dots, \theta_T^{(1)} & \text{Scenario 1} \\ \theta_t, \theta_{t+1}^{(2)}, \theta_{t+2}^{(2)}, \dots, \theta_T^{(2)} & \text{Scenario 2} \\ \vdots & \vdots & & \end{array}$$

$$\theta_t, \theta_{t+1}^{(m)}, \theta_{t+2}^{(m)}, \dots, \theta_T^{(m)}$$
 Scenario m

For each scenario k, compute the **compensation** for each action in A_t



State (Budget) B_t and feasible set of actions A_t

Request $\theta_t = (r_t, c_t)$ arrives at time t

Simulate multiple request scenarios

$$\theta_t, \theta_{t+1}^{(1)}, \theta_{t+2}^{(1)}, ..., \theta_T^{(1)}$$
 Scenario 1
 $\theta_t, \theta_{t+1}^{(2)}, \theta_{t+2}^{(2)}, ..., \theta_T^{(2)}$ Scenario 2
:

$$\theta_t, \theta_{t+1}^{(m)}, \theta_{t+2}^{(m)}, \dots, \theta_T^{(m)}$$
 Scenario m

For each scenario k, compute the **compensation** for each action in A_t



Compensation(scenario k, a) =

(Max total reward in scenario k) – (Max total reward in scenario k if action a is taken at time t)

State (Budget) B_t and feasible set of actions A_t

Request $\theta_t = (r_t, c_t)$ arrives at time t

Simulate multiple request scenarios

$$\theta_t, \theta_{t+1}^{(1)}, \theta_{t+2}^{(1)}, ..., \theta_T^{(1)}$$
 Scenario 1 $\theta_t, \theta_{t+1}^{(2)}, \theta_{t+2}^{(2)}, ..., \theta_T^{(2)}$ Scenario 2 :

$$\theta_t, \theta_{t+1}^{(m)}, \theta_{t+2}^{(m)}, \dots, \theta_T^{(m)}$$
 Scenario m

For each scenario k, compute the **compensation** for each action in A_t

Take the **action** with the **minimum compensation** averaged over m scenarios



Compensation(scenario k, a) =

(Max total reward in scenario k) – (Max total reward in scenario k if action a is taken at time t)

State (Budget) B_t and feasible set of actions A_t

Request $\theta_t = (r_t, c_t)$ arrives at time t

Simulate multiple request scenarios

$$\theta_t, \theta_{t+1}^{(1)}, \theta_{t+2}^{(1)}, ..., \theta_T^{(1)}$$
 Scenario 1 $\theta_t, \theta_{t+1}^{(2)}, \theta_{t+2}^{(2)}, ..., \theta_T^{(2)}$ Scenario 2 :

$$\theta_t, \theta_{t+1}^{(m)}, \theta_{t+2}^{(m)}, \dots, \theta_T^{(m)}$$
 Scenario m

For each scenario k, compute the **compensation** for each action in A_t

Take the **action** with the **minimum compensation** averaged over *m* scenarios

Repeat the process



Compensation(scenario k, a) =

(Max total reward in scenario k) – (Max total reward in scenario k if action a is taken at time t)

RAMS minimizes hindsight-based regret

RAMS minimizes hindsight-based regret

Informal Meta Theorem [RAMS inherits guarantees of near-optimal algos].

Given a dynamic resource allocation setting, if there exists an algorithm **ALG** satisfying certain technical conditions, then

Regret(RAMS) ≤ Regret Upper Bound of **ALG** + Sampling Error

RAMS minimizes hindsight-based regret

Informal Meta Theorem [RAMS inherits guarantees of near-optimal algos].

Given a dynamic resource allocation setting, if there exists an algorithm **ALG** satisfying certain technical conditions, then

Regret(RAMS) ≤ Regret Upper Bound of **ALG** + Sampling Error

Proof of the Informal Meta Theorem.

$$\operatorname{Regret}(\operatorname{RAMS}) = \sum_{t=1}^{T} \mathbb{E}[\operatorname{Comp}_{t}(a_{t}^{\operatorname{RAMS}})] \leq \sum_{t=1}^{T} \mathbb{E}[\operatorname{Comp}_{t}(a_{t}^{\operatorname{ALG}})]$$

Compensated Coupling or Performance Diff. Lemma

RAMS chooses the action with the minimum compensation

RAMS is on-par with SOTA

RAMS is on-par with SOTA

Corollary of the Meta Theorem.

Polynomial regret for multi-secretary problem under different type distributions [this work]

Bounded regret for Network Revenue Management and Online Matching for a **few types** [Vera and Banerjee '21]

Logarithmic regret for Network Revenue Management with many types and nondegeneracy assumps. [Bray '23]

Log-Squared regret for Network Revenue Management with many types and w/o nondegeneracy assumps. [Jiang et. al '22]

RAMS is on-par with SOTA

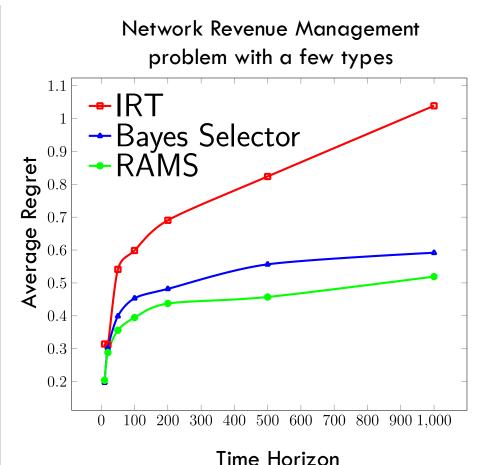
Corollary of the Meta Theorem

Polynomial regret for multi-secretary problem under different type distributions [this work]

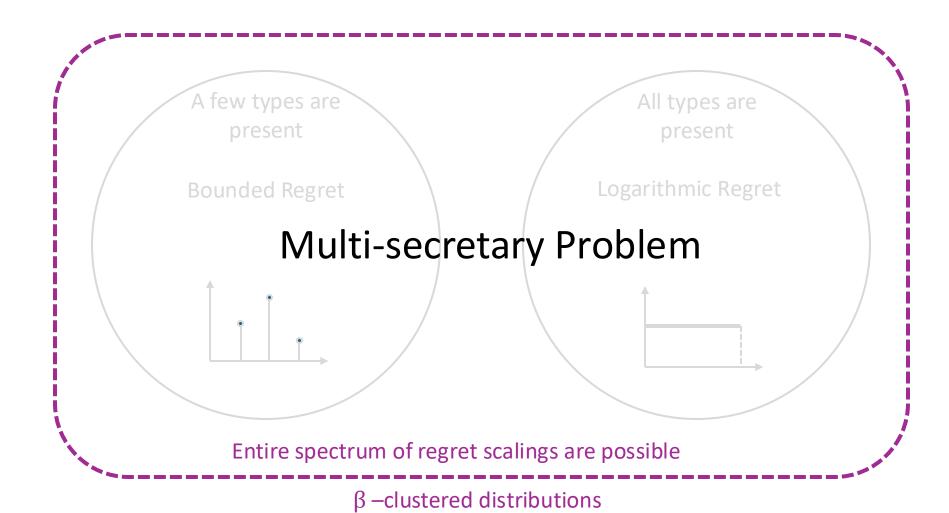
Bounded regret for Network Revenue Management and Online Matching for a **few types** [Vera and Banerjee '21]

Logarithmic regret for Network Revenue Management with many types and non-degeneracy assumps. [Bray '23]

Log-Squared regret for Network Revenue Management with many types and w/o nondegeneracy assumps. [Jiang et. al '22]

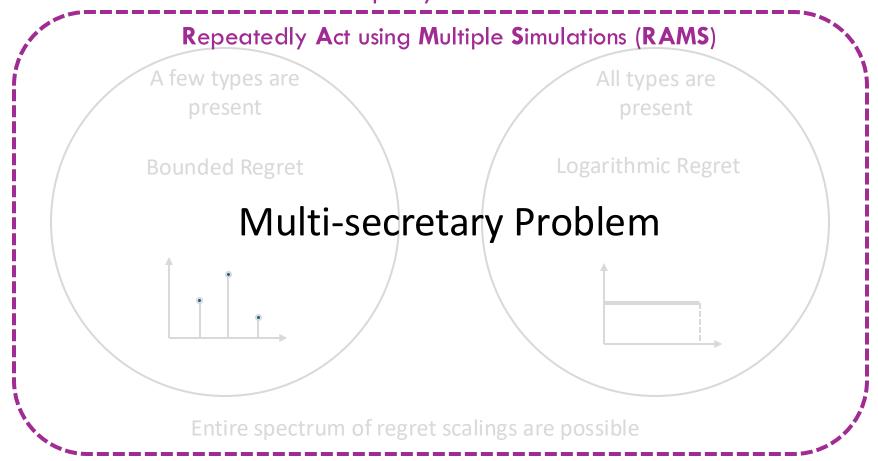


What is the interplay between the distribution of request types and achievable algorithmic performance?

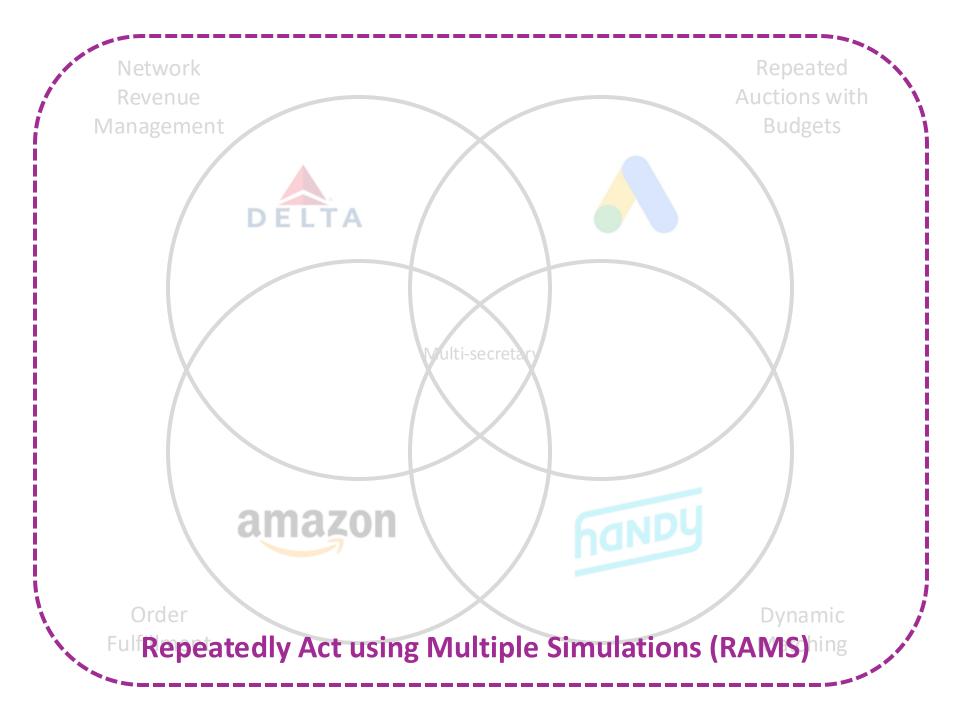


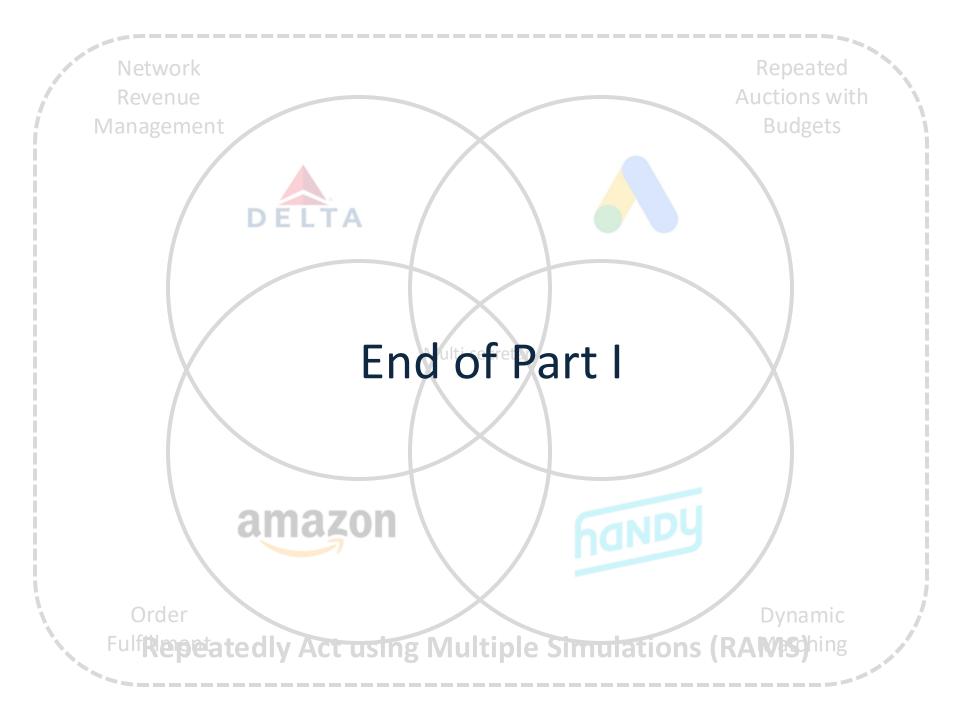
Can we design a **unified**, **simple** and **near-optimal** algorithms which works for all type distributions?

one policy to solve them all



 β –clustered distributions

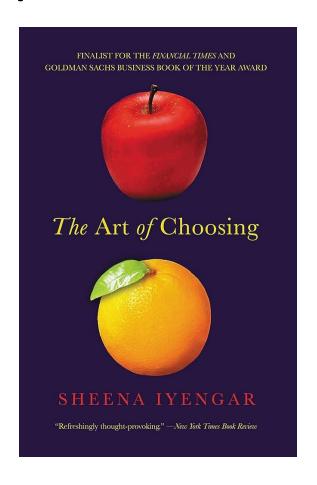




Part II

The Impact of Rankings and Personalized Recommendations in Marketplaces

People have ill-formed preferences

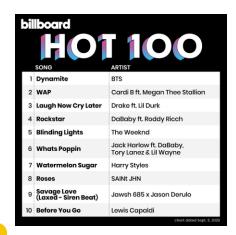




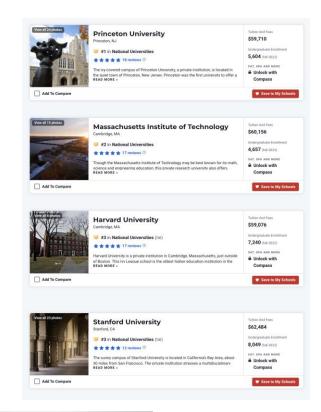
"Majority of Americans would pursue a different degree, institution or major if given a chance to do it all over again"

Rankings offer a plausible solution to aid decision-making





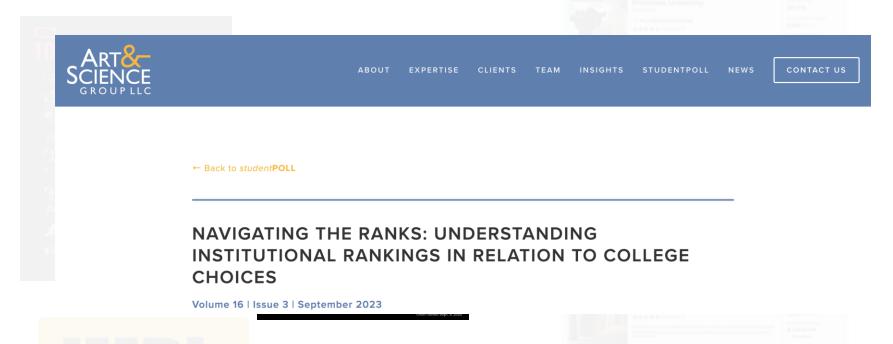








Rankings offer a plausible solution to aid decision-making



58% of high school seniors actively considered college rankings in their decision-making process

Rankings are not personalized to individual tastes







See the rating distribution

A new overall rating chart shows you a breakdown of a home's 1 to 5 star reviews.

Sort by recency and rating

Now you can sort reviews to read the latest and greatest from prior guests.

Find relevant reviews

New details, like type of trip or length of stay, make it simple to pick out relevant reviews.

Significant work on personalized recos in uncapacitated settings







See the rating distribution

A new overall rating chart shows you a breakdown of a home's 1 to 5 star reviews.

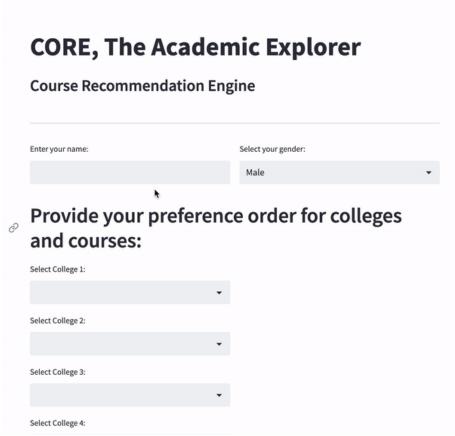
Sort by recency and rating

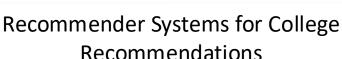
Now you can sort reviews to read the latest and greatest from prior guests.

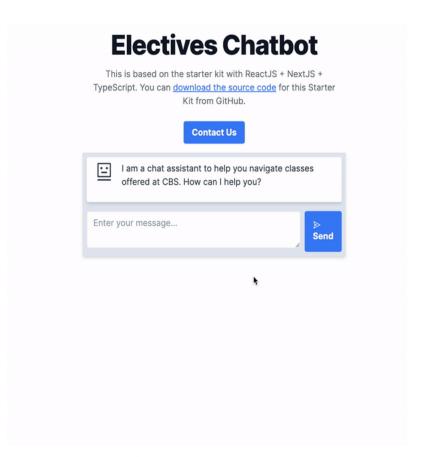
Find relevant reviews

New details, like type of trip or length of stay, make it simple to pick out relevant reviews.

My vision for the future







Chatbots for Course Recommendations

Research Questions

What are the implications of providing personalized recommendations in marketplaces with capacity constraints?

How does this contrast with marketplaces with no capacity constraints?

In a Nutshell



In a Nutshell



We study a prototypical utility model which comprises of public and private utility components



In a Nutshell



We study a prototypical utility model which comprises of public and private utility components



Identify subtle interplay of personalized recommendations and capacity constraints on social welfare



In a Nutshell



We study a prototypical utility model which comprises of public and private utility components



Identify subtle interplay of personalized recommendations and capacity constraints on social welfare



Personalized recos may have *little to no* benefit in markets with uncapacitated supply (think movies, songs)



In a Nutshell



We study a prototypical utility model which comprises of public and private utility components



Identify subtle interplay of personalized recommendations and capacity constraints on social welfare



Personalized recos may have *little to no* benefit in markets with uncapacitated supply (think movies, songs)



Personalized recos unlock *significant* social welfare in capacitated supply settings (think AirBnB, college admits)



Model

n individuals



n items



$$u(2, 6) = \rho \times 6 + (1 - \rho) \times 6$$

Common term

Depends only on



Private term

Depends on both



Known through publicly available rankings

Apriori unknown, can be known using personalized recos

Model

n individuals







$$= \rho$$



$$+(1-\rho)\times$$

Uncapacitated supply setting



Many-to-One matching

Individuals simply choose the item they want to consume

Capacitated supply setting

One-to-One matching

Individuals are matched by centralized clearinghouse using some matching mechanism

Model

n individuals



n items

$$u(2, 6) = \rho \times 6 + (1 - \rho) \times 6$$

Uncapacitated supply setting

Capacitated supply setting

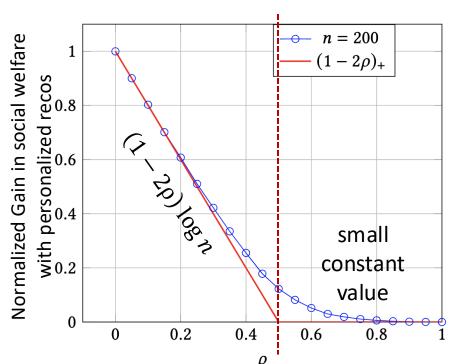
We care about the social welfare which is the average (across the n individuals) utility obtained by the individuals

Results

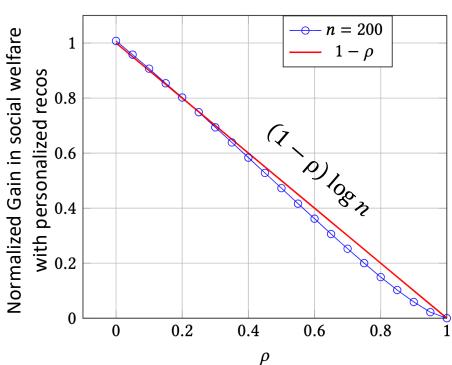
Common term is Exponentially distributed with rate 1

Private idiosyncratic term is Exponentially distributed with rate 1

Uncapacitated Supply Setting



Capacitated Supply Setting







We study prototypical models of dynamic resource allocation problem (focus on the multisecretary prob)





We study prototypical models of dynamic resource allocation problem (focus on the multisecretary prob)



Shape of the reward distribution is a fundamental driver of algorithmic performance





We study prototypical models of dynamic resource allocation problem (focus on the multisecretary prob)



Shape of the reward distribution is a fundamental driver of algorithmic performance



Workhorse policies can be highly suboptimal and nearoptimal algorithms are overly specified





We study prototypical models of dynamic resource allocation problem (focus on the multisecretary prob)



Shape of the reward distribution is a fundamental driver of algorithmic performance



Workhorse policies can be highly suboptimal and nearoptimal algorithms are overly specified



Design a simple and near-optimal policy called Repeatedly Act using Multiple Simulations (RAMS)







We study a prototypical utility model which comprises of public and private utility components





We study a prototypical utility model which comprises of public and private utility components



Identify subtle interplay of personalized recommendations and capacity constraints on social welfare





We study a prototypical utility model which comprises of public and private utility components



Identify subtle interplay of personalized recommendations and capacity constraints on social welfare



Personalized recos may have *little to no* benefit in markets with uncapacitated supply (think movies, songs)





We study a prototypical utility model which comprises of public and private utility components



Identify subtle interplay of personalized recommendations and capacity constraints on social welfare



Personalized recos may have *little to no* benefit in markets with uncapacitated supply (think movies, songs)



Personalized recos unlock *significant* social welfare in capacitated supply settings (think AirBnB, college admits)



So long and Thanks for all the fish