Coded Slotted ALOHA with Power Control and its Application to Realistic Channel Models

Akshit Kumar Parikshit Hegde Amira Alloum
Cedric Adjih Rahul Vaze

Internship Talk

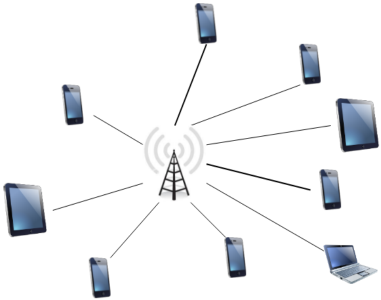
NOKIA Bell Labs
Indian Institute of Technology Madras
Inria inventeurs du monde numérique
tifr

Acknowledgements

- Amira
- Cedric
- Rahul
- Lou
- Maths team

Motivation

Scenario: IOT Networks



Large number of random devices
$\Rightarrow$ Uncoordinated Algos

Small Packets
Low Communication Overheads
$\Rightarrow$ Open Loop Algos

Connectionless Protocols

Motivation

Scenario: IOT Networks

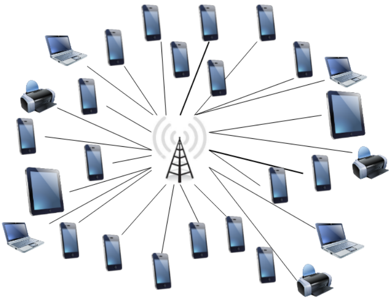
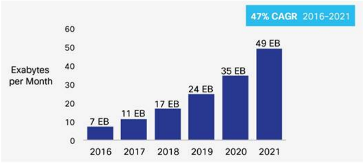


Figure 2. Cisco Forecasts 49 Exabytes per Month of Mobile Data Traffic by 2021



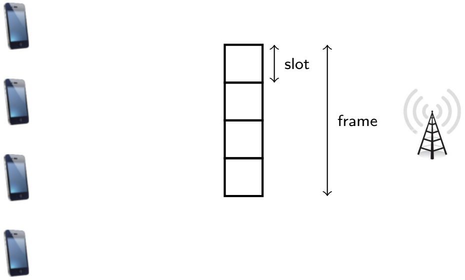
Source: Cisco VNI Mobile, 2017

Exponential increase in the number of devices!

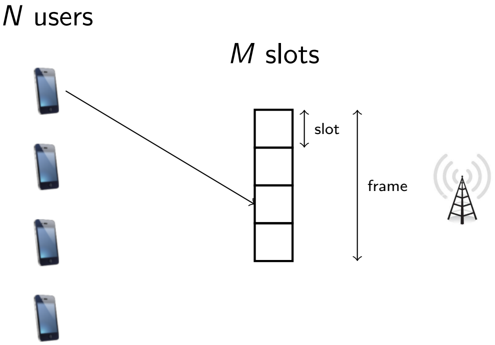
Problem Setup(Slotted ALOHA)

N users

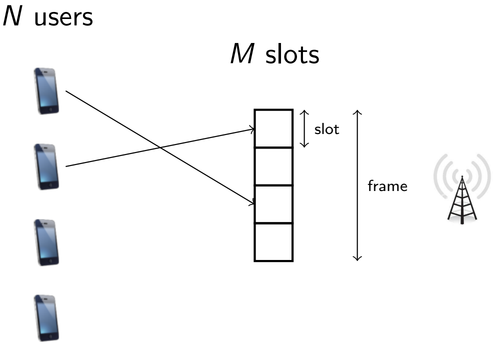
M slots



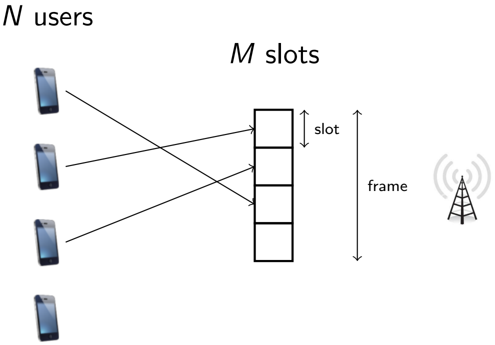
Problem Setup(Slotted ALOHA)



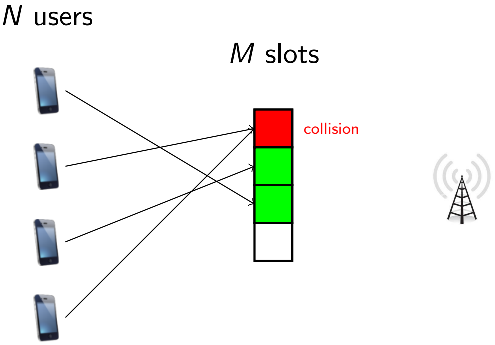
Problem Setup(Slotted ALOHA)



Problem Setup(Slotted ALOHA)



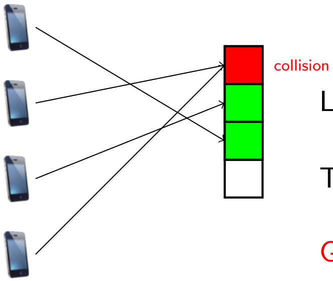
Problem Setup(Slotted ALOHA)



Problem Setup(Slotted ALOHA)

N users

M slots



Load: $g = \frac{N}{M} \quad (= \frac{4}{4})$

Throughput$= \frac{\text{Avg. no. of recovered packets}}{M}$

Goal: Maximize Throughput
(Apply load accordingly)

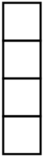
For SA: max Throughput $=$ 0.37, when g=1

Problem Setup: Coded Slotted ALOHA

[Casini et. al., '07][Liva '11]

N users

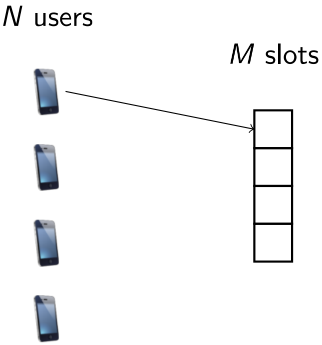
M slots



repetition	1	2
Λ	0.5	0.5

Problem Setup: Coded Slotted ALOHA

[Casini et. al., '07][Liva '11]



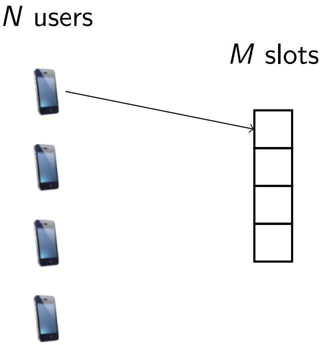
repetition	1	2
Λ	0.5	0.5

Sample

1

Problem Setup: Coded Slotted ALOHA

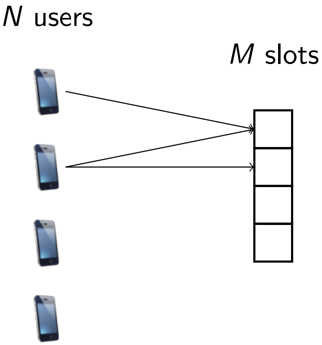
[Casini et. al., '07][Liva '11]



repetition	1	2
Λ	0.5	0.5

Problem Setup: Coded Slotted ALOHA

[Casini et. al., '07][Liva '11]



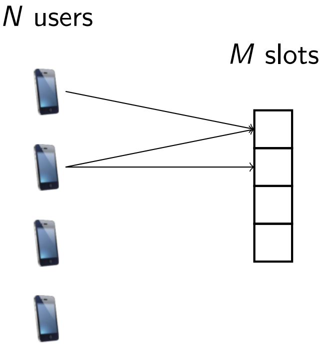
repetition	1	2
Λ	0.5	0.5

Sample

2

Problem Setup: Coded Slotted ALOHA

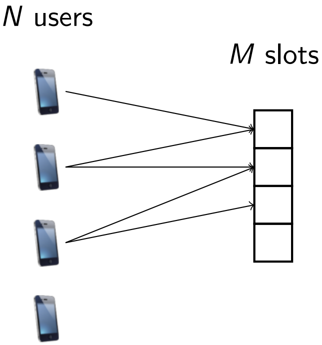
[Casini et. al., '07][Liva '11]



repetition	1	2
Λ	0.5	0.5

Problem Setup: Coded Slotted ALOHA

[Casini et. al., '07][Liva '11]



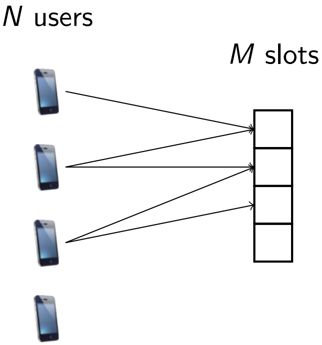
repetition	1	2
Λ	0.5	0.5

Sample

2

Problem Setup: Coded Slotted ALOHA

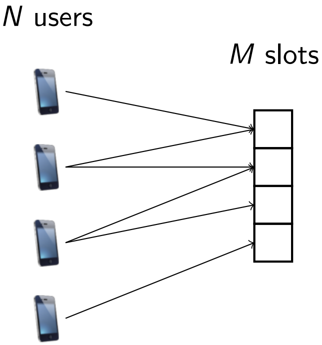
[Casini et. al., '07][Liva '11]



repetition	1	2
Λ	0.5	0.5

Problem Setup: Coded Slotted ALOHA

[Casini et. al., '07][Liva '11]



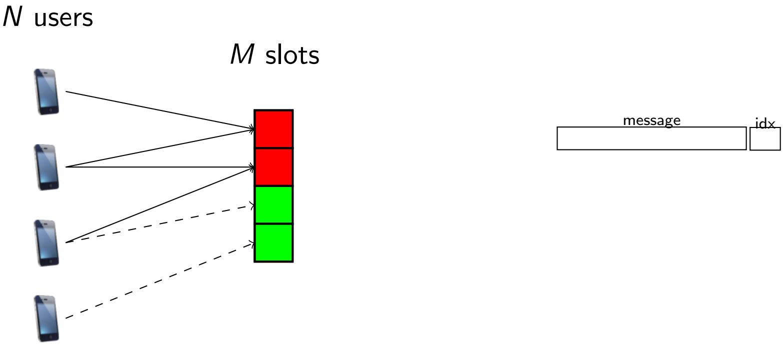
repetition	1	2
Λ	0.5	0.5

Sample

1

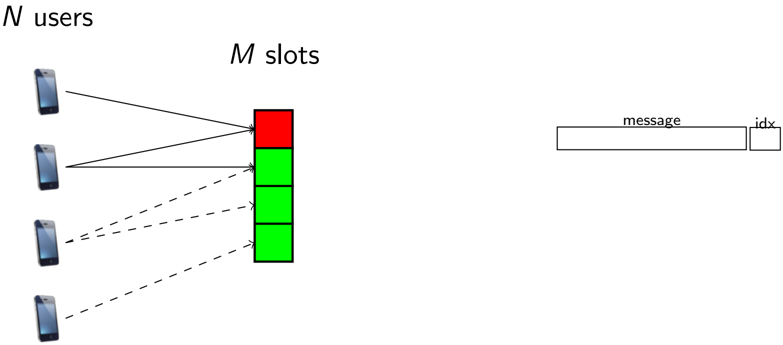
Problem Setup: Coded Slotted ALOHA

[Casini et. al., '07][Liva '11]



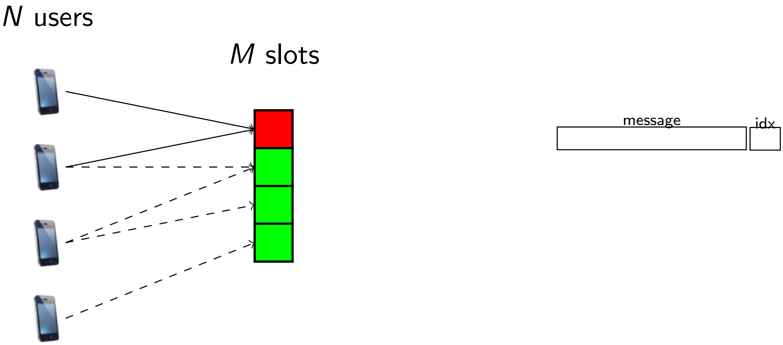
Problem Setup: Coded Slotted ALOHA

[Casini et. al., '07][Liva '11]



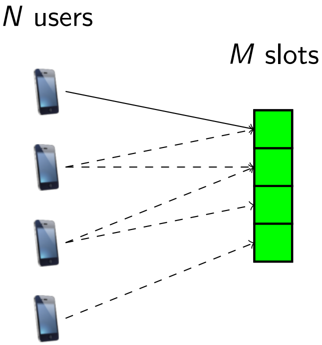
Problem Setup: Coded Slotted ALOHA

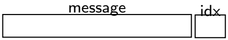
[Casini et. al., '07][Liva '11]



Problem Setup: Coded Slotted ALOHA

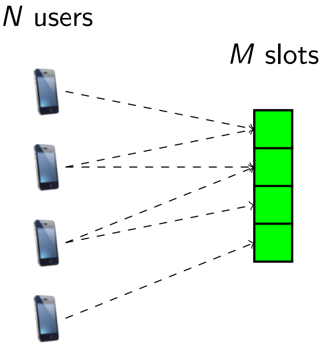
[Casini et. al., '07][Liva '11]

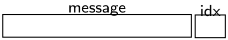




Problem Setup: Coded Slotted ALOHA

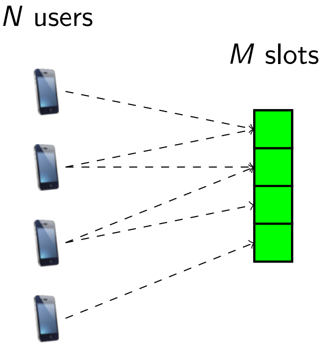
[Casini et. al., '07][Liva '11]





Problem Setup: Coded Slotted ALOHA

[Casini et. al., '07][Liva '11]

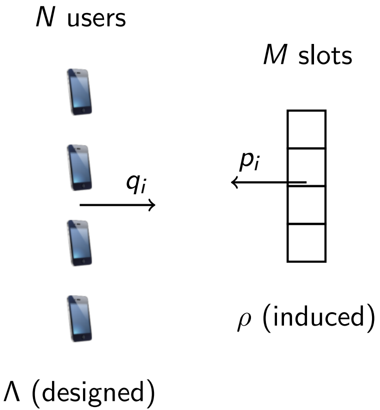


"Inter-Slot SIC"
(Successive Interference Cancellation)

Iterative Algorithm

Bipartite Graph of LDPC Codes
Peeling Decoder over Erasure Channel

Analysis of CSA: Density Evolution



Asymptotic Analysis: $N, M \to \infty$

At iteration i

q_i: prob that packet replica is associated to unknown user

(given p_i from other slots of the packet)

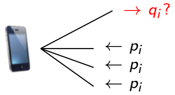
p_i: prob of packet still in collision in its slot

(given q_{i-1} from other packets associated to the slot)

Analysis of CSA: Density Evolution

q_i: prob that packet replica is associated to unknown user

(given p_i from other slots of the packet)



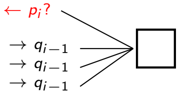
$$q_i = p_i^3$$

$$\lambda(x) = \lambda_1 + \lambda_2 x + \lambda_3 x^2 \ldots \lambda_{d_{max}} x^{d_{max}-1}$$

$$q_i = \lambda(p_i)$$

p_i: prob of packet still in collision in its slot

(given q_{i-1} from other packets associated to the slot)



$$1 - p_i = (1 - q_{i-1})^3$$

$$\rho(x) = \rho_1 + \rho_2 x + \rho_3 x^2 \ldots \rho_N x^{N-1}$$

$$p_i = 1 - \rho(1 - q_{i-1})$$

Analysis of CSA: Density Evolution

$q_{i-1} \to p_i \to q_i$

$q_{i-1} \to q_i = f_{DE}(q_{i-1})$

$\lambda(x) = \lambda_1 + \lambda_2 x + \lambda_3 x^2 \ldots \lambda_{d_{max}} x^{d_{max}-1}$

$q_i = \lambda(p_i)$

$q_0 = 1$

$q_0 \to q_1 \to \ldots q_\infty \to p_\infty$

Throughput $= g(1 - \Lambda(p_\infty))$

Liva '11: Throughput $= 0.97$
$\Lambda(x) = 0.5x^2 + 0.28x^3 + 0.22x^8$

Narayanan, Pfister '12: Soliton Distribution is optimal

Throughput $\to 1$

$\rho(x) = \rho_1 + \rho_2 x + \rho_3 x^2 \ldots \rho_N x^{N-1}$

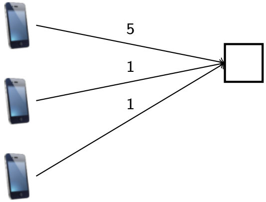
$p_i = 1 - \rho(1 - q_{i-1})$

Capture using Power Control

$SIR_1 = \frac{5}{2}$

$SIR_2 = \frac{1}{6}$

$SIR_3 = \frac{1}{6}$



If $SIR > \beta$

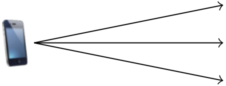
$\Downarrow$

Packet recovered in Physical Layer

Say, $\beta = 1$

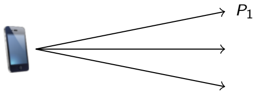
"Intra-Slot SIC"

CSA with Power Control



Power	P_1	P_2
δ	0.5	0.5

CSA with Power Control

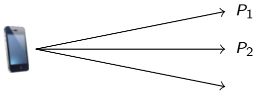


Power	P_1	P_2
δ	0.5	0.5

$\downarrow$ Sample

P_1

CSA with Power Control

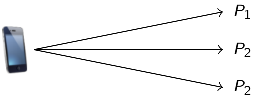


Power	P_1	P_2
δ	0.5	0.5

$\downarrow$ Sample

P_2

CSA with Power Control

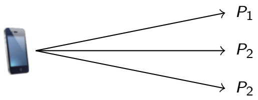


Power	P_1	P_2
δ	0.5	0.5

$$\Big\downarrow \text{Sample}$$

$$P_2$$

CSA with Power Control



Power	P_1	P_2	$\ldots$	P_n
δ	δ_1	δ_2	$\ldots$	δ_n

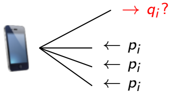
Required: $P_i > 5\beta P_{i+1}$ (for theoretical tractability)

Density Evolution Analysis

q_i: prob that packet replica is associated to unknown user

(given p_i from other slots of the packet)

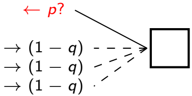
$$q_i = p_i^3$$



p_i: prob of packet still in collision in its slot

(given q_{i-1} from other packets associated to the slot)

$$1 - p = (1 - q)^3$$

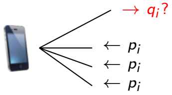


Density Evolution Analysis

q_i: prob that packet replica is associated to unknown user

(given p_i from other slots of the packet)

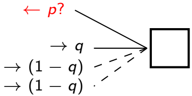
$$q_i = p_i^3$$



p_i: prob of packet still in collision in its slot

(given q_{i-1} from other packets associated to the slot)

$$1 - p = (1-q)^3 + \binom{3}{1} q(1-q)^2 \delta_1 \delta_2 + \binom{3}{1} q(1-q)^2 \delta_2 \delta_1$$

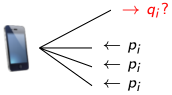


Density Evolution Analysis

q_i: prob that packet replica is associated to unknown user

(given p_i from other slots of the packet)

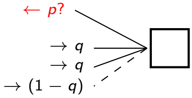
$$q_i = p_i^3$$



p_i: prob of packet still in collision in its slot

(given q_{i-1} from other packets associated to the slot)

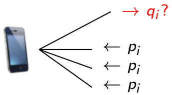
$$1 - p = (1-q)^3 + \binom{3}{1} q(1-q)^2 \delta_1 \delta_2 + \binom{3}{1} q(1-q)^2 \delta_2 \delta_1 + \binom{3}{2} q^2 (1-q) \delta_1 \delta_2^2$$



Density Evolution Analysis

q_i: prob that packet replica is associated to unknown user

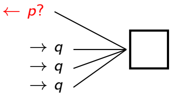
(given p_i from other slots of the packet)



$$q_i = p_i^3$$

p_i: prob of packet still in collision in its slot

(given q_{i-1} from other packets associated to the slot)



$$\begin{aligned} 1 - p = (1-q)^3 & \\ & + \binom{3}{1} q(1-q)^2 \delta_1 \delta_2 + \binom{3}{1} q(1-q)^2 \delta_2 \delta_1 \\ & + \binom{3}{2} q^2 (1-q) \delta_1 \delta_2^2 \\ & + q^3 \delta_1 \delta_2^3 \end{aligned}$$

Density Evolution Analysis

Power	P_1	P_2
δ	δ_1	δ_2

$$q = \lambda(p)$$

$$p = 1 - \rho_1 - \sum_{l=2}^{N} \rho_l \left((1-q)^{l-1} + \sum_{t=1}^{\min\{5,l-1\}} \delta_1 \delta_2^t \binom{l-1}{t} q^t (1-q)^{l-1-t} + (l-1)\,\delta_1 \delta_2 q (1-q)^{l-2} \right)$$

$$q_0 = 1$$
$$q_{i+1} = f_{DE}(q_i)$$
$$q_0 \to q_1 \to \ldots \to q_\infty \to p_\infty$$
$$\text{Throughput} = g(1 - \Lambda(p_\infty))$$

Results

- $\delta_1 = 0.4, \delta_2 = 0.6$
- $\Lambda(x) = 0.5x^2 + 0.28x^3 + 0.22x^8$
- $M = 1000$

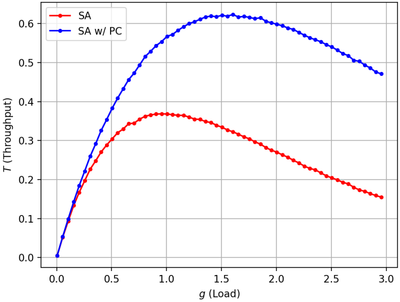


Figure: Slotted Aloha

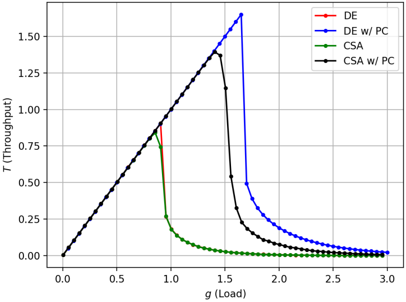


Figure: Coded Slotted Aloha

Results

- $\Lambda(x) = 0.5x^2 + 0.28x^3 + 0.22x^8$
- $M = 1000$

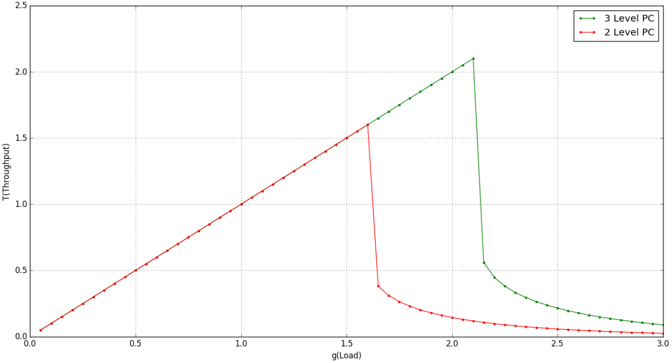


Figure: Comparison of 2 and 3 level Power Control

Realistic Channel Model : Path Loss Model

- [Khalegi et al., 2017] studied the capture effect with Path Loss Model.

Realistic Channel Model : Path Loss Model

- [Khalegi et al., 2017] studied the capture effect with Path Loss Model.
- Exact density evolution (DE) eqn. formulation is difficult.

Realistic Channel Model : Path Loss Model

- [Khalegi et al., 2017] studied the capture effect with Path Loss Model.
- Exact density evolution (DE) eqn. formulation is difficult.
- Gave a DE formulation with coefficients derived using Monte-carlo simulation.

Realistic Channel Model : Path Loss Model

- [Khalegi et al., 2017] studied the capture effect with Path Loss Model.
- Exact density evolution (DE) eqn. formulation is difficult.
- Gave a DE formulation with coefficients derived using Monte-carlo simulation.
- No explicit DE eqn. $\implies$ Optimizing Λ-distribution is difficult

Path Loss Model

- P_r : Power received by the base station
- P_t : Power transmitted by the user
- $r_{\max}$: Maximum distance of the user from the base station

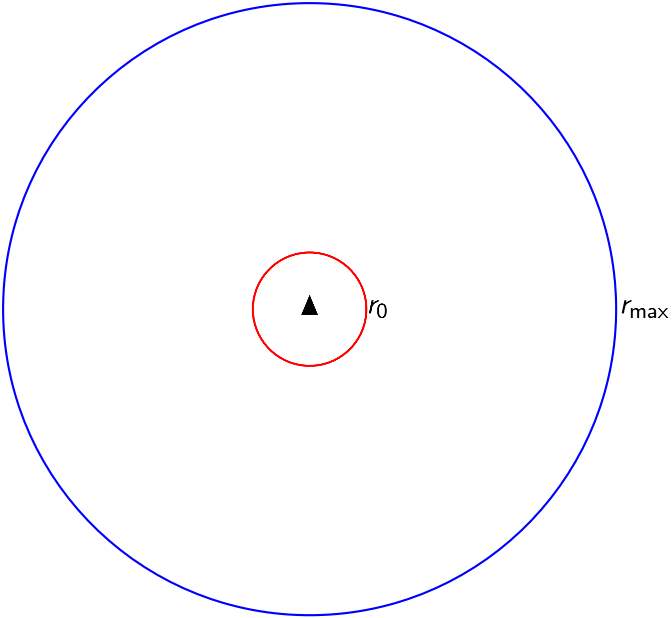
Path Loss Model

- P_r : Power received by the base station
- P_t : Power transmitted by the user
- $r_{\max}$: Maximum distance of the user from the base station

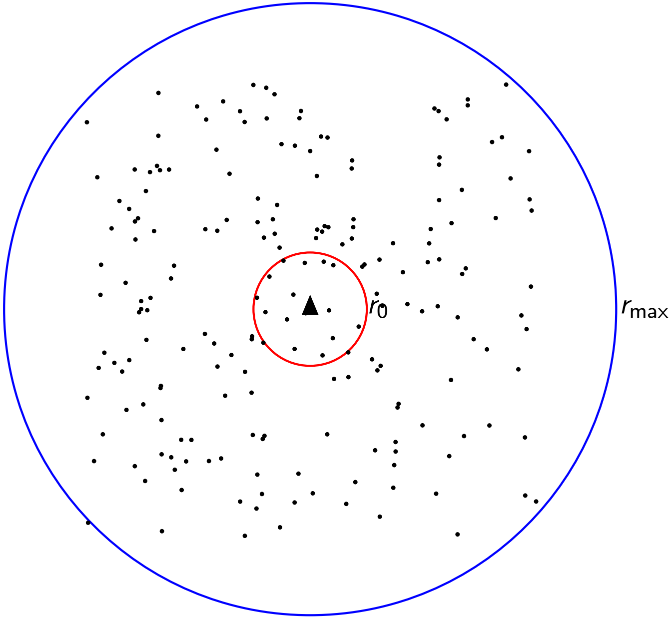
$$P_r = \begin{cases} P_t, & \text{if } r \leq r_0 \\ P_t \left(\frac{r}{r_0}\right)^{\gamma}, & \text{if } r_0 < r \leq r_{\max} \end{cases}$$

where γ is the path loss exponent.

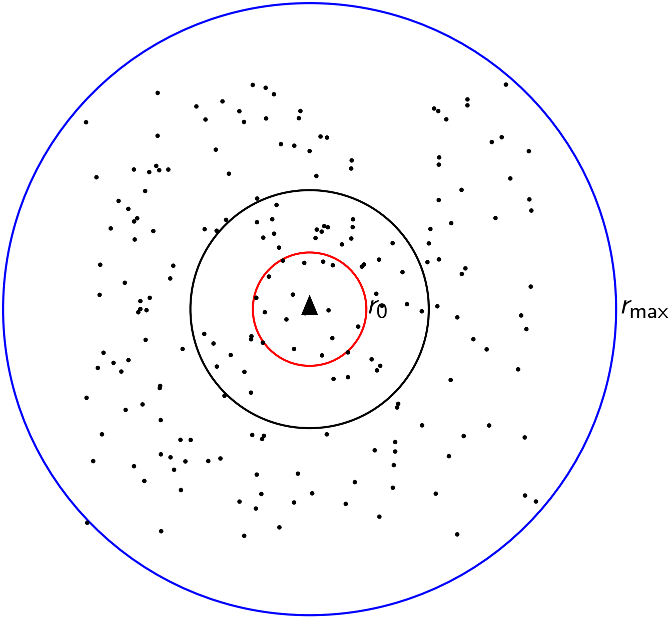
Approximating the Path Loss Model



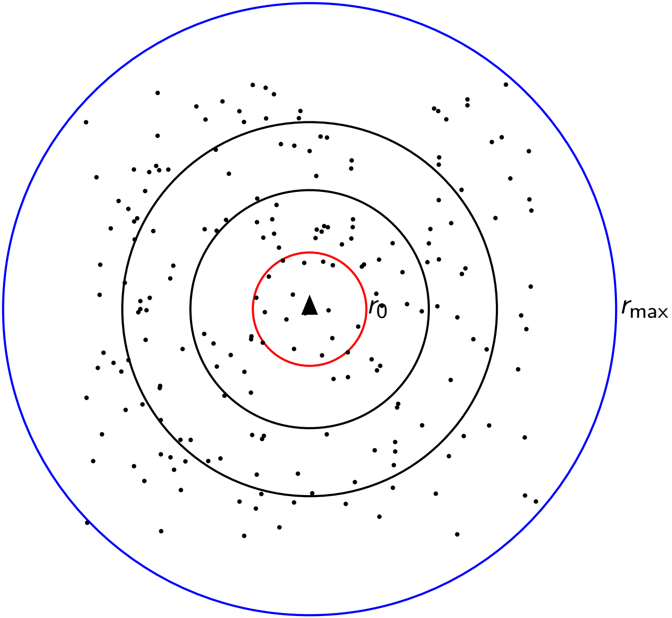
Approximating the Path Loss Model



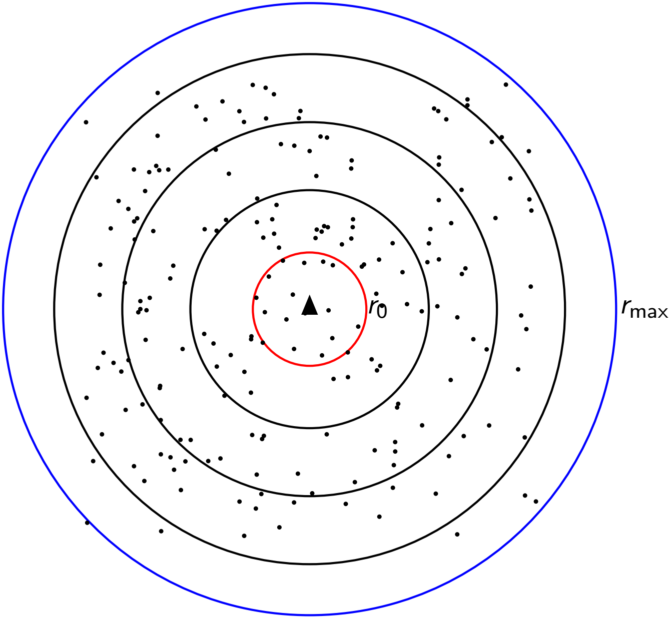
Approximating the Path Loss Model



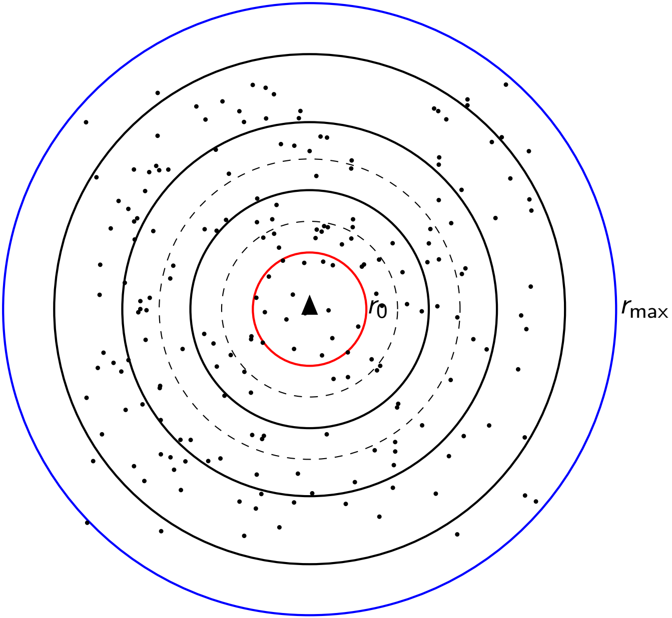
Approximating the Path Loss Model



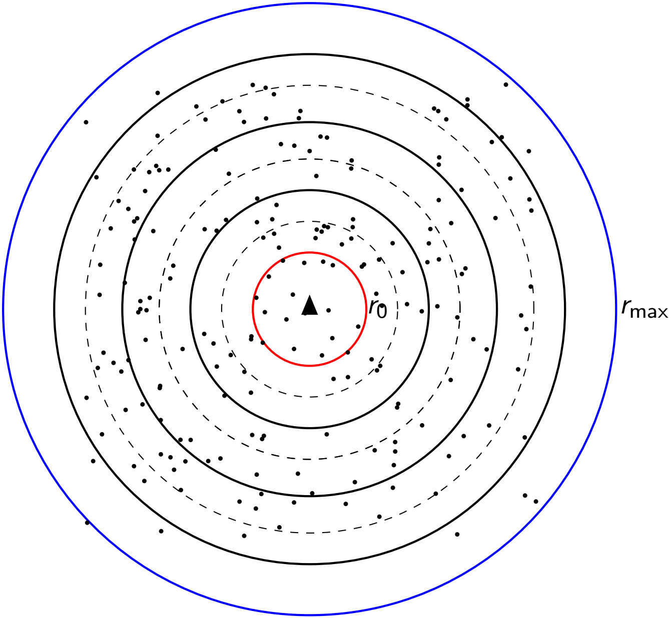
Approximating the Path Loss Model



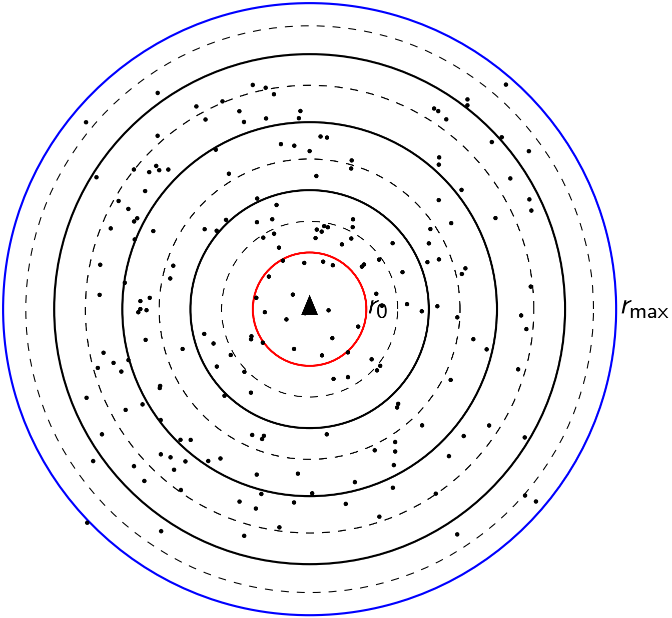
Approximating the Path Loss Model



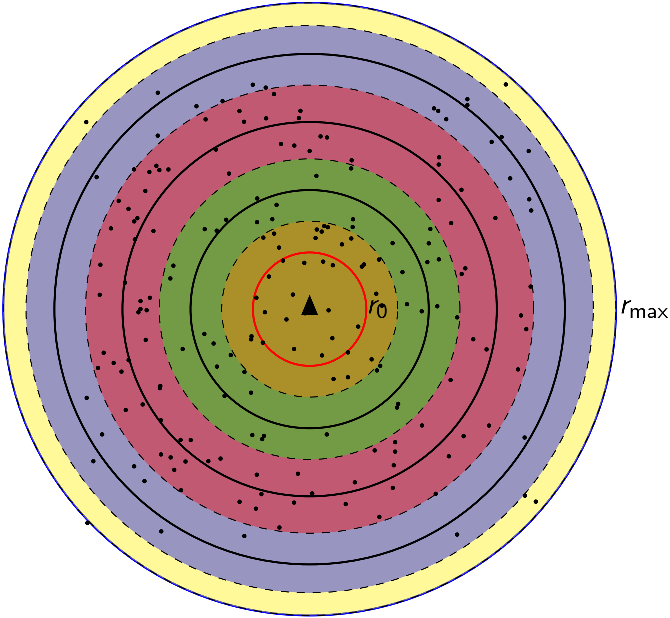
Approximating the Path Loss Model



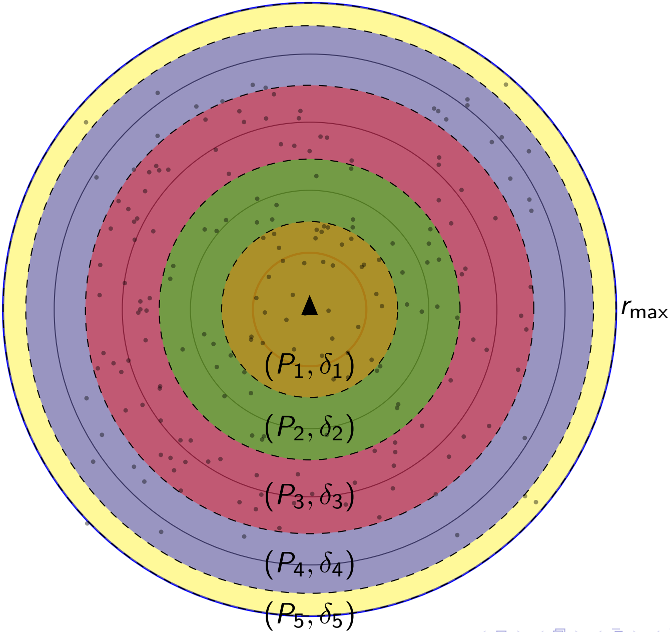
Approximating the Path Loss Model



Approximating the Path Loss Model



Approximating the Path Loss Model



Relating Approx. Path Loss Model to n-level power control

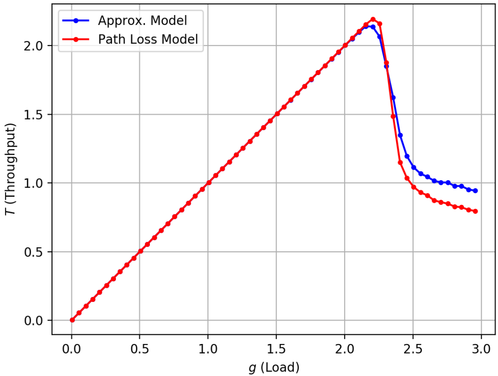
- Discretization of received power in n level $\iff$ n level power control

Relating Approx. Path Loss Model to *n*-level power control

- Discretization of received power in n level $\iff$ n level power control
- Spatial distribution of users $\iff$ $\bar{\delta}$ in n level power control

How close is the approximation?

- Choose $n = 10$
- $r_{\text{max}} = 1000 r_0$
- $\gamma = -3$
- $M = 1000$



Optimization Framework

Optimizing the Λ-distribution for path loss model is difficult

Optimization Framework

Optimizing the Λ-distribution for path loss model is difficult

$$\Downarrow$$

Optimization Framework

Optimizing the Λ-distribution for path loss model is difficult

$$\Downarrow$$

Approximate the path loss model by discretization of distances

Optimization Framework

Optimizing the Λ-distribution for path loss model is difficult

$\Downarrow$

Approximate the path loss model by discretization of distances

$\Updownarrow$

Optimization Framework

Optimizing the Λ-distribution for path loss model is difficult

$$\Downarrow$$

Approximate the path loss model by discretization of distances

$$\Updownarrow$$

n - level power control

Optimization Framework

Optimizing the Λ-distribution for path loss model is difficult

$$\Downarrow$$

Approximate the path loss model by discretization of distances

$$\Updownarrow$$

n - level power control

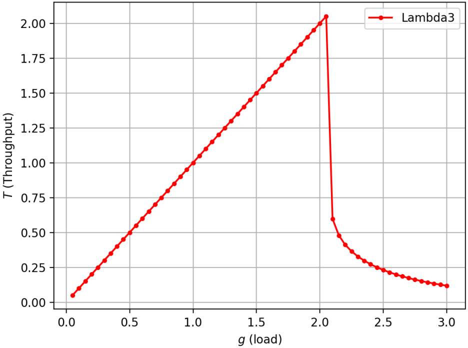
$$\boxed{\text{Conjecture : } \mathtt{OPT}_{\text{path loss}} \geq \mathtt{OPT}_{\text{n-level}}}$$

Optimization Problem

Objective : Maximize throughput $(T = g\,(1 - \Lambda(p)))$

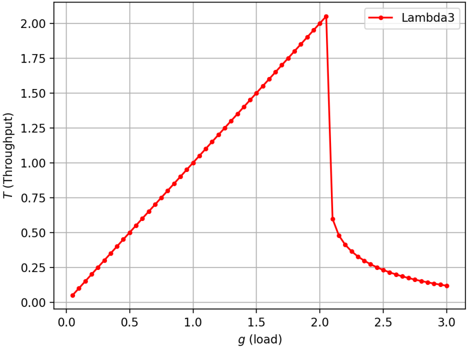
Optimization Problem

$$\text{Objective : Maximize throughput } (T = g\,(1 - \Lambda(p)))$$



Optimization Problem

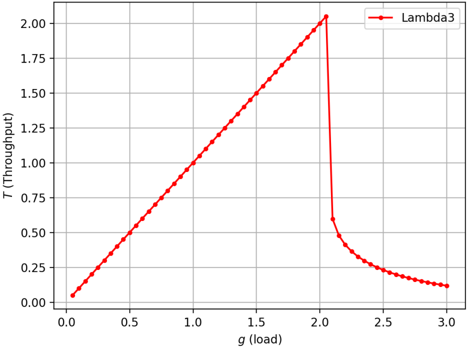
Objective : Maximize throughput ($T = g\,(1 - \Lambda(p))$)



$$\Downarrow$$

Optimization Problem

Objective : Maximize throughput $\left(T = g\left(1 - \Lambda(p)\right)\right)$

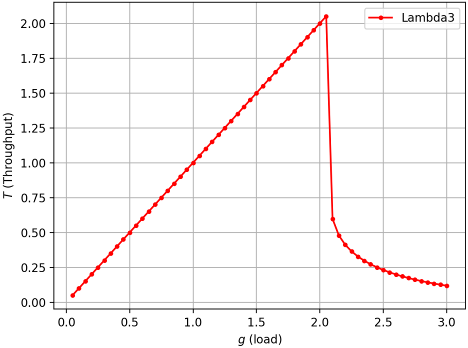


$$\Downarrow$$

$$\max \quad g\left(1 - \Lambda(p)\right)$$

Optimization Problem

$$\text{Objective}: \text{ Maximize throughput } (T = g\,(1 - \Lambda(p)))$$



$$\Downarrow$$

$$\max \quad g\,(1 - \Lambda(p)) \iff \max \quad g$$

Formulation of optimization problem

- Input :

Formulation of optimization problem

- Input :
  - Spatial distribution of users $\iff \bar{\delta}$

Formulation of optimization problem

- Input :
  - Spatial distribution of users $\iff \bar{\delta}$
  - Discretization level : $n \iff \bar{P}$

Formulation of optimization problem

- Input :
  - Spatial distribution of users $\iff \bar{\delta}$
  - Discretization level : $n \iff \bar{P}$

$$
\begin{aligned}
\underset{g,\bar{\lambda}}{\text{maximize}} \quad & g \\
\text{subject to} \quad & \\
& \lambda_i \geq 0,\; i = 2, \ldots, d_{\max}, \\
& \sum \lambda_i = 1, \\
& q < f_{DE}(q),\; \forall q \in (0, 1]
\end{aligned}
$$

Formulation of optimization problem

- Input :
  - Spatial distribution of users $\iff \bar{\delta}$
  - Discretization level : $n \iff \bar{P}$

$$
\begin{aligned}
\underset{g,\bar{\lambda}}{\text{maximize}} \quad & g \\
\text{subject to} \quad & \\
& \lambda_i \geq 0,\ \ i = 2, \ldots, d_{\max}, \\
& \sum \lambda_i = 1, \\
& q < f_{DE}(q),\ \forall q \in (0, 1]
\end{aligned}
$$

- Output : $\Lambda^\star, g^\star$
  - For eg. $\bar{\delta} = [0.33980118, 0.39227827, 0.26792056], n = 3$
    $g^\star = 2.1025, \Lambda^\star(x) \approx 0.674x^2 + 0.122x^3 + 0.204x^8$

Solving the optimization problem

$$\begin{aligned}
\underset{g,\bar{\lambda}}{\text{maximize}} \quad & g \\
\text{subject to} \quad & \\
& \lambda_i \geq 0,\ i = 2, \ldots, d_{\max}, \\
& \sum \lambda_i = 1, \\
& q < f_{DE}(q),\ \forall q \in (0, 1]
\end{aligned}$$

- Constraint $q < f_{DE}(q),\ \forall q \in (0, 1]$ is a *non-convex* constraint
- Use Differential Evolution as a black box solver to solve for a general optimization problem
- No convergence guarantees to the global optimal.

Connecting the dots

- $\bar{\delta} = [0.33980118, 0.39227827, 0.26792056]$

Connecting the dots

- $\bar{\delta} = [0.33980118, 0.39227827, 0.26792056]$
- $\Lambda_3(x) = 0.5x^2 + 0.28x^3 + 0.22x^8$

Connecting the dots

- $\bar{\delta} = [0.33980118, 0.39227827, 0.26792056]$
- $\Lambda_3(x) = 0.5x^2 + 0.28x^3 + 0.22x^8$
- $\Lambda_{opt}(x) \approx 0.674x^2 + 0.122x^3 + 0.204x^8$

Connecting the dots

- $\bar{\delta} = [0.33980118, 0.39227827, 0.26792056]$
- $\Lambda_3(x) = 0.5x^2 + 0.28x^3 + 0.22x^8$
- $\Lambda_{opt}(x) \approx 0.674x^2 + 0.122x^3 + 0.204x^8$

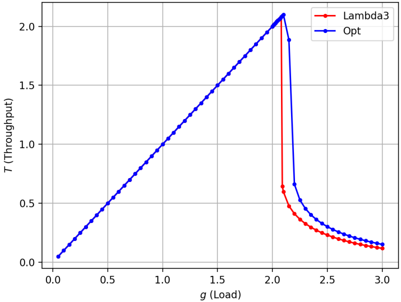
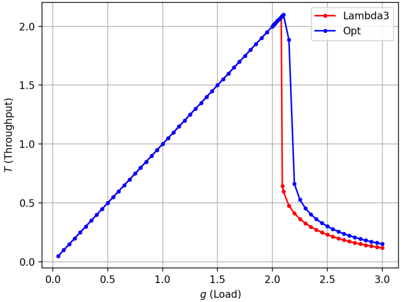


Figure: *n*-level power control

Connecting the dots

- $\bar{\delta} = [0.33980118, 0.39227827, 0.26792056]$
- $\Lambda_3(x) = 0.5x^2 + 0.28x^3 + 0.22x^8$
- $\Lambda_{opt}(x) \approx 0.674x^2 + 0.122x^3 + 0.204x^8$

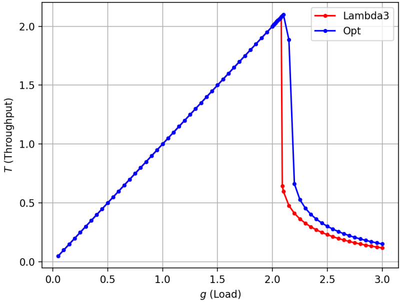


$\Rightarrow$

Figure: *n*-level power control

Connecting the dots

- $\bar{\delta} = [0.33980118, 0.39227827, 0.26792056]$
- $\Lambda_3(x) = 0.5x^2 + 0.28x^3 + 0.22x^8$
- $\Lambda_{opt}(x) \approx 0.674x^2 + 0.122x^3 + 0.204x^8$



$\Rightarrow$

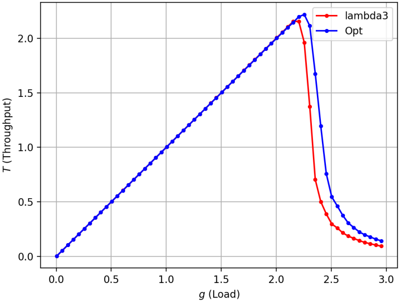


Figure: *n*-level power control

Figure: Path Loss Model

Summary

- Introduced power control in Slotted Aloha and Coded Slotted Aloha scheme
- Used this as a framework to study the Path Loss Model and optimize the Λ-distribution for the same
- Further work:
  - Jointly optimize the power scheme and repetition scheme.

Merci.

References

Ehsan Khaleghi, Cedric Adjih, Amira Alloum, Paul Muhlethaler (2017)
Near-Far Effect on Coded Slotted ALOHA
PIMRC 2017 - IEEE 28th Annual International Symposium on Personal, Indoor, and Mobile Radio Communications - Workshop WS-07 on "The Internet of Things (IoT), the Road Ahead: Applications, Challenges, and Solutions", Oct 2017, Montreal, Canada. 2017