

Robotics Assignment 1

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Solution 1

Suppose we are given two frames, denoted by frames F_0 and F_1 . We can assume the two frames have two additional features, namely

1. The axis x_1 is perpendicular to the axis z_0 .
2. The axis x_1 intersects the axis z_0 .

Under these conditions we claim that there exist unique numbers a, d, θ, α such that

$$A = R_{z,\theta} Trans_{z,d} Trans_{x,a} R_{x,\alpha}$$

If the first condition is satisfied, then x_1 is perpendicular to z_0 and we have $x_1 \cdot z_0 = 0$

$$\text{This implies } x_1^T \cdot z_0 = \begin{bmatrix} r_{11} & r_{21} & r_{31} \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = r_{31} = 0$$

Since each row and column of R_0^1 must have unit length, $r_{31} = 0$, implies that

$$r_{11}^2 + r_{21}^2 = 1$$

$$r_{32}^2 + r_{33}^2 = 1$$

Hence there exists unique α, θ such that $(r_{11}, r_{21}) = (\cos\theta, \sin\theta)$ & $(r_{33}, r_{32}) = (\cos\alpha, \sin\alpha)$.

Using the fact that R_0^1 is a rotation matrix, it can be shown that remaining elements of R_0^1 will be trigonometric functions of α, θ .

Therefore we can obtain the R_0^1 rotation matrix as follows:

$$R_0^1 = \begin{bmatrix} \cos\theta & -\sin\theta\cos\alpha & \sin\theta\cos\alpha \\ \sin\theta & \cos\theta\cos\alpha & -\cos\theta\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix}$$

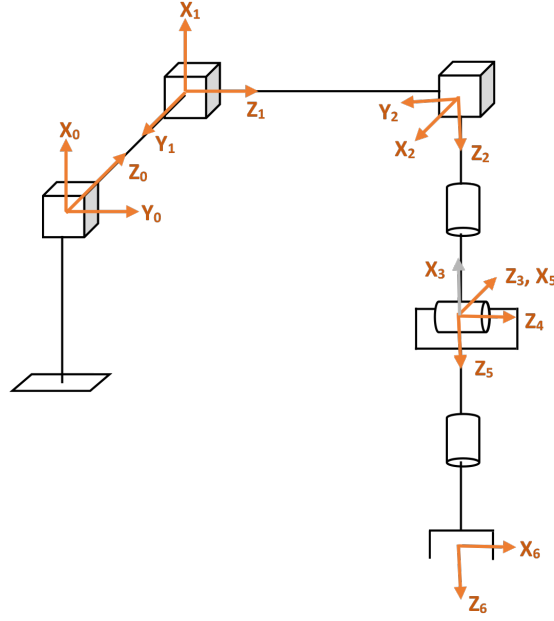
If the second condition is satisfied, then the origin of the two frames can be related by a linear combination of the vectors z_0 and x_1 . Thus we obtain the following relationship

$$P_0^1 = P_0^0 + dz_0^0 + ax_1^0$$
$$P_0^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + d \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + a \begin{bmatrix} \cos\theta \\ \sin\theta \\ 0 \end{bmatrix} = \begin{bmatrix} a\cos\theta \\ a\sin\theta \\ d \end{bmatrix}$$

Combining the above results, we see that four parameters are sufficient to specify any homogeneous transformation. Therefore there exist unique DH pa-

rameters such that the homogeneous transformation can be expressed as a combination of 2 rotation and 2 translation matrices.

Solution 2



S.No	d	a	θ	α
1	d_1	0	0°	-90°
2	d_2	0	90°	-90°
3	d_3	0	0°	0°
4	0	0	θ_4	-90°
5	0	0	θ_5	90°
6	1	0	θ_6	0°

i. The table above contains the DH parameters of the manipulators.

The transformation matrix is given as follows:

$$T_{i-1}^i = \begin{bmatrix} \cos\theta_i & -\sin\theta_i\cos\alpha_i & \sin\theta_i\sin\alpha_i & a_i\cos\theta_i \\ \sin\theta_i & \cos\theta_i\cos\alpha_i & -\cos\theta_i\sin\alpha_i & a_i\sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The individual transformation matrices are as follows:

$$T_0^1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{aligned}
T_1^2 &= \begin{bmatrix} 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
T_2^3 &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
T_3^4 &= \begin{bmatrix} \cos\theta_4 & 0 & \sin\theta_4 & 0 \\ \sin\theta_4 & 0 & -\cos\theta_4 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
T_4^5 &= \begin{bmatrix} \cos\theta_5 & 0 & -\sin\theta_5 & 0 \\ \sin\theta_5 & 0 & \cos\theta_5 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
T_5^6 &= \begin{bmatrix} \cos\theta_6 & 0 & -\sin\theta_6 & 0 \\ \sin\theta_6 & 0 & \cos\theta_6 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
\text{ii. } T_0^5 &= T_0^1.T_1^2.T_2^3.T_3^4.T_4^5 = \begin{bmatrix} -\sin\theta_5 & 0 & -\cos\theta_5 & -d_3 \\ -\sin\theta_4\cos\theta_5 & -\cos\theta_4 & \sin\theta_4\cos\theta_5 & d_2 \\ -\cos\theta_4\cos\theta_5 & \sin\theta_4 & \cos\theta_4\sin\theta_5 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
T_0^6 &= T_0^1.T_1^2.T_2^3.T_3^4.T_4^5.T_5^6 \\
T_0^6 &= \begin{bmatrix} -\sin\theta_5\cos\theta_6 & \sin\theta_5\sin\theta_6 & -\cos\theta_5 & -\cos\theta_5 - d_3 \\ -\sin\theta_4\cos\theta_5\cos\theta_6 - \cos\theta_4\sin\theta_6 & \sin\theta_4\cos\theta_5\sin\theta_6 - \cos\theta_4\cos\theta_6 & \sin\theta_4\sin\theta_5 & \sin\theta_4\sin\theta_5 + d_2 \\ -\cos\theta_4\cos\theta_5\cos\theta_6 + \sin\theta_4\sin\theta_6 & \cos\theta_4\cos\theta_5\sin\theta_6 + \sin\theta_4\cos\theta_6 & \cos\theta_4\sin\theta_5 & \cos\theta_4\sin\theta_5 + d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
\text{iii. For } d_1 = 2, d_2 = 2, d_3 = 3, d_6 = 1 \text{ and } \theta_4 = 0, \theta_5 = 0, \theta_6 = \pi \\
T_0^6 &= T_0^1.T_1^2.T_2^3.T_3^4.T_4^5.T_5^6 = \begin{bmatrix} 0 & 0 & -1 & -4 \\ 0 & 1 & 0 & 2 \\ 1 & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{aligned}$$

Solution 3

Python Program Source Code

The Python Program is as follows:

```

# Importing the necessary libraries for the program

# Importing numpy to perform matrix multiplications
import numpy as np

```

```

# Importing pandas to read data as a data frame
import pandas as pd

# Importing math to perform trigonometric operations
import math

MIN_VALUE = 6.12323400e-17 # Setting the min value

# Reading the DH parameters of robot as a CSV file
df = pd.read_csv('dh_parameters.csv')

# Function to make the Transformation Matrix for d,a,alpha and theta
def transformationMatrix(d,a,alpha,theta):

    # Converting theta from degrees to radians
    theta = math.radians(theta)

    # Converting alpha from degrees to radians
    alpha = math.radians(alpha)

    # Calculating the first row of matrix
    element_11 = math.cos(theta) \
    if abs(math.cos(theta)) > MIN_VALUE else 0.00

    element_12 = -1*math.sin(theta)*math.cos(alpha)\
    if abs(-1*math.sin(theta)*math.cos(alpha)) > MIN_VALUE else 0.00

    element_13 = math.sin(theta)*math.sin(alpha) \
    if abs(math.sin(theta)*math.sin(alpha)) > MIN_VALUE else 0.00

    element_14 = a*math.cos(theta)

    # Calculating the second row of matrix
    element_21 = math.sin(theta) \
    if abs(math.sin(theta)) > MIN_VALUE else 0.00

    element_22 = math.cos(theta)*math.cos(alpha) \
    if abs(math.cos(theta)*math.cos(alpha)) > MIN_VALUE else 0.00

    element_23 = -1*math.cos(theta)*math.sin(alpha) \
    if abs(math.cos(theta)*math.sin(alpha)) > MIN_VALUE else 0.00

    element_24 = a*(math.sin(theta))

```

```

# Calculating the third row of matrix
element_31 = 0

element_32 = math.sin(alpha) \
if abs(math.sin(alpha)) > MIN_VALUE else 0.00

element_33 = math.cos(alpha) \
if abs(math.cos(alpha)) > MIN_VALUE else 0.00

element_34 = d

# Calculating the fourth row of matrix
element_41 = 0

element_42 = 0

element_43 = 0

element_44 = 1

# Returning the transformation matrix
return np.matrix([[element_11,element_12,element_13,element_14],
[element_21,element_22,element_23,element_24],
[element_31,element_32,element_33,element_34],
[element_41,element_42,element_43,element_44]])

# Calculating the number of rows in DH Table
number_of_rows, number_of_columns = df.shape

# Setting the Identity matrix as initial Transformation Matrix
T_On = np.matrix([[1,0,0,0],
[0,1,0,0],
[0,0,1,0],
[0,0,0,1]])

# Iterating through all the rows in the DH Table to obtain pairwise
# Transformation Matrices
for index,row in df.iterrows():
    print "Printing the Transformation Matrix T_" + str(index) + str

    temp = transformationMatrix(row['d'],row['a'],row['alpha'],row['
    print temp

```

```

T_0n = T_0n * temp

print "Printing the Transformation Matrix T_0" + str(number_of_rows)
print T_0n

# Calculating the position vector of the tool tip
position_vector = np.array([[T_0n.item((0,3))],[T_0n.item((1,3))],[T_0n.
print "Position Vector of end effector"
print position_vector

# Calculating the rotation matrix of the tool tip
rot_mat = np.matrix([[T_0n.item((0,0)),T_0n.item((0,1)),T_0n.item((0,2))],
                    [T_0n.item((1,0)),T_0n.item((1,1)),T_0n.item((1,2))],
                    [T_0n.item((2,0)),T_0n.item((2,1)),T_0n.item((2,2))]])
print "Rotation Matrix of end effector"
print rot_mat

```

Input to the Program

The input file to the program is provided as a CSV file. Sample input file for this program is as follows:

```

d,a,alpha,theta
0,0,0,60
0,0,-90,30
5,5,0,30
5,5,-90,60
0,0,90,30
0,0,-90,30

```

Output of the Program

The output of the program obtained is as follows:

```

Printing the Transformation Matrix T_01
[[ 0.5      -0.8660254  0.      0.      ]
 [ 0.8660254  0.5      0.      0.      ]
 [ 0.         0.         1.      0.      ]
 [ 0.         0.         0.      1.      ]]
Printing the Transformation Matrix T_12
[[ 0.8660254  0.      -0.5      0.      ]
 [ 0.5         0.      0.8660254  0.      ]
 [ 0.         -1.      0.         0.      ]
 [ 0.         0.         0.         1.      ]]
Printing the Transformation Matrix T_23

```

```

[[ 0.8660254  -0.5      0.      4.33012702]
 [ 0.5      0.8660254  0.      2.5      ]
 [ 0.      0.      1.      5.      ]
 [ 0.      0.      0.      1.      ]]
Printing the Transformation Matrix T_34
[[ 0.5      0.      -0.8660254  2.5      ]
 [ 0.8660254  0.      0.5      4.33012702]
 [ 0.      -1.      0.      5.      ]
 [ 0.      0.      0.      1.      ]]
Printing the Transformation Matrix T_45
[[ 0.8660254  0.      0.5      0.      ]
 [ 0.5      0.      -0.8660254  0.      ]
 [ 0.      1.      0.      0.      ]
 [ 0.      0.      0.      1.      ]]
Printing the Transformation Matrix T_56
[[ 0.8660254  0.      -0.5      0.      ]
 [ 0.5      0.      0.8660254  0.      ]
 [ 0.      -1.      0.      0.      ]
 [ 0.      0.      0.      1.      ]]
Printing the Transformation Matrix T_06
[[ 4.33012702e-01  8.66025404e-01  -2.50000000e-01  -1.00000000e+01]
 [ -5.00000000e-01  -3.03318571e-16  -8.66025404e-01  4.33012702e+00]
 [ -7.50000000e-01  5.00000000e-01  4.33012702e-01  -7.50000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  1.00000000e+00]]
Position Vector of end effector
[[ -10.      ]
 [ 4.33012702]
 [ -7.5      ]]
Rotation Matrix of end effector
[[ 4.33012702e-01  8.66025404e-01  -2.50000000e-01]
 [ -5.00000000e-01  -3.03318571e-16  -8.66025404e-01]
 [ -7.50000000e-01  5.00000000e-01  4.33012702e-01]]

```

References

While answering the assignment questions, the following references were made use of:

<http://robotics.stackexchange.com/questions/7570/homogenous-transformation-matrix-for-dh-parameters>

<http://www.cs.duke.edu/brd/Teaching/Bio/asmb/current/Papers/chap3-forward-kinematics.pdf>