Robotics Assignment 1

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4th October 2016

Solution 1

Suppose we are given two frames, denoted by frames $F_0$and F_1. We can assume the two frames have two additional features, namely

1. The axis $x_1$is perpendicular to the axis z_0.
2. The axis $x_1$intersects the axis z_0.

Under these conditions we claim that there exist unique numbers a, d, θ, α such that

$A = R_{z,\theta} Trans_{z,d} Trans_{x,a} R_{x,\alpha}$

If the first condition is satisfied, then $x_1$is perpendicular to $z_0$and we have $x_1.z_0= 0$

This implies $x_1^T.z_0= \begin{bmatrix} r_{11} & r_{21} & r_{31} \end{bmatrix} . \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = r_{31} = 0$

Since each row and column of R_0^1must have unit length, $r_{31}= 0$, implies that

$r_{11}^2 + r_{21}^2= 1$

$r_{32}^2 + r_{33}^2= 1$

Hence there exists unique α, θ such that $(r_{11}, r_{21}) = (cos\theta, sin\theta)$ & $(r_{33}, r_{32}) = (cos\alpha, sin\alpha)$.

Using the fact that R_0^1 is a rotation matrix, it can be shown that remaining elements of R_0^1will be trigonometric functions of α, θ.

Therefore we can obtain the R_0^1rotation matrix as follows:

$$R_0^1 = \begin{bmatrix} cos\theta & -sin\theta cos\alpha & sin\theta cos\alpha \\ sin\theta & cos\theta cos\alpha & -cos\theta sin\alpha \\ 0 & sin\alpha & cos\alpha \end{bmatrix}$$

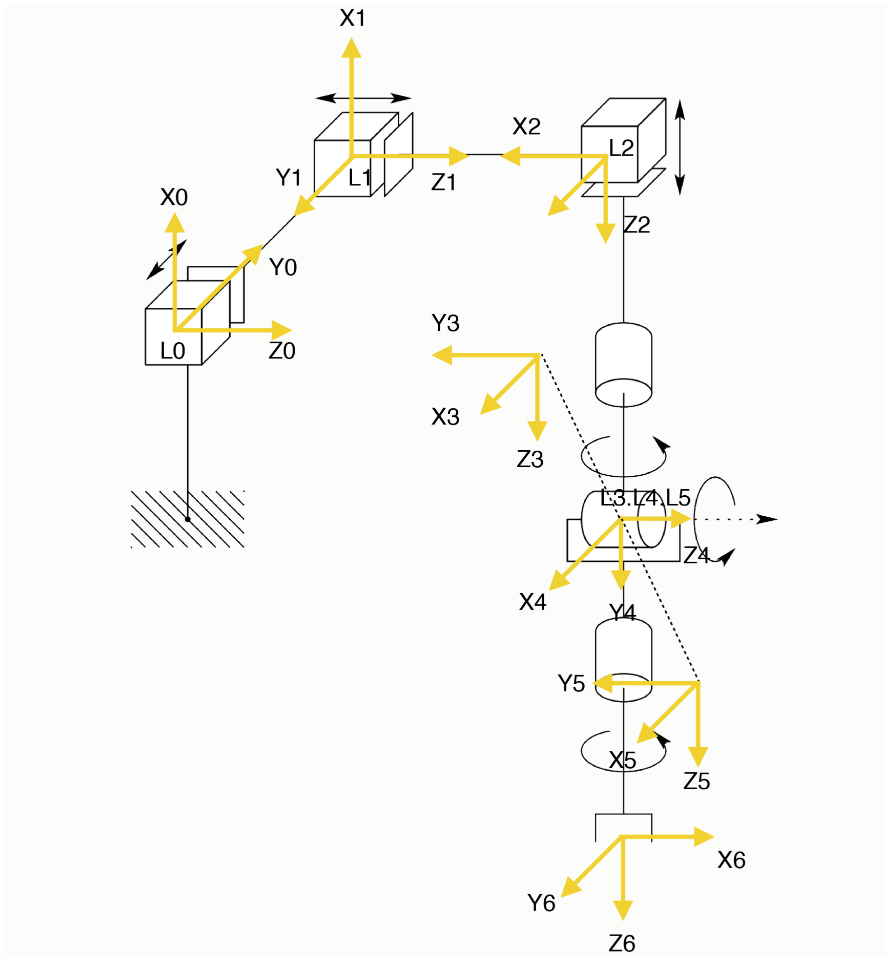
If the second condition is satisfied, then the origin of the two frames can be related by a linear combination of the vectors $z_0$and x_1. Thus we obtain the following relationship

$P_0^1 = P_0^0 + dz_0^0 + ax_1^0$

$$P_0^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + d \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + a \begin{bmatrix} cos\theta \\ sin\theta \\ 0 \end{bmatrix} = \begin{bmatrix} acos\theta \\ asin\theta \\ d \end{bmatrix}$$

Combining the above results, we see that four parameters are sufficient to specify any homogeneous transformation. Therefore there exist unique DH parameters such that the homogeneous transformation can be expressed as a combination of 2 rotation and 2 translation matrices.

Solution 2



i. The table below contains the DH parameters of the manipulators.

S.No	d	a	θ	α
1	d_1	0	0^o	-90^o
2	d_2	0	90^o	-90^o
3	d_3	0	0^o	0^o
4	0	0	θ_4	-90^o
5	0	0	θ_5	90^o
6	1	0	θ_6	0^o

The transformation matrix is given as follows:

$$T_{i-1}^{i}= \begin{bmatrix} cos\theta_i & -sin\theta_i cos\alpha_i & sin\theta_i sin\alpha_i & a_i cos\theta_i \\ sin\theta_i & cos\theta_i cos\alpha_i & -cos\theta_i sin\alpha_i & a_i sin\theta_i \\ 0 & sin\alpha_i & cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The individual trandformation matrices are as follows:

$$T_0^1= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_1^2= \begin{bmatrix} 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^3= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^4= \begin{bmatrix} cos\theta_4 & 0 & sin\theta_4 & 0 \\ sin\theta_4 & 0 & -cos\theta_4 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_4^5= \begin{bmatrix} cos\theta_5 & 0 & -sin\theta_5 & 0 \\ sin\theta_5 & 0 & cos\theta_5 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_5^6= \begin{bmatrix} cos\theta_6 & 0 & -sin\theta_6 & 0 \\ sin\theta_6 & 0 & cos\theta_6 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

ii. $T_0^5 = T_0^1.T_1^2.T_2^3.T_3^4.T_4^5 = \begin{bmatrix} -sin\theta_5 & 0 & -cos\theta_5 & -d_3 \\ -sin\theta_4 cos\theta_5 & -cos\theta_4 & sin\theta_4 cos\theta_5 & d_2 \\ -cos\theta_4 cos\theta_5 & sin\theta_4 & cos\theta_4 sin\theta_5 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$T_0^6 = T_0^1.T_1^2.T_2^3.T_3^4.T_4^5.T_5^6$

$$T_0^6 = \begin{bmatrix} -sin\theta_5 cos\theta_6 & sin\theta_5 sin\theta_6 & -cos\theta_5 & -cos\theta_5 - d_3 \\ -sin\theta_4 cos\theta_5 cos\theta_6 - cos\theta_4 sin\theta_6 & sin\theta_4 cos\theta_5 sin\theta_6 - cos\theta_4 cos\theta_6 & sin\theta_4 sin\theta_5 & sin\theta_4 sin\theta_5 + d_2 \\ -cos\theta_4 cos\theta_5 cos\theta_6 + sin\theta_4 sin\theta_6 & cos\theta_4 cos\theta_5 sin\theta_6 + sin\theta_4 cos\theta_6 & cos\theta_4 sin\theta_5 & cos\theta_4 sin\theta_5 + d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

iii. For $d_1 = 2, d_2 = 2, d_3 = 3, d_6 = 1$ and $\theta_4 = 0, \theta_5 = 0, \theta_6 = \pi$

$$T_0^6 = T_0^1.T_1^2.T_2^3.T_3^4.T_4^5.T_5^6 = \begin{bmatrix} 0 & 0 & -1 & -4 \\ 0 & 1 & 0 & 2 \\ 1 & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Solution 3

Python Program Source Code

The Python Program is as follows:

```
# Importing the necessary libraries for the program

# Importing numpy to perform matrix multiplications
import numpy as np

# Importing pandas to read data as a data frame
import pandas as pd

# Importing math to perform trigonometric operations
import math

MIN_VALUE = 6.12323400e-17 # Setting the min value

# Reading the DH parameters of robot as a CSV file
df = pd.read_csv('dh_parameters.csv')

# Function to make the Transformation Matrix for d,a,alpha and theta
def transformationMatrix(d,a,alpha,theta):

        # Converting theta from degrees to radians
        theta = math.radians(theta)

        # Converting alpha from degrees to radians
        alpha = math.radians(alpha)

        # Calculating the first row of matrix
        element_11 = math.cos(theta) \
        if abs(math.cos(theta)) > MIN_VALUE else 0.00

        element_12 = -1*math.sin(theta)*math.cos(alpha)\
        if abs(-1*math.sin(theta)*math.cos(alpha)) > MIN_VALUE else 0.00

        element_13 = math.sin(theta)*math.sin(alpha) \
        if abs(math.sin(theta)*math.sin(alpha)) > MIN_VALUE else 0.00

        element_14 = a*math.cos(theta)



        # Calculating the second row of matrix
```

```
        element_21 = math.sin(theta) \
        if abs(math.sin(theta)) > MIN_VALUE else 0.00

        element_22 = math.cos(theta)*math.cos(alpha) \
        if abs(math.cos(theta)*math.cos(alpha)) > MIN_VALUE else 0.00

        element_23 = -1*math.cos(theta)*math.sin(alpha) \
        if abs(math.cos(theta)*math.sin(alpha)) > MIN_VALUE else 0.00

        element_24 = a*(math.sin(theta))



        # Calculating the third row of matrix
        element_31 = 0

        element_32 = math.sin(alpha) \
        if abs(math.sin(alpha)) > MIN_VALUE else 0.00

        element_33 = math.cos(alpha) \
        if abs(math.cos(alpha)) > MIN_VALUE else 0.00

        element_34 = d


        # Calculating the fourth row of matrix
        element_41 = 0

        element_42 = 0

        element_43 = 0

        element_44 = 1

        # Returning the transformation matrix
        return np.matrix([[element_11,element_12,element_13,element_14],
        [element_21,element_22,element_23,element_24],
        [element_31,element_32,element_33,element_34],
        [element_41,element_42,element_43,element_44]])

# Calculating the number of rows in DH Table
number_of_rows, number_of_columns = df.shape

# Setting the Identity matrix as initial Transformation Matrix
T_0n = np.matrix([[1,0,0,0],
                 [0,1,0,0],
```

```
                    [0,0,1,0],
                    [0,0,0,1]])

    # Iterating through all the rows in the DH Table to obtain pairwise
    # Transformation Matrices
    for index,row in df.iterrows():
            print "Printing the Transformation Matrix T_" + str(index) + str

            temp = transformationMatrix(row['d'],row['a'],row['alpha'],row['

            print temp

            T_0n = T_0n * temp

    print "Printing the Transformation Matrix T_0" + str(number_of_rows)
    print T_0n

    # Calculating the position vector of the tool tip
    position_vector = np.array([[T_0n.item((0,3))],[T_0n.item((1,3))],[T_0n.
    print "Position Vector of end effector"
    print position_vector

    # Calculating the rotation matrix of the tool tip
    rot_mat = np.matrix([[T_0n.item((0,0)),T_0n.item((0,1)),T_0n.item((0,2))]
                    [T_0n.item((1,0)),T_0n.item((1,1)),T_0n.item((1,2))],
                    [T_0n.item((2,0)),T_0n.item((2,1)),T_0n.item((2,2))]])
    print "Rotation Matrix of end effector"
    print rot_mat
```

Input to the Program

The input file to the program is provided as a CSV file. Sample input file for this program is as follows:

```
d,a,alpha,theta
0,0,0,60
0,0,-90,30
5,5,0,30
5,5,-90,60
0,0,90,30
0,0,-90,30
```

Output of the Program

The output of the program obtained is as follows:

```
Printing the Transformation Matrix T_01
```

```
[[ 0.5        -0.8660254  0.         0.        ]
 [ 0.8660254  0.5        0.         0.        ]
 [ 0.         0.         1.         0.        ]
 [ 0.         0.         0.         1.        ]]
Printing the Transformation Matrix T_12
[[ 0.8660254  0.        -0.5        0.        ]
 [ 0.5        0.         0.8660254  0.        ]
 [ 0.        -1.         0.         0.        ]
 [ 0.         0.         0.         1.        ]]
Printing the Transformation Matrix T_23
[[ 0.8660254  -0.5         0.          4.33012702]
 [ 0.5         0.8660254   0.          2.5       ]
 [ 0.          0.          1.          5.        ]
 [ 0.          0.          0.          1.        ]]
Printing the Transformation Matrix T_34
[[ 0.5         0.         -0.8660254   2.5       ]
 [ 0.8660254   0.          0.5         4.33012702]
 [ 0.         -1.          0.          5.        ]
 [ 0.          0.          0.          1.        ]]
Printing the Transformation Matrix T_45
[[ 0.8660254  0.         0.5        0.        ]
 [ 0.5        0.        -0.8660254  0.        ]
 [ 0.         1.         0.         0.        ]
 [ 0.         0.         0.         1.        ]]
Printing the Transformation Matrix T_56
[[ 0.8660254  0.        -0.5        0.        ]
 [ 0.5        0.         0.8660254  0.        ]
 [ 0.        -1.         0.         0.        ]
 [ 0.         0.         0.         1.        ]]
Printing the Transformation Matrix T_06
[[  4.33012702e-01   8.66025404e-01  -2.50000000e-01  -1.00000000e+01]
 [ -5.00000000e-01  -3.03318571e-16  -8.66025404e-01   4.33012702e+00]
 [ -7.50000000e-01   5.00000000e-01   4.33012702e-01  -7.50000000e+00]
 [  0.00000000e+00   0.00000000e+00   0.00000000e+00   1.00000000e+00]]
Position Vector of end effector
[[-10.        ]
 [  4.33012702]
 [ -7.5       ]]
Rotation Matrix of end effector
[[  4.33012702e-01   8.66025404e-01  -2.50000000e-01]
 [ -5.00000000e-01  -3.03318571e-16  -8.66025404e-01]
 [ -7.50000000e-01   5.00000000e-01   4.33012702e-01]]
```

References

While answering the assignment questions, the following references were made use of:

http://robotics.stackexchange.com/questions/7570/homogenous-transformation-matrix-for-dh-parameters

http://www.cs.duke.edu/brd/Teaching/Bio/asmb/current/Papers/chap3-forward-kinematics.pdf