Preparation of TP GTVS

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The nonlinear system Σ considered here is given by the following equations

$$\Sigma : \begin{cases} \dfrac{dx\left(t\right)}{dt} = -x\left(t\right) + u\left(t\right), \\ y\left(t\right) = \tanh\left(x\right), \end{cases} \tag{1}$$

with $\tanh\left(x\right) = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$.

A) Write the state variable $x\left(t\right)$ and the input $u\left(t\right)$ in function of the output $y\left(t\right)$ and its derivatives, i.e.,

$$x\left(t\right) = f_1\left(y\left(t\right), y^{(1)}\left(t\right), \cdots, y^{(n)}\left(t\right)\right),$$
$$u\left(t\right) = f_2\left(y\left(t\right), y^{(1)}\left(t\right), \cdots, y^{(m)}\left(t\right)\right).$$

B) Is the system (1) flat with the output $y\left(t\right)$? Why ?

C) Given a desired trajectory $y_d\left(t\right)$, find the corresponding input $u_d\left(t\right)$.

D) In order to obtain an LTV system from (1), the later has to be linearised around a desired trajectory. Let

$$y_d\left(t\right) = \alpha\left(1 - e^{-\frac{t}{T}}\right), \tag{2}$$

with $\alpha = 0.9$ and $T = 1$, be the desired trajectory.

i) Linearise (1) around the trajectory (2), i.e., around $x\left(t\right) = x_d\left(t\right) + x_\delta\left(t\right)$, $u\left(t\right) = u_d\left(t\right) + u_\delta\left(t\right)$ and $y\left(t\right) = y_d\left(t\right) + y_\delta\left(t\right)$.

ii) Give the associated tangent LTV system Σ_{T_d} around the trajectory (2), i.e. which satisfy

$$\Sigma_{T_d} : \begin{cases} \dfrac{x_\delta\left(t\right)}{dt} = A\left(t\right) x_\delta\left(t\right) + B\left(t\right) u_\delta\left(t\right), \\ y_\delta\left(t\right) = C\left(t\right) x_\delta\left(t\right) + D\left(t\right) u_\delta\left(t\right). \end{cases} \tag{3}$$

iii) Give the time-varying transfer function $H\left(\partial\right)$ of (3).

iv) Deduce a right-coprime factorization $H\left(\partial\right) = B_R\left(\partial\right) A_R\left(\partial\right)^{-1}$ of $H\left(\partial\right)$.

E) To control the nonlinear system (1), locally around the desired trajectory (2), one propose a Two-Degrees-Of-Freedom ($2DOF$) control structure of Fig. 1.

u^* and y^* are, respectively, u_d and y_d. The transfer matrices $\mathcal{R}\left(\partial\right)$, $\mathcal{S}\left(\partial\right)$ and $\mathcal{T}\left(\partial\right)$ are computed to ensure the desired performances of the closed-loop system. Let $R_L\left(\partial\right)$, $S_L\left(\partial\right)$ and $T\left(\partial\right)$ be three left-comprime polynomials so that $\mathcal{R}\left(\partial\right) = R_L\left(\partial\right)$, $\mathcal{S}\left(\partial\right) = S_L^{-1}\left(\partial\right)$ and $\mathcal{T}\left(\partial\right) = T\left(\partial\right)$.

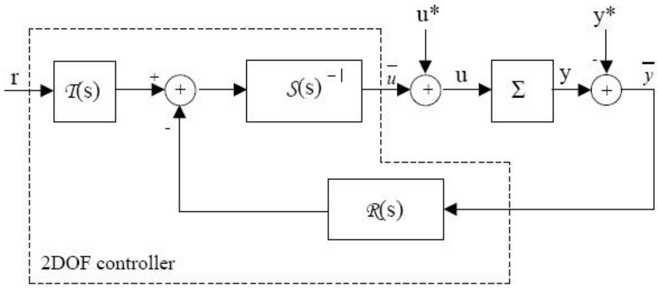

FIGURE 1 – 2DOF closed-loop system

a) Write the *Diophantine* equation allowing to find the polynomials $R_L(\partial)$ and $S_L(\partial)$ which can place the poles of the closed-loop system at $\Lambda_{cl} = \{-1, -1\}$, i.e., $A_{cl}(\partial) = (\partial + 1)^2$.

b) Choose $S_L(\partial) = s_1(t)\,\partial$ (no free term, i.e., integral effect), $R_L(\partial) = r_1(t)\,\partial + r_0(t)$ and solve (by hand) the system of algebraic equations (to find the coefficients $s_1(t)$, $r_1(t)$ and $r_0(t)$) resulting from the Diophantine equation.

c) As the transfer from the reference r to the error $e = (r - y)$ is given by

$$H_{re}(\partial) = B_R(\partial)\,A_{cl}(\partial)^{-1}\,T(\partial) - 1,$$

find T (take it as a constant polynomial) in order to *asymptotically* track a constant reference $r = const.$

In order to implement the previous 2DOF controller, one has to ensure its causality. For this, the transfers $\mathcal{R}(\partial)$, $\mathcal{S}(\partial)$ and $\mathcal{T}(\partial)$ are implemented as

$$\begin{aligned}
\mathcal{R}(\partial) &= V^{-1}(\partial)\,R_L(\partial)\,,\\
\mathcal{S}(\partial) &= S_L^{-1}(\partial)\,V(\partial)\,,\\
\mathcal{T}(\partial) &= V^{-1}(\partial)\,T(\partial)\,,
\end{aligned} \tag{4}$$

where $V(\partial)$ is a given polynomial having coefficients in $\mathbb{R}$ and roots in the left-half complex plane. It does not affect the dynamic of the closed-loop system and it is used just to ensure the causality of the synthesised controller in the implementation step.