# Ecole Centrale de Nantes

## NOLCO Lab

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**System definition.** Consider the following nonlinear system, which is based on the model of PVTOL [1]

$$\begin{array}{rcl} \ddot{x} & = & -\sin(\theta)u_1 + \varepsilon\cos(\theta)u_2 \\ \ddot{z} & = & \cos(\theta)u_1 + \varepsilon\sin(\theta)u_2 - 1 \\ \ddot{\theta} & = & u_2 \end{array} \tag{1}$$

One assumes that the parameter is small, $\varepsilon = 10^{-3}$.
**1.** Supposing that the outputs which must be stabilized at 0 are defined as $x$ and $z$,

$$y_1 = x, \;\; y_2 = z$$

design a control law allowing to decouple and to linearize, by an input-output point-of-view, the nominal system (without perturbation neither uncertainties). What are the relative degrees ? Conclusion on the presence (or not) of internal dynamics ? Note that the control law $u$ will read as

$$u \;\; = \;\; \left[ \begin{array}{c} u_1 \\ u_2 \end{array} \right] = a(x,z) + b(x,z) \cdot w, \tag{2}$$

with the "new" control input $w$ being designed as a linear state feedback[1]

$$w \;\; = \;\; \left[ \begin{array}{c} w_1 \\ w_2 \end{array} \right] = \left[ \begin{array}{c} -k_{11}\dot{y}_1 - k_{12}y_1 \\ -k_{21}\dot{y}_2 - k_{22}y_2 \end{array} \right]. \tag{3}$$

The linear controller is tuned in order to have, for the linear representation (second order system), a damping coefficient equal to 1 and a "sufficiently" fast response. Simulate the closed-loop system under Simulink (take care for the selection of integration algorithm and the step size). Plot Figures with

- coordinates $x$ and $z$ versus time,
- angle $\theta$ versus time,
- control inputs $u_1$ and $u_2$.

[1] Note that $\dot{y}_1 = \dot{x}$ and $\dot{y}_2 = \dot{z}$.

Conclusions. Comment the behavior of the internal dynamics. Is it possible to prove its (un)stability ?

**2.** Consider now the previous system with $\varepsilon = 0$; furthermore, suppose that some uncertainties can appear on $\theta$-dynamics through the time varying function $\delta(t)$. The dynamics of the system reads now as

$$\begin{array}{rcl} \ddot{x} & = & -u_1 \sin(\theta) \\ \ddot{z} & = & u_1 \cos(\theta) - 1 \\ \ddot{\theta} & = & u_2 + \delta(t) \end{array} \tag{4}$$

**Consider firstly $\delta(t) = 0$.** By stating $y_1 = x$ and $y_2 = z$, prove that the previous system can be written as

$$\begin{bmatrix} \ddot{y}_1 \\ \ddot{y}_2 \end{bmatrix} = \alpha + \beta u \tag{5}$$

Analyze the structure of the matrix $\beta$. Conclusion.

Due to the previous analysis, it is necessary to use a dynamical state feedback controller, which implies that one need to consider that $u_1$ and $\dot{u}_1$ have to be viewed as new state variables. Then, from the system (4), derive a new state system and show that one gets the following input-output representation (with $u_* = [\ddot{u}_1 \;\; u_2]^T$)

$$\begin{bmatrix} y_1^{(4)} \\ y_2^{(4)} \end{bmatrix} = \alpha_* + \beta_* \cdot u_* \tag{6}$$

Simulate the closed-loop system under Simulink with NO uncertainties. Plot Figures with

- coordinates $x$ and $z$ versus time,
- angle $\theta$ versus time,
- control inputs $u_1$ and $u_2$.

Conclusions. Add now the terms $\delta(t) = 200$, then $\delta(t) = 200\sin(t)$ *only* in the model, the control law being the same. Conclusions. What would be the solution to improve the performance of the closed loop system ?

**3.** From the previous simulations, it is clear that the used controller is not robust. A solution is to increase the robustness by using specific methodology as **sliding mode control** [2, 3]. Detail the design methodology (sliding variable definition, gain evaluation, ...). Simulate the closed-loop system under Simulink. Conclusions.

**4.** One consider now the control law based on adaptive sliding mode theory. The objective consists in using a dynamical gain which will be adapted, *online*, with respect to the establishment (or not) of a sliding motion. A very recent solution [4] reads as ($i \in \{1,2\}$, $\sigma_i$ being the sliding variable)

$$w_i = -K_i \cdot \text{sign}(\sigma_i) \tag{7}$$

with the gain $K_i(t)$ defined such that

$$\dot{K}_i = \begin{cases} \bar{K} \cdot |\sigma_i| \cdot \text{sign}(|\sigma_i| - \mu_i) & \text{if } K_i > \eta_i \\ \eta_i & \text{if } K_i \leq \eta_i \end{cases} \tag{8}$$

with $K_i(0) > 0$, $\bar{K}_i > 0$, $\eta_i > 0$ and $\mu_i > 0$ very small. The parameter $\eta_i$ is introduced in order to get only positive values for $K_i$. In the sequel, for discussion and proof, and without loss of generality but for a sake of clarity, one supposes that $K_i(t) > \eta_i$ for all $t > 0$.

**4.1** Analyze the control algorithm (how does it work ?). In particular, what is the role of the parameter $\mu_i$ ?

**4.2** Tune by simulation the different parameters, the objective being to obtain accuracy, robustness and stability. Plot the gain ; conclusion.

**4.3** What would be the “best” tuning for $\mu_i$ (please justify the answer) ? Does it work when applied on the simulator ? Show that there exists a minimal value for $\mu_i$, this minimal value depending on $K_i$ and the sampling period ?

## References


[1] Hauser, J.E., “Approximate tracking for nonlinear systems with applications to flight control”, Memorandum no.UCB/ERL/M89/99, Electronics Research Laboratory, University of Berkeley, p.79-107, 1989.

[2] V.I. Utkin, Sliding mode in control and optimization, Springer, 1992.

[3] V.I. Utkin, J. Guldner, and J. Shi, Sliding mode in control in electromechanical systems, Taylor & Francis, 1999.

[4] F. Plestan, Y. Shtessel, V. Brégeault, and A. Poznyak, “New methodologies for adaptive sliding mode control”, *International Journal of Control*, Vol.83, No.9, pp.1907-1919, 2010.