IT300 Exercise Set 1

Problem 1. State if the following statements are True/False. If the statement is True give a formal argument why this is the case. If it is False, give a counter-example.

i) *"In every instance of the Stable Matching Problem, there is a stable matching containing a pair (m, w) such that m is ranked first on the preference list of w and w is ranked first on the preference list of m "*

ii) "In every Stable Marriage problem instance, there always exists a stable matching where for every matched pair (m, w), it is true that both m and w do not have the other as their least preferred partner."

iii) Exercise 2 in KT Chapter 1.

Problem 3. Let $n \geq 1$ be an integer. Given n men and n women, recall that a perfect matching in this context is a way to pair every man with a unique woman and vice-versa. Prove that the total number of perfect matchings with n men and n women is n!.
(Hint: Use can use induction or a more direct combinatorial proof.)

Problem 2. Solved Exercise 1 in [KT] Chapter 1. (S*olve without consulting the solution!*)

Problem 4. Exercise 4 in [KT] Chapter 1.

Problem 5. Solved Exercise 2 in [KT] Chapter 1**.** (S*olve without consulting the solution!*)