Probability and Statistics (IT302)

3rd August 2020 (9:45AM-10:15AM) Class

Sample Space

The set of all possible outcomes of a statistical experiment is called the **sample space** and is represented by the symbol S.

Each outcome in a sample space is called an **element** or a **member** of the sample space, or simply a **sample point**.

If the sample space has a finite number of elements, members generally separated by commas and enclosed in braces. Thus, the sample space S, of possible outcomes when a coin is flipped, may be written S = {H, T}, where H and T correspond to heads and tails, respectively.

Source : Probability & Statistics for Engineers & Scientists, by Ronald E. Walpole, Raymond H. Myers, Sharon L. Myers, Keying Ye, 9^{th} Edition, Prentice Hall

Sample Space Contd.

Example-1

Consider the experiment of tossing a die. If observed that the number that shows on the top face, the sample space is S1={1, 2, 3, 4, 5, 6}.

If observed only on whether the number is even or odd, the sample space is simply S2 = {even, odd}.

It illustrates the fact that more than one sample space can be used to describe the outcomes of an experiment. In this case, *S*1 provides more information than S2.

In some experiments, it is helpful to list the elements of the sample space systematically by means of a **Tree diagram**.

Source : Probability & Statistics for Engineers & Scientists, by Ronald E. Walpole, Raymond H. Myers, Sharon L. Myers, Keying Ye, 9^{th} Edition, Prentice Hall

Sample Space Contd.

Example-2

An experiment consists of flipping a coin and then flipping it a second time if a head occurs. If a tail occurs on the first flip, then a die is tossed once. To list the elements of the sample space providing the most information, **Tree diagram** constructed is shown in Figure 1.

The various paths along the branches of the tree give the distinct sample points. Starting with the top left branch and moving to the right along the first path, we get the sample point HH, indicating the possibility that heads occurs on two successive flips of the coin. Likewise, the sample point T3 indicates the possibility that the coin will show a tail followed by a 3 on the toss of the die. By proceeding along all paths, one can see that the sample space is S = {HH, HT, T1, T2, T3, T4, T5, T6}.

Source : Probability & Statistics for Engineers & Scientists, by Ronald E. Walpole, Raymond H. Myers, Sharon L. Myers, Keying Ye, 9th Edition, Prentice Hall

Sample Space Contd.

Example-2 Contd.

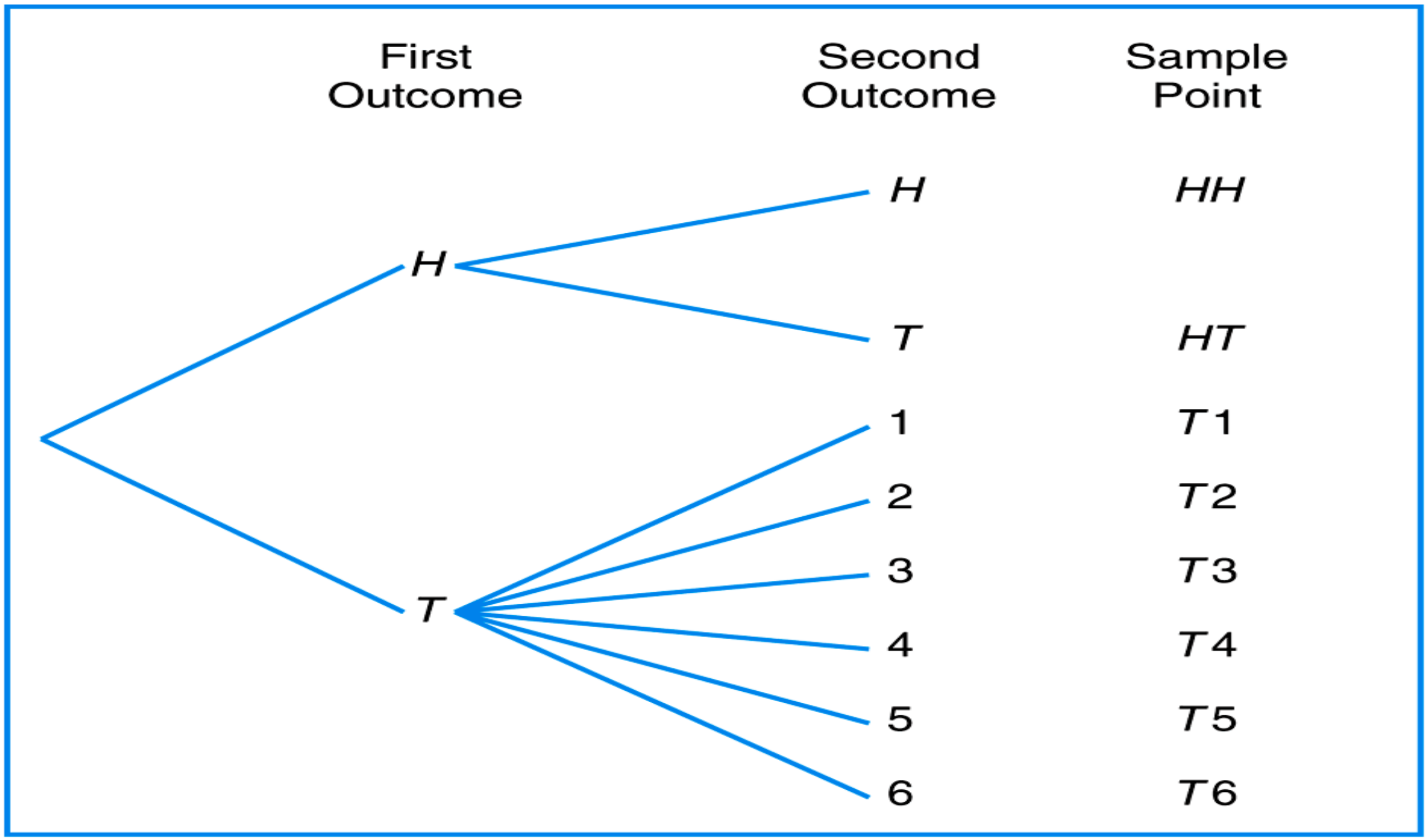

Figure 1 Tree diagram for Example 2

Source : Probability & Statistics for Engineers & Scientists, by Ronald E. Walpole, Raymond H. Myers, Sharon L. Myers, Keying Ye, 9th Edition, Prentice Hall

Sample Space Contd.

Example-3

Suppose that three items are selected at random from a manufacturing process. Each item is inspected and classified defective, D, or non-defective, N. Sample Space (Tree diagram) that provides the most information shown in Figure 2.

The various paths along the branches of the Tree give the distinct sample points. Starting with the first path, we get the sample point DDD, indicating the possibility that all three items inspected are defective. As we proceed along the other paths, we see that the sample space is

S = {DDD, DDN, DND, DNN, NDD, NDN, NND, NNN}.

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Sample Space Contd.

Example-3 Contd.

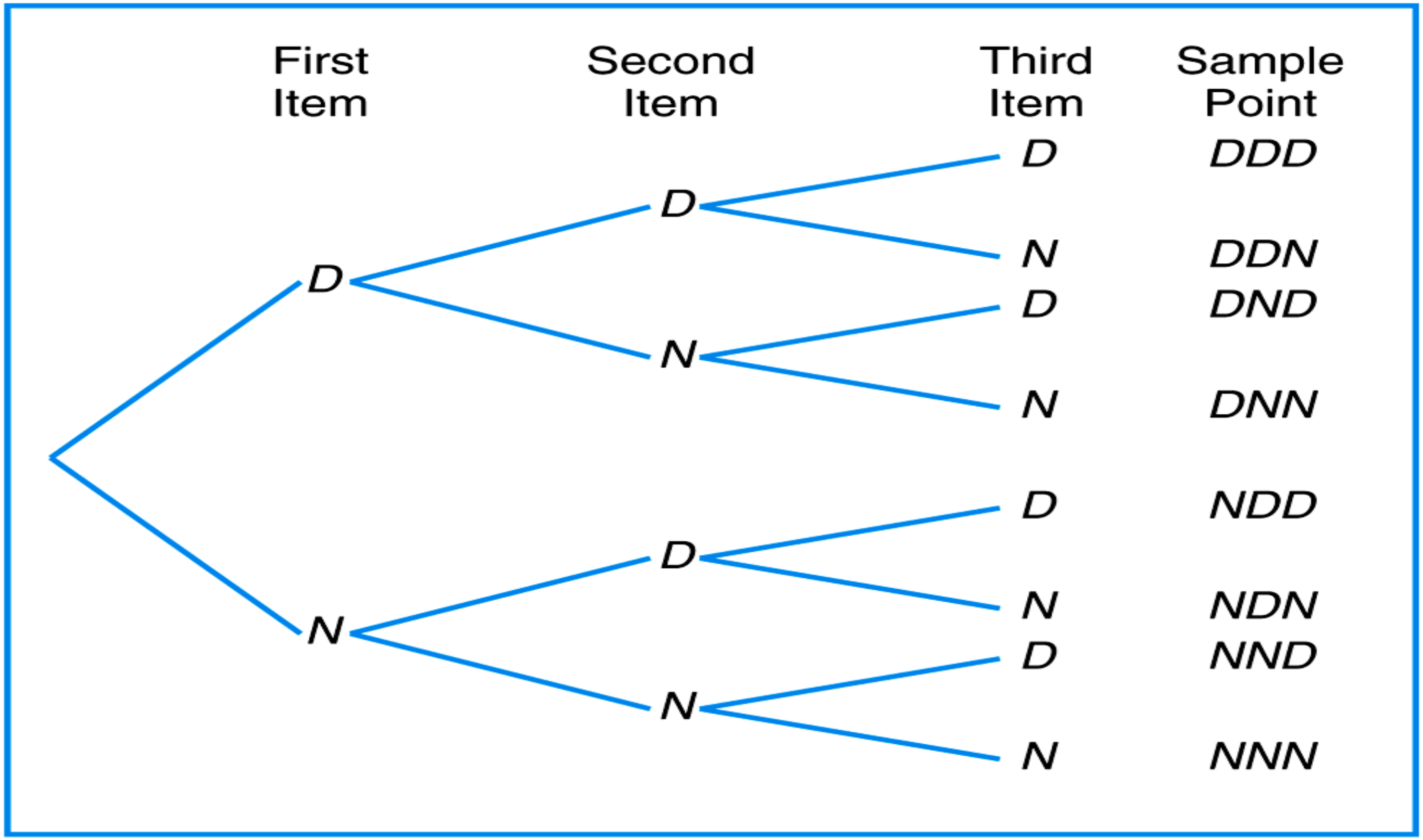

Figure 2 Tree diagram for Example 3

Source : Probability & Statistics for Engineers & Scientists, by Ronald E. Walpole, Raymond H. Myers, Sharon L. Myers, Keying Ye, 9th Edition, Prentice Hall

Sample Spaces with a Large or Infinite Number of Sample Points

Sample spaces with a large or infinite number of sample points are best described by a statement or **rule method**. For example, if the possible outcomes of an experiment are the set of cities in the world with a population over 1 million, our sample space is written

S = {x | x is a city with a population over 1 million}, which reads “S is the set of all ‘x’ such that ‘x’ is a city with a population over 1 million.” The vertical bar is read “such that.”

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Sample Spaces with a Large or Infinite Number of Sample Points Contd.

If S is the set of all points (x, y) on the boundary or the interior of a circle of radius 2 with center at the origin, then the rule $S = \{(x, y) \mid x^2 + y^2 \leq 4\}$.

Whether the sample space described by the **rule method** or by listing the elements depend on the specific problem at hand.

The **rule method** has practical advantages, particularly for many experiments where listing becomes a tedious chore.

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Exercises-1

Q1: List the elements of each of the following sample spaces:

a) The set of integers between 1 and 50 divisible by 8

Ans: S = {8, 16, 24, 32, 40, 48}.

b) The set $S = \{x \mid x^2 + 4x - 5 = 0\}$

Ans: For $x^2 + 4x - 5 = (x + 5)(x - 1) = 0$, the only solutions are $x = -5$ and $x = 1$. $S = \{-5, 1\}$.

c) The set of outcomes when a coin is tossed until a tail or three heads appear;

Ans: S = {T, HT, HHT, HHH}.

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Exercises-1 Contd.

List the elements of each of the following sample spaces:

d) The set $S = \{x \mid x \text{ is a continent}\}$

Ans: S = {N. America, S. America, Europe, Asia, Africa, Australia, Antarctica}.

e) The set $S = \{x \mid 2x - 4 \geq 0 \text{ and } x < 1\}$

Ans: Solving $2x - 4 \geq 0$ gives $x \geq 2$. Since we must also have $x < 1$, it follows that $S = \varphi$.

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Event

An **event** is a subset of a sample space.

For any given experiment, we may be interested in the occurrence of certain **events** rather than in the occurrence of a specific element in the sample space.

For instance, we may be interested in the event A that the outcome when a die is tossed is divisible by 3. This will occur if the outcome is an element of the subset A = {3, 6} of the sample space S1 in Example-1.

As a further illustration, we may be interested in the event B that the number of defectives is greater than 1 in Example-3. This will occur if the outcome is an element of the subset B = {DDN, DND, NDD, DDD}
of the sample space S.

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Event Contd.

Given the sample space $S = \{t \mid t \geq 0\}$, where t is the life in years of a certain electronic component, then the event A that the component fails before the end of the fifth year is the subset $A = \{t \mid 0 \leq t < 5\}$.

It is conceivable that an event may be a subset that includes the entire sample space S or a subset of S called the **null set** and denoted by the symbol φ, which contains no elements at all.

For instance, if we let A be the event of detecting a microscopic organism by the naked eye in a biological experiment, then $A = \varphi$.

Also, if $B = \{x \mid x \text{ is an even factor of } 7\}$, then B must be the null set, since the only possible factors of 7 are the odd numbers 1 and 7.

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Complement of an Event

The **complement** of an event A with respect to S is the subset of all elements of S that are not in A. Complement of A denoted *by* the symbol A'.

Example : Let R be the event that a red card is selected from an ordinary deck of 52 playing cards, and let *S* be the entire deck. Then R' is the event that the card selected from the deck is not a red card but a black card.

Example : Consider the sample space

S = {book, cell phone, mp3, paper, stationery, laptop}.

Let A = {book, stationery, laptop, paper}.

Then the complement of A is A' = {cell phone, mp3}.

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Intersection of Two Events A and B

The **intersection** of two events A and B, denoted by the symbol $A \cap B$, is the event containing all elements that are common to A and B.

Example : Let E be the event that a person selected at random in a classroom is majoring in engineering, and let F be the event that the person is female. Then $E \cap F$ is the event of all female engineering students in the classroom.

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Mutually Exclusive Events

Two events A and B are **mutually exclusive**, or **disjoint**, if $A \cap B = \varphi$, that is, **if A and B have no elements in common**.

A cable television company offers programs on eight different channels, three of which are affiliated with ABC, two with NBC, and one with CBS. The other two are an educational channel and the ESPN sports channel. Suppose that a person subscribing to this service turns on a television set without first selecting the channel.

Let A be the event that the program belongs to the NBC network and B the event that it belongs to the CBS network. Since a television program cannot belong to more than one network, the events A and B have no programs in common. Therefore, the intersection $A \cap B$ contains no programs, and consequently the **events A and B are mutually exclusive**.

Union of the Two Events

The **union** of the two events A and B, denoted by the symbol $A \cup B$, is the event containing all the elements that belong to A or B or both.

Example : Let $A = \{a, b, c\}$ and $B = \{b, c, d, e\}$; then

$$A \cup B = \{a, b, c, d, e\}.$$

Example : Let P be the event that an employee selected at random from an oil drilling company smokes cigarettes. Let Q be the event that the employee selected drinks alcoholic beverages. Then the **event $P \cup Q$** is the set of all employees who either drink or smoke or do both.

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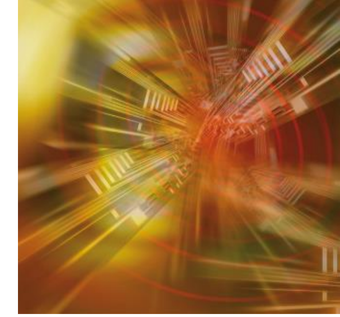

Exercises-2

Q2: Use the rule method to describe the sample space S consisting of all points in the first quadrant inside a circle of radius 3 with center at the origin.

Ans: $S = \{(x, y) \mid x^2 + y^2 < 9;\ x \geq 0,\ y \geq 0\}$.

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Exercises-3

Q3: Which of the following events are equal?

a) $A = \{1, 3\}$;

b) $B = \{x \mid x \text{ is a number on a die}\}$;

c) $C = \{x \mid x^2 - 4x+3 = 0\}$;

d) $D = \{x \mid x \text{ is the number of heads when six coins are tossed}\}$.

Ans: (a) $A = \{1, 3\}$.

(b) $B = \{1, 2, 3, 4, 5, 6\}$.

(c) $C = \{x \mid x^2 - 4x +3 = 0\} = \{x \mid (x - 1)(x - 3) = 0\} = \{1, 3\}$.

(d) $D = \{0, 1, 2, 3, 4, 5, 6\}$. Clearly, $A = C$.

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Exercises-4

Q4: An experiment involves tossing a pair of dice, one green and one red, and recording the numbers that come up. If x equals the outcome on the green die and y the outcome on the red die, describe the sample space S

(a) by listing the elements (x, y);

Ans: S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6),(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5,4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}.

(b) by using the rule method.

Ans: $S = \{(x, y) \mid 1 \leq x, y \leq 6\}$.

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Exercises-5

Q5: Two jurors are selected from 4 alternates to serve at a murder trial. Using the notation A_1A_3, for example, to denote the simple event that alternates 1 and 3 are selected, list the 6 elements of the sample space S.

Ans: $S = \{A_1A_2, A_1A_3, A_1A_4, A_2A_3, A_2A_4, A_3A_4\}$.

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