Probability and Statistics (IT302) Class No. 20
30^{th} September 2020 Wednesday 11:15AM - 11:45AM

Multinomial Experiment

A **multinomial experiment** is a statistical experiment that has the following properties:

- The experiment consists of ***n*** repeated trials.
- Each trial has a discrete number of possible outcomes.
- On any given trial, the probability that a particular outcome will occur is constant.
- The trials are independent; that is, the outcome on one trial does not affect the outcome on other trials.

Consider the following statistical experiment. You **toss two dice three times**, and record the outcome on each toss. This is a multinomial experiment because:

- The experiment consists of repeated trials. We toss the dice three times.
- Each trial can result in a discrete number of outcomes - 2 through 12.
- The probability of any outcome is constant; it does not change from one toss to the next.
- The trials are independent; that is, getting a particular outcome on one trial does not affect the outcome on other trials.

Source : https://stattrek.com/probability-distributions/multinomial.aspx Accessed on 29th September 2020

Multinomial Experiments Contd.

- The **binomial experiment becomes a multinomial experiment if each trial have more than two possible outcomes**.

- The classification of a manufactured **product as being light, heavy, or acceptable** and the recording of accidents at a certain intersection according to the day of the week constitute multinomial experiments.

- The **drawing of a card from a deck with replacement is also a multinomial experiment** if the 4 suits are the outcomes of interest.

Source : Probability & Statistics for Engineers & Scientists, by Ronald E. Walpole, Raymond H. Myers, Sharon L. Myers, Keying Ye, 9th Edition, Prentice Hall.

Multinomial Distribution

A multinomial distribution is the probability distribution of the outcomes from a multinomial experiment. The multinomial formula defines the probability of any outcome from a multinomial experiment.

Multinomial Formula. Suppose a multinomial experiment consists of **n** trials, and each trial can result in any of **k possible outcomes**: $E_1, E_2, \ldots, E_k$. Suppose, further, that each possible outcome can occur with probabilities $p_1, p_2, \ldots, p_k$. Then, the probability (P) that E_1 occurs n_1 times, E_2 occurs n_2 times, . . . , and E_k occurs n_k times is:

$$P = [\ n! / (\ n_1! * n_2! * \ldots n_k!\ )\ ] * (\ p_1^{n1} * p_2^{n2} * \ldots * p_k^{nk}\ )$$

where $n = n_1 + n_2 + \ldots + n_k$.

Source : https://stattrek.com/probability-distributions/multinomial.aspx Accessed on 29th September 2020

Example-1

Suppose a card is drawn randomly from an ordinary deck of playing cards, and then put back in the deck. This exercise is repeated five times. What is the probability of drawing 1 spade, 1 heart, 1 diamond, and 2 clubs?

Solution: To solve this problem, apply the multinomial formula with the known values

The experiment consists of 5 trials, so **$n = 5$**. The 5 trials produce 1 spade, 1 heart, 1 diamond, and 2 clubs; so $n_1 = 1$, $n_2 = 1$, $n_3 = 1$, and $n_4 = 2$. On any particular trial, the probability of drawing a spade, heart, diamond, or club is 0.25, 0.25, 0.25, and 0.25, respectively. Thus, **$p_1 = 0.25$**, **$p_2 = 0.25$**, **$p_3 = 0.25$**, and **$p_4 = 0.25$**. Use these inputs into the multinomial formula, as shown below:

$$P = [\ n! / (\ n_1! * n_2! * \ldots n_k!\ )\ ] * (\ p_1^{n1} * p_2^{n2} * \ldots * p_k^{nk}\ )$$

$$P = [\ 5! / (\ 1! * 1! * 1! * 2!\ )\ ] * [\ (0.25)^1 * (0.25)^1 * (0.25)^1 * (0.25)^2\ ]$$

$$P = 0.05859$$

Thus, **if five cards are drawn with replacement from an ordinary deck of playing cards, the probability of drawing 1 spade, 1 heart, 1 diamond, and 2 clubs is 0.05859.**

Source : https://stattrek.com/probability-distributions/multinomial.aspx Accessed on 29th September 2020

Example-2

Suppose we have a bowl with 10 marbles - 2 red marbles, 3 green marbles, and 5 blue marbles. We randomly select 4 marbles from the bowl, with replacement. What is the probability of selecting 2 green marbles and 2 blue marbles?

Solution: To solve this problem, apply the multinomial formula with known values

The experiment consists of 4 trials, so **n=4**. The 4 trials produce 0 red marbles, 2 green marbles, and 2 blue marbles; so **nred=0**, **ngreen=2**, and **nblue=2**. On any particular trial, the probability of drawing a red, green, or blue marble is 0.2, 0.3, and 0.5, respectively. Thus, pred = 0.2, pgreen = 0.3, and pblue = 0.5. Use these inputs into the multinomial formula, as shown below:

$$P = [\ n! / (\ n_1! * n_2! * \ldots n_k!\ )\ ] * (\ p_1^{n1} * p_2^{n2} * \ldots * p_k^{nk}\ )$$

$$P = [\ 4! / (\ 0! * 2! * 2!\ )\ ] * [\ (0.2)^0 * (0.3)^2 * (0.5)^2]$$

$$P = 0.135$$

Thus, **if 4 marbles are drawn with replacement from the bowl, the probability of drawing 0 red marbles, 2 green marbles, and 2 blue marbles is 0.135.**

Source : https://stattrek.com/probability-distributions/multinomial.aspx Accessed on 29th September 2020

What is a Multinomial Experiment?

A multinomial experiment is a statistical experiment that has the following characteristics:

- The experiment involves one or more trials.
- Each trial has a discrete number of possible outcomes.
- On any given trial, the probability that a particular outcome will occur is constant.
- All of the trials in the experiment are independent.

Tossing a pair of dice is a perfect example of a multinomial experiment. Suppose we toss a pair of dice three times. Each toss represents a trial, so this experiment would have 3 trials. Each toss also has a discrete number of possible outcomes - 2 through 12.

The probability of any particular outcome is constant; for example, the probability of rolling a 12 on any particular toss is always 1/36. Finally, the outcome on any toss is not affected by previous or succeeding tosses; so the trials in the experiment are independent.

Source : https://stattrek.com/probability-distributions/multinomial.aspx Accessed on 29th September 2020

What is a Multinomial Distribution?

A multinomial distribution is a probability distribution. It refers to the probabilities associated with each of the possible outcomes in a multinomial experiment.

For example, suppose we flip three coins and count the number of coins that land on heads. This multinomial experiment has four possible outcomes: 0 heads, 1 head, 2 heads, and 3 heads. Probabilities associated with each possible outcome are an example of a multinomial distribution, as shown below.

Outcome	Probability
0 heads	0.125
1 head	0.375
2 heads	0.375
3 heads	0.125

Above values completely defines the probabilities associated with every possible outcome from this multinomial experiment. **It is the multinomial distribution for this experiment.**

Source : https://stattrek.com/probability-distributions/multinomial.aspx Accessed on 29th September 2020

Multinomial Distribution

If a given trial can result in the k outcomes $E_1, E_2, \ldots, E_k$ with probabilities $p_1, p_2, \ldots, p_k$, then the probability distribution of the random variables $X_1, X_2, \ldots, X_k$, representing the number of occurrences for $E_1, E_2, \ldots, E_k$ in n independent trials, is

$$f(x_1, x_2, \ldots, x_k; p_1, p_2, \ldots, p_k, n) = \binom{n}{x_1, x_2, \ldots, x_k} p_1^{x_1} p_2^{x_2} \cdots p_k^{x_k},$$

with

$$\sum_{i=1}^{k} x_i = n \text{ and } \sum_{i=1}^{k} p_i = 1.$$

The multinomial distribution derives its name from the fact that the terms of the multinomial expansion of $(p_1 + p_2 + \cdots + p_k)^n$ correspond to all the possible values of $f(x_1, x_2, \ldots, x_k; p_1, p_2, \ldots, p_k, n)$.

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Example 5.7

The complexity of arrivals and departures of planes at an airport is such that computer simulation is often used to model the "ideal" conditions. For a certain airport with three runways, it is known that in the ideal setting the following are the probabilities that the individual runways are accessed by a randomly arriving commercial jet:

Runway 1: $p_1 = 2/9$, Runway 2: $p_2 = 1/6$, Runway 3: $p_3 = 11/18$.

What is the probability that 6 randomly arriving airplanes are distributed in the following fashion?

Runway 1: 2 airplanes, Runway 2: 1 airplane, Runway 3: 3 airplanes

Solution: Using the multinomial distribution, we have

$$f\left(2,1,3;\frac{2}{9},\frac{1}{6},\frac{11}{18},6\right) = \binom{6}{2,1,3}\left(\frac{2}{9}\right)^2\left(\frac{1}{6}\right)^1\left(\frac{11}{18}\right)^3$$
$$= \frac{6!}{2!\,1!\,3!}\cdot\frac{2^2}{9^2}\cdot\frac{1}{6}\cdot\frac{11^3}{18^3} = 0.1127.$$

Source : Probability & Statistics for Engineers & Scientists, by Ronald E. Walpole, Raymond H. Myers, Sharon L. Myers, Keying Ye, 9th Edition, Prentice Hall.

Exercise Problem 5.9

In testing a certain kind of truck tire over rugged terrain, it is found that 25% of the trucks fail to complete the test run without a blowout. Of the next 15 trucks tested, find the probability that

(a) from 3 to 6 have blowouts;

(b) fewer than 4 have blowouts;

(c) more than 5 have blowouts.

Solution: For $n = 15$ and $p = 0.25$, we have

(a) $P(3 \le X \le 6) = P(X \le 6) - P(X \le 2) = 0.9434 - 0.2361 = 0.7073.$

(b) $P(X < 4) = P(X \le 3) = 0.4613.$

(c) $P(X > 5) = 1 - P(X \le 5) = 1 - 0.8516 = 0.1484.$

Exercise Problem 5.15

It is known that 60% of mice inoculated with a serum are protected from a certain disease. If 5 mice are inoculated, find the probability that

(a) none contracts the disease;

(b) fewer than 2 contract the disease;

(c) more than 3 contract the disease.

Solution: $p = 0.4$ and $n = 5$.

(a) $P(X = 0) = 0.0778.$

(b) $P(X < 2) = P(X \leq 1) = 0.3370.$

(c) $P(X > 3) = 1 - P(X \leq 3) = 1 - 0.9130 = 0.0870.$

Source : Probability & Statistics for Engineers & Scientists, by Ronald E. Walpole, Raymond H. Myers, Sharon L. Myers, Keying Ye, 9th Edition, Prentice Hall.

Example Problem 5.19

As a student drives to school, he encounters a traffic signal. This traffic signal stays green for 35 seconds, yellow for 5 seconds, and red for 60 seconds. Assume that the student goes to school each weekday between 8:00 and 8:30 a.m. Let X1 be the number of times he encounters a green light, X2 be the number of times he encounters a yellow light, and X3 be the number of times he encounters a red light. Find the joint distribution of X1, X2, and X3.

Solution:

Let X1 = number of times encountered green light with P(Green) = 0.35,
X2 = number of times encountered yellow light with P(Yellow) = 0.05, and
X3 = number of times encountered red light with P(Red) = 0.60. Then

$$f(x_1, x_2, x_3) = \binom{n}{x_1, x_2, x_3} (0.35)^{x_1} (0.05)^{x_2} (0.60)^{x_3}$$

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What is the number of outcomes?

The number of outcomes refers to the number of different results that could occur from a multinomial experiment. For example, suppose we roll a die. Each roll of the die can have six possible outcomes - 1, 2, 3, 4, 5, or 6. Similarly, the roll of two dice can have eleven possible outcomes - the numbers from 2 to 12.

What is the probability of an outcome?

Each trial in a multinomial experiment can have a discrete number of outcomes. The likelihood that a particular outcome will occur in a single trial is the probability of the outcome.

For example, suppose we toss two dice. The probability of tossing a 2 is 1/36; the probability of tossing a 3 is 2/36, the probability of tossing 4 is 3/36, etc.

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What is the frequency of an outcome?

In a multinomial experiment, the frequency of an outcome refers to the number of times that an outcome occurs.

For example, suppose we toss a single die. This experiment has 6 possible outcomes; the die could land on 1, 2, 3, 4, 5, or 6. Suppose that we roll the die four times and observe the following outcomes: we roll a 1, a 3, and a two 5's? The frequency for each outcome is shown in the Table below.

Outcome	Frequency
1	1
2	0
3	1
4	0
5	2
6	0

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What is the relation between a multinomial and a binomial experiment?

- A binomial experiment is actually a special case of a multinomial experiment.
- The binomial experiment is a multinomial experiment, in which each trial can have only two possible outcomes.
- The flip of a coin is a good example of a binomial experiment, since a coin flip can have only two possible outcomes - heads or tails.

Source : https://stattrek.com/probability-distributions/multinomial.aspx Accessed on 29th September 2020