Probability and Statistics (IT302)

5th August 2020 (11:15AM-11:45AM) Class

Probability Models

Probability models allow us to find the probability of any collection of outcomes.

An **event** is any collection of outcomes from some chance process. That is, an event is a subset of the sample space. Events are usually designated by capital letters, like *A*, *B*, *C*, and so on.

If *A* is any event, we write its probability as $P(A)$.

In the dice-rolling example, suppose we define event *A* as "sum is 5."

There are 4 outcomes that result in a sum of 5. Since each outcome has probability 1/36, $P(A) = 4/36$.

Suppose event *B* is defined as "sum is not 5." What is $P(B)$?

$$P(B) = 1 - 4/36 = 32/36$$

Source : https://www.goldenvalleyhs.org/apps/pages/index.jsp?uREC_ID=322884&type=u&pREC_ID=740733

The Practice of Statistics, 5th Edition.

Basic Rules of Probability

- The probability of any event is a number between 0 and 1.
- All possible outcomes together must have probabilities whose sum is exactly 1.
- If all outcomes in the sample space are equally likely, the probability that event A occurs can be found using the formula

$$P(A) = \frac{\text{number of outcomes corresponding to event } A}{\text{total number of outcomes in sample space}}$$

- The probability that an event does not occur is 1 minus the probability that the event does occur.
- If two events have no outcomes in common, the probability that one or the other occurs is the sum of their individual probabilities.

Two events A and B are **mutually exclusive** (**disjoint**) if they have no outcomes in common and so can never occur together—that is, if $P(A \text{ and } B) = 0$.

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The Practice of Statistics, 5th Edition.

Basic Rules of Probability

We can summarize the basic probability rules more concisely in symbolic form.

Basic Probability Rules
•For any event A, $0 \le P(A) \le 1$.
•If S is the sample space in a probability model, $$P(S) = 1.$$
•In the case of equally likely outcomes, $$P(A) = \frac{\text{number of outcomes corresponding to event } A}{\text{total number of outcomes in sample space}}$$
•**Complement rule:** $P(A^C) = 1 - P(A)$
•**Addition rule for mutually exclusive events:** If A and B are mutually exclusive, $$P(A \text{ or } B) = P(A) + P(B).$$

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The Practice of Statistics, 5th Edition.

Two-Way Tables and Probability

When finding probabilities involving two events, a two-way table can display the sample space in a way that makes probability calculations easier.

Consider the example on page 309. Suppose we choose a student at random. Find the probability that the student

	Pierced Ears?		
Gender	**Yes**	**No**	**Total**
Male	19	71	**90**
Female	84	4	**88**
Total	**103**	**75**	**178**

a) has pierced ears.

b) is a male with pierced ears.

c) is a male or has pierced ears.

Define events *A*: is male and *B*: has pierced ears.

(a) Each student is equally likely to be chosen. 103 students have pierced ears. So, *P*(pierced ears) =*P*(*B*) = 103/178.

Source : https://www.goldenvalleyhs.org/apps/pages/index.jsp?uREC_ID=322884&type=u&pREC_ID=740733
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Total	**103**	**75**	**178**

a) has pierced ears.

b) is a male with pierced ears.

c) is a male or has pierced ears.

Define events *A*: is male and *B*: has pierced ears.

(b) We want to find *P*(male *and* pierced ears), that is, *P*(*A* and *B*). Look at the intersection of the "Male" row and "Yes" column. There are 19 males with pierced ears. So, *P*(*A* and *B*) = 19/178.

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a) has pierced ears.

b) is a male with pierced ears.

c) is a male or has pierced ears.

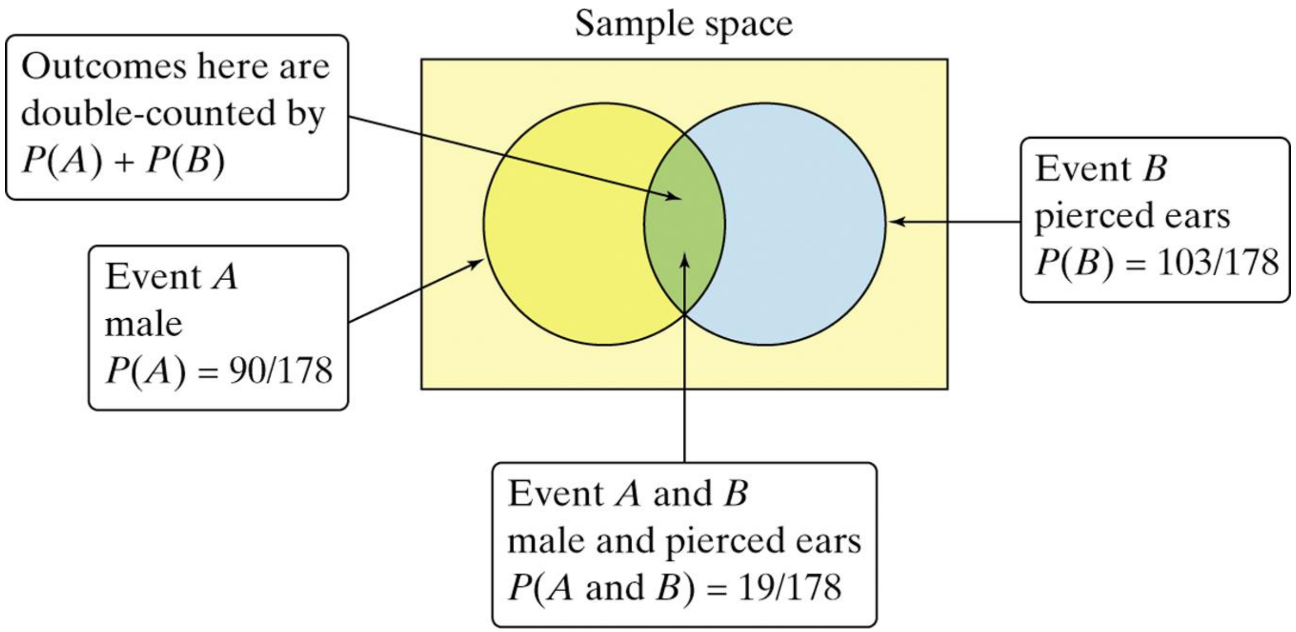
Define events *A*: is male and *B*: has pierced ears.

(c) We want to find *P*(male *or* pierced ears), that is, *P*(*A* or *B*). There are 90 males in the class and 103 individuals with pierced ears. However, 19 males have pierced ears – don't count them twice!
P(*A* or *B*) = (19 + 71 + 84)/178. So, *P*(*A* or B) = 174/178

Source : https://www.goldenvalleyhs.org/apps/pages/index.jsp?uREC_ID=322884&type=u&pREC_ID=740733
The Practice of Statistics, 5th Edition.

General Addition Rule for Two Events

We can’t use the addition rule for mutually exclusive events unless the events have no outcomes in common.



General Addition Rule for Two Events
If A and B are any two events resulting from some chance process, then $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

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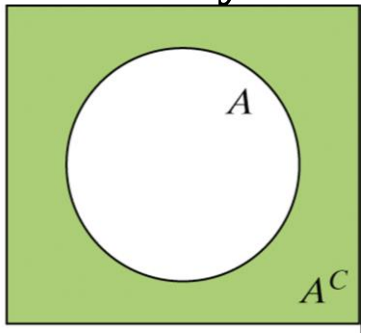
The Practice of Statistics, 5th Edition.

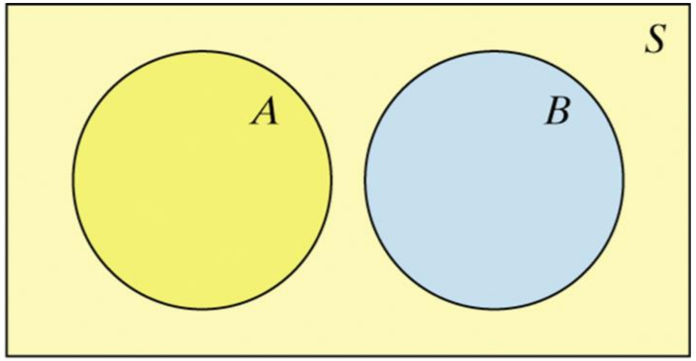
Venn Diagrams and Probability

Because Venn diagrams have uses in other branches of mathematics, some standard vocabulary and notation have been developed.

The complement A^C contains exactly the outcomes that are not in A.



The events *A* and *B* are mutually exclusive (disjoint) because they do not overlap. That is, they have no outcomes in common.



Source : https://www.goldenvalleyhs.org/apps/pages/index.jsp?uREC_ID=322884&type=u&pREC_ID=740733
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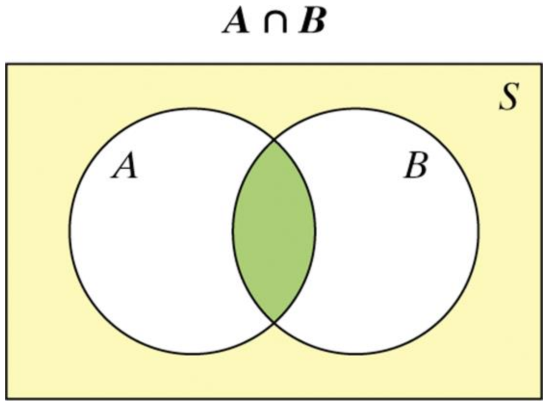
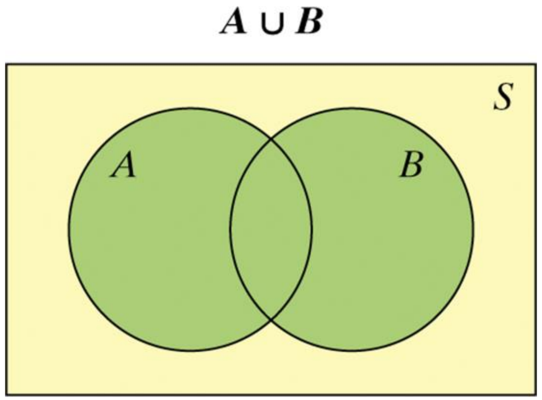
Venn Diagrams and Probability

The intersection of events *A* and *B* ($A \cap B$) is the set of all outcomes in both events *A and B*.



The union of events *A* and *B* ($A \cup B$) is the set of all outcomes in either event *A or B*.



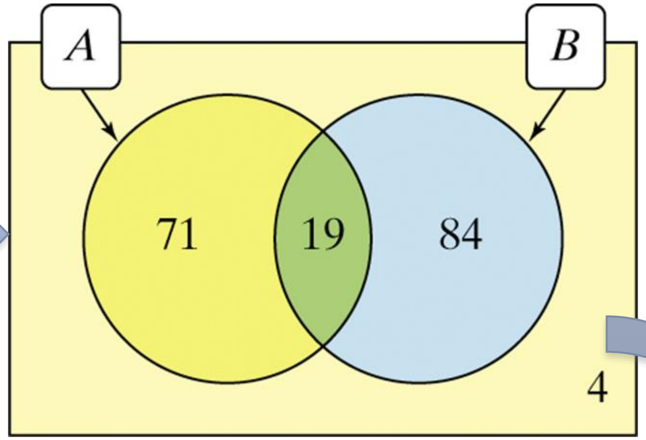
Hint: To keep the symbols straight, remember ∪ for **u**nion and ∩ for i**n**tersection.

Source : https://www.goldenvalleyhs.org/apps/pages/index.jsp?uREC_ID=322884&type=u&pREC_ID=740733
The Practice of Statistics, 5th Edition.

Venn Diagrams and Probability

Recall the example on gender and pierced ears. We can use a Venn diagram to display the information and determine probabilities.

	Pierced Ears?		
Gender	**Yes**	**No**	**Total**
Male	19	71	**90**
Female	84	4	**88**
Total	**103**	**75**	**178**



Define events *A*: is male and *B*: has pierced ears.

Region in Venn diagram	In words	In symbols	Count
In the intersection of two circles	Male and pierced ears	$A \cap B$	19
Inside circle *A*, outside circle *B*	Male and no pierced ears	$A \cap B^C$	71
Inside circle *B*, outside circle *A*	Female and pierced ears	$A^C \cap B$	84
Outside both circles	Female and no pierced ears	$A^C \cap B^C$	4

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The Practice of Statistics, 5th Edition.

Probability of an Event

To find the probability of an event A, we sum all the probabilities assigned to the sample points in A. This sum is called the **probability** of A and is denoted by $P(A)$.

The **probability** of an event A is the sum of the weights of all sample points in A. Therefore,

$$0 \leq P(A) \leq 1, \quad P(\phi) = 0, \quad \text{and} \quad P(S) = 1.$$

Furthermore, if $A_1, A_2, A_3, \ldots$ is a sequence of mutually exclusive events, then

$$P(A_1 \cup A_2 \cup A_3 \cup \cdots) = P(A_1) + P(A_2) + P(A_3) + \cdots .$$

The likelihood of the occurrence of an event resulting from such a statistical experiment is evaluated by means of a set of real numbers, called weights or probabilities, ranging from 0 to 1.

Source : Probability & Statistics for Engineers & Scientists, by Ronald E. Walpole, Raymond H. Myers, Sharon L. Myers, Keying Ye, 9th Edition, Prentice Hall

Example for Probability of an Event

Example 2.24: A coin is tossed twice. What is the probability that at least 1 head occurs?

***Solution* :** The sample space for this experiment is $S = \{HH, HT, TH, TT\}$. If the coin is balanced, each of these outcomes is equally likely to occur. Therefore, we assign a probability of ω to each sample point. Then $4\omega = 1$, or $\omega = 1/4$. If A represents the event of at least 1 head occurring, then

$$A = \{HH, HT, TH\} \text{ and } P(A) = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}.$$

Example for Probability of an Event

Example 2.25: A die is loaded in such a way that an even number is twice as likely to occur as an odd number. If E is the event that a number less than 4 occurs on a single toss of the die, find P(E).

Solution: The sample space is $S = \{1, 2, 3, 4, 5, 6\}$. We assign a probability of w to each odd number and a probability of $2w$ to each even number. Since the sum of th probabilities must be 1, we have $9w = 1$ or $w = 1/9$. Hence, probabilities of 1/9 and 2/9 are assigned to each odd and even number, respectively. Therefore

$$E = \{1, 2, 3\} \text{ and } P(E) = \frac{1}{9} + \frac{2}{9} + \frac{1}{9} = \frac{4}{9}$$

Source : Probability & Statistics for Engineers & Scientists, by Ronald E. Walpole, Raymond H. Myers, Sharon L. Myers, Keying Ye, 9th Edition, Prentice Hall

Example for Probability of an Event

Example 2.26: In Example 2.25, let A be the event that an even number turns up and let B be the event that a number divisible by 3 occurs. Find P(A ∪ B) and P(A ∩ B).

Solution: For the events A={2,4,6} and B={3,6}, we have A∪B ={2,3,4,6} and A∩B = {6}. By assigning a probability of 1/9 to each odd number and 2/9 to each even number, we have

$$P(A \cup B) = \frac{2}{9} + \frac{1}{9} + \frac{2}{9} + \frac{2}{9} = \frac{7}{9} \quad \text{and} \quad P(A \cap B) = \frac{2}{9}$$

If the sample space for an experiment contains *N* elements, all of which are equally likely to occur, we assign a probability equal to 1/*N* to each of the *N* points. The probability of any event *A* containing *n* of these *N* sample points is then the ratio of the number of elements in *A* to the number of elements in *S*.

Source : Probability & Statistics for Engineers & Scientists, by Ronald E. Walpole, Raymond H. Myers, Sharon L. Myers, Keying Ye, 9th Edition, Prentice Hall

Rule 2.3

If an experiment can result in any one of N different equally likely outcomes, and if exactly n of these outcomes correspond to event A, then the probability of event A is

$$P(A) = \frac{n}{N}.$$

Source : Probability & Statistics for Engineers & Scientists, by Ronald E. Walpole, Raymond H. Myers, Sharon L. Myers, Keying Ye, 9th Edition, Prentice Hall

Additive Rules

Additive Rule applies to unions of events.

If A and B are two events, then $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

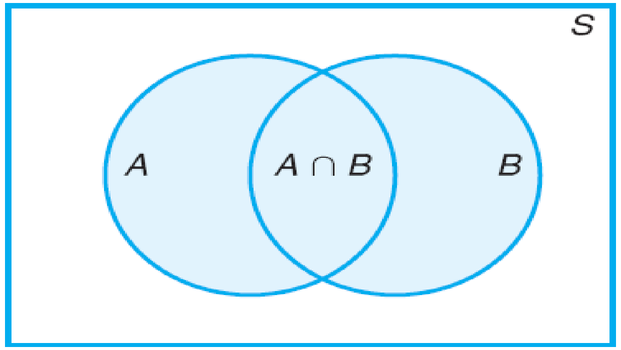


Figure 2.7: Additive rule of probability

Proof **:** Consider the Venn diagram in Figure 2.7. The $P(A \cup B)$ is the sum of the probabilities of the sample points in $A \cup B$. Now $P(A) + P(B)$ is the sum of all the probabilities in A plus the sum of all the probabilities in B. Therefore, we have added the probabilities in $(A \cap B)$ twice. Since these probabilities add up to $P(A \cap B)$, we must subtract this probability once to obtain the sum of the probabilities in $A \cup B$.

Source : Probability & Statistics for Engineers & Scientists, by Ronald E. Walpole, Raymond H. Myers, Sharon L. Myers, Keying Ye, 9th Edition, Prentice Hall

Corollary

If A and B are mutually exclusive, then $P(A \cup B) = P(A) + P(B)$.

If A and B are mutually exclusive, $A \cap B = 0$ and then $P(A \cap B) = P(\varphi) = 0$.

If $A_1, A_2, \ldots, A_n$ are mutually exclusive, then

$$P(A_1 \cup A_2 \cup \cdot \cdot \cdot \cup A_n) = P(A_1) + P(A_2) + \cdot \cdot \cdot + P(A_n).$$

If $A_1, A_2, \ldots, A_n$ is a partition of sample space S, then

$$P(A_1 \cup A_2 \cup \cdot \cdot \cdot \cup A_n) = P(A_1) + P(A_2) + \cdot \cdot \cdot + P(A_n) = P(S) = 1.$$

Theorem 2.8

For three events A, B, and C,

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C).$$

Source : Probability & Statistics for Engineers & Scientists, by Ronald E. Walpole, Raymond H. Myers, Sharon L. Myers, Keying Ye, 9th Edition, Prentice Hall

Additive Rule Example

Example 2.29: John is going to graduate from an industrial engineering department in a university by the end of the semester. After being interviewed at two companies he likes, he assesses that his probability of getting an offer from company A is 0.8, and his probability of getting an offer from company B is 0.6. If he believes that the probability that he will get offers from both companies is 0.5, what is the probability that he will get at least one offer from these two companies?

Solution : Using the additive rule, we have $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.8 + 0.6 - 0.5 = 0.9$.

Source : Probability & Statistics for Engineers & Scientists, by Ronald E. Walpole, Raymond H. Myers, Sharon L. Myers, Keying Ye, 9th Edition, Prentice Hall

Additive Rule Example

Example 2.31: If the probabilities are, respectively, 0.09, 0.15, 0.21, and 0.23 that a person purchasing a new automobile will choose the color green, white, red, or blue, what is the probability that a given buyer will purchase a new automobile that comes in one of those colors?

Solution **:** Let G, W, R, and B be the events that a buyer selects, respectively, a green, white, red, or blue automobile. Since these four events are mutually exclusive, the probability is $P(G \cup W \cup R \cup B) = P(G) + P(W) + P(R) + P(B) = 0.09 + 0.15 + 0.21 + 0.23 = 0.68$.

Source : Probability & Statistics for Engineers & Scientists, by Ronald E. Walpole, Raymond H. Myers, Sharon L. Myers, Keying Ye, 9th Edition, Prentice Hall

Theorem 2.9

Theorem 2.9: If A and A are complementary events, then P(A) + P(A$'$) = 1.

Proof **:** Since A ∪ A = S and the sets A and A are disjoint, 1 = P(S) = P(A ∪ A$'$) = P(A) + P(A$'$).

Example

Example 2.32: If the probabilities that an automobile mechanic will service 3, 4, 5, 6, 7, or 8 or more cars on any given workday are, respectively, 0.12, 0.19, 0.28, 0.24, 0.10, and 0.07, what is the probability that he will service at least 5 cars on his next day at work?

Solution **:** Let E be the event that at least 5 cars are serviced. Now, $P(E) = 1 - P(E)$, where E is the event that fewer than 5 cars are serviced. Since $P(E) = 0.12 + 0.19 = 0.31$, it follows from Theorem 2.9 that $P(E) = 1 - 0.31 = 0.69$.

Source : Probability & Statistics for Engineers & Scientists, by Ronald E. Walpole, Raymond H. Myers, Sharon L. Myers, Keying Ye, 9th Edition, Prentice Hall

Exercise

Question 2.51 A box contains 500 envelopes, of which 75 contain \$100 in cash, 150 contain \$25, and 275 contain \$10. An envelope may be purchased for \$25. What is the sample space for the different amounts of money? Assign probabilities to the sample points and then find the probability that the first envelope purchased contains less than \$100.

Solution : $S = \{\$10, \$25, \$100\}$ with weights 275/500 = 11/20, 150/500 = 3/10, and 75/500 = 3/20, respectively. The probability that the first envelope purchased contains less than \$100 is equal to 11/20 + 3/10 = 17/20

Source : Probability & Statistics for Engineers & Scientists, by Ronald E. Walpole, Raymond H. Myers, Sharon L. Myers, Keying Ye, 9th Edition, Prentice Hall

Exercise

Question 2.55 If each coded item in a catalog begins with 3 distinct letters followed by 4 distinct nonzero digits, find the probability of randomly selecting one of these coded items with the first letter a vowel and the last digit even.

Solution: By Theorem 2.2, there are N = (26)(25)(24)(9)(8)(7)(6) = 47,174,400 possible ways to code the items of which n = (5)(25)(24)(8)(7)(6)(4) = 4,032,000 begin with a vowel and end with an even digit. Therefore, n/N= 10/117.

Exercise

Question 2.57: If a letter is chosen at random from the English alphabet, find the probability that the letter

a) is a vowel exclusive of *y*;

b) is listed somewhere ahead of the letter *j*;

c) is listed somewhere after the letter *g*.

Solution :

a) Since 5 of the 26 letters are vowels, we get a probability of 5/26.

b) Since 9 of the 26 letters precede j, we get a probability of 9/26.

c) Since 19 of the 26 letters follow g, we get a probability of 19/26.

Source : Probability & Statistics for Engineers & Scientists, by Ronald E. Walpole, Raymond H. Myers, Sharon L. Myers, Keying Ye, 9th Edition, Prentice Hall

Exercise

Question 2.64: Interest centers around the life of an electronic component. Suppose it is known that the probability that the component survives for more than 6000 hours is 0.42. Suppose also that the probability that the component survives *no longer than* 4000 hours is 0.04.

(a) What is the probability that the life of the component is less than or equal to 6000 hours?

(b) What is the probability that the life is greater than 4000 hours?

Solution :

(a) $1 - 0.42 = 0.58$;

(b) $1 - 0.04 = 0.96$.

Source : Probability & Statistics for Engineers & Scientists, by Ronald E. Walpole, Raymond H. Myers, Sharon L. Myers, Keying Ye, 9th Edition, Prentice Hall

Exercise

Question 2.63: According to Consumer Digest (July/August 1996), the probable location of personal computers (PC) in the home is as follows: Adult bedroom: 0.03,

Child bedroom: 0.15, Other bedroom: 0.14, Office or den: 0.40, Other rooms: 0.28

(a) What is the probability that a PC is in a bedroom?

(b) What is the probability that it is not in a bedroom?

Solution :

a) 0.32 (0.03+0.15+0.14)
b) 0.68 (1-0.32)

Source : Probability & Statistics for Engineers & Scientists, by Ronald E. Walpole, Raymond H. Myers, Sharon L. Myers, Keying Ye, 9th Edition, Prentice Hall

What is Conditional Probability?

The probability we assign to an event can change if we know that some other event has occurred. This idea is the key to many applications of probability.

When we are trying to find the probability that one event will happen under the condition that some other event is already known to have occurred, we are trying to determine a conditional probability.

The probability that one event happens given that another event is already known to have happened is called a **conditional probability**.

Suppose we know that event A has happened. Then the probability that event *B* happens given that event *A* has happened is denoted by $P(B \mid A)$.

Read | as "given that" or "under the condition that"

Source : https://www.goldenvalleyhs.org/apps/pages/index.jsp?uREC_ID=322884&type=u&pREC_ID=740733
The Practice of Statistics, 5th Edition.

Definition of Conditional Probability

The probability of an event B occurring when it is known that some event A has occurred is called a **Conditional Probability** and is denoted by $P(B|A)$.

The symbol $P(B|A)$ is usually read "the probability that B occurs given that A occurs" or simply "the probability of B, given A."

The conditional probability of B, given A, denoted by $P(B|A)$, is defined by

$$P(B|A) = \frac{P(A \cap B)}{P(A)}, \quad \text{provided} \quad P(A) > 0.$$

Source : Probability & Statistics for Engineers & Scientists, by Ronald E. Walpole, Raymond H. Myers, Sharon L. Myers, Keying Ye, 9th Edition, Prentice Hall

Calculating Conditional Probabilities

Calculating Conditional Probabilities

To find the conditional probability $P(A \mid B)$, use the formula

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

The conditional probability $P(B \mid A)$ is given by

$$P(B \mid A) = \frac{P(B \cap A)}{P(A)}$$

Source : https://www.goldenvalleyhs.org/apps/pages/index.jsp?uREC_ID=322884&type=u&pREC_ID=740733
The Practice of Statistics, 5th Edition.

As an additional illustration, suppose that our sample space S is the population of adults in a small town who have completed the requirements for a college degree. We shall categorize them according to gender and employment status. The data are given in Table 2.1.

Table 2.1: Categorization of the Adults in a Small Town

	Employed	Unemployed	Total
Male	460	40	500
Female	140	260	400
Total	600	300	900

One of these individuals is to be selected at random for a tour throughout the country to publicize the advantages of establishing new industries in the town. We shall be concerned with the following events: M: a man is chosen, E: the one chosen is employed. Using the reduced sample space E, we find that **$P(M|E) = 460/600 = 23/30$**

Source : Probability & Statistics for Engineers & Scientists, by Ronald E. Walpole, Raymond H. Myers, Sharon L. Myers, Keying Ye, 9th Edition, Prentice Hall

Continuation from Previous Slide

Let $n(A)$ denote the number of elements in any set A. Using this notation, since each adult has an equal chance of being selected, we can write

$$P(M|E) = \frac{n(E \cap M)}{n(E)} = \frac{n(E \cap M)/n(S)}{n(E)/n(S)} = \frac{P(E \cap M)}{P(E)},$$

where $P(E \cap M)$ and $P(E)$ are found from the original sample space S. To verify this result, note that

$$P(E) = \frac{600}{900} = \frac{2}{3} \quad \text{and} \quad P(E \cap M) = \frac{460}{900} = \frac{23}{45}.$$

Hence,

$$P(M|E) = \frac{23/45}{2/3} = \frac{23}{30},$$

Source : Probability & Statistics for Engineers & Scientists, by Ronald E. Walpole, Raymond H. Myers, Sharon L. Myers, Keying Ye, 9th Edition, Prentice Hall

Calculating Conditional Probabilities

Consider the two-way table on page 321 of "The Practice of Statistics, 5th Edition"

Define events E: the grade comes from an Engineering and Physical Science (EPS) Course and L: the grade is lower than a B.

	Grade Level			
School	**A**	**B**	**Below B**	**Total**
Liberal Arts	2,142	1,890	2,268	**6300**
Engineering and Physical Sciences	368	432	800	**1600**
Health and Human Services	882	630	588	**2100**
Total	**3392**	**2952**	**3656**	***10000***

Find *P*(*L*)

$$P(L) = 3656 / 10000 = 0.3656$$

Find *P*(*E* | *L*)

$$P(E \mid L) = 800 / 3656 = 0.2188$$

Find *P*(*L* | *E*)

$$P(L \mid E) = 800 / 1600 = 0.5000$$

Source :

- https://www.goldenvalleyhs.org/apps/pages/index.jsp?uREC_ID=322884&type=u&pREC_ID=740733
- The Practice of Statistics, 5th Edition.

The General Multiplication Rule

General Multiplication Rule

The probability that events *A* and *B* both occur can be found using the **general multiplication rule**

$$P(A \cap B) = P(A) \bullet P(B \mid A)$$

where $P(B \mid A)$ is the conditional probability that event *B* occurs given that event *A* has already occurred.

In words, this rule says that for both of two events to occur, first one must occur, and then given that the first event has occurred, the second must occur.

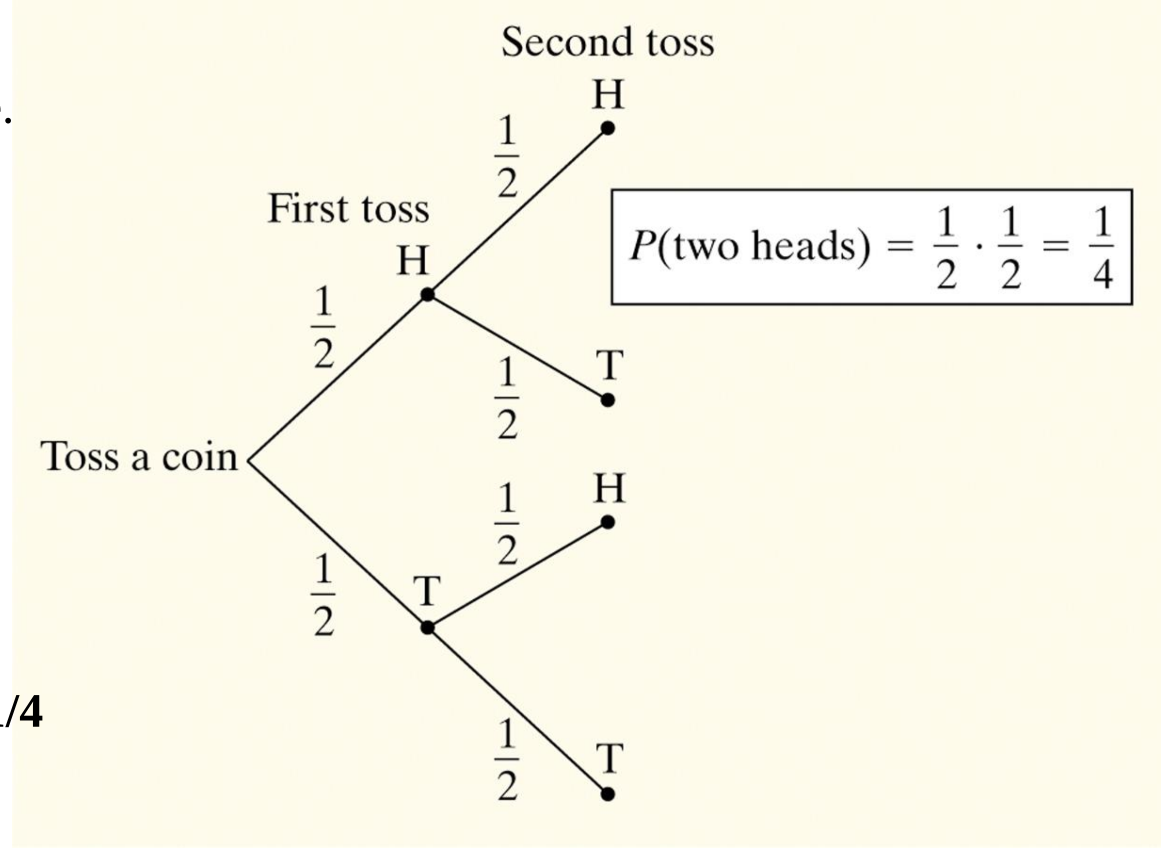
Source : https://www.goldenvalleyhs.org/apps/pages/index.jsp?uREC_ID=322884&type=u&pREC_ID=740733
The Practice of Statistics, 5th Edition.

Tree Diagrams

The general multiplication rule is especially useful when a chance process involves a sequence of outcomes. In such cases, we can use a **tree diagram** to display the sample space.

Consider flipping a coin twice.

What is the probability of getting two heads?



Sample Space:
HH HT TH TT

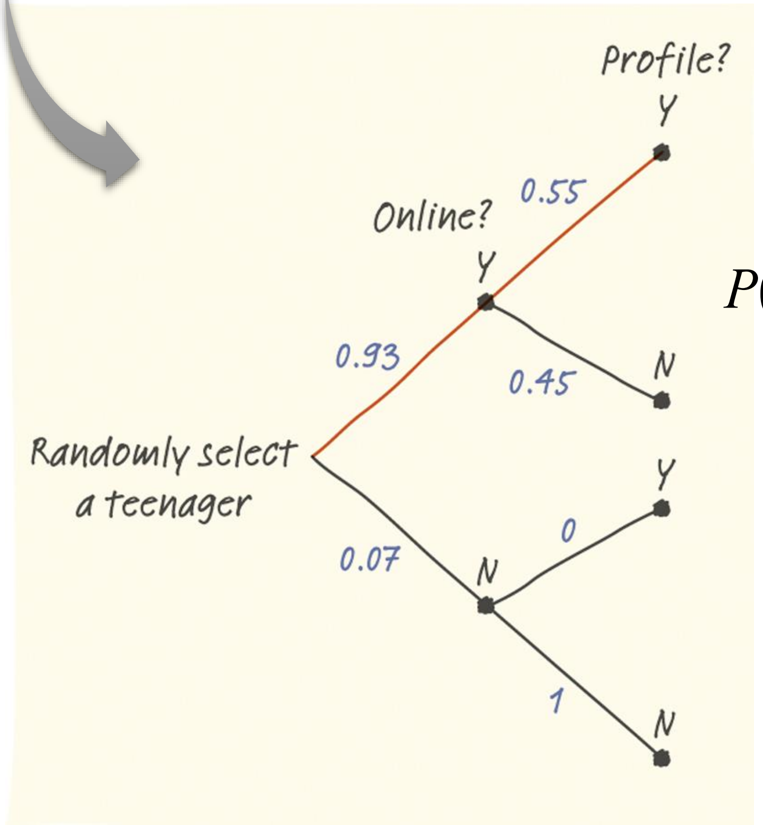
So, *P*(two heads) = *P*(HH) = 1/4

Source : https://www.goldenvalleyhs.org/apps/pages/index.jsp?uREC_ID=322884&type=u&pREC_ID=740733
The Practice of Statistics, 5th Edition.

Example: Tree Diagrams

The Pew Internet and American Life Project finds that 93% of teenagers (ages 12 to 17) use the Internet, and that 55% of online teens have posted a profile on a social-networking site.

What percent of teens are online and have posted a profile?



$$P(\text{online}) = 0.93$$

$$P(\text{profile} \mid \text{online}) = 0.55$$

$$\begin{aligned} P(\text{online and have profile}) &= P(\text{online}) \times P(\text{profile} \mid \text{online}) \\ &= (0.93)(0.55) \\ &= 0.5115 \end{aligned}$$

51.15% of teens are online *and* have posted a profile.

Source : https://www.goldenvalleyhs.org/apps/pages/index.jsp?uREC_ID=322884&type=u&pREC_ID=740733
The Practice of Statistics, 5th Edition.

Exercise

Question 2.73: If *R* is the event that a convict committed armed robbery and *D* is the event that the convict pushed dope, state in words what probabilities are expressed by

a) $P(R|D)$

b) $P(D'|R)$

c) $P(R'|D')$

Solution :

a) The probability that a convict who pushed dope, also committed armed robbery.

b) The probability that a convict who committed armed robbery, did not push dope.

c) The probability that a convict who did not push dope also did not commit armed robbery.

Source : Probability & Statistics for Engineers & Scientists, by Ronald E. Walpole, Raymond H. Myers, Sharon L. Myers, Keying Ye, 9th Edition, Prentice Hall

Exercise

Question 2.74: A class in advanced physics is composed of 10 juniors, 30 seniors, and 10 graduate students. The final grades show that 3 of the juniors, 10 of the seniors, and 5 of the graduate students received an “AA” for the course. If a student is chosen at random from this class and is found to have earned an “AA”, what is the probability that he or she is a senior?

Solution : P (S |A) = 10/18 = 5/9.

Source : Probability & Statistics for Engineers & Scientists, by Ronald E. Walpole, Raymond H. Myers, Sharon L. Myers, Keying Ye, 9th Edition, Prentice Hall

Independent Events

Two events A and B are **independent** if and only if

$$P(B|A) = P(B) \quad \text{or} \quad P(A|B) = P(A),$$

assuming the existences of the conditional probabilities. Otherwise, A and B are **dependent**.

Theorem 2.10

If in an experiment the events A and B can both occur, then

$$P(A \cap B) = P(A)P(B|A), \text{ provided } P(A) > 0.$$

The probability that both A and B occur is equal to the probability that A occurs multiplied by the conditional probability that B occurs, given that A occurs. Since the events $A \cap B$ and $B \cap A$ are equivalent, it follows from **Theorem 2.10** that we can also write

$$P(A \cap B) = P(B \cap A) = P(B)P(A|B).$$

In other words, it does not matter which event is referred to as A and which event is referred to as B.

Source : Probability & Statistics for Engineers & Scientists, by Ronald E. Walpole, Raymond H. Myers, Sharon L. Myers, Keying Ye, 9th Edition, Prentice Hall

Example

Example 2.36: Suppose that we have a fuse box containing 20 fuses, of which 5 are defective. If 2 fuses are selected at random and removed from the box in succession without replacing the first, what is the probability that both fuses are defective?

Solution : We shall let A be the event that the first fuse is defective and B the event that the second fuse is defective; then we interpret $A \cap B$ as the event that A occurs and then B occurs after A has occurred. The probability of first removing a defective fuse is 1/4; then the probability of removing a second defective fuse from the remaining 4 is 4/19. Hence,

$$P(A \cap B) = \left(\frac{1}{4}\right)\left(\frac{4}{19}\right) = \frac{1}{19}.$$

Source : Probability & Statistics for Engineers & Scientists, by Ronald E. Walpole, Raymond H. Myers, Sharon L. Myers, Keying Ye, 9th Edition, Prentice Hall