SM 300

Engineering Economics

Demand Forecasting

Major Learning Objectives-

- Elements/properties of a good forecast.
- Qualitative and Quantitative Approaches to Forecasting
- Some forecasting techniques
- Forecast Accuracy

Forecast

- A *statement about the future value* of a variable of interest.

i.e. *predictions about the future*. Eg. Demand.

- Better predictions → More informed decisions.
- Generally, forecasts can be:
  - Long-range: Covering several years. Egs:

Planned power plant that will operate for next 20 years; flyovers, bridges or even profit potential of a new product/service to make tech investments

  - Short-range: A day or week. Egs:

Weather, Weekly sales forecasts

One may also have medium-range forecasts which are typically for a few months to an year

Elements of a Good Forecast

1. Timely

Forecasting horizon must cover the time necessary to implement possible changes--- Inventory levels cannot be changed immediately

2. Accurate [as much as possible]

Degree of accuracy must be stated so that management plans for errors and compares alternative forecasts

3. Reliable

Should work consistently. A technique which sometimes provides very good forecast and sometime very bad will make the management nervous

Elements of a Good Forecast (Contd.)

4. Meaningful Units

Expressed in units which can be understood by the user. Eg. For Financial Mangers forecast should be in currency terms

5. Written

Written format will increase the likelihood of using same information and also help in evaluating later on once the results are in.

6. Simple to Understand and Use

The circumstances under which the technique can be used and limitation of techniques should be clear; otherwise misuse

7. Cost-Effective

Benefits should outweigh costs

Approaches to Forecasts

Qualitative

- Mainly subjective inputs, which may defy precise numerical description
- Permit inclusion of *soft* data like human factors, personal opinions
- May get contaminated with personal biases

and Quantitative

- Either (a) *projection of historical data* [time series] or

(b) *development of associative models* that attempt to utilize *causal (explanatory) variables* to make forecasts

- Mainly analyze objective or *hard* data
- Usually avoid personal biases

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Forecasting Techniques

Judgmental Forecasts

- Uses Qualitative Approach
- Relies on subjective inputs like consumer surveys, the sales staff, managers and executives, and panel of experts.
- Frequently give insights that are not otherwise available
- Generally used for quick forecasts or when reliable data not available

Time-series Forecasts

- Simply attempt to project past experience into the future using historical data with assumption that future will be similar to past
- Some models smooth out random variations in historical data; others identify specific patterns and project patterns into future *without looking into causes* of the pattern

Associative Models

- Use equations consisting of one or more *explanatory* variables.

Eg. Case study of Gary's Detailing Job that was explained in earlier class.

Judgmental Forecasts

1. Executive opinions

- A small group of upper-level managers (eg. In marketing, operations and finance) may meet and collectively develop a forecast
- Often used as a part of *long-range planning and new product* development
- Brings together the *knowledge and talents of various managers*.
- Risk ----- view of one person may prevail
- Sometimes not so good forecast since responsibility is diffused

Judgmental Forecasts (Contd.)

2. Sales force opinions

- Information sources are the members of sales staff or customer service staff due to direct contact with consumers

- Often aware of future customer plans…. However, may not distinguish…. what consumers

  like to do vs. *will* do.

- May get influenced by recent experiences

(overly pessimistic or overly optimistic) and

- May have *conflict of interest*….

when forecasts used to establish sales quotas for them → advantage (for self) in providing low sales forecasts

Judgmental Forecasts (Contd.)

3. Consumer surveys

- Consumers determine demand… therefore solicit i/p from them
- In some cases *every* customer or potential customer *can be contacted*……

However *usually too many customers* or *no way to identify* all potential customers

- Hence, use *consumer surveys* that *sample* *consumer opinions*

Advantage: may *get untapped useful info* … open-ended Qs… *especially important for new product launches*

Disadvantage:

Knowledge and skill required to construct survey, administer, correctly interpret results;

Often time consuming and expensive;

Consumers may show irrational behavior while shopping (Eg. influenced by kids etc.)

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Smoothing Techniques for Time-Series Forecasting

Historical data typically contain an amount of *random variation* [*White Noise*] that obscure systematic movements in the data.

Averaging techniques *smooth fluctuations* in a time series ….because individual highs and lows in data offset each other when combined into average.

A forecast based on an average tends to *exhibit less variability* than the original data.

Techniques for Averaging (Contd.)

Moving average-

A technique that *averages a number of recent actual values*…. updated as new values become available.

$$F_t = MA_n = \frac{A_{t-n} + \ldots A_{t-2} + A_{t-1}}{n}$$

where

F_t = Forecast for time period t

Ma_n = n period moving average

A_{t-1} = Actual value in period t-1

n = Number of periods (data points) in the moving average

Note: As each new actual value becomes available, the forecast is updated by adding the newest value and dropping the oldest and then re-computing the average.

Example Moving Average

Compute a three-period moving average forecast given demand for shopping carts for the last five periods.

Period	Demand
1	42
2	40
3	43
4	40
5	41

$$F_6 = \frac{43 + 40 + 41}{3} = 41.33$$

If actual value in period 6 turns out to be 38 then what would be the moving average forecast for period 7?

Example Moving Average (Contd.)

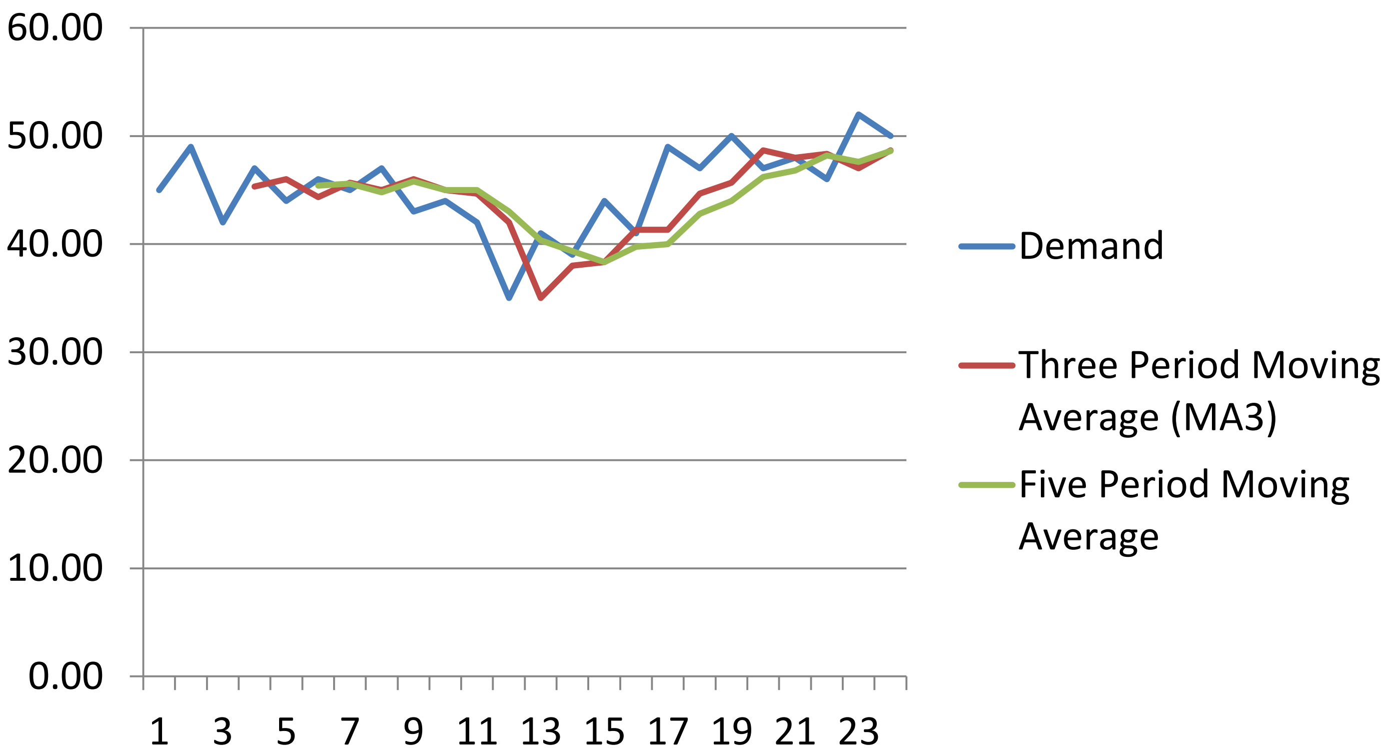
If actual value in period 6 is 38 then three-period moving average forecast in period 7 would be:

Period	Demand
1	42
2	40
3	43
4	40
5	41
6	38

$$F_7 = \frac{40 + 41 + 38}{3} = 39.67$$

Thus, the forecast "moves" by reflecting only the most recent values

More About Moving Averages



A moving average forecast tends to smooth and lag changes in the data

The more periods in a moving average, the greater the forecast will lag changes in the data (i.e. more smooth but less responsive to real changes)

Disadvantage:

All values (oldest as well as the most recent) in the average are weighted equally

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Techniques for Averaging (Contd.)

- **Weighted moving average:**

More recent values in a series are given more weight in computing forecast

$$F_t = W_n A_{t-n} + \ldots W_2 A_{t-2} + W_1 A_{t-1}$$

Note:

The weights must always sum to 1.00

In simple moving average the weights are 1/n for each value.

In weighted moving average the heaviest weights are assigned to the most recent values.

Example Weighted Moving Average

Given the following demand data,

a. Compute a weighted average forecast using a weight of 0.40 for the most recent period, 0.30 for the next most recent, 0.20 for the next, and 0.10 for the next.

b. If the actual demand for period 6 is 39, forecast demand for period 7 using the same weight as in part a.

Period	Demand
1	42
2	40
3	43
4	40
5	41

Answer Example Weighted Moving Average

a. $F_6 = 0.10(40) + 0.20(43) + 0.30(40) + 0.40(41) = 41.0$

b. $F_7 = 0.10(43) + 0.20(40) + 0.30(41) + 0.40(39) = 40.2$

Note: If four weights are used, only the *four recent* demands are used to prepare the forecast.

Advantage over simple moving average:

More reflective of the most recent occurrences

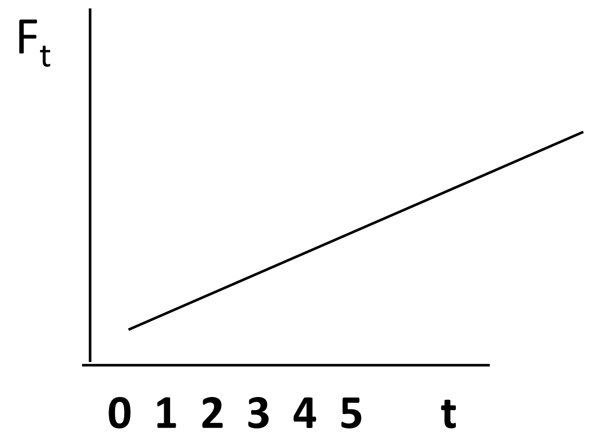
Disadvantage of/challenge in the technique:

Choice of weights is arbitrary and generally involves the use of trial and error to find a suitable weighting scheme.

Trend Equations

A *linear* trend equation has the form

$$F_t = a + bt$$



Where F_t = Forecast for period t

t = Specified number of time periods

a = Value of F_t at t = 0 [Intercept]

b = Slope of the line

Techniques for Linear Trend (Contd.)

Linear Trend Line [Can be derived using Calculus in the same way as derived for an OLS regression line with t == x & F_t = y]

$$b = \frac{n \sum (ty) - \sum t \sum y}{n \sum t^2 - (\sum t)^2}$$

$$a = \frac{\sum y - b \sum t}{n}$$

Where,

n = Number of periods

y = Value of the time series

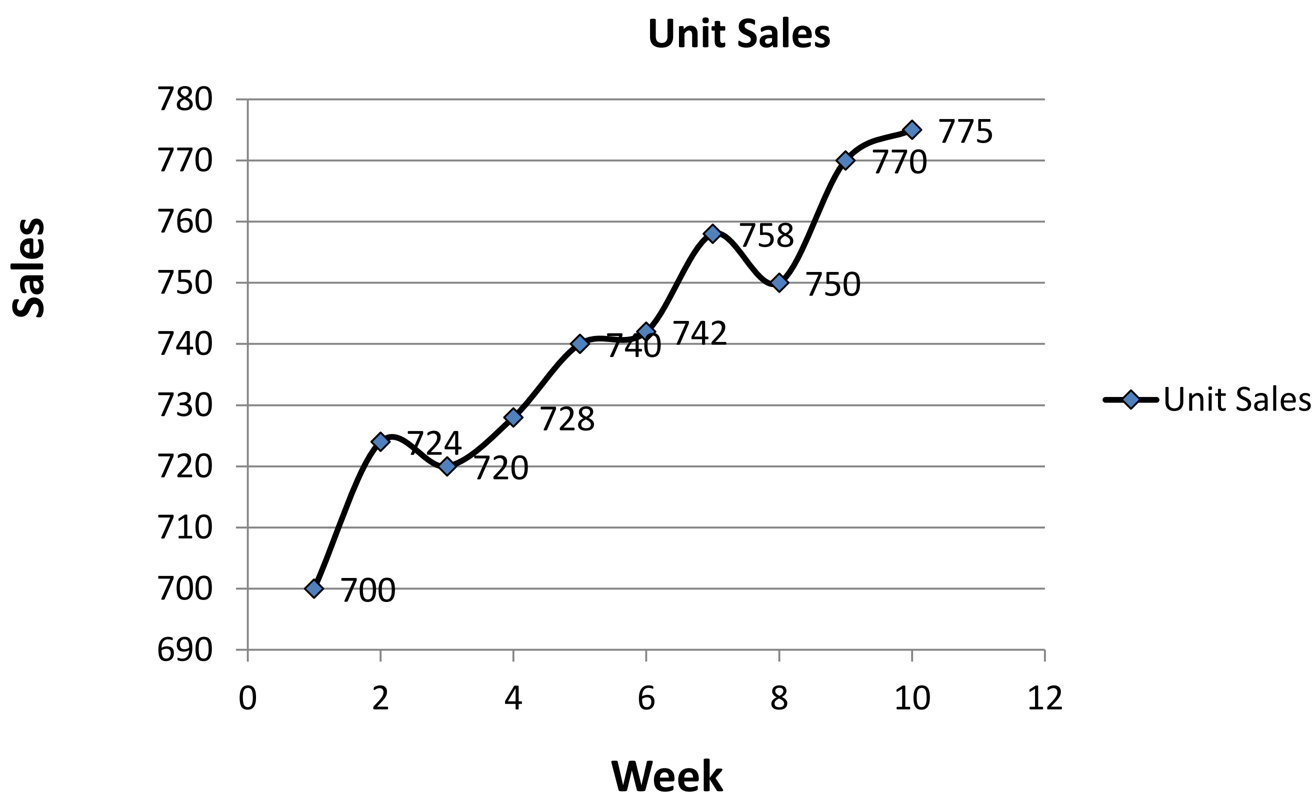
Example Linear Trend Line

Q. Cell phone sales for a firm over the last 10 weeks are shown in the table below. Plot the data, and visually check to see if a linear trend line would be appropriate. Then determine the equation of the trend line, and predict sales for weeks 11 and 12.

Week	Unit Sales
1	700
2	724
3	720
4	728
5	740
6	742
7	758
8	750
9	770
10	775

Example Linear Trend Line (Contd.)

a. The plot is:



The plot suggests that linear trend line would be appropriate

Example Linear Trend Line (Contd.)

b. To calculate **a** and **b**:

n = 10 and

t	y	t^2	ty
1	700	1	700
2	724	4	1448
3	720	9	2160
4	728	16	2912
5	740	25	3700
6	742	36	4452
7	758	49	5306
8	750	64	6000
9	770	81	6930
10	775	100	7750
$\Sigma t = 55$ $(\Sigma t)^2 = 3025$	$\Sigma y = 7407$	$\Sigma t^2 = 385$	$\Sigma ty = 41358$

b = [10*41358 – 55*7407] / [10 *385 – 3025] = 6195/825 = **7.51**

a = [7407- 7.51 * 55] / 10 = **699.40**

Therefore, Trend Line Equation is given as:

$\mathbf{F_t = 699.40 + 7.51\ t}$

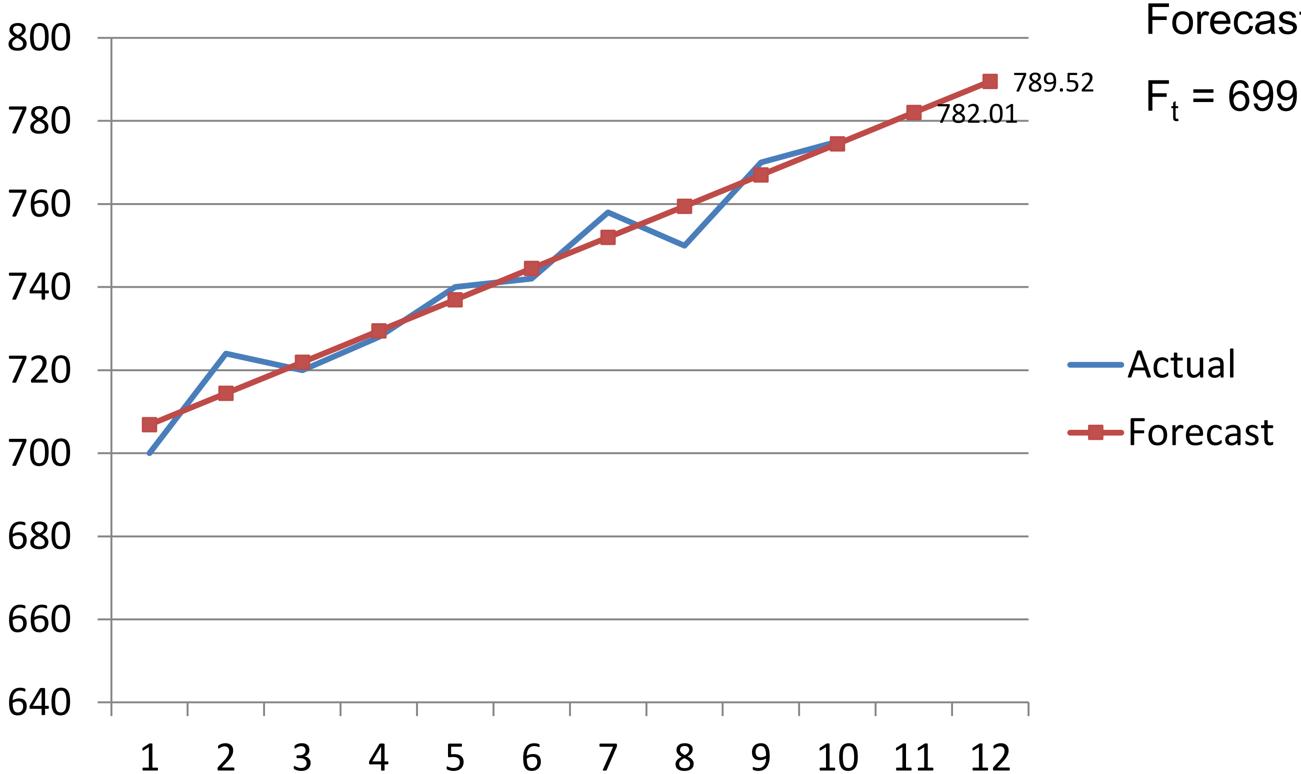
Example Linear Trend Line (Contd.)

c. The forecast for the next two periods are:

F_{11} = 699.40 + 7.51 * 11 = **782.01**

and

F_{12} = 699.40 + 7.51 * 12 = **789.52**



Forecast

F_t = 699.40 + 7.51 t

Actual	Forecast
700	706.91
724	714.42
720	721.93
728	729.44
740	736.95
742	744.46
758	751.97
750	759.48
770	766.99
775	774.50
	782.01
	789.52

Associative Forecasting Techniques

- Rely on identification of related variables that can be used to predict values of the variable of interest… cause and effect relationships

Eg. Sales of a product may be a function of Price of the product, price of the competitor's product, advertising intensity etc.

- The essence of associative techniques is the development of an equation that summarizes the effects of **predictor variables**.
- The primary method of analysis is known as **regression**

Associative Forecasting Techniques (Contd.)

Simple Linear Regression

- It is a two-variable relationship based on least square estimation technique

$$y = a + bx$$

where:

y = Value of the dependent variable

x = Value of the independent variable

a = y-intercept

b = Slope of the regression line

$$b = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - (\sum x)^2}$$

$$a = \frac{\sum y - b\sum x}{n}$$

OR

$$a = \bar{y} - b\bar{x}$$

Where n= number of paired observations

Comments on Use of Linear Regression Analysis

<u>Assumptions</u>:

- Variations around the line are random
- Deviations around the line normally distributed
- Predictions are being made only within the range of observed values
- For best results:
  - Always plot the data to verify linearity
  - Check for data being time-dependent- If yes, use time-series or time as an independent variable as part of multiple regression analysis
  - Small correlation may imply that other variables are important

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Forecast Accuracy

- Error =

Difference between actual value and predicted value

- Mean Absolute Deviation (MAD)
  - Average absolute error
- Mean Squared Error (MSE)
  - Average of squared error

Forecast Accuracy (Contd.)

For summation over t

Mean Absolute Deviation (MAD) $= \dfrac{\sum \left| \text{Actual} - \text{forecast} \right|}{\text{n}}$

Mean Square Error (MSE) $= \dfrac{\sum (\text{Actual} - \text{forecast})^2}{\text{n - 1}}$

Forecast Accuracy (Example)

Period	Actual	Forecast
1	217	215
2	213	216
3	216	215
4	210	214
5	213	211
6	219	214
7	216	217
8	212	216

Forecast Accuracy (Example)

Period	Actual	Forecast	(A-F)	|A-F|	(A-F)^2	(|A-F|/Actual)*100
1	217	215	2	2	4	0.92
2	213	216	-3	3	9	1.41
3	216	215	1	1	1	0.46
4	210	214	-4	4	16	1.9
5	213	211	2	2	4	0.94
6	219	214	5	5	25	2.28
7	216	217	-1	1	1	0.46
8	212	216	-4	4	16	1.89
			-2	22	76	10.26

MAD= 2.750

MSE= 10.857

Methods of Economic Analysis in Engineering

Time Value of Money:

Would you prefer to have Rs. 1 million now or Rs. 1 million 10 years from now?

- A dollar received today is worth more than a dollar received tomorrow
  - This is because a dollar received today can be invested to earn interest
  - The amount of interest earned depends on the rate of return that can be earned on the investment
  - In an inflationary period a rupee today represents a greater *real purchasing power* than a rupee a year hence.
- Time value of money quantifies the value of a dollar/rupee through time

Interest and Rate of Return (ROR)

- **Interest** is a manifestation of time value of money.
- Calculated as difference between an ending amount and a beginning amount of money

➢ Interest = end amount – original amount

- Interest rate is interest over specified time period based on original amount

$$\textbf{Interest rate (\%)} = \frac{\text{interest accrued per time unit}}{\text{original amount}} \times 100\%$$

- Interest rate and rate of return (ROR) have same numeric value, but different interpretations

Interest Rate and ROR Interpretations

Borrower's perspective

Take loan of $5,000 for one year; repay $5,350

Interest paid = $350

Interest rate = 350/5,000

= 7%

INTEREST RATE

Investor's perspective

Invest (or lend) $5,000 for one year; receive $5,350

Interest earned = $350

Rate of return = 350/5,000

= 7%

RATE OF RETURN

Minimum Attractive Rate of Return (MARR)

- The Minimum Attractive Rate of Return (MARR) is a *reasonable rate of return* established for the evaluation and selection of alternatives.
- A project is not economically viable unless it is expected to return at least the MARR.
- MARR is also referred to as the *hurdle rate, cutoff rate, benchmark rate, and minimum acceptable rate of return.*
- The MARR is established by (financial) managers and is used as a criterion against which an alternative's ROR is measured, when making the accept/reject investment decision.

Eg. In the United States, the current U.S. Treasury Bill return is sometimes used as the *benchmark safe rate*. The MARR will always be higher than this, or a similar, safe rate.

ROR and MARR

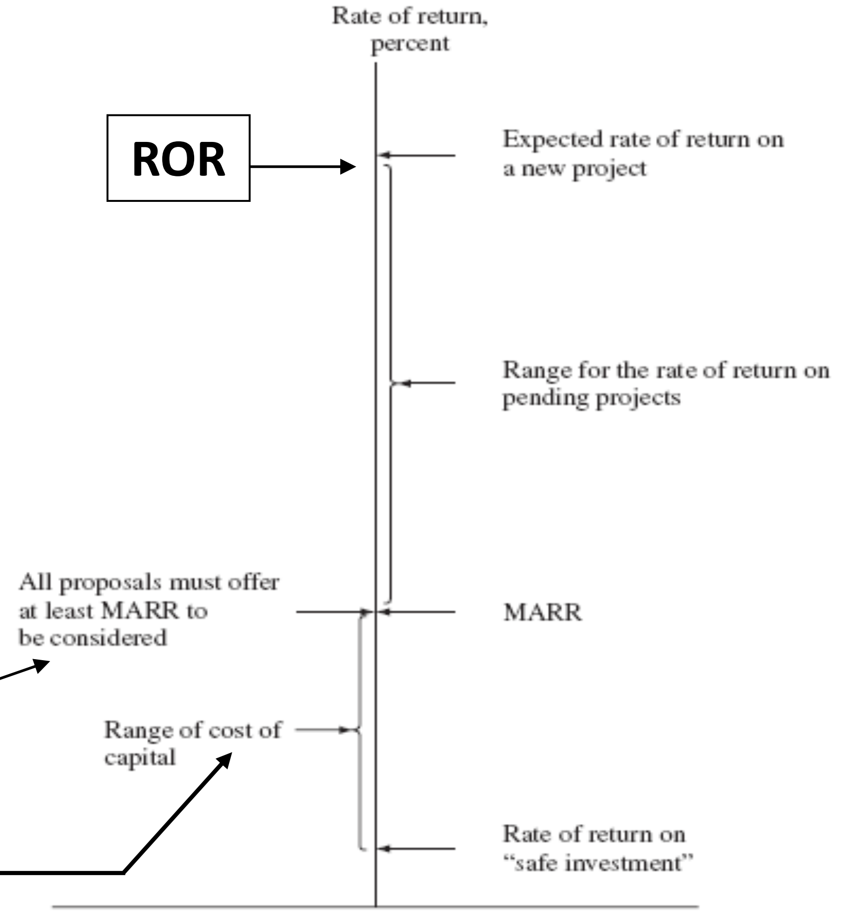
Cost of capital (COC) – interest rate paid for funds to finance projects

MARR – Minimum ROR needed for an alternative to be justified and economically acceptable. MARR ≥ COC.

If COC = 5% and 6% must be realized, MARR = 11%

Always, for acceptable projects

ROR ≥ MARR > COC



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Equivalence

- Before we delve into the economic aspects, think of the many types of equivalency we may utilize daily by transferring from one scale to another. Eg.

Length- 12 Inches = 1 Feet

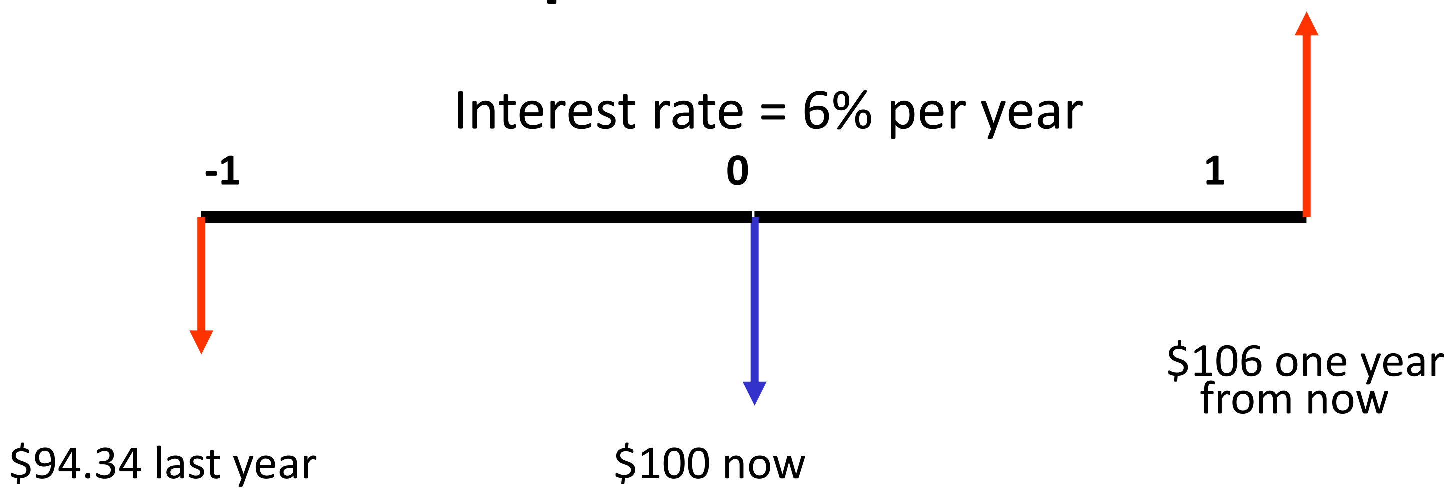
100 centimetres = 1 meter

- Economic equivalence is a fundamental concept upon which engineering economy computations are based.

Economic equivalence is established, in general, when we are indifferent between a future payment, or series of future payments, and a present sum of money.

Equivalence

Different sums of money at different times may be equal in economic value



Interpretation:

$94.34 last year, $100 now, and $106 one year from now are equivalent *only* at an interest rate of 6% per year

Simple Interest

Simple interest is always based on the original amount, which is also called the principal

Interest per period = (principal)(interest rate)

Total interest = (principal)(n periods)(interest rate)

Example: Invest $250,000 in a bond at 5% per year simple

Interest each year =

$$250{,}000(0.05) = \$12{,}500$$

Interest over 3 years =

$$250{,}000(3)(0.05) = \$37{,}500$$

Compound Interest

Compound interest is based on the principal plus all accrued interest

Interest per period = (principal + accrued interest)(interest rate)

Total interest = (principal)(1+interest rate)$^{n \text{ periods}}$ – principal

Example: Invest $250,000 at 5% per year compounded

Interest, year 1 = 250,000(0.05) = $12,500

Interest, year 2 = 262,500(0.05) = $13,125

Interest, year 3 = 275,625(0.05) = $13,781

Interest over 3 years = 250,000(1.05)^3 – 250,000 = $39,406

Illustration of Simple versus Compound Interest

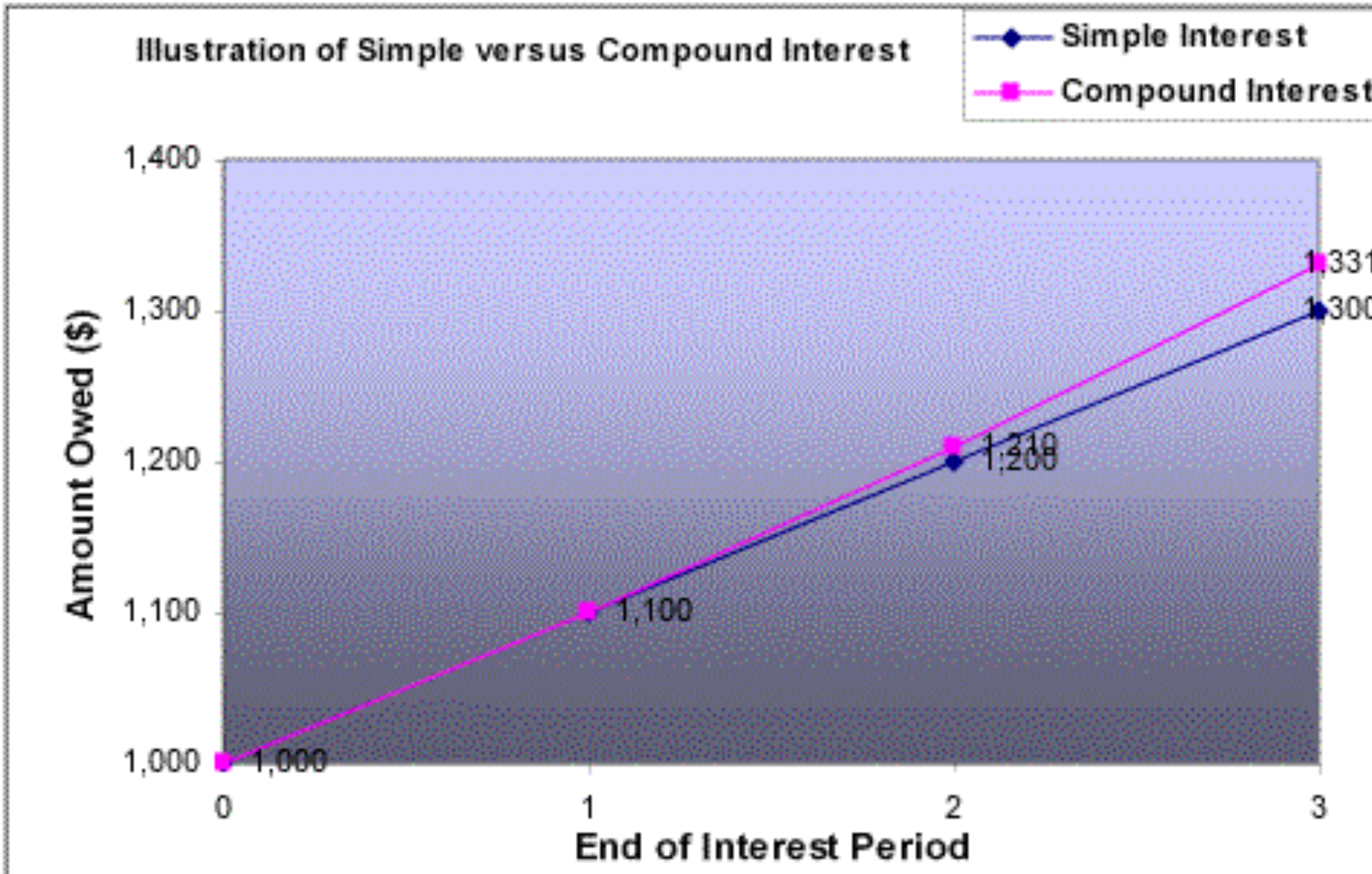


Illustration of Simple versus Compound Interest

Last Canadian Bank

Simple Interest Calculation	Simple Interest	Accumulated Year-End Balance
Year 1 $1,000.00 × 9%	$ 90.00	$1,090.00
Year 2 $1,000.00 × 9%	90.00	$1,180.00
Year 3 $1,000.00 × 9%	90.00	$1,270.00
	$270.00	

First Canadian Bank

Compound Interest Calculation	Compound Interest	Accumulated Year-End Balance
Year 1 $1,000.00 × 9%	$ 90.00	$1,090.00
Year 2 $1,090.00 × 9%	98.10	$1,188.10
Year 3 $1,188.10 × 9%	106.93	$1,295.03
	$295.03	

$25.03 Difference

Terminology and Symbols

- ✓ t = time index in periods; years, months, etc.
- ✓ P = present sum of money at time t = 0; $
- ✓ F = sum of money at a future time t; $
- ✓ A = series of equal, end-of-period cash flows; currency per period, $ per year
- ✓ n = total number of periods; years, months
- ✓ i = compound interest rate or rate of return; % per year

Terminology and Symbols

Example: Borrow \$5,000 today and repay annually for 10 years starting next year at 5% per year compounded. Identify all symbols.

Given: P = \$5,000

i = 5% per year

n = 10 years

t = year 1, 2, ..., 10

Find: A = ? per year

(F not used here)

Cash Flow Estimates

Cash inflow – receipt, revenue, income, saving

Cash outflows – cost, expense, disbursement, loss

Net cash flow (NCF) = inflow – outflow

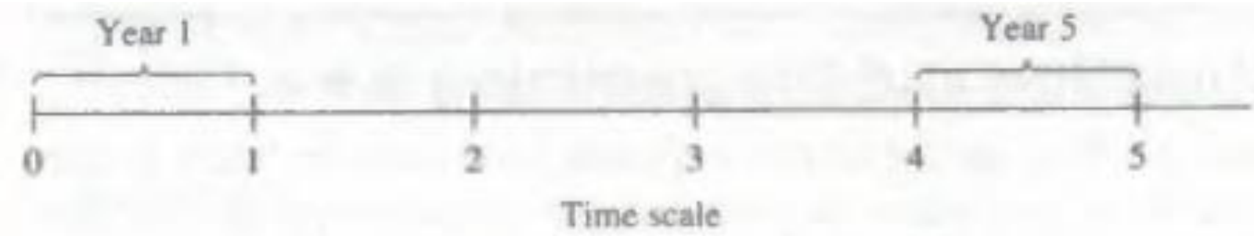
End-of-period convention: all cash flows and NCF occur at the end of an interest period

Cash Flow Diagrams

- The cash flow diagram is a very important tool in an economic analysis, especially when the cash flow series is complex.
- It is a graphical representation of *cash flows drawn on the y axis* with a *time scale on the x axis*.
- The diagram includes *what is known*, *what is estimated*, *and what is* needed.
- That is, once the cash flow diagram is complete, another person should be able to work the problem by looking at the diagram.

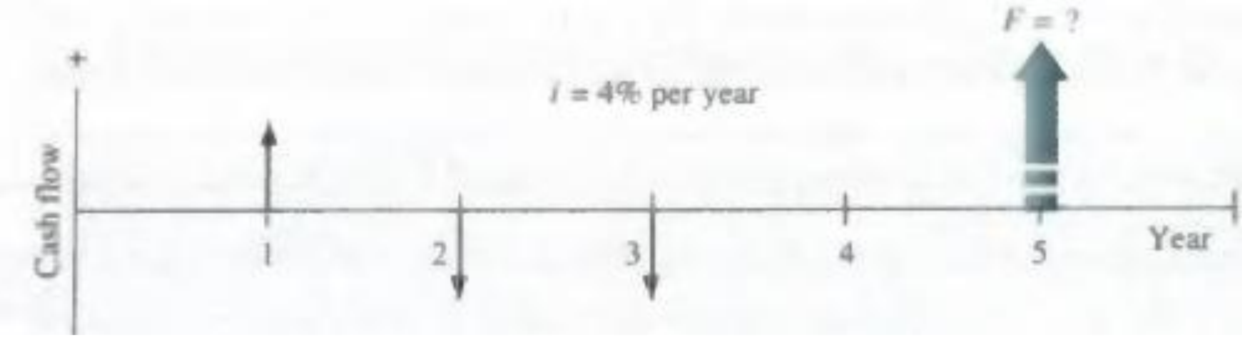
Cash Flow Diagrams (Contd.)

Figure 1–4
A typical cash flow time scale for 5 years.



- Cash flow diagram time **t = 0 is the present**, and ***t = 1 is the end of time period 1***. *We assume* that the periods are in years for now.
- Since the end-of-year convention places cash flows at the ends of years, the "1" marks the end of year 1.

Figure 1–5
Example of positive and negative cash flows.



The direction of the arrows on the diagram is important to differentiate income from outgo.

A vertical arrow pointing up indicates a positive cash flow. Conversely, a down-pointing arrow indicates a negative cash flow.

Cash Flow Diagrams (Contd.)

- Before the diagramming of cash flows, a perspective or vantage point must be determined so that + or - signs can be assigned and the economic analysis performed correctly. Eg.
- Assume you borrow $8500 from a bank today to purchase an $8000 used car for cash next week, and you plan to spend the remaining $500 on a new paint job for the car two weeks from now.
- There are several perspectives possible when developing the cash flow diagram—those of the borrower (that's you), the banker, the car dealer, or the paint shop owner.

Cash Flow Diagrams (Contd.)

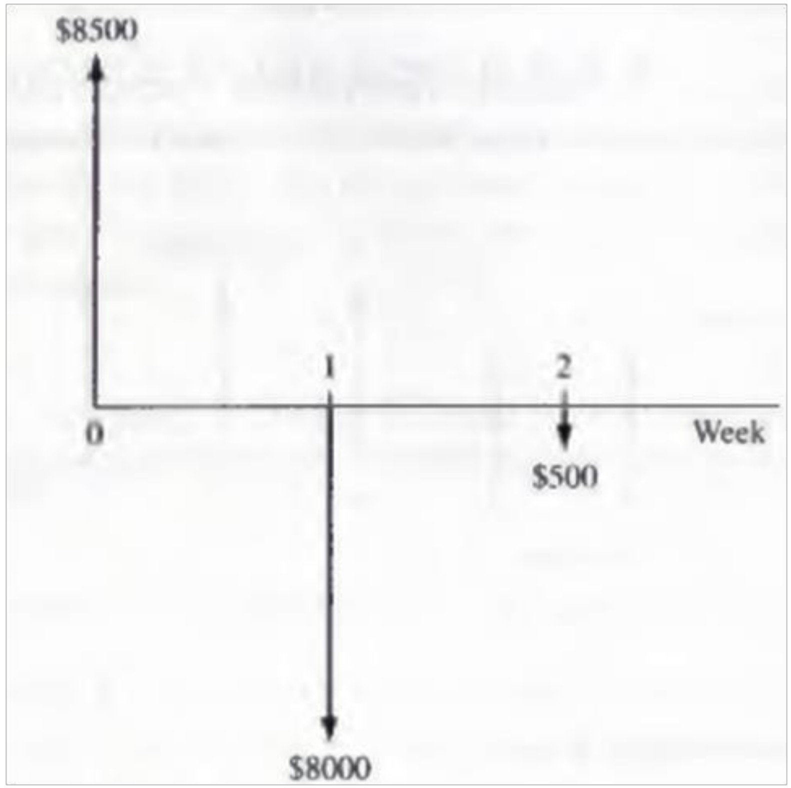
- The cash flow signs and amounts for these perspectives are as follows.

Perspective	Activity	Cash flow with Sign, $	Time, week
You	Borrow	+8500	0
	Buy car	−8000	1
	Paint job	−500	2
Banker	Lender	−8500	0
Car dealer	Car sale	+8000	1
Painter	Paint job	+500	2

- *One, and only one, of the perspectives is selected to develop the diagram.*
- *For your perspective,* all three cash flows are involved in the diagram.

Cash Flow Diagrams (Contd.)

- *For your perspective,* the diagram appears as shown below with a time scale of *weeks*.



Applying the end-of-period convention, you have a receipt of +$8500 now (time 0) and cash outflows of —$8000 at the end of week 1, followed by —$500 at the end of week 2.

Cash Flow Diagram Another Example

An electrical engineer wants to deposit an amount *P now such that she can withdraw an equal* annual amount of *A1 = \$2000 per year for the first 5 years, starting 1 year after the deposit, and* a different annual withdrawal of *A2 = \$3000 per year for the following 3 years. How would the* cash flow diagram appear if *i = 8.5% per year?*

Example: Rate of Return

In 1970, when Wal-Mart Stores, Inc. went public, an investment of 100 shares cost \$1,650. That investment would have been worth \$12,283,904 in the year 2002.

What is the rate of return on that investment?

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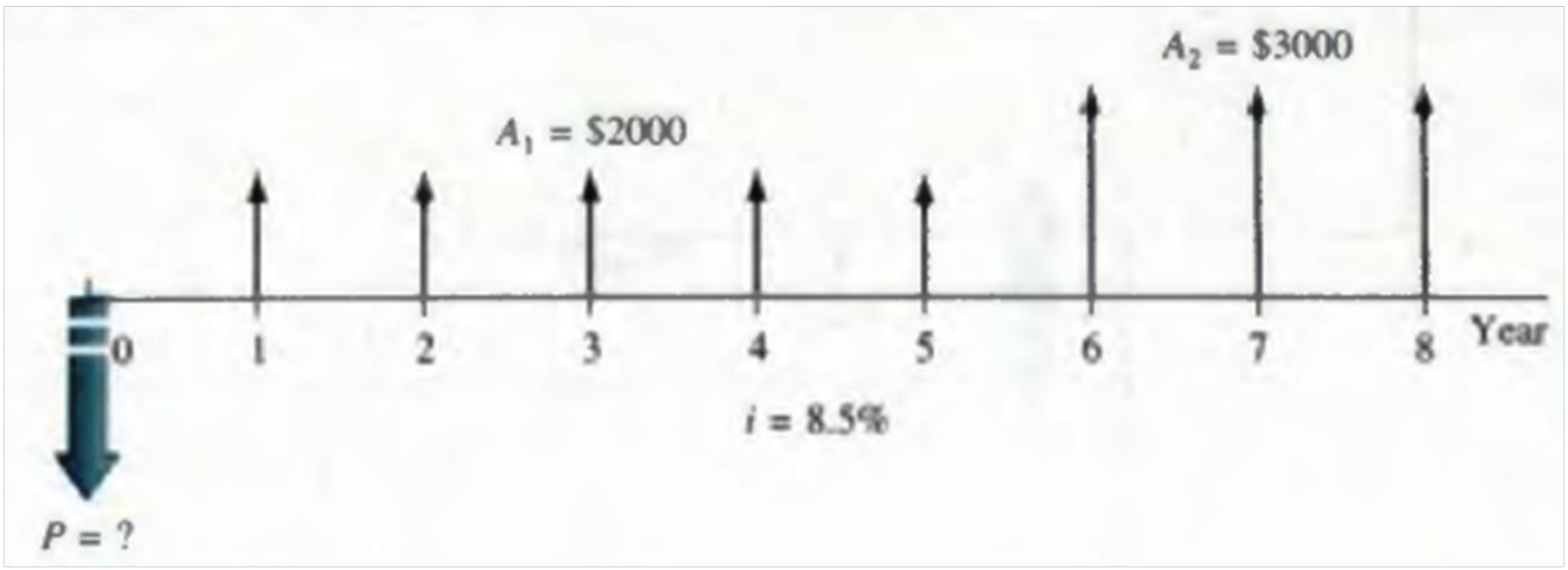

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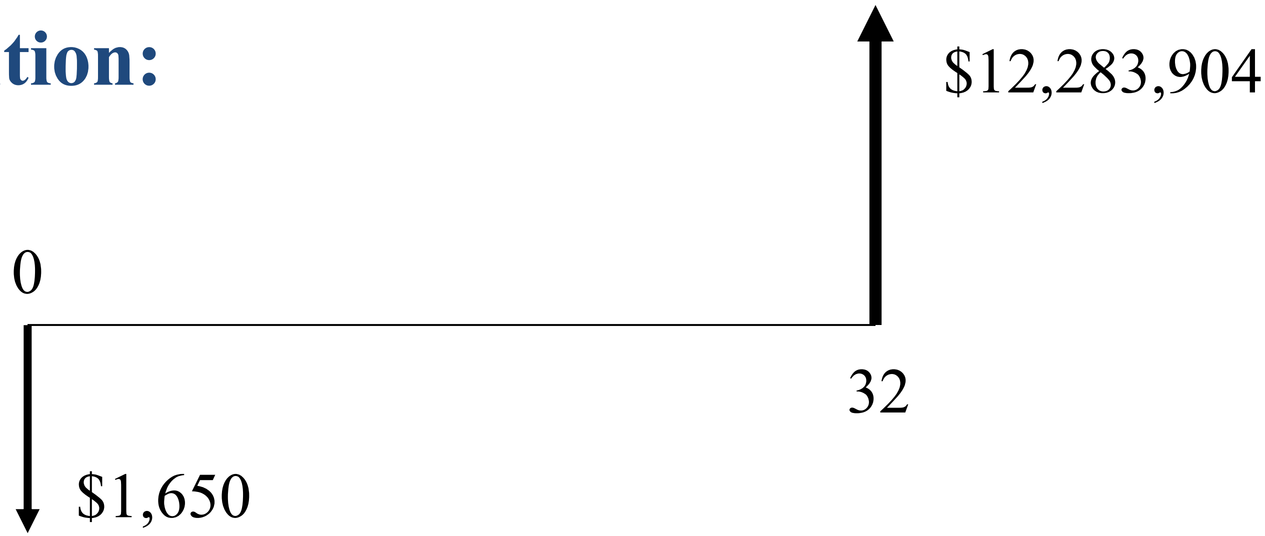
Solution:



Given: $P = \$1{,}650$

$F = \$12{,}283{,}904$

$N = 32$

Find i:

$$F = P(1+i)^N$$

$$\$12{,}283{,}904 = \$1{,}650\,(1 + i\,)^{32}$$

$i = 32.13\%$ Rate of Return

Suppose that in 1970 you invested that amount ($1,650) in a savings account at 6% per year. Then, you could have only $10,648 on January, 2002.

$F = \$1{,}650\,(1 + 0.06)^{32}$

What is the meaning of this 6% interest here?

This would have been your opportunity cost if putting money in savings account was the best you could do at that time!

Thus, in 1970, as long as you earn more than 6% interest in another investment, you will take that investment.

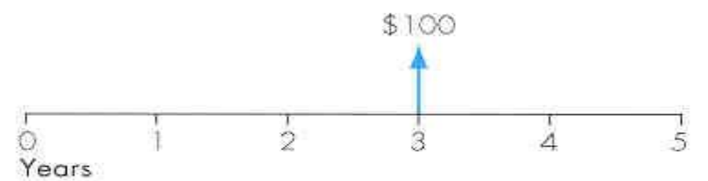
Therefore, 6% can be viewed as a minimum attractive rate of return (or required rate of return).

So, you can apply the following decision rule, to see if the proposed investment is a good one.

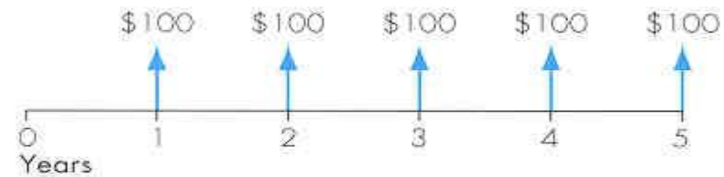
ROR (32.13%) > MARR(6%)

The Five Types of Cash Flows

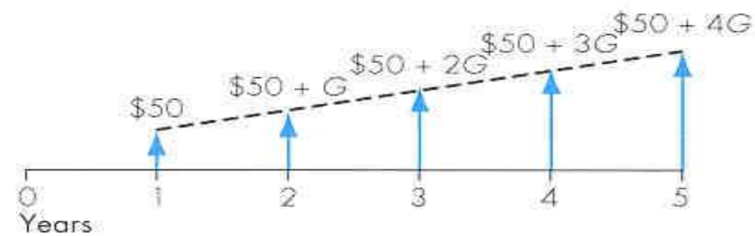
(a) Single cash flow



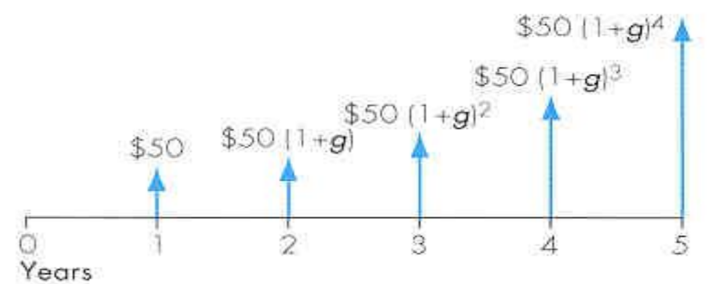
(b) Equal (uniform) payment series



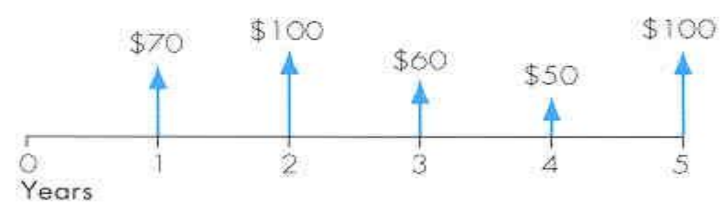
(c) Linear gradient series



(d) Geometric gradient series



(e) Irregular payment series



Summary of compound-interest factors

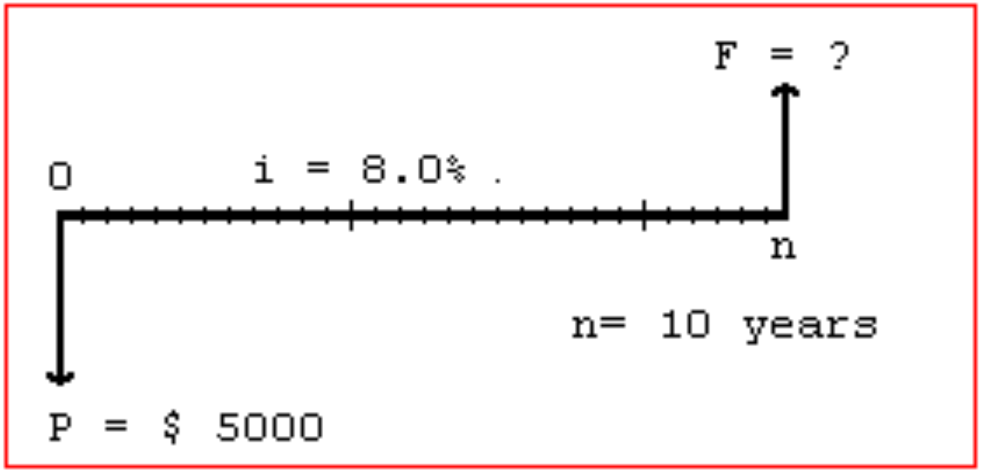
Name	Abbreviation	Notation	Formula
Single-payment present worth factor	SPPWF	$(P/F, i, n)$	$(1+i)^{-n}$
Single-payment compound-amount factor	SPCAF	$(F/P, i, n)$	$(1+i)^{n}$

Example: Finding Future Value (F)

A person deposits \$5000 into an account which pays interest at a rate of 8% per year. The amount in the account after 10 years is closest to:

(A) \$2,792 (B) \$9,000 (C) \$10,795 (D) \$12,165

The cash flow diagram is:



Solution:

F = P(F/P,i,n)

= 5000(F/P,8%,10)

= 5000(2.1589)

= \$10,794.50

Answer is (C)

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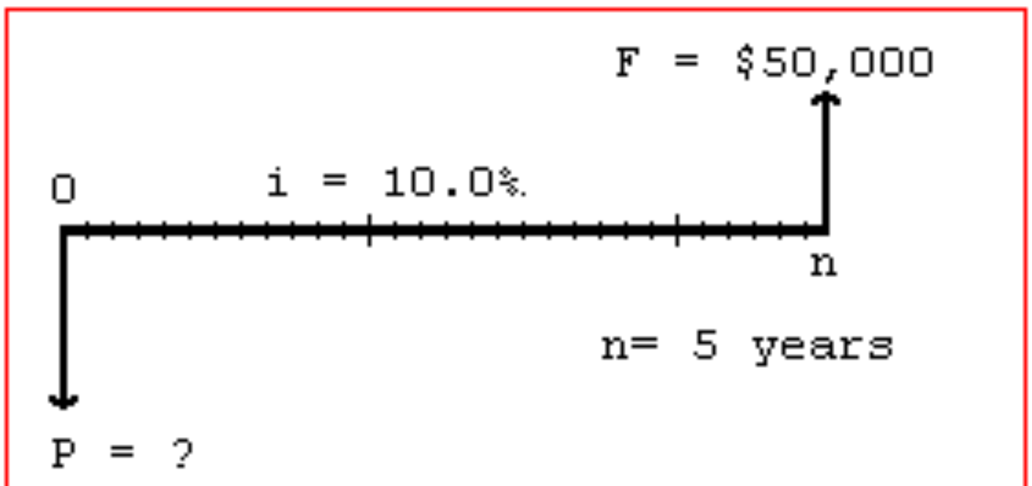
Example: Finding Present Value (P)

A small company wants to make a single deposit now so it will have enough money to purchase a backhoe costing $50,000 five years from now. If the account will earn interest of 10% per year, the amount that must be deposited now is nearest to:

(A) $10,000 (B) $ 31,050 (C) $ 33,250 (D) $319,160

The cash flow diagram is:



Solution:

P = F(P/F,i,n)

= 50,000(P/F,10%,5)

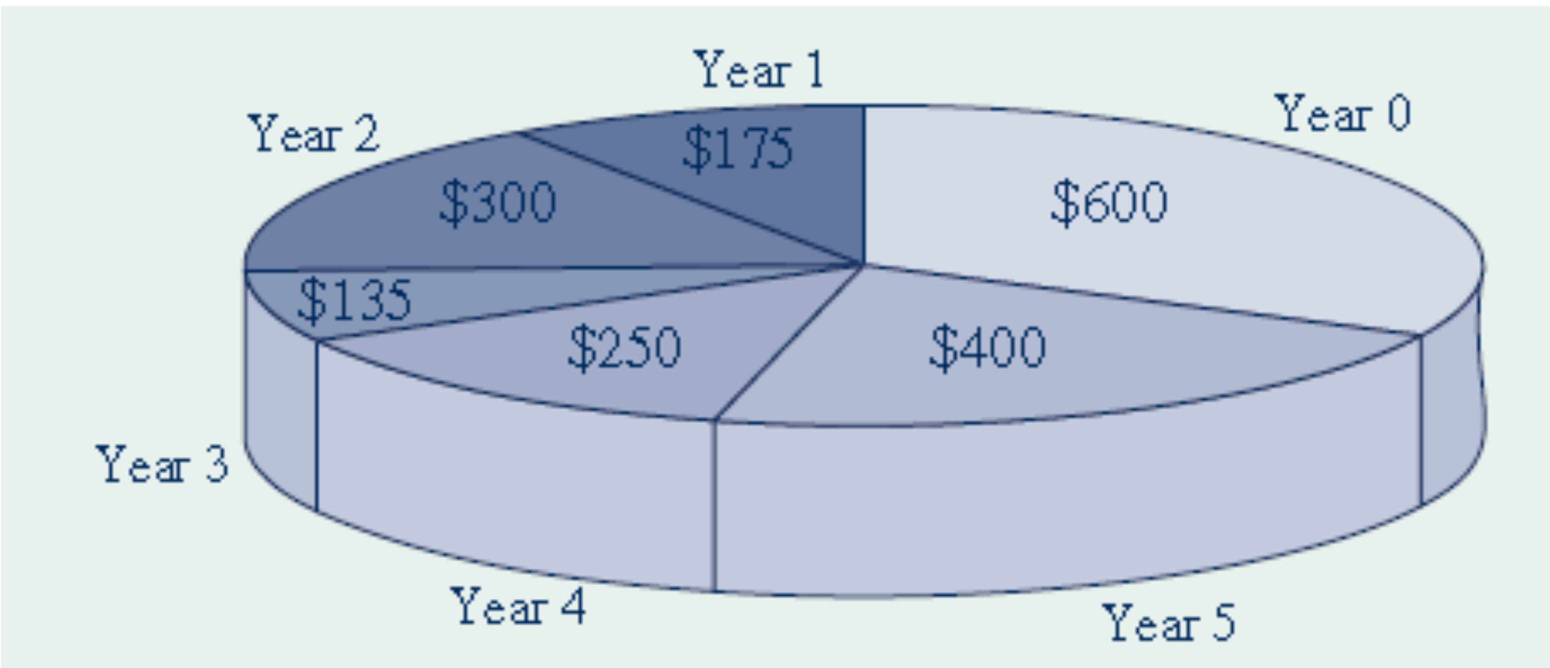
= 50,000(0.6209)

= $31,045

Answer is (B)

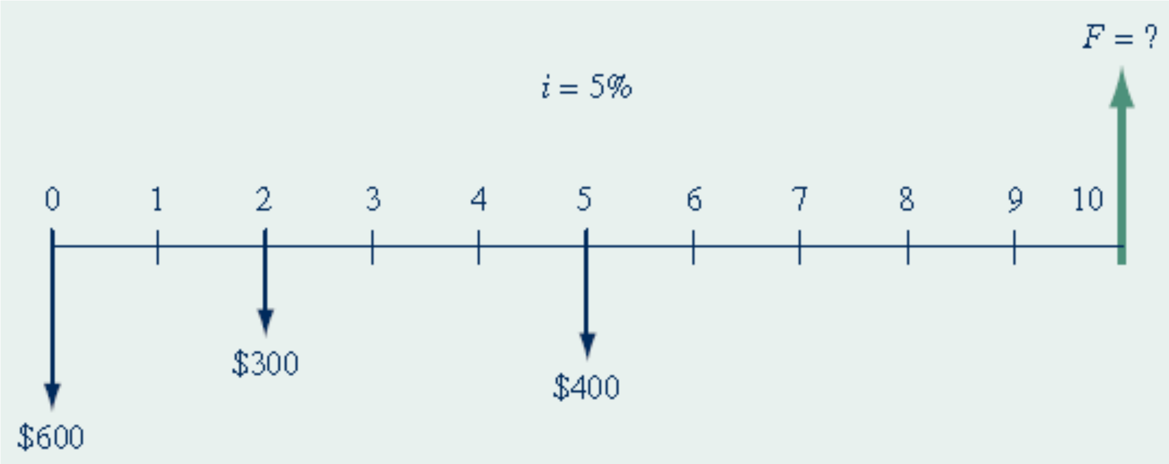
Example 2: Finding Future Value (F)

An independent engineering consultant reviewed records and found that the cost of office supplies varied as shown in the pie chart. If the engineer wants to know the equivalent value in year 10 of only the three largest amounts, what is it at an interest rate of 5% per year?



Example 2: Finding Future Value (F) (Soln)

The Cash Flow- Diagram for Future Worth in Year 10

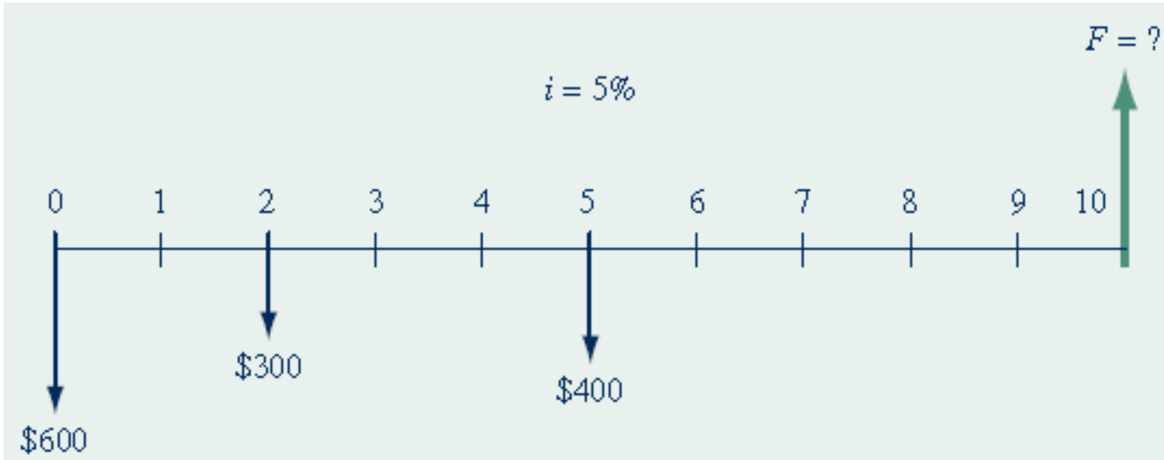


Use *F/P factors to find F in year 10.*

$$F = 600(F/P,5\%,10) + 300(F/P,5\%,8) + 400(F/P,5\%,5)$$
$$= 600(1.6289) + 300(1.4775) + 400(1.2763)$$
$$= \$1931.11$$

Finding Future Value (F) (Soln)

The Cash Flow- Diagram for Future Worth in Year 10



The problem could also be solved by finding the present worth in year 0 of the $300 and $400 costs using the P/F factors and then finding the future worth of the total in year 10.

Solve the problem in the above mentioned alternative way.

Ans: $P_{0,2,5} = 600 + 300 \times (P/F, 5\%,2) + 400 \times (P/F, 5\%,5)$

$P_{0,2,5} = 600 + 300 \times 0.9070 + 400 \times 0.7835$

$P_{0,2,5} = 600 + 272.1 + 313.4 = 1185.5$

$F = P_{0,2,5} \times (F/P, 5\%,10) = 1185.5 \times 1.629 = \$1931.17$

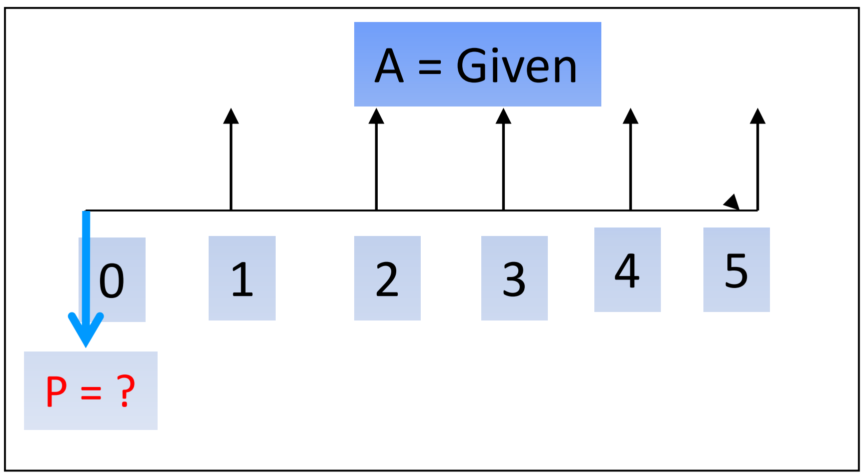
Uniform Series Involving P/A and A/P

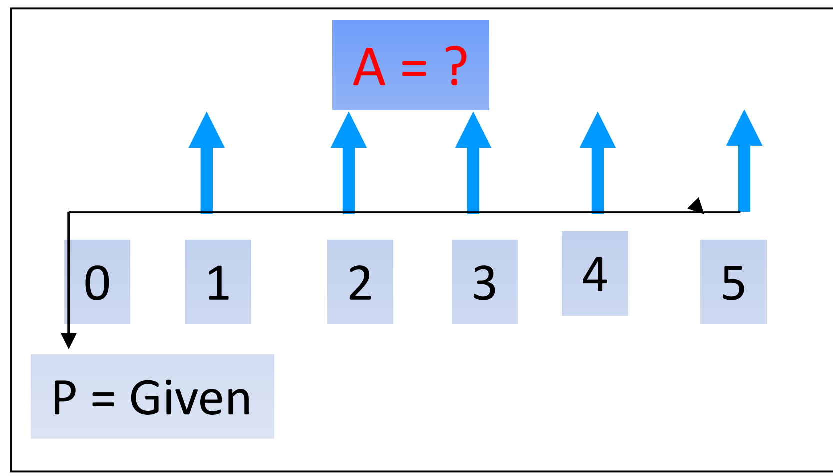
The uniform series factors that involve **P and A** are derived as follows:

(1) Cash flow occurs in ***consecutive*** interest periods

(2) Cash flow amount is ***same*** in each interest period

The cash flow diagrams are:





P = A(P/A,i,n) ⟵ Standard Factor Notation ⟶ A = P(A/P,i,n)

Note: P is one period *Ahead* of first A value

Summary of compound-interest factors again

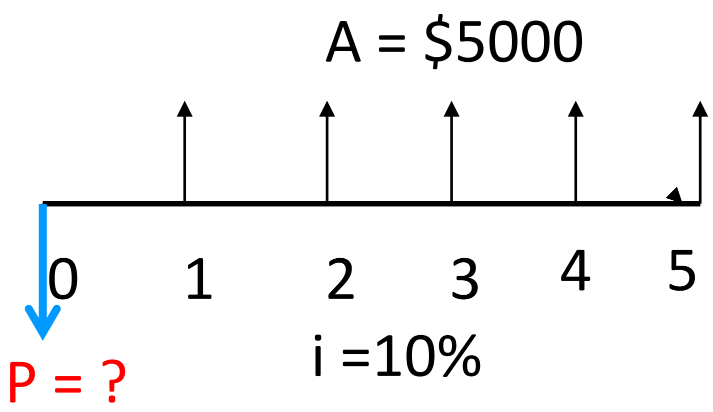
Name	Abbreviation	Notation	Formula
Single-payment present worth factor	SPPWF	$(P/F, i, n)$	$(1+i)^{-n}$
Single-payment compound-amount factor	SPCAF	$(F/P, i, n)$	$(1+i)^{n}$
Uniform-series compound-amount factor	USCAF	$(F/A, i, n)$	$\frac{(1+i)^{n}-1}{i}$
Sinking fund factor	SFF	$(A/F, i, n)$	$\frac{i}{(1+i)^{n}-1}$
Uniform-series present-worth factor	USPWF	$(P/A, i, n)$	$\frac{1-(1+i)^{-n}}{i}$
Capital recovery factor	CRF	$(A/P, i, n)$	$\frac{i}{1-(1+i)^{-n}}$

Example: Uniform Series Involving P/A

A chemical engineer believes that by modifying the structure of a certain water treatment polymer, his company would earn an extra \$5000 per year. At an interest rate of 10% per year, how much could the company afford to spend now to just break even over a 5 year project period?

(A) \$11,170 **(B) 13,640** **(C) \$15,300** **(D) \$18,950**

The cash flow diagram is as follows:



Solution:

P = 5000(P/A,10%,5)

= 5000(3.7908)

= \$18,954

Answer is (D)

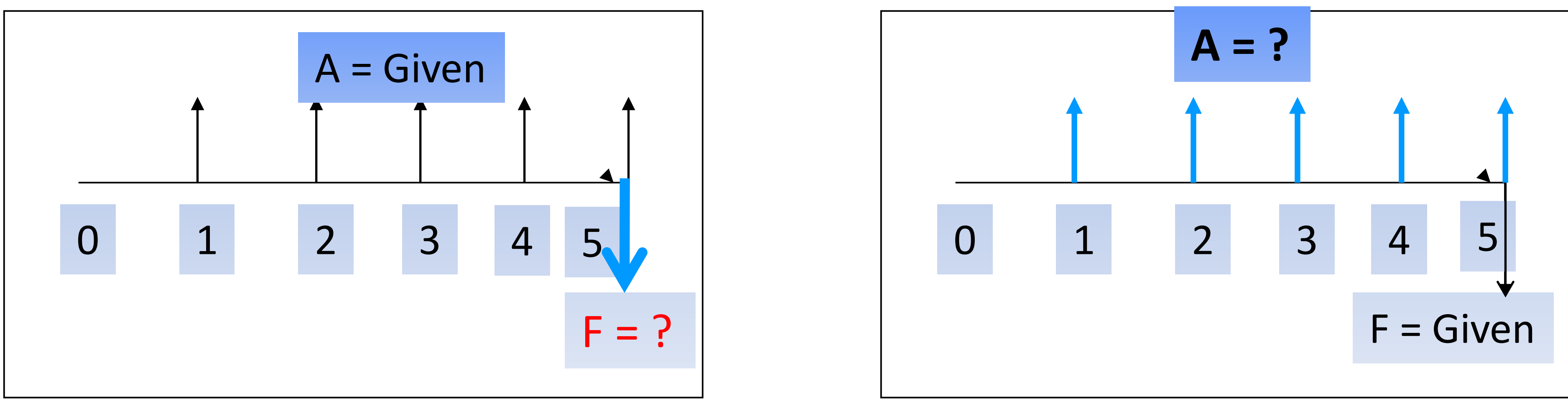
Uniform Series Involving F/A and A/F

The uniform series factors that involve **F and A** are derived as follows:

(1) Cash flow occurs in ***consecutive*** interest periods

(2) Last cash flow occurs in ***same*** period as F

Cash flow diagrams are:



F = A(F/A,i,n) ⟵ Standard Factor Notation ⟶ A = F(A/F,i,n)

Note: F takes place in the *same* period as last A

Summary of compound-interest factors again

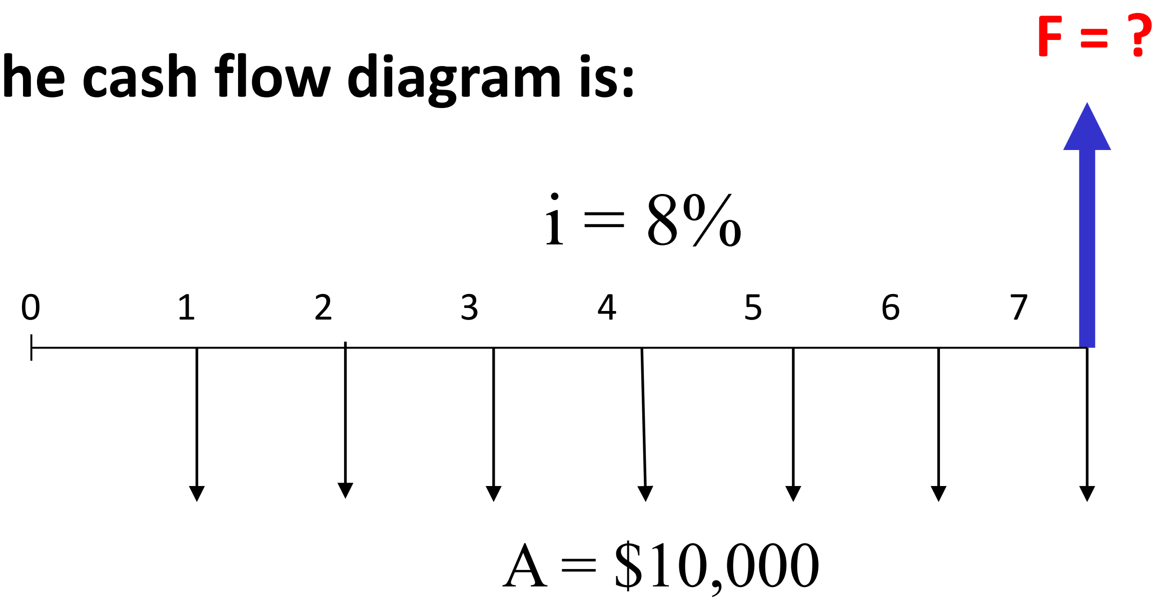
Name	Abbreviation	Notation	Formula
Single-payment present worth factor	SPPWF	$(P/F, i, n)$	$(1+i)^{-n}$
Single-payment compound-amount factor	SPCAF	$(F/P, i, n)$	$(1+i)^{n}$
Uniform-series compound-amount factor	USCAF	$(F/A, i, n)$	$\frac{(1+i)^{n}-1}{i}$
Sinking fund factor	SFF	$(A/F, i, n)$	$\frac{i}{(1+i)^{n}-1}$
Uniform-series present-worth factor	USPWF	$(P/A, i, n)$	$\frac{1-(1+i)^{-n}}{i}$
Capital recovery factor	CRF	$(A/P, i, n)$	$\frac{i}{1-(1+i)^{-n}}$

Example: Uniform Series Involving F/A

An industrial engineer made a modification to a chip manufacturing process that will save her company $10,000 per year. At an interest rate of 8% per year, how much will the savings amount to in 7 years?

(A) $45,300 **(B) $68,500** **(C) $89,228** **(D) $151,500**

The cash flow diagram is:



Solution:

$$F = 10{,}000(F/A, 8\%, 7)$$
$$= 10{,}000(8.9228)$$
$$= \$89{,}228$$

Answer is (C)

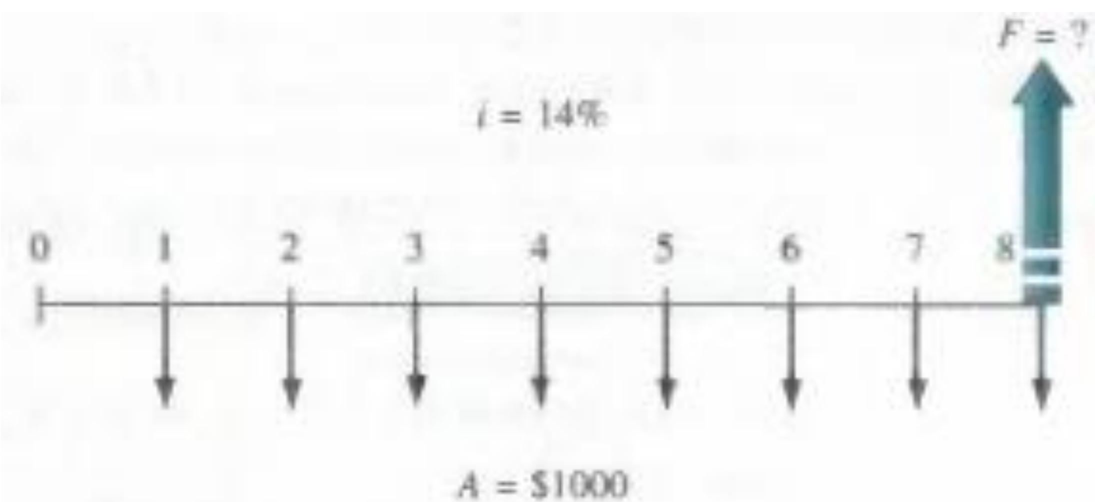
Note: In this case savings is NOT inflow. Savings will earn interest only when invested… hence it is outflow

Example2: Uniform Series Involving F/A

The president of Ford Motor Company wants to know the equivalent future worth of a $1000 capital investment each year for 8 years, starting 1 year from now. Ford capital earns at a rate of 14% per year.

Solution:

The cash flow diagram is:



$$F = 1000(F/A,14\%,8)$$
$$= 1000(13.2328)$$
$$= \$13,232.80$$

$$\frac{(1+i)^n - 1}{i}$$

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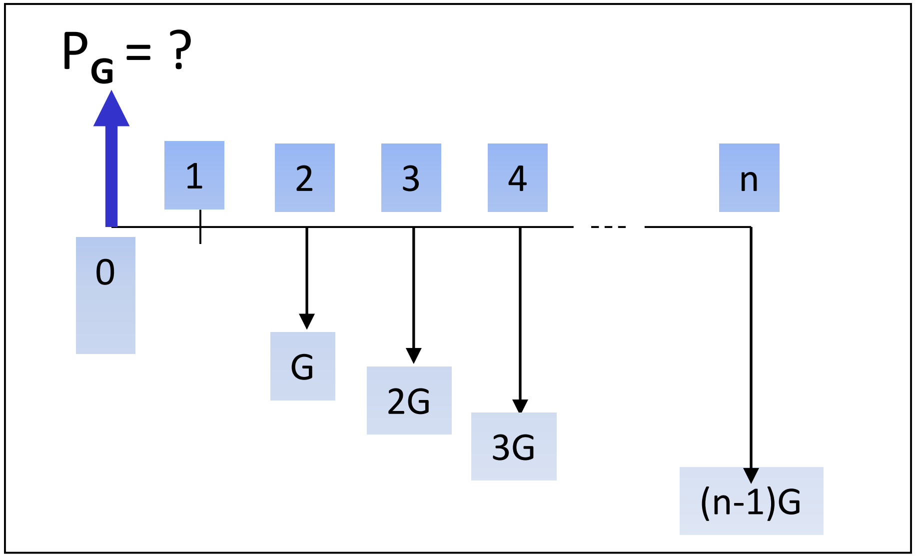
Engineering Economics

Arithmetic Gradients

Arithmetic gradients *change* by the *same amount* *e*ach period

The cash flow diagram for the P_G of an arithmetic gradient is:



G starts b/w periods 1 and 2 (not between 0 and 1)

This is because cash flow in year 1 is usually not equal to G and is handled separately as a *base amount* (shown on next slide)

Standard factor notation is

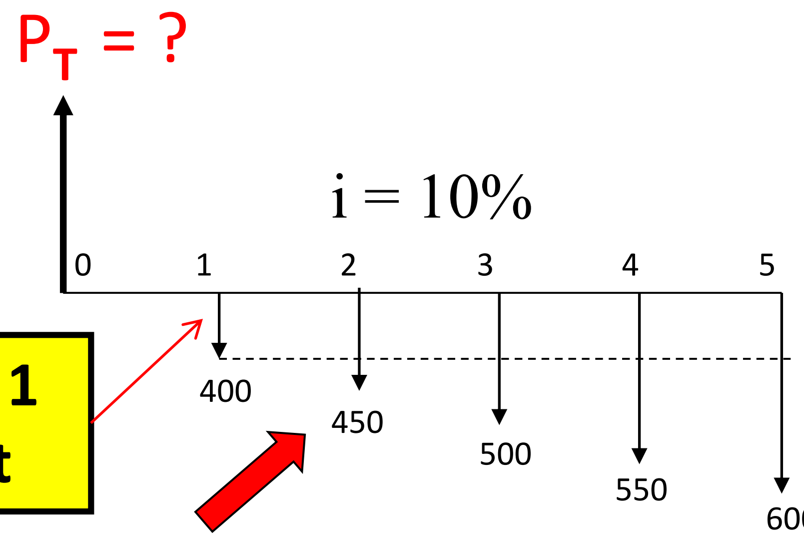
$$P_G = G(P/G,i,n)$$

Note that P_G is located Two Periods Ahead of the first change that is equal to G

P/G Factor

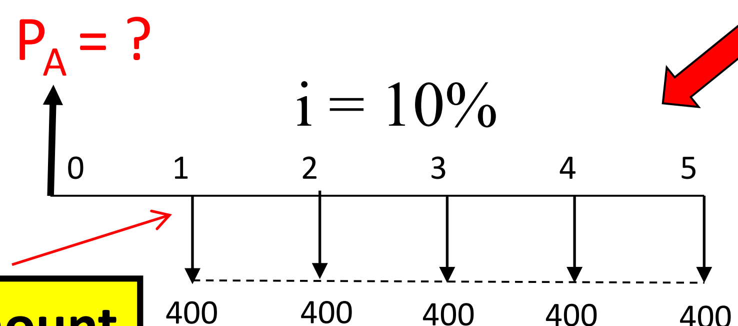
$$(P/G, i, n) = \frac{1}{i}\left[\frac{(1+i)^n - 1}{i(1+i)^n} - \frac{n}{(1+i)^n}\right]$$

Typical Arithmetic Gradient Cash Flow

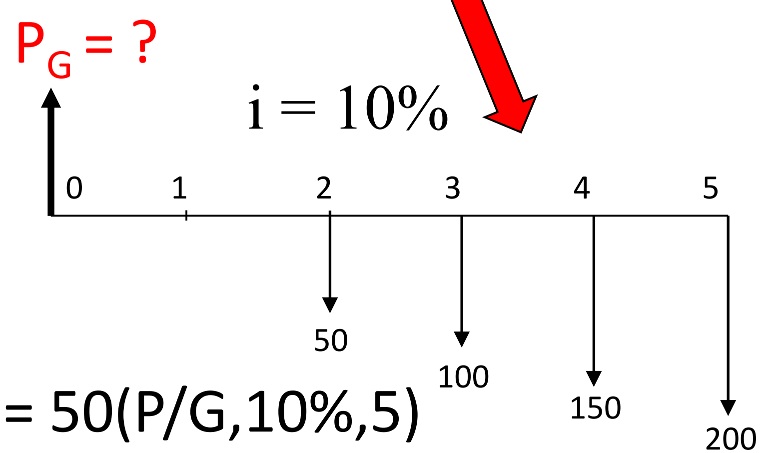


Amount in year 1 is base amount

This diagram = this *base amount* plus this gradient



+



Amount in year 1 is base amount

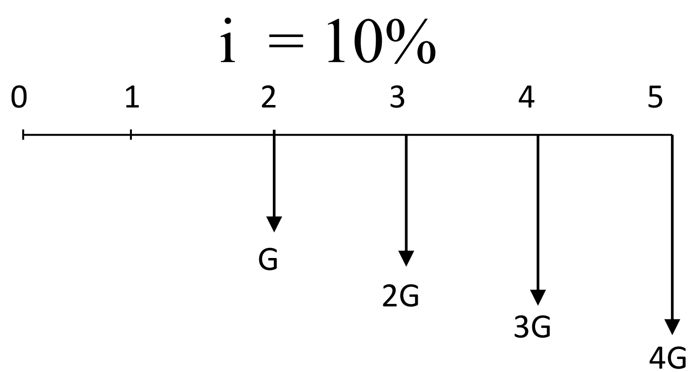
P_A = 400(P/A,10%,5)

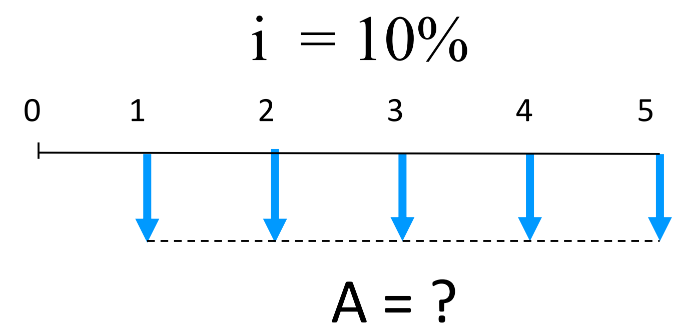
P_G = 50(P/G,10%,5)

$P_T = P_A + P_G$ = 400(P/A,10%,5) + 50(P/G,10%,5)

Converting Arithmetic Gradient to A

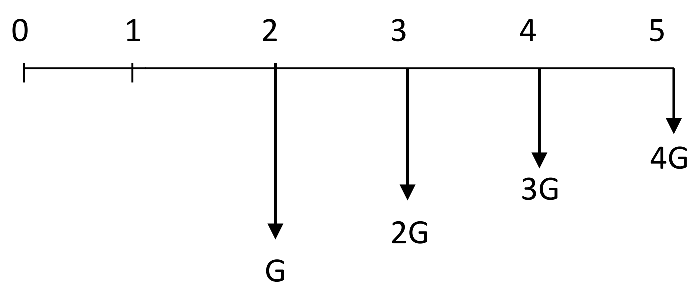
Arithmetic gradient can be converted into equivalent A value using G(A/G,i,n)





General equation when ***base amount is involved*** is

A = base amount + G(A/G,i,n)



For decreasing gradients, change plus sign to minus

A = base amount - G(A/G,i,n)

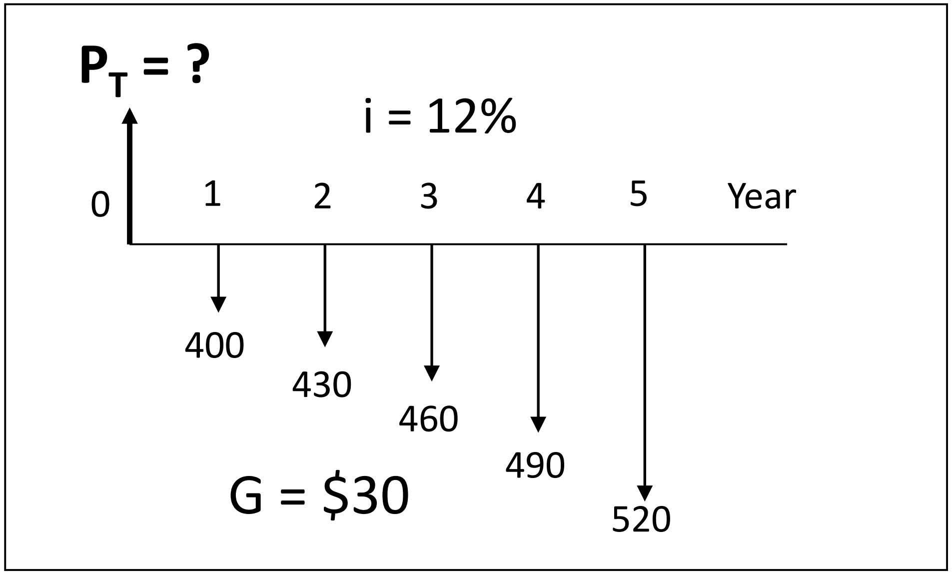
A/G Factor

$$(A/G, i, n) = \left[\frac{1}{i} - \frac{n}{(1+i)^n - 1}\right]$$

Example: Arithmetic Gradient

The present worth of \$400 in year 1 and amounts increasing by \$30 per year through year 5 at an interest rate of 12% per year is closest to: **(A) \$1532** **(B) \$1,634** **(C) \$1,744** **(D) \$1,829**

Cash Flow Diagram



Solution:

$\mathbf{P_T}$ = 400(P/A,12%,5) + 30(P/G,12%,5)

= 400(3.6048) + 30(6.3970)

= \$1,633.83 Answer is (B)

The cash flow could also be converted into an **A** value as follows:

A = 400 + 30(A/G,12%,5)

= 400 + 30(1.7746)

= \$453.24

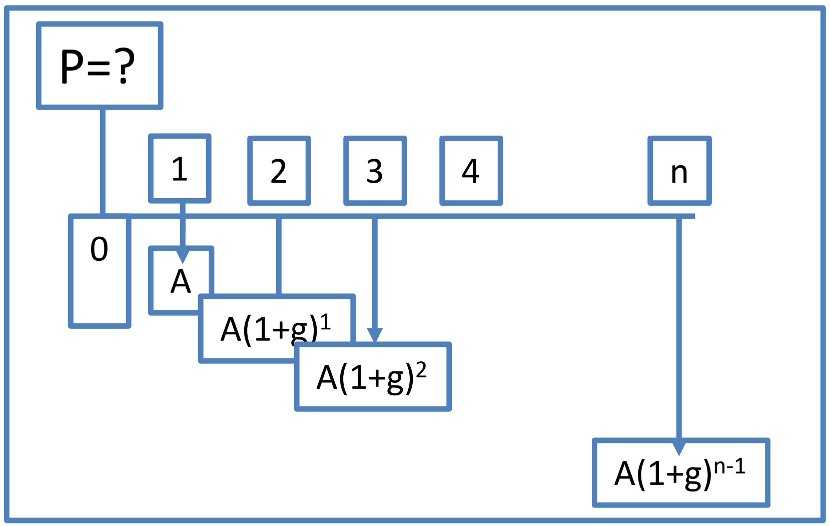
P_T = A(P/A,12%,5)

= \$453.24 * 3.605 = 1633.93

Geometric Gradients

Geometric Gradients Change by the *Same Percentage* Each Period

Cash Flow Diagram for Present Worth of Geometric Gradients is as follows:



There are no Tables for Geometric Factors
Use Following Equation:

$$P=A\{1-[(1+g)/(1+i)]^n\}/(i-g)$$

Where: A=Cash Flow in Period 1
g=Rate of increase

If g=i, $P=A\ \{n/(1+i)\}$

Note: If g is negative, change signs in front of both g's

Geometric Gradient Example

Example: Find the present worth of \$1,000 in year 1 and amounts increasing by 7% per year through year 10. Use an interest rate of 12% per year.

(a) \$5,670 (b) \$7,335 (c) \$12,670 (d)\$13,550

Solution: $P = 1000\{1-[(1+0.07)/(1+0.12)]^{10}\}/(0.12-0.07)$

$= \$7,333$

Answer is (b)

Geometric Gradient Example 2

A coal-fired power plant has upgraded an emission control valve. The modification costs only \$8000 and is expected to last 6 years with a \$200 salvage value. The maintenance cost is expected to be high at \$1700 the first year, increasing by 11% per year thereafter. Determine the equivalent present worth of the modification and maintenance cost at 8% per year.

$$P_T = -8000 - P_g + 200(P/F,8\%,6)$$
$$= -8000 - 1700\left[\frac{1-(1.11/1.08)^6}{0.08-0.11}\right] + 200(P/F,8\%,6)$$
$$= -8000 - 1700(5.9559) + 126 = \$-17{,}999$$

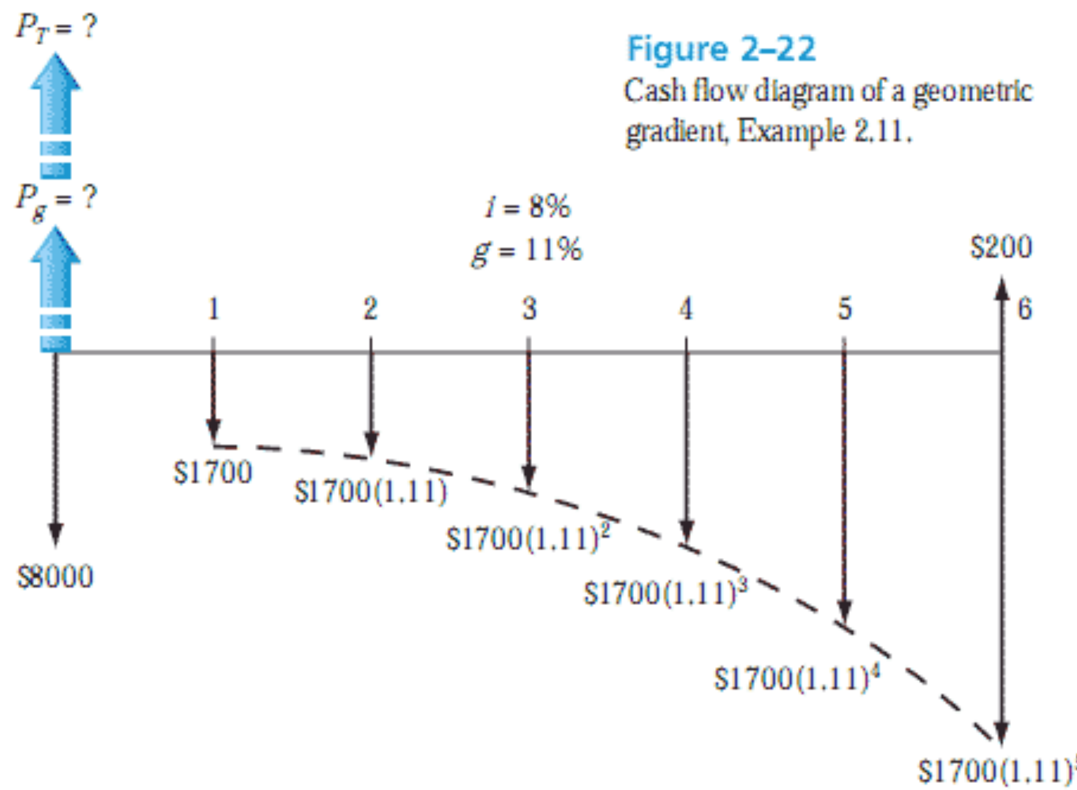


Figure 2–22
Cash flow diagram of a geometric gradient, Example 2.11.

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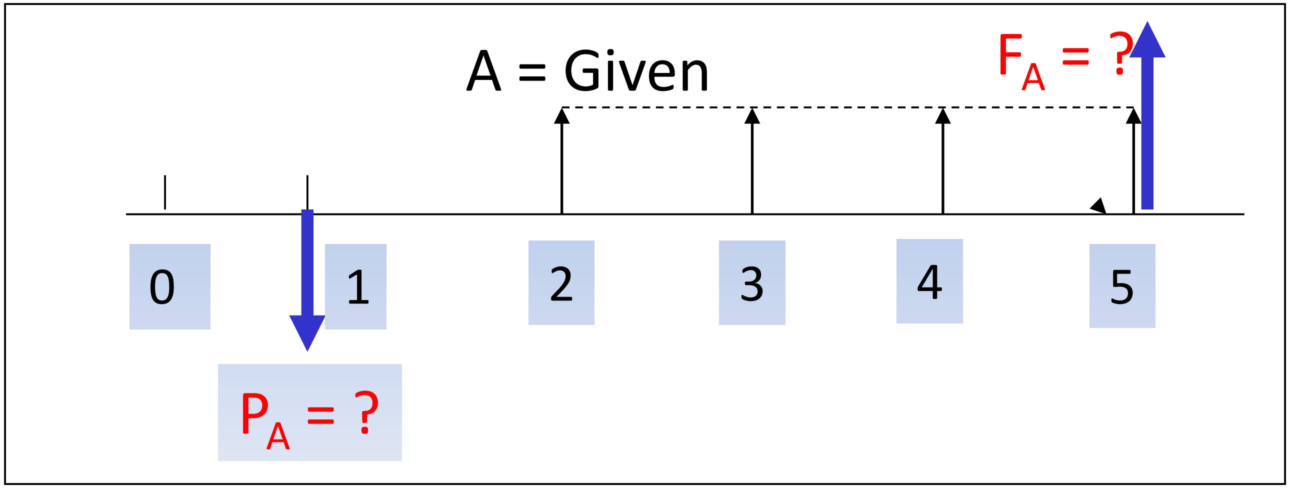
Engineering Economics

Shifted Uniform Series

Shifted uniform series starts at a time *other than period 1*

The cash flow diagram below is an example of a shifted series

Series starts in period 2, not period 1



Shifted series usually require the use of *multiple factors*

Remember:

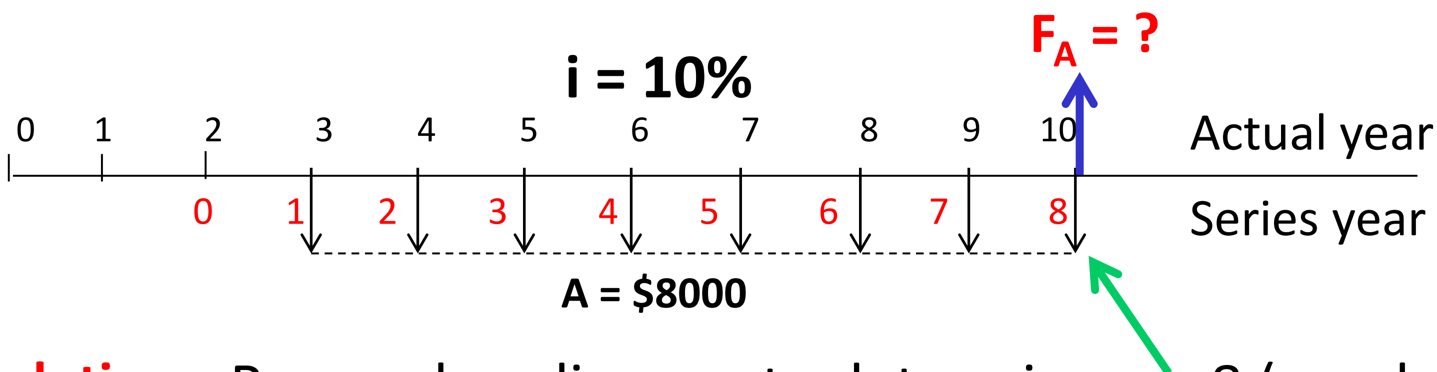
When using P/A or A/P factor, P_A is always *one year ahead* of first A

When using F/A or A/F factor, F_A is in *same year as last A*

Example Using F/A Factor: Shifted Uniform Series

How much money would be available in year 10 if \$8000 is deposited each year in years 3 through 10 at an interest rate of 10% per year?

Cash flow diagram is:



Solution: Re-number diagram to determine n = 8 (number of arrows)

$$
\begin{aligned}
F_A &= 8000(F/A,10\%,8) \\
&= 8000(11.4359) \\
&= \mathbf{\$91,487}
\end{aligned}
$$

Shifted Series and Random Single Amounts

For cash flows that include *uniform series* **and** *randomly placed single amounts*:

Uniform series procedures* are applied to the *series amounts

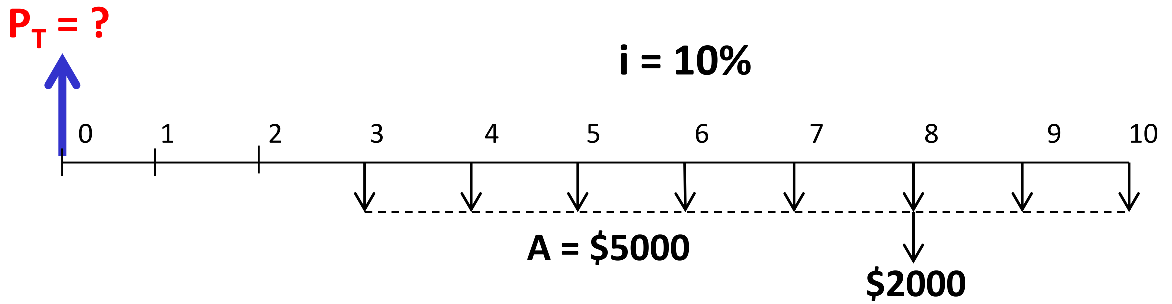
Single amount formulas* are applied to the *one-time cash flows

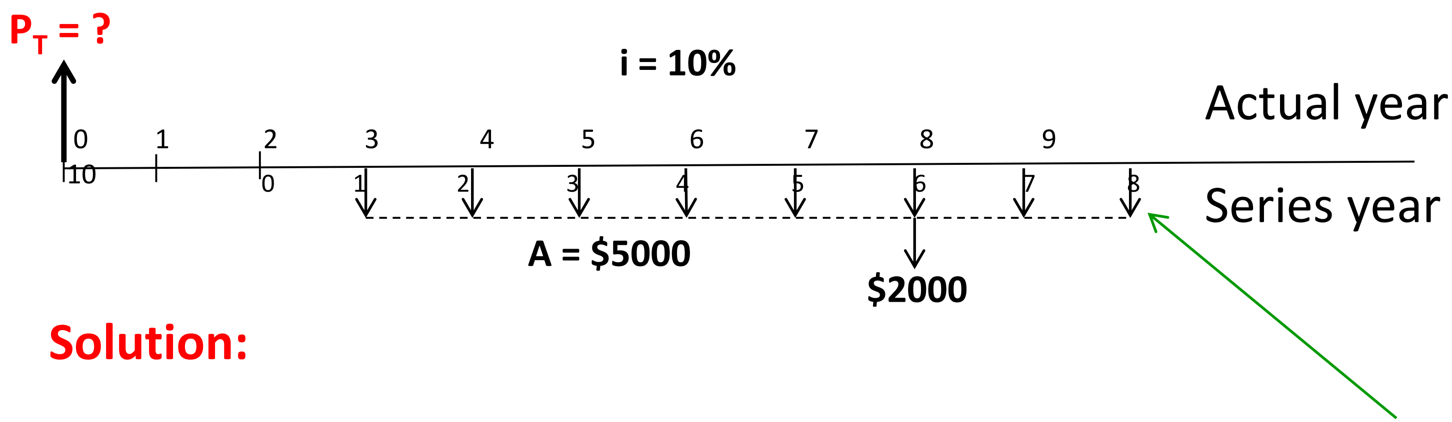
The resulting values are then *combined* per the problem statement

The following slides illustrate the procedure

Example: Series and Random Single Amounts

Find the present worth in year 0 for the cash flows
shown using an interest rate of 10% per year.

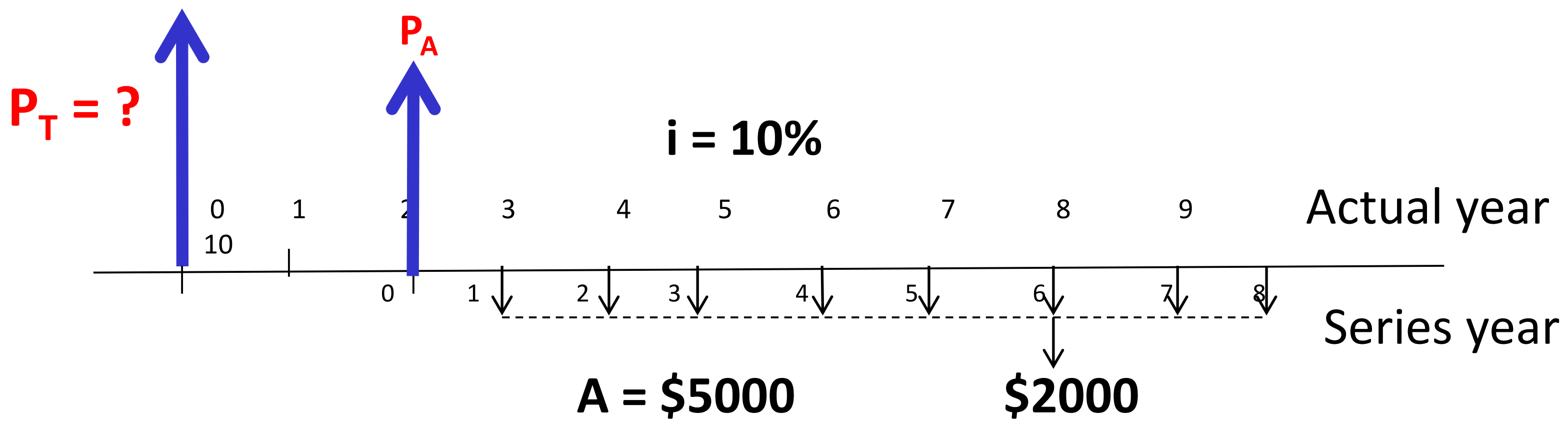




Solution:

First, re-number cash flow diagram to get n for uniform series: n = 8

Example: Series and Random Single Amounts



Use P/A to get P_A in year 2: P_A = 5000(P/A,10%,8) = 5000(5.3349) = $26,675

Move P_A back to year 0 using P/F: P_0 = 26,675(P/F,10%,2) = 26,675(0.8264) = $22,044

Move $2000 single amount back to year 0: P_{2000} = 2000(P/F,10%,8) = 2000(0.4665) = $933

Now, add P_0 and P_{2000} to get P_T i.e. P_T = 22,044 + 933 = $22,977

In short, P_T = 5000 × (P/A,10%,8) × (P/F,10%,2) + 2000 ×(P/F,10%,8)

Example

A construction firm is considering the purchase of an air compressor.

The compressor has the following expected end of year maintenance costs:

Year 1	***$800***
Year 2	***$800***
Year 3	***$900***
Year 4	***$1000***
Year 5	***$1100***
Year 6	***$1200***
Year 7	***$1300***
Year 8	***$1400***

What is the present equivalent maintenance cost if the interest rate is 12% per year compounded annually?

Alt Soln 1

GIVEN:

MAINT $COST_{1\text{-}8}$ PER DIAGRAM

i = 12%/YR, CPD ANNUALLY

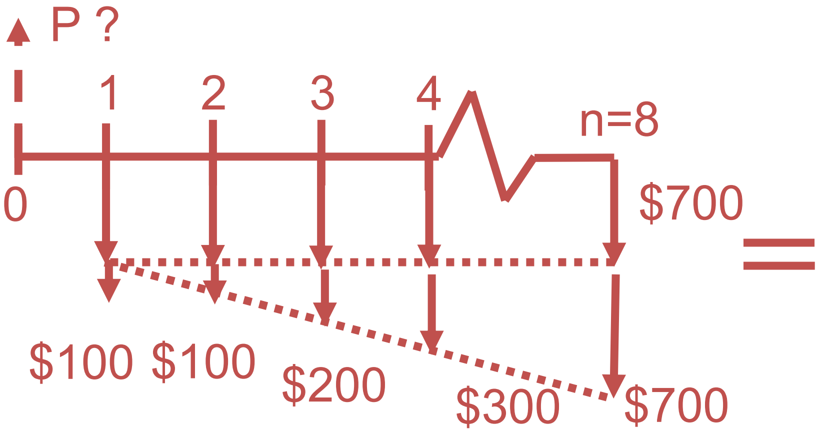
FIND P:

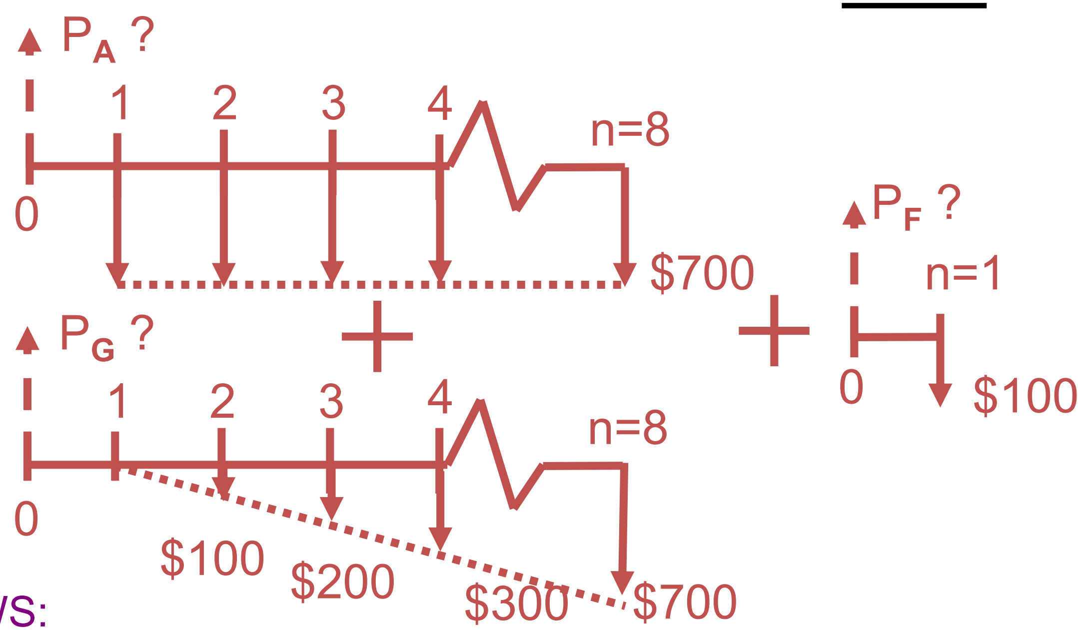
$$P = P_A + P_G + P_F = A(P/A,i,n) + G(P/G,i,n) + F(P/F,i,n)$$

$$= \$700(P/A,12\%,8) + \$100(P/G,12\%,8) + \$100(P/F,12\%,1)$$

$$= \$700(4.9676) + \$100(14.4715) + \$100(0.8929) = \underline{\underline{\$5014}}$$

DIAGRAM:





NOTE: CAN BREAK INTO 3 CASH FLOWS:
ANNUAL, LINEAR GRADIENT, AND FUTURE

Alt Soln 2

GIVEN:

MAINT COST$_{1\text{-}8}$ PER DIAGRAM

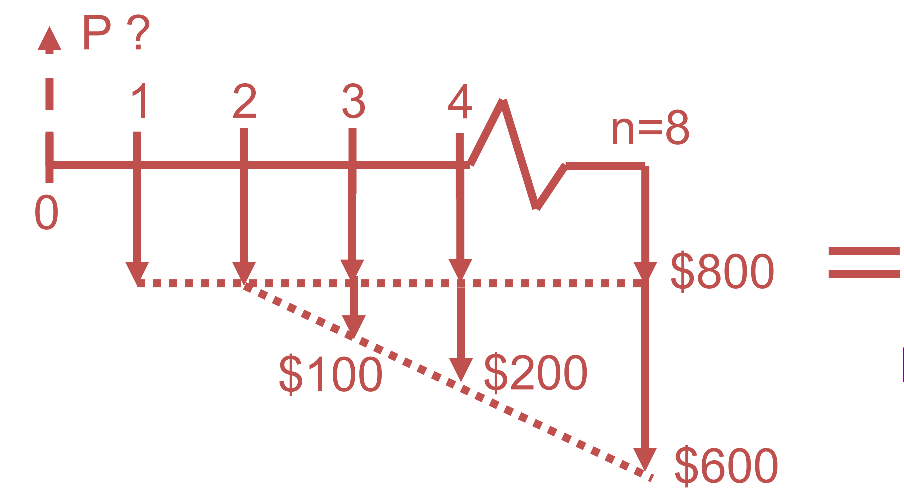
i = 12%/YR, CPD ANNUALLY

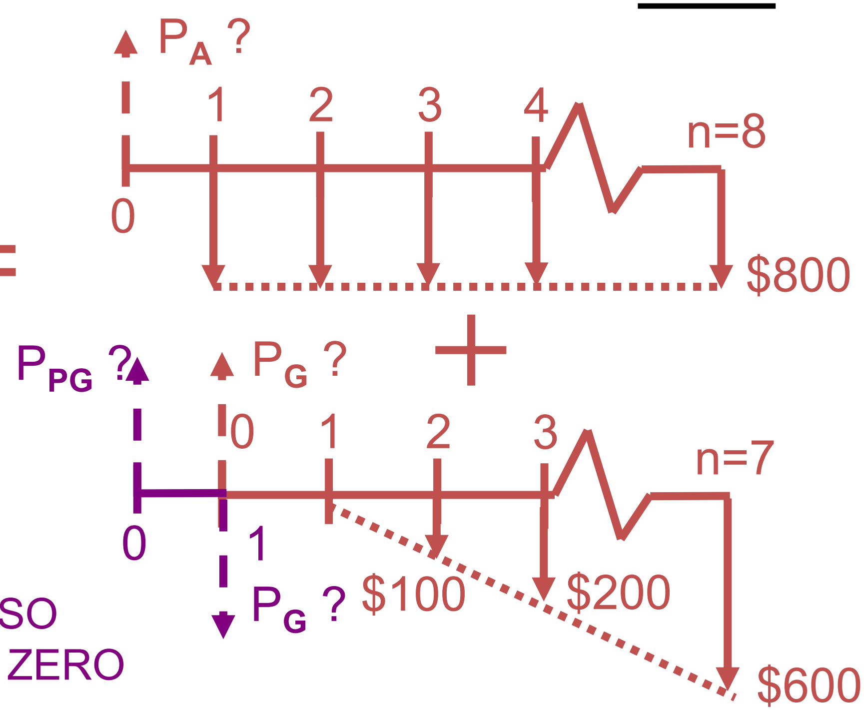
FIND P: $P = P_A + P_G(P_{PG}) = A(P/A,i,n) + G(P/G,i,n-1)(P/F,i,1)$

$= \$800(P/A,12\%,8) + \$100(P/G,12\%,7)(P/F,12\%,1)$

DIAGRAM:

$= \$800(4.9676) + \$100(11.6443)(0.8929) = \underline{\underline{\$5014}}$





NOTE: P_G MUST BE OFFSET ONE YEAR – SO BRING THE OFFSET YEAR BACK TO TIME ZERO

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Engineering Economics

Shifted Geometric Gradients

Shifted gradient begins at a time other than between periods 1 and 2

Equation yields P_g for *all* cash flows (base amount A_1 is included)

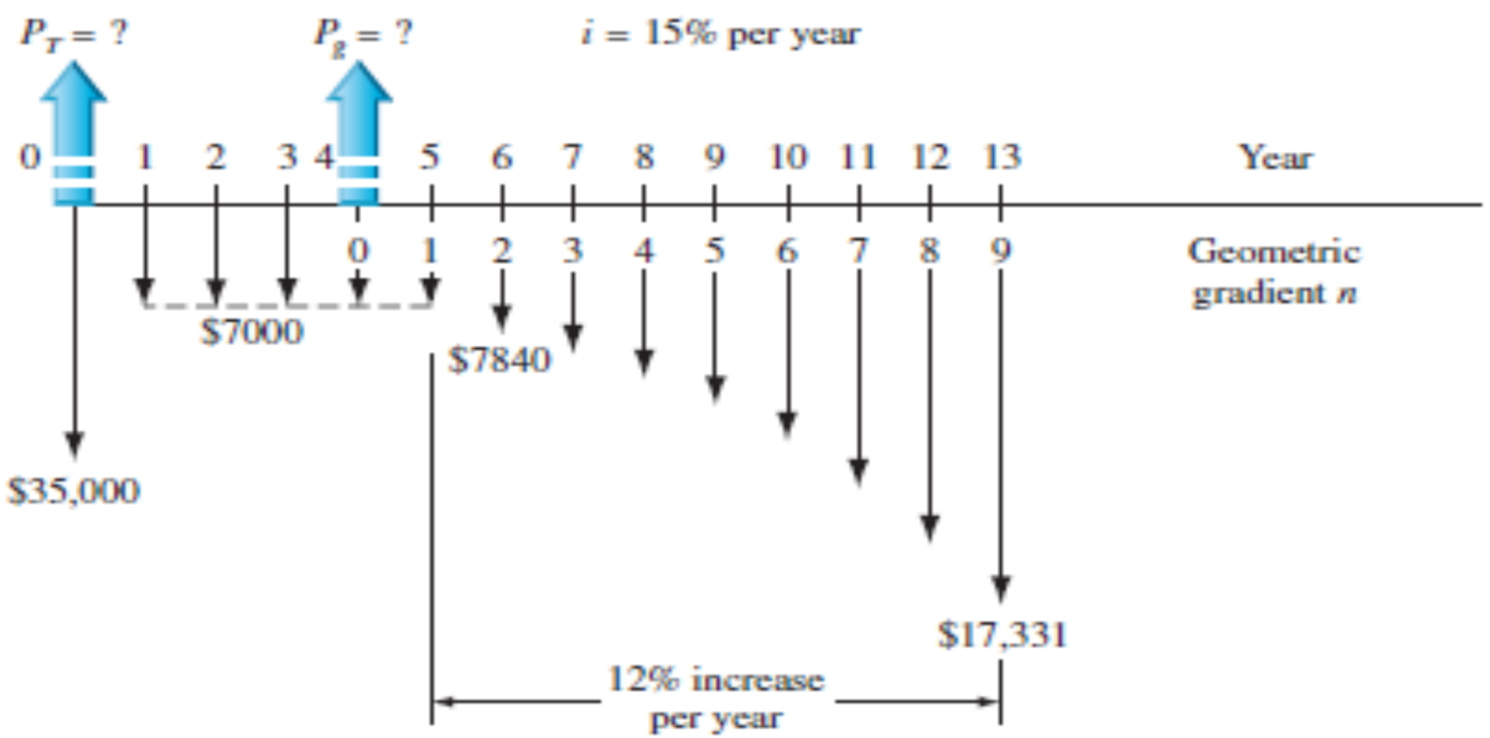
Equation ($i \neq g$): $P_g = A_1\{\{1 - [(1+g)/(1+i)]^n\}/(i-g)\}$

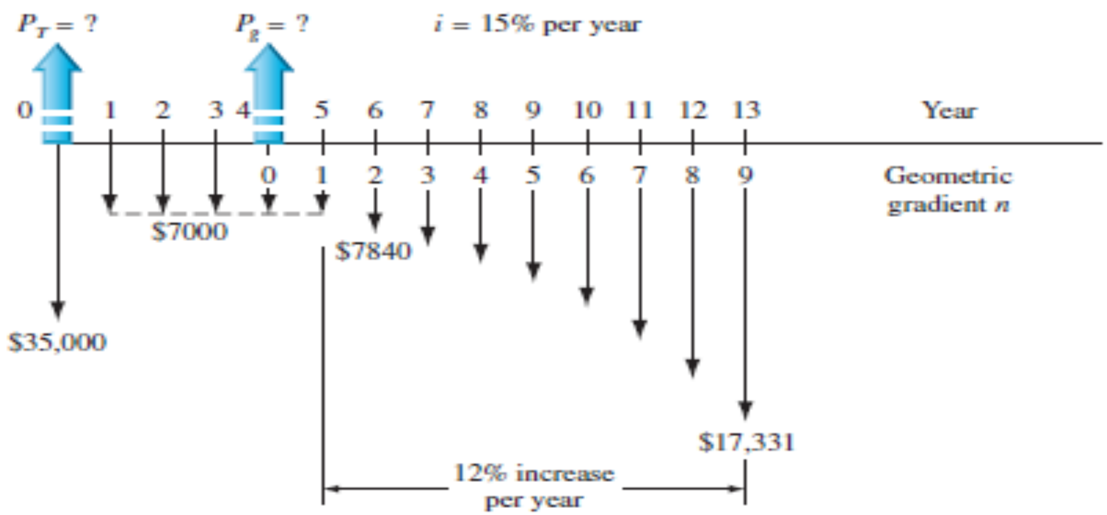
For negative gradient, change signs on both g values

There are no tables for geometric gradient factors

Example: Shifted Geometric Gradient

Weirton Steel signed a 5-year contract to purchase water treatment chemicals from a local distributor for $7000 per year. When the contract ends, the cost of the chemicals is expected to increase by 12% per year for the next 8 years. If an initial investment in storage tanks is $35,000, determine the equivalent present worth in year 0 of all of the cash flows at i = 15% per year.





Gradient starts between actual years 5 and 6; these are gradient years 1 and 2.
P_g is located in gradient year 0, which is actual year 4

$P_g = 7000\{[1-[(1+0.12)/(1+0.15)]^9\ ]/(0.15-0.12)\} = \$49,401$

Move P_g and other cash flows to year 0 to calculate P_T

That is, find P_A for the \$7000 amounts for years 1 through 4

Next, find P_T at year 0 ,

$\mathbf{P_T = 35,000 + 7000(P/A,15\%,4) + 49,401(P/F,15\%,4) = \$83,232}$

Negative Shifted Gradients

For negative arithmetic gradients, change sign on G term from + to -

General equation for determining P: **P = present worth of base amount - P_G**

Changed from + to -

For negative geometric gradients, change signs on both g values

Changed from + to -

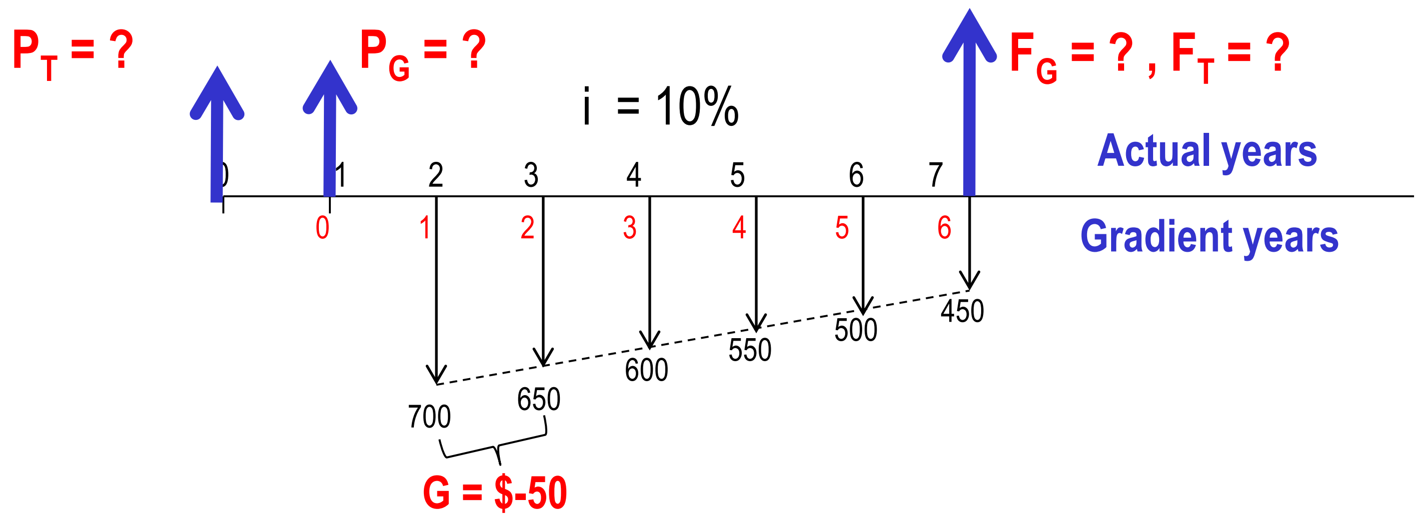
$$P_g = A_1\{1-[(1-g)/(1+i)]^n/(i+g)\}$$

Changed from - to +

All other procedures are the same as for positive gradients

Example: Negative Shifted Arithmetic Gradient

For the cash flows shown, find the future worth in year 7 at i = 10% per year



Solution: Gradient G first occurs between actual years 2 and 3; these are gradient years 1 and 2

P_G is located in gradient year 0 (actual year 1); base amount of $700 is in gradient years 1-6

1) $P_G = 700(P/A,10\%,6) – 50(P/G,10\%,6) = 700(4.3553) – 50(9.6842) = \$2565$

$F_G = P_G(F/P,10\%,6) = 2565(1.7716) = \$4544$

2) $P_T = P_G\ (P/F,10\%,1)$ Then, $F_T = P_T\ (F/P,10\%,7)$

$F_T = 2565 * 0.9091 * 1.949 = \$4544.759$

Unknown Recovery Period n

Unknown recovery period problems involve solving for n, given i and 2 other values (P, F, or A)

Procedure: Set up equation with all symbols involved and solve for n

A contractor purchased equipment for $60,000 that provided income of $8,000 per year. At an interest rate of 10% per year, the length of time required to recover the investment was closest to:

(a) 10 years **(b) 12 years** **(c) 15 years** **(d) 18 years**

Solution: Can use either the P/A or A/P factor. Using A/P:

$$60{,}000(A/P,10\%,n) = 8{,}000$$
$$(A/P,10\%,n) = 0.13333$$

From A/P column in i = 10% interest tables, n is between 14 and 15 years

Answer is (c)

Unknown Recovery Period n (From Factor Table)

10%			Compound Interest Factors						10%
	Single Payment		Uniform Payment Series				Arithmetic Gradient		
	Compound Amount Factor Find *F* Given *P*	Present Worth Factor Find *P* Given *F*	Sinking Fund Factor Find *A* Given *F*	Capital Recovery Factor Find *A* Given *P*	Compound Amount Factor Find *F* Given *A*	Present Worth Factor Find *P* Given *A*	Gradient Uniform Series Find *A* Given *G*	Gradient Present Worth Find *P* Given *G*	
n	*F/P*	*P/F*	*A/F*	*A/P*	*F/A*	*P/A*	*A/G*	*P/G*	*n*
1	1.100	.9091	1.0000	1.1000	1.000	0.909	0	0	1
2	1.210	.8264	.4762	.5762	2.100	1.736	0.476	0.826	2
3	1.331	.7513	.3021	.4021	3.310	2.487	0.937	2.329	3
4	1.464	.6830	.2155	.3155	4.641	3.170	1.381	4.378	4
5	1.611	.6209	.1638	.2638	6.105	3.791	1.810	6.862	5
6	1.772	.5645	.1296	.2296	7.716	4.355	2.224	9.684	6
7	1.949	.5132	.1054	.2054	9.487	4.868	2.622	12.763	7
8	2.144	.4665	.0874	.1874	11.436	5.335	3.004	16.029	8
9	2.358	.4241	.0736	.1736	13.579	5.759	3.372	19.421	9
10	2.594	.3855	.0627	.1627	15.937	6.145	3.725	22.891	10
11	2.853	.3505	.0540	.1540	18.531	6.495	4.064	26.396	11
12	3.138	.3186	.0468	.1468	21.384	6.814	4.388	29.901	12
13	3.452	.2897	.0408	.1408	24.523	7.103	4.699	33.377	13
14	3.797	.2633	.0357	.1357	27.975	7.367	4.996	36.801	14
15	4.177	.2394	.0315	.1315	31.772	7.606	5.279	40.152	15
16	4.595	.2176	.0278	.1278	35.950	7.824	5.549	43.416	16
17	5.054	.1978	.0247	.1247	40.545	8.022	5.807	46.582	17

$(A/P,10\%,n) = 0.1333$ lies between $n = 14$ and $n = 15$

Unknown Interest Rate i

Unknown interest rate problems involve solving for i, given n and 2 other values (P, F, or A)

(Usually requires a trial & error solution and/or interpolation in interest tables)

Procedure: Set up equation with all symbols involved and solve for i

A contractor purchased equipment for $60,000 which provided income of $16,000 per year for 10 years. The annual rate of return of the investment was closest to:

(a) 15% (b) 18% (c) 20% (d) 23%

Solution: Can use either the P/A or A/P factor. Using A/P:

$$60{,}000(A/P, i\%, 10) = 16{,}000$$
$$(A/P, i\%, 10) = 0.26667$$

From A/P column at n = 10 in the interest tables, i is between 20% and 25%

Answer is (d)

Unknown Interest Rate i [From factor tables]

20% Compound Interest Factors 20%

	Single Payment		Uniform Payment Series				Arithmetic Gradient		
	Compound Amount Factor Find F Given P	Present Worth Factor Find P Given F	Sinking Fund Factor Find A Given F	Capital Recovery Factor Find A Given P	Compound Amount Factor Find F Given A	Present Worth Factor Find P Given A	Gradient Uniform Series Find A Given G	Gradient Present Worth Find P Given G	
n	*F/P*	*P/F*	*A/F*	*A/P*	*F/A*	*P/A*	*A/G*	*P/G*	*n*
1	1.200	.8333	1.0000	1.2000	1.000	0.833	0	0	1
2	1.440	.6944	.4545	.6545	2.200	1.528	0.455	0.694	2
3	1.728	.5787	.2747	.4747	3.640	2.106	0.879	1.852	3
4	2.074	.4823	.1863	.3863	5.368	2.589	1.274	3.299	4
5	2.488	.4019	.1344	.3344	7.442	2.991	1.641	4.906	5
6	2.986	.3349	.1007	.3007	9.930	3.326	1.979	6.581	6
7	3.583	.2791	.0774	.2774	12.916	3.605	2.290	8.255	7
8	4.300	.2326	.0606	.2606	16.499	3.837	2.576	9.883	8
9	5.160	.1938	.0481	.2481	20.799	4.031	2.836	11.434	9
10	6.192	.1615	.0385	.2385	25.959	4.192	3.074	12.887	10

25% Compound Interest Factors 25%

	Single Payment		Uniform Payment Series				Arithmetic Gradient		
	Compound Amount Factor Find F Given P	Present Worth Factor Find P Given F	Sinking Fund Factor Find A Given F	Capital Recovery Factor Find A Given P	Compound Amount Factor Find F Given A	Present Worth Factor Find P Given A	Gradient Uniform Series Find A Given G	Gradient Present Worth Find P Given G	
n	*F/P*	*P/F*	*A/F*	*A/P*	*F/A*	*P/A*	*A/G*	*P/G*	*n*
1	1.250	.8000	1.0000	1.2500	1.000	0.800	0	0	1
2	1.563	.6400	.4444	.6944	2.250	1.440	0.444	0.640	2
3	1.953	.5120	.2623	.5123	3.813	1.952	0.852	1.664	3
4	2.441	.4096	.1734	.4234	5.766	2.362	1.225	2.893	4
5	3.052	.3277	.1218	.3718	8.207	2.689	1.563	4.204	5
6	3.815	.2621	.0888	.3388	11.259	2.951	1.868	5.514	6
7	4.768	.2097	.0663	.3163	15.073	3.161	2.142	6.773	7
8	5.960	.1678	.0504	.3004	19.842	3.329	2.387	7.947	8
9	7.451	.1342	.0388	.2888	25.802	3.463	2.605	9.021	9
10	9.313	.1074	.0301	.2801	33.253	3.571	2.797	9.987	10

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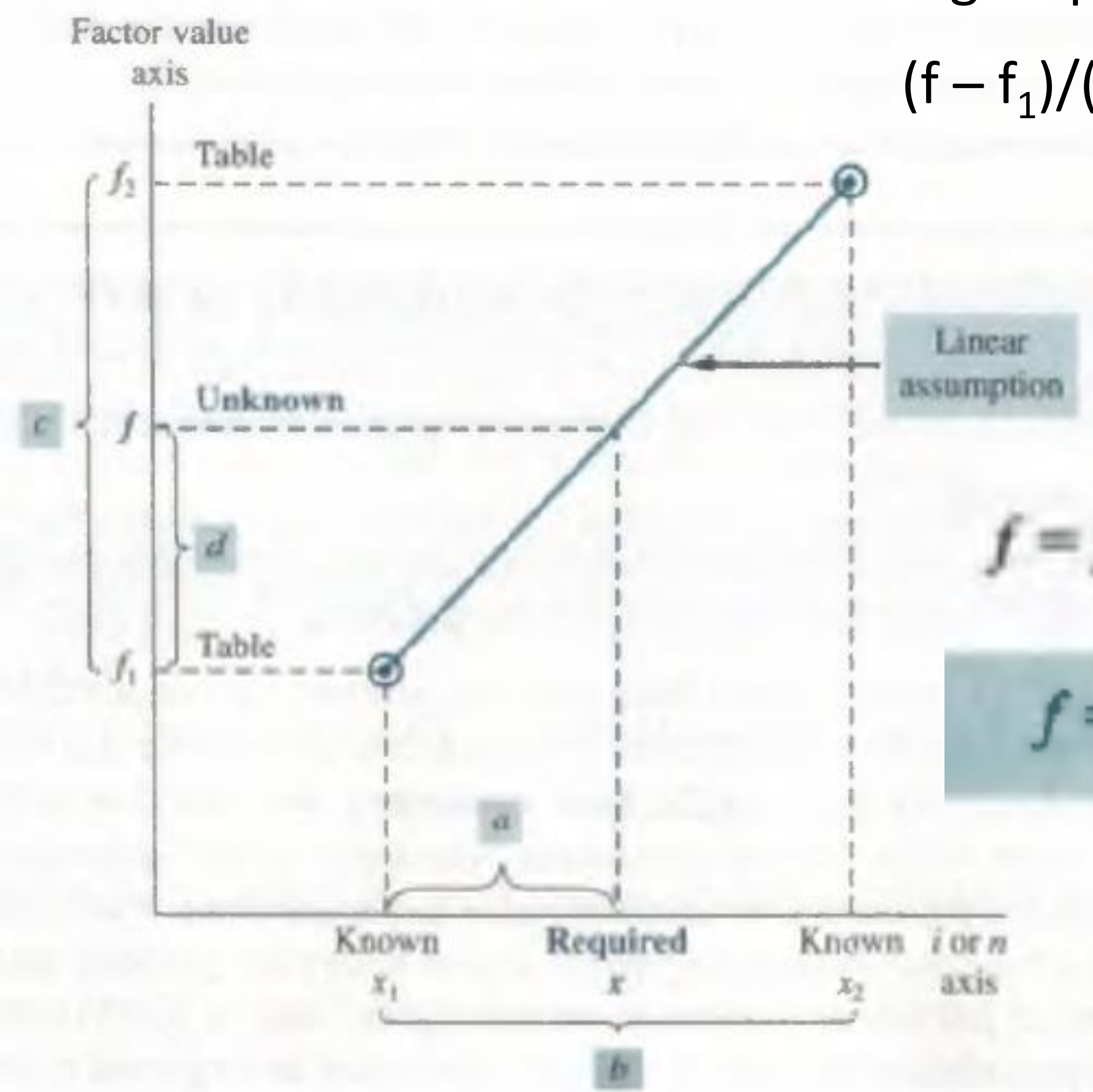
Engineering Economics

Linear Interpolation in Factor Value Tables

Using Slope Formulae

$(f - f_1)/(x - x_1) = (f_2 - f_1)/(x_2 - x_1)$



$$f = f_1 + \frac{(x - x_1)}{(x_2 - x_1)}(f_2 - f_1)$$

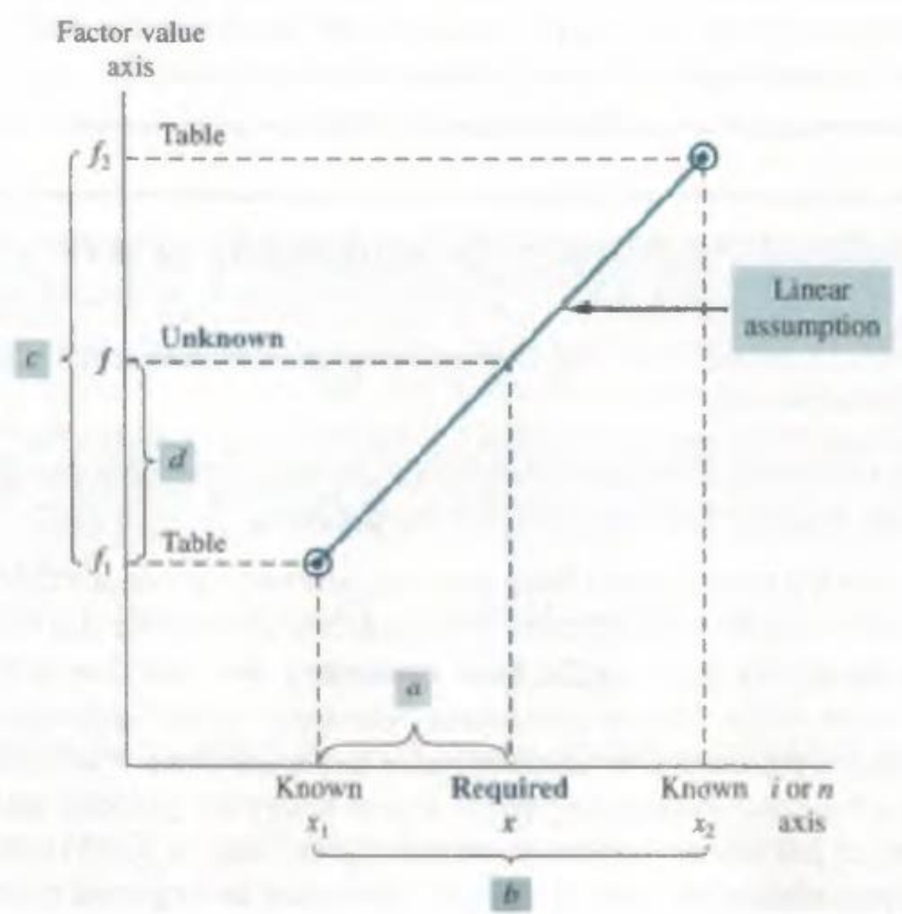
$$f = f_1 + \frac{a}{b}c = f_1 + d$$

Note: c/b = d/a

Hence, a/b * c = d

Linear Interpolation in Factor Value Tables (Contd.)

Determine the *P/A factor value for i = 7.75% and n = 10 years*



Here, the ***unknown is P/A factor*** with ***x*** = interest rate ***i*** with a value of 7.75%.

The bounding interest rates are i_1 = 7% and i_2 = 8%, and the corresponding P/A factor values are f_1 = (P/A,7%, 10) = 7.0236 and f_2 = (P/A,8%,10) = 6.7101. With 4-place accuracy,

$$f = f_1 + \frac{(i - i_1)}{(i_2 - i_1)}(f_2 - f_1) = 7.0236 + \frac{(7.75 - 7)}{(8 - 7)}(6.7101 - 7.0236)$$

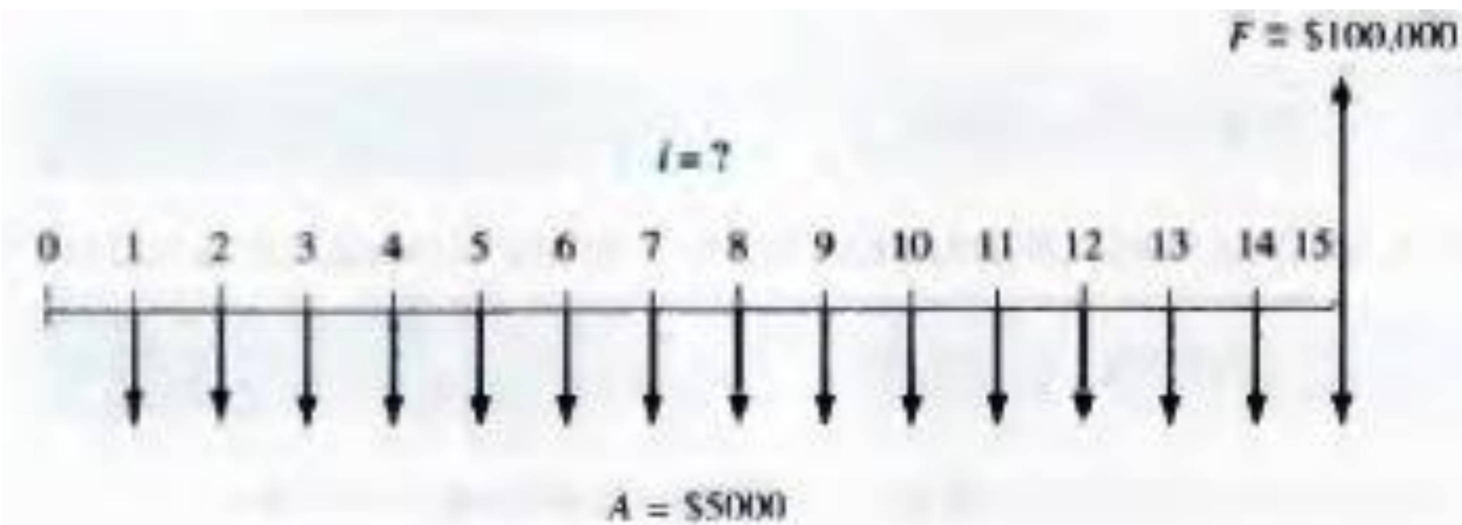
$$= 7.0236 + (0.75)(-0.3135) = 7.0236 - 0.2351$$

$$= 6.7885$$

Linear Interpolation in Factor Value Tables Example 2

Pyramid Energy requires that for each of its offshore wind power generators, \$5000 per year be placed into a capital reserve fund to cover unexpected major rework on field equipment.
In one case, \$5000 was deposited for 15 years and covered a rework costing \$100,000 in year 15. What rate of return did this practice provide to the company? Use interpolation.

Solution: The Cash Flow



Either the *A/F or F/A factor can be used.*

Linear Interpolation in Factor Value Tables Example 2 Soln.

Using A/F, $\quad A = F(A/F,i,n)$

$5000 = 100{,}000\ (A/F,i,15)$

$(A/F, i, 15) = 0.0500$

From the A/F interest tables for 15 years, the value 0.0500 lies between 3% and 4% whose factor values (A/F) are 0.0538 and 0.0499 respectively.

Thus, using

$$f = f_1 + \frac{(i - i_1)}{(i_2 - i_1)}(f_2 - f_1)$$

Here,

$0.0500 = 0.0538 + (i – 3)/ (4 -3) * (0.0499 -0.0538)$

i.e. $\quad i – 3 = (0.0500 -0.0538)/(0.0499 – 0.0538)$

i.e. $\quad i = 3 + 0.974 = 3.974$

Thus, by interpolation, i = 3.97%.

Values of Interest Factors When *n Equals Infinity*

Single Payment:

$(F/P, i, \infty) = \infty$

$(P/F, i, \infty) = 0$

Arithmetic Gradient Series:

$(A/G, i, \infty) = 1/i$

$(P/G, i, \infty) = 1/i^2$

Uniform Payment Series:

$(A/F, i, \infty) = 0$

$(A/P, i, \infty) = i$

$(F/A, i, \infty) = \infty$

$(P/A, i, \infty) = 1/i$

Nominal and Effective Interest Rates

- The '***Nominal***' and '***Effective***' interest rates have the same basic relationship as simple and compound interest rates.
- The difference here is that the concepts of nominal and effective are used when interest is ***compounded more than once each year***.
- The term *APR (**Annual Percentage Rate**)* is often stated as the annual interest rate for credit cards, loans, and house mortgages. This is the same as the ***nominal rate***. An APR of 15% is the same as nominal 15% per year or a nominal 1.25% per month.
- Also, the term *APY (**Annual Percentage Yield**)* is a commonly stated annual rate of return for investments, certificates of deposit, and savings accounts. This is the same as an ***effective rate***.
- The ***nominal rate never exceeds the effective rate***, and similarly APR < APY.

Nominal and Effective Interest Rates- Terminologies

- Interest period (t) – period of time over which interest is expressed. For example, 1% *per month.*
- Compounding period (CP) – Shortest time unit over which interest is charged or earned. For example, 10% per year *compounded monthly.*
- Compounding frequency (m) – Number of times compounding occurs within the interest period t. For example, at i = 10% per year, compounded monthly, interest would be *compounded 12 times* during the one year interest period.
- Payment period (PP) – Length of time between cash flows. For example, a deposit of *$500 every 6 months* at interest rate of 1% per month.

Nominal and Effective Interest Rates (Contd.)

- Before discussing the conversion from nominal to effective rates, it is important to *identify* a stated rate as either nominal or effective.

TABLE 3.1 Various Interest Statements and Their Interpretations

(1) Interest Rate Statement	(2) Interpretation	(3) Comment
i = 12% per year i = 1% per month i = 3½% per quarter	i = *effective* 12% per year compounded yearly i = *effective* 1% per month compounded monthly i = *effective* 3½% per quarter compounded quarterly	When no compounding period is given, interest rate is an effective rate, with compounding period assumed to be equal to stated time period.
i = 8% per year, compounded monthly i = 4% per quarter compounded monthly i = 14% per year compounded semiannually	i = *nominal* 8% per year compounded monthly i = *nominal* 4% per quarter compounded monthly i = *nominal* 14% per year compounded semiannually	When compounding period is given without stating whether the interest rate is nominal or effective, it is assumed to be nominal. Compounding period is as stated.
i = APY of 10% per year compounded monthly i = effective 6% per quarter i = effective 1% per month compounded daily	i = *effective* 10% per year compounded monthly i = *effective* 6% per quarter compounded quarterly i = *effective* 1% per month compounded daily	If interest rate is stated as an effective or APY rate, then it is an effective rate. If compounding period is not given, compounding period is assumed to coincide with stated time period.

Nominal and Effective Interest Rates (Contd.)

Examples:

TABLE 3.2 Specific Examples of Interest Statements and Interpretations

(1) Interest Rate Statement	(2) Nominal or Effective Interest	(3) Compounding Period
15% per year compounded monthly	Nominal	Monthly
15% per year	Effective	Yearly
Effective 15% per year compounded monthly	Effective	Monthly
20% per year compounded quarterly	Nominal	Quarterly
Nominal 2% per month compounded weekly	Nominal	Weekly
2% per month	Effective	Monthly
2% per month compounded monthly	Effective	Monthly
Effective 6% per quarter	Effective	Quarterly
Effective 2% per month compounded daily	Effective	Daily
1% per week compounded continuously	Nominal	Continuously

Understanding Interest Rate Terminology

A *nominal interest rate (r)* is obtained by multiplying an interest rate that is expressed over a short time period by the number of periods in a longer time period: That is:

r = interest rate per period x number of periods

Example: If i = 1% per month, nominal rate per year is

r = (1)(12) = 12% per year

A nominal interest rate can be found for any time period that is longer than the compounding period.

Effective interest rates (i) take compounding into account (effective rates can be obtained from nominal rates via a formula discussed later).

IMPORTANT:

Nominal interest rates are essentially simple interest rates. Therefore, they can *never* be used in interest formulas.

Effective rates must *always* be used hereafter in all interest formulas.

Effective Interest Rates

Nominal rates can be converted into effective rates **for any time period** via the following equation:

$$i \text{ per period} = (1 + r / m)^m - 1$$

where i = *effective* interest rate for a certain period, say 6 months

r = *nominal* interest rate for the same time period (6 months here)

m = number of times that interest is *compounded* [as per the numerical] *in that same period* (six months here)

Thus, *effective interest rates* can also be *calculated for any time period longer* than the compounding period of a given interest rate.

Example:

a. A Visa credit card issued through Frost Bank carries an interest rate of 1% per month on the unpaid balance. Calculate the effective rate per semiannual and annual periods.

b. If the card's interest rate is stated as 3.5% per quarter, find the effective semiannual and annual rates.

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Effective Interest Rates- Recap

Nominal rates can be converted into effective rates **for any time period** via the following equation:

$$\boldsymbol{i \text{ per period} = (1 + r / m)^m - 1}$$

where i = *effective* interest rate for a certain period, say 6 months

r = *nominal* interest rate for the same time period (6 months here)

m = number of times that interest is *compounded* [as per the numerical] *in that same period* (six months here)

Thus, *effective interest rates* can also be *calculated for any time period longer* than the compounding period of a given interest rate.

Example:

a. A Visa credit card issued through Frost Bank carries an interest rate of 1% per month on the unpaid balance. Calculate the effective rate per semiannual and annual periods.

b. If the card's interest rate is stated as 3.5% per quarter, find the effective semiannual and annual rates.

Effective Interest Rates (Contd.)

Solution

a. The compounding period is monthly. For the effective interest rate per semiannual period, the r in Equation [3.2] must be the nominal rate per 6 months.

$$\begin{aligned} r &= 1\% \text{ per month} \times 6 \text{ months per semiannual period} \\ &= 6\% \text{ per semiannual period} \end{aligned}$$

The m in Equation [3.2] is equal to 6, since the frequency with which interest is compounded is 6 times in 6 months. The effective semiannual rate is

$$\begin{aligned} i \text{ per 6 months} &= \left(1 + \frac{0.06}{6}\right)^6 - 1 \\ &= 0.0615 \qquad (6.15\%) \end{aligned}$$

For the effective annual rate, $r = 12\%$ per year and $m = 12$. By Equation [3.2],

$$\text{Effective } i \text{ per year} = \left(1 + \frac{0.12}{12}\right)^{12} - 1 = 0.1268 \qquad (12.68\%)$$

Effective Interest Rates (Contd.)

b. For an interest rate of 3.5% per quarter, the compounding period is a quarter. In a semiannual period, $m = 2$ and $r = 7\%$.

$$\begin{aligned} i \text{ per 6 months} &= \left(1 + \frac{0.07}{2}\right)^2 - 1 \\ &= 0.0712 \qquad (7.12\%) \end{aligned}$$

The effective interest rate per year is determined using $r = 14\%$ and $m = 4$.

$$\begin{aligned} i \text{ per year} &= \left(1 + \frac{0.14}{4}\right)^4 - 1 \\ &= 0.1475 \qquad (14.75\%) \end{aligned}$$

Comment: Note that the term r/m in Equation [3.2] is always the effective interest rate per compounding period. In part (*a*) this is 1% per month, while in part (*b*) it is 3.5% per quarter.

Continuous Compounding

If we allow compounding to occur more and more frequently, the compounding period becomes shorter and shorter. Then *m*, the number of compounding periods per payment period, increases.

Take limit as m → ∞ to find the effective continuous interest rate

$$\mathbf{i = e^r - 1}$$

Example 1: If the nominal annual r = 15% *per year,* the effective continuous rate *per year* is

Solution: **Effective rate** i % = $e^{0.15} - 1$ = 16.183%

Example 2

a. For an interest rate of 18% per year compounded continuously, calculate the effective monthly and annual interest rates.

b. An investor requires an *effective* return of at least 15%. What is the minimum annual nominal rate that is acceptable for continuous compounding?

Continuous Compounding (Contd.)

Solution

a. The nominal monthly rate is $r = 18\%/12 = 1.5\%$, or 0.015 per month. By Equation [3.3], the effective monthly rate is

$$i\% \text{ per month} = e^r - 1 = e^{0.015} - 1 = 1.511\%$$

Similarly, the effective annual rate using $r = 0.18$ per year is

$$i\% \text{ per year} = e^r - 1 = e^{0.18} - 1 = 19.72\%$$

b. Solve Equation [3.3] for r by taking the natural logarithm.

$$\begin{aligned} e^r - 1 &= 0.15 \\ e^r &= 1.15 \\ \ln e^r &= \ln 1.15 \\ r\% &= 13.976\% \end{aligned}$$

Therefore, a nominal rate of 13.976% per year compounded continuously will generate an effective 15% per year return.

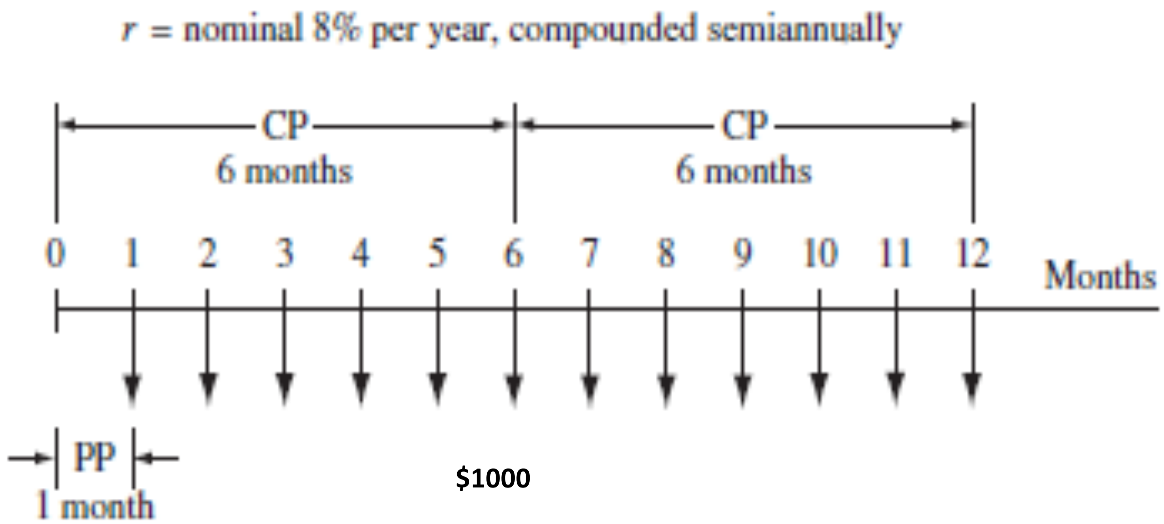
Comment: The general formula to find the nominal rate, given the effective continuous rate i, is $r = \ln(1 + i)$

When to use Continuous Compounding in Business

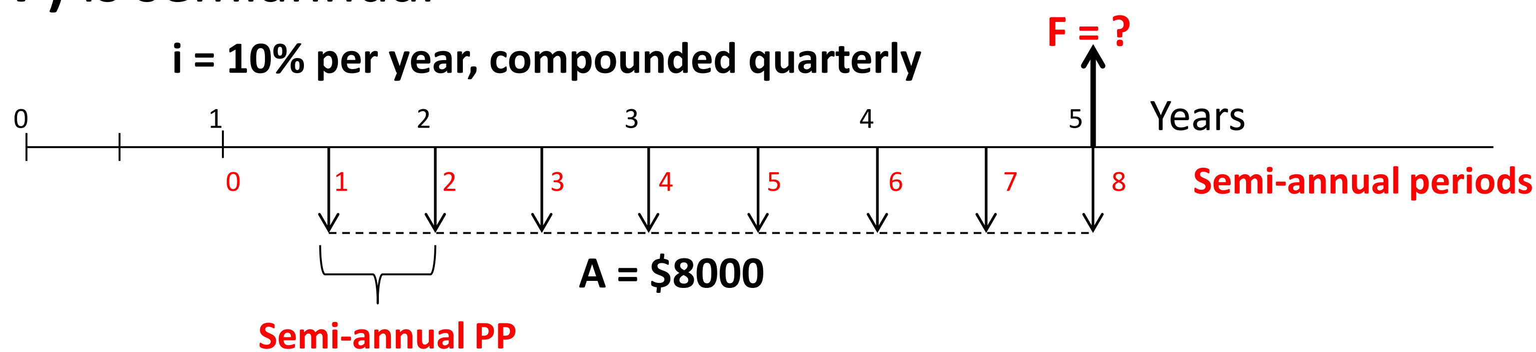
- For some business activities, cash flows occur throughout the day.
- Examples of such costs are energy and water costs, inventory costs, and labour costs.
- A realistic model for these activities is to increase the frequency of the cash flows to become continuous.
- In these cases, the economic analysis can be performed for **continuous cash flow (also called continuous funds flow)** and the continuous compounding of interest as discussed earlier.
- Different expressions must be derived for the factors for these cases.
- However, the *monetary differences for continuous* cash flows *relative to the discrete cash flow* and discrete compounding assumptions *are usually not large*.
- Accordingly, most engineering economy studies do not require the analyst to utilize these mathematical forms to make a sound economic decision.

Equivalence Relations: PP and CP

In the diagram below, the compounding period (CP) is semiannual and the payment period (PP) is monthly



Similarly, for the diagram below, the CP is quarterly and the **payment period (PP)** is semiannual



Equivalence Relations: PP and CP (Contd.)

In general, there are three steps to be followed in the cases where the compounding period (CP) and the payment period (PP) do not coincide.

1. Compare the lengths of PP and CP.
2. Identify the cash-flow series as involving only single amounts (P and F) or series amounts (A, G, or g).
3. Select the proper i and n values (as discussed in the following slides)

Single Amount Factors

For problems involving single amounts (*F/P* and *P/F*), there are an infinite number of i and n combinations that can be used, with only two restrictions:

(1) The i must be an *effective* interest rate, and

(2) The time units on n must be *the same* as that on i
(i.e., if i is a rate per quarter, then n is the number of quarters between P and F)

In standard factor notation, the single-payment equations can be generalized.

$P = F$(P/F, effective i per period, number of periods)

OR $F =$ P(F/P, effective i per period, number of periods)

Thus, for a nominal interest rate of 12% per year compounded monthly, any of the *i* and corresponding *n* values shown in Table in the next slide could be used.

Single Amount Factors (Contd.)

For example, if an effective quarterly interest rate is used for *i*, that is, $(1.01)^3 - 1 =$ 3.03%, then the *n* time unit is 4 quarters in a year.

TABLE 3.4 Various *i* and *n* Values for Single-Amount Equations Using *r* = 12% per Year, Compounded Monthly

Effective Interest Rate, *i*	Units for *n*
1% per month	Months
3.03% per quarter	Quarters
6.15% per 6 months	Semiannual periods
12.68% per year	Years
26.97% per 2 years	2-year periods

Example 1: Sherry expects to deposit \$1000 now, \$3000 in 4 years from now, and \$1500 in 6 years from now and earn at a rate of 12% per year compounded semiannually through a company-sponsored savings plan. What amount can she withdraw 10 years from now?

Single Amount Factors (Contd.)

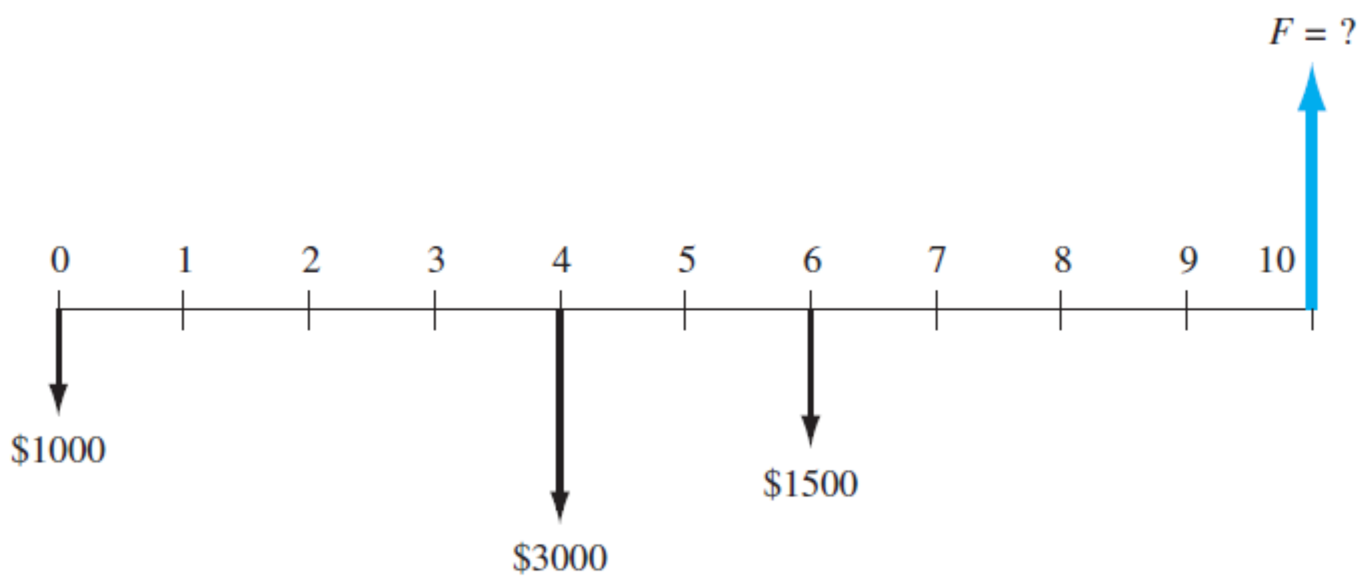
Solution

Only single-amount P and F values are involved (Figure 3.2). Since only effective rates can be present in the factors, use an effective rate of 6% per semiannual compounding period and semiannual payment periods. The future worth is calculated using Equation [3.5].

$$F = 1000(F/P,6\%,20) + 3000(F/P,6\%,12) + 1500(F/P,6\%,8)$$
$$= \$11.634$$

An alternative solution strategy is to find the effective annual rate by Equation [3.2] and express n in years as determined from the problem statement.

$$\text{effective } i \text{ per year} = \left(1 + \frac{0.12}{2}\right)^2 - 1 = 0.1236 \qquad (12.36\%)$$



Example 2: Single Amounts

How much money will be in an account in 5 years if \$10,000 is deposited now at an interest rate of 1% per month? Use three different interest rates: (a) monthly, (b) quarterly , and (c) yearly.

(a) For monthly rate, 1% is effective [n = (5 years)×(12 CP per year) = 60]

F = 10,000(F/P,1%,60) = \$18,167

months

effective i per month

i and n must *always* have same time units

(b) For a quarterly rate, effective i/quarter = $(1 + 0.03/3)^3 - 1 = 3.03\%$

F = 10,000(F/P,3.03%,20) = \$18,167

quarters

effective i per quarter

i and n must *always* have same time units

(c) For an annual rate, effective i/year = $(1 + 0.12/12)^{12} - 1 = 12.683\%$

F = 10,000(F/P,12.683%,5) = \$18,167

years

effective i per year

i and n must *always* have same time units

Series with PP ≥ CP

For series cash flows, *first step* is to determine *relationship* between Payment Period (PP) and Compounding Period (CP)

Determine if PP ≥ CP, or if PP < CP

When **PP ≥ CP**, the *only* procedure (2 steps) that can be used is as follows:

Step 1. Count the number of payments and use that number as *n*. For example, if payments are made quarterly for 5 years, *n* is 20.

Step 2. Find the *effective* interest rate over the *same time period* as *n* in step 1. For example, if *n* is expressed in quarters, then the effective interest rate per quarter *must* be used.

Use these values for *n* and *i* (**and only these!**) in the factors, functions, or formulas.

Examples of n and i values for series with PP ≥ CP

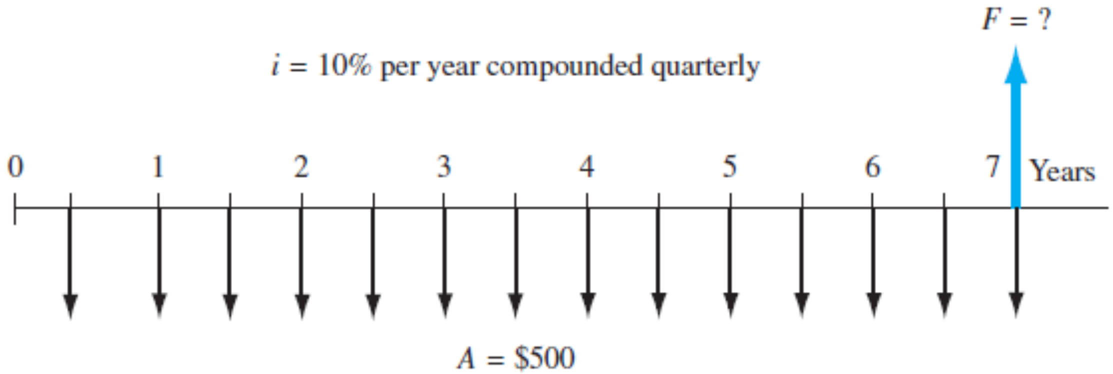
TABLE 3.5 Examples of *n* and *i* Values Where PP = CP or PP > CP

(1) Cash-flow Sequence	(2) Interest Rate	(3) What to Find; What is Given	(4) Standard Notation
\$500 semiannually for 5 years	8% per year compounded semiannually	Find P; given A	$P = 500(P/A,4\%,10)$
\$75 monthly for 3 years	12% per year compounded monthly	Find F; given A	$F = 75(F/A,1\%,36)$
\$180 quarterly for 15 years	5% per quarter	Find F; given A	$F = 180(F/A,5\%,60)$
\$25 per month increase for 4 years	1% per month	Find P; given G	$P = 25(P/G,1\%,48)$
\$5000 per quarter for 6 years	1% per month	Find P; given A	$P = 5000(P/A,3.03\%,24)$

Example 1: Series with PP ≥ CP

For the past 7 years, a quality manager has paid \$500 every 6 months for the software maintenance contract on a laser-based measuring instrument. What is the equivalent amount after the last payment, if these funds are taken from a pool that has been returning 10% per year compounded quarterly?

Solution:



The cash flow diagram is shown in Figure 3.3. The payment period (6 months) is longer than the compounding period (quarter); that is, PP > CP. Applying the guideline, determine an effective semiannual interest rate. Use Equation [3.2] or Table 3.3 with $r = 0.05$ per 6-month period and $m = 2$ quarters per semiannual period.

$$\text{Effective } i\% \text{ per 6-months} = \left(1 + \frac{0.05}{2}\right)^2 - 1 = 5.063\%$$

The value $i = 5.063\%$ is reasonable, since the effective rate should be slightly higher than the nominal rate of 5% per 6-month period. The number of semiannual periods is $n = 2(7) = 14$. The future worth is

$$F = A(F/A,5.063\%,14) \quad = 500\ (19.6845) = \$9842$$

Example 2: Series with PP ≥ CP

How much money will be accumulated in 10 years from a deposit of \$500 every 6 months if the interest rate is 1% per month?

Solution: First, find relationship between PP and CP

PP = ***six months***, CP = ***one month***; Therefore, **PP > CP**

Since PP > CP, find effective i per PP of six months

Step 1. i **per 6 months** $= (1 + 0.06/6)^6 - 1 = 6.15\%$

Next, determine n (number of 6-month periods)

Step 2: $n = 10(2) = 20$ **six month periods**

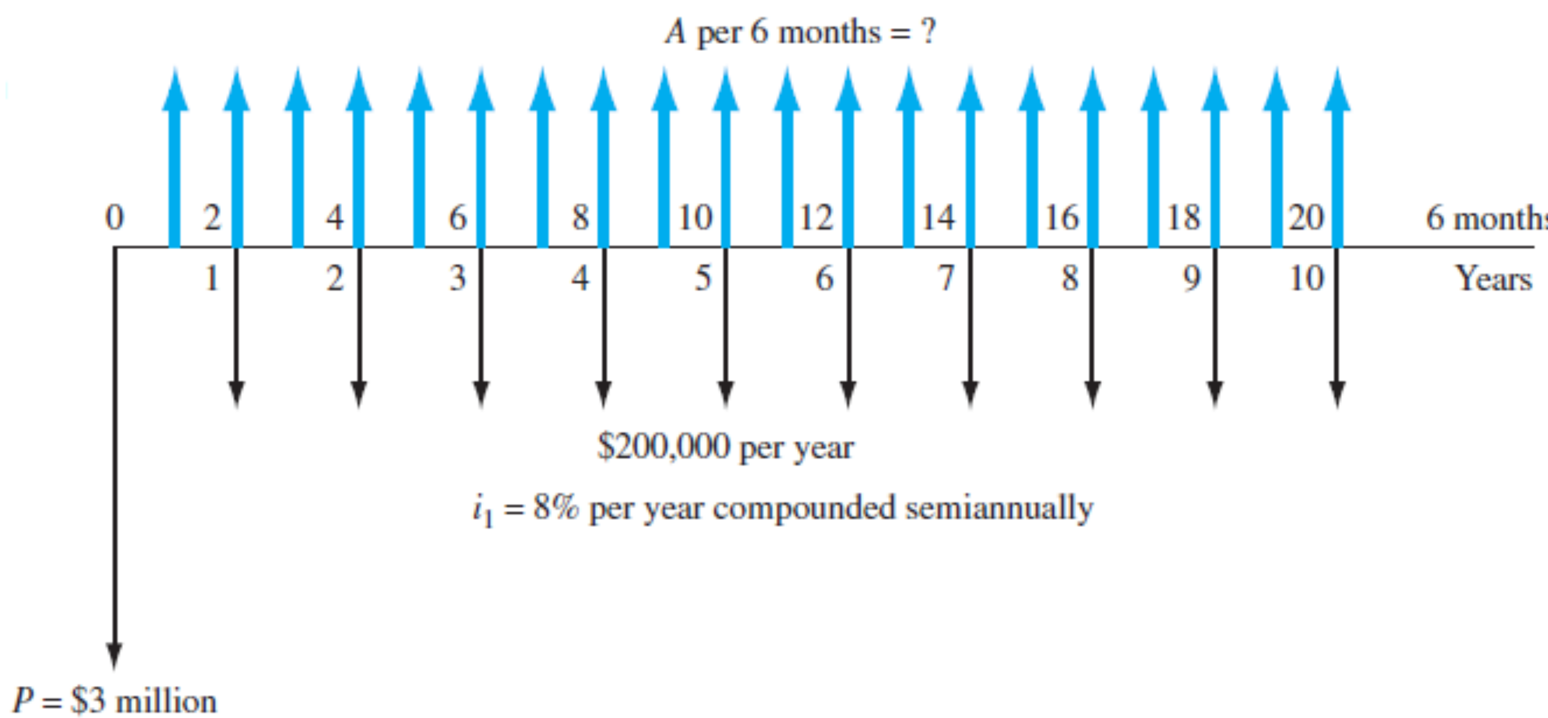
Finally, set up equation and solve for F

$F = 500(F/A,6.15\%,20) = \$18,692$ (by factor interpolation or formula)

Example 3: Single Amount and Series with PP ≥ CP

ExxonMobil is using a recently installed remotely controlled system to detect underwater leakage from offshore platforms. Assume this system costs \$3 million to install and an estimated \$200,000 per year for all materials, operating, personnel, and maintenance costs. The expected life is 10 years. An engineer wants to estimate the total revenue requirement for each 6-month period that is necessary to recover the investment, interest, and annual costs. Find this semiannual A value if capital funds are evaluated at 8% per year compounded semiannually.

Solution



Example 3: Single Amount and Series with PP ≥ CP

There are several ways to solve this problem, but the most straightforward one is a two-stage approach. First, convert all cash flows to a *P* at time 0, then find the *A* over the 20 semiannual periods.

For stage 1, recognize that PP > CP, that is, 1 year > 6 months.

According to the procedure for types 1 and 2 cash flows, ***n* = 10**,the number of annual payments. Now, find the effective *i per year* and use it to find *P*.

$$i\% \text{ per year} = (1 + 0.08/2)^2 - 1 = 8.16\%$$

$$\begin{aligned} P &= 3{,}000{,}000 + 200{,}000(P/A{,}8.16\%{,}10) \\ &= 3{,}000{,}000 + 200{,}000(6.6620) \\ &= \$4{,}332{,}400 \end{aligned}$$

For stage 2, *P* is converted to a semiannual *A* value. Now, PP = CP = 6 months, and $n = 20$ *semiannual* payments. The effective semiannual *i* for use in the A/P factor is determined directly from the problem statement using r/m.

$$i\% \text{ per 6 months} = 8\%/2 = 4\%$$

$$\begin{aligned} A &= 4{,}332{,}400(A/P{,}4\%{,}20) \\ &= \$318{,}778 \end{aligned}$$

In conclusion, $318,778 every 6 months will repay the initial and annual costs, if money is worth 8% per year compounded semiannually.

Series with PP < CP

Two policies:

(1) interperiod cash flows earn *no interest* (most common)

(2) Interperiod cash flows earn *compound interest*

For policy (1), *positive cash flows* are moved *to beginning of the interest period* in which they occur and *negative cash flows are moved to the end of the interest period*

Note: The condition of PP < CP with no interperiod interest is the ***only situation in which*** the actual cash flow diagram is changed

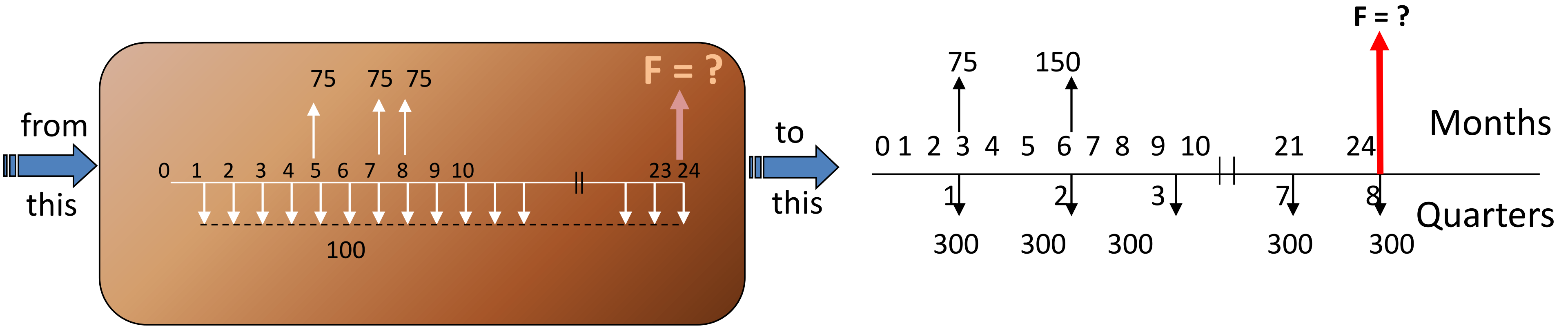
For policy (2), cash flows are *not moved* and equivalent P, F, and A values are determined using the *effective interest rate per payment period*

Series with PP < CP (Contd.)

A person deposits \$100 per month into a savings account for 2 years. If \$75 is withdrawn in months 5, 7 and 8 (in addition to the deposits), **construct the cash flow diagram** to determine how much will be in the account after 2 years at i = 6% per year, compounded quarterly. Assume there is no interperiod interest.

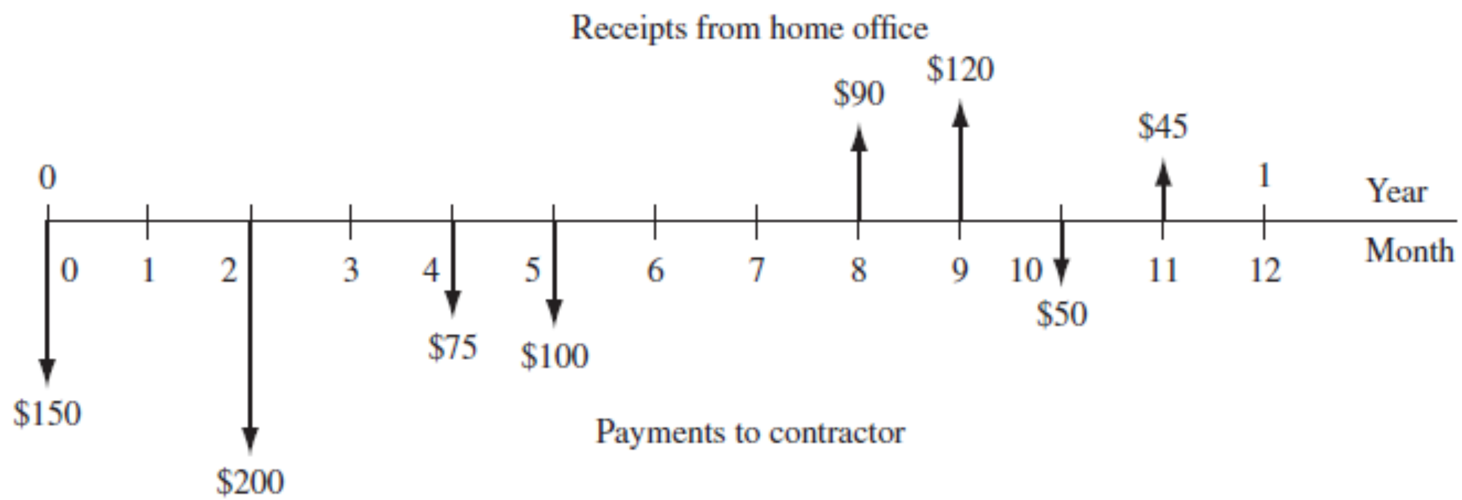
Solution:

Since PP < CP with no interperiod interest, the cash flow diagram must be *changed using quarters as the time periods*



Example Series with PP < CP

Rob is the on-site coordinating engineer for Alcoa Aluminum, where an under-renovation mine has new ore refining equipment being installed by a local contractor. Rob developed the cash flow diagram in Figure 3.5*a* in $1000 units from the project perspective. Included are payments to the contractor he has authorized for the current year and approved advances from Alcoa's home office. He knows that the interest rate on equipment "field projects" such as this is 12% per year compounded quarterly, and that Alcoa does not bother with interperiod compounding of interest. Will Rob's project finances be in the "red" or the "black" at the end of the year? By how much?



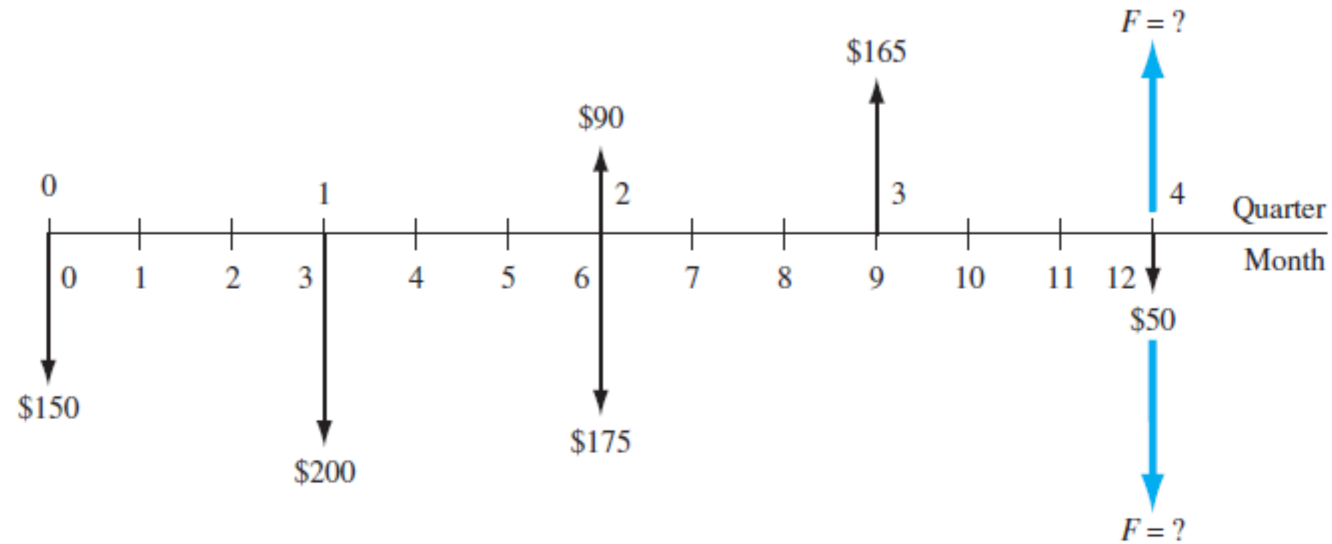
(*a*)

Example Series with PP < CP

With no interperiod interest considered, Figure b reflects the moved cash flows.

The future worth after four quarters requires an F at an effective rate per quarter such that **PP = CP = 1 quarter**.

Therefore, the ***effective i*** = 12%/4 = **3%**.



(b)

Figure b shows all negative cash flows (payments to contractor) moved to the end of the respective quarter, and all positive cash flows (receipts from home office) moved to the beginning of the respective quarter. Calculate the F value at 3%.

$$F = 1000[-150(F/P,3\%,4) - 200(F/P,3\%,3) + (-175 + 90)(F/P,3\%,2) + 165(F/P,3\%,1) - 50] = \$-357{,}592$$

Rob can conclude that the on-site project finances will be in the red about $357,600 by the end of the year.

Varying Rates

When interest rates vary over time, use the interest rates associated with their respective time periods to find P

Example: Find the present worth of \$2500 deposits in years 1 through 8 if the interest rate is 7% per year for the first five years and 10% per year thereafter.

Solution: $P = 2{,}500(P/A,7\%,5) + 2{,}500(P/A,10\%,3)(P/F,7\%,5)$

$= \$14{,}683$

An equivalent annual worth value can be obtained by replacing each cash flow amount with ‘A’ and setting the equation equal to the calculated P value

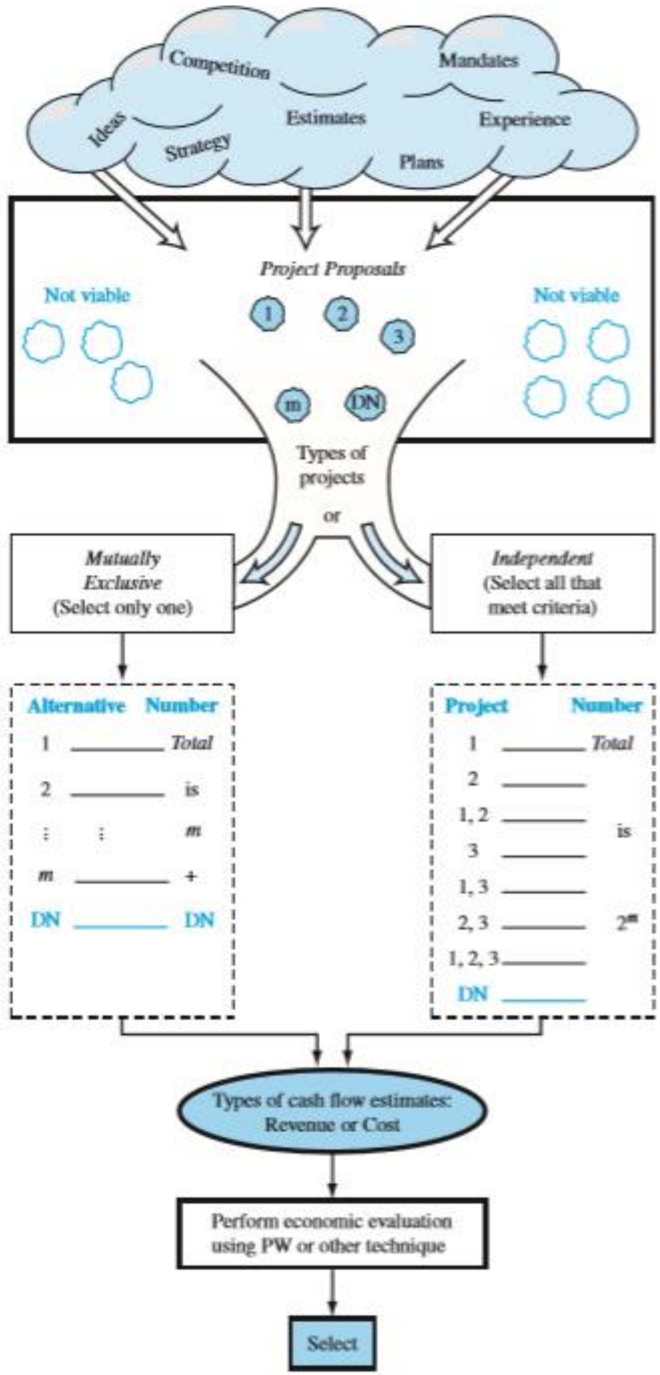
$$14{,}683 = A(P/A,7\%,5) + A(P/A,10\%,3)(P/F,7\%,5)$$

$A =$ **\$2500 per year**

SM 300

Engineering Economics

Logical progression from proposals to alternatives to selection.



Formulating Alternatives

Two types of economic proposals

- Mutually Exclusive (ME) Alternatives:
  Only one can be selected; Compete against each other. Default is DN, when no alternative is economically justifiable

- Independent Projects:
  More than one can be selected; Compete only against DN.
  (There may be dependent projects requiring a particular project to be selected before another, and/or contingent projects where one project may be substituted for another.)

Do Nothing (DN) – An ME alternative or independent project to maintain the current approach; no new costs, revenues or savings

Formulating Alternatives (Contd.)

Two types of cash flow estimates

- Revenue: Alternatives include estimates of costs (cash outflows) *and* revenues (cash inflows)

- Cost: Alternatives include *only* costs; revenues and savings assumed equal for all alternatives; also called *service alternatives*

Basis for Comparison of Alternatives- Present Worth Analysis

Present Worth (PW) Analysis of Alternatives

- Convert all cash flows to PW using Minimum Acceptable Rate of Return (MARR). This converts all future cash flows into present dollar equivalents. This makes it easy to determine the economic advantage of one alternative over another.

- Precede *costs* by *minus* sign; *receipts* by *plus* sign

EVALUATION

- For one project, if PW > 0, it is financially viable.

- For mutually exclusive alternatives, select one with *numerically largest PW* **[less negative or more positive]**

- For independent projects, select all with *PW > 0*

Selection of Alternatives by PW

For the alternatives shown below, which should be selected if they are (a) mutually exclusive; (b) independent?

Project ID	**Present Worth**
A	\$30,000
B	\$12,500
C	\$-4,000
D	\$ 2,000

Solution:

(a) **Select numerically largest PW; alternative A**

(b) **Select all with PW > 0; projects A, B & D**

Example: PW Evaluation of Equal-Life ME Alts.

Alternative X has a first cost of $20,000, an operating cost of $9,000 per year, and a $5,000 salvage value after 5 years. Alternative Y will cost $35,000 with an operating cost of $4,000 per year and a salvage value of $7,000 after 5 years. At an MARR of 12% per year, which should be selected?

Solution: Find PW at MARR and select numerically larger PW value

$$PW_X = -20{,}000 - 9000(P/A,12\%,5) + 5000(P/F,12\%,5)$$
$$= -\$49{,}606$$

$$PW_Y = -35{,}000 - 4000(P/A,12\%,5) + 7000(P/F,12\%,5)$$
$$= -\$45{,}447$$

Select alternative Y

PW of Different-Life Alternatives

Must compare alternatives for ***equal service***
(i.e., alternatives must **end at the same time**)

Two ways to compare equal service:

- **Least common multiple (LCM) of lives**
- **Specified study period**

(The LCM procedure is used unless otherwise specified)

Assumptions of LCM approach

- Service provided is needed over the LCM or more years

- Selected alternative can be repeated over each life cycle of LCM in exactly the same manner

- Cash flow estimates are the same for each life cycle (i.e., change in exact accord with the inflation or deflation rate)

Example: Different-Life Alternatives

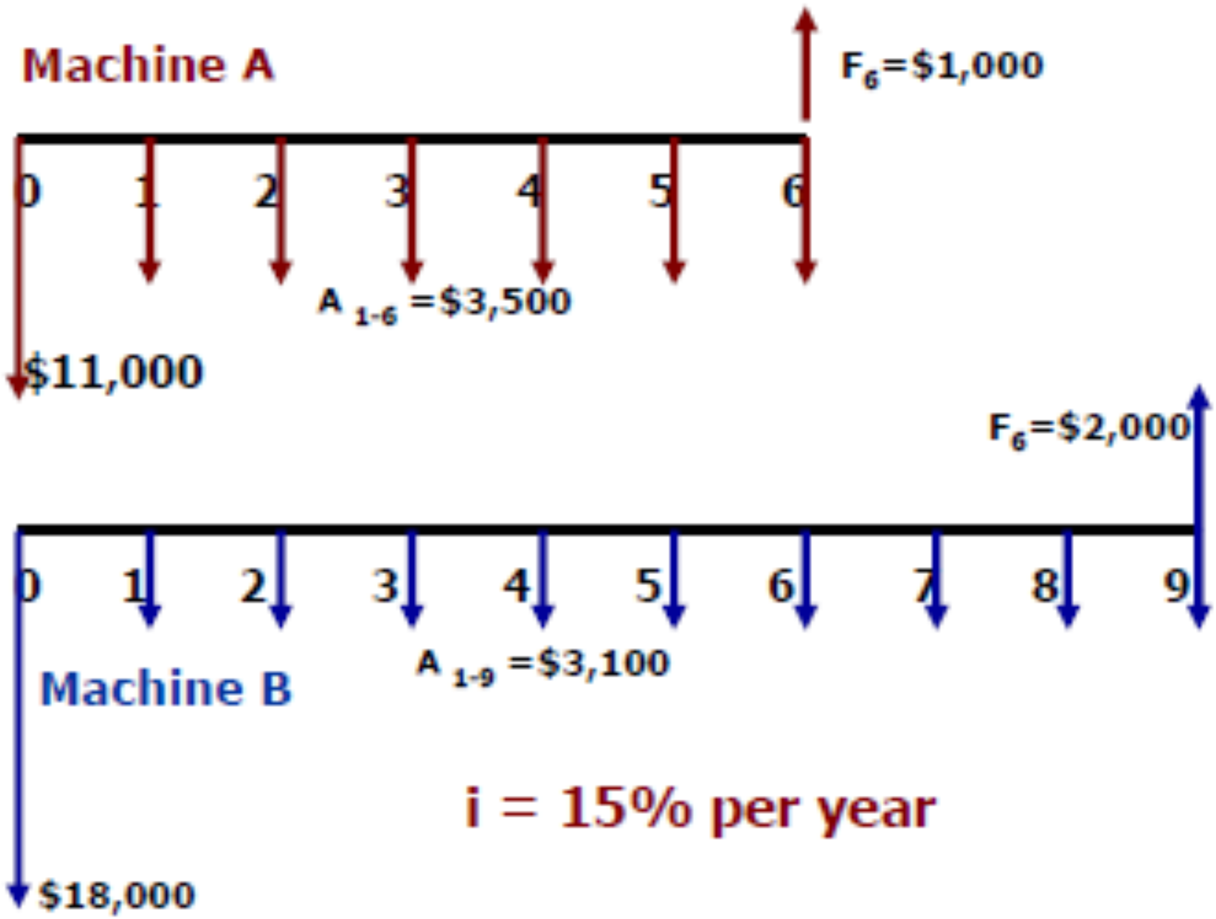
Which machine to choose?

	Machine A	**Machine B**
First Cost	$11,000	$18,000
Annual Operating Cost	3,500	3,100
Salvage Value	1,000	2,000
Life	6 years	9 years

i = 15% per year

Example: Different-Life Alternatives (Solution)

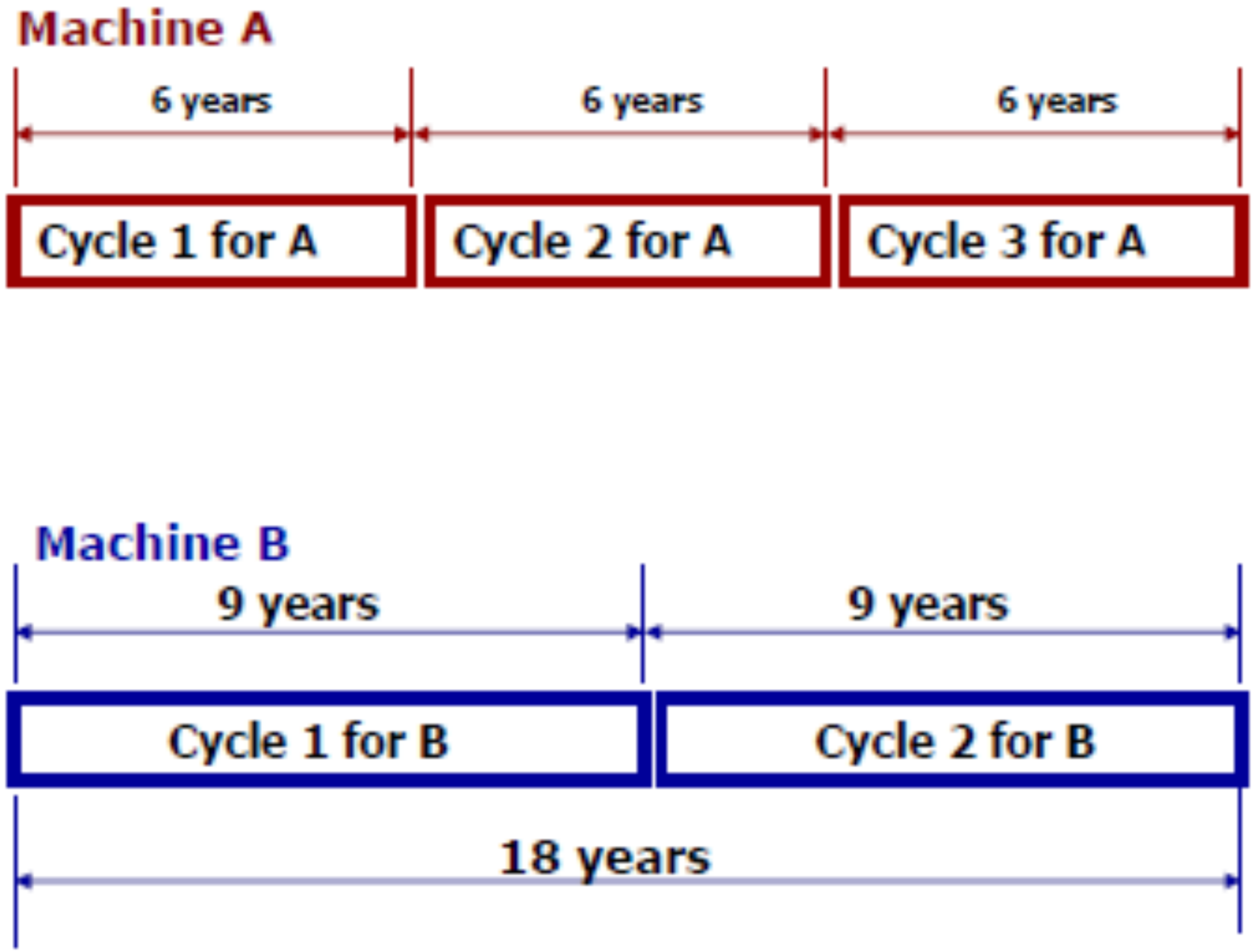
Cash Flow Diagram for Each:



LCM(6,9) = 18 year study period will apply for present worth

Example: Different-Life Alternatives (Solution) (contd.)

Cash Flow Diagram for LCM duration:



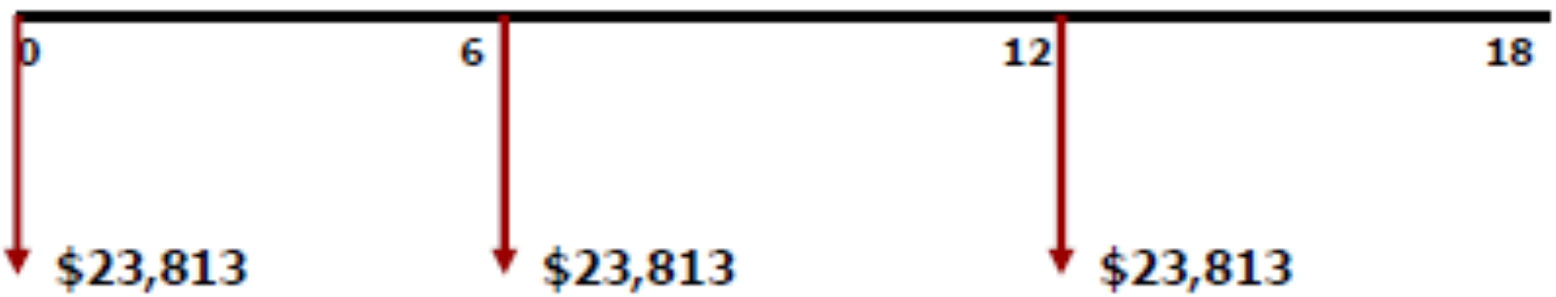
Example: Different-Life Alternatives (Solution) (contd.)

Calculate the present worth of a 6-year cycle for alternative A

PWA = -11,000 - 3,500 (P|A, 15%, 6) + 1,000 (P|F, 15%, 6)

= -11,000 - 3,500 (3.7845) + 1,000 (.4323)

= $ -23,813 which occurs at time 0, 6 and 12



Now, for the 18 years:

PWA= - 23,813- 23,813 (P|F, 15%, 6)- 23,813 (P|F, 15%, 12)

= - 23,813 - 10,294 – 4,451 = $ -38,558

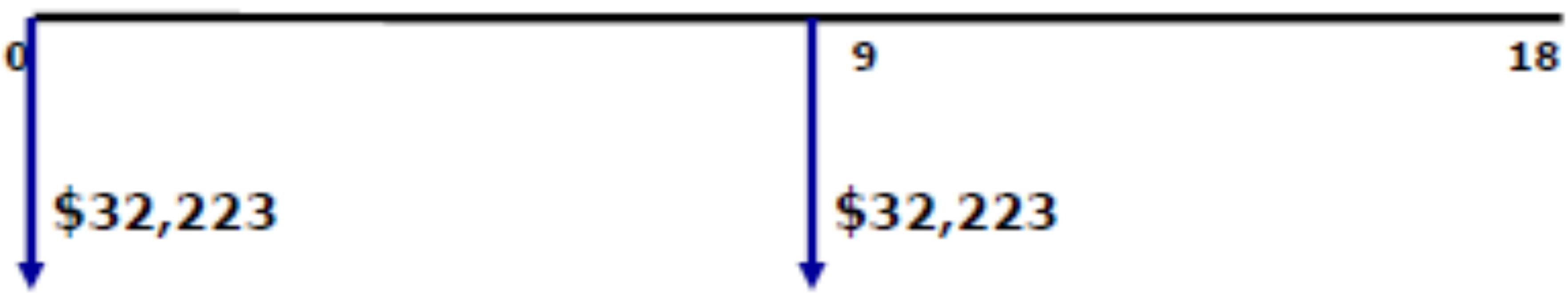
Example: Different-Life Alternatives (Solution) (contd.)

Calculate the Present Worth of a 9-year cycle for alternative B

PWB = -18,000-3,100(P|A, 15%, 9) + 2,000(P|F, 15%, 9)

= -18,000 - 3,100(4.7716) + 2,000(.2843)

= \$ - 32,223 which occurs at time 0 and 9



Now, for the 18 years:

PWB = - 32,223 – 32,223(P|F, 15%, 9)

= - 32,223 - 32,223(.2843) = \$ - 41,384

PWA > PWB, so the company should choose machine A.

PW Evaluation Using a Study Period

- Once a study period is specified, all cash flows after this time are ignored

- Salvage value is the estimated market value *at the end of <u>study period</u>*

Short study periods are often defined by management when business goals are short-term

Study periods are commonly used in <u>*equipment replacement analysis*</u>

Example: Study Period PW Evaluation

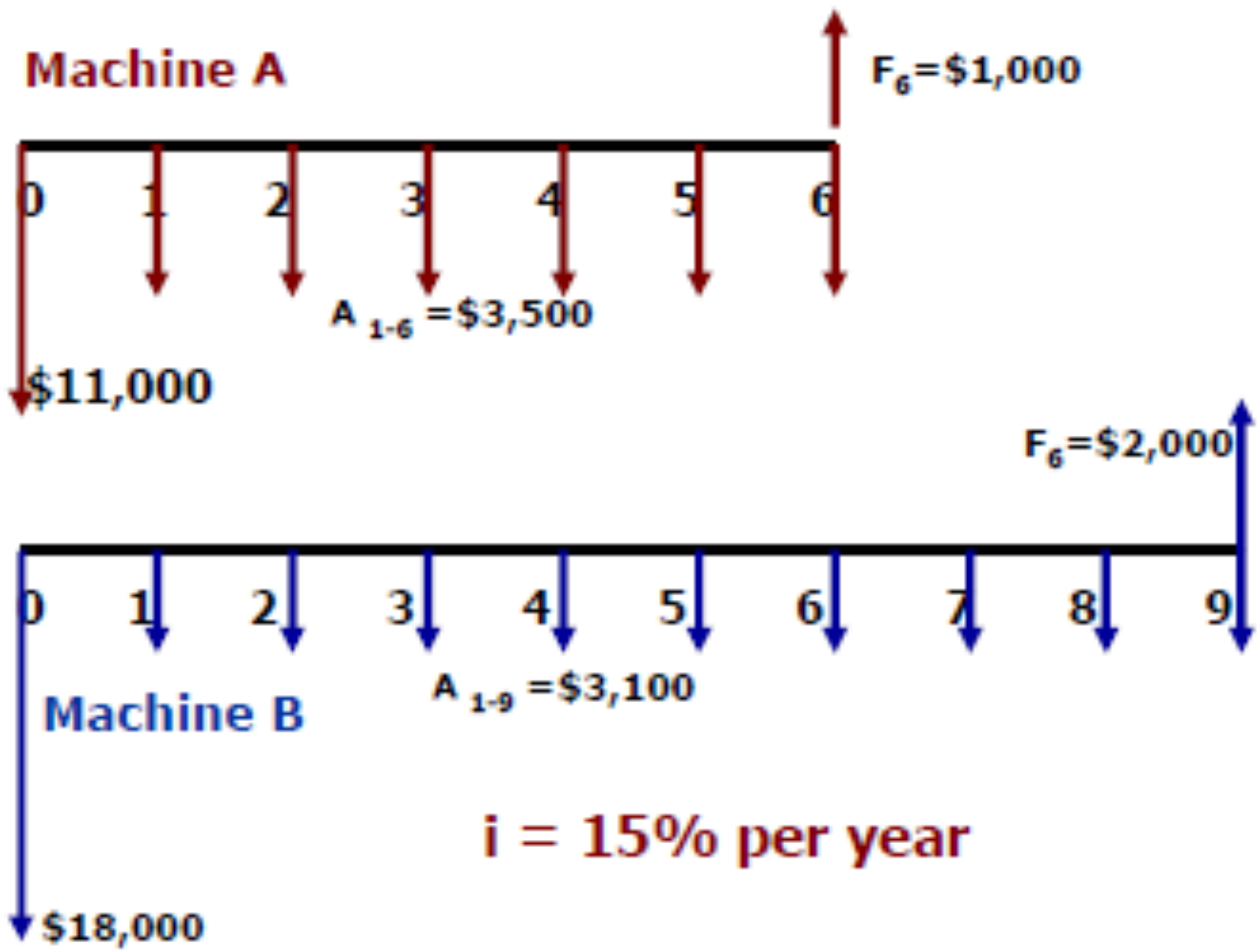
Continue with previous example, which machine should the company choose if the interest rate is 15% per year, and the study period is 5 years? ***Assume*** **that salvage values at the end of the study period are same as end of life.**

	Machine A	**Machine B**
First Cost	\$11,000	\$18,000
Annual Operating Cost	3,500	3,100
Salvage Value	1,000	2,000
Life	6 years	9 years

i = 15% per year

Example: Study Period PW Evaluation (Solution)

Cash Flow Diagrams:



Note: Study Period is 5 years

Example: Study Period PW Evaluation (Solution) contd.

Machine A:

PWA = -11,000 – 3,500(P/A,15%,5) + 1,000(P/F,15%,5)

= -11,000 – 3,500(3.3522) + 1,000(0.4972)

= $- 22,235.50

Machine B:

PWB = -18,000 – 3,100(P/A,15%,5) + 2,000(P/F,15%,5)

= -18,000 – 3,100(3.3522) + 2,000(0.4972)

= $- 27,397.42

Decision:

PWA > PWB, so the company should choose machine A.

Future Worth Analysis

FW evaluation of alternatives is especially applicable for LARGE capital investment situations when **maximizing the future worth of a corporation** is important.

FW is exactly like PW analysis, except calculate FW

FW > 0 means the MARR is met or exceeded. For two or more mutually exclusive alternatives, select the one with the numerically largest FW value.

Applications:

Projects that become live only at the end of the investment period, like

- Toll Roads
- Electric generation facilities
- Hotels
- Commercial Buildings

FW of Equal-Life Example

There are two alternatives for purchasing a concrete mixer. Both the alternatives have same useful life. The cash flow details of alternatives are as follows;

Alternative-1: Initial purchase cost = Rs.300000, Annual operating and maintenance cost = Rs.20000, Expected salvage value = Rs.125000, Useful life = 5 years.

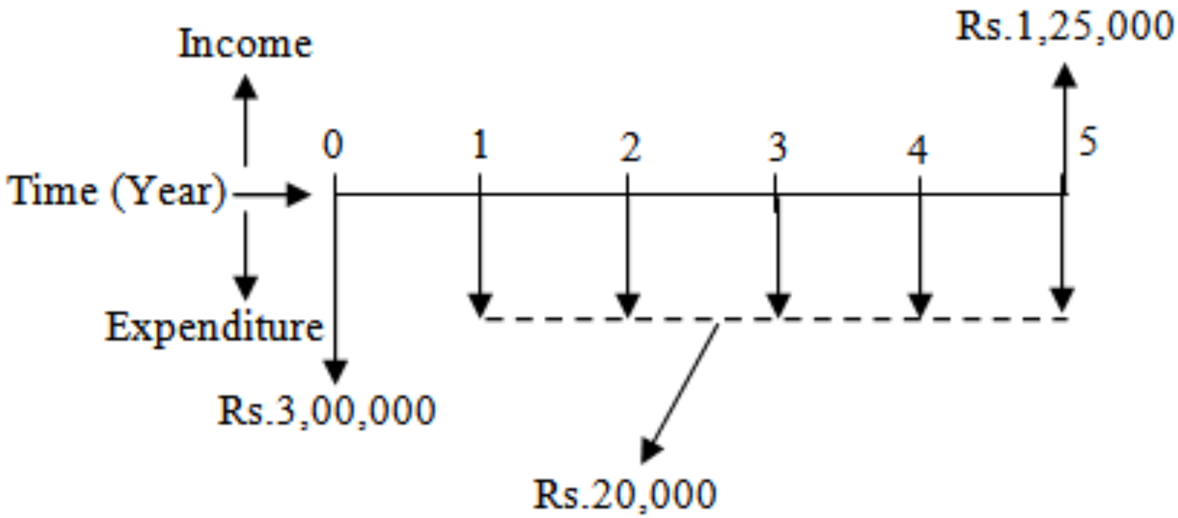
Alternative-2: Initial purchase cost = Rs.200000, Annual operating and maintenance cost = Rs.35000, Expected salvage value = Rs.70000, Useful life = 5 years.

Using future worth method, find out which alternative should be selected, if the rate of interest is 10% per year.

FW of Equal-Life Example (Soln.)

Cash Flow and FW

Alternative 1



$$FW_1 = -300000(F/P,10\%,5) - 20000(F/A,10\%,5) + 125000$$

$$FW_1 = -300000 \times 1.6105 - 20000 \times 6.1051 + 125000$$

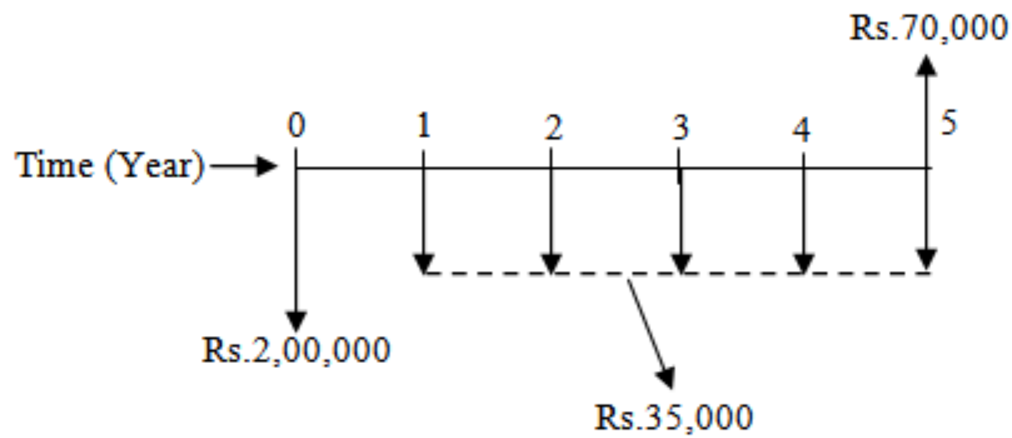
$$FW_1 = -483150 - 122102 + 125000$$

***FW$_1$* = -Rs.480252**

FW of Equal-Life Example (Soln.)

Cash Flow and FW

Alternative 2



$$FW_2 = -200000(F/P,10\%,5) - 35000(F/A,10\%,5) + 70000$$

$$FW_2 = -200000 \times 1.6105 - 35000 \times 6.1051 + 70000$$

$$FW_2 = -322100 - 213679 + 70000$$

$$\boldsymbol{FW_2 = -Rs.465779}$$

Alternative-2 will be selected as it shows lower negative equivalent future worth as compared to Alternative-1

FW of Different-Life Alternatives

Must compare alternatives for ***equal service***
(i.e. alternatives must ***end*** at the same time)

Two ways to compare equal service:

- **Least common multiple (LCM) of lives**
- **Specified study period**

(The LCM procedure is used unless otherwise specified)

Annual Worth (AW) Analysis

Advantages of AW Analysis

Note: AW needs to be calculated for only one life cycle

Assumptions:

- Services needed for ***at least the LCM*** of lives of alternatives
- Selected alternative ***will be repeated*** in succeeding life cycles in same manner as for the first life cycle
- All cash flows ***will be same*** in every life cycle (i.e., will change by only inflation or deflation rate)

Alternatives usually have the following cash flow estimates

- Initial investment, P – First cost of an asset
- Salvage value, S – Estimated value of asset at end of useful life
- Annual amount, A – Cash flows associated with asset, such as annual operating cost (AOC), etc.

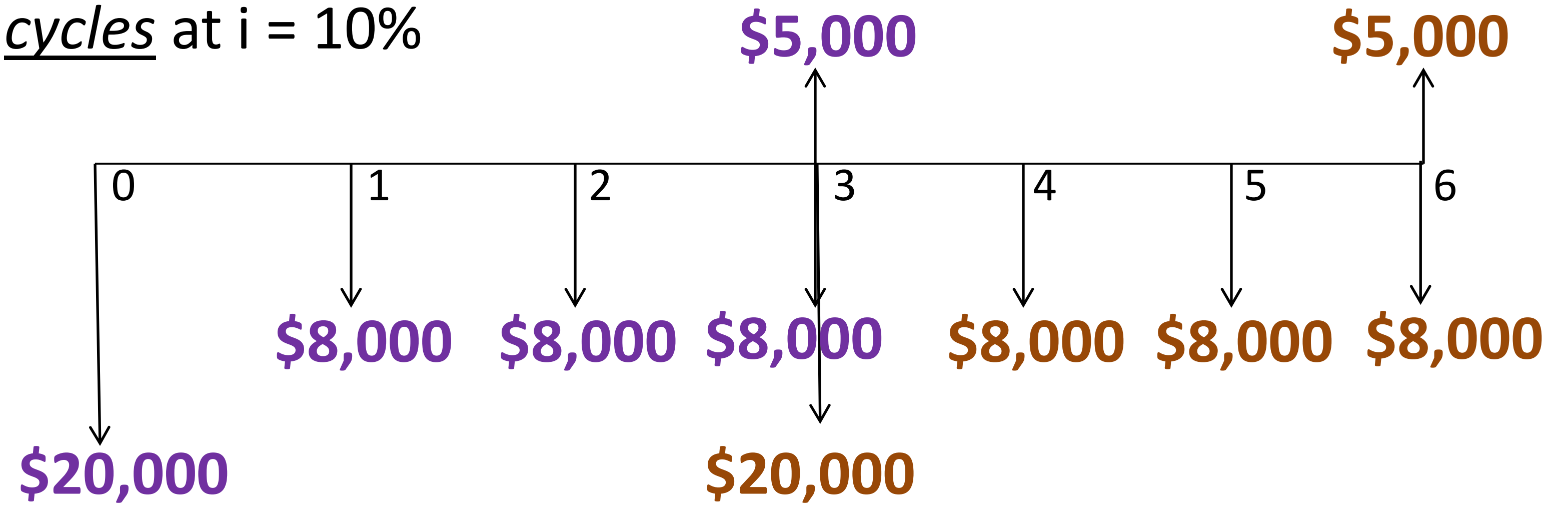
Relationship between AW, PW and FW

$$AW = PW * (A/P,i\%,n) = FW * (A/F,i\%,n)$$

n is years for equal-service comparison (value of LCM or specified study period)

Calculation of Annual Worth

An asset has a first cost of \$20,000, an annual operating cost of \$8000 and a salvage value of \$5000 after 3 years. Calculate the AW over a period corresponding to (i) one *life cycle* and (ii) two *life cycles* at i = 10%



$$AW_{one} = -\ 20{,}000(A/P,10\%,3) - 8000 + 5000(A/F,10\%,3)$$
$$= \$-14{,}532$$

$$AW_{two} = -\ 20{,}000(A/P,10\%,6) - 8000 - 15{,}000(P/F,10\%,3)(A/P,10\%,6) + 5000(A/F,10\%,6)$$

$$= \$-14{,}532$$

AW for one life cycle is the *same for all* life cycles!!

Selection Guidelines for Annual Worth Analysis

One alternative: If AW ≥ 0, the requested MARR is met or exceeded and the alternative is economically justified.

Two or more alternatives: Select the alternative with the AW that is **numerically largest,** that is, less negative or more positive. This indicates a lower AW of cost for cost alternatives or a larger AW of net cash flows for revenue alternatives.

Mutually Exclusive (ME) Alternative Evaluation by Annual Worth (AW)

A company is considering two machines.

Machine X has a first cost of $30,000, Average Operating Cost (AOC) of $18,000, and Salvage of $7000 after 4 years.

Machine Y will cost $50,000 with an AOC of $16,000 and Salvage of $9000 after 6 years.

Which machine should the company select at an interest rate of 12% per year?

Solution:

$$AW_X = -30{,}000(A/P,12\%,4) - 18{,}000 + 7{,}000(A/F,12\%,4)$$
$$= \$-26{,}412$$
$$AW_Y = -50{,}000(A/P,12\%,6) - 16{,}000 + 9{,}000(A/F,12\%,6)$$
$$= \$-27{,}052$$

Select Machine X; it has the numerically larger AW value

Note: For AW there is already an assumption that the services of the alternative is needed for the LCM period.

Capitalized Cost (CC) Analysis

CC refers to the present worth of a project with a very long life, that is, PW as n becomes *infinite*

Basic equation is: $CC = P = \dfrac{A}{i}$

"A" essentially represents the *interest* on a perpetual investment

Note: The factor (P/A, i,∞) = 1/i

For example, in order to be able to withdraw $50,000 per year forever at i = 10% per year, the amount of capital required is 50,000/0.10 = $500,000

- **For *finite life* alternatives, convert all cash flows into an A value over *one life cycle* and then divide by i**
  ➔ Assumption that the same alternative will continue for infinite life cycles

Example: Capitalized Cost

Compare the machines shown below on the basis of their capitalized cost. Use i = 10% per year

	Machine 1	Machine 2
First cost,$	-20,000	-100,000
Annual cost,$/year	-9000	-7000
Salvage value, $	4000	-----
Life, years	3	∞

Solution:

Convert cash flows into A and then divide by i

$$A_1 = -20{,}000(A/P,10\%,3) - 9000 + 4000(A/F,10\%,3) = \$-15{,}834$$

$$CC_1 = -15{,}834 / 0.10 = \$-158{,}340$$

$$A_2 = -100{,}000\ (A/P,\ 10\%,\ \infty\ ) - 7000$$

$$CC_2 = -100{,}000 - 7000/\ 0.10 = \$-170{,}000$$

Select machine 1

Note: (A/P, i, infinity) = i

Hence,

$A_2 = -100{,}000 * i - 7000$

and

$CC_2 = A_2 / i$

AW of Permanent Investment

Use A = Pi for AW of *infinite* life alternatives
Find AW over *one life cycle* for *finite* life alternatives

Compare the alternatives below using AW and i = 10% per year

	C	**D**
First Cost, $	-50,000	-250,000
Annual operating cost, $/year	-20,000	-9,000
Salvage value, $	5,000	75,000
Life, years	5	∞

Solution: Find AW of C over 5 years and AW of D using relation A = Pi

AW_C = -50,000(A/P,10%,5) – 20,000 + 5,000(A/F,10%,5)
= $-32,371

AW_D = P**i** + AOC = -250,000(0.10) – 9,000
= $-34,000

Select alternative C

Note:
(A/P, i, infinity) = i
And
(A/F, i, infinity) = 0

Capital Recovery (CR) and Annual Worth (AW)

Capital recovery (CR) is the equivalent annual amount that an asset, process, or system must earn (new revenue) each year to just recover the first cost plus a stated rate of return over its expected life. Salvage value is considered when calculating CR. [**Note:** Annual operating costs (AOC) are not included]

$$\mathbf{CR = -P(A/P,i\%,n) + S(A/F,i\%,n)}$$

Example:

An asset has a first cost of \$20,000, an annual operating cost of \$8000 and a salvage value of \$5000 after 3 years. (At i =10%)

[Note: Annual Operating Cost (AOC) not included in CR]

$$\mathbf{CR = -20{,}000(A/P,10\%,3) + 5000(A/F,10\%,3) = \$ - 6532 \text{ per year}}$$

i.e. Net revenue from the asset **for 3 years *must be*** at least **\$6532 per year** <u>to recover</u> *initial investment* and *10% p.a.* rate of return.

Then, AW = CR + AOC (along with sign)

$$\mathbf{AW = -6532 - 8000 = \$ - 14{,}532}$$

SM 300

Engineering Economics

ROR Analysis
for
Comparing Alternate Economic Proposals

Why use ROR for Comparing Alternative Investment Options?

The PW, FW and AW methods used so far to compare the alternatives assumed that the ***value of i was known***.

In real world ***i*** fluctuates widely depending upon various factors like:

- Who is the investor (individual, corporation, government agency)
- How much money is being invested.
- How long the money can be committed.
- The credit rating of the investor.

Thus, it may be wise and advantageous to *analyze a cash flow of alternative investment to determine what 'i' it yields* and ***compare it to*** *possible available i* in real world.

Rate of Return Analysis- ROR Calculation and Project Evaluation

- To determine ROR, find the *i** value in the relation

$$PW = 0 \quad \text{or} \quad AW = 0 \quad \text{or} \quad FW = 0$$

- Alternatively, a relation like the following finds i*

$$PW_{outflow} = PW_{inflow} \text{ [Without sign]}$$

- *Note:* ROR may also be called as **Internal Rate of Return** (IROR) since the resulting interest rate depends only on the cash flows themselves

---

- For evaluation, a project is economically viable if

$$i^* \geq MARR$$

ROR Calculation Using PW, AW or FW Relation

ROR is the unique i* rate at which a PW, FW, or AW relation equals exactly 0

Steps

- Set PW/AW/FW = 0 and solve by trial-and-error
- If PW/AW/FW > 0, i* needs to be higher; else i* needs to be lower
- Once you’ve bracketed the solution, use linear interpolation to get the actual value of i*

ROR Calculation Using PW, FW or AW Relation

ROR is the unique i* rate at which a PW, FW, or AW relation equals exactly 0

Example 1: Consider the following cash flow and compute the ROR using a present worth equation. [Note: There is only **ONE** sign change in net cash flow and hence only one ROR value]

Year	0	1	2	3	4	5
Net Cash Flow	-1,000	0	0	+500	0	+1,500

Solution:

$PW_{receipts} + PW_{disbursements} = 0$

$500(P/F,i\%,3) + 1{,}500(P/F,i\%,5) - 1{,}000 = 0$

<u>By trial and error:</u>

$i = 16\%: 500(0.6407) + 1{,}500(0.4761) - 1{,}000 = 34.5 > 0$ ➔ **$i > 16\%$**

$i = 18\%: 500(0.6086) + 1{,}500(0.4371) - 1{,}000 = -40.05 < 0$

<u>By interpolation</u>

$0 = 34.5 + (i - 16)/(18 - 16)\ (-40.05 - 34.5)$

$$f = f_1 + \frac{(i - i_1)}{(i_2 - i_1)}(f_2 - f_1)$$

i=16.9%

ROR Calculation Using PW- Example 2

Consider the following cash flow and compute the ROR using a present worth equation. [Note: There is only **ONE** sign change in net cash flow and hence only one ROR value]

Year	0	1 to 10	10
Net Cash Flow	-$5,000	$100	$7,000

Solution:

$PW_{receipts} + PW_{disbursements} = 0$

$100(P/A,i\%,10) + 7{,}000(P/F,i\%,10) - 5{,}000 = 0$

<u>By trial and error</u>:

For i=5%: $100(7.7217) + 7{,}000(0.6139) - 5{,}000 = 69.47 > 0$ ➔ **i > 5%**

For i=6%: $100(7.3601) + 7{,}000(0.5584) - 5{,}000 = -355.19 < 0$

<u>By interpolation</u>

$0 = 69.47 + (i - 5)/(6 - 5)\ (-355.19 - 69.47)$

$$f = f_1 + \frac{(i - i_1)}{(i_2 - i_1)}(f_2 - f_1)$$

i=5.16%

ROR Calculation Using AW- Same Example 2

Consider the following cash flow and compute the ROR using a annual worth equation. [Note: There is only **ONE** sign change in net cash flow and hence only one ROR value]

Year	0	1 to 10	10
Cash Flow	-$5,000	$100	$7,000

Solution:

$AW_{receipts} + AW_{disbursements} = 0$

100 + 7,000 (A/F,i%,10) – 5,000 (A/P,i%,10) = 0

By trial and error:

For i=5%: 100 + 7,000(0.0795) –5,000(0.12950) = 9 > 0 ➔ **i > 5%**

For i=6%: 100 + 7,000(0.07587) –5,000(0.13587) = -48.26 < 0

By interpolation

$$f = f_1 + \frac{(i - i_1)}{(i_2 - i_1)}(f_2 - f_1)$$

0 = 9 + (i – 5)/(6 – 5) (-48.26 – 9)

i=5.157% ~ 5.16% [Same as PW method]

ROR Calculation Using PW Relation

<u>Example 3</u>:

An investment of \$20,000 in new equipment will generate income of \$7000 per year for 3 years, at which time the machine can be sold for an estimated \$8000. If the company's MARR is 15% per year, should it buy the machine?

Solution:: The ROR equation, based on a PW relation, is:

$$0 = -20{,}000 + 7000(P/A, i^*, 3) + 8000(P/F, i^*, 3)$$

Solve for i* by trial and error.

Let first i* be at MARR 15%

$i^* = 18.2\%$ per year

Since i* > MARR = 15%, ***the company should buy the machine***

Conventional and Nonconventional Cash Flows

- For some cash flow series (net for one project or incremental for two alternatives) it is possible that more than one unique rate of return i* exists.
- This is referred to as multiple i* values.
- It is difficult to complete the economic evaluation when multiple i* values are present, since none of the values may be the correct rate of return.
- In actuality, finding the rate of return is solving for the root(s) of an n^{th} order polynomial.
- ***Conventional or simple cash flows*** have only one sign change over the entire series. Commonly, this is negative in year 0 to positive at some time during the series. There is a unique, real number i* value for a conventional series.
- A ***nonconventional series*** has more than one sign change and multiple roots may exist.

Conventional and Nonconventional Cash Flows (Contd.)

TABLE 6.10 **Examples of Conventional and Nonconventional Net or Incremental Cash Flows for a 6-year Period**

	Sign on Cash Flow							
Type of Series	0	1	2	3	4	5	6	Number of Sign Changes
Conventional	−	+	+	+	+	+	+	1
Conventional	−	−	−	+	+	+	+	1
Conventional	+	+	+	+	+	−	−	1
Nonconventional	−	+	+	+	−	−	−	2
Nonconventional	+	+	−	−	−	+	+	2
Nonconventional	−	+	−	−	+	+	+	3

Multiple ROR Values

Multiple i* values may exist when there is more than one sign change in net cash flow (NCF) series.

Such CF series are called non-conventional

Two tests for multiple i* values:

Descarte's rule of signs: Total number of real i* values *is ≤* the number of sign changes in *net cash flow series.*

Norstom's criterion: There is *one real-number, positive i* value* if the cumulative cash flow series $S_0, S_1, \ldots, S_n$ changes sign only once and $S_0 < 0$. *Note*:

- If cumulative amount in the end is zero, i=0% trivially solves the problem
- -,0,0,…,0,+ (or) +,0,0,…,- is ONE sign change.
- Norstom's Criteria is only a *sufficient condition, not necessary.*
  - *IF it is satisfied THEN there exists a unique positive root.*
  - *IF it is not satisfied … there still may be a unique positive root !*

  Example: Net cash flow of {-1,2,-2,4} has a unique i=1, although the cumulative cash flow {-1,1,-1,3} changes sign three times.*

Example: How many i* Values

Determine the maximum number of i* values for the cash flow shown below

Year	Expense	Income	**Net cash flow**	**Cumulative CF**
0	-12,000	-	-12,000	-12,000
1	-5,000	+ 3,000	-2,000	-14,000
2	-6,000	+ 9,000	+3,000	-11,000
3	-7,000	+15,000	+8,000	-3,000
4	-8,000	+16,000	+8,000	+5,000
5	-9,000	+8,000	-1,000	+4,000

Solution:

The sign on the net cash flow changes twice, indicating two possible i* values

The cumulative cash flow begins negatively with *one sign change*

Therefore, there is only one i* value (i* = 8.7%)

Example: How many i* Values (Soln.)

Using Present Worth: $PW_{receipts} + PW_{disbursements} = 0$

i.e. 3000(P/F, i*, 2) + 8000 (P/F, i*,3) + 8000 (P/F, i*, 4) – 12000

- 2000(P/F, i*, 1) – 1000(P/F, i*, 5) = 0

<u>Using Trial and Error</u>

For i = 8% : 3000 (0.8573) + 8000 (0.7938) + 8000 (0.7350) -12000 - 2000 (0.9259) – 1000 (0.6806) = 269.9 ➔ **i > 8%**

For i = 9% : 3000 (0.8417) + 8000 (0.7722) + 8000 (0.7084) -12000 - 2000 (0.9174) – 1000 (0.6499) = -114.8

<u>By interpolation</u>

0 = 269.9 + (i – 8)/(9 – 8) (-114.8 – 269.9)

$$f = f_1 + \frac{(i - i_1)}{(i_2 - i_1)}(f_2 - f_1)$$

i* = 8.7016%

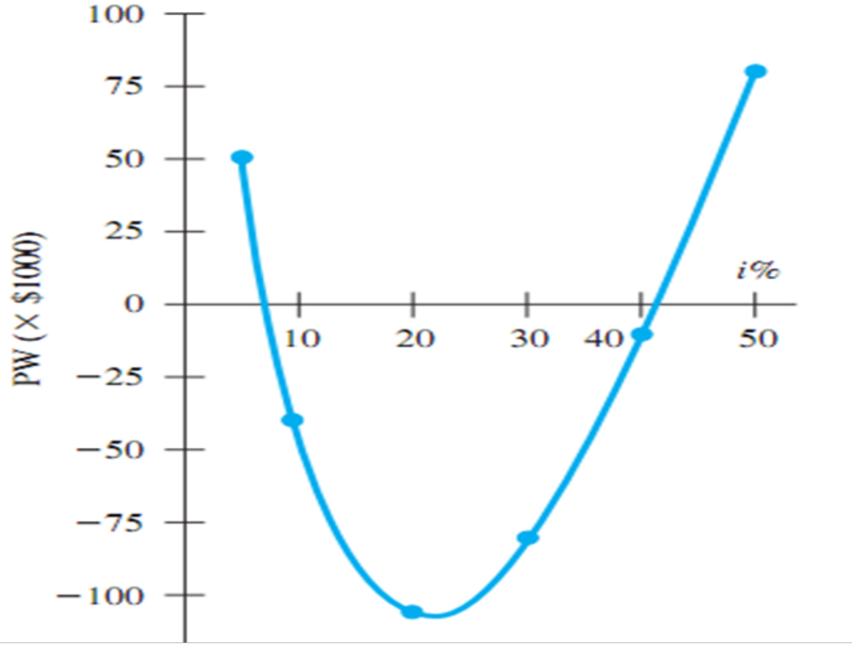
Plot of PW for CF Series with Multiple ROR Values

Year	Cash Flow ($1000)	Sequence Number	Cumulative Cash Flow ($1000)
0	+2000	S_0	+2000
1	−500	S_1	+1500
2	−8100	S_2	−6600
3	+6800	S_3	+200

Two sign changes for cash flows, then *maximum* two values for i* as per Descarte's rule

Two sign changes for cumulative cash flows, so *Norstrom is inconclusive*

i%	5	10	20	30	40	50
PW ($1000)	+51.44	−39.55	−106.13	−82.01	−11.83	+81.85



Present Worth of Flows at Several *i* Rates

i* values at ~8% and ~41%

Dealing With Multiple i* Values--- Computing External ROR (EROR)

These techniques are used under the following conditions:

- The PW or AW value at the MARR is determined and could be used to make the decision, but information on the ROR is deemed necessary to finalize the economic decision, and
- The two tests of cash flow sign changes (Descartes' and Norstrom's) indicate multiple roots (i* values) are possible, and
- More than one positive *i* value or all negative i* values are obtained when the PW graph and* IRR function are developed, and
- A single, reliable rate of return value is required by management or engineers to make a clear economic decision.

Note: The result of follow-up analysis to obtain a single ROR value when multiple, non-useful *i* values* are present **does not determine the internal rate of return (IROR)** [since it does not depend *only on* the internal cash flows] for nonconventional net cash flow series. Hence, called the external rate of return (**EROR**)

Dealing With Multiple i* Values (Contd.)

Two new interest rates to consider for External Rate of Return:

- ***Borrowing rate i_b*** – rate at which funds are borrowed *from an external source* to provide funds to the project [Some years, the net cash flow (NCF) will be negative and you must borrow funds from some source to continue.]
- ***Investment rate i_i*** – rate at which extra funds are *invested external* to the project. [Some years produce positive NCF, and you want to invest the excess money at a good rate of return.]

Two approaches to determine External ROR (EROR)

(1) Modified ROR (MIRR)

(2) Return on Invested Capital (ROIC)

Modified ROR Approach (MIRR)

This is the easier approach to apply. However, the investment and borrowing rates must be reliably estimated, since the results may be quite sensitive to them. The symbol i' will identify the result.

Four step Procedure:

- **Determine PW in *year 0* of all negative NCF at i_b**
- **Determine FW in *year n* of all positive NCF at i_i**
- **Calculate EROR = i' by $PW(F/P,i',n) + FW = 0$**
- **If $i' \geq$ MARR, project is economically justified**

Example: EROR Using MIRR Method

For the NCF shown below, find the EROR by the MIRR method if MARR = 9%, i_b = 8.5%, and i_i = 12%

Year	0	1	2	3
NCF	+2000	-500	-8100	+6800

Solution:

$$PW_0 = -500(P/F,8.5\%,1) - 8100(P/F,8.5\%,2)$$
$$= \$-7342$$

$$FW_3 = 2000(F/P,12\%,3) + 6800$$
$$= \$9610$$

$$PW_0(F/P,i',3) + FW_3 = 0$$
$$-7342(1 + i')^3 + 9610 = 0$$

$i' = 0.0939$ (9.39%)

Since i' > MARR of 9%, project is justified

Return on Invested Capital (ROIC) Approach

- Measure of how effectively project uses funds that ***remain internal to project***
- ROIC rate, i'', is determined using ***net-investment procedure***

Three step Procedure

(1) Develop series of FW relations for each year t (t= 1, 2, …, n years):

$$FW_t = FW_{t-1}(1 + k) + NCF_t \qquad [6.5]$$

where FW_t = future worth in year t based on previous year and time value of money

NCF_t = net cash flow in year t

$$k = \begin{cases} i_i & \text{if } FW_{t-1} > 0 \quad \text{(extra funds available)} \\ i'' & \text{if } FW_{t-1} < 0 \quad \text{(project uses all available funds)} \end{cases}$$

(2) Set future worth relation for last year n equal to 0 (i.e., FW_n= 0); solve for i''

(3) If i'' ≥ MARR, *project is justified*; otherwise, *reject*

ROIC Example

For the NCF shown below, find the EROR by the ROIC method if MARR = 9% and i_i = 12%

Year	0	1	2	3
NCF	+2000	-500	-8100	+6800

Solution:

Year 0: $F_0 = \$+2000$ **$F_0 > 0$; invest in year 1 at i_i = 12%**

Year 1: $F_1 = 2000(1.12) - 500 = \$+1740$ **$F_1 > 0$; invest in year 2 at i_i = 12%**

Year 2: $F_2 = 1740(1.12) - 8100 = \$-6151$ **$F_2 < 0$; use i'' for year 3**

Year 3: $F_3 = -6151(1 + i'') + 6800$ **Set $F_3 = 0$ and solve for i''**

$$-6151(1 + i'') + 6800 = 0$$
$$i'' = 10.55\%$$

Since i'' > MARR of 9%, project is justified

Multiple ROR values- Another Example

Large oil exploration corporations are using better machinery and technology to cap offshore oil spills before they become major disasters. Marine Wells, a company experienced in providing containment response equipment, has estimated the net annual savings shown below (in $ million of cash flow) over the current and next 3 years if their equipment is contracted for by international offshore exploration corporations such as BP, Exxon-Mobil, Chevron, and Total SA. The negative amount in year 1 assumes no oil spill is experienced; the cost is that of the annual contract. Find a unique rate of return using (a) the MIRR method, and (b) the ROIC method, if the following external rates are estimated.

MARR = 12% per year

Borrowing rate for extra funds = 10% per year

Reinvestment rate for excess funds = 15% per year

Year	Cash Flow, $ million
0	50
1	−200
2	50
3	100

Multiple ROR values- Another Example (Solution)

This is a nonconventional cash flow series and does have multiple i^* values as indicated by the cash flow sign test (2 changes) and cumulative cash flow test (inconclusive and $S_0 > 0$). Mathematically, two positive i^* values can be determined: 0% and 256%, neither of which is useful for economic decision making.

a. For the MIRR method, reinvestment is at $i_r = 15\%$ for any excess fund years, and the borrowing rate is $i_b = 10\%$. For a hand solution of the external rate of return i', use the MIRR procedure and utilize Figure 6.5 as a general reference.

1. Negative NCF in year 1 at borrowing rate:

$$PW_0 = -200(P/F,10\%,1) = \$-181.82$$

2. Positive NCF in years 0, 2 and 3 at reinvestment rate:

$$FW_3 = 50[(F/P,15\%,3) + (F/P,15\%,1)] + 100 = \$233.54$$

3. Per Equation [6.4], set $FW_3 = PW_0$ considering the time value of money and solve for i'.

$$233.54 = 181.82(F/P,i',3)$$
$$181.82(1 + i')^3 = 233.54$$
$$(1 + i')^3 = 1.2845$$
$$i' = 0.0870 \qquad (8.70\%)$$

4. The external rate of return of $i' = 8.70\%$ is less than MARR = 12%. The project is not economically viable.

Multiple ROR values- Another Example (Solution)

b. The ROIC method uses the reinvestment rate of $i_r = 15\%$

1. Develop the FW relations for years 0 through 3 using $i_r = 15\%$ only when $FW_{t-1} > 0$.

Year 0:	$FW_0 = \$50$	(Reinvest at 15%)
Year 1:	$FW_1 = 50(1.15) - 200$ $= \$-142.50$	(Figure 6.6*b*; use i'' for year 2)
Year 2:	$FW_2 = -142.50(1 + i'') + 50$	(Figure 6.6*c*; use i'' for year 3)
Year 3:	$FW_3 = [-142.50(1 + i'') + 50] \times (1 + i'') + 100$	(Figure 6.6*d*)

2. Set $FW_3 = 0$ and solve for i'' using the quadratic equation.

$$-142.50(1 + i'')^2 + 50(1 + i'') + 100 = 0$$

The two roots for $1 + i''$ are -0.68 and 1.0313. This translates into the rates of -168% and 3.13%. Discard the negative 168%, since it is below the -100% lower limit for a rate of return. We conclude that the EROR is $i'' = 3.13\%$.

3. Since 3.13% is much less than the MARR of 12%, again the project is not economically viable.

Rate of Return (ROR) Analysis in the situation of Multiple Alternatives or Projects

For independent projects, select all alternatives that have ROR ≥ MARR (no incremental analysis is necessary)

However, when only a few projects can be selected Incremental Analysis is Necessary because…

- Selecting the alternative with highest ROR may not yield highest return on ***available capital***
- Must consider ***weighted average*** of total capital available
- Capital ***not*** invested in a project is assumed to ***earn at MARR***

Why Incremental Analysis is Necessary

Example: Assume \$90,000 is available for investment and MARR = 16% per year. If alternative A would earn 35% per year on investment of \$50,000, and B would earn 29% per year on investment of \$85,000, the weighted averages are:

Overall ROR_A = [50,000(0.35) + 40,000(0.16)]/90,000 = 26.6%

Overall ROR_B = [85,000(0.29) + 5,000(0.16)]/90,000 = 28.3%

Which investment is better, economically?

Why Incremental Analysis is Necessary (Contd.)

If selection basis is higher ROR:

Select alternative A (wrong answer)

If selection basis is higher overall ROR:

Select alternative B

Conclusion: Must use an **incremental ROR analysis** to make a consistently correct selection

Unlike PW, AW, and FW values, if not analyzed correctly, ROR values can lead to an incorrect alternative selection. This is called the **ranking inconsistency problem**

Calculation of Incremental CF

Incremental cash flow = cash flow$_B$ – cash flow$_A$
where *<u>larger initial investment</u>* is Alternative B

Example: Either of the cost alternatives shown below can be used in a grinding process. Tabulate the incremental cash flows.

	A	**B**	**B - A**
First cost, $	-40,000	- 60,000	-20,000
Annual cost, $/year	-25,000	-19,000	+6000
Salvage value, $	8,000	10,000	+2000

The incremental CF is shown in the (B-A) column

The ROR on the extra $20,000 investment in B determines which alternative to select.

Interpretation of ROR on Extra Investment

Based on the concept that any *avoidable investment* that does not yield at least the MARR should not be made.

Once a lower-cost alternative *has been economically* justified, the ROR on the *extra investment (i.e., additional amount of money* associated with a higher first-cost alternative) must also yield a ROR ≥ MARR (because the extra investment *is avoidable* by selecting the economically-justified lower-cost alternative).

This incremental ROR is identified as Δi^*

ROR Evaluation for Two ME Alternatives

(1) Order alternatives by *increasing initial investment cost*

(2) *Develop incremental cash flow* (CF) *series* using LCM of years

(3) Draw incremental *cash flow diagram*, if needed

(4) Count sign changes to see if *multiple Δi* values exist*

(5) Set up PW, AW, or FW = 0 relation and *find* $\Delta i^*_{B\text{-}A}$

***Note**: Incremental ROR analysis requires equal-service comparison. The LCM of lives must be used in the relation*

(6) If $\Delta i^*_{B\text{-}A} < MARR$, *select A*; otherwise, select B

If multiple **Δi*** values exist, ***find EROR*** using either MIRR or ROIC approach.

Example 1: Incremental ROR Evaluation

Either of the cost alternatives shown below can be used in a chemical refining process. If the company's MARR is 15% per year, determine which should be selected on the basis of ROR analysis?

	A	B
First cost ,$	-40,000	-60,000
Annual cost, $/year	-25,000	-19,000
Salvage value, $	8,000	10,000
Life, years	5	5

Example 1: ROR Evaluation of Two Alternatives

Solution, using procedure:

	A	B	B - A
First cost , $	-40,000	-60,000	-20,000
Annual cost, $/year	-25,000	-19,000	+6000
Salvage value, $	8,000	10,000	+2000
Life, years	5	5	

Order by first cost **and** ***find incremental cash flow*** **B - A**

Write ROR equation **(in terms of PW, AW, or FW) on incremental CF**

$$0 = -20{,}000 + 6000(P/A,\Delta i^*,5) + 2000(P/F,\Delta i^*,5)$$

Solve **for Δi* [Use Descart's and Norstrom's Rules] and** ***compare*** **to MARR**

$\boldsymbol{\Delta i^*_{B\text{-}A}} = 17.2\% > \text{MARR of } 15\%$

ROR on $20,000 extra investment is acceptable: **Select B**

Example 2: Incremental ROR Evaluation

As Ford Motor Company retools an old truck assembly plant in Michigan to produce a fuel-efficient economy model, the Ford Focus. Ford and its suppliers are seeking additional sources for light, long-life transmissions. Automatic transmission component manufacturers use highly finished dies for precision forming of internal gears and other moving parts. Two United States–based vendors make the required dies. Use the per unit estimates below and a MARR of 12% per year to select the more economical vendor bid. Show both hand and spreadsheet solutions.

	A	B
Initial cost, $	−8,000	−13,000
Annual costs, $ per year	−3,500	−1,600
Salvage value, $	0	2,000
Life, years	10	5

Solution

These are cost alternatives, since all cash flows are costs. Use the procedure described above to determine Δi^*_{B-A}.

1. Alternatives A and B are correctly ordered with the higher first-cost alternative in column 2 of Table 8–4.
2. The cash flows for the LCM of 10 years are tabulated.

Example 2: Incremental ROR Evaluation- Soln.

TABLE 8–4 Incremental Cash Flow Tabulation, Example 8.3

Year	Cash Flow A (1)	Cash Flow B (2)	Incremental Cash Flow (3) = (2) − (1)
0	$ −8,000	$ −13,000	$ −5,000
1–5	−3,500	−1,600	+1,900
5	—	+2,000 −13,000	−11,000
6–10	−3,500	−1,600	+1,900
10	—	+2,000	+2,000
	$ −43,000	$ −38,000	$ +5,000

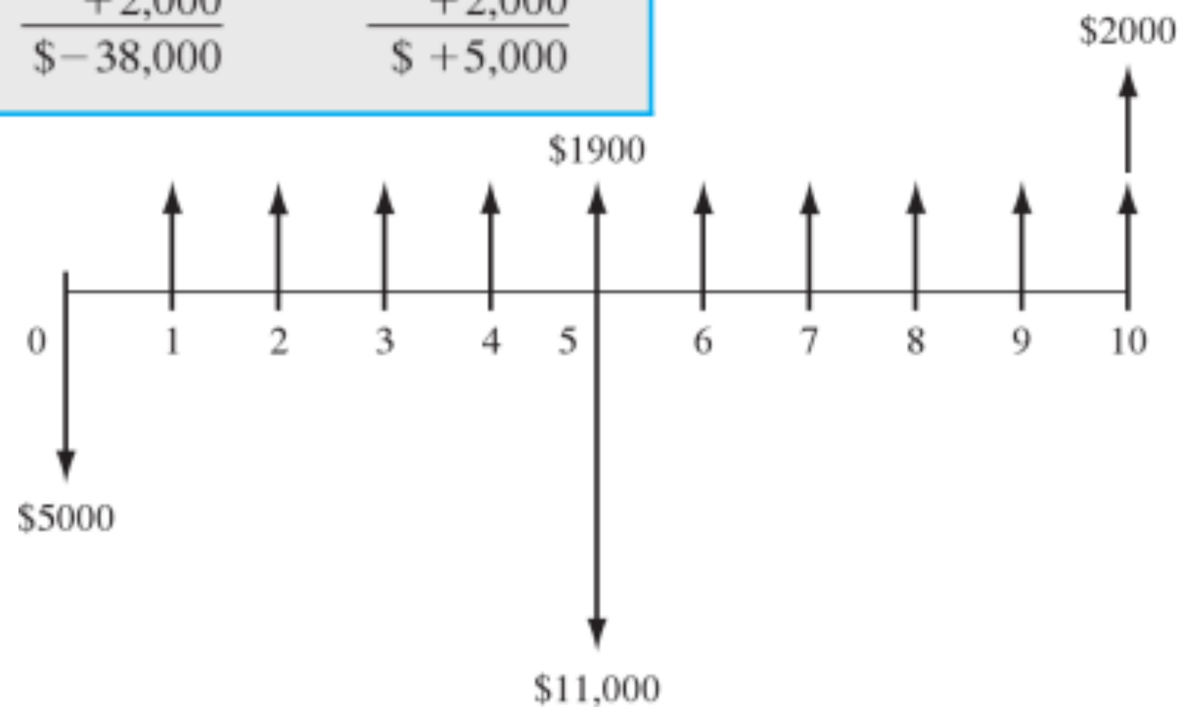


Diagram of incremental cash flows, Example 8.3.

Example 2: Incremental ROR Evaluation- Soln.

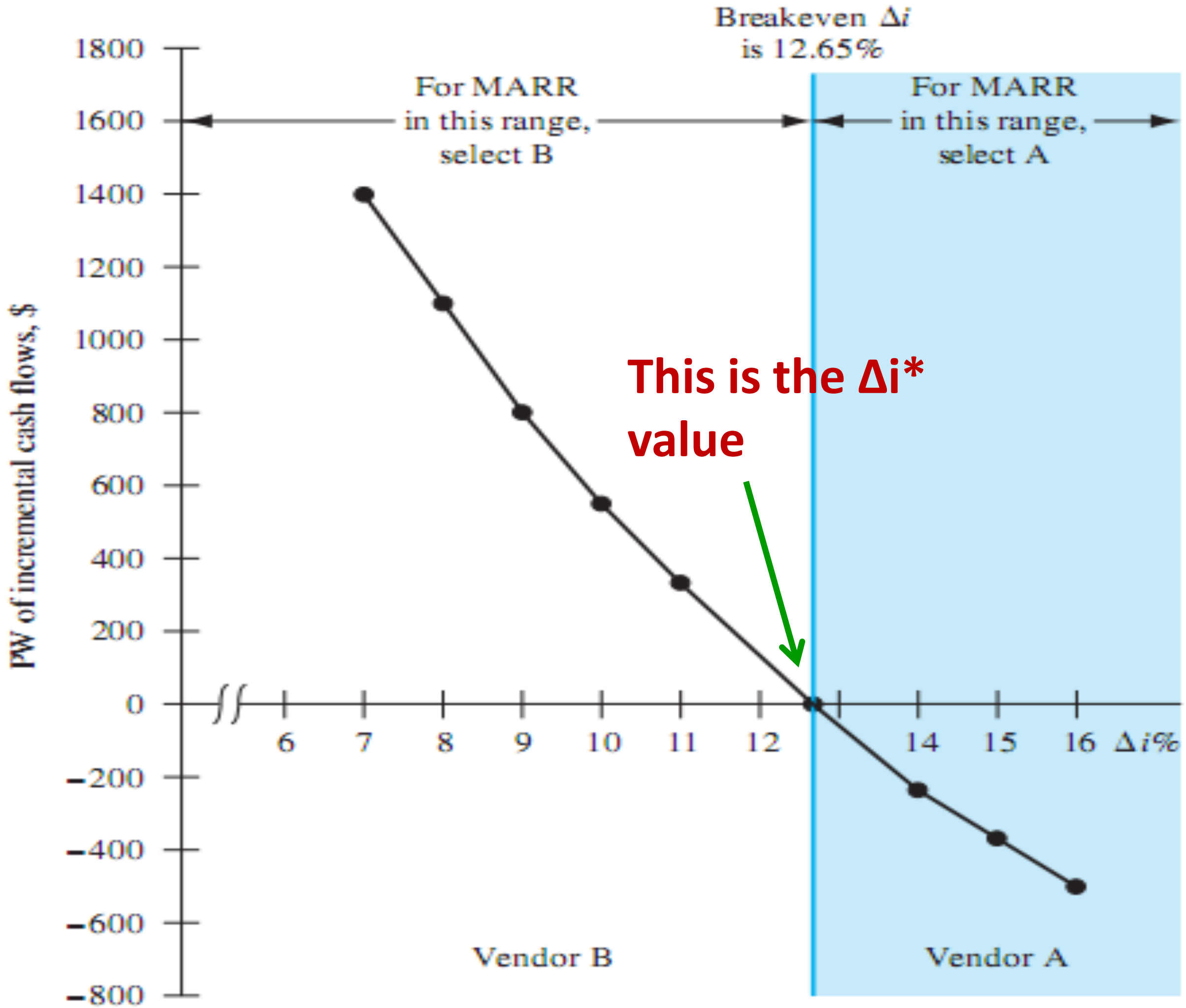
3. The incremental cash flow diagram is shown in Figure 8–2.
4. There are three sign changes in the incremental cash flow series, indicating as many as three roots. There are also three sign changes in the cumulative incremental series, which starts negatively at $S_0 = \$-5000$ and continues to $S_{10} = \$+5000$, indicating that more than one positive root may exist.
5. The rate of return equation based on the PW of incremental cash flows is

$$0 = -5000 + 1900(P/A,\Delta i^*,10) - 11{,}000(P/F,\Delta i^*,5) + 2000(P/F,\Delta i^*,10) \qquad [8.2]$$

   In order to resolve any multiple-root problem, we can assume that the investment rate i_i in the ROIC technique will equal the Δi^* found by trial and error. Solution of Equation [8.2] for the first root discovered results in Δi^* between 12% and 15%. By interpolation $\Delta i^* = 12.65\%$.
6. Since the rate of return of 12.65% on the extra investment is greater than the 12% MARR, the higher-cost vendor B is selected.

Breakeven ROR Value

An ROR at which the PW, AW or FW values of *incremental* cash flows between two alternatives are equal.



If MARR > breakeven ROR, select lower-investment alternative

ROR Analysis – Multiple Alternatives

Six-Step Procedure for Mutually Exclusive (Equal-Life) Alternatives

(1) Order alternatives from *smallest to largest initial investment*

(2) **Only for revenue alternatives**, calculate **i*** (vs. DN) and *eliminate all with i* < MARR*; remaining alternative with lowest cost is ***defender*. F**or cost alternatives, go to step (3)

(3) Determine incremental CF between ***defender*** and ***next lowest-cost*** alternative (known as the *challenger*). Set up ROR relation

(4) Calculate **Δi*** on incremental CF between *two alternatives from step (3)*

(5) If **Δi* ≥ MARR,** *eliminate defender* and *challenger becomes new defender* against next alternative on list

(6) Repeat steps (3) through (5) *until only one alternative* remains. ***Select it***.

For Independent Projects

Compare each alternative vs. DN and select ***all with ROR ≥ MARR***

ROR Analysis – Multiple Alternatives- Eg.

Caterpillar Corporation wants to build a spare parts storage facility in the Phoenix, Arizona, vicinity. A plant engineer has identified four different location options. The initial cost of earthwork and prefab building and the annual net cash flow estimates are detailed in Table 8–5. The annual net cash flow series vary due to differences in maintenance, labor costs, transportation charges, etc. If the MARR is 10%, use incremental ROR analysis to select the one economically best location.

TABLE 8–5 Estimates for Four Alternative Building Locations, Example 8.6

	A	B	C	D
Initial cost, $	−200,000	−275,000	−190,000	−350,000
Annual cash flow, $ per year	+22,000	+35,000	+19,500	+42,000
Life, years	30	30	30	30

Solution

All sites have a 30-year life, and they are revenue alternatives. The procedure outlined above is applied.

ROR Analysis – Multiple Alternatives- Eg. Soln.

TABLE 8–6 Computation of Incremental Rate of Return for Four Alternatives, Example 8.6

	C (1)	A (2)	B (3)	D (4)
Initial cost, $	−190,000	−200,000	−275,000	−350,000
Cash flow, $ per year	+19,500	+22,000	+35,000	+42,000
Alternatives compared	C to DN	A to DN	B to A	D to B
Incremental cost, $	−190,000	−200,000	−75,000	−75,000
Incremental cash flow, $	+19,500	+22,000	+13,000	+7,000
Calculated $(P/A,\Delta i^*,30)$	9.7436	9.0909	5.7692	10.7143
Δi^*,%	9.63	10.49	17.28	8.55
Increment justified?	No	Yes	Yes	No
Alternative selected	DN	A	B	B

1. The alternatives are ordered by increasing initial cost in Table 8–6.
2. Compare location C with the do-nothing alternative. The ROR relation includes only the P/A factor.

Use Interpolation

$$0 = -190{,}000 + 19{,}500(P/A,i^*,30)$$

Table 8–6, column 1, presents the calculated $(P/A,\Delta i^*,30)$ factor value of 9.7436 and $\Delta i_c^* = 9.63\%$. Since $9.63\% < 10\%$, location C is eliminated. Now the comparison is A to DN, and column 2 shows that $\Delta i_A^* = 10.49\%$. This eliminates the do-nothing alternative; the defender is now A and the challenger is B.

ROR Analysis – Multiple Alternatives- Eg. Soln.

3. The incremental cash flow series, column 3, and Δi^* for *B-to-A comparison* are determined from

$$0 = -275{,}000 - (-200{,}000) + (35{,}000 - 22{,}000)(P/A,\Delta i^*,30)$$
$$= -75{,}000 + 13{,}000(P/A,\Delta i^*,30)$$

4. From the interest tables, look up the P/A factor at the MARR, which is $(P/A,10\%,30) = 9.4269$. Now, any P/A value greater than 9.4269 indicates that the Δi^* will be less than 10% and is unacceptable. The P/A factor is 5.7692, so B is acceptable. For reference purposes, $\Delta i^* = 17.28\%$.

5. Alternative B is justified incrementally (new defender), thereby eliminating A.

6. Comparison D-to-B (steps 3 and 4) results in the PW relation $0 = -75{,}000 + 7000(P/A,\Delta i^*,30)$ and a P/A value of 10.7143 ($\Delta i^* = 8.55\%$). Location D is eliminated, and **only alternative B remains; it is selected.**

An alternative must *always* be incrementally compared with an acceptable alternative, and the do-nothing alternative can end up being the only acceptable one. Since C was not justified in this example, location A was not compared with C. Thus, *if* the B-to-A comparison had not indicated that B was incrementally justified, then the D-to-A comparison would be correct instead of D-to-B.

ROR of Bond Investment

Bond is *IOU* with face value **(V)**, coupon rate **(b)**, no. of payment periods/year **(c)**, dividend **(I)**, and maturity date **(n).** Amount paid for the bond is **P.**

$$I = Vb/c$$

General equation for i*: $0 = - P + I(P/A,i^*,n_xc) + V(P/F,i^*,n_xc)$

A $10,000 bond with 6% interest payable quarterly is purchased for $8000. If the bond matures in 5 years, what is the ROR (a) per quarter, (b) per year?

Solution: **(a)** $I = 10,000(0.06)/4 = \$150$ per quarter

ROR equation is: $\mathbf{0 = -8000 + 150(P/A,i^*,20) + 10,000(P/F,i^*,20)}$

By trial and error or spreadsheet: $i^* = 2.8\%$ per quarter

(b) Nominal i^* per year = 2.8(4) = 11.2% per year

Effective i^* per year = $(1 + 0.028)^4 - 1$ = 11.7% per year

Effective i per year = $(1 + 0.028)^4 - 1$ = 11.7% per year

SM 300

Engineering Economics

Benefit/Cost Analysis

Differences: Public vs. Private Projects (contd.)

	Private	Public
Purpose	Provide goods or services at a profit; maximize profit or minimize cost	Protect health; protect lives and property; provide services (at no profit); provide jobs Schools, Hospitals, Highways etc.
Sources of capital	Private investors and lenders	Taxation; private lenders
Method of financing	Individual ownership; partnerships; corporations	Direct payment of taxes; loans without interest; loans at low interest; self-liquidating bonds; indirect subsidies; guarantee of private loans
Multiple purposes	Moderate	Common (e.g., reservoir project for flood control, electrical power generation, irrigation, recreation, education)
Project life	Usually relatively short (5 to 10 years)	Usually relatively long (20 to 60 years)
Relationship of suppliers of capital to project	Direct	Indirect, or none
Nature of benefits	Monetary or relatively easy to equate to monetary terms	Often nonmonetary, difficult to quantify, difficult to equate to monetary terms
Beneficiaries of project	Primarily, entity undertaking project	General public
Conflict of purposes	Moderate	Quite common (dam for flood control versus environmental preservation)
Conflict of interests	Moderate	Very common (between agencies)
Effect of politics	Little to moderate	Frequent factors; short-term tenure for decision makers; pressure groups; financial and residential restrictions; etc.
Measurement of efficiency	Rate of return on capital	Very difficult; no direct comparison with private projects

Public Projects- Cash Flow Classifications and B/C Relations

Must identify each cash flow as either benefit, disbenefit, or cost

Benefit (B) -- Advantages to the *public* Eg. Highway construction- connecting villages to towns/cities

Disbenefit (D) – (Expected) Disadvantages to the *public* Eg. Loss of pond/land areas where highway is constructed

Cost (C) -- Expenditures by the *government* Eg. Construction of the highway, Maintenance cost for the highway

Note: Savings to government are subtracted from costs

Conventional B/C ratio = (B–D) / C

Modified B/C ratio = [(B–D) – M&O] / Initial Investment

Profitability Index = NCF / Initial Investment

Note 1: **All terms must be expressed in *same units*, i.e., PW, AW, or FW**

Note 2: ***Do not use minus sign* ahead of costs**

Decision Guidelines for B/C and PI

Benefit/cost analysis

If B/C ratio ≥ 1.0, project is economically justified at discount rate applied

If B/C ratio < 1.0, project is not economically acceptable

Profitability Index analysis of revenue projects

If PI ≥ 1.0, project is economically justified at discount rate applied

If PI < 1.0, project is not economically acceptable

B/C Analysis – Single Project

Note: **All terms must be expressed in *same units*, i.e., PW, AW, or FW**

$$\left.\begin{array}{l} \textbf{Conventional B/C ratio} = \dfrac{B - D}{C} \\[2ex] \textbf{Modified B/C ratio} = \dfrac{B - D - M\&O}{\text{Initial Investment}} \end{array}\right\}$$

If B/C ratio ≥ 1.0, accept project; otherwise, reject

$$PI = \frac{\text{PW of } NCF_t}{\text{PW of initial investment}}$$

If PI ≥ 1.0, accept project; otherwise, reject

Example: B/C Analysis – Single Project

A flood control project will have a first cost of \$1.4 million with an annual maintenance cost of \$40,000 and a 10 year life. Reduced flood damage is expected to amount to \$175,000 per year. Lost income to farmers is estimated to be \$25,000 per year. At an interest rate of 6% per year, should the project be undertaken?

Solution: Express all values in AW terms (since most of the values are given per year) and find B/C ratio

B = \$175,000

D = \$25,000

C = 1,400,000(A/P,6%,10) + \$40,000 = \$230,218

B/C Ratio = (B-D)/C = (175,000 – 25,000)/230,218

= 0.65 < 1.0

Do not undertake flood control project

B/C Analysis of Independent Projects

- Independent projects comparison does not require incremental analysis
- Compare each alternative's overall B/C ratio with Do-Nothing (DN) option
- No budget limit: Accept all alternatives with B/C ratio ≥ 1.0

However, Mutually Exclusive Alternatives may require Incremental B/C Analysis.

Choosing ME Alternatives based on B/C
Defender, Challenger and Do Nothing Alternatives

When selecting from two or more ME alternatives, there is a:

✓ Defender – in-place system or currently selected alternative

✓ Challenger – Alternative challenging the defender

✓ Do-nothing option – Status quo system

General approach for incremental B/C analysis of two ME alternatives:

- Lower total cost alternative is first compared to *Do-nothing (DN)*
- If B/C ratio for the lower cost alternative is < 1.0, the DN option is compared to incremental B/C of the higher-cost alternative
- If both alternatives lose out to DN option, DN prevails, unless overriding needs requires selection of one of the alternatives

Alternative Selection Using Incremental B/C Analysis – Two or More ME Alternatives

(1) Determine *equivalent total cost* for each alternative

(2) Order alternatives by increasing total cost

(3) Identify *B and D for each alternative*, if given, or go to step 5

(4) Calculate B/C ratio for each alternative and eliminate all with B/C ratio < 1.0

(5) Determine incremental costs and benefits for first two alternatives

(6) Calculate incremental B/C ratio; if >1.0, higher cost alternative becomes defender

(7) Repeat steps 5 and 6 until only one alternative remains

Example: Incremental B/C Analysis

Compare two alternatives using i = 10% and B/C ratio

Alternative	X	Y
First cost, $	320,000	540,000
M&O costs, $/year	45,000	35,000
Benefits, $/year	110,000	150,000
Disbenefits, $/year	20,000	45,000
Life, years	10	20

Solution: First, calculate equivalent total cost

[**Note**: Disbenefits are ***Not*** Costs]

AW of $costs_X$ = 320,000(A/P,10%,10) + 45,000 = $97,080

AW of $costs_Y$ = 540,000(A/P,10%,20) + 35,000 = $98,428

Order of analysis is X, then Y

X vs. DN: (B-D)/C = (110,000 – 20,000) / 97,080 = 0.93 Eliminate X

Y vs. DN: (150,000 – 45,000) / 98,428 = 1.07 Eliminate DN

Select Y

Example: ΔB/C Analysis; Selection Required

Must select one of two alternatives using i = 10% and ΔB/C ratio

Alternative	X	Y
First cost, $	320,000	540,000
M&O costs, $/year	45,000	35,000
Benefits, $/year	110,000	150,000
Disbenefits, $/year	20,000	45,000
Life, years	10	20

Solution: (Note: Cost is already in AW terms)

AW of costs_X = \$97,080 AW of costs_Y = \$98,428

Must select X or Y; DN not an option, *compare Y to X*

Incremental values: ΔB = 150,000 – 110,000 = \$40,000

ΔD = 45,000 – 20,000 = \$25,000

ΔC = 98,428 – 97,080 = \$1,348

Y vs. X: (ΔB - ΔD) / ΔC = (40,000 – 25,000) / 1,348 = 11.1 Eliminate X

Select Y

Another Example- Incremental Benefit Cost Analysis (Assume: All the values are in AW terms)

	A	B	C	D	E	F	G
Benefits	174	180	136	80	136	178	89
Disbenefits	4	60	10	8	10	2	14
Costs	100	120	90	80	70	110	50
$\frac{B-D}{C}$	*1.7*	*1.0*	*1.4*	*0.9*	*1.8*	*1.6*	*1.5*

Not Feasible (D)

Cost Order: G E C A F B

G – DN: (From feasibility, (B – D) / C = ***1.5*** — Upgrade to **G**

E – G: $\frac{(136-89)-(10-14)}{(70-50)} = \frac{51}{20} = \mathbf{2.55}$ — Upgrade to **E**

C – E: $\frac{(136-136)-(10-10)}{(90-70)} = \frac{0}{20} = \mathbf{0}$ — Keep Proj. **E**

A – E: $\frac{(174-136)-(4-10)}{(100-70)} = \frac{44}{30} = \mathbf{1.47}$ — Upgrade to **A**

Another Example- Incremental Benefit Cost Analysis (Assume: All the values are in AW terms)

	A	B	C	D	E	F	G
Benefits	174	180	136	80	136	178	89
Disbenefits	4	60	10	8	10	2	14
Costs	100	120	90	80	70	110	50
$\frac{B-D}{C}$	*1.7*	*1.0*	*1.4*	*0.9*	*1.8*	*1.6*	*1.5*

Not Feasible (D)

Cost Order: G E C A F B

F – A: $\dfrac{(178-174)-(2-4)}{(110-100)} = \dfrac{6}{10} = \mathbf{\mathit{0.6}}$

Keep Proj. **A**

B – A: $\dfrac{(180-174)-(60-4)}{(120-100)} = \dfrac{-50}{20} = \mathbf{\mathit{-2.5}}$

Best is Proj **A**

Replacement Study

Replacement Study Basics

Reasons for replacement study

1. Reduced performance- Due to physical deterioration
2. Altered requirements- New requirements of speed, accuracy etc.
3. Obsolescence- International competition and rapidly changing tech.

Terminology

Defender – *Currently installed* asset or the *in-place* asset

Challenger – *Potential replacement* for defender

Market value (MV) – Value of defender if *sold in open market*

Economic service life – No. of years at which *lowest AW* of cost occurs

Defender first cost – *MV of defender*; used as its first cost (P) in analysis

Challenger first cost – *Capital to recover* for challenger (usually its P value)

Sunk cost – Prior expenditure *not recoverable from challenger cost*

Non-owner's viewpoint – *Outsider's (consultant's) viewpoint* for objectivity. This is followed for replacement analysis.

Example: Replacement Basics

An asset purchased 2 years ago for $40,000 is harder to maintain than expected. It can be sold now for $12,000 or kept for a maximum of 2 more years, in which case its operating cost will be $20,000 each year, with a salvage value of $9,000 two years from now. A suitable challenger will have a first cost of $60,000 with an annual operating cost of $4,100 per year and a salvage value of $15,000 after 5 years. Determine the values of P, A, n, and S for the defender and challenger for an AW analysis.

Solution:

Defender: P = $-12,000; A = $-20,000; n = 2; S = $9,000

Challenger: P = $-60,000; A = $-4,100; n = 5; S = $15,000

Overview of a Replacement Study

- Replacement studies are *applications of* the **AW method**

AW values that are used are often referred to as EUAC (Equivalent Uniform Annual Cost) where,

EUAC would be the negative of AW, as it treats costs as positive and revenues/salvage as negative

- Study periods (planning horizons) are either specified or unlimited
- *Assumptions* for unlimited study period:
  1. Services provided for indefinite future
  2. Challenger is best available now and for future, and will be repeated in future life cycles
  3. Cost estimates for each life cycle for defender and challenger remain the same
- If study period is specified, assumptions do not hold
- Replacement study procedures differ for the two cases

Economic Service Life

Economic service life (ESL) refers to the asset retention time (n) where AW (with sign) is maximized
i.e. Equivalent Uniform Annual Cost (EUAC) is minimized

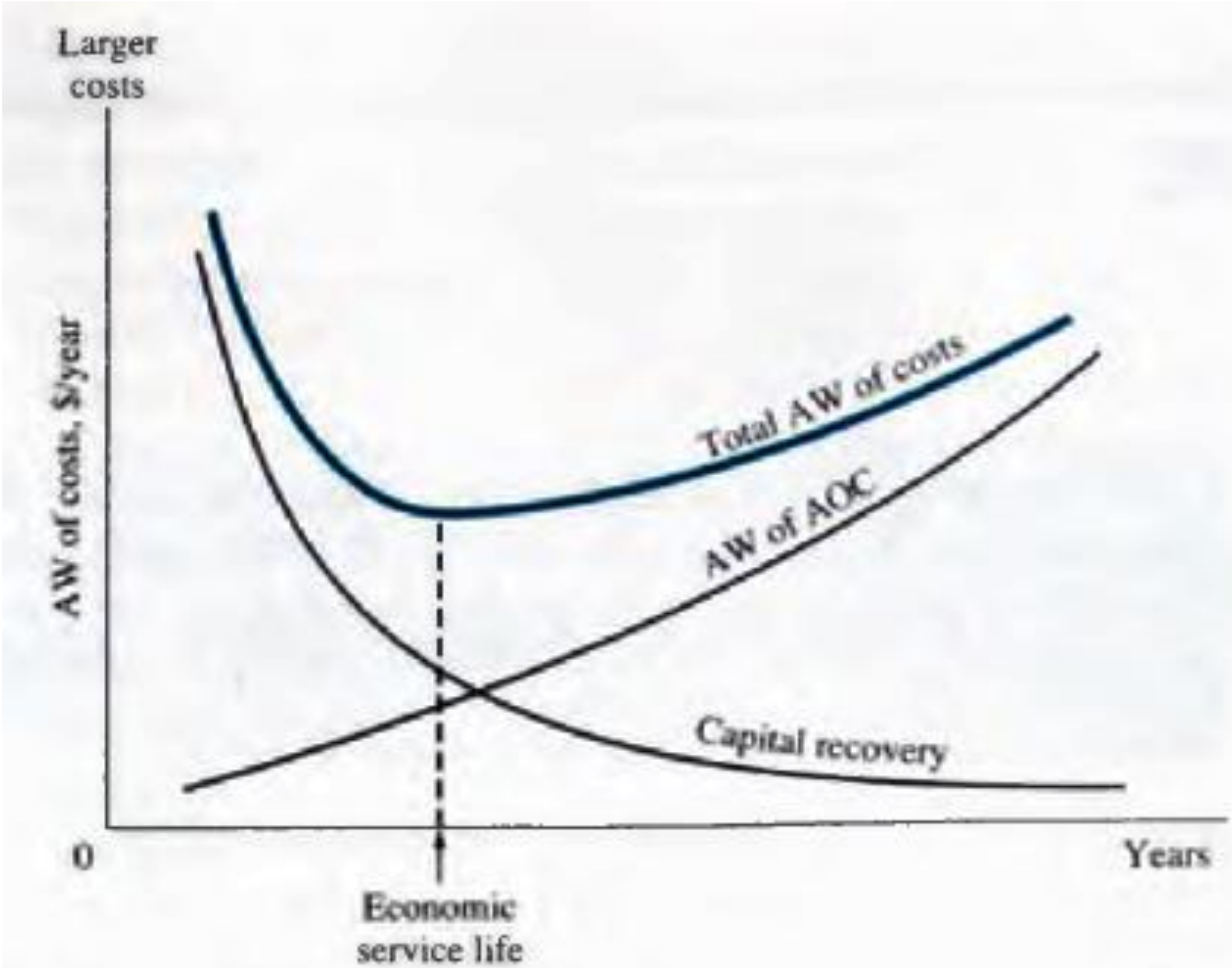
Determined by calculating AW for 1, 2, 3,…n years

General equation is:

Total AW = capital recovery – AW of annual operating costs
= CR – AW of AOC

Note: AW for different length of years can be different.
However, AW for one Life Cycle will be same for all Life Cycles

Economic Service Life (Contd.)



Annual worth curves of cost (ignore negative sign) elements that determine the economic service life.

Example: Economic Service Life

Determine the ESL of an asset which has the costs shown below.

Let i = 10%

Year	Cost,$/year	Salvage value,$
0	- 20,000	-
1	-5,000	10,000
2	-6,500	8,000
3	- 9,000	5,000
4	-11,000	5,000
5	-15,000	3,000

Solution:

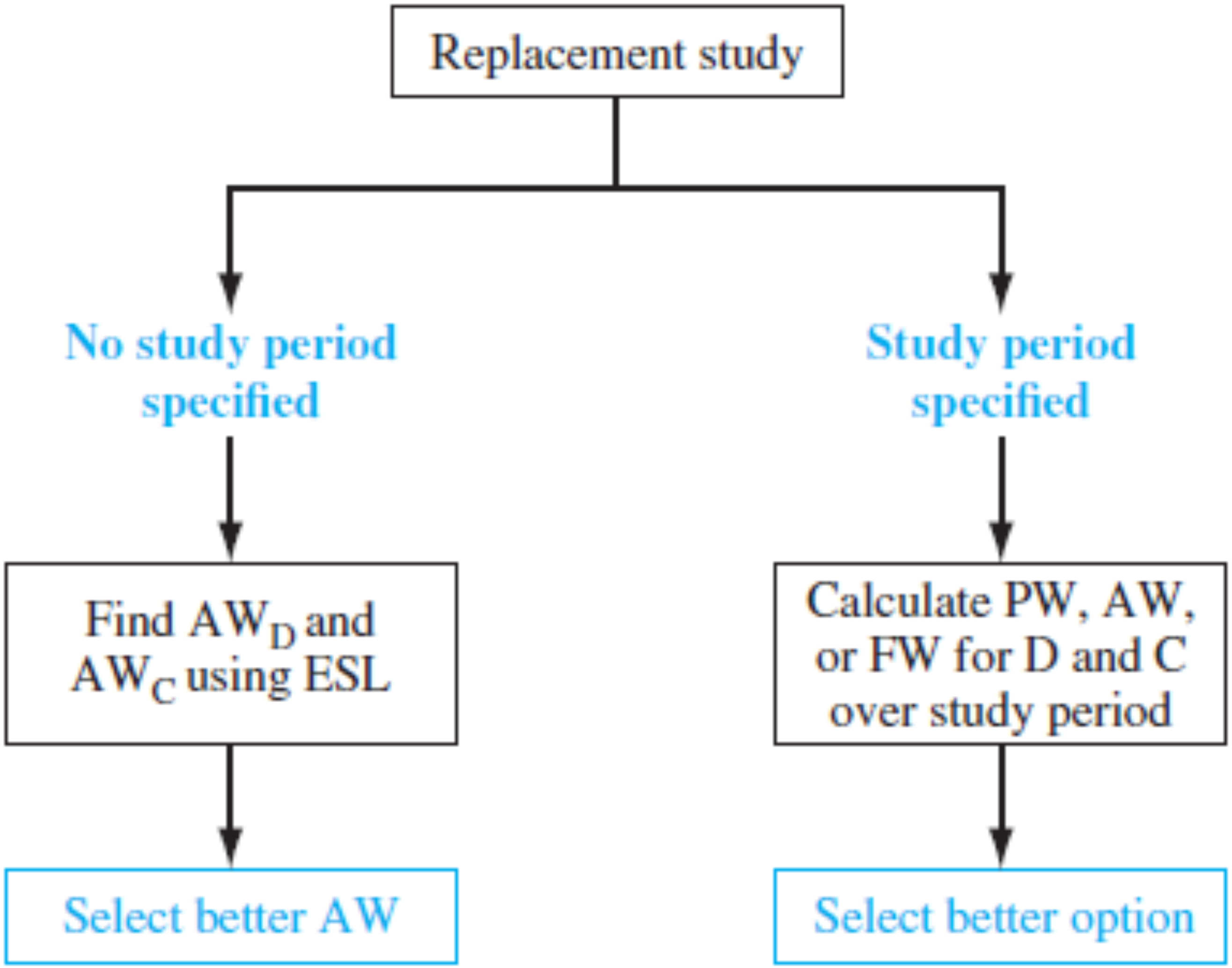
AW_1 = [- 20,000 – 5000(P/F,10%,1)] (A/P,10%,1) + 10,000(A/F,10%,1) = $ -17,000

AW_2 = [- 20,000 – 5000(P/F,10%,1) - 6500(P/F,10%,2)] (A/P,10%,2) + 8000(A/F,10%,2) = $ -13,429

Similarly, AW_3 = $ -13,239 AW_4 = **$ -12,864** AW_5 = $ -13,623

Economic service life is 4 years

Performing a Replacement Study



Performing a Replacement Study -- Unlimited Study Period

1. Calculate AW_D and AW_C *based on their ESL;* select lower AW
2. If AW_C was selected in step (1), keep for n_C years (i.e., economic service life of challenger); if AW_D was selected, keep defender one more year and then repeat analysis (i.e., one-year-later analysis)
3. As long as all estimates remain current in succeeding years, keep defender until n_D is reached, and then replace defender with best challenger
4. If any estimates change before n_D is reached, repeat steps (1) through (4)

Note: If study period is specified, perform steps (1) through (4) *only through end of study period* (discussed later)

Example: Replacement Analysis

An asset purchased 2 years ago for \$40,000 is harder to maintain than expected. It can be sold now for \$12,000 or kept for a maximum of 2 more years, in which case its operating cost will be \$20,000 each year, with a salvage value of \$10,000 after 1 year or \$9000 after two years. A suitable challenger will have an annual worth of \$-24,000 per year. At an interest rate of 10% per year, should the defender be replaced now, one year from now, or two years from now?

Solution: **First, determine ESL for defender**

[Note: Defender first cost = *MV of defender*; used as its first cost (P) in analysis]

$$\mathbf{AW_{D1}} = -12{,}000(A/P,10\%,1) - 20{,}000 + 10{,}000(A/F,10\%,1) = \$-23{,}200$$

$$\mathbf{AW_{D2}} = -12{,}000(A/P,10\%,2) - 20{,}000 + 9{,}000(A/F,10\%,2) = \$-22{,}629$$

ESL is n = 2 years; AW_D = \$-22,629

AW_C = \$-24,000

Lower AW = \$-22,629 **Replace defender in 2 years**

Example 2: Replacement Analysis

Two years ago, Toshiba Electronics made a $15 million investment in new assembly line machinery. It purchased approximately 200 units at $70,000 each and placed them in plants in 10 different countries. The equipment sorts, tests, and performs insertion-order kitting on electronic components in preparation for special-purpose printed circuit boards. A new international industry standard requires a $16,000 additional cost next year (year 1 of retention) on each unit in addition to the expected operating cost. Due to the new standards, coupled with rapidly changing technology, a new system is challenging these 2-year-old machines. The chief engineer at Toshiba USA has asked that a replacement study be performed this year and each year in the future, if need be. At i = 10% and with the estimates below, do the following:

a. Determine the AW values and economic service lives necessary to perform the replacement study.

(Contd.)

Example 2: Replacement Analysis (Contd.)

Challenger: First cost: $50,000
Future market values: decreasing by 20% per year
Estimated retention period: no more than 5 years
AOC estimates: $5000 in year 1 with increases of $2000 per year thereafter

Defender: Current international market value: $15,000
Future market values: decreasing by 20% per year
Estimated retention period: no more than 3 more years
AOC estimates: $4000 next year, increasing by $4000 per year thereafter, plus the extra $16,000 next year

b. Perform the replacement study now.
c. After 1 year, it is time to perform the follow-up analysis. The challenger is making large inroads into the market for electronic components assembly equipment, especially with the new international standards features built in. The expected market value for the defender is still $12,000 this year, but it is expected to drop to virtually nothing in the future—$2000 next year on the worldwide market and zero after that. Also, this prematurely outdated equipment is more costly to keep serviced, so the estimated AOC next year has been increased from $8000 to $12,000 and to $16,000 two years out. Perform the follow-up replacement study analysis.

Example 2: Replacement Analysis (Contd.)

Solution

a. The results of the ESL analysis, shown in Table 9.2, include all the market values and AOC estimates for the challenger in the top of the table. Note that P = \$50,000 is also the market value in year 0. The total AW of costs is shown by year, should the challenger be placed into service for that number of years. As an example, if the challenger is kept for 4 years, AW_4 is

$$\begin{aligned}\text{Total AW}_4 &= -50{,}000(A/P,10\%,4) + 20{,}480(A/F,10\%,4)\\ &\quad -[5000 + 2000(A/G,10\%,4)]\\ &= \$-19{,}123\end{aligned}$$

The defender costs are analyzed in the same way in Table 9.2 (bottom) up to the maximum retention period of 3 years.

The lowest AW cost (numerically largest) values for the replacement study are

Challenger: $AW_C = \$-19{,}123$ for $n_C = 4$ years

Defender: $AW_D = \$-17{,}307$ for $n_D = 3$ years

Defender				
Defender Year k	**Market Value**	**AOC**	**Total AW If Retained k Years**	
0	\$15,000	—	—	
1	12,000	\$−20,000	\$−24,500	
2	9,600	−8,000	−18,357	
3	7,680	−12,000	−17,307	ESL

Example 2: Replacement Analysis (Contd.)

Challenger				
Challenger Year k	Market Value	AOC	Total AW If Owned k Years	
0	\$50,000	—	—	
1	40,000	\$ −5,000	\$−20,000	
2	32,000	−7,000	−19,524	
3	25,600	−9,000	−19,245	
4	20,480	−11,000	−19,123	ESL
5	16,384	−13,000	−19,126	

b. To perform the replacement study now, apply only the first step of the procedure. Select the defender because it has the better AW of costs (\$−17,307), and expect to retain it for 3 more years. Prepare to perform the one-year-later analysis 1 year from now.

c. One year later, the situation has changed significantly for the equipment Toshiba retained last year. Apply the steps for the one-year-later analysis:

Example 2: Replacement Analysis (Contd.)

2. After 1 year of defender retention, the challenger estimates are still reasonable, but the defender market value and AOC estimates are substantially different. Go to step 3 to perform a new ESL analysis for the defender.
3. The defender estimates in Table 9.2 (bottom) are updated below, and new AW values are calculated. There is now a maximum of 2 more years of retention, 1 year less than the 3 years determined last year.

Year k	Market Value	AOC	Total AW If Retained k More Years	
0	\$12,000	—	—	
1	2,000	\$−12,000	\$−23,200	
2	0	−16,000	−20,819	ESL

The defender ESL is 2 years. The AW and n values for the new replacement study are:

Challenger: unchanged at $AW_C = \$-19{,}123$ for $n_C = 4$ years

Defender: new $AW_D = \$-20{,}819$ for $n_D = 2$ more years

Now select the challenger based on its favorable AW value. Therefore, replace the defender now, not 2 years from now. Expect to keep the challenger for 4 years, or until a better challenger appears on the scene.

Summary- Replacement Analysis Over Unlimited or Infinite Study Period or Planning Horizon

- Compute the economic lives of both defender and challenger. Let's use $\mathbf{N_{D*}}$ and $\mathbf{N_{C*}}$ to indicate the economic lives of the defender and the challenger, respectively. The annual equivalent cost for the defender and the challenger at their respective economic lives are indicated by $\mathbf{AE_{D*}}$ and $\mathbf{AE_{C*}}$.

- Compare $\mathbf{AE_{D*}}$ and $\mathbf{AE_{C*}}$. If $\mathbf{AE_{D*}}$ is bigger than $\mathbf{AE_{C*}}$, we know that it is more costly to keep the defender than to replace it with the challenger. Thus, the challenger should replace the defender now.

- If the defender should not be replaced now, when should it be replaced? First, we need to continue to use until its economic life is over. Then, we should calculate the cost of running the defender for one more year after its economic life. If this cost is greater than $\mathbf{AE_{C*}}$ the defender should be replaced at the end of is economic life. This process should be continued until you find the optimal replacement time. This approach is called **marginal analysis**, that is, to calculate the incremental cost of operating the defender for just one more year.

Replacement Value

Oftentimes it is helpful to know the minimum defender market value that, if exceeded, will make the challenger the better alternative. This defender value, called its replacement value (RV), yields a breakeven value between challenger and defender.

- *Replacement value* (RV) is market/trade-in value of defender that renders AW_D and AW_C equal to each other
- It is the minimum market (trade-in) value required to make the challenger economically attractive.

Set up equation $\mathbf{AW_D = AW_C}$ except **use RV in place of P** for the defender; **then solve for RV**

If defender can be sold for amount > RV, *challenger is the better option,* because it will have a lower AW value

That is, if the *actual* market trade-in ***exceeds*** the *breakeven* replacement value, the *challenger is the better* alternative and should *replace the defender now*.

Example: Replacement Value

An asset purchased 2 years ago for $40,000 is harder to maintain than expected. It can be sold now for $12,000 or kept for a maximum of 2 more years, in which case its operating cost will be $20,000 each year, with a salvage value of $10,000 at the end of year two. A suitable challenger will have an initial cost of $65,000, an annual cost of $15,000, and a salvage value of $18,000 after its 5 year life. Determine the RV of the defender that will render its AW equal to that of the challenger, using an interest rate of 10% per year. Recommend a course of action.

Solution: **Set $AW_D = AW_C$**

$$- RV(A/P,10\%,2) - 20{,}000 + 10{,}000(A/F,10\%,2) = - 65{,}000(A/P,10\%,5) - 15{,}000 + 18{,}000(A/F,10\%,5)$$

RV = $24,228

Thus, if market value of defender > $24,228, *select challenger*

Replacement Analysis Over Specified Study Period

Same procedure as before, except *calculate AW values over study period* instead of over Economic Service Life (ESL) years of n_D and n_C

- It is necessary to develop *all viable defender-challenger combinations* and calculate AW or PW for each combination *over study period*

- Select option with lowest cost or highest income

Example: Replacement Analysis [Coterminated Assumption]

Amoco Canada has oil field equipment placed into service 5 years ago for which a replacement study has been requested. Due to its special purpose, it has been decide that the current equipment will have to serve for 2, 3, or 4 more years before replacement. The equipment has a current market value of $100,000, which is expected to decrease by $25,000 per year. The AOC is constant now, and is expected to remain so, at $25,000 per year. The replacement challenger is a fixed price contract to provide the same services at $60,000 per year for a minimum of 2 years and a maximum of 5 years. Use MARR 12% per year to perform a replacement study over a 6-year period to determine when to sell the current equipment and purchase the contract services.

Solution:

Since the defender will be retained for 2, 3, or 4 years, there are three viable options (X, Y, and Z).

Option	Defender Retained	Challenger Serves
X	2 years	4 years
Y	3	3
Z	4	2

Example: Replacement Analysis **Solution (contd.):**

The defender annual worth values are identified with subscripts D2, D3, and D4 for the number of years retained.

$$AW_{D2} = -100{,}000(A/P,12\%,2) + 50{,}000(A/F,12\%,2) - 25{,}000 = \$-60{,}585$$

$$AW_{D3} = -100{,}000(A/P,12\%,3) + 25{,}000(A/F,12\%,3) - 25{,}000 = \$-59{,}226$$

$$AW_{D4} = -100{,}000(A/P,12\%,4) - 25{,}000 = \$-57{,}923$$

Salvage Values after 2 and 3 years resp.

For all options, the challenger has an annual worth of

$$AW_C = \$-60{,}000$$

Option	Time in Service, Years: Defender	Challenger	AW Cash Flows for Each Option, \$/Year: 1	2	3	4	5	6	Option PW, \$
X	2	4	−60,585	−60,585	−60,000	−60,000	−60,000	−60,000	−247,666
Y	3	3	−59,226	−59,226	−59,226	−60,000	−60,000	−60,000	−244,817
Z	4	2	−57,923	−57,923	−57,923	−57,923	−60,000	−60,000	−240,369

A sample PW computation for option Y is

$$PW_Y = -59{,}226(P/A,12\%,3) - 60{,}000(F/A,12\%,3)(P/F,12\%,6) = \$-244{,}817$$

Option Z has the lowest cost PW value (\$240,369). Keep the defender all 4 years, then replace it. Same answer will result if the annual worth or future worth of each option is calculated at the MARR (instead of last column PW)

SM 300

Engineering Economics

Depreciation Methods

[After-Tax Replacement Studies]

What is Depreciation?

- Depreciation means decrease in worth of an asset.
- Almost everything depreciates as time proceeds; however, land is considered a nondepreciable asset.
- Depreciation can be defined as:

➢ A decline in the market value (MV) of an asset (deterioration)

➢ A decline in the value of an asset to its owner (obsolescence).

➢ Allocation of the cost of an asset over its depreciable or useful life. *Accountants usually use this definition*, and it is *employed in economic analysis* for income-tax computation purposes…. Therefore, depreciation is a way to claim over time an already paid expense for a depreciable asset.

Requirements

- For an asset to be depreciated, it must satisfy three requirements:
    1. The asset must be used for business purposes to generate income.
    2. The asset must have a useful life that can be determined and is longer than 1 year.
    3. The asset must be one that decays, gets used up, wears out, becomes obsolete, or loses value to the owner over time as a result of natural causes.

Depreciation Terminology

Definition: ***Book (noncash) method*** to represent **decrease in value** of a tangible asset over time

Two types: Book depreciation and Tax depreciation

Book depreciation: Used for *internal accounting* to track value of assets

***Tax depreciation*:** Used to determine *taxes due* based on tax laws

In USA only **tax depreciation** *must be calculated* ***using MACRS***; **book depreciation** *can be calculated* ***using any method***

Common Depreciation Terms

First cost *P* or unadjusted basis *B*: Total installed cost of asset = Initial purchase price + all costs incurred in placing the asset in service

Book value BV_t: Remaining undepreciated capital investment on the books after total amount of depreciation charges to date have been subtracted from the basis.

Recovery period *n*: Depreciable life of asset in years- often set by law

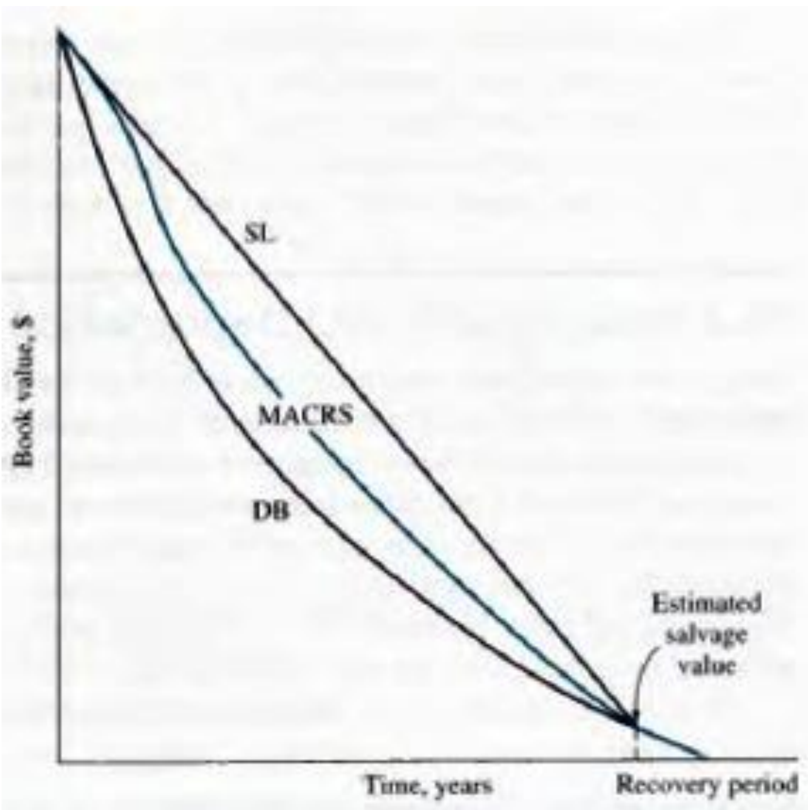
Market value *MV*: Amount realizable if asset were sold in open market… depending on the asset, can be drastically different from BV

Salvage value *S*: Estimated trade-in or MV at end of asset's useful life

Depreciation rate d_t: Fraction of first cost or basis removed each year

Depreciation Methods

- Classical methods for book depreciation (Used in India)
  - ➢Straight Line (SL) Model
  - ➢Declining Balance (DB) Model
- Modified Accelerated Cost Recovery System *(MACRS) Methods*--- Method for tax depreciation in the U.S.



General shape of book value curves for different depreciation methods.

Straight Line Depreciation

➡ **Book value decreases *linearly with time***

$$D_t = \frac{B - S}{n}$$

Where: D_t = annual depreciation charge
t = year
B = first cost or unadjusted basis
S = salvage value
n = recovery period

Note: *Useful life* <u>can be more</u> than *recovery period*

$$BV_t = B - tD_t$$

Where: BV_t = book value after t years

SL depreciation rate is **constant** for each year: $d = d_t = 1/n$

Example: SL Depreciation

An argon gas processor has a first cost of \$20,000 with a \$5,000 salvage value after 5 years. Find (a) D_3 and (b) BV_3 for year three. (c) Plot book value vs. time.

Solution: **(a) $D_3 = (B - S)/n$**

$= (20{,}000 - 5{,}000)/5$

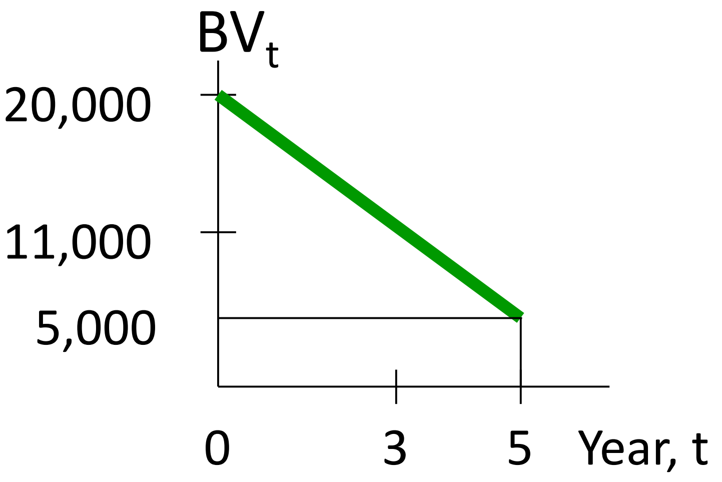
$= \$3{,}000$

(b) $BV_3 = B - tD_t$

$= 20{,}000 - 3(3{,}000)$

$= \$11{,}000$

(c) Plot BV vs. time



Declining Balance (DB) and Double Declining Balance (DDB) Depreciation

- DDB is an accelerated depreciation method
- Provides greater depreciation amounts in the early time periods over straight line
- Fixed percentage or Uniform percentage method where a fixed percentage is removed annually

Declining Balance (DB) and Double Declining Balance (DDB) Depreciation

- **Determined by multiplying Book Value (BV) at beginning of year *by fixed percentage d***
  - **Max rate for d is twice straight line rate, i.e., $d \le 2/n$**
  - **Cannot depreciate below salvage value**

Book value for year t is given by:

$$(BV)_t = B(1 - d)^t$$

Depreciation for year t is obtained by either relation:

$$D_t = d\ (BV)_{t-1} = d\,B\,(1 - d)^{t-1}$$

Where: D_t = depreciation for year t

d = uniform depreciation rate (2/n for DDB)

B = first cost or unadjusted basis

$(BV)_{t-1}$ = book value at end of previous year

Example: Double Declining Balance

A depreciable construction truck has a first cost of \$20,000 with a \$4,000 salvage value after 5 years. Find the (a) depreciation, and (b) book value after 3 years using DDB depreciation.

Solution:

(a) $d = 2/n = 2/5 = 0.4$

$$D_3 = d * B(1 - d)^{t-1}$$
$$= 0.4(20{,}000)(1 - 0.40)^{3-1}$$
$$= \$2880$$

(b) $BV_3 = B * (1 - d)^t$

$$= 20{,}000 * (1 - 0.4)^3$$
$$= \$4320$$

Switching Between Depreciation Methods

- Switching between depreciation methods may assist in accelerated reduction of the book value
- It also maximizes the present value of accumulated and total depreciation over the recovery period.
- Therefore, switching usually increases the tax advantage in years where the depreciation is larger.

General rules of switching

1. Switching is recommended when the depreciation for year t by the currently used method is less than that for a new method. The selected depreciation D_t is the larger amount.
2. Only one switch can take place during the recovery period.
3. Regardless of the (classical) depreciation methods, the book value cannot go below the estimated salvage value. When switching from a DB method, the estimated salvage value, not the DB-implied salvage value, is used to compute the depreciation for the new method; we assume $S = 0$ in all cases. (This does not apply to MACRS, since it already includes switching.)
4. The undepreciated amount, that is, BV_t, is used as the new adjusted basis to select the larger D_t for the next switching decision.

In all situations, the criterion is to **maximize the present worth of the total depreciation PW_D.** The combination of depreciation methods that results in the largest present worth is the best switching strategy.

$$PW_D = \sum_{t=1}^{t=n} D_t(P/F,i,t) \qquad [16A.5]$$

Procedure to Switch from DDB to SL

The procedure to switch from DDB to SL depreciation is as follows:

1. For each year t, compute the two depreciation charges.

   For DDB: $$D_{\mathrm{DDB}} = d(\mathrm{BV}_{t-1}) \qquad [16A.7]$$

   For SL: $$D_{\mathrm{SL}} = \frac{\mathrm{BV}_{t-1}}{n - t + 1} \qquad [16A.8]$$

   Time Remaining in Recovery Period after t-1

2. Select the larger depreciation value. The depreciation for each year is

$$D_t = \max[D_{\mathrm{DDB}}, D_{\mathrm{SL}}] \qquad [16A.9]$$

3. If needed, determine the present worth of total depreciation, using Equation [16A.5].

Example

The Outback Steakhouse main office has purchased a \$100,000 online document imaging system with an estimated useful life of 8 years and a tax depreciation recovery period of 5 years. Compare the present worth of total depreciation for (a) the SL method, (b) the DDB method, (c) DDB-to-SL switching. Use a rate of i = 15% per year.

Soln.

(*a*) Equation [16.1] determines the annual SL depreciation.

$$D_t = \frac{100{,}000 - 0}{5} = \$20{,}000$$

Since D_t is the same for all years, the P/A factor replaces P/F to compute PW_D.

$$PW_D = 20{,}000(P/A,15\%,5) = 20{,}000(3.3522) = \$67{,}044$$

(*b*) For DDB, $d = 2/5 = 0.40$. The results are shown in Table 16A–2. The value $PW_D =$ \$69,915 exceeds \$67,044 for SL depreciation. As is predictable, the accelerated depreciation of DDB increases PW_D. Sample calculations for DDB:

For Year 1: $D = d\,(BV_0) = 0.40 \times 100{,}000 = 40{,}000$ & $BV_1 = B\,(1- d) = 100{,}000\,(1-0.4) = 60{,}000$

For Year 3: $D = d\,(BV_2) = 0.40 \times 36{,}000 = 14{,}400$ & $BV_3 = B\,(1-d)^3 = 100{,}000\,(1 – 0.4)^3 = 21{,}600$

Example (Contd.)

TABLE 16A–2 DDB Model Depreciation and Present Worth Computations, Example 16A.3*b*

Year t	D_t, \$	BV_t, \$	$(P/F,15\%,t)$	Present Worth of D_t, \$
0		100,000		
1	40,000	60,000	0.8696	34,784
2	24,000	36,000	0.7561	18,146
3	14,400	21,600	0.6575	9,468
4	8,640	12,960	0.5718	4,940
5	5,184	7,776	0.4972	2,577
Totals	92,224			69,915

(*c*) Use the DDB-to-SL switching procedure.

1. The DDB values for D_t in Table 16A–2 are repeated in Table 16A–3 for comparison with the D_{SL} values from Equation [16A.8]. The D_{SL} values change each year because BV_{t-1} is different. Only in year 1 is D_{SL} = \$20,000, the same as computed in part (*a*). For illustration, compute D_{SL} values for years 2 and 4. For $t = 2$, BV_1 = \$60,000 by the DDB method and

$$D_{SL} = \frac{60{,}000 - 0}{5 - 2 + 1} = \$15{,}000$$

For $t = 4$, BV_3 = \$21,600 by the DDB method and

$$D_{SL} = \frac{21{,}600 - 0}{5 - 4 + 1} = \$10{,}800$$

Example (Contd.)

TABLE 16A–3 Depreciation and Present Worth for DDB-to-SL Switching, Example 16A.3c

Year	DDB Method, $			SL Method	Larger	P/F	Present Worth of
t	D_{DDB}	BV_t		D_{SL}, $	D_t, $	Factor	D_t, $
0	—	100,000	/5				
1	40,000	60,000	/4	20,000	40,000	0.8696	34,784
2	24,000	36,000	/3	15,000	24,000	0.7561	18,146
3	14,400	21,600	/2	12,000	14,400	0.6575	9,468
4*	8,640	12,960	/1	10,800	10,800	0.5718	6,175
5	5,184	7,776		12,960	10,800	0.4972	5,370
Totals	92,224				100,000		73,943

*Indicates year of switch from DDB to SL depreciation.

Switched in Y4, so same D as Y4 for SL

2. The column "Larger D_t" indicates a switch in year 4 with D_4 = \$10,800. The D_{SL} = \$12,960 in year 5 would apply *only* if the switch occurred in year 5. Total depreciation with switching is \$100,000 compared to the DDB amount of \$92,224.
3. With switching, PW_D = \$73,943, which is an increase over both the SL and DDB methods.

Example DB and DDB

Albertus Natural Stone Quarry purchased a computer-controlled face-cutter saw for $80,000. The unit has an anticipated life of 5 years and a salvage value of $10,000. (a) Compare the schedules for annual depreciation and book value using two methods: DB at 150% of the straight line rate and at the DDB rate. (b) How is the estimated $10,000 salvage value used?

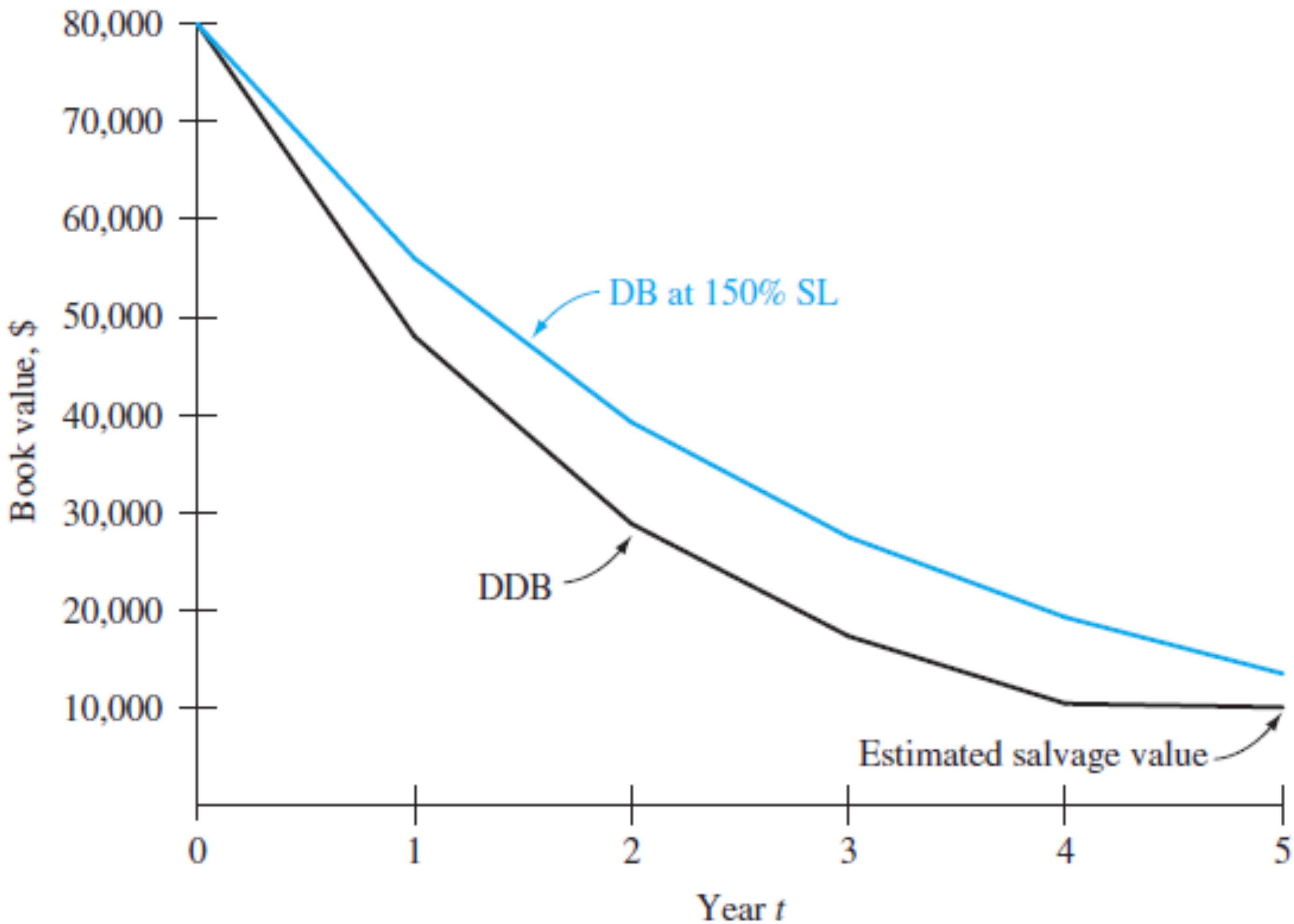
Solution

a. The DB depreciation rate is $d = 1.5/5 = 0.30$ while the DDB rate is $d_{\max} = 2/5 = 0.40$. Table 12.1 and Figure 12.3 present the comparison of

TABLE 12.1 **Annual Depreciation and Book Value, Example 12.2**

	Declining Balance, $d = 0.30$		Double Declining Balance, $d = 0.40$	
Year, t	D_t	BV_t	D_t	BV_t
0		$80,000		$80,000
1	$24,000	56,000	$32,000	48,000
2	16,800	39,200	19,200	28,800
3	11,760	27,440	11,520	17,280
4	$8,232	19,208	6,912	10,368
5	5,762	13,446	368	10,000

Example DB and DDB (Contd)



Example DB and DDB (Contd)

depreciation and book value. Example calculations of depreciation and book value for each method follow.

150% DB for year 2 by Equation [12.5]
with $d = 0.30$ $\quad D_2 = 0.30(56{,}000) = \$16{,}800$

by Equation [12.6] $\quad BV_2 = 80{,}000(0.70)^2 = \$39{,}200$

DDB for year 3 by Equation [12.5]
with $d = 0.40$ $\quad D_3 = 0.40(28{,}800) = \$11{,}520$

by Equation [12.6] $\quad BV_3 = 80{,}000(0.60)^3 = \$17{,}280$

The DDB depreciation is considerably larger during the first years, causing the book values to decrease faster, as indicated in Figure 12.3.

b. The \$10,000 salvage value is not utilized by the 150% DB method since the book value is not reduced this far. However, the DDB method reduces book value to \$10,368 in year 4. Therefore, not all of the calculated depreciation for year 5, $D_5 = 0.40(10{,}368) = \$4147$, can be removed; only the \$368 above S can be written off.

MODIFIED ACCELERATED COST RECOVERY SYSTEM (MACRS)

MACRS determines annual depreciation amounts using the relation

$$D_t = d_t B \qquad [12.10]$$

where the depreciation rate is tabulated in Table 12.2. (Tab this page for future reference.) The book value in year t is determined by either subtracting the annual depreciation from the previous year's book value, or by subtracting the accumulated depreciation from the first cost.

$$BV_t = BV_{t-1} - D_t \qquad [12.11]$$

$$= B - \sum_{j=1}^{j=t} D_j \qquad [12.12]$$

The first cost is always completely depreciated, since MACRS assumes that $S = 0$, even though there may be an estimated positive salvage.

The MACRS recovery periods are standardized to the values of 3, 5, 7, 10, 15, and 20 years for personal property. Note that all MACRS depreciation rates (Table 12.2) are presented for 1 year longer than the recovery period, and that the extra-year rate is one-half of the previous year's rate.

MODIFIED ACCELERATED COST RECOVERY SYSTEM (MACRS)

TABLE 12.2 MACRS Depreciation Rates Applied to the Basis

	Depreciation Rate (%)					
Year	$n = 3$	$n = 5$	$n = 7$	$n = 10$	$n = 15$	$n = 20$
1	33.33	20.00	14.29	10.00	5.00	3.75
2	44.45	32.00	24.49	18.00	9.50	7.22
3	14.81	19.20	17.49	14.40	8.55	6.68
4	7.41	11.52	12.49	11.52	7.70	6.18
5		11.52	8.93	9.22	6.93	5.71
6		5.76	8.92	7.37	6.23	5.29
7			8.93	6.55	5.90	4.89
8			4.46	6.55	5.90	4.52
9				6.56	5.91	4.46
10				6.55	5.90	4.46
11				3.28	5.91	4.46
12					5.90	4.46
13					5.91	4.46
14					5.90	4.46
15					5.91	4.46
16					2.95	4.46
17–20						4.46
21						2.23

Payback Period Analysis

Payback period: Estimated amount of time (n_p) for cash inflows to recover an initial investment (P) plus a stated rate of return (i%)

Types of payback analysis: **No-return** and **discounted** payback

1. **No-return payback** means rate of return is ZERO (i = 0%)
2. **Discounted payback** considers time value of money (i > 0%)

Caution: *Payback period analysis is a good initial screening tool, rather than the primary method to justify a project or select an alternative*

Note: ***Don't Get Confused*** with Capital recovery (CR) which is the equivalent annual amount that an asset, process, or system must earn (new revenue) each year to just recover the first cost plus a stated rate of return over its expected life.

Points to Remember About Payback Analysis

- **No-return payback neglects time value of money**, so no return is expected for the investment made
- **No cash flows after the payback period are considered** in the analysis. Return may be higher if these cash flows are expected to be positive.

- Approach of payback analysis is different from PW, AW, ROR and B/C analysis. A different alternative may be selected using payback.
- Rely on payback as a ***supplemental tool***; use PW or AW at the MARR for a reliable decision
- Discounted payback ($i > 0\%$) gives a good sense of the ***risk*** involved

Payback Period Computation

Formula to determine payback period (n_p) varies with type of analysis.

NCF = Net Cash Flow per period t

No return, $i = 0\%$; NCF_t varies annually:	$0 = -P + \sum_{t=1}^{t=n_p} NCF_t$	Eqn. 1
No return, $i = 0\%$; annual uniform NCF:	$n_p = \frac{P}{NCF}$	Eqn. 2
Discounted, $i > 0\%$; NCF_t varies annually:	$0 = -P + \sum_{t=1}^{t=n_p} NCF_t(P/F,i,t)$	Eqn. 3
Discounted, $i > 0\%$; annual uniform NCF:	$0 = -P + NCF(P/A,i,n_p)$	Eqn. 4

Example: Payback Analysis

	System 1	System 2
First cost, $	12,000	8,000
NCF, $ per year	3,000	1,000 (year 1-5)
		3,000 (year 6-14)
Maximum life, years	7	14

Problem: **Use (a) no-return payback, (b) discounted payback at 15%, and (c) PW analysis at 15% to select a system. Comment on the results.**

Solution: (a) Use Eqns. 2 and 1

n_{p1} = 12,000 / 3,000 = **4 years**

n_{p2} : $0 = -P + \Sigma_1^{np2} NCF$

= -8,000 + 5(1,000) + 1(3,000) = **0 [Trial and Error]**

Hence, 5 + 1 = 6 years

Since no-return payback for System 1 is 4 years < 6 years for System 2

Select System 1

Example: Payback Analysis (continued)

	System 1	System 2
First cost, $	12,000	8,000
NCF, $ per year	3,000	1,000 (year 1-5)
		3,000 (year 6-14)
Maximum life, years	7	14

Solution: (b) Use Eqns. 4 and 3

System 1: $0 = -12{,}000 + 3{,}000(P/A,15\%,n_{p1}) \rightarrow (P/A,15\%,n_{p1}) = 4$

$n_{p1} =$ **6.6 years** **[Use Interpolation in P/A factor column]**

System 2: $0 = -8{,}000 + 1{,}000(P/A,15\%,5) + 3{,}000(P/A,15\%,n_{p2} - 5)^{*}(P/F,15\%,5)$

$n_{p2} =$ **9.5 years [Use factor table and Interpolation]**

Select system 1

Note: Here, it is assumed that n_{p2} occurs during year 6-14.

(c) Find PW over LCM of 14 years

$PW_1 =$ **$663**

$PW_2 =$ **$2470**

Select system 2

Comment: **PW method considers cash flows after payback period. Selection changes from system 1 to 2**

SM 300

Engineering Economics

Breakeven Analysis

Breakeven Point

Value of a parameter that makes two elements equal

The parameter (or variable) can be an amount of revenue, cost, supply, demand, etc. for one project or between two alternatives

- **One project** - Breakeven point is identified as Q_{BE}. Determined using linear or non-linear math relations for revenue and cost
- **Between two alternatives** - Determine one of the parameters **P, A, F, i, or n** with others constant

Solution is by methods:

- **Direct solution of relations**
- **Trial and error**

Cost-Revenue Model — One Project

Quantity, Q — An amount of the variable or parameter in question, e.g., units/year, hours/month

Breakeven value is Q_{BE}

Fixed cost, FC — Costs not directly dependent on the variable or parameter, e.g., buildings, fixed overhead, insurance, minimum workforce cost

Variable cost, VC — Costs that change with parameters such as production level and workforce size. These are labor, material and marketing costs. Variable cost per unit is v

Total cost, TC — Sum of fixed and variable costs, TC = FC + VC

Revenue, R — Amount is dependent on quantity sold

Revenue per unit is r

Profit, P — Amount of revenue remaining after costs

P = R – TC = R – (FC+VC)

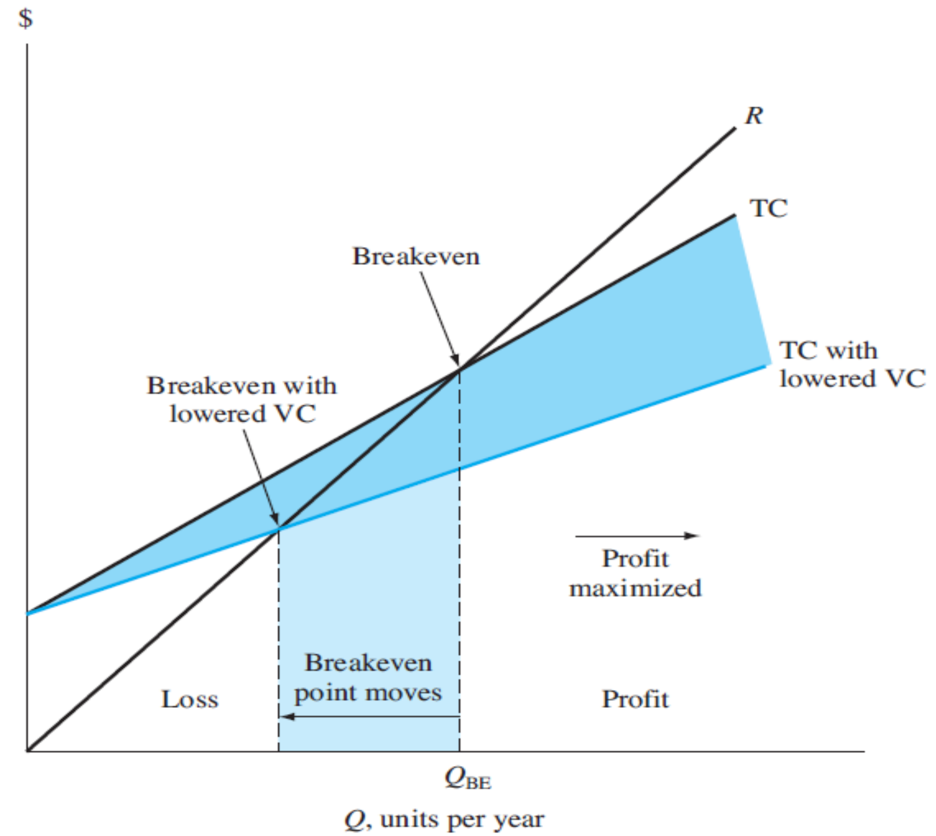
Breakeven for linear R and TC

Set R = TC and solve for $Q = Q_{BE}$

$$R = TC$$
$$rQ = FC + vQ$$

$$Q_{BE} = \frac{FC}{r - v}$$

When variable cost, v, is lowered, Q_{BE} decreases (moves to left)



Example: One Project Breakeven Point

A plant produces 15,000 units/month. Find breakeven level if FC = \$75,000 /month, revenue is \$8/unit and variable cost is 2.50/unit. Determine expected monthly profit or loss.

Solution: Find Q_{BE} and compare to 15,000; calculate Profit

$$Q_{BE} = 75{,}000 / (8.00-2.50) = 13{,}636 \text{ units/month}$$

Production level is above breakeven $\Rightarrow$ **Profit**

$$\text{Profit} = R - (FC + VC)$$
$$= rQ - (FC + vQ) = (r-v)Q - FC$$
$$= (8.00 - 2.50)(15{,}000) - 75{,}000$$
$$= \$\ 7500/\text{month}$$

Breakeven Between Two Alternatives

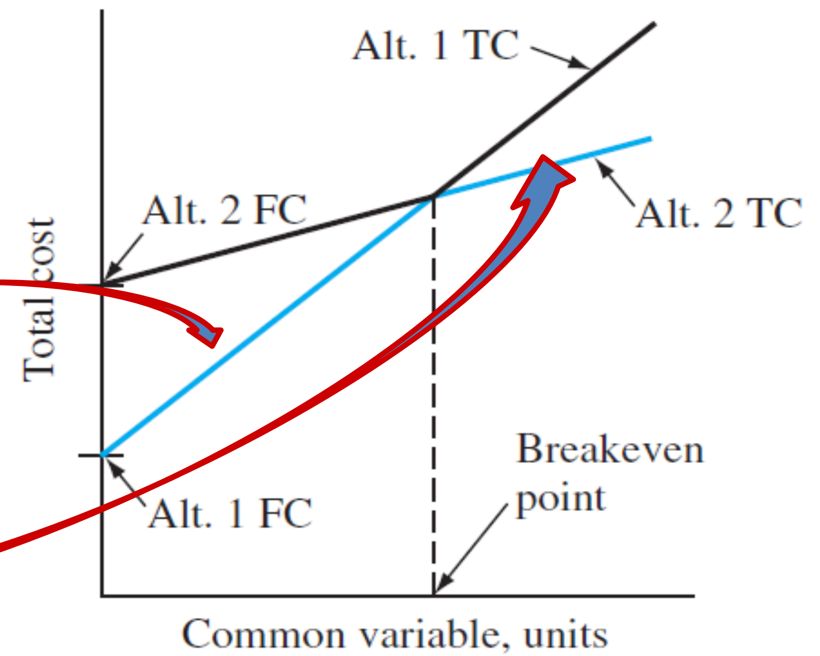
To determine value of common variable between 2 alternatives, do the following:

1. Define the common variable
2. Develop equivalence PW, AW or FW relations as function of common variable for each alternative
3. Equate the relations; solve for variable. **This is breakeven value**

Selection of alternative is based on anticipated value of common variable:

✓ Value BELOW breakeven; select higher variable cost

✓ Value ABOVE breakeven; select lower variable cost



Example: Two Alternative Breakeven Analysis

Perform a make/buy analysis where the common variable is X, the number of units produced each year. AW relations are:

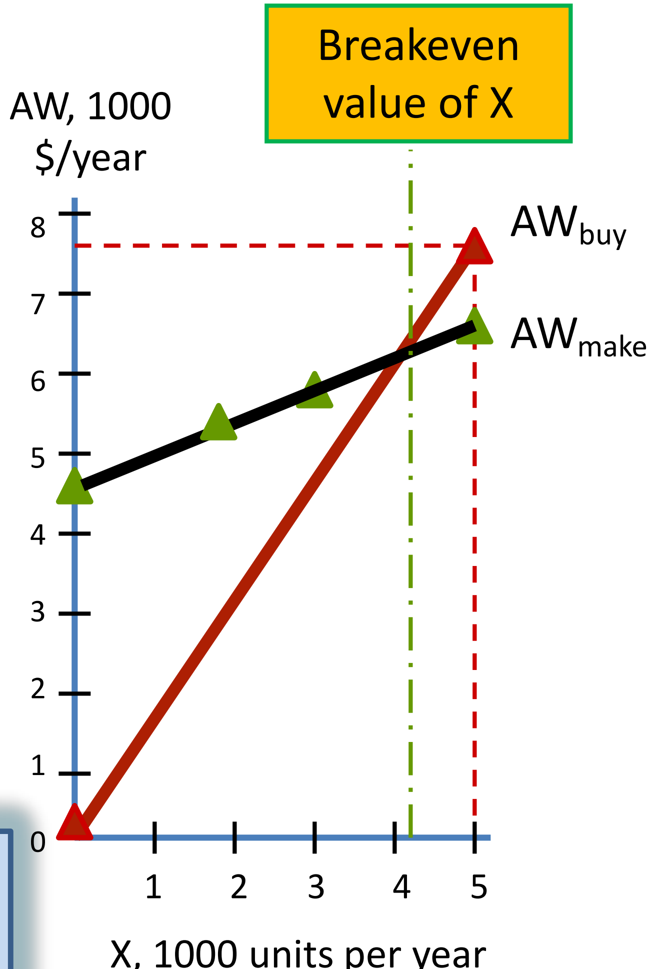
$$AW_{make} = -18{,}000(A/P,15\%,6) + 2{,}000(A/F,15\%,6) – 0.4X$$

$$AW_{buy} = -1.5X$$

Solution: Equate AW relations, solve for X

$$-1.5X = -4528 - 0.4X$$
$$X = 4116 \text{ per year}$$

If anticipated production > 4116, select make alternative (lower variable cost of 0.4 for Make compared to 1.5 for Buy)



Example 2: Breakeven Analysis

Two methods of weed control in an irrigation ditch are under consideration. Method A involves lining the ditch at a cost of \$30,000. The lining is expected to last 20 years. Maintenance with this method will cost \$2 per mile per year. Method B involves spraying a chemical which costs \$45 per gallon, with one gallon capable of treating 10 miles. Spraying equipment will cost \$2,500 and will have a life of 3 years with no salvage value. At an interest rate of 10% per year, (a) how many miles of ditch must require treatment in order for the two methods to breakeven, and (b) if 400 miles of ditch must be treated each year, which method should be selected?

(a) Set the annual worth for the two methods equal to each other and solve for x miles/yr:

$$-30{,}000\,(A/P, 10\%, 20) - 2x = -2500\,(A/P, 10\%, 3) - (45/10)x$$
$$-30{,}000\,(0.11746) - 2x = -2500\,(0.40211) - 4.5x$$
$$2.5x = 2518.53$$
$$x = 1{,}007 \text{ miles/year}$$

(b) At 400 miles per year, the spray method has the lower cost (check by replacing x with 400 in each equation and get -\$4324 vs \$-2805)

Application: ***Managerial uses of Break-Even Analysis:***

The break-even analysis can be used for the following purposes:

1) Margin of Safety:

It reflects the difference between the actual volume of sales and the break-even volume of sales. It reveals the percentage increase in sales necessary to reach the BEP so as atleast to avoid losses.

2) Volume needed to Attain Target Profit:

BEA may be utilized for the purpose of determining the volume of sales necessary to achieve a target profit.

3) Change in Price:

BEA will help the management to know the required volume of sales to maintain the previous level of profit. And on the basis of its knowledge and experience, it will be much easier for the management to judge whether the required increase in sales will be feasible

4) Change in Costs:

BEA helps the management to decide the total sales volume to maintain present profits without any change in price even if variable cost or fixed cost changes.

5) To Expand Capacity or Not:

The management might often be interested in knowing whether to expand production capacity or not through the installation of additional equipment. Through break-even analysis, it would be possible to examine the various implications of this proposal.

6) Effect of Alternative Prices:

The BE chart can be modified to show the profit position at different price levels.

7) Drop and/ or Add Decision:

BEA is helps the management in taking decision about the addition of a new product or dropping of a particular product and it also helps to know consequent effects on revenue and cost.

8) Make or Buy Decision:

Many business firms often have the option of making certain components or ingredients which are part of their finished products or purchasing them from outside suppliers. BEA helps in taking right decision.

9) Choosing Promotion-Mix:

Sellers often use several modes of sales promotion e.g. personal selling, advertising etc. BEA helps to select the right mode of promotion mix.

10) Equipment Selection:

BEA analysis can also be used to compare different ways of doing jobs.

11) Production Planning:

BEA can also help in production planning to as to give maximum contribution towards profit and fixed costs.

12) Improving Profit Performance:

There are 4 specific ways in which profit performance of a business can be improved:

(a) Increasing the volume of sales (b) Increasing the selling price

(c) Reducing variable cost per unit (d) Reducing the fixed cost.

Location Decision

Factors Affecting Location Decisions

Government	a. Policies on foreign ownership of production facilities Local content requirements Import restrictions Currency restrictions Environment regulations Local product standards Liability laws b. Stability issues
Cultural differences	Living circumstances for foreign workers and their dependents Ways of doing business Religious holidays/traditions
Customer preferences	Possible “buy locally” sentiment
Labor	Level of training and education of workers Work ethic Wage rates Possible regulations limiting the number of foreign employees Language differences
Resources	Availability and quality of raw materials, energy, transportation infra.
Financial	Financial incentives, tax rates, inflation rates, interest rates
Technological	Rate of technological change, rate of innovations
Market	Market potential, competition
Safety	Crime, terrorism threat

Evaluating Location Alternatives

There are different techniques that can be used to evaluate location alternatives

1. Factor Rating
2. Centre of Gravity Method

Evaluating Location Alternatives (Contd.)

Factor Rating

Technique can be applied to a wide range of decisions including location analysis.

It establishes a ***composite*** value for each alternative that summarizes all the related factors.

Factor rating enables decision makers to incorporate their personal opinions (qualitative) and quantitative information.

Evaluating Location Alternatives (Contd.)

Procedure:

1. Determine which factors are relevant
2. Assign a weight to each factor that indicates its relative importance compared with all other factors (Weights typically sum to 1.00)
3. Decide on a common scale for all factors, and set a minimum acceptable score if necessary
4. Score each location alternative
5. Multiply the factor weight by the score for each factor, and sum the results for each location alternative
6. Choose the alternative that has the highest composite score, unless it fails to meet the minimum acceptable score

Evaluating Location Alternatives (Contd.)

Eg. Factor Rating

A photo-processing company intends to open a new branch store. The following table contains information on two potential locations (Alt1 and Alt2). Which is better?

		Scores (Out of 100)	
Factor	Weight	Alt 1	Alt 2
Proximity to existing source	.10	100	60
Traffic volume	.05	80	80
Rental costs	.40	70	90
Size	.10	86	92
Layout	.20	40	70
Operating Cost	.15	80	90
	1.00		

Evaluating Location Alternatives (Contd.)

Eg. Factor Rating Solution

		Scores (Out of 100)		Weighted Scores	
Factor	Weight	Alt 1	Alt 2	Alt 1	Alt 2
Proximity to existing source	.10	100	60	.10(100) = 10.0	.10(60) = 6.0
Traffic volume	.05	80	80	.05(80) = 4.0	.05(80) = 4.0
Rental costs	.40	70	90	.40(70) = 28.0	.40(90) = 36.0
Size	.10	86	92	.10(86) = 8.6	.10(92) = 9.2
Layout	.20	40	70	.20(40) = 8.0	.20(70) = 14.0
Operating Cost	.15	80	90	.15(80) = 12.0	.15(90) = 13.5
	1.00			70.6	82.7

Alt. 2 is better since it has higher composite factor rating.

Evaluating Location Alternatives (Contd.)

The Centre of Gravity Method

It is a method to determine the location of a facility that will minimize shipping costs or travel time to various destinations.

Eg. Community planners may use this method to determine the location of fire and public safety centers, schools, etc.

For Distribution Problems- In this method, a map is used and locations are marked on the coordinate system to determine the relative locations.

For equal quantities to be shipped to each location:

Centre of Gravity = (Avg. of X CoOrds, Avg. of Y CoOrds)

For unequal quantities to be shipped to each location:

Centre of Gravity = Weighted Avg. of CoOrdinates

$$\bar{x} = \frac{\Sigma x_i Q_i}{\Sigma Q_i}$$

$$\bar{y} = \frac{\Sigma y_i Q_i}{\Sigma Q_i}$$

Where

Q_i = Quantity to be shipped to destination *i*

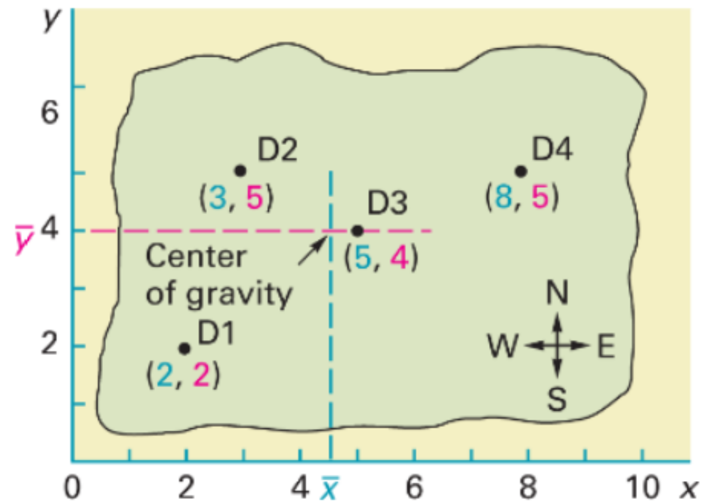
x_i = x coordinate of destination *i*

y_i = y coordinate of destination *i*

Evaluating Location Alternatives (Contd.)

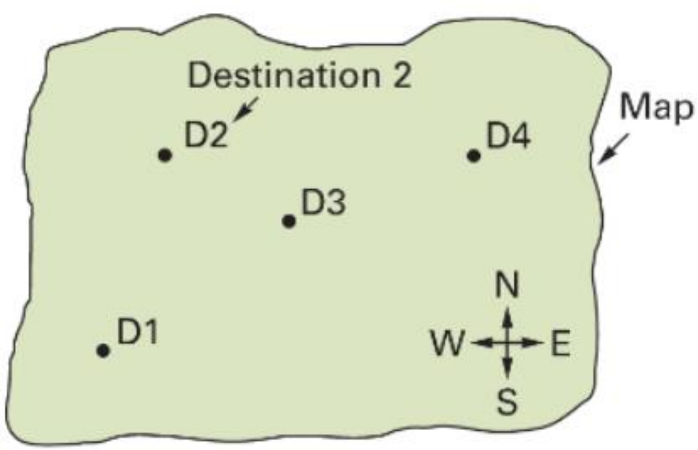
Eg. Centre of Gravity Method

Calculate Center of Gravity where the warehouse should be located when equal quantity of shipment is to be sent from warehouse to each of the destination.

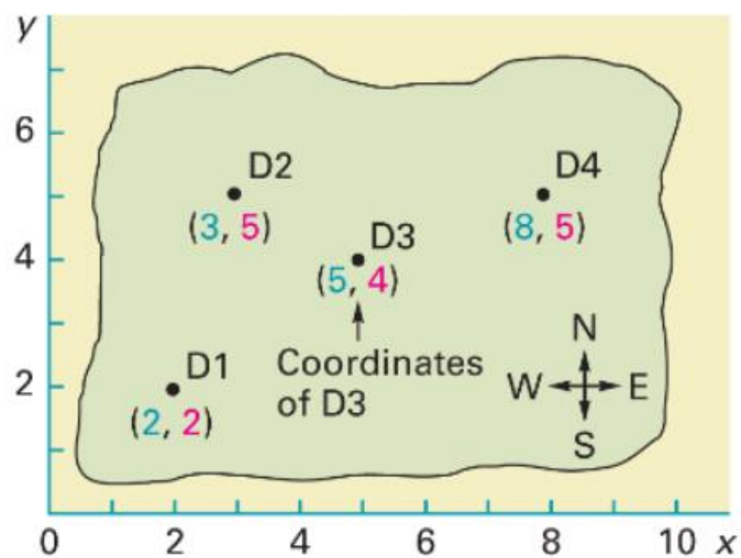


C. Center of gravity

Ans. (4.5, 4)



A. Map showing destinations

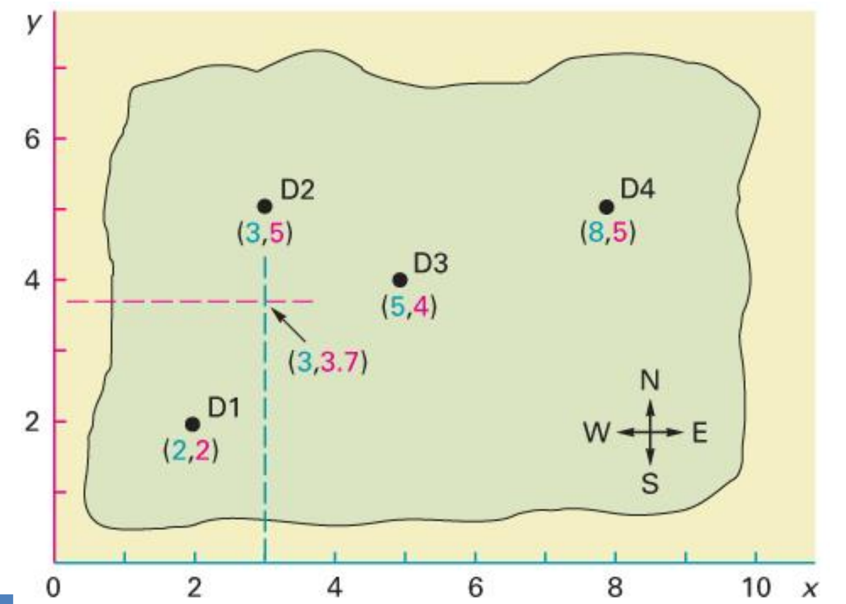


B. Add a coordinate system

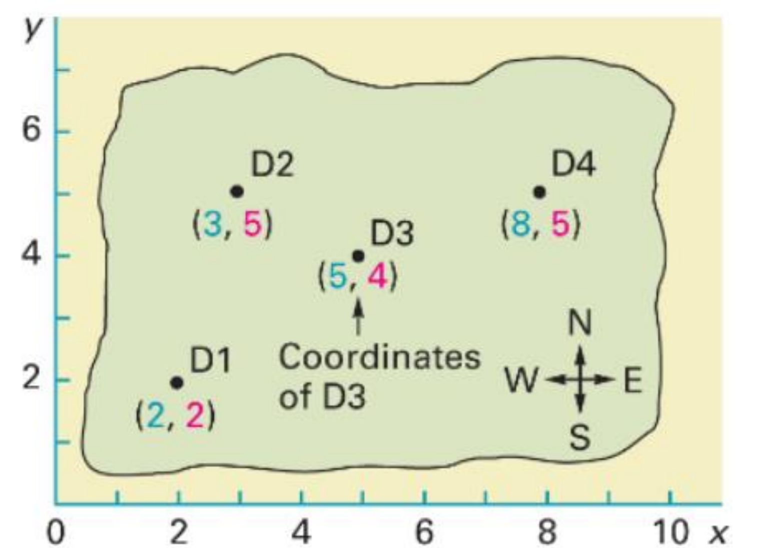
Evaluating Location Alternatives (Contd.)

Eg. Centre of Gravity Method

Calculate Center of Gravity where the warehouse should be located when quantity of shipment to be sent to each of the destination from warehouse is as given in the Table



Destination	X, Y	Weekly Quantity
D1	2,2	800
D2	3,5	900
D3	5,4	200
D4	8,5	100
		2,000



Ans. (3, 3.7)