CAT QUESTIONS

1. To reach ekapathshaala a student has to take 100 steps. He can take of steps of 3 or 5 only. How many different steps of 5 can he take, if he must take at least one step of each type?

$3x + 5y = 100$ $\quad x = 1, 2, 3, 4, \ldots\ldots$

* No. of steps can't be in points

$x = 1$
$3(1) + 5y = 100$
$3 + 5y = 100$
$5y = 97$
$y = 19.4$

$x = 2$
$3(2) + 5y = 100$
$6 + 5y = 100$
$5y = 94$
$y = 18.8$

$x = 3$
$3(3) + 5y = 100$
$9 + 5y = 100$
$5y = 91$
$y = 18.2$

$x = 5$
$3(5) + 5y = 100$
$15 + 5y = 100$
$5y = 85$
$\boxed{y = 17}$

$x = 6$
$3(6) + 5y = 100$
$18 + 5y = 100$
$5y = 82$
$y = 16.4$

$x = 7$
$3(7) + 5y = 100$
$21 + 5y = 100$
$5y = 79$
$y = 15.8$

$x = 10$
$3(10) + 5y = 100$
$30 + 5y = 100$
$5y = 70$
$y = 14$

$x = 15$
$3(15) + 5y = 100$
$45 + 5y = 100$
$5y = 55$
$y = 11$

$x = 20$
$3(20) + 5y = 100$
$60 + 5y = 100$
$5y = 40$
$y = 8$

$x = 25$

$3(25) + 5y = 100$

$75 + 5y = 100$

$5y = 25$

$y = 5$

$x = 30$

$3(30) + 5y = 100$

$90 + 5y = 100$

$5y = 10$

$y = 2$

$x = 35$

$3(35) + 5y = 100$

$105 + 5y = 100$

$5y = 100 - 105$

$y = \frac{-5}{5} = -1$

∴ We will take multiples of 5.

$x = 5$, $y = 17$

$x = 10$, $y = 14$

$x = 15$, $y = 11$

$x = 20$, $y = 8$

$x = 25$, $y = 5$

$x = 30$, $y = 2$.

[We can't take y as negative ∴ 6]

∴ Total 6 values

2. PQR × QS = PQRS

PQR = 3 digit no.

QS = 2 digit no.

PQRS = 4 digit no.

Find D.

QS = 10 (∵ smallest 2 digit no)

∴ Q = 1 , S = 0

P Q R × Q S = P Q R S
4 1 9 1 0 4 1 9 0

∴ Q = 1

3. What is the no. of zeroes at end of 150!
[CAT 1993]

* Concept of Trailing zeroes:- [Jinke piche zeroes lage ho]

Eg:- 23 × 10 = 230 [1 zero]
23 × 10 × 10 = 2300 [2 zero]
2035 → [No zeroes]

150 × 149 × 148 × ..... × 4 × 3 × 2 × 1

Only for prime no's:-

$\frac{150}{2} = \frac{75}{2} = \frac{37}{2} = \frac{18}{2} = \frac{9}{2} = \frac{4}{2} = \frac{2}{2} = 1$

No. of 2's = 2^{146}

$\frac{150}{5} = \frac{30}{5} = \frac{6}{5} = 1$

No. of 5's = 5^{37}

$10 \times 10 \times 10 \times ..... \times 10 = 10^{37}$

∴ 37 zeroes

4. Find the power of 2 in 100!
Find the power of 3 in 100!
Find the power of 6 in 100!
Find the power of 10 in 100!

$2 :- \frac{100}{2} = \frac{50}{2} = \frac{25}{2} = \frac{12}{2} = \frac{6}{2} = \frac{3}{2} = 1$

$= 2^{97}$

$3 :- \frac{100}{3} = \frac{33}{3} = \frac{11}{3} = \frac{3}{3} = 1$

$= 3^{48}$

$6 :- \frac{100}{3} = \frac{33}{3} = \frac{11}{3} = \frac{3}{3} = 1$ [(2 x 3) will take highes

$= 6^{48}$

$10 :- \frac{100}{5} = \frac{20}{5} = \frac{4}{5} = 1$ [(2x5) take highest]

$= 5^{24}$

5. What is the greatest power of 7 that will divide 100! completly?

$\frac{100}{7} = \frac{14}{7} = 2$

7^{16}

6. $167! = 5^P \times Y$. What is the greatest value of P (given Y is not a multiple of 5)? 1

$$\frac{167}{5} = \frac{33}{5} = \frac{6}{5} = 1$$

$$= 5^{40}$$

7. Find the smallest no. that should 198! so that it becomes odd number.

$$\frac{198}{2} = \frac{99}{2} = \frac{49}{2} = \frac{24}{2} = \frac{12}{2} = \frac{6}{2} = \frac{3}{2} = 1$$

$$2^{194}$$

$$\frac{198}{2^{194}}$$

8. When 2 numbers are divided by a divisor, 19 & 32 are obtained as remainders. When the sum of these 2 no's is divided by the same divisor, the remainder is 8. Find divisor.

* Property:- Whatever can be done with a no. can be done same with the remainder.

Eg:- $\frac{104}{5} = 4$ $\quad \frac{109}{5} = 3$ $\quad \frac{207}{5} = 2$

$\frac{4}{5} = 4$ $\quad \frac{3}{5} = 3$ $\quad \frac{7}{5} = 2$

19 32 remainders
19+32 = 51 (but remainder is 8)
43 cut kar diya

∴ Divisor = 43

9. The remainder when 2^{8491} is divided by 7 is
(i) 2 (iii) 6 (v) 3
(ii) 1 (iv) 5

* Remainder Cycle:-

1. $\frac{2^{8491}}{7}$

$\frac{2}{7} = 2$

$\frac{4}{7} = 4$ ∴ steps = 3

$\frac{8}{7} = 1$ power ÷ step

$$
\begin{array}{r|l|l}
3 & 8491 & 283 \\
 & 6 & \\
\hline
 & 24 & \\
 & 24 & \\
\hline
 & 9 & \\
 & 9 & \\
\hline
 & 1 &
\end{array}
$$

∴ Ans = 2

Ans:- (i) 2

Remainder = 1

2. $\frac{4^{1986}}{9}$

$\frac{4}{9} = 4$

$\frac{16}{9} = 7$

$\frac{28}{9} = 1$

No. of steps = 3

power ÷ step

3	1986	662
	18	
	18	
	18	
	6	
	6	
	0	

Remainder = 0 $\therefore$ Ans = 1

3. $\frac{7^{1986}}{9}$

$\frac{7}{9} = 7$

$\frac{49}{9} = 4$

$\frac{28}{9} = 1$

steps = 3

power ÷ step

3	1986	662
	18	
	18	
	18	
	6	
	6	
	0	

Remainder = 0 $\therefore$ Ans = 1

[CAT] 4. $\frac{3^{1987}}{19}$

$\frac{3}{19} = 3$ $\quad$ $\frac{12}{19} = 12$

$\frac{9}{19} = 9$ $\quad$ $\frac{36}{19} = 17$

$\frac{27}{19} = 8$ $\quad$ $\frac{51}{17} = 13$

$\frac{24}{19} = 5$ $\quad$ $\frac{39}{19} = 1$

$\frac{15}{19} = 15$

$\frac{45}{19} = 7$

$\frac{21}{19} = 2$

$\frac{6}{19} = 6$

$\frac{18}{19} = 18$

$\frac{54}{19} = 16$

$\frac{48}{19} = 10$

$\frac{30}{19} = 11$

$\frac{33}{19} = 14$

$\frac{42}{19} = 4$

Steps = 18

power ÷ step

$$\begin{array}{r|l|l} 18 & 1987 & 11 \\ & 18 & \\ \hline & 18 & \\ & 18 & \\ \hline & 7 & \end{array}$$

Remainder = 7

∴ Ans :- 2

5. $\frac{3^{187654!}}{5}$

$\frac{3}{5} = 3$

$\frac{9}{5} = 4$

$\frac{12}{5} = 2$

$\frac{6}{5} = 1$

Steps = 4

power ÷ steps

Ans :- 1.

$187654 \times 187653 \times \ldots\ldots \times 4 \times 3 \times 2 \times 1.$

* Fermat Rule :-

$\frac{a^{even}}{a+1} = 1$

$\frac{a^{odd}}{a+1} = a$

$\frac{(a+1)^{any}}{a} = 1$

6. $\frac{4^{96067}}{6}$

$\frac{4}{6} = 4$

$\frac{16}{6} = 4$

As remainder is repeating.

$\frac{16}{6} = 4$

$\therefore$ Ans = 4

7. $\frac{2^{4^{6^{8^{10^{11}}}}}}{3}$

$= \frac{a^{even}}{a+1}$

$= 1$

8. $\frac{2^{3^{4^{5^{6^{\cdots^{1001}}}}}}}{3}$

$= \frac{a^{odd}}{a+1} = a$

$= 2$

9. $\frac{3^{4^{5^{6^{7^{8^{\cdots}}}}}}}{2}$

$\frac{(a+1)^{any}}{a} = 1$

$= \underline{1}$

* Property:-

(i) $x^n - y^n \div x-y, x+y, n = even$

Eg:- $x^2 - y^2 = (x-y)(x+y)$

$4^2 - 2^2 = (4-2),(4+2)$

$2, 6$

(ii) $x^n - y^n = x-y, n = odd$

Eg:- $x^3 - y^3 = (x-y)(x^2+y^2+xy)$

$3^3 - 1^3 = 27-1 = 26 \rightarrow 2$

(iii) $x^n + y^n = x+y, n = odd$

Eg:- $x^3 + y^3 = (x+y)(x^2-xy+y^2)$

$3^3 + 1^3 = 27+1 = 28$

$\swarrow$

4

10. $\frac{63^{27} - 31^{27}}{32}$

$x^n - y^n$, n = odd

$(x-y)$

32

Remainder = 0

11. $\frac{63^{27} + 31^{27}}{94}$

$x^n + y^n$, n = odd

$(x+y)$

94

Remainder = 0

12. $\frac{63^{24} - 31^{24}}{32}$

$63-31, 63+31$

$32, 94$

Remainder = 0

13. $\frac{63^{24} - 31^{24}}{94}$

$63-31, 63+31$

$32, 94$

Remainder = 0

14. Find remainder:-

$\frac{7^{11^{12^{13^{14}}}}}{6}$

Ans:-1.

15: $\frac{5^{12^{13^{14^{15}}}}}{6}$

Ans:-1.

16. $\frac{5^{13^{14^{17^{21}}}}}{6}$

Ans:-5.