

Assignment 1

Problem - 2.1:

a).

$$P(O, X=HHH) = (0.0).(0.4).(0.7).(0.1).(0.7).(0.5) = 0.0$$

$$P(O, X= HHC) = (0.0).(0.4).(0.7).(0.1).(0.3).(0.1) = 0.0$$

$$P(O, X= HCH) = (0.0).(0.4).(0.3).(0.7).(0.4).(0.5) = 0.0$$

$$P(O, X= HCC) = (0.0).(0.4).(0.3).(0.7).(0.6).(0.1) = 0.0$$

$$P(O, X= CHC) = (1.0).(0.2).(0.4).(0.1).(0.3).(0.1) = 0.00024$$

$$P(O, X= CHH) = (1.0).(0.2).(0.4).(0.1).(0.7).(0.5) = 0.0028$$

$$P(O, X= CCH) = (1.0).(0.2).(0.6).(0.7).(0.4).(0.5) = 0.0168$$

$$P(O, X= CCC) = (1.0).(0.2).(0.6).(0.7).(0.6).(0.1) = 0.00504$$

$$\text{Final} = 0.0+0.0+0.0+0.0+0.00024+0.0028+0.0168+0.00504=0.02488$$

b). Considering state 0 as H, state 1 as C, Computing $P(o|\lambda)$

$$= 0.0*0.4=0.0$$

$$= 1.0*0.2=0.2$$

$$= [(0.0*0.7)+(0.2*0.4)]*0.1=0.008$$

$$= [(0.0*0.3)+(0.2*0.6)]*0.7=0.084$$

$$= [(0.008*0.7)+(0.084*0.4)]*0.5=0.0196$$

$$= [(0.008*0.3)+(0.084*0.6)]*0.1=0.00528$$

$$= 0.0196+0.00528=0.02488$$

c). work factor for the method in part a) – For Direct Computation, $O(N^T T)$ -> Exponential

For Example $T=3$ Observations, $N=2$ Hidden States

then number of hidden paths $= 2^3 * 3 = 24$ path evaluations

To multiply 6 numbers, we need 5 multiplication steps $= 8 * 5 = 40$ Multiplications ($N^T \times (2T-1)$ multiplications)

work factor for the method in part b) – For Forward algorithm, $O(N^2 T)$ -> Polynomial

$$2^2 * 3 = 12 \text{ multiplications}$$

Problem - 2.3:

a) observation sequence $\mathcal{O} = (\mathcal{O}_0, \mathcal{O}_1, \mathcal{O}_2, \mathcal{O}_3)$, where $\mathcal{O}_i \in \{0, 1, 2\}$. Verify that $\sum P(\mathcal{O} | \lambda) = 1$, the sum is over the observation sequences of length four

I wrote a Python program to calculate the probability and included the code in the assignment folder. I started by setting up the A, B, and π matrices with the given values. Then, using itertools, I

generated all possible observation sequences of length 4 along with their state transitions. The program outputs the probability for each sequence, and when I add them all together, the total comes out to 1.

Below are 81 observation sequences of length 4:

for (0, 0, 0, 0) observation sequence is 0.02840236

for (0, 0, 0, 1) observation sequence is 0.01835428

for (0, 0, 0, 2) observation sequence is 0.01744736

for (0, 0, 1, 0) observation sequence is 0.0146524

for (0, 0, 1, 1) observation sequence is 0.0131824

for (0, 0, 1, 2) observation sequence is 0.0138572

for (0, 0, 2, 0) observation sequence is 0.01260644

for (0, 0, 2, 1) observation sequence is 0.01300292

for (0, 0, 2, 2) observation sequence is 0.01409464

for (0, 1, 0, 0) observation sequence is 0.0147616

for (0, 1, 0, 1) observation sequence is 0.0099688

for (0, 1, 0, 2) observation sequence is 0.0096296

for (0, 1, 1, 0) observation sequence is 0.0102496

for (0, 1, 1, 1) observation sequence is 0.0101728

for (0, 1, 1, 2) observation sequence is 0.0109376

for (0, 1, 2, 0) observation sequence is 0.0099968

for (0, 1, 2, 1) observation sequence is 0.0110024

for (0, 1, 2, 2) observation sequence is 0.0120808

for (0, 2, 0, 0) observation sequence is 0.01274924

for (0, 2, 0, 1) observation sequence is 0.00880052

for (0, 2, 0, 2) observation sequence is 0.00856624

for (0, 2, 1, 0) observation sequence is 0.010022

for (0, 2, 1, 1) observation sequence is 0.0102608

for (0, 2, 1, 2) observation sequence is 0.0111052

for (0, 2, 2, 0) observation sequence is 0.01016356

for (0, 2, 2, 1) observation sequence is 0.01138708

for (0, 2, 2, 2) observation sequence is 0.01254536
for (1, 0, 0, 0) observation sequence is 0.0194936
for (1, 0, 0, 1) observation sequence is 0.0126728
for (1, 0, 0, 2) observation sequence is 0.0120736
for (1, 0, 1, 0) observation sequence is 0.01052
for (1, 0, 1, 1) observation sequence is 0.009632
for (1, 0, 1, 2) observation sequence is 0.010168
for (1, 0, 2, 0) observation sequence is 0.0092584
for (1, 0, 2, 1) observation sequence is 0.0096712
for (1, 0, 2, 2) observation sequence is 0.0105104
for (1, 1, 0, 0) observation sequence is 0.0142016
for (1, 1, 0, 1) observation sequence is 0.0097568
for (1, 1, 0, 2) observation sequence is 0.0094816
for (1, 1, 1, 0) observation sequence is 0.01088
for (1, 1, 1, 1) observation sequence is 0.011072
for (1, 1, 1, 2) observation sequence is 0.011968
for (1, 1, 2, 0) observation sequence is 0.0109504
for (1, 1, 2, 1) observation sequence is 0.0122272
for (1, 1, 2, 2) observation sequence is 0.0134624
for (1, 2, 0, 0) observation sequence is 0.0140728
for (1, 2, 0, 1) observation sequence is 0.0098344
for (1, 2, 0, 2) observation sequence is 0.0096128
for (1, 2, 1, 0) observation sequence is 0.0118
for (1, 2, 1, 1) observation sequence is 0.012256
for (1, 2, 1, 2) observation sequence is 0.013304
for (1, 2, 2, 0) observation sequence is 0.0121832
for (1, 2, 2, 1) observation sequence is 0.0137576
for (1, 2, 2, 2) observation sequence is 0.0151792
for (2, 0, 0, 0) observation sequence is 0.01893724

for (2, 0, 0, 1) observation sequence is 0.01233652
for (2, 0, 0, 2) observation sequence is 0.01176224
for (2, 0, 1, 0) observation sequence is 0.0103756
for (2, 0, 1, 1) observation sequence is 0.0095536
for (2, 0, 1, 2) observation sequence is 0.0100988
for (2, 0, 2, 0) observation sequence is 0.00919796
for (2, 0, 2, 1) observation sequence is 0.00964628
for (2, 0, 2, 2) observation sequence is 0.01049176
for (2, 1, 0, 0) observation sequence is 0.0151648
for (2, 1, 0, 1) observation sequence is 0.0104584
for (2, 1, 0, 2) observation sequence is 0.0101768
for (2, 1, 1, 0) observation sequence is 0.0118624
for (2, 1, 1, 1) observation sequence is 0.0121312
for (2, 1, 1, 2) observation sequence is 0.0131264
for (2, 1, 2, 0) observation sequence is 0.0120128
for (2, 1, 2, 1) observation sequence is 0.0134504
for (2, 1, 2, 2) observation sequence is 0.0148168
for (2, 2, 0, 0) observation sequence is 0.01546076
for (2, 2, 0, 1) observation sequence is 0.01082948
for (2, 2, 0, 2) observation sequence is 0.01059376
for (2, 2, 1, 0) observation sequence is 0.013118
for (2, 2, 1, 1) observation sequence is 0.0136592
for (2, 2, 1, 2) observation sequence is 0.0148348
for (2, 2, 2, 0) observation sequence is 0.01358644
for (2, 2, 2, 1) observation sequence is 0.01536292
for (2, 2, 2, 2) observation sequence is 0.01695464

Sum of all the above probabilities are equals to $0.9999999999999997 \approx 1.0$

By executing the code snippet provided in the folder, it gives above output. Hence is verified

b). Using Forward algorithm:

I wrote a Python program to calculate the probability and included the code in the assignment folder. I started by setting up the A, B, and π matrices with the given values. Then, using itertools, I generated all possible observation sequences of length 4 along with their state transitions. The program outputs the probability for each sequence, and when I add them all together, the total comes out to 1.

Observation sequence (0, 0, 0, 0), Probability is 0.028409

Observation sequence (0, 0, 0, 1), Probability is 0.01836

Observation sequence (0, 0, 0, 2), Probability is 0.01745

Observation sequence (0, 0, 1, 0), Probability is 0.01465

Observation sequence (0, 0, 1, 1), Probability is 0.01318

Observation sequence (0, 0, 1, 2), Probability is 0.013859

Observation sequence (0, 0, 2, 0), Probability is 0.0126

Observation sequence (0, 0, 2, 1), Probability is 0.01300

Observation sequence (0, 0, 2, 2), Probability is 0.01409

Observation sequence (0, 1, 0, 0), Probability is 0.01477

Observation sequence (0, 1, 0, 1), Probability is 0.00997

Observation sequence (0, 1, 0, 2), Probability is 0.00963

Observation sequence (0, 1, 1, 0), Probability is 0.01025

Observation sequence (0, 1, 1, 1), Probability is 0.01017

Observation sequence (0, 1, 1, 2), Probability is 0.01094

Observation sequence (0, 1, 2, 0), Probability is 0.009989

Observation sequence (0, 1, 2, 1), Probability is 0.011

Observation sequence (0, 1, 2, 2), Probability is 0.0120799

Observation sequence (0, 2, 0, 0), Probability is 0.012750

Observation sequence (0, 2, 0, 1), Probability is 0.0088

Observation sequence (0, 2, 0, 2), Probability is 0.00856

Observation sequence (0, 2, 1, 0), Probability is 0.010020

Observation sequence (0, 2, 1, 1), Probability is 0.01026

Observation sequence (0, 2, 1, 2), Probability is 0.01111

Observation sequence (0, 2, 2, 0), Probability is 0.010159
Observation sequence (0, 2, 2, 1), Probability is 0.01139
Observation sequence (0, 2, 2, 2), Probability is 0.012549
Observation sequence (1, 0, 0, 0), Probability is 0.019489
Observation sequence (1, 0, 0, 1), Probability is 0.01267
Observation sequence (1, 0, 0, 2), Probability is 0.01207
Observation sequence (1, 0, 1, 0), Probability is 0.01052
Observation sequence (1, 0, 1, 1), Probability is 0.00964
Observation sequence (1, 0, 1, 2), Probability is 0.01017
Observation sequence (1, 0, 2, 0), Probability is 0.00925
Observation sequence (1, 0, 2, 1), Probability is 0.00967
Observation sequence (1, 0, 2, 2), Probability is 0.01051
Observation sequence (1, 1, 0, 0), Probability is 0.0142
Observation sequence (1, 1, 0, 1), Probability is 0.00976
Observation sequence (1, 1, 0, 2), Probability is 0.009479
Observation sequence (1, 1, 1, 0), Probability is 0.010879
Observation sequence (1, 1, 1, 1), Probability is 0.011080
Observation sequence (1, 1, 1, 2), Probability is 0.01197
Observation sequence (1, 1, 2, 0), Probability is 0.0109500
Observation sequence (1, 1, 2, 1), Probability is 0.01223
Observation sequence (1, 1, 2, 2), Probability is 0.01346
Observation sequence (1, 2, 0, 0), Probability is 0.0140700
Observation sequence (1, 2, 0, 1), Probability is 0.00983
Observation sequence (1, 2, 0, 2), Probability is 0.00962
Observation sequence (1, 2, 1, 0), Probability is 0.0118
Observation sequence (1, 2, 1, 1), Probability is 0.01226
Observation sequence (1, 2, 1, 2), Probability is 0.0133

Observation sequence (1, 2, 2, 0), Probability is 0.01218
Observation sequence (1, 2, 2, 1), Probability is 0.01376
Observation sequence (1, 2, 2, 2), Probability is 0.01519
Observation sequence (2, 0, 0, 0), Probability is 0.01893
Observation sequence (2, 0, 0, 1), Probability is 0.01234
Observation sequence (2, 0, 0, 2), Probability is 0.01176
Observation sequence (2, 0, 1, 0), Probability is 0.01037
Observation sequence (2, 0, 1, 1), Probability is 0.00956
Observation sequence (2, 0, 1, 2), Probability is 0.0101
Observation sequence (2, 0, 2, 0), Probability is 0.0092
Observation sequence (2, 0, 2, 1), Probability is 0.00964999
Observation sequence (2, 0, 2, 2), Probability is 0.01049
Observation sequence (2, 1, 0, 0), Probability is 0.01517000
Observation sequence (2, 1, 0, 1), Probability is 0.01046
Observation sequence (2, 1, 0, 2), Probability is 0.01017
Observation sequence (2, 1, 1, 0), Probability is 0.01185999
Observation sequence (2, 1, 1, 1), Probability is 0.01213
Observation sequence (2, 1, 1, 2), Probability is 0.01312
Observation sequence (2, 1, 2, 0), Probability is 0.01202
Observation sequence (2, 1, 2, 1), Probability is 0.01345
Observation sequence (2, 1, 2, 2), Probability is 0.01482
Observation sequence (2, 2, 0, 0), Probability is 0.01546
Observation sequence (2, 2, 0, 1), Probability is 0.01083
Observation sequence (2, 2, 0, 2), Probability is 0.0106
Observation sequence (2, 2, 1, 0), Probability is 0.01312
Observation sequence (2, 2, 1, 1), Probability is 0.01366
Observation sequence (2, 2, 1, 2), Probability is 0.01483
Observation sequence (2, 2, 2, 0), Probability is 0.013579
Observation sequence (2, 2, 2, 1), Probability is 0.0153599

Observation sequence (2, 2, 2, 2), Probability is 0.01695

Final sum is 0.9999907800000002 \approx 1.0

Problem – 2.10

a). Choose $N = 2$ hidden states. What do the hidden states represent?

a. How did Mr. Beato determine that the "new artist" referred to in the title of the video was AI-generated?

Beato used Logic Pro, which is an Apple version of AI track splitter. With old songs recorded in studio, it should be able to split out the vocals, guitars, drums and keyboards. If it's an AI generated tune, it should have a problem doing this and we will hear a ton of artifacts in different parts. We can hear some artifacts in the reverbs that show song that was recorded on analog tapes 55 years ago. If you have an AI song, they are full of artifacts. It's trained on low quality MP3s where they feed in entire mix. So, it can't separate out the parts.

b. What could be done to make it more difficult to detect AI generated music?

They need multitrack recordings so they can hear the voices and instruments. Even if you have multitrack, you don't have the effects that were used in the mix unless they were printed in the session, which is possible. But for old songs, this is not the case, you can hear how the system really struggles to separate out the guitar parts, and the keyboard parts. Some vocal parts have a little weird artifact in them, but guitars and keyboards are separated poorly.