Assignment II (ICSE Class 12 2019)

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3b) If $\sec^{-1} x = \csc^{-1} y$, show that $\frac{1}{x^2} + \frac{1}{y^2} = 1$

Solution: Given $\sec^{-1} x = \csc^{-1} y$

The range of $\sec^{-1} x$ is $[0, \pi] - \{\frac{\pi}{2}\}$

The range of $\csc^{-1} y$ is $[-\frac{\pi}{2}, \frac{\pi}{2}] - \{0\}$

Let

$$\begin{aligned} \sec^{-1} x = \csc^{-1} y &= \theta \\ \implies x &= \sec\theta \quad (1) \\ \implies y &= \csc\theta \quad (2) \end{aligned}$$

From all the above statements we can conclude that range of θ is $\left(0, \frac{\pi}{2}\right)$.

Then

$$\frac{1}{x^2} + \frac{1}{y^2} = \frac{1}{\sec^2\theta} + \frac{1}{\csc^2\theta} \quad (3)$$

As

$$\begin{aligned} \frac{1}{\sec\theta} &= \cos\theta \\ \frac{1}{\csc\theta} &= \sin\theta \\ \implies \frac{1}{x^2} + \frac{1}{y^2} &= \cos^2\theta + \sin^2\theta \quad (4) \\ &= 1 \end{aligned}$$

Hence proved.

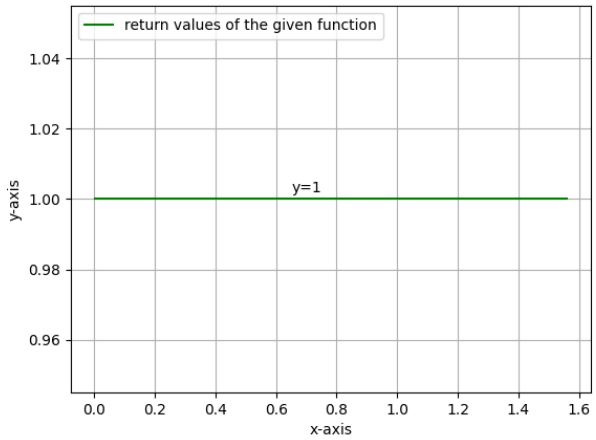

Fig. 1. proof for the condition