

# Assignment II (ICSE Class 12 2019)

Kola Akshitha - AI21BTECH11017

**3b)** If  $\sec^{-1} x = \csc^{-1} y$ , show that  $\frac{1}{x^2} + \frac{1}{y^2} = 1$

**Solution:** Given  $\sec^{-1} x = \csc^{-1} y$

The range of  $\sec^{-1} x$  is  $[0, \pi] - \{\frac{\pi}{2}\}$

The range of  $\csc^{-1} y$  is  $[-\frac{\pi}{2}, \frac{\pi}{2}] - \{0\}$

Let

$$\sec^{-1} x = \csc^{-1} y = \theta$$

$$\implies x = \sec \theta \quad (1)$$

$$\implies y = \csc \theta \quad (2)$$

From all the above statements we can conclude that range of  $\theta$  is  $(0, \frac{\pi}{2})$ .

Then

$$\frac{1}{x^2} + \frac{1}{y^2} = \frac{1}{\sec^2 \theta} + \frac{1}{\csc^2 \theta} \quad (3)$$

As

$$\frac{1}{\sec \theta} = \cos \theta$$

$$\frac{1}{\csc \theta} = \sin \theta$$

$$\implies \frac{1}{x^2} + \frac{1}{y^2} = \cos^2 \theta + \sin^2 \theta \quad (4)$$

$$= 1$$

Hence proved.