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Assignment II (ICSE Class 12 2019)

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3b) If
$$\sec^{-1}x=\csc^{-1}y$$
, show that $\frac{1}{x^2}+\frac{1}{y^2}=1$ **Solution:** Given $\sec^{-1}x=\csc^{-1}y$ The range of $\sec^{-1}x$ is $[0,\pi]-\{\frac{\pi}{2}\}$ The range of $\csc^{-1}y$ is $[-\frac{\pi}{2},\frac{\pi}{2}]-\{0\}$ Let

$$\sec^{-1} x = \csc^{-1} y = \theta$$

$$\implies x = \sec \theta \tag{1}$$

$$\implies y = \csc \theta \tag{2}$$

From all the above statements we can conclude that range of θ is $\left(0, \frac{\pi}{2}\right)$.

Then

$$\frac{1}{x^2} + \frac{1}{y^2} = \frac{1}{\sec^2 \theta} + \frac{1}{\csc^2 \theta} \tag{3}$$

As

$$\frac{1}{\sec \theta} = \cos \theta$$

$$\frac{1}{\csc \theta} = \sin \theta$$

$$\implies \frac{1}{x^2} + \frac{1}{y^2} = \cos^2 \theta + \sin^2 \theta$$

$$= 1$$
(4)

Hence proved.

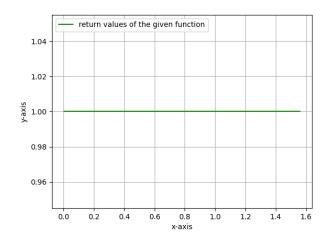


Fig. 1. proof for the condition