Assignment 7 : Papoulis Textbook

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Outline

Question

Chapter 8 Example 8.28

We are given an N (η, 1) random variable x and we wish to test the simple hypothesis $\eta = \eta_0$ against $\eta \neq \eta_0$

Solution

In this problem $\eta_{m0} = \eta_0$ and

$$f(X,\eta) = \frac{1}{\sqrt{(2\pi)^n}} exp\left\{-\frac{1}{2}\sum(x_i-\eta)^2\right\} \tag{1}$$

The above expression is maximum if the sum

$$\sum(x_i-\eta)^2 = \sum(x_i-\bar{x})^2 + n(\bar{x}-\eta)^2 \tag{2}$$

is minimum, that is $\eta = \bar{x}$. Hence $\eta_m = \bar{x}$.

$$\lambda = \frac{exp\left\{-\frac{1}{2}\sum(x_i-\eta)^2\right\}}{exp\left\{-\frac{1}{2}\sum(x_i-\bar{x})^2\right\}} = exp\left\{-\frac{n}{2}(\bar{x}-\eta_0)^2\right\} \tag{3}$$

$$\mathbf{w} = -2log\lambda = n(\bar{x} - \eta_0)^2 = (\bar{x} - \eta_0)^2 = \frac{(\bar{x} - \eta_0)^2}{\frac{1}{\sqrt{n}}} \tag{4}$$

The right side is a random variable with $(\chi)^2(1)$ distribution. Hence the random variable **w** has a $(\chi)^2(m - m_0)$ distribution not only asymptotically, but for any n.