Assignment 1

AI21BTECH11017

3.6.(c) Determine the inverse Z transform of the following.

$$X[z] = \frac{1 - \frac{1}{2}z^{-1}}{1 + \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}} \qquad |z| > \frac{1}{2} \tag{1}$$

Solution: Given

$$X[z] = \frac{1 - \frac{1}{2}z^{-1}}{1 + \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}} \qquad |z| > \frac{1}{2} \tag{2}$$

$$\implies X[z] = \frac{4z(2z-1)}{8z^2 + 6z + 1} \tag{3}$$

$$\implies \frac{X[z]}{4z} = \frac{(2z-1)}{8z^2 + 6z + 1} \tag{4}$$

$$\implies \frac{X[z]}{4z} = \frac{-3}{4z+1} + \frac{4}{2z+1} \tag{5}$$

$$\implies X[z] = \frac{-3}{1 + \frac{1}{4}z^{-1}} + \frac{4}{1 + \frac{1}{2}z^{-1}} \tag{6}$$

$$\tag{7}$$

$$= -3\sum_{n=0}^{\infty}\left(\frac{-1}{4}\right)^n z^{-n} + 4\sum_{n=0}^{\infty}\left(\frac{-1}{2}\right)^n z^{-n} \qquad |z| > \frac{1}{2} \tag{8}$$

$$= \sum_{n=0}^{\infty}\left(4\left(\frac{-1}{2}\right)^n - 3\left(\frac{-1}{4}\right)^n\right)z^{-n} \qquad |z| > \frac{1}{2} \tag{9}$$

Hence inverse Z transform is

$$x(n) = \begin{cases} 4\left(\frac{-1}{2}\right)^n - 3\left(\frac{-1}{4}\right)^n & n \geq 0 \\ 0 & n < 0 \end{cases} \tag{10}$$