TUTORIAL - 6

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Q→ What do you mean by Minimum Spanning tree? What are the applications of MST?

Ans → Minimum Spanning tree is a subset of edges of a connected edge-weight undirected graph that connects all the vertices together without any cycle & with minimum possible edge weight.

APPLICATION:

i) Consider n stations are to be linked using a communication network and lying of communication link between any two stations involves a cost. The ideal solution would be to extract a subgraph termed as minimum cost spanning tree.

ii) Designing LAN

iii) Suppose you want to construct highways or railroads spanning several cities, then we can use concept of MST.

2. Analyze time and space complexity of Prim, Kruskal, Dijakstra and Bellman ford Algorithm.

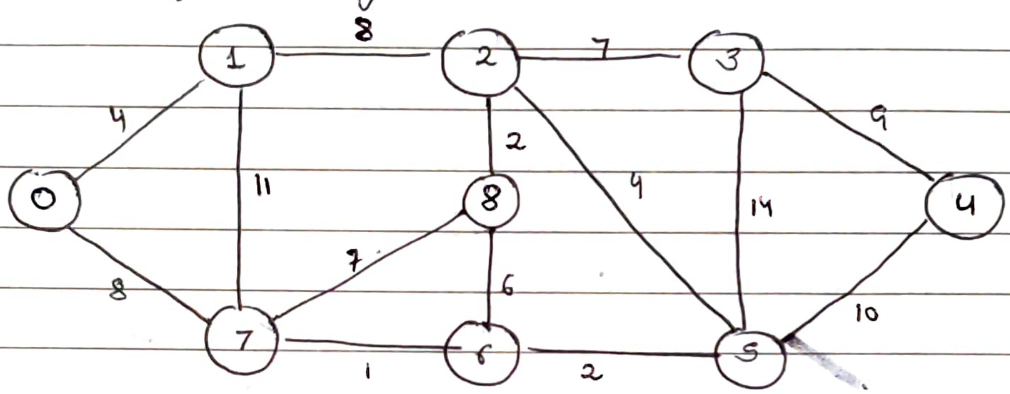
Ans →

T.C of Prim's algorithm : $O(|E| \log |V|)$

S.C of Prim's algorithm : $O(|V|)$

$\Rightarrow$ T.C of Kruskal's Algorithm : $O(E \log E)$
$\Rightarrow$ S.C of Kruskal's Algorithm : $O(V)$
$\Rightarrow$ T.C of Dijkstra's Algorithm : $O(V^2)$
$\Rightarrow$ S.C of Dijkstra's Algorithm : $O(V^2)$
$\Rightarrow$ TC of Bellman Ford's Algorithm : $O(VE)$
$\Rightarrow$ SC of Bellman Ford's Algorithm : $O(E)$

3. Apply Krushkal's and Prism's Algorithm on given graph to compute MST & its weight



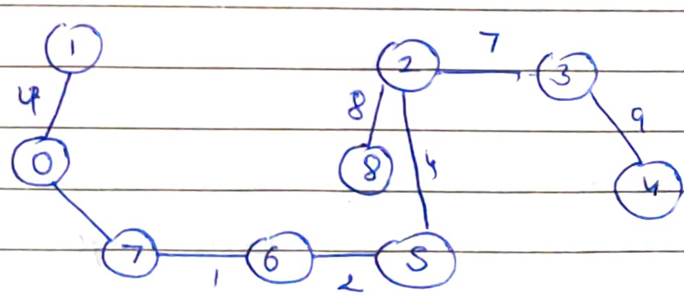
Ans $\rightarrow$

Krushkal's algorithm

O	V	M	
6	7	1	✓
5	6	2	✓
2	8	2	✓
0	1	4	✓
2	5	4	✓
6	8	6	✗
2	3	7	✓
7	8	7	
0	7	8	✗
1	2	8	✓ ✗
4	3	9	✓
4	5	10	✗
1	7	11	✗
3	5	14	✗

Prism Algorithm

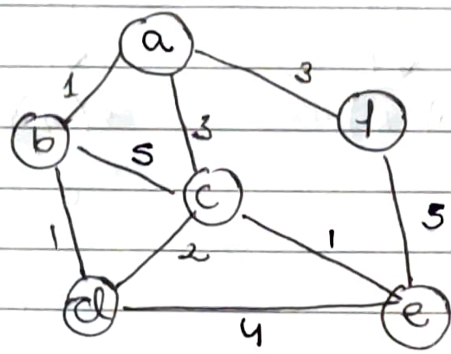
Weight = 4+8+2+4+2+7+9+3
= 37



Weight : 1+2+2+4+4+7+8+9
= 37

Q→ Given a directed weighted graph. You are also given the shortest path from a source vertex 's' to a destination vertex 't'. Does the shortest path remain same in following case:

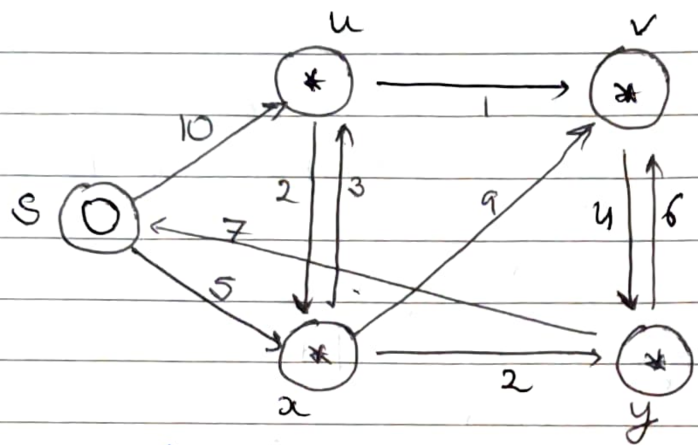
i) If weight of every edge is increased by 10 units.
ii) If weight of every edge is multiplied by 10 units.



Ans → i) The shortest path may change. The reason is that there may be different no. of edges in different paths from 's' to 't'. For eg: Let the shortest path of weights 15 and has edges 5. Let there are another path with 2 edges and total weight 25.
The weight of shortest path is increased by 5*10 and becomes 15+50. Weight of other path is increased by 2*10 and becomes 26+20. So, the shortest path changes to other path with weight as 45.

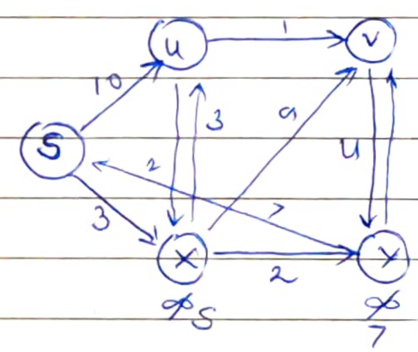
ii) If we multiply all edges weight by 10, the shortest path doesnot change. The reason is that weights of all paths from 's' to 't' gets multiplied by same unit. The number of edges of path doesn't matter.

Qs. Apply Dijkstra & Bellmann Ford algorithm on graph given right side to compute shortest path to all nodes from node S.

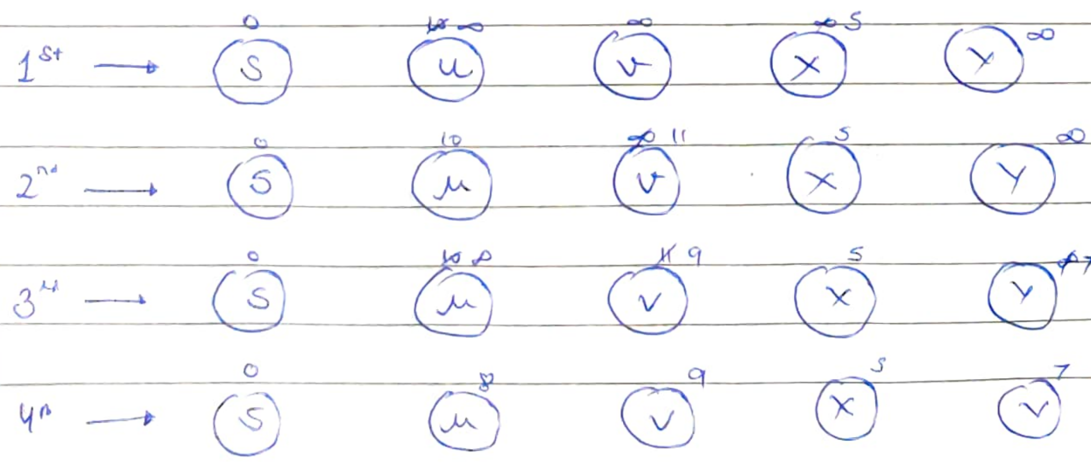


Ans → Dijastra's Algorithm :-

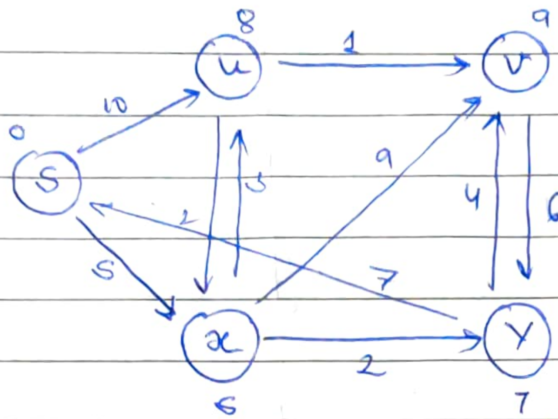
NODE	SHORTEST DIST FROM SOURCE NODE
u	8
x	5
v	9
y	7



Bellman Ford Algorithm →



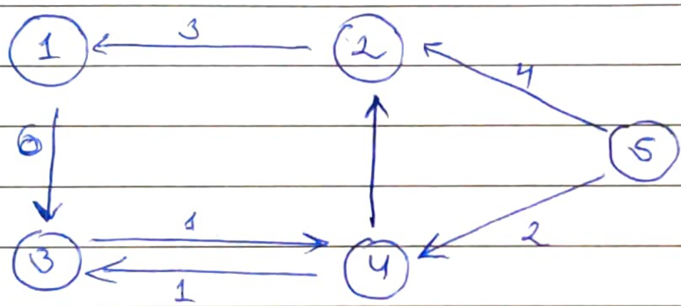
graph does not have -ve cycle.



→ Final graph

6) Apply all pair shortest path algorithm. Floyed Warshall on below mentioned graph. Also analyse space & time complexity of it.

Ans →



$$
\begin{array}{c}
\\ 1 \\ 2 \\ 3 \\ 4 \\ 5
\end{array}
\begin{array}{c}
\begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\
\begin{bmatrix}
0 & \infty & 6 & 3 & \infty \\
2 & 0 & \infty & \infty & \infty \\
\infty & \infty & 0 & 2 & \infty \\
\infty & 1 & 1 & 0 & \infty \\
\infty & 4 & \infty & 2 & 0
\end{bmatrix}
\end{array}
$$

$$
\begin{array}{c}
\\ 1 \\ 2 \\ 3 \\ 4 \\ 5
\end{array}
\begin{array}{c}
\begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\
\begin{bmatrix}
0 & \infty & 6 & 3 & \infty \\
2 & 0 & 8 & 5 & \infty \\
\infty & \infty & 0 & 2 & \infty \\
\infty & 1 & 1 & 0 & \infty \\
\infty & 4 & \infty & 2 & 0
\end{bmatrix}
\end{array}
$$

$$= \begin{array}{cc} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & \infty & 6 & 3 & \infty \\ 2 & 0 & 8 & 5 & \infty \\ \infty & \infty & 0 & 2 & \infty \\ 3 & 1 & 1 & 0 & \infty \\ 6 & 4 & 12 & 2 & 0 \end{bmatrix} \end{array}$$

$$= \begin{array}{cc} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & \infty & 6 & 3 & \infty \\ 2 & 0 & 8 & 5 & \infty \\ \infty & \infty & 0 & 2 & \infty \\ 3 & 1 & 1 & 0 & \infty \\ 6 & 4 & 12 & 2 & 0 \end{bmatrix} \end{array}$$

Time complexity $\rightarrow O(|V|^3)$

Space complexity $\rightarrow O(|V|^2)$