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Design and Analysis of Algorithms

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SECTION : F

ROLL NO: 42

COURSE: B. Tech CSE

Tutorial-4

1. $T(n) = 3T(n/2) + n^2$

Ans - $a = 3, b = 2$

$n^{\log_b a} = n^{\log_2 3}$

Comparing $n^{\log_2 3}$ and n^2

$n^{\log_2 3} < n^2$ (Case 3)

∴ according to master's Theorem,

$T(n) = O(n^2)$

2. $T(n) = 4T(n/2) + n^2$

$a = 4, b = 2$

$n^{\log_b a} = n^{\log_2 4} = n^2 = f(n)$ (Case 2)

∴ according to master's theorem $T(n) = O(n^2 \log n)$

3. $T(n) = T(n/2) + 2^n$

$a = 1, b = 2$

$n^{\log_2 1} = n^0 = 1$

$1 < 2^n$ (Case 3)

∴ According to master's theorem $T(n) = O(2^n)$

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4. $T(n) = 2^n T(n/2) + n^n$

∴ Master's theorem is not applicable as a is function.

5. $T(n) = 16T(n/4) + n$

$a = 16$, $b = 4$, $f(n) = n$

$n^{\log_b a} = n^{\log_4 16} = n^2$

$n^2 > f(n)$ (Case 1)

$\therefore T(n) = \Theta(n^2)$

6. $T(n) = 2T(n/2) + n \log n$

$a = 2$, $b = 2$, $f(n) = n \log n$

$n^{\log_b a} = n^{\log_2 2} = n$

Now, $f(n) > n$

∴ According to master's $T(n) = \Theta(n \log n)$

7. $T(n) = 2T(n/2) + \frac{n}{\log n}$

$a = 2, b = 2$, $f(n) = \frac{n}{\log n}$

$n^{\log_b a} = n^{\log_2 2} = n$

$n > f(n)$

∴ According to master's theorem $T(n) = \Theta(n)$

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8. $T(n) = 2T\left(\frac{n}{4}\right) + n^{0.51}$

$a = 2, \; b = 4, \; f(n) = n^{0.51}$

$n^{\log_b a} = n^{\log_4 2} = n^{0.5}$

$n^{0.5} < f(n)$

∴ According to master's theorem $T(n) = \Theta(n^{0.5})$

9. $T(n) = 0.5T\left(\frac{n}{2}\right) + \frac{1}{n}$

∴ Master's not applicable as $a < 1$

10. $T(n) = 16T\left(\frac{n}{4}\right) + n!$

$a = 16, \; b = 4, \; f(n) = n!$

$n^{\log_b a} = n^{\log_4 16} = n^2$

$n^2 < n!$

∴ According to master's method, $T(n) = \Theta(n!)$.

11. $T(n) = 4T\left(\frac{n}{2}\right) + \log n$

Ans – $a = 4, \; b = 2, \; f(n) = \log n$

$n^{\log_b a} = n^{\log_2 4} = n^2$

$n^2 > f(n)$

∴ According to master's, $T(n) = \Theta(n^2)$

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12. $T(n) = Sqrt(n) + (n/2) + \log n$

∴ Master's Not applicable as a is not constant

13. $T(n) = 3T(n/2) + n$

$a = 3, b = 2 \quad , f(n) = n$

$n^{\log_b a} = n^{\log_2 3} = n^{1.58}$

$n^{1.58} > f(n)$

∴ According to master's Theorem, $T(n) = O(n^{\log_2 3})$

14. $T(n) = 3T(n/3) + \sqrt{n}$

Ans ⇒ $a = 3, b = 3 \quad , f(n) = \sqrt{n}$

$n^{\log_b a} = n^{\log_3 3} = n$

$n > \sqrt{n}$

∴ According to master's theorem, $T(n) = \Theta(n)$

15. $T(n) = 4T(n/2) + Cn$

Ans ⇒ $a = 4 \quad , b = 2 \quad , f(n) = C * n$

$n^{\log_b a} = n^{\log_2 4} = n^2$

$n^2 > C \cdot n$

∴ According to master's theorem, $T(n) = \Theta(n^2)$

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16. $T(n) = 3T(n/4) + n\log n$

Ans $a = 3, b = 4, f(n) = n\log n$

$n^{\log_b a} = n^{\log_4 3} = n^{0.79}$

$n^{0.79} < n\log n$

∴ According to master's theorem, $T(n) = \Theta(n\log n)$

17. $T(n) = 3T(n/3) + n/2$

Ans → $a = 3, b = 3, f(n) = n/2$

$n^{\log_b a} = n^{\log_3 3} = n$

$\Theta(n) = \Theta(n/2)$

∴ According to master's theorem

$T(n) = \Theta(n\log n)$

18. $T(n) = 6T(n/3) + n^2\log n$

Ans → $a = 6, b = 3, f(n) = n^2\log n$

$n^{\log_b a} = n^{\log_3 6} \cdot n^{1.63}$

$n^{1.63} < n^2\log n$

∴ According to master's theorem $T(n) = \Theta(n^2\log n)$

19. $T(n) = 4T(n/2) + n/\log n$.

Ans $a = 4$, $b = 2$, $f(n) = n/\log n$

$n^{\log_b a} = n^{\log_2 4} = n^2$

$n^2 > n/\log n$

∴ According to Master's theorem $T(n) = \Theta(n^2)$

20. $T(n) = 64T(n/8) - n^2 \log n$

Ans Master's theorem is not applicable as $f(n)$ is not increasing function.

21. $T(n) = 7T(n/3) + n^2$

Ans

$a = 7$, $b = 3$, $f(n) = n^2$

$n^{\log_b a} = n^{\log_3 7} = n^{1.7}$

$n^{1.7} < n^2$

∴ According to master's, $T(n) = \Theta(n^2)$

22. $T(n) = T(n/2) + n(2 - \cos n)$

Ans Master's method isn't applicable since regular condition is isolated in Case 3.