11.4.24

Logics and Proofs

Logic is the discipline that deals with the method of reasoning. The aim of logic is to provide rules by which we can determine whether the particular reasoning is valids.

Proposition:

Proposition is a declarative statement that is either true or false but not both. The sentences which are Interogative, imperative and exclamatory are not proposition.

If proposition is true, then the truth value is one. If the proposition is false. then the truth value is ~~false~~ zero.

classify which of the following statement are propation and determine their truth value:

1. water boils at 100°c ✓ 1
2. 2 + 5 = 8 ✓ 0
3. Delhi is the capital of India ✓ 1
4. Get out of the class ×
5. who is the Rep of the class? ×
6. $x - y = k$ ×

Proposition which don't contain any of the logical operator or connectives are called atomic or primary or primitive proposition. The statements which can be constructed by combining one or more atomic statements using connectives are called molecular or ~~atoms~~ compound proposition.

Negation (not p):

If p is a proposition, $\neg p$ or $\sim p$ or p' or $\bar{p}$

p is true, negation p is false

p is false, negation p is true

If there are n proposition,

In the 1st column,

$\frac{1}{2} \times 2^n \rightarrow T$ $\quad$ $\frac{1}{2} \times 2^n \rightarrow F$

In the 2nd column,

$\frac{1}{4} \times 2^n \rightarrow T$ $\quad$ $\frac{1}{4} \times 2^n \rightarrow F$ and so on

conjunction: [∪, ∨]

$p \vee q$

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Disjunction: [∩, ∧]

$p \wedge q$

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

* p : Rama killed Ravana

q : Rama is the king of Ayothi

$p \vee q$: Rama killed Ravana or Rama is the king of Ayothi

$p \wedge q$: Rama killed Ravana and Rama is the king of Ayothi

$\sim p$: Rama did not kill Ravana

$\sim q$: Rama is not the king of Ayothi

2.8.24 Conditional Statement :

$p \rightarrow q$ if p, then q

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Biconditional Statement :

$p \leftrightarrow q$ if p, then q
if q, then p

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Precedence of connectives :

- negation ($\sim$) 1
- disjunction ($\wedge$) 2
- conjunction ($\vee$) 3
- conditional ($\rightarrow$) 4
- Biconditional ($\leftrightarrow$) 5

1. Truth Table for $(p \vee \sim q) \rightarrow (p \wedge q)$

p	q	$\sim q$	$p \wedge q$	$p \vee \sim q$	$(p \vee \sim q) \rightarrow (p \wedge q)$
T	T	F	T	T	T
T	F	T	F	T	F
F	T	F	F	F	T
F	F	T	F	T	F

2. Truth Table : $(p \leftrightarrow q) \leftrightarrow (p \wedge q) \vee (\sim p \wedge \sim q)$

p	q	$\sim p$	$\sim q$	$p \wedge q$	$\sim p \wedge \sim q$	$(p \wedge q) \vee (\sim p \wedge \sim q)$
T	T	F	F	T	F	T
T	F	F	T	F	F	F
F	T	T	F	F	F	F
F	F	T	T	F	T	T

$(p \leftrightarrow q)$	$(p \leftrightarrow q) \leftrightarrow (p \wedge q) \vee (\sim p \wedge \sim q)$
T	T
F	T
F	T
T	T

3. Truth Table : $(p \vee q) \rightarrow (p \wedge q)$

p	q	$p \wedge q$	$p \vee q$	$(p \vee q) \rightarrow (p \wedge q)$
T	T	T	T	T
T	F	F	T	F
F	T	F	T	F
F	F	F	F	T

* A compound proposition is always true, no matter what the truth value of the proposition that occurs in known as ~~tauto~~ tautology.

* A compound proposition that is always false is called contradiction.

* A compound proposition which is neither tautology nor contradiction is known as contingency.

Logical equivalence:

compound propositions that have the same truth value in all possible cases are called logical equivalence.

The compound propositions p and q are called equivalent if p is tautology.

It denotes $(p \equiv q)$ in same cases we use $\Leftrightarrow$.

4. $p \to q$ $\sim p \vee q$. PT logical equivalence

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

p	q	$\sim p$	$\sim p \vee q$
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

$$(p \to q) \equiv (\sim p \vee q)$$

Hence Proved

* This property can be known as conditional disjunction equivalence

$$(p \to q) \equiv (\sim p \vee q)$$

5. $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ (Prove) are logically

p	q	r	$q \wedge r$	$p \vee (q \wedge r)$
T	T	T	T	T
T	T	F	F	T
T	F	T	F	T
T	F	F	F	T
F	T	T	T	T
F	T	F	F	F
F	F	T	F	F
F	F	F	F	F

p	q	r	$p \vee q$	$p \vee r$	$(p \vee q) \wedge (p \vee r)$
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	T	T	T
T	F	F	T	T	T
F	T	T	T	T	T
F	T	F	T	F	F
F	F	T	F	T	F
F	F	F	F	F	F

$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$$

* $p \rightarrow q$

- p only if q
- q whenever p
- q if p
- q is necessary for p
- necessary condition for p is q
- q follows from p
- q when p
- p is sufficient for q or a sufficient condition for p is q
- p implies q
- q unless ~q

* ~ tautology = contradiction

* ~ contradiction = tautology

Laws of logic:

* Demorgan's law:

$$\sim(p \wedge q) \equiv \sim p \vee \sim q$$

$$\sim(p \vee q) \equiv \sim p \wedge \sim q$$

* Identity law:

$$p \wedge T \equiv p$$

$$p \vee F \equiv p$$

* commutative law:

$$p \vee q \equiv q \vee p$$

$$p \wedge q \equiv q \wedge p$$

* Domination law:

$$p \vee T \equiv T$$

$$p \wedge F \equiv F$$

* Idempotent law:

$$p \vee p \equiv p$$

$$p \wedge p \equiv p$$

* Associative law:

$$(p \vee q) \vee r \equiv p \vee (q \vee r)$$

* Distributive law:

$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$$

$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$$

* Absorption law:

$$p \vee (p \wedge q) \equiv p$$

$$p \wedge (p \vee q) \equiv p$$

* Double negation law:

$$\sim(\sim p) \equiv p$$

* Negation law:

$$p \wedge \sim p \equiv F$$

$$p \vee \sim p \equiv T$$

8. Show that $\sim(p \vee (\sim p \wedge q))$ and $\sim p \wedge \sim q$ are logically equivalence

$\sim(p \vee (\sim p \wedge q)) \equiv \sim p \wedge \sim(\sim p \wedge q)$ [demorgan's law]

$\equiv \sim p \wedge (p \vee \sim q)$ (demorgan's law)

$\equiv (\sim p \wedge p) \vee (\sim p \wedge \sim q)$ (distributive law)

$\equiv F \vee (\sim p \wedge \sim q)$

$\equiv (\sim p \wedge \sim q) \vee F$ (commutative law)

$\equiv \sim p \wedge \sim q$ (Identity law)

9. Show that $(p \vee q) \wedge \sim(\sim p \wedge q) \equiv p$

$(p \vee q) \wedge \sim(\sim p \wedge q) \equiv (p \vee q) \wedge (p \vee \sim q)$ [demorgan]

$\equiv p \vee (q \wedge \sim q)$ [Negation]

$\equiv p \vee F$ [Identity]

$\equiv p$

Hence proved.

10. Show that $\sim(\sim(p \vee q) \wedge r) \vee \sim q) \equiv q \wedge r$

$\sim(\sim(p \vee q) \wedge r) \vee \sim q) \equiv \sim((\sim p \wedge \sim q) \wedge r) \vee \sim q)$

$\equiv \sim((\sim p \wedge \sim q) \wedge r) \wedge q$

$\equiv (\sim(\sim p \wedge \sim q) \vee \sim r) \wedge q$

$\equiv ((p \vee q) \vee \sim r) \wedge q$

demorgan's

$\equiv ((p \vee \sim r) \vee q) \wedge q$

$\equiv ((p \vee \sim r) \wedge q) \vee (q \wedge q)$

$\equiv ((p \vee \sim r) \wedge q) \vee q$

$\equiv ((p \wedge q) \vee (p \sim r \wedge q)) \vee q$

Logical equivalence is involving conditional equivalence:

* $p \to q \equiv \sim p \vee q$
* $p \to q \equiv \sim q \to \sim p$
* $p \vee q \equiv \sim p \to q$
* $p \wedge q \equiv \sim (p \to \sim q)$
* $\sim (p \to q) \equiv p \wedge \sim q$
* $(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$
* $(p \to r) \wedge (q \to r) \equiv (p \vee q) \to r$
* $(p \to q) \vee (p \to r) \equiv p \to (q \vee r)$
* $(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$
* $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$
* $p \leftrightarrow q \equiv \sim p \leftrightarrow \sim q$
* $p \leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$
* $\sim (p \leftrightarrow q) \equiv p \leftrightarrow \sim q$

11. Prove that $(p \wedge q) \to (p \vee q)$ is a tautology

$\equiv \sim (p \wedge q) \vee (p \vee q)$ [conditional equivalence]

$\equiv (\sim p \vee \sim q) \vee (p \vee q)$ [de morgan's]

$\equiv (\sim p \vee p) \vee (\sim q \vee q)$ [asso]

$\equiv T \vee T$ [negation]

$\equiv T$

12. $\sim(p\to q)$ and $p\wedge\sim q$ are equivalence

$\sim(p\to q) \equiv \sim(\sim p\vee q)$ [cond. equi]

$\equiv p\wedge\sim q$ [demorgan's]

Hence proved.

13. Show that $p\to(q\vee r) \equiv \sim r\to(p\to q) \equiv (p\wedge\sim q)\to r$

$p\to(q\vee r) \equiv \sim p\vee(q\vee r)$ [cond. equivalence]

— ①

$\sim r\to(p\to q) \equiv r\vee(p\to q)$

$\equiv r\vee(\sim p\vee q)$

$\equiv \sim p\vee(q\vee r)$ — 2

$(p\wedge\sim q)\to r \equiv \sim(p\wedge\sim q)\vee r$

$\equiv (\sim p\vee q)\vee r$

$\equiv \sim p\vee(q\vee r)$ — ③

① ≡ ② ≡ ③

Hence proved.

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14. ST $p \to (q \to r) \equiv (p \wedge q) \to r$

$p \to (q \to r) \equiv \sim p \vee (q \to r)$ [conditional equivalence]

$\equiv \sim p \vee (\sim q \vee r)$ [conditional equivalence]

$\equiv (\sim p \vee \sim q) \vee r$ [association law]

$\equiv \sim (p \wedge q) \vee r$ [de morgan's law]

$\equiv (p \wedge q) \to r$ [conditional equivalence]

15. PT $(\sim p \wedge (\sim q \wedge r)) \vee (q \wedge r) \vee (p \wedge r) \equiv r$

$(\sim p \wedge (\sim q \wedge r)) \vee (q \wedge r) \vee (p \wedge r) \equiv ((\sim p \wedge \sim q) \wedge r) \vee (q \wedge r) \vee (p \wedge r)$
[associative law]

$\equiv (\sim (p \vee q) \wedge r) \vee ((p \vee q) \wedge r)$ [distributive law]

$\equiv (\sim (p \vee q) \vee (p \vee q)) \wedge r$ [distributive law]

$\equiv T \wedge r$ [Identity law]

$\equiv r$

16. PT $\underbrace{(p \vee q) \wedge \sim (\sim p \wedge (\sim q \vee \sim r))}_{\mathrm{I}} \vee \underbrace{(\sim p \wedge \sim q) \vee (\sim p \wedge \sim r)}_{\mathrm{II}}$

$\sim p \wedge (\sim q \vee \sim r)$

$\sim p \wedge \sim (q \vee r)$ $\quad$ $\sim p \wedge \sim (q \wedge r)$

II $\equiv \sim p \wedge (\sim q \vee \sim r)$ [distributive law] $\quad$ $\sim (p \vee (q \wedge r))$

$\equiv (\sim p \wedge \sim q) \vee \sim r$ [associative law]

$\equiv \sim (p \vee q) \vee \sim r$ [de morgan's law]

$\equiv \sim ((p \vee q) \wedge r)$ [de morgan's law]

$\equiv \sim ($

I $\equiv (p \vee q) \wedge (p \vee \sim (\sim q \vee \sim r))$ [de morgan's law]

$\equiv (p \vee q) \wedge (p \vee (q \wedge r))$ [de morgan's law]

$\equiv \cancel{(p \wedge (q \wedge r)) \vee (p}$ $p \vee (q \wedge (q \wedge r))$ [distributive law)

$\equiv p \vee ((q \wedge q) \wedge r)$ [associative law]

$\equiv p \vee (q \wedge r)$ [idempotent law]

$I \vee II \equiv \sim(p \vee (q \wedge r)) \vee (p \vee (q \wedge r))$ [negation law]

$\equiv T$

17. $p \to (q \vee r) \equiv (p \to q) \vee (p \to r)$

$p \to (q \vee r) \equiv \sim p \vee (q \vee r)$ [cond. equi.]

$(p \to q) \vee (p \to r) \equiv (\sim p \vee q) \vee (\sim p \vee r)$ [cond. equi.]

$\equiv$

Logical implication or Tautological implication:

A statement formula A logically implies another statement formula B if and only if $A \to B$ is a tautology. therefore $A \Rightarrow B$.

So, if $A \Rightarrow B$, then A is called antecedent and B is called consequent. Further $A \Rightarrow B$ guarantees that B has the Truth value T, where A has the truth value T.

Therefore, inorder to show any of the given implication it is sufficient to show that the assignment of truth value T to the antecedent of the given condition leads to the truth value T for the consequent.

18. $p \to (q \to r) \Rightarrow (p \to q) \to (p \to r)$

p	q	r	$q \to r$	[A] $p \to (q \to r)$	$(p \to q)$	$(p \to r)$	[B] $(p \to q) \to (p \to r)$
T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	F
T	F	T	T	T	F	T	T
T	F	F	T	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	F	T	T	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T			

$A \to B$
T
T
T
T
T
T
T
T

$A \to B$ is a tautology

$\therefore A \Rightarrow B$.

19. P.T $(p \to q) \wedge \sim q \Rightarrow \sim p$

Antecedent : $(p \to q) \wedge \sim q$

consequent : $\sim p$

Assume that antecedent has the truth value T, $\sim q$ and $p \to q$ both are true,

Truth value of q is F and truth value of p is also F, then consequent $\sim p$ is true.

20. $p \wedge (p \to q) \Rightarrow q$

Antecedent : $p \wedge (p \to q)$

consequent : q

Assume the antecedent $p \wedge (p \to q)$ is true, then

p and $p \to q$ both are true. If then

truth value of p is true and truth

value of q is true.

consequent : q is T

21. P.T $(p \to q) \wedge \sim q \Rightarrow \sim p$

To show:

$((p \to q) \wedge \sim q) \longrightarrow \sim p \equiv T$

$\equiv ((\sim p \vee q) \wedge \sim q) \to \sim p$

$\equiv \sim((\sim p \vee q) \wedge \sim q) \vee \sim p$

$\equiv (\sim(\sim p \vee q) \vee q) \vee \sim p$

$\equiv ((p \wedge \sim q) \vee q) \vee \sim p$

$\equiv ((p \vee q) \wedge (\sim q \vee q)) \vee \sim p$

$\equiv ((p \vee q) \wedge T) \vee \sim p$

$\equiv ((p \wedge T) \vee (q \wedge T)) \vee \sim p$

$\equiv (p \vee q) \vee \sim p$

$\equiv (p \vee \sim p) \vee q$

$\equiv T \vee q$

$\equiv T$

22. $(p \vee q) \wedge (p \to r) \wedge (q \to r) \Rightarrow r$

To show:

$$((p \vee q) \wedge (p \to r) \wedge (q \to r)) \to r$$

$$\equiv ((p \vee q) \wedge (\sim p \vee r) \wedge (\sim q \vee r)) \to r$$

$$\equiv \sim((p \vee q) \wedge (\sim p \vee r) \wedge (\sim q \vee r)) \vee r$$

$$\equiv \sim((p \vee q) \wedge ((\sim p \wedge \sim q) \vee r)) \vee r$$

$$\equiv \sim((p \vee q) \wedge (\sim (p \vee q) \vee r)) \vee r$$

$$\equiv (\sim(p \vee q) \vee \sim(\sim(p \vee q) \vee r)) \vee r$$

$$\equiv (\sim(p \vee q) \vee ((p \vee q) \wedge \sim r)) \vee r$$

$$\equiv (F \wedge \sim r) \vee r$$

$$\equiv (F \vee r) \wedge (r \vee \sim r)$$

$$\equiv r \wedge T$$

$$\equiv T$$

23. $(p \to (q \to s)) \wedge ((\sim r \vee p) \wedge q) \Rightarrow (r \to s)$

p	q	r	s	$q \to s$	(a) $p \to (q \to s)$	$\sim r$	$(\sim r \vee p)$	(b) $(\sim r \vee p) \wedge q$	$a \wedge b$
T	T	T	T	T	T	F	T	T	T
T	T	T	F	F	F	F	T	T	F
T	T	F	T	T	T	T	T	T	T
T	T	F	F	F	F	T	T	T	F
T	F	T	T	T	T	F	T	F	F
T	F	T	F	T	T	F	T	F	F
T	F	F	T	T	T	T	T	F	F
T	F	F	F	T	T	T	T	F	F
F	T	T	T	T	T	F	F	F	F
F	T	T	F	F	T	F	F	F	F
F	T	F	T	T	T	T	T	T	T
F	T	F	F	F	T	T	T	T	T
F	F	T	T	T	T	F	F	F	F
F	F	T	F	T	T	F	F	F	F
F	F	F	T	T	T	T	T	F	F
F	F	F	F	T	T	T	T	F	F

$r \to s$	$(a \wedge b) \to (r \to s)$
T	T
F	T
T	T
T	T
T	T
F	T
T	T
T	T
T	T
F	T
T	T
T	T
T	T
F	T
T	T
T	T

Since $(a \wedge b) \to (r \to s)$ is a tautology,

$\therefore (a \wedge b) \Rightarrow (r \to s)$

Hence Proved.

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Duality law:

Two formulas A and A^* are set to be the duals of each other if either one can be obtained from other by replacing 'or' by 'and' and 'and' by 'or' and the connectives $\wedge$ and $\vee$ are ares also called duals of each other. If the formula A contains spiral character such as T and F, then the duals can be obtained by repalcing T by F and F by T.

24. Write the duals of the following

* $(p \wedge q) \vee r$: $(p \vee q) \wedge r$

* $(p \vee q) \wedge F$: $(p \wedge q) \vee T$

* $\sim(p \wedge q) \vee (p \wedge (q \vee \sim s))$: $\sim(p \wedge q) \wedge (p \vee (q \wedge \sim s))$

Theory of Inference:

Inference Theory is concerned with the inferreing of conclusion from certain hypothesis from basic assumption called premises by applying certain principle of reasoning called rules of inference.

When a conclusion derived from a set of premises by using rules of inferences. the process of such derivation is called formal proof.

The Rules are inference only means used to draw a conclusion from a set of premises in a finite sequence of steps called argument.

Any conclusion which is arrived at by following these rules is called a valid conclusion and argument is called a valid argument.

* Premise / hypothesis:
Premise is a statement which is assumed to be true

Formal Proof:

* Direct Proof
* Indirect Proof

* conditional conclusion.

Rules of Inference:

A set of Premises $H_1, H_2, \dots, H_n$ and a conclusion c are given, we assume that $H_1, H_2, \dots, H_n$ are all true, we want to conclude the conclusion c, (i.e) we want to prove the conclusion c is true.

* Rule P:

A premise may be introduced at any point in derivation

* Rule T:

A formula S may be introduced in a derivation, if S is tautology imply any one or more of the preceding formula in the derivation.

Rule of Inference	Tautology	Name of Rules
	$[p \wedge (p \to q)] \Rightarrow q$ $[(p \to q) \wedge p] \Rightarrow q$	Modus ponens [detachment rule]
	$(p \to q) \wedge \sim q \Rightarrow \sim p$ $(\sim q) \wedge (p \to q) \Rightarrow \sim p$	Modus Tollens [contra positive]
	$(p \to q) \wedge (q \to r) \Rightarrow (p \to r)$ $(p \to q) \wedge (q \to r) \Rightarrow p \to (q \wedge r)$	chain rule / hypothetical syllogism
	$p \wedge q \Rightarrow p$; $p \wedge q \Rightarrow q$	Simplification
	$p \Rightarrow p \vee q$; $q \Rightarrow p \vee q$	Addition

	$(p \vee q) \wedge \sim p \Rightarrow q$ $(p \vee q) \wedge \sim q \Rightarrow p$	disjunctive simplification
	$(p \rightarrow q) \equiv \sim q \rightarrow \sim p \equiv \sim p \vee q$	conditional equivalence
	$(p \leftrightarrow q) \equiv (p \rightarrow q) \wedge (q \rightarrow p) \equiv$ $(p \wedge q) \vee (\sim q \wedge \sim p)$	

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25. Demonstrate r is a valid inference from the premises $p \rightarrow q$, $q \rightarrow r$, p

$H_1 = p \rightarrow q \quad H_2 = q \rightarrow r \quad H_3 = p \quad C = r$

To prove: $H_1 \wedge H_2 \wedge H_3 \Rightarrow C$

$((p \rightarrow q) \wedge (q \rightarrow r) \wedge p) \Rightarrow r$

Step No.	Premise	Rule	Reason
1.	$p \rightarrow q$	P	given Premise
2.	p	P	given Premise
3.	$[(p \rightarrow q) \wedge p] \rightarrow q$	T	Modus Ponens (1) and (2)
4.	$q \rightarrow r$	P	given Premise
5.	$[(q \rightarrow r) \wedge q] \rightarrow r$	T	Modus Ponens (3) and (4)

26. conclude d from the premise $(a\to b)\wedge(a\to c)$, $\sim(b\wedge c)$, $d\vee a$,

To Prove:

$$[[(a\to b)\wedge(a\to c)]\wedge(\sim(b\wedge c))\wedge(d\vee a)]\Rightarrow r$$

Steps	Premise	Rule	Reason
1.	$(a\to b)\wedge(a\to c)$	P	given Premise
2.	$a\to(b\wedge c)$	T	chain rule
3.	$\sim(b\wedge c)$	P	given Premise
4.	$(a\to(b\wedge c))\wedge\sim(b\wedge c)$ $\Rightarrow\ \sim a$	T	modus Tollens
5.	$d\vee a$	P	given Premise
6.	$(d\vee a)\wedge\sim a\Rightarrow d$	T	disjunctive Simplification

27. Prove the following using direct and indirect proof

$a\to b$, $c\to b$, $d\to(a\vee c)$, $d\Rightarrow b$

Step No	Premise	Rule	Reason
1.	$a\to b$	P	given Premise
2.	$\sim a\vee b$	T	cond. equi
3.	$c\to b$	P	given Premise
4.	$\sim c\vee b$	T	cond. equi.
5.	$(\sim a\vee b)\wedge(\sim c\vee b)$ $(\sim a\wedge\sim c)\vee b$	T	distributive law
6.	$\sim(a\vee c)\vee b$	T	demorgan's law
7.	$d\to(a\vee c)$	P	given Premise

8.	d	P	given Premise
9.	$(d \to (a \vee c)) \wedge d \Rightarrow a \vee c$	T	Modus Ponens (7) (8)
10.	$[(\sim(a \vee c) \vee b) \wedge (a \vee c)] \Rightarrow b$	T	disjunctive simplification

Indirect Proof:

Step No	Premise	Rule	Reason
1.	$\sim b$	T	Negated conclusion
2.	$a \to b$	P	given Premise
3.	$(a \to b) \wedge \sim b \Rightarrow \sim a$	T	Modus Tollens (1) (2)
4.	$c \to b$	P	given Premise
5.	$(a \to b) \wedge (c \to b) \Rightarrow (a \wedge c) \to b$	T	cond. statement
6.	$d \to (a \vee c)$	P	given Premise
7.	$(d \to (a \vee c)) \wedge ((a \wedge c) \to b)$ $d \to b$	T	chain Rule (5) (6)
8.	d	P	given Premise
9.	b	T	Modus Pons (7) (8)
10.	$b \wedge \sim b$	T	conjunction
11	F		contradiction

Arguments:

It consist of premises together with a conclusion. The argument is said to be valid $(H_1 \wedge H_2 \wedge H_3 \ldots H_n) \Rightarrow C$, otherwise the argument is invalid

→ direct → indirect → conditional

If H_1, H_2, H_3 are premises and c is a conclusion, then $(H_1 \wedge H_2 \wedge \ldots H_n) \Rightarrow C$ holds

* Indirect: $[(\sim C) \wedge (H_1 \wedge H_2 \wedge \cdots H_n) \Rightarrow F]$ contradiction

Then, negated conclusion are taken as one of the premises along with the given premises and contradiction concluded finally.

* Conditional: [Rule CP]

If we derive S from R and a set of premises, then we can derive $r \to s$ from the set of premises alone. (i.e) The Rule states if $R \to S$ is a conclusion, obtained from the given set of premises $H_1, H_2, \ldots, H_n$, It is enough to conclude that S from $H_1, H_2, H_3, \ldots, H_n$ and R

88. Derive using conditional proof:

$$\sim p \vee q, \sim q \vee r, r \to s \Rightarrow p \to s$$

It is enough to prove that

$$(\sim p \vee q) \wedge (\sim q \vee r) \wedge (r \to s) \wedge p \Rightarrow s$$

Step No	Premise	Rule	Reason
1.	$\sim p \vee q$	P	given Premise
2.	$p \to q$	T	cond. equi
3.	$\sim q \vee r$	P	given Premise
4.	$q \to r$	T	cond. equi
5.	$(p \to q) \wedge (q \to r) \Rightarrow p \to r$	T	chain rule (2) (4)
6.	$r \to s$	P	given premise
7.	$(p \to r) \wedge (r \to s) \Rightarrow p \to s$	T	chain rule (5) (6)
8.	p	P	additional premise
9.	$[p \wedge (p \to s)] \Rightarrow s$	T	Modus Ponens (7) (8)

29. $p, p \to (q \to (r \wedge s)) \Rightarrow q \to s$ conditional proof

It is enough to Prove that

$$p \wedge (p \to (q \to (r \wedge s))) \wedge q \Rightarrow s$$

Step No	Premise	Rule	Reason
1.	$p \to (q \to (r \wedge s))$	P	given Premise
2.	$\sim p \vee (q \to (r \wedge s))$	T	Demorgan's
3.	$\sim p \vee (\sim q \vee (r \wedge s))$	T	Demorgan's
4.	$(\sim p \vee \sim q) \vee (r \wedge s)$	T	Associative
5.			

Step No	Premise	Rule	Reason
1.	$p \to (q \to (r \wedge s))$	P	given Premise
2.	p	P	given Premise
3.	$p \wedge (p \to (q \to (r \wedge s))) \Rightarrow q \to (r \wedge s)$	T	Modus Pons (1) (2)
4.	q	P	add. Premise
5.	$q \wedge (q \to (r \wedge s)) \Rightarrow r \wedge s$	T	Modus Pons (3) (4)
6.	s	T	simplification

30. $p \to q$, $q \to r$, $\sim(p \vee r)$, $p \vee r \Rightarrow r$

Direct:

Steps	Premise	Rule	Reason
1.	$p \to q$	P	given premise
2.	$q \to r$	P	" "
3.	$(\sim p \vee q)$ $(\sim q \vee r)$	T	chain rule
4.	$\sim(p \vee r)$	P	given
5.	$(p \vee r)$	P	" "
			negation
6.	$(\sim p \vee q) \wedge (\sim q \vee r)$		