1. Which of the following statements are not preposition? Justify.

The declarative statement is said to be an preposition not the imperative, exclamatory or other statements. In the above questions (A), (B) and (C) is not a preposition.

2. Let P: we should be honest, Q: we should be dedicated. R: we should be overconfident. Then "We should be honest or dedicated but not overconfident" is represented by. Justify.

D) $P \vee Q \wedge \sim R$

We should be honest or dedicated $\Rightarrow P \vee Q$

but not overconfident $\Rightarrow \wedge \sim R$

Answer : $P \vee Q \wedge \sim R$

3. What is tautology? Give an Example

A compound proposition (i.e) always true no matter what the truth value of the proposition that occur in it.

Ex : $(p \wedge q) \rightarrow (p \vee q)$

p	q	$p \wedge q$	$p \vee q$	$(p \wedge q) \rightarrow (p \vee q)$
F	F	F	F	T
F	T	F	T	T
T	F	F	T	T
T	T	T	T	T

4. State whether rule of inference is used in the argument "if it is today, college will close. The college is not closed today. Therefore it didn't rain today". Justify

P : It rains today

Q: The college will close

If it rains today, the college will close $\Rightarrow p \to q$

The college is not closed today $\Rightarrow \sim q$

conclusion: It didn't rain today $\Rightarrow \sim p$

$$((p \to q) \wedge \sim q) \Rightarrow \sim p$$

The rule of inference used in the statement is modus tollens

5. consider the statement $\sim p \to \sim q$ is equivalent to which of the statement. justify (i) $p \to q$ (ii) $q \to p$ (iii) $\sim q \vee p$

$$\sim p \to \sim q \equiv \sim(\sim p) \vee \sim q$$

$$\sim p \to \sim q \equiv p \vee \sim q \to ①$$

$$q \to p \equiv \sim q \vee p - ②$$

$$① \equiv ②$$

D) (ii) + (iii) only

6. Let $p(x)$ denote the statement '$x > 3$'. The truth value of $p(4)$ is. justify

$$p(x) = x > 3$$

$$p(4) = 4 > 3$$

$p(x)$ denotes the statement '$x > 3$'. $p(4)$ denotes $4 > 3$ (4 is greater than 3) is true then the truth value of $p(4)$ is true.

7. Verify wether $q \vee (p \wedge \sim q) \vee (\sim p \wedge \sim q)$ is tautological

To prove: $q \vee (p \wedge \sim q) \vee (\sim p \wedge \sim q) \equiv T$

$$q \vee (p \wedge \sim q) \vee (\sim p \wedge \sim q) \equiv (q \vee p) \wedge (q \vee \sim q) \vee (\sim p)$$ [distributive law]

$$\equiv (q \vee p) \wedge T \vee (\sim p \wedge \sim q)$$

$$\equiv (q \vee p) \wedge T \vee \sim(p \vee q)$$ [De Morgan's law]

$\equiv (p \vee q) \wedge T \vee \sim(p \vee q)$ [commutative law]

$\equiv (p \vee q) \vee \sim(p \vee q) \wedge T$ [negation law]

$\equiv T \wedge T \equiv T$

Hence Proved

8. Show that $p \to r,\ \sim p \to q,\ q \to s \Rightarrow \sim r \to s$

Step	Premise	Rule	Reason
1.	$p \to r$	P	given Premise
2.	$\sim p \to q$	P	given Premise
3.	$q \to s$	P	given Premise
4.	$(\sim p \to q) \wedge (q \to s) \equiv (\sim p \to s)$	T	chain Rule (2)(3)
5.	$\sim p \to s \equiv p \vee s$	T	conditional law
6.	$p \to r \equiv \sim p \vee r$	T	conditional equivalence
7.	$(p \vee s) \wedge (\sim p \vee r) \equiv (s \vee r)$	T	Resolution (5)(6)
8.	$(s \vee r) \equiv (r \vee s)$	T	commutative law
9.	$(r \vee s) \equiv \sim r \to s$	T	conditional equivalence

9. Without using truth table, prove that

$$\underset{(1)}{p \to (q \vee r)} \Leftrightarrow \underset{(2)}{\sim r \to (p \to q)} \Leftrightarrow \underset{(3)}{(p \wedge \sim q) \to r}$$

$p \to (q \vee r) \equiv \sim p \vee (q \vee r)$ [conditional equivalence] — ①

$\sim r \to (p \to q) \equiv r \vee (p \to q)$ [conditional equivalence]

$\equiv r \vee (\sim p \vee q)$ [conditional equivalence]

$\equiv (\sim p \vee q) \vee r$ [commutative law]

$\equiv \sim p \vee (q \vee r)$ [associative law] — ②

$(p \wedge \sim q) \rightarrow r \equiv \sim(p \wedge \sim q) \vee r$ [conditional equivalence]

$\equiv (\sim p \vee q) \vee r$ [De morgan's theorem]

$\equiv \sim p \vee (q \vee r)$ [Associative law] — 3

1 ≡ 2 ≡ 3

Hence Proved

10. Using rules of inference show that $s \vee r$ is tautologicall[y] implied by $p \vee q$, $p \rightarrow r$ and $q \rightarrow s$

Step	Premise	Rule	Reason
1.	$p \vee q$	P	given Premise
2.	$p \vee q \equiv \sim p \rightarrow q$	T	conditional equivalence
3.	$p \rightarrow r$	P	given Premise
4.	$q \rightarrow s$	P	given Premise
5.	$\sim p \vee r$	T	conditional equivalence (3)
6.	$(p \vee q) \wedge (\sim p \vee r) \equiv q \vee r$	T	Resolution (1) (5)
7.	$\sim q \vee s$	T	conditional equivalence (4)
8.	$(q \vee r) \wedge (\sim q \vee s) \equiv r \vee s$	T	Resolution (6) (7)
9.	$r \vee s \equiv s \vee r$	T	commutative law

11. Show that the following premises are inconsistent

A) If Nirmala misses many classes through illness, then she fails high school.

B) If Nirmala fails high school, then she is uneducated

c) If Nirmala reads a lot of books, then she is not uneducated.

D) Nirmala misses many classes through illness and reads a lot of books p : Nirmala misses many classes through illness

q : She fails high school

r : She is uneducated

s : Nirmala reads a lot of books

$p \to q,\ q \to r,\ s \to \sim r,\ p \wedge s \Rightarrow F$

Step	Premise	Rule	Reason
1.	$p \to q$	P	given Premise
2.	$q \to r$	P	given Premise
3.	$(p \to q) \wedge (q \to r) \equiv (p \to r)$	T	chain Rule (1) (2)
4.	$s \to \sim r$	P	given Premise
5.	$p \wedge s$	P	given Premise
6.	$\sim s \vee \sim r$	T	conditional equivalence (4)
7.	$\sim p \vee r$	T	conditional equivalence (3)
8.	$\sim s \vee \sim r \equiv \sim r \vee \sim s$	T	commutative law (6)
9.	$\sim p \vee r \equiv r \vee \sim p$	T	commutative law (7)
10.	$(r \vee \sim p) \wedge (\sim r \vee \sim s) \equiv (\sim p \vee \sim s)$	T	Resolution (8) (9)
11.	$(\sim p \vee \sim s) \equiv \sim (p \wedge s)$	T	De morgan's law (10)
12.	$(p \wedge s) \wedge \sim (p \wedge s) \equiv F$	T	Negation law

12. Verify whether the following $(p \to q) \wedge (r \to s)$, $(q \to t) \wedge (s \to u)$, $\sim (t \wedge u)$, $p \to r$ imply the conclusion $\sim p$.

Step	Premise	Rule	Reason
1.	$(p \to q) \wedge (r \to s)$	P	given Premise
2.	$(q \to r) \wedge (s \to u)$	P	given Premise
3.	$(p \to q) \wedge (r \to s) \wedge (q \to r) \wedge (s \to u) \equiv (p \to T) \wedge (r \to u)$	T	chain Rule (1)(2)
4.	$\sim (t \wedge u)$	P	given Premise
5.	$(\sim t \vee \sim u)$	T	de morgan's law
6.	$p \to r$	P	given Premise
7.	$\sim p \vee r$	T	conditional equivalence (6)
8.	$(p \to t) \wedge (r \to u) \equiv (\sim p \vee t) \wedge (\sim r \vee u)$	T	conditional equivalence (3)
9.	$(\sim p \vee t) \wedge (\sim r \vee u) \wedge (\sim p \vee r) \equiv \sim p \vee (t \wedge r) \wedge (\sim r \vee u)$	T	distributive law (8)(7)
10.	$\sim p \vee (t \wedge r) \wedge (\sim r \vee u) \wedge (\sim t \vee \sim u) \equiv \sim p \vee (t \wedge r) \wedge (\sim r \vee \sim t)$	T	Resolution (9)(3)
11.	$\sim p \vee (t \wedge r) \wedge \sim (r \wedge t)$	T	de morgan's law (10)
12.	$\sim p \vee (t \wedge r) \wedge \sim (t \wedge r)$	T	commutative law (11)
13.	$\sim p \vee (t \wedge r) \wedge \sim (t \wedge r) \equiv \sim p \vee F$	T	Negation (12)
14.	$\sim p \vee F \cong \sim p$	T	Domination law (13)