31. $p \to \sim q,\ \sim r \to p,\ q \Rightarrow r$

Step No	Premise	Rule	Reason
1.	$p \to \sim q$	P	given Premise
2.	$\sim r \to p$	P	gvn Premise
3.	$\sim r \to \sim q$	T	chain rule (2) (1)
4.	$q \to r$	T	cond. equi (3)
5.	q	P	given Premise
6	r	T	Modus Ponens (4) (5)

Inconsistent Premises:

A set premises $H_1, H_2, H_3 \cdots H_n$ are said to be inconsistent, if their conjunction implies a contraction

$$(H_1 \wedge H_2 \wedge, \ldots \wedge H_n) \Rightarrow F$$

A set of premises are said to be consistent, if their are not consistent.

32. $p \to q,\ p \to r,\ q \to \sim r,\ p$; P.T inconsistent

Step No	Premise	Rule	Reason
1.	$p \to q$	P	gn Premise
2.	$q \to \sim r$	P	gn Premise
3.	$(p \to q) \wedge (q \to \sim r)\ p \to \sim r$	T	chain rule (1) (2)
4.	p	P	gn Premise
5.	$p \to r$	P	given Premise

6.	$p \wedge (p \to r)$ r	T	Modus Ponens (4)(2)
7.	$p \wedge (p \to \sim r) \sim r$	T	Modus Ponens (4)(5)
8.	$r \wedge \sim r$ F	T	Negation

33. $a \to (b \to c)$, $d \to (b \wedge \sim c)$, $a \wedge d$ P.T inconsistent

Step No	Premise	Rule	Reason
1.	$a \to (b \to c)$	P	Given Premise
2.	$\cancel{a \to (\sim b \wedge c)}$ $d \to (b \wedge \sim c)$	P	Given Premise
3.	$d \to (\sim (b \to c))$	T	cond. equi
4.	$(b \to c) \to \sim d$	T	cond. equi.
5.	$a \to \sim d$	T	chain rule (1) (4)
6.	$a \wedge d$	P	Given Premise
7.	$\sim a \vee \sim d$	T	demorgan's (5)
8.	$\sim (a \wedge d)$	T	demorgan's
9.	F	T	Negation (6)(8)

34. Show that the premises are inconsistent

- If Jack misses many classes, then he fails high school
- If Jack fails high school, then he is uneducated
- If Jack reads lot of books, then he is not uneducated
- If Jack misses many class and reads lot of books

p: Jack misses many class

q: He fails high school

r: He is uneducated

s: Jack reads lot of books

$p \to q$, $q \to r$, $s \to \sim r$, $p \wedge s$

$\sim(p \to \sim s)$

Step No	Premise	Rule	Reason
1.	$p \to q$	P	Given Premise
2.	$q \to r$	P	Given Premise
3.	$(p \to q) \wedge (q \to r) \Rightarrow p \to r$	T	chain Rule (1)(2)
4.	$\sim p \to \sim r$	T	cond. equi (3)
5.	$(\sim p \to \sim r) \wedge (s \to \sim r) \cdot (\sim p \vee s) \to \sim r$	T	cond. equi
6.	$\sim(\sim p \vee s) \vee \sim r$	T	cond. equi.
7.	$(p \wedge \sim s) \vee \sim r$	T	cond. equi
8.	$p \wedge s$	P	Given. Premise

35. Question at Back

p : It is sunny this afternoon

q : It is colder than yesterday

r : we will go swimming

s : we will take a trip

t : we will be home by sunset

$\sim p \wedge q,\ r \to p,\ \sim r \to s,\ s \to t \Rightarrow t$

Step No	Premise	Rule	Reason
1.	$\sim r \to s$	P	gn Premise
2.	$s \to t$	P	gn Premise
3.	$\sim r \to t$	T	chain rule (1) (2)
4.	$\sim p \wedge q$	P	gn Premise
5.	$\sim (p \vee \sim q)$		

Predicate calculus

36. All students are intelligent. ~~are~~ Some intelligent student like music. Everyone who like music are stupid student. Only intelligent student like music.

P(x) : x is a student

Q(x) : x is intelligent

R(x) : x likes music

1. $\forall x\,(P(x) \rightarrow Q(x))$
2. $\exists x\,(P(x) \wedge Q(x) \wedge R(x))$
3. $\forall x\,(R(x) \rightarrow (\sim Q(x) \wedge P(x)))$
4. $\exists x\,(Q(x) \wedge P(x) \rightarrow R(x))$

37. Express the negation of statement using Quantifiers

- If the teacher is absent then some students do not keep quiet
- All the student keep quiet and the teacher is present
- Some of the students don't keep quiet or the teacher is absent

T : teacher is present

Q(x) : student's x keeps quiet

- $\sim T \rightarrow \exists x \sim Q(x) \equiv \sim T \rightarrow \sim \forall x Q(x)$

$$\equiv T \vee \sim \forall x\, Q(x)$$

$$\sim(\sim T \rightarrow \exists x \sim Q(x)) \equiv \sim(T \vee \sim \forall x\, Q(x))$$

$$\equiv \sim T \wedge \forall x\, Q(x)$$

The negation is,

If the teacher is absent and all the students keep Quiet

Inference Theory of Predicate calculus:

Rule 1: universal specification or universal instantiation

US is the rule of inference which states that one can conclude that $P(c)$ is true if $\forall x P(x)$ is true where c is arbitary value (Rule US).

Rule 2: Existential specification

ES is the rule which allows us to conclude that $P(c)$ is true, if $\exists x P(x)$ is true (Rule ES)

Rule 3: Universal generalisation

UG is the rule which states that $\forall x P(x)$ is true if $P(c)$ is true (Rule UG)

Rule 4: Existential generalisation

EG is the rule which state that $\exists x P(x)$ is true if $P(c)$ is true (Rule EG)

Logical Equivalence and Implication of Quantified statements:

Let $P(x)$ and $Q(x)$ be open statement defined for a given universe.

* Logical equivalence:

The two statements $P(x)$ and $Q(x)$ are said to be logically equivalent then $P(a) \leftrightarrow Q(a)$ is true for each replacement 'a' from the universe

$$\forall x\,(P(x) \Longleftrightarrow Q(x)) \quad \text{or} \quad \forall x\,(P(x) \equiv Q(x))$$

* Logical Implication:

If the implication $P(a) \rightarrow Q(a)$ is true for each 'a' in the universe then we write

$$\forall x\,(P(x) \Rightarrow Q(x))$$

In general, $\forall x\, P(x) \equiv Q(x)$ if $\forall x\,(P(x) \rightarrow Q(x))$

and $\forall x\,(Q(x) \rightarrow P(x))$

* For the statement : $\forall x\,(P(x) \rightarrow q(x))$
* converse : $\forall x\,(q(x) \rightarrow P(x))$
* Inverse : $\forall x\,(\sim P(x) \rightarrow \sim q(x))$
* contrapositive : $\forall x\,(\sim q(x) \rightarrow \sim p(x))$

The statements and its contrapositive are logically equivalent

$$\forall x\,(P(x) \rightarrow Q(x)) \equiv \forall x\,(\sim Q(x) \rightarrow \sim P(x))$$

converse and Inverse statements are logically equivalent

$$\forall x\,(Q(x) \rightarrow P(x)) \equiv \forall x\,(\sim P(x) \rightarrow \sim Q(x))$$

Equivalence:

* $\exists x\,(A(x) \vee B(x)) \equiv \exists x\, A(x) \vee \exists x\, B(x)$
* $\exists x\,(A(x) \wedge B(x)) \equiv \exists x\, A(x) \wedge \exists x\, B(x)$
* $\sim (\exists x)\, A(x) \equiv (x) \sim A(x)$
* $\sim x\, A(x) \equiv \exists x \sim A(x)$
* $(x)\, A \vee B(x) \equiv A \vee (x) B(x)$
* $(\exists x)\,[A \wedge B(x)] \equiv A \wedge (\exists x) B(x)$
* $(x) A(x) \rightarrow B \equiv (\exists x)\,(A(x) \rightarrow B)$
* $\exists x\,(A(x) \rightarrow B) \equiv (x)\,(A(x) \rightarrow B)$
* $A \rightarrow x\, B(x) \equiv (x)(A \rightarrow B(x))$
* $A \rightarrow (\exists x) B(x) \equiv (\exists x)\,(A \rightarrow B(x))$

Implication:

* $(x)A(x) \vee (x)B(x) \Rightarrow (x)[A(x) \vee B(x)]$

* $\exists x [A(x) \wedge B(x)] \Rightarrow (\exists x) A(x) \wedge (\exists x) B(x)$

* $[B \rightarrow A(x)] \Rightarrow B \rightarrow (x) A(x)$

Logical equivalence and Implification for quantified statement in one variable:

For any set of open statements in the variable x and for proscribed universe, we have the following logical implication and equivalence.

* $\exists x [P(x) \wedge q(x)] \equiv \exists x P(x) \wedge \exists x q(x)$

* $\forall x [P(x) \vee q(x)] \equiv (\forall x) p(x) \vee (\forall x)(q(x))$

* $\forall x [P(x) \wedge q(x)] \equiv (\forall x) p(x) \wedge (\forall x) q(x)$

* $\forall x p(x) \vee \forall x q(x) \equiv \forall x [p(x) \vee q(x)]$

* $\forall x [p(x) \wedge q(x) \wedge r(x)] \equiv \forall x [p(x) \wedge q(x) \wedge r(x)]$

* $\exists x [p(x) \rightarrow q(x)] \equiv \exists x (\sim p(x) \vee q(x))$

* $\forall x \sim(\sim p(x)) \equiv \forall x p(x)$

* $\forall x \sim(p(x) \wedge q(x)) \equiv \forall x (\sim p(x) \vee \sim q(x))$

* $\forall x \sim(p(x) \vee q(x)) \equiv \forall x (\sim p(x) \wedge q(x))$

* $(\forall x)[p(x) \rightarrow q(x)] \equiv [\forall x p(x) \rightarrow \forall x q(x)]$

Negation of quantified statement:

* $\sim(\forall x P(x)) \equiv (\exists x)(\sim P(x))$

* $\sim[(\exists x) p(x)] \equiv \forall x (\sim p(x))$

* $\sim(\forall x \sim p(x)) \equiv (\exists x)(p(x))$

* $\sim(\exists x \sim p(x)) \equiv (\forall x)(p(x))$

Theory of Inference and valid arguments:

The rules of universal specification and generalisation

* US rule

$(\forall x)(P(x)) \Rightarrow P(y)$ y is new

$[(\forall x) P(x) \Rightarrow P(x), P(y), P(a), P(z) \cdots]$

* UG rule

$P(y) \Rightarrow \forall x P(x)$

from $P(y)$, we can derive $x P(x)$

* ES rule

$\exists x (P(x)) \Rightarrow P(a)$ or $P(b)$ or $P(c)$

$\exists x (P(x)) \Rightarrow P(y)$

* EG rule

$P(y) \Rightarrow \exists x P(x)$

from $P(x)$ or $P(a)$, we can derive $(\exists x) P(x)$

38. Show that premises "One students in the class knows how to write programs in Java", "Everyone who knows how to write a program in Java can get a high paying job."

Imply the conclusion "Some one in this class can get a high paying job".

$C(x)$: x is a student in the class

$P(x)$: x knows to write program in Java

$J(x)$: x can get high paying job

P: $\exists x (P(x) \wedge C(x))$, $\forall x (P(x) \rightarrow J(x))$

C: $\exists x (C(x) \wedge J(x))$

Steps	Premise	Rules	Reason
1.	$\exists x(P(x)\wedge L(x))$	P	given Premise
2.	$P(a)\wedge L(a)$		ES(1)
3.	$P(a)$	T	Simplification (2)
4.	$L(a)$	T	Simplification (2)
5.	$\forall x(P(x)\rightarrow J(x))$	P	given Premise
6.	$P(a)\rightarrow J(a)$	T	US(5)
7.	$J(a)$	T	Modus Ponens (3) (6)
8.	$L(a)\wedge J(a)$	T	conjunction (4) (7)
9.	$\exists x(L(x)\wedge J(x))$		EG (8)

39. Show that premise "The Student in this class has not read the book," "Everyone in the class passed the first exams". Imply the conclusion "Someone who passed the first exam has not read the book".

$C(x)$: x is in the class

$R(x)$: x not read the book

$E(x)$: x passed the 1st exam

P: $\exists x(C(x)\wedge R(x))$, ~~$\forall x(E(x)\wedge$~~ $\forall x(C(x)\rightarrow E(x))$

C: $\exists x(E(x)\wedge R(x))$

40.

40. Check the validity of the argument. "All humming birds are richly coloured", "No large birds like honey" "Birds that don't like honey are dull in colour" Imply "Humming Birds are small"

P(x): x is a humming bird

Q(x) : x is richly coloured

R(x) : x is large likes honey

S(x) : x is large

P : $\forall(x)(P(x) \to Q(x))$, $\forall x(\sim S(x)$

41. Show by indirect method of proof

$\forall x (P(x) \vee Q(x))$ conclusion : $\forall x P(x) \vee \exists x P(x)$

Steps	Premise	Rule	Reason
1.	$\sim(\forall x P(x) \vee \exists x Q(x))$	T	negated conclusion
2.	$\sim(\forall x P(x)) \wedge \sim(\exists x Q(x))$	T	demorgan's law
3.	$\sim(\forall x P(x))$	T	Simplification (2)
4.	$\sim(\exists x Q(x))$	T	Simplification (2)
5.	$\exists x \sim P(x)$	T	negation (3)
6.	$\forall x \sim Q(x)$	T	negation (4)
7.	$\sim P(a)$		ES(5)
8.	$\sim Q(a)$		US(6)

9.	$\sim P(a) \wedge \sim Q(a)$	T	conjunction (7) (8)
10.	$\sim (P(a) \vee Q(a))$	T	demorgan's (9)
11.	$\forall x (P(x) \vee Q(x))$	P	given Premise
12.	$P(a) \vee Q(a)$		US
13.	$\sim (P(a) \vee Q(a)) \wedge (P(a) \vee Q(a)) \equiv F$	T	contradiction

42. Show that the conclusion $\forall x (P(x) \to \sim Q(x))$ follows from the Premise $\exists (x) (P(x) \wedge Q(x)) \to \forall y (R(y) \to S(y))$, $\exists y (R(y) \wedge \sim S(y))$

Step No	Premise	Rule	Reason
1.	$\exists y (R(y) \wedge \sim S(y))$	P	given Premise
2.	$R(a) \wedge \sim S(a)$		ES(1)
3.	$\sim (R(a) \to S(a))$	T	cond. equi.
4.	$\exists y (\sim R(y) \to S(y))$		EG(3)
5.	$\forall y (R(y) \to S(y))$	T	Negation equi
6.	$\exists x (P(x) \wedge Q(x)) \to \forall y (R(y) \to S(y))$	P	given Premise
7.	$\sim \exists x (P(x) \wedge Q(x))$	T	Modus Ponens (5)(6)
8.	$\forall x \sim (P(x) \wedge Q(x))$	T	Negation equi (7)
9.	$\sim (P(b) \wedge Q(b))$		US (8)
10.	$\sim P(b) \vee \sim Q(b)$	T	demorgan's
11.	$P(b) \to \sim Q(b)$	T	cond equi
12.	$\forall x (P(x) \to \sim Q(x))$		UG