Data Communication

Lecture Notes

Lecture: 6

Week: 6

Topic: Data & Signals (Part 2)

Lecture Outline:

6.1 Analog to Digital Conversion

6.2 Pulse Code Modulation (PCM)

6.1 Analog to Digital Conversion

Sometimes we may want to change an analog signal in our hand to a digital signal (analog to digital conversion) before transmission due to digital signal's superiority over analog signal. Some of the reasons why digital signal is preferable over analog signal are as follows:

- Digital is more robust than analog to noise and interference
- Digital is more viable to using regenerative repeaters
- Digital hardware more flexible by using microprocessors and VLSI
- Can be coded to yield extremely low error rates with error correction
- Easier to multiplex several digital signals than analog signals
- Digital is more efficient in trading off SNR for bandwidth

Strictly speaking, it might be more correct to refer to this as a process of converting analog data into digital data; this process is known as digitization. Once analog data have been converted into digital data, several things can happen. The three most common are as follows:

1. The digital data can be transmitted using NRZ-L. In this case, we have in fact gone directly from analog data to a digital signal.

2. The digital data can be encoded as a digital signal using a code other than NRZ-L. Thus, an extra step is required.

3. The digital data can be converted into an analog signal, using suitable modulation technique.

Figure 1: Digitizing analog data

This last, seemingly curious, procedure is illustrated in Figure 1, which shows voice data that are digitized and then converted to an analog signal. This allows digital transmission. The voice data, because they have been digitized, can be treated as digital data, even though transmission requirements (e.g., use of microwave) dictate that an analog signal be used.

The device used for converting analog data into digital form for transmission, and subsequently recovering the original analog data from the digital, is known as a codec (coder-decoder). In this section we examine the two principal techniques used in codecs, pulse code modulation and delta modulation. The section closes with a discussion of comparative performance.

6.2 Pulse Code Modulation (PCM)

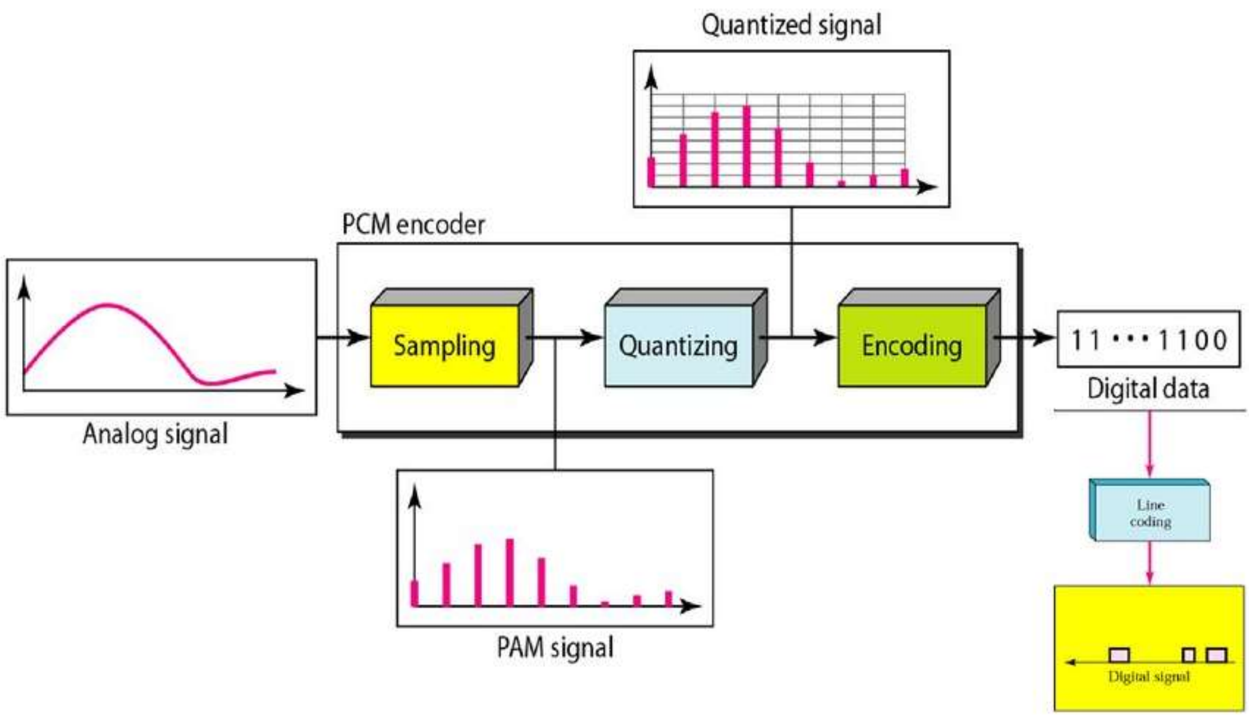


Figure 2: Components of PCM encoder

The most common technique to change an analog signal to digital data (digitization) is called pulse code modulation (PCM). A PCM encoder has three processes, as shown in figure 2.

A PCM encoder has three processes:

- The analog signal is sampled.
- The sampled signal is quantized.

- The quantized values are encoded as streams of bits.

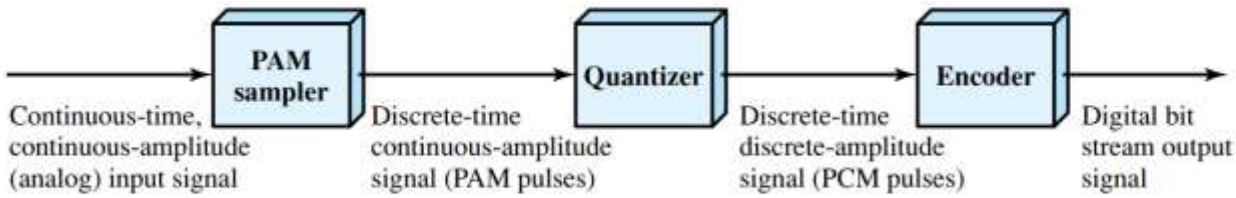


Figure 3: PCM block diagram

Thus, PCM starts with a continuous-time, continuous-amplitude (analog) signal, from which a digital signal is produced (Figure 3). The digital signal consists of blocks of n bits, where each n-bit number is the amplitude of a PCM pulse. On reception, the process is reversed to reproduce the analog signal. Notice, however, that this process violates the terms of the sampling theorem. By quantizing the PAM pulse, the original signal is now only approximated and cannot be recovered exactly. This effect is known as quantizing error or quantizing noise.

Sampling

The first step in PCM is sampling. The analog signal is sampled every T_s s, where T_s is the sample interval or period. The inverse of the sampling interval is called the sampling rate or sampling frequency and denoted by f_s, where $f_s = 1/T_s$. There are three sampling methods—ideal, natural, and flat-top—as shown in Figure 4.

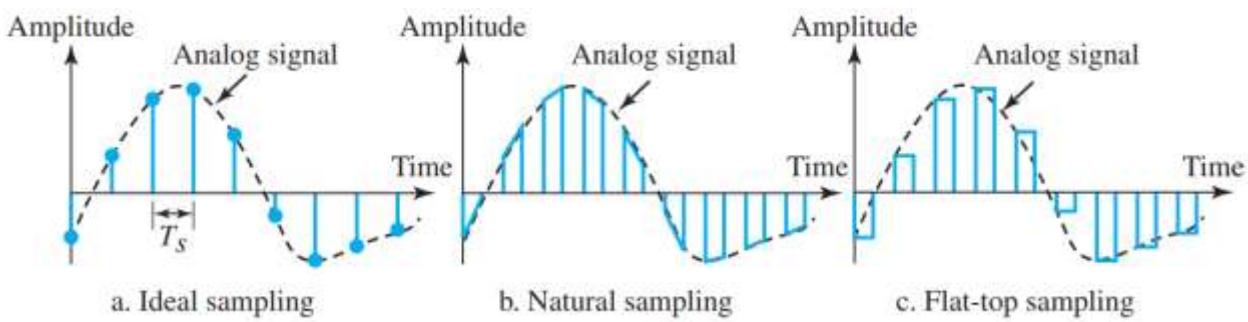


Figure 4: Three different sampling methods for PCM

In ideal sampling, pulses from the analog signal are sampled. This is an ideal sampling method and cannot be easily implemented. In natural sampling, a high-speed switch is turned on for only the small period of time when the sampling occurs. The result is a sequence of samples that retains the shape of the analog signal. The most common sampling method, called sample and hold, however, creates flat-top samples by using a circuit.

The sampling process is sometimes referred to as pulse amplitude modulation (PAM). We need to remember, however, that the result is still an analog signal with nonintegral values.

Sampling Rate

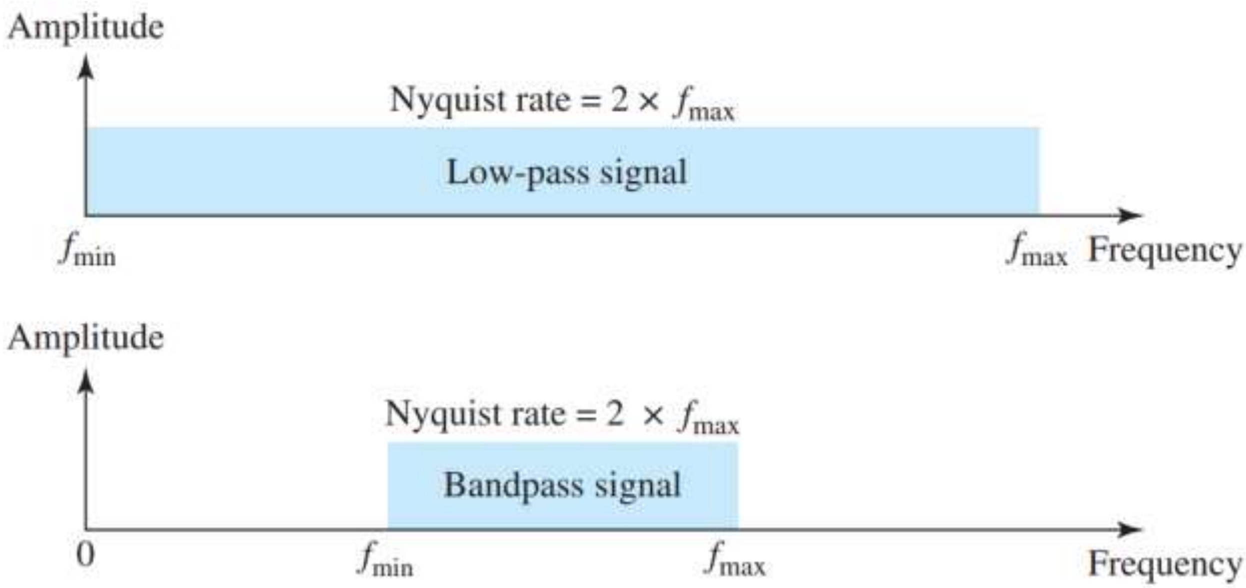


Figure 5: Nyquist sampling rate for low-pass and bandpass signals

One important consideration is the sampling rate or frequency. What are the restrictions on T_s? This question was elegantly answered by Nyquist. According to the Nyquist theorem, to reproduce the original analog signal, one necessary condition is that the sampling rate be at least twice the highest frequency in the original signal.

We need to elaborate on the theorem at this point. First, we can sample a signal only if the signal is band-limited. In other words, a signal with an infinite bandwidth cannot be sampled. Second, the sampling rate must be at least 2 times the highest frequency, not the bandwidth. If the analog signal is low-pass, the bandwidth and the highest frequency are the same value. If the analog signal is bandpass, the bandwidth value is lower than the value of the maximum frequency. Figure 5 shows the value of the sampling rate for two types of signals.

Example 1: For an intuitive example of the Nyquist theorem, let us sample a simple sine wave at three sampling rates: $f_s = 4f$ (2 times the Nyquist rate), $f_s = 2f$ (Nyquist rate), and $f_s = f$ (one-half the Nyquist rate). Figure 6 shows the sampling and the subsequent recovery of the signal.

It can be seen that sampling at the Nyquist rate can create a good approximation of the original sine wave (part a). Oversampling in part b can also create the same approximation, but it is redundant and unnecessary. Sampling below the Nyquist rate (part c) does not produce a signal that looks like the original sine wave.

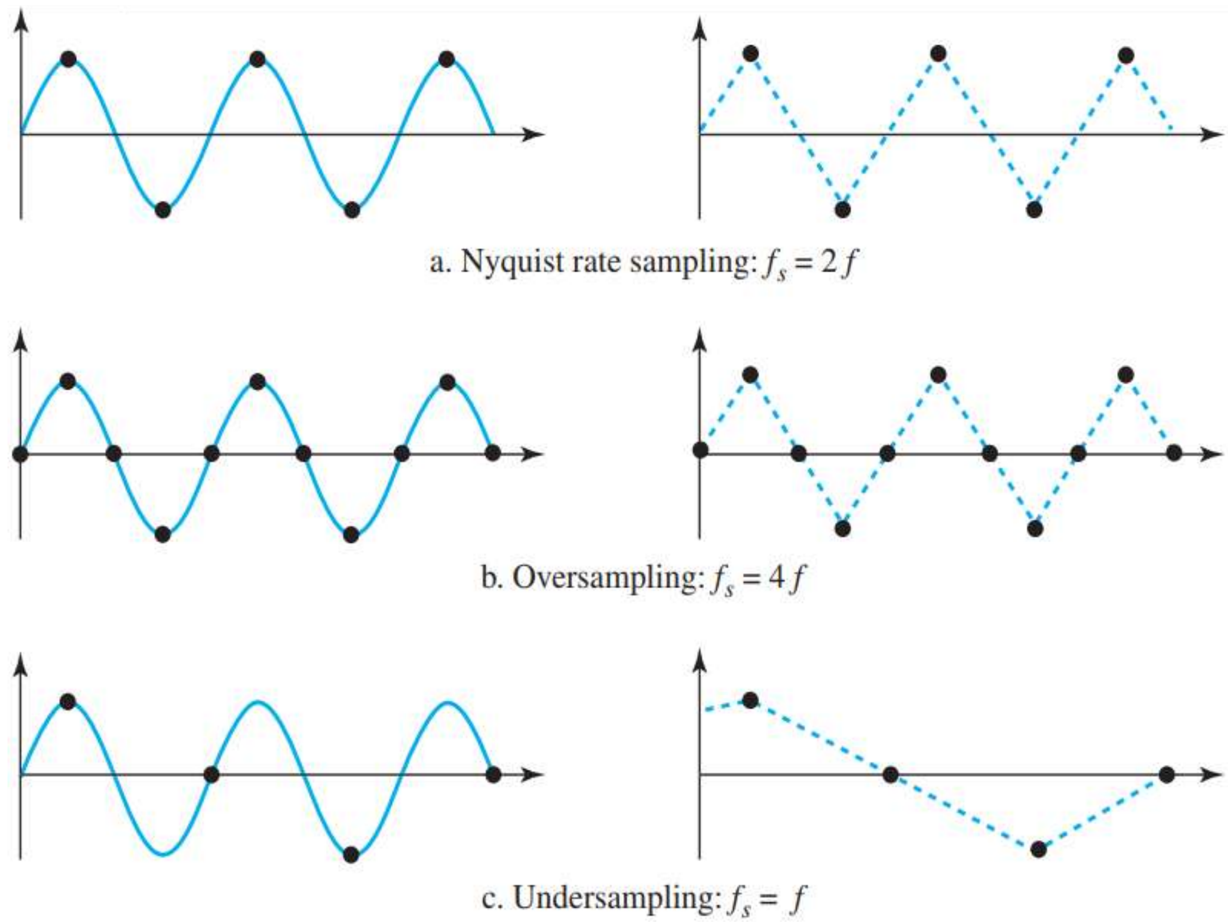


Figure 6: Recovery of a sampled sine wave for different sampling rates

Example 2: As an interesting example, let us see what happens if we sample a periodic event such as the revolution of a hand of a clock. The second hand of a clock has a period of 60 s. According to the Nyquist theorem, we need to sample the hand (take and send a picture) every 30 s ($T_s = ½T$ or $f_s = 2f$). In Figure 7a, the sample points, in order, are 12, 6, 12, 6, 12, and 6. The receiver of the samples cannot tell if the clock is moving forward or backward. In part b, we sample at double the Nyquist rate (every 15 s). The sample points, in order, are 12, 3, 6, 9, and 12. The clock is moving forward. In part c, we sample below the Nyquist rate ($T_s = T$ or $f_s = f$). The sample points, in order, are 12, 9, 6, 3, and 12. Although the clock is moving forward, the receiver thinks that the clock is moving backward.

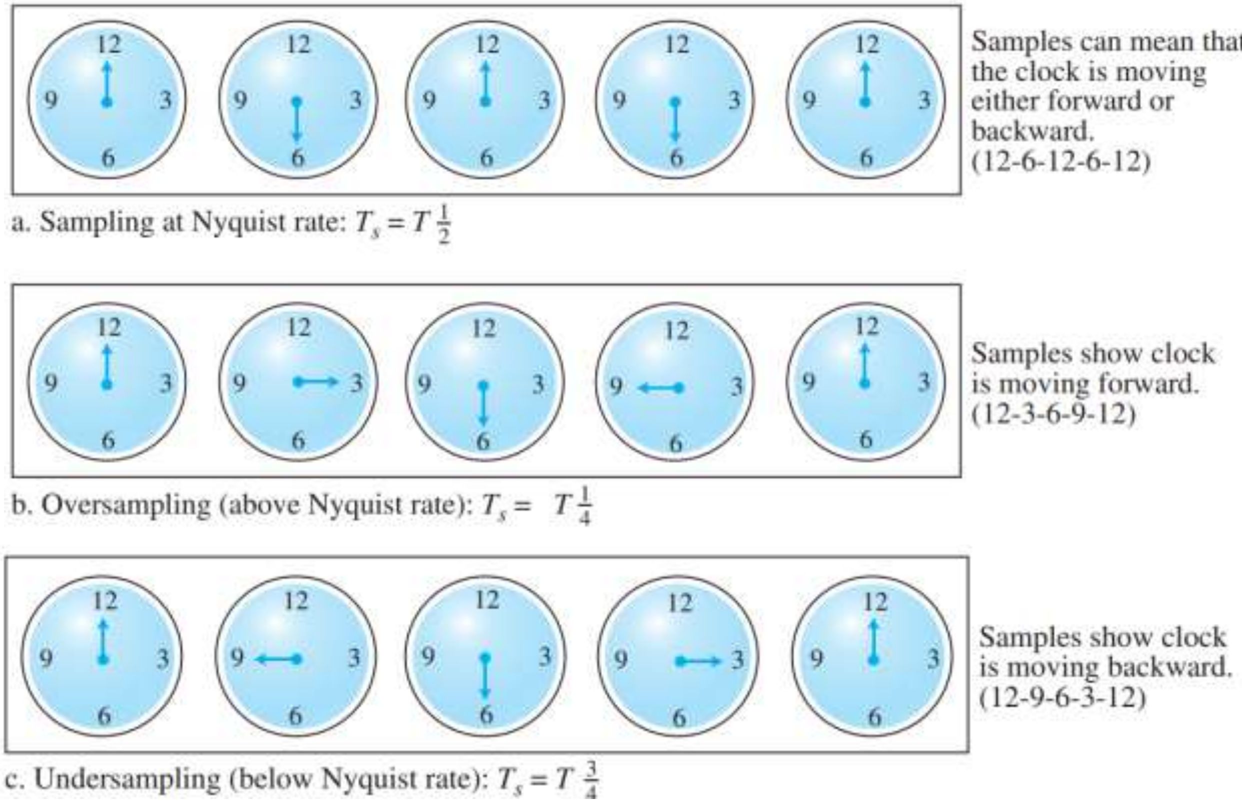


Figure 7: Sampling of a clock with only one hand

Example 3: An example related to Example 2 is the seemingly backward rotation of the wheels of a forward moving car in a movie. This can be explained by undersampling. A movie is filmed at 24 frames per second. If a wheel is rotating more than 12 times per second, the undersampling creates the impression of a backward rotation.

Example 4: Telephone companies digitize voice by assuming a maximum frequency of 4000 Hz. The sampling rate therefore is 8000 samples per second.

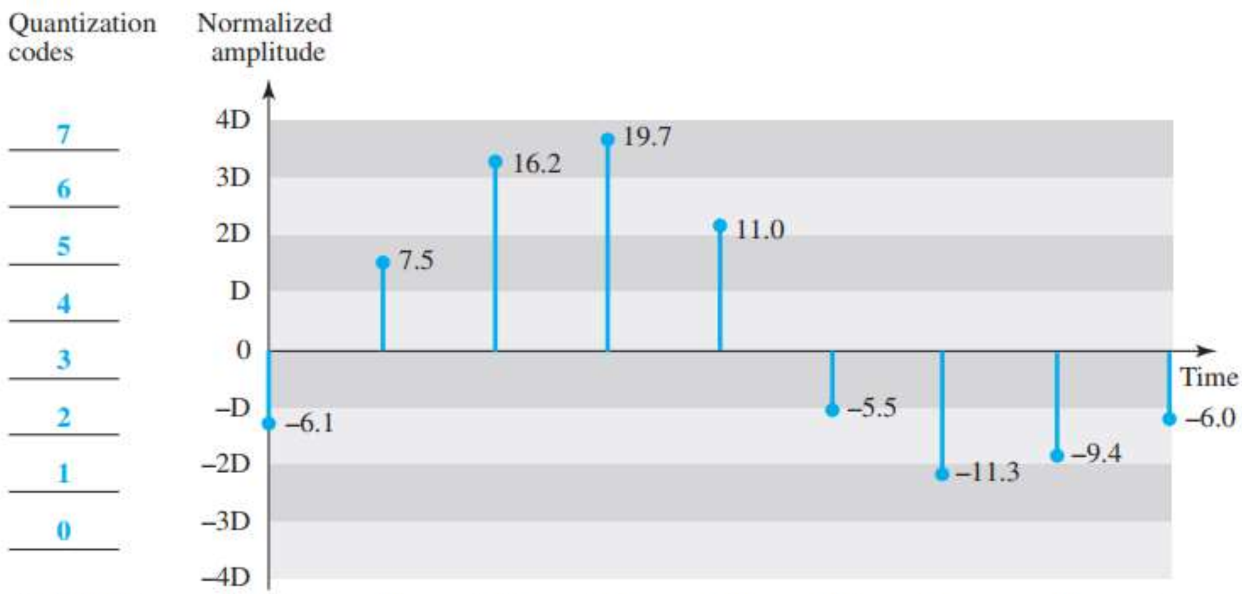
Example 5: A complex low-pass signal has a bandwidth of 200 kHz. What is the minimum sampling rate for this signal?

Solution: The bandwidth of a low-pass signal is between 0 and f, where f is the maximum frequency in the signal. Therefore, we can sample this signal at 2 times the highest frequency (200 kHz). The sampling rate is therefore 400,000 samples per second.

<u>Example 6</u>: A complex bandpass signal has a bandwidth of 200 kHz. What is the minimum sampling rate for this signal?

<u>Solution</u>: We cannot find the minimum sampling rate in this case because we do not know where the bandwidth starts or ends. We do not know the maximum frequency in the signal.

Quantization



Normalized PAM values	–1.22	1.50	3.24	3.94	2.20	–1.10	–2.26	–1.88	–1.20
Normalized quantized values	–1.50	1.50	3.50	3.50	2.50	–1.50	–2.50	–1.50	–1.50
Normalized error	–0.38	0	+0.26	–0.44	+0.30	–0.40	–0.24	+0.38	–0.30
Quantization code	2	5	7	7	6	2	1	2	2
Encoded words	010	101	111	111	110	010	001	010	010

Figure 8: Quantization and encoding of a sampled signal

The result of sampling is a series of pulses with amplitude values between the maximum and minimum amplitudes of the signal. The set of amplitudes can be infinite with non-integral values between the two limits. These values cannot be used in the encoding process. The following are the steps in quantization:

1. We assume that the original analog signal has instantaneous amplitudes between V_{min} and V_{max}.
2. We divide the range into L zones, each of height Δ (delta).

$$\Delta = \frac{V_{max} - V_{min}}{L}$$

3. We assign quantized values of 0 to $L - 1$ to the midpoint of each zone.
4. We approximate the value of the sample amplitude to the quantized values. As a simple example, assume that we have a sampled signal and the sample amplitudes are between −20 and +20 V. We decide to have eight levels (L = 8). This means that Δ = 5 V. Figure 8 shows this example.

We have shown only nine samples using ideal sampling (for simplicity). The value at the top of each sample in the graph shows the actual amplitude. In the chart, the first row is the normalized value for each sample (actual amplitude/Δ). The quantization process selects the quantization value from the middle of each zone. This means that the normalized quantized values (second row) are different from the normalized amplitudes. The difference is called the normalized error (third row). The fourth row is the quantization code for each sample based on the quantization levels at the left of the graph. The encoded words (fifth row) are the final products of the conversion.

Quantization Levels

In the previous example, we showed eight quantization levels. The choice of L, the number of levels, depends on the range of the amplitudes of the analog signal and how accurately we need to recover the signal. If the amplitude of a signal fluctuates between two values only, we need only two levels; if the signal, like voice, has many amplitude values, we need more quantization levels. In audio digitizing, L is normally chosen to be 256; in video it is normally thousands. Choosing lower values of L increases the quantization error if there is a lot of fluctuation in the signal.

Quantization Error

One important issue is the error created in the quantization process. (Later, we will see how this affects high-speed modems.) Quantization is an approximation process. The input values to the quantizer are the real values; the output values are the approximated values. The output values are chosen to be the middle value in the zone. If the input value is also at the middle of the zone, there is no quantization error; otherwise, there is an error. In the previous example, the normalized amplitude of the third sample is 3.24, but the normalized quantized value is 3.50. This means that there is an error of +0.26. The value of the error for any sample is less than $\Delta/2$. In other words, we have $-\Delta/2 \leq \text{error} \leq \Delta/2$.

The quantization error changes the signal-to-noise ratio of the signal, which in turn reduces the upper limit capacity according to Shannon.

It can be proven that the contribution of the quantization error to the SNR_{dB} of the signal depends on the number of quantization levels L, or the bits per sample n_b, as shown in the following formula:

$SNR_{dB} = 6.02n_b + 1.76$ dB

Example 7: What is the SNR_{dB} in the example of Figure 8?

Solution: We can use the formula to find the quantization. We have eight levels and 3 bits per sample, so

$SNR_{dB} = 6.02n_b + 1.76$ dB = 19.82 dB. Increasing the number of levels increases the SNR.

Example 8: A telephone subscriber line must have an SNR_{dB} above 40. What is the minimum number of bits per sample?

Solution: We can calculate the number of bits as

$$n_b = \frac{SNR_{dB} - 1.76}{6.02} = \frac{40 - 1.76}{6.02} = 6.35 \approx 7$$

Telephone companies usually assign 7 or 8 bits per sample.

Encoding

The last step in PCM is encoding. After each sample is quantized and the number of bits per sample is decided, each sample can be changed to an n_b-bit code word. In Figure 8 the encoded words are shown in the last row. A quantization code of 2 is encoded as 010; 5 is encoded as 101; and so on. Note that the number of bits for each sample is determined from the number of quantization levels. If the number of quantization levels is L, the number of bits is $n_b = \log_2 L$. In our example L is 8 and n_b is therefore 3. The bit rate can be found from the formula

Bit rate = sampling rate x number of bits per sample = $fs \times nb$

Example 9: We want to digitize the human voice. What is the bit rate, assuming 8 bits per sample?

Solution: The human voice normally contains frequencies from 0 to 4000 Hz. So, the sampling rate and bit rate are calculated as follows:

Sampling rate = 4000 x 2 = 8000 samples/s

Bit rate = 8000 x 8 = 64,000 bps = 64 kbps

Original Signal Recovery

The recovery of the original signal requires the PCM decoder. The decoder first uses circuitry to convert the code words into a pulse that holds the amplitude until the next pulse. After the staircase signal is completed, it is passed through a low-pass filter to smooth the staircase signal into an analog signal. The filter has the same cutoff frequency as the original signal at the sender. If the signal has been sampled at (or greater than) the Nyquist sampling rate and if there are enough quantization levels, the original signal will be recreated. Note that the maximum and minimum values of the original signal can be achieved by using amplification. Figure 9 shows the simplified process.

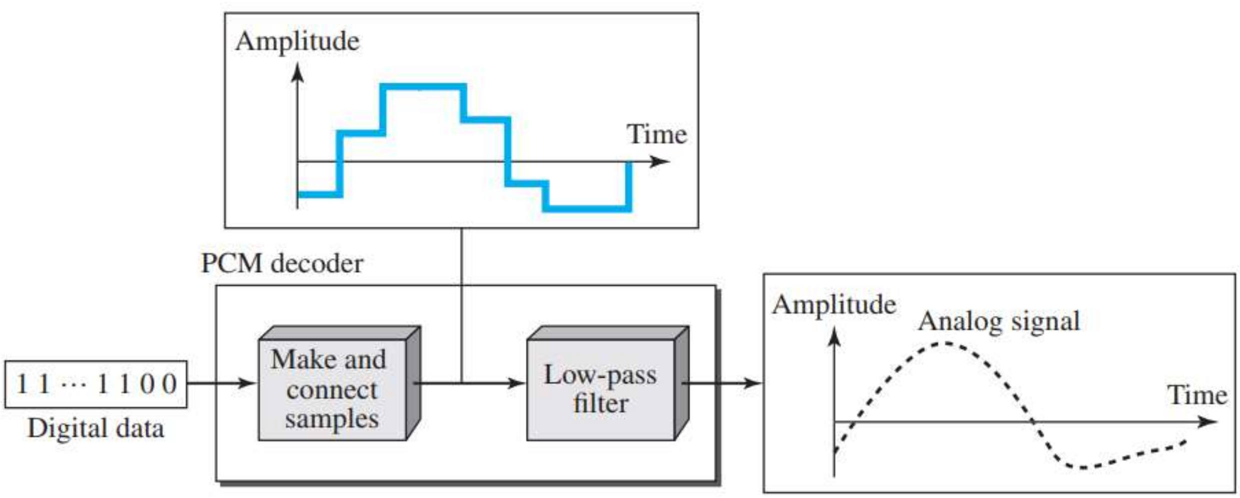


Figure 9: Components of a PCM decoder

PCM Bandwidth

Suppose we are given the bandwidth of a low-pass analog signal. If we then digitize the signal, what is the new minimum bandwidth of the channel that can pass this digitized signal? We have said that the minimum bandwidth of a line-encoded signal is $B_{min} = c \times N \times (1/r)$. We substitute the value of N in this formula:

$$B_{min} = c \ x \ N \ x \ \frac{1}{r} = c \ x \ f_s x \ n_b \ x \ \frac{1}{r}$$

$$= c \ x \ n_b x \ 2 \ x \ B_{analog} \ x \ \frac{1}{r}$$

When $\frac{1}{r} = 1$ (for an NRZ or bipolar signal) and $c = \frac{1}{2}$ (the average situation),

The minimum bandwidth is

$$B_{min} = n_b x \ B_{analog}$$

This means the minimum bandwidth of the digital signal is n_b times greater than the bandwidth of the analog signal. This is the price we pay for digitization.

<u>Example 10</u>: We have a low-pass analog signal of 4 kHz. If we send the analog signal, we need a channel with a minimum bandwidth of 4 kHz. If we digitize the signal and send 8 bits per sample, we need a channel with a minimum bandwidth of 8 x 4 kHz = 32 kHz.

References:

1. Forouzan, B. A. "Data Communication and Networking. Tata McGraw." (2005).
2. William Stallings, "Data and Computer Communications", Pearson