① $5x+6=x^2$; $x^2-5x+6=0$

$\left.\begin{matrix} a=1 \\ b=-5 \\ c=6 \end{matrix}\right\}$ $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-(-5)\pm\sqrt{(-5)^2-4\cdot1\cdot6}}{2\cdot1}=$

$=\frac{5\pm\sqrt{25-16}}{2}=\frac{5\pm\sqrt{9}}{2}=\frac{5\pm3}{2}<\begin{matrix}\frac{5+3}{2}=\frac{8}{2}=4 \\ \frac{5-3}{2}=\frac{2}{2}=1\end{matrix}$

$\boxed{\begin{matrix} x_1=4 \\ x_2=1 \end{matrix}}$

② $3x=x^2-40$; $x^2-3x-40=0$ $\left\{\begin{matrix} a=1 \\ b=-3 \\ c=-40 \end{matrix}\right.$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-(-3)\pm\sqrt{(-3)^2-4\cdot1\cdot(-40)}}{2\cdot1}=$

$=\frac{3\pm\sqrt{9+160}}{2}=\frac{3\pm\sqrt{169}}{2}=\frac{3\pm13}{2}$

$=<\begin{matrix}\frac{3+13}{2}=8 \\ \frac{3-13}{2}=-5\end{matrix}$ $\boxed{\begin{matrix} x_1=8 \\ x_2=-5 \end{matrix}}$

③ $\left.\begin{matrix} x+y=6 \\ x\cdot y=8 \end{matrix}\right\}$ $y=6-x$; $x(6-x)=8$

$6x-x^2=8$; $-x^2+6x-8=0$ $\left[\begin{matrix} a=-1 \\ b=6 \\ c=-8 \end{matrix}\right.$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-6\pm\sqrt{6^2-4\cdot(-1)(-8)}}{2\cdot(-1)}=$

$=\frac{-6\pm\sqrt{36-32}}{-2}=\frac{-6\pm\sqrt{4}}{-2}=\frac{-6\pm2}{-2}=$

$=<\begin{matrix}\frac{-6+2}{-2}=\frac{-4}{-2}=\textcircled{2} \\ \frac{-6-2}{-2}=\frac{-8}{-2}=\textcircled{4}\end{matrix}$

④ $(13+x)\cdot(27-x) = (13\cdot 27) - 51$

$351 - 13x + 27x - x^2 = 300$

$-x^2 + 14x + 351 - 300 = 0$

$-x^2 + 14x + 51 = 0$ ——— $\begin{cases} a = -1 \\ b = 14 \\ c = 51 \end{cases}$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-14 \pm \sqrt{14^2 - 4(-1)51}}{2\cdot(-1)} =$

$= \frac{-14 \pm \sqrt{196 + 204}}{-2} = \frac{-14 \pm \sqrt{400}}{-2} = \frac{-14 \pm 20}{-2} =$

$= \begin{cases} \frac{-14+20}{-2} = \frac{6}{-2} = -\frac{6}{2} = -3 \\ \frac{-14-20}{-2} = \frac{-34}{-2} = 17 \end{cases}$

$\boxed{\begin{array}{l} x_1 = -3 \\ x_2 = 17 \end{array}}$

⑤ número impar → $2x+1$

número impar consecutivo → $(2x+1)+2 = 2x+3$

$(2x+1)^2 + (2x+3)^2 = 394$

igualdades notables

$(2x)^2 + 1^2 + 2\cdot 2x\cdot 1 + (2x)^2 + 3^2 + 2\cdot 2x\cdot 3 = 394$

$4x^2 + 1 + 4x + 4x^2 + 9 + 12x = 394$

$8x^2 + 16x + 10 = 394$

$8x^2 + 16x + 10 - 394 = 0$; $8x^2 + 16x - 384 = 0$

$\boxed{\begin{array}{l} a = 8 \\ b = 16 \\ c = -384 \end{array}} \rightarrow x = \frac{-16 \pm \sqrt{16^2 - 4\cdot 8\,(-384)}}{2\cdot 8} =$

$= \frac{-16 \pm \sqrt{256 + 12288}}{16} = \frac{-16 \pm \sqrt{12544}}{16} =$

$= \frac{-16 \pm 112}{16} = \begin{cases} \frac{-16+112}{16} = 6 \\ \frac{-16-112}{16} = -8 \end{cases}$

$\boxed{\begin{array}{l} x_1 = 6 \\ x_2 = -8 \end{array}}$

⑥ $15x + 100 = x^2 \quad ; -x^2 + 15x + 100 = 0$

$\left.\begin{array}{l} a = -1 \\ b = 15 \\ c = 100 \end{array}\right\} \quad x = \dfrac{-15 \pm \sqrt{15^2 - 4 \cdot (-1) \cdot 100}}{2(-1)} =$

$= \dfrac{-15 \pm \sqrt{225 + 400}}{-2} = \dfrac{-15 \pm \sqrt{625}}{-2} = \dfrac{-15 \pm 25}{-2}$

$=$ $\begin{cases} \dfrac{-15+25}{-2} = \dfrac{10}{-2} = \dfrac{-10}{2} = \underline{\underline{-5}} \\ \dfrac{-15-25}{-2} = \dfrac{-40}{-2} = \underline{\underline{20}} \end{cases}$

$\boxed{\begin{array}{l} x_1 = -5 \\ x_2 = 20 \end{array}}$

⑦ $x^2 + (x+1)^2 + (x+2)^2 = 365$

$x^2 + x^2 + 1^2 + 2 \cdot 1 \cdot x + x^2 + 2^2 + 2 \cdot 2x = 365$

$x^2 + x^2 + 1 + 2x + x^2 + 4 + 4x - 365 = 0$

$3x^2 + 6x - 360 = 0$

$\left.\begin{array}{l} a = 3 \\ b = 6 \\ c = -360 \end{array}\right\} \quad x = \dfrac{-6 \pm \sqrt{6^2 - 4 \cdot 3 \cdot (-360)}}{2 \cdot 3} =$

$= \dfrac{-6 \pm \sqrt{36 + 4320}}{6} = \dfrac{-6 \pm \sqrt{4356}}{6} =$

$= \dfrac{-6 \pm 66}{6}$ $\begin{cases} \dfrac{-6+66}{6} = \underline{\underline{10}} \\ \dfrac{-6-66}{6} = \underline{\underline{-12}} \end{cases}$

$\boxed{\begin{array}{l} x_1 = 10 \\ x_2 = -12 \end{array}}$

⑧ $x \cdot y = 216 \longrightarrow \frac{2}{3}y \cdot y = 216$

$\frac{x}{y} = \frac{2}{3} \longrightarrow x = \frac{2}{3}y$

$\frac{2}{3}y^2 = 216 \longrightarrow \frac{2}{3}y^2 - 216 = 0$

"y" es una variable como "x"

$a = \frac{2}{3}$, $b = 0$, $c = -216$

$$x = \pm\sqrt{-\frac{c}{a}} = \pm\sqrt{\frac{-(-216)}{\frac{2}{3}}} = \pm\sqrt{216 \cdot \frac{3}{2}} = \pm\sqrt{324} = \pm 18$$

$y_1 = 18$, $y_2 = -18$ $\longrightarrow$ $x_1 = \frac{2}{3} \cdot 18 = 12$

$x_2 = \frac{2}{3}(-18) = -12$

⑨ $3x^2 + 2x = 85$; $3x^2 + 2x - 85 = 0$

$a = 3$, $b = 2$, $c = -85$

$$x = \frac{-2 \pm \sqrt{2^2 - 4 \cdot 3 \cdot (-85)}}{2 \cdot 3} = \frac{-2 \pm \sqrt{4 + 1020}}{6} = \frac{-2 \pm 32}{6}$$

$\frac{-2+32}{6} = \frac{30}{6} = 5$

$\frac{-2-32}{6} = \frac{-34}{6} = \frac{-17}{3}$

$x_1 = 5$, $x_2 = \frac{-17}{3}$

(10) $x + \frac{1}{3} = \frac{1}{x} + 3$

$$\frac{3x^2}{3x} + \frac{x}{3x} = \frac{3}{3x} + \frac{9x}{3x}$$

$$3x^2 + x = 3 + 9x$$

$$3x^2 + x - 9x - 3 = 0$$

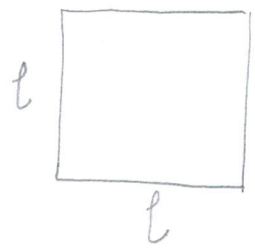
$$3x^2 - 8x - 3 = 0 \quad \begin{cases} a = 3 \\ b = -8 \\ c = -3 \end{cases}$$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \cdot 3 \cdot (-3)}}{2 \cdot 3} = \frac{8 \pm \sqrt{64 + 36}}{6} =$$

$$= \frac{8 \pm \sqrt{100}}{6} \quad \begin{cases} \frac{8+10}{6} = \underline{\underline{3}} \\ \frac{8-10}{6} = \frac{-2}{6} = \underline{\underline{\frac{-1}{3}}} \end{cases}$$

$$\boxed{\begin{aligned} x_1 &= 3 \\ x_2 &= \frac{-1}{3} \end{aligned}}$$

(11)



Area $= l^2$

Perimetro $= 4 \cdot l$

$l = 4x - 1$

$l^2 = 49 \longrightarrow (4x-1)^2 = 49$ (igualdades notables)

$$(4x)^2 + 1^2 - 2 \cdot 4x \cdot 1 = 49$$

$$16x^2 + 1 - 8x - 49 = 0$$

$$16x^2 - 8x - 48 = 0 \quad \begin{cases} a = 16 \\ b = -8 \\ c = -48 \end{cases}$$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \cdot 16 \cdot (-48)}}{2 \cdot 16} = \frac{8 \pm \sqrt{64 + 3072}}{32} =$$

$$= \frac{8 \pm \sqrt{3136}}{32} = \frac{8 \pm 56}{32} \quad \begin{cases} \frac{8+56}{32} = \underline{\underline{2}} \\ \frac{8-56}{32} = \frac{-48}{32} = \frac{-12}{8} = \underline{\underline{\frac{-6}{4}}} \end{cases}$$

$$\boxed{\begin{aligned} x_1 &= 2 \\ x_2 &= \frac{-6}{4} \end{aligned}}$$

(12) $x^2+(x+1)^2=41$

$x^2+x^2+1^2+2x\cdot 1=41$

$x^2+x^2+1+2x-41=0$

$2x^2+2x-40=0$

$\left.\begin{array}{l} a=2 \\ b=2 \\ c=-40 \end{array}\right\}\ x=\dfrac{-2\pm\sqrt{2^2-4\cdot 2\cdot(-40)}}{2\cdot 2}=$

$=\dfrac{-2\pm\sqrt{4+320}}{4}=\dfrac{-2\pm\sqrt{324}}{4}=\dfrac{-2\pm 18}{4}=$

$=\begin{cases} \dfrac{-2+18}{4}=\underline{\underline{4}} \\ \dfrac{-2-18}{4}=\underline{\underline{-5}} \end{cases}$ $\boxed{\begin{array}{l} x_1=4 \\ x_2=-5 \end{array}}$

- Los números buscados pueden ser $\begin{cases} 4 \text{ y } 5 \\ -5 \text{ y } -4 \end{cases}$