① $2x(3x+2)=0$

FORMA 1

$2x=0 \longrightarrow x_1=\frac{0}{2}$; $\boxed{x_1=0}$

$3x+2=0 \longrightarrow 3x=-2$; $\boxed{x=\frac{-2}{3}}$

FORMA 2

$2x(3x+2)=0$; $6x^2+4x=0$ $\begin{cases} a=6 \\ b=4 \\ c=0 \end{cases}$

$\boxed{x_1=0}$

$x_2=\frac{-b}{a}=\frac{-4}{6}=\frac{-2}{3}$; $\boxed{x_2=\frac{-2}{3}}$

② $x(x+5)=0$

$\boxed{x_1=0}$

$x+5=0 \longrightarrow \boxed{x_2=-5}$

③ $(2x+1)^2=1+(x+1)(x-1)$

• Aplicar igualdades notables

$(2x)^2+1^2+2\cdot1\cdot2x=1+x^2-1^2$

$4x^2+1+4x=1+x^2-1$

$4x^2-x^2+4x+1-1+1=0$

$3x^2+4x+1=0$

$a=3$
$b=4$
$c=1$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-4\pm\sqrt{4^2-4\cdot3\cdot1}}{2\cdot3}=\frac{-4\pm\sqrt{16-12}}{6}=$

$=\frac{-4\pm\sqrt{4}}{6}=\frac{-4\pm2}{6}$ $\begin{cases} \frac{-4+2}{6}=\frac{-2}{6}=\frac{-1}{3} \\ \frac{-4-2}{6}=\frac{-6}{6}=-1 \end{cases}$

$\boxed{\begin{array}{l} x_1=\frac{-1}{3} \\ x_2=-1 \end{array}}$

④ $3x(x-2)+4 = 2x^2-1$

$3x^2-6x+4 = 2x^2-1$

$3x^2-2x^2-6x+4+1 = 0$

$x^2-6x+5 = 0$ ——— $\begin{cases} a=1 \\ b=-6 \\ c=5 \end{cases}$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-(-6) \pm \sqrt{(-6)^2-4\cdot1\cdot5}}{2\cdot1} =$

$= \frac{6 \pm \sqrt{36-20}}{2} = \frac{6 \pm \sqrt{16}}{2} = \frac{6 \pm 4}{2}$ $\begin{cases} \frac{6+4}{2} = 5 \\ \frac{6-4}{2} = 1 \end{cases}$

$\boxed{\begin{array}{l} x_1 = 5 \\ x_2 = 1 \end{array}}$

⑤ $\frac{x^2-1}{3} = \frac{x^2-2x+1}{2}$

$\frac{2x^2-2}{6} = \frac{3x^2-6x+3}{6}$; $2x^2-2 = 3x^2-6x+3$

$2x^2-3x^2+6x-2-3 = 0$; $-x^2+6x-5 = 0$ $\begin{cases} a=-1 \\ b=6 \\ c=-5 \end{cases}$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-6 \pm \sqrt{6^2-4\cdot(-1)\cdot(-5)}}{2\cdot(-1)} =$

$= \frac{-6 \pm \sqrt{36-20}}{-2} = \frac{-6 \pm \sqrt{16}}{-2} = \frac{-6 \pm 4}{-2}$ $\begin{cases} \frac{-6+4}{-2} = 1 \\ \frac{-6-4}{-2} = 5 \end{cases}$

$\boxed{\begin{array}{l} x_1 = 1 \\ x_2 = 5 \end{array}}$

⑥ $\frac{x^2}{2} + \frac{5x}{3} = x - \frac{1}{6}$; $\frac{3x^2}{6} + \frac{10x}{6} = \frac{6x}{6} - \frac{1}{6}$

$3x^2+10x-6x+1 = 0$; $3x^2+4x+1 = 0$ — $\begin{cases} a=3 \\ b=4 \\ c=1 \end{cases}$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-4 \pm \sqrt{4^2-4\cdot3\cdot1}}{2\cdot3} =$

$= \frac{-4 \pm \sqrt{16-12}}{6} = \frac{-4 \pm \sqrt{4}}{6} = \frac{-4 \pm 2}{6}$ $\begin{cases} \frac{-4+2}{6} = \frac{-2}{6} = \frac{-1}{3} \\ \frac{-4-2}{6} = -1 \end{cases}$

$\boxed{x_1 = \frac{-1}{3} \; ; \; x_2 = -1}$